

# Chapter 5

**SET :** Collection of well defined and distinct objects is called a set.

**OR SET :** A set is an aggregate, class or Collection of distinct objects or numbers, is called a set.

**MEMBERS OR ELEMENTS OF A SET:-** Articles contained in a set are called its members or elements of a set. Members of a set can be written in any order. e.g.  $\{a, b, c\}, \{b, c, a\}$  are same set.

**OR MEMBERS OR ELEMENTS OF A SET:-** An object involved in a set is called its elements.

**PRESENTATION OF SET:**

**1. DESCRIPTIVE METHOD:**

In this method we write the set in sentence form for example first five natural numbers.

**2. TABULAR METHOD:** In this method we write the set in the form for example  $\{1, 2, 3, 4, 5\}$

**3. SET BUILDER NOTATION:** In this method we write the set in the form

$\{x : x \in N, x \leq 5\}$ . We read is as A is the set consisting of elements of the type x, where x belongs to N and X is less then or equal to five.

**KINDS OF SET:**

**NULL SET OR EMPTY SET:** A set having no element is called null set. Denoted by  $\phi$  or  $\{ \}$ .

**OR NULL SET OR EMPTY SET:**

A set with out elements is called an empty set. It is denoted by  $\phi$  or  $\{ \}$ .

**FINITE SET:** If the members of a set can be counted i.e it contains limited number of members, then it is called Finite set. For example

$A = \{1, 2, 3, \dots, 10\}$  is finite set,

**OR FINITE SET:** A set having finite number of elements is called a finite set.

**INFINITE SET:** If the numbers of members of a set is not limited, it is called infinite set. For example  $A = \{1, 2, 3, \dots\}$ .

**OR INFINITE SET:** A set having infinite number of elements is called a infinite set.

**SUBSET:** If two sets A and B are given such that every member of A is also a member of set B then A is called a subset of B. Symbolically it is written as  $A \subseteq B$  and read as A is a subset of B. For example:

If  $A = \{1, 2, 3, \dots, 10\}$   $A \subseteq B$  and

$B = \{1, 2, 3, \dots, 11\}$  then  $A \subseteq B$ .

**OR SUBSET:** If each elements of A is also an element of an other set B, then A is said to be a subset of the set B.

**SUPERSET:** If two sets A and B are given such that  $A \subseteq B$  then B is called the superset of A and is also written as  $B \supseteq A$ . For example: If

$A = \{1, 2, \dots, 10\}$  and  $B = \{1, 2, \dots, 15\}$  then B is a superset of A.

**PROPER SUBSET:**

If two sets A and B are given such that

i). A is a subset of B, and

ii). There exists at least one member in B which is not a member of A, then A is a proper subset of B. symbolically it is written as  $A \subset B$  and read as A is a proper subset of B.

**OR PROPER SUBSET:** If A is a subset of B, and B has atleast one element which does not belong to the set A is called proper subset of B.

**OR PROPER SUBSET:** If two sets A and B are given such that, A is a subset of B, and  $A \neq B$

**IMPROPER SUBSET:**

If two sets A and B are given such that

1)  $A \subseteq B$ , and

2) B does not contain a member which is not a member of A then set A is an improper subset B. It is written as  $A \subseteq B$ . It means that every set is an improper subset of itself.

**OR IMPROPER SUBSET:** If A is the subset of B and B does not have an element which does not belong to the set A, then A is called improper subset of B.

**EQUAL OR NOT EQUAL SETS:** If A and B are two sets such that  $A \subseteq B$  and  $B \subseteq A$  then we say that A and B are equal sets which is written as  $A = B$ . If they are not equal, we write as  $A \neq B$ .

After defining equal sets we see that if set A is a proper subset of set B then it can be defined as:  $A \subset B$  and  $A \neq B$ .

**OR EQUAL SETS:**

Two set A and B are said to be equal if each element of A is an element of B and each element of B is an element of A.

**EQUIVALENT SETS:**

If two sets A and B are in 1 – 1 correspondence then these sets are called Equivalent sets.

If  $A = \{a, b, c\}$ ,  $B = \{1, 2, 3\}$ , and  $C = \{a, b, 3, 4\}$

then A and B are Equivalent sets whereas A and C are non-equivalent sets. B and C are also non-equivalent.

**OR EQUIVALENT SETS:**

Two sets A and B are said to be Equivalent if they have the same number of elements.

**OVERLAPPING SETS:** If two sets are given such that none of them is a subset of the other set and there is atleast one element in both the sets which is common to both the sets, then the sets are called 'Overlapping sets'.

For example, if  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 5, 6, 7\}$

are overlapping sets because 3 is common element of both sets A and B and  $B \not\subset A$ , or  $A \not\subset B$ .

**OR OVERLAPPING SETS:**

Two sets are said to be overlapping if

- atleast one element in both the sets
- none of them is a subset of the other.

**DISJOINT SETS:** Two sets having no common element are called disjoint sets. For example  $A = \{5, 8, 9\}$  and  $B = \{1, 2, 3\}$  are Disjoint sets.

**OR DISJOINT SETS:** Two sets are said to be disjoint if they have no common elements.

**POWER SET:** If A is any set, the set containing of all the subsets of the set A is called the power set of A. It is denoted by  $P(A)$ .

**OR POWER SET:** For a set A, the set of all possible subsets of the set A is called the power set of A.

**UNIVERSAL SET:**

A set containing of all elements of U which are not the elements of the sets under consideration is called Universal set and is denoted by U.

**OR UNIVERSAL SET:** A set containing the entire element under consideration in particular problem is called Universal set and is denoted by U.

**OPERATION ON SETS:**

**1. UNION OF TWO SETS:** Union set  $A \cup B$  of two sets A and B is a set which contains members of set A or set B.  $A \cup B = \{x | x \in A \text{ or } x \in B\}$

Exp1: if  $A = \{1, 2, 3\}$ ,  $B = \{3, 4, 5, 6\}$  then

$$A \cup B = \{1, 2, 3\} \cup \{3, 4, 5, 6\} \quad \text{or}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

**2. INTERSECTION OF TWO SETS:**

Intersection set  $A \cap B$  of two sets A and B is a set which contains the members of A and B that are common to both sets.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

For example if  $A = \{4, 5, 6, 8\}$ ,  $B = \{3, 5, 8, 10\}$  then

$$A \cap B = \{4, 5, 6, 8\} \cap \{3, 5, 8, 10\} = \{5, 8\}$$

**3. DIFFERENCE OF TWO SETS:-** Difference set  $A - B$  of two sets A and B is a set which contains element of set A that are not contained in set B.  $A - B$  is some times written as  $A/B$ .

$$A/B = \{x | x \in A \text{ and } x \notin B\}$$

Exp3: if  $A = \{5, 6, 7, 8\}$  and  $B = \{7, 8, 9, 10\}$  then

$$A/B = \{5, 6\} \text{ and } B/A = \{9, 10\}.$$

**4. COMPLEMENT SET:-** If U is a universal set then  $U - A$  is called complement set of A and is denoted by  $A'$  or  $A^c$ .

Exp4: if  $U = \{1, 2, 3, 4, 5, 6\}$ ,  $A = \{3, 4, 5\}$  and  $B = \phi$  then  $A' = U - A = \{1, 2, 6\}$

$$B' = \phi' = U - \phi = \{1, 2, 3, 4, 5, 6\} = U$$

**OR COMPLEMENT SET:-**

A set consisting of the elements of U which are not the elements of the set A is called complement of A.

**5. 1-1 CORRESPONDENCE OR ONE TO ONE CORRESPONDENCE** A correspondence between two set for which each element of either set is paired with **exactly one element** of the other set is called one to one correspondence.

**SOME IMPORTANT SETS AND THEIR NOTATIONS OR SETS OF NUMBERS:**

The following notation are commonly used for different sets of numbers:

Set of natural numbers :  $N = \{1, 2, 3, \dots\}$

Set of whole numbers:  $W = \{0, 1, 2, \dots\}$

Set of integers:  $Z = \{0, \pm 1, \pm 2, \dots\}$

Set of prime numbers:  $P = \{2, 3, 5, 7, \dots\}$

Set of odd integers:  $O = \{\pm 1, \pm 3, \pm 5, \dots\}$

Set of even integers:  $E = \{0, \pm 2, \pm 4, \dots\}$

Set of Rational numbers:  $Q = \left\{ \frac{p}{q}, q \neq 0, p, q \in Z \right\}$

Set of Irrational numbers:

$$Q' = \left\{ \sqrt{p} : p \text{ is not perfect square} \right\} \quad \text{or}$$

$$Q' = \left\{ \sqrt[n]{p} : p \neq q^n; p, q, n > 1 \right\}$$

Set of Real numbers:  $R = Q \cup Q'$

### Exercise 5.1

Q1: If  $A = \{1, 2, 3\}$ ,  $B = \{0, 1\}$  and  $C = \{1, 3, 4\}$

then find i).  $A \cup B$

$$\text{Solution: } A \cup B = \{1, 2, 3\} \cup \{0, 1\}$$

$$A \cup B = \{0, 1, 2, 3\}$$

ii).  $A \cap B$

$$\text{Solution: } A \cap B = \{1, 2, 3\} \cap \{0, 1\}$$

$$A \cap B = \{1\}$$

iii).  $A \cup C$

$$\text{Solution: } A \cup C = \{1, 2, 3\} \cup \{1, 3, 4\}$$

$$A \cup C = \{1, 2, 3, 4\}$$

iv).  $A \cap C$

$$\text{Solution: } A \cap C = \{1, 2, 3\} \cap \{1, 3, 4\}$$

$$A \cap C = \{1, 3\}$$

v).  $B \cup C$

$$\text{Solution: } B \cup C = \{0, 1\} \cup \{1, 3, 4\}$$

$$B \cup C = \{0, 1, 3, 4\}$$

vi).  $A \cap A$

$$\text{Solution: } A \cap A = \{1, 2, 3\} \cap \{1, 2, 3\}$$

$$A \cap A = \{1, 2, 3\}$$

Q2: Find  $A \setminus B$  and  $B \setminus A$  when

$$\text{i). } A = \{1, 3, 5, 7\}, B = \{3, 4, 5, 6, 7, 8\}$$

$$\text{Solution: } A \setminus B = \{1, 3, 5, 7\} \setminus \{3, 4, 5, 6, 7, 8\}$$

$$A \setminus B = \{1\}$$

$$\text{Now } B \setminus A = \{3, 4, 5, 6, 7, 8\} \setminus \{1, 3, 5, 7\}$$

$$B \setminus A = \{4, 6, 8\}$$

$$\text{ii). } A = \{0, \pm 1, \pm 2, \pm 3\}, B = \{-1, -2, -3\}$$

$$\text{Solution: } A \setminus B = \{0, \pm 1, \pm 2, \pm 3\} \setminus \{-1, -2, -3\}$$

$$A \setminus B = \{0, 1, 2, 3\}$$

$$\text{Now } B \setminus A = \{-1, -2, -3\} \setminus \{0, \pm 1, \pm 2, \pm 3\}$$

$$B \setminus A = \{ \}$$

$$\text{iii). } A = \{1, 2, 3, 4, \dots\}, B = \{1, 3, 5, 7, \dots\}$$

Solution:  $A \setminus B = \{1, 2, 3, 4, \dots\} \setminus \{1, 3, 5, 7, \dots\}$

$$A \setminus B = \{2, 4, 6, \dots\}$$

Now  $B \setminus A = \{1, 3, 5, 7, \dots\} \setminus \{1, 2, 3, 4, \dots\}$

$$B \setminus A = \{ \}$$

Q3: If  $U = \{1, 2, 3, \dots, 20\}$ ,  $A = \{2, 4, 6, \dots, 20\}$ ,

$B = \{1, 3, 5, \dots, 19\}$  and  $C = \phi = \{ \}$  then find

i).  $A'$

Solution:

$$A' = U - A = \{1, 2, 3, \dots, 20\} - \{2, 4, 6, \dots, 20\}$$

$$A' = \{1, 3, 5, \dots, 19\} \dots (1)$$

ii).  $B'$

Solution:

$$B' = U - B = \{1, 2, 3, \dots, 20\} - \{1, 3, 5, \dots, 19\}$$

$$B' = \{2, 4, 6, \dots, 20\} \dots (2)$$

iii).  $C'$

Solution:  $C' = U - C = \{1, 2, 3, \dots, 20\} - \{ \}$

$$C' = \{1, 2, 3, \dots, 20\} \dots (1)$$

iv).  $A' \cup B'$

Solution: we have from equations (1) & (2)

$$A' \cup B' = \{1, 3, 5, \dots, 19\} \cup \{2, 4, 6, \dots, 20\}$$

$$A' \cup B' = \{1, 2, 3, \dots, 20\}$$

v).  $A' \cap B'$

Solution: we have from equations (1) & (2)

$$A' \cap B' = \{1, 3, 5, \dots, 19\} \cap \{2, 4, 6, \dots, 20\}$$

$$A' \cap B' = \{ \}$$

vi).  $A' \cap B$

Solution: we have from equation (1)

$$A' \cap B = \{1, 3, 5, \dots, 19\} \cap \{1, 3, 5, \dots, 19\}$$

$$A' \cap B = \{1, 3, 5, \dots, 19\}$$

vii).  $A' \cup C'$

Solution: we have from equations (1) & (3)

$$A' \cup C' = \{1, 3, 5, \dots, 19\} \cup \{1, 2, 3, \dots, 20\}$$

$$A' \cup C' = \{1, 2, 3, \dots, 20\}$$

viii).  $A \cap C'$

Solution: we have from equation (3)

$$A \cap C' = \{2, 4, 6, \dots, 20\} \cap \{1, 2, 3, \dots, 20\}$$

$$A \cap C' = \{2, 4, 6, \dots, 20\}$$

ix).  $C' \cap C$

Solution: we have from equation (3)

$$C' \cap C = \{1, 2, 3, \dots, 20\} \cap \{ \}$$

$$C' \cap C = \{ \}$$

x).  $B' \cup C'$

Solution: we have from equations (2) & (3)

$$B' \cup C' = \{2, 4, 6, \dots, 20\} \cup \{1, 2, 3, \dots, 20\}$$

$$B' \cup C' = \{1, 2, 3, \dots, 20\}$$

Q4. If  $U$  = Set of Natural numbers upto 15

And  $A$  = Set of even numbers upto 15

And  $B$  = Set of odd numbers upto 15

$$U = \{1, 2, 3, \dots, 15\}$$

$$A = \{2, 4, 6, \dots, 14\}, B = \{1, 3, 5, \dots, 15\}$$

Then find

i).  $A' \cup B'$

Solution: First we have to find  $A'$  &  $B'$

$$A' = U - A = \{1, 2, 3, \dots, 15\} - \{2, 4, 6, \dots, 14\}$$

$$A' = \{1, 3, 5, \dots, 15\} \dots (1)$$

Now

$$B' = U - B = \{1, 2, 3, \dots, 15\} - \{1, 3, 5, \dots, 15\}$$

$$B' = \{2, 4, 6, \dots, 14\} \dots (2)$$

Now using equations (1) and (2) we get

$$A' \cup B' = \{1, 3, 5, \dots, 15\} \cup \{2, 4, 6, \dots, 14\}$$

$$A' \cup B' = \{1, 2, 3, \dots, 15\}$$

ii).  $A' \cap B'$

Solution: using equations (1) and (2) we get

$$A' \cap B' = \{1, 3, 5, \dots, 15\} \cap \{2, 4, 6, \dots, 14\}$$

$$A' \cap B' = \{ \}$$

iii).  $U'$

Solution:

$$U' = U - U = \{1, 2, 3, \dots, 15\} - \{1, 2, 3, \dots, 15\}$$

$$U' = \{ \}$$

iv).  $\phi'$

$$\text{Solution: } \phi' = U - \phi = \{1, 2, 3, \dots, 15\} - \{ \}$$

$$\phi' = \{1, 2, 3, \dots, 15\} \dots (3)$$

v).  $B \cap A'$

Solution: Using equation (1)

$$B \cap A' = \{1, 3, 5, \dots, 15\} \cap \{1, 3, 5, \dots, 15\}$$

$$B \cap A' = \{1, 3, 5, \dots, 15\}$$

vi).  $B \cup B'$

Solution: Using equation (2)

$$B \cup B' = \{1, 3, 5, \dots, 15\} \cup \{2, 4, 6, \dots, 14\}$$

$$B \cup B' = \{1, 2, 3, \dots, 15\}$$

vii).  $A \cap A'$

Solution: Using equation (1)

$$A \cap A' = \{2, 4, 6, \dots, 14\} \cap \{1, 3, 5, \dots, 15\}$$

$$A \cap A' = \{ \}$$

viii).  $A \cup B'$

Solution: Using equation (2)

$$A \cup B' = \{2, 4, 6, \dots, 14\} \cup \{2, 4, 6, \dots, 14\}$$

$$A \cup B' = \{2, 4, 6, \dots, 14\}$$

#### PROPERTIES OF UNION AND INTERSECTION OF TWO SETS:-

##### 1. COMMUTATIVE PROPERTY OF UNION:-

If  $A$  and  $B$  are any two sets then  $A \cup B = B \cup A$

Proof: Let  $x \in A \cup B$

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \in B \text{ or } x \in A$$

$$\Rightarrow x \in B \cup A$$

Hence  $A \cup B \subseteq B \cup A \dots \dots \dots (1)$

Conversely Let  $x \in B \cup A$

$$\Rightarrow x \in B \text{ or } x \in A$$

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \in A \cup B$$

Hence  $B \cup A \subseteq A \cup B$  .....(2)

From eq (1) and (2) we get  $A \cup B = B \cup A$

## 2.COMMUTATIVE PROPERTY OF INTERSECTION:

If A and B are any two sets then  $A \cap B = B \cap A$

Proof: Let  $x \in A \cap B$

$$\Rightarrow x \in A \text{ and } x \in B$$

$$\Rightarrow x \in B \text{ and } x \in A$$

$$\Rightarrow x \in B \cap A$$

Hence  $A \cap B \subseteq B \cap A$  .....(1)

Conversely Let  $x \in B \cap A$

$$\Rightarrow x \in B \text{ and } x \in A$$

$$\Rightarrow x \in A \text{ and } x \in B$$

$$\Rightarrow x \in A \cap B$$

Hence  $B \cap A \subseteq A \cap B$  .....(2)

From eq (1) and (2) we get  $A \cap B = B \cap A$

## 3.ASSOCIATIVE PROPERTY OF UNION:

If A ,B and C are any three sets then

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Proof: Let  $x \in (A \cup B) \cup C$

$$\Rightarrow x \in (A \cup B) \text{ or } x \in C$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ or } x \in C$$

$$\Rightarrow x \in A \text{ or } x \in B \cup C$$

$$\Rightarrow x \in A \cup (B \cup C)$$

Hence  $(A \cup B) \cup C \subseteq A \cup (B \cup C)$  .....(1)

Conversely Let  $x \in A \cup (B \cup C)$

$$\Rightarrow x \in A \text{ or } x \in B \cup C$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ or } x \in C$$

$$\Rightarrow x \in A \cup B \text{ or } x \in C$$

$$\Rightarrow x \in (A \cup B) \cup C$$

Hence  $A \cup (B \cup C) \subseteq (A \cup B) \cup C$  .....(2)

From eq (1) and (2) we get

$$A \cup (B \cup C) = (A \cup B) \cup C$$

## 4.ASSOCIATIVE PROPERTY OF INTERSECTION:

If A ,B and C are any three sets then

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Proof: Let  $x \in (A \cap B) \cap C$

$$\Rightarrow x \in (A \cap B) \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ and } x \in B \cap C$$

$$\Rightarrow x \in A \cap (B \cap C)$$

Hence  $(A \cap B) \cap C \subseteq A \cap (B \cap C)$  .....(1)

Conversely Let  $x \in A \cap (B \cap C)$

$$\Rightarrow x \in A \text{ and } x \in B \cap C$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C$$

$$\Rightarrow x \in A \cap B \text{ and } x \in C$$

$$\Rightarrow x \in (A \cap B) \cap C$$

Hence  $A \cap (B \cap C) \subseteq (A \cap B) \cap C$  .....(2)

From eq (1) and (2) we get

$$A \cap (B \cap C) = (A \cap B) \cap C$$

## 5.DISTRIBUTIVE PROPERTY OF UNION OVER INTERSECTION:

If A ,B and C are any three sets then  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: Let  $x \in A \cup (B \cap C)$

$$\Rightarrow x \in A \text{ or } x \in B \cap C$$

$$\Rightarrow x \in A \text{ or } \{x \in B \text{ and } x \in C\}$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in A \text{ or } x \in C$$

$$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C)$$

Hence  $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$  .....(1)

Conversely Let  $x \in (A \cup B) \cap (A \cup C)$

$$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in A \text{ or } x \in C$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ or } x \in B \cap C$$

$$\Rightarrow x \in A \cup (B \cap C)$$

Hence  $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$  .....(2)

From eq (1) and (2) we get

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

## 6.DISTRIBUTIVE PROPERTY OF INTERSECTION OVER UNION:

If A ,B and C are any three sets then

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Proof: Let  $x \in A \cap (B \cup C)$

$$\Rightarrow x \in A \text{ and } x \in B \cup C$$

$$\Rightarrow x \in A \text{ and } \{x \in B \text{ or } x \in C\}$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ or } x \in A \text{ and } x \in C$$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

Hence  $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$  .....(1)

Conversely Let  $x \in (A \cap B) \cup (A \cap C)$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ or } x \in A \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ or } x \in C$$

$$\Rightarrow x \in A \text{ and } x \in B \cup C$$

$$\Rightarrow x \in A \cap (B \cup C)$$

Hence  $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$  .....(2)

From eq (1) and (2) we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

## 7. DE MORGAN'S LAWS

1.If A and B are any two sets then

$$(A \cup B)' = A' \cap B' \text{ or } (A \cup B)^c = A^c \cap B^c$$



Proof: Let  $x \in (A \cup B)'$

$$x \notin A \cup B$$

$$x \notin A \text{ or } x \notin B$$

$$x \in A' \text{ and } x \in B'$$

$$x \in A' \cap B'$$

$$(A \cup B)' \subseteq A' \cap B' \dots\dots\dots(1)$$

Conversely  $x \in A' \cap B'$

$$x \in A' \text{ and } x \in B'$$

$$x \notin A \text{ or } x \notin B$$

$$x \notin A \cup B$$

$$x \in (A \cup B)'$$

$$A' \cap B' \subseteq (A \cup B)' \dots\dots\dots(2)$$

From eq (1) and (2) we get  $(A \cup B)' = A' \cap B'$

## 7. DE MORGAN'S LAWS

2.If A and B are any two sets then

$$(A \cap B)' = A' \cup B' \text{ or } (A \cap B)^c = A^c \cup B^c$$

Proof: Let  $x \in (A \cap B)'$

$$x \notin A \cap B$$

$$x \notin A \text{ and } x \notin B$$

$$x \in A' \text{ or } x \in B'$$

$$x \in A' \cup B'$$

$$(A \cap B)' \subseteq A' \cup B' \dots\dots\dots(1)$$

Conversely  $x \in A' \cup B'$

$$x \in A' \text{ or } x \in B'$$

$$x \notin A \text{ and } x \notin B$$

$$x \notin A \cap B$$

$$x \in (A \cap B)'$$

$$A' \cup B' \subseteq (A \cap B)' \dots\dots\dots(2)$$

From eq (1) and (2) we get  $(A \cap B)' = A' \cup B'$

Exp5: if  $A = \{1, 2, 3\}, B = \{4, 5, 6\}$  then verify commutative property of Union.

Sol: Given  $A = \{1, 2, 3\}, B = \{4, 5, 6\}$

$$A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Now  $B \cup A = \{4, 5, 6\} \cup \{1, 2, 3\}$

$$B \cup A = \{1, 2, 3, 4, 5, 6\}$$

Hence  $A \cup B = B \cup A$

Exp6: if  $A = \{a, b, c\}, B = \{b, c, d, e\}$  then verify commutative property of intersection.

Sol: Given  $A = \{a, b, c\}, B = \{b, c, d, e\}$

$$A \cap B = \{a, b, c\} \cap \{b, c, d, e\}$$

$$A \cap B = \{b, c\}$$

Now  $B \cap A = \{b, c, d, e\} \cap \{a, b, c\}$

$$B \cap A = \{b, c\} \text{ Hence } A \cap B = B \cap A$$

Exp7: If  $A = \{3, 4, 5\}, B = \{5, 6, 7\}, C = \{8, 9, 10\}$

Then prove that  $A \cup (B \cap C) = (A \cup B) \cap C$

Sol:  $A = \{3, 4, 5\}, B = \{5, 6, 7\}, C = \{8, 9, 10\}$

Taking  $A \cup (B \cap C) = \{3, 4, 5\} \cup (\{5, 6, 7\} \cap \{8, 9, 10\})$

$$A \cup (B \cap C) = \{3, 4, 5\} \cup \{5, 6, 7, 8, 9, 10\}$$

$$A \cup (B \cap C) = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

Now  $(A \cup B) \cap C = (\{3, 4, 5\} \cup \{5, 6, 7\}) \cap \{8, 9, 10\}$

$$(A \cup B) \cap C = \{3, 4, 5, 6, 7\} \cap \{8, 9, 10\}$$

$$(A \cup B) \cap C = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

Hence  $A \cup (B \cap C) = (A \cup B) \cap C$

Exp8: if  $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{3, 4, 5\}$

Then prove that  $(A \cap B) \cap C = A \cap (B \cap C)$

Sol: Given  $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{3, 4, 5\}$

$$(A \cap B) \cap C = (\{1, 2, 3\} \cap \{2, 3, 4\}) \cap \{3, 4, 5\}$$

$$(A \cap B) \cap C = \{2, 3\} \cap \{3, 4, 5\}$$

$$(A \cap B) \cap C = \{3\}$$

$$A \cap (B \cap C) = \{1, 2, 3\} \cap (\{2, 3, 4\} \cap \{3, 4, 5\})$$

$$A \cap (B \cap C) = \{1, 2, 3\} \cap \{3, 4\}$$

$$A \cap (B \cap C) = \{3\}$$

Hence  $A \cup (B \cap C) = (A \cup B) \cap C$

Exp9: If  $A = \{1, 2, 3, 4\}, B = \{5, 6, 7\}, C = \{7, 8, 9\}$

Then prove that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup (\{5, 6, 7\} \cap \{7, 8, 9\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{7\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 7\}$$

$$(A \cup B) \cap (A \cup C) =$$

$$(\{1, 2, 3, 4\} \cup \{5, 6, 7\}) \cap (\{1, 2, 3, 4\} \cup \{7, 8, 9\})$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 7, 8, 9\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 7\}$$

Hence  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Exp10: If  $A = \{a, b, c\}, B = \{c, d, e\}, C = \{e, f, g\}$

Then prove that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol: Given  $A = \{a, b, c\}, B = \{c, d, e\}, C = \{e, f, g\}$

$$A \cap (B \cup C) = \{a, b, c\} \cap (\{c, d, e\} \cup \{e, f, g\})$$

$$A \cap (B \cup C) = \{a, b, c\} \cap \{c, d, e, f, g\}$$

$$A \cap (B \cup C) = \{c\}$$

$$(A \cap B) \cup (A \cap C) =$$

$$(\{a, b, c\} \cap \{c, d, e\}) \cup (\{a, b, c\} \cap \{e, f, g\})$$

$$(A \cap B) \cup (A \cap C) = \{c\} \cup \{c\}$$

$$(A \cap B) \cup (A \cap C) = \{c\}$$

Hence  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Exp11: if  $U = \{1, 2, 3, 4, 5, 6\}, A = \{2, 3\}, B = \{3, 4, 5\}$

Then show that  $(A \cup B)' = A' \cap B'$

Sol:  $U = \{1, 2, 3, 4, 5, 6\}$   $A = \{2, 3\}$ ,  $B = \{3, 4, 5\}$

$$A \cup B = \{2, 3\} \cup \{3, 4, 5\}$$

$$A \cup B = \{2, 3, 4, 5\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$(A \cup B)' = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4, 5\}$$

$$(A \cup B)' = \{1, 6\} \dots\dots\dots(1)$$

Now  $A' = \{1, 2, 3, 4, 5, 6\} - \{2, 3\}$

$$A' = \{1, 4, 5, 6\}$$

And  $B' = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5\}$

$$B' = \{1, 2, 6\}$$

$$A' \cap B' = \{1, 4, 5, 6\} \cap \{1, 2, 6\}$$

$$A' \cap B' = \{1, 6\} \dots\dots\dots(2)$$

From eq (1) and (2)  $(A \cup B)' = A' \cap B'$

Exp11: if  $U = \{1, 2, 3, 4, 5, 6\}$   $A = \{2, 3\}$ ,  $B = \{3, 4, 5\}$

Then show that  $(A \cap B)' = A' \cup B'$

Sol:  $U = \{1, 2, 3, 4, 5, 6\}$   $A = \{2, 3\}$ ,  $B = \{3, 4, 5\}$

$$A \cap B = \{2, 3\} \cap \{3, 4, 5\}$$

$$A \cap B = \{3\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, 4, 5, 6\} - \{3\}$$

$$(A \cap B)' = \{1, 2, 4, 5, 6\} \dots\dots\dots(1)$$

Now  $A' = \{1, 2, 3, 4, 5, 6\} - \{2, 3\}$

$$A' = \{1, 4, 5, 6\}$$

And  $B' = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5\}$

$$B' = \{1, 2, 6\}$$

$$A' \cup B' = \{1, 4, 5, 6\} \cup \{1, 2, 6\}$$

$$A' \cup B' = \{1, 2, 4, 5, 6\} \dots\dots\dots(2)$$

From eq (1) and (2)  $(A \cap B)' = A' \cup B'$

### Exercise 5.2

Q1: Verify commutative property of union and intersection for the following sets.

i).  $A = \{1, 2, 3, \dots, 12\}$ ,  $B = \{2, 4, 5, 8, 10, 12\}$

Sol:  $A \cup B = \{1, 2, 3, \dots, 12\} \cup \{2, 4, 5, 8, 10, 12\}$

$$A \cup B = \{1, 2, 3, \dots, 12\} \dots\dots(1)$$

Now  $B \cup A = \{2, 4, 5, 8, 10, 12\} \cup \{1, 2, 3, \dots, 12\}$

$$B \cup A = \{1, 2, 3, \dots, 12\} \dots\dots(2)$$

from equations (1) and (2) we get

$$A \cup B = B \cup A \text{ Hence}$$

commutative law with respect to Union holds

Now For Intersection

$$A \cap B = \{1, 2, 3, \dots, 12\} \cap \{2, 4, 5, 8, 10, 12\}$$

$$A \cap B = \{2, 4, 5, 8, 10, 12\} \dots\dots(3)$$

Now  $B \cap A = \{2, 4, 5, 8, 10, 12\} \cap \{1, 2, 3, \dots, 12\}$

$$B \cap A = \{2, 4, 5, 8, 10, 12\} \dots\dots(4)$$

from equations (3) and (4) we get

$$A \cap B = B \cap A \text{ Hence}$$

commutative law with respect to Intersection holds

ii).  $A = N = \{1, 2, 3, \dots\}$ ,

$$B = \{x/x \in N \wedge x \text{ is an even integer}\} = \{2, 4, 6, \dots\}$$

Sol:  $A \cup B = \{1, 2, 3, \dots\} \cup \{2, 4, 6, \dots\}$

$$A \cup B = \{1, 2, 3, \dots\} \dots\dots(1)$$

Now  $B \cup A = \{2, 4, 6, \dots\} \cup \{1, 2, 3, \dots\}$

$$B \cup A = \{1, 2, 3, \dots\} \dots\dots(2)$$

from equations (1) and (2) we get

$$A \cup B = B \cup A \text{ Hence}$$

commutative law with respect to Union holds

Now For Intersection

$$A \cap B = \{1, 2, 3, \dots\} \cap \{2, 4, 6, \dots\}$$

$$A \cap B = \{2, 4, 6, \dots\} \dots\dots(3)$$

Now  $B \cap A = \{2, 4, 6, \dots\} \cap \{1, 2, 3, \dots\}$

$$B \cap A = \{2, 4, 6, \dots\} \dots\dots(4)$$

from equations (3) and (4) we get

$$A \cap B = B \cap A \text{ Hence}$$

commutative law with respect to Intersection holds

iii).  $A =$  set of first ten prime numbers

$$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

$B =$  Set of first ten composite numbers

$$B = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$$

Sol:

$$A \cup B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \cup \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$$

$$A \cup B = \{2, 3, 4, \dots, 19, 23, 29\} \dots\dots(1)$$

Now

$$B \cup A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

$$B \cup A = \{2, 3, 4, \dots, 19, 23, 29\} \dots\dots(2)$$

from equations (1) and (2) we get

$$A \cup B = B \cup A \text{ Hence}$$

commutative law with respect to Union holds

Now For Intersection

$$A \cap B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\} \cap \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$$

$$A \cap B = \{ \} \dots\dots(3)$$

Now

$$B \cap A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

$$B \cap A = \{ \} \dots\dots(4)$$

from equations (3) and (4) we get

$$A \cap B = B \cap A \text{ Hence}$$

commutative law with respect to Intersection holds

Q2: Verify associative property of union and intersection for the following sets.

i)  $A = \{a, b, c, \dots, z\}, B = \{a, e, i, o, u\}, C = \{a, d, i, l, m, n, o\}$

Sol: For Union  $A \cup (B \cup C) = (A \cup B) \cup C$

Take LHS

$$A \cup (B \cup C) = \{a, b, c, \dots, z\} \cup (\{a, e, i, o, u\} \cup \{a, d, i, l, m, n, o\})$$

$$A \cup (B \cup C) = \{a, b, c, \dots, z\} \cup \{a, d, e, i, l, m, n, o, u\}$$

$$A \cup (B \cup C) = \{a, b, c, \dots, z\} \dots (1)$$

Take RHS

$$(A \cup B) \cup C = (\{a, b, c, \dots, z\} \cup \{a, e, i, o, u\}) \cup \{a, d, i, l, m, n, o\}$$

$$(A \cup B) \cup C = \{a, b, c, \dots, z\} \cup \{a, d, i, l, m, n, o\}$$

$$(A \cup B) \cup C = \{a, b, c, \dots, z\} \dots (2)$$

From equations (1) and (2) we get

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Now for intersection  $A \cap (B \cap C) = (A \cap B) \cap C$

Take LHS

$$A \cap (B \cap C) = \{a, b, c, \dots, z\} \cap (\{a, e, i, o, u\} \cap \{a, d, i, l, m, n, o\})$$

$$A \cap (B \cap C) = \{a, b, c, \dots, z\} \cap \{a, i, o\}$$

$$A \cap (B \cap C) = \{a, i, o\} \dots (3)$$

Take RHS

$$(A \cap B) \cap C = (\{a, b, c, \dots, z\} \cap \{a, e, i, o, u\}) \cap \{a, d, i, l, m, n, o\}$$

$$(A \cap B) \cap C = \{a, e, i, o, u\} \cap \{a, d, i, l, m, n, o\}$$

$$(A \cap B) \cap C = \{a, i, o\} \dots (4)$$

From equations (3) and (4) we get

$$A \cap (B \cap C) = (A \cap B) \cap C$$

ii).  $A = \{1, 2, 3, \dots, 100\}, B = \{2, 4, 6, \dots, 100\}, C = \{1, 3, 5, \dots, 99\}$

Sol: For Union  $A \cup (B \cup C) = (A \cup B) \cup C$

Take LHS

$$A \cup (B \cup C) = \{1, 2, 3, \dots, 100\} \cup (\{2, 4, 6, \dots, 100\} \cup \{1, 3, 5, \dots, 99\})$$

$$A \cup (B \cup C) = \{1, 2, 3, \dots, 100\} \cup \{1, 3, 5, \dots, 99\}$$

$$A \cup (B \cup C) = \{1, 2, 3, \dots, 100\} \dots (1)$$

Take RHS

$$(A \cup B) \cup C = (\{1, 2, 3, \dots, 100\} \cup \{2, 4, 6, \dots, 100\}) \cup \{1, 3, 5, \dots, 99\}$$

$$(A \cup B) \cup C = \{1, 2, 3, \dots, 100\} \cup \{1, 3, 5, \dots, 99\}$$

$$(A \cup B) \cup C = \{1, 2, 3, \dots, 100\} \dots (2)$$

From equations (1) and (2) we get

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Now for intersection  $A \cap (B \cap C) = (A \cap B) \cap C$

Take LHS

$$A \cap (B \cap C) = \{1, 2, 3, \dots, 100\} \cap (\{2, 4, 6, \dots, 100\} \cap \{1, 3, 5, \dots, 99\})$$

$$A \cap (B \cap C) = \{1, 2, 3, \dots, 100\} \cap \{ \}$$

$$A \cap (B \cap C) = \{ \} \dots (3)$$

Take RHS

$$(A \cap B) \cap C = (\{1, 2, 3, \dots, 100\} \cap \{2, 4, 6, \dots, 100\}) \cap \{1, 3, 5, \dots, 99\}$$

$$(A \cap B) \cap C = \{2, 4, 6, \dots, 100\} \cap \{1, 3, 5, \dots, 99\}$$

$$(A \cap B) \cap C = \{ \} \dots (4)$$

From equations (3) and (4) we get

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Q3: Verify distributive property of union over intersection and intersection over union

i).  $A = \{0, 1, 2\}, B = \{0\}, C = \{ \} = \phi$

Solution: For Union over intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Take LHS

$$A \cup (B \cap C) = \{0, 1, 2\} \cup (\{0\} \cap \{ \})$$

$$A \cup (B \cap C) = \{0, 1, 2\} \cup \{ \}$$

$$A \cup (B \cap C) = \{0, 1, 2\} \dots (1)$$

Now Take RHS

$$(A \cup B) \cap (A \cup C) = (\{0, 1, 2\} \cup \{0\}) \cap (\{0, 1, 2\} \cup \{ \})$$

$$(A \cup B) \cap (A \cup C) = \{0, 1, 2\} \cup \{0, 1, 2\}$$

$$(A \cup B) \cap (A \cup C) = \{0, 1, 2\} \dots (2)$$

From equations (1) and (2) we get

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Now for Intersection over Union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Take LHS

$$A \cap (B \cup C) = \{0, 1, 2\} \cap (\{0\} \cup \{ \})$$

$$A \cap (B \cup C) = \{0, 1, 2\} \cap \{0\}$$

$$A \cap (B \cup C) = \{0\} \dots (3)$$

Now Take RHS

$$(A \cap B) \cup (A \cap C) = (\{0, 1, 2\} \cap \{0\}) \cup (\{0, 1, 2\} \cap \{ \})$$

$$(A \cap B) \cup (A \cap C) = \{0\} \cup \{ \}$$

$$(A \cap B) \cup (A \cap C) = \{0\} \dots (4)$$

From equations (3) and (4) we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

ii).  $A = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\},$

$$B = \{-1, -2, -3, -4, -5\}, C = \{-1, -2, +3, +4\}$$

Solution: For Union over intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Take LHS

$$A \cup (B \cap C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup (\{-1, -2, -3, -4, -5\} \cap \{-1, -2, +3, +4\})$$

$$A \cup (B \cap C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup \{-1, -2\}$$

$$A \cup (B \cap C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \dots (1)$$

Now Take RHS

$$(A \cup B) \cap (A \cup C) = (\{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup \{-1, -2, -3, -4, -5\})$$

$$\cap (\{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cup \{-1, -2, +3, +4\})$$

$$(A \cup B) \cap (A \cup C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cap \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$$

$$(A \cup B) \cap (A \cup C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \dots (2)$$

From equations (1) and (2) we get

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Now for Intersection over Union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Take LHS

$$A \cap (B \cup C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$$

$$\cap (\{-1, -2, -3, -4, -5\} \cup \{-1, -2, +3, +4\})$$

$$A \cap (B \cup C) = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cap \{-1, -2, \pm 3, \pm 4, -5\}$$

$$A \cap (B \cup C) = \{-1, -2, \pm 3, \pm 4, -5\} \dots (3)$$

Now Take RHS

$$(A \cap B) \cup (A \cap C) = (\{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cap \{-1, -2, -3, -4, -5\}) \cup (\{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\} \cap \{-1, -2, +3, +4\})$$

$$(A \cap B) \cup (A \cap C) = \{-1, -2, -3, -4, -5\} \cup \{-1, -2, +3, +4\}$$

$$(A \cap B) \cup (A \cap C) = \{-1, -2, \pm 3, \pm 4, -5\} \dots (4)$$

From equations (3) and (4) we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Q4: Verify De. Morgan's laws for set of following sets.

i).  $U = \{x/x \in \mathbb{N} \wedge 1 \leq x \leq 20\}$

$$A = \{2, 3, 5, 7, 11, 12, 13, 17\} \quad B = \{1, 4, 6, 8, 10, 14, 17, 18\}$$

Solution: First we find  $A', B'$

$$A' = U - A = \{1, 2, 3, \dots, 20\} - \{2, 3, 5, 7, 11, 12, 13, 17\}$$

$$A' = \{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\} \dots (1)$$

$$B' = U - B = \{1, 2, 3, \dots, 20\} - \{1, 4, 6, 8, 10, 14, 17, 18\}$$

$$B' = \{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\} \dots (2)$$

Now

$$A \cup B = \{2, 3, 5, 7, 11, 12, 13, 17\} \cup \{1, 4, 6, 8, 10, 14, 17, 18\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 17, 18\}$$

$$(A \cup B)' = U - A \cup B$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 17, 18\}$$

$$(A \cup B)' = \{9, 15, 16, 19, 20\} \dots (3)$$

$$A' \cap B' = \{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\} \cap \{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$$

$$A' \cap B' = \{9, 15, 16, 19, 20\} \dots (4)$$

From equations (3) and (4) we get

$$(A \cup B)' = A' \cap B'$$

Now  $A \cap B = \{2, 3, 5, 7, 11, 12, 13, 17\} \cap \{1, 4, 6, 8, 14, 17, 18\}$

$$A \cap B = \{17\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, \dots, 20\} - \{17\}$$

$$(A \cap B)' = \{1, 2, 3, \dots, 16, 18, 19, 20\} \dots (5)$$

$$A' \cup B' = \{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\} \cup \{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$$

$$A' \cup B' = \{1, 2, \dots, 16, 18, 19, 20\} \dots (6)$$

From equations (5) and (6) we get

$$(A \cap B)' = A' \cup B'$$

ii)  $U = \{1, 2, 3, \dots, 10\}, A = \{2, 4, 6, 8, 10\}, B = \{1, 3, 5, 7, 9\}$

Solution: First we find  $A', B'$

$$A' = U - A = \{1, 2, 3, \dots, 10\} - \{2, 4, 6, 8, 10\}$$

$$A' = \{1, 3, 5, 7, 9\} \dots (1)$$

$$B' = U - B = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$B' = \{2, 4, 6, 8, 10\} \dots (2)$$

Now  $A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 3, 5, 7, 9\}$

$$A \cup B = \{1, 2, 3, \dots, 10\}$$

And  $(A \cup B)' = U - (A \cup B)$

$$(A \cup B)' = \{1, 2, 3, \dots, 10\} - \{1, 2, 3, \dots, 10\} = \{ \} \dots (3)$$

$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\}$$

$$A' \cap B' = \{ \} \dots (4)$$

From equations (3) and (4) we get

$$(A \cup B)' = A' \cap B'$$

Now

$$A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$$

$$A \cap B = \{ \}$$

$$(A \cap B)' = U - A \cap B$$

$$= \{1, 2, 3, \dots, 10\} - \{ \}$$

$$(A \cap B)' = \{1, 2, \dots, 10\} \dots (5)$$

$$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$$

$$A' \cup B' = \{1, 2, 3, \dots, 10\} \dots (6)$$

From equations (5) and (6) we get

$$(A \cap B)' = A' \cup B'$$

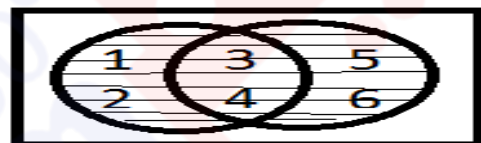
Exp 14i). If  $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\},$

$C = \{3, 4, 7, 8\}$  the verify  $A \cup B = B \cup A$  with the help of Venn diagram.

Sol:  $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\},$

$$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$



$$B \cup A = \{3, 4, 5, 6\} \cup \{1, 2, 3, 4\}$$

$$B \cup A = \{1, 2, 3, 4, 5, 6\}$$



Thus  $A \cup B = B \cup A$

Exp 14ii). If  $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\},$

$C = \{3, 4, 7, 8\}$  the verify  $A \cap B = B \cap A$  with the help of Venn diagram.

Sol:  $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\},$

$$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$$

$$A \cap B = \{3, 4\}$$



$$B \cap A = \{3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$$

$$B \cap A = \{3, 4\}$$



Thus  $A \cap B = B \cap A$



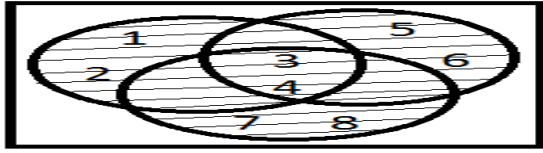
Exp 14iii). If  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  
 $C = \{3, 4, 7, 8\}$  the verify with the help of Venn diagram.  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol:  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{3, 4, 7, 8\}$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6\} \cap \{3, 4, 7, 8\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{3, 4\}$$

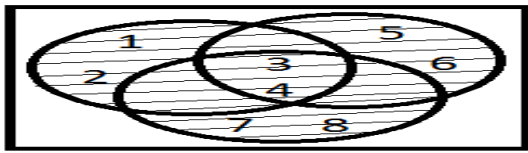
$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7, 8\}$$



$$(A \cup B) \cap C = (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}) \cap \{3, 4, 7, 8\}$$

$$(A \cup B) \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 7, 8\}$$

$$(A \cup B) \cap C = \{3, 4\}$$



Hence  $A \cup (B \cap C) = (A \cup B) \cap C$

Exp 14iv). If  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  
 $C = \{3, 4, 7, 8\}$  the verify with the help of Venn diagram.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol:  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{3, 4, 7, 8\}$

Sol:  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{3, 4, 7, 8\}$

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6\} \cup \{3, 4, 7, 8\})$$

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{3, 4\}$$

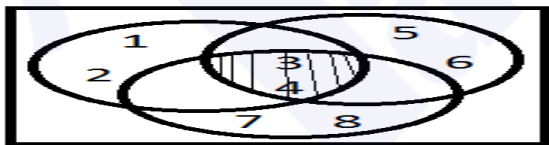
$$A \cap (B \cup C) = \{3, 4\}$$



$$(A \cap B) \cup (A \cap C) = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}) \cup (\{1, 2, 3, 4\} \cap \{3, 4, 7, 8\})$$

$$(A \cap B) \cup (A \cap C) = \{3, 4\} \cup \{3, 4\}$$

$$(A \cap B) \cup (A \cap C) = \{3, 4\}$$



Hence  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

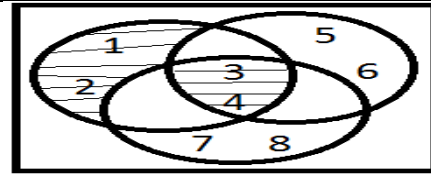
Exp 14v). If  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  
 $C = \{3, 4, 7, 8\}$  the verify with the help of Venn diagram.  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol:  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{3, 4, 7, 8\}$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6\} \cap \{3, 4, 7, 8\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{3, 4\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

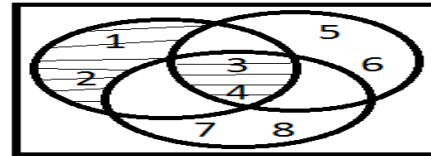


$$(A \cup B) \cap (A \cup C) =$$

$$(\{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}) \cap (\{1, 2, 3, 4\} \cup \{3, 4, 7, 8\})$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 7, 8\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4\}$$



Hence  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

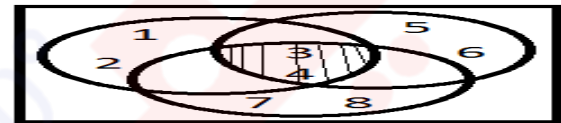
Exp 14vi). If  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  
 $C = \{3, 4, 7, 8\}$  the verify with the help of Venn diagram.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol:  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{3, 4, 7, 8\}$

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6\} \cup \{3, 4, 7, 8\})$$

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}$$

$$A \cap (B \cup C) = \{3, 4\}$$

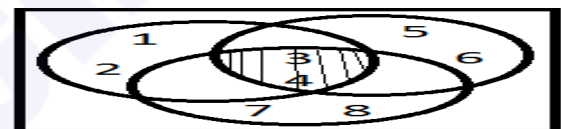


$$(A \cap B) \cup (A \cap C) =$$

$$(\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}) \cup (\{1, 2, 3, 4\} \cap \{3, 4, 7, 8\})$$

$$(A \cap B) \cup (A \cap C) = \{3, 4\} \cup \{3, 4\}$$

$$(A \cap B) \cup (A \cap C) = \{3, 4\}$$



Hence  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Exp15i)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $A \cup B$

Sol: Given  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

$$A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$$

$$A \cup B = \{1, 2, 3, 5, 6\}$$



Exp15ii)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $A \cap B$

Sol: Given  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

$$A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$$

$$A \cap B = \{2\}$$



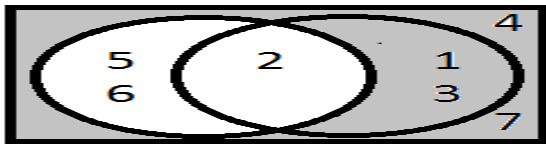
Exp15iii)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$

Then draw Venn diagram for  $A'$

Sol: Given  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$

$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{2, 5, 6\}$$

$$A' = \{1, 3, 4, 7\}$$



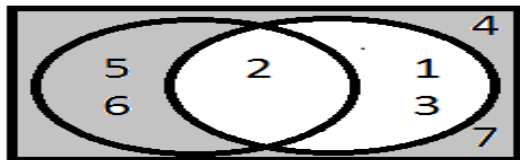
Exp15iv)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $B'$

Sol: Given  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $B = \{1, 2, 3\}$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3\}$$

$$B' = \{4, 5, 6, 7\}$$



Exp15v)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $(A \cup B)'$

Sol: Given  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

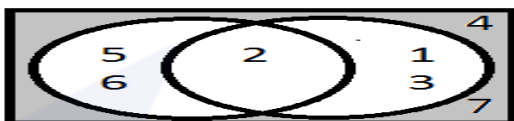
$$A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$$

$$A \cup B = \{1, 2, 3, 5, 6\}$$

Now  $(A \cup B)' = U - (A \cup B)$

$$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 5, 6\}$$

$$(A \cup B)' = \{4, 7\}$$



Ep15vi)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $A' \cap B'$

Sol: Given  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{2, 5, 6\}$$

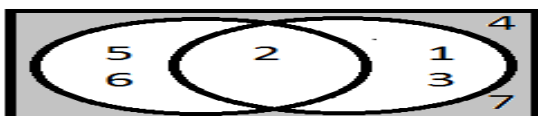
$$A' = \{1, 3, 4, 7\}$$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3\}$$

$$B' = \{4, 5, 6, 7\}$$

$$A' \cap B' = \{1, 3, 4, 7\} \cap \{4, 5, 6, 7\}$$

$$A' \cap B' = \{4, 7\}$$



Ep15vii)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $(A \cap B)'$

Sol: Given  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

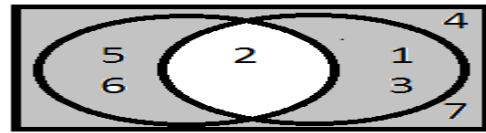
$$A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$$

$$A \cap B = \{2\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{2\}$$

$$(A \cap B)' = \{1, 3, 4, 5, 6, 7\}$$



E15viii)  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Then draw Venn diagram for  $A' \cup B'$

Sol: Given  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{2, 5, 6\}$$

$$A' = \{1, 3, 4, 7\}$$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3\}$$

$$B' = \{4, 5, 6, 7\}$$

$$A' \cup B' = \{1, 3, 4, 7\} \cup \{4, 5, 6, 7\}$$

$$A' \cup B' = \{1, 3, 4, 5, 6, 7\}$$



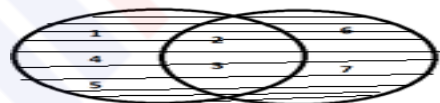
### Exercise 5.3

Q1i): If  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{2, 3, 6, 7\}$

then draw the Venn diagram for  $A \cup B$

Sol:  $A \cup B = \{1, 2, 3, 4, 5\} \cup \{2, 3, 6, 7\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

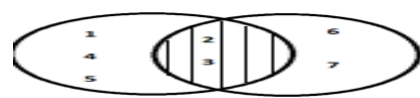


Q1ii): If  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{2, 3, 6, 7\}$

then draw the Venn diagram for  $A \cap B$

Sol:  $A \cap B = \{1, 2, 3, 4, 5\} \cap \{2, 3, 6, 7\}$

$$A \cap B = \{2, 3\}$$



Q2i). If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$

and  $C = \{5, 6, 9, 10\}$  then verify with the help of

Venn diagrams.  $A \cup (B \cap C) = (A \cup B) \cap C$

Sol: If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$  and

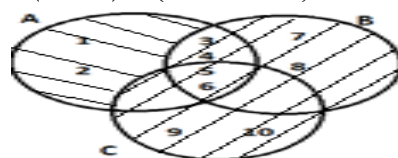
$$C = \{5, 6, 9, 10\}$$

Take LHS

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 9, 10\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A \cup (B \cap C) = \{1, 2, 3, \dots, 10\}$$



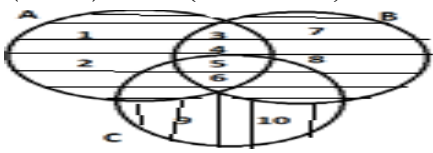
.....(1)

Now RHS

$$(A \cup B) \cup C = (\{1, 2, 3, 4, 5, 6\} \cup \{3, 4, 5, 6, 7, 8\}) \cup \{5, 6, 9, 10\}$$

$$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{5, 6, 9, 10\}$$

$$(A \cup B) \cup C = \{1, 2, 3, \dots, 10\}$$



.....(2)

From equation (1) and (2) we get

$$A \cup (B \cap C) = (A \cup B) \cup C$$

Q2ii). If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$

and  $C = \{5, 6, 9, 10\}$  then verify with the help of

Venn diagrams.  $A \cap (B \cap C) = (A \cap B) \cap C$

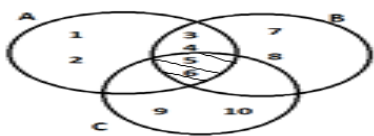
Sol: If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$  and  $C = \{5, 6, 9, 10\}$

Take LHS

$$A \cap (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cap (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 9, 10\})$$

$$A \cap (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cap \{5, 6\}$$

$$A \cap (B \cap C) = \{5, 6\}$$



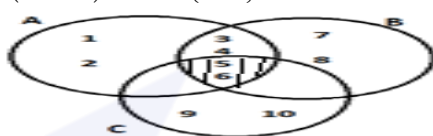
.....(1)

Now RHS

$$(A \cap B) \cap C = (\{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 5, 6, 7, 8\}) \cap \{5, 6, 9, 10\}$$

$$(A \cap B) \cap C = \{3, 4, 5, 6\} \cap \{5, 6, 9, 10\}$$

$$(A \cap B) \cap C = \{5, 6\}$$



.....(2)

From equations (1) and (2) we get

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Q2iii). If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$

and  $C = \{5, 6, 9, 10\}$  then verify with the help of

Venn diagrams  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

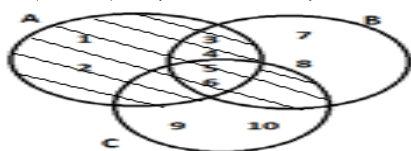
Sol: If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$  and  $C = \{5, 6, 9, 10\}$

Take LHS

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 9, 10\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$$



.....(1)

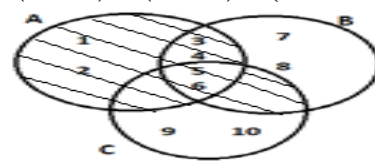
Now RHS

$$(A \cup B) \cap (A \cup C) = (\{1, 2, 3, 4, 5, 6\} \cup \{3, 4, 5, 6, 7, 8\})$$

$$\cap (\{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 9, 10\})$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{1, 2, 3, 4, 5, 6, 9, 10\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\}$$



.....(2)

From equations (1) and (2) we obtained

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Q2iv). If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$

and  $C = \{5, 6, 9, 10\}$  then verify with the help of

Venn diagrams.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .

Sol: If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{3, 4, 5, 6, 7, 8\}$  and  $C = \{5, 6, 9, 10\}$

Solution: Take LHS

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cap (\{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 9, 10\})$$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A \cap (B \cup C) = \{3, 4, 5, 6\}$$



.....(1)

Now RHS

$$(A \cap B) \cup (A \cap C) = (\{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 5, 6, 7, 8\})$$

$$\cup (\{1, 2, 3, 4, 5, 6\} \cap \{5, 6, 9, 10\})$$

$$(A \cap B) \cup (A \cap C) = \{3, 4, 5, 6\} \cup \{5, 6\}$$

$$(A \cap B) \cup (A \cap C) = \{3, 4, 5, 6\}$$



.....(2)

From equations (1) and (2) we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Q3: If  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 2, 3, 4\}$  and

$B = \{3, 4, 5\}$  then draw the Venn diagrams for

$A'$ ,  $B'$ ,  $A' \cup B'$ ,  $A' \cap B'$  and also verify that

Sol:  $A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4\}$

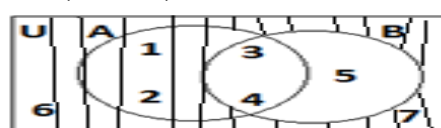
$$A' = \{5, 6, 7\} \dots (1)$$



.....(1)

Now  $B' = U - B = \{1, 2, 3, 4, 5, 6, 7\} - \{3, 4, 5\}$

$$B' = \{1, 2, 6, 7\} \dots (2)$$

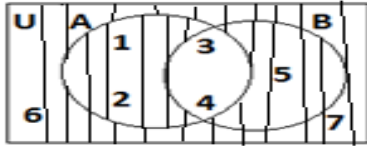


.....(2)

Using equation (1) and (2)

$$A' \cup B' = \{5, 6, 7\} \cup \{1, 2, 6, 7\}$$

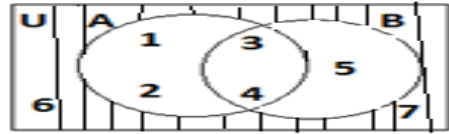
$$A' \cup B' = \{1, 2, 5, 6, 7\}$$



Using equation (1) and (2)

$$A' \cap B' = \{5, 6, 7\} \cap \{1, 2, 6, 7\}$$

$$A' \cap B' = \{1, 2, 5, 6, 7\}$$



Q3i): If  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 2, 3, 4\}$  &  $B = \{3, 4, 5\}$  then draw Venn diagrams for

$$(A \cup B)' = A' \cap B'$$

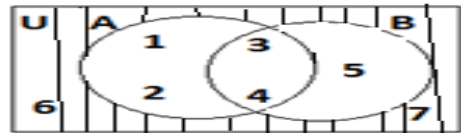
Sol: LHS  $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5\}$

$$A \cup B = \{1, 2, 3, 4, 5\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4, 5\}$$

$$(A \cup B)' = \{6, 7\}$$



$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4\}$$

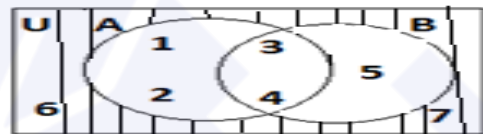
$$A' = \{5, 6, 7\}$$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7\} - \{3, 4, 5\}$$

$$B' = \{1, 2, 6, 7\}$$

$$A' \cap B' = \{5, 6, 7\} \cap \{1, 2, 6, 7\}$$

$$A' \cap B' = \{6, 7\}$$



From (1) and (2) we get  $(A \cup B)' = A' \cap B'$

Q3i): If  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 2, 3, 4\}$  &  $B = \{3, 4, 5\}$  then draw Venn diagrams for

$$(A \cap B)' = A' \cup B'$$

Sol:  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 2, 3, 4\}$   $B = \{3, 4, 5\}$

$$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5\}$$

$$A \cap B = \{3, 4\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{3, 4\}$$

$$(A \cap B)' = \{1, 2, 5, 6, 7\}$$



$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4\}$$

$$A' = \{5, 6, 7\}$$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7\} - \{3, 4, 5\}$$

$$B' = \{1, 2, 6, 7\}$$

$$A' \cup B' = \{5, 6, 7\} \cup \{1, 2, 6, 7\}$$

$$A' \cup B' = \{1, 2, 5, 6, 7\}$$



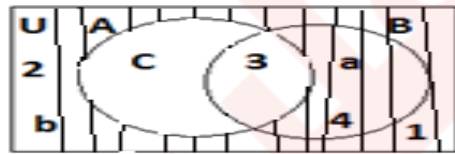
Hence  $(A \cap B)' = A' \cup B'$

Q4: If  $U = \{a, b, c, 1, 2, 3, 4\}$ ,  $A = \{c, 3\}$  &

$B = \{a, 3, 4\}$  then draw Venn diagrams for  $A'$ ,  $B'$ ,  $A \setminus B$ ,  $B \setminus A$

Sol:  $A' = U - A = \{a, b, c, 1, 2, 3, 4\} - \{c, 3\}$

$$A' = \{a, b, 1, 2, 4\}$$



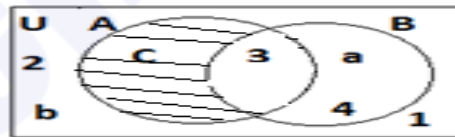
$$B' = U - B = \{a, b, c, 1, 2, 3, 4\} - \{a, 3, 4\}$$

$$B' = \{b, c, 1, 2\}$$



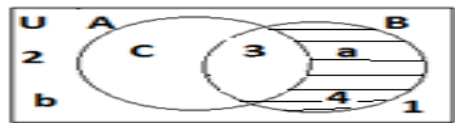
Now  $A \setminus B = \{c, 3\} - \{a, 3, 4\}$

$$A \setminus B = \{c\}$$



Now  $B \setminus A = \{a, 3, 4\} \setminus \{c, 3\}$

$$B \setminus A = \{a, 4\}$$



Q5. If  $U = \{a, b, c, d, e, f, g\}$ ,  $A = \{a, b, c\}$ , and

$B = \{c, d, e\}$  then verify De-Morgan's laws with the help of Venn diagrams

Sol:  $A' = U - A = \{a, b, c, d, e, f, g\} - \{a, b, c\}$

$$A' = \{d, e, f, g\} \dots (1)$$

$$B' = U - B = \{a, b, c, d, e, f, g\} - \{c, d, e\}$$

$$B' = \{a, b, f, g\} \dots (2)$$

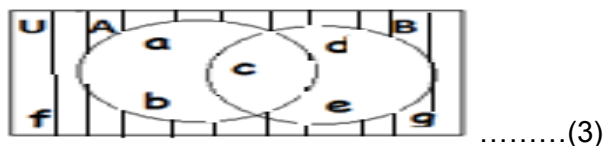
Now  $A \cup B = \{a, b, c\} \cup \{c, d, e\}$

$$A \cup B = \{a, b, c, d, e\}$$

$$(A \cup B)' = U - (A \cup B) = \{a, b, c, d, e, f, g\} - \{a, b, c, d, e\}$$



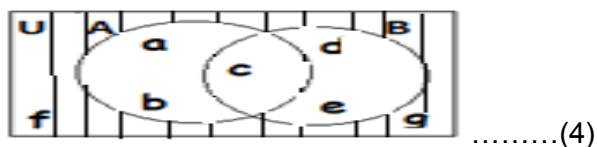
$$(A \cup B)' = \{f, g\} \dots\dots\dots(3)$$



Using equation (1) and (2)

$$A' \cap B' = \{d, e, f, g\} \cap \{a, b, f, g\}$$

$$A' \cap B' = \{f, g\} \dots\dots\dots(4)$$

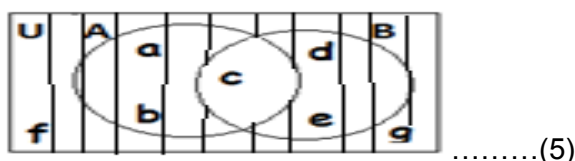


$$\text{Now } A \cap B = \{a, b, c\} \cap \{c, d, e\}$$

$$A \cap B = \{c\}$$

$$\text{Now } (A \cap B)' = U - (A \cap B) = \{a, b, c, d, e, f, g\} - \{c\}$$

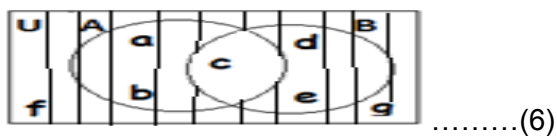
$$(A \cap B)' = \{a, b, d, e, f, g\} \dots\dots\dots(5)$$



Using equation (1) and (2)

$$A' \cup B' = \{d, e, f, g\} \cup \{a, b, f, g\}$$

$$A' \cup B' = \{a, b, d, e, f, g\} \dots\dots\dots(6)$$



From equations (3) and (4) we get

$$(A \cup B)' = A' \cap B'$$

From equations (5) and (6) we get

$$(A \cap B)' = A' \cup B'$$

**ORDERED PAIRS:** Ordered pair is the pair of elements in which order of the elements is kept maintained. For example  $(x, y)$  is an ordered pair whose first element is  $x$  and second element is  $y$ .

**OR ORDERED PAIRS:** Ordered pair is the pair of elements in which order of the elements is maintained.

**EXP16:** Find  $x$  &  $y$  given  $(2x, x + y) = (6, 2)$

Sol: Given  $(2x, x + y) = (6, 2)$  two order pairs are equal iff corresponding components are equal. Hence  $2x = 6$   $x + y = 2$

$$\begin{aligned} x = 3 & \Rightarrow 3 + y = 2 \\ & y = 2 - 3 \\ & y = -1 \end{aligned}$$

#### CARTESIAN PRODUCT OF SETS:

Cartesian product of two sets  $A$  and  $B$  is a set consisting of all the ordered pairs  $(a, b)$  such that  $a \in A$  and  $b \in B$ . This Cartesian product is denoted by  $A \times B$  and is read as  $A$  cross  $B$ .  $A \times B = \{(a, b) : a \in A, b \in B\}$ . Remember that  $A \times \phi = \phi \times A = \phi$

**BINARY RELATION:** If  $A$  and  $B$  are two sets, then every subset of  $R$  of the set  $A \times B$  is called binary relation from  $A$  to  $B$ .

**OR BINARY RELATION:** A subset of  $A \times B$  is said to be binary relation from  $A$  to  $B$ .

**NOTE THAT** if  $(a, b) \in R$  then we write  $a R b$

If  $(a, b) \notin R$  then we write  $a \not R b$

Exp17: If  $A = \{a, b\}$  &  $B = \{1, 2\}$  then find  $A \times B$  and write all binary relations from  $A$  to  $B$

Sol: Given  $A = \{a, b\}$  &  $B = \{1, 2\}$

$$\text{Thus } A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

$$\text{Number of elements } n(A \times B) = 2 \times 2 = 4$$

$$\text{Number of binary relations/ possible subset} = 2^4 = 16$$

And binary relations are

$$\begin{aligned} R_1 &= \phi & R_2 &= \{(a, 1)\} \\ R_3 &= \{(a, 2)\} & R_4 &= \{(b, 1)\} \\ R_5 &= \{(b, 2)\} & R_6 &= \{(a, 1), (a, 2)\} \\ R_7 &= \{(a, 1), (b, 1)\} & R_8 &= \{(a, 1), (b, 2)\} \\ R_9 &= \{(a, 2), (b, 1)\} & R_{10} &= \{(a, 2), (b, 2)\} \\ R_{11} &= \{(b, 1), (b, 2)\} & R_{12} &= \{(a, 1), (a, 2), (b, 1)\} \\ R_{13} &= \{(a, 1), (b, 1), (b, 2)\} \\ R_{14} &= \{(a, 1), (a, 2), (b, 2)\} \\ R_{15} &= \{(a, 2), (b, 1), (b, 2)\} \\ R_{16} &= \{(a, 1), (a, 2), (b, 1), (b, 2)\} \end{aligned}$$

Similarly, total number of binary relation in  $B \times A = 2^4 = 16$

Exp18: Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3\}$  then find  $A \times B$  and write any five relations from  $A$  to  $B$

Sol: Given  $A = \{1, 2\}$ ,  $B = \{1, 2, 3\}$

$$A \times B = \{1, 2\} \times \{1, 2, 3\}$$

$$A \times B = \{(1, 1), (1, 2), (1, 3),$$

$$(2, 1), (2, 2), (2, 3)\}$$

Number of binary relations  $= 2^6 = 64$  but according to Question five are

$$R_1 = \{(1, 1)\}$$

$$R_2 = \{(1, 2)\}$$

$$R_3 = \{(2, 1), (2, 2)\}$$

$$R_4 = \{(2, 2), (2, 3)\}$$

$$R_5 = \{(2, 3)\}$$

**DOMAIN OF BINARY RELATION:** If set  $R$  is the binary relation, then the set consisting of first elements of all ordered pairs of  $R$  is called the Domain of  $R$ .

**OR DOMAIN OF BINARY RELATION:** A set consisting of first elements of all ordered pairs of a binary relation is called the Domain of the relation.

**RANGE OF BINARY RELATION:** A set consisting of second elements of all ordered pairs of a binary relation is called the Range of the relation.

**OR RANGE OF BINARY RELATION:** If set R is binary relation, then set consisting of second elements of all ordered pairs of R is called Range of R.

**Exp 19:** Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3\}$  Define a relation from A to B as  $R = \{(a, b) \in A \times B | a < b\}$

then i). find the ordered pairs in R

ii). Find the domain and range of R

iii). Is  $1R3$ ,  $2R2$ ?

Sol: Given  $A \times B = \{1, 2\} \times \{1, 2, 3\}$

$$A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$$

Given that  $R = \{(a, b) \in A \times B | a < b\}$

So  $R = \{(1, 2), (1, 3), (2, 3)\}$

Domain of  $R = \{1, 2\}$  and Range of  $R = \{2, 3\}$

$1R3$  means  $(1, 3) \in R$  true

$2R2$  means  $(2, 2) \notin R$  thus  $2 \not R 2$

### Exercise 5.4

Q1: If  $A = \{1, 2, 3\}$ ,  $B = \{4, 5\}$  then

i). write 3 binary relations from A to B

Solution:  $n(A) = 3, n(B) = 2$

$$A \times B = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$$

Possible binary relations  $2^{m \times n} = 2^{3 \times 2} = 2^6 = 64$

$$R_1 = \{(1, 4), (2, 4)\}$$

$$R_2 = \{(1, 4), (3, 4)\}$$

$$R_3 = \{(1, 5), (2, 4)\}$$

ii). write 4 binary relations from B to A

Solution:  $n(A) = 3, n(B) = 2$

$$B \times A = \{(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$$

Possible binary relations  $2^{n \times m} = 2^{2 \times 3} = 2^6 = 64$

$$R_1 = \{(4, 1), (5, 1)\}$$

$$R_2 = \{(4, 1), (4, 2), (4, 3)\}$$

$$R_3 = \{(4, 2), (4, 3), (5, 1), (5, 2)\}$$

$$R_4 = \{(4, 1), (4, 3), (5, 3)\}$$

iii). write 4 binary relations in A

Solution:  $n(A) = 3$

$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Possible binary relations  $2^{m \times m} = 2^{3 \times 3} = 2^9$

$$R_1 = \{(1, 1), (1, 2)\}$$

$$R_2 = \{(1, 2), (1, 3), (2, 3)\}$$

$$R_3 = \{(3, 1), (1, 3), (2, 2)\}$$

$$R_4 = \{(1, 1), (1, 3), (3, 3), (2, 1)\}$$

iv). write 2 binary relations in B

Solution:  $n(B) = 2$

$$B \times B = \{(4, 4), (4, 5), (5, 4), (5, 5)\}$$

Possible binary relations  $2^{n \times n} = 2^{2 \times 2} = 2^4 = 16$

$$R_1 = \{(4, 4), (4, 5)\}$$

$$R_2 = \{(5, 4)\}$$

Q2. If  $A = \{1, 2, 3, 4\}$ ,  $B = \{1, 3, 5\}$  &

$R = \{(x, y) | y < x\}$  is a binary relation from A to B, write it in tabular form.

Solution: Given the relation  $R = \{(x, y) | y < x\}$

$$A \times B = \{(1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$$

With condition  $x > y$ ,  $x \in A, y \in B$  i.e.,

$$R = \{(2, 1), (3, 1), (4, 1), (4, 3)\}$$

Q3. Domain of a binary relation  $\{(x, y) | y = 2x\}$

in set w is  $\{0, 4, 8\}$ , find Range of R

Solution: Given the relation  $\{(x, y) | y = 2x\}$

With condition  $y = 2x$ ,  $x \in \{0, 4, 8\}$

We have to find the corresponding values of y

When  $x = 0$   $y = 2(0) = 0$

When  $x = 4$   $y = 2(4) = 8$

When  $x = 8$   $y = 2(8) = 16$

Thus the range =  $\{0, 8, 16\}$

Q4: Domain of a binary relation

$R = \{(x, y) | y + 1 = 2x^2\}$  is set N. Find its range

Solution: Given the relation

$$R = \{(x, y) | y + 1 = 2x^2\}$$

With condition

$$y + 1 = 2x^2$$

$$y = 2x^2 - 1 \quad x \in N = \{1, 2, 3, \dots\}$$

Since we know that Set N is infinite so we find few corresponding values of y

$$\text{When } x = 1 \quad y = 2(1)^2 - 1 = 2 - 1 = 1$$

$$\text{When } x = 2 \quad y = 2(2)^2 - 1 = 8 - 1 = 7$$

$$\text{When } x = 3 \quad y = 2(3)^2 - 1 = 18 - 1 = 17$$

$$\text{When } x = 4 \quad y = 2(4)^2 - 1 = 32 - 1 = 31$$

And so on

Thus the range =  $\{1, 7, 17, 31, \dots\}$

**FUNCTION:** A binary relation  $f$  between the two non-empty sets A and B such that

1.  $Dom(f) = A$

2. there is no repetition in the first elements of any two ordered pairs of  $f$ .

Then  $f$  is said to be function from  $A$  to  $B$ . It is written as  $f : A \rightarrow B$ .

Exp20i): If  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  then  $f_1 = \{(1, a), (2, b)\}$  is function or not

Sol: Given  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  and  $f_1 = \{(1, a), (2, b)\}$

$f_1$  is not a function because  $\text{Dom } f_1 = \{1, 2\} \neq A$

Exp20ii): If  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  then  $f_2 = \{(1, a), (2, b), (3, c), (3, d)\}$  is function or not

Sol: Given  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  and  $f_2 = \{(1, a), (2, b), (3, c), (3, d)\}$

$f_2$  is not a function because  $\text{Dom } 3 \in A$  is repeated

Exp20iii): If  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  then  $f_3 = \{(1, a), (2, b), (3, c)\}$  is function or not

Sol: Given  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  and  $f_3 = \{(1, a), (2, b), (3, c)\}$

$f_3$  is a function because  $\text{Dom } f_3 = A$  And there is no repetition in Dom of  $f_3$  so,  $f_3 : A \rightarrow B$

Exp20iv): If  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  then  $f_4 = \{(1, a), (2, a), (3, d)\}$  is function or not

Sol: Given  $A = \{1, 2, 3\}, B = \{a, b, c, d\}$  and  $f_4 = \{(1, a), (2, a), (3, d)\}$

$f_4$  is a function because  $\text{Dom } f_4 = A$  And there is no repetition in Dom of  $f_4$  so,  $f_4 : A \rightarrow B$

#### DOMAIN, CO-DOMAIN & RANGE OF A FUNCTION

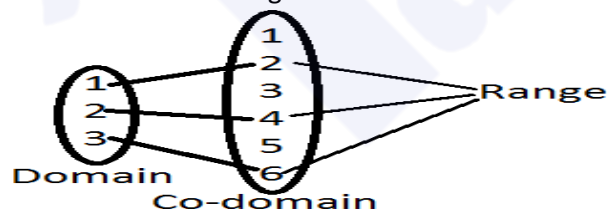
Let  $f : A \rightarrow B$  then set  $A$  is called Domain

The set  $B$  is called Codomain

And the set of second elements of all ordered pair contained in  $f$  is called range e.g.  $A = \{1, 2, 3\}$

And  $B = \{1, 2, 3, 4, 5, 6\}$  then  $f : A \rightarrow B$

As shown in arrow diagram



#### INTO FUNCTION:

If  $f : A \rightarrow B$  such that  $\text{Range}(f) \subset B$ . i.e.

$\text{Range}(f) \neq B$ , then  $f$  is called an into function.

#### ONTO FUNCTION OR SURJECTIVE FUNCTION:

If  $f : A \rightarrow B$  such that  $\text{Range}(f) = B$ , then  $f$  is called an onto function.

#### ONE TO ONE FUNCTION OR 1-1 FUNCTION

**OR INJECTIVE FUNCTION:** If  $f : A \rightarrow B$  such that there is no repetition in the Second elements of

any its two ordered pairs, then  $f$  is called one to one function or injective function.

Exp: if  $A = \{1, 2, 3\}$  and  $B = \{a, b, c\}$  then  $f = \{(1, a), (2, b), (3, c)\}$  is one-one function

#### BIJECTIVE FUNCTION:

If  $f : A \rightarrow B$  such that

1.  $f$  is **onto** function
2.  $f$  is **one to one** function.

Then  $f$  is called bijective function.

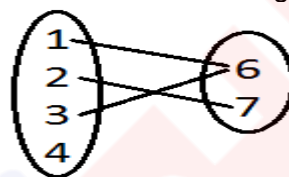
**OR BIJECTIVE FUNCTION:** If  $f : A \rightarrow B$  is bijective function such that  $f$  is both **one to one** and **onto** function.

### Exercise 5.5

Q1:  $A = \{1, 2, 3, 4\}$  and  $B = \{6, 7\}$  and following are the relations from  $A$  to  $B$ , then state whether these are function or not? If these are function then state which kind of functions are they?

i).  $R_1 = \{(1, 6), (2, 7), (3, 6)\}$

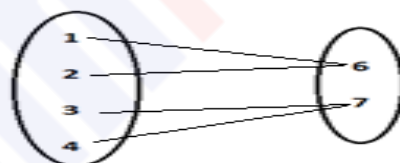
Solution: the Venn diagram of the relation



Not a function because  $\text{Dom } R_1 \neq A$

ii).  $R_2 = \{(1, 6), (2, 6), (3, 7), (4, 7)\}$

Solution: the Venn diagram of the relation



**Domain**

**Range**

$R_2$  is a function

$R_2$  is onto function or surjective function

iii).  $R_3 = \{(1, 6), (2, 6), (3, 6), (4, 6)\}$

Solution: the Venn diagram of the relation



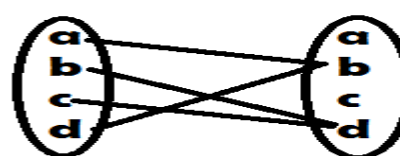
**Domain**

**Codomain**

$R_3$  is a into function

Q2: Which of following relations on set  $\{a, b, c, d\}$  function? State the kind of functions as well?

i).  $\{(a, b), (c, d), (b, d), (d, b)\}$

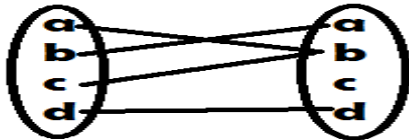


**Domain**

**Codomain**

Given relation is into function

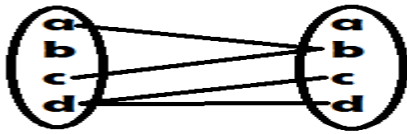
ii).  $\{(b, a), (c, b), (a, b), (d, d)\}$



**Domain Codomain**

Given relation is a function

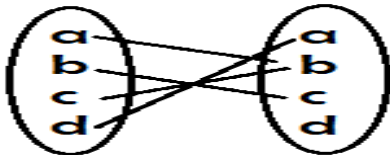
iii).  $\{(d, c), (c, b), (a, b), (d, d)\}$



**Domain Codomain**

Given relation is not a function because d is repeated in domain and  $b \notin$  domain

iv).  $\{(a, b), (b, c), (c, b), (d, a)\}$



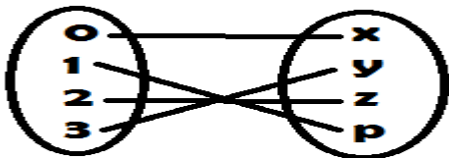
Given relation is into function

Q3: If  $A = \{0, 1, 2, 3\}$  and  $B = \{x, y, z, p\}$  then

state whether following relations show that there exists one-one corresponding between the elements of set A and B. If not give the reason

i).  $\{(0, x), (2, z), (3, y), (1, p)\}$

Sol: the Venn diagram of the given relation

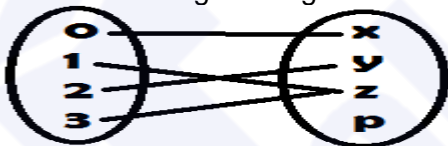


**Domain Codomain**

The given relation is bijective function and show the one-one corresponding

ii).  $\{(0, x), (1, z), (2, y), (3, z)\}$

Sol: the Venn diagram of given relation is



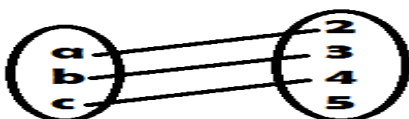
**Domain Codomain**

given relation is function, there is not one-one corresponding because in range z is repeated

Q4: If  $C = \{a, b, c\}$  and  $D = \{2, 3, 4, 5\}$  then state,

whether the following relations shows that there exists one-one correspondence between the elements of set C and D if not what kind of these relations?

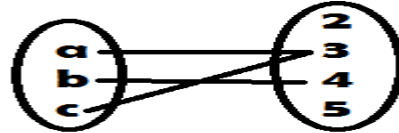
i).  $\{(a, 2), (b, 3), (c, 4)\}$



**Domain Codomain**

The given relation show one one function but not one-one corresponding

ii).  $\{(a, 3), (b, 4), (c, 3)\}$



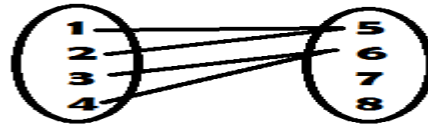
**Domain Codomain**

The given relation is function, there is not one-one corresponding because in range 3 is repeated

Q5: If  $X = \{1, 2, 3, 4\}$  &  $Y = \{5, 6, 7, 8\}$  then write

i). a function from x to y

Sol:  $f = \{(1, 5), (2, 5), (3, 6), (4, 6)\}$  Venn diagram

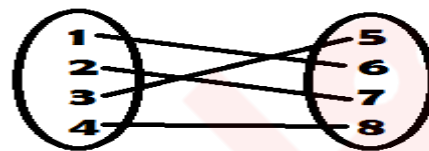


**Domain Codomain**

f is into function

ii). a one-one function from X to Y

Sol:  $f = \{(1, 6), (2, 7), (3, 5), (4, 8)\}$  Venn diagram



**Domain Codomain**

f is one-one function

iii). a relation which shows that there exists one-one correspondence between X to Y

Sol:  $f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$  & Venn diagram

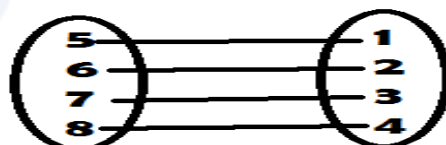


**Domain Codomain**

f is one-one function and there exists one-one correspondence

iv). A function which is onto from Y to X

Solution:  $f = \{(5, 1), (6, 2), (7, 3), (8, 4)\}$

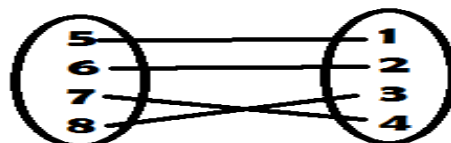


**Domain Codomain**

f is onto function

v). a bijective function from Y to X

Solution:  $f = \{(5, 1), (6, 2), (7, 4), (8, 3)\}$

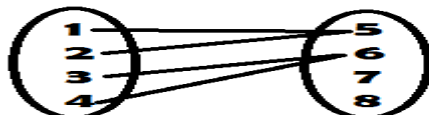


**Domain Codomain**

f is bijective function

vi). a function from X to Y which is neither one one nor onto.

Sol:  $f = \{(1, 5), (2, 5), (3, 6), (4, 6)\}$

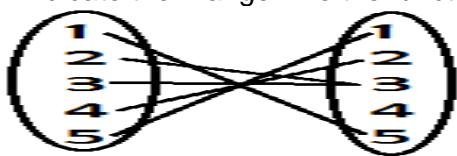


**Domain Codomain**

f is neither one-one nor onto function

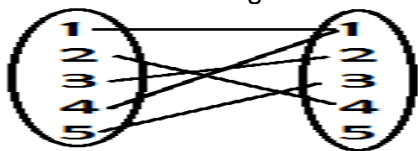


Q6i) Let  $A = \{1, 2, 3, 4, 5\}$  check whether  $\{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)\}$  is function on A. indicate their range. Are the function is onto.



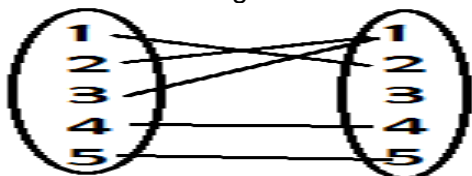
Given relation is a function Dom =  $\{1, 2, 3, 4, 5\}$   
And Range =  $\{1, 2, 3, 5\}$  but function is not onto because Range  $\neq A$

Q6ii) Let  $A = \{1, 2, 3, 4, 5\}$  check whether  $\{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$  is function on A. indicate their range. Are the function is onto.



Given relation is a function Dom =  $\{1, 2, 3, 4, 5\}$   
And Range =  $\{1, 2, 3, 4\}$  but function is not onto because Range  $\neq A$

Q6iii) Let  $A = \{1, 2, 3, 4, 5\}$  check whether  $\{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$  is function on A. indicate their range. Are the function is onto.



Given relation is a function Dom =  $\{1, 2, 3, 4, 5\}$   
And Range =  $\{1, 2, 3, 5\}$  but function is not onto because Range  $\neq A$

### Review Exercise 5

Q1. Full in the correct circle

i). The relation in  $A = \{1, 2, 3\}, B = \{4, 5\}$  is

$R = \{(1, 4), (2, 5), (3, 4)\}$  the R is

- ☐ one-one function      ☐ function from A to A  
☐ Not a function      ☐ onto function from A to B

ii). If A has 2 elements and B has 3 elements then the number of binary relations in  $A \times B$  is

- ☐  $2 \times 3$       ☐  $2^3$   
☐  $2^6$       ☐  $2^2$

iii). Which one of following are disjoint sets

- ☐  $\{0, 1, 2, 3\} \{3, 2, 1, 0\}$       ☐  $\{0, 2, 4, 6\} \{2, 4, 6, 8\}$   
☐  $\{0, 3, 6, 9\} \{9, 16, 25, 36\}$       ☐  $\{0, 4, 8, 12\} \{6, 10, 14, 18\}$

iv). If universal set  $U = \{x | x \text{ is a positive odd integer less than } 30\}$   $R = \{1, 5, 7\}, S = \{1, 3, 7, 11, 13\}$

how many elements are in  $(R \cap S)'$

- ☐ 15      ☐ 13  
☐ 7      ☐ 2

v). If  $f$  is a function from A to B, then  $f$  is onto function if

- ☐ Range  $f = B$       ☐ Range  $f \neq A$   
☐ Dom  $f = A$       ☐ 2<sup>nd</sup> element of all order pair is not repeated

vi) If  $R = \{(0, 0), (8, 2), (10, 3), (14, 12)\}$  then Dom R

- ☐  $\{0, 8, 10, 14\}$       ☐  $\{0, 2, 3, 12\}$   
☐  $\{8, 10, 4\}$       ☐  $\{0, 10\}$

Q2i). If  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Then find  $A' \cup B'$

Sol: if  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Given set are in tabular form

$U = \{1, 2, 3, \dots, 100\}, A = \{2, 4, 6, \dots, 100\}$

$B = \{1, 3, 5, \dots, 99\}$  Now

$A' = U - A$

$A' = \{1, 2, 3, \dots, 100\} - \{2, 4, 6, \dots, 100\}$

$A' = \{1, 3, 5, \dots, 99\}$

$B' = U - B$

$B' = \{1, 2, 3, \dots, 100\} - \{1, 3, 5, \dots, 99\}$

$B' = \{2, 4, 6, \dots, 100\}$

Now  $A' \cup B' = \{1, 3, 5, \dots, 99\} \cup \{2, 4, 6, \dots, 100\}$

$A' \cup B' = \{1, 2, 3, \dots, 100\}$

Q2ii). If  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Then find  $A' \cap B'$

Sol: if  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Given set are in tabular form

$U = \{1, 2, 3, \dots, 100\}, A = \{2, 4, 6, \dots, 100\}$

$B = \{1, 3, 5, \dots, 99\}$  Now

$A' = U - A$

$A' = \{1, 2, 3, \dots, 100\} - \{2, 4, 6, \dots, 100\}$

$A' = \{1, 3, 5, \dots, 99\}$

$B' = U - B$

$B' = \{1, 2, 3, \dots, 100\} - \{1, 3, 5, \dots, 99\}$

$B' = \{2, 4, 6, \dots, 100\}$

Now  $A' \cap B' = \{1, 3, 5, \dots, 99\} \cap \{2, 4, 6, \dots, 100\}$

$A' \cap B' = \{\}$

Q2iii). If  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Then find  $A \cap B'$

Sol: if  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Given set are in tabular form

$$U = \{1, 2, 3, \dots, 100\}, A = \{2, 4, 6, \dots, 100\}$$

$$B = \{1, 3, 5, \dots, 99\} \text{ Now}$$

$$B' = U - B$$

$$B' = \{1, 2, 3, \dots, 100\} - \{1, 3, 5, \dots, 99\}$$

$$B' = \{2, 4, 6, \dots, 100\}$$

$$\text{Now } A \cap B' = \{2, 4, 6, \dots, 100\} \cap \{2, 4, 6, \dots, 100\}$$

$$A \cap B' = \{2, 4, 6, \dots, 100\}$$

Q2iv). If  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Then find  $A' \cap B$

Sol: if  $U = \{\text{Natural number up to } 100\}$

$A = \{\text{Positive even number up to } 100\}$

$B = \{\text{Positive odd number up to } 100\}$

Given set are in tabular form

$$U = \{1, 2, 3, \dots, 100\}, A = \{2, 4, 6, \dots, 100\}$$

$$B = \{1, 3, 5, \dots, 99\} \text{ Now}$$

$$A' = U - A$$

$$A' = \{1, 2, 3, \dots, 100\} - \{2, 4, 6, \dots, 100\}$$

$$A' = \{1, 3, 5, \dots, 99\}$$

$$\text{Now } A' \cap B = \{1, 3, 5, \dots, 99\} \cap \{1, 3, 5, \dots, 99\}$$

$$A' \cap B = \{1, 3, 5, \dots, 99\}$$

Q3i). If  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

Then verify associative property of union.

Sol: Given  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

We have to verify that  $A \cup (B \cup C) = (A \cup B) \cup C$

$$A \cup (B \cup C) = \{1, 2, 3, 5, 7\} \cup (\{2, 4, 6\} \cup \{2, 5, 9\})$$

$$A \cup (B \cup C) = \{1, 2, 3, 5, 7\} \cup \{2, 4, 5, 6, 9\}$$

$$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7, 9\} \dots (1)$$

$$(A \cup B) \cup C = (\{1, 2, 3, 5, 7\} \cup \{2, 4, 6\}) \cup \{2, 5, 9\}$$

$$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7\} \cup \{2, 5, 9\}$$

$$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\} \dots (2)$$

From eq (1) and eq (2) we get

Hence  $A \cup (B \cup C) = (A \cup B) \cup C$  Proved

Q3ii). If  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

Then verify associative property of Intersection.

Sol: Given  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

We have to verify that  $A \cap (B \cap C) = (A \cap B) \cap C$

$$A \cap (B \cap C) = \{1, 2, 3, 5, 7\} \cap (\{2, 4, 6\} \cap \{2, 5, 9\})$$

$$A \cap (B \cap C) = \{1, 2, 3, 5, 7\} \cap \{2\}$$

$$A \cap (B \cap C) = \{2\} \dots (1)$$

$$(A \cap B) \cap C = (\{1, 2, 3, 5, 7\} \cap \{2, 4, 6\}) \cap \{2, 5, 9\}$$

$$(A \cap B) \cap C = \{2\} \cap \{2, 5, 9\}$$

$$(A \cap B) \cap C = \{2\} \dots (2)$$

From eq (1) and eq (2) we get

Hence  $A \cap (B \cap C) = (A \cap B) \cap C$  Proved

Q3iii). If  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

verify Distributive property of union over intersection

Sol: Given  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

To verify that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = \{1, 2, 3, 5, 7\} \cup (\{2, 4, 6\} \cap \{2, 5, 9\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 5, 7\} \cup \{2\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 5, 7\} \dots (1)$$

$$(A \cup B) \cap (A \cup C) =$$

$$(\{1, 2, 3, 5, 7\} \cup \{2, 4, 6\}) \cap (\{1, 2, 3, 5, 7\} \cup \{2, 5, 9\})$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 5, 7, 9\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 5, 7\} \dots (2)$$

From eq (1) and eq (2) we get

Hence  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$  Proved

Q3iv). If  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

verify Distributive property of intersection over union

Sol: Given  $A = \{1, 2, 3, 5, 7\}, B = \{2, 4, 6\}, C = \{2, 5, 9\}$

To verify that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cap (B \cup C) = \{1, 2, 3, 5, 7\} \cap (\{2, 4, 6\} \cup \{2, 5, 9\})$$

$$A \cap (B \cup C) = \{1, 2, 3, 5, 7\} \cap \{2, 4, 5, 6, 9\}$$

$$A \cap (B \cup C) = \{2, 5\} \dots (1)$$

$$(A \cap B) \cup (A \cap C) =$$

$$(\{1, 2, 3, 5, 7\} \cap \{2, 4, 6\}) \cup (\{1, 2, 3, 5, 7\} \cap \{2, 5, 9\})$$

$$(A \cap B) \cup (A \cap C) = \{2\} \cup \{2, 5\}$$

$$(A \cap B) \cup (A \cap C) = \{2, 5\} \dots (2)$$

From eq (1) and eq (2) we get

Hence  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  Proved

Q4. If  $U = \{x | x \in N \wedge 1 \leq x \leq 40\}$

$$A = \{1, 6, 11, 16, 21, 26, 31\},$$

$$B = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$$

Then verify De Morgan's Laws.

Sol: Given  $U = \{1, 2, 3, \dots, 40\}$  and

$$A = \{1, 6, 11, 16, 21, 26, 31\},$$

$$B = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$$

Take

$$A \cup B = \{1, 2, 5, 6, 8, 11, 14, 16, 17, 20, 21, 26, 31, 32\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$(A \cup B)' = \{3, 4, 7, 9, 10, 12, 13, 15, 18, 19, 22, 23, 24, 25, 27, 28, 29, 30, 33, 34, \dots, 40\} \dots (1)$$

$$A \cap B = \{11\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, \dots, 10, 12, 13, 14, \dots, 40\} \dots (2)$$

$$A' = U - A$$

$$A' = \{2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19,$$

20, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, ..., 40}

$$B' = U - B$$

$$B' = \{1, 3, 4, 6, 7, 9, 10, 12, 13, 15, 16, 18, 19, 21, 22$$

$$24, 25, 27, 28, 30, 31, 33, 34, 35, \dots, 40\}$$

$$A' \cap B' = \{3, 4, 7, 9, 10, 12, 13, 15, 18, 19, 22, 24,$$

$$25, 27, 28, 30, 33, 34, 35, \dots, 40\} \dots (3)$$

$$A' \cup B' = \{1, 2, 3, \dots, 10, 12, 13, 14, \dots, 40\} \dots (4)$$

From eq (1) & eq (3)      From eq (2) & eq (4)

$$(A \cup B)' = A' \cap B' \quad (A \cap B)' = A' \cup B'$$

Q5 If  $U = \{1, 2, 3, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

Sol:  $U = \{1, 2, 3, 5, 6, 7\}$ ,  $A = \{2, 5, 6\}$ ,  $B = \{1, 2, 3\}$

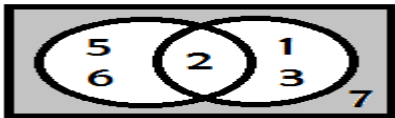
$$A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$$

$$A \cup B = \{1, 2, 3, 5, 6\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$(A \cup B)' = \{1, 2, 3, 5, 6, 7\} - \{1, 2, 3, 5, 6\}$$

$$(A \cup B)' = \{7\} \dots (1)$$



$$A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$$

$$A \cap B = \{2\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, 5, 6, 7\} - \{2\}$$

$$(A \cap B)' = \{1, 3, 5, 6, 7\} \dots (2)$$



$$A' = U - A$$

$$A' = \{1, 2, 3, 5, 6, 7\} - \{2, 5, 6\}$$

$$A' = \{1, 3, 7\}$$

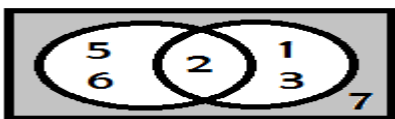
$$B' = U - B$$

$$B' = \{1, 2, 3, 5, 6, 7\} - \{1, 2, 3\}$$

$$B' = \{5, 6, 7\}$$

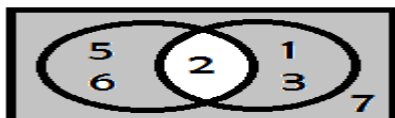
$$A' \cap B' = \{1, 3, 7\} \cap \{5, 6, 7\}$$

$$A' \cap B' = \{7\} \dots (3)$$



$$A' \cup B' = \{1, 3, 7\} \cup \{5, 6, 7\}$$

$$A' \cup B' = \{1, 3, 5, 6, 7\} \dots (4)$$



From eq (1) & eq (3)      From eq (2) & eq (4)

$$(A \cup B)' = A' \cap B' \quad (A \cap B)' = A' \cup B'$$

Q6. If  $U = \{1, 2, 3, \dots, 10\}$ ,  $A = \{1, 2, 3, 4\}$ ,

$B = \{3, 4, 5, 6\}$ ,  $C = \{3, 4, 7, 8\}$  then verify

distributive laws with help of Venn Diagram.

Sol:  $U = \{1, 2, 3, \dots, 10\}$ ,  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$

$$C = \{3, 4, 7, 8\}$$

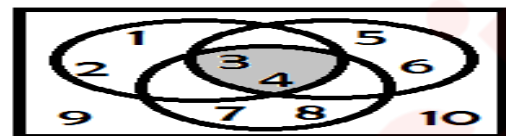
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Take

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6\} \cup \{3, 4, 7, 8\})$$

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}$$

$$A \cap (B \cup C) = \{3, 4\} \dots (1)$$

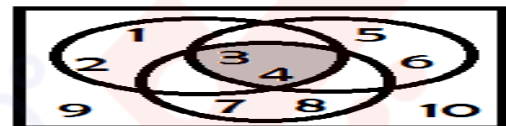


$$(A \cap B) \cup (A \cap C) = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\})$$

$$\cup (\{1, 2, 3, 4\} \cap \{3, 4, 7, 8\})$$

$$(A \cap B) \cup (A \cap C) = \{3, 4\} \cup \{3, 4\}$$

$$(A \cap B) \cup (A \cap C) = \{3, 4\} \dots (2)$$



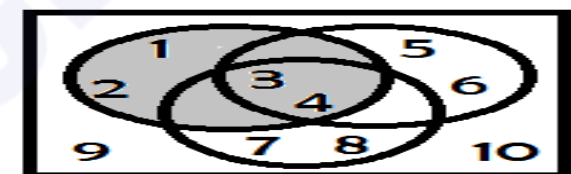
From eq (1) and (2) we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6\} \cap \{3, 4, 7, 8\})$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{3, 4\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4\} \dots (3)$$

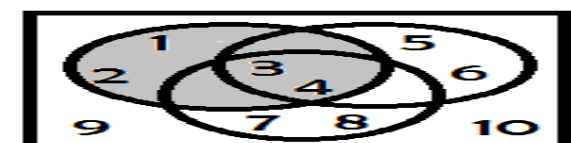


$$(A \cup B) \cap (A \cup C) = (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6\})$$

$$\cap (\{1, 2, 3, 4\} \cup \{3, 4, 7, 8\})$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 7, 8\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4\} \dots (4)$$



From eq (3) and eq (4) we get

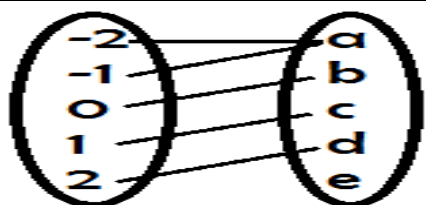
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Q7i) If  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$  are

relation  $\{(-2, a), (-1, a), (0, b), (1, c), (2, d)\}$  is function mention the type also

Sol: Given sets  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$

And relation  $\{(-2, a), (-1, a), (0, b), (1, c), (2, d)\}$



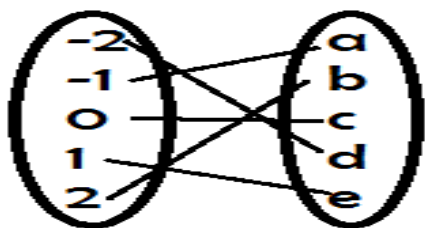
Domain Codomain

Given relation is a function but neither one-one nor onto function.

Q7ii) If  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$  are relation  $\{(-1, a), (1, e), (-2, d), (0, c), (2, b)\}$  is function mention the type also

Sol: Given sets  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$

And relation  $\{(-1, a), (1, e), (-2, d), (0, c), (2, b)\}$



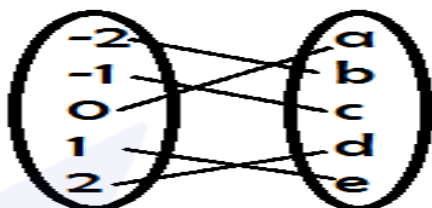
Domain Codomain

Given relation is a function and one-one also onto function thus it is bijective function.

Q7iii) If  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$  are relation  $\{(2, d), (0, a), (-2, b), (-1, c), (1, e)\}$  is function mention the type also

Sol: Given sets  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$

And relation  $\{(2, d), (0, a), (-2, b), (-1, c), (1, e)\}$



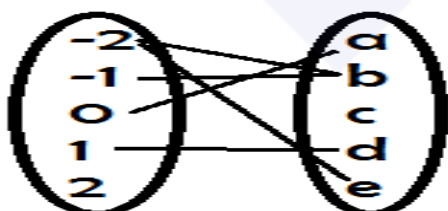
Domain Codomain

Given relation is a function and one-one also onto function thus it is bijective function.

Q7iv) If  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$  are relation  $\{(-2, b), (-1, b), (0, a), (1, d), (-2, e)\}$  is function mention the type also

Sol: Given sets  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$

And relation  $\{(-2, b), (-1, b), (0, a), (1, d), (-2, e)\}$



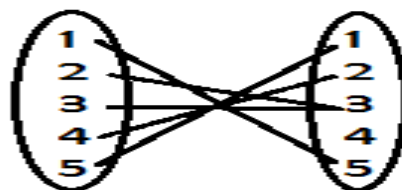
Domain Codomain

Given relation is a function and one-one also onto function thus it is bijective function.

Q8i). If  $A = \{1, 2, 3, 4, 5\}$  whether the relation  $\{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)\}$  are function. Indicate their range. Are function is onto.

Sol: Given  $A = \{1, 2, 3, 4, 5\}$  and relation in A is

$\{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)\}$



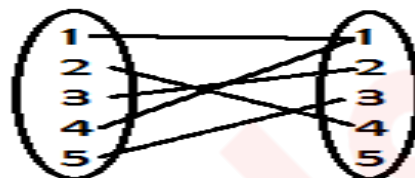
Domain Codomain

Given relation is a function and its range is  $\{1, 2, 3, 5\}$  but function is not onto

Q8ii). If  $A = \{1, 2, 3, 4, 5\}$  whether the relation  $\{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$  are function. Indicate their range. Are function is onto.

Sol: Given  $A = \{1, 2, 3, 4, 5\}$  and relation in A is

$\{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$



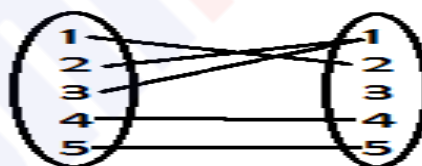
Domain Codomain

Given relation is a function and its range is  $\{1, 2, 3, 4\}$  but function is not onto

Q8iii). If  $A = \{1, 2, 3, 4, 5\}$  whether the relation  $\{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$  are function. Indicate their range. Are function is onto.

Sol: Given  $A = \{1, 2, 3, 4, 5\}$  and relation in A is

$\{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$



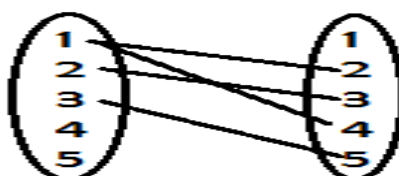
Domain Codomain

Given relation is a function and its range is  $\{1, 2, 4, 5\}$  but function is not onto

Q8iv). If  $A = \{1, 2, 3, 4, 5\}$  whether the relation  $\{(1, 2), (2, 3), (1, 4), (3, 5)\}$  are function. Indicate their range. Are function is onto.

Sol: Given  $A = \{1, 2, 3, 4, 5\}$  and relation in A is

$\{(1, 2), (2, 3), (1, 4), (3, 5)\}$



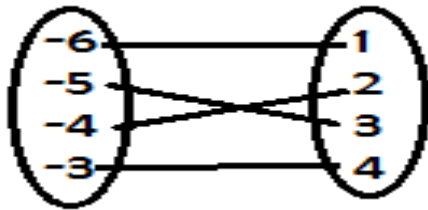
Domain Codomain

Given relation is not a function and its range is  $\{2, 3, 4, 5\}$

Q9i). If  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$  then write one-one function from X to Y.

Sol: Given  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$

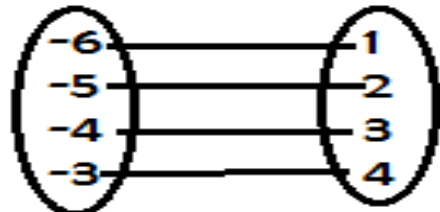




One-one function from X to Y is  
 $\{(-6,1), (-5,3), (-4,2), (-3,4)\}$

Q9ii). If  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$  then write onto function from X to Y.

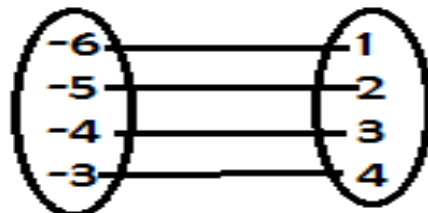
Sol: Given  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$



Onto function from X to Y is  
 $\{(-6,1), (-5,2), (-4,3), (-3,4)\}$

Q9iii). If  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$  then write one-one function from X to Y.

Sol: Given  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$



One-one function from X to Y is  
 $\{(-6,1), (-5,2), (-4,3), (-3,4)\}$

Q9iv). If  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$  then write function from X to Y which is neither one-one nor onto.

Sol: Given  $X = \{-6, -5, -4, -3\}$ ,  $Y = \{1, 2, 3, 4\}$



function from X to Y which is neither one-one nor onto  $\{(-6,1), (-5,2), (-4,4), (-3,4)\}$

Project:  $(-2,5), (-1,0), (0,1)$  and  $(2,5)$  is a function. If the x-values and y-values of this function are switched. The relation that results is not a function. Provide an example of a function that will still be a function if the x-values and y-values are switched.

Sol: Given  $(-2,5), (-1,0), (0,1)$  &  $(2,5)$



Given relation is onto function but do not defined Domain and codomain  
 When the x-values and y-values of this function are switched



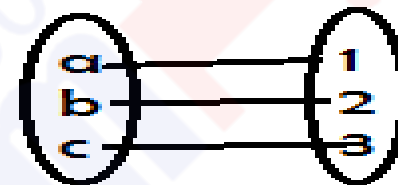
New switched relation is not a function because in domain 5 is repeated

Example Let  $A = \{1, 2, 3\}$  to  $B = \{a, b, c\}$



A function from A to B  $f = \{(1,a), (2,b), (3,c)\}$   
 $f$  is a bijective function

When the x-values and y-values of this function are switched



New function  $f^{-1} = \{(a,1), (b,2), (c,3)\}$

Therefore  $f^{-1}$  is also a bijective function.

**Note that** in a case of switched only bijective function remains a function.