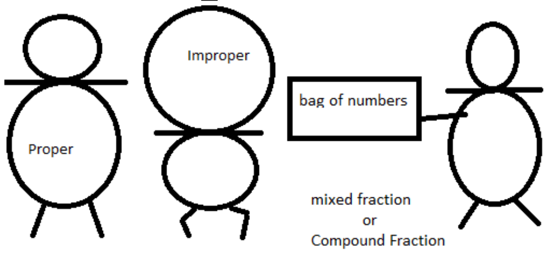
Chapter 4



Identity: An equation which is true for all values of the variable, for example $\sin^2\theta+\cos^2\theta=1$ is trigonometry identity & $5x+3=3x+1+2x+2$ is Algebraic Identity.

Improper Fraction:

A rational fraction $\dfrac{p(x)}{q(x)}$ is said to be Improper fraction if degree of $p(x)\geq$ the degree of $q(x)$ then it is called improper fraction.

Proper Fraction: A rational fraction $\dfrac{p(x)}{q(x)}$ is said to be proper fraction if degree of $p(x)<$ degree of $q(x)$ then it is called proper fraction.

Division of Polynomial:

$$Divisor\overset{\displaystyle Quotient}{\overline{\left)\begin{array}{c} Divident \\ \underline{Divisor\times Quotient} \\ Remainder\end{array}\right.}}$$

its General Formula

$$Divident=(Divisor)(Quotient)+Remainder$$

This result is known as "**Division Algorithm**"

Converting improper into sum of polynomial and proper fraction

$$\frac{20}{3}=6\frac{2}{3}=6+\frac{2}{3}$$

$$3\overset{\displaystyle 6}{\overline{\left)\begin{array}{c} 20 \\ \underline{18} \\ 2\end{array}\right.}}$$

Case 1:When Q(x) has non repeated linear factors

$$\frac{4x+5}{(x+a)(x+b)}=\frac{A}{x+a}+\frac{B}{x+b}$$

Exp1:Resolve $\dfrac{1}{(x+1)(x+2)}$ into partial fraction

Sol: Given $\dfrac{1}{(x+1)(x+2)}=\dfrac{A}{x+1}+\dfrac{B}{x+2}$ ….(1)

Multiply both sides by $(x+1)(x+2)$ we get

$1=A(x+2)+B(x+1)$ ………………….(2)

Put $x+1=0$ or $x=-1$ in eq (2)

$1=A(-1+2)+B(-1+1)$

$\Rightarrow 1=A$

Put $x+2=0$ or $x=-2$ in eq (2)

$1=A(-2+2)+B(-2+1)$

$1=0-B$

$\Rightarrow B=-1$

Now putting these values of A and B in eq (1)

$$\frac{1}{(x+1)(x+2)}=\frac{1}{x+1}+\frac{-1}{x+2}$$

Or $\dfrac{1}{(x+1)(x+2)}=\dfrac{1}{x+1}-\dfrac{1}{x+2}$

Exp2: Find the partial fraction of $\dfrac{3x+2}{x^2-x-2}$

Sol: Since x^2-x-2

$=x^2-2x+1x-2$

$=x(x-2)+1(x-2)$

$=(x+1)(x-2)$ Therefore,

$\dfrac{3x+2}{x^2-x-2}=\dfrac{3x+2}{(x+1)(x-2)}=\dfrac{A}{x+1}+\dfrac{B}{x-2}$ ….(1)

Multiply each term by $(x+1)(x-2)$ we get

$3x+2=A(x-2)+B(x+1)$ …………(2)

Put $x+1=0$ or $x=-1$ in eq (2)

$3(-1)+2=A(-1-2)+B(-1+1)$

$-3+2=A(-3)+0$

$-1=-3A$

$\Rightarrow A=\dfrac{1}{3}$

Put $x-2=0$ or $x=2$ in eq (2)

$3(2)+2=A(2-2)+B(2+1)$

$6+2=0+3B$

$\Rightarrow B=\dfrac{8}{3}$

Now putting these values of A and B in eq (1)

$$\frac{3x+2}{x^2-x-2}=\frac{1}{3(x+1)}+\frac{8}{3(x-2)}$$

Case 2: When Q(x) has repeated linear factors.

$$\frac{x}{(x+a)^2}=\frac{A}{x+a}+\frac{B}{(x+a)^2}$$

Exp3: Find the partial fractions of $\dfrac{x}{(x+1)^2}$

Sol: Given $\dfrac{x}{(x+1)^2}$ has linear repeated factors

$\dfrac{x}{(x+1)^2}=\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$ …………..(1)

Multiply each term by $(x+1)^2$

$x=A(x+1)+B$ ………………(2)

Put $x+1=0$ or $x=-1$ in eq (2)

$-1=A(-1+1)+B$

$-1=0+B$

$\Rightarrow B=-1$

Choose $x=0$ in eq (2) & using $B=-1$

$0=A(0+1)+(-1)$

$0=A-1$

$\Rightarrow 1=A$

Now putting these values of A and B in eq (1)

$$\frac{x}{(x+1)^2}=\frac{1}{x+1}+\frac{-1}{(x+1)^2}$$

Exp4: Find partial fractions of $\dfrac{2x^2+1}{(x-2)^2(x+3)}$

Sol: $\dfrac{2x^2+1}{(x-2)^2(x+3)}$ has linear repeated factors

$$\frac{2x^2+1}{(x-2)^2(x+3)}=\frac{A}{x+3}+\frac{B}{x-2}+\frac{C}{(x-2)^2} \quad .....(1)$$

Multiply each term by $(x-2)^2(x+3)$

$$2x^2+1=A(x-2)^2+B(x+3)(x-2)+C(x+3)$$

Put $x+3=0$ or $x=-3$ in eq (2)

$$2(-3)^2+1=A(-3-2)^2+B(-3+3)(-3-2)+C(-3+3)$$

$$2(9)+1=A(-5)^2+0+0$$

$$19=25A \qquad \Rightarrow A=\frac{19}{25}$$

Put $x-2=0$ or $x=2$ in eq (2)

$$2(2)^2+1=A(2-2)^2+B(2+3)(2-2)+C(2+3)$$

$$2(4)+1=0+0+C(5)$$

$$9=5C \qquad \Rightarrow C=\frac{9}{5}$$

Choose $x=0$ in eq (2) & using $B=-1$

$$2(0)^2+1=A(0-2)^2+B(0+3)(0-2)+C(0+3)$$

$$0+1=A(-2)^2+B(-6)+C(3) \text{ using } A=\tfrac{19}{25}, C=\tfrac{9}{5}$$

$$1=\frac{19}{25}(-2)^2-6B+\frac{9}{5}(3)$$

$$1=\frac{76}{25}-6B+\frac{27}{5}$$

$$6B=\frac{76}{25}+\frac{27}{5}\times\frac{5}{5}-\frac{1}{1}\times\frac{25}{25}$$

$$6B=\frac{76+135-25}{25}$$

$$6B=\frac{186}{25} \qquad \Rightarrow B=\frac{31}{25}$$

Now putting these values of A,B and C in eq (1)

$$\frac{2x^2+1}{(x-2)^2(x+3)}=\frac{19}{25(x+3)}+\frac{31}{25(x-2)}+\frac{9}{5(x-2)^2}$$

Exercise 4.1

Q1. $\dfrac{3x-2}{2x^2-x}$

Sol: $\therefore$ fraction is proper and have linear factor

$\dfrac{3x-2}{2x^2-x}=\dfrac{3x-2}{x(2x-1)}$ With two linear factors

$$\frac{3x-2}{x(2x-1)}=\frac{A}{x}+\frac{B}{2x-1}\cdots(1)$$

Multiply equation (1) by $x(2x-1)$ we get

$$3x-2=A(2x-1)+Bx\cdots(2)$$

Put x = 1/2 in equation (2) we get

$$3\left(\frac{1}{2}\right)-2=A\left(2\left(\frac{1}{2}\right)-1\right)+B\left(\frac{1}{2}\right)$$

$$\frac{3}{2}-2=A(1-1)+\frac{B}{2}$$

$$\frac{3-4}{2}=A(0)+\frac{B}{2}$$

$$\frac{-1}{2}=0+\frac{B}{2} \qquad \Rightarrow B=-1$$

Put x = 0 in equation (2) we get

$$3(0)-2=A[2(0)-1]+B(0)$$

$$-2=-A+0 \qquad \Rightarrow A=2$$

Putting the values of A and B in eq (1)

$$\frac{3x-2}{x(2x-1)}=\frac{2}{x}-\frac{1}{2x-1}$$

Q2. $\dfrac{x-1}{x^2+6x+5}$

Sol: $\dfrac{x-1}{x^2+6x+5}=\dfrac{x-1}{x^2+5x+1x+5}$

$$\frac{x-1}{x^2+6x+5}=\frac{x-1}{x(x+5)+1(x+5)}$$

$$\frac{x-1}{x^2+6x+5}=\frac{x-1}{(x+1)(x+5)}$$

With two linear factors

$$\frac{x-1}{(x+1)(x+5)}=\frac{A}{x+1}+\frac{B}{x+5}\cdots(1)$$

Multiply equation (1) by $(x+1)(x+5)$ we get

$$x-1=A(x+5)+B(x+1) \;............(2)$$

Put x = - 5 in equation (2) we get

$$-5-1=A(-5+5)+B(-5+1)$$

$$-6=A(0)+B(-4)$$

$$-6=0-4B$$

$$\Rightarrow B=\frac{-6}{-4} \qquad \Rightarrow B=\frac{3}{2}$$

Put x = - 1 in equation (2) we get

$$-1-1=A(-1+5)+B(-1+1)$$

$$-2=A(4)+B(0)$$

$$-2=4A+0$$

$$\Rightarrow A=\frac{-2}{4} \qquad \Rightarrow A=\frac{-1}{2}$$

Putting the values of A and B in eq (1)

$$\frac{x-1}{(x+1)(x+5)}=\frac{-1}{2(x+1)}+\frac{3}{2(x+5)}$$

Q3:Resolve $\dfrac{1}{x^2-1}$ into partial fraction

Sol: $\dfrac{1}{x^2-1}=\dfrac{1}{(x-1)(x+1)}=\dfrac{A}{x-1}+\dfrac{B}{x+1}$ ....(1)

Multiply both sides by $(x-1)(x+1)$ we get

$$1=A(x+1)+B(x-1) \;.....................(2)$$

Put $x+1=0$ or $x=-1$ in eq (2)

$1=A(-1+1)+B(-1-1)$

$1=-2B \quad \Rightarrow B=\frac{-1}{2}$

Put $x-1=0$ or $x=1$ in eq (2)

$1=A(1+1)+B(1-1)$

$1=2A+0 \quad \Rightarrow A=\frac{1}{2}$

Now putting these values of A and B in eq (1)

$$\frac{1}{x^2-1}=\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$$

Q4: $\frac{x}{x^2+4x-5}$

Sol: $\frac{x}{x^2+4x-5}$

Since $x^2+4x-5=x^2+5x-1x-5$

$=x(x+5)-1(x+5)$

$=(x-1)(x+5)$

Therefore $\frac{x}{(x-1)(x+5)}=\frac{A}{x-1}+\frac{B}{x+5}$ .....(1)

Multiply each term by $(x-1)(x+5)$

$x=A(x+5)+B(x-1)$ ....(2)

Put $x-1=0$ or $x=1$ in eq (2)

$1=A(1+5)+B(1-1)$

$1=6A \quad \Rightarrow A=\frac{1}{6}$

Put $x+5=0$ or $x=-5$ in eq (2)

$-5=A(-5+5)+B(-5-1)$

$-5=0-6B \quad \Rightarrow B=\frac{5}{6}$

Putting the values of A and B in eq (1)

$$\frac{x}{(x-1)(x+5)}=\frac{1}{6(x-1)}+\frac{5}{6(x+5)}$$

Q5. $\frac{4x+2}{(x+2)(2x-1)}$

Sol: $\frac{4x+2}{(x+2)(2x-1)}=\frac{A}{x+2}+\frac{B}{2x-1}$ ......(1)

Multiply each term by $(x+2)(2x-1)$

$4x+2=A(2x-1)+B(x+2)$ ....(2)

Put $2x-1=0$ or $x=\frac{1}{2}$ in eq (2)

$4\left(\frac{1}{2}\right)+2=A\left(2\left(\frac{1}{2}\right)-1\right)+B\left(\frac{1}{2}+2\right)$

$2+2=0+\frac{5}{2}B$

$4=\frac{5}{2}B \quad \Rightarrow B=\frac{8}{5}$

Put $x+2=0$ or $x=-2$ in eq (2)

$4(-2)+2=A(2(-2)-1)+B(-2+2)$

$-8+2=A(-4-1)+0$

$-6=-5A \quad \Rightarrow A=\frac{6}{5}$

Now putting these values of A and B in eq (1)

$$\frac{4x+2}{(x+2)(2x-1)}=\frac{6}{5(x+2)}+\frac{8}{5(2x-1)}$$

Q6. $\frac{x^2+5x+3}{(x^2-1)(x+1)}$

Sol: Given fraction is proper

$$\frac{x^2+5x+3}{(x^2-1^2)(x+1)}=\frac{x^2+5x+3}{(x-1)(x+1)(x+1)}$$

$$\frac{x^2+5x+3}{(x^2-1^2)(x+1)}=\frac{x^2+5x+3}{(x-1)(x+1)^2}$$

with repeating linear factors

$$\frac{x^2+5x+3}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\cdots(1)$$

Multiply equation (1) by $(x-1)(x+1)^2$ we get

$$\frac{x^2+5x+3}{(x-1)(x+1)^2}(x-1)(x+1)^2=\frac{A}{x-1}(x-1)(x+1)^2$$

$$+\frac{B}{x+1}(x-1)(x+1)^2+\frac{C}{(x+1)^2}(x-1)(x+1)^2$$

$x^2+5x+3=A(x+1)^2+B(x+1)(x-1)+C(x-1)\cdots(2)$

Put x = 1 in equation (2) we get

$1^2+5(1)+3=A(1+1)^2+B(1+1)(1-1)+C(1-1)$

$1+5+3=A(2)^2+0+0$

$9=A(4) \quad \Rightarrow A=\frac{9}{4}$

Put x = - 1 in equation (2) we get

$(-1)^2+5(-1)+3=A(-1+1)^2+B(-1+1)(-1-1)+C(-1-1)$

$1-5+3=A(0)+B(0)+C(-2)$

$-1=0+0-2c \quad \Rightarrow C=\frac{1}{2}$

Choose x = 0 and Putting A=9/4& C = 1/2 in eq (2)

$(0)^2+5(0)+3=A(0+1)^2+B(0+1)(0-1)+C(0-1)$

$0+0+3=\frac{9}{4}(1)+B(1)(-1)+\frac{1}{2}(-1)$

$3=\frac{9}{4}-B-\frac{1}{2}$

$B=\frac{9}{4}-\frac{1}{2}-3$

$B=\frac{9-2-12}{4} \quad \Rightarrow B=\frac{-5}{4}$

Putting the values of A,B and C in eq (1)

$$\frac{x^2+5x+3}{(x-1)(x+1)^2}=\frac{9}{4(x-1)}-\frac{5}{4(x+1)}+\frac{1}{2(x+1)^2}$$

Q7. $\frac{x^2+2}{(x+2)(x^2+5x+6)}$

Sol: Take denominator $(x+2)(x^2+5x+6)$

$=(x+2)(x^2+3x+2x+6)$

$=(x+2)[x(x+3)+2(x+3)]$

$=(x+2)(x+2)(x+3)$

with repeating linear factors

$$\frac{x^2+2}{(x+2)^2(x+3)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+3}\cdots(1)$$

Multiply equation (1) by $(x+2)^2(x+3)$ we get

$x^2+2=A(x+2)(x+3)+B(x+3)+C(x+2)^2$.(2)

Put x = - 2 in equation (2) we get

$(-2)^2+2=A(-2+2)(-2+3)+B(-2+3)+C(-2+2)^2$

$4+2=A(0)(1)+B(1)+C(0)^2$

$6=0+B+0$

$\Rightarrow B=6$

Put x = - 3 in equation (2) we get

$(-3)^2+2=A(-3+2)(-3+3)+B(-3+3)+C(-3+2)^2$

$9+2=A(-1)(0)+B(0)+C(-1)^2$

$11=0+0+C$

$\Rightarrow C=11$

Choose x=0 & Putting B=6 & C=11 in eq (2) we get

$0^2+2=A(0+2)(0+3)+6(0+3)+11(0+2)^2$

$0+2=A(2)(3)+6(3)+11(4)$

$2=6A+18+44$

$-6A=18+44-2$

$-6A=60$ $\qquad A=-10$

Putting the values of A,B and C in eq (1)

$$\frac{x^2+2}{(x+2)(x^2+5x+6)}=\frac{-10}{x+2}+\frac{6}{(x+2)^2}+\frac{11}{x+3}$$

Q8. $\dfrac{2x-1}{x(x-3)^2}$

Sol: fraction is proper with repeating linear factors

$$\frac{2x-1}{x(x-3)^2}=\frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}\cdots(1)$$

Multiply equation (1) by $x(x-3)^2$ we get

$$\frac{2x-1}{x(x-3)^2}x(x-3)^2=\frac{A}{x}x(x-3)^2$$
$$+\frac{B}{x-3}x(x-3)^2+\frac{C}{(x-3)^2}x(x-3)^2$$

$2x-1=A(x-3)^2+Bx(x-3)+Cx\cdots(2)$

Put x = 0 in equation (2) we get

$2(0)-1=A(0-3)^2+B(0)(0-3)+C(0)$

$0-1=A(9)+0+0$

$-1=9A$ $\qquad \Rightarrow A=\dfrac{-1}{9}$

Put x = 3 in equation (2) we get

$2(3)-1=A(3-3)^2+B(3)(3-3)+C(3)$

$6-1=A(0)+B(3)(0)+3C$

$5=0+0+3C$ $\qquad \Rightarrow C=\dfrac{5}{3}$

Choose x=1 and Put A=-1/9 & C=5/3 in eq (2)

$2(1)-1=\dfrac{-1}{9}(1-3)^2+B(1)(1-3)+\dfrac{5}{3}(1)$

$2-1=\dfrac{-1}{9}(-2)^2+B(1)(-2)+\dfrac{5}{3}$

$1=\dfrac{-1}{9}(4)-2B+\dfrac{5}{3}$

$2B=\dfrac{-4}{9}+\dfrac{5}{3}-1$

$2B=\dfrac{-4+15-9}{9}=\dfrac{2}{9}$ $\qquad \Rightarrow B=\dfrac{1}{9}$

Putting the values of A,B and C in eq (1)

$$\frac{2x-1}{x(x-3)^2}=\frac{-1}{9x}+\frac{1}{9(x-3)}+\frac{5}{3(x-3)^2}$$

Q9. $\dfrac{x^2}{x^2+2x+1}$

Sol: Given $\dfrac{x^2}{x^2+2x+1}$ is improper

$$\begin{array}{r|l} & \quad 1 \\ \hline x^2+2x+1 & x^2 \\ & \pm x^2\pm 2x\pm 1 \\ \hline & \quad -2x-1 \end{array}$$

So, $\dfrac{x^2}{x^2+2(x)(1)+1^2}=1+\dfrac{-2x-1}{(x+1)^2}$ ......(1)

Take $\dfrac{-2x-1}{(x+1)^2}=\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$ ....(2)

Multiply each term by $(x+1)^2$

$-2x-1=A(x+1)+B$ ....(3)

Put $x+1=0$ or $x=-1$ in eq (3)

$-2(-1)-1=A(-1+1)+B$

$2-1=0+B$ $\qquad \Rightarrow 1=B$

Choose $x=0$ in eq (3) & Using B=1

$-2(0)-1=A(0+1)+1$

$-1=A+1$ $\qquad \Rightarrow A=-2$

Putting the values of A and B in eq (2)

$\dfrac{-2x-1}{(x+1)^2}=\dfrac{-2}{x+1}+\dfrac{1}{(x+1)^2}$ Now put in (1)

$$\frac{x^2}{x^2+2(x)(1)+1^2}=1-\frac{2}{x+1}+\frac{1}{(x+1)^2}$$

Q10. $\dfrac{x^2}{(x+1)(x-1)^2}$

Sol: $\dfrac{x^2}{(x+1)(x-1)^2}$ has linear repeated factors

$$\frac{x^2}{(x+1)(x-1)^2}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}\quad .....(1)$$

Multiply each term by $(x-1)^2(x+1)$

$x^2=A(x-1)^2+B(x+1)(x-1)+C(x+1)\ldots(2)$

Put $x+1=0$ or $x=-1$ in eq (2)

$(-1)^2=A(-1-1)^2+B(-1+1)(-1-1)+C(-1+1)$

$1=A(-2)^2+0+0$

$1=4A$ $\qquad \Rightarrow A=\dfrac{1}{4}$

Put $x-1=0$ or $x=1$ in eq (2)

$$(1)^2 = A(1-1)^2 + B(1+1)(1-1) + C(1+1)$$
$$1 = 0 + 0 + 2C$$
$$1 = 2C \qquad \Rightarrow C = \frac{1}{2}$$

Choose $x = 0$ in eq (2) & using A and C

$$(0)^2 = A(0-1)^2 + B(0+1)(0-1) + C(0+1)$$
$$0 = A - B + C$$

$B = A + C$ Using $A = \frac{1}{4}, C = \frac{1}{2}$

$$B = \frac{1}{4} + \frac{1}{2} \times \frac{2}{2} \qquad \Rightarrow B = \frac{3}{4}$$

Now putting these values of A,B and C in eq (1)

$$\frac{x^2}{(x+1)(x-1)^2} = \frac{1}{4(x+1)} + \frac{3}{4(x-1)} + \frac{1}{2(x-1)^2}$$

$\boxed{\text{Case 3:}}$ When Q(x) has non repeated irreducible quadratic factors.

$$\frac{Bx+C}{(ax^2+bx+c)(dx^2+ex+f)} = \frac{Ax+B}{(ax^2+bx+c)} + \frac{Cx+D}{(dx^2+ex+f)}$$

Exp5: $\dfrac{1}{(x+1)(x^2+2)}$

Sol: $\dfrac{1}{(x+1)(x^2+2)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+2}$ ....(1)

Multiply each fraction by $(x+1)(x^2+2)$

$1 = A(x^2+2) + (Bx+C)(x+1)$ ....(2)

Put $x+1=0$ or $x=-1$ in eq (2)

$$1 = A((-1)^2+2) + (B(-1)+C)(-1+1)$$
$$1 = A(1+2) + (-B+C)(0)$$
$$1 = 3A \qquad \Rightarrow A = \frac{1}{3}$$

Again from eq (2)

$1 = Ax^2 + 2A + Bx^2 + Bx + Cx + C$ Rearranging

$$1 = Ax^2 + Bx^2 + Bx + Cx + 2A + C$$
$$1 = (A+B)x^2 + (B+C)x + 2A + C$$

Comparing the coefficients

Coefficients of x^2

$$A + B = 0$$
$$B = -A$$
$$B = -\frac{1}{3}$$

Coefficients of x

$$B + C = 0$$
$$C = -B$$
$$C = -\left(-\tfrac{1}{3}\right)$$
$$C = \frac{1}{3}$$

Putting the values of A,B and C in eq (1)

$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} - \frac{x-1}{3(x^2+2)}$$

Exp6: $\dfrac{4x^2-28}{x^4-x^2-6}$

Sol: Take $x^4 - x^2 - 6 = x^4 - 3x^2 + 2x^2 - 6$

$$= x^2(x^2-3) + 2(x^2-3)$$
$$= (x^2-3)(x^2+2)$$

So $\dfrac{4x^2-28}{x^4-x^2-6} = \dfrac{4x^2-28}{(x^2-3)(x^2+2)}$

$\dfrac{4x^2-28}{(x^2-3)(x^2+2)} = \dfrac{Ax+B}{x^2-3} + \dfrac{Cx+D}{x^2+2}$ .....(1)

Multiply each fraction by $(x^2-3)(x^2+2)$

$$4x^2 - 28 = (Ax+B)(x^2+2) + (Cx+D)(x^2-3)$$
$$4x^2 - 28 = Ax^3 + 2Ax + Bx^2 + 2B + Cx^3 - 3Cx + Dx^2 - 3D$$

Rearranging the terms according to power of x

$$4x^2 - 28 = Ax^3 + Cx^3 + Bx^2 + Dx^2 + 2Ax - 3Cx + 2B - 3D$$
$$4x^2 - 28 = (A+C)x^3 + (B+D)x^2 + (2A-3C)x + 2B - 3D$$

Comparing the coefficients

Coefficients of x^3

$A + C = 0$

$C = -A$........(i)

coefficients of x^2

$B + D = 4$

$D = 4 - B$...... .....(ii)

Coefficients of x

$2A - 3C = 0$ ......(iii)

Putting value of C

$$2A - 3(-A) = 0$$
$$2A + 3A = 0$$
$$5A = 0$$
$$\Rightarrow A = 0$$

Constant

$2B - 3D = -28$ ..... (iv)

Putting value of D

$$2B - 3(4-B) = 28$$
$$2B - 12 + 3B = 28$$
$$5B = 12 + 28$$
$$\Rightarrow B = 8$$

Put B=8 in eq (ii)

$$D = 4 - 8$$
$$D = -4$$

Putting the values of A,B,C and D in eq (1)

$$\frac{4x^2-28}{(x^2-3)(x^2+2)} = \frac{0x+8}{x^2-3} + \frac{0x+(-4)}{x^2+2}$$
$$\frac{4x^2-28}{(x^2-3)(x^2+2)} = \frac{8}{x^2-3} - \frac{4}{x^2+2}$$

Exp7: $\dfrac{1}{(x-1)(x^2+1)^2}$

Sol: $\dfrac{1}{(x-1)(x^2+1)^2} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$ ..(1)

Multiply each fractions by $(x-1)(x^2+1)^2$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$
$$1 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1)$$

Put $x-1=0$ or $x=1$ in eq (2)

$$1 = A(1^2+1)^2 + (B.1+C)(0)(1+1) + (D+E)(0)$$
$$1 = A(2)^2 + 0 + 0 \qquad \Rightarrow A = \frac{1}{4}$$

Form eq (2)

$$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x-x^2-1) + (Dx+E)(x-1)$$
$$1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 - Bx^3 - Bx + Cx^3 + Cx - Cx^2 - C + Dx^2 - Dx + Ex - E$$

Rearranging according to powers of x

$$1 = Ax^4 + Bx^4 - Bx^3 + Cx^3 + 2Ax^2 + Bx^2 - Cx^2 + Dx^2 - Bx + Cx - Dx + Ex + A - C - E$$

$$1 = (A+B)x^4 + (-B+C)x^3 + (2A+B-C+D)x^2 + (-B+C-D+E)x + A - C - E$$

Comparing the coefficients

Coefficients of x^4

$A + B = 0$

$B = -A$

$B = \frac{-1}{4}$

coefficients of x^3

$-B + C = 0$

$C = B$

$C = \frac{-1}{4}$

Constant

$A - C - E = 1$

$\frac{1}{4} - \left(\frac{-1}{4}\right) - E = 1$

$\frac{1}{2} - E = 1$

$E = \frac{1}{2} - 1 = \frac{-1}{2}$

coefficients of x

$-B + C - D + E = 0$

$-\left(\frac{-1}{4}\right) + \left(\frac{-1}{4}\right) + \left(\frac{-1}{2}\right) = D$

$D = \frac{-1}{2}$

Putting the values of A,B,C,D and E

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} - \frac{x+1}{2(x^2+1)^2}$$

Exercise 4.2

Q1. $\frac{1}{x(x^2+1)}$

Sol: Given fraction is proper with linear factor and Quadratic factor

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \cdots (1)$$

Multiply equation (1) by $x(x^2+1)$ we get

$$\frac{1}{x(x^2+1)}x(x^2+1) = \frac{A}{x}x(x^2+1) + \frac{Bx+C}{x^2+1}x(x^2+1)$$

$1 = A(x^2+1) + (Bx+C)x$

$1 = Ax^2 + A + Bx^2 + Cx$

$1 = Ax^2 + Bx^2 + Cx + A$

$1 = (A+B)x^2 + Cx + A \cdots (2)$

Comparing the coefficients of equation (2)

constant terms

1 = A

coefficients of x

C = 0

coefficients of x^2

A + B = 0

B = -A or B = - 1

Putting the values of A,B and C in eq (1)

$$\frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-1x+0}{x^2+1}$$

$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

Q2. $\frac{x^2+3x+1}{(x-1)(x^2+3)}$

Sol: $\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3} \cdots (1)$

Multiply equation (1) by $(x-1)(x^2+3)$ we get

$x^2 + 3x + 1 = A(x^2+3) + (Bx+C)(x-1) \quad \cdots\cdots (2)$

$x^2 + 3x + 1 = Ax^2 + 3A + Bx^2 - Bx + Cx - C$

$x^2 + 3x + 1 = Ax^2 + Bx^2 - Bx + Cx - C + 3A$

$x^2 + 3x + 1 = (A+B)x^2 + (-B+C)x + (-C+3A) \quad \cdots (3)$

Put x = 1 in eq (2) we get

$1^2 + 3(1) + 1 = A\left((1)^2 + 3\right) + \left(B(1) + C\right)(1-1)$

$1 + 3 + 1 = A(1+3) + (B+C)(0)$

$5 = 4A + 0 \quad \Rightarrow A = \frac{5}{4}$

Comparing coefficients of eq (3)

coefficients of x^2

1 = A + B

1 – A = B

$B = 1 - \frac{5}{4}$

$B = \frac{-1}{4}$

coefficients of x

-B + C = 3

C = 3 + B

$C = 3 + \frac{-1}{4}$

$C = \frac{12-1}{4}$

$C = \frac{11}{4}$

Putting the values of A,B and C in eq (1)

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} + \frac{-x+11}{4(x^2+3)}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} - \frac{x-11}{4(x^2+3)}$$

Q3. $\frac{2x+1}{(x-1)(x^2+1)}$

proper with linear factor

Sol: $\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} \cdots (1)$

Multiply each term by $(x-1)(x^2+1)$

$2x + 1 = A(x^2+1) + (Bx+C)(x-1) \cdots (2)$

Put x = 1 in eq (2) we get

$2(1) + 1 = A\left((1)^2 + 1\right) + \left(B(1) + C\right)(1-1)$

$2 + 1 = A(1+1) + (B+C)(0)$

$3 = 2A + 0 \quad \Rightarrow A = \frac{3}{2}$

From eq (2)

$2x + 1 = Ax^2 + A + Bx^2 - Bx + Cx - C$

$2x + 1 = Ax^2 + Bx^2 - Bx + Cx + A - C$

$2x + 1 = (A+B)x^2 + (-B+C)x + A - C \cdots (3)$

Comparing coefficients

coefficients of x^2

$A + B = 0$

$B = -A$

$B = \frac{-3}{2}$

coefficients of x

$-B + C = 2$

$C = 2 + B$

$C = 2 - \frac{3}{2} = \frac{1}{2}$

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{3}{2(x-1)} + \frac{-3x+1}{2(x^2+1)}$$

Q4. $\dfrac{-3}{x^2\left(x^2+5\right)}$

Solution: we have proper fraction with linear repeating factor and Quadratic factor

$$\frac{-3}{x^2\left(x^2+5\right)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+5} \quad \text{............(1)}$$

Multiply equation (1) by $x^2\left(x^2+5\right)$ we get

$$-3=Ax\left(x^2+5\right)+B\left(x^2+5\right)+\left(Cx+D\right)x^2 \quad \text{......(2)}$$

Put $x=0$ in eq (2)

$$-3=A0\left(0^2+5\right)+B\left(0^2+5\right)+\left(C.0+D\right)\left(0\right)^2$$

$$-3=0+5B+0 \qquad \Rightarrow B=\frac{-3}{5}$$

From eq (2)

$$-3=Ax^3+3Ax+Bx^2+3B+Cx^3+Dx^2$$
$$-3=Ax^3+Cx^3+Bx^2+Dx^2+3Ax+3B$$
$$-3=\left(A+C\right)x^3+\left(B+D\right)x^2+3Ax+3B\cdots\left(3\right)$$

Comparing the coefficient of eq (3)

coefficients of x
$$3A=0$$
$$A=0$$

coefficients of x^3
$$A+C=0$$
$$C=0 \quad \therefore A=0$$

coefficients of x^2
$$B+D=0$$
$$D=-B \qquad \Rightarrow D=\frac{3}{5}$$

Putting the values of A,B and C in eq (1)

$$\frac{1}{x^2\left(x^2+3\right)}=\frac{0}{x}+\frac{3}{5x^2}+\frac{0.x+\frac{3}{5}}{x^2+3}$$

$$\frac{1}{x^2\left(x^2+3\right)}=\frac{-3}{5x^2}+\frac{3}{5\left(x^2+3\right)}$$

Q5. $\dfrac{3x-2}{\left(x+4\right)\left(3x^2+1\right)}$

Solution: we have proper fraction with linear factor and Quadratic factor

$$\frac{3x-2}{\left(x+4\right)\left(3x^2+1\right)}=\frac{A}{x+4}+\frac{Bx+C}{3x^2+1} \quad \text{.........(1)}$$

Multiply equation (1) by $\left(x+4\right)\left(3x^2+1\right)$

$$3x-2=A\left(3x^2+1\right)+\left(Bx+C\right)\left(x+4\right) \quad \text{.........(2)}$$

Put $x+4=0$ or $x=-4$

$$3\left(-4\right)-2=A\left(3\left(-4\right)^2+1\right)+\left(B\left(-4\right)+C\right)\left(-4+4\right)$$
$$-12-2=A\left(48+1\right)+\left(-4B+C\right)\left(0\right)$$
$$-14=49A \qquad \Rightarrow A=\frac{-2}{7}$$

$$3x-2=3Ax^2+A+Bx^2+4Bx+Cx+4C$$
$$3x-2=3Ax^2+Bx^2+4Bx+Cx+4C+A$$
$$3x-2=\left(3A+B\right)x^2+\left(4B+C\right)x+\left(4C+A\right)\ldots\left(3\right)$$

Comparing the coefficients of equation (3)

Coefficients of x^2
$$3A+B=0$$
$$B=-3A$$
$$B=-3\left(\frac{-2}{7}\right)=\frac{6}{7}$$

Coefficients of x
$$4B+C=3$$
$$C=3-4B$$
$$C=3-4\left(\frac{6}{7}\right)$$
$$C=\frac{21-24}{7}=\frac{-3}{7}$$

Putting the values of A,B and C in eq (1)

$$\frac{3x-2}{\left(x+4\right)\left(3x^2+1\right)}=\frac{-2}{7\left(x+4\right)}+\frac{6x-3}{7\left(3x^2+1\right)}$$

Q6. $\dfrac{5x}{\left(x+1\right)\left(x^2-2\right)^2}$

Sol: $$\frac{5x}{\left(x+1\right)\left(x^2-2\right)^2}=\frac{A}{x+1}+\frac{Bx+C}{x^2-2}+\frac{Dx+E}{\left(x^2-2\right)^2} \quad (1)$$

Multiply equation (1) by $\left(x+1\right)\left(x^2-2\right)^2$

$$5x=A\left(x^2-2\right)^2+\left(Bx+C\right)\left(x+1\right)\left(x^2-2\right)+\left(Dx+E\right)\left(x+1\right)$$

put $x+1=0$ or $x=-1$ in eq (2)

$$5\left(-1\right)=A\left(\left(-1\right)^2-2\right)^2+\left(B\left(-1\right)+C\right)\left(0\right)\left(\left(-1\right)^2-2\right)+\left(D\left(-1\right)+E\right)\left(0\right)$$
$$-5=A\left(1-2\right)^2+0+0 \qquad \Rightarrow A=-5$$

From eq (2)

$$5x=A\left(x^4-4x^2+4\right)+\left(Bx+C\right)\left(x^3-2x+x^2-2\right)$$
$$+Dx^2+Dx+Ex+E$$
$$5x=Ax^4-4Ax^2+4A+Bx^4-2Bx^2+Bx^3-2Bx$$
$$+Cx^3-2Cx+Cx^2-2C+Dx^2+Dx+Ex+E$$
$$5x=Ax^4+Bx^4+Bx^3+Cx^3-4Ax^2-2Bx^2+Cx^2+Dx^2$$
$$-2Bx-2Cx+Dx+Ex+4A-2C+E$$
$$5x=\left(A+B\right)x^4+\left(B+C\right)x^3+\left(-4A-2B+C+D\right)x^2$$
$$\left(-2B-2C+D+E\right)x+\left(4A-2C+E\right)\cdots\left(3\right)$$

Comparing the coefficients in equation (3)

coefficients of x^4
$$A+B=0$$
$$B=-A=-\left(-5\right)$$
$$B=5$$

coefficients of x^3
$$B+C=0$$
$$C=-B$$
$$C=-5$$

Constant
$$4A-2C+E=0$$
$$4\left(-5\right)-2\left(-5\right)+E=0$$
$$-20+10+E=0$$
$$E-10=0$$
$$E=10$$

coefficients of x^2
$$-4A-2B+C+D=0$$
$$-4\left(-5\right)-2\left(5\right)+\left(-5\right)+D=0$$
$$20-10-5+D=0$$
$$D+5=0$$
$$D=-5$$

Putting the values of A,B,C,D and E in eq (1)

$$\frac{5x}{\left(x+1\right)\left(x^2-2\right)^2}=\frac{-5}{x+1}+\frac{5x-5}{x^2-2}+\frac{-5x+10}{\left(x^2-2\right)^2}$$

Q7. $\dfrac{5x^2-4x+8}{\left(x^2+1\right)^2\left(x-2\right)}$

Sol: we have proper fraction with linear factor and Quadratic repeating factors

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-2}\cdots(1)$$

Multiply equation (1) by $(x^2+1)^2(x-2)$

$$5x^2-4x+8=(Ax+B)(x^2+1)(x-2)+(Cx+D)(x-2)+E(x^2+1)^2\cdots(2)$$

$$5x^2-4x+8=(Ax+B)(x^3-2x^2+x-2)$$
$$+Cx^2-2Cx+Dx-2D+E(x^4+2x^2+1)$$

$$5x^2-4x+8=Ax^4-2Ax^3+Ax^2-2Ax+Bx^3-2Bx^2$$
$$+Bx-2B+Cx^2-2Cx+Dx-2D+Ex^4+2Ex^2+E$$

$$5x^2-4x+8=(A+E)x^4+(-2A+B)x^3+(A-2B+C+2E)x^2$$
$$+(-2A+B-2C+D)x+(-2B-2D+E)\cdots(3)$$

put x = 2 in equation (2) we get

$$5(4)-4(2)+8=(2A+B)(4+1)(0)+(2C+D)(0)+E(4+1)^2$$
$$20-8+8=0+0+E(25)$$
$$20=25E$$
$$E=\frac{20}{25}\qquad\Rightarrow E=\frac{4}{5}$$

Comparing the coefficients in equation (3)

coefficients of x^4

$$A+E=0$$
$$A=-E$$
$$A=\frac{-4}{5}$$

coefficients of x^3

$$-2A+B=0$$
$$B=2A$$
$$B=2\left(\frac{-4}{5}\right)=\frac{-8}{5}$$

Constant

$$-2B-2D+E=8$$
$$-2B+E-8=2D$$
$$-2\left(\frac{-8}{5}\right)+\frac{4}{5}-8=2D$$
$$\frac{16+4-40}{5}=2D$$
$$\frac{-20}{10}=D\qquad\Rightarrow D=-2$$

coefficients of x

$$-2A+B-2C+D=-4$$
$$-2\left(\frac{-4}{5}\right)+\left(\frac{-8}{5}\right)+(-2)+4=2C$$
$$2=2C\qquad\Rightarrow C=1$$

Putting the values of A,B,C,D and E in eq (1)

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{-4x-8}{5(x^2+1)}+\frac{x-2}{(x^2+1)^2}+\frac{4}{5(x-2)}$$

Q8. $\dfrac{4x-5}{(x^2+4)^2}$

Sol: $$\frac{4x-5}{(x^2+4)^2}=\frac{Ax+B}{x^2+4}+\frac{Cx+D}{(x^2+4)^2}\quad\ldots\ldots(1)$$

Multiply each term by $(x^2+4)^2$

$$4x-5=(Ax+B)(x^2+4)+Cx+D$$
$$4x-5=Ax^3+4Ax+Bx^2+4B+Cx+D$$
$$4x-5=Ax^3+Bx^2+(4A+C)x+4B+D$$

Comparing the coefficients

Coefficients of x^3: $A=0$

coefficients of x^2: $B=0$

Coefficients of x

$$4A+C=4$$
$$4(0)+C=4$$
$$C=4$$

Constant terms

$$4B+D=-5$$
$$4(0)+D=-5$$
$$D=-5$$

Putting A,B,C and D in eq (1)

$$\frac{4x-5}{(x^2+4)^2}=\frac{0x+0}{x^2+4}+\frac{4x+(-5)}{(x^2+4)^2}$$
$$\frac{4x-5}{(x^2+4)^2}=\frac{4x-5}{(x^2+4)^2}$$

Q9. $\dfrac{8x^2}{(x^2+1)(1-x^4)}$

Sol: $$\frac{8x^2}{(x^2+1)(1-x^4)}=\frac{8x^2}{(x^2+1)(1+x^2)(1-x)(1+x)}$$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{A}{1+x}+\frac{B}{1-x}+\frac{Cx+D}{x^2+1}+\frac{Ex+F}{(x^2+1)^2}\quad(1)$$

Multiply each term by $(x^2+1)^2(1+x)(1-x)$

$$8x^2=A(1-x)(x^2+1)^2+B(x^2+1)^2(1+x)$$
$$+(Cx+D)(x^2+1)(1-x^2)+(Ex+F)(1-x^2)\quad(2)$$

Put $x=1$ in eq (2)

$$8(1)^2=A(0)(1+1)^2+B(1+1)^2(1+1)$$
$$+(C+D)(1+1)(0)+(E+F)(0)$$
$$8=8B\qquad\Rightarrow B=1$$

Put $x=-1$ in eq (2)

$$8(-1)^2=A(2)(1+1)^2+B(1+1)^2(0)$$
$$+(-C+D)(1+1)(0)+(-E+F)(0)$$
$$8=8A\qquad\Rightarrow A=1$$

From eq (2)

$$8x^2=A(1-x)(x^4+2x^2+1)+B(x^4+2x^2+1)(1+x)$$
$$+(Cx+D)(1+x^2)(1-x^2)+(Ex+F)(1-x^2)$$

$$8x^2=A(x^4+2x^2+1-x^5-2x^3-x)$$
$$+B(x^4+2x^2+1+x^5+2x^3+x)$$
$$+(Cx+D)(1-x^4)+(Ex+F)(1-x^2)$$

$$8x^2=Ax^4+2Ax^2+A-Ax^5-2Ax^3-Ax$$
$$+Bx^4+2Bx^2+B+Bx^5+2Bx^3+Bx$$
$$+Cx-Cx^5+D-Dx^4+Ex-Ex^3+F-Fx^2$$

Rearranging terms according to powers of x

$$8x^2=Bx^5-Ax^5-Cx^5+Ax^4+Bx^4-Dx^4$$
$$-2Ax^3+2Bx^3-Ex^3+2Ax^2+2Bx^2-Fx^2$$
$$-Ax+Bx+Cx+Ex+A+B+D+F$$

$$8x^2=(B-A-C)x^5+(A+B-D)x^4$$
$$+(-2A+2B-E)x^3+(2A+2B-F)x^2$$
$$+(-A+B+C+E)x+A+B+D+F$$

Comparing the coefficients

Coefficients of x^5

$$B-A-C=0$$
$$1-1-C=0$$
$$C=0$$

coefficients of x^4

$$A+B-D=0$$
$$1+1-D=0$$
$$D=2$$

Coefficients of x^3 coefficients of x^2

$$-2A+2B-E=0$$
$$-2+2-E=0$$
$$E=0$$

$$2A+2B-F=8$$
$$2+2-F=8$$
$$4-8=F$$
$$F=-4$$

Putting A,B,C,D,E,F in eq (1)

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{1}{1+x}+\frac{1}{1-x}+\frac{0x+2}{x^2+1}+\frac{0x-4}{(x^2+1)^2}$$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{1}{1+x}+\frac{1}{1-x}+\frac{2}{x^2+1}-\frac{4}{(x^2+1)^2}$$

Q10. $\dfrac{2x^2+4}{(x^2+1)^2(x-1)}$

Sol: we have proper fraction with linear factor and Quadratic repeating factors

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}\cdots(1)$$

Multiply equation (1) by $(x^2+1)^2(x-1)$

$$2x^2+4=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2\cdots(2)$$

$$2x^2+4=(Ax+B)(x^3-x^2+x-1)$$
$$+Cx^2-Cx+Dx-D+E(x^4+2x^2+1)$$

$$2x^2+4=Ax^4-Ax^3+Ax^2-Ax+Bx^3-Bx^2$$
$$+Bx-B+Cx^2-Cx+Dx-D+Ex^4+2Ex^2+E$$

$$2x^2+4=(A+E)x^4+(-A+B)x^3+(A-B+C+2E)x^2$$
$$+(-A+B-C+D)x+(-B-D+E)\cdots(3)$$

put x = 1 in equation (2) we get

$$2(1)+4=(A+B)(1+1)(0)+(C+D)(0)+E(1+1)^2$$
$$2+4=0+0+4E$$
$$6=4E$$
$$\Rightarrow E=\tfrac{6}{4} \qquad \Rightarrow E=\tfrac{3}{2}$$

Comparing the coefficients in equation (3)

coefficients of x^4

$$A+E=0$$
$$A=-E$$
$$A=\tfrac{-3}{2}$$

coefficients of x^3

$$-A+B=0$$
$$B=A$$
$$B=\tfrac{-3}{2}$$

Constant

$$-B-D+E=4$$
$$-\left(\tfrac{-3}{2}\right)+\tfrac{3}{2}-4=D$$
$$3-4=D$$
$$\Rightarrow D=-1$$

coefficients of x

$$-A+B-C+D=0$$
$$-\left(\tfrac{-3}{2}\right)+\left(\tfrac{-3}{2}\right)+(-1)=C$$
$$-1=C$$

Putting the values of A,B,C,D and E in eq (1)

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{\frac{-3}{2}x+\frac{-3}{2}}{x^2+1}+\frac{-1x+(-1)}{(x^2+1)^2}+\frac{\frac{3}{2}}{x-1}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=-\frac{3x+3}{2(x^2+1)}-\frac{x+1}{(x^2+1)^2}+\frac{3}{2(x-1)}$$

Review Exercise 4

Q1 Fill in the correct circle only

i). $\dfrac{1}{x^2-1}=$..............................

O $\frac{1}{x+1}-\frac{1}{x-1}$ O $\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$

O $\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$ O $\frac{2}{x-1}-\frac{1}{2(x+1)}$

ii). If $P(x)$ and $Q(x)$ are two polynomial then $\dfrac{P(x)}{Q(x)}, Q(x)\neq 0$ is ....fraction

O Rational O Irrational

O Proper O Improper

iii). $\dfrac{x^2+2}{x^2+2x+2}$ is.................... fraction

O Proper O Improper

O Irrational O None of these

iv). What is quotient when $x^3-8x^2+16x-5$ is divided by $x-5$

O x^2-x+5 O x^2-3x+2

O x^2-3x+1 O $x^2+13x-49+\frac{240}{x+5}$

Q2. Resolve into partial fractions.

Q2i). $\dfrac{2x^2+1}{(x+1)(x-1)}$

Solution: improper so $\dfrac{2x^2+1}{(x+1)(x-1)}=\dfrac{2x^2+1}{x^2-1}$

$$\begin{array}{r|l} & \ \ 2 \\ \hline x^2-1 & \ 2x^2+1 \\ & \underline{\pm 2x^2 \mp 2} \\ & \ \ \ 3 \end{array}$$

$$\frac{2x^2+1}{(x+1)(x-1)}=2+\frac{3}{x^2-1}$$

$$\frac{2x^2+1}{(x+1)(x-1)}=2+\frac{3}{(x+1)(x-1)}\cdots(1)$$

Take

$$\frac{3}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}\cdots(2)$$

Multiply equation (2) by $(x+1)(x-1)$

$$\frac{3}{(x+1)(x-1)}(x+1)(x-1)=\frac{A}{x+1}(x+1)(x-1)$$
$$+\frac{B}{x-1}(x+1)(x-1)$$

$$3=A(x-1)+B(x+1)\cdots(3)$$

Put x = 1 in equation (3)

$$3=A(1-1)+B(1+1)$$
$$3=A(0)+B(2)$$
$$3=0+2B \qquad \Rightarrow B=\frac{3}{2}$$

Put x = - 1 in equation (3)

$$3 = A(-1-1) + B(-1+1)$$
$$3 = A(-2) + B(0)$$
$$3 = -2A + 0 \qquad \Rightarrow A = \frac{-3}{2}$$

Putting the value of A and B in (2)

$$\frac{3}{(x+1)(x-1)} = \frac{-3}{2(x+1)} + \frac{3}{2(x-1)}$$

Thus equation (1) becomes

$$\frac{2x^2+1}{(x+1)(x-1)} = 2 + \frac{-3}{2(x+1)} + \frac{3}{2(x-1)}$$

Q2ii). $\dfrac{2x^3-3x^2+9x+8}{x^2-3x+2}$

Solution: improper so

$$\begin{array}{r|l} & \quad 2x+3 \\ \hline x^2-3x+2 & 2x^3-3x^2+9x+8 \\ & \underline{\pm 2x^3 \mp 6x^2 \pm 4x} \\ & \qquad 3x^2+5x+8 \\ & \qquad \underline{\pm 3x^2 \mp 9x \pm 6} \\ & \qquad\qquad 14x+2 \end{array}$$

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2} = 2x+3+\frac{14x+2}{x^2-3x+2}\cdots(1)$$

Take $\dfrac{14x+2}{x^2-3x+2} = \dfrac{14x+2}{x^2-2x-x+2}$

$$\frac{14x+2}{x^2-3x+2} = \frac{14x+2}{x(x-2)-1(x-2)}$$
$$\frac{14x+2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}\cdots(2)$$

Multiply equation (2) by $(x-1)(x-2)$

$$\frac{14x+2}{(x-1)(x-2)}(x-1)(x-2) = \frac{A}{x-1}(x-1)(x-2) + \frac{B}{x-2}(x-1)(x-2)$$
$$14x+2 = A(x-2) + B(x-1)\cdots(3)$$

Put x = 1 in equation (3)

$$14(1)+2 = A(1-2) + B(1-1)$$
$$14+2 = A(-1) + B(0)$$
$$16 = -A \qquad \Rightarrow A = -16$$

Put x = 2 in equation (3)

$$14(2)+2 = A(2-2) + B(2-1)$$
$$28+2 = A(0) + B(1)$$
$$30 = B \qquad \Rightarrow B = 30$$

Putting the value of A and B in (2)

$$\frac{14x+2}{(x-1)(x-2)} = \frac{-16}{x-1} + \frac{30}{x-2}$$

Thus equation (1) becomes

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2} = 2x+3+\frac{-16}{x-1}+\frac{30}{x-2}$$

Q2iii). $\dfrac{3x-1}{x^3-2x^2+x}$

Sol: $\dfrac{3x-1}{x^3-2x^2+x} = \dfrac{3x-1}{x\{x^2-2x+1\}}$

$$\frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x-1)^2}$$

with linear factors with linear repeating

$$\frac{3x-1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}\cdots(1)$$

Multiply equation (1) by $x(x-1)^2$ we get

$$3x-1 = A(x-1)^2 + Bx(x-1) + Cx\cdots(2)$$

Put x = 1 in equation (2) we get

$$3(1)-1 = A(1-1)^2 + B(1)(1-1) + C(1)$$
$$3-1 = A(0) + B(1)(0) + C$$
$$\Rightarrow C = 2$$

Put x = 0 in equation (2) we get

$$3(0)-1 = A(0-1)^2 + B(0)(0-1) + C(0)$$
$$0-1 = A(1) + B(0)(-1) + 0$$
$$\Rightarrow A = -1$$

Choose x = 2 and using A = - 1 & C = 2 in (2)

$$3(2)-1 = -1(2-1)^2 + B(2)(2-1) + 2(2)$$
$$6-1 = -1(1) + B(2)(1) + 4$$
$$5 = -1 + 2B + 4$$
$$5+1-4 = 2B$$
$$2 = 2B \qquad \Rightarrow B = 1$$

Putting the values of A and B in eq (1)

$$\frac{3x-1}{x(x-1)^2} = \frac{-1}{x} + \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

Q2iv). $\dfrac{x+1}{(x-1)^2}$

Sol: $\dfrac{x+1}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}\cdots(1)$

Multiply equation (1) by $(x-1)^2$ we get

$$x+1 = A(x-1) + B\cdots(2)$$

Put x = 1 in equation (2) we get

$$1+1 = A(1-1) + B$$
$$2 = A(0) + B \qquad \Rightarrow B = 2$$

Choose x = 2 and using B = 2 in (2)

$$2+1 = A(2-1) + 2$$
$$3 = A(1) + 2$$
$$A = 3-2 \qquad \Rightarrow A = 1$$

Putting the values of A and B in eq (1)

$$\frac{x+1}{(x-1)^2} = \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

Q2v). $\frac{2x^2}{x^4-4}$

Sol $\frac{2x^2}{x^4-4}=\frac{2x^2}{\left(x^2\right)^2-\left(2\right)^2}$

$$\frac{2x^2}{x^4-4}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{x^2-2}\cdots(2)$$

Multiply equation (1) by $\left(x^2-2\right)\left(x^2+2\right)$

$$2x^2=\left(Ax+B\right)\left(x^2+2\right)+\left(Cx+D\right)\left(x^2-2\right)$$
$$2x^2=Ax^3+2Ax+Bx^2+2B+Cx^3-2Cx+Dx^2-2D$$
$$2x^2=Ax^3+Cx^3+Bx^2+Dx^2+2Ax-2Cx+2B-2D$$
$$2x^2=\left(A+C\right)x^3+\left(B+D\right)x^2+\left(2A-2C\right)x+\left(2B-2D\right)\cdots(2)$$

Comparing the coefficients in equation (3)

Constant	coefficients of x
$2B-2D=0$	$2A-2C=0$
$B-D=0$	$A-C=0$
$B=D$	$A=C$
coefficients of x^2	coefficients of x^3
$B+D=2$	$A+C=0$
$B+B=2 \quad \therefore B=D$	$A+A=0 \quad \therefore A=C$
$2B=2$	$2A=0$
$\Rightarrow B=1=D$	$\Rightarrow A=0=C$

Putting the values of A,B,C and D in eq (1)

$$\frac{2x^2}{x^4-4}=\frac{0x+1}{x^2+2}+\frac{0x+1}{x^2-2}$$
$$\frac{2x^2}{x^4-4}=\frac{1}{x^2+2}+\frac{1}{x^2-2}$$

Qvi). $\frac{3x^2+3x+2}{x^4-1}$

Sol: $\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{\left(x^2\right)^2-1^2}$

$$\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{\left(x^2-1^2\right)\left(x^2+1\right)}$$
$$\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{\left(x+1\right)\left(x-1\right)\left(x^2+1\right)}$$

with two linear and one Quadratic factor

$$\frac{3x^2+3x+2}{\left(x+1\right)\left(x-1\right)\left(x^2+1\right)}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+1}\cdots(1)$$

Multiply equation (1) by $\left(x-1\right)\left(x+1\right)\left(x^2+1\right)$

$$3x^2+3x+2=A\left(x-1\right)\left(x^2+1\right)+B\left(x+1\right)\left(x^2+1\right)+\left(Cx+D\right)\left(x^2-1\right)\cdots(2)$$
$$3x^2+3x+2=A\left(x^3+x-x^2-1\right)+B\left(x^3+x+x^2+1\right)\; Cx^3-Cx+Dx^2-D$$
$$3x^2+3x+2=Ax^3+Ax-Ax^2-A+Bx^3+Bx+Bx^2+B+Cx^3-Cx+Dx^2-D$$
$$3x^2+3x+2=Ax^3+Bx^3+Cx^3-Ax^2+Bx^2+Dx^2+Ax+Bx-Cx-A+B-D$$
$$3x^2+3x+2=\left(A+B+C\right)x^3+\left(-A+B+D\right)x^2+\left(A+B-C\right)x+\left(-A+B-D\right)\cdots(3)$$

Put x = 1 in equation (2)

$$3\left(1\right)^2+3\left(1\right)+2=A\left(1-1\right)\left(1^2+1\right)+B\left(1+1\right)\left(1^2+1\right)+\left(C\left(1\right)+D\right)\left(1^2-1\right)$$
$$3+3+2=A\left(0\right)\left(2\right)+B\left(2\right)\left(2\right)+\left(C+D\right)\left(0\right)$$
$$8=0+4B+0 \qquad \Rightarrow B=\frac{8}{4}=2$$

Put x = 1 in equation (2)

$$3\left(-1\right)^2+3\left(-1\right)+2=A\left(-1-1\right)\left(\left(-1\right)^2+1\right)+B\left(-1+1\right)\left(\left(-1\right)^2+1\right)+\left(C\left(-1\right)+D\right)\left(\left(-1\right)^2-1\right)$$
$$3-3+2=A\left(-2\right)\left(2\right)+B\left(0\right)\left(2\right)+\left(-C+D\right)\left(0\right)$$
$$2=-4A+0+0 \qquad \Rightarrow A=\frac{-1}{2}$$

Comparing the coefficients in equation (3)

Constant	coefficients of x
$-A+B-D=2$	$A+B-C=3$
$-\left(\frac{-1}{2}\right)+2-2=D$	$\frac{-1}{2}+2-3=C$
$D=\frac{1}{2}$	$C=\frac{-1}{2}-1 \qquad \Rightarrow C=\frac{-3}{2}$

Putting the values of A,B,C and D in eq (1)

$$\frac{3x^2+3x+2}{\left(x+1\right)\left(x-1\right)\left(x^2+1\right)}=\frac{-1}{2\left(x+1\right)}+\frac{2}{x-1}+\frac{-3x+1}{2\left(x^2+1\right)}$$

Q7. $\frac{x^3+3x^2+1}{\left(x^2+1\right)^2}$

Sol: $\frac{x^3+3x^2+1}{\left(x^2+1\right)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{\left(x^2+1\right)^2}\cdots(1)$

Multiply equation (1) by $\left(x^2+1\right)^2$ we get

$$x^3+3x^2+1=\left(Ax+B\right)\left(x^2+1\right)+\left(Cx+D\right).....(2)$$
$$x^3+3x^2+1=Ax^3+Ax+Bx^2+B+Cx+D$$
$$x^3+3x^2+1=Ax^3+Bx^2+\left(A+C\right)x+B+D\cdots(3)$$

Comparing the coefficients in equation (3)

coefficients of x^3	coefficients of x^2
$A=1$	$B=3$
coefficients of x	Constant
$A+C=0$	$B+D=1$
$C=-A$	$D=1-B$
$C=-1$	$D=1-3$
	$D=-2$

Putting the values of A,B,C and D in eq (1)

$$\frac{x^3+3x^2+1}{\left(x^2+1\right)^2}=\frac{x+3}{x^2+1}+\frac{-x-2}{\left(x^2+1\right)^2}$$

Q2viii). $\frac{2x^3-1}{x^3+x^2}$

Sol: $\frac{2x^3-1}{x^3+x^2}$ is an improper fraction

$$\begin{array}{r|l} & \quad 2 \\ \hline x^3+x^2 & \; 2x^3 \quad -1 \\ & \pm 2x^3 \pm 2x^2 \\ \hline \end{array}$$

$$-2x^2-1$$

$$\frac{2x^3-1}{x^3+x^2}=2+\frac{-2x^2-1}{x^2(x+1)} \quad \text{........(1)}$$

$$\frac{-2x^2-1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1} \quad \text{.......(2)}$$

Multiply each term by $x^2(x+1)$

$$-2x^2-1=Ax(x+1)+B(x+1)+Cx^2 \quad \text{.....(3)}$$

Put $x=0$ in eq (3)

$$-2(0)^2-1=A(0)(0+1)+B(0+1)+C(0)^2$$
$$0-1=0+B+0$$
$$B=-1$$

Put $x=-1$ in eq (3)

$$-2(-1)^2-1=A(-1)(-1+1)+B(-1+1)+C(-1)^2$$
$$-2-1=0+0+C$$
$$C=-3$$

Choose $x=2$

$$-2(2)^2-1=A(2)(2+1)+B(2+1)+C(2)^2$$

$-8-1=6A+3B+4C$ using $B=-1, C=-3$

$$-9=6A+3(-1)+4(-3)$$
$$-9=6A-3-12$$
$$-9+3+12=6A$$
$$6=6A \qquad \Rightarrow A=1$$

Putting the values of A,B and C in eq (2)

$$\frac{-2x^2-1}{x^2(x+1)}=\frac{1}{x}+\frac{-1}{x^2}+\frac{-3}{x+1}$$ Put in eq (1)

$$\frac{2x^3-1}{x^3+x^2}=2+\frac{1}{x}-\frac{1}{x^2}-\frac{3}{x+1}$$

Q2ix). $\dfrac{4x^2+3x+14}{x^3-8}$

Sol: $\dfrac{4x^2+3x+14}{x^3-2^3}=\dfrac{4x^2+3x+14}{(x-2)(x^2+2x+4)}$

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+2x+4} \quad \text{...(1)}$$

Multiply each term by $(x-2)(x^2+2x+4)$

$$4x^2+3x+14=A(x^2+2x+4)+(Bx+C)(x-2) \text{(2)}$$

Put $x=2$ in eq (2)

$$4(2)^2+3(2)+14=A((2)^2+2(2)+4)+(2B+C)(0)$$
$$4(4)+6+14=A(4+4+4)+0$$
$$16+20=12A$$
$$36=12A \qquad \Rightarrow A=3$$ Form eq (2)

$$4x^2+3x+14=Ax^2+2Ax+4A$$
$$+Bx^2-2Bx+Cx-2C$$
$$4x^2+3x+14=(A+B)x^2+(2A-2B+c)x+4A-2C$$

Comparing the coefficients

Coefficients of x^2 | constant

$A+B=4$ | $4A-2C=14$

$3+B=4$ | $4(3)-14=2C$

$B=1$ | $-2=2C \quad \Rightarrow C=-1$

Putting the values of A,B and C in eq (1)

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}=\frac{3}{x-2}+\frac{x-1}{x^2+2x+4}$$

Q3. Resolve $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Sol: $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$

Multiply each term by $(x+1)(x^2+1)^2$

$$x^4+3x^2+x+1=A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)$$
$$+(Dx+E)(x+1) \quad \text{...............(2)}$$

$$x^4+3x^2+x+1=A(x^4+2x^2+1)$$
$$+(Bx+C)(x^3+x+x^2+1)$$
$$+(Dx+E)(x+1)$$

$$x^4+3x^2+x+1=Ax^4+2Ax^2+A+Bx^4+Bx^2+Bx^3$$
$$+Bx+Cx^3+Cx+Cx^2+C+Dx^2+Dx+Ex+E$$

$$x^4+3x^2+x+1=(A+B)x^4+(B+C)x^3$$
$$+(2A+B+C+D)x^2+(B+C+D+E)x$$
$$+A+C+E \quad \text{....................(3)}$$

Put $x=-1$ in eq (2)

$$1+3-1+1=A(1+1)^2+(-B+C)(0)(1+1)$$
$$+(-D+E)(0)$$
$$4=4A+0+0 \qquad \Rightarrow A=1$$

Comparing the coefficients of eq (3)

coefficients of x^4 | coefficients of x^3

$A+B=1$ | $B+C=0$

$1+B=1$ | $0+C=0$

$B=0$ | $C=0$

Constant | coefficients of x^2

$A+C+E=1$ | $2A+B+C+D=3$

$1+0+E=1$ | $2+0+0+D=3$

$E=0$ | $D=1$

Putting the values of A,B,C,D & E in eq (1)

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\frac{1}{x+1}+\frac{0x+0}{x^2+1}+\frac{1x+0}{(x^2+1)^2}$$

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\frac{1}{x+1}+\frac{x}{(x^2+1)^2}$$