

# Chapter 3

**Ratio:** comparison of two quantities with same units same type. The ratio of two quantities  $a$  and  $b$  is denoted as  $a:b$  and is defined as  $a:b = a \div b = \frac{a}{b}, b \neq 0$

Exp1i): write 3:12 in simplified form

Sol: Given 3:12 divided by 3  
1:4

Exp1ii): write  $6a:18b$  in simplified form

Sol: Given  $6a:18b$  divided by 6  
 $a:3b$

Exp2 Divide Rs 5070 among three persons in the ratio 2:5:6

Sol: Total Amount = Rs 5070

Given ratio 2:5:6

Sum of ratio =  $2+5+6=13$

Share of 1<sup>st</sup> person =  $\frac{2}{13} \times 5070 = \text{Rs. } 780$

Share of 2<sup>nd</sup> person =  $\frac{5}{13} \times 5070 = \text{Rs. } 1950$

Share of 3<sup>rd</sup> person =  $\frac{6}{13} \times 5070 = \text{Rs. } 2340$

**Proportion:** A statement of equality of two ratios of same type and same unit is called proportion. If four quantities  $a, b, c, d$  are in proportion then mathematically these are written as

1.  $a:b::c:d$

2.  $a:b=c:d$

3. Product of means = Product of extremes

i.e.  $a \times d = b \times c$

4.  $\frac{a}{c} = \frac{b}{d}$  or  $\frac{a}{b} = \frac{c}{d}$

Exp3: if  $a^3 - b^3, a^2 - b^2, a^2 + ab + b^2, x$  are in proportion. Find the value of  $x$

Sol: Given  $a^3 - b^3 : a^2 - b^2 = a^2 + ab + b^2 : x$

Using Product of means = Product of extremes

$$(a^2 - b^2)(a^2 + ab + b^2) = x(a^3 - b^3)$$

$$\frac{(a^2 - b^2)(a^2 + ab + b^2)}{a^3 - b^3} = x$$

$$\text{Or } x = \frac{(a-b)(a+b)(a^2 + ab + b^2)}{(a-b)(a^2 + ab + b^2)}$$

$$x = a + b$$

**Variations:**

There are two types of Variations.

1. Direct variations 2. Inverse variations

**Direct Variation:** A relation between two quantities such that an increase or decrease causes an increase or decrease in the other quantity in the same ratio is called direct variation.

If two quantities  $x$  and  $y$  vary directly, then we write  $y \propto x$  (read as  $y$  varies directly with  $x$ ) or  $y = kx$  ( $k$  is constant of variation).

Exp4i):  $y$  varies directly as  $x$  &  $y=27$  when  $x=3$   
Find an equation connecting  $x$  and  $y$

Solution: we have  $y \propto x$  or

$$y = kx \text{ using } y = 27 \text{ \& } x = 3$$

$$27 = k(3)$$

$$\Rightarrow k = 9$$

Exp4ii):  $y$  varies directly as  $x$  &  $y=27$  when  $x=3$

Find the value of  $y$  when  $x=11$

Solution: we have  $y \propto x$  or

$$y = kx \text{ using } y = 27 \text{ \& } x = 3$$

$$27 = k(3)$$

$$\Rightarrow k = 9$$

Now to find  $y$  when  $x=11$  and using  $k=9$

$$y = 9(11)$$

$$y = 99$$

Exp5: If  $y \propto x$  then complete the following table

$x$	4	5	8		
$y$	6			18	22.5

Sol: we have  $y \propto x$

Or  $y = kx$  using  $x=4$  &  $y=6$

$$6 = k(4)$$

$$k = \frac{3}{2}$$

Now to find  $y$  when  $x=5$  and using  $k=3/2$

$$y = \frac{3}{2}(5) = \frac{15}{2} = 7.5$$

Now to find  $y$  when  $x=8$  and using  $k=3/2$

$$y = \frac{3}{2}(8) = \frac{24}{2} = 12$$

Now to find  $x$  when  $y=18$  and using  $k=3/2$

$$18 = \frac{3}{2}x$$

$$x = 18 \times \frac{2}{3} = 12$$

Now to find  $x$  when  $y=22.5$  and using  $k=3/2$

$$22.5 = \frac{3}{2}x$$

$$x = \frac{225}{10} \times \frac{2}{3} = \frac{75 \times 2}{10}$$

$$x = \frac{150}{10} = 15$$

So complete table is given as under

$x$	4	5	8	12	15
$y$	6	7.5	12	18	22.5

**Inverse Variation:** A relation between two quantities such that an increase or decrease causes an increase or decrease in the other quantity in same ratio is called inverse variation. If two quantities  $x$  and  $y$  vary inversely, then we write  $y \propto \frac{1}{x}$  (read as  $y$  varies inversely with  $x$ ) or  $y = \frac{k}{x}$  ( $k$  is constant of variation).

Exp6: if  $x$  varies inversely as  $y$  &  $x=3$  when  $y=12$   
Find the value of  $y$  when  $x=6$

Sol: Given  $y \propto \frac{1}{x}$  or simply  $y = \frac{k}{x}$

Using  $x = 3$  when  $y = 12$

$$12 = \frac{k}{3}$$

$$\Rightarrow k = 36$$

Now to find  $y$  when  $x = 6$  using  $k = 36$

$$y = \frac{k}{x} = \frac{36}{6}$$

$$y = 6$$

Exp7: Given that Pressure "P" on the quantity of gas in a container varies inversely to its volume V. When Pressure is  $10\text{N/m}^2$  its volume is  $25\text{m}^3$ . Find the pressure when the volume is  $20\text{m}^3$ .

Sol: Given  $P \propto \frac{1}{v}$  or  $P = \frac{k}{v}$

Using  $P = 10\text{ N/m}^2$  and  $v = 25\text{m}^3$

$$10 = \frac{k}{25}$$

$$k = 250$$

Now to find P when  $V = 20\text{m}^3$  and using  $k = 250$

$$P = \frac{250}{20}$$

$$P = 12.5\text{ N/m}^2$$

### Exercise 3.1

Q1: which is greater ratio, 5:7 and 151:208

Sol: we have 5:7 and 151:208

$$\text{First ratio } \frac{5}{7} = 0.714285$$

$$\text{Second ratio } \frac{151}{208} = 0.72596153$$

Clearly second ratio is greater.

Q2. Gold and silver are mixed in the ratio 7:4. If 36 grams of silver are used How much gold is used.

Sol: Let amount of Gold =  $x$

Amount of silver = 36 grams

Given ratio Gold : Silver = 7 : 4

Using proportion  $7 : 4 = x : 36$

Product of mean = Product of extreme

$$4x = 36 \times 7$$

$$x = \frac{36 \times 7}{4}$$

$$x = 63$$

Amount of Gold = 63 grams

Q3. Divide the annual profit of Rs.40,000 of a factory among 3 partners in the ratio 5:8:12

Sol: Given ratio of 3 partners = 5:8:12

$$\text{Sum of ratio} = 5 + 8 + 12 = 25$$

Total amount = 40 000

$$\begin{aligned} \text{Share of 1st Partner} &= \frac{5}{25} \times 40000 \\ &= \text{Rs } 8000 \end{aligned}$$

$$\begin{aligned} \text{Share of 1st Partner} &= \frac{8}{25} \times 40000 \\ &= \text{Rs } 12800 \end{aligned}$$

$$\begin{aligned} \text{Share of 1st Partner} &= \frac{12}{25} \times 40000 \\ &= \text{Rs } 19200 \end{aligned}$$

Q4. If  $11 : x - 1 = 22 : 27$ , find the value of  $x$

Sol: Given  $11 : x - 1 = 22 : 27$

Using Product of Mean = Product of extreme

$$22 \times (x - 1) = 11 \times 27$$

$$\frac{22(x - 1)}{11} = \frac{11 \times 27}{11}$$

$$2(x - 1) = 27$$

$$2x - 2 = 27$$

$$2x = 27 + 2$$

$$x = \frac{29}{2} = 14.5$$

Q5i). There is direct variation between  $x^2$  and  $y$ , when  $x = 7$ , when  $y = 49$  Find  $y$  when  $x = 9$

Sol: Given  $y \propto x^2 \Rightarrow y = kx^2 \dots (1)$

When  $y = 49$ , and  $x = 7$  put in (1)

$$49 = k(7)^2$$

$$49 = 49k$$

$$k = 1$$

Now to find  $y$  when  $x = 9$  using  $k = 1$  put in (1)

$$y = 1(9)^2 = 81$$

Q5ii). There is direct variation between  $x^2$  and  $y$ , when  $x = 7$ , when  $y = 49$  Find  $x$  when  $y = 100$

Sol: Given  $y \propto x^2 \Rightarrow y = kx^2 \dots (1)$

When  $y = 49$ , and  $x = 7$  put in (1)

$$49 = k(7)^2$$

$$49 = 49k$$

$$k = 1$$

Now to find  $x$  when  $y = 100$  using  $k = 1$  put in (1)

$$100 = 1 \cdot x^2$$

$$x^2 = 100$$

$$\Rightarrow \sqrt{x^2} = \pm \sqrt{100}$$

$$\Rightarrow x = \pm 10$$

Q6i).  $X$  varies inversely as  $y$ , and when  $x = 4$ ,  $y = 24$ . Find  $y$  when  $x = 12$

Sol: Given  $y \propto \frac{1}{x}$   $y = \frac{k}{x} \dots (1)$

When  $x = 4$  and  $y = 24$  put in (1)

$$24 = \frac{k}{4}$$

$$24 \times 4 = k$$

$$96 = k$$

$$\Rightarrow k = 96$$

Now to find  $y$  when  $x = 12$ , using  $k = 96$  in (1)

$$y = \frac{96}{12}$$

$$y = 8$$

Q6ii).  $X$  varies inversely as  $y$ , and when  $x = 4$ ,  $y = 24$ . Find  $x$  when  $y = 24$

Sol: Given  $y \propto \frac{1}{x}$   $y = \frac{k}{x} \dots (1)$

When  $x = 4$  and  $y = 24$  put in (1)

$$24 = \frac{k}{4}$$

$$k = 24 \times 4 = 96$$

Now to find  $x$  when  $y = 24$ , using  $k = 96$  in (1)

$$24 = \frac{96}{x}$$

$$24x = 96$$

$$x = \frac{96}{24}$$

$$x = 4$$

Q7i).  $r \propto \frac{1}{p^3}$  &  $p=9$  when  $r=2$ . Find  $r$  when  $p = 3$

Sol: Given  $r \propto \frac{1}{p^3}$   $r = \frac{k}{p^3} \dots\dots(1)$

when  $p = 9$  when  $r = 2$ , put in (1)

$$2 = \frac{k}{9^3}$$

$$2 \times 9^3 = k$$

$$\Rightarrow k = 2 \times 729$$

$$\Rightarrow k = 1458$$

Now to find  $r$  when  $p = 3$ , using  $k = 1458$  in (1)

$$r = \frac{1458}{3^3}$$

$$r = \frac{1458}{27}$$

$$r = 54$$

Q7ii).  $r \propto \frac{1}{p^3}$  &  $p=9$  when  $r=2$ . Find  $p$  when  $r = \frac{1}{4}$

Sol: Given  $r \propto \frac{1}{p^3}$   $r = \frac{k}{p^3} \dots\dots(1)$

when  $p = 9$  when  $r = 2$ , put in (1)

$$2 = \frac{k}{9^3}$$

$$2 \times 9^3 = k$$

$$\Rightarrow k = 2 \times 729$$

$$\Rightarrow k = 1458$$

Now to find  $p$  when  $r = \frac{1}{4}$ , using  $k = 1458$  in (1)

$$\frac{1}{4} = \frac{1458}{p^3}$$

$$p^3 = 4 \times 1458$$

$$p^3 = 5832$$

$$\Rightarrow p^3 = (18)^3$$

$$\Rightarrow \sqrt[3]{p^3} = \sqrt[3]{(18)^3}$$

$$p = 18$$

Q8. If  $y \propto x$ , then complete the following tables.

x.	4	6		15
y.	2		3.5	

Sol: Given  $y \propto x \Rightarrow y = kx \dots(1)$

When  $y = 2$ , and  $x = 4$  put in (1)

$$2 = k(4)$$

$$k = \frac{1}{2}$$

When  $y = ?$  and  $x = 6$ , using  $k = 1/2$  put in (1)

$$y = \frac{1}{2}(6) = 3$$

When  $x = ?$  and  $y = 3.5$  using  $k = 1/2$  put in (1)

$$3.5 = \frac{1}{2}x$$

$$3.5 \times 2 = x$$

$$\Rightarrow x = 7$$

When  $y = ?$  and  $x = 15$ , using  $k = 1/2$  put in (1)

$$y = \frac{1}{2}(15) = 7.5$$

Putting the values in the table

x.	4	6	7	15
y.	2	3	3.5	7.5

**Continued Proportional:** If three quantities  $a, b$  and  $c$  are linked as  $a:b::b:c$  then we say that these quantities are in continued proportional.  $b$  is the **mean proportional** (or **Geometric mean**) while  $a$  and  $c$  are first and third proportional.

**Mean Proportional:** If three quantities  $a, b$  and  $c$  are linked as  $a:b::b:c$  then  $b$  is called mean proportional.

Then Product of means = Product of extremes

$$\text{i.e. } b \times b = a \times c \quad \Rightarrow b^2 = a \times c$$

$$\Rightarrow b = \sqrt{a \times c}$$

$$\therefore \text{Mean proportional} = \sqrt{\text{product of extremes}}$$

Exp 8: Find mean proportion of 5 and 15

Sol: Let mean proportion =  $x$

So 5,  $x$ , 15 form continued proportion, So

$$5 : x = x : 15$$

Product of mean = Product of extreme

$$x^2 = 5 \times 15$$

$$x^2 = 5 \times 5 \times 3 \text{ Taking square root}$$

$$x = \pm 5\sqrt{3}$$

**Exp9:** Find the third proportion of  $a^2b^2$  &  $abc$

Sol: Let third proportion =  $x$

So  $a^2b^2, abc, x$  are in continued proportion, so

$$a^2b^2 : abc = abc : x$$

Product of extreme = product of mean

$$a^2b^2x = abc \times abc$$

$$a^2b^2x = a^2b^2c^2$$

$$\Rightarrow x = c^2$$

Exp 10. Find the fourth proportion of  $a^3 - b^3$ ,

$$a+b \text{ and } a^2 + ab + b^2$$

Sol: Let fourth proportion =  $x$

$$\text{So } a^3 - b^3 : a+b = a^2 + ab + b^2 : x$$

Product of extreme = Product of mean

$$(a^3 - b^3)x = (a+b)(a^2 + ab + b^2)$$

$$x = \frac{(a+b)(a^2 + ab + b^2)}{(a^3 - b^3)}$$

$$x = \frac{(a+b)(a^2 + ab + b^2)}{(a-b)(a^2 + ab + b^2)}$$

$$x = \frac{a+b}{a-b}$$

**Theorems on Proportional:**

1). If  $a:b=c:d$ , then  $a:c=b:d$  is called

**Alternendo property.** e.g.  $3:5=6:10$

Then  $3:6=5:10$

2). If  $a:b=c:d$ , then  $b:a=d:c$  is called **Invertendo property**. e.g.  $6:10=9:15$   
Then  $10:6=15:9$

3). If  $a:b=c:d$ , then  $a+b:b=c+d:d$  or  $\frac{a+b}{b} = \frac{c+d}{d}$  is called **componendo property**.

Or then  $a:a+b=c:c+d$  or  $\frac{a}{a+b} = \frac{c}{c+d}$  is called **componendo property**. e.g.  $4:5=8:10$

Then  $\frac{4+5}{5} = \frac{8+10}{10}$  or  $(4+5):5=(8+10):10$

4). If  $a:b=c:d$ , then  $a-b:b=c-d:d$  or  $\frac{a-b}{b} = \frac{c-d}{d}$  is called **Dividendo property**.

Or If  $a:b=c:d$ , then  $a:a-b=c:c-d$  or  $\frac{a}{a-b} = \frac{c}{c-d}$  is called **Dividendo property**.

e.g.  $5:4=10:8$  then  $(5-4):4=(10-8):8$

2). If  $a:b=c:d$ , then  $a+b:a-b=c+d:c-d$  or  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$  called **componendo- dividendo property**.

Or If  $a:b=c:d$ , then  $a-b:a+b=c-d:c+d$  or  $\frac{a-b}{a+b} = \frac{c-d}{c+d}$  called **dividend- componendo property**.

e.g.  $7:3=14:6$  then  $\frac{7+3}{7-3} = \frac{14+6}{14-6}$

Exp11: if  $\frac{a}{b} = \frac{c}{d}$  then prove that  $\frac{2a+3b}{b} = \frac{2c+3d}{d}$

Sol: Given  $\frac{a}{b} = \frac{c}{d}$  multiply both sides by  $\frac{2}{3}$

$$\frac{2a}{3b} = \frac{2c}{3d} \text{ using componendo property}$$

$$\frac{2a+3b}{3b} = \frac{2c+3d}{3d} \text{ Multiply by 3}$$

$$\frac{2a+3b}{b} = \frac{2c+3d}{d} \text{ which is required result}$$

Exp12. If  $\frac{3a-4b}{3a+4b} = \frac{3c-4d}{3c+4d}$  then  $\frac{a}{b} = \frac{c}{d}$

Sol: Given  $\frac{3a-4b}{3a+4b} = \frac{3c-4d}{3c+4d}$

using componendo Dividendo property

$$\frac{(3a-4b)+(3a+4b)}{(3a-4b)-(3a+4b)} = \frac{(3c-4d)+(3c+4d)}{(3c-4d)-(3c+4d)}$$

$$\frac{3a-4b+3a+4b}{3a-4b-3a-4b} = \frac{3c-4d+3c+4d}{3c-4d-3c-4d}$$

$$\frac{6a}{-8b} = \frac{6c}{-8d}$$

Thus  $\frac{a}{b} = \frac{c}{d}$

Exp13: if  $\frac{(x+3)^2+(x-4)^2}{(x+3)^2-(x-4)^2} = \frac{13}{12}$  then find x

Sol: Given  $\frac{(x+3)^2+(x-4)^2}{(x+3)^2-(x-4)^2} = \frac{13}{12}$

using componendo Dividendo property

$$\frac{(x+3)^2+(x-4)^2+(x+3)^2-(x-4)^2}{(x+3)^2+(x-4)^2-\{(x+3)^2-(x-4)^2\}} = \frac{13+12}{13-12}$$

$$\frac{2(x+3)^2}{2(x-4)^2} = \frac{25}{1} \text{ Taking square root}$$

$$\frac{x+3}{x-4} = \pm 5$$

$$x+3 = \pm 5(x-4)$$

Either

$$x+3 = -5(x-4)$$

$$x+3 = -5x+20$$

$$x+5x = 20-3$$

$$6x = 17$$

$$x = \frac{17}{6}$$

or

$$x+3 = 5(x-4)$$

$$x+3 = 5x-20$$

$$3+20 = 5x-x$$

$$23 = 4x$$

$$x = \frac{23}{4}$$

$$\text{Solution set} = \left\{ \frac{17}{6}, \frac{23}{4} \right\}$$

### Exercise 3.2

Q1i): Are 4,12,36 in continued proportional?

Sol: Given 4,12,36

For continued proportional we can write

$$4:12=12:36$$

Using Product of extreme = Product of Mean

$$4 \times 36 = 12 \times 12$$

$$144 = 144 \text{ Satisfied}$$

So given numbers are in continued proportional

Q1ii): Are 3,12,39 in continued proportional?

Sol: Given 3, 12, 39

For continued proportional we can write

$$3:12=12:39$$

Using Product of Mean = Product of extreme

$$12 \times 12 = 3 \times 39$$

$$144 = 117 \text{ Not Satisfied}$$

So given numbers are not in continued proportional

Q1iii): Are 72,24,8 in continued proportional?

Sol: Given 72,24,8

For continued proportional we can write

$$72:24=24:8$$

Using Product of extreme = Product of Mean

$$24 \times 24 = 72 \times 8$$

$$576 = 576 \text{ Satisfied}$$

So given numbers are in continued proportional

Q2: Find the mean proportional of 12,3

Solution: Let the mean proportional = x

So 12,x,3 are in continued proportion

For continued proportional we can write

$$12:x=x:3$$

Using Product of extreme = Product of Mean

$$x \times x = 12 \times 3$$

$$x^2 = 36$$

Taking square root on both side

$$x = \pm 6$$

Q3: If 5,15,x are in continued proportional, find the value of x

Solution: we have 5,15,x

For continued proportional we can write

$$5 : 15 = 15 : x$$

Using Product of extreme = Product of Mean

$$5 \times x = 15 \times 15$$

$$x = \frac{15 \times 15}{5} = 3 \times 15 = 45$$

Q4: If  $3x - 1, 4, 35$  are in continued proportion, find the value of  $x$

Solution: we have  $3x - 1, 4, 35$

For continued proportional we can write

$$3x - 1 : 4 = 4 : 35$$

Using Product of extreme = Product of Mean

$$35 \times (3x - 1) = 4 \times 4$$

$$105x - 35 = 16$$

$$105x = 16 + 35$$

$$x = \frac{51}{105} = \frac{17}{35}$$

Q5: Find mean proportional of  $a^2 - b^2$  &  $\frac{a+b}{a-b}$

Solution: Let the mean proportional =  $x$

So  $a^2 - b^2, x, \frac{a+b}{a-b}$  are in continued proportion

For continued proportional we can write

$$a^2 - b^2 : x = x : \frac{a+b}{a-b}$$

Using Product of extreme = Product of Mean

$$x \times x = (a^2 - b^2) \times \frac{a+b}{a-b}$$

$$x^2 = (a+b)(a-b) \times \frac{a+b}{a-b}$$

$$x^2 = (a+b)^2$$

Taking square root on both side

$$\sqrt{x^2} = \pm \sqrt{(a+b)^2}$$

$$x = \pm(a+b)$$

Q6: If  $\frac{a}{b} = \frac{c}{d}$ , then prove that  $\frac{ac+bd}{ac-bd} = \frac{a^2-b^2}{a^2+b^2}$

Sol: Given  $\frac{a}{b} = \frac{c}{d}$  it can be written as

$$\frac{c}{d} = \frac{a}{b} \text{ multiply both side by } \frac{a}{b} \text{ we get}$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{a}{b} \times \frac{a}{b}$$

$$\frac{ac}{bd} = \frac{a^2}{b^2}$$

Using componendo-Dividendo property

$$\frac{ac+bd}{ac-bd} = \frac{a^2+b^2}{a^2-b^2}$$

Q7i): Solve  $\frac{\sqrt{3x+2} + \sqrt{x}}{\sqrt{3x+2} - \sqrt{x}} = \frac{4}{1}$

Sol: Given  $\frac{\sqrt{3x+2} + \sqrt{x}}{\sqrt{3x+2} - \sqrt{x}} = \frac{4}{1}$

Using componendo-Dividendo property

$$\frac{\sqrt{3x+2} + \sqrt{x} + \sqrt{3x+2} - \sqrt{x}}{\sqrt{3x+2} + \sqrt{x} - (\sqrt{3x+2} - \sqrt{x})} = \frac{4+1}{4-1}$$

$$\frac{\sqrt{3x+2} + \sqrt{x} + \sqrt{3x+2} - \sqrt{x}}{\sqrt{3x+2} + \sqrt{x} - \sqrt{3x+2} + \sqrt{x}} = \frac{5}{3}$$

$$\frac{2\sqrt{3x+2}}{2\sqrt{x}} = \frac{5}{3}$$

$$\frac{\sqrt{3x+2}}{\sqrt{x}} = \frac{5}{3}$$

Cross multiplying  $3\sqrt{3x+2} = 5\sqrt{x}$

Squaring both sides

$$(3\sqrt{3x+2})^2 = (5\sqrt{x})^2$$

$$9(3x+2) = 25x$$

$$27x + 18 = 25x$$

$$27x - 25x = -18$$

$$2x = -18$$

$$x = \frac{-18}{2}$$

$$x = -9$$

Q7ii): Solve  $\frac{(x-1)^2 + (x+2)^2}{(x-1)^2 - (x+2)^2} = \frac{-17}{8}$

Sol: Given  $\frac{(x-1)^2 + (x+2)^2}{(x-1)^2 - (x+2)^2} = \frac{-17}{8}$

Using componendo-Dividendo property

$$\frac{\text{Numerator} + \text{Denominator}}{\text{Numerator} - \text{Denominator}} = \frac{\text{Numerator} + \text{Denominator}}{\text{Numerator} - \text{Denominator}}$$

$$\frac{(x-1)^2 + (x+2)^2 + (x-1)^2 - (x+2)^2}{(x-1)^2 + (x+2)^2 - \{(x-1)^2 - (x+2)^2\}} = \frac{-17+8}{-17-8}$$

$$\frac{2(x-1)^2}{(x-1)^2 + (x+2)^2 - (x-1)^2 + (x+2)^2} = \frac{-9}{-25}$$

$$\frac{2(x-1)^2}{2(x+2)^2} = \frac{9}{25}$$

$$\frac{(x-1)^2}{(x+2)^2} = \frac{9}{25}$$

Taking Square root on both sides

$$\sqrt{\frac{(x-1)^2}{(x+2)^2}} = \pm \sqrt{\frac{9}{25}}$$

$$\frac{x-1}{x+2} = \frac{\pm 3}{5}$$

Cross multiplying

$$5(x-1) = \pm 3(x+2)$$

Either

$$5(x-1) = 3(x+2)$$

$$5x - 5 = 3x + 6$$

$$5x - 3x = 6 + 5$$

$$2x = 11$$

$$x = \frac{11}{2}$$

or

$$5(x-1) = -3(x+2)$$

$$5x - 5 = -3x - 6$$

$$5x + 3x = -6 + 5$$

$$8x = -1$$

$$x = \frac{-1}{8}$$

Q7iii): Solve  $\frac{\sqrt{x^2+a^2} - \sqrt{x^2-a^2}}{\sqrt{x^2+a^2} + \sqrt{x^2-a^2}} = \frac{1}{3}$

Sol: Given  $\frac{\sqrt{x^2+a^2} - \sqrt{x^2-a^2}}{\sqrt{x^2+a^2} + \sqrt{x^2-a^2}} = \frac{1}{3}$

Using componendo-Dividendo property



$$\frac{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2} + \sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2} - \{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}\}} = \frac{1+3}{1-3}$$

$$\frac{2\sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2} - \sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}} = \frac{4}{-2}$$

$$\frac{2\sqrt{x^2 + a^2}}{-2\sqrt{x^2 - a^2}} = -2$$

$$\frac{\sqrt{x^2 + a^2}}{-\sqrt{x^2 - a^2}} = -2$$

$$\sqrt{x^2 + a^2} = 2\sqrt{x^2 - a^2}$$

Squaring both sides

$$(\sqrt{x^2 + a^2})^2 = (2\sqrt{x^2 - a^2})^2$$

$$x^2 + a^2 = 4(x^2 - a^2)$$

$$x^2 + a^2 = 4x^2 - 4a^2$$

$$4a^2 + a^2 = 4x^2 - x^2$$

$$5a^2 = 3x^2$$

$$x^2 = \frac{5}{3}a^2$$

Taking square root on both sides

$$x = \pm \sqrt{\frac{5}{3}} a$$

#### Joint variation:

A direct variation with two or more quantities  
e.g. if  $x$  is jointly proportional to  $y$  and  $z$ . so  
 $x \propto yz$

Or  $x = k yz$

Exp14: if  $y$  varies jointly as  $x$  and  $z$  and  $y=12$   
when  $x=9$  and  $z=3$  find  $z$  when  $y=6$  &  $x=15$

Sol: As variation  $y \propto xz$  or simply  $y = kxz$

Putting  $y=12$  when  $x=9$  and  $z=3$

$$12 = k(9)(3)$$

$$12 = 27k$$

$$k = \frac{12}{27} = \frac{4}{9}$$

Now to find  $z$  when  $y=6$  &  $x=15$  using  $k = 4/9$

$$6 = \frac{4}{9}(15)z$$

$$\frac{6 \times 9}{4 \times 15} = z \Rightarrow z = \frac{9}{10}$$

### Exercise 3.3

Q1: If  $y$  varies jointly as  $x$  &  $z$ . if  $y=33$  when  
 $x=9$  &  $z=12$ . Find  $y$  when  $x=16$  &  $z=22$

Sol: Given If  $y$  varies jointly as  $x$  &  $z$ . So,  
 $y \propto xz$

$$y = kxz$$

If  $y=33$  when  $x=9$  &  $z=12$

$$33 = k(9)(12)$$

$$\frac{33}{9 \times 12} = k$$

$$\Rightarrow k = \frac{11}{36}$$

Now find  $y$  when  $x=16$  &  $z=22$  using  $k=11/36$

$$y = \frac{11}{36}(16)(22)$$

Thus

$$y = \frac{968}{9}$$

Q2. If  $f$  varies jointly as  $g$  and cube of  $h$  if  
 $f=200$  when  $g=5$  &  $h=4$ . Find  $f$  when  
 $g=3$  &  $h=6$

Sol: Given  $f$  varies jointly as  $g$  & cube of  $h$

$$f \propto gh^3$$

$$f = kgh^3$$

if  $f=200$  when  $g=5$  &  $h=4$

$$200 = k(5)(4)^3$$

$$\frac{200}{(5)(64)} = k$$

$$\Rightarrow k = \frac{5}{8}$$

Now to Find  $f$  when  $g=3$  &  $h=6$  using  $k=5/8$

$$f = \frac{5}{8}(3)(6)^3$$

$$f = 405$$

Q3. Suppose  $a$  is jointly proportional to  $b$  &  $c$  if  
 $a=4$  when  $b=8$  &  $c=9$ . Then what value of  
 $a$  when  $b=2$  &  $c=18$

Sol: Given  $a$  is jointly proportional to  $b$  &  $c$

$$a \propto bc$$

$$a = kbc$$

if  $a=4$  when  $b=8$  &  $c=9$ .

$$4 = k(8)(9)$$

$$\frac{4}{(8)(9)} = k$$

$$\Rightarrow k = \frac{1}{18}$$

Now to Find  $a$  if  $b=2$  &  $c=18$  using  $k=1/18$

$$a = \frac{1}{18}(2)(18)$$

$$a = 2$$

Q4. If  $p$  varies jointly as  $q$  &  $r^2$ . if  $p=225$   
when  $q=4$  &  $r=3$ . Find  $p$  when  $q=6$  &  $r=8$

Sol: Given If  $p$  varies jointly as  $q$  &  $r$ .

$$p \propto qr^2$$

$$p = kqr^2$$

if  $p=225$  when  $q=4$  &  $r=3$ .

$$225 = k(4)(3)^2$$

$$\frac{225}{4 \times 9} = k \Rightarrow k = \frac{25}{4}$$

Now Find  $p$  when  $q=6$  &  $r=8$  using  $k=25/4$

$$p = \frac{25}{4}(6)(8)^2$$

Thus

$$p = 2400$$

Q5. if  $a$  varies jointly  $b$  cubed &  $c$  if  $a = 36$  when  $b = 4$  &  $c = 6$ . find  $a$  when  $b = 2$  &  $c = 14$

Sol: if  $a$  varies jointly  $b$  cubed &  $c$

$$a \propto b^3 c$$

$$a = k b^3 c$$

if  $a = 36$  when  $b = 4$  &  $c = 6$ .

$$36 = k(4)^3(6)$$

$$\frac{36}{(64)(6)} = k$$

$$\Rightarrow k = \frac{3}{32}$$

Now find  $a$  when  $b = 2$  &  $c = 14$  using  $k = 3/32$

$$a = \frac{3}{32}(2)^3(14)$$

Thus

$$a = \frac{21}{2}$$

Q6. If  $z$  varies jointly as  $x$  &  $y$ . if  $z = 12$  when  $x = 2$  &  $y = 4$  find constant of variation.

Sol: given If  $z$  varies jointly as  $x$  &  $y$

$$z \propto x y$$

$$z = k x y$$

if  $z = 12$  when  $x = 2$  &  $y = 4$

$$12 = k(2)(4)$$

$$\frac{12}{8} = k$$

Thus constant of variation  $k = 3/2$

and variation  $z = \frac{3}{2} x y$

Q7. If  $y$  varies jointly as  $x^2$  &  $z$ . If  $y = 6$  when  $x = 4$  &  $z = 9$ . Write  $y$  as a function of  $x$  &  $z$ . and determine the value of  $y$ , when  $x = -8$  &  $z = 12$

Sol: Given If  $y$  varies jointly as  $x^2$  &  $z$

$$y \propto x^2 z$$

$$y = k x^2 z$$

If  $y = 6$  when  $x = 4$  &  $z = 9$ .

$$6 = k(4)^2(9)$$

$$\frac{6}{16 \times 9} = k$$

$$k = \frac{1}{24}$$

Therefore given variation becomes

$$\text{Thus } y = \frac{x^2 z}{24}$$

Now find  $y$ , when  $x = -8$  &  $z = 12$

$$y = \frac{(-8)^2(12)}{24}$$

Thus

$$y = 32$$

Q8. If  $p$  varies jointly as  $q$  &  $r^2$  and inversely as  $s$  &  $t^2$ ,  $p = 40$  when  $q = 8, r = 5, s = 3, t = 2$   
Find  $p$  in terms of  $q, r, s, t$ . Also find value of  $p$

when  $q = -2, r = 4, s = 3$  &  $t = -1$

Sol: Given If  $p$  varies jointly as  $q$  &  $r^2$  and inversely as  $s$  &  $t^2$

$$p \propto \frac{qr^2}{st^2}$$

$$p = k \frac{qr^2}{st^2}$$

If  $p = 40$  when  $q = 8, r = 5, s = 3, t = 2$

$$40 = k \frac{(8)(5)^2}{(3)(2)^2}$$

$$40 = k \frac{50}{3}$$

$$k = \frac{3}{50} \times 40$$

$$\text{Thus } k = \frac{12}{5}$$

$$\text{Therefore } p = \frac{12qr^2}{5st^2}$$

Now find  $p$  when  $q = -2, r = 4, s = 3$  &  $t = -1$

$$p = \frac{12(-2)(4)^2}{5(3)(-1)^2}$$

$$p = \frac{-128}{5}$$

**K- Method:** Let  $a:b=c:d$  be a proportion.

Then  $\frac{a}{b} = \frac{c}{d} = k$  (say). Thus  $a = bk, c = dk$

These eqs are used to evaluate certain expressions more easily. This method is called K-Method.

Exp15: If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$  then each of these ratio is

equal to  $\frac{la+mc+ne}{lb+md+nf}$

Sol: Let  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$  .....(1)

Then each of ratio equal to  $k$

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k \text{ or}$$

$$a = bk, \quad c = dk, \quad e = fk$$

Now taking  $\frac{la+mc+ne}{lb+md+nf}$  putting  $a, b, c$

$$= \frac{lbk + mdk + nfk}{lb + md + nf}$$

$$= \frac{k(lb + md + nf)}{lb + md + nf}$$

$$= k \text{ .....(2)}$$

From eq (1) and eq (2) we get

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{la+mc+ne}{lb+md+nf}$$

Exp16: Prove that  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$

Sol: Let  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$  .....(1)

Then each of ratio equal to k

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k \text{ or}$$

$$a = bk, \quad c = dk, \quad e = fk$$

Adding  $a + c + e = bk + dk + fk$

$$a + c + e = k(b + d + f)$$

$$\frac{a + c + e}{b + d + f} = k \text{ .....(2)}$$

From eq (1) and eq (2) we get

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a + c + e}{b + d + f} \text{ Hence proved}$$

Exp17 Prove that  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$

Sol: Let  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$  .....(1)

each ratio equal to k

$$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k \text{ or}$$

$$a = bk, \quad c = dk, \quad e = fk \text{ Squaring}$$

$$a^2 = b^2k^2, \quad c^2 = d^2k^2, \quad e^2 = f^2k^2 \text{ adding}$$

$$a^2 + c^2 + e^2 = b^2k^2 + d^2k^2 + f^2k^2$$

$$a^2 + c^2 + e^2 = k^2(b^2 + d^2 + f^2)$$

$$\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2} = k^2 \text{ taking square root on bs}$$

$$\sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}} = \sqrt{k^2}$$

$$\sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}} = k \text{ .....(2)}$$

From eq (1) and (2) we get

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}} \text{ Hence proved}$$

Exp18q if  $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$  then prove that

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

Sol: Let  $\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = k$  then each ratio equal to k

$$\frac{a}{x} = k, \quad \frac{b}{y} = k, \quad \frac{c}{z} = k \text{ taking reciprocal}$$

$$\frac{x}{a} = \frac{1}{k}, \quad \frac{y}{b} = \frac{1}{k}, \quad \frac{z}{c} = \frac{1}{k} \text{ .....(1) Taking cubes}$$

$$\frac{x^3}{a^3} = \frac{1}{k^3}, \quad \frac{y^3}{b^3} = \frac{1}{k^3}, \quad \frac{z^3}{c^3} = \frac{1}{k^3} \text{ Adding}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{1}{k^3} + \frac{1}{k^3} + \frac{1}{k^3}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3}{k^3} \text{ .....(2)}$$

From eq (1) Product of LHS= Product of RHS

$$\frac{x}{a} \cdot \frac{y}{b} \cdot \frac{z}{c} = \frac{1}{k} \cdot \frac{1}{k} \cdot \frac{1}{k}$$

$$\frac{xyz}{abc} = \frac{1}{k^3} \text{ Multiply 3 on both sides}$$

$$\frac{3xyz}{abc} = \frac{3}{k^3} \text{ .....(3) From eq (2) \& (3)}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

### Exercise 3.4

Q1i). If  $\frac{a}{b} = \frac{c}{d}$  then prove that

$$\frac{2a + 3b}{2a - 3b} = \frac{2c + 3d}{2c - 3d}$$

Sol: Given  $\frac{a}{b} = \frac{c}{d}$  multiply by  $\frac{2}{3}$

$$\frac{2a}{3b} = \frac{2c}{3d} \text{ using componendo Dividendo}$$

$$\frac{2a + 3b}{2a - 3b} = \frac{2c + 3d}{2c - 3d}$$

Q1ii). If  $\frac{a}{b} = \frac{c}{d}$  then prove that

$$\frac{pa + qb}{ma - nb} = \frac{pa + qd}{mc - nd}$$

Sol: Let  $\frac{a}{b} = \frac{c}{d} = k$  then each ratio equal to k

$$\frac{a}{b} = k, \quad \frac{c}{d} = k$$

$$a = bk, \quad c = dk$$

Taking LHS and putting value of a

$$\begin{aligned} \frac{pa + qb}{ma - nb} &= \frac{pbk + qb}{mbk - nb} \\ &= \frac{b(pk + q)}{b(mk - n)} \\ &= \frac{pk + q}{mk - n} \text{ .....(1)} \end{aligned}$$

Taking LHS and putting value of c

$$\begin{aligned} \frac{pc + qd}{mc - nd} &= \frac{pdk + qd}{mdk - nd} \\ &= \frac{d(pk + q)}{d(mk - n)} \\ &= \frac{pk + q}{mk - n} \text{ .....(2)} \end{aligned}$$

Form equations (1) and (2) we get

$$\frac{pa + qb}{ma - nb} = \frac{pa + qd}{mc - nd}$$

Q2.  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{pa^2 + qc^2 + e^2}{pb^2 + qd^2 + f^2}}$

Sol: Given  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$  .....(1)

Then each of these fraction equal to k



$$\text{i.e., } \frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$$

$$\text{or } a = bk, \quad c = dk, \quad e = fk$$

Squaring both sides

$$a^2 = b^2k^2, \quad c^2 = d^2k^2, \quad e^2 = f^2k^2$$

$$pa^2 = pb^2k^2, \quad qc^2 = qd^2k^2, \quad e^2 = f^2k^2$$

Now adding

$$pa^2 + qc^2 + e^2 = pb^2k^2 + qd^2k^2 + f^2k^2$$

$$pa^2 + qc^2 + e^2 = (pb^2 + qd^2 + f^2)k^2$$

$$\frac{pa^2 + qc^2 + e^2}{pb^2 + qd^2 + f^2} = k^2$$

$$\sqrt{\frac{pa^2 + qc^2 + e^2}{pb^2 + qd^2 + f^2}} = \sqrt{k^2}$$

$$\sqrt{\frac{pa^2 + qc^2 + e^2}{pb^2 + qd^2 + f^2}} = k \dots (2)$$

From equations (1) and (2) we get

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{pa^2 + qc^2 + e^2}{pb^2 + qd^2 + f^2}}$$

Q3. If  $\frac{x-y}{z} = \frac{y-z}{x} = \frac{z-x}{y}$  then prove that

$x = y = z$ , where  $x, y, z$  are non-zero numbers and  $x + y + z \neq 0$

Sol: Given  $\frac{x-y}{z} = \frac{y-z}{x} = \frac{z-x}{y} = k \dots (1)$

Then each of these fraction equal to  $k$

$$\frac{x-y}{z} = k, \quad \frac{y-z}{x} = k, \quad \frac{z-x}{y} = k$$

$$x - y = zk, \quad y - z = xk, \quad z - x = yk \dots (A)$$

Now adding

$$x - y + y - z + z - x = zk + xk + yk$$

$$0 = k(z + x + y)$$

$$0 = k(x + y + z)$$

$$\text{Since } x + y + z \neq 0 \Rightarrow k = 0$$

Put  $k = 0$  in system (A) we get

$$x - y = z(0), \quad y - z = x(0), \quad z - x = y(0)$$

$$x - y = 0, \quad y - z = 0, \quad z - x = 0$$

$$x = y, \quad y = z, \quad z = x$$

By transitive property  $x = y = z$

Q4. If  $\frac{2y+2z-x}{a} = \frac{2z+2x-y}{b} = \frac{2x+2y-z}{c}$ ,

then prove that  $\frac{x}{2b+2c-a} = \frac{y}{2c+2a-b} = \frac{z}{2a+2b-c}$

Sol:  $\frac{2y+2z-x}{a} = \frac{2z+2x-y}{b} = \frac{2x+2y-z}{c} = k$

Then each of these fraction equal to  $k$

$$\frac{2y+2z-x}{a} = k, \quad \frac{2z+2x-y}{b} = k, \quad \frac{2x+2y-z}{c} = k$$

$$2y+2z-x = ak, \quad 2z+2x-y = bk, \quad 2x+2y-z = ck$$

Arranging

$$-x + 2y + 2z = ak \dots (1)$$

$$2x - y + 2z = bk \dots (2)$$

$$2x + 2y - z = ck \dots (3)$$

Multiply (2) by 2

Multiply (3) by 2

Multiply (1) by -1

Adding three eq

$$4x - 2y + 4z = 2bk$$

$$4x + 4y - 2z = 2ck$$

$$x - 2y - 2z = -ak$$

$$9x = 2bk + 2ck - ak$$

$$9x = k(2b + 2c - a)$$

$$\frac{x}{2b + 2c - a} = \frac{k}{9} \dots (4)$$

Multiply (3) by 2

Multiply (1) by 2

Multiply (2) by -1

Adding three eq

$$4x + 4y - 2z = 2ck$$

$$-2x + 4y + 4z = 2ak$$

$$-2x + y - 2z = -bk$$

$$9y = 2ck + 2ak - bk$$

$$9y = k(2c + 2a - b)$$

$$\frac{y}{2c + 2a - b} = \frac{k}{9} \dots (5)$$

Multiply (1) by 2

Multiply (2) by 2

Multiply (1) by -1

Adding three eq

$$-2x + 4y + 4z = 2ak$$

$$4x - 2y + 4z = 2bk$$

$$-2x - 2y + z = -ck$$

$$9z = 2ak + 2bk - ck$$

$$9z = k(2a + 2b - c)$$

$$\frac{z}{2a + 2b - c} = \frac{k}{9} \dots (6)$$

From equations (4), (5) and (6) we get

$$\frac{x}{2b + 2c - a} = \frac{y}{2c + 2a - b} = \frac{z}{2a + 2b - c}$$

Q5. Prove that each of its fraction in

$$\frac{x+y}{a+b} = \frac{y+z}{b+c} = \frac{z+x}{c+a} \text{ is equal to } \frac{x+y+z}{a+b+c}$$

Sol: Given  $\frac{x+y}{a+b} = \frac{y+z}{b+c} = \frac{z+x}{c+a} = k \dots (1)$

Then each of these fraction equal to  $k$

$$\frac{x+y}{a+b} = k, \quad \frac{y+z}{b+c} = k, \quad \frac{z+x}{c+a} = k$$

$$x + y = k(a + b), \quad y + z = k(b + c), \quad z + x = k(c + a)$$

Adding

$$x + y + y + z + z + x = k(a + b + b + c + c + a)$$

$$2x + 2y + 2z = k(2a + 2b + 2c)$$

$$2(x + y + z) = 2k(a + b + c)$$

$$\frac{x + y + z}{a + b + c} = \frac{2k}{2}$$

$$\frac{x + y + z}{a + b + c} = k \dots (2)$$

$$\frac{x + y + z}{a + b + c} = k \dots (2)$$

From eqs (1) & (2)

$$\frac{x+y}{a+b} = \frac{y+z}{b+c} = \frac{z+x}{c+a} = \frac{x+y+z}{a+b+c}$$

Q6: if  $\frac{bz+cy}{b-c} = \frac{cx+az}{c-a} = \frac{ay+bx}{a-b}$  then

$$(a+b+c)(x+y+z) = ax+by+cz$$

Sol: Let  $\frac{bz+cy}{b-c} = \frac{cx+az}{c-a} = \frac{ay+bx}{a-b} = k$  then

$$\frac{bz+cy}{b-c} = k, \quad \frac{cx+az}{c-a} = k, \quad \frac{ay+bx}{a-b} = k$$

$$bz+cy = k(b-c) \text{ or } bz+cy = kb-kc \dots (1)$$

$$cx+az = k(c-a) \text{ or } cx+az = kc-ka \dots (2)$$

$$ay + bx = k(a - b) \text{ or } ay + bx = ka - kb \dots (3)$$

Adding eqs (1), (2) and (3) we get

$$bz + cy + cx + az + ay + bx = kb - kc + kc - ka + ka - kb$$

Rearranging

$$ay + az + bx + bz + cx + cy = kb - kb - kc + kc - ka + ka$$

$$ay + az + bx + bz + cx + cy = 0$$

adding  $ax, by, cz$  on both sides

$$ax + ay + az + bx + by + bz + cx + cy + cz = ax + by + cz$$

$$a(x + y + z) + b(x + y + z) + c(x + y + z) = ax + by + cz$$

$$(a + b + c)(x + y + z) = ax + by + cz$$

Q7 if  $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$  then

$$(b-c)x + (c-a)y + (a-b)z = 0$$

Sol: Let  $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = k$  then

$$\frac{x}{b+c-a} = k, \frac{y}{c+a-b} = k, \frac{z}{a+b-c} = k \text{ or}$$

$$x = kb + kc - ka \dots (1)$$

$$y = ka + kc - kb \dots (2)$$

$$z = ka + kb - kc \dots (3)$$

Taking LHS  $(b-c)x + (c-a)y + (a-b)z$

Putting values of  $x, y, z$

$$= (b-c)(kb + kc - ka) + (c-a)(ka + kc - kb) + (a-b)(ka + kb - kc)$$

$$= kb^2 + kbc - kab - kbc - kc^2 + kac$$

$$+ kac + kc^2 - kbc - ka^2 - kac + kab$$

$$+ ka^2 + kab - kac - kab - kb^2 + kbc$$

Rearranging the terms

$$= kb^2 - kb^2 + kbc - kbc - kab + kab$$

$$- kbc + kbc - kc^2 + kc^2 - kac + kac$$

$$- ka^2 + ka^2 + kab - kab + kac - kac$$

$$= 0 = \text{RHS hence proved}$$

8. if  $2x+3y:3y+4z:4z+5x=4a-5b:3b-a:2b-3a$

Then  $7x+6y+8z=0$

Sol  $2x+3y:3y+4z:4z+5x=4a-5b:3b-a:2b-3a$

Or  $\frac{2x+3y}{4a-5b} = \frac{3y+4z}{3b-a} = \frac{4z+5x}{2b-3a} = k$  then

$$\frac{2x+3y}{4a-5b} = k, \frac{3y+4z}{3b-a} = k, \frac{4z+5x}{2b-3a} = k \text{ Or}$$

$$2x+3y = k(4a-5b) \text{ or } 2x+3y = 4ak-5bk \dots (1)$$

$$3y+4z = k(3b-a) \text{ or } 3y+4z = 3bk-ka \dots (2)$$

$$4z+5x = k(2b-3a) \text{ or } 4z+5x = 2bk-3ak \dots (3)$$

Adding eqs (1), (2) and (3)

$$2x+3y+3y+4z+4z+5x$$

$$= 4ak - 5bk + 3bk - ka + 2bk - 3ak$$

$$7x+6y+8z = 4ak - ka - 3ak$$

$$-5bk + 3bk + 2bk$$

$$7x+6y+8z = 0 \text{ Hence proved.}$$

Q9. If  $\frac{a-b}{d-e} = \frac{b-c}{e-f}$  then each of them is equal

$$\text{to } \frac{b\{(f-d)+(cd-af)\}}{e(f-d)}$$

Sol: Let  $\frac{a-b}{d-e} = \frac{b-c}{e-f} = k \dots (1)$  then

$$\frac{a-b}{d-e} = k, \quad \frac{b-c}{e-f} = k \quad \text{or}$$

$$a-b = k(d-e), \quad b-c = k(e-f)$$

$$a-b = kd - ke \dots (2) \quad b-c = ke - kf \dots (3)$$

X eq (2) by  $f$  X eq (3) by  $d$

$$af - bf = kdf - kef \quad bd - cd = ked - kdf$$

Adding

$$af - bf + bd - cd = kdf - kef + ked - kdf$$

Multiply by -1

$$bf - bd + cd - af = kef - ked$$

$$b(f-d) + (cd-af) = ke(f-d)$$

$$\frac{b(f-d) + (cd-af)}{e(f-d)} = k \dots (2)$$

From eqs (1) and (2) we get

$$\frac{a-b}{d-e} = \frac{b-c}{e-f} = \frac{b(f-d) + (cd-af)}{e(f-d)}$$

Exp19. A stone is dropped from the top of hill. The distance it falls is proportional to the square of the time of fall. The stone falls 19.6m after 2seconds. How far does it fall after 3seconds?

Sol: Let distance covered =  $d$

And time spends =  $t$

According to condition  $d \propto t^2$

$$\text{Or } d = k t^2$$

When  $d = 19.6\text{m}$  then  $t = 2 \text{ sec}$

$$19.6 = k(2)^2$$

$$19.6 = k(4)$$

$$\text{Or } k = 4.9$$

Now to find  $d$  when  $t = 3\text{sec}$  using  $k = 4.9$

$$d = 4.9(3)^2$$

$$d = 4.9(9)$$

$$d = 44.1\text{m}$$

Exp20: Height of an image  $y$  on a screen varies directly as distance  $x$  of the projector from the screen. Height of an image is 20cm when distance of the projector from screen is 100cm. At what distance should projector be kept from the screen so that height of an image on screen be 15cm.

Sol: Let Height of an image =  $y$

Distance of projector =  $x$

So,  $y \propto x$

$$\text{Or } y = kx$$

When height  $y = 20\text{cm}$  projector's distance  $x = 100\text{cm}$

$$20 = k(100)$$

$$k = \frac{20}{100}$$

$$\text{Or } k = \frac{1}{5}$$

When height  $y = 15\text{cm}$  projector's distance  $x = ?$

$$15 = \frac{1}{5}x$$

$$15 \times 5 = x$$

Or Distance of projector from screen  $x = 75\text{cm}$

Exp21: The ratio of the mass of sand to cement in a particular type of concrete is 4.8:2. If 6kg of sand are used, how much cement is needed?

Sol: Sand : Cement

Mass 6kg : x

Ratio 4.8 : 2

For direct proportion

$$\frac{6}{4.8} = \frac{x}{2}$$

Or 
$$\frac{6 \times 2}{4.8} = x$$

Mass of cement  $x = 2.5\text{ kg}$

Exp22i): 4 people can paint a fence in 3 hours. How long will it take 6 people to paint it.

Sol: Number of people =  $p$

Time taken to complete the work =  $t$

According to condition  $t \propto \frac{1}{p}$

Or 
$$t = \frac{k}{p}$$

When  $p = 4$  and  $t = 3$  hours so,

$$3 = \frac{k}{4}$$

$$\Rightarrow k = 12$$

Now to find  $t = ?$  when  $p = 6$  and using  $k=12$

$$t = \frac{12}{6}$$

$$t = 2 \text{ hours}$$

Exp22: 4 people can paint a fence in 3 hours. How many people are needed to complete the job in half an hour?

Sol: Number of people =  $p$

Time taken to complete the work =  $t$

According to condition  $t \propto \frac{1}{p}$

Or 
$$t = \frac{k}{p}$$

When  $p = 4$  and  $t = 3$  hours so,

$$3 = \frac{k}{4}$$

$$\Rightarrow k = 12$$

Now to find  $p = ?$  when  $t = \frac{1}{2}$  hours and using  $k=12$

$$\frac{1}{2} = \frac{12}{p} \text{ cross multiplication}$$

Number of persons  $p = 24$

### Exercise 3.5

Q1. A hedge is made of wooden planks. The thickness of the hedge varies directly as number of planks. Four planks make 12cm thick hedge.

i). Find Thickness of hedge when number of planks is 6

ii). Number of planks when thickness of hedge is 9

Sol: Given Thickness of hedge =  $x$

Number of planks =  $y$

According to given condition

$$y \propto x \Rightarrow y = kx \dots (1)$$

When  $y = 4$  and  $x = 12\text{cm}$

$$4 = k(12)$$

$$k = \frac{1}{3}$$

i). Find  $x$  when  $y = 6$ , using  $k = \frac{1}{3}$  put in (1)

$$6 = \frac{1}{3}x$$

$$6 \times 3 = x$$

$$18 = x$$

$$\Rightarrow x = 18\text{cm}$$

ii). Find  $y$  when  $x = 9\text{cm}$

$$y = \frac{1}{3}(9)$$

$$y = 3$$

Q2. A fountain, pressure 'P' of water at any internal point varies directly as depth 'd' from surface.

Pressure is 51 newton/ $\text{cm}^2$  when depth is 3cm.

Find pressure when depth is 7cm.

Sol: Given  $d \propto P \Rightarrow d = kP \dots (1)$

When  $P = 51 \text{ N/cm}^2$  and  $d = 3\text{cm}$

$$3 = k(51)$$

$$k = \frac{3}{51} = \frac{1}{17}$$

Find  $P$  when  $d = 7\text{cm}$  using  $k = \frac{1}{17}$  put in (1)

$$7 = \frac{1}{17}P$$

$$7 \times 17 = P$$

$$\Rightarrow P = 119 \text{ N/cm}^2$$

Q3. Pressure of gas in container varies directly as temperature. When pressure is 50  $\text{N/m}^2$ , temperature is  $75^\circ\text{C}$ . Find pressure when temperature is  $150^\circ$

Sol: Given Pressure of gas =  $x$

Temperature =  $y$

According to condition

$$y \propto x \Rightarrow y = kx \dots (1)$$

When  $y = 75^\circ\text{C}$  and  $x = 50 \text{ N/m}^2$  put in (1)

$$75 = k(50)$$

$$k = \frac{3}{2}$$

Find  $x$  when  $y = 150^\circ$  using  $k = \frac{3}{2}$  in (1)

$$150 = \frac{3}{2}x$$

$$150 \times \frac{2}{3} = x$$

$$50 \times 2 = x$$

$$x = 100$$

Q4. If 8 persons complete a work in 10 days then how many days would 10 persons take to complete same work?

Sol: Given Number of persons =  $x$

Number of days =  $y$

According to condition

$$y \propto \frac{1}{x} \quad y = \frac{k}{x} \dots\dots(1)$$

When  $x = 8$  and  $y = 10$  put in (1)

$$10 = \frac{k}{8}$$

$$k = 10 \times 8 = 80$$

Find  $y = ?$  and  $x = 10$ , using  $k = 80$  in (1)

$$y = \frac{80}{10}$$

$$y = 8$$

Q5. Volume of gas 'V' varies inversely as pressure 'P'. when  $P = 300 \text{ N/m}^2$  and  $V = 4\text{m}^3$ .

Find the pressure when  $V = 3\text{m}^3$

Solution: According to condition

$$P \propto \frac{1}{V} \quad P = \frac{k}{V} \dots\dots\dots(1)$$

when  $P = 300 \text{ N/m}^2$  and  $V = 4\text{m}^3$

$$300 = \frac{k}{4}$$

$$k = 300 \times 4$$

$$k = 1200$$

Find  $P$  when  $V = 3\text{m}^3$  Using  $k = 1200$  in (1)

$$P = \frac{1200}{3}$$

$$P = 400 \text{ N/m}^2$$

Q6. Attraction force 'F' between two magnets vary inversely as square of the distance 'd' between them. F is 18 newton when d is 2cm. Find distance when attraction force is 2 newton.

Sol: Given  $d^2 \propto \frac{1}{F} \quad d^2 = \frac{k}{F} \dots\dots\dots(1)$

When  $F = 18$  and  $d = 2\text{cm}$  put in (1)

$$2^2 = \frac{k}{18}$$

$$k = 4 \times 18 = 72$$

When  $d = ?$  and  $F = 2 \text{ N}$ , using  $k = 72$  in (1)

$$d^2 = \frac{72}{2} = 36$$

$$d = 6$$

Q7. The volume of a right circular cylinder varies jointly as the height and the square of the radius.

The volume of a right circular cylinder, with radius 4cm and height 7cm is  $352\text{cm}^3$ . Find the volume of another cylinder with radius 8cm and height 14cm.

Sol: Let Volume of right circular cylinder = V

Height of right circular cylinder = h

Radius of right circular cylinder = r

According to condition  $V \propto hr^2$

Or  $V = khr^2$

When  $r = 4\text{cm}$ ,  $h = 7\text{cm}$  and  $V = 352\text{cm}^3$

Putting  $352 = k(7)(4)^2$

$$k = \frac{352}{112} = \frac{22}{7}$$

Now to find  $V = ?$  when  $r = 8\text{cm}$  and  $h = 14\text{cm}$

$$V = \frac{22}{7}(14)(8)^2$$

Hence volume of cylinder  $V = 2816\text{cm}^3$

### Review Exercise 3

Q1. Fill in the correct circle

i). Direct variation between a and b can be expressed as

☐  $a = b$

☐  $a = \frac{1}{b}$

☐  $a \propto b$

☐  $a \propto \frac{1}{b}$

ii). If  $m \propto \frac{1}{n}$  then

☐  $m = kn$

☐  $n = km$

☐  $\frac{m}{n} = k$

☐  $mn = k$

iii). Identify the item that does not have the same ratio as the other three

☐  $\frac{30}{45}$

☐ 4 to 6

☐ 2 : 3

☐ 3 to 2

iv). If  $\frac{a}{b} = \frac{c}{d}$  then by alternendo property

☐  $\frac{a-b}{b} = \frac{c-d}{d}$

☐  $\frac{a}{a+b} = \frac{c}{c+d}$

☐  $\frac{a}{c} = \frac{b}{d}$

☐  $\frac{b}{a} = \frac{d}{c}$

v). if  $7:9::x:27$

☐  $x = 21$

☐  $x = 3$

☐  $x = 7$

☐  $x = 81$

vi). The third proportional of x and y is

☐  $xy$

☐  $\frac{x}{y}$

☐  $\frac{y^2}{x}$

☐ None of these

vii). If  $x \propto \frac{1}{y}$  and  $y \propto \frac{1}{z}$  then

☐  $y \propto \frac{1}{z}$

☐  $x \propto z$

☐  $xy \propto z$

☐  $xz \propto y$

viii). If  $2a+1:21::4:7$  then  $a =$

☐  $13/2$

☐  $11/2$

☐ 10

☐  $9/2$

ix). if  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$  then each fraction is equal to

☐  $\frac{la+mb+ne}{ld+me+nf}$

☐  $\frac{la+mc+ne}{lb+md+nf}$

☐  $\frac{la+mc+ne}{md+nd+ef}$

☐  $\frac{la+mb+nc}{lb+mc+nf}$

x). which of the following is a situation in which x varies directly as y

☐  $x = \frac{4}{y}$

☐  $x = \frac{7}{16}y$

☐  $x = xy$

☐  $xy = 6$

Q2. Find the constant of variation, when  $s \propto t^2$  and  $t = 10$  when  $s = 5$

Sol: Given  $s \propto t^2 \Rightarrow s = kt^2 \dots\dots(1)$

When  $s = 5$  and  $t = 10$  put in (1)

$$5 = k(10)^2$$

$$5 = k(100)$$

$$\frac{5}{100} = k$$

$$\Rightarrow k = \frac{1}{20}$$

Q3.  $y \propto \frac{1}{x^2}$ ,  $y = 4$  when  $x = 3$ . Find x when  $y = 9$

Sol: Given  $y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2} \dots\dots(1)$

$y = 4$  when  $x = 3$  put in (1)

$$4 = \frac{k}{3^2}$$

$$k = 4 \times 9 = 36$$

Find  $x$  when  $y = 9$  using  $k = 36$  in (1)

$$9 = \frac{36}{x^2}$$

$$9x^2 = 36$$

$$x^2 = \frac{36}{9}$$

$$x^2 = 4$$

$$\sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm 2$$

Q4. Pressure of gas in the closed vessel varies directly with the temperature. If pressure is 150 unit the temperature is 70 units. What will be the pressure if temperature is 140 units.

Solution: Assume that Pressure of a gas =  $P$

Temperature of a gas =  $T$

According to condition

$$T \propto P \quad T = kP \dots (1)$$

When  $P = 150$  unit and  $T = 70$  unit put in (1)

$$70 = k(150)$$

$$\frac{70}{150} = k$$

$$\Rightarrow k = \frac{7}{15}$$

Find  $P$  when  $T = 140$  unit using  $k = \frac{7}{15}$  in (1)

$$140 = \frac{7}{15}P$$

$$140 \times \frac{15}{7} = P$$

$$20 \times 15 = P$$

$$\Rightarrow P = 300 \text{ unit}$$

Q5. In a electric circuit, current varies inversely as resistance. When current is 44 amp, the resistance is 30 ohm. How much current will flow if resistance becomes 22 ohm.

Sol: Let Electric current =  $I$

And Resistance =  $R$

According to condition

$$R \propto \frac{1}{I}, \quad R = \frac{k}{I} \dots (1)$$

When  $I = 44$  amp,  $R = 30$  ohm put in (1)

$$30 = \frac{k}{44}$$

$$30 \times 44 = k$$

$$\Rightarrow k = 1320$$

Find  $I$  when  $R = 22$  ohm using  $k = 1320$  in (1)

$$22 = \frac{1320}{I}$$

$$22I = 1320$$

$$I = \frac{1320}{22}$$

$$I = 60 \text{ amp}$$

Q6. If  $a$  varies jointly as  $b$  and square root of  $c$ . If  $a = 21$  when  $b = 5$  &  $c = 36$ , Find  $a$  when  $b = 9$  &  $c = 225$

Sol: If  $a$  varies jointly as  $b$  and square root of  $c$ .

So,  $a \propto b\sqrt{c}$

$$\text{Or } a = kb\sqrt{c}$$

If  $a = 21$  when  $b = 5$  &  $c = 36$ ,

$$21 = k(5)\sqrt{36}$$

$$\frac{21}{(5)(6)} = k$$

$$\Rightarrow k = \frac{7}{10}$$

Find  $a$  when  $b = 9$  &  $c = 225$  using  $k = 7/2160$

$$a = \frac{7}{10}(9)\sqrt{225}$$

$$a = \frac{945}{10} = 94.5$$

Q7. What number should be added to each of number 3, 8, 11 and 20 to make them in proportion?

Solution: Suppose the number =  $x$

According to condition

$$3 + x : 8 + x = 11 + x : 20 + x$$

Product of mean = Product of extreme

$$(8 + x)(11 + x) = (3 + x)(20 + x)$$

$$88 + 8x + 11x + x^2 = 60 + 3x + 20x + x^2$$

$$88 + 19x = 60 + 23x$$

$$88 - 60 = 23x - 19x$$

$$28 = 4x$$

$$\Rightarrow x = 7$$

Thus the number = 7

And the proportion  $10 : 15 = 18 : 27$

Q8. What number should be subtracted to each of the number 6, 8, 7 and 11 so that the remaining number are in proportion?

Solution: suppose the number =  $x$

According to condition

$$6 - x : 8 - x = 7 - x : 11 - x$$

Product of mean = Product of extreme

$$(8 - x)(7 - x) = (6 - x)(11 - x)$$

$$56 - 8x - 7x + x^2 = 66 - 6x - 11x + x^2$$

$$56 - 15x = 66 - 17x$$

$$17x - 15x = 66 - 56$$

$$2x = 10$$

$$\Rightarrow x = 5$$

Thus the number = 5

And the proportion  $1 : 3 = 2 : 6$

Q9. The ratio between two numbers is 8 : 3 and their difference is 20. Find the numbers.

Solution: difference of their ratio =  $8 - 3 = 5$

$$\text{First number} = \frac{8}{5} \times 20$$

$$= 8 \times 4$$

$$= 32$$

$$\text{Second number} = \frac{3}{5} \times 20$$

$$= 3 \times 4$$

$$= 12$$

Q10. Find the numbers in continued proportion such that their sum is 14 and sum of their squares is 84.

Sol: Let 3 numbers are in continued proportion



First number =  $x$       Second number =  $y$

Third number =  $z$

So continued proportion

$$x : y = y : z$$

$$\Rightarrow y^2 = xz \dots (1)$$

According to conditions

$$\text{Sum} = 14 \quad \text{sum of squares} = 84$$

$$x + y + z = 14 \quad x^2 + y^2 + z^2 = 84$$

$$x + z = 14 - y \dots (2) \quad x^2 + z^2 = 84 - y^2 \dots (3)$$

$$\text{Squaring } (x + z)^2 = (14 - y)^2$$

$$x^2 + z^2 + 2xz = 14^2 + y^2 - 2(14)y$$

Using equation (1) and (3) we get

$$84 - y^2 + 2y^2 = 196 + y^2 - 28y \quad \therefore y^2 = xz$$

$$84 + y^2 = 196 + y^2 - 28y \quad \therefore x^2 + z^2 = 84 - y^2$$

$$28y = 196 - 84 + y^2 - y^2$$

$$28y = 112$$

$$y = \frac{112}{28}$$

$$y = 4$$

Put  $y = 4$  in equation (1)

$$xz = 4^2$$

$$xz = 16$$

$$z = \frac{16}{x} \dots (4)$$

Put  $y = 4$  in equations (2) and (3) we get

$$x + z = 14 - 4 \quad x^2 + z^2 = 84 - 4^2$$

$$x + z = 10 \quad x^2 + z^2 = 84 - 16$$

$$z = 10 - x \dots (5) \quad x^2 + z^2 = 68 \dots (6)$$

Put the value of  $z = 10 - x$  in equation (6)

$$x^2 + (10 - x)^2 = 68$$

$$x^2 + 10^2 + x^2 - 20x = 68$$

$$2x^2 - 20x + 100 - 68 = 0$$

$$2x^2 - 20x + 32 = 0$$

$$2(x^2 - 10x + 16) = 0$$

$$x^2 - 8x - 2x + 16 = 0$$

$$x(x - 8) - 2(x - 8) = 0$$

$$(x - 2)(x - 8) = 0$$

Either

or

$$x - 2 = 0$$

$$x - 8 = 0$$

$$x = 2$$

$$x = 8$$

put in equation (4)

$$z = \frac{16}{2}$$

$$z = \frac{16}{8}$$

$$z = 8$$

$$z = 2$$

Therefore the three numbers are

$x$	$y$	$z$
either 2	4	8
or 8	4	2

Q11. The mean proportion of two number is 6 and their sum is 13. Find the number

Sol: Given two numbers are

First Number =  $x$

Second Number =  $y$

And mean proportion = 6

then  $x, 6, y$  are continued proportion so

$$x : 6 = 6 : y$$

$$xy = 36$$

$$y = \frac{36}{x} \dots (1)$$

According to condition

$$x + y = 13$$

$$x + \frac{36}{x} = 13 \quad \therefore y = \frac{36}{x}$$

$$x \cdot x + \frac{36}{x} \cdot x = 13 \cdot x$$

$$x^2 + 36 = 13x$$

$$x^2 - 13x + 36 = 0$$

$$x^2 - 9x - 4x + 36 = 0$$

$$x(x - 9) - 4(x - 9) = 0$$

$$(x - 4)(x - 9) = 0$$

Either

or

$$x - 4 = 0$$

$$x - 9 = 0$$

$$x = 4$$

$$x = 9$$

Put in equation (1)

$$y = \frac{36}{4}$$

$$y = \frac{36}{9}$$

$$y = 9$$

$$y = 4$$

So first number = 4

Or First number = 9

Second number = 9

Second number = 4

Q12. Find angle of a triangle which are in ratio 3 : 4 : 5

Solution: Since triangle has three angles

Suppose that First angle =  $x$

Second angle =  $y$

Third angle =  $z$

Sum of angles = 180

$$x + y + z = 180 \dots (1)$$

according to condition

$$x : y : z = 3 : 4 : 5$$

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$$

Then each of its fraction equal to  $k$

$$\frac{x}{3} = k,$$

$$\frac{y}{4} = k,$$

$$\frac{z}{5} = k$$

$$x = 3k$$

$$y = 4k$$

$$z = 5k \dots (2)$$

Now adding and using equation (1)

$$x + y + z = 3k + 4k + 5k$$

$$180 = 12k$$

$$\Rightarrow k = \frac{180}{12}$$

$$\Rightarrow k = 15$$

Putting the value of  $k$  in equation (2)

$$x = 3(15), \quad y = 4(15), \quad z = 5(15)$$

$$x = 45,$$

$$y = 60,$$

$$z = 75$$

Q13. If  $\frac{a}{b} = \frac{c}{d}$  then prove that

$$\frac{ac(a+c)}{bd(b+d)} = \frac{(a+c)^3}{(b+d)^3}$$

$$\text{Sol: Given } \frac{a}{b} = \frac{c}{d} = k \dots (1)$$

Then each of its fraction equal to  $k$

$$\frac{a}{b} = k$$

$$\frac{c}{d} = k$$

$$a = bk$$

$$c = dk$$

Take LHS and putting the value of  $a$  and  $c$

$$\frac{ac(a+c)}{bd(b+d)} = \frac{(bk)(dk)(bk+dk)}{bd(b+d)}$$

$$\frac{ac(a+c)}{bd(b+d)} = \frac{k^2 bd(bk+dk)}{bd(b+d)}$$

$$\frac{ac(a+c)}{bd(b+d)} = \frac{k^2 \cdot k(b+d)}{(b+d)}$$

$$\frac{ac(a+c)}{bd(b+d)} = k^3 \dots (2)$$

Now Take RHS and putting the value of a & c

$$\frac{(a+c)^3}{(b+d)^3} = \frac{(bk+dk)^3}{(b+d)^3}$$

$$\frac{(a+c)^3}{(b+d)^3} = \frac{\{k(b+d)\}^3}{(b+d)^3}$$

$$\frac{(a+c)^3}{(b+d)^3} = \frac{k^3(b+d)^3}{(b+d)^3}$$

$$\frac{(a+c)^3}{(b+d)^3} = k^3 \dots (3)$$

From equations (2) and (3) we get

$$\frac{ac(a+c)}{bd(b+d)} = \frac{(a+c)^3}{(b+d)^3}$$

Q14. If a, b, c are in continued proportion then

prove that  $\frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a^2-b^2}{b^2-c^2}$

Solution: Since a,b,c are in continued

proportion so  $a:b=b:c$   
 $\frac{a}{b} = \frac{b}{c} = k \dots (1)$

Then each of its fraction equal to k

$$\frac{a}{b} = k \quad \frac{b}{c} = k$$

$$a = bk \quad b = ck$$

Take

$$\frac{a}{c} = \frac{bk}{c} \quad \therefore a = bk$$

$$\frac{a}{c} = \frac{(ck)k}{c} \quad \therefore b = ck$$

$$\frac{a}{c} = k^2 \dots (2)$$

Now take

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{(bk)^2 + (bk)(ck) + b^2}{(ck)^2 + (ck)c + c^2}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{b^2k^2 + bck^2 + (ck)^2}{c^2k^2 + c^2k + c^2}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{(ck)^2 k^2 + (ck)ck^2 + c^2k^2}{c^2k^2 + c^2k + c^2}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{c^2k^2k^2 + c^2k^3 + c^2k^2}{c^2k^2 + c^2k + c^2}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{c^2k^4 + c^2k^3 + c^2k^2}{c^2k^2 + c^2k + c^2}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{c^2(k^4 + k^3 + k^2)}{c^2(k^2 + k + 1)}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{k^2(k^2 + k + 1)}{(k^2 + k + 1)}$$

$$\frac{a^2+ab+b^2}{b^2+bc+c^2} = k^2 \dots (3)$$

Now Take

$$\frac{a^2-b^2}{b^2-c^2} = \frac{(bk)^2 - (ck)^2}{(ck)^2 - c^2}$$

$$\frac{a^2-b^2}{b^2-c^2} = \frac{b^2k^2 - c^2k^2}{c^2k^2 - c^2}$$

$$\frac{a^2-b^2}{b^2-c^2} = \frac{(ck)^2 k^2 - c^2k^2}{c^2k^2 - c^2}$$

$$\frac{a^2-b^2}{b^2-c^2} = \frac{c^2k^2k^2 - c^2k^2}{c^2k^2 - c^2}$$

$$\frac{a^2-b^2}{b^2-c^2} = \frac{c^2k^2(k^2 - 1)}{c^2(k^2 - 1)}$$

$$\frac{a^2-b^2}{b^2-c^2} = k^2 \dots (4)$$

From equations (2), (3) and (4) we get

$$\frac{a}{c} = \frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a^2-b^2}{b^2-c^2}$$

Q15. If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$  then prove that

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$$

Sol: Given  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \dots (1)$

Then each of these fraction equal to k

$$\text{i.e., } \frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$$

or  $a = bk, \quad c = dk, \quad e = fk$

Take LHS and putting the value of a,c and e

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{(bk)^3 + (dk)^3 + (fk)^3}{b^3 + d^3 + f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{b^3k^3 + d^3k^3 + f^3k^3}{b^3 + d^3 + f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{k^3(b^3 + d^3 + f^3)}{b^3 + d^3 + f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = k^3 \dots (2)$$

Now Take RHS and putting the value of a,c & e

$$\frac{ace}{bdf} = \frac{(bk)(dk)(fk)}{bdf}$$

$$\frac{ace}{bdf} = \frac{bdfk^3}{bdf}$$

$$\frac{ace}{bdf} = k^3 \dots (3)$$

From equations (2) and (3) we get

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$$