Chapter 2

Quadratic Formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Nature of the roots of quadratic equation depends upon discriminant i.e., b^2-4ac

Roots are Real have 3 cases

i). if $b^2-4ac=0$

i.e., $x=\frac{-b\pm\sqrt{0}}{2a}=\frac{-b\pm0}{2a}=\frac{-b}{2a}$

roots are Real { rational } and equal

ii). if b^2-4ac is perfect square

Let $b^2-4ac=d^2$

i.e., $x=\frac{-b\pm\sqrt{d^2}}{2a}=\frac{-b\pm d}{2a}$

roots are Real { Rational } and unequal

iii). if b^2-4ac is positive number e

Let $b^2-4ac=e$

i.e., $x=\frac{-b\pm\sqrt{e}}{2a}$

roots are Real { irrational } and unequal

iv) roots are Imaginary/ complex conjugate

if b^2-4ac is negative number i.e., $-f$

Let $b^2-4ac=-f$

i.e., $x=\frac{-b\pm\sqrt{-f}}{2a}=\frac{-b\pm i\sqrt{f}}{2a}$

roots are imaginary / complex conjugate

Exp 1. Find discriminant of $x^2+9x+2=0$

Sol: we have $x^2+9x+2=0$ by comparing

$a=1, b=9, c=2$

Discriminant $=b^2-4ac$

$=9^2-4(1)(2)$

$=81-8$

$=73$

Exp2i). Find nature of roots of $x^2-8x+16=0$

Sol: Given $x^2-8x+16=0$ by comparing

$a=1, b=-8, c=15$

Discriminant $=b^2-4ac$

$=(-8)^2-4(1)(16)$

$=64-64$

$=0$

Therefore, roots of the given equation are real {rational} and equal

Exp2ii). Find nature of roots of $x^2+9x+2=0$

Sol: Given $x^2+9x+2=0$ by comparing

$a=1, b=9, c=2$

Discriminant $=b^2-4ac$

$=(9)^2-4(1)(2)$

$=81-8$

$=73>0$

Therefore, roots of the given equation are real {irrational} and unequal

Exp2iii) Find nature of roots of $6x^2-x-15=0$

Sol: Given $6x^2-x-15=0$ by comparing

$a=6, b=-1, c=-15$

Discriminant $=b^2-4ac$

$=(-1)^2-4(6)(-15)$

$=1+360$

$=361$

$=19^2$

Therefore, roots of the given equation are real {rational} and unequal

Exp2iv) Find nature of roots of $4x^2+x+1=0$

Sol: Given $4x^2+x+1=0$ by comparing

$a=4, b=1, c=1$

Discriminant $=b^2-4ac$

$=(1)^2-4(4)(1)$

$=1-16$

$=-15<0$

Therefore, roots of the given equation are complex conjugate {imaginary} and unequal

Exp3i Determine nature of roots of $x^2-6x+9=0$

And verify the result by solving them

Sol: Given $x^2-6x+9=0$ by comparing

$a=1, b=-6, c=9$

Discriminant $=b^2-4ac$

$=(-6)^2-4(1)(9)$

$=36-36$

$=0$

Therefore, roots of the given equation are real {rational} and equal. Now verification

$x^2-6x+9=0$

$x^2-3x-3x+9=0$

$x(x-3)-3(x-3)=0$

$(x-3)(x-3)=0$

Either $x-3=0$ or $x-3=0$

$x=3$ $x=3$

so roots of given equation are rational & equal

Exp3i Determine nature of roots of $x^2+5x+6=0$

And verify the result by solving them

Sol: Given $x^2+5x+6=0$ by comparing

$a=1, b=5, c=6$

Discriminant $=b^2-4ac$

$=(5)^2-4(1)(6)$

$=25-24$

$=1=1^2$

Therefore, roots of the given equation are real {rational} and unequal. Now verification

$x^2+5x+6=0$

$x^2+3x+2x+6=0$

$x(x+3)+2(x+3)=0$

$(x+2)(x+3)=0$

Either $x+2=0$ or $x+3=0$

$x=-2$ $x=-3$

so roots of given equation are rational & unequal

Exp4 without solving determine nature of roots of $3x^2-4x+6=0$

Sol: Given $3x^2-4x+6=0$ by comparing

$a=3, b=-4, c=6$

Discriminant $=b^2-4ac$

$=(-4)^2-4(3)(6)$

$=16-72$

$=-56<0$

Therefore, roots of the given equation are complex conjugate or imaginary and unequal

Exp5 without solving determine nature of roots of $2x^2-7x=-1$

Sol: Given $2x^2-7x=-1$

Or $2x^2-7x+1=0$ by comparing

$a=2, b=-7, c=1$

Discriminant $=b^2-4ac$

$=(-7)^2-4(2)(1)$

$=49-8$

$=41>0$

Therefore, roots of the given equation are real {Irrational} and unequal

Exp6i). Determine the value of k for which given $kx^2+4x+1=0$ have real roots.

Sol: we have $kx^2+4x+1=0$ by comparing

$a=k, b=4, c=1$

Discriminant $=b^2-4ac\geq 0$

$(4)^2-4(k)(1)\geq 0$

$16-4k\geq 0$

$16\geq 4k$

$4\geq k$

Or $k\leq 4$

Exp6ii). Determine the value of k for which given $2x^2+kx+3=0$ have real roots.

Sol: we have $2x^2+kx+3=0$ by comparing

$a=2, b=k, c=3$

Discriminant $=b^2-4ac\geq 0$

$(k)^2-4(2)(3)\geq 0$

$k^2-24\geq 0$

$k^2\geq 24$

$\sqrt{k^2}\geq\sqrt{24}$

$|k|\geq 2\sqrt{6}$

$\pm k\geq 2\sqrt{6}$

Either $k\geq 2\sqrt{6}$ or $-k\geq 2\sqrt{6}$

$k\leq -2\sqrt{6}$

Exercise 2.1

Q1i). Find discriminant of $x^2-4x+13=0$

Sol: Since $x^2-4x+13=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we get $a=1$, $b=-4, c=13$

$\therefore$ Discriminant $=b^2-4ac$

$=(-4)^2-4(1)(13)$

$=16-52$

$=-36$

Q1ii). Find discriminant of $4x^2-5x+1=0$

Sol: Since $4x^2-5x+1=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=4, b=-5, c=1$

$\therefore$ Discriminant $=b^2-4ac$

$=(-5)^2-4(4)(1)$

$=25-16$

$=9$

Q1iii). Find discriminant of $x^2+x+1=0$

Sol: Since $x^2+x+1=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=1$, $b=1, c=1$

$\therefore$ Discriminant $=b^2-4ac$

$=(1)^2-4(1)(1)$

$=1-4$

$=-3$

Q2i). Examine nature of roots of $3x^2-5x+1=0$

Sol: Since $3x^2-5x+1=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=3$, $b=-5, c=1$

$\therefore$ Discriminant $=b^2-4ac$

$=(-5)^2-4(3)(1)$

$=25-12$

$=13>0$

As $b^2-4ac>0$, but not a perfect square, therefor, roots are real {irrational} & Unequal

Q2ii). Examine nature of roots of $6x^2+x-2=0$

Sol: Since $6x^2+x-2=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=6, b=1, c=-2$

$\therefore$ Discriminant $=b^2-4ac$

$=(1)^2-4(6)(-2)$

$=1+48$

$=49$

$=7^2$

As b^2-4ac is a perfect square, Therefor, roots are real;{rational} & Unequal

Q2iii)Examine nature of roots of $3x^2+2x+1=0$

Sol: Since $3x^2+2x+1=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=3$, $b=2, c=1$

$\therefore$ Discriminant $=b^2-4ac$

$=(2)^2-4(3)(1)$

$=4-12$

$=-8<0$

As $b^2-4ac<0$, Therefor, roots are imaginary & unequal or Complex conjugate

Q3i). For what value of k the roots of $x^2+kx+9=0$ are equal.

Sol: Comparing $x^2+kx+9=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1, b=k, c=9$

As the roots are equal, therefore

$\therefore$ Discriminant $=b^2-4ac=0$

$$\Rightarrow (k)^2-4(1)(9)=0$$
$$\Rightarrow k^2-36=0$$
$$\Rightarrow k^2=36$$
$$\Rightarrow k=\pm 6$$

Q3ii). For what value of k the roots of $12x^2+kx+3=0$ are equal.

Sol: Comparing $12x^2+kx+3=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=12, b=k, c=3$

As the roots are equal, therefore

$\therefore$ Discriminant $=b^2-4ac=0$

$$\Rightarrow (k)^2-4(12)(3)=0$$
$$\Rightarrow k^2-144=0$$
$$\Rightarrow k^2=144$$
$$\Rightarrow k=\pm 12$$

Q3iii). For what value of k the roots of $x^2-5x+k=0$ are equal.

Sol: Comparing $x^2-5x+k=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1, b=-5, c=k$

As the roots are equal, therefore

$\therefore$ Discriminant $=b^2-4ac=0$

$$\Rightarrow (-5)^2-4(1)(k)=0$$
$$\Rightarrow 25-4k=0$$
$$\Rightarrow 25=4k$$
$$\Rightarrow k=\frac{25}{4}$$

Q4i). Determine whether $x^2+5x+5=0$ have real roots and if so, find roots.

Sol: Comparing $x^2+5x+5=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1, b=5, c=5$

To check the nature of the roots

$\therefore$ Discriminant $=b^2-4ac$

$$=(5)^2-4(1)(5)$$
$$=25-20$$
$$=5>0$$

As $b^2-4ac>0$, but not a perfect square, therefore, roots are real; Unequal and irrational

Since the roots are real so we will solve further

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ Putting values of a,b & c

$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(5)}}{2(1)}$$

$$x=\frac{-5\pm\sqrt{25-20}}{2}$$

$$x=\frac{-5\pm\sqrt{5}}{2}$$

$$S.S=\left\{\frac{-5+\sqrt{5}}{2},\frac{-5-\sqrt{5}}{2}\right\}$$ Unequal & irrational

Q4ii). Determine whether $4x^2+12x+9=0$ have real roots and if so, find roots.

Sol: Comparing $4x^2+12x+9=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=4, b=12, c=9$

To check the nature of the roots

$\therefore$ Discriminant $=b^2-4ac$

$$=(12)^2-4(4)(9)$$
$$=144-144$$
$$=0$$

As $b^2-4ac=0$, therefor, the roots are real { rational } and Equal { Repeated }

Since roots are real so we will solve further

$$4x^2+12x+9=0$$
$$(2x)^2+2(2x)(3)+(3)^2=0$$
$$(2x+3)^2=0$$
$$(2x+3)(2x+3)=0$$

Either $2x+3=0$, $2x=-3$, $x=\frac{-3}{2}$ or $2x+3=0$, $2x=-3$, $x=\frac{-3}{2}$

Solution Set $=\left\{\frac{-3}{2}\right\}$ Repeated

Q4iii). Determine whether $6x^2+x-2=0$ have real roots and if so, find roots.

Sol: Comparing $6x^2+x-2=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=6, b=1, c=-2$

To check the nature of the roots

$\therefore$ Discriminant $=b^2-4ac$

$$=(1)^2-4(6)(-2)$$
$$=1+48$$
$$=49$$
$$=7^2$$

As b^2-4ac is a perfect square, therefor, the roots are real "Unequal and rational"

Since roots are real so we will solve further

$$6x^2+x-2=0$$
$$6x^2+4x-3x-2=0$$
$$2x(3x+2)-1(3x+2)=0$$
$$(2x-1)(3x+2)=0$$

Either $2x-1=0$, $2x=1$, $x=\frac{1}{2}$ or $3x+2=0$, $3x=-2$, $x=\frac{-2}{3}$

Solution Set $=\left\{\frac{1}{2},\frac{-2}{3}\right\}$ Unequal and rational

Q5i). Determine nature of roots of
$3x^2-10x+3=0$ & verify result by solving them.
Sol: To check the nature of the roots
Comparing $3x^2-10x+3=0$
with the quadratic equation $ax^2+bx+c=0$
we have $a=3$, $b=-10$, $c=3$
$\therefore$ Discriminant $=b^2-4ac$

$$=(-10)^2-4(3)(3)$$
$$=100-36$$
$$=64$$
$$=8^2$$

As b^2-4ac is a perfect square, therefor, roots are real "Unequal & rational"

Verification

$$3x^2-10x+3=0$$
$$3x^2-9x-x+3=0$$
$$3x(x-3)-1(x-3)=0$$
$$(3x-1)(x-3)=0$$

Either
$3x-1=0$
$3x=1$
$x=\frac{1}{3}$

Or
$x-3=0$
$x=3$

Solution Set $=\left\{3,\frac{1}{3}\right\}$ Unequal and rational

Q5ii). Determine nature of roots of
$x^2-6x+4=0$ & verify result by solving them.
Sol: To check the nature of the roots
Comparing $x^2-6x+4=0$
with the quadratic equation $ax^2+bx+c=0$
we have $a=1$, $b=-6$, $c=4$
$\therefore$ Discriminant $=b^2-4ac$

$$=(-6)^2-4(1)(4)$$
$$=36-16$$
$$=20>0$$

As $b^2-4ac>0$, but not a perfect square, therefor, the roots are real "Unequal and irrational"
Verification Using quadratic formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Putting values of a,b & c

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$$
$$x=\frac{6\pm\sqrt{36-20}}{2}$$
$$x=\frac{6\pm\sqrt{20}}{2}=\frac{6\pm2\sqrt{5}}{2}$$
$$x=3\pm\sqrt{5}$$

SS $=\left\{3+\sqrt{5},3-\sqrt{5}\right\}$ roots are Unequal & irrational

Q5iii). Determine nature of roots of $x^2-3=0$ & verify result by solving them.
Sol: To check the nature of the roots
Comparing $x^2-3=0$ with the quadratic equation $ax^2+bx+c=0$ so $a=1$, $b=0$, $c=-3$
$\therefore$ Discriminant $=b^2-4ac$

$$=(0)^2-4(1)(-3)$$
$$=0+12$$
$$=12$$

As $b^2-4ac>0$, but not a perfect square, therefor, roots are real "Unequal and irrational"
Verification Since we have $x^2-3=0$

$$x^2=3$$
$$\Rightarrow x=\pm\sqrt{3}$$

S Set $=\left\{\sqrt{3},-\sqrt{3}\right\}$ roots are Unequal & irrational

Q6i). For what value of k the roots of
$2x^2+3x+k=0$ are (a) Real (b) Imaginary
Sol: Comparing $2x^2+3x+k=0$
with the quadratic equation $ax^2+bx+c=0$
we have $a=2$, $b=3$, $c=k$
(a) if roots are real then $b^2-4ac\ge0$

$$\Rightarrow(3)^2-4(2)(k)\ge0$$
$$\Rightarrow9-8k\ge0$$
$$\Rightarrow9\ge8k$$
$$\Rightarrow\frac{9}{8}\ge k \quad Or \quad k\le\frac{9}{8}$$

(b) if roots are imaginary then $b^2-4ac<0$

$$\Rightarrow(3)^2-4(2)(k)<0$$
$$\Rightarrow9-8k<0$$
$$\Rightarrow9<8k$$
$$\Rightarrow\frac{9}{8}<k \quad Or \quad k>\frac{9}{8}$$

Solution Set $=\begin{cases}\text{Real} & k\le\frac{9}{8}\\ \text{Imaginary} & k>\frac{9}{8}\end{cases}$

Q6ii). For what value of k the roots of
$kx^2+2x+1=0$ are (a) Real (b) Imaginary
Sol: Comparing $kx^2+2x+1=0$
with the quadratic equation $ax^2+bx+c=0$
we have $a=k$, $b=2$, $c=1$
(a) if the roots are real then $b^2-4ac\ge0$

$$\Rightarrow(2)^2-4(k)(1)\ge0$$
$$\Rightarrow4-4k\ge0$$
$$\Rightarrow4\ge4k$$
$$\Rightarrow1\ge k \quad Or \quad k\le1$$

(b) if roots are imaginary then $b^2-4ac<0$

$$\Rightarrow(2)^2-4(k)(1)<0$$
$$\Rightarrow4-4k<0$$
$$\Rightarrow4<4k$$
$$\Rightarrow1<k \quad Or \quad k>1$$

Solution Set $=\begin{cases}\text{Real} & k\le1\\ \text{Imaginary} & k>1\end{cases}$

Q6iii). For what value of k the roots of
$x^2+5x+k=0$ are (a) Real (b) Imaginary
Sol: Comparing $x^2+5x+k=0$
with the quadratic equation $ax^2+bx+c=0$
we have $a=1$, $b=5$, $c=k$
(a) if the roots are real then $b^2-4ac\ge0$

$\Rightarrow (5)^2 - 4(1)(k) \geq 0$
$\Rightarrow 25 - 4k \geq 0$
$\Rightarrow 25 \geq 4k$
$\Rightarrow \frac{25}{4} \geq k$ Or $k \leq \frac{25}{4}$

(b) if roots are imaginary then $b^2 - 4ac < 0$

$\Rightarrow (5)^2 - 4(1)(k) < 0$
$\Rightarrow 25 - 4k < 0$
$\Rightarrow 25 < 4k$
$\Rightarrow \frac{25}{4} < k$ Or $k > \frac{25}{4}$

Solution Set $= \left\{ \begin{array}{ll} \text{Real} & k \leq \frac{25}{4} \\ \text{Imaginary} & k > \frac{25}{4} \end{array} \right\}$

Cube root of unity

Let x be a cube root of unity, then

$$x = \sqrt[3]{1} = (1)^{\frac{1}{3}}$$
$$(x)^3 = \left(1^{\frac{1}{3}}\right)^3$$

$\Rightarrow x^3 = 1$
$\Rightarrow x^3 - 1 = 0$
$\Rightarrow x^3 - 1^3 = 0$
$\Rightarrow (x-1)(x^2 + x.1 + 1^2) = 0$
$\Rightarrow (x-1)(x^2 + x + 1) = 0$

Either $x - 1 = 0$, $x = 1$ Or $x^2 + x + 1 = 0$

Comparing with the quadratic equation $ax^2 + bx + c = 0$ we have $a = 1$, $b = 1$, $c = 1$

Using quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Putting values of a,b & c

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$
$$x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2} \qquad \therefore i = \sqrt{-1}$$

Solution Set $= \left\{1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$

Let $\omega = \frac{-1+i\sqrt{3}}{2} \Rightarrow \omega^2 = \frac{-1-i\sqrt{3}}{2}$ then

Solution Set $= \{1, \omega, \omega^2\}$

Sum of the cube roots of unity

1). $1 + \omega + \omega^2 = 0$

Sol: since $\omega = \frac{-1+i\sqrt{3}}{2} \Rightarrow \omega^2 = \frac{-1-i\sqrt{3}}{2}$

Taking LHS and putting the values

$$1 + w + w^2 = 1 + \frac{-1+\sqrt{-3}}{2} + \frac{-1-\sqrt{3}}{2}$$
$$= \frac{2 - 1 + \sqrt{-3} - 1 - \sqrt{3}}{2} = \frac{0}{2} = 0 = \text{RHS}$$

Product of the cube roots of unity

Since $\omega = \frac{-1+i\sqrt{3}}{2} \Rightarrow \omega^2 = \frac{-1-i\sqrt{3}}{2}$

Taking $1.w.w^2 = (1)\left(\frac{-1+\sqrt{-3}}{2}\right)\left(\frac{-1-\sqrt{3}}{2}\right)$

$$= \frac{(-1)^2 - \left(\sqrt{-3}\right)^2}{2 \times 2}$$
$$= \frac{1-(-3)}{4}$$
$$= \frac{1+3}{4} = \frac{4}{4}$$
$$w^3 = 1$$

Reciprocal of the cube roots of unity

Since $w^3 = 1$

Taking $w = w.\frac{w^2}{w^2}$

$$= \frac{w^3}{w^2} \qquad \therefore w^3 = 1$$
$$w = \frac{1}{w^2}$$

And $w^2 = w^2.\frac{w}{w}$

$$= \frac{w^3}{w} \qquad \therefore w^3 = 1$$
$$w^2 = \frac{1}{w}$$

Exp 7 Show that

$x^3 + y^3 = (x+y)(x+wy)(x+w^2y)$

Sol: Taking RHS $= (x+y)(x+wy)(x+w^2y)$

$$= (x+y)(x^2 + wxy + w^2xy + w^3y^2)$$
$$= (x+y)(x^2 + (w+w^2)xy + 1.y^2)$$
$$= (x+y)(x^2 + (-1)xy + y^2)$$
$$= (x+y)(x^2 - xy + y^2)$$
$$= x^3 + y^3$$

Exp8.Evaluate $w^{15}, w^{24}, w^{90}, w^{101}, w^{-2}, w^{-13}$

Sol: Given $w^{15}, w^{24}, w^{90}, w^{101}, w^{-2}, w^{-13}$

$$w^{15} = (w^3)^5 = (1)^5 = 1$$
$$w^{24} = (w^3)^8 = (1)^8 = 1$$
$$w^{90} = (w^3)^{30} = (1)^{30} = 1$$
$$w^{101} = w^{99}.w^2 = (w^3)^{33}.w^2 = 1.w^2 = w^2$$
$$w^{-2} = w^{-2}.1 = w^{-2}.w^3 = w^{-2+3} = w$$
$$w^{-13} = w^{-13}.1 = w^{-13}.1^5 = w^{-13}.w^{3\times5} = w^{-13+15} = w^2$$

Exp9 Show that $\left(-1+i\sqrt{3}\right)^3 + \left(-1-i\sqrt{3}\right)^3 = 16$

Sol: since $\omega = \frac{-1+i\sqrt{3}}{2} \Rightarrow \omega^2 = \frac{-1-i\sqrt{3}}{2}$

$2w=-1+i\sqrt{3},\quad 2w^2=-1-i\sqrt{3}$

Taking LHS $\left(-1+i\sqrt{3}\right)^3+\left(-1-i\sqrt{3}\right)^3$

$=\left(2w\right)^3+\left(2w^2\right)^3$

$=2^3w^3+2^3\left(w^3\right)^2$

$=8(1)+8.(1)^2$

$=8+8$

$=16$ RHS

Exercise 2.2

Q1i). Find cube root of -1

Sol: Let x be a cube root of -1, then

$x=\sqrt[3]{-1}=(-1)^{\frac{1}{3}}$

$(x)^3=\left\{(-1)^{\frac{1}{3}}\right\}^3$

$\Rightarrow\quad x^3=-1$

$\Rightarrow\quad x^3+1=0$

$\Rightarrow\quad x^3+1^3=0$

$\Rightarrow\quad (x+1)\left(x^2-x.1+1^2\right)=0$

$\Rightarrow\quad (x+1)\left(x^2-x+1\right)=0$

Either $x+1=0$, $x=-1$ Or $x^2-x+1=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=1$, $b=-1$, $c=1$

Using quadratic formula

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ Putting values of a,b & c

$x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$x=\dfrac{1\pm\sqrt{1-4}}{2}$

$x=\dfrac{1\pm\sqrt{-3}}{2}=\dfrac{1\pm i\sqrt{3}}{2}\qquad \therefore i=\sqrt{-1}$

$x=-\dfrac{\left(-1\mp i\sqrt{3}\right)}{2}$

Either $x=-\dfrac{\left(-1-i\sqrt{3}\right)}{2}$, $x=-\omega^2$ Or $x=-\dfrac{\left(-1+i\sqrt{3}\right)}{2}$, $x=-\omega$

Solution Set $=\left\{-1,-\omega,-\omega^2\right\}$

Q1ii). Find cube root of 8

Sol: Let x be a cube root of 8, then

$x=\sqrt[3]{8}=(8)^{\frac{1}{3}}$

$(x)^3=\left\{(8)^{\frac{1}{3}}\right\}^3$

$\Rightarrow\quad x^3=8$

$\Rightarrow\quad x^3-8=0$

$\Rightarrow\quad x^3-2^3=0$

$\Rightarrow\quad (x-2)\left(x^2+x.2+2^2\right)=0$

$\Rightarrow\quad (x-2)\left(x^2+2x+4\right)=0$

Either $x-2=0$, $x=2$ Or $x^2+2x+4=0$

Comparing with the quadratic equation

$ax^2+bx+c=0$ we have $a=1$, $b=2, c=4$

Using quadratic formula

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ Putting values of a,b & c

$x=\dfrac{-(2)\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$

$x=\dfrac{-2\pm\sqrt{4-16}}{2}=\dfrac{-2\pm\sqrt{-12}}{2}$

$x=\dfrac{-2\pm\sqrt{-3\times4}}{2}=\dfrac{-2\pm2i\sqrt{3}}{2}\qquad \therefore i=\sqrt{-1}$

$x=2\dfrac{\left(-1\pm i\sqrt{3}\right)}{2}$

Either $x=2\dfrac{\left(-1+i\sqrt{3}\right)}{2}$, $x=2\omega$ Or $x=2\dfrac{\left(-1-i\sqrt{3}\right)}{2}$, $x=2\omega^2$

Solution Set $=\left\{2,2\omega,2\omega^2\right\}$

Q1iii). Find cube root of - 27

Sol: Let x be a cube root of - 27, then

$x=\sqrt[3]{-27}=(-27)^{\frac{1}{3}}$

$\Rightarrow\quad x^3=-27$

$x^3+27=0$

$x^3+3^3=0$

$(x+3)\left(x^2-x.3+3^2\right)=0$

Either $x+3=0$, $x=-3$ Or $x^2-3x+9=0$

Comparing with $ax^2+bx+c=0$ we have $a=1, b=-3\,c=9$ Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$

$x=\dfrac{3\pm\sqrt{9-36}}{2}=\dfrac{3\pm\sqrt{-27}}{2}$

$x=\dfrac{3\pm\sqrt{9(-3)}}{2}=\dfrac{3\pm3\sqrt{-3}}{2}$

$x=-3\left(\dfrac{-1\mp\sqrt{-3}}{2}\right)$

Either $x=-3\left(\dfrac{-1-\sqrt{-3}}{2}\right)$, $x=-3w^2$ Or $x=-3\left(\dfrac{-1+\sqrt{-3}}{2}\right)$, $x=-3w$

Solution Set $=\left\{-3,-3w,-3w^2\right\}$

Q2i). Evaluate $\omega^{12}+\omega^{58}+\omega^{95}$

Sol: Since $\omega^{12}+\omega^{58}+\omega^{95}$

$=\omega^{12}+\omega^{57+1}+\omega^{93+2}$

$=\omega^{3\times4}+\omega^{3\times19}.\omega^1+\omega^{3\times31}.\omega^2$

$=\left(\omega^3\right)^4+\left(\omega^3\right)^{19}.\omega+\left(\omega^3\right)^{31}.\omega^2\qquad \therefore \omega^3=1$

$=(1)^4+(1)^{19}.\omega+(1)^{31}.\omega^2$

$=1+1.\omega+1.\omega^2\qquad \therefore 1+\omega+\omega^2=0$

$=1+\omega+\omega^2$

$=0$

Q2ii). Evaluate $\left(1+\omega-\omega^2\right)^7$

Sol: Since $\left(1+\omega-\omega^2\right)^7$

$=\left(\{1+\omega\}-\omega^2\right)^7 \qquad \therefore 1+\omega+\omega^2=0$

$=\left(-\omega^2-\omega^2\right)^7 \qquad 1+\omega=-\omega^2$

$=\left(-2\omega^2\right)^7=\left(-2\right)^7\omega^{2\times7}$

$=-128\omega^{14}=-128\omega^{12+2}$

$=-128\left(\omega^3\right)^4\omega^2=-128.\left(1\right)^4\omega^2$

$=-128\omega^2$

Q2iii). Evaluate $\left(1+3\omega-\omega^2\right)\left(1+\omega-2\omega^2\right)$

Sol: Since $\left(1+3\omega-\omega^2\right)\left(1+\omega-2\omega^2\right)$

$=\left(\{1+\omega\}+2\omega-\omega^2\right)\left(\{1+\omega\}-2\omega^2\right)$

$=\left(-\omega^2+2\omega-\omega^2\right)\left(-\omega^2-2\omega^2\right) \qquad \therefore 1+\omega=-\omega^2$

$=\left(-\omega^2+2\omega-\omega^2\right)\left(-\omega^2-2\omega^2\right)$

$=\left(2\omega-2\omega^2\right)\left(-3\omega^2\right)$

$=\left(2\right)\left(\omega-\omega^2\right)\left(-3\right)\left(\omega^2\right)$

$=-6\left(\omega-\omega^2\right)\left(\omega^2\right)$

$=-6\left(\omega.\omega^2-\omega^2.\omega^2\right)$

$=-6\left(\omega^3-\omega^3\omega\right) \qquad \therefore \omega^3=1$

$=-6\left(1-\omega\right)$

Q2iii). Evaluate $\left(1+3w+w^2\right)\left(1+w-2w^2\right)$

Sol: Since $\left(1+3w+w^2\right)\left(1+w-2w^2\right)$

$=\left(1+w+w^2+2w\right)\left(-w^2-2w^2\right)$

$=\left(0+2w\right)\left(-3w^2\right)$

$=-6w^3$

$=-6$

Q3i). Prove that $\left(1+2\omega\right)\left(1+2\omega^2\right)\left(1-\omega-\omega^2\right)=6$

Sol: Take LHS $\left(1+2\omega\right)\left(1+2\omega^2\right)\left(1-\omega-\omega^2\right)$

$=\left\{1\left(1+2\omega^2\right)+2\omega\left(1+2\omega^2\right)\right\}\left(1-\left\{\omega+\omega^2\right\}\right)$

$=\left\{1+2\omega^2+2\omega+4\omega^3\right\}\left(1-\{-1\}\right)$

$=\left\{1+2\left(\omega^2+\omega\right)+4.1\right\}\left(1+1\right)$

$=\left\{1+2\left(-1\right)+4\right\}\left(2\right)$

$=\left(1-2+4\right)\left(2\right)$

$=\left(3\right)\left(2\right)$

$=6=RHS$

Hence $\left(1+2\omega\right)\left(1+2\omega^2\right)\left(1-\omega-\omega^2\right)=6$

Q3ii). Prove that $\left(-1+i\sqrt{3}\right)^4\left(-1-i\sqrt{3}\right)^5=512\omega^2$

Sol: As $\omega=\dfrac{-1+i\sqrt{3}}{2}$ and $\omega^2=\dfrac{-1-i\sqrt{3}}{2}$

$\Rightarrow \qquad 2\omega=-1+i\sqrt{3}$ and $\qquad 2\omega^2=-1-i\sqrt{3}$

Take LHS $\left(-1+i\sqrt{3}\right)^4\left(-1-i\sqrt{3}\right)^5=\left(2\omega\right)^4.\left(2\omega^2\right)^5$

$=2^4\omega^4.2^5\omega^{10}$

$=16\times32\omega^{4+10}$

$=512\omega^{14}$

$=512\omega^{12}.\omega^2$

$=512\left(\omega^3\right)^4\omega^2$

$=512\times1.\omega^2$

$=512\omega^2$

Q4i). Show that $x^3-y^3=\left(x-y\right)\left(x-\omega y\right)\left(x-\omega^2y\right)$

Sol: Take RHS $\left(x-y\right)\left(x-\omega y\right)\left(x-\omega^2y\right)$

$=\left(x-y\right)\left\{x\left(x-\omega^2y\right)-\omega y\left(x-\omega^2y\right)\right\}$

$=\left(x-y\right)\left\{x^2-\omega^2xy-\omega xy+\omega^3y^2\right\}$

$=\left(x-y\right)\left\{x^2-\left(\omega^2+\omega\right)xy+\omega^3y^2\right\}$

$=\left(x-y\right)\left\{x^2-\left(-1\right)xy+1.y^2\right\}$

$=\left(x-y\right)\left\{x^2+xy+y^2\right\}$

$=x^3-y^3=LHS$

Hence $x^3-y^3=\left(x-y\right)\left(x-\omega y\right)\left(x-\omega^2y\right)$

Q4ii). Show that $\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)=1$

Sol: Take LHS $\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)$

$=\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^3\omega\right)\left(1+\omega^6\omega^2\right)$

$=\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega\right)\left(1+\omega^2\right) \qquad \therefore \omega^3=1$

Rearranging the same factors

$=\left(1+\omega\right)\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^2\right)$

$=\left(1+\omega\right)^2\left(1+\omega^2\right)^2$

$=\left\{\left(1+\omega\right)\left(1+\omega^2\right)\right\}^2$

$=\left\{1\left(1+\omega^2\right)+\omega\left(1+\omega^2\right)\right\}^2$

$=\left\{1+\omega^2+\omega+\omega^3\right\}^2$

$=\left\{0+1\right\}^2$

$=1^2$

$=1=RHS$

Hence $\left(1+\omega\right)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)=1$

$\boxed{\textbf{Relation between the roots}}$ (Solutions) and the coefficients of the quadratic equation

Let α , β be the roots of the quadratic equation $ax^2+bx+c=0, \quad a\neq0$ so that

$\alpha=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta=\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$\boxed{\text{The Sum of the roots}}$

$\text{Sum}=\alpha+\beta=\dfrac{-b+\sqrt{b^2-4ac}}{2a}+\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$=\dfrac{\left(-b+\sqrt{b^2-4ac}\right)+\left(-b-\sqrt{b^2-4ac}\right)}{2a}$

$=\dfrac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-b-b+\sqrt{b^2-4ac}-\sqrt{b^2-4ac}}{2a}$

$= \frac{-2b}{2a}$

$\alpha + \beta = \frac{-b}{a}$

The Product of the roots

$Product = \alpha.\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$= \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}$

$= \frac{+b^2-(b^2-4ac)}{4a^2}$

$= \frac{b^2-b^2+4ac}{4a.a}$

$= \frac{4ac}{4a.a}$

$= \frac{c}{a}$

Exp10i). Without solving find the sum and product of the roots of $2x^2-3x-4=0$

Sol: Given $2x^2-3x-4=0$ by comparing

$a=2, b=-3, c=-4$

$\therefore$ sum of roots $S = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$

$\therefore$ Product of roots $P = \frac{c}{a} = \frac{-4}{2} = -2$

Exp10ii). Without solving find the sum and product of the roots of $3x^2+6x-2=0$

Sol: Given $3x^2+6x-2=0$ by comparing

$a=3, b=6, c=-2$

$\therefore$ sum of roots $S = \frac{-b}{a} = \frac{-6}{3} = -2$

$\therefore$ Product of roots $P = \frac{c}{a} = \frac{-2}{3}$

Exp11. Find value of k so that sum of roots of $2x^2+kx+6=0$ is equal to three times product of roots

Sol: Given $2x^2+kx+6=0$ by comparing

$a=2, b=k, c=6$ according to condition

sum of roots = $3\times$ Product of roots

$S=3P$

$\frac{-b}{a} = \frac{3c}{a}$

$-b=3c$ putting the values

$-k=3(6)$

$k=-18$

Exp12. Find value of a if sum of square of roots $x^2-3ax+a^2=0$ is 7

Sol: Given $\alpha^2+\beta^2=7$ and $x^2-3ax+a^2=0$

by comparing $A=1, B=-3a, C=a^2$

$\therefore\ P=\alpha\beta = \frac{c}{a} = \frac{a^2}{1} = a^2$

$\therefore\ S=\alpha+\beta = \frac{-b}{a} = \frac{-(-3a)}{1} = 3a$

$(\alpha+\beta)^2 = (3a)^2$

$\alpha^2+\beta^2+2\alpha\beta = 9a^2$ Putting the values

$7+2(a^2)=9a^2$

$7=9a^2-2a^2$

$7=7a^2$

$a^2=1$

$\Rightarrow a=\pm1$

Exp13. Find value of k if roots of $x^2-7x+k=0$ differ by unity

Sol: Given roots are differ by unit Assume that $\alpha, \alpha+1$ be the roots of $x^2-7x+k=0$

$\therefore\ S=\alpha+\alpha+1 = \frac{-b}{a} = \frac{-(-7)}{1} = 7$

$2\alpha+1=7$

$2\alpha=7-1$

$2\alpha=6$

$\alpha=3$

$\therefore\ P=\alpha(\alpha+1) = \frac{c}{a} = \frac{k}{1} = k$ putting $\alpha=3$

$3(3+1)=k$

$k=12$

Exp13. If α, β are roots of $9x^2-27x+k=0$, Find the value of k such that $2\alpha+5\beta=7$

Sol: Given $9x^2-27x+k=0$

$\alpha+\beta = \frac{-(-27)}{9}$ $\quad \alpha\beta = \frac{k}{9}$ ....(2)

$\alpha+\beta = \frac{27}{9}$

$\alpha+\beta=3$.........(1)

Given condition $2\alpha+5\beta=7$

$2\alpha+2\beta+3\beta=7$

$2(\alpha+\beta)+3\beta=7$ putting

$2(3)+3\beta=7$

$6+3\beta=7$

$3\beta=7-6$

$3\beta=1$

$\beta=\frac{1}{3}$

Put in eq (1) $\quad \alpha+\frac{1}{3}=3$

$\alpha = 3-\frac{1}{3} = \frac{9-1}{3} = \frac{8}{3}$

eq (2) $\quad \frac{k}{9}=\alpha\beta$ $\quad$ Putting

$\frac{k}{9} = \left(\frac{8}{3}\right)\left(\frac{1}{3}\right)$

$\frac{k}{9} = \frac{8}{9}$

$\Rightarrow k=8$

Exp15. Find value of m and n if both sum and product of roots of $mx^2-5x+n=0$

Sol: Given $mx^2-5x+n=0$ by comparing

$a=m, b=-5, c=n$ according to condition

$\therefore \alpha+\beta=10$ $\quad\quad$ $\therefore \alpha\beta=10$

$\frac{-(-5)}{m}=10$ $\quad\quad$ $\frac{n}{m}=10$

$5=10m$ $\quad\quad$ $n=10m$

$\frac{5}{10}=m$ $\quad\quad$ $n=10\left(\frac{1}{2}\right)$ $\quad \therefore m=\frac{1}{2}$

$m=\frac{1}{2}$ $\quad\quad$ $n=5$

Exercise 2.3

Q1i). Without solving equation, find the sum & product of roots of $4x^2-4x-3=0$

Sol: Comparing $4x^2-4x-3=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=4$, $b=-4$, $c=-3$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting values of a and b

$\alpha+\beta=\frac{-(-4)}{4}=\frac{4}{4}=1$

Product of the roots

$\alpha.\beta=\frac{c}{a}$ Putting values

$\alpha.\beta=\frac{-3}{4}$

Q1ii). Without solving equation, find the sum & product of roots of $2x^2+5x+6=0$

Sol: Comparing $2x^2+5x+6=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=2$, $b=5$, $c=6$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting the values of a and b

$\alpha+\beta=\frac{-(5)}{2}=\frac{-5}{2}$

Product of the roots $\alpha.\beta=\frac{c}{a}$ Putting values

$\alpha.\beta=\frac{6}{2}=3$

Q1iii). Without solving equation, find the sum & product of roots of $3x^2+2x-5=0$

Sol: Comparing $3x^2+2x-5=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=3$, $b=2$, $c=-5$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting the values of a and b

$\alpha+\beta=\frac{-(2)}{3}=\frac{-2}{3}$

Product of the roots $\alpha.\beta=\frac{c}{a}$ Putting values

$\alpha.\beta=\frac{-5}{3}$

Q2. Find the value of k if sum of the roots of $2x^2+kx+6=0$ is equal to product of roots.

Sol: Comparing $2x^2+kx+6=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=2$, $b=k$, $c=6$

According to the given condition

Sum of the roots = Product of the roots i.e.,

$$\alpha+\beta=\alpha.\beta$$

$$\frac{-b}{a}=\frac{c}{a} \quad \times\text{by } a$$

$$-b=c$$

Substituting the values of b and c

$$-k=6$$

$$\Rightarrow \quad k=-6$$

Q3. Find the value of k if the sum of square of the roots of $x^2-5kx+6k^2=0$ is equal to 13.

Sol: Comparing $x^2-5kx+6k^2=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-5k$, $c=6k^2$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting the values of a and b

$\alpha+\beta=\frac{-(-5k)}{1}=\frac{5k}{1}=5k$

Product of the roots

$\alpha.\beta=\frac{c}{a}$ Putting the values of a and c

$\alpha.\beta=\frac{6k^2}{1}=6k^2$

According to the given condition

$\alpha^2+\beta^2=13$

$\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta=13$

$(\alpha+\beta)^2-2\alpha\beta=13$ $\quad\quad \therefore a^2+b^2+2ab=(a+b)^2$

Putting the values of $\alpha+\beta$ and $\alpha.\beta$ we get

$$(5k)^2-2(6k^2)=13$$

$$25k^2-12k^2=13$$

$$13k^2=13$$

$$\Rightarrow k^2=1$$

$$\Rightarrow k=\pm 1$$

Q4. For what value of k the roots of the equation $x^2-5x+k=0$ differ by unity.

Sol: Comparing $x^2-5x+k=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-5$, $c=k$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting the values of a and b

$\alpha+\beta=\frac{-(-5)}{1}=\frac{5}{1}=5$

Product of the roots

$\alpha.\beta=\frac{c}{a}$ Putting the values of a and c

$\alpha.\beta=\frac{k}{1}=k$

According to the given condition Differ by unity

$\alpha-\beta=1$

Using formula

$4\alpha\beta=(\alpha+\beta)^2-(\alpha-\beta)^2$ putting the values

$4k=(5)^2-(1)^2$

$4k=25-1$

$4k=24$

$k=\frac{24}{4}=6$

Q4. For what value of k the roots of the equation $x^2-5x+k=0$ differ by unity.

Sol: Comparing $x^2-5x+k=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-5$, $c=k$

Let the roots are $\alpha,\beta=\alpha+1$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting the values of a, b & β

$\alpha+\alpha+1=\frac{-(-5)}{1}=5$

$2\alpha=5-1$

$\alpha=\frac{4}{2}=2$

Product of the roots

$\alpha.\beta=\frac{c}{a}$ Putting the values of a, c, α, & β

$\alpha(\alpha+1)=\frac{k}{1}$

$2(2+1)=k$

$k=6$

Q5. For what value of k the roots of the equation $x^2-9x+k+2=0$ differ by three.

Sol: Comparing $x^2-9x+k+2=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-9$, $c=k+2$

Sum of the roots

$\alpha+\beta=\frac{-b}{a}$ Putting the values of a and b

$\alpha+\beta=\frac{-(-9)}{1}=\frac{9}{1}=9$

Product of the roots

$\alpha.\beta=\frac{c}{a}$ Putting the values of a and c

$\alpha.\beta=\frac{k+2}{1}=k+2$

According to given condition

Roots are Differ by three $\alpha-\beta=3$

Using formula

$4\alpha\beta=(\alpha+\beta)^2-(\alpha-\beta)^2$ putting values

$4(k+2)=(9)^2-(3)^2$

$4k+8=81-9$

$4k=72-8$

$k=\frac{64}{4}=16$

Q5. For what value of k the roots of the equation $x^2-9x+k+2=0$ differ by three.

Sol: Comparing $x^2-9x+k+2=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-9$, $c=k+2$

According to condition Let roots are $\alpha,\alpha+3$

Sum of the roots

$\alpha+\alpha+3=\frac{-(-9)}{1}$

$2\alpha=9-3$

$\alpha=\frac{6}{2}=3$

Product of the roots

$\alpha(\alpha+3)=\frac{k+2}{1}$ putting the value of α

$3(3+3)=k+2$

$18=k+2$

$k=18-2$

$k=16$

Q6. If α,β be the roots of $x^2-5x+k=0$, find k such that 3 α + 2 β = 12

Sol: Comparing $x^2-5x+k=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-5$, $c=k$

So, $\alpha+\beta=\frac{-b}{a}$ Putting the values of a and b

$\alpha+\beta=\frac{-(-5)}{1}=\frac{5}{1}=5$

Now $\alpha.\beta=\frac{c}{a}$ Putting the values of a and c

$\alpha.\beta=\frac{k}{1}=k$

According to the given condition

$3\alpha+2\beta=12$

$\alpha+2\alpha+2\beta=12$

$\alpha+2(\alpha+\beta)=12$

$\alpha+2(5)=12$ $\quad\therefore \alpha+\beta=5$

$\alpha+10=12$

$\alpha=12-10$

$\alpha=2$

$\therefore \alpha+\beta=5$

$2+\beta=5$ $\quad\therefore \alpha=2$

$\beta=5-2=3$

Now using $\alpha.\beta=k$

$(2)(3)=k$ $\quad\therefore \alpha=2,\beta=3$

$k=6$

Q7. Find the value of m and n if both sum and product of the roots of the quadratic equation $mx^2-3x-n=0$ are equal to $\frac{3}{5}$.

Sol: Comparing $mx^2-3x-n=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=m$, $b=-3$, $c=-n$

Sum of the roots = $\frac{3}{5}$

$\alpha+\beta=\frac{3}{5}$

$\frac{-b}{a}=\frac{3}{5}$

Putting the values of a and b

$\frac{-(-3)}{m}=\frac{3}{5}$

$15=3m$

$\Rightarrow m=5$

Product of the roots = $\frac{3}{5}$

$\alpha.\beta=\frac{3}{5}$

$\frac{c}{a}=\frac{3}{5}$

Putting the values of a and c

$\frac{-n}{m}=\frac{3}{5}$

$\frac{-n}{5}=\frac{3}{5}$ $\quad\therefore m=5$

$\Rightarrow n=-3$

Exp16i): If α,β are roots of $ax^2+bx+c=0$ then find value of symmetric function $\alpha+\beta$

Sol: As α,β are roots of $ax^2+bx+c=0$

$\alpha+\beta=\frac{-b}{a}$

Exp16ii): If α,β are roots of $ax^2+bx+c=0$ then find value of symmetric function $\alpha\beta$

Sol: As α,β are roots of $ax^2+bx+c=0$

$\alpha\beta=\frac{c}{a}$

Exp16iii): If α,β are roots of $ax^2+bx+c=0$ then find value of symmetric function $\alpha^2+\beta^2$

Sol: As α,β are roots of $ax^2+bx+c=0$

$\alpha^2+\beta^2=\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta$

$=(\alpha+\beta)^2-2\alpha\beta$ putting the values

$=\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)$

$=\frac{b^2}{a^2}-\frac{2c}{a}\times\frac{a}{a}$

$=\frac{b^2-2ac}{a^2}$

Exp16iv): If α,β are roots of $ax^2+bx+c=0$ then find value of symmetric function $\alpha^3+\beta^3$

Sol: As α,β are roots of $ax^2+bx+c=0$

$\therefore(\alpha+\beta)^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ put values

$=\left(\frac{-b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)$

$=\frac{-b^3}{a^3}+\frac{3bc}{a^2}\times\frac{a}{a}$

$=\frac{-b^3+3abc}{a^3}$

Exp16v): If α,β are roots of $ax^2+bx+c=0$ then find value of symmetric function $\frac{1}{\alpha}+\frac{1}{\beta}$

Sol: As α,β are roots of $ax^2+bx+c=0$

$\therefore\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta}{\beta}\times\frac{1}{\alpha}+\frac{1}{\beta}\times\frac{\alpha}{\alpha}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ put values

$=\frac{-b}{a}\div\frac{c}{a}$

$=\frac{-b}{a}\times\frac{a}{c}$

$=\frac{-b}{c}$

Exp16vi): If α,β are roots of $ax^2+bx+c=0$ then find value of symmetric function $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Sol: As α,β are roots of $ax^2+bx+c=0$

$\therefore\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2}{\beta^2}\times\frac{1}{\alpha^2}+\frac{1}{\beta^2}\times\frac{\alpha^2}{\alpha^2}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$ put values

$=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha^2\beta^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$ putting values

$=\left[\left(\frac{-b}{a}\right)^2-2\frac{c}{a}\right]\div\left(\frac{c}{a}\right)^2$

$=\left[\frac{b^2}{a^2}-\frac{2c}{a}\times\frac{a}{a}\right]\times\frac{a^2}{c^2}$

$=\frac{b^2-2ac}{a^2}\times\frac{a^2}{c^2}$

$=\frac{b^2-2ac}{c^2}$

Formation of quadratic equation through roots

Since $ax^2+bx+c=0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$x^2-\left(-\frac{b}{a}\right)x+\frac{c}{a}=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\boxed{x^2-Sx+P=0}$

Exp 17 Form a quadratic equation whose roots are $1+\sqrt{5}$ and $1-\sqrt{5}$

Sol: Roots of quadratic eq are $1+\sqrt{5}$ &$1-\sqrt{5}$

Sum of roots $=S=1+\sqrt{5}+1-\sqrt{5}=2$

Product of roots $P=(1+\sqrt{5})(1-\sqrt{5})$

$P=1^2-(\sqrt{5})^2$

$P=1-5=-4$

Since $x^2-Sx+P=0$ putting values

$x^2-2x+(-4)=0$

$x^2-2x-4=0$

Exp 18i) Form a quadratic equation whose roots are 2a + 1 and 2b +1

Sol: Roots of quadratic eq are 2a + 1& 2b +1

Sum of roots $S = 2a + 1 + 2b + 1$

$$S = 2a + 2b + 2$$

Product of roots $P = (2a+1)(2b+1)$

$$P = 4ab + 2a + 2b + 1$$

Since $x^2 - Sx + P = 0$ putting values

$x^2 - (2a + 2b + 2)x + (4ab + 2a + 2b + 1) = 0$

Exp 18ii) Form a quadratic equation whose roots are a^2 and b^2

Sol: Roots of quadratic eq are a^2 and b^2

Sum of roots $S = a^2 + b^2$

Product of roots $P = a^2b^2$

Since $x^2 - Sx + P = 0$ putting values

$x^2 - (a^2 + b^2)x + a^2b^2 = 0$

Exp 18iii) Form a quadratic equation whose roots are $\frac{1}{a}$ and $\frac{1}{b}$

Sol: Roots of quadratic eq are $\frac{1}{a}$ and $\frac{1}{b}$

Sum of roots $S = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

Product of roots $P = \frac{1}{a} \times \frac{1}{b} = \frac{1}{ab}$

Since $x^2 - Sx + P = 0$ putting values

$x^2 - \left(\frac{a+b}{ab}\right)x + \frac{1}{ab} = 0$ multiply by ab

$abx^2 - (a+b)x + 1 = 0$

Exp 18iv) Form a quadratic equation whose roots are $\frac{2}{3}$ and $\frac{3}{2}$

Sol: Roots of quadratic eq are $\frac{2}{3}$ and $\frac{3}{2}$

Sum of roots $S = \frac{2}{3} + \frac{3}{2} = \frac{4+9}{6} = \frac{13}{6}$

Product of roots $P = \frac{2}{3} \times \frac{3}{2} = 1$

Since $x^2 - Sx + P = 0$ putting values

$x^2 - \frac{13}{6}x + 1 = 0$ multiply each term by 6

$6x^2 - 13x + 6 = 0$

Exp 18v) Form a quadratic equation whose roots are $\frac{2}{5}$ and $\frac{5}{2}$

Sol: Roots of quadratic eq are $\frac{2}{5}$ and $\frac{5}{2}$

Sum of roots $S = \frac{2}{5} + \frac{5}{2} = \frac{4+25}{10} = \frac{29}{10}$

Product of roots $P = \frac{2}{5} \times \frac{5}{2} = 1$

Since $x^2 - Sx + P = 0$ putting values

$x^2 - \frac{29}{10}x + 1 = 0$ multiply each term by 10

$10x^2 - 29x + 10 = 0$

Exercise 2.4

Q1i). If α,β be the roots of $ax^2 + bx + c = 0$, find the value of $\alpha^3\beta + \beta^3\alpha$

Sol: Sum of the roots $\alpha + \beta = \frac{-b}{a}$

Product of the roots $\alpha.\beta = \frac{c}{a}$

According to the given condition

$$\alpha^3\beta + \beta^3\alpha = \alpha\beta(\alpha^2 + \beta^2)$$
$$= \alpha\beta(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta)$$
$$= \alpha\beta\left(\{\alpha + \beta\}^2 - 2\alpha\beta\right)$$

Putting the values α + β and α,β

$$\alpha^3\beta + \beta^3\alpha = \frac{c}{a}\left(\left\{\frac{-b}{a}\right\}^2 - 2\left(\frac{c}{a}\right)\right)$$
$$= \frac{c}{a}\left(\frac{b^2}{a^2} - \frac{2c}{a}.\frac{a}{a}\right)$$
$$= \frac{c}{a}\left(\frac{b^2 - 2ac}{a^2}\right)$$
$$= \frac{c(b^2 - 2ac)}{a^3}$$

Q1ii). If α,β be the roots of $ax^2 + bx + c = 0$, find the value of $(\alpha - \beta)^2$

Sol: As we know $\alpha + \beta = \frac{-b}{a}$ & $\alpha.\beta = \frac{c}{a}$

According to the given condition

Using formula

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

Putting the values α + β and α,β

$$(\alpha - \beta)^2 = \left(\frac{-b}{a}\right)^2 - 4\left(\frac{c}{a}\right)$$
$$(\alpha - \beta)^2 = \frac{b^2}{a^2} - \frac{4c}{a}.\frac{a}{a}$$
$$(\alpha - \beta)^2 = \frac{b^2 - 4ac}{a^2}$$

Q2i). Find quadratic eq whose roots are $1, \frac{1}{2}$

Sol: Sum of the roots

$$\alpha + \beta = 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$$

Product of the roots

$$\alpha.\beta = (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

The required equation is given by

$$x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$$

Putting the values α + β and α,β

$$x^2 - \frac{3}{2}x + \frac{1}{2} = 0 \qquad \times \text{by } 2$$
$$2x^2 - 2.\frac{3}{2}x + 2.\frac{1}{2} = 2.0$$
$$2x^2 - 3x + 1 = 0$$

Q2ii).Find quadratic eq whose roots are $-3, 4$

Sol: Sum of the roots $\alpha + \beta = -3 + 4 = 1$

Product of the roots $\alpha.\beta = (-3)(4) = -12$

The required equation is given by

$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$

Putting the values α + β and α,β

$x^2-(1)x+(-12)=0$

$x^2-x-12=0$

Q2iii). Find quadratic eq whose roots are $3+\sqrt{2},3-\sqrt{2}$

Sol: Sum of the roots

$$\begin{aligned}\alpha+\beta&=(3+\sqrt{2})+(3-\sqrt{2})\\&=3+\sqrt{2}+3-\sqrt{2}\\&=3+3+\sqrt{2}-\sqrt{2}\\&=6\end{aligned}$$

Product of the roots

$\alpha.\beta=(3+\sqrt{2})(3-\sqrt{2})$

$\alpha.\beta=(3)^2-(\sqrt{2})^2$

$\alpha.\beta=9-2$

$\alpha.\beta=7$

The required equation is given by

$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$

Putting the values α + β and α,β

$x^2-6x+7=0$

Q2iv).Find quadratic eq whose roots are $a,-2a$

Sol: Sum of the roots
$$\begin{aligned}\alpha+\beta&=(a)+(-2a)\\&=a-2a\\&=-a\end{aligned}$$

Product of the roots
$$\alpha.\beta=(a)(-2a)$$
$$\alpha.\beta=-2a^2$$

The required equation is given by

$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$

Putting the values α + β and α,β

$x^2-(-a)x+(-2a^2)=0$

$x^2+ax-2a^2=0$

Q3. Form a quadratic eq whose roots are square of roots of eq $ax^2+bx+c=0$ $a\neq 0$

Sol: Since α,β be the roots of $ax^2+bx+c=0$

$\alpha+\beta=\frac{-b}{a}$ & $\alpha.\beta=\frac{c}{a}$

The roots of the required equation are α^2,β^2

∴ Sum of the roots $=\alpha^2+\beta^2$

$$\begin{aligned}&=\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta\\&=(\alpha+\beta)^2-2\alpha\beta\\&=\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)\\&=\frac{b^2}{a^2}-\frac{2c}{a}.\frac{a}{a}\\&=\frac{b^2-2ac}{a^2}\end{aligned}$$

∴ Product of the roots $=\alpha^2.\beta^2$

$=(\alpha\beta)^2$ Putting

$$\begin{aligned}&=\left(\frac{c}{a}\right)^2\\&=\frac{c^2}{a^2}\end{aligned}$$

The required equation is given by

$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$

$$x^2-\left(\frac{b^2-2ac}{a^2}\right)x+\left(\frac{c^2}{a^2}\right)=0$$

$\Rightarrow a^2x^2-(b^2-2ac)x+(c^2)=0$

Q4i). If α, β be roots of $2x^2+3x+1=0$, then find value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

Sol: Comparing $2x^2+3x+1=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=2, b=3, c=1$

Since α,β be the roots of $2x^2+3x+1=0$

$\alpha+\beta=\frac{-b}{a}=\frac{-3}{2}$ and $\alpha.\beta=\frac{c}{a}=\frac{1}{2}$

Now
$$\begin{aligned}\frac{\alpha}{\beta}+\frac{\beta}{\alpha}&=\frac{\alpha}{\alpha}.\frac{\alpha}{\beta}+\frac{\beta}{\alpha}.\frac{\beta}{\beta}\\&=\frac{\alpha^2+\beta^2}{\alpha\beta}\\&=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}\\&=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\end{aligned}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\left[\left(\frac{-3}{2}\right)^2-2\left(\frac{1}{2}\right)\right]\div\frac{1}{2}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\left[\frac{9}{4}-\frac{2}{2}\times\frac{2}{2}\right]\times\frac{2}{1}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\left[\frac{9-4}{4}\right]\times\frac{2}{1}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{5}{2}$$

Q4ii). If α, β be roots of $2x^2+3x+1=0$, then find value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Sol: Comparing $2x^2+3x+1=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=2, b=3, c=1$

Since α,β be the roots of $2x^2+3x+1=0$

$\alpha+\beta=\frac{-b}{a}=\frac{-3}{2}$ and $\alpha.\beta=\frac{c}{a}=\frac{1}{2}$ Now

$$\begin{aligned}\frac{1}{\alpha^2}+\frac{1}{\beta^2}&=\frac{\beta^2}{\beta^2}.\frac{1}{\alpha^2}+\frac{1}{\beta^2}.\frac{\alpha^2}{\alpha^2}\\&=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}\\&=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha^2\beta^2}\\&=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}\end{aligned}$$

Putting the values

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left[\left(\frac{-3}{2}\right)^2-2\left(\frac{1}{2}\right)\right]\div\left(\frac{1}{2}\right)^2$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\left[\frac{9}{4}-\frac{2}{2}\times\frac{2}{2}\right]\times\frac{4}{1}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{9-4}{4}\times\frac{4}{1}=5$$

Q4iii). If α, β be roots of $2x^2+3x+1=0$, then find value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Sol: Comparing $2x^2+3x+1=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=2$, $b=3$, $c=1$

Since α,β be the roots of $2x^2+3x+1=0$

$$\alpha+\beta=\frac{-b}{a}=\frac{-3}{2} \text{ and } \alpha.\beta=\frac{c}{a}=\frac{1}{2} \text{ Now}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha}{\alpha}.\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}.\frac{\beta}{\beta}$$

$$=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$\therefore (a+b)^3=a^3+b^3+3ab(a+b)$ Putting values

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\left[\left(\frac{-3}{2}\right)^3-3\left(\frac{1}{2}\right)\left(\frac{-3}{2}\right)\right]\div\frac{1}{2}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\left[\frac{-27}{8}+\frac{9}{4}\times\frac{2}{2}\right]\times\frac{2}{1}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{-27+18}{8}\times\frac{2}{1}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{-9}{4}$$

Q5. If α, β be roots of $3x^2-2x+5=0$, then find the equation whose roots are $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$

Sol: Comparing $3x^2-2x+5=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=3$, $b=-2$, $c=5$

Since α,β be the roots of $ax^2+bx+c=0$

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-2)}{3}=\frac{2}{3} \text{ and } \alpha.\beta=\frac{c}{a}=\frac{5}{3}$$

The roots of the required equation are $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$

∴ Sum of the roots

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha}{\alpha}.\frac{\alpha}{\beta}+\frac{\beta}{\alpha}.\frac{\beta}{\beta}$$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\left[\left(\frac{2}{3}\right)^2-2\left(\frac{5}{3}\right)\right]\div\frac{5}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\left[\frac{4-30}{9}\right]\times\frac{3}{5}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{-26}{15}$$

∴ Product of the roots $\frac{\alpha}{\beta}.\frac{\beta}{\alpha}=1$ so

$$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$$

$$x^2-\left(\frac{-26}{15}\right)x+(1)=0$$

$$\Rightarrow 15x^2-(-26)x+15=0$$

$$\Rightarrow 15x^2+26x+15=0$$

Q6. If α, β be roots of $x^2-4x+2=0$, then find equation whose roots are $\alpha+\frac{1}{\alpha},\ \beta+\frac{1}{\beta}$

Sol: Comparing $x^2-4x+2=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-4$, $c=2$

Since α,β be the roots of $ax^2+bx+c=0$

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-4)}{1}=\frac{4}{1}=4 \text{ and}$$

$$\alpha.\beta=\frac{c}{a}=\frac{2}{1}=2$$

The roots of the required equation are

$$\alpha+\frac{1}{\alpha},\ \beta+\frac{1}{\beta}$$

∴ Sum of the roots

$$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=\alpha+\beta+\frac{\beta}{\beta}\frac{1}{\alpha}+\frac{1}{\beta}\frac{\alpha}{\alpha}$$

$$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=\alpha+\beta+\frac{\beta+\alpha}{\alpha\beta}$$

Putting the values

$$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=4+\frac{4}{2}$$

$$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=4+2=6$$

∴ Product of the roots

$$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{\alpha}{\alpha}\frac{\alpha}{\beta}+\frac{\beta}{\alpha}.\frac{\beta}{\beta}+\frac{1}{\alpha\beta}$$

$$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{1}{\alpha\beta}+\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{1}{\alpha\beta}+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

Putting the values

$$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=2+\frac{1}{2}+\frac{(4)^2-2(2)}{2}$$

$$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\frac{2}{2}.\frac{2}{1}+\frac{1}{2}+\frac{16-4}{2}$$

$$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\frac{4+1+16-4}{2}=\frac{17}{2}$$

The required equation is given by

$$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$$

$$x^2-(6)x+\left(\frac{17}{2}\right)=0$$

$$\Rightarrow 2x^2-12x+17=0$$

Division of Polynomial:

$$\begin{array}{r|l} & \textit{Quotient} \\ \hline \textit{Divisor} & \textit{Divident} \\ & \textit{Divisor}\times\textit{Quotient} \\ \hline & \textit{Remainder} \end{array}$$

its General Formula

$$\textit{Divident}=(\textit{Divisor})(\textit{Quotient})+\textit{Remainder}$$

This result is known as "**Division Algorithm**"

Remainder Theorem:
A polynomial of degree $n \geq 1$, divided by $(x-a)$ until a constant is obtained called remainder denoted by $f(a)$

Factor Theorem: A polynomial $f(x)$ has a factor $(x-a)$ iff $f(a)=0$

Synthetic Division: The method of synthetic division for dividing the polynomial of degree n $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x^1 + a_o$ by $x-a$ is illustrated and described as follow:

$$\begin{array}{c|cccccc l}
a & a_n & a_{n-1} & a_{n-2} & \cdots & a_1 & a_o & \text{Row one} \\
 & \downarrow & \overset{+}{aa_n} & \overset{+}{ab} & \overset{+}{ac} & \overset{+}{ad} & \overset{+}{ae} & \text{Row Two} \\
\hline
 & a_n & b & c & \cdots d & e & f & \text{Row Three} \\
 & \multicolumn{5}{c}{\text{Coefficients of Quotient}} & \text{Remainder} &
\end{array}$$

where $b = a_{n-1} + aa_n$, $c = a_{n-2} + ab$, $\cdots d = \cdots + ac$ and $e = a_1 + ad$, $f = a_o + ae$

Quotient should be $a_n x^{n-1} + bx^{n-2} + cx^{n-3} + \cdots + dx + e$ of degree $(n-1)$ if Remainder is zero or $f=0$

And Quotient is also known as Depressed polynomial.

Exp19: Use synthetic division to find the Quotient Q(x) and the remainder R when $3x^3 - 2x^2 - 150$ is divided by $x-4$

Sol: Given $P(x) = 3x^3 - 2x^2 + 0x - 150$

And divisor $x-4=0 \quad \Rightarrow x=4$

Then by synthetic division

4	3	-2	0	-150
		12	40	160
	3	10	40	10

Therefore Q(x) = $3x^2 + 10x + 40$ & $R = 10$

Exp20: use synthetic division to find the value of k if 2 is a zero of $2x^4 + x^3 + kx^2 - 8$

Sol: Given $P(x) = 2x^4 + x^3 + kx^2 + 0x - 8$

And given zero is 2

Then by synthetic division

2	2	1	K	0	-8
		4	10	2k+20	4k+40
	2	5	k+10	2k+20	4k+32

Since 2 is a zero of Polynomial so

$$4k + 32 = 0$$
$$4k = -32$$
$$k = -8$$

Exp21: use synthetic division to find the value of m and n if $x-1 \,\&\, x+2$ are factors of $x^3 - mx^2 + nx + 12$

Sol: Given $P(x) = x^3 - mx^2 + nx + 12$

Given $x-1$ and $x+2$ are factors. There zeros $x-1=0 \Rightarrow x=1$ & $x+2=0 \Rightarrow x=-2$

Then by synthetic division

1	1	-m	n	12
		1	1-m	1-m+n
	1	1-m	1-m+n	13-m+n
-2		-2	2+2m	
	1	-1-m	3+m+n	

Since 1 &-2 is a zero of Polynomial so we have

$13 - m + n = 0$ …………(1)

$3 + m + n = 0$ …………(2)

$16 + 2n = 0$ by adding eq(1) and eq (2)

$\Rightarrow n = -8$ put in eq (2)

$$3 + m + (-8) = 0$$
$$m - 5 = 0$$
$$m = 5$$

Thus $m=5$ & $n=-8$

Exp22:if -1 and 2 are roots of quartic equation $x^4 - 5x^2 + 4 = 0$ use synthetic division to find other roots

Solution: Given quartic eq $x^4 - 5x^2 + 4 = 0$

And roots are -1 and 2

Then by synthetic division

-1	1	0	-5	0	4
		-1	1	4	-4
	1	-1	-4	4	0
2		2	2	-4	
	1	1	-2	0	

$Q(x) = x^2 + x - 2 = 0$ having other factors

$$x^2 + 2x - 1x - 2 = 0$$
$$x(x+2) - 1(x+2) = 0$$
$$(x+2)(x-1) = 0$$

Either $x+2=0$ or $x-1=0$

$x=-2$ $\quad x=1$

Thus other two roots are $x=-2$ & $x=1$

Exercise 2.5

Q1i). Use synthetic division to find the quotient $Q(x)$ and remainder R when $3x^3 + 2x^2 - x - 1$ is divided by $x+3$

Sol: Since $3x^3 + 2x^2 - x - 1$

And $x+3=0 \quad \Rightarrow x=-3$

-3	3	2	-1	-1
		-9	21	-60
	3	-7	20	-61

Therefore $Q(x) = 3x^2 - 7x + 20$ & $R = -61$

Q1ii). Use synthetic division to find the quotient $Q(x)$ and remainder R when $2x^3 - 7x^2 + 12x - 27$ is divided by $x-3$

Sol: Since $2x^3 - 7x^2 + 12x - 27$

And $x-3=0 \quad \Rightarrow x=3$

3	2	-7	12	-27
		6	-3	27
	2	-1	9	0

Therefore $Q(x) = 2x^2 - x + 9$ and $R = 0$

Q1iii). Use synthetic division to find the quotient $Q(x)$ and remainder R when $2x^4 - 3x^2 + 5x - 7$ is divided by $x+2$

Sol: Since $2x^4 - 3x^2 + 5x - 7$

$= 2x^4 + 0.x^3 - 3x^2 + 5x - 7$

And $x+2=0 \quad \Rightarrow x=-2$

-2	2	0	-3	5	-7
		-4	8	-10	10
	2	-4	5	-5	3

Therefore $Q(x)=2x^3-4x^2+5x-5$ & $R=3$

Q2. Use synthetic division to find the value of k if -2 is a zero of x^3+4x^2+kx+8

Sol: Since x^3+4x^2+kx+8

Given zero $x=-2$ of polynomial

-2	1	4	K	8
		-2	-4	-2k+8
	1	2	k-4	-2k+16=R

Since -2 is a zero of the given polynomial

Or x + 2 is a factor so its Remainder = 0

Therefore
$$R=-2k+16=0$$
$$\Rightarrow -2k=-16$$
$$\Rightarrow k=\frac{-16}{-2}=8$$

Q3. Use synthetic division to find the value of p and q if $x+1$ and $x-2$ are the factors of x^3+px^2+qx+6

Sol: Since x^3+px^2+qx+6

Factor $x+1=0 \quad \Rightarrow x=-1$

Factor $x-2=0 \quad \Rightarrow x=2$

-1	1	P	q	6
		-1	-p+1	p-q-1
	1	p-1	-p+q+1	p-q+5=0
2		2	2p+2	
	1	P+1	P+q+3=0	

Since $x+1$ and $x-2$ are the factors of the given polynomial so the remainder = 0

$p-q+5=0\cdots(1)$

$p+q+3=0\cdots(2)$

$2p+\quad 8=0$

$2p=-8$

$p=\frac{-8}{2}=-4$

Put in eq (2) we get

$-4+q+3=0$

$q-4+3=0$

$q-1=0$

$q=1$

Hence $p=-4, q=1$

Q4. If $x+1$ and $x-2$ are the factors of the polynomial x^3+ax^2+bx+2, then using synthetic division, find the values of a and b.

Sol: Since x^3+ax^2+bx+2

Factor $x+1=0 \quad \Rightarrow x=-1$

Factor $x-2=0 \quad \Rightarrow x=2$

-1	1	a	b	2
		-1	-a+1	a-b-1
	1	a-1	-a+b+1	a-b+1=0
2		2	2a+2	
	1	a+1	a+b+3=0	

Since $x+1$ and $x-2$ are the factors of the given polynomial so the remainder = 0

$a-b+1=0\cdots(1)$

$a+b+3=0\cdots(2)$

$2a+\quad 4=0$

$2a=-4$

$a=\frac{-4}{2}=-2$

Put in equation (2) we get

$-2+b+3=0$

$b-2+3=0$

$b+1=0$

$b=-1$

Hence $a=-2, b=-1$

Q5. One root of the cubic equation $x^3-7x-6=0$ is 3. Use synthetic division to find the other roots.

Sol: Since $x^3-7x-6=0$

Or $x^3+0.x^2-7x-6=0$

3	1	0	-7	-6
		3	9	6
	1	3	2	0

Therefore $Q(x)=x^2+3x+2$ and $R=0$

To find the other roots take Q (x) = 0

$x^2+3x+2=0$

$x^2+2x+1x+2=0$

$x(x+2)+1(x+2)=0$

$(x+1)(x+2)=0$

Either or

$x+1=0 \qquad x+2=0$

$x=-1 \qquad x=-2$

Hence the other roots are $-1, -2$

Q6. If – 1 and 2 are roots of the quartic equation $x^4-5x^3+3x^2+7x-2=0$. Use synthetic division to find the other roots.

Sol: Since $x^4-5x^3+3x^2+7x-2=0$

-1	1	-5	3	7	-2
		-1	6	-9	2
2	1	-6	9	-2	0
		2	-8	2	
	1	-4	1	0	

Therefore $Q(x)=x^2-4x+1$

To find the other roots take Q (x) = 0

$x^2-4x+1=0$ Comparing with

$ax^2+bx+c=0$ we have $a=1$, $b=-4$, $c=1$

using quadratic formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Putting values of a, b & c

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}$$

$$x=\frac{4\pm\sqrt{4\times 3}}{2}=\frac{4\pm 2\sqrt{3}}{2}$$

$$x=\frac{4}{2}\pm\frac{2\sqrt{3}}{2}=2\pm\sqrt{3}$$

Hence the other roots are $2+\sqrt{3},\ 2-\sqrt{3}$

Simultaneous Equation: A system of equation which have a common solution is called simultaneous system of equation.i.e.

$$\left.\begin{aligned} ax+by&=c \\ dx+ey&=f \end{aligned}\right\} \rightarrow \text{Simultaneous Linear equation}$$

Exp23: Solve $\begin{aligned} 2x+y&=10 \\ 4x^2+y^2&=68 \end{aligned}$ the system

Sol; Given system $2x+y=10$ …………..(1)

$4x^2+y^2=68$ …………….(2)

Form eq (1) $y=10-2x$ ……..(3) put in eq (2)

$4x^2+(10-2x)^2=68$

$4x^2+100-40x+4x^2-68=0$

$8x^2-40x+32=0$ divided by 8

$x^2-5x+4=0$

$x^2-4x-1x+4=0$

$x(x-4)-1(x-4)=0$

$(x-1)(x-4)=0$

Either $x-1=0$ or $x-4=0$

$x=1$ | $x=4$ put in eq (3)

$y=10-2(1)$ | $y=10-2(4)$

$y=10-2$ | $y=10-8$

$y=8$ | $y=2$

Solution of the given system $=\{(1,8),(4,2)\}$

Exp24: Solve $\begin{aligned} x-y&=7 \\ x^2+3xy+y^2&=-1 \end{aligned}$ the system

Sol: Given system $x-y=7$ …………(1)

And $x^2+3xy+y^2=-1$ ……….(2)

From eq (1) $x=7+y$ ………..(3) put in (2)

$(7+y)^2+3(7+y)y+y^2=-1$

$49+14y+y^2+21y+3y^2+y^2=-1$

$y^2+3y^2+y^2+14y+21y+49+1=0$

$5y^2+35y+50=0$ divided by 5

$y^2+7y+10=0$

$y^2+5y+2y+10=0$

$y(y+5)+2(y+5)=0$

$(y+2)(y+5)=0$

Either $y+2=0$ or $y+5=0$

$y=-2$ | $y=-5$ Put in (3)

$x=7+(-2)$ | $x=7+(-5)$

$x=5$ | $x=2$

Solution of given system $=\{(5,-2),(2,-5)\}$

Exp25: solve $\begin{aligned} x^2+y^2&=4 \\ 2x^2-y^2&=8 \end{aligned}$ the system

Sol: Given system $x^2+y^2=4$ ………(1)

$2x^2-y^2=8$ ……...(2)

Adding (1) & (2) $3x^2=12$

$x^2=4$ taking square root

$x=\pm 2$

Put $x^2=4$ in eq (1)

$4+y^2=4$

$y^2=0$

$\Rightarrow y=0$

Solution of given system $=\{(-2,0),(2,0)\}$

Exercise 2.6

Q1i). solve $2x-y=3$ the system

$x^2+y^2=2$

Sol: Since $2x-y=3\cdots(1)$

$x^2+y^2=2\cdots(2)$

Form equation (1)

$2x-y=3$

$2x-3=y\cdots(3)$

Putting the value of y in equation (2) we get

$x^2+(2x-3)^2=2$

$x^2+(2x)^2-2(2x)(3)+(3)^2=2$

$x^2+4x^2-12x+9-2=0$

$5x^2-12x+7=0$

$5x^2-7x-5x+7=0$

$x(5x-7)-1(5x-7)=0$

$(x-1)(5x-7)=0$

Either | Or
$x-1=0$ | $5x-7=0$
$x=1$ | $5x=7$
 | $x=\frac{7}{5}$

Substituting these values in equation (3)

When $x=1$ | $x=7/5$

$y=2(1)-3$ | $y=2(7/5)-3$

$y=2-3$ | $y=14/5-3$

$y=-1$ | $y=\frac{14-15}{5}=\frac{-1}{5}$

Solution Set $=\left\{(1,-1),\left(\frac{7}{5},\frac{-1}{5}\right)\right\}$

Q1ii). Solve $x+2y=0$ the system

$x^2+4y^2=32$

Sol: Since $x+2y=0\cdots(1)$

$x^2+4y^2=32\cdots(2)$

Form equation (1) we have

$x+2y=0$

$x=-2y\cdots(3)$

Putting the value of y in equation (2) we get

$(-2y)^2+4y^2=32$

$4y^2+4y^2=32$

$8y^2=32$

$y^2=4$ $\Rightarrow y=\pm 2$

Substituting these values in equation (3)

When $y=2$ $\quad$ $y=-2$

$x=-2(2)$ $\quad$ $x=-2(-2)$

$x=-4$ $\quad$ $x=4$

Solution Set $=\{(-4,2),(4,-2)\}$

Q1iii). Solve the system $2x-y=-8$
$x^2+4x=y$

Sol: Since $2x-y=-8\cdots(1)$

$x^2+4x=y\cdots(2)$

Form equation (1)

$2x-y=-8$

$2x+8=y\cdots(3)$

Putting the value of y in equation (2) we get

$x^2+4x=2x+8$

$x^2+4x-2x-8=0$

$x(x+4)-2(x+4)=0$

$(x-2)(x+4)=0$

Either $\quad$ Or

$x-2=0$ $\quad$ $x+4=0$

$x=2$ $\quad$ $x=-4$

Substituting these values in equation (3)

When x = 2 $\quad$ x = - 4

$y=2(2)+8$ $\quad$ $y=2(-4)+8$

$y=4+8$ $\quad$ $y=-8+8$

$y=12$ $\quad$ $y=0$

Solution Set $=\{(2,12),(-4,0)\}$

Q1iv). Solve the system $2x+y=4$
$x^2-2x+y^2=3$

Sol: Since $2x+y=4\cdots(1)$

$x^2-2x+y^2=3\cdots(2)$

Form equation (1)

$2x+y=4$

$y=4-2x\cdots(3)$

Putting the value of y in equation (2) we get

$x^2-2x+(4-2x)^2=3$

$x^2-2x+16-16x+4x^2=3$

$5x^2-18x+13=0$

$5x^2-13x-5x+13=0$

$x(5x-13)-1(5x-13)=0$

$(x-1)(5x-13)=0$

Either $\quad$ Or

$x-1=0$ $\quad$ $5x-13=0$

$x=1$ $\quad$ $5x=13$

$x=\frac{13}{5}$

Substituting these values in equation (3)

When x = 1 $\quad$ $x=\frac{13}{5}$

$y=4-2(1)$ $\quad$ $y=4-2\left(\frac{13}{5}\right)$

$y=4-2$ $\quad$ $y=\frac{20-26}{5}$

$y=2$ $\quad$ $y=\frac{-6}{5}$

Solution Set $=\left\{(1,2),\left(\frac{13}{5},\frac{-6}{5}\right)\right\}$

Q1v). Solve the system $4x^2+5y^2=4$
$3x^2+y^2=3$

Sol: Since $4x^2+5y^2=4\cdots(1)$

$3x^2+y^2=3\cdots(2)$

From equation (2) we get

$3x^2+y^2=3$

$y^2=3-3x^2\cdots(3)$

Putting the value of y in equation (1) we get

$4x^2+5(3-3x^2)=4$

$4x^2+15-15x^2=4$

$4x^2-15x^2=4-15$

$-11x^2=-11$

$x^2=\frac{-11}{-11}$

$x^2=1$

Substituting $x^2=1$ in equation (3) we get

$y^2=3-3(1)$

$y^2=3-3=0$

Therefore $x^2=1$ and $y^2=0$

Taking square root

$x=\pm1$ and $y=0$

Solution set $=\{(\pm1,0)\}=\{(1,0),(-1,0)\}$

Q1vi). Solve the system $5x^2=y^2+9$
$x^2=-y^2+45$

Sol: Since $5x^2=y^2+9\cdots(1)$

$x^2=-y^2+45\cdots(2)$

Adding equations (1) and (2)

$$\begin{array}{l} 5x^2=y^2+9 \\ \underline{x^2=-y^2+45} \\ 6x^2=\quad +54 \end{array}$$

$x^2=\frac{54}{6}=9$

Substituting $x^2=9$ in equation (1) we get

$5(9)=y^2+9$

$45-9=y^2$

$y^2=36$

Thus $x^2=9$ & $y^2=36$ Taking square root

$x=\pm3$ & $y=\pm6$

S.S $=\{(\pm3,\pm6)\}=\{(3,6),(-3,6),(3,-6),(-3,-6)\}$

Q1vii). Solve the system $4x^2+3y^2-5=0$
$2x^2+3y^2-4=0$

Sol: Since $4x^2+3y^2-5=0\cdots(1)$

$2x^2+3y^2-4=0\cdots(2)$

Subtracting

$$\begin{array}{l} 4x^2+3y^2-5=0 \\ \underline{\pm 2x^2 \pm 3y^2 \mp 4=0} \\ 2x^2 \qquad -1=0 \end{array}$$

$2x^2=1$

$x^2=\dfrac{1}{2}$

Substituting $x^2=\dfrac{1}{2}$ in equation (1) we get

$4\left(\dfrac{1}{2}\right)+3y^2-5=0$

$2+3y^2-5=0$

$3y^2-3=0$

$3y^2=3$

$y^2=\dfrac{3}{3}=1$

Therefore $x^2=\dfrac{1}{2}$ and $y^2=1$

Taking square root

$x=\pm\dfrac{1}{\sqrt{2}}$ and $y=\pm 1$

Solution set $=\left\{\left(\dfrac{\pm 1}{\sqrt{2}},\pm 1\right)\right\}$

Q2i) Solve $\begin{array}{l} x+y=9 \\ x^2+3xy+2y^2=0 \end{array}$ The system

Sol: Given system $x+y=9$ ……….(1)

$x^2+3xy+2y^2=0$ …………(2)

From (1) $y=9-x$ …………(3) put in (2)

$x^2+3x(9-x)+2(9-x)^2=0$

$x^2+27x-3x^2+2\left(81-18x+x^2\right)=0$

$x^2-3x^2+27x+162-36x+2x^2=0$

$x^2-3x^2+2x^2+27x-36x+162=0$

$-9x+162=0$

$-9x=-162$

$x=18$

Put in eq (3) $y=9-18=-9$

Solution set $=\{(18,-9)\}$

Q2ii). Solve $\begin{array}{l} y-x=4 \\ 2x^2+xy+y^2=8 \end{array}$ the system

Sol: Given system $y-x=4$ ………..(1)

$2x^2+xy+y^2=8$ ……….(2)

From (1) $y=4+x$ ……..(3) Put in (2)

$2x^2+x(4+x)+(4+x)^2=8$

$2x^2+4x+x^2+16+8x+x^2=8$

$2x^2+x^2+x^2+4x+8x+16-8=0$

$4x^2+12x+8=0$ divided by 4

$x^2+3x+2=0$

$x^2+2x+1x+2=0$

$x(x+2)+1(x+2)=0$

$(x+1)(x+2)=0$

Either $x+1=0$ or $x+2=0$

$x=-1$ $\qquad x=-2$ put in (3)

$y=4+(-1)$ $\qquad y=4+(-2)$

$y=3$ $\qquad y=2$

Solution set $=\{(-1,3),(-2,2)\}$

Exp26: A rectangular shed is being build that has an area of 120 square feet and is 7 Feet longer than it is wide. Determine its dimensions.

Sol: Let width = x feet

Length = $x+7$ feet

Given that area = 120 square feet

$x(x+7)=120$

$x^2+7x-120=0$

$x^2+15x-8x-120=0$

$x(x+15)-8(x+15)=0$

$(x-8)(x+15)=0$

Either $x-8=0$ or $x+15=0$

$x=8$ $\qquad x=-15$ which is impossible

Therefore width of rectangle = 8 feets

And Length of rectangle = 8 + 7 = 15 feets

Exp27: A men purchased a number of shares of stock for an amount of Rs. 6000 if he had paid Rs. 20 less per share, number of share that could have been purchased for amount of money would have increased by 10. How many share did he buy?

Sol: Let number of share = x

The amount paid per share = y

Total amount = 6000

$xy=6000$ ………………(1)

$y=\dfrac{6000}{x}$ ……………(2)

According to condition

$(x+10)(y-20)=6000$

$xy-20x+10y-200=6000$ put (1)

$6000-20x+10y-200=6000$

$-20x+10y-200=0$ put value of x

$-20x+10\left(\dfrac{6000}{x}\right)-200=0$ multiply by x

$-20x^2+60000-200x=0$ divided by -20

$x^2+10x-3000=0$

$x^2+60x-50x-3000=0$

$x(x+60)-50(x+60)=0$

$(x-50)(x+60)=0$

Either $x-50=0$ or $x+60=0$

$x=50$ $\qquad x=-60$ is not admissible

Thus number of share purchased is 50

Exercise 2.7

Q1. Find two consecutive positive integers whose product is 72.
Sol: Consider consecutive integers
First positive integer = x
Second positive integer = x + 1
Then according to the given condition

$$x(x+1)=72$$
$$x^2+x-72=0$$
$$x^2+9x-8x-72=0$$
$$x(x+9)-8(x+9)=0$$
$$(x-8)(x+9)=0$$

Either $x-8=0$, $x=8$ Or $x+9=0$, $x=-9$

We take only positive integer as given in question
First integer = 8
Second integer = 8 + 1 = 9

Q2. The sum of the square of three consecutive integer is 50. Find the integers.
Sol: Consider consecutive integers
First integer = x
Second integer = x + 1
Third integer = x + 2
Then according to the given condition

$$x^2+(x+1)^2+(x+2)^2=50$$
$$x^2+x^2+2.x.1+1^2+x^2+2.x.2+2^2-50=0$$
$$x^2+x^2+x^2+2x+4x+1+4-50=0$$
$$3x^2+6x-45=0 \quad \div \text{ by } 3$$
$$x^2+2x-15=0$$
$$x^2+5x-3x-15=0$$
$$x(x+5)-3(x+5)=0$$
$$(x-3)(x+5)=0$$

Either $x-3=0$, $x=3$ Or $x+5=0$, $x=-5$

Take x = 3
1^{st} integer = 3
2^{nd} integer = 3 + 1 = 4
3^{rd} integer =3+2=5

Take x = - 5
1^{st} integer = - 5
2^{nd} integer =- 5+1= - 4
3^{rd} integer =-5+2= - 3

Q3. The length of prayer hall is 5 meter more than its width. If the area of the hall is 36 square meter. Find length and width of hall.
Sol: Consider dimension of the hall
Width of the hall = x
Length of the hall = x + 5
Then according to condition Area = 36 m^2

$$x(x+5)=36$$
$$x^2+5x-36=0$$
$$x^2+9x-4x-36=0$$
$$x(x+9)-4(x+9)=0$$
$$(x-4)(x+9)=0$$

Either $x-4=0$, $x=4$ Or $x+9=0$, $x=-9$

We take only positive integer because distance between any two points should be positive
Width of the hall = 4 meter
Length of the hall = 4 + 5 = 9 meter

Q4. The sum of two numbers is 11 and sum of their squares is 65. Find the numbers.
Sol: Consider two numbers
First number = x
Second number = y
Sum of two numbers is 11

$$x+y=11\cdots(1)$$

Sum of square of two numbers is 65

$$x^2+y^2=65\cdots(2)$$

From equation (1)

$$x+y=11$$
$$y=11-x\cdots(3)$$

Putting the value of y in equation (2)

$$x^2+(11-x)^2=65$$
$$x^2+11^2-2.11.x+x^2-65=0$$
$$x^2+x^2-22x+121-65=0$$
$$2x^2-22x+56=0 \quad \div \text{ by } 2$$
$$x^2-11x+28=0$$
$$x^2-7x-4x+28=0$$
$$x(x-7)-4(x-7)=0$$
$$(x-4)(x-7)=0$$

Either $x-4=0$, $x=4$ Or $x-7=0$, $x=7$

Putting the value of x in equation (3)

$y=11-4$, $y=7$; $y=11-7$, $y=4$

When x = 4
1^{st} number = 4
2^{nd} number = 7

When x = 7
1^{st} number = 7
2^{nd} number = 4

Q5. The sum of square of two numbers is 100. One number is two more than the other. Find the numbers.
Sol: Consider two numbers according to condition (one number is 2 more than other)
First number = x
Second number = x + 2
Sum of square of two numbers is 100

$$x^2+(x+2)^2=100$$
$$x^2+x^2+2.x.2+2^2-100=0$$
$$2x^2+4x+4-100=$$
$$2x^2+4x-96=0 \quad \div \text{by } 2$$
$$x^2+2x-48=0$$
$$x^2+8x-6x-48=0$$
$$x(x+8)-6(x+8)=0$$
$$(x-6)(x+8)=0$$

Either $x-6=0$, $x=6$ Or $x+8=0$, $x=-8$

When x = 6
1^{st} number = 6
2^{nd} number=6+2= 8

When x = - 8
1^{st} number = -8
2^{nd} number= -8+2= -6

Q6. The area of a rectangular field is 252 square meter. The length of its side is 9 meter longer than its width. Find its sides

Sol: Consider dimension of rectangular field

Width of the hall = x

Length of the hall = x + 9

Then according to the given condition

Area = 252 m^2

$$x(x+9)=252$$
$$x^2+9x-252=0$$
$$x^2+21x-12x-252=0$$
$$x(x+21)-12(x+21)=0$$
$$(x-12)(x+21)=0$$

Either Or

$x-12=0$ $\quad$ $x+21=0$

$x=12$ $\quad$ $x=-21$

We take only positive integer because distance between any two points should be positive

Width of the rectangular field = 12 meter

Length of the rectangular field = 12 +9=21 meter

Q7. One side of a rectangle is 3 centimeter less than twice the other. If the area of the rectangle is 54 square centimeters, then find the sides of rectangle.

Sol: Let width of rectangle = x

Than according to condition

Length of the rectangle = $2x-3$

Given that Area = 54 cm^2

$$x(2x-3)=54$$
$$2x^2-3x-54=0$$
$$2x^2-12x+9x-54=0$$
$$2x(x-6)+9(x-6)=0$$
$$(2x+9)(x-6)=0$$

Either Or

$2x+9=0$ $\quad$ $x-6=0$

$2x=-9$ $\quad$ $x=6$

$x=\frac{-9}{2}$

We take only positive integer because distance between any two points should be positive

Width of the rectangular field = 6 centimeter

Length of the rectangular field = $=2(6)-3$

$=12-3$

$=9$ cm

Q8. The length of one side of right triangle exceeds the length of other by 3 centimeters. If the hypotenuse is 15 centimeters, then find the length of the sides of triangle

Sol: since the triangle has three sides

Let Base = x Perpendicular = x + 3

And hypotenuses = 15 centimeter

Using Pythagoras theorem

Base^2 + perpendicular^2= hypotenuses^2

$$x^2+(x+3)^2=15^2$$
$$x^2+x^2+2.x.3+3^2=225$$
$$2x^2+6x+9-225=0$$
$$2x^2+6x-216=0 \quad \div \text{ by } 2$$
$$x^2+3x-108=0$$
$$x^2+12x-9x-108=0$$
$$x(x+12)-9(x+12)=0$$
$$(x-9)(x+12)=0$$

Either $x-9=0$ Or $x+12=0$

$x=9$ $\quad$ $x=-12$

We take only positive integer because distance between any two points should be positive

Base = 9 Perpendicular = 9 + 3=12

Q9. The sides of a right triangle in cm are $(x-1),x,(x+1)$. Find the sides of triangle.

Sol: Since in right angled triangle the longest side is hypotenuses, so

Hypotenuses = x + 1

Base = x

Perpendicular = x – 1

Using Pythagoras theorem

Base^2 + perpendicular^2= hypotenuses^2

$$x^2+(x-1)^2=(x+1)^2$$
$$x^2+x^2-2.x.1+1^2=x^2+2.x.1+1^2$$
$$2x^2-2x+1-x^2-2x-1=0$$
$$2x^2-x^2-2x-2x+1-1=0$$
$$x^2-4x=0$$
$$x(x-4)=0$$

Either $x=0$ Or $x-4=0$

$x=4$

When x = 0

Perpendicular = 0 – 1 = -1

Base = 0

Hypotenuses = 0 + 1 = 1

We take only positive measurement because distance between any two points should be positive, so we neglect the above dimensions

When x = 4

Perpendicular =4 – 1 = 3

Base = 4

Hypotenuses = 4 + 1 = 5

Q10. A farmer bought some goats for Rs.9000. If he had paid Rs. 100 less for each, he would have got 3 goats more for the same amount of money. How many goats did he buy, when the rate in each case is uniform?

Sol: since cost each goat is uniform

Let the cost of each goat = x

Number of goats = y

Former bought some goats for Rs. 9000

i.e., $xy=9000$

$x=\frac{9000}{y}$ 1000 less for each, he got 3 goats more for same amount

$$(x-100)(y+3)=9000$$
$$xy+3x-100y-300=9000$$

Putting the values of x and xy

$$9000+3\left(\frac{9000}{y}\right)-100y-300=9000$$
$$\frac{27000}{y}-100y-300=0 \quad \div \text{by } 100$$
$$\frac{270}{y}-y-3=0 \quad \times\text{by "y"}$$

$\frac{270}{y}.y - y.y - 3.y = 0.y$

$270 - y^2 - 3y = 0$

$-y^2 - 3y + 270 = 0$

$y^2 + 3y - 270 = 0$

$y^2 + 18y - 15y - 270 = 0$

$y(y+18) - 15(y+18) = 0$

$(y-15)(y+18) = 0$

Either $y - 15 = 0$, $y = 15$ or $y + 18 = 0$, $y = -18$

Since number of goats cannot be negative
So number of goats y = 15

Review Exercise 2

Q1. Fill in the correct circle only

i). if the sum of roots of $(a+1)x^2 + (2a+3)x + (3a+4) = 0$ is - 1 then product of roots

O 0 O 1
O 2 O 2

ii). The sum of the roots of quadratic eq is 2 & sum of cubes of roots is 98 the eq

O $x^2 - 2x - 15 = 0$ O $x^2 - 2x + 15 = 0$
O $x^2 - 4x + 15 = 0$ O None of these

iii). If a,b,c positive real numbers, then both roots of eq $ax^2 + bx + c = 0$ are always

O Imaginary O Irrational
O Rational O All of these

iv). If a and b are the roots of $4x^2 - 3x + 7 = 0$ then the value of $\frac{1}{a} + \frac{1}{b}$ is

O -3/4 O 3/7
O -3/7 O 4/7

Q2. For what value of k the roots of the equation $3x^2 - 5x + k = 0$ are equal

Sol: Comparing $3x^2 - 5x + k = 0$
with the quadratic equation $ax^2 + bx + c = 0$
we have $a = 3$, $b = -5$, $c = k$
According to the given condition

Discriminant $= b^2 - 4ac = 0$

$\Rightarrow (-5) - 4(3)(k) = 0$

$\Rightarrow 25 - 12k = 0$

$\Rightarrow 25 = 12k$

$\Rightarrow k = \frac{25}{12}$

Q3. Evaluate $\left(-1 + i\sqrt{3}\right)^7 + \left(-1 - i\sqrt{3}\right)^7$

Sol: As $\omega = \frac{-1 + i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1 - i\sqrt{3}}{2}$

$\Rightarrow$ $2\omega = -1 + i\sqrt{3}$ and $2\omega^2 = -1 - i\sqrt{3}$

Take $\left(-1 + i\sqrt{3}\right)^7 + \left(-1 - i\sqrt{3}\right)^7 = (2\omega)^7 + (2\omega^2)^7$

$= 2^7\omega^7 + 2^7\omega^{14}$

$= 2^7\left(\omega^6\omega^1 + \omega^{12}\omega^2\right)$

$= 128\left(\left(\omega^3\right)^2\omega^1 + \left(\omega^3\right)^4\omega^2\right)$

$= 128\left((1)^2\omega^1 + (1)^4\omega^2\right)$ $\therefore \omega^3 = 1$

$= 128\left(\omega + \omega^2\right)$ $\therefore 1 + \omega + \omega^2 = 0$

$= 128(-1)$

$= -128$

Q4i). Without solving the equation. Find the sum and product of the roots of $4x^2 - 1 = 0$

Sol: Comparing $4x^2 + 0.x - 1 = 0$
with the quadratic equation $ax^2 + bx + c = 0$
we have $a = 4$, $b = 0$, $c = -1$
Sum of the roots

$\alpha + \beta = \frac{-b}{a}$ Putting the values of a and b

$\alpha + \beta = \frac{-(0)}{4} = \frac{0}{4} = 0$

Product of the roots

$\alpha.\beta = \frac{c}{a}$ Putting the values of a and c

$\alpha.\beta = \frac{-1}{4}$

Q4ii). Without solving the equation. Find the sum and product of the roots of $3x^2 + 4x = 0$

Sol: Comparing $3x^2 + 4x + 0 = 0$
with the quadratic equation $ax^2 + bx + c = 0$
we have $a = 3$, $b = 4$, $c = 0$
Sum of the roots

$\alpha + \beta = \frac{-b}{a}$ Putting the values of a and b

$\alpha + \beta = \frac{-4}{3}$

Product of the roots

$\alpha.\beta = \frac{c}{a}$ Putting the values of a and c

$\alpha.\beta = \frac{0}{3} = 0$

Q5. Find the value of k so that sum of the roots of $3x^2 + (2k+1)x + k - 5 = 0$ is equal to the product of the roots.

Sol: Comparing $3x^2 + (2k+1)x + k - 5 = 0$
with the quadratic equation $ax^2 + bx + c = 0$
we have $a = 3$, $b = 2k + 1$, $c = k - 5$
According to the given condition
Sum of the roots = Product of the roots i.e.,

$\alpha + \beta = \alpha.\beta$

$\frac{-b}{a} = \frac{c}{a}$ $\times$by a

$-b = c$

Substituting the values of b and c

$-(2k+1) = k - 5$

$\Rightarrow$ $-2k - 1 = k - 5$

$\Rightarrow$ $+5 - 1 = k + 2k$

$\Rightarrow$ $4 = 3k$

$\Rightarrow$ $k = \frac{4}{3}$

Q6. Find value of k if roots of $x^2-3x+k+1=0$ differ by unity.

Sol: Comparing $x^2-3x+k+1=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=1$, $b=-3$, $c=k+1$

Sum of the roots $\alpha+\beta=\frac{-b}{a}$ Putting

$\alpha+\beta=\frac{-(-3)}{1}=\frac{3}{1}=3$

Product of the roots $\alpha.\beta=\frac{c}{a}$ Putting

$\alpha.\beta=\frac{k+1}{1}=k+1$

According to the given condition

Differ by unity $\alpha-\beta=1$

Using formula

$4\alpha\beta=(\alpha+\beta)^2-(\alpha-\beta)^2$ putting the values

$4(k+1)=(3)^2-(1)^2$

$4k+4=9-1$

$4k=8-4$

$4k=4$

$k=\frac{4}{4}=1$

Q7 Find quadratic eq whose roots multiplicative inverse of roots of $12x^2-17x+6=0$

Sol: Comparing $12x^2-17x+6=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=12$, $b=-17$, $c=6$

$\alpha+\beta=\frac{-b}{a}=\frac{-(-17)}{12}=\frac{17}{12}$

$\alpha.\beta=\frac{c}{a}=\frac{6}{12}$

According to condition

$S=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$ Putting the values

$S=\frac{\frac{17}{12}}{\frac{6}{12}}=\frac{17}{6}$

Product of the roots

$P=\frac{1}{\alpha}.\frac{1}{\beta}=\frac{1}{\alpha\beta}$ Putting the values

$P=1\div\frac{6}{12}=\frac{12}{6}=2$

The required equation is given by

$x^2-(\text{Sum of the roots})x+(\text{Product of the roots})=0$

$x^2-\left(\frac{17}{6}\right)x+(2)=0 \qquad \times \text{by } 6$

$\Rightarrow 6x^2-17x+12=0$

Q8. If one of the root of quadratic equation $2x^2+kx+4=0$ is 2, find the other root. Also find the value of k

Sol: Comparing $2x^2+kx+4=0$

with the quadratic equation $ax^2+bx+c=0$

we have $a=2$, $b=k$, $c=4$

$\alpha.\beta=\frac{c}{a}$ Putting the values of a, c and α

$2\beta=\frac{4}{2}$

$2\beta=2$

$\beta=\frac{2}{2}=1$

Sum $\alpha+\beta=\frac{-b}{a}$ Putting values of a,b,α & β

$2+1=\frac{-k}{2}$

$3=\frac{-k}{2}$

$-k=3\times 2$

$\Rightarrow k=-6$

Q9. One root of cubic equation $x^3+6x^2+11x+6=0$ is - 3. Use synthetic division to find the other roots.

Sol: Since $x^3+6x^2+11x+6=0$

-3	1	6	11	6
		-3	-9	-6
	1	3	2	0

Therefore $Q(x)=x^2+3x+2$ and $R=0$

To find the other roots take Q (x) = 0

$x^2+3x+2=0$

$x^2+2x+1x+2=0$

$x(x+2)+1(x+2)=0$

$(x+1)(x+2)=0$

Either $x+1=0$, $x=-1$ or $x+2=0$, $x=-2$

Hence the other roots are $-1, -2$

Q10i). Solve the system $x+y=3$, $x^2-3xy+y^2=29$

Sol: Since $x+y=3\cdots(1)$

$x^2-3xy+y^2=29\cdots(2)$

Form equation (1)

$x+y=3$

$y=3-x\cdots(3)$

Putting the value of y in equation (2) we get

$x^2-3x(3-x)+(3-x)^2=29$

$x^2-9x+3x^2+(3)^2-2(3)(x)+(x)^2=29$

$x^2-9x+3x^2+9-6x+x^2=29$

$x^2+3x^2+x^2-9x-6x+9-29=0$

$5x^2-15x-20=0 \qquad \div \text{by } 5$

$x^2-3x-4=0$

$x^2-4x+1x-4=0$

$x(x-4)+1(x-4)=0$

$(x+1)(x-4)=0$

Either $x+1=0$, $x=-1$ Or $x-4=0$, $x=4$

Substituting these values in equation (3)

$y=3-(-1)$, $y=3+1$, $y=4$

$y=3-4$, $y=-1$

Solution Set $=\{(-1,4),(4,-1)\}$

Q10ii). Solve the system $7x^2-4=5y^2$
$3x^2+2=4y^2$

Sol: Since $7x^2-4=5y^2 \cdots(1)$
$3x^2+2=4y^2 \cdots(2)$

$\times$ eq (1) by $4 \Rightarrow \quad 28x^2-16= \quad 20y^2$
$\times$ eq (2) by $5 \Rightarrow \underline{\pm 15x^2 \pm 10 = \pm 20y^2}$ Subtract
$13x^2-26=0$

$13x^2=26$
$x^2=2$

Putting the value of x^2 in equation (1)

$5y^2=7(2)-4$
$5y^2=14-4$
$5y^2=10$
$y^2=\frac{10}{5}=2$

Thus $x^2=2$ $\qquad y^2=2$

Taking square root on both sides

$x=\pm\sqrt{2}$ $\qquad y=\pm\sqrt{2}$

Solution set $=\left\{\left(\pm\sqrt{2},\pm\sqrt{2}\right)\right\}$

Q11. Area of a rectangle is 48 cm^2 if length and width are each increased by 4cm, area of the larger rectangle is 120cm^2 Find length and width of the original rectangle.

Sol: Let width of the rectangle = x
Length of the rectangle = y
Area of rectangle = 48cm^2

$xy=48$

$y=\frac{48}{x}$

After increasing
Width of new rectangle = x + 4
Length of new rectangle = y + 4
Area of new rectangle =120cm^2

$(x+4)(y+4)=120$
$x(y+4)+4(y+4)=120$
$xy+4x+4y+16=120$

Putting the value of xy and y

$48+4x+4\left(\frac{48}{x}\right)+16=120$

$4x+\frac{192}{x}+48+16-120=0$

$4x+\frac{192}{x}-56=0 \qquad \times by\ \frac{x}{4}$

$x^2+48-14x=0$
$x^2-8x-6x+48=0$
$x(x-8)-6(x-8)=0$
$(x-6)(x-8)=0$

Either $x-6=0$, $x=6$ $\qquad$ Or $x-8=0$, $x=8$

When width x = 6 cm

So length $=\frac{48}{6}=8$ cm

Activity 1: Find the error in $x^2+x+11=0$ for the real solution

Sol: Given $x^2+x+11=0$ by comparing
$a=1, b=1, c=11$

Discriminant $=b^2-4ac$ putting the values
$=1^2-4(1)(11)$
$=1-44$
$=-43<0$ roots are imaginary

But according to question roots should be real

So take small change $x^2+x-11=0$ to get positive discriminant

Discriminant $=b^2-4ac$ putting the values
$=1^2-4(1)(-11)$
$=1+44$
$=45>0$ So Roots are real

Using Quadratic formula

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-11)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{45}}{2}=\frac{-1\pm\sqrt{9\times5}}{2}$$

$$x=\frac{-1\pm3\sqrt{5}}{2}$$

Solution set $=\left\{\frac{-1-3\sqrt{5}}{2},\frac{-1+3\sqrt{5}}{2}\right\}$

Activity 2: Divide x^5+x^2+5x+7 by $x+2$ and find quotient and Remainder. Verify your answer by using long division.

Sol: Given $P(x)=x^5+0x^4+0x^3+x^2+5x+7$

And divisor $x+2=0$ or $x=-2$

Then by synthetic division

-2	1	0	0	1	5	7
		-2	4	-8	14	-38
	1	-2	4	-7	19	-31

Q(x)= $x^4-2x^3+4x^2-7x+19$ and $R=-31$

By using Long division

$$\begin{array}{r|l}
 & x^4-2x^3+4x^2-7x+19 \\ \hline
x+2 & x^5+0x^4+0x^3+x^2+5x+7 \\
 & \pm x^5 \pm 2x^4 \\ \hline
 & -2x^4+0x^3 \\
 & \mp 2x^4 \mp 4x^3 \\ \hline
 & 4x^3+x^2 \\
 & \pm 4x^3 \pm 8x^2 \\ \hline
 & -7x^2+5x \\
 & \mp 7x^2 \mp 14x \\ \hline
 & 19x+7 \\
 & \pm 19x \pm 38 \\ \hline
 & R=-33
\end{array}$$