Chapter 1

Quadratic Equations

Quadratic equation in one variable or Second degree equation:- An equation of the form $ax^2+bx+c=0$, $a,b,c\in R$ where $a\neq 0$ is called quadratic equation. Or
An eq of degree 2 is called quadratic equation.

Solution set of the equation: The set of those values of variable for which the equation is true, is called solutions/roots set of the equation.

Roots of the equation: Those values of the variables in which the given equation is true is called roots of the equation.

Solution of Quadratic equation:
We may solve a quadratic equation by any one of the following three methods:

1. By Factorization
2. By Completing the Square
3. By the Quadratic Formula

Solution of Quadratic equation By Factorization:
In Quadratic eq $ax^2+bx+c=0$ we find out product of a (coefficient of x^2) and c (constant term) i.e. $a.c$. Then we find two numbers b_1 and b_2 such that $b_1.b_2=a.c$ and $b_1+b_2=b$.
Hence $ax^2+bx+c=ax^2+\left(b_1+b_2\right)x+c$ then we factorize after grouping.

Or Solution of Quadratic equation By Factorization:
1.Write the equation in standard form.
2.Factorize the equation.
3.Equate each factor to zero separately.
4.Solve equations for values of given variables.

Exp 1i). Solve $2x^2+2x-11=1$ by factorization
Solution: we have $2x^2+2x-11=1$

$$2x^2+2x-11-1=0$$
$$2x^2+2x-12=0 \text{ divided by 2}$$
$$x^2+x-6=0 \quad \text{Standard form}$$
$$x^2+3x-2x-6=0$$
$$x\left(x+3\right)-2\left(x+3\right)=0$$
$$\left(x-2\right)\left(x+3\right)=0$$

$\therefore x-2=0$ or $x+3=0$
$x=2$ $\qquad x=-3$

The solution set $=\left\{2,-3\right\}$

Exp 1ii). Solve $12t^2=t+1$ by factorization
Solution: we have $12t^2=t+1$ form

$$12t^2-t-1=0 \quad \text{Standard form}$$
$$12t^2-4t+3t-1=0$$
$$4t\left(3t-1\right)+1\left(3t-1\right)=0$$
$$\left(4t+1\right)\left(3t-1\right)=0$$

$\therefore 4t+1=0$ or $3t-1=0$
$4t=-1$ $\qquad 3t=1$
$t=\frac{-1}{4}$ $\qquad t=\frac{1}{3}$

Solution set $=\left\{\frac{-1}{4},\frac{1}{3}\right\}$

Exp 2: A ball is thrown straight up, from 3m above the ground with a velocity of 14m/s. Gravity pulls it down as 5m per square second. When does it hit the ground.
Solution: Height start at 3m $= 3$
With Velocity 14m/sec $= 14t$
De-acceleration 5m/sec^2 $= -5t^2$
Add them up and height h at any time t is

$$h=3+14t-5t^2$$

And ball will hit the ground when height is zero

$$3+14t-5t^2=0 \quad \text{or}$$
$$5t^2-14t-3=0$$
$$5t^2-15t+t-3=0$$
$$5t\left(t-1\right)+1\left(t-3\right)=0$$
$$\left(5t+1\right)\left(t-1\right)=0$$

Either $5t+1=0$ or $t-1=0$
$5t=-1$ $\qquad t=1$
$t=\frac{-1}{5}$ which is impossible

Therefore ball hits the ground after time $t=3$ sec

Solution of Quadratic equation By Completing square:
1.Write the equation in standard form.
2.Divide the equation by coefficient of x^2 (if other then 1)
3.Shift the constant to right hand side
4.Add square of half of coefficient of x both sides
5.Simplify R.H.S to make L.H.S as a perfect square.
6.Take square root on both sides & solve

Exp3:Solve $x^2-8x+9=0$ by completing square
Solution: we have $x^2-8x+9=0$

$$x^2-8x=-9$$
$$x^2-\left(x\right)\left(8\right)=-9$$
$$x^2-2\left(x\right)\left(\frac{8}{2}\right)=-9$$
$$x^2-2\left(x\right)\left(4\right)+\left(4\right)^2=-9+\left(4\right)^2$$
$$\left(x-4\right)^2=-9+16$$
$$\left(x-4\right)^2=7$$

Taking square root on both sides

$$\sqrt{\left(x-4\right)^2}=\sqrt{7}$$
$$x-4=\pm\sqrt{7}$$
$$x=4\pm\sqrt{7}$$

Solution set $=\left\{4+\sqrt{7},4-\sqrt{7}\right\}$

Solution of Quadratic equation By Formula:
1.Write the equation in standard form.
2.Take the values of a,b,c by comparing the standard equation.
Put in the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and solve the R.H.S to get roots.

Derivation of Quadratic Formula

We have Quadratic equation

$ax^2+bx+c=0, a\neq 0$

$ax^2+bx=-c$ or

$x^2+\frac{b}{a}x=\frac{-c}{a}$ dividing by a

$x^2+(x)\left(\frac{b}{a}\right)=\frac{-c}{a}$

$x^2+2(x)\left(\frac{b}{2a}\right)=\frac{-c}{a}$

$x^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}\times\frac{4a}{4a}$

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$

$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$

$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ required formula

Exp4: Solve $3x^2-6x+2=0$ by quadratic formula

Solution: We have $3x^2-6x+2=0$

By comparing with $ax^2+bx+c=0$

Here $a=3, b=-6, c=2$ using

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(2)}}{2(3)}$

$x=\frac{6\pm\sqrt{36-24}}{6}$

$x=\frac{6\pm\sqrt{12}}{6}$

$x=\frac{6\pm\sqrt{4\times 3}}{6}$

$x=\frac{6\pm 2\sqrt{3}}{6}=\frac{2\left(3\pm\sqrt{3}\right)}{6}$

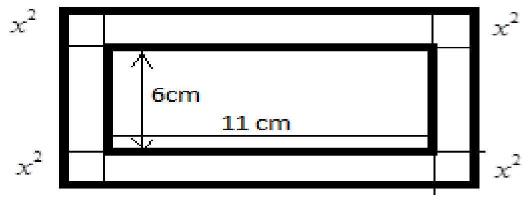
$x=\frac{3\pm\sqrt{3}}{3}=\frac{3}{3}\pm\frac{\sqrt{3}}{3}$

$x=1\pm\frac{\sqrt{3}}{3}$

The solution set $=\left\{1+\frac{\sqrt{3}}{3}, 1-\frac{\sqrt{3}}{3}\right\}$

Exp5: A company is making frames as part of a new product they are launching. The frame will be cut of a piece of steel. To keep the weight down, the final area should be 28 cm^2. The inside of a frame has to be 11cm by 6cm. what should the width x of a metal be?

Solution: Let width of frame = x cm



According to condition

Length $=11+2x$ width $=6+2x$

So Area covered by frame $=(11+2x)(6+2x)$

$=66+22x+12x+4x^2$

$=66+34x+4x^2$

Total area = Area of inside rectangle + area of frame

$=11\times 6+28$

$=66+28$

$=94$

So, both area are equal

$4x^2+34x+66=94$

$4x^2+34x+66-94=0$

$4x^2+34x-28=0$

$2\left(2x^2+17x-14\right)=0$

Or $2x^2+17x-14=0$

By comparing with $ax^2+bx+c=0$

Here $a=2, b=17, c=-14$ using

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$x=\frac{-(17)\pm\sqrt{(17)^2-4(2)(-14)}}{2(2)}$

$x=\frac{-17\pm\sqrt{289+112}}{4}$

$x=\frac{-17\pm\sqrt{401}}{4}$

$\therefore x=\frac{-17-\sqrt{401}}{4}$ or $x=\frac{-17+\sqrt{401}}{4}$

$x=-9.3$ approximate $x=0.8$ approximate

As distance can not be negative so

width of frame = 0.8 cm

Exercise 1.1

Q1. Solve $x^2+5x+4=0$ by factorization

Sol: Since a=+1, b=+5, c=+4

$x^2+5x+4=0$

$x^2+4x+x+4=0$

$x(x+4)+1(x+4)=0$

$(x+1)(x+4)=0$

R.W	+	a.c
Sign of b	×	Result
	+	4
+	×	+
4	×	1

Either $x+1=0$ Or $x+4=0$

$x=-1$ $x=-4$

Solution Set $=\{-1,-4\}$

Q1ii). Solve $(x-3)^2=4$ by factorization

Sol: Since $(x-3)^2=4$

$x^2-6x+9=4$

$x^2-6x+9-4=0$

$x^2-6x+5=0$

So a = + 1, b = - 6, c = + 5

$x^2-6x+5=0$

$x^2-5x-1x+5=0$

$x(x-5)-1(x-5)=0$

$(x-1)(x-5)=0$

R.W.	+	5
-	×	-
5	×	1

Either x - 1 = 0 Or x - 5 =0

x = 1 x= 5

Solution Set = $\{1,5\}$

Q1iii). Solve $x^2+3x-10=0$ by factorization

Sol: Since a = + 1, b = + 3, c = - 10

$x^2+3X-10=0$

$x^2+5x-2x-10=0$

$x(x+5)-2(x+5)=0$

$(x-2)(x+5)=0$

R.W.	-	10
+	×	-
4	×	1
5	×	2

Either x – 2 = 0 Or x + 5 =0

x =2 x= - 5

Solution Set = { 2, - 5 }

Q1iv).Solve $6x^2-13x+5=0$ by factorization

Sol: Since a = + 6, b = - 13, c = + 5

$6x^2-13x+5=0$

$6x^2-10x-3x+5=0$

$2x(3x-5)-1(3x-5)=0$

$(2x-1)(3x-5)=0$

R.W.	+	30
-	×	-
12	×	1
11	×	2
10	×	3

Either 2x – 1 = 0 Or 3x - 5 =0

2x =1 3x= 5

$x=\frac{1}{2}$ $x=\frac{5}{3}$

Solution Set $=\left\{\frac{1}{2},\frac{5}{3}\right\}$

Q1v) Solve $3(x^2-1)=4(x+1)$ by factorization

Sol: Since $3(x^2-1)=4(x+1)$

$3x^2-3=4x+4$

$3x^2-4x-4-3=0$

$3x^2-4x-7=0$

So a = + 3, b = - 4, c = - 7

$3x^2-4x-7=0$

$3x^2-7x+3x-7=0$

$x(3x-7)+1(3x-7)=0$

$(x+1)(3x-7)=0$

R.W.	-	21
-	×	+
5	×	1
6	×	2
7	×	3

Either x + 1 = 0 Or 3x - 7 =0

x = - 1 3x= 7

Solution Set $=\left\{-1,\frac{7}{3}\right\}$ $x=\frac{7}{3}$

1vi Solve $x(3x-5)=(x-6)(x-7)$ by factorization

Sol: Since $x(3x-5)=(x-6)(x-7)$

$3x^2-5x=x^2-7x-6x+42$

$3x^2-5x=x^2-13x+42$

$3x^2-x^2-5x+13x-42=0$

$2x^2+8x-42=0$ Divided by 2

$x^2+4x-21=0$

So a = + 1, b = + 4, c = - 21

$x^2+4x-21=0$

$x^2+7x-3x-21=0$

$x(x+7)-3(x+7)=0$

$(x-3)(x+7)=0$

R.W.	-	21
+	×	-
5	×	1
6	×	2
7	×	3

Either x + 7 = 0 Or x - 3 =0

x = - 7 x= 3

Solution Set = $\{3,-7\}$

Q2i).Solve $x^2+6x-40=0$ by completing square

Sol: Since $x^2+6x-40=0$

$x^2+6x=40$

$(x)^2+(x)(6)=40$

$(x)^2+2(x)\left(\frac{6}{2}\right)=40$

$(x)^2+2(x)(3)=40$

$(x)^2+2(x)(3)+(3)^2=40+(3)^2$

$(x+3)^2=40+9$

$(x+3)^2=49$

Taking square root on both sides

$\sqrt{(x+3)^2}=\pm\sqrt{49}$

$x+3=\pm 7$

$x=-3\pm 7$

Either or

X = -3 + 7 x = - 3 – 7

X = 4 x = -10

Solution set $\{4,-10\}$

Q2ii).Solve $x^2-10x+11=0$ by completing square

Sol: Since $x^2-10x+11=0$

$x^2-10x=-11$

$(x)^2-(x)(10)=-11$

$(x)^2-2(x)\left(\frac{10}{2}\right)=-11$

$(x)^2-2(x)(5)=-11$

$(x)^2-2(x)(5)+(5)^2=-11+(5)^2$

$(x-5)^2=-11+25$

$(x-5)^2=14$

Taking square root on both sides

$\sqrt{(x-5)^2}=\pm\sqrt{14}$

$x-5=\pm\sqrt{14}$

$x=5\pm\sqrt{14}$

Either X = 5 + $\sqrt{14}$ or x = 5 - $\sqrt{14}$

Solution set $\{5+\sqrt{14},5-\sqrt{14}\}$

Q2iii).Solve $4x^2+12x=0$ by completing square

Sol: Since $4x^2+12x=0$ Divided by 4

$$x^2+3x=0$$
$$(x)^2+(x)(3)=0$$
$$(x)^2+2(x)\left(\frac{3}{2}\right)=0$$
$$(x)^2+2(x)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2=0+\left(\frac{3}{2}\right)^2$$
$$\left(x+\frac{3}{2}\right)^2=\left(\frac{3}{2}\right)^2$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{3}{2}\right)^2}=\pm\sqrt{\left(\frac{3}{2}\right)^2}$$
$$x+\frac{3}{2}=\pm\frac{3}{2}$$
$$x=-\frac{3}{2}\pm\frac{3}{2}$$

Either

$$x=-\frac{3}{2}+\frac{3}{2}$$
$$x=0$$

or

$$x=-\frac{3}{2}-\frac{3}{2}$$
$$x=\frac{-6}{2}=-3$$

Solution set = $\{0,-3\}$

Q2ivSolve $5x^2-10x-840=0$ completing square

Sol: Since $5x^2-10x-840=0$ Divided by 5

$$x^2-2x-168=0$$
$$x^2-2x=168$$
$$(x)^2-(x)(2)=168$$
$$(x)^2-2(x)\left(\frac{2}{2}\right)=168$$
$$(x)^2-2(x)(1)=168$$
$$(x)^2-2(x)(1)+(1)^2=168+(1)^2$$
$$(x-1)^2=168+1$$
$$(x-1)^2=169$$

Taking square root on both sides

$$\sqrt{(x-1)^2}=\pm\sqrt{169}$$
$$x-1=\pm13$$
$$x=1\pm13$$

Either

X = 1 +13

X = 14

or

x = 1 - 13

x= -12

Solution set = $\{-12,14\}$

Q2v Solve $9x^2-6x+\frac{5}{9}=0$ completing square

Sol: Since $9x^2-6x+\frac{5}{9}=0$ Divided by 9

$$x^2-\frac{6}{9}x=-\frac{5}{9\times9}$$
$$x^2-\frac{2}{3}x=-\frac{5}{81}$$
$$(x)^2-(x)\left(\frac{2}{3}\right)=-\frac{5}{81}$$
$$(x)^2-2(x)\left(\frac{2}{2\times3}\right)=-\frac{5}{81}$$
$$(x)^2-2(x)\left(\frac{1}{3}\right)=-\frac{5}{81}$$
$$(x)^2-2(x)\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2=-\frac{5}{81}+\left(\frac{1}{3}\right)^2$$
$$\left(x-\frac{1}{3}\right)^2=-\frac{5}{81}+\frac{1}{9}\times\frac{9}{9}$$
$$\left(x-\frac{1}{3}\right)^2=\frac{-5+9}{81}=\frac{4}{81}$$
$$\sqrt{\left(x-\frac{1}{3}\right)^2}=\pm\sqrt{\frac{4}{81}}$$ Taking square root
$$x-\frac{1}{3}=\pm\frac{2}{9}$$
$$x=\frac{1}{3}\pm\frac{2}{9}=\frac{3\pm2}{9}$$

Either

$$x=\frac{3-2}{9}$$
$$x=\frac{1}{9}$$

or

$$x=\frac{3+2}{9}$$
$$x=\frac{5}{9}$$

Solution set $=\left\{\frac{1}{9},\frac{5}{9}\right\}$

Q2vi). Solve $(x-1)(x+3)=5(x+2)-3$ by completing square

Sol: Since $(x-1)(x+3)=5(x+2)-3$

$$x^2+3x-x-3=5x+10-3$$
$$x^2+3x-x-5x-10+3-3=0$$
$$x^2-3x-10=0$$
$$(x)^2-(x)(3)=10$$
$$(x)^2-2(x)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2=10+\left(\frac{3}{2}\right)^2$$
$$\left(x-\frac{3}{2}\right)^2=10+\frac{9}{4}=\frac{40+9}{4}$$
$$\left(x-\frac{3}{2}\right)^2=\frac{49}{4}$$ Taking square root on bs
$$\sqrt{\left(x-\frac{3}{2}\right)^2}=\sqrt{\frac{49}{4}}$$
$$x-\frac{3}{2}=\pm\frac{7}{2}$$
$$x=\frac{3}{2}\pm\frac{7}{2}=\frac{3\pm7}{2}$$

Either $x=\frac{3-7}{2}$ or $x=\frac{3+7}{2}$

$$x=\frac{-4}{2}=-2 \qquad x=\frac{10}{2}=5$$

Solution set $=\{-2,5\}$

Q3i.Sovle $x^2-8x+15=0$ by quadratic formula

Sol: Since $x^2-8x+15=0$ Comparing we the general quadratic eq i.e., $ax^2+bx+c=0$ we get $a=1,\quad b=-8, c=15$

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values of a,b and c we get

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(15)}}{2(1)}$$

$$x=\frac{8\pm\sqrt{64-60}}{2}$$

$$x=\frac{8\pm\sqrt{4}}{2}=\frac{8\pm2}{2}$$

$$x=\frac{8}{2}\pm\frac{2}{2}=4\pm1$$

Either or

$x=4+1=5$ $\quad$ $x=4-1=3$

Solution set $=\{5,3\}$

Q3ii.Solve $x^2-2x-4=0$ by quadratic formula

Sol: Since $x^2-2x-4=0$ Comparing with $ax^2+bx+c=0$ we get $a=1,\quad b=-2, c=-4$

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values of a,b and c we get

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4+16}}{2}=\frac{2\pm\sqrt{20}}{2}$$

$$x=\frac{2\pm\sqrt{4\times5}}{2}=\frac{2\pm\sqrt{4}\sqrt{5}}{2}$$

$$x=\frac{2\pm2\sqrt{5}}{2}=1\pm\sqrt{5}$$

Solution Set $=\{1+\sqrt{5},1-\sqrt{5}\}$

Q3iii. Solve $4x^2+3x=0$ by quadratic formula

Sol: Since $4x^2+3x+0=0$ Comparing with $ax^2+bx+c=0$ we get $a=4,\quad b=3,\quad c=0$

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values of a,b and c we get

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(4)(0)}}{2(4)}$$

$$x=\frac{-3\pm\sqrt{(3)^2}}{8}=\frac{-3\pm3}{8}$$

Either or

$x=\dfrac{-3-3}{8}$ $\quad$ $x=\dfrac{-3+3}{8}$

$x=\dfrac{-6}{8}=\dfrac{-3}{4}$ $\quad$ $x=\dfrac{0}{8}=0$

Solution set $=\left\{\dfrac{-3}{4},0\right\}$

Q3iv. Solve $3x(x-2)+1=0$ by quadratic formula

Sol: Since $3x(x-2)+1=0$

$$3x^2-6x+1=0$$

Comparing we the general quadratic equation i.e., $ax^2+bx+c=0$ we get $a=3,b=-6,c=1$

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values of a,b and c we get

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(1)}}{2(3)}$$

$$x=\frac{6\pm\sqrt{36-12}}{6}=\frac{6\pm\sqrt{24}}{6}$$

$$x=\frac{6\pm\sqrt{4\times6}}{6}=\frac{6\pm\sqrt{4}\sqrt{6}}{6}$$

$$x=\frac{6\pm2\sqrt{6}}{6}=\frac{6}{6}\pm\frac{2\sqrt{6}}{6}$$

$$x=1\pm\frac{\sqrt{6}}{3}$$

Solution set $=\left\{1+\dfrac{\sqrt{6}}{3},1-\dfrac{\sqrt{6}}{3}\right\}$

Q3v.Solve $6x^2-17x+12=0$ by quadratic formula

Sol: Since $6x^2-17x+12=0$

Comparing we the general quadratic equation i.e., $ax^2+bx+c=0$ we get $a=6,\quad b=-17, c=12$

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values of a,b and c we get

$$x=\frac{-(-17)\pm\sqrt{(-17)^2-4(6)(12)}}{2(6)}$$

$$x=\frac{17\pm\sqrt{289-288}}{12}=\frac{17\pm\sqrt{1}}{12}=\frac{17\pm1}{12}$$

Either or

$x=\dfrac{17-1}{12}$ $\quad$ $x=\dfrac{17+1}{12}=\dfrac{18}{12}$

$x=\dfrac{16}{12}=\dfrac{4}{3}$ $\quad$ $x=\dfrac{3}{2}$

Solution set $=\left\{\dfrac{4}{3},\dfrac{3}{2}\right\}$

Q3vi.Solve $\dfrac{x^2}{3}-\dfrac{x}{12}=\dfrac{1}{24}$ by quadratic formula

Sol: Since $\dfrac{x^2}{3}-\dfrac{x}{12}=\dfrac{1}{24}$

Multiply each term by 24 then

$$24.\frac{x^2}{3}-24.\frac{x}{12}=24.\frac{1}{24}$$

$$8x^2-2x=1$$

$$8x^2-2x-1=0$$

Comparing we general quadratic equation i.e., $ax^2+bx+c=0$ we get $a=8,b=-2,c=-1$

Using quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values of a,b and c we get

$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(8)(-1)}}{2(8)}$

$x = \frac{2\pm\sqrt{4+32}}{16} = \frac{2\pm\sqrt{36}}{16} = \frac{2\pm 6}{16}$

Either

$x = \frac{2-6}{16} = \frac{-4}{16}$

$x = \frac{-1}{4}$

Or

$x = \frac{2+6}{16} = \frac{8}{16}$

$x = \frac{1}{2}$

Solution Set $= \left\{\frac{-1}{4}, \frac{1}{2}\right\}$

Q4i. Find all solution of $t^2 - 8t + 7 = 0$

Solution: we have $t^2 - 8t + 7 = 0$

$t^2 - 7t - t + 7 = 0$

$t(t-7) - 1(t-7) = 0$

$(t-1)(t-7) = 0$

$t-1=0$ or $t-7=0$

$t=1$ $t=7$

Solution set $= \{1,7\}$

Q4ii. Find all solution of $72 + 6x = x^2$

Solution; We have $72 + 6x = x^2$

Or $x^2 - 6x - 72 = 0$

$x^2 - 12x + 6x - 72 = 0$

$x(x-12) + 6(x-12) = 0$

$(x+6)(x-12) = 0$

$\therefore x+6=0$ or $x-12=0$

$x=-6$ $x=12$

Solution set $= \{-6,12\}$

Q4iii. Find all solution of $r^2 + 4r + 1 = 0$

Solution; We have $r^2 + 4r + 1 = 0$

Comparing we the general quadratic equation i.e. $ax^2 + bx + c = 0$ we get $a=1, b=4, c=1$

using $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$x = \frac{-4\pm\sqrt{4^2-4(1)(1)}}{2(1)}$

$x = \frac{-4\pm\sqrt{16-4}}{2}$

$x = \frac{-4\pm\sqrt{12}}{2} = \frac{-4\pm\sqrt{4\times 3}}{2}$

$x = \frac{-4\pm 2\sqrt{3}}{2} = \frac{2\left(-2\pm\sqrt{3}\right)}{2}$

$x = -2\pm\sqrt{3}$

Solution set $= \left\{-2-\sqrt{3}, -2+\sqrt{3}\right\}$

Q4vi Find all solution of $x(x+10) = 10(-10-x)$

Solution: we have $x(x+10) = 10(-10-x)$

$x^2 + 10x = -100 - 10x$

$x^2 + 10x + 10x + 100 = 0$

$x^2 + 20x + 100 = 0$

$(x)^2 + 2(x)(10) + (10)^2 = 0$

$(x+10)^2 = 0$

$\Rightarrow x+10 = 0$

$x = -10$

Solution set $= \{-10\}$

Q5. The equation $(y+13)(y+a)$ has no linear term. Find the value of a

Solution; we have $(y+13)(y+a)$

Or $y^2 + ay + 13y + 13a$

$y^2 + (a+13)y + 13a$

According to condition { no linear term }

Means $a+13=0$ or $a=-13$

Q6. The equation $ax^2 + 5x = 3$ has $x=1$ as a solution. What is the other solution.

Sol: we have $ax^2 + 5x = 3$ has solution $x=1$

$a(1)^2 + 5(1) = 3$

$a + 5 = 3$

$a = 3 - 5$

$a = -2$

Therefore given equation becomes

$-2x^2 + 5x = 3$

$-2x^2 + 5x - 3 = 0$

Or $2x^2 - 5x + 3 = 0$

$2x^2 - 3x - 2x - 3 = 0$

$x(2x-3) - 1(2x-3) = 0$

$(2x-3)(x-1) = 0$

Either $2x-3=0$ or $x-1=0$

$2x = 3$

$x = \frac{3}{2}$ $x=1$

Solution set $= \left\{\frac{3}{2}, 1\right\}$

Q7. What is the positive difference of the roots of $x^2 - 7x - 9 = 0$

Solution we have $x^2 - 7x - 9 = 0$

Comparing we the general quadratic equation i.e. $ax^2 + bx + c = 0$ we get $a=1, b=-7, c=9$

using $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$x = \frac{-(-7)\pm\sqrt{(-7)^2-4(1)(-9)}}{2(1)}$

$x = \frac{7\pm\sqrt{49+36}}{2}$

$x = \frac{7\pm\sqrt{85}}{2}$

Therefore roots are $\frac{7-\sqrt{85}}{2}, \frac{7+\sqrt{85}}{2}$

Difference of roots are $\frac{7+\sqrt{85}}{2}-\frac{7-\sqrt{85}}{2}$

$$=\frac{7+\sqrt{85}-7+\sqrt{85}}{2}$$

$$=\frac{2\sqrt{85}}{2}$$

Difference of roots are $=\sqrt{85}$

Solution of equations reducible to quadratic equation in one variable:

Type 1:Equation of the form, $ax^{2n}+bx^n+c=0$

1.Put $x^n=y$

2.Find the value of y from the new equation

3Put the values of y in supposition to get roots.

Exp 6 Solve $12x^4-11x^2+2=0$

Solution we have $12x^4-11x^2+2=0$

Let $y=x^2 \Rightarrow y^2=x^4$ given equation becomes

$$12y^2-11y+2=0$$
$$12y^2-8y-3y+2=0$$
$$4y(3y-2)-1(3y-2)=0$$
$$(4y-1)(3y-2)=0$$

Either $4y-1=0$ or $3y-2=0$

$y=\frac{1}{4}$ $\qquad y=\frac{2}{3}$

Putting back value of $y=x^2$ therefore,

$x^2=\frac{1}{4}$ $\qquad x^2=\frac{2}{3}$

$x=\pm\frac{1}{2}$ $\qquad x=\pm\sqrt{\frac{2}{3}}$

Solution set $=\left\{\pm\frac{1}{2},\pm\sqrt{\frac{2}{3}}\right\}$

Type 2: The Equation of the form, $ax+\frac{b}{x}=c$

(one term is reciprocal to the other)

1. Suppose one term = y
2. Take reciprocal of the equation then put in question and solve to get value of y.
3. Put y values in supposition to get roots

Exp7: Solve $2x+\frac{4}{x}=9$

Solution we have $2x+\frac{4}{x}=9$

Multiply each term by x

$$x\left(2x+\frac{4}{x}\right)=9x$$
$$2x^2+4=9x$$
$$2x^2-9x+4=0$$
$$2x^2-8x-1x+4=0$$
$$2x(x-4)-1(x-4)=0$$
$$(2x-1)(x-4)=0$$

Either $2x-1=0$ or $x-4=0$

$x=\frac{1}{2}$ $\qquad x=4$

Solution set $=\left\{\frac{1}{2},4\right\}$

Exp 8: Solve $\frac{x-1}{x+3}+\frac{x+3}{x-1}=\frac{13}{6}$

Solution We have $\frac{x-1}{x+3}+\frac{x+3}{x-1}=\frac{13}{6}$

Let $y=\frac{x-1}{x+3} \Rightarrow \frac{1}{y}=\frac{x+3}{x-1}$

Therefore $y+\frac{1}{y}=\frac{13}{6}$

Multiply each term by 6y

$$6y^2+6=13y$$

Or
$$6y^2-13y+6=0$$
$$6y^2-9y-4y+6=0$$
$$3y(2y-3)-2(2y-3)=0$$
$$(3y-2)(2y-3)=0$$

$3y-2=0$ $\qquad 2y-3=0$

Either $y=\frac{2}{3}$ **or** $y=\frac{3}{2}$

Putting back the value of y

$\frac{x-1}{x+3}=\frac{2}{3}$ $\qquad \frac{x-1}{x+3}=\frac{3}{2}$

By cross multiplication

$3(x-1)=2(x+3)$ $\qquad 2(x-1)=3(x+3)$

$3x-3=2x+6$ $\qquad 2x-2=3x+9$

$3x-2x=6+3$ $\qquad 2x-3x=9+2$

$x=9$ $\qquad -x=11$

$\qquad x=-11$

Solution set $=\{-11,9\}$

Type 3 $a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$

1 Let $y=x+\frac{1}{x}$

2 taking square on bs to get the value of $x^2+\frac{1}{x^2}$

3 putting these substitution in given equation

Exmple 9i). $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$

Sol: we have $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$

Let $y=x+\frac{1}{x}$ $\Rightarrow y^2=x^2+\frac{1}{x^2}+2$

$$y^2-2=x^2+\frac{1}{x^2}$$

Thus given equation becomes

$$2(y^2-2)-9y+14=0$$
$$2y^2-4-9y+14=0$$
$$2y^2-9y+10=0$$

$2y^2-5y-4y+10=0$

$y(2y-5)-2(2y-5)=0$

$(y-2)(2y-5)=0$

Either $y-2=0$ or $2y-5=0$

Putting back value of y

$x+\frac{1}{x}-2=0$ or $2\left(x+\frac{1}{x}\right)-5=0$

Multiply each term by x

$x^2+1-2x=0$ | $2x^2+2-5x=0$

$x^2-2x+1=0$ | $2x^2-5x+2=0$

$(x)^2-2(x)(1)+(1)^2=0$ | $2x^2-4x-x+2=0$

$(x-1)^2=0$ | $2x(x-2)-1(x-2)=0$

$\Rightarrow x-1=0$ | $(2x-1)(x-2)=0$

$x=1$ | $2x-1=0$, $x=\frac{1}{2}$ | $x-2=0$, $x=2$

Solution set $=\left\{\frac{1}{2},1,2\right\}$

Exp 9ii). $8\left(x^2+\frac{1}{x^2}\right)-42\left(x-\frac{1}{x}\right)+29=0$

Sol: we have $8\left(x^2+\frac{1}{x^2}\right)-42\left(x-\frac{1}{x}\right)+29=0$

Let $y=x-\frac{1}{x}$ $\Rightarrow$ $y^2=x^2+\frac{1}{x^2}-2$

$y^2+2=x^2+\frac{1}{x^2}$

Thus given equation becomes

$8(y^2+2)-42y+29=0$

$8y^2+16-42y+29=0$

$8y^2-42y+45=0$

$8y^2-30y-12y+45=0$

$2y(4y-15)-3(4y-15)=0$

$(2y-3)(4y-15)=0$

Either $2y-3=0$ or $4y-15=0$

Putting back value of y

$2\left(x-\frac{1}{x}\right)-3=0$ or $4\left(x-\frac{1}{x}\right)-15=0$

Multiply each term by x

$2x^2-2-3x=0$ | $4x^2-4-15x=0$

$2x^2-3x-2=0$ | $4x^2-15x-4=0$

$2x^2-4x+1x-2=0$ | $4x^2-16x+1x-4=0$

$2x(x-2)+1(x-2)=0$ | $4x(x-4)+1(x-4)=0$

$(2x+1)(x-2)=0$ | $(4x+1)(x-4)=0$

$2x+1=0$, $x=\frac{-1}{2}$ | $x-2=0$, $x=2$ | $4x+1=0$, $x=\frac{-1}{4}$ | $x-4=0$, $x=4$

Solution set $=\left\{\frac{-1}{4},\frac{-1}{2},2,4\right\}$

Type 4: The Equation of the form, $a^{2x}+a^x+b=0$

1. Put $a^x=y$
2. Find y & the find x by using $\log$.

Example 10 solve $4.2^{2x}-10.2^x+4=0$

Let $y=2^x\Rightarrow y^2=2^{2x}$ given equation becomes

$4y^2-10y+4=0$

$4y^2-8y-2y+4=0$

$4y(y-2)-2(y-2)=0$

$(4y-2)(y-2)=0$

$4y-2=0$ | $y-2=0$

$y=\frac{2}{4}=\frac{1}{2}$ | $y=2$

Putting back value of y

$2^x=2^{-1}$ | $2^x=2^1$

$\Rightarrow x=-1$ | $\Rightarrow x=1$

Solution set $=\{-1,1\}$

Example 11. Solve $2^{2+x}+2^{2-x}=10$

Solution: we have $2^{2+x}+2^{2-x}=10$

$2^2 2^x+2^2 2^{-x}=10$

$4.2^x+\frac{4}{2^x}=10$

Let $y=2^x$ given equation becomes

$4y+\frac{4}{y}=10$ multiply each term by y

$4y^2+4=10y$

$4y^2-10y+4=0$ divided by 2

$2y^2-5y+2=0$

$2y^2-4y-1y+2=0$

$2y(y-2)-1(y-2)=0$

$(2y-1)(y-2)=0$

Either $2y-1=0$, $y=\frac{1}{2}$ or $y-2=0$, $y=2$

Putting back value of y

$2^x=2^{-1}$ | $2^x=2^1$

$\Rightarrow x=-1$ | $\Rightarrow x=1$

Solution set $=\{-1,1\}$

Type 5: The Equation of the form, $(x+a)(x+b)(x+c)(x+d)=k$

1.Multiply those factors such that $a+b=c+d$

2.Put the common terms = y

3.Find the value of y from the new equation

4Put the values of y in supposition to get roots.

Example 12: Solve $(x+1)(x+3)(x-2)(x-4)=24$

Sol: given $(x+1)(x+3)(x-2)(x-4)=24$

Here $1-2=3-4$

$(x+1)(x-2)(x+3)(x-4)=24$

$(x^2-2x+1x-2)(x^2-4x+3x-12)=24$

$(x^2-x-2)(x^2-x-12)=24$

Let $y=x^2-x$ so

$(y-2)(y-12)=24$

$y^2-12y-2y+24=24$

$y^2-14y=0$

$y(y-14)=0$

$y=0 \qquad y-14=0$

Putting back value of y

$x^2-x=0 \qquad x^2-x-14=0$

$x(x-1)=0 \qquad x=\dfrac{1\pm\sqrt{(-1)^2-4.1(-14)}}{2(1)}$

$x=0 \qquad x-1=0 \qquad x=\dfrac{1\pm\sqrt{1+56}}{2}$

$x=1 \qquad x=\dfrac{1\pm\sqrt{57}}{2}$

Solution set $=\left\{0,1,\dfrac{1+\sqrt{57}}{2},\dfrac{1-\sqrt{57}}{2}\right\}$

Exercise 1.2

Q1i). Solve $x^4-5x^2+4=0$

Sol: Since $x^4-5x^2+4=0$

Suppose that $y=x^2$ then $y^2=x^4$ so

$y^2-5y+4=0$

$y^2-4y-y+4=0$

$y(y-4)-1(y-4)=0$

$(y-1)(y-4)=0$

R.W	+	a.c
Sign of b	×	Result
	+	4
-	×	-
	×	
4		1

Either $y-1=0$ Or $y-4=0$

$y=1 \qquad y=4$

Putting back value of $y=x^2$

$x^2=1 \qquad x^2=4$

$\Rightarrow x=\pm1 \qquad \Rightarrow x=\pm2$

Solution Set $=\{\pm1,\pm2\}$

Q1ii). Solve $x^4-7x^2+12=0$

Sol: Since $x^4-7x^2+12=0$

Suppose that $y=x^2$ then $y^2=x^4$ so

$y^2-7y+12=0$

$y^2-4y-3y+12=0$

$y(y-4)-3(y-4)=0$

$(y-3)(y-4)=0$

R.W	+	12
-	×	-
6	×	1
5	×	2
4	×	3

Either $y-3=0$ Or $y-4=0$

$y=3 \qquad y=4$

Putting back value of $y=x^2$

$x^2=3 \qquad x^2=4$

$\Rightarrow x=\pm\sqrt{3} \qquad \Rightarrow x=\pm2$

Solution Set $=\{\pm\sqrt{3},\pm2\}$

Q1iii). Solve $6x^4-13x^2+5=0$

Sol: Since $6x^4-13x^2+5=0$

Suppose that $y=x^2$ then $y^2=x^4$ so

$6y^2-13y+5=0$

$6y^2-10y-3y+5=0$

$2y(3y-5)-1(3y-5)=0$

$(2y-1)(3y-5)=0$

R.W	+	30
-	×	-
12	×	1
11	×	2
10	×	3

Either $2y-1=0$ Or $3y-5=0$

$2y=1 \qquad 3y=5$

$y=\dfrac{1}{2} \qquad y=\dfrac{5}{3}$

Putting back value of $y=x^2$

$x^2=\dfrac{1}{2} \qquad x^2=\dfrac{5}{3}$

$\Rightarrow x=\dfrac{\pm1}{\sqrt{2}} \qquad \Rightarrow x=\pm\sqrt{\dfrac{5}{3}}$

Solution Set $=\left\{\dfrac{\pm1}{\sqrt{2}},\pm\sqrt{\dfrac{5}{3}}\right\}$

Q1iv). Solve $x+2-\dfrac{1}{x+2}=\dfrac{3}{2}$

Sol: Since $x+2-\dfrac{1}{x+2}=\dfrac{3}{2}$

Let $y=x+2$ so $y-\frac{1}{y}=\frac{3}{2}$

Multiply each term by 2y we get

$2y.y-2y.\frac{1}{y}=2y.\frac{3}{2}$

$2y^2-2=3y$

$2y^2-3y-2=0$

$2y^2-4y+1y-2=0$

$2y(y-2)+1(y-2)=0$

$(y-2)(2y+1)=0$

R.W	-	4
	- ×	+
4	×	1

Either $y-2=0$ Or $2y+1=0$

$y=2 \qquad 2y=-1$

$y=\dfrac{-1}{2}$

Putting back value of $y=x+2$ we get

$x+2=2 \qquad x+2=\dfrac{-1}{2}$

$x=2-2 \qquad x=\dfrac{-1}{2}-2$

$x=0 \qquad x=\dfrac{-5}{2}$

Solution Set $=\left\{0,\dfrac{-5}{2}\right\}$

Q1v). Solve $x-\dfrac{4}{x}=2$

Sol: Since $x-\dfrac{4}{x}=2$ Multiply each term by x

$x^2-4=2x$

$x^2-2x-4=0$ Comparing with

$ax^2+bx+c=0$ so $a=1,\ b=-2,c=-4$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting

$x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$

$x=\dfrac{2\pm\sqrt{4+16}}{2}=\dfrac{2\pm\sqrt{20}}{2}$

$x=\dfrac{2\pm\sqrt{4}\sqrt{5}}{2}$

$x=\dfrac{2}{2}\pm\dfrac{2\sqrt{5}}{2}$

$x=1\pm\sqrt{5}$

Solution set $=\{1+\sqrt{5},1-\sqrt{5}\}$

Q1vi). Solve $\frac{x+2}{x-2}-\frac{x-2}{x+2}=\frac{5}{6}$

Solution we have $\frac{x+2}{x-2}-\frac{x-2}{x+2}=\frac{5}{6}$

Let $y=\frac{x+2}{x-2} \Rightarrow \frac{1}{y}=\frac{x-2}{x+2}$

Thus $y-\frac{1}{y}=\frac{5}{6}$ Multiply each term by 6y

$$6y^2-6=5y$$
$$6y^2-5y-6=0$$
$$6y^2-9y+4y-6=0$$
$$3y(2y-3)+2(2y-3)=0$$
$$(3y+2)(2y-3)=0$$

Either $3y+2=0$ or $2y-3=0$

$y=\frac{-2}{3}$ $\qquad$ $y=\frac{3}{2}$

Putting back the value of y

$\frac{x+2}{x-2}=\frac{-2}{3}$ $\qquad$ $\frac{x+2}{x-2}=\frac{3}{2}$

By cross multiplication

$3(x+2)=-2(x-2)$ $\qquad$ $2(x+2)=3(x-2)$

$3x+6=-2x+4$ $\qquad$ $2x+4=3x-6$

$3x+2x=4-6$ $\qquad$ $2x-3x=-6-4$

$5x=-2$ $\qquad$ $-x=-10$

$x=\frac{-2}{5}$ $\qquad$ $x=10$

Solution set $=\left\{\frac{-2}{5},10\right\}$

Q1vii). $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0$

Sol: Since $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0 \ldots(1)$

Let $y=x+\frac{1}{x}$ $\qquad$ $y^2=\left(x+\frac{1}{x}\right)^2$

$$y^2=x^2+\tfrac{1}{x^2}+2.x.\tfrac{1}{x}$$
$$y^2=x^2+\tfrac{1}{x^2}+2$$
$$y^2-2=x^2+\tfrac{1}{x^2}$$

Putting into equation (1) we get

$$3\left(y^2-2\right)-16y+26=0$$
$$3y^2-6-16y+26=0$$
$$3y^2-16y+20=0$$
$$3y^2-10y-6y+20=0$$
$$y(3y-10)-2(3y-10)=0$$
$$(y-2)(3y-10)=0$$

R.W.	+	60
-	×	-
15	×	1
14	×	2
13	×	3
12	×	4
11	×	5
10	×	6

Either $\qquad$ or

y - 2 = 0 $\qquad$ 3y - 10 =0

Putting back value of $y=x+\frac{1}{x}$ we get

$x+\frac{1}{x}-2=0$ $\qquad$ $3\left(x+\frac{1}{x}\right)-10=0$

Multiply each term by x we get

$x.x+\frac{1}{x}.x-2.x=0$ $\qquad$ $3x.x+\frac{3}{x}.x-10.x=0$

$x^2+1-2x=0$ $\qquad$ $3x^2-10x+3=0$

$3x^2-9x-1x+3=0$

$x^2-2.x.1+1^2=0$ $\qquad$ $3x(x-3)-1(x-3)=0$

$(x-1)^2=0$ $\qquad$ $(x-3)(3x-1)=0$

$x-3=0$ $\qquad$ $3x-1=0$

$\Rightarrow x-1=0$ $\qquad$ $x=3$ $\qquad$ $3x=1$

$x=1$ $\qquad$ $x=\frac{1}{3}$

Solution Set = $\left\{1,3,\frac{1}{3}\right\}$

Q1viii). $\left(x+\frac{1}{x}\right)^2-10\left(x+\frac{1}{x}\right)+16=0$

Let $y=x+\frac{1}{x}$ $\qquad \Rightarrow y^2=\left(x+\frac{1}{x}\right)^2$

Thus given equation becomes

$$y^2-10y+16=0$$
$$y^2-8y-2y+16=0$$
$$y(y-8)-2(y-8)=0$$
$$(y-2)(y-8)=0$$

Either $y-2=0$ or $y-8=0$

Putting back value of y

$x+\frac{1}{x}-2=0$ $\qquad$ $x+\frac{1}{x}-8=0$

Multiply each term by x

$x^2+1-2x=0$ $\qquad$ $x^2+1-8x=0$

$x^2-8x+1=0$

$x^2-2x+1=0$ $\qquad$ $x=\frac{8\pm\sqrt{64-4}}{2}$

$x^2-2.x.1+1^2=0$ $\qquad$ $x=\frac{8\pm\sqrt{60}}{2}$

$(x-1)^2=0$ $\qquad$ $x=\frac{8\pm\sqrt{4\times15}}{2}$

$\Rightarrow x-1=0$ $\qquad$ $x=\frac{8\pm2\sqrt{15}}{2}$

$x=1$ $\qquad$ $x=\frac{2\left(4\pm\sqrt{15}\right)}{2}$

$x=4\pm\sqrt{15}$

Solution set $=\left\{1,4\pm\sqrt{15}\right\}$

Q1ix). $\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-4=0$

Sol: Since $\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-4=0$ ……(1)

Let $y=x-\frac{1}{x}$ $\qquad$ $y^2=\left(x-\frac{1}{x}\right)^2$

$$y^2=x^2+\tfrac{1}{x^2}-2.x.\tfrac{1}{x}$$
$$y^2=x^2+\tfrac{1}{x^2}-2$$
$$y^2+2=x^2+\tfrac{1}{x^2}$$

Putting into equation (1) we get

$(y^2+2)-y-4=0$

$y^2-y-2=0$

$y^2-2y+1y-2=0$

$y(y-2)+1(y-2)=0$

$(y-2)(y+1)=0$

R.W. - 2 / - × + / × / 2 1

Either $y-2=0$ or $y+1=0$

Putting back value of $y=x-\frac{1}{x}$ we get

$x-\frac{1}{x}-2=0$ $\qquad$ $x-\frac{1}{x}+1=0$

Multiply each term by x we get

$x.x-\frac{1}{x}.x-2.x=0$ $\qquad$ $x.x-\frac{1}{x}.x+1.x=0$

$x^2-2x-1=0$ $\qquad$ $x^2+x-1=0$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$ $\qquad$ $x=\frac{-1\pm\sqrt{1^2-4(1)(-1)}}{2(1)}$

$x=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm\sqrt{8}}{2}$ $\qquad$ $x=\frac{-1\pm\sqrt{1+4}}{2}$

$x=\frac{2\pm\sqrt{4\times2}}{2}=\frac{2\pm\sqrt{4}\sqrt{2}}{2}$ $\qquad$ $x=\frac{-1\pm\sqrt{5}}{2}$

$x=1\pm\sqrt{2}$

Solution Set = $\left\{1\pm\sqrt{2},\frac{-1\pm\sqrt{5}}{2}\right\}$

Q1x). $3^{2x}-10.3^x+9=0$

Sol: Since $3^{2x}-10.3^x+9=0\cdots(1)$

Let $y=3^x$ $\qquad \Rightarrow y^2=3^{2x}$

Putting into equation (1) we get

$y^2-10y+9=0$

$y^2-9y-1y+9=0$

$y(y-9)-1(y-9)=0$

$(y-1)(y-9)=0$

R.W. + 9 / - × - / × / 9 1

Either $y-9=0$, $y=9$ or $y-1=0$, $y=1$

Putting back value of $y=3^x$ we get

$3^x=9$ $\qquad$ $3^x=1$

$3^x=3^2$ $\qquad$ $3^x=3^0$

$\Rightarrow x=2$ $\qquad$ $\Rightarrow x=0$

Solution Set = $\{0,2\}$

Q1xi). $3.3^{2x+1}-10.3^x+1=0$

Sol: Since $3.3^{2x+1}-10.3^x+1=0$ ....(1)

$3.3^{2x}.3-10.3^x+1=0$

$9.3^{2x}-10.3^x+1=0\cdots(1)$

Let $y=3^x$ $\qquad \Rightarrow y^2=3^{2x}$

Putting into equation (1) we get

$9y^2-10y+1=0$

$9y^2-9y-1y+1=0$

$9y(y-1)-1(y-1)=0$

$(9y-1)(y-1)=0$

R.W. + 9 / - × - / × / 9 1

Either $9y-1=0$, $9y=1$, $y=\frac{1}{9}$ or $y-1=0$, $y=1$

Putting back value of $y=3^x$ we get

$3^x=\frac{1}{9}$ $\qquad$ $3^x=1$

$3^x=\frac{1}{3^2}=3^{-2}$ $\qquad$ $3^x=3^0$

$\Rightarrow x=-2$ $\qquad$ $\Rightarrow x=0$

Solution Set = $\{-2,0\}$

Q1xii). $5^{x+1}+5^{2-x}=126$

Sol: Since $5^{x+1}+5^{2-x}=126$

$5^x.5^1+5^25^{-x}=126$

$5.5^x+\frac{25}{5^x}-126=0\cdots(1)$

Let $y=5^x$ Then we have

$5y+\frac{25}{y}-126=0$

Multiply each term by y

$5y.y+\frac{25}{y}.y-126.y=0$

Putting into equation (1) we get

$5y^2-126y+25=0$

$5y^2-125y-1y+25=0$

$5y(y-25)-1(y-25)=0$

$(5y-1)(y-25)=0$

R.W. + 125 / - × - / × / 125 1

Either $5y-1=0$, $y=\frac{1}{5}=5^{-1}$ or $y-25=0$, $y=25=5^2$

Putting back value of $y=5^x$ we get

$5^x=5^{-1}$ $\qquad$ $5^x=5^2$

$\Rightarrow x=-1$ $\qquad$ $\Rightarrow x=2$

Solution Set = $\{-1,2\}$

Q1xiii). $(x-3)(x+9)(x+5)(x-7)=385$

Sol: Rearranging with respect to a + b = c + d

$(x-3)(x+5)(x+9)(x-7)=385$

$\{x(x+5)-3(x+5)\}\{x(x-7)+9(x-7)\}-385=0$

$(x^2+5x-3x-15)(x^2-7x+9x-63)-385=0$

$(x^2+2x-15)(x^2+2x-63)-385=0$

Let $y=x^2+2x$ then we have

$(y-15)(y-63)-385=0$

$y(y-63)-15(y-63)-385=0$

$y^2-63y-15y+945-385=0$

$y^2-78y+560=0$

$y^2-70y-8y+560=0$

$y(y-70)-8(y-70)=0$

$(y-8)(y-70)=0$

R.W. + 560 / - × - / 77 × 1 / 75 × 3 / 70 × 8

Either $y-8=0$ Or $y-70=0$

Putting back value of $y=x^2+2x$ then we have

$x^2+2x-8=0$

$x^2+4x-2x-8=0$

$x(x+4)-2(x+4)=0$

$(x-2)(x+4)=0$

either Or

$x-2=0$ $x+4=0$

$x=2$ $x=-4$

$x^2+2x-70=0$

$x=\dfrac{-(2)\pm\sqrt{(2)^2-4(1)(70)}}{2(1)}$

$x=\dfrac{-2\pm\sqrt{4+280}}{2}$

$x=\dfrac{-2\pm\sqrt{284}}{2}$

$x=\dfrac{-2\pm\sqrt{4\times71}}{2}$

$x=\dfrac{-2\pm2\sqrt{71}}{2}=\dfrac{-2}{2}\pm\dfrac{2\sqrt{71}}{2}$

$x=-1\pm\sqrt{71}$

Solution set $=\{2,-4,-1\pm\sqrt{71}\}$

Q1xiv). $(x+1)(x+2)(x+3)(x+4)+1=0$

Solution: Rearranging with respect to a+b =c+d

$(x+1)(x+4)(x+2)(x+3)+1=0$

$\{x(x+4)+1(x+4)\}\{x(x+3)+2(x+3)\}+1=0$

$(x^2+4x+1x+4)(x^2+3x+2x+6)+1=0$

$(x^2+5x+4)(x^2+5x+6)+1=0$

Let $y=x^2+5x$ then we have

$(y+4)(y+6)+1=0$

$y(y+6)+4(y+6)+1=0$

$y^2+6y+4y+24+1=0$

$y^2+10Y+25=0$

$y^2+2.Y.5+5^2=0$

$(Y+5)^2=0$

$\Rightarrow Y+5=0$

Putting back value of $y=x^2+5x$ then we have

$x^2+5x+5=0$

$x=\dfrac{-(5)\pm\sqrt{(5)^2-4(1)(5)}}{2(1)}$

$x=\dfrac{-5\pm\sqrt{25-20}}{2}$

$x=\dfrac{-5\pm\sqrt{5}}{2}$

Solution set $=\left\{\dfrac{-5-\sqrt{5}}{2},\dfrac{-5+\sqrt{5}}{2}\right\}$

Q1xv). $(x+1)(x+3)(x+5)(x+7)+16=0$

Sol: Since $(x+1)(x+3)(x+5)(x+7)+16=0$

$(x+1)(x+7)(x+3)(x+5)+16=0$

$\{x(x+7)+1(x+7)\}\{x(x+5)+3(x+5)\}+16=0$

$(x^2+7x+1x+7)(x^2+5x+3x+15)+16=0$

$(x^2+8x+7)(x^2+8x+15)+16=0$

Let $y=x^2+8x$ Then we have

$(y+7)(y+15)+16=0$

$y(y+15)+7(y+15)+16=0$

$y^2+15y+7y+105+16=0$

$y^2+22y+121=0$

$y^2+2.y.11+11^2=0$

$(y+11)^2=0$

$\Rightarrow y+11=0$

Putting back value of $y=x^2+8x$, then

$x^2+8x+11=0$ Comparing with

$ax^2+bx+c=0$ so $a=1,\ b=8,\ c=11$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting

$x=\dfrac{-(8)\pm\sqrt{(8)^2-4(1)(11)}}{2(1)}$

$x=\dfrac{-8\pm\sqrt{64-44}}{2}$

$x=\dfrac{-8\pm\sqrt{20}}{2}=\dfrac{-8\pm\sqrt{4\times5}}{2}$

$x=\dfrac{-8\pm2\sqrt{5}}{2}=\dfrac{2(-4\pm\sqrt{5})}{2}$

$x=-4\pm\sqrt{5}$

Solution Set $=\{-4-\sqrt{5},-4+\sqrt{5}\}$

Q2 Solve $x^4-2x^3-2x^2+2x+1=0$

Solution we have $x^4-2x^3-2x^2+2x+1=0$

Dividing each term by x^2

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0 \text{ rearranging}$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad \ldots\ldots(1)$$

Let $y=x-\dfrac{1}{x}$ $\quad y^2=\left(x-\dfrac{1}{x}\right)^2$

$y^2=x^2+\frac{1}{x^2}-2.x.\frac{1}{x}$

$y^2=x^2+\frac{1}{x^2}-2$

$y^2+2=x^2+\frac{1}{x^2}$

Putting into equation (1) we get

$y^2+2-2y-2=0$

$y^2-2y=0$

$y(y-2)=0$

Either $y=0$ or $y-2=0$

Putting back value of y

$x-\dfrac{1}{x}=0$ $\quad x-\dfrac{1}{x}-2=0$

Multiplying each term by x

$x^2-1=0$ $\quad x^2-1-2x=0$

$x^2=1$ $\quad x^2-2x-1=0$

$x=\pm1$ $\quad a=1,b=-2,c=-1$

$$x=\frac{-(-2)\pm\sqrt{4-4(1)(-1)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm2\sqrt{2}}{2}$$

$$x=1\pm\sqrt{2}$$

Solution set $=\{\pm1,1\pm\sqrt{2}\}$

Radical equations:

An equation in which the variable occurs in one or more radical sign. e.g. $\sqrt{x+a}=\sqrt{x}+b$

Radical equations

Type 1: $\sqrt{ax+b}=cx+d$

Step1: Squaring both sides

Example 13 solve $\sqrt{27-3x}=x-3$

Solution: we have $\sqrt{27-3x}=x-3$

Taking square on both sides

$$\left(\sqrt{27-3x}\right)^2=(x-3)^2$$

$$27-3x=x^2-6x+9$$

$$0=x^2-6x+3x+9-27$$

$$x^2-3x-18=0$$

$$x^2-6x+3x-18=0$$

$$x(x-6)+3(x-6)=0$$

$$(x+3)(x-6)=0$$

Either $x+3=0$ or $x-6=0$

$x=-3$ $\quad$ $x=6$

To verify the roots for given equation

put $x=-3$

$\sqrt{27-3(-3)}=(-3)-3$

$\sqrt{27+9}=-3-3$

$\sqrt{36}=-6$

$6=-6$ False

put $x=6$

$\sqrt{27-3(6)}=(6)-3$

$\sqrt{27-18}=6-3$

$\sqrt{9}=3$

$3=3$ true

Therefore $x=-3$ is a extraneous root

And $x=6$ is a real root

Solution set $=\{6\}$

Type 2: $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

Step 1, squaring both sides

Step 2, separate the radical term

Step 3, Again squaring both sides

Example 14 Solve $\sqrt{x+2}+\sqrt{x+7}=\sqrt{x+23}$

Sol: Since $\sqrt{x+2}+\sqrt{x+7}=\sqrt{x+23}$ squaring

$$\left(\sqrt{x+2}+\sqrt{x+7}\right)^2=\left(\sqrt{x+23}\right)^2$$

$$x+2+x+7+2\sqrt{x+2}\sqrt{x+7}=x+23$$

$$2x+9+2\sqrt{x^2+7x+2x+14}=x+23$$

$$2\sqrt{x^2+9x+14}=x-2x+23-9$$

$$2\sqrt{x^2+9x+14}=-x+14$$

$$2^2\left(\sqrt{x^2+9x+14}\right)^2=(14-x)^2$$

$$4\left(x^2+9x+14\right)=14^2-2(14)(x)+x^2$$

$$4x^2-x^2+36x+28x+56-196=0$$

$$3x^2+64x-140=0$$

$$3x^2+70x-6x-140=0$$

$$x(3x+70)-2(3x+70)=0$$

$$(x-2)(3x+70)=0$$

Either $x-2=0$ or $3x+70=0$

$x=2$ $\quad$ $x=\frac{-70}{3}$

To verify put $x=2$ in given equation

$$\sqrt{2+2}+\sqrt{2+7}=\sqrt{2+23}$$

$$\sqrt{4}+\sqrt{9}=\sqrt{25}$$

$$2+3=5$$

$5=5$ true so $x=2$ is a real root

Now to verify $x=\frac{-70}{3}$ in given equation

$$\sqrt{\frac{-70}{3}+2}+\sqrt{\frac{-70}{3}+7}=\sqrt{\frac{-70}{3}+23}$$

$$\sqrt{\frac{-70+6}{3}}+\sqrt{\frac{-70+21}{3}}=\sqrt{\frac{-70+69}{3}}$$

$\sqrt{\frac{-64}{3}}+\sqrt{\frac{-49}{3}}=\sqrt{\frac{1}{3}}$ false so $x=\frac{-70}{3}$ is extraneous root. Therefore Solution set $=\{2\}$

Type 3: $\sqrt{ax^2+bx+c}+\sqrt{ax^2+bx+d}=e$

Step 1: Rearranging

$$\sqrt{ax^2+bx+c}=e-\sqrt{ax^2+bx+d}$$

Step 2, separate the radical term

Step 3, Again squaring both sides

Exp 15 Solve $\sqrt{x^2+3x+5}+\sqrt{x^2+3x+1}=2$

Sol: we have $\sqrt{x^2+3x+5}+\sqrt{x^2+3x+1}=2$

$$\sqrt{x^2+3x+5}=2-\sqrt{x^2+3x+1}$$

Taking square on both sides

$$\left(\sqrt{x^2+3x+5}\right)^2=\left(2-\sqrt{x^2+3x+1}\right)^2$$

$$x^2+3x+5=4+x^2+3x+1-4\sqrt{x^2+3x+1}$$

$$4\sqrt{x^2+3x+1}=0$$

$$\sqrt{x^2+3x+1}=0$$

$x^2+3x+1=0$ Again squaring

$$x=\frac{-3\pm\sqrt{9-4}}{2}$$

$$x=\frac{-3\pm\sqrt{5}}{2}$$

Solution set $=\left\{\frac{-3\pm\sqrt{5}}{2}\right\}$

Exercise 1.3

Q1i). Solve $\sqrt{5x+21}=x+3$

Sol: Since $\sqrt{5x+21}=x+3$

Taking square on both sides we get

$$\left(\sqrt{5x+21}\right)^2=(x+3)^2$$
$$5x+21=x^2+2.x.3+3^2$$
$$x^2+6x-5x+9-21=0$$
$$x^2+x-12=0$$
$$x^2+4x-3x-12=0$$
$$x(x+4)-3(x+4)=0$$
$$(x-3)(x+4)=0$$

Either $x-3=0$, $x=3$ or $x+4=0$, $x=-4$

Now it is necessary to verify the value of x in the given radical equation. $\sqrt{5x+21}=x+3$

If x = 3, then
$$\sqrt{5.3+21}=3+3$$
$$\sqrt{15+21}=6$$
$$\sqrt{36}=6$$
$$6=6 \text{ True}$$
Thus $x=3$ is a Real root

If x = - 4, then
$$\sqrt{5(-4)+21}=-4+3$$
$$\sqrt{-20+21}=-1$$
$$\sqrt{1}=-1$$
$$1=-1 \quad \text{False}$$
thus $x=-4$ is an extraneous root

Solution Set $=\{3\}$

Q1ii). Solve $\sqrt{2x-1}=x-2$

Sol: Since $\sqrt{2x-1}=x-2$

Taking square on both sides we get

$$\left(\sqrt{2x-1}\right)^2=(x-2)^2$$
$$2x-1=x^2-2.x.2+2^2$$
$$x^2-4x-2x+4+1=0$$
$$x^2-6x+5=0$$
$$x^2-5x-1x+5=0$$
$$x(x-5)-1(x-5)=0$$
$$(x-1)(x-5)=0$$

Either $x-1=0$, $x=1$ or $x-5=0$, $x=5$

Now it is necessary to verify the value of x in the given radical equation. $\sqrt{2x-1}=x-2$

If x = 1, then
$$\sqrt{2.1-1}=1-2$$
$$\sqrt{2-1}=-1$$
$$\sqrt{1}=-1$$
$$1=-1 \text{ False}$$
Thus x = 1 is an extraneous root so

If x = 5, then
$$\sqrt{2.5-1}=5-2$$
$$\sqrt{10-1}=3$$
$$\sqrt{9}=3$$
$$3=3 \text{ True}$$
thus s $x=$ 5 is a real root

Solution Set $=\{5\}$

Q1iii). $\sqrt{4x+5}=2x-5$

Sol: Since $\sqrt{4x+5}=2x-5$

Taking square on both sides we get

$$\left(\sqrt{4x+5}\right)^2=(2x-5)^2$$
$$4x+5=(2x)^2-2(2x).5+5^2$$
$$4x^2-20x-4x+25-5=0$$
$$4x^2-24x+20=0 \qquad \div by 4$$
$$x^2-6x+5=0$$
$$x^2-5x-1x+5=0$$
$$x(x-5)-1(x-5)=0$$
$$(x-1)(x-5)=0$$

Either $x-1=0$, $x=1$ or $x-5=0$, $x=5$

Now it is necessary to verify the value of x in the given radical equation. $\sqrt{4x+5}=2x-5$

If x = 1, then
$$\sqrt{2.1-1}=1-2$$
$$\sqrt{2-1}=-1$$
$$\sqrt{1}=-1$$
$$1=-1 \text{ False}$$
Thus $x=1$ is a Real root

If x = 5, then
$$\sqrt{2.5-1}=5-2$$
$$\sqrt{10-1}=3$$
$$\sqrt{9}=3$$
$$3=3 \text{ True}$$
Thus x = 1 is an extraneous root

Solution Set $=\{5\}$

Q1iv). Solve $\sqrt{29-4x}=2x+3$

Sol: Since $\sqrt{29-4x}=2x+3$

Taking square on both sides we get

$$\left(\sqrt{29-4x}\right)^2=(2x+3)^2$$
$$29-4x=4x^2+2.2x.3+3^2$$
$$4x^2+12x+4x+9-29=0$$
$$4x^2+16x-20=0 \qquad \div by\, 4+$$
$$x^2+4x-5=0$$
$$x^2+5x-1x-5=0$$
$$x(x+5)-1(x+5)=0$$
$$(x-1)(x+5)=0$$

Either $x-1=0$, $x=1$ or $x+5=0$, $x=-5$

Now it is necessary to verify the value of x in the given radical equation. $\sqrt{29-4x}=2x+3$

If x = 1, then
$$\sqrt{29-4.1}=2.1+3$$
$$\sqrt{29-4}=2+3$$
$$\sqrt{25}=5$$
$$5=5 \quad \text{True}$$
Thus $x=1$ is Real root

If x = - 5, then
$$\sqrt{29-4(-5)}=2(-5)+3$$
$$\sqrt{29+20}=-10+3$$
$$\sqrt{49}=-7$$
$$7=-7 \quad \text{False}$$
Thus x = - 5 is an extraneous root

Solution Set $=\{1\}$

Q1v). Solve $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Sol: Since $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Taking square on both sides

$$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2=\left(\sqrt{6x+13}\right)^2$$
$$\left(\sqrt{x+7}\right)^2+\left(\sqrt{x+2}\right)^2+2\left(\sqrt{x+7}\right)\left(\sqrt{x+2}\right)=6x+13$$
$$x+7+x+2+2\sqrt{x(x+2)+7(x+2)}=6x+13$$
$$2x+9+2\sqrt{x^2+2x+7x+14}=6x+13$$
$$2\sqrt{x^2+9x+14}=6x-2x+13-9$$
$$2\sqrt{x^2+9x+14}=4x+4 \qquad \div by2$$
$$\sqrt{x^2+9x+14}=2x+2$$

Taking square on both sides

$$\left(\sqrt{x^2+9x+14}\right)^2=(2x+2)^2$$
$$x^2+9x+14=4x^2+2.2x.2+4$$
$$4x^2-x^2+8x-9x+4-14=0$$
$$3x^2-x-10=0$$
$$3x^2-6x+5x-10=0$$
$$3x(x-2)+5(x-2)=0$$
$$(3x+5)(x-2)=0$$

Either $3x+5=0$, $3x=-5$, $x=\frac{-5}{3}$ Or $x-2=0$, $x=2$

Now it is necessary to verify value of x in given radical equation. $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

If $x=\frac{-5}{3}$, then $\sqrt{\frac{-5}{3}+7}+\sqrt{\frac{-5}{3}+2}=\sqrt{6\left(\frac{-5}{3}\right)+13}$

$$\sqrt{\frac{-5+21}{3}}+\sqrt{\frac{-5+6}{3}}=\sqrt{2(-5)+13}$$
$$\sqrt{\frac{16}{3}}+\sqrt{\frac{1}{3}}=\sqrt{-10+13}$$
$$\frac{4}{\sqrt{3}}+\frac{1}{\sqrt{3}}=\sqrt{3}$$
$$\frac{5}{\sqrt{3}}=\sqrt{3} \qquad \text{false}$$

Thus $x=\frac{-5}{3}$ is an extraneous root,

If x = 2, then

$$\sqrt{2+7}+\sqrt{2+2}=\sqrt{6(2)+13}$$
$$\sqrt{9}+\sqrt{4}=\sqrt{12+13}$$
$$3+2=\sqrt{25}$$
$$5=5 \qquad \text{True}$$

Thus $x=2$ is a real root

Solution Set $=\{2\}$

Q1vi). $\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$

Sol: Since $\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$

Taking square on both sides

$$\left(\sqrt{x}+\sqrt{3x+1}\right)^2=\left(\sqrt{5x+1}\right)^2$$
$$\left(\sqrt{x}\right)^2+\left(\sqrt{3x+1}\right)^2+2\left(\sqrt{x}\right)\left(\sqrt{3x+1}\right)=5x+1$$
$$x+3x+1+2\sqrt{x(3x+1)}=5x+1$$
$$4x+1+2\sqrt{3x^2+x}=5x+1$$
$$2\sqrt{3x^2+x}=5x-4x+1-1$$
$$2\sqrt{3x^2+x}=x$$

Taking square on both sides

$$\left(2\sqrt{3x^2+x}\right)^2=(x)^2$$
$$4\left(3x^2+x\right)=x^2$$
$$12x^2-x^2+4x=0$$
$$11x^2+4x=0$$
$$x(11x+4)=0$$

Either $x=0$ Or $11x+4=0$, $11x=-4$, $x=\frac{-4}{11}$

Now it is necessary to verify the value of x in the given radical equation. $\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$

If $x=\frac{-4}{11}$, then $\sqrt{\frac{-4}{11}}+\sqrt{3\left(\frac{-4}{11}\right)+1}=\sqrt{5\left(\frac{-4}{11}\right)+1}$

$$\sqrt{\frac{-4}{11}}+\sqrt{\frac{-12+11}{11}}=\sqrt{\frac{-20+11}{11}}$$
$$\sqrt{\frac{-4}{11}}+\sqrt{\frac{-1}{11}}=\sqrt{\frac{-9}{11}} \qquad \text{False}$$

Thus $x=\frac{-4}{11}$ is an extraneous root

If x = 0, then

$$\sqrt{0}+\sqrt{3(0)+1}=\sqrt{5(0)+1}$$
$$0+\sqrt{0+1}=\sqrt{0+1}$$
$$0+\sqrt{1}=\sqrt{1}$$
$$1=1 \qquad \text{True}$$

Thus $x=0$ is an real root

Solution Set $=\{0\}$

Q1vii). Solve $\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$

Sol: Since $\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$

Or $\sqrt{6x+40}=\sqrt{x+5}+\sqrt{x+21}$

Taking square root on both sides

$$\left(\sqrt{6x+40}\right)^2=\left(\sqrt{x+5}+\sqrt{x+21}\right)^2$$
$$\left(\sqrt{6x+40}\right)^2=\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)$$
$$6x+40=x+5+x+21+2\sqrt{x(x+21)+5(x+21)}$$
$$6x+40=2x+26+2\sqrt{x^2+21x+5x+105}$$
$$6x-2x+40-26=2\sqrt{x^2+26x+105}$$

$$4x+14=2\sqrt{x^2+26x+105} \qquad \div by2$$
$$2x+7=\sqrt{x^2+26x+105}$$
Taking square on both sides
$$\left(\sqrt{x^2+26x+105}\right)^2=(2x+7)^2$$
$$x^2+26x+105=4x^2+2.2x.7+49$$
$$4x^2-x^2+28x-26x+49-105=0$$
$$3x^2+2x-56=0$$
$$3x^2+14x-12x-56=0$$
$$x(3x+14)-4(3x+14)=0$$
$$(3x+14)(x-4)=0$$
Either $3x+14=0$, $3x=-14$, $x=\dfrac{-14}{3}$ Or $x-4=0$, $x=4$

Now it is necessary to verify value of x in given radical equation. $\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$

If $x=\dfrac{-14}{3}$ then
$$\sqrt{6\left(\frac{-14}{3}\right)+40}-\sqrt{\frac{-14}{3}+21}=\sqrt{\frac{-14}{3}+5}$$
$$\sqrt{\frac{-84+120}{3}}-\sqrt{\frac{-14+63}{3}}=\sqrt{\frac{-14+15}{3}}$$
$$\sqrt{\frac{36}{3}}-\sqrt{\frac{49}{3}}=\sqrt{\frac{1}{3}}$$
$$\frac{6}{\sqrt3}-\frac{7}{\sqrt3}=\frac{1}{\sqrt3}$$
$$\frac{-1}{\sqrt3}=\frac{1}{\sqrt3} \qquad \text{False}$$
Thus $x=\dfrac{-14}{3}$ is an extraneous root so

If x = 4, then
$$\sqrt{6(4)+40}-\sqrt{4+21}=\sqrt{4+5}$$
$$\sqrt{24+40}-\sqrt{25}=\sqrt9$$
$$\sqrt{64}-5=3$$
$$8-5=3$$
$$3=3 \qquad \text{True}$$
Thus $x=4$ is a real root.

Solution Set $=\{4\}$

Q1viii). $\sqrt{2x-3}+\sqrt{2x+4}=\sqrt{6x+13}$

Sol: Since $\sqrt{2x-3}+\sqrt{2x+4}=\sqrt{6x+13}$

Taking square root on both sides
$$\left(\sqrt{2x-3}+\sqrt{2x+4}\right)^2=\left(\sqrt{6x+13}\right)^2$$
$$\left(\sqrt{2x-3}\right)^2+\left(\sqrt{2x+4}\right)^2+2\left(\sqrt{2x-3}\right)\left(\sqrt{2x+4}\right)=6x+13$$
$$2x-3+2x+4+2\sqrt{2x(2x+4)-3(2x+4)}=6x+13$$
$$4x+1+2\sqrt{4x^2+8x-6x-12}=6x+13$$
$$2\sqrt{4x^2+2x-12}=6x-4x+13-1$$
$$2\sqrt{4x^2+2x-12}=2x+12 \qquad \div by2$$
$$\sqrt{4x^2+2x-12}=x+6$$
Taking square on both sides
$$\left(\sqrt{4x^2+2x-12}\right)^2=(x+6)^2$$
$$4x^2+2x-12=x^2+2.x.6+36$$
$$4x^2-x^2+2x-12x-12-36=0$$
$$3x^2-10x-48=0$$
$$3x^2-18x+8x-48=0$$
$$3x(x-6)+8(x-6)=0$$
$$(3x+8)(x-6)=0$$
Either $3x+8=0$, $3x=-8$, $x=\dfrac{-8}{3}$ Or $x-6=0$, $x=6$

Now it is necessary to verify value of x in given radical equation. $\sqrt{2x-3}+\sqrt{2x+4}=\sqrt{6x+13}$

If $x=\dfrac{-8}{3}$, then
$$\sqrt{2\left(\frac{-8}{3}\right)-3}+\sqrt{2\left(\frac{-8}{3}\right)+4}=\sqrt{6\left(\frac{-8}{3}\right)+13}$$
$$\sqrt{\frac{-16-9}{3}}+\sqrt{\frac{-16+12}{3}}=\sqrt{\frac{-48}{3}+13}$$
$$\sqrt{\frac{-25}{3}}+\sqrt{\frac{-4}{3}}=\sqrt{\frac{-48+39}{3}}$$
$$5\sqrt{\frac{-1}{3}}+2\sqrt{\frac{-1}{3}}=\sqrt{\frac{-9}{3}} \qquad \text{False}$$
Thus $x=\dfrac{-8}{3}$ is an extraneous root

If x = 6, then
$$\sqrt{2(6)-3}+\sqrt{2(6)+4}=\sqrt{6(6)+13}$$
$$\sqrt{12-3}+\sqrt{12+4}=\sqrt{36+13}$$
$$\sqrt9+\sqrt{16}=\sqrt{49}$$
$$3+4=7$$
$$7=7 \qquad \text{True}$$
Thus $x=6$ is a real root so

Solution Set $=\{6\}$

Q1ix). $\sqrt{x^2+2x+4}+\sqrt{x^2+2x+9}=5$

Sol: Since $\sqrt{x^2+2x+4}+\sqrt{x^2+2x+9}=5$

Or $\sqrt{x^2+2x+4}=5-\sqrt{x^2+2x+9}$

Taking square on both sides
$$\left(\sqrt{x^2+2x+4}\right)^2=\left(5-\sqrt{x^2+2x+9}\right)^2$$
$$x^2+2x+4=25+\left(\sqrt{x^2+2x+9}\right)^2-2.5.\sqrt{x^2+2x+9}$$
$$x^2+2x+4=25+x^2+2x+9-10\sqrt{x^2+2x+9}$$
$$4=34-10\sqrt{x^2+2x+9}$$
$$10\sqrt{x^2+2x+9}=30 \qquad \div by10$$
$$\sqrt{x^2+2x+9}=3$$
Again taking square both sides
$$\left(\sqrt{x^2+2x+9}\right)^2=3^2$$

$x^2+2x+9=9$
$x^2+2x=9-9$
$x(x+2)=0$

Either $x=0$ or $x+2=0$
$x=-2$

Now it is necessary to verify value of x in given radical equation. $\sqrt{x^2+2x+4}+\sqrt{x^2+2x+9}=5$

If $x=0$, $\sqrt{(0)^2+2(0)+4}+\sqrt{(0)^2+2(0)+9}=5$
$\sqrt{4}+\sqrt{9}=5$
$2+3=5$
$5=5$ True
Thus $x=0$ is a real root
If $x=-2$, then
$\sqrt{(-2)^2+2(-2)+4}+\sqrt{(-2)^2+2(-2)+9}=5$
$\sqrt{4-4+4}+\sqrt{4-4+9}=5$
$\sqrt{4}+\sqrt{9}=5$
$5=5$ True
Thus $x=-2$ is a real root
Solution set $=\{0,-2\}$

Q1x). $\sqrt{2x^2+3x+5}+\sqrt{2x^2+3x+1}=2$
Solution: $\sqrt{2x^2+3x+5}+\sqrt{2x^2+3x+1}=2$
OR $\sqrt{2x^2+3x+5}=2-\sqrt{2x^2+3x+1}$
Taking square on both sides
$\left(\sqrt{2x^2+3x+5}\right)^2=\left(2-\sqrt{2x^2+3x+1}\right)^2$
$2x^2+3x+5=4+\left(\sqrt{2x^2+3x+1}\right)^2-2.2.\sqrt{2x^2+3x+1}$
$2x^2+3x+5=4+2x^2+3x+1-4\sqrt{2x^2+3x+1}$

$0=-2\sqrt{2x^2+3x+1}$ $\div$by -2
$\sqrt{2x^2+3x+1}=0$
Again taking square both sides
$\left(\sqrt{2x^2+3x+1}\right)^2=0^2$
$2x^2+3x+1=0$
$2x^2+2x+x+1=0$
$2x(x+1)+1(x+1)=0$
$(2x+1)(x+1)=0$
Either or
$x+1=0$ $2x+1=0$
$x=-1$ $2x=-1$
$x=\dfrac{-1}{2}$

Now it is necessary to verify value of x in given radical equation. $\sqrt{2x^2+3x+5}+\sqrt{2x^2+3x+1}=2$
If $x=-1$, then
$\sqrt{2(-1)^2+3(-1)+5}+\sqrt{2(-1)^2+3(-1)+1}=2$
$\sqrt{2-3+5}+\sqrt{2-3+1}=2$
$\sqrt{4}+\sqrt{0}=2$
$2+0=2$ True

Thus $x=-1$ is a real root
If $x=\frac{-1}{2}$, then
$\sqrt{2\left(\frac{-1}{2}\right)^2+3\left(\frac{-1}{2}\right)+5}+\sqrt{2\left(\frac{-1}{2}\right)^2+3\left(\frac{-1}{2}\right)+1}=2$
$\sqrt{\frac{1}{2}-\frac{3}{2}+5}+\sqrt{\frac{1}{2}-\frac{3}{2}+1}=2$
$\sqrt{\frac{1-3}{2}+5}+\sqrt{\frac{1-3}{2}+1}=2$
$\sqrt{\frac{-2}{2}+5}+\sqrt{\frac{-2}{2}+1}=2$
$\sqrt{-1+5}+\sqrt{-1+1}=2$
$\sqrt{4}+\sqrt{0}=2$
$2=2$ True
Thus $x=\dfrac{-1}{2}$ is a real root
Solution set $=\{-1,\frac{-1}{2}\}$

Q2 Find x if $2x+5$ satisfies
$\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$
Solution: we have $\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$
Given that $2x+5=0$ satisfies given equation
$x=\dfrac{-5}{2}$ putting $\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$
$\sqrt{40-9\left(\frac{-5}{2}\right)}-2\sqrt{7-\left(\frac{-5}{2}\right)}=\sqrt{-\left(\frac{-5}{2}\right)}$
$\sqrt{40+\frac{45}{2}}-2\sqrt{7+\frac{5}{2}}=\sqrt{\frac{5}{2}}$
$\sqrt{\frac{80+45}{2}}-2\sqrt{\frac{14+5}{2}}=\sqrt{\frac{5}{2}}$
$\sqrt{\frac{125}{2}}-2\sqrt{\frac{19}{2}}=\sqrt{\frac{5}{2}}$ which is false
Thus $2x+5$ do not satisfies the given equation
Or $\sqrt{40-9x}-\sqrt{-x}=2\sqrt{7-x}$
Taking square on both sides
$\left(\sqrt{40-9x}-\sqrt{-x}\right)^2=\left(2\sqrt{7-x}\right)^2$
$\left(\sqrt{40-9x}\right)^2+\left(\sqrt{-x}\right)^2-2\sqrt{40-9x}.\sqrt{-x}=4(7-x)$
$40-9x+(-x)-2\sqrt{-40x+9x^2}=28-4x$
$40-28-9x-x+4x=2\sqrt{9x^2-40x}$
$12-6x=2\sqrt{9x^2-40x}$ divided by 2
$6-3x=\sqrt{9x^2-40x}$ taking square
$(6-3x)^2=\left(\sqrt{9x^2-40x}\right)^2$
$36+9x^2-2(6)(3x)=9x^2-40x$
$36-36x=-40x$
$40x-36x=-36$
$4x=-36$
$x=-9$
Put in given equation
$\sqrt{40-9(-9)}-\sqrt{-(-9)}=2\sqrt{7-(-9)}$
$\sqrt{40+81}-\sqrt{9}=2\sqrt{7+9}$

$\sqrt{121}-\sqrt{9}=2\sqrt{16}$

$11-3=2(4)$

$8=8$ satisfies

Therefore $x=-9$ is a real root

S.S $=\{-9\}$

Review Exercise 1

Q1. Fill the correct circle only

i). if $(x+1)(x-5)=0$ then solutions are

O $x=1,-5$ O $x=1,5$

O $x=-1,-5$ O $x=-1,5$

ii). If $x^2-x-1=0$ then $x=$ ……………

O $\frac{-1\pm\sqrt{5}}{2}$ O $-1\pm\frac{\sqrt{5}}{2}$

O $\frac{1\pm\sqrt{5}}{2}$ O $1\pm\frac{\sqrt{5}}{2}$

iii). $\frac{-1\pm\sqrt{5}}{2}$ in simplified form is

O $1\pm\sqrt{24}$ O $1\pm\sqrt{6}$

O $2\pm\sqrt{6}$ O Cannot be simplified

iv). Apply the quadratic formula to $2x^2-x=3$

O $a=2,b=-1,c=3$ O $a=2,b=1,c=3$

O $a=1,b=-1,c=-3$ O $a=1,b=-1,c=0$

v). if $x^2-3x-4=0$ then the solution are

O $x=4,-1$ O $x=-4,1$

O $x=4,1$ O $x=-4,-1$

vi). If $2x^2-4x+9=0$ solutions are

O $x=\frac{2\pm\sqrt{22}}{2}$ O $x=\frac{-2\pm\sqrt{22}}{2}$

O $x=2\pm\frac{\sqrt{22}}{2}$ O $x=-2\pm\frac{\sqrt{22}}{2}$

vii). If $x^2-\frac{1}{4}=0$ the solutions are

O $x=\pm\frac{1}{2}$ O $x=\pm\frac{1}{4}$

O $x=\pm\frac{1}{8}$ O $x=\pm\frac{1}{16}$

viii). What are the solution of $x^2+7x-18=0$

O 2 or -9 O -2 or 9

O -2 or -9 O 2 or 9

ix). Roots of $x^2-8x+15=0$ are

O 1 or -7 O 2 or 4

O -2 or 4 O 3 or 5

Q2. Solve $2w^4-5w^2+2=0$

Solution: we have $2w^4-5w^2+2=0$

Let $y=w^2 \Rightarrow y^2=w^4$ Given equation becomes

$2y^2-5y+2=0$

$2y^2-4y-1y+2=0$

$2y(y-2)-1(y-2)=0$

$(2y-1)(y-2)=0$

Either $2y-1=0$ or $y-2=0$

$y=\frac{1}{2}$ $y=2$

Putting back the value of y

$w^2=\frac{1}{2}$ $w^2=2$

$w=\frac{\pm1}{\sqrt{2}}$ $w=\pm\sqrt{2}$

Solution set $=\left\{\frac{\pm1}{\sqrt{2}},\pm\sqrt{2}\right\}$

Q3. Find the constant a and b such that $x=1$ and $x=-1$ are the solutions to $ax^2+bx+2=0$

Sol: since $ax^2+bx+2=0$ have solution $x=1$

So $a(1)^2+b(1)+2=0$

Or $a+b+2=0$ ……………………..(1)

And $ax^2+bx+2=0$ have solution $x=-1$

So $a(-1)^2+b(-1)+2=0$

Or $a-b+2=0$ ……………………..(2)

Adding eq (1) and eq (2)

$a+b+2=0$

$\underline{a-b+2=0}$

$2a+4=0$

$2a=-4$

$a=-2$ put in eq (1)

$-2+b+2=0$

$\Rightarrow b=0$

Q4. Find all solutions of x such that x^2+5x+6 and $x^2+19x+32$ are equal

Sol: according to condition

$x^2+19x+32=x^2+5x+6$

$x^2-x^2+19x-5x=6-32$

$13x=-26$

$x=-2$

Q5. Find the solution to $49x^2-316x+132=0$

Solution we have $49x^2-316x+132=0$

$49x^2-316x+132=0$

$49x^2-294x-22x+132=0$

$49x(x-6)-22(x-6)=0$

$(49x-22)(x-6)=0$

Either $49x-22=0$ $x-6=0$

$x=\frac{22}{49}$ $x=6$

Solution set $=\left\{6,\frac{22}{49}\right\}$