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Chapter 1

NUMBER SYSTEMS

EXERCISE 1.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Name the property used in. (Rwp 11 G-I)
(a) $4+9 = 9+4$

Ans. (a) Commutative law of additive.

Q.2 Prove that $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$
(Swl 13, Mtn 13 G-I, A.J.K 17)

Ans. $\frac{a}{b} = \frac{c}{d}$

Multiplying with bd, we get

$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$

$ad = bc$

Q.3 Name the property used in
$a(b-c) = ab - ac$. (Bpr 16)(Mtn 16 G-II)

Ans. Distributivity of multiplication over subtraction.

Q.4 Does the set {1, − 1} posses closure property w.r.t. addition and multiplication?
(Bpr 12)(Fbd 11, 19 G-II), (Mtn 12 G-I) (Gjw 09)
(Sgd 17, 19)(Swl 17)(Lhr 18 G-I)

Ans. $\{1, -1\}$

As $1 + 1 = 2 \notin \{1, -1\}$

$\therefore$ The set is not closed w.r.t. addition.

and $1 \times 1 = 1 \in \{1, -1\}$

$-1 \times -1 = 1 \in \{1, -1\}$

$-1 \times 1 = -1 \in \{1, -1\}$

$\therefore$ The set has closed property w.r.t multiplication.

Q.5 Does the set {0, −1} posses closure property w.r.t. addition and multiplication? Give the reason. (Bpr 14)(Lhr 17 G-I)(Swl 19)
(Rwp 17 G-II), (Mtn 2009 G-I, 12 G-II, 17 G-II)

Ans. $\{0, -1\}$

$\because$ $0 + (-1) = 0 - 1 = -1 \in \{0, -1\}$

$0 + 0 = 0 \in \{0, -1\}$

$-1 + (-1) = -1 - 1 = -2 \notin \{0, -1\}$

$\therefore$ The set $\{0, -1\}$ does not have closed property w.r.t. addition.

and $-1 \times -1 = 1 \notin \{0, -1\}$

$\therefore$ The set $\{0, -1\}$ is not closed w.r.t. multiplication.

Q.6 Prove that $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$.
(Fbd 14)(Lhr 08, 19 G-II)

Ans. L.H.S

$= \frac{a}{c} + \frac{b}{c}$

$= \frac{1}{c} \cdot a + \frac{1}{c} \cdot b$ (Division rule)

$= \frac{1}{c}(a+b)$ (Left distributive law)

$= \frac{a+b}{c}$ (Division rule)

Q.7 Prove the following rule of addition:
$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

Ans. Left Side $= \frac{a}{b} + \frac{c}{d}$

$= \frac{ad}{bd} + \frac{bc}{bd}$ $\boxed{\text{Golden rule of fractions}}$

$= ad \times \frac{1}{bd} + bc \times \frac{1}{bd}$ $\boxed{\text{Definition of division}}$

$= (ad + bc) \times \frac{1}{bd}$ $\boxed{\text{Right distributive property}}$

$= \frac{ad+bc}{bd}$ = Right Side

Q.8 Prove that $-\frac{7}{12} - \frac{5}{18} = \frac{-21-10}{36}$
(Mtn 18-GII)(Fbd 17)

Ans. Left Side $= -\frac{7}{12} - \frac{5}{18}$

$= -\frac{7}{12} \times \frac{3}{3} - \frac{5}{18} \times \frac{2}{2}$ $\boxed{\text{Golden rule of fractions}}$

$= -\frac{21}{36} - \frac{10}{36}$

$= -21 \times \frac{1}{36} - 10 \times \frac{1}{36}$ $\boxed{\text{Definition of division}}$

$= (-21 - 10) \times \frac{1}{36}$ $\boxed{\text{Right distributive property}}$

$= \frac{-21-10}{36}$ $\boxed{\text{Definition of division}}$

= Right Side

Q.9 Name the properties used in these equations.
(a) $4 + 9 = 9 + 4$ (Fbd 19)

Ans. (a) $4 + 9 = 9 + 4$ Commutative law of addition

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If n is a prime number the $\sqrt{n}$ is:
(Lhr 09, 06 G-I)(Mtn 15 G-I), (Gjw 15)(Fbd 15) (Mtn 18 G-I)(D.G.K 17 G-II)
(a) an irrational number
(b) a rational number
(c) a prime number (d) none of these

Q.2 The number $\sqrt{3}$ is an: (Fbd, Lhr 08, Gjw 10)
(a) Natural (b) Rational
(c) Irrational (d) Integer

Q.3 $Q \cup Q'$ equal to: (Bpr G-I, Mtn 09 G-I)
(a) C (b) $\Re$
(c) Z (d) N

Q.4 Every rational number is also:
(a) Irrational number (b) Prime number
(c) Whole number (d) Real number

Q.5 Every recurring decimal represents a ——— number: (Swl 13 G-I, Mtn 11 G-I)
(a) Rational (b) Irrational
(c) Even (d) Odd

Q.6 $\frac{22}{7}$ is called a:
(a) Irrational (b) Rational
(c) Integer (d) None of these

Q.7 $\forall$ a, b $\in$ R $\Rightarrow$ ab $\in$ R is called: (D.G.K 12 G-I)
(a) Closure law (b) Commutative law
(c) Associative law (d) Distributive law

Q.8 Name the property which is used in the inequality $-3 < -2 \Rightarrow 0 < 1$: (Bpr 12 G-I)
(a) Additive property (b) Multiplicative property
(c) Both (a) and (b) (d) Reflexive

Q.9 The set {0, 1} is closed under:
(Lhr 18 G-I, Bpr Board 11 G-I, Fbd Board 12-13 G-I)
(a) Addition (b) Multiplication
(c) Subtraction (d) Division

Q.10 $\sqrt{-1}$ belongs to set of: (Rwp 13 G-I)(Sgd 17)
(a) Real numbers (b) Complex numbers
(c) Prime numbers (d) Even numbers

Q.11 $a > 0 \Rightarrow$ (Lhr 13 G-II)
(a) $-a > 0$ (b) $2a < 0$
(c) $\frac{1}{a} > 0$ (d) $1 + \frac{1}{a} < 0$

Q.12 Number 0.1010010001 ——— is: (D.G.K 14 G-II)
(a) Rational (b) Irrational
(c) Integer (d) Terminating

Q.13 $a(b - c) = ab - ac$ is called: (Mtn 16 G-II)
(a) Multiplicative property
(b) Associative property
(c) Trichotomy property
(d) Distributivity of multiplicative over subtraction

Q.13 {1, −1} is closed w.r.t: (Sgd 13)
(a) Addition (b) Multiplication
(c) Subtraction (d) Division

Q.14 Trichotomy is property of: (Lhr 10)
(a) Inequality (b) Equality
(c) Division (d) Subtraction

Q.15 A number which cannot be written in the form $\frac{p}{q}$, where p and q are relatively prime integers and $q \neq 0$ is called the
(a) rational number (b) irrational number
(c) natural number (d) whole number

Q.16 Irrational numbers are
(a) terminating decimals
(b) non- terminating decimals
(c) non- terminating , repeating decimals
(d) non- terminating , non-repeating decimals

Q.17 π is
(Mtn 11 G-II)(Lhr 04) (Sar 18, Mtn 17 G-II)
(a) integers (b) natural number
(c) rational number (d) irrational number

Q.18 π is defined as
(a) ratio of diameter of a circle to its circumference
(b) ratio of the circumference of a circle to its diameter
(c) ratio of area of a circle to its circumference
(d) ratio of the circumference of a circle to its area

Q.19 Zero is
(a) a natural number (b) a whole number
(c) a positive integer (d) a negative integer

Q.20 Let a, b, c and d $\in$ R. Then $a = b, c = d \Rightarrow$
(a) $a + c = b + d$ (b) $a + b = c + d$
(c) $a - b = c - d$ (d) none of these

Q.21 If a, b, c $\in$ R and $a > b \Rightarrow ac < bc$, then
(a) $c > 0$ (b) $c < 0$
(c) $c \geq 0$ (d) $c \leq 0$

Q.22 $a > b \Rightarrow -a < -b$

Name of the property used in the above inequation is

(a) Additive property
(b) Multiplicative property
(c) Reflexive property
(d) Transitive property

Q.23 $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$, $a \neq 0$, $b \neq 0$

Name of the property used in the above inequation is

(a) additive property
(b) multiplicative inverse property
(c) additive property
(d) transitive property

Q.24 The additive identity of real numbers is

(a) 0 (b) 1
(c) 2 (d) – 1

Q.25 For all x, y, z ∈ R $z + x = z + y \Rightarrow x = y$

What is above property called?

(a) Cancellation property w.r.t. Multiplication
(b) Cancellation property w.r.t. Addition
(c) Multiplicative property
(d) Additive property

Q.26 If x, y, z ∈ R, then name the property used in the equation given below?

$x (yz) = (xy) z$

(a) Closure property w.r.t. Multiplication.
(b) Commutative property w.r.t. Multiplication.
(c) Associative property w.r.t. Multiplication.
(d) Trichotomy property

Q.27 If a, b ∈ R, where R is a set of real numbers, then the property used in the equation: $a + b = b + a$ is called

(a) Closure property
(b) Associative property
(c) Commutative property
(d) Trichotomy property

Q.28 If x, y ∈ R, where R is a set of real numbers, then the property used in the equation $xy = yx$ is called

(a) Closure property
(b) Trichotomy property
(c) Commutative property
(d) Additive Inverse

Q.29 Name the property used in the equation:

$2 + 3 = 3 + 2$?

(a) Closure property w.r.t. Multiplication
(b) Commutative property w.r.t. Multiplication
(c) Associative property w.r.t. Multiplication
(d) Commutative Property w.r.t. Addition.

Q.30 If a, b, c ∈ R, where R is a set of real numbers, then the property used in the equation: $a + (b + c) = (a + b) + c$ is called

(a) Closure property
(b) Associative property
(c) Commutative property
(d) Additive inverse

Q.31 If a ∈ R, where R is a set of real numbers, then the property used in the equation $a + 0 = 0 + a = a$ is called

(a) Closure property
(b) Trichotomy property
(c) Commutative property
(d) Additive Identity

Q.32 For any x, y ∈ R, where R is a set of real numbers, then the property used in the equation $x(y + z) = xy + xz$ is called

(a) Closure property
(b) Associative property
(c) Commutative property
(d) Distributive Property

Q.33 For any a, b ∈ R, where R is a of real numbers. Then either $a < b$ or $a = b$ or $a > b$. The property used is called. (Fbd 19)

(a) Trichotomy Property
(b) Archmidean Property
(c) Transitive Property
(d) Multiplicative Property

Q.34 For any x, y, z ∈ R, where R is a set of real numbers. $x < y$ and $y < z \Rightarrow x < z$ The property used is called

(a) Trichotomy Property
(b) Archmidean Property
(c) Transitive Property
(d) Multiplicative Property

Q.35 The number 1 is a. (Lhr 08 G-I)

(a) Prime number (b) Irrational number
(c) Even number (d) Odd number

Q.36 The additive property of order of the real numbers is

(a) $\forall$ a, b, c $\in$ R, a < b $\Rightarrow$ a + c < b + c
(b) $\forall$ a, b, c $\in$ R, a < b $\Rightarrow$ a + c = b + c
(c) $\forall$ a, b, c $\in$ R, a < b $\Rightarrow$ a + c > b + c
(d) $\forall$ a, b, c $\in$ R, a < b $\Rightarrow$ a + c < b − c

Q.37 The reflexive property of equality of real numbers is

(a) $a = a \ \forall a \in R$ (b) $a \neq a \ \forall a \in R$
(c) $a \approx a \ \forall a \in R$ (d) $a \geq a \ \forall a \in R$

Q.38 The multiplicative property of equality of real number is

(a) $a = b \Rightarrow ac = bc \quad \forall a, b, c \in R$
(b) $a = b \Rightarrow ac = b \quad \forall a, b, c \in R$
(c) $a = b \Rightarrow a^2 = c^2 \quad \forall a, b, c \in R$
(d) $a = b \Rightarrow a = bc \quad \forall a, b, c \in R$

Q.39 The left distributive property of real numbers is

(a) $(b + c)\,a = a + b + c \ \forall a, b, c \in R$
(b) $(a + b)\,c = ac + bc \ \forall a, b, c \in R$
(c) $a\,(b + c) = ab + ac \ \forall a, b, c \in R$
(d) $(a + b)\,c = ab + c \ \forall a, b, c \in R$

Q.40 The transitive property of equality of real numbers is

(a) $a = b \wedge b = c \Rightarrow b = -c \quad \forall a, b, c \in R$
(b) $a = b \wedge b = c \Rightarrow a = c \ \forall a, b, c \in R$
(c) $a = b \wedge b = c \Rightarrow a = 1 \ \forall a, b, c \in R$
(d) $a = b \wedge b = c \Rightarrow a^2 = b \forall a, b, c \in R$

Q.41 The symmetric property of equality of real numbers is

(a) $a = b \Rightarrow b = a \ \forall a, b \in R$
(b) $a = a \Rightarrow b = b \ \forall a \in R$
(c) $a = b \Rightarrow b = a^2 \ \forall a, b \in R$
(d) $a = b \Rightarrow a - b = 0 \ \forall a, b \in R$

Q.42 The cancellation property with respect to addition of equality of the real numbers is

(a) $a + c = b + c \Rightarrow a \neq b \ \forall a, b, c \in R$
(b) $a + c = b + c \Rightarrow a = b \ \forall a, b, c \in R$
(c) $a + c = b + c \Rightarrow a = c \ \forall a, b, c \in R$
(d) $a + c = b + c \Rightarrow c = b \ \forall a, b, c \in R$

Q.43 The cancellation property with respect to multiplication of equality of the real numbers is

(a) $ac = bc \Rightarrow a = c \quad \forall a, b, c \in R, c \neq 0$
(b) $ac = bc \Rightarrow b = c \quad \forall a, b, c \in R, c \neq 0$
(c) $ac = bc \Rightarrow a \neq b \quad \forall a, b, c \in R, c \neq 0$
(d) $ac = bc \Rightarrow a = b \quad \forall a, b, c \in R, c \neq 0$

Q.44 The transitive property of order of the real numbers is

(a) $\forall a, b, c \in R, a < b \wedge b < c \Rightarrow a < c$
(b) $\forall a, b, c \in R, a < b \wedge b < c \Rightarrow a \in c$
(c) $\forall a, b, c \in R, a < b \wedge b < c \Rightarrow a = c$
(d) $\forall a, b, c \in R, a < b \wedge b < c \Rightarrow a > c$

Q.45 The transitive property of order of the real numbers is

(a) $\forall a, b, c \in R, a > b \wedge b > c \Rightarrow a > c$
(b) $\forall a, b, c \in R, a > b \wedge b > c \Rightarrow a \geq c$
(c) $\forall a, b, c \in R, a > b \wedge b > c \Rightarrow a = c$
(d) $\forall a, b, c \in R, a > b \wedge b > c \Rightarrow a < c$

Q.46 Golden rule of fractions is that for: (Fbd 14)

$k \neq 0, \dfrac{a}{b} =$

(a) $\dfrac{ab}{k}$ (b) $\dfrac{k}{ab}$
(c) $\dfrac{kb}{ka}$ (d) $\dfrac{ka}{kb}$

Q.47 $\sqrt{2}$ is (Mtn 08 G-I)

(a) integer (b) rational number
(c) irrational number (d) natural number

Q.48 If n is not a perfect square, then $\sqrt{n}$ is

(a) integer (b) rational number
(c) irrational number (d) natural number

Q.49 The property $\forall a \in R, a = a$ is called (Gjw Board 16)

(a) Reflexive (b) Symmetric
(c) Transitive (d) Commutative

Q.50 Additive inverse of 2 is: (DGK 12, 14 G-I)

(a) 0 (b) 1
(c) −2 (d) $\dfrac{1}{2}$

Q.51 Number $\dfrac{1}{\pi}$ is. (Swl Board 17)

(a) rational (b) irrational
(c) prim (d) whole

Q.52 $\sqrt{\dfrac{5}{16}}$ is. (Lhr 18 G-II)

(a) Rational number (b) Irrational number
(c) Prime number (d) Whole number

Q.53 Additive inverse of $\dfrac{x}{y}$ is

(a) $-\dfrac{x}{y}$ (b) $\dfrac{x}{y}$
(c) $-\dfrac{y}{x}$ (d) 0

Q.54 $\sqrt{\dfrac{6}{19}}$ **is:** **(Lhr 10 G-II)**

(a) Rational number (b) Irrational number
(c) Prime number (d) Whole number

Q.55 Division of a natural number by another natural number gives

(a) always a natural number
(b) always an integer
(c) always a rational number
(d) always an irrational number

Q.56 If $x, y \in R$ and $x\,y = 0$, then

(a) $x = 0$ (b) $y = 0$
(c) $x = 0$ and $y = 0$ (d) $x = 0$ or $y = 0$.

Q.57 The multiplicative identity of real numbers is: **(D.G.K 15, 19)**

(a) 0 (b) 1
(c) 2 (d) 3

Q.58 $Q = \{x \mid x = \dfrac{p}{q}$ **where** $p, q \in Z \wedge q \neq 0\}$ **is a set of**

(a) Rational numbers
(b) Irrational numbers
(c) Set of natural numbers
(d) Set of integers

Q.59 Additive inverse of $a \in R$ is: **(Gjw 19 G-II)**

(a) 2 (b) 1
(c) $\dfrac{1}{a}$ (d) $-a$

Q.60 The property used in $(a + 1) + \dfrac{3}{4} = a + \left(1 + \dfrac{3}{4}\right)$ **is.** (Bpr 19)

(a) Closure (b) Associative
(c) Commutative (d) Additive

Q.61 The property $\forall\, a, b \in \mathbb{R}$, $a = b \Rightarrow b = a$ is called: (Lhr 19 G-II)

(a) Commutative (b) Transitive
(c) Symmetric (d) Reflexive

EXERCISE 1.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define complex numbers.

Ans. The numbers of the form $x + i\,y$, where $x, y \in P$, are called complex numbers, here x is called real part and y is called imaginary part of the complex number.

For example, $3 + 2\,i$, $4 - \dfrac{2}{7}\,i$ etc. are complex numbers.

Q.2 Simplify: (2, 6) · (3, 7) **(Mtn 13 G-I) (Swl 13)**

Ans. $(2, 6) \cdot (3, 7) = [2(3) - 6(7), 2(7) + 6(3)]$
$= (6 - 42, 14 + 18) = (-36, 32)$

Q.3 Express $(2+\sqrt{-3})\,(3 + \sqrt{-3})$ in the form a + bi. **(Rwp 11, Rwp 13)**

Ans. $(2 + \sqrt{-3})\,(3 + \sqrt{-3})$
$= 6 + 2\sqrt{-3} + 3\sqrt{-3} + (\sqrt{-3})^2$
$= 6 + 5\sqrt{-3} - 3 = 3 + 5\sqrt{-3}i$ Ans.

Q.4 Simplify (2, 6) ÷ (3, 7) **(Mtn 09 G-II)**

Ans. $(2, 6) \div (3, 7) = \dfrac{2 + 6i}{3 + 7i}$

$= \dfrac{(2 + 6i)}{(3 + 7i)} \times \dfrac{(3 - 7i)}{(3 - 7i)}$

$= \dfrac{(2 + 6i)\,(3 - 7i)}{(3 + 7i)(3 - 7i)}$

$= \dfrac{6 - 14i + 18i + 42}{(3)^2 - (7i)^2} = \dfrac{48 + 4i}{9 + 49}$

$= \dfrac{48 + 4i}{58} = \left(\dfrac{48}{58}, \dfrac{4}{58}\right) = \left(\dfrac{24}{29}, \dfrac{2}{29}\right)$

Q.5 Separate real and imaginary parts of the number $\dfrac{2 - 7i}{4 + 5i}$. **(Mtn 08 G-II)**
(D.G.K 17 G-I, Rwp 18, Bpr 18)(Bpr 19)

Ans. $\dfrac{2 - 7i}{4 + 5i} = \dfrac{2 - 7i}{4 + 5i} \times \dfrac{4 - 5i}{4 - 5i}$

$= \dfrac{(2 - 7i)\,(4 - 5i)}{(4 + 5i)(4 - 5i)} = \dfrac{8 - 10i - 28i - 35}{(4)^2 - (5i)^2}$

$= \dfrac{8 - 35 - 38i}{16 + 25} = \dfrac{-27 - 38i}{41}$

$= \dfrac{-27 - 38i}{41} = -\dfrac{27}{41} - \dfrac{38}{41}\,i$

Q.6 Simplify $(-1)^{\frac{-21}{2}}$.
(Sgd 16)(Lhr 07 G-I)(Gjw 17)(Fbd 17)

Ans. $(-1)^{\frac{-21}{2}} = \left[(-1)^{\frac{1}{2}}\right]^{-21} = i^{-21} = i^{-24}\,.\,i^3 = (i^4)^{-6}(-i)$
$= (1)^{-6}(-i) = 1(-i) = -i$

Q.7 Find the multiplicative inverse of: (−4, 7)
(Lhr 18 G-I, Rwp 16)(Mtn 19 G-I)

Ans. Multiplicative inverse of $(-4, 7) = \dfrac{1}{(-4, 7)}$

$= \dfrac{1}{-4 + 7\,i} = \dfrac{1}{-4 + 7\,i} \times \dfrac{-4 - 7\,i}{-4 - 7\,i}$

$= \dfrac{-4 - 7\,i}{(-4)^2 - (7\,i)^2} = \dfrac{-4 - 7\,i}{16 + 49} = \dfrac{-4 - 7\,i}{65}$

$= \left(\dfrac{-4}{65}\right) + \left(\dfrac{-7}{65}\right) i = \left(\dfrac{-4}{65}, \dfrac{-7}{65}\right)$

Q.8 Find the multiplicative inverse of $\left(\sqrt{2}, -\sqrt{5}\right)$.

(Bpr 11, 16)(DGK 14 G-I)(Gjw 09)
(Fbd 11, 19 G-II)(Lhr 19 G-II)

Ans. We know that the multiplicative inverse of (a, b) is $\left(\dfrac{a}{a^2+b^2}, -\dfrac{b}{a^2+b^2}\right)$, so the multiplicative inverse of $\left(\sqrt{2}, -\sqrt{5}\right)$ is

$$\left(\frac{\sqrt{2}}{\left(\sqrt{2}\right)^2+\left(-\sqrt{5}\right)^2}, -\frac{-\sqrt{5}}{\left(\sqrt{2}\right)^2+\left(-\sqrt{5}\right)^2}\right) = \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right).$$

Q.9 Factorize $a^2 + 4b^2$

(Mtn 10 G-I, 16 G-I), (Gjw 16)

Ans. $a^2 + 4b^2 = a^2 - (-4b^2) = a^2 - (4\,i^2 b^2)$
$= (a)^2 - (2ib)^2 = (a + 2ib)\,(a - 2ib)$
$= (a + 2ib)(a - 2ib)$

Q.10 Factorize: $3x^2 + 3y^2$

(Mtn 12 G-I)(Sgd 13)(Bpr 12)(Mtn 14)(Lhr 17 G-II)

Ans. $3x^2 + 3y^2 = 3\,x^2 - 3\,i^2\,y^2$ (As $i^2 = -1$)
$= 3\{(x)^2 - (i\,y)^2\} = 3(x - i\,y)(x + i\,y)$

Q.11 Factorize $9a^2 + 16b^2$. **(Mtn 19 G-I)(Rwp 19)**
(Swl 17)(D.G.K 13)(Lhr 16G-I)(Mtn 17G-II)(Fbd-18)

Ans. $9a^2 + 16b^2$
$= 9a^2 - (-16b^2) = (3a)^2 - (4ib)^2$
$= (3a + 4bi)(3a - 4bi)$

Q.12 Simplify the following (5, –4) ÷ (–3, –8).

(Mtn 18 G-II)

Ans. $(5, -4) \div (-3, -8)$

$$= \frac{(5, -4)}{(-3, -8)}$$

$$= \frac{5 - 4i}{-3 - 8i} \times \frac{-3 + 8i}{-3 + 8i}$$

$$= \frac{-15 + 40i + 12i - 32i^2}{(-3)^2 + (-8)^2}$$

$$= \frac{-15 + 32 + 52i}{9 + 64}$$

$$= \frac{17 + 52i}{73} \quad \text{Ans.}$$

Q.13 Separate into real and imaginary parts.
$\dfrac{(-2 + 3i)^2}{1 + i}$ **(Swl 19)**

Ans. $\dfrac{(-2 + 3i)^2}{(1 + i)} = \dfrac{4 + 9i^2 - 12i}{1 + i} = \dfrac{4 - 9 - 12i}{1 + i}$

$$= \frac{-5 - 12i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$= \frac{-5 + 5i - 12i + 12i^2}{1 - i^2} = \frac{-5 - 7i - 12}{1 + 1}$$

$$= \frac{-17 - 7i}{2} = -\frac{17}{2} - \frac{7}{2}i$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 What is multiplicative inverse of $1 - 2i$:

(Bpr 18)(Sgd 19)

(a) $\dfrac{1 + 2i}{5}$ (b) $\dfrac{1 - 2i}{5}$

(c) $\dfrac{1 + 2i}{\sqrt{5}}$ (d) $\dfrac{1 - 2i}{\sqrt{5}}$

Q.2 If $z = x + iy$, $x, y \in R$ then $|z|$ equals:

(Lhr 09 G-I)

(a) $x^2 + y^2$ (b) $\sqrt{x^2 + y^2}$

(c) $\sqrt{x^2 - y^2}$ (d) $\sqrt{x + y}$

Q.3 $(-i)^{19}$ equal to:

(Mtn 08 G-II),(Lhr 15 G-I),(Mtn 08 G-II)

(a) $-i$ (b) i

(c) 1 (d) –1

Q.4 Additive inverse of complex number of (a, b) is:

(Mtn 11 G-II)

(a) (–a, b) (b) (–a, –b)

(c) (a, –b) (d) (a, b)

Q.5 If $z = -2 + 3i$, then $|z| =$ **(D.G.K 13 G-II)**

(a) 1 (b) 13

(c) $\sqrt{13}$ (d) $-2 - 3i$

Q.6 Modulus of a complex no. $-5i$ is: **(Bpr 12 G-I)**

(a) ± 5 (b) –5

(c) $\sqrt{5}$ (d) 5

Q.7 Conjugate of $-2 + 3i$: (Fbd 12 G-I, Bpr 11 G-I)

(a) $-2 + 3i$ (b) $+2 - 3i$

(c) $2 + 3i$ (d) $-2 - 3i$

Q.8 Multiplicative identity of complex numbers is:

(Rwp 11 G-I, Rwp 13 G-II)

(a) (0, 0) (b) (1, 0)

(c) (0, 1) (d) (1, 1)

Q.9 If Z is a complex number, the $|z|^2$ is equal to:

(Sgd 13, Fbd 11 G-I), (D.G K 17 G-I)

(a) Z^2 (b) $\bar{Z}^2$

(c) $Z\bar{Z}$ (d) Z

Q.10 Multiplicative inverse of (1, 0) is:

(Lhr 13 G-II)(Fbd 19)

(a) (–1, 0) (b) (0, –1)

(c) (0, 1) (d) (1, 0)

Q.11 If $z = 3 - 4i$, then $|\bar{z}|$ is: **(Mtn 14 G-I)**

(a) 4 (b) 5

(c) –5 (d) 1

Q.12 i^{14} equal to: (Bpr 14), (Lhr 17G-I)
(a) 1 (b) −1
(c) i (d) $-i$

Q.13 Multiplicative inverse of non-zero real number "a" is: (Rwp 14)
(a) 0 (b) a
(c) −a (d) $\frac{1}{a}$

Q.14 The modulus of a complex number $3 - 4i$ is: (Lhr 15 G-II)
(a) 4 (b) 5
(c) −5 (d) 0

Q.15 Multiplicative inverse of $-i$: (Lhr 14, Lhr 10 G-I), (D.G.K 15 G-II)
(a) i (b) $-i$
(c) 1 (d) −1

Q.16 (7, 9) + (3, −5) = (Lhr 11)
(a) (7, 9) (b) (3, −5)
(c) (10, 4) (d) (4, 10)

Q.17 $\left|-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right| =$ (Mtn 2014 G-II)
(a) 3 (b) 2
(c) 1 (d) 0

Q.18 If $z = 20 + 15i$, then |z|.
(a) 20 (b) 15
(c) 25 (d) 30

Q.19 The number $\sqrt{-1}$ is called: (Gjw 18)
(a) Real number (b) Natural number
(c) Complex number (d) Rational number

Q.20 The simplified form of i^{101} is: (D.G.K 15 G-II)
(a) −1 (b) 1
(c) i (d) $-i$

Q.21 The multiplicative inverse of $(\sqrt{2}, -\sqrt{5})$ (Lhr 10 G-I)
(a) $\left(\frac{\sqrt{2}}{\sqrt{7}}, \frac{\sqrt{5}}{\sqrt{7}}\right)$ (b) $\left(\frac{\sqrt{2}}{7}, \frac{-\sqrt{5}}{7}\right)$
(c) $\left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$ (d) $\left(\frac{-\sqrt{2}}{7}, \frac{-\sqrt{5}}{7}\right)$

Q.22 The product of two conjugate complex numbers is
(a) always > 0 (b) always < 0
(c) either > 0 or < 0 (d) none of these

Q.23 Product of a complex number and its conjugate is
(a) a real number (b) irrational number
(c) a complex number
(d) either real number or complex number.

Q.24 Factors of $9a^2 + 25b^2$ in complex number system are
(a) $(3a - 5bi)(3a + 5bi)$
(b) $(3a - 5b)(3a + 5b)$
(c) $(3a - 5bi)(3a + 5b)$
(d) $(3a - 5b)(3a + 5bi)$

Q.25 The real part of the complex number $a + bi$ is
(a) b (b) −b
(c) a (d) −a

Q.26 The imaginary part of the complex number $a + bi$ is
(a) b (b) bi
(c) a (d) none of these

Q.27 i^{14} (Mtn 17 G-I)
(a) 1 (b) −1
(c) i (d) $-i$

Q.28 Let $x, y \in R$, then $x + iy$ is purely imaginary if
(a) $x \neq 0, y = 0$ (b) $x = 0, y = 0$
(c) $x = 0, y \neq 0$ (d) $x \neq 0, y \neq 0$

Q.29 $-a - ib$ equals. (Rwp 17 G-I)
(a) $a + ib$ (b) $-a + ib$
(c) $a - ib$ (d) $-a - bi$

Q.30 Which of the following is correct:
(a) $2 + 7i > 10 + i$ (b) $1 + i > 1 - i$
(c) $4 + 3i > 1 + 3i$ (d) None of these.

Q.31 Modulus |z| of complex number $z = a + ib$ is the distance of a point from:
(a) x - axis (b) y - axis
(c) origin (d) infinity

Q.32 Modulus of complex number $z = a + ib$ is:
(Rwp 12 G-I)(Mtn Board 2015 G-II)
(a) $\sqrt{a^2 + b^2}$ (b) $\sqrt{a^2 - b^2}$
(c) $\sqrt{b^2 - a^2}$ (d) None of the above

Q.33 Modulus of $15i + 20$ is
(a) 20 (b) 15
(c) 25 (d) None of the above

Q.34 Conjugate of complex number (−a, −b) is
(a) (−a, b) (b) (−a, −b)
(c) (a, −b) (d) None of these

Q.35 Conjugate of $a + ib$ is
(a) $-a + ib$ (b) $a + ib$
(c) $-a - ib$ (d) $a - ib$

Q.36 Conjugate of $a - ib$ is
(a) $b + ia$ (b) $-a + ib$
(c) $-a - ib$ (d) $a + ib$

Q.37 Conjugate of $-3-2i$ is

(a) $3+2i$ (b) $-3+2i$

(c) $2+3i$ (d) $-2+3i$

Q.38 If $z_1 = (a, b)$, $z_2 = (c, d)$ are in X, then which expression defines the sum of z_1 and z_2.

(a) $(a+c, b+d)$ (b) $(a+b, c+d)$

(c) $(a+d, b+c)$ (d) $(b+d, a+c)$

Q.39 If $z_1 = 4i$ and $z_2 = 3-9i$, then $z_1 + z_2 =$

(a) $3-5i$ (b) $3i-5$

(c) $7-9i$ (d) $3+5i$

Q.40 If $z = \dfrac{a+ib}{a-ib}$ then $|z| =$

(a) a^2+b^2 (b) a^2-b^2

(c) 1 (d) $\sqrt{a^2+b^2}$

Q.41 The identity element with respect to subtraction is

(a) 0 (b) 1

(c) -1 (d) 0 and 1

Q.42 The additive inverse of a real number a is

(a) 0 (b) $-a$

(c) a (d) $\dfrac{1}{a}$

Q.43 The multiplicative inverse of a non-zero real number a is

(a) 0 (b) $-a$

(c) a (d) $\dfrac{1}{a}$

Q.44 The multiplicative inverse of 3 is

(a) 0 (b) 1

(c) -3 (d) $\dfrac{1}{3}$

Q.45 $z = (a, b)$, then $z^{-1} =$ (Gjw, Fbd 09)

(a) $\left(\dfrac{1}{a}, \dfrac{1}{b}\right)$

(b) $(-a, -b)$

(c) $\left(\dfrac{a}{a^2+b^2}, \dfrac{b}{a^2+b^2}\right)$

(d) $\left(\dfrac{a}{a^2+b^2}, -\dfrac{b}{a^2+b^2}\right)$

Q.46 If z_1 and z_2 are complex numbers, then $\overline{z_1 . z_2} =$

(a) $\overline{z_1}.\overline{z_2}$ (b) $\overline{z_1}.z_2$

(c) $z_1.\overline{z_2}$ (d) $\overline{\overline{z_1.z_2}}$

Q.47 The modulus of $1-\sqrt{3}\,i$ is equal to (Mtn 16 G-I)

(a) -2 (b) 2

(c) $-\sqrt{2}$ (d) $\sqrt{10}$

Q.48 i^{17} is equal to (Fbd Board 16)

(a) -1 (b) 1

(c) i (d) $-i$

Q.49 $i^9 =$ (Mtn 11 G-I)

(a) 1 (b) -1

(c) i (d) $-i$

Q.50 Modulus of $5-3i$ is. (Fsd 17)

(a) 4 (b) 16

(c) 34 (d) $\sqrt{34}$

Q.51 $|a+bi|$ is equal to. (Lhr 17 G-II)

(a) $\sqrt{a^2+b^2}$ (b) $\sqrt{a^2-b^2}$

(c) $\sqrt{-a^2-b^2}$ (d) $\sqrt{b^2-a^2}$

Q.52 For a complex number $(0, 1)^2 =$ .... (A.J.K 17)

(a) 1 (b) -1

(c) i (d) $-i$

Q.53 If $i = \sqrt{-1}$, then $i^{14} =$: (Mtn 19 G-I)

(a) 1 (b) -1

(c) i (d) $-i$

Q.54 If $z = \cos\theta + i\sin\theta$, then $|Z|$ is equal to: (Rwp 19)

(a) 0 (b) 1

(c) 2 (d) 3

EXERCISE 1.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Prove that product of any two conjugate complex numbers is a real number.

(DGK 13 G-II)(Grw 19 G-II)

Ans. Let, $Z = a+ib \in C$

$$\overline{Z} = a-ib$$

$$Z+\overline{Z} = a+ib+a-1b$$
$$= 2a \in R$$

Also, $Z\overline{Z} = (a+ib)(a-ib) = (a)^2-(16)^2$
$$= a^2-a^2b^2$$
$$= a^2-(-1)b^2$$
$$= a^2+b^2$$

Q.2 If $Z_1 = 2 + i$, $z_2 = 3 - 2i$, $z_3 = 1 + 3i$ then express $\dfrac{\overline{z}_1\,\overline{z}_3}{Z_2}$ in the form a + b*i*. (Bpr 2013)

Ans.
$$\frac{\overline{z}_1\,\overline{z}_3}{Z_2} = \frac{(2-i)(1-3i)}{3-2i}$$
$$= \frac{2-6i-i+3i^2}{3-2i} + \frac{2-7i+3(-1)}{3-2i}$$
$$= \frac{-1-7i}{3-2i} \times \frac{3+2i}{3+2i}$$
$$= \frac{-3-2i-21i-14i^2}{3^2-(2i)^2}$$
$$= \frac{-3+14-23i}{9-(-4)}$$
$$= \frac{11-23i}{13} = \frac{11}{13} - \frac{23}{13}i$$

Q.3 Simplify by expressing in the form a+b*i*: $\dfrac{2}{\sqrt{5}+\sqrt{-8}}$
(Mtn 11, 16, 17 G-I)

Ans.
$$\frac{2}{\sqrt{5}+\sqrt{-8}} = \frac{2}{\sqrt{5}+\sqrt{8}\sqrt{-1}}$$
$$= \frac{2}{\sqrt{5}+\sqrt{8}\,i}$$
$$= \frac{2}{\sqrt{5}+\sqrt{8}\,i} \times \frac{\sqrt{5}-\sqrt{8}\,i}{\sqrt{5}-\sqrt{8}\,i}$$
$$= \frac{2\sqrt{5}-2\sqrt{8}\,i}{(\sqrt{5})^2+(\sqrt{8}\,i)^2}$$
$$= \frac{2\sqrt{5}-2\times 2\sqrt{2}\,i}{5-8\,i^2}$$
$$= \frac{2\sqrt{5}-4\sqrt{2}\,i}{5-8(-1)}$$
$$= \frac{2\sqrt{5}-4\sqrt{2}\,i}{5+8}$$
$$= \frac{2\sqrt{5}-4\sqrt{2}\,i}{13}$$
$$= \frac{2\sqrt{5}}{13} - \frac{4\sqrt{2}}{13}i$$

Q.4 Simplify in form of a + ib of $\dfrac{1}{\sqrt{6}-\sqrt{-12}}$.
(A.J.K 17)

Ans.
$$\frac{1}{\sqrt{6}-\sqrt{-12}}$$
$$= \frac{1}{\sqrt{6}-2\sqrt{3}\,i} = \frac{1}{\sqrt{6}-2\sqrt{3}\,i} \times \frac{\sqrt{6}+2\sqrt{3}i}{\sqrt{6}+2\sqrt{3}i}$$
$$= \frac{\sqrt{6}+2\sqrt{3}i}{(\sqrt{6})^2+(2\sqrt{3})^2} = \frac{\sqrt{6}+2\sqrt{3}i}{6+12} = \frac{\sqrt{6}+2\sqrt{3}i}{18}$$
$$= \frac{\sqrt{6}}{18} + \frac{2\sqrt{3}}{18}i$$

Q.5 Show that $\forall\, z \in C$, $z^2 + \overline{z}^{\,2}$ is a real number.
(Sgd 13 G-II), (Bpr 11, 16, 18), (Fbd-18)
(Rwp 12 G-I, 17 G-I), (Lhr 16 G-I)
(Mtn 17 G-II, 18 G-I)(Gjw 19 G-I)

Ans. Let $z = a + ib$, then $\overline{z} = a - ib$

$\Rightarrow$ $z^2 = a^2 - b^2 + 2ab\,i$,

$(\overline{z})^2 = a^2 - b^2 - 2abi$

$z^2 + (\overline{z})^2$

$= (a^2 - b^2 + 2ab\,i) + (a^2 - b^2 - 2ab\,i)$

$= 2a^2 - 2b^2$ = a real number

Q.6 Show that $\forall\, z \in C$, $(z - \overline{z})^2$ is a real number.
(Rwl 17 G-II)(Swl 17, Gjw 18)

Ans. Let $z = a + i\,b$, then $\overline{z} = a - i\,b$

$z - \overline{z} = (a + i\,b) - (a - i\,b)$

$= a + i\,b - a + i\,b = 2\,i\,b$

$(z - \overline{z})^2 = (2\,i\,b)^2 = 4\,i^2\,b^2$

$= 4\,(-1)\,b^2 = -4\,b^2$ = a real number

Q.7 Simplify $\left(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^3$. (Mtn 16 G-I)

Ans. $\left(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^3$

$$= \left(-\frac{1}{2}\right)^3 + \left(\frac{\sqrt{3}}{2}i\right)^3 + 3\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$
$$= -\frac{1}{8} + \frac{(3^{1/2})^3}{8}i^3 - \frac{3\sqrt{3}\,i}{4}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$
$$= -\frac{1}{8} + \frac{3^{3/2}}{8}(-i) + \frac{3\sqrt{3}\,i}{8} - \frac{3(\sqrt{3})^2}{8}i^2$$
$$= -\frac{1}{8} - \frac{3\sqrt{3}\,i}{8} + \frac{3\sqrt{3}\,i}{8} - \frac{9}{8}(-1) = -\frac{1}{8} + \frac{9}{8} = \frac{8}{8} = 1$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $(-1)^{-21/2}$ is equal to: (Lhr 07 G-I)

(a) $-i$ (b) i

(c) 1 (d) -1

Q.2 Multiplicative inverse of $-3i$ is: (Mtn 10 G-I)

(a) $3i$ (b) $\dfrac{1}{3}i$

(c) $-\dfrac{1}{3}i$ (d) $-3i$

Q.3 If $\overline{z} = -z$, then
(a) z is purely real
(b) z is any complex number
(c) z is purely imaginary
(d) real part of z = imaginary part of z

Q.4 Real part of $\frac{i}{1+i}$ is
(a) $\frac{1}{2}$ (b) 1
(c) 0 (d) $\frac{1}{2}i$

Q.5 Imaginary part of $\frac{i}{1+i}$ is
(a) $\frac{1}{2}$ (b) 1
(c) i (d) $\frac{1}{2}i$

Q.6 The ordered pairs (2, 5) and (5, 2) are
(a) Not equal (b) Equal
(c) Disjoint (d) Empty

Q.7 $i^2 + 1 =$
(a) -1 (b) 0
(c) i (d) 1

Q.8 If $z = x + i\,y = r(\cos\theta + i\sin\theta)$, then modulus of z is
(a) $|x^2 + y^2|$ (b) $\cos^2\theta + \sin^2\theta$
(c) r (d) $\sqrt{x^2 - y^2}$

Q.9 If $z = x + i\,y = r(\cos\theta + i\sin\theta)$, then arg z is
(a) $\tan\theta$ (b) $\cos^2\theta + \sin^2\theta$
(c) r (d) θ

Q.10 If $z = r(\cos\theta + i\sin\theta)$, then z is expressed in
(a) polar form
(b) modulus argument form
(c) trigonometric form
(d) all of these

Q.11 $(-\sqrt{3} + i)^2 =$
(a) $2\{2 - i\sqrt{3}\}$ (b) $2\{1 - i\sqrt{3}\}$
(c) $2\{1 + i\sqrt{3}\}$ (d) $3\{1 - i\sqrt{2}\}$

Q.12 Polar form of $-3\,i$ is
(a) $3\left\{\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right\}$
(b) $3\left\{\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right\}$
(c) $3\left\{\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right\}$
(d) $3\{\cos(-\pi) + i\sin(-\pi)\}$

Q.13 $\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$ in Cartesian form is
(a) 0 (b) i
(c) $-i$ (d) 1

Q.14 De Moivre's theorem is
(a) $(\sin\theta + i\cos\theta)^n = \sin n\theta - i\cos n\theta$
(b) $(\sin\theta + i\cos\theta)^n = \sin\theta + i\cos\theta$
(c) $(\cos\theta + i\sin\theta)^n = \cos n\theta - i\sin n\theta$
(d) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Q.15 The multiplicative inverse of (0, −1) is equal to (Sgd 16)
(a) (1, 0) (b) (0, 1)
(c) (−1, 0) (d) (0, 0)

Q.16 If z is a complex number, then $|z|^2$ i. (Rwp 16)
(a) z^2 (b) $(\overline{z})^2$
(c) $z\overline{z}$ (d) $\frac{z}{z}$

Q.17 If $z = 1 - i$, then $|z| =$ (Lhr 16 G-I)
(a) 2 (b) -2
(c) $-\sqrt{2}$ (d) $\sqrt{2}$

Q.18 The real part of $(a + bi)^2$ is: (Mtn 09 G-II)
(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $b^2 - a^2$ (d) 2ab

Q.19 The multiplicative inverse of $-3 - 5i$ is. (Lhr 18 G-II)
(a) $\frac{3}{34} + \frac{5}{34}i$ (b) $\frac{-3}{34} - \frac{5}{34}i$
(c) $\frac{-3}{34} + \frac{5}{34}i$ (d) $\frac{-3}{\sqrt{34}} + \frac{5}{\sqrt{34}}i$

Q.20 Multiplicative inverse of complex number $(\sqrt{2}, -\sqrt{5})$ is: (Mtn 18 G-II)
(a) $\left(\frac{\sqrt{2}}{\sqrt{7}}, \frac{\sqrt{5}}{\sqrt{7}}\right)$ (b) $\left(\frac{-\sqrt{2}}{\sqrt{7}}, \frac{-\sqrt{5}}{\sqrt{7}}\right)$
(c) $\left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{\sqrt{7}}\right)$ (d) $\left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$

Q.21 If $z = a + ib$, what is the value of $\cos\theta$: (Rwp 18)
(a) $\frac{a}{|z|}$ (b) $\frac{b}{|z|}$
(c) $\frac{a}{b}$ (d) $\frac{b}{a}$

Q.22 1. $(1+i)^8 =$ (Swl 19)
(a) 2 (b) 4
(c) 8 (d) 16

Chapter 2

SETS, FUNCTIONS AND GROUPS

EXERCISE 2.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define Power set.

Ans. The set of all possible subsets of a set A is called the power set of A and it is denoted by the symbol P(A)

Q.2 Write two proper subset of {a, b, c}. **(Mtn 13 G-I, Rwp 11) (Mtn 09 G-II, Swl 13, Lhr 18 G-II)**

Ans. $\{a\}, \{b, c\}$

Q.3 Write down the power set of {+, −, ×, ÷}. **(Mtn 12 G-II)**

Ans. Let, $A = \{+, -, \times, \div\}$.

$p(A) = \{\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

Q.4 Write the power set of {a, {b, c}}. **(Gjw 13)(Mtn 13 G-II), (Lhr 16 G-I, 19 G-II) (Mtn 17 G-II)(Gjw 18, Fbd 18)(Sgd 19)**

Ans. $A = \{a, \{b, c\}\}$

$p(A) = \{\phi, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

Q.5 Write the power set of A = {0, 1}. **(Gjw 14, Lhr 14 G-II, Bpr 18, Rwp 18)**

Ans. $P(A) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$

Q.6 Write two proper subsets of the set $\{x \mid x \in Q \wedge 0 < x \leq 2\}$. **(Fbd 17, 19 G-II)**

Ans. Given set is $\{x \mid x \in Q \wedge 0 < x \leq 2\}$ two of its proper subsets are $\{1\}, \{2\}$

Q.7 Write the set $\{x \mid x \in Q \wedge x^2 = 2\}$ in tabular form. **(Fbd 16)**

Ans. $\{\ \}$

As there is no rational number whose square is 2.

Q.8 Write the set $\{x \mid x \in p \wedge x < 12\}$ in descriptive and tabular form. **(Swl 19)**

Ans. Descriptive form: set of all prime number less than 12.

Tabular form: $\{2, 3, 5, 7, 11\}$

MULTIPLE CHOICE QUESTIONS

❑ *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 If A = Set of all alphabets, then A is called: (Mtn 11 G-II)

(a) Finite set (b) Infinite set
(c) Empty set (d) Power set

Q.2 The set {ϕ} is called: (Mtn Board 2012 G-II) (D.G.K 17 G-II)

(a) Infinite set (b) Empty set
(c) Singleton (d) Solution set

Q.3 If A = {0}, then p(A) (Power set of set A) is: (Fbd 12 G-I, Bpr 11 G-I)

(a) $\{0\}$ (b) $\{0, \phi\}$
(c) $\{\phi, \{0\}\}$ (d) $\{\{0\}, \{\phi\}\}$

Q.4 The number of ways in which a set can be described is: (Rwp 13 G-II, 11 G-I, Bpr 13 G-I)

(a) 1 (b) 3
(c) 2 (d) 4

Q.5 Set {0} is called: (Gjw 11 G-I)

(a) Null set (b) Power set
(c) Singleton (d) Infinite set

Q.6 The power set of empty set is:

(a) Empty set (b) Non-empty set
(c) Proper set (d) Improper set

Q.7 If A = { }, then p(A) = (Rwp 14, Mtn 14 G-II) (Lhr 04)

(a) $\{\ \}$ (b) $\{0\}$
(c) $\{\phi\}$ (d) $\{0, 1\}$

Q.8 The number of subsets of a set of 4 elements is: (DGK 14 G-I)(Sgd 16)(Fbd 19)

(a) 16 (b) 18
(c) 4 (d) 6

Q.9 $\{x \mid x \in E$ and $4 < x < 6\}$ equals: (Lhr 10 G-II, Fbd 18, Lhr 18 G-2)

(a) $\{4\}$ (b) $\{5\}$
(c) $\{6\}$ (d) ϕ

Q.10 Every set is a ________ subset of itself: (Lhr 08 G-I)

(a) Proper (b) Improper
(c) Finite (d) Infinite

Q.11 A set is defined as
(a) Collection of same objects.
(b) Well defined collection of same objects.
(c) Well defined collection of distinct objects.
(d) None of these.

Q.12 Distinct objects means
(a) Identical objects (b) Not identical
(c) Similar (d) None of these

Q.13 The objects in a set are called
(a) Elements (b) Sub-sets
(c) Whole numbers (d) Overlapping sets

Q.14 A set can be described by
(a) One way (b) Two ways
(c) Several ways (d) Three ways

Q.15 If a set is described in words, the method is called
(a) Tabular form (b) Descriptive form
(c) Set builder notation (d) Non-tabular method

Q.16 If a set is described by listing its elements within brackets is called
(a) Set builder notation
(b) Tabular form
(c) Descriptive method
(d) None of these

Q.17 If a set is described as $\{x \mid x \in N \wedge x \leq 100\}$ is the
(a) Set builder notation
(b) Tabular form
(c) Descriptive method
(d) Non-set builder method

Q.18 $a \in A$ means
(a) a is an element of set A
(b) a is subset of A
(c) a is a whole number
(d) a contains A

Q.19 Two sets A and B are said to be equivalent if
(a) $n(A) = n(B)$
(b) $n(A) \neq n(B)$
(c) A and B have same elements number of elements
(d) None of these

Q.20 If set $A = \{1,2,3\}$ and $B = \{2,1,3\}$ then sets A and B are
(a) Not equal (b) Equal
(c) Disjoint (d) Overlapping

Q.21 The well defined collection of disjoint object is a
(a) Complex number (b) Rational number
(c) Whole numbers (d) Set

Q.22 $A \subseteq B$ (i.e., $A \subset B$ and $A = B$) then
(a) A is improper subset of B
(b) A is proper subset of B
(c) B is improper subset of A
(d) B is proper subset of A

Q.23 If $A \subseteq B$ and $B \subseteq A$ then
(a) $A = \varnothing$ (b) $A = B$
(c) $B = \varnothing$ (d) A I $B = \varnothing$

Q.24 $A \supseteq B$ means
(a) A is super set of B
(b) B is supper set of A
(c) A is subset of B
(d) A is equivalent to B

Q.25 If $n(S) = 3$ then $n\{P(S)\} =$
(a) 2 (b) 4
(c) 8 (d) 16

Q.26 The number of elements of the set $\{x : x \in N, x^2 = 1\}$, where N is the set of all natural numbers, is
(a) 0 (b) 1
(c) 2 (d) infinite

Q.27 A set having no element is called
(a) Null set (b) Subset
(c) Singleton (d) Super set

Q.28 The proper subset E of a set F is denoted by.
(a) $F \subset E$ (b) $E \subset F$
(c) $F \subseteq E$ (d) $E \sim F$

Q.29 If $A = \{\{5\}\}$, then $P(A)$ is equal to
(a) $\{\{\varnothing, \{5\}\}\}$ (b) $\{\varnothing, \{5\}\}$
(c) $\{\varnothing, \{\{5\}\}\}$ (d) $\{\{\varnothing\}, \{5\}\}$

Q.30 If a set has 6 elements, then what is number of elements in the power set of set.
(a) 62 (b) 32
(c) 128 (d) 64

Q.31 If the number of elements in finite sets A and B are the same, they are called
(a) super sets (b) null sets
(c) equivalent sets (d) equal sets

Q.32 If the number of elements in a set X are n, then number of elements in P(X) are
(a) 2^n (b) 3^n
(a) n^n (d) n^2

Q.33 The symbol ~ is used for
(a) Equal sets (b) Equivalent sets
(c) Overlapping sets (d) None of these

Q.34 If A is a set then P(A) the set of all possible subsets of A is
(a) Power set of A (b) Subset of A
(c) Null set (d) Infinite set

Q.35 {a} is called
(a) Singlton set (b) Null set
(c) Subset (d) None of these

Q.36 { } is called
(a) Empty set (b) Not a set
(c) Subset (d) None of these

Q.37 A set A is said to be subset of B if
(a) Each element of B is an element of A
(b) Each element of A is an element of B
(c) If some elements of A is not in B
(d) If no element of A is the element of B

Q.38 $A \subset B$ means
(a) A is improper subset of B
(b) A is proper subset of B
(c) B is improper subset of A
(d) B is proper subset of A

Q.39 If A = {a, {a,b}}, the n(P(A)) = (Bpr 15)
(a) 2 (b) 3
(c) 4 (d) 6

Q.40 The power set of the empty set is (Lhr 15 G-II)
(a) Empty set (b) Non-empty set
(c) Proper set (d) Improper set

Q.41 If A = { } , the P(A) is equal to (Mtn 15 G-II)
(a) Infinite set (b) Empty set
(c) Singleton set (d) Null set

Q.42 If A is a subset of B and A = B then A is (Rwp 15)
(a) Improper subset of B
(b) Super set of B
(c) Improper subset of A
(d) Proper subset of A

Q.43 If A = { } , the P(A) = (Gjw 15)
(a) Empty set (b) {0}
(c) { ϕ } (d) 0

Q.44 Tabular form of $\{x / x \in p \wedge x < 12\}$ (Fbd 15)
(a) {1, 2, 3, 5, 7, 11} (b) {2, 3, 5, 7, 9, 11}
(c) {2, 3, 7, 11} (d) {2, 3, 5, 7, 11}

Q.45 The tabular form of a set $\{x \mid x \in Q \wedge x^2 = 2\}$ is equal to. (Mtn 16 G-I)
(a) $\{\sqrt{2}, -\sqrt{2}\}$ (b) {4}
(c) { } (d) {2, −2}

Q.46 The number of subsets of three elements is equal to. (Sgd 17)
(a) 4 (b) 6
(c) 8 (d) 10

Q.47 If every element of a set A is also an element of set B, then **(Sgd 18)**
(a) $A \subseteq B$ (b) $B \subseteq A$
(c) $A \cap B = \phi$ (d) $A \cap B = B$

Q.48 Which symbol is used for the membership of a set? (Bpr 16)
(a) ^ (b) v
(c) ∈ (d) ~

Q.49 If two sets P and Q are equivalent, they are denoted by
(a) $P \in Q$ (b) $P \leftrightarrow Q$
(c) $P \sim Q$ (d) $P = Q$

Q.50 The set of all rational numbers between 2, 3 is.
(a) an empty set (b) an infinite set
(c) a finite set (d) a power set

Q.51 if A = {a, {a, b}}, then number of elements in P(A) is. (Bpr 19)
(a) 2 (b) 3
(c) 4 (d) 8

Q.52 The tabular form of the set $\{x \mid x \in Q \wedge x^2 = 2\}$ is: (DGK 19)
(a) $\{\sqrt{2}, -\sqrt{2}\}$ (b) {4}
(c) { } (d) {4 , −4}

EXERCISE 2.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define Union of two sets. **(Mtn 19 G-I)**

Ans. The Union of two sets A and B, written as $A \cup B$, is the set of all elements, which belong either to A or to B or to both A and B.
Symbolically $A \cup B = \{x \mid x \in A \vee x \in B\}$

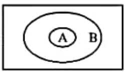
Q.2 Define Intersection of two sets and give an example. **(Bpr 19)**

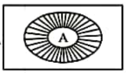
Ans. The Intersection of two sets A and B, written as $A \cap B$, is the set of all elements, which are common to both A and B.
Symbolically $A \cap B = \{x \mid x \in A \wedge x \in B\}$

Example: A = {1, 2, 3}, B = {3, 4, 5}
$A \cap B = \{3\}$

Q.3 Show that A − B and B − A by Venn diagram in case of $A \subseteq B$. **(Bpr 13)**

Ans.

A − B


B − A


Q.4 If U = {1, 2, 3, …… 20} and A = {1,3,5,…, 19}. Verify that (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

Ans. $A' = U - A = \{1,2,3, \dots, 20\} - \{1,3,5, \dots, 19\}$
$= \{2, 4, 6, \dots, 20\}$

$A \cup A' = \{1, 3, 5, \dots, 19\} \cup \{2, 4, 6, \dots 20\}$
$= \{1, 2, 3, 4, \dots, 19, 20\}$
$= U$

$A \cap A' = \{1, 3, 5, \dots, 19\} \cap \{2, 4, 6, \dots, 20\}$
$= \{\ \} = \phi$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If A' is complement of the set A then $A \cap A'$ equal: (Mtn 09 G-II)
(a) A (b) A'
(c) U (d) ϕ

Q.2 If $A \subseteq B$, then $A \cup B$ = (Mtn 11 G-II)
(a) A (b) B
(c) A^C (d) B^C

Q.3 If $A \cap A' = A$, then (DGK 13 G-II)
(a) $A = \phi$ (b) $A = U$
(c) $A = B$ (d) $A = \{0\}$

Q.4 If $B - A = B$, then: (Mtn 14 G-I)
(a) $A \cap B = A$ (b) $A \cap B = \phi$
(c) $A \cup B = \phi$ (d) $A \cap B \neq \phi$

Q.5 $A \cup \phi$ = (Swl 14)
(a) A (b) ϕ
(c) X (d) B

Q.6 If every element of a set A is also element of set B, then: (Fbd 14)
(a) $A \subseteq B$ (b) $B \subseteq A$
(c) $A \cap B = \phi$ (d) $A \cap B = A'$

Q.7 If $A \not\subset B$, $B \not\subset A$ and A and B have at least one element common, they are called
(a) equal sets (b) null sets
(c) overlapping sets (d) subsets

Q.8 If $A \cap B = \varnothing$, then $n(A - B)$ is equal to: (Lhr 10 G-I)
(a) $n(A)$ (b) $n(A \cap B)$
(c) $n(B)$ (d) $n(A \cup B)$

Q.9 If U is a universal set and $A \subseteq U$ then $A \cup A'$ equals: (Lhr 09 G-I)
(a) A (b) U
(c) $A \cap A'$ (d) A'

Q.10 If $A \cup B = \varnothing$, and $A = \varnothing$ then
(a) $B = \varnothing$ (b) $B = \{\varnothing\}$
(c) $B = A$ (d) $B \neq A$

Q.11 If sets A and B are equal then
(a) $A \supset B$ (b) $B \supset A$
(c) $A = B$ (d) $A \neq B$

Q.12 Which is the commutative law?
(a) $A \cap B' = B \cap A'$ (b) $A \cap B = B \cap A$
(c) $A \cap B = B' \cap A$ (d) $A \cap B = B \cap A'$

Q.13 If $A \subseteq B$ and $B \subseteq A$, then
(a) A and B are power sets
(b) A and B are disjoint sets
(c) A and B are super sets
(d) A and B are equal sets.

Q.14 If $A \subset B$, then $A \cap B$ is equal to
(a) A (b) B
(c) $\varnothing$ (d) $A \cup B$

Q.15 ïf $A \subset B$, then $A \cup B$ is equal to
(a) A (b) B
(c) $\varnothing$ (d) $A \cap B$

Q.16 $A - B$ is a subset of
(a) A (b) B
(c) $A \cap B$ (d) $A \cup B$

Q.17 $B - A$ is a subset of
(a) A (b) B
(c) $A \cap B$ (d) $A \cup B$

Q.18 $A \cap \varnothing$ =
(a) A (b) $\varnothing$
(c) A' (d) $\varnothing'$

Q.19 $A \cup \varnothing$ =
(a) A (b) $\varnothing$
(c) A' (d) $\varnothing'$

Q.20 $A \cap A^c$ =
(a) U (b) $\{0\}$
(c) A (d) $\varnothing$

Q.21 $A \cup A^c$ =
(a) U (b) $\{0\}$
(c) A (d) $\varnothing$

Q.22 $A \cup (B \cap C)$ =
(a) $(A \cap B) \cup (A \cap C)$
(b) $(A \cup B) \cap (A \cup C)$
(c) $(A \cap B) \cup (A \cup C)$
(d) $(A \cap B) \cap (A \cup C)$

Q.23 If A and B are two sets, then $A \cup (A \cap B)$ is equal to. (Sgd 11 G-II)
(a) B (b) A
(c) $\varnothing$ (d) $A \cap B$

Q.24 If A and B are two sets, then $A \cap (A \cup B)$ is equal to **(Mtn 18 G-II)**
(a) A (b) B
(c) $\varnothing$ (d) $A \cup B$

Q.25 $A \cap (B - A)$ =
(a) A (b) $\{\varnothing\}$
(c) B (d) $\varnothing$

Q.26 A set containing finite numbers of elements is called
(a) null set (b) super set
(c) finite set (d) infinite set

Q.27 If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$ and $A \cap B$ is
(a) $\{1, 2, 3\}$ (b) $\{5, 6, 7\}$
(c) $\{4\}$ (d) $\varnothing$

Q.28 If $W = \{0, 1, 2, 3, 4, ...\}$, $N = \{1, 2, 3, 4, ...\}$ then $N - W = ?$

(a) W (b) $\{0\}$
(c) $\varnothing$ (d) none of these.

Q.29 If $A = \{ 1, 2, 7, 9 \}$, $B = \{ 1, 4, 7, 11 \}$ then A and B are called

(a) disjoint sets (b) equal sets
(c) overlapping sets (d) complementary sets

Q.30 If two sets have no element common, they are called

(a) disjoint (b) over lapping
(c) dissimilar (d) exhaustive

Q.31 If A^C is complement of a set A, then $A \cap A^C$ is equal (Mtn Board 2015 G-I)

(a) A (b) A^C
(c) $\cup$ (d) ϕ

Q.32 If $A \subseteq B$, then complement of A in B is.

(a) $A - B$ (b) $B - A$
(c) $A \cap B$ (d) $A \cup B$

Q.33 If A and B are any two sets, then

(a) $A \cap B \subseteq B$ (b) $A \cap B \nsubseteq B$
(c) $A \cap B \subseteq B'$ (d) $A \cap B \nsubseteq B'$

Q.34 If $A \subseteq B$, then $A - B =$

(a) A (b) $\varnothing$
(c) B (d) $\{\varnothing\}$

Q.35 Set containing A or B is denoted by: (Bpr 15)

(a) $A \cap B$ (b) $A \cup B$
(c) $A \subseteq B$ (d) $B \subseteq A$

Q.36 $A \cup (A \cap B) =$ (Fbd 17)

(a) B (b) $A \cap B$
(c) Φ (d) A

Q.37 If $A \subseteq B$, then $n(A - B) =$ ______ (A.J.K 17)

(a) 0 (b) n(A)
(c) n(B) (d) $n(A \cap B)$

Q.38 If A and B are disjoint sets, then $A \cap B$ is equal to: (Mtn 08 G-I)

(a) A (b) B
(c) ϕ (d) U

Q.39 Set containing elements of A or B is denoted by: (Mtn 08 G-II)

(a) $A \cap B$ (b) $A \subset B$
(c) $B \subset A$ (d) $B \cup A$

Q.40 If $A \cap B = \varphi$, then n (A - B) is equal to: (Fbd 19 G-II)

(a) $n(A \cup B)$ (b) $n(A \cap B)$
(c) n(A) (d) n(B)

Q.41 If A and B are two sets, then $A - B =$: (Lhr 19 G-II)

(a) $A \cup B^c$ (b) $A \cap B^c$
(c) $(A \cup B)^c$ (d) $(A \cap B)^c$

EXERCISE 2.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 From suitable properties of union and intersection, prove that.
$A \cap (A \cup B) = A \cup (A \cap B)$

Ans. $A \cap (A \cup B)$
$= (A \cap A) \cup (A \cap B)$ (Distributive law)
$= A \cup (A \cap B)$ ($\because A \cap A = A$)

Q.2 What are De Morgan's Laws?

Ans. (i) $(A \cup B)' = A' \cap B'$
(ii) $(A \cap B)' = A' \cup B'$

LONG QUESTIONS

Q.6 Verify De Morgan's laws for the given sets:
$U = \{1, 2, 3, ..., 20\}$, $A = \{2, 4, 6, ..., 20\}$
$B = \{1, 3, 5, ... , 19\}$ **(Rwp, Swl 19)**

Ans. $A = \{2, 4, 6, ..., 20\}$, $B = \{1, 3, 5, ..., 19\}$

(a) $(A \cup B)^c = A^c \cap B^c$

$A \cup B = \{2, 4, 6, ..., 20\} \cup \{1, 3, 5, ..., 19\}$
$= \{1, 2, 3, ..., 20\} = U$

$A^c = U - A = \{1, 2, 3, ..., 20\} - \{2, 4, 6, ..., 20\}$
$= \{1, 5, 7, ..., 19\} = B$

$B^c = U - B = \{1, 2, 3, ..., 20\} - \{1, 3, 5, ..., 19\}$
$= \{2, 4, 6, ..., 20\} = A$

$L.H.S = (A \cup B)^c = U - (A \cup B) = U - U = \phi$

$R.H.S = A^c \cap B^c = \{1,3,5,...,9\} \cap \{2, 4, 6, ... ,20\} = \phi$

Hence, $\boxed{(A \cup B)^c = A^c \cap B^c}$

(b) $(A \cap B)^c = A^c \cup B^c$

$A \cap B = \{2, 4, 6, ..., 20\} \cap \{1, 3, 5, ..., 19\} = \phi$

$A^c = U - A = \{1, 2, 3, ..., 20\} - \{2, 4, 6, ..., 20\}$
$= \{1, 3, 5, ..., 19\}$

$B^c = U - B = \{1, 2, 3, ..., 20\} - \{1, 3, 5, ..., 19\}$
$= \{2, 4, 6, ..., 20\}$

$L.H.S = (A \cap B)^c = U - (A \cap B) = \{1, 2, 3, ..., 20\} - \phi$
$= \{1, 2, 3, ..., 20\}$

$R.H.S = A^c \cup B^c = \{1, 3, 5, ..., 19\} \cup \{2, 4, 6, ..., 20\}$
$= \{1, 2, 3, ..., 20\}$

Hence, $\boxed{(A \cap B)^c = A^c \cup B^c}$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 De-Morgan's law is: (Fbd 11 G-I)

(a) $(A \cup B)' = A' \cap B$
(b) $(A \cup B)' = A \cap B$
(c) $(A \cup B)' = A' \cup B'$
(d) $(A \cup B)' = A \cup B$

Q.2 **$A \cup B$ =** (Mtn 14 G-II, Lhr 13 G-I)

(a) $A \cap A$ (b) $A \cap B$
(c) ϕ (d) $B \cup A$

Q.3 **If A and B are two sets, then $A \cap B = B \cap A$ represent the**

(a) Commutative property of intersection
(b) Commutative property of union
(c) Associative property of intersection
(d) Associative property of union

Q.4 **$A \cap (A \cup B)$ =** (D.G.K 2015 G-II)

(a) B (b) $A \cup B$
(c) A (d) $A \cap B$

Q.5 **For any two sets A and B, $(A \cap B)'$ is equal to.** (Rwp 16)

(a) A^c (b) B^c
(c) $A' \cup B'$ (d) $A \cap B$

Q.6 **If A, B are non-empty sets, then $A \cup B$ equals:** (Lhr 07 G-I)

(a) A (b) $B \cup A$
(c) $\varnothing$ (d) B

Q.7 **For any two sets A = B if and only if $A \cup B$ =**

(a) A' (b) B'
(c) $A \cap B$ (d) $B \cup A$

Q.8 **For any two subsets A and B of set $\cup$, then $(A \cup B)$ is equal to:** (Rwp 19)

(a) $A \cup B'$ (b) $A \cap B'$
(c) $A' \cup B'$ (d) $A' \cap B'$

EXERCISE 2.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 **Show that statement $(p \wedge q) \to p$ is a tautology.** (Lhr 13 G-II)(Fbd 16)

Ans.

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore$ $(p \wedge q) \to p$ is a tautology.

Q.2 **Determine whether $q \vee (\sim q \vee p)$ is a tautology, absurdity or a contigency?** (Mtn 10 G-I)

Ans.

p	q	$\sim q$	$\sim q \vee p$	$q \wedge (\sim p \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	F

Q.3 **Write inverse and converse of $\sim p \to \sim q$.** (Mtn 11 G-II)(Sgd 16)(Gjw 18)

Ans. Inverse of $\sim p \to \sim q$ is $p \to q$
Converse of $\sim p \to \sim q$ is $\sim q \to \sim p$

Q.4 **Write inverse and converse of $\sim p \to q$.** (Mtn 12 G-II)(Gjw 16)(Sgd 17)

Ans. Inverse of $\sim p \to q$ is $p \to \sim q$
Converse of $\sim p \to q$ is $q \to \sim p$

Q.5 **Show that $p \to (p \vee q)$ is a tautology?** (Gjw 13, Fbd 12, DGK 11 G-I, Bpr 18) (Lhr 19 G-II)(Swl 19)

Ans.

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

No, it is not a tautology.

Q.6 **Construct truth table of $(p \wedge \sim p) \to q$.** (Rwp 13, Mtn 18 G-II)

Ans.

p	$\sim p$	q	$p \wedge \sim p$	$(p \wedge \sim p) \to q$
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T

Q.7 **Show that $p \to (p \vee q)$ is a tautology.** (Lhr 09 G-I)

Ans. Truth table for $p \to (p \vee q)$ is given below:

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

From the truth table we can see that the given statement is true for all the values of the variables involved. Hence it is a tautology.

LONG QUESTIONS

Q.1 **Construct the truth table of $[(p \to q) \wedge p] \to q$** (Mtn 12 G-I)

Ans. $[(p \to q) \wedge p] \to q$ (Given)

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If $\sim p \rightarrow q$ be a given conditional then its inverse is: (Mtn 10 G-I)
(a) $\sim p \rightarrow \sim q$ (b) $q \rightarrow p$
(c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow \sim q$

Q.2 Proportion $p \longleftrightarrow q$ represents: (Mtn 11 G-II)
(a) Conjunction (b) Disjunction
(c) Conditional (d) Biconditional

Q.3 The converse of $p \rightarrow q$ is:(Rwp 11 G-I)(Guj 18)
(a) $q \rightarrow p$ (b) $\sim p \rightarrow \sim q$
(c) $\sim p \rightarrow q$ (d) $p \rightarrow q$

Q.4 Proposition $q \rightarrow p$ is called converse of: (Rwp 12 G-I)
(a) $\sim p \rightarrow \sim q$ (b) $\sim q \rightarrow \sim p$
(c) $p \rightarrow q$ (d) $\sim q \rightarrow p$

Q.5 Inverse of proposition $p \rightarrow q$ is: (Gjw 11 G-I)
(a) $p \rightarrow \sim q$ (b) $p \rightarrow q$
(c) $\sim p \rightarrow q$ (d) $\sim p \rightarrow \sim q$

Q.6 The converse of $\sim p \rightarrow q$ is: (Mtn 14 G-I) (Lhr 11 G-I)
(a) $\sim p \rightarrow q$ (b) $q \rightarrow \sim p$
(c) $\sim q \rightarrow p$ (d) $p \rightarrow \sim q$

Q.7 The contrapositive of $p \rightarrow q$ is: (Lhr 10 G-II)
(a) $\sim p \rightarrow \sim q$ (b) $\sim q \rightarrow \sim p$
(c) $q \rightarrow p$ (d) $p \rightarrow q$

Q.8 To draw general conclusions from well-known facts is called
(a) logic (b) proposition
(c) induction (d) deduction

Q.9 A declarative statement which is either true or false but not both is called
(a) logic (b) proposition
(c) induction (d) deduction

Q.10 A biconditional is written in symbols as
(a) $p \leftrightarrow q$ (b) $p \vee q$
(c) $p \rightarrow q$ (d) $p \wedge q$

Q.11 $(p \rightarrow q) \wedge (q \rightarrow p)$ is logically equivalent to
(a) $p \leftrightarrow q$ (b) $q \rightarrow p$
(c) $p \rightarrow q$ (d) $\sim p \rightarrow \sim q$

Q.12 Which is the converse of the sentence $\sim p \rightarrow q$?
(a) $q \rightarrow p$ (b) $p \rightarrow \sim q$
(c) $q \rightarrow \sim p$ (d) $\sim q \rightarrow p$

Q.13 If $\sim p \rightarrow q$ be a given conditional, then its inverse is
(a) $\sim p \rightarrow \sim q$ (b) $q \rightarrow p$
(c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow \sim q$

Q.14 If $q \rightarrow p$ be a given conditional, then its inverse is
(a) $\sim p \rightarrow \sim q$ (b) $q \rightarrow p$
(c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow \sim q$

Q.15 If $p \rightarrow q$ be a given conditional, then its contrapositive is **(D.G.K 17 G-I)**
(a) $\sim p \rightarrow \sim q$ (b) $q \rightarrow p$
(c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow \neg q$

Q.16 If $\sim p \rightarrow \sim q$ be a given conditional, then its contrapositive is
(a) $\sim p \rightarrow \sim q$ (b) $q \rightarrow p$
(c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow \sim q$

Q.17 The conjunction of two statements p and q is denoted by
(a) $p \sim q$ (b) $p \rightarrow q$
(c) $p \wedge q$ (d) $p \vee q$

Q.18 The sentence $p \wedge q$ is true if and only if
(a) p is false and q is true
(b) both p and q are false
(c) p is true and q is false
(d) both p and q are true

Q.19 The sentence $p \vee q$ is false if and only if (DGK 14 G-II)
(a) p is false and q is true
(b) both p and q are false
(c) p is true and q is false
(d) both p and q are true

Q.20 The disjunction of two statements p and q is denoted by
(a) $p \sim q$ (b) $p \rightarrow q$
(c) $p \wedge q$ (d) $p \vee q$

Q.21 When $p \rightarrow q$ is true, which related conditional must be true?
(a) $q \rightarrow p$ (b) $\sim p \rightarrow \sim q$
(c) $\sim q \rightarrow \sim p$ (d) $p \rightarrow \sim q$

Q.22 Symbol used for universal quantifier is
(a) $\exists$ (b) $\in$
(c) $\leftrightarrow$ (d) $\forall$

Q.23 Symbol used for existential quantifier is
(a) $\exists$ (b) $\in$
(c) $\forall$ (d) $\rightarrow$

Q.24 The phrase, " For all x in S", is abbreviated as
(a) $\exists x \in S$ (b) $x \in S$
(c) $\forall x \in S$ (d) $\forall x \notin S$

Q.25 The phrase, " There exist an x in S", is abbreviated as
(a) $\exists x \in S$ (b) $x \in S$
(c) $\forall x \in S$ (d) $\forall x \notin S$

Q.26 A statement which is true for all possible values of the variables involved in it, is called a

(a) tautology (b) conditional
(c) implication (d) absurdity

Q.27 A compound statement of the form "if p then q" is called an

(a) tautology (b) conditional
(c) consequent (d) absurdity

Q.28 Drawing conclusion from premises believed to be true is called. (Mtn 16 G-II)

(a) Induction (b) Deduction
(c) Proposition (d) Conclusion

Q.29 The conjunction of two statements p and q is denoted by. (Lhr 16 G-I)(Swl 19)

(a) $p \vee q$ (b) $p \rightarrow q$
(c) $q \rightarrow p$ (d) $p \wedge q$

Q.30 Proposition $q \rightarrow p$ is converse of (Mtn 15 G-II)

(a) $p \rightarrow q$ (b) $\sim q \rightarrow \sim p$
(c) $q \rightarrow \sim p$ (d) $\sim q \rightarrow p$

Q.31 $q \rightarrow p$ is called converse of. (Mtn 17 G-I)

(a) $\sim p \rightarrow q$ (b) $p \rightarrow q$
(c) $q \rightarrow p$ (d) $\sim q \rightarrow p$

Q.32 In the conditional $p \rightarrow q$, p is called

(a) antecedent (b) consequent
(c) domain (d) range

Q.33 In the conditional $p \rightarrow q$, q is called

(a) antecedent (b) consequent
(c) domain (d) range

Q.34 Which of the following sentences is equivalent to $\sim(p \vee q)$?

(a) $\sim p \vee \sim q$ (b) $\sim p \wedge \sim q$
(c) $\sim p \rightarrow q$ (d) $\sim p \vee q$

Q.35 The symbol used to denote a biconditional between two propositions is: (Mtn 19 G-I)

(a) $\rightarrow$ (b) $\wedge$
(c) $\leftrightarrow$ (d) $\vee$

EXERCISE 2.5

SHORT ANSWERS TO THE QUESTIONS

Q.1 Convert $(A \cap B)' = A' \cup B'$ in logical form. **(Rwp 17 G-I)**

Ans. The corresponding logical form of $(A \cap B)'$

$= A' \cup B'$ is.

$\sim(p \wedge q) = \sim p \vee \sim q$

LONG QUESTIONS

Q.1 Convert the given theorem to logical form and prove by constructing truth table.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Sgd 17)

Ans. Given that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

The logical form is:

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F
T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	F	F	F	T	F	F
F	F	F	F	F	F	F	F

Hence,

$$\boxed{A \cup (B \cap C) = (A \cap B) \cup (A \cap C)}$$

Q.2 Give logical proof of the theorem

$(A \cap B)' = A' \cup B'$. (Mtn 12 G-II)(Rwp 18)

Ans. Given that: $(A \cap B)' = A' \cup B'$

The logical form is: $\sim(p \wedge q) = \sim p \vee \sim q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Hence, $\boxed{(A \cap B)' = A' \cup B'}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If A and B are two sets, then logical form of $(A \cup B)'$ is: (DGK 12 G-I)

(a) $\sim(p \wedge q)$ (b) $p \wedge q$
(c) $p \vee q$ (d) $\sim(p \vee q)$

Q.2 Write the logical form of $(A \cup B)' = A' \cap B'$

(a) $\sim(p \wedge q) = \sim p \vee \sim q$
(b) $\sim(p \vee q) = \sim p \wedge \sim q$
(c) $\sim(p \wedge q) = \sim p \wedge \sim q$
(d) $\sim(p \vee q) = \sim p \vee \sim q$

Q.3 Write the logical form of
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(a) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
(b) $p \wedge (q \vee r) = (p \wedge q) \vee (p \vee r)$
(c) $p \wedge (q \wedge r) = (p \wedge q) \wedge (p \vee r)$
(d) $p \vee (q \wedge r) = (p \wedge q) \vee (p \wedge r)$

EXERCISE 2.6

SHORT ANSWERS TO THE QUESTIONS

Q.1 For A = {1, 2, 3, 4}, find the relate {(x, y)/x + y>5}. (Mtn 12 G-I)(Mtn 17 G-II)

Ans. r = {(4, 4), (4, 3), (4, 2), (3, 4), (3, 3), (2, 4)}

Q.2 Let A = {1, 2, 3, 4}, then find relation in A for {(x, y)/x + y < 5, and x, y ∈ A}. (Mtn 18 G-II) (Mtn 14 G-I, Lhr 14 G-II, Gjw 12)

Ans. {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}.

Q.3 Write the inverse of (Rwp 19)
{(1, 3) , (2, 5) , (3, 7) , (4, 9) , (5, 11)}

Ans. Let r = {(1, 3) , (2, 5) , (3, 7) , (4, 9) , (5, 11)}
$r = \{(3, 1), (5, 2), (7, 3), (9, 4), (11, 5)\}$

Q.4 Find the inverse of the relation R = $\{(x, y): y^2 = 4ax, a > 0, x \geq 0\}$. State whether the relation and its inverse are functions? (D.G.K 17)

Ans. $R^{-1} = \{(x, y): x^2 = 4ay, a > 0, x \geq 0\}$. R is not a function whereas R^{-1} is a function.

Q.5 Find the inverse of the relation R = $\{(x, y): x^2 + y^2 = 9, |x| \leq 3, |y| \leq 3\}$. State whether the relation and its inverse are functions?

Ans. $R^{-1} = \{(x, y): x^2 + y^2 = 9, |x| \leq 3, |y| \leq 3\}$. None of R and R^{-1} are functions.

MULTIPLE CHOICE QUESTIONS

❑ *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 If A = {1, 2, 3, 4}, then domain of {(x, y)/x + y<5} is: (Mtn 11 G-I)
(a) {1, 2, 3} (b) {1, 2, 3, 4}
(c) {2, 3, 4} (d) None

Q.2 The set {(1, 2)} is called: (Mtn 12 G-I)
(a) Infinite set (b) Singleton
(c) Empty set (d) Ordered pair

Q.3 If a function f = A → B is such that range f = B, then f is called. (Lhr 17 G-II)
(a) Injective (b) Surjective
(c) Into (d) Periodic

Q.4 A function f : X → Y is called onto if: (Bpr 11 G-I)
(a) Dom f = X (b) Range f = Y
(c) Range f = X (d) Dom f = Range f

Q.5 The domain of f = {(a, 1), (b, 1), (c, 1)}:
(a) {a, b, c} (b) {a}
(c) {1} (d) {b, c}

Q.6 If f : A → B is function such that range f ≠ B, the f is called: (Fbd 11 G-I)
(a) Onto (b) Injective
(c) Bijective (d) Into

Q.7 The set {(a, b)} is called: (Bpr 14)
(a) Infinite set (b) Singleton set
(c) Empty set (d) Set with two elements

Q.8 If A = { 1 , –1 } then number of elements in A×A are
(a) 2 (b) 6
(c) 4 (d) 8

Q.9 If a set A has m elements and a set B has n elements, then number of elements in A × B is
(a) m × n (b) m
(c) n (d) None of these

Q.10 R is a relation from A to B if and only if R ⊆
(a) B × A (b) A × A
(c) B × B (d) A × B

Q.11 If (x – 2, 2) = (3, 2), then
(a) x = 5 (b) x = 2
(c) x = –5 (d) x = 3

Q.12 In a binary relation, the set consisting of all the second elements of the ordered pairs is called
(a) function (b) range
(c) domain (d) conclusion

Q.13 The inverse of the linear function {(x, y): y = mx + c} is
(a) {(x, y): x = my + c} (b) {(x, y): y = mx+c}
(c) {(x, y): y = mx – c} (d) {(x, y): y = mx+d}

Q.14 The graph of the quadratic function $\{(x, y): y = ax^2 + bx + c\}$ is a
(a) straight line (b) line segment
(c) parabola (d) circle

Q.15 The graph of the function f = {(x,y) / y = mx+c} is (Lhr 15 G-II)
(a) Parabola (b) Straight line
(c) Circle (d) Ellipse

Q.16 A function which is onto is called. (Fbd 16)(D.G.K 12 G-I)
(a) Injective (b) Surjective
(c) Bjective (d) Bijective

Q.17 In a binary relation, the set consisting of all the first elements of the ordered pairs is called
(a) function (b) range
(c) domain (d) antecedent

Q.18 The domain of relation f = {(a, 1), (b, 1), (c, 1)} is (Gjw 19 G-II)
(a) {a, b, c} (b) {a}
(c) {b} (d) {1}

EXERCISE 2.7

SHORT ANSWERS TO THE QUESTIONS

Q.1 Prepare a table of addition of the elements of the set of residue classes modulo 4. (Gjw 18)

Ans.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 An element $b \in S$ is said to be an inverse of $a \in S$ w.r.t * if $a * b = b * a =$

(a) 1 (b) e
(c) – 1 (d) None of the above

Q.2 Which of the following is unary operation:
(D.G.K. 15)(Swl 15 G-I)(Mtn 13 G-I)(Lhr 09 G-I)

(a) Addition (b) Multiplication
(c) Square root (d) Division

Q.3 Which of the following is not binery operation.
(Bpr 16)

(a) Division (b) Addition
(c) Multiplication (d) Square root

EXERCISE 2.8

LONG QUESTIONS

Q.1 If G is a group under ※ and a, b ∈ G, find the solution of the equations a ※ x = b, x ※ a=b.
(Bpr 12 G-I)(Lhr 19 G-II)(Sgd 16)

Ans. Since $a \in G$, there exists $a^{-1} \in G$ such that

$$a ※ a^{-1} = a^{-1} ※ a = e$$

Given that: $a ※ x = b$

$a^{-1} \times (a ※ x) = a^{-1} ※ b$

$\Rightarrow$ $(a^{-1} ※ a) ※ x = a^{-1} ※ b$

$e ※ x = a^{-1} ※ b \Rightarrow \boxed{x = a^{-1} ※ b}$

Again, Given that: $x ※ a = b$

$(x ※ a) ※ a^{-1} = b ※ a^{-1}$

$\Rightarrow$ $x ※ (a ※ a^{-1}) = b ※ a^{-1}$

$x ※ e = b ※ a^{-1} \Rightarrow \boxed{x = b ※ a^{-1}}$

Q.2 Show that the set $\{1, \omega, \omega\}^2$, when $\omega^3 = 1$, is an Abelian group w.r.t ordinary multiplication.
(Mtn 13 G-I, 17, 18 G-II, 19 G-I)(Swl 13 G-I, 17)
(Sgd 13 G-I, 18, 19)(Lhr 13 G-I)(A.J.K 17)

Ans. Let $G = \{1, \omega, \omega^2\}$, ※ = ×

※	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

Ordinary multiplication obeys associative and commutative laws.

1 is multiplicative identity

1 is multiplicative inverse of 1

w is multiplicative inverse of w^2

w^2 is multiplicative inverse of w

We see that all the five properties of an abelian group are satisfied. Then the given set is an abelian group.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If S = {1, –1, i, –i} is a group under multiplication then multiplicative inverse of –i is:
(Mtn 10, 15 G-I)

(a) 1 (b) –1
(c) i (d) –i

Q.2 Set N of natural numbers is monoid:

(a) + (b) –
(c) × (d) ÷

Q.3 If S = N, then under addition S is a:
(DGK 14 G-II)

(a) Group (b) Semi-group
(c) Grouped (d) Monoid

Q.4 Number of identity elements in any group are:
(Lhr 06 G-I)(Sgd 19)

(a) 1 (b) 2
(c) 3 (d) none of these

Q.5 S = {1, –1, 2, –2} is a group under

(a) multiplication (b) subtraction
(c) addition (d) None of these.

Q.6 S = {1 , ω , ω²} where ω is a cube root of unity form an abelian group with respect to

(a) multiplication (b) division
(c) addition (d) subtraction

Q.7 $S = \{1, -1, i, -i\}$ where $i = \sqrt{-1}$ form an abelian group with respect to

(a) multiplication (b) division

(c) addition (d) subtraction

Q.8 The set M of all square matrices of order 2 forms an abelian group with respect to

(a) ordinary multiplication

(b) matrix division

(c) matrix addition

(d) None of these.

Q.9 The set $S = \{0, 1\}$ has closure property w.r.t.

(a) $+$ (b) $-$

(c) $\div$ (d) $\times$

Q.10 An element $e \in S$ is said to be an identity element of S w.r.t * if $a * e = e * a =$

(a) 1 (b) 0

(c) a (d) None of the above

Q.11 The identity element in a group is

(a) unique (b) infinite

(c) both a and b (d) not possible

Q.12 In a group G, if $b * b = b$, then $b =$

(a) 1 (b) e

(c) -1 (d) $\{ e \}$

Q.13 Inverse of an element in a group is

(a) infinite (b) finite

(c) unique (d) not possible

Q.14 A groupoid (S) is called ------------ if it is associative in S.

(a) group (b) abelian-group

(c) semi-group (d) associative -group

Q.15 If a, b, $\in$ G and G is a group, then $(ab)^{-1}$ is equal to: (Rwp 15)(Mtn, Rwp 17 G-II)(Mtn 18 G-I)

(a) $a^{-}b^{-1}$ (b) $b^{-1}a^{-1}$

(c) $\dfrac{-1}{ab}$ (d) $\dfrac{1}{(ab)^{-1}}$

Q.16 Set of integers is a group w.r.t (Gjw 15)(Swl 17)(Lhr 17 G-I)

(a) $+$ (b) $\div$

(c) $\times$ (d) $-$

Q.17 The multiplicative identity in the set of real numbers is. (Rwp 16)

(a) Zero (b) 1

(c) 3 (d) 2

Q.18 (Z, +) has no identity after than. (Rwp17 G-I)

(a) 1 (b) -1

(c) 1 (d) 0

Q.19 How many inverse elements correspond to each element of a group. (Bpr 18)

(a) At least one (b) Only one

(c) Two (d) At least two

Chapter 3

MATRICES AND DETERMINANTS

EXERCISE 3.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 If $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, then find x and y **(Mtn 17 G-II)**

Ans. $x + 3 = 2 \Rightarrow x = -1$

$3y - 4 = 2 \Rightarrow 3y = 6 \Rightarrow y = 2$

Q.24 Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$.

(Swl 17)(Sgd 18)(Mtn 18 G-II)(Gjw 19 G-II) (Fbd 19 G-I + II)

Ans. $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

$x + 3 = y$ ... (i)

$3y - 4 = 2x$ ... (ii)

Putting equation (i) in equation (ii)

$3(x + 3) - 4 = 2x$

$3x + 9 - 4 = 2x$

$3x + 5 = 2x$

$3x - 2x = -5$

$x = -5$

Putting in equation (i)

$-5 + 3 = y$

$y = -2$

Q.2 If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the values of a and b. **(Sgd 16)(Lhr 09 G-I) (Rwp 17 G-I)(Gjw 17)(Fbd 18)(Swl 19)**

Ans. $A^2 = A\,A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$

$= \begin{bmatrix} (1)(1)+(2)(a) & (1)(2)+(2)(b) \\ (a)(1)+(b)(a) & (a)(2)+(b)(b) \end{bmatrix}$

$= \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} \longrightarrow$ (i)

But $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow$ (ii)

From (i) and (ii), we have

$\begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

We know that when two matrices are equal, then their corresponding elements are equal.

Comparing the element a_{11}, we have

$1 + 2a = 0$

$\Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$

Comparing the element a_{12}, we have

$2 + 2b = 0$

$\Rightarrow 2b = -2 \Rightarrow b = -1$

Thus $a = -\frac{1}{2}$, $b = -1$

Q.3 If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the values of a and b. **(Lhr 16 G-I, 19 G-II) (Rwp 17 G-II)(Sgd 17)(Bpr 18)(Gjw 18)**

Ans. $A^2 = A.\,A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$

$= \begin{bmatrix} (1)(1)+(-1)(a) & (1)(-1)+(-1)(b) \\ (a)(1)+(b)(a) & (a)(-1)+b(b) \end{bmatrix}$

$= \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix} \longrightarrow$ (i)

But $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow$ (ii)

From (i) and (ii), we have

$\therefore \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We know that when two matrices are equal, then their corresponding elements are equal.

Comparing the element a_{11}, we have

$1 - a = 1 \Rightarrow a = 1 - 1 \Rightarrow a = 0$

Comparing the element a_{12}, we have

$-1 - b = 0 \quad \Rightarrow \quad b = -1$

Thus $a = 0$, $b = -1$

Q.4 Find the matrix X if;

$X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

Ans. Suppose $XA = B \longrightarrow$ (i)

where $A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

Post multiplying (i) by A^{-1}, we have

$$(XA)A^{-1} = BA^{-1}$$

$\Rightarrow X(AA^{-1}) = BA^{-1}$ ($\because$ Matrix multiplication is associative)

$\Rightarrow XI_2 = BA^{-1}$ ($\because AA^{-1} = I_2$)

$\Rightarrow X = BA^{-1} \longrightarrow$ (ii)

Now we will find A^{-1}

$$A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix} = (5)(1) - (2)(-2)$$

$$= 5 + 4 = 9 \neq 0$$

Since $|A| \neq 0$, A^{-1} exists.

and $\text{Adj. } A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

Now $A^{-1} = \dfrac{1}{|A|}\text{ Adj. } A = \dfrac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

Putting A^{-1} and B in eq. (ii), we have

$$X = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}\left\{\frac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}\right\} = \frac{1}{9}\begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} (-1)(1) + (5)(2) & (-1)(-2) + (5)(5) \\ (12)(1) + (3)(2) & (12)(-2) + (3)(5) \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} -1 + 10 & 2 + 25 \\ 12 + 6 & -24 + 15 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 27 \\ 18 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \times \frac{1}{9} & 27 \times \frac{1}{9} \\ 18 \times \frac{1}{9} & -9 \times \frac{1}{9} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Q.5 Find the matrix X if;

$$\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$ **(Mtn 16 G-I)**

Ans. Suppose $AX = B \longrightarrow$ (i)

where $A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

Premultiplying (i) by A^{-1}, we have

$$A^{-1}(AX) = A^{-1}B$$

$\Rightarrow (A^{-1}A)X = A^{-1}B$ ($\because$ Matrix multiplication is associative)

$\Rightarrow I_2X = A^{-1}B$ ($\because AA^{-1} = I_2$)

$\Rightarrow X = A^{-1}B \longrightarrow$ (ii)

Now we will find A^{-1}

$$A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix} = (5)(1) - (2)(-2)$$

$$= 5 + 4 = 9 \neq 0$$

Since $|A| \neq 0$, A^{-1} exists.

and $\text{Adj. } A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

Now $A^{-1} = \dfrac{1}{|A|}\text{ Adj. } A = \dfrac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

Substituting A^{-1} and B in equation (ii),

we have $X = \dfrac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

$$= \frac{1}{9}\begin{bmatrix} (1)(2) + (-2)(5) & (1)(1) + (-2)(10) \\ (2)(2) + (5)(5) & (2)(1) + (5)(10) \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 2 - 10 & 1 - 20 \\ 4 + 25 & 2 + 50 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -8 & -19 \\ 29 & 52 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{8}{9} & -\frac{19}{9} \\ \frac{29}{9} & \frac{52}{9} \end{bmatrix}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If $A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$, then |A| is equal to: (Gjw 14)

(a) 5 (b) 20
(c) 14 (d) 6

Q.2 If A is non-singular square matrix, then AA^{-1} equals. (Lhr 14 G-I)(Mtn 09 G-I)(Lhr 18 G-I)

(a) A (b) A^{-1}
(c) 0 (d) I

Q.3 If order of a matrix is $m \times n$ then order of A^t is: (Gjw 12)

(a) $m \times n$ (b) $n \times n$
(c) $n \times m$ (d) $m \times m$

Q.4 If $\begin{bmatrix} \lambda & 1 \\ -2 & 1 \end{bmatrix}$ is singular, then λ. (Gjw 11)

(a) 1 (b) 2
(c) –2 (d) –1

Q.5 For matrix equation $\begin{bmatrix} 3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -3 & 2 \end{bmatrix}$ the value of y = (Lhr 10 G-I)

(a) 1 (b) 2
(c) 3 (d) 4

Q.6 **If $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$, the $|A|$ =** (D.G.K 14, D.G.K 15 G-I)

(a) 1 (b) 0
(c) –1 (d) 12

Q.7 **Let A be a square matrix then $|A^t|$ = ______** (Bpr 12, DGK 14)

(a) $|A|$ (b) A
(c) A^{-1} (d) $\frac{1}{|A^t|}$

Q.8 **$A = \begin{bmatrix} 1 & 2 & 5 & 3 \\ 1 & 1 & 1 & 4 \end{bmatrix}$ is matrix of order:**(Swl 14)

(a) 2×3 (b) 2×4
(c) 4×2 (d) 0

Q.9 **If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj A =** (Fbd 14)

(a) $\begin{bmatrix} d & b \\ -c & a \end{bmatrix}$ (b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
(c) $\begin{bmatrix} d & +b \\ c & a \end{bmatrix}$ (d) $\begin{bmatrix} d & -b \\ c & a \end{bmatrix}$

Q.10 **$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ is equal to:** (Rwp 14)

(a) –2 (b) 2
(c) 0 (d) 3

Q.11 **If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then order of A^t is.** (Mtn 10 G-I)

(a) 3×1 (b) 1×1
(c) 1×3 (d) 3×3

Q.12 **Inverse of a scalar matrix is:** (Mtn 10 G-II)

(a) Unit matrix (b) Scalar matrix
(c) Null matrix (d) Column matrix

Q.13 **If $\begin{vmatrix} 4 & x+1 \\ 0 & x-1 \end{vmatrix} = 0$, then x =** (Mtn 11 G-I)

(a) 0 (b) 1
(c) –1 (d) 4

Q.14 **A square matrix X is non-singular if:** (Mtn 11 G-II)

(a) $|X| = 0$ (b) $|X| \neq 0$
(c) $|X| = 1$ (d) $|X| \neq -1$

Q.15 **A square matrix M is singular if:** (Mtn 12 G-I)

(a) $|M| = 1$ (b) $|M| = -1$
(c) $|M| = 0$ (d) $|M| = M$

Q.16 **If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, then A – B =** (Mtn 13 G-II)

(a) $\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 7 \\ 2 & 9 \end{bmatrix}$

Q.17 **If $A = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $B = [5\ \ 0]$, then AB is equal to:** (Fbd 12, Bpr 11)

(a) $\begin{bmatrix} 10 & 0 \\ 5 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$
(c) $[10\ \ 0]$ (d) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Q.18 **If A is a matrix of order $m \times n$, then the number of elements in each row of A is:** (Lhr 09, 10 G-II)

(a) m (b) n
(c) $m + n$ (d) $m - n$

Q.19 **The order of the matrix $\begin{bmatrix} 2 & 9 & 4 \\ 1 & 3 & 4 \end{bmatrix}$ is ....**

(a) 3×3 (b) 3×2
(c) 2×1 (d) 2×3

Q.20 **What is called a matrix of order $m \times 1$?**

(a) Row matrix (b) Column matrix
(c) Identity matrix (d) Scalar matrix

Q.21 **$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ is a**

(a) Row matrix (b) Column matrix
(c) Identity matrix (d) Scalar matrix

Q.22 **If A is a matrix of order $m \times n$ such that $m \neq n$, then what is A called?**

(a) A rectangular matrix (b) A square matrix
(c) A null matrix (d) An identity matrix

Q.23 **If A is a matrix of order $m \times n$ such that $m = n$, then what is A called?**

(a) A rectangular matrix (b) A square matrix
(c) A null matrix (d) An identity matrix

Q.24 **What a matrix is called in which all the elements are zero?** (Gjw 16)

(a) Square matrix (b) Null matrix
(c) Identity matrix (d) Rectangular matrix

Q.25 **$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a (an)**

(a) null matrix (b) diagonal matrix
(c) scalar matrix (d) identity matrix

Q.26 $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ **is a (an)**

(a) scalar matrix (b) null matrix
(c) diagonal matrix (d) identity matrix

Q.27 The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **is (Mtn 17 G-II)**

(a) singular (b) non-singular
(c) rectangular (d) null

Q.28 If A is a non-singular square matrix of order n, then the rank of A is

(a) equal to n (b) less than n
(c) greater than n (d) does not exist

Q.29 If I_n is the identity matrix of order n, then rank of I_n is

(a) equal to n (b) less than n
(c) greater than n (d) does not exist

Q.30 If AB = BA = I, then A and B are

(a) equal to each other.
(b) multiplicative inverse of each other.
(c) additive inverse of each other.
(d) both singular.

Q.31 If A, B, C are three matrices such that AB = AC ⇒ B = C, then A is

(a) singular matrix (b) null matrix
(c) non-singular matrix (d) none of these

Q.32 If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **such that ad – bc ≠ 0 then A^{-1} is**

(a) $\dfrac{1}{ad-bc}\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$ (b) $\dfrac{1}{ad-bc}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$

(c) $\dfrac{1}{bc-ad}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (d) $\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Q.33 $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ **is: (Mtn 15 G-I)**

(a) Null matrix (b) Identity matrix
(c) Diagonal matrix (d) Scalar matrix

Q.34 If order of a matrix A is m × n and order of B is n × p then order of AB: (Mtn 15 G-I)

(a) m × n (b) m × p
(c) p × m (d) n × p

Q.35 [0] is ________ matrix: (Gjw 15)

(a) Square (b) Unit
(c) Rectangular (d) Scalar

Q.36 $(AB)^t$ = (Bpr 16)

(a) AB (b) BA
(c) A^tB^t (d) B^tA^t

Q.37 Matrix of order 1 × m is called. (Bpr 16)

(a) Row matrix (b) Column matrix
(c) Null matrix (d) Diagonal matrix

Q.38 If ord(A) = m × n, ordB = n × p, then ord$(AB)^t$ is. (D.G.K 17 G-I)

(a) m × n (b) m × m
(c) p × m (d) p × n

Q.39 The inverse of a square matrix exists if A is. (Lhr 17 G-II)

(a) Singular (b) Non-Singular
(c) Symmetric (d) Rectangular

Q.40 [8] is a. (Rwp 17 G-I)

(a) Square matrix (b) Unit matrix
(c) Scalar matrix (d) Rectangular matrix

Q.41 If a matrix A is of order m × n then A has number of elements. (Swl 17)

(a) m × n (b) m + n
(c) m – n (d) m

Q.42 $\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ **is a (an)**

(a) identity matrix (b) diagonal matrix
(c) scalar matrix (d) null matrix

Q.43 If A = $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$**, B =** $\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}$**, then A + B = (Swl 14)**

(a) $\begin{bmatrix} 0 & 5 \\ 9 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

Q.44 If A is a matrix of order 3 × 4, then order of AA^t is. (Lhr 17 G-I)(Fbd 16)

(a) 4 × 3 (b) 3 × 4
(c) 4 × 4 (d) 3 × 3

Q.45 If A is a matrix of order 3 × 4, then order of A^t is equal to: (Mtn 15 G-II)

(a) 4 × 3 (b) 4 × 4
(c) 3 × 3 (d) 3 × 4

Q.46 The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ **is called: (Gjw 18)**

(a) Identity (b) Null
(c) Scalar (d) Diagonal

Q.47 If A is a matrix of order 4 × 3, then number of elements in each column of A is: (Bpr 18)

(a) 2 (b) 3
(c) 4 (d) 5

Q.48 The matrix [a b c d] is: (Bpr 19)
(a) Square (b) Unit
(c) Null (d) Row

Q.49 If order of x = 3 × 2 and that of A = 2 × 2, then order of XA = : (Bpr 19)
(a) 3×2 (b) 2×3
(c) 2×2 (d) 3×3

Q.50 If A is a matrix of order 3 × 2 then A^t A is of order: (Fbd 19 G-I)
(a) 3×3 (b) 2×3
(c) 2×2 (d) 3×2

Q.51 If $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ **then the value of y will be:** (Fbd 19 G-II)
(a) 2 (b) -2
(c) 4 (d) -4

Q.52 The additive inverse of a matrix A is:(DGK 19)
(a) A (b) $-A$
(c) A^2 (d) $\dfrac{\text{adj}(A)}{|A|}$

EXERCISE 3.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Find A^{-1} if $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$.
(Lhr 13)(Sgd 13)(Gjw 14)(Mtn 09)

Ans. $|A| = \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = 5 - 3 = 2$

$\text{adj } A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|}\text{adj } A = \dfrac{1}{2}\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}$

Q.2 Find the inverses of the matrix $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$
(Lhr 09, 17, 18 G-II)(Fbd 16)(Sgd 19)

Ans. Let $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} = (2)(3) - (1)(6) = 6 - 6 = 0$

Since $|A| = 0$, A^{-1} does not exists.

Q.3 Find the inverse of the matrix $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$.
(Rwp 18)

Ans. Let $A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} -2 & 3 \\ -4 & 5 \end{vmatrix}$

$= -10 + 12 = 2 \neq 0$

and $\text{Adj. } A = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$

Thus $A^{-1} = \dfrac{1}{|A|}\text{Adj. } A = \dfrac{1}{2}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & -\frac{3}{2} \\ 2 & -1 \end{bmatrix}$

Q.4 If A and B are square matrices of the same order, then explain why in general $(A + B)(A - B) \neq A^2 - B^2$ **(Fbd 19 G-I, II)**

Ans. $(A + B)(A - B) = (A - B)(A - B) = AA - AB + BA - BB$

$= A^2 - AB + BA - B^2$

$\neq A^2 - AB + AB - B^2$ ($\because BA \neq AB$ in general)

$\neq A^2 - B^2$

Hence $(A + B)(A - B) \neq A^2 - B^2$

Q.5 Solve the system of linear equations.
$4x_1 + 3x_2 = 5$, $3x_1 - 2x_2 = 7$ **(Rwp 19)**

Ans. $4x_1 + 3x_2 = 5$

$3x_1 - 2x_2 = 7$

Writing equations in matrix form

$\begin{bmatrix} 4 & 3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$|A| = \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} = -8 - 9 = -17$

$\text{Adj } A = \begin{bmatrix} -2 & -3 \\ -3 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|}\text{ Adj } A$

$= \dfrac{1}{-17}\begin{bmatrix} -2 & -3 \\ -3 & 4 \end{bmatrix}$

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{17}\begin{bmatrix} -2 & -3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$= -\dfrac{1}{17}\begin{bmatrix} -10-21 \\ -15+28 \end{bmatrix} = -\dfrac{1}{17}\begin{bmatrix} -31 \\ 13 \end{bmatrix} = \begin{bmatrix} \frac{31}{17} \\ \frac{-13}{17} \end{bmatrix}$

$\Rightarrow x = \dfrac{31}{17}, y = \dfrac{-13}{17}$

Q.6 Solve the matrix equation for 2X − 3A = B of $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$.
(DGK 14 G-I)

Ans. $2X - 3A = B$

$2X = B + 2A$

$X = \dfrac{1}{2}(B + 3A)$

$$X = \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + 3\begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & -1-3 & 0+6 \\ 4-6 & 2+12 & 1+15 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$$

Q.7 Find the inverse of matrix $A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$.

(Mtn 19 G-I)

Ans. Let $A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -2 & 3 \\ -4 & 5 \end{vmatrix} = -10 + 12 = 2$$

$$\text{Adj } A = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A = \frac{1}{2}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times \frac{1}{2} & -3 \times \frac{1}{2} \\ 4 \times \frac{1}{2} & -2 \times \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{-3}{2} \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow \boxed{A^{-1} = \begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}}$$

LONG QUESTIONS

Q.1 Solve the matrix equation for matrix A.

$$\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix} A - \begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$$ **(Fbd 12)**

Ans. Given that: $A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$

$$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$$

$$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -1+2 & 2+0 \\ 3-1 & 1+5 \end{bmatrix}$$

$$\Rightarrow \quad A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3a+4b & a+2b \\ 3c+4d & c+2d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

According to definition of equal matrices, so

$3a + 4b = 1$ ...... (1)

$a + 2b = 2 \Rightarrow a = 2 - 2b$ ...... (2)

Putting value of eq. (2) in eq. (1):

$3(2 - 2b) + 4b = 1 \Rightarrow 6 - 6b + 4b = 1$

$\Rightarrow 6 - 2b = 1$

$2b = 6 - 1 \Rightarrow 2b = 5 \Rightarrow \boxed{b = \frac{5}{2}}$

Putting value of b in eq. (1):

$3a + 4\left(\frac{5}{2}\right) = 1 \Rightarrow 3a + 2(5) = 1 \Rightarrow 3a + 10 = 1$

$3a = 1 - 10 \Rightarrow 3a = -9 \Rightarrow a = \frac{-9}{3}$

$\Rightarrow \boxed{a = -3}$

$3c + 4d = 2$ ...... (3)

$c + 2d = 6 \Rightarrow c = 6 - 2d$ ...... (4)

Putting value of eq. (4) in eq. (3):

$3(6 - 2d) + 4d = 2 \Rightarrow 18 - 6d + 4d = 2$

$\Rightarrow 18 - 2d = 2$

$2d = 18 - 2 \Rightarrow 2d = 16 \Rightarrow d = \frac{16}{2}$

$\Rightarrow \boxed{d = 8}$

Putting value of d in eq. (4):

$c = 6 - 2(8) = 6 - 16 \Rightarrow \boxed{c = -10}$

$$\Rightarrow \boxed{A = \begin{bmatrix} -3 & 5/2 \\ -10 & 8 \end{bmatrix}}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If $A = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, then order of A^t is:

(DGK 13, Gjw 14, Lhr 14 G-II)

(a) 3×2 (b) 2×3

(c) 2×2 (d) 3×3

Q.2 If the matrix $\begin{bmatrix} 2 & 3 \\ 4 & x \end{bmatrix}$ is singular, then x is.

(Lhr 13 G-I)

(a) 3 (b) 4

(c) 2 (d) 6

Q.3 For any non-singular matrix A, A^{-1} is: (Mtn 08 G-II)(Fbd 14)(Rwp 17 G-II)(Gjw 18)

(a) $|A|\,\text{adj }A$ (b) $\dfrac{1}{|A|\,\text{adj }A}$
(c) $\dfrac{\text{adj }A}{|A|}$ (d) $\dfrac{|A|}{\text{adj }A}$

Q.4 If A is non-singular matrix, then $(\text{adj }A)^{-1}$ equals to: (Mtn 09 G-I)

(a) $\dfrac{A}{|A|}$ (b) $|A|\,A$
(c) A (d) A^{-1}

Q.5 If A is a square and non-singular matrix, then AA^{-1} equal: (Rwp 11)

(a) A (b) A^{-1}
(c) $\dfrac{1}{A}$ (d) I

Q.6 If A is a matrix of order 3×2, the order of A^t is: (Rwp 12)

(a) 2×3 (b) 3×3
(c) 2×2 (d) 3×2

Q.7 If $A=\begin{bmatrix}1 & 7\\ 6 & 4\end{bmatrix}$, then $|A|$ equal to: (Rwp 13)

(a) 46 (b) −46
(c) 38 (d) −38

Q.8 If $\begin{bmatrix}\lambda & 1\\ -2 & 1\end{bmatrix}$ is singular, then $\lambda=$ (Gjw 13)

(a) 1 (b) 2
(c) −2 (d) −1

Q.9 $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ is a: (Sgd 13 G-II)

(a) Unit matrix (b) Diagonal matrix
(c) Scalar matrix (d) Square matrix

Q.10 If A and B are two square matrices of same order, $(A+B)^2=$

(a) $A^2+2AB+B^2$ (b) $A^2+2BA+B^2$
(c) $A^2+AB+BA+B^2$
(d) A^2+B^2

Q.11 If A and B are two square matrices of same order and $(A+B)^2=A^2+2AB+B^2$, then

(a) $A=B^t$ (b) $AB=BA$
(c) $A=-B$ (d) $A=B$

Q.12 If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then matrix A is singular if

(a) $ab-cd=0$ (b) $ac-bd=0$
(c) $ad-bc=1$ (d) $ad-bc=0$

Q.13 If A, B, C are three non-singular matrices such that AB = C then B = ...

(a) $\dfrac{C}{A}$ (b) CA^{-1}
(c) $A^{-1}C$ (d) $C^{-1}A$

Q.14 The order of $\begin{bmatrix}l & m & n\end{bmatrix}\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ is

(a) 1×1 (b) 2×2
(c) 3×3 (d) None of these.

Q.15 If A is a non-singular matrix, then $(A^{-1})^{-1}=$

(a) A^{-2} (b) A
(c) $\dfrac{1}{A}$ (d) A^{-1}

Q.16 If matrix A is non-singular, then (Rwp 17 G-II)

(a) $A^{-1}=\dfrac{\text{Adj. }A}{A}$ (b) $A^{-1}=\dfrac{\text{Adj. }|A|}{A}$
(c) $A^{-1}=\dfrac{\text{Adj. }A}{\text{order of }A}$ (d) $A^{-1}=\dfrac{\text{Adj. }A}{|A|}$

Q.17 The additive inverse of A is: (Bpr 15 G-I)

(a) A^{-1} (b) A
(c) $\dfrac{\text{adj }A}{|A|}$ (d) $-A$

Q.18 Every diagonal matrix is also. (Mtn 16 G-II)

(a) Triangular Matrix (b) Scalar Matrix
(c) Rectangular Matrix
(d) Symmetric Matrix

Q.19 If the Matrix $\begin{bmatrix}\lambda & -4\\ 1 & 2\end{bmatrix}$ is singular then $\lambda=$ (Fbd 16)

(a) 2 (b) 4
(c) 1 (d) $\dfrac{1}{2}$

Q.20 Inverse of a square matrix A does not exist if A is: (Lhr 15 G-I)

(a) Singular (b) Non-singular
(c) Unit (d) Diagonal

Q.21 If the matrix $\begin{bmatrix}r & 4\\ 3 & 2\end{bmatrix}$ is singular, then r equals: (Swl 13, Mtn 13 G-I)

(a) 2 (b) 4
(c) 6 (d) 8

Q.22 If $\begin{bmatrix}k & 4\\ 4 & k\end{bmatrix}=0$, then k = (Mtn 14 G-I)(Lhr 17 G-I)(Sgd 18, 19)

(a) 16 (b) 0
(c) ±4 (d) 8

Q.23 If $A = \begin{bmatrix} d & b \\ -c & a \end{bmatrix}$, then adj (A) = __________

(Mtn 17 G-I)

(a) $\begin{bmatrix} a & -b \\ c & d \end{bmatrix}$ (b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(c) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (d) $\begin{bmatrix} d & -b \\ c & a \end{bmatrix}$

Q.24 If A and B are non-singular matrices, then $(AB)^{-1}$ =

(Rwp 17 G-I)(Lhr 09 G-I)(D.G.K 15 G-I)(Fbd 17)

(a) $A^{-1}B^{-1}$ (b) A^{-1}

(c) B^{-1} (d) $B^{-1}A^{-1}$

Q.25 Let $A = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$ then |A| equals:(Lhr 07 G-I)

(a) 4 (b) 3

(c) 5 (d) 1

Q.26 The order of the matrix A is 3×2, then order of A^tA is: (Fbd 18)

(a) 3×2 (b) 3×3

(c) 2×2 (d) 2×3

Q.27 If matrix $\begin{bmatrix} x & 4 \\ 2 & 8 \end{bmatrix}$ is singular, then x =

(Lhr 18 G-I)

(a) 0 (b) -1

(c) 2 (d) 1

Q.28 If $\begin{vmatrix} 2 & \lambda \\ 3 & 7 \end{vmatrix} = 2$, then λ = (Mtn 18 G-II)

(a) 1 (b) 2

(c) 3 (d) 4

Q.29 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then A^t (the transpose of A) is: (Sgd 18)

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 5 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \\ 4 & 6 \end{bmatrix}$

Q.30 If $\begin{bmatrix} \gamma & 3 \\ 4 & 2 \end{bmatrix}$ is singular then λ is equal to:

(Gjw 19 G-II)

(a) 2 (b) 6

(c) 4 (d) 8

Q.31 For a non singular matrix *A*, if *AX* = *B*, then *X* = (Mtn 19 G-I)

(a) $A^{-1}B$ (b) BA^{-1}

(c) $(AB)^{-1}$ (d) $(BA)^{-1}$

Q.32 The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is (Sgd 19)

(a) Null matrix (b) Identity matrix

(c) Diagonal matrix (d) Scalar matrix

Q.33 If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 2$ then $\begin{vmatrix} c & d \\ a & d \end{vmatrix}$ = (Swl 19)

(a) 2 (b) -2

(c) ± 2 (d) 0

EXERCISE 3.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Evaluate the determinant of

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}.$$ **(Rwp 17 G-II)**

Ans. $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & 1 \\ 4 & -3 & 2 \end{vmatrix}$

$= 1(6+3) + 2(4-4) + 3(-6-12)$

$= 1(9) + 2(0) + 3(-18)$

$= 9 + 0 - 54 = -45$

Q.2 Evaluate $\begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$ **(Gjw 17)**

Ans. $\begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$

$= 5(-2+3) + 2(6-6) - 4(3-2)$

$= 5(1) + 2(0) - 4(1)$

$= 5 + 0 - 4 = 1$ Ans.

Q.3 Without expansion show that $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$

(Lhr 16 G-I)(A.J.K 17)(Mtn 18 G-II)

Ans. Left Side $= \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 6 & 6 & 6 \end{vmatrix}$ By $R_2 - R_1$, $R_3 - R_1$

$= 2\begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix}$ (By taking 2 common from R_3)

$= 2 \times 0$ (As R_2 and R_3 are identical)

$= 0 =$ Right Side

Q.4 Without expansion show that:

(Gjw 18)(Lhr 18 G-II)

$$\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

Ans. Left Side $= \begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix}$

$= \begin{vmatrix} 6 & 1 & 2 \\ 3 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix}$ By $C_2 - C_1$, $C_3 - C_1$

$= 2\begin{vmatrix} 6 & 1 & 1 \\ 3 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ (By taking 2 common from C_3)

$= 2 \times 0$ (As C_2 and C_3 are identical)

$= 0 =$ Right Side

Q.5 Without expansion show that

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0.$$

Ans. L.H.S $= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$ $C_2 + C_3$

$= \begin{vmatrix} 2 & 2 & -1 \\ 1 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix}$ C_1 and C_2 are identical

$= 0 =$ R.H.S

Q.6 Without expansion, show that:

(Srd 18)(Fbd 18) (DGK 11)(Mtn 16 G-II)

$$\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix} = 0$$

Ans. L.H.S $= \begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix}$

Multiplying R_2 with abc.

$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ bc & ca & ab \\ a & b & c \end{vmatrix}$ R_1 and R_2 are identical.

$= \frac{1}{abc}(0) = 0 =$ R.H.S

Q.7 Without expansion show that

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

Ans. L.H.S. $= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix}$

$= \frac{xyz}{xyz}\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = \frac{1}{xyz}\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ xyz & xyz & xyz \end{vmatrix}$ By xC_1, yC_2, zC_3

$= \frac{xyz}{xyz}\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$ By taking xyz common from R_3

$= -\begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ x^2 & y^2 & z^2 \end{vmatrix}$ By interchanging R_2 and R_3

$= (-)(-)\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ By interchanging R_1 and R_2

$= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} =$ R.H.S.

Q.8 Without expansion verify that:

$$\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$$

(Rwp 14)(Mtn 10 G-II)(Lhr 14 G-II)(Bpr 13)
(Gjw 16, 19 G-I)(Mtn 18 G-I)(Gjw 19 G-I)

Ans. L.H.S

$= \begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix}$

$= \begin{vmatrix} \alpha+\beta+\gamma & \beta+\gamma & 1 \\ \alpha+\beta+\gamma & \gamma+\alpha & 1 \\ \alpha+\beta+\gamma & \alpha+\beta & 1 \end{vmatrix}$

$\underbrace{C_1 + C_2}$

$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta+\gamma & 1 \\ 1 & \gamma+\alpha & 1 \\ 1 & \alpha+\beta & 1 \end{vmatrix}$

taking $\alpha+\beta+\gamma$ common from c_1.

$= (\alpha+\beta+\gamma)(c) = 0 =$ R.H.S

Q.9 Show that $\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = 9\begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$.

(Lhr 09 G-II)

Ans. L.H.S. $= \begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 2 & 3 & 0 \\ 1 & 3 & 2 \\ 2 & 15 & 1 \end{vmatrix}$ By taking 3 common from R_2

$= 3 \times 3\begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$ By taking 3 common from C_2

$= 9\begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix} =$ R.H.S.

Q.10 If $B = \begin{bmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & -1 \end{bmatrix}$**, then find** B_{21}, B_{22}

(Lhr 17 G-II, 09 G-I, 19 G-II)

Ans. $B_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 5 \\ 1 & -2 \end{vmatrix} = (-1)^3(4-5)$

$= (-1)(-1) = 1$

$B_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 5 \\ -2 & -2 \end{vmatrix} = (-1)^4(-10+10)$

$= 1(0) = 0$

Q.11 Find value of λ if $\begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ **is singular.**

(DGK 14 G-II)(DGK 17 G-I)
(Mtn 18 G-I)(Rwp 16 G-II, 19)

Ans. A is singular if $|A| = 0$

$$\begin{vmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$4(3-18) - \lambda(7-12) + 3(21-6) = 0$$
$$4(-15) - \lambda(-5) + 3(15) = 0$$
$$-60 + 5\lambda + 45 = 0$$
$$-15 + 5\lambda = 0$$
$$5\lambda = 15$$
$$\boxed{\lambda = 3}$$

Q.12 Find x if $\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = 0$.

(Bpr 14)(Lhr 16 G-I)

Ans. $\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$

$$3(0-4) - 1(0-4x) + x(-1-3x) = -30$$
$$-12 + 4x - x - 3x^2 = -30$$
$$-3x^2 + 3x - 12 = -30$$
$$-3x^2 + 3x - 12 + 30 = 0$$
$$-3x^2 + 3x + 18 = 0$$
$$-3(x^2 - x - 6) = 0$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0 \Rightarrow x = 3, -2$$

Q.13 Find x if $\begin{vmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 3 & 6 & x \end{vmatrix} = 0$.

(Gjw 14)(Bpr 16, 18)(Rwp 19)

Ans. $\begin{vmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 3 & 6 & x \end{vmatrix} = 0$

$$1(x^2-12) - 2(2x-6) + 1(12-3x) = 0$$
$$x^2 - 12 - 4x + 12 + 12 - 3x = 0$$
$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$

$x - 3 = 0$ | $x - 4 = 0$
$x = 3$ | $x = 4$

Q.14 Without expansion show that

$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$: **(Bpr 19)**

Ans. L.H.S

$= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$ $C_2 + C_3$

$= \begin{vmatrix} 2 & 2 & -1 \\ 1 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix}$ C_1 is lenticels to C_2

$= 0$

$=$ R.H.S

LONG QUESTIONS

Q.1 Show that:

$$\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$$

(Mtn 16 G-II)(A.J.K 17)(Lhr 17 G-I)(Fbd 19)

Ans. L.H.S. $= \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$

$= \begin{vmatrix} a+b+c & a & a^2 \\ a+b+c & b & b^2 \\ a+b+c & c & c^2 \end{vmatrix}$ (Applying $C_1 + C_2$)

$= (a+b+c)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$\left(\text{Applying } \dfrac{1}{a+b+c} C_1\right)$

$= (a+b+c)\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$

(Applying $R_2 - R_1$ and $R_3 - R_1$)

$= (a+b+c)\begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} b-a & (b-a)(b+a) \\ c-a & (c-a)(c+a) \end{vmatrix}$

$= (a+b+c)(b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$

$= (a+b+c)(b-a)(c-a)(c+a-b-a)$

$= (a+b+c)(b-a)(c-a)(c-b)$

$= (a+b+c)(a-b)(c-a)(b-c)$

$= (a+b+c)(a-b)(b-c)(c-a) =$ R.H.S.

Q.2 Without expansion verify that

$$\begin{vmatrix} 1 & a^2 & \dfrac{a}{bc} \\ 1 & b^2 & \dfrac{b}{ca} \\ 1 & c^2 & \dfrac{c}{ab} \end{vmatrix} = 0. \qquad \textbf{(Swl 17)}$$

Ans. L.H.S. $= \begin{vmatrix} 1 & a^2 & a/bc \\ 1 & b^2 & b/ca \\ 1 & c^2 & c/ab \end{vmatrix}$

$= \dfrac{1}{abc}\begin{vmatrix} 1 & a^2 & a^2 \\ 1 & b^2 & b^2 \\ 1 & c^2 & c^2 \end{vmatrix}$

(Applying abc C_3)

$= \dfrac{1}{abc} \times 0$

($\because$ each elements of the second and the third columns are identical)

$= 0 =$ R.H.S.

Q.3 Show that $\begin{vmatrix} a+\lambda & a & a \\ a & a+\lambda & a \\ a & a & a+\lambda \end{vmatrix} = \lambda^2(3a+\lambda).$

(Rwp 13 G-I)(D.G.K 15 G-I)(Fbd 17)(Lhr 19 G-II)

Ans. L.H.S. $= \begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix}$

$= \begin{vmatrix} 3a+l & a & a \\ 3a+l & a+l & a \\ 3a+l & a & a+l \end{vmatrix}$

(Applying $C_1 + (C_2 + C_3)$)

$= (3a+l)\begin{vmatrix} 1 & a & a \\ 1 & a+l & a \\ 1 & a & a+l \end{vmatrix}$

$\left(\text{Applying } \dfrac{1}{3a+l} C_1\right)$

$= (3a+l)\left[1\begin{vmatrix} a+l & a \\ a & a+l \end{vmatrix} - a\begin{vmatrix} 1 & a \\ 1 & a+l \end{vmatrix} + a\begin{vmatrix} 1 & a+l \\ 1 & a \end{vmatrix}\right]$

$= (3a+l)\,[(a+l)^2 - a^2 - a(a+l-a) + a(a-a-l)]$

$= (3a+l)\,[a^2 + 2al + l^2 - a^2 - a(l) + a(-l)]$

$= (3a+l)\,[2al + l^2 - al - al] = (3a+l)\,[l^2]$

$= l^2(3a+l) =$ R.H.S.

Q.4 Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$

(Mtn 12, 13 G-II)(Sgd 19)

Ans. L.H.S. $= \begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix}$

$= \begin{vmatrix} x+3 & 1 & 1 & 1 \\ x+3 & x & 1 & 1 \\ x+3 & 1 & x & 1 \\ x+3 & 1 & 1 & x \end{vmatrix}$

(Applying $C_1 + (C_2 + C_3 + C_4)$)

$$= (x+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix}$$

$\left(\text{Applying } \dfrac{1}{x+3}\, C_1\right)$

$$= (x+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x-1 & 0 & 0 \\ 0 & 0 & x-1 & 0 \\ 0 & 0 & 0 & x-1 \end{vmatrix}$$

(Applying $R_2 - R_1, R_3 - R_1, R_4 - R_1$)

$$= (x+3)\begin{vmatrix} x-1 & 0 & 0 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

Applying the property of determinant:

$$= (x+3)(x-1)(x-1)(x-1)$$
$$= (x+3)(x-1)^3 = \text{R.H.S.}$$

Q.5 If $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$ then verify $(AB)^{-1} = B^{-1}A^{-1}$. **(D.G.K 11 G-I)**

Ans. Given that: $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+8 & 1-2 \\ 3+0 & -1+0 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & -1 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 5 & -1 \\ 3 & -1 \end{vmatrix} = -5+3 = -2 \neq 0$$

$$\text{adj of }(AB) = \begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|}\text{adj of }(AB) = \frac{1}{-2}\begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{3}{2} & \frac{-5}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \Rightarrow |A| = 0+2 = 2 \neq 0$$

$$\text{adj of } A = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj of } A = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

and $\quad B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} -3 & 1 \\ 4 & -1 \end{vmatrix}$

$$= 3-4 = -1 \neq 0$$

$$\text{adj of } B = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$$

$$\therefore \quad B^{-1} = \frac{1}{|B|}\text{adj of } B = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0+\frac{1}{2} & -1+\frac{1}{2} \\ 0+\frac{3}{2} & -4+\frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{3}{2} & \frac{-5}{2} \end{bmatrix}$$

Hence, $\boxed{(AB)^{-1} = B^{-1}A^{-1}}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If A is a square matrix of order 2, then |kA| equals. (Lhr 14, 19 G-II)(D.G.K 17 G-II)

(a) $k|A|$ (b) $\frac{1}{k}|A|$
(c) $2k\,|A|$ (d) $k^2\,|A|$

Q.2 If two rows of any square matrix are identical, then the value of determinant is: (Lhr 14 G-I)(Mtn 15 G-II)

(a) 1 (b) –1
(c) 0 (d) 2

Q.3 If $\begin{vmatrix} -1 & 3 \\ x & 1 \end{vmatrix} = 0$, then x = (Lhr 15 G-II) (Mtn 14 G-I)

(a) a (b) –3
(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Q.4 If the matrices A and B are conformable for multiplication, the $(AB)^t$ equals to: (Bpr 13-14, Mtn 12 G-I)(Swl 17)(Lhr 17 G-II)

(a) B^tA^t (b) AB
(c) BA (d) A^tB^t

Q.5 If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{bmatrix}$, then A_{32} equal to: (Bpr 14)

(a) 2 (b) 0
(c) –1 (d) –3

Q.6 The value of determinant $\begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix}$ equals (Mtn 10 G-II)

(a) abc + xyz (b) $\frac{abc}{xyz}$

(c) abc − xyz (d) zero

Q.7 The value of $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$ is: (DGK 12)

(a) 0 (b) 2

(c) −1 (d) −3

Q.8 If in a square matrix, two columns are identical the |A| = (Fbd 11)

(a) −1 (b) 0

(c) 1 (d) 2

Q.9 If A is a square matrix of order 3 × 3 then | kA | equals: (Lhr 09)(Mtn 18 G-I)
(Lhr 09-10 G-II)(Bpr 15)(Sgd 16)(D.G.K 17 G-I)

(a) k |A| (b) k^2 |A|

(c) k^3 |A| (d) k^4 |A|

Q.10 If A = $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$ then cofactor of 6 is: (Lhr 08 G-I)(Fbd 17)

(a) 1 (b) −1

(c) −6 (d) 3

Q.11 If |A| is the determinant of a square matrix A then |A| is: (Lhr 08 G-I)

(a) Always positive (b) Modulus of A

(c) Always −ve (d) May be +ve or −ve

Q.12 If two rows (or two columns) in a square matrix are identical (i.e. corresponding elements are equal), the value of the determinant is

(a) 0 (b) 1

(c) − 1 (d) ± 1

Q.13 If each element in any row or each element in any column of a square matrix is zero, then value of the determinant is

(a) 0 (b) 1

(c) −1 (d) none of these.

Q.14 If any two rows of a square matrix are interchanged, the determinant of the resulting matrix

(a) is zero. (b) $\frac{1}{|A|}$

(c) −|A| (d) none of these.

Q.15 Minors and co-factors of the elements in a determinant are equal in magnitude but they may differ in

(a) order (b) position

(c) sign (d) symmetry

Q.16 If A is a square matrix such that $A^2 = I$, then A^{-1} =

(a) − A (b) A

(c) A + I (d) None of these.

Q.17 For a square matrix A, |A| equals. (Lhr 08)

(a) A^t (b) $|A^t|$

(c) $-|A^t|$ (d) $-A^t$

Q.18 If each element of a 3 × 3 matrix A is multiplied by 3, then the determinant of the resulting matrix is

(a) $|A|^3$ (b) 27 |A|

(c) 3 |A| (d) 9 |A|

Q.19 If A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ =

(a) 0 (b) |A|

(c) |adj. A| (d) none of these

Q.20 If A = $\begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$, then A_{33} equals:

(a) −1 (b) 1

(c) 7 (d) −7

Q.21 The co-factor of an element a_{ij} denoted by A_{ij} = (Mtn 16 G-II)(D.G.K 17 G-II, 19)

(a) $(-1)^{ij} M_{ij}$ (b) $(-1)^{i-j} M_{ij}$

(c) $(-1)^{i+j} M_{ij}$ (d) $(1)^{i+j} M_{ij}$

Q.22 If A = $\begin{bmatrix} a & b & c \\ d & e & f \\ a & b & c \end{bmatrix}$, then det A is equal to. (Mtn 15 G-I)

(a) 0 (b) 1

(c) (a − b)(c − a) (d) $a^2 - ab + ac + b^2$

Q.23 $(A^{-1})^t$ is equal to. (Lhr 16 G-I)

(a) A (b) $A^{-1}A^t$

(c) $(A^t)^{-1}$ (d) $-A^t$

Q.24 If A is non-singular matrix then $(A^t)^t$ equals. (Gjw 16)

(a) A^t (b) A

(c) $-A^t$ (d) −A

Q.25 M_{21} of $\begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}$ is. (Mtn 16 G-II)

(a) 1 (b) 2
(c) 3 (d) −3

Q.26 If |A| is the determinant of a square matrix A then |A| is: (Lhr 08 G-I)

(a) Always positive (b) Modulus of A
(c) Always −ve (d) May be +ve or −ve

Q.27 If A and B are non-singular matrices, then $(AB)^{-1}$ = (Rwp 15, 16)(Fbd 15)

(a) $B^{-1}A^{-1}$ (b) $A^{-1}B^{-1}$
(c) $(BA)^{-1}$ (d) none of these.

Q.28 If A is a square matrix, then

(a) $|A^t| = A$ (b) $|A^t| = -A$
(c) $|A^t| = |A|$ (d) $A^t = A$

Q.29 The value of determinant of an identity matrix is equal to. (Sgd 17)

(a) 0 (b) 1
(c) −1 (d) 2

Q.30 If two rows are interchanged, the determinant of resulting matrix. (A.J.K 17)

(a) 0 (b) −|A|
(c) |A| (d) In

Q.31 If A = $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$**, then cofactor of "4" is:** (Mtn 18 G-I)

(a) 1 (b) −1
(c) −4 (d) 3

Q.32 The determinant of any unit matrix has value. (Rwp 18)

(a) Less than 1 (b) Greater than 1
(c) 1 (d) 0

Q.33 If A and B are non-singular matrices then $(AB)^{-1}$ (Fbd 19 G-I)

(a) $A^{-1}B^{-1}$ (b) $\frac{1}{AB}$
(c) $B^{-1}A^{-1}$ (d) $(BA)^{-1}$

Q.34 If A is 4 × 4 matrix then | KA | = : (Fbd 19 G-II)

(a) $K|A|^4$ (b) $K^2|A|^2$
(c) $K^3|A|$ (d) $K^4|A|$

Q.35 For any two matrices A and B then $(AB)^t$ equals (Gjw 19 G-II)

(a) AB (b) A^tB^t
(c) B^tA^t (d) BA

Q.36 If A = $[a_{ij}]_{2\times2}$, then |kA| =: (Lhr 19 G-II)

(a) $|A|$ (b) $k^2|A|$
(c) $k|A|$ (d) $k|A|^2$

Q.37 If A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 4 & 5 & 2 \end{bmatrix}$**, then M_{13} =:** (Mtn 19 G-I)

(a) 13 (b) 0
(c) 10 (d) 7

Q.30 If "A" is a square matrix and $(\bar{A})^t = -A$, then "A" is called: (Rwp 19)

(a) Skew Symmetric (b) Symmetric
(c) Skew Hermitian (d) Hermitian

Q.31 If |A| = 5, then $|A^t|$ = (Swl 19)

(a) −5 (b) $\frac{1}{5}$
(c) 0 (d) 5

EXERCISE 3.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define row rank of a matrix.
(Mtn 17 G-II)(Rwp 18, 19)(Fbd 19 G-II)(DGK 19)

Ans. Number of non-zero rows in echelon form or reduced echelon form is called the rank of the matrix.

For example;

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & +1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank A = 2.

Q.2 Define a Symmetric Matrix. **(Mtn 18 G-I)**

Ans. A square matrix A is said to be symmetric if $A^t = A$.

Q.3 Define a Skew Symmetric Matrix. **(Mtn 18 G-I)(Swl 19)**

Ans. A square matrix A is said to be skew symmetric if $A^t = -A$.

Q.4 Define Hermitian Matrix. **(Bpr 16)(Fbd 19)**

Ans. A square matrix A over the field of complex numbers is called Hermitian matrix if $(\overline{A})^t = A$

Q.5 Define Skew Hermitian Matrix.

Ans. A square matrix A over the field of complex numbers is called Skew Hermitian if $(\overline{A})^t = -A$

Q.6 Define an Upper Triangular Matrix. **(Gjw 19 G-II)**

Ans. A square matrix in which each element below the diagonal is zero is called an upper triangular matrix.

Q.7 If $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$**, Find** $\left(\overline{A}\right)^t$

Ans. $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$

$\overline{A} = \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$ (Replacing i by $-i$)

$\left(\overline{A}\right)^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$

Q.8 If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$**, find** $A\,(\bar{A})^t$

(Lhr 10 G-I)(Sgd 16)

Ans. $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} 1 \\ 1-i \\ -i \end{bmatrix}$

$\Rightarrow (\bar{A})^t = [\,1 \quad 1-i \quad -i\,]$

Now $A(\bar{A})^t = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix} [\,1 \quad 1-i \quad -i\,]$

$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 1-i^2 & -i-i^2 \\ i & i-i^2 & -i^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 1-(-1) & -i-(-1) \\ i & i-(-1) & -(-1) \end{bmatrix}$

$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 1+1 & -i+1 \\ i & i+1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1-i & -i \\ 1+i & 2 & 1-i \\ i & 1+i & 1 \end{bmatrix}$

LONG QUESTIONS

Q.1 Find the rank of the matrix $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$.

(Bahawalpur Board 2013 G-I, 2016)

Ans. Given that: $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & -4 & -7 \\ 0 & 3 & 15 \\ 0 & 2 & 10 \\ 0 & 5 & 25 \end{bmatrix}$

(Applying $R_2 - 2R_1$, $R_3 - R_1$ and $R_4 - 3R_1$)

$\underset{\sim}{R} \begin{bmatrix} 1 & -4 & -7 \\ 0 & 1 & 5 \\ 0 & 2 & 10 \\ 0 & 5 & 25 \end{bmatrix}$ $\left(\text{Applying } \frac{1}{3} R_2\right)$

$\underset{\sim}{R} \begin{bmatrix} 1 & -4 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (Applying $R_3 - 2R_2$ and $R_4 - 5R_2$)

Here, Number of non-zero rows = 2

$\Rightarrow$ $\boxed{\text{Rank of matrix} = 2}$

Q.2 Define rank of a matrix and find rank of given matrix. $\begin{bmatrix} 1 & -1 & 2 & -3 \\ 2 & 0 & 7 & -7 \\ 3 & 1 & 12 & -11 \end{bmatrix}$

Ans. Rank number of non-zero rows of a matrix when it is reduced to Echelon form.

$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 2 & 0 & 7 & -7 \\ 3 & 1 & 12 & -11 \end{bmatrix}$

$\underset{\sim}{\substack{R_2 - 2R_1 \\ R_3 - 3R_1}} \begin{bmatrix} 1 & -2 & 2 & -3 \\ 0 & 2 & 3 & -1 \\ 0 & 4 & 6 & -2 \end{bmatrix}$

$\underset{\sim}{1/2\, R_2} \begin{bmatrix} 1 & -2 & 2 & -3 \\ 0 & 1 & 3/2 & -1 \\ 0 & 4 & 6 & -2 \end{bmatrix}$

$\underset{\sim}{R_3 - 4R_2} \begin{bmatrix} 1 & -2 & 2 & -3 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank = 2

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Rank of matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is ______

(Sgd 13)(Lhr 15 G-I)(Rwp 15)

(a) 1 (b) 2
(c) 3 (d) 4

Q.2 The rank of $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is: (Lhr 13 G-I)

(a) 0 (b) 1
(c) 2 (d) 3

Q.3 A square matrix is Skew Hermitian if $(\bar{A})^t$ =

(Lhr 06 G-I)(Mtn 18 G-II)

(a) A (b) $\bar{A}$
(c) A^t (d) $-A$

Q.4 A square matrix is symmetric if A^t equals:

(Lhr Board 2007 G-I) (Sgd Board 2017)

(a) A (b) $-A$
(c) A^2 (d) $\frac{A}{2}$

Q.5 Rank of a null matrix is

(a) 0 (b) 1
(c) does not exist
(d) equal to its number of rows

Q.6 If a matrix A is symmetric as well as skew symmetric, then

(a) A is null matrix
(b) A is unit matrix
(c) A is triangular matrix
(d) A is diagonal matrix

Q.7 A square matrix A is Hermitian if $(\overline{A})^t$ =

(Rwp 17 G-II)

(a) A (b) $-A^t$
(c) $-A$ (d) $\overline{A}$

Q.8 The matrix A is skew Hermitian if $(\overline{A})^t$ =

(Lhr 15 G-II)(Mtn 17 G-II)

(a) A (b) $-A^t$
(c) $-A$ (d) $\overline{A}$

Q.9 In a diagonal matrix, all elements except those of the diagonal are ....

(a) equal (b) not equal
(c) one (d) zero

Q.10 If A is a square matrix, then $A + A^t$ is

(a) null matrix (b) unit null matrix
(c) symmetric matrix (d) skew symmetric matrix

Q.11 If A is a square matrix, then $A - A^t$ is

(a) null matrix (b) unit null matrix
(c) symmetric matrix (d) skew symmetric matrix

Q.12 Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is

(a) 1 (b) 2
(c) 3 (d) 4

Q.13 Trivial solution of the homogenous linear equation is equal to. (Sgd 16)

(a) (0, 0, 0) (b) (1, 0, 0)
(c) (0, 1, 0) (d) (0, 0, 1)

Q.14 A square matrix $A = [a_{ij}]$ with complex entries is called skew Hermitran if $(\bar{A})^t$ is equal to.

(Sgd 16)

(a) A (b) $-A$
(c) $|A|$ (d) $-|A|$

Q.15 A square matrix A is symmetric if A^t =

(Lhr 12 G-I)(Lhr 18 G-I)

(a) $-A$ (b) A
(c) A^t (d) A^2

Q.16 A matrix X is skew symmetric if: (DGK 11)

(a) $X^t = X$ (b) $X^t = -X$
(c) $X = X^{-1}$ (d) $X = -X^t$

Q.17 An element a_{ij} of a square matrix $A = [a_{ij}]$ is said to be above the diagonal if

(a) $i = j$ (b) $i < j$
(c) $i > j$ (d) $i \neq j$

Q.18 An element a_{ij} of a square matrix $A = [a_{ij}]$ is said to be below the diagonal if

(a) $i = j$ (b) $i < j$
(c) $i > j$ (d) $i \neq j$

Q.19 An element a_{ij} of a square matrix $A = [a_{ij}]$ is not a diagonal element if

(a) $i = j$ (b) $i < j$
(c) $i > j$ (d) $i \neq j$

Q.20 $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ is
(a) scalar matrix (b) diagonal matrix
(c) lower triangular matrix
(d) upper triangular matrix

Q.21 $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ is.
(a) scalar matrix (b) diagonal matrix
(c) lower triangular matrix
(d) upper triangular matrix

Q.22 $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 4 \\ 5 & 6 & 0 \end{bmatrix}$ is
(a) scalar matrix (b) diagonal matrix
(c) triangular matrix (d) none of these

Q.23 An element a_{ij} of a square matrix $A = [a_{ij}]$ is said to be a diagonal element if
(a) $i = j$ (b) $i < j$
(c) $i > j$ (d) $i \neq j$

Q.24 The matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 6 \end{bmatrix}$ **is:** (Lhr 13 G-I)
(a) Diagonal (b) Scalar
(c) Triangular (d) Singular

Q.25 The value of determinant $\begin{vmatrix} 1 & 12 & 25 \\ 0 & 3 & 15 \\ 0 & 0 & 8 \end{vmatrix}$ **is:**
(Lhr 12 G-II)(Fbd 18)
(a) 0 (b) 1
(c) 8 (d) 24

Q.26 The rank of [1 0 3] is. (Mtn 17 G-I)
(a) Zero (b) 1
(c) 2 (d) 3

Q.27 A matrix A is said to be symmetric if.
(Lhr 18 G-I)
(a) $A^t = A$ (b) $A^t = -A$
(c) $(\bar{A})^t = A$ (d) $(\bar{A})^t = -A$

Q.28 A square matrix A is skew symmetric. If A^t is equal to. (Bpr 18)(Rwp 18)
(a) A (b) −A
(c) $-A^t$ (d) $\bar{A}$

Q.29 A square matrix $A = [a_{ij}]$ in which $a_{ij} = 0$ for $i > j$ is called: (Lhr 19 G-II)
(a) Upper triangular (b) Lower triangular
(c) Symmetric (d) Skew – symmetric

Q.30 if A= $\begin{bmatrix} 4 & x & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ **is a singular matrix, then 'x' is equal to:**
(a) 3 (b) 4
(c) 6 (d) 7

EXERCISE 3.5

SHORT ANSWERS TO THE QUESTIONS

Q.1 Differentiate non-homogeneous and homogeneous linear equation.

Ans. An equation of the form $ax + by + cz = d$ with $d \neq 0$, is called non-homogenous linear equation but an equation of the form $ax + by + cz = 0$ is called homogenous linear equation in there variables.

Q.2 What is augmented matrix?

Ans. The matrix [A : B] obtained from a system of linear equations by placing the constant column matrix B to the right of the co-efficient matrix A is called the augmented matrix.

Q.3 Differentiate trivial and non-trivial solution.

Ans. For a homogenous system of linear equation

$AX = 0$

A solution in which all variables of system are equal to zero exist and is called trivial solution. Any other solution of the system is called non-trivial solution.

LONG QUESTIONS

Q.1 Use Cramer's Rule to solve the system.

$$2x + 2y + z = 3,\ 3x - 2y - 2z = 1,\ 5x + y - 3z = 2$$

(Mtn 14 G-II, 12, 15 G-I,)(Rwp 14 G-II)(Sgd 13 G-I)
(Lhr 15 G-I)(Gjw 16)(Mtn 13,15,16, G-I)
(Swl 13 G-I) (D.G.K 14 G-II)(Fbd 19 G-II)

Ans. Given that: $\left.\begin{aligned} 2x + 2y + z &= 3 \\ 3x - 2y - 2z &= 1 \\ 5x + y - 3z &= 2 \end{aligned}\right]$ ...... (1)

This can be written as:

$$\begin{bmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \Rightarrow AX = B$$

Now, $A = \begin{bmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 2 & 1 \\ 3 & -2 & -2 \\ 5 & 1 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$$

$$= 2(6+2) - 2(-9+10) + 1(3+10)$$

$$= 2(8) - 2(1) + 1(13)$$

$$= 16 - 2 + 13 = 27 \neq 0$$

So, $$x = \frac{\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & -3 \end{vmatrix}}{|A|}$$

$$= \frac{3\begin{vmatrix} -2 & -2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}}{27}$$

$$= \frac{3(6+2) - 2(-3+4) + 1(1+4)}{27}$$

$$= \frac{3(8) - 2(1) + 1(5)}{27} = \frac{24 - 2 + 5}{27} = \frac{27}{27} = 1$$

$$y = \frac{\begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}}{|A|}$$

$$= \frac{2\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} - 3\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}}{27}$$

$$= \frac{2(-3+4) - 3(-9+10) + 1(6-5)}{27}$$

$$= \frac{2(1) - 3(1) + 1(1)}{27}$$

$$= \frac{2 - 3 + 1}{27} = \frac{0}{27} = 0$$

and $$z = \frac{\begin{vmatrix} 2 & 2 & 3 \\ 3 & -2 & 1 \\ 5 & 1 & 2 \end{vmatrix}}{|A|}$$

$$= \frac{2\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}}{27}$$

$$= \frac{2(-4-1) - 2(6-5) + 3(3+10)}{27}$$

$$= \frac{2(-5) - 2(1) + 3(13)}{27}$$

$$= \frac{-10 - 2 + 39}{27} = \frac{27}{27} = 1$$

$$\Rightarrow \boxed{x = 1,\ y = 0,\ z = 1}$$

Q.2 Solve the following system of linear equations by Cramer's Rule $\left.\begin{aligned} 2x_1 - x_2 + x_3 &= 8 \\ x_1 + 2x_2 + 3x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 1 \end{aligned}\right\}$.

(Bpr 12 G-I, 19)(Swl 13 G-I)(Fbd 14)(Sgd 17)

Ans. Given that: $\left.\begin{aligned} 2x_1 - x_2 + x_3 &= 8 \\ x_1 + 2x_2 + 2x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 1 \end{aligned}\right]$ ...... (1)

This can be written as:

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 1 \end{bmatrix} \Rightarrow AX = B$$

$$\Rightarrow \quad X = A^{-1}B \quad ...... (2)$$

Now, $$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 2 \\ -2 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix}$$

$$= 2(-2+4) + 1(-1-2) + 1(-2-2)$$

$$= 2(2) + 1(-3) + 1(-4)$$

$$= 4 - 3 - 4 = -3 \neq 0$$

So, $$x = \frac{\begin{vmatrix} 8 & -1 & 1 \\ 6 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix}}{|A|}$$

$$= \frac{8\begin{vmatrix} 2 & 2 \\ -2 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 2 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 6 & 2 \\ 1 & -2 \end{vmatrix}}{-3}$$

$$= \frac{8(-2+4) + 1(-6-2) + 1(-12-2)}{-3}$$

$$= \frac{8(2) + 1(-8) + 1(-14)}{-3}$$

$$= \frac{16 - 8 - 14}{-3} = \frac{16 - 22}{-3} = \frac{-6}{-3} = 2$$

$$x_2 = \frac{\begin{vmatrix} 2 & 8 & 1 \\ 1 & 6 & 2 \\ 1 & 1 & -1 \end{vmatrix}}{|A|}$$

$$= \frac{2\begin{vmatrix} 6 & 2 \\ 1 & -1 \end{vmatrix} - 8\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 6 \\ 1 & 1 \end{vmatrix}}{-3}$$

$$= \frac{2(-6-2) - 8(-1-2) + 1(1-6)}{-3}$$

$$= \frac{2(-8) - 8(-3) + 1(-5)}{-3}$$

$$= \frac{-16 + 24 - 5}{-3} = \frac{-21 + 24}{-3} = \frac{3}{-3} = -1$$

and $x_3 = \dfrac{\begin{vmatrix} 2 & -1 & 8 \\ 1 & 2 & 6 \\ 1 & -2 & 1 \end{vmatrix}}{|A|}$

$$= \frac{2\begin{vmatrix} 2 & 6 \\ -2 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 6 \\ 1 & 1 \end{vmatrix} + 8\begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix}}{-3}$$

$$= \frac{2(2+12) + 1(1-6) + 8(-2-2)}{-3}$$

$$= \frac{2(14) + 1(-5) + 8(-4)}{-3}$$

$$= \frac{28 - 5 - 32}{-3} = \frac{28 - 37}{-3} = \frac{-9}{-3} = 3$$

$\therefore \quad \boxed{x_1 = 2,\ x_2 = -1,\ x_3 = 3}$

Q.3 Solve the system of linear equation.
$3x_1 + x_2 - x_3 = -4$
$x_1 + x_2 - 2x_3 = -4$ by crammer's rule.
$-x_1 + 2x_2 - x_3 = 1$

(Rwp 11, 13)(Fbd 16)(Sgd 16)(Gjw 19 G-II)

Ans. Given that: $3x_1 + x_2 - x_3 = -4$, $x_1 + x_2 - 2x_3 = -4$, $-x_1 + 2x_2 - x_3 = 1$

This system of equations can be written as:

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 1 \end{bmatrix}$$

$AX = B \quad \Rightarrow \quad X = A^{-1}B \quad \ldots\ldots (1)$

Here,

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= 3(-1+4) - 1(-1-2) - 1(2+1)$$

$$= 3(3) - 1(-3) - 1(3)$$

$$= 9 + 3 - 3 = 9 \neq 0$$

Now,

$x_1 = \dfrac{\begin{vmatrix} -4 & 1 & -1 \\ -4 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix}}{|A|}$

$$= \frac{-4\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} -4 & 1 \\ 1 & 2 \end{vmatrix}}{9}$$

$$= \frac{-4(-1+4) - 1(4+2) - 1(-8-1)}{9}$$

$$= \frac{-4(3) - 1(6) - 1(-9)}{9}$$

$$= \frac{-12 - 6 + 9}{9} = \frac{-9}{9} = -1$$

$x_2 = \dfrac{\begin{vmatrix} 3 & -4 & -1 \\ 1 & -4 & -2 \\ -1 & 1 & -1 \end{vmatrix}}{|A|}$

$$= \frac{3\begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -4 \\ -1 & 1 \end{vmatrix}}{9}$$

$$= \frac{3(4+2) - 1(1-4) - 4(2+1)}{9}$$

$$= \frac{3(6) - 1(-3) - 4(3)}{9}$$

$$= \frac{18 + 3 - 12}{9} = \frac{9}{9} = 1$$

$\Rightarrow \boxed{x_1 = -1,\ x_2 = 1,\ x_3 = 2}$

$x_3 = \dfrac{\begin{vmatrix} 3 & 1 & -4 \\ 1 & 1 & -4 \\ -1 & 2 & 1 \end{vmatrix}}{|A|}$

$$= \frac{3\begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -4 \\ -1 & 1 \end{vmatrix} + (-4)\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}}{9}$$

$$= \frac{3(1+8) - 1(1-4) - 4(2+1)}{9}$$

$$= \frac{3(9) - 1(-3) - 4(3)}{9}$$

$$= \frac{27 + 3 - 12}{9} = \frac{18}{9} = 2$$

$\Rightarrow \boxed{x_1 = -1,\ x_2 = 1,\ x_3 = 2}$

Q.4 Find the value of λ for which the following system does not possess a unique solution. Also solve the system for the value of λ.

$$\mathbf{x_1 + 4x_2 + \lambda x_3 = 2}$$
$$\mathbf{2x_1 + x_2 - 2x_3 = 11}$$
$$\mathbf{3x_1 + 2x_2 - 2x_3 = 16}$$

Ans. Given that:
$$\left.\begin{aligned} x_1 + 4x_2 + \lambda x_3 &= 2 \\ 2x_1 + x_2 - 2x_3 &= 11 \\ 3x_1 + 2x_2 - 2x_3 &= 16 \end{aligned}\right] \quad \ldots\ldots (1)$$

This can be written as:
$$\begin{bmatrix} 1 & 4 & \lambda \\ 2 & 1 & -2 \\ 3 & 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \\ 16 \end{bmatrix}$$

The augmented matrix is:
$$\begin{bmatrix} 1 & 4 & \lambda & M & 2 \\ 2 & 1 & -2 & M & 11 \\ 3 & 2 & -2 & M & 16 \end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 4 & \lambda & M & 2 \\ 0 & -7 & -2-2\lambda & M & 7 \\ 0 & -10 & -2-3\lambda & M & 10 \end{bmatrix}$$

(Applying $R_2 - 2R_1$ and $R_3 - 3R_1$)

$$\underset{\sim}{R}\begin{bmatrix} 1 & 4 & \lambda & M & 2 \\ 0 & 1 & \frac{2+2\lambda}{7} & M & -1 \\ 0 & -10 & -2-3\lambda & M & 10 \end{bmatrix} \quad \left(\text{Applying } \frac{1}{-7}R_2\right)$$

Applying $R_3 + 10R_2$:

$$\underset{\sim}{R}\begin{bmatrix} 1 & 4 & \lambda & M & 2 \\ 0 & 1 & \frac{2+2\lambda}{7} & M & -1 \\ 0 & 0 & \frac{6-\lambda}{7} & M & 0 \end{bmatrix} \ldots (2)$$

$$\boxed{\begin{aligned} &-2-3\lambda+\frac{20+20\lambda}{7} \\ &\frac{-14-21\lambda+20+20\lambda}{7} \\ &\frac{6-\lambda}{7} \end{aligned}}$$

The system does not possess a unique solution if:

$$\frac{6-\lambda}{7} = 0 \Rightarrow 6-\lambda = 0 \Rightarrow \lambda = 6$$

Putting the value of λ in eq. (2):

$$\begin{bmatrix} 1 & 4 & 6 & M & 2 \\ 0 & 1 & \frac{2+2(6)}{7} & M & -1 \\ 0 & 0 & 0 & M & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 6 & M & 2 \\ 0 & 1 & \frac{2+12}{7} & M & -1 \\ 0 & 0 & 0 & M & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 6 & M & 2 \\ 0 & 1 & \frac{14}{7} & M & -1 \\ 0 & 0 & 0 & M & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 6 & M & 2 \\ 0 & 1 & 2 & M & -1 \\ 0 & 0 & 0 & M & 0 \end{bmatrix}$$

$$\therefore \quad x_2 + 2x_3 = -1 \Rightarrow \boxed{x_2 = -2x_3 - 1}$$

and $x_1 + 4x_2 + 6x_3 = 2 \Rightarrow x_1 + 4(-2x_3 - 1) + 6x_3 = 2$

$$x_1 - 8x_3 - 4 + 6x_3 = 2 \Rightarrow x_1 - 2x_3 - 4 = 2$$

$$\Rightarrow \quad x_1 = 2x_3 + 4 + 2 \Rightarrow \boxed{x_1 = 2x_3 + 6}$$

$$\Rightarrow \quad \boxed{x_3 = t,\ x_2 = -2t - 1,\ x_1 = 2t + 6}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The trivial solution of homogeneous system of linear equation in three variables is:

(Sgd 13 G-I)

(a) (0, 0, 1) (b) (0, 1, 0)
(c) (1, 0, 0) (d) (0, 0, 0)

Q.2 The (0, 0) solution of the system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ is called. (Lhr 16 G-I)

(a) Constant (b) Inconsistent
(c) Non-trivial (d) Trivial

Q.3 The system of linear equations $ax + by = 0$, $cx + dy = 0$ has a non-trivial solution, if

(a) $ad - bc > 0$ (b) $ad - bc < 0$
(c) $ad - bc = 0$ (d) $ad - bc = 1$

Chapter 4

QUADRATIC EQUATION

EXERCISE 4.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define quadratic equation.

Ans. An equation of the form $ax^2 + bx + c = 0$ is called quadratic equation where a, b, c are real numbers and $a \neq 0$.

Q.2 State two basic techniques for solving quadratic equation. **(Sgd 18)**

Ans. There are two basic techniques for solving a quadratic equations:

(i) Factorization.

(ii) Completing square method.

Q.3 Solve $x^2 + 7x + 12 = 0$

Ans. $x^2 + 7x + 12 = 0$

$x^2 + 3x + 4x + 12 = 0$

$x(x + 3) + 4(x + 3) = 0$

$(x + 3)(x + 4) = 0$

Either $x + 3 = 0$ or $x + 4 = 0$

$x = -3$ $\quad$ $x = -4$

Solution Set = $\{-3, -4\}$

Q.4 Solve the equation $x(x + 7) = (2x - 1)(x + 4)$

(Gjw 16)(D.G.K 17 G-I)(Mtn 18 G-II)(Gjw 14)

Ans. $x(x + 7) = (2x - 1)(x + 4)$

$x^2 + 7x = 2x^2 + 8x - x - 4$

$0 = 2x^2 - x^2 + 7x - 7x - 4$

$0 = x^2 - 4$

$0 = x^2 - 2^2$

$0 = (x - 2)(x + 2)$

Either $x - 2 = 0$ or $x + 2 = 0$

$x = 2$ $\quad$ $x = -2$

Solution set = $\{2, -2\}$

Q.5 Solve the equation $\dfrac{a}{ax - 1} + \dfrac{b}{bx - 1} = a + b$

(Lhr 08 G-I)

Ans. $\dfrac{a}{ax - 1} + \dfrac{b}{bx - 1} = a + b$

$$\left\{\frac{a}{ax - 1} - b\right\} + \left\{\frac{b}{bx - 1} - a\right\} = 0$$

$$\left\{\frac{a - b(ax - 1)}{ax - 1}\right\} + \left\{\frac{b - a(bx - 1)}{bx - 1}\right\} = 0$$

Multiplying both sides by $(ax - 1)(bx - 1)$ to get the standard form.

$$(ax - 1)(bx - 1)\left[\left\{\frac{a - b(ax - 1)}{ax - 1}\right\} + \left\{\frac{b - a(bx - 1)}{bx - 1}\right\}\right] = 0$$

$(bx - 1)\{a - abx + b\} + (ax - 1)\{b - abx + a\} = 0$

$(bx - 1)(a + b - abx) + (ax - 1)(a + b - abx) = 0$

$(a + b - abx)\{(bx - 1) + (ax - 1)\} = 0$

$(a + b - abx)(ax + bx - 2) = 0$

$a + b - abx = 0$	$ax + bx - 2 = 0$
$abx = a + b$	$x(a + b) = 2$
$x = \dfrac{a + b}{ab}$	$x = \dfrac{2}{a + b}$

Hence solution set is $\left\{\dfrac{a + b}{ab}, \dfrac{2}{a + b}\right\}$

Q.6 Solve by completing the square:

$x^2 - 2x - 899 = 0$.

By compiling square method.

Ans. $x^2 - 2x - 899 = 0$

$x^2 - 2x(1) - 899 = 0$

$x^2 - 2x(1) + (1)^2 - 899 - (1)^2 = 0$

$\{x^2 - 2(x)(1) + (1)^2\} - 900 = 0$

$(x - 1)^2 = 900$

$(x - 1)^2 = (30)^2$

$x - 1 = \pm 30$

$x - 1 = 30$	or $x - 1 = -30$
$x = 30 + 1$	$x = -30 + 1$
$x = 31$	$x = -29$

Hence solution set is $\{-29, 31\}$

Q.7 Solve the equation by completing square.

$x^2 - 3x - 648 = 0$. (Rwp 17 G-II)

Ans. $x^2 - 3x - 648 = 0$

$(x)^2 - 2 \cdot \dfrac{3}{2}x = 648$

$$x^2 - 2\cdot\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 = 648 + \left(\frac{3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = 648 + \frac{9}{4}$$

$$= \frac{2592 + 9}{4} = \frac{2601}{4} = \left(\frac{51}{2}\right)^2$$

Taking square root

$x-\frac{3}{2}=\pm\frac{51}{2}$ or $x-\frac{3}{2}=\frac{-51}{2}$

Either $x-\frac{3}{2}=\frac{51}{2}$

$x=\frac{3}{2}+\frac{51}{2}$

$=\frac{3+51}{2}$

$=\frac{54}{2}=27$

or $x-\frac{3}{2}=\frac{-51}{2}$

$x=\frac{3}{2}-\frac{51}{2}$

$=\frac{3-51}{2}$

$=\frac{-48}{2}=-24$

Solution set = $\{27, -24\}$

Q.8 Solve $5x^2+2ax-a^2=0$ by quadratic formula. (Fbd 16)(Swl 19)

Ans. $5x^2+2ax-a^2=0$

By quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2a\pm\sqrt{(2a)^2-4.5(-a)^2}}{2\times 5}$$

$$=\frac{-2a\pm\sqrt{4a^2+20a^2}}{10}$$

$$=\frac{-2a\pm\sqrt{24a^2}}{10}$$

$$=\frac{-2a\pm 2a\sqrt{6}}{10}=\frac{2a(-1\pm\sqrt{6})}{10}$$

$$x=\frac{a(-1\pm\sqrt{6})}{5}$$

Q.9 Solve by quadratic formula $15x^2+2ax-a^2=0$ (Mtn 11 G-II)(Rwp 17 G-I)(Rwp 18)

Ans. $15x^2+(2a)x-a^2=0$

Applying quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(2a)\pm\sqrt{(2a)^2-4(15)(-a^2)}}{2(15)}$$

$$=\frac{-2a\pm\sqrt{4a^2+60a^2}}{30}=\frac{-2a\pm\sqrt{64a^2}}{30}$$

$$=\frac{-2a\pm 8a}{30}$$

$x=\frac{-2a-8a}{30}$ | $x=\frac{-2a+8a}{30}$

$x=\frac{-10a}{30}$ | $x=\frac{6a}{30}$

$x=-\frac{1}{3}a$ | $x=\frac{1}{5}a$

Hence solution set is $\left\{-\frac{1}{3}a, \frac{1}{5}a\right\}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The graph of quadratic function is: (Mtn 13 G-I)

(a) Triangle (b) Parabola
(c) Square (d) None of these

Q.2 The quadratic formula is: (Mtn 13 G-II)

(a) $x=\frac{-b\pm\sqrt{4ac-b^2}}{2a}$

(b) $x=\frac{-b\pm\sqrt{b^2-4a}}{2a}$

(c) $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(d) $x=\frac{-b\pm\sqrt{b^2-4abc}}{2a}$

Q.3 An equation of the form $ax^2+bx+c=0$ is called quadratic if: (Rwp 15) (Sgd 13)

(a) $a=0$ (b) $a\neq 0$
(c) $b=0$ (d) $b\neq 0$

Q.4 Solution set of the equation $x^2-3x+2=0$ is

(a) $\{-1, 2\}$ (b) $\{1, -2\}$
(c) $\{-1, -2\}$ (d) $\{1, 2\}$

Q.5 A quadratic equation $ax^2+bx+c=0$ becomes linear equation if. (Mtn 16 G-I)

(a) $a=0$ (b) $b=0$
(c) $c=0$ (d) $a=b$

Q.6 A quadratic equation has degree. (Lhr 17 G-I, II)

(a) 0 (b) 1
(c) 2 (d) 3

Q.7 Roots of the equation $x^2-7x+10=$ are. (Lhr 18 G-II)

(a) $(2, -5)$ (b) $(-2, 5)$
(c) $(2, 5)$ (d) $(-2, -5)$

Q.8 The degree of the equation $x^3+3x^2+4x+5=0$ is:

(a) 4 (b) 3
(c) 2 (d) 1

Q.9 A quadratic equation has degree.

(a) 0 (b) 1
(c) 2 (d) 3

EXERCISE 4.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 What is an Exponential equation?

(Sgd 13 G-II)(Rwp 16)

Ans. Equation, in which the variable occurs in the power, is called exponential equation.

Q.2 Define reciprocal equation.

Ans. An equation which remains unchanged when x is replaced by $\frac{1}{x}$.

Q.3 Solve the equation $x^{1/2} - x^{1/4} - 6 = 0$.

(Mtn 18, 19 G-I)

Ans. $x^{1/2} - x^{1/4} - 6 = 0$

Let $y = x^{1/4} \Rightarrow y^2 = (x^{1/4})^2 = x^{1/2}$

$y^2 - y - 6 = 0$

$y^2 - 3y + 2y - 6 = 0$

$y(y-3) + 2(y-3) = 0$

$(y-3)(y+2) = 0$

$y - 3 = 0$	$y + 2 = 0$
$y = 3$	$y = -2$
$x^{1/4} = 3$	$x^{1/4} = -2$
$(x^{1/4})^4 = (3)^4$	$(x^{1/4})^4 = (-2)^4$
$x = 81$	$x = 16$

Solution set = {81, 16}

LONG QUESTIONS

Q.1 Solve the equation (x+1)(x+2)(x+3)(x+4) = 24.

(Fbd 14)

Ans. Given that: $(x+1)(x+2)(x+3)(x+4) = 24$

Here, we see that: $1 + 4 = 2 + 3 \Rightarrow 5 = 5$ (True)

$\therefore$ $[(x+1)(x+4)][(x+2)(x+3)] = 24$

$[x^2 + 4x + 1x + 4][x^2 + 3x + 2x + 6] = 24$

$[x^2 + 5x + 4][x^2 + 5x + 6] = 24$

Put $x^2 + 5y = y$, so

$[y+4][y+6] = 24 \Rightarrow y^2 + 6y + 4y + 24 = 24$

$y^2 + 10y = 24 - 24 \Rightarrow y^2 + 10y = 0$

$\Rightarrow$ $y(y+10) = 0$

$y = 0, -10$

When $y = 0 \Rightarrow x^2 + 5x = 0 \Rightarrow x(x+5) = 0$

Either $x = 0$ or $x + 5 = 0 \Rightarrow x = -5$

When $y = -10 \Rightarrow x^2 + 5x = -10$

$\Rightarrow$ $x^2 + 5x + 10 = 0$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 - 40}}{2} = \frac{-5 \pm \sqrt{-15}}{2}$$

S. Set= $\left\{0, -5, \frac{-5 \pm \sqrt{-15}}{2}\right\}$

Q.2 Solve $3^{2x-1} - 12.3^x + 81 = 0$ (Lhr 17 G-II)

Ans. Given that: $3^{2x-1} - 12.3^x + 81 = 0 \Rightarrow 3^{2x}.3^{-1} - 12.3^x + 81 = 0$

$3^{2x}.\frac{1}{3} - 12.3^x + 81 = 0 \Rightarrow \frac{3^{2x} - 36.3^x + 243}{3} = 0$

$3^{2x} - 36.3^x + 243 = 0$

Put $3^x = y \Rightarrow 3^{2x} = y^2$, so

$\therefore$ $y^2 - 36y + 243 = 0 \Rightarrow y^2 - 27y - 9y + 243 = 0$

$y(y-27) - 9(y-27) = 0 \Rightarrow (y-27)(y-9) = 0 \Rightarrow y = 27, 9$

When $y = 27 \Rightarrow 3^x = 27 \Rightarrow 3^x = 3^3 \Rightarrow \boxed{x = 3}$

When $y = 9 \Rightarrow 3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow \boxed{x = 2}$

S. Set = {3, 2}

Q.3 Solve the equation $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$.

(Swl 14)

Ans. Given that: $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$

Put $x + \frac{1}{x} = y$, so

$\therefore$ $y^2 - 3y - 4 = 0 \Rightarrow y^2 - 4y + y - 4 = 0$

$y(y-4) + 1(y-4) = 0 \Rightarrow (y-4)(y+1) = 0$

$\Rightarrow$ $y = -1, 4$

When $y = -1 \Rightarrow x + \frac{1}{x} = -1 \Rightarrow \frac{x^2+1}{x} = -1$

$x^2 + 1 = -x \Rightarrow x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

When $y = 4 \Rightarrow x + \frac{1}{x} = 4 \Rightarrow \frac{x^2+1}{x} = 4$

$\Rightarrow$ $x^2 + 1 = 4x$

$x^2 - 4x + 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2} = \frac{2(2 \pm \sqrt{3})}{2} = 2 \pm \sqrt{3}$$

S. Set = $\left\{\frac{-1 \pm \sqrt{-3}}{2}, 2 \pm \sqrt{3}\right\}$

Q.4 Solve the equation.

$$\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)^2 = 0$$

(Mtn 12 G-I)(Bpr 13)(D.G.K 15 G-I)(Sgd 19)

Ans. Given that: $\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right) = 0 \Rightarrow x^2+\frac{1}{x^2}$

$$-2+3\left(x+\frac{1}{x}\right) = 0$$

$$\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)-2 = 0$$

Put $x+\frac{1}{x} = y \Rightarrow \left(x+\frac{1}{x}\right)^2 = y^2 \Rightarrow x^2+\frac{1}{x^2}+2 =$

$y^2 \Rightarrow x^2+\frac{1}{x^2} = y^2-2$

$\therefore \quad y^2-2+3y-2 = 0 \Rightarrow y^2+3y-4 = 0$

$\Rightarrow \quad y^2+4y-y-4 = 0$

$y(y+4)-1(y+4) = 0 \Rightarrow (y+4)(y-1) = 0$

$\Rightarrow \quad y = -4, 1$

When $y = -4 \Rightarrow x+\frac{1}{x} = -4 \Rightarrow \frac{x^2+1}{x} = -4$

$x^2+1 = -4x \Rightarrow x^2+4x+1 = 0$

$$x = \frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)} = \frac{-4\pm\sqrt{16-4}}{2}$$

$$= \frac{-4\pm\sqrt{12}}{2}$$

$$= \frac{-4\pm2\sqrt{3}}{2} = 2\cdot\frac{-2\pm\sqrt{3}}{2} = -2\pm\sqrt{3}$$

When $y = 1 \Rightarrow x+\frac{1}{x} = 1 \Rightarrow \frac{x^2+1}{x} = 1$

$x^2+1 = x \Rightarrow x^2-x+1 = 0$

$$x = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)} = \frac{1\pm\sqrt{1-4}}{2}$$

$$= \frac{1\pm\sqrt{-3}}{2}$$

S. Set $= \left\{-2\pm\sqrt{3}, \frac{1\pm\sqrt{-3}}{2}\right\}$

Q.5 Solve $4.2^{2x+1}-9.2^x+1 = 0$ (Gjw 19 G-II)

Ans. $4.2^{2x+1}-9.2^x+1 = 0$

$4.2^{2x}.2^1-9.2^x+1 = 0$

$\Rightarrow \quad 8.(2^x)^2-9.\,2^x+1 = 0 \quad \ldots (1)$

Let, $y = 2^x$

(1) $\Rightarrow \quad 8y^2-9y+1 = 0$

$$y = \frac{-(-9)\pm\sqrt{(-9)^2-4(8)(1)}}{2(8)}$$

$$y = \frac{9\pm\sqrt{81-32}}{16} = \frac{9\pm\sqrt{49}}{16} = \frac{9\pm7}{16}$$

$$y = \frac{9+7}{16} = 1 \quad , \quad y = \frac{9-7}{16} = \frac{1}{8}$$

When, $y = 1 \Rightarrow 2^x = 1 \Rightarrow 2^x = 2^0 \Rightarrow x = 0$

When, $y = \frac{1}{8} \Rightarrow 2^x = \frac{1}{8} \Rightarrow 2^x = \frac{1}{2^3}$

$\Rightarrow \quad 2^x = 2^{-3} \Rightarrow x = -3$

S.S $= \{-3, 0\}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Equation $2^{2x} - 3.2^x + 32 = 0$ is ________ equation: (Mtn 11, 15 G-I)

(a) Reciprocal (b) Radical
(c) Exponential (d) Linear

Q.2 The conditional equation $\frac{1}{x} = 3$ is true only if: (Mtn 11 G-I)

(a) $x = \frac{1}{3}$ (b) $x = 3$
(c) $x = -\frac{1}{3}$ (d) $x = -3$

Q.3 An equation $16(x^4+1) - 8(x^3+x) + 9x^2 = 0$ is of the form: (Fbd 15)

(a) Exponential (b) Radical
(c) Rational (d) Reciprocal

Q.4 If $4^x = \frac{1}{2}$, then x is equal to: (Lhr 13 G-I) (Fbd 11, 17)

(a) $-\frac{1}{2}$ (b) -2
(c) $\frac{1}{2}$ (d) 2

Q.5 $x+\frac{3}{x} = 4$ is. (Mtn 16 G-II)

(a) Reciprocal Equation
(b) Transcendental Equation
(c) Quadratic Equation
(d) Identity

Q.6 If $4^{3x} = \frac{1}{2}$, then x equals. (D.G.K 17 G-II)

(a) $\frac{1}{6}$ (b) $-\frac{1}{6}$
(c) 6 (d) -6

Q.7 If $4^{-x} = \frac{1}{2}$, then x = (Swl 17)

(a) 1 (b) $-\frac{1}{2}$
(c) -1 (d) $\frac{1}{2}$

Q.8 A reciprocal equation remains unchanged when x variable is replaced by. (Rwp 16)(Mtn 18 G-II)

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$

(c) $\frac{1}{x^2}$ (d) $-x$

Q.9 If $3^x + 2^{2x} = 5^x$, then the value of x is. (Bpr 18)

(a) 0 (b) 1

(c) 2 (d) 3

Q.10 A reciprocal equation remains unchanged when variable x is replaced by (Gjw 19 G-II)

(a) $-\frac{1}{x}$ (b) $\frac{1}{x}$

(c) $\frac{1}{x^2}$ (d) $-x$

Q.11 An equation which remains unchanged when x is replaced by $\frac{1}{x}$ is. (Bpr 19)

(a) Exponential (b) Radical

(c) Reducible (d) Reciprocal

Q.12 The number of roots of polynomial $8x^6 - 19x^3 - 27 = 0$ are: (Mtn 19 G-I)

(a) 2 (b) 4

(c) 6 (d) 8

EXERCISE 4.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define radical equation.

Ans. Equations involving radical expression of the variable are called radical equation.

Q.2 Define extraneous roots. (DGK 19)

Ans. To solve a radical equation, we first obtain an equation free from radical. Every solution of radical equation is also a solution of the radical free equation but the new equation have solution that are not solution of original radical equation, such roots are called extraneous or extra roots.

EXERCISE 4.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 Find three cube roots of unity. (Rwp 13 G-I, 16)

Ans. Let x be a cube root of unity

$\therefore \quad x = \sqrt[3]{1} = (1)^{\frac{1}{3}}$

$\Rightarrow \quad x^3 = 1$

$\Rightarrow \quad x^3 - 1 = 0$

$\Rightarrow \quad (x-1)(x^2 + x + 1) = 0$

$\therefore \quad x - 1 = 0 \Rightarrow x = 1$

and $x^2 + x + 1 = 0$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{3}\,i}{2} \quad (\because \sqrt{-1} = i)$

Thus the cube roots of unity are

$1, \frac{-1+\sqrt{-3}}{2}$ and $\frac{-1-\sqrt{-3}}{2}$

or $1, \frac{-1+\sqrt{3}\,i}{2}$ and $\frac{-1-\sqrt{3}\,i}{2}$

Q.2 Prove that each complex cube root of unity is square of the other (Sgd Board 2017)(Lhr 09 G-II)

Ans. We know that complex cube root of unity are

$\frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}$

$$\left(\frac{-1+\sqrt{3}\,i}{2}\right)^2 = \frac{(-1)^2 + (\sqrt{3}\,i)^2 + 2(-1)(\sqrt{3}\,i)}{4}$$

$$= \frac{1-3-2\sqrt{3}\,i}{4} = \frac{-2-2\sqrt{3}\,i}{4} = 2\left(\frac{-1-\sqrt{3}\,i}{4}\right)$$

$$= \frac{-1-\sqrt{3}\,i}{2}$$

$$\left(\frac{-1-\sqrt{3}\,i}{2}\right)^2 = \left[-\frac{(1+\sqrt{3}\,i)}{2}\right]^2$$

$$= \frac{(1)^2 + (\sqrt{3}\,i)^2 + 2(1)(\sqrt{3}\,i)}{4} = \frac{1-3+2\sqrt{3}\,i}{4}$$

$$= \frac{-2+2\sqrt{3}\,i}{4} = 2\left(\frac{-1+\sqrt{3}\,i}{4}\right) = \frac{-1+\sqrt{3}\,i}{2}$$

Hence each complex cube root of unity is square of the other.

Q.3 Prove that $1 + w + w^2 = 0$ or show that sum of cube root of unity is zero.

(Lhr 17 G-I, II)(Mtn 12 G-II)

Ans. We know that cube root of unity are

$1, w = \frac{-1+\sqrt{-3}}{2}$ and $w^2 = \frac{-1-\sqrt{-3}}{2}$

$\therefore \quad 1 + w + w^2$

$$= 1 + \frac{-1+\sqrt{-3}}{2} + \frac{-1-\sqrt{-3}}{2}$$

$$= \frac{2-1-\sqrt{-3}-1-\sqrt{3}}{2} = \frac{0}{2} = 0$$

Q.4 Show that $\left(\frac{-1-\sqrt{3}\,i}{2}\right)^2 = \frac{-1+\sqrt{3}\,i}{2}$

(Sgd 13 G-I)

Ans. $\left(\frac{-1-\sqrt{3}\,i}{2}\right)^2 = \left[-\frac{(1+\sqrt{3}\,i)}{2}\right]^2$

$$= \frac{(1)^2 + (\sqrt{3}\,i)^2 + 2(1)(\sqrt{3}\,i)}{4} = \frac{1-3+2\sqrt{3}\,i}{4}$$

$$= \frac{-2+2\sqrt{3}\,i}{4} = 2\left(\frac{-1+\sqrt{3}\,i}{4}\right) = \frac{-1+\sqrt{3}\,i}{2}$$

Q.5 Prove that **(Rwp 17 G-I)(Fbd 18)**

$$\left(\frac{1+\sqrt{-3}}{2}\right)^9+\left(\frac{1-\sqrt{-3}}{2}\right)^9=-2$$

Ans. L.H.S

$$=\left(\frac{1+\sqrt{-3}}{2}\right)^9+\left(\frac{1-\sqrt{-3}}{2}\right)^9$$
$$=(-w)^9+(-w^2)^9$$
$$=-w^9-w^{18}$$
$$=-(w^3)^3-(w^3)^6$$
$$=-1^3-1^6$$
$$=-1-1=-2=\text{L.H.S}$$

Q.6 Prove that $(-1+\sqrt{-3})^4+(-1-\sqrt{-3})^4=-16$
(Lhr 08, 09 G-I)(Fbd 17)(Gjw 19 G-I)(Mtn 17 G-II)

Ans. L.H.S. $=(-1+\sqrt{-3})^4+(-1-\sqrt{-3})^4$

$$=\left[2\left(\frac{-1+\sqrt{-3}}{2}\right)\right]^4+\left[2\left(\frac{-1-\sqrt{-3}}{2}\right)\right]^4$$
$$=[2\omega]^4+[2\omega^2]^4=16\,\omega^4+16\,\omega^8$$
$$=16\left\{\omega\cdot\omega^3+(\omega^3)^2\cdot\omega^2\right\}$$
$$=16\left\{\omega\cdot 1+(1)^2\cdot\omega^2\right\}\ ;\text{using } \omega^3=1$$
$$=16\{\omega+\omega^2\}=16(-1)\ ;\text{using } 1+\omega+\omega^2=0$$
$$=-16=\text{R.H.S.}$$

Q.7 Show that $x^3-y^3=(x-y)(x-wy)(x-w^2y)$
(Mtn 18 G-II)(Gjw 19 G-II)

Ans. L.H.S

$$=(x-y)(x-wy)(x-w^2y)$$
$$=(x-y)[x^2-w^2xy-wxy+w^3y^2]$$
$$=(x-y)[x^2-(w2+w)xy+1.y^2]$$
$$=(x-y)[x^2-(-1)xy+y^2]$$
$$=(x-y)(x^2+xy+y^2)$$
$$=x^3-y^3=\text{R.H.S}$$

Q.8 If w is a root of $x^2+x+1=0$, show that its root w^2. (Mtn 17 G-I)

Ans. $x^2+x+1=0$

$$x=\frac{-1\pm\sqrt{1^2-4.1.1}}{2.1}=\frac{-1\pm\sqrt{-3}}{2}$$

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^2=\frac{(-1)^2+(\sqrt{-3})^2+2(-1)\sqrt{-3}}{4}$$
$$=\frac{1-3-2\sqrt{-3}}{4}=\frac{-2-2\sqrt{-3}}{4}$$
$$=2\left(\frac{-1-\sqrt{-3}}{4}\right)=\frac{-1-\sqrt{-3}}{2}$$

Also

$$\left(\frac{-1-\sqrt{-3}}{2}\right)^2=\frac{(-1)2+(\sqrt{-3})2+2(-1)(-\sqrt{-3})}{4}$$
$$=\frac{1-3+2\sqrt{-3}}{4}=\frac{-2+2\sqrt{-3}}{4}$$
$$=\frac{2(-1+\sqrt{-3})}{4}=\frac{-1+\sqrt{-3}}{2}$$

$\therefore$ Both roots are square of each other

if $w=\dfrac{-1+\sqrt{-3}}{2}$, then $w^2=\dfrac{-1-\sqrt{-3}}{2}$

if $w=\dfrac{-1-\sqrt{-3}}{2}$, then $w^2=\dfrac{-1+\sqrt{-3}}{2}$

if w is a root of $x^2+x+1=0$, then
w^2 is another root of $x^2+x+1=0$

Q.9 Find four fourth roots of unity. **(Lhr 18 G-I)**

Ans. suppose $x=1^{\frac{1}{4}}$ or $x^4=1$

$\Rightarrow$ $x^4-1=0$
$\Rightarrow$ $(x^2)^2-1^2=0$
$\Rightarrow$ $(x^2-1)(x^2+1)=0$
$\Rightarrow$ $(x-1)(x+1)(x^2+1)=0$

Either $x-1=0$, $x=1$ or $x+1=0$, $x=-1$

or $x^2+1=0$
$x^2=-1$
$x+\pm i$

and sum $=1+(-1)+i+(-i)=0$
product $=1(-1)(i)(-i)=(-1)(-i^2)=i^2=-1$

Q.10 Write two properties of the cube roots of unity. **(Fbd 19)**

Ans.
1. Sum of three cube root of unity is zero.
2. product of three cube root of unit is one.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If w is complex cube root of unity, the value of w^{15} is: (Bpr 14) (Mtn 08)
(a) 1 (b) zero
(c) $-w$ (d) w

Q.2 Factors of x^3-y^3 are: (Mtn 08 G-II)
(a) $(x-y)(x^2+y^2+xy)$ (b) $(x-y)(x^2+y^2)$
(c) $(x-y)(x^2-y^2)$ (d) $(x+y)(x^2+y^2)$

Q.3 If w is a complex cube root of unity then the equation with roots 2w and $2w^2$ is: (Mtn 09 G-II)
(a) $x^2+2x+4=0$ (b) $x^2-2x+4=0$
(c) $x^2+2x-4=0$ (d) $x^2-2x-4=0$

Q.4 The quadratic equation with one root i is: (Mtn 10 G-I)

(a) $x^2 + 1 = 0$ (b) $x^2 - 1 = 0$
(c) $x^2 - i = 0$ (d) $x^2 + i = 0$

Q.5 If w is complex cube root of unity, then conjugate of w is (Lhr 15 G-II)

(a) w^2 (b) $-w^2$
(c) $-w$ (d) i

Q.6 The product of all four fourth root of unity is: (Mtn 12 G-I, 17 G-II)(Lhr 18 G-I)

(a) 1 (b) -1
(c) 0 (d) 2

Q.7 If w is a cube root of unity, then w^3 = ? (Mtn 12 G-II)(DGK 19)

(a) w (b) w^2
(c) -1 (d) 1

Q.8 If w is a cube root of unity, then $w^{29} + w^{28} + 1$ equals:
(Mtn 13 G-I) (DGK 15 G-II) (Swl 13)(Rwp 17 G-II)

(a) 0 (b) 1
(c) 2 (d) 3

Q.9 Four fourth roots of 81 are:

(a) $\pm 4, \pm 4i$ (b) $\pm 2, \pm 2i$
(c) $\pm 3, \pm 3i$ (d) $\pm 9, \pm 9i$

Q.10 If w is a cube root of unity, then $1 + w + w^2$ = (Lhr 11 G-I)(DGK 13 G-II)

(a) 0 (b) 1
(c) -1 (d) $-w$

Q.11 If w is complex cube root of unity, then w^2 = (Mtn 14 G-I)

(a) 0 (b) 1
(c) w^2 (d) $\frac{1}{w}$

Q.12 $1 + w + w^2 = 0$: (Mtn 14 G-II) (Bpr 14)

(a) w (b) 1
(c) w^2 (d) 0

Q.13 Four fourth roots of unity are: (Fbd 14)(Mtn 17 G-I)

(a) $-1, +1, i, -i$ (b) $-2, 2, -2i, 2i$
(c) $-3, 3, 3i, -3i$ (d) $\pm 4, \pm 4i$

Q.14 If w is the imaginary cube root of unity then w^2 = (Lhr 10 G-II)(Mtn 17 G-I)

(a) 1 (b) -1
(c) w^{-1} (d) w^{-2}

Q.15 Sum of all three cube root of unity is: (Lhr 14 G-II)(Swl 17)

(a) 1 (b) -1
(c) 3 (d) 0

Q.16 The sum of the four fourth roots of unity is: (Lhr 14 G-II)

(a) 0 (b) +1
(c) -1 (d) i

Q.17 $1 + w^4$ = ? (Lhr 13 G-I & II)

(a) w (b) $-w$
(c) w^2 (d) $-w^2$

Q.18 The cube roots of unity are 1, ω, ω² where ω = -----------

(a) $\frac{1 - i\sqrt{-3}}{2}$ (b) $\frac{-1 + i\sqrt{3}}{2i}$
(c) $\frac{-1 + i\sqrt{3}}{2}$ (d) $\frac{1 + i\sqrt{3}}{2}$

Q.19 Complex cube root of –1 are (Mtn 16 G-II)

(a) w, w^2 (b) $1, w, w^2$
(c) $-1, -w, -w^2$ (d) $-w, -w^2$

Q.20 $1 - w + w^2$ = (Fbd 16)

(a) -1 (b) 0
(c) $-w$ (d) $-2w$

Q.21 Product of all cube roots of unity is. (Fbd 17)(Sgd 17)(Lhr 14 G-I)(Gjw 15)

(a) 1 (b) 0
(c) -1 (d) w

Q.22 Sum of complex roots of unity equals. (Rwp 17 G-I)

(a) 0 (b) -1
(c) 1 (d) w

Q.23 w^3 equals. (Rwp 17 G-I)

(a) 0 (b) -1
(c) i (d) 1

Q.24 If w is complex cube root of unity, then w^{12} = (A.J.K 17)

(a) w (b) $-w$
(c) 1 (d) -1

Q.25 The four fourth roots of 16 are. (Mtn 17 G-II)

(a) $-1, 1, i, -i$ (b) $-2, 2, 2i, -2i$
(c) $-3, 3, 3i - 3i$ (d) $-4, 4, 4i - 4i$

Q.26 If w is the cube root of unity then w = (Lhr 16 G-I)

(a) w^2 (b) 1
(c) w^{-2} (d) $1 + w$

Q.27 Cube roots of unity are. (Fbd 18)

(a) $-1, -2, 1$ (b) $1, -1, w$
(c) $1, w, w^2$ (d) $-1, -w, -w^2$

Q.28 If w be the cube root of unity, then $w^2 =$

(Lhr 18 G-II)

(a) $\frac{-1-\sqrt{3}i}{2}$ (b) $\frac{1-\sqrt{3}i}{2}$

(c) 1 (d) $\frac{1+\sqrt{3}i}{2}$

Q.29 The polynomial $ax^3 + bx^2 + 8$ has degree:

(Fbd 19 G-I)

(a) 8 (b) 3

(c) a + b (d) 5

Q.30 If ω is cube root of unity then $\omega^{29} + \omega^{28} + 1 =$:

(Fbd 19 G-II)

(a) 1 (b) 2

(c) 0 (d) -1

Q.31 If ω is complex cube root of unity then ω^{15} equals. (Gjw 19 G-II)

(a) 1 (b) zero

(c) ω (d) $-\omega$

Q.32 The sum of the four fourth roots of 81 is:

(a) 0 (b) 81

(c) -81 (d) 3

Q.33 If "w" is a cube root of unity, then $(1 + w - w^2)(1 - w + w^2)$ will be equal to:

(Rwp 19)

(a) 3 (b) 4

(c) 2 (d) 1

Q.34 If w is cube root of unity, then $(1 + \omega - \omega^2)^3 =$

(Swl 19)

(a) -8ω (b) 8ω

(c) 8 (d) -8

EXERCISE 4.5

SHORT ANSWERS TO THE QUESTIONS

Q.1 State remainder theorem.

(Bpr 16)(Mtn 10 G-I)(Mtn 11 G-II)

Ans. Statement: If a polynomial f(x) of degree $n \geq 1$, n is non-negative integer is divided by $x - a$ till no x-term exist in the remainder, then f(a) is the remainder.

Q.2 State Factor Theorem:

(Lhr 10 G-I)(Mtn 10 G-II)(Sgd 16)

Ans. The polynomial $x - a$ is a factor of the polynomial f(x) if and only if $f(a) = 0$ i.e., $(x - a)$ is a factor of f(x) if and only if $x = a$ is root of the polynomial equation $f(x) = 0$.

Q.3 Use the remainder theorem to find the remainder when $x^2 + 3x + 7$ is divided by $x + 1$. (Mtn 08 G-I)(Mtn 18 G-II)

Ans. Let $f(x) = x^2 + 3x + 7$.

$x + 1 = 0 \Rightarrow x = -1$

Putting $x = -1$, we have

By remainder theorem Remainder $= f(-1)$

$= (-1)^2 + 3(-1) + 7$

$= 1 - 3 + 7 = 5$

Q.4 When $x^4 + 2x^3 + kx^2 + 3$ is divided by $x - 2$, the remainder is 1. Find the value of k.

(Rwp 13 G-I)(Bpr 18)(Fbd 18, 19 G-II)

Ans. Let $f(x) = x^4 + 2x^3 + kx^2 + 3$

By remainder theorem, we have

Remainder

$= f(2) = (2)^4 + 2(2)^3 + k(2)^2 + 3$

$= 16 + 16 + 4k + 3$

$= 35 + 4k$

But given that remainder is 1.

Thus $35 + 4k = 1$

$\Rightarrow 4k = -34$

$\Rightarrow k = -\frac{17}{2}$.

Q.5 Find the values of a and b if -2 and 2 are the roots of the Polynomial. (Federal 14)(Fbd 17) **$x^3 - 4x^2 + ax + b$.**

Ans. Let $f(x) = x^3 - 4x^2 + ax + b$

Since -2 and 2 are roots of f (x).

$f(-2) = 0$

$\Rightarrow (-2)^3 - 4(-2)^2 + a(-2) + b = 0$

$\Rightarrow -8 - 16 - 2a + b = 0$

$\Rightarrow -24 - 2a + b = 0$

$\Rightarrow 2a - b = -24$ ....(i)

and $f(2) = 0$

$\Rightarrow (2)^3 - 4(2)^2 + a(2) + b = 0$

$\Rightarrow 8 - 16 + 2a + b = 0$

$\Rightarrow -8 + 2a + b = 0$

$\Rightarrow 2a + b = 8$ ......(ii)

Adding eqs. (i) and (ii), we get

$4a = -16$

$\Rightarrow a = -4$

Putting $a = -4$ in eq. (ii), we get

$2(-4) + b = 8$

$\Rightarrow -8 + b = 8 \Rightarrow b = 16$

Hence $a = -4$ and $b = 16$.

LONG QUESTIONS

Q.1 Use synthetic division to find the values of p and q if (x + 1) and (x − 2) are the factors of the polynomial $x^3 + px^2 + qx + 6$.
(Mtn 13 G-I)(Bpr 12)(Bpr 18)(Swl 13)(Mtn 11 G-II)

Ans. Let $P(x) = x^3 + px^2 + qx + 6$

& $x + 1 = 0 \Rightarrow x = -1$; $x - 2 = 0 \Rightarrow x = 2$

-1	1	p	q	6
	$\downarrow$	-1	$-p+1$	$p-q-1$
2	1	$p-1$	$-p+q+1$	$p-q+5$
	$\downarrow$	2	$2p+2$	
	1	$p+1$	$p+q+3$	

Since x + 1 and x = 2 are factors of P(x), so

$p - q + 5 = 0$

$p = q - 5$ ...... (1) & $p+q+3 = 0$ ...... (2)

Putting value of eq. (1) in eq. (2):

$q - 5 + q + 3 = 0 \Rightarrow 2q - 2 = 0 \Rightarrow 2q = 2$

$\Rightarrow q = \frac{2}{2} \Rightarrow \boxed{q = 1}$

Putting value of q in eq. (1):

$p = 1 - 5 \Rightarrow \boxed{p = -4}$

Q.2 Find the values of a and b if −2 and 2 are roots of the polynomial $x^3 - 4x^2 + ax + b$.
(Sgd 13 G-II)(Fbd, Bpr 19)

Ans. Let $f(x) = x^3 - 4x^2 + ax + b$

Since −2 and 2 are roots of f (x).

$f(-2) = 0$

$\Rightarrow (-2)^3 - 4(-2)^2 + a(-2) + b = 0$

$\Rightarrow -8 - 16 - 2a + b = 0$

$\Rightarrow -24 - 2a + b = 0$

$\Rightarrow 2a - b = -24$ ....(i)

and $f(2) = 0$

$\Rightarrow (2)^3 - 4(2)^2 + a(2) + b = 0$

$\Rightarrow 8 - 16 + 2a + b = 0$

$\Rightarrow -8 + 2a + b = 0$

$\Rightarrow 2a + b = 8$ ....(ii)

Adding eqs. (i) and (ii), we get

$4a = -16$

$\Rightarrow a = -4$

Putting $a = -4$ in eq. (ii), we get

$2(-4) + b = 8$

$\Rightarrow -8 + b = 8 \Rightarrow b = 16$

Hence $a = -4$ and $b = 16$.

Q.3 When the polynomial $x^3 + 2x^2 + kx + 4$ is divided by x − 2, the remainder is 14. Find the value of k. **(Mtn 17 G-I)**

Ans. $f(x) = x^3 + 2x^2 + kx + 4$

Put $x = 2$

$f(2) = 2^3 + 2(2)^2 + k(2) + 4$

$= 8 + 8 + 2k + 4$

$= 20 + 2k$

By remainder theorem $f(2) = 14$

$14 = 20 + 2k$

$14 - 20 = 2k$

$2k = -6 \Rightarrow k = -2$

Q.4 Find the value of k if the polynomial. $x^3 + kx^2 - 7x + 6$ has a remainder of −4 when divided by x + 2. **(Mtn 16 G-I)(Lhr 17 G-I)**

Ans. Let

$f(x) = x^3 + kx^2 - 7x + 6$

Put $x = -2$

$f(-2) = (-2)^3 + k(-2)^2 - 7(-2) + 6$

$= -8 + 4k + 14 + 6$

$= 12 + 4k$

But remainder, $f(-2) = -4$

$-4 = 12 + 4k$

$-16 = 4k$

$\Rightarrow \boxed{k = -4}$

MULTIPLE CHOICE QUESTIONS

❑ *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 The factor of $x^2 + 4x - 5$ is: (Mtn 09 G-II)

(a) $x + 1$ (b) $x + 2$
(c) $x - 2$ (d) $x - 1$

Q.2 If the polynomial f(x) is divided by a linear factor (divisor) ax − 1, the remainder is: (Mtn 13 G-II)

(a) $f\left(\frac{1}{a}\right)$ (b) $f\left(-\frac{1}{a}\right)$
(c) $f(a)$ (d) $f(-a)$

Q.3 The polynomial $ax^3 + bx^2 + 8$ has degree: (DGK 13 G-I)

(a) 8 (b) 3
(c) $a + b$ (d) 4

Q.4 $-x^2 - x - 1$ is divided by x − 1, remainder is: (DGK 13 G-II)

(a) 3 (b) −3
(c) 0 (d) 1

Q.5 The degree of constant polynomial is: (Fbd 12) (Bpr 11 G-I)

(a) 0 (b) 1
(c) 2 (d) 3

Q.6 If a polynomial $f(x) = x^3 + 4x^2 - 2x + 5$ is divided by $x - 1$, then the remainder is: (DGK 15 G-II) (Mtn 15 G-II) (Bpr 13) (Gjw 14)(Mtn 18 G-I)

(a) 4 (b) 8
(c) 5 (d) −2

Q.7 $2x^3 + 2x + 1$ is polynomial of degree: (Rwp 11)

(a) 0 (b) 1
(c) 2 (d) 3

Q.8 If a polynomial f(x) is divided by $x - \frac{1}{a}$, then remainder is ——— (Sgd 13)

(a) $f(-a)$ (b) $f\left(-\frac{1}{a}\right)$
(c) $f\left(\frac{1}{a}\right)$ (d) $f(a)$

Q.9 If −2 and −3 are roots of a quadratic equation, then equation is: (Sgd 13 G-II)

(a) $x^2 + 5x - 6 = 0$ (b) $x^2 - 5x + 6 = 0$
(c) $x^2 - 5x - 6 = 0$ (d) $x^2 + 5x + 6 = 0$

Q.10 Synthetic division is a process of: (Mtn 14 G-I)

(a) Addition (b) Subtraction
(c) Multiplication (d) Division

Q.11 The degree of the polynomial $7x^5 + 5x^2 + 9 = 0$ is: (Bpr 15)(Swl 14)

(a) 2 (b) 6
(c) 5 (d) 7

Q.12 If $x - 2$ is a factor of $ax^2 - 12x + 4$, then a = (Fbd 14)

(a) 2 (b) 5
(c) 7 (d) 0

Q.13 If $x^2 + 3x + 7$ is divided by $x + 1$, the remainder is: (Rwp 14)

(a) −5 (b) 11
(c) 5 (d) −11

Q.14 When $3x^4 + 4x^3 + x - 5$ is divided by $x + 1$, then remainder is: (Lhr 12 G-I)(Mtn 18 G-II)

(a) 7 (b) 6
(c) −6 (d) −7

Q.15 If $x + 3$ is a factor of $3x^3 + kx^2 - 22x + 24$, then value of k is

(a) 1 (b) 2
(c) − 1 (d) None of these

Q.16 The value of the polynomial $x^3 - 4x^2 + 5x - 2 = 0$ at $x = 2$ is

(a) 3 (b) 2
(c) 1 (d) 0

Q.17 If − 1 is a root of $x^3 - 5x^2 + 7x + 13 = 0$ the depressed equation is

(a) $-5x^2 + 7x+13 = 0$ (b) $x^2 - 6x + 12 = 0$
(c) $x^2 - 6x + 13 = 0$ (d) $5x^2 + 6x + 13 = 0$

Q.18 The value of the polynomial $4x^4 + 6x^3 + x^2 + x - 3$ at $x = -2$ is

(a) 9 (b) 12
(c) 15 (d) None of these.

Q.19 A quadratic equation with 1 and − 5 as roots is

(a) $x^2 + 4x - 5 = 0$ (b) $x^2 - 4x - 5 = 0$
(c) $x^2 + 4x + 5 = 0$ (d) None of these.

Q.20 The value of the polynomial $3x^3 + 4x^2 - 5x + 6$ at $x = -1$ is

(a) 8 (b) 9
(c) 12 (d) None of these.

Q.21 A quadratic equation with 1 and 2 as roots is

(a) $x^2 + 3x - 2 = 0$ (b) $x^2 - 3x + 2 = 0$
(c) $x^2 + 3x + 2 = 0$ (d) None of these

Q.22 The degree of the polynomial $6x^4 + 3x^2 + 6x + 5 = 0$ is

(a) 1 (b) 2
(c) 3 (d) 4

Q.23 If $x^3 + 3x^2 - 6x + 2$ i divided by $x + 2$, then the remainder is. (Sgd 16)

(a) −18 (b) 9
(c) −9 (d) 18

Q.24 The roots of equation $x^2 - 5x + 6$ are. (Rwp 16)(Lhr 17 G-I)

(a) 2, −3 (b) −2, −3
(c) 2, 3 (d) −2, 3

Q.25 The polynomial $3x^2 + 2x + 1$ has degree. (Gjw 16)

(a) 0 (b) 3
(c) 2 (d) 4

Q.26 What is the value of $3x^2 - 7x + 1$ at $x = -1$ is. (Fbd 16)

(a) −3 (b) 3
(c) 9 (d) 11

Q.27 What is the degree of $x^3 - 4x^4y^2 + 7y^5$? (Bpr 16)

(a) 3 (b) 4
(c) 5 (d) 6

Q.28 If $x - 1$ is a factor of $5x^2 + 10x - k$, the k equal to: (Mtn 09 G-II)

(a) 10 (b) −10
(c) 15 (d) −15

Q.29 $x^3 - 3x^2 + 2x - 6$ has factor. (D.G.K 17 G-I)

(a) $x - 4$ (b) $x - 3$
(c) $x + 3$ (d) $x + 2$

Q.30 One root of the equation $x^2 - 3x + a = 0$ is 2, then a is. (Lhr 17 G-II)

(a) -2 (b) 2
(c) 3 (d) -3

Q.31 If $x - a$ is a factor of a polynomial f(x), then f(a) equals to. (Sgd 17)

(a) 1 (b) 0
(c) 2 (d) -1

Q.32 If 2 and 3 are roots of a quadratic equation, the equation is: (Gjw 13)

(a) $x^2 + 5x - 6 = 0$ (b) $x^2 - 5x + 6 = 0$
(c) $x^2 - 5x - 6 = 0$ (d) $x^2 + 5x + 6 = 0$

Q.33 When $x^3 - 2x^2 + 3x + 3$ is divided by $x - 3$, the remainder is. (Bpr 19)

(a) -21 (b) 21
(c) -51 (d) 51

Q.34 The polynomial $3x^2 + 2x+1$ has degree: (DGK 19)

(a) 0 (b) 3
(c) 2 (d) 4

Q.35 When $P(x) = x^3 + 4x^2 - 2x + 5$ is divided by (x - 1), remainder is (Sgd 19)

(a) 10 (b) -10
(c) 8 (d) -8

EXERCISE 4.6

SHORT ANSWERS TO THE QUESTIONS

Q.1 What is the relation between the roots and the coefficients of a quadratic equation. (Rwp 11 G-I)(Mtn 08 G-I)

Ans. Let α, β be the roots of $ax^2 + bx + c = 0$

then $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad \alpha + \beta = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} + \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a} = -\dfrac{2b}{2a} = -\dfrac{b}{a}$

and $\alpha\beta = \left(\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$

$= \dfrac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{4a^2}$

$= \dfrac{b^2 - b^2 + 4ac}{4a^2} = \dfrac{4ac}{4a^2} = \dfrac{c}{a}$

Thus Sum of the roots $= -\dfrac{b}{a}$ and Product of the roots $= \dfrac{c}{a}$

Q.2 What is quadratic equation whose roots are α,β. (Sgd 13 G-I)

Ans. Quadratic equation whose roots are α, β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Q.3 If α, β are the roots of $3x^2 - 2x + 4 = 0$. Find the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ (Gjw 13 G-I, 2016)(Rwp 16)(Swl 19)

Ans. Since α, β are the roots of

$3x^2 - 2x + 4 = 0$

$\therefore \quad \alpha + \beta = \dfrac{2}{3}$ and $\alpha\beta = \dfrac{4}{3}$

$\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

$= \dfrac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha^2\beta^2}$

$= \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \dfrac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2} = \dfrac{\frac{4}{9} - \frac{8}{3}}{\frac{16}{9}} = \dfrac{4 - 24}{9} \times \dfrac{9}{16} = -\dfrac{20}{16} = -\dfrac{5}{4}$

Q.4 If α, β are the roots of $3x^2 - 2x + 4 = 0$, find $\alpha^2 - \beta^2$. (Sgd 13 G-II)(Lhr 16 G-I)

Ans. $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$

$= (\alpha + \beta)\sqrt{(\alpha - \beta)^2}$

$= (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \left(\dfrac{2}{3}\right)\sqrt{\left(\dfrac{2}{3}\right)^2 - 4\left(\dfrac{4}{3}\right)} = \left(\dfrac{2}{3}\right)\sqrt{\dfrac{4}{9} - \dfrac{16}{3}}$

$= \left(\dfrac{2}{3}\right)\sqrt{\dfrac{4 - 48}{9}} = \left(\dfrac{2}{3}\right)\sqrt{\dfrac{-44}{9}} = \left(\dfrac{2}{3}\right)\dfrac{\sqrt{-44}}{3}$

$= \left(\dfrac{2}{3}\right)\dfrac{\sqrt{-1 \times 4 \times 11}}{3} = \left(\dfrac{2}{3}\right)\dfrac{2\sqrt{11}\,i}{3} = \dfrac{4\sqrt{11}\,i}{9}$

Q.5 If α, β are the roots of $3x^2 - 2x + 4 = 0$, find the values of $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ (Lhr 14 G-I)(Rwp 17 G-II)(Gjw 19 G-I)

Ans. $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \dfrac{\{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta\}}{\alpha\beta} = \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \dfrac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}} = \dfrac{\frac{4}{9} - \frac{8}{3}}{\frac{4}{3}} = \dfrac{4 - 24}{9} \times \dfrac{3}{4} = -\dfrac{20}{9} \times \dfrac{3}{4} = -\dfrac{5}{4}$

LONG QUESTIONS

Q.1 If α, β are the roots of $5x^2 - x - 2 = 0$ form the equation whose roots are $\dfrac{3}{\alpha}$ and $\dfrac{3}{\beta}$. (Sgd 18) (Mtn 11 G-I)(Gjw 13)(Rwp 17 G-II)(Lhr 19 G-II)

Ans. Given that: One root $= \dfrac{3}{\alpha}$; Other root $= \dfrac{3}{\beta}$

Since α, β are the roots of $5x^2 - x - 2 = 0$. Then

$$\alpha + \beta = -\frac{-1}{5} = \frac{1}{5} \quad \ldots\ldots (1) \quad \&$$

$$\alpha\beta = \frac{-2}{5} \quad \ldots\ldots (2)$$

Now, $$S = \frac{3}{\alpha} + \frac{3}{\beta} = \frac{3\beta + 3\alpha}{\alpha\beta} = \frac{3(\beta + \alpha)}{\alpha\beta}$$

$$= \frac{3(\alpha + \beta)}{\alpha\beta} = \frac{3\left(\frac{1}{5}\right)}{\frac{-2}{5}}$$

$$= \frac{\frac{3}{5}}{\frac{-2}{5}} = -\frac{3}{2}$$

and $$P = \frac{3}{\alpha} \cdot \frac{3}{\beta} = \frac{9}{\alpha\beta} = \frac{9}{\frac{-2}{5}} = \frac{9 \times 5}{-2} = -\frac{45}{2}$$

The required equation is:

$$x^2 - Sx + P = 0 \Rightarrow x^2 - \frac{-3}{2}x + \frac{-45}{2} = 0$$

$$\Rightarrow \quad x^2 + \frac{3}{2}x - \frac{45}{2} = 0$$

Multiplying by 2, so $\boxed{2x^2 + 3x - 45 = 0}$

Q.2 If the roots of equation $Px^2 + qx + q = 0$ are 'α' and 'β' then prove that $\sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}} + \sqrt{\dfrac{q}{p}} = 0$ (D.G.K 13 G-II)(Rwp 14)(Rwp 17 G-I, 19)(Sgd 17)

Ans. Since α and β are the roots of $px^2 + qx + q = 0$. Then

$$\alpha + \beta = -\frac{q}{p} \quad \ldots\ldots (1)$$

$$\alpha\beta = \frac{q}{p} \Rightarrow \sqrt{\alpha\beta}$$

$$= \sqrt{\frac{q}{p}} \Rightarrow \sqrt{\alpha}\sqrt{\beta}$$

$$= \sqrt{\frac{q}{p}} \quad \ldots\ldots (2)$$

Dividing eq. (1) by eq. (2):

$$\frac{\alpha + \beta}{\sqrt{\alpha} \cdot \sqrt{\beta}} = \frac{-\frac{q}{p}}{\sqrt{\frac{q}{p}}} \Rightarrow \frac{\alpha}{\sqrt{\alpha}\sqrt{\beta}} + \frac{\beta}{\sqrt{\alpha}\sqrt{\beta}}$$

$$= -\sqrt{\frac{q}{p}}$$

$$\frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} = -\sqrt{\frac{q}{p}} \Rightarrow \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$$

$$= -\sqrt{\frac{q}{p}}$$

$$\boxed{\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0}$$

Q.3 If α, β are the roots of equation $ax^2 + bx + c = 0$, then find the equation whose roots are $\dfrac{1}{\alpha^3}, \dfrac{1}{\beta^3}$.

Ans. Since a, b are the roots of $ax^2 + bx + c = 0$.

Then, $\alpha + \beta = -\dfrac{b}{a} \ldots\ldots (1) \quad \& \quad \alpha\beta = \dfrac{c}{a} \quad \ldots\ldots (2)$

Now, $$S = \frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\beta^3 + \alpha^3}{\alpha^3\beta^3} = \frac{(\beta + \alpha)(\beta^2 + \alpha^2 - \alpha\beta)}{(\alpha\beta)^3}$$

$$= \frac{(\alpha + \beta)(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta - \alpha\beta)}{(\alpha\beta)^3}$$

$$= \frac{(\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)}{(\alpha\beta)^3} = \frac{-\frac{b}{a}\left(\left(-\frac{b}{a}\right)^2 - 3\left(\frac{c}{a}\right)\right)}{\left(\frac{c}{a}\right)^3}$$

$$= \frac{-\frac{b}{a}\left(\frac{b^2}{a^2} - \frac{3c}{a}\right)}{\frac{c^3}{a^3}} = \frac{-\frac{b}{a}\left(\frac{b^2 - 3ac}{a^2}\right)}{\frac{c^3}{a^3}}$$

$$= \frac{-\frac{b^3 - 3abc}{a^3}}{\frac{c^3}{a^3}} = -\frac{b^3 - 3abc}{c^3}$$

and $$P = \frac{1}{\alpha^3} \cdot \frac{1}{\beta^3} = \frac{1}{\alpha^3\beta^3} = \frac{1}{(\alpha\beta)^3} = \frac{1}{\left(\frac{c}{a}\right)^3} = \frac{1}{\frac{c^3}{a^3}}$$

$$= \frac{a^3}{c^3}$$

The required equation is:

$$x^2 - Sx + P = 0 \Rightarrow x^2 - \left(-\frac{b^3 - 3abc}{c^3}\right)x + \frac{a^3}{c^3} = 0$$

$$x^2 + \frac{b^3 - 3abc}{c^3}x + \frac{a^3}{c^3} = 0$$

Multiplying by c^3, so $\boxed{c^3x^2 + (b^3 - 3abc)x + a^3 = 0}$

Q.4 If α, β are the roots of $x^2 - px - p - c = 0$ then prove that $(1 + \alpha)(1 + \beta) = 1 - c$ (Rwp 18)

Ans. See Short Question 3

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 **Let α, β are values of $4x^2 + 5x - 6 = 0$, the value of $4\alpha + 4\beta$ is:** (Mtn 08 G-II)(Rwp 17 G-II)

(a) $-\frac{4}{5}$ (b) -5

(c) $\frac{5}{2}$ (d) $-\frac{2}{5}$

Q.2 **Let α, β be the roots of $ax^2 + bx + c = 0, a \neq 0$, then $\alpha\beta$ is:** (Bpr 12)

(a) $-\frac{b}{a}$ (b) $\frac{c}{a}$

(c) $-\frac{c}{a}$ (d) $\frac{a}{c}$

Q.3 **Product of roots of $2x^2 - 4x + 2 = 0$ is:** (Rwp 12)

(a) -1 (b) 1

(c) 2 (d) -2

Q.4 **If α, β are the roots of the equation $x^2 - 4x + 5 = 0$, then $\alpha\beta$ is equal to:** (Rwp 13)

(a) 5 (b) -4

(c) 2 (d) 4

Q.5 **If one root of $x^2 - 3x + a = 0$ is 2 then a = ____** (Gjw 15)

(a) -2 (b) 2

(c) 0 (d) -3

Q.6 **The product of roots of $5x^2 - x + 2 = 0$ is:** (Mtn 14 G-II)

(a) $\frac{5}{2}$ (b) $-\frac{5}{2}$

(c) $-\frac{2}{5}$ (d) $\frac{2}{5}$

Q.7 **Roots of $ax^2 + bx + c = 0$ are rational, if $b^2 - 4ac$ is:** (D.G.K 14 G-II)

(a) Positive (b) Negative

(c) Zero (d) Perfect square

Q.8 **If α, β are the roots of $x^2 + x + 1 = 0$, then $\alpha + \beta$:** (Swl 14)

(a) -1 (b) 2

(c) 1 (d) 0

Q.9 **For quadratic equation $ax^2 + bx + c = 0$, the sum of roots is:** (Rwp 14)(Lhr 18 G-I)

(a) $\frac{b}{a}$ (b) $-\frac{b}{a}$

(c) $\frac{c}{a}$ (d) $\frac{a}{c}$

Q.10 **The roots of the equation $x^2 + px + q = 0$ are additive inverse of one another then:** (Lhr 10 G-I)(Sgd 16)

(a) $p = 1$ (b) $q = 1$

(c) $q = 0$ (d) $p = 0$

Q.11 **The number of roots of quadratic equation is:** (Gjw 14)

(a) 1 (b) 2

(c) 3 (d) 4

Q.12 **If α, β are the roots of $x^2 - px - p - c = 0$, then $\alpha\beta$.**

(a) $p + c$ (b) $-p + c$

(c) $p - c$ (d) $-(p + c)$

Q.13 **The product of roots of equation $ax^2 + bx - c = 0$ is:** (Gjw 12)

(a) $\frac{c}{a}$ (b) $\frac{b}{a}$

(c) $-\frac{b}{a}$ (d) $-\frac{c}{a}$

Q.14 **If the roots of the equation $\frac{a}{x-a} + \frac{b}{x-b} = 1$ are equal in magnitude and opposite in signs then**

(a) $a + b = 1$ (b) $a - b = 0$

(c) $a - b = 1$ (d) $a + b = 0$

Q.15 **For what value of k, the sum of the roots of the equation $x^2 + kx + 4 = 0$ is equal to the product of its roots:**

(a) ± 1 (b) 4

(c) ± 4 (d) -4

Q.16 **If one root of the equation $a x^2 + b x + c = 0$ be reciprocal of other, then.**

(a) $a = 0, c \neq 0$ (b) $b = c$

(c) $a \neq 0, c \neq 0$ (d) $a = c$.

Q.17 **Only one of the roots of $a x^2 + b x + c = 0, a \neq 0$, is zero if**

(a) $c = 0$ (b) $b = 0, c = 0$

(c) $b = 0, c \neq 0$ (d) $b \neq 0, c = 0$

Q.18 **Both the roots of the equation $a x^2 + b x + c = 0$, are zero if**

(a) $a = 0$ and $b = 0$ (b) $a = 0$ and $c = 0$

(c) $b = 0$ and $c = 0$ (d) $a = b = c = 0$

Q.19 **The product of the roots of the equation $ax^2 + bx + c = 0$.** (F-18)

(a) b/a (b) $-$b/a

(c) c/a (d) $-$c/a

Q.20 **If one solution of the equation $x^2 + ax + 2 = 0$ is $x = 1$, then a = .** (Lhr 16 G-I)

(a) -7 (b) 7

(c) -3 (d) $d = 0$

Q.21 If α, β are the roots of equation $x^2 - 4x + 5 = 0$, then αβ equals to. (Gjw 16)(Fbd 19)

(a) 5 (b) −4
(c) 2 (d) 4

Q.22 If α, β are the roots of equation $x^2 - x + 1 = 0$, then α + β equals to. (Gjw 18)

(a) −1 (b) 0
(c) 1 (d) 2

Q.23 If α, β are the roots of $3x^2 - 2x + 4 = 0$ then (α + 1) (β + 1) = : (Fbd 19 G-II)

(a) $\frac{3}{4}$ (b) $\frac{-4}{3}$
(c) $\frac{4}{3}$ (d) 3

Q.24 If α and β are roots of $ax^2 + bx + c = 0$, then a. β is equal to: (Rwp 19)

(a) −b/a (b) a/b
(c) c/a (d) a/c

Q.25 The product of roots of the equation $3x^2 + 4x = 0$ is (Sgd 19)

(a) $\frac{-4}{3}$ (b) $\frac{4}{3}$
(c) 0 (d) 4

EXERCISE 4.7

SHORT ANSWERS TO THE QUESTIONS

Q.1 What is Discriminant of $ax^2 + bx + c = 0$ (Sgd 13 G-I)

Ans. Discriminant of $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

Q.2 Explain how one can determine nature of roots with the help of discriminant?

Ans.

(i) If $b^2 - 4ac = 0$, then roots will be real and repeated equal

(ii) If $b^2 - 4ac < 0$, then roots will be imaginary / complex and distinct unequal.

(iii) If $b^2 - 4ac > 0$, then roots will be real and distinct / unequal. However, if $b^2 - 4ac$ is a perfect square then roots are rational.

Q.3 Discuss nature of roots of equation.
$x^2 - 5x + 6 = 0$ (Fbd 16)(Sgd 16)(Lhr 19 G-II)

Ans. Take $b^2 - 4ac$

$= (-5)^2 - 4(1)(6)$

$= 25 - 24$

$= 1 > 0$ and a perfect square.

Q.4 Discuss the nature of the roots of the quadratic equation $2x^2 + 5x - 1 = 0$ (Lhr 09 G-I)(Fbd 19 G-II)

Ans. $2x^2 - 5x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0$

$a = 2, b = 5, c = -1$

Discriminant $= b^2 - 4ac = (5)^2 - 4(2)(1)$

$= 25 - 8 = 17$

⇒ Disc. > 0, but not a perfect square.

Therefore, the roots are irrational and unequal.

Q.5 Show that the roots of $(p + q) x^2 - px - q = 0$ are rational. (Mtn 16 G-II)(Gjw 19 G-II)(Swl 17)(Rwp 17 G-I, II)(Rwp 17, 18)

Ans. Take discriminate

$= (-p)^2 - 4(p + q)(-q)$

$= p^2 + 4pq + 4q^2$

$= (p + 2q)^2 \geq 0$

∴ Roots are rational.

Q.6 Show that the roots of the equation (Gjw 17)
$x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0, m \neq 0$ will be real.

Ans. See Long Question 6

Q.7 Show that $\frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2$. (Lhr 08 G-I)

Ans. Let α, α^2 be the roots, the

$\alpha + \alpha^2 = -\frac{b}{a}$ ... (i)

$\alpha . \alpha^2 = \frac{c}{a}$

$\alpha^3 = \frac{c}{a}$

$a = \left(\frac{c}{a}\right)^{\frac{1}{3}}$ ... (ii)

Put in equation (i)

$$\left(\frac{c}{a}\right)^{\frac{1}{3}} + \left[\left(\frac{c}{a}\right)^{\frac{1}{3}}\right]^2 = -\frac{b}{a}$$

Taking cube

$$\left(\left(\frac{c}{a}\right)^{\frac{1}{3}} + \left(\frac{c}{a}\right)^{\frac{2}{3}}\right)^3 = \left(-\frac{b}{a}\right)^3$$

$$\frac{c}{a} + \left(\frac{c}{a}\right)^2 + 3 . \left(\frac{c}{a}\right)^{\frac{1}{3}}\left(\frac{c}{a}\right)^{\frac{2}{3}}\left(\left(\frac{c}{a}\right)^{\frac{1}{3}} + \left(\frac{c}{a}\right)^{\frac{2}{3}}\right) = -\frac{b^3}{a^3}$$

$$\frac{c}{a} + \frac{c^2}{a^2} + 3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3} \quad \text{By equation (i)}$$

Multiplying with a^3

$ca^2 + c^2a - 2abc = -b^3$

Therefore, the roots are irrational and unequal.

LONG QUESTIONS

Q.1 Show that roots of $x^2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2(1 + m^2)$. (A.J.K 17)

(D.G.K 11, 12, 13 G-I, 15 G-II)

(Sgd 13 G-I, 16)(Gjw 16)(Fbd 19 G-II)(Swl 19)

Ans. Given that: $x^2 + (mx + c)^2 = a^2 \Rightarrow x^2 + m^2x^2 + 2mcx + c^2 = a^2$

$(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

Here, we can take: $a = 1 + m^2,\ b = 2mc,\ c = c^2 - a^2$

The roots will be equal if:

$b^2 - 4ac = 0 \Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - a^2m^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4a^2m^2 = 0$

$-4c^2 + 4a^2 + 4a^2m^2 = 0 \Rightarrow -4[c^2 - a^2 - a^2m^2] = 0$

$c^2 - a^2 - a^2m^2 = 0 \Rightarrow c^2 = a^2 + a^2m^2$

$\Rightarrow \boxed{c^2 = a^2(1 + m^2)}$

Q.2 Prove that $\dfrac{x^2}{a^2} + \dfrac{(mx + c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2$, $a \neq 0$, $b \neq 0$.

(Mtn 14, 16 G-II)(D.G.K 14 G-II)

Ans. $\dfrac{x^2}{a^2} + \dfrac{(mx + c)^2}{b^2} = 1 \Rightarrow \dfrac{b^2x^2 + a^2(mx + c)^2}{a^2b^2} = 1$

$b^2x^2 + a^2(mx + c)^2 = a^2b^2$

$b^2x^2 + a^2(m^2x^2 + 2mcx + c^2) - a^2b^2 = 0$

$b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$

$(a^2m^2 + b^2)x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$

Comparing it with $Ax^2 + Bx + C = 0$, we have

$A = (a^2m^2 + b^2)$, $B = 2a^2mcx$, $C = a^2c^2 - a^2b^2$

Now Disc. $= B^2 - 4AC$

$= 4a^4m^2c^2 - 4(a^2m^2 + b^2)(a^2c^2 - a^2b^2)$

For equal roots Disc. = 0

$4a^4m^2c^2 - 4(a^2m^2 + b^2)(a^2c^2 - a^2b^2) = 0$

$a^4m^2c^2 - (a^4m^2c^2 - a^4b^2m^2 + a^2b^2c^2 - a^2b^4) = 0$

(dividing by 4)

$a^4m^2c^2 - a^4m^2c^2 + a^4b^2m^2 - a^2b^2c^2 + a^2b^4 = 0$

$a^4b^2m^2 - a^2b^2c^2 + a^2b^4 = 0$

$a^4b^2m^2 + a^2b^4 = a^2b^2c^2$

$a^2b^2c^2 = a^4b^2m^2 + a^2b^4$

$a^2b^2c^2 = a^2b^2(a^2m^2 + b^2)$

$\Rightarrow c^2 = a^2m^2 + b^2$

Q.3 Show that the roots of the equation:

$x^2 - 2\left(m + \dfrac{1}{m}\right)x + 3 = 0$, $m \neq 0$ are real.

(Mtn 18 G-I)

Ans. Take $b^2 - 4ac$

$= \left[-2\left(m + \dfrac{1}{m}\right)\right]^2 - 4.1.3 = 4\left(m^2 + \dfrac{1}{m^2} + 2\right) - 12$

$= 4m^2 + \dfrac{4}{m^2} + 8 - 12 = 4m^2 + \dfrac{4}{m^2} - 4$

$= 4m^2 + \dfrac{4}{m^2} - 8 + 4 = 4\left(m^2 + \dfrac{1}{m^2} - 2\right) + 4$

$= 4\left(m - \dfrac{1}{m}\right)^2 + 4 > 0$

$\therefore$ Roots are real

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The roots of $x^2 + 2x + 3 = 0$ are:

(Mtn 08)(Rwp 15)

(a) Imaginary (b) Real equal
(c) Real unequal (d) Rational

Q.2 Nature of roots of quadratic equation can be found with the help of: (Mtn 11 G-II)

(a) Quadratic formula (b) Remainder theorem
(c) Factor theorem (d) Discriminant

Q.3 The roots of equation $x^2 - x - 2 = 0$ are:

(Mtn 12 G-I)

(a) 2, −1 (b) 1, 2
(c) 1 (d) −2

Q.4 The roots of $ax^2 + bx + c = 0$ are complex if:

(Mtn 12 G-II)

(a) $b^2 - 4ac < 0$ (b) $b^2 - 4ac = 0$
(c) $b^2 - 4ac > 0$ (d) $b^2 - 4ac \neq 0$

Q.5 In a quadratic equation $ax^2 - bx + c = 0$ if $b^2 - 4ac < 0$, then the roots are:

(Mtn 15 G-I, G-II, 10 G-II)(Lhr 12)(Rwp 13)

(DGK 11, 17 G-II)

(a) Rational (b) Irrational
(c) Equal (d) Complex

Q.6 Roots of equation $ax^2 + bx + c = 0$ are real and equal if: (Swl 13)

(a) $b^2 - 4ac = 0$ (b) $b^2 - 4ac \neq 0$
(c) $b^2 - 4ac < 0$ (d) $4ac - b^2 > 0$

Q.7 $x^2 + x - 6 = 0$ have roots: (Sgd 13 G-II)

(a) Real (b) Equal
(c) Complex (d) Trivial

Q.8 The discriminant of $x^2 + x + 1 = 0$ is: (D.G.K 14 G-I)
(a) 3 (b) 2
(c) -2 (d) -3

Q.9 If $b^2 - 4ac > 0$ and perfect square then roots are: (Lhr 14)
(a) Rational (b) Irrational
(c) Equal (d) Complex

Q.10 If the roots of $px^2 + qx + 1 = 0$ are equal, then: (Gjw 13)(DGK 17 G-I)
(a) $p^2 + 4q = 0$ (b) $p^2 - 4q = 0$
(c) $q^2 + 4p = 0$ (d) $q^2 - 4p = 0$

Q.11 Roots of equation $x^2 - 4x + 8 = 0$ are: (Gjw 11)
(a) Imaginary (b) Real
(c) Rational (d) Equal

Q.12 The discriminant for equal roots is: (Lhr 11 G-II)
(a) > 0 (b) < 0
(c) $= 0$ (d) Perfect square

Q.13 For what values of k, the roots of the equation $x^2 + kx + 4 = 0$ are equal:
(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

Q.14 The roots of the equation $x^2 + x + 1 = 0$ are
(a) Complex (b) Irrational
(c) Rational (d) None of these

Q.15 If the roots of $ax^2 + b = 0$ are real and unequal, then. (Mtn 16 G-II)
(a) $ab > 0$ (b) $ab < 0$
(c) $ab = 0$ (d) $ab \geq 0$

Q.16 For real and repeated roots $b^2 - 4ac$ will be. (Bpr 16)
(a) Positive (b) Negative
(c) Zero (d) Imaginary

Q.17 The roots of equation $ax^2 + bx + c = 0$ are rational if $b^2 - 4ac$ is. **(Guj 18)**
(a) Positive (b) Perfect square
(c) Negative (d) 0

Q.18 Nature of roots of the equation $2x^2 + 5x - 1 = 0$ are. **(Mtn 18 G-I)**
(a) Irrational and unequal (b) Rational and equal
(c) Imaginary (d) Rational and unequal

Q.19 The discriminant of $ax^2 + bx + c = 0, a \neq 0$ is. **(Rwp 18)**
(a) $b^2 + 4ac$ (b) $4ac - b^2$
(c) $b^2 - 4ac$ (d) $a^2 - 4bc$

Q.20 The roots of the equation $x^2 + x - 6 = 0$ are. **(Sgd 18)**
(a) Real (b) Eqaul
(c) Complex (d) Irrational

Q.21 Roots of equation $x^2 + x + 2 = 0$ are. (Bpr 18)
(a) Real, equal (b) Real, unequal
(c) Equal (d) Imaginary

Q.22 If b^2 - 4ac > 0 but not a perfect square. then roots are: (Lhr 19 G-II)
(a) Equal (b) Complex
(c) Rational (d) Irrational

Q.23 If s = sum of roots and p = product of roots, then quadratic equation can be written as: (Mtn 19 G-I)
(a) $x^2 + s\,x + p = 0$ (b) $s^2 - s\,x - p = 0$
(c) $x^2 - s\,x + p = 0$ (d) $s\,x^2 - s\,x + p = 0$

EXERCISE 4.8

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define system of siMtneous equation.

Ans. When we have two or more variables, then we need two or more than two equations. Such set of equations is called system of siMtneous equation.

Q.2 Solve the systems of equations $2x - y = 4$; $2x^2 - 4xy - y^2 = 6$ **(Gjw 13 G-I)**

Ans. $2x - y = 4$ (i)

$2x^2 - 4xy - y^2 = 6$ (ii)

From eq. (i) $y = 2x - 4$

Putting in eq. (ii), we have

$$2x^2 - 4x\,(2x - 4) - (2x - 4)^2 = 6$$
$$\Rightarrow 2x^2 - 8x^2 + 16x - \left(4x^2 - 16\,x + 16\right) = 6$$
$$\Rightarrow -6x^2 + 16x - 4x^2 + 16x - 16 - 6 = 0$$
$$\Rightarrow -10\,x^2 + 32x - 22 = 0$$
$$\Rightarrow 5x^2 - 16x + 11 = 0$$
$$\Rightarrow 5x^2 - 11x - 5x + 11 = 0$$
$$\Rightarrow x(5x - 11) - 1\,(5x - 11) = 0$$
$$\Rightarrow (x - 1)\,(5x - 11) = 0$$
$$\Rightarrow (x - 1) = 0,\ (5x - 11) = 0$$
$$\Rightarrow x = 1,\ x = \frac{11}{5}$$

when $x = 1$, $y = 2x - 4 = 2(1) - 4 = -2$

$\Rightarrow (1, -2)$

when $x = \frac{11}{5}$, $y = 2x - 4 = 2\left(\frac{11}{5}\right) - 4$

$$= \frac{22 - 20}{5} = \frac{2}{5}$$

$$\Rightarrow \left(\frac{11}{5}, \frac{2}{5}\right)$$

$$\therefore \text{S. S.} = \left\{(1, -2), \left(\frac{11}{5}, \frac{2}{5}\right)\right\}$$

LONG QUESTIONS

Q.1 Solve the system of equation:

$$3x + 4y = 25$$

$$\frac{3}{x} + \frac{4}{y} = 2 \qquad \textbf{(Lhr 16 G-I)(D.G.K 17 G-I)}$$

Ans. Given that: $3x + 4y = 25 \Rightarrow 4y = 25 - 3x$

$$\Rightarrow \quad y = \frac{25-3x}{4} \quad \ldots\ldots (1)$$

$$\frac{3}{x} + \frac{4}{y} = 2 \Rightarrow \frac{3y+4x}{xy} = 2$$

$$\Rightarrow \quad 3y + 4x = 2xy \quad \ldots\ldots (2)$$

Putting the value of eq. (1) in eq. (2):

$$3\left(\frac{25-3x}{4}\right) + 4x = 2x\left(\frac{25-3x}{4}\right)$$

$$\Rightarrow \quad \frac{75-9x}{4} + 4x = \frac{50x - 6x^2}{4}$$

Multiplying both sides by 4:

$$75 - 9x + 16x = 50x - 6x^2$$

$$\Rightarrow \quad 6x^2 - 9x + 16x - 50x + 75 = 0$$

$$6x^2 - 43x + 75 = 0$$

$$x = \frac{-(-43) \pm \sqrt{(43)^2 - 4(6)(75)}}{2(6)}$$

$$= \frac{43 \pm \sqrt{1849 - 1800}}{12}$$

$$= \frac{43 \pm \sqrt{49}}{12} = \frac{43 \pm 7}{12} = \frac{43+7}{12}, \frac{43-7}{12}$$

$$= \frac{50}{12}, \frac{36}{12} = \frac{25}{6}, 3$$

When $x = \frac{25}{6}$, then from eq. (1):

$$y = \frac{25 - 3\left(\frac{25}{6}\right)}{4}$$

$$= \frac{25 - \frac{25}{2}}{4}$$

$$= \frac{\frac{50-25}{2}}{4}$$

$$= \frac{50-25}{2\times 4} = \frac{25}{8}$$

$$\therefore \quad \left(\frac{25}{6}, \frac{25}{8}\right)$$

When $x = 3$, then from eq. (1):

$$y = \frac{25 - 3(3)}{4}$$

$$= \frac{25-9}{4} = \frac{16}{4} = 4$$

$$\therefore \quad (3, 4)$$

$$\text{S. Set} = \left\{(3, 4), \left(\frac{25}{6}, \frac{25}{8}\right)\right\}$$

EXERCISE 4.10

SHORT ANSWERS TO THE QUESTIONS

Q.1 The sum of a number and its reciprocal is $\frac{26}{5}$. Find the number. (Mtn 11, 19 G-I)(Lhr 17 G-II)

Ans. Let x be a number then according to the given statement.

$$x + \frac{1}{x} = \frac{26}{5}$$

$$\frac{x^2+1}{x} = \frac{26}{5}$$

$$5(x^2 + 1) = 26x$$

$$5x^2 - 26x + 5 = 0$$

$$5x^2 - 25x - x + 5 = 0$$

$$5x(x-5) - 1(x-5) = 0$$

$$(x-5)(5x-1) = 0$$

$$x - 5 = 0 \text{ and } 5x - 1 = 0$$

$$\Rightarrow \quad x = 5 \Rightarrow x = \frac{1}{5}$$

Hence $x = 5, \frac{1}{5}$ are the required numbers.

Q.2 Find two consecutive numbers, whose product is 132. (Lhr 10 G-I)

Ans. Let two consecutive numbers be x and x + 1. The according to the given statement

$$x(x+1) = 132$$

$$\Rightarrow \quad x^2 + x - 132 = 0$$

$$\Rightarrow \quad x^2 + 12x - 11x - 132 = 0$$

$$\Rightarrow \quad x(x+12) - 11(x+12) = 0$$

$$\Rightarrow \quad (x+12)(x-11) = 0$$

$$\Rightarrow \quad x + 12 = 0 \text{ and } x - 11 = 0$$

$$\Rightarrow \quad x = -12 \Rightarrow x = 11$$

$$x + 1 = -12 + 1 = -11$$

and $x + 1 = 11 + 1 = 12$

Hence the required numbers are 11, 12 or −12, − 11.

Q.3 Find two consecutive numbers, whose product is 132. (Mtn 12, 16 G-I, 11 G-II)

Ans. Let two consecutive numbers be x and x + 1. The according to the given statement

$x(x+1) = 132$

$\Rightarrow \quad x^2 + x - 132 = 0$

$\Rightarrow \quad x^2 + 12x - 11x - 132 = 0$

$\Rightarrow \quad x(x+12) - 11(x+12) = 0$

$\Rightarrow \quad (x+12)(x-11) = 0$

$\Rightarrow \quad x + 12 = 0 \quad \text{and} \quad x - 11 = 0$

$\Rightarrow \quad x = -12 \quad \Rightarrow x = 11$

$x + 1 = -12 + 1 = -11$

and $\quad x + 1 = 11 + 1 = 12$

Hence the required numbers are 11, 12 or −12, − 11.

Q.4 A number exceeds its square root by 56. Find the number. (Rwp 11 G-I)(Lhr 16 G-I)(Sgd 17)

Ans. Let x be a number. Then according to the given statement

$x = \sqrt{x} + 56$

$\Rightarrow \quad x - 56 = \sqrt{x}$

$\Rightarrow \quad (x-56)^2 = (\sqrt{x})^2$

$\Rightarrow \quad x^2 - 112x + 3136 = x$

$\Rightarrow \quad x^2 - 113x + 3136 = 0$

$\Rightarrow \quad x^2 - 64x - 49x + 3136 = 0$

$\Rightarrow \quad x(x-64) - 49(x-64) = 0$

$\Rightarrow \quad (x-64)(x-49) = 0$

$\Rightarrow \quad x - 64 = 0 \quad \text{and} \quad x - 49 = 0$

$\Rightarrow \quad x = 64 \quad \Rightarrow x = 49$

Hence x = 64 is the required number which satisfied the given condition.

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Two natural numbers whose sum is 15 and difference is 5, are

(a) 15 , 5 (b) 2 , 10
(c) 9 , 4 (d) 5 , 10

Q.2 A has 3 times as many rupees as B. If they have 28 in all, then B has

(a) Rs. 21 (b) Rs. 7
(c) Rs. 22 (d) Rs. 27

Chapter 5

PARTIAL FRACTION

EXERCISE 5.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define a Rational fraction. **(D.G.K 12 G-I)**

Ans. The quotient of two polynomials $\frac{P(x)}{Q(x)}$ where $Q(x) \neq 0$, with no common factors, is called a Rational Function. A rational function is of two types.

Q.2 List the names of two types of a rational fraction. **(Fbd 11 G-I)**

Ans. Two types of a rational fraction are

(a) Proper rational fraction

(b) Improper rational fraction

Q.3 Define partial fraction.
(MTN 18 G-II, D.G. K 13 G-I)(Mtn 18 G-II)

Ans. Expressing a single rational function as a sum of two or more single rational functions is called partial fraction.

Q.4 Define proper rational fraction with example.
(Mtn 11, 12, 17, 19 G-I)(Lhr 16 G-I)(D.G.K 16 G-I)
(Lhr 17, 19 G-II)(A.J.K 17)(Swl 19)(Gjw 19 G-II)

Ans. A rational fraction $\frac{P(x)}{Q(x)}$ is called a Proper Rational Fraction if the degree of the polynomial P(x) in the numerator is less than the degree of the polynomial Q(x) in the denominator. For example, $\frac{2}{3x+5}$, $\frac{x}{x^2+1}$ and $\frac{x^2+1}{x^3+1}$ are proper rational fractions or proper fractions.

Q.5 Define Improper Rational Function.
(Mtn 08 G-II)(Bpr 16, 19)(A.J.K 17)(Sgd 19)

Ans. A rational function $\frac{P(x)}{Q(x)}$ is called an Improper Rational Function if the degree of the polynomial P(x) in the numerator is equal to or greater than the degree of the polynomial Q(x) in the denominator.

e.g. $\frac{x^2-1}{x^2+1}, \frac{x^3-1}{x^2-2x+5}$

Q.5 Resolve $\frac{1}{(x-1)(2x-1)}$. **(Rwp 16)**

Ans. $\frac{1}{(x-1)(2x-1)} = \frac{A}{x-1} + \frac{B}{2x-1}$

$1 = A(2x-1) + B(x-1)$

Put $x = 1$

$1 = A(2(1)-1) + B(1-1)$

$1 = A(1) + B(0)$ $\boxed{A = 1}$

Put $2x - 1 = 0$ or $x = \frac{1}{2}$

$1 = A(0) + B\left(\frac{1}{2} - 1\right) \Rightarrow 1 = B\left(-\frac{1}{2}\right)$ $\boxed{-2 = B}$

$\therefore \frac{1}{(x-1)(2x-1)} = \frac{1}{x-1} - \frac{2}{2x-1}$

Q.7 If $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$

Find value of B. **(DGK 19)**

Ans. $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$

Multiplying with $(x+3)(x+4)$, we get

$7x + 25 = A(x+4) + B(x+3)$

Put $x + 4 = 0$ or $x = -4$

$7(-4) + 25 = A(0) + B(-4+3)$

$-28 + 25 = B(-1)$

$-3 = -B$

$B = 3$

Q.8 Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions.
(Gjw 19 G-II)

Ans. $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$

$7x + 25 = A(x+4) + B(x+3)$

Put $x + 4 = 0$ or $x = -4$

$7(-4) + 25 = B(-4+3)$

$\Rightarrow -28 + 25 = -B \Rightarrow \boxed{B = 3}$

Put $x + 3 = 0$ or $x = -3$

$7(-3) + 25 = A(-3+4)$

$\Rightarrow -21 + 25 = A \Rightarrow \boxed{A = 4}$

$\frac{7x+25}{(x+3)(x+4)} = \frac{4}{x+3} + \frac{3}{x+4}$

LONG QUESTIONS

Q.1 Resolve into partial fraction.

$$\frac{1}{(1-ax)(1-bx)(1-cx)}$$ **(Lhr13 G-II)(Mtn15 G-I)**

Ans. $\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{A}{1-ax} + \frac{B}{1-bx} + \frac{C}{1-cx}$ (1)

Multiplying both sides of eq. (1) by $(1-ax)(1-bx)(1-cx)$ we have

$$(1-ax)(1-bx)(1-cx)\,\frac{1}{(1-ax)(1-bx)(1-cx)}$$

$$= \frac{A}{(1-ax)}(1-ax)(1-bx)(1-cx) + \frac{B}{1-bx}(1-ax)(1-bx)(1-cx) + \frac{C}{1-cx}(1-ax)(1-bx)(1-cx)$$

$$1 = A(1-bx)(1-cx) + B(1-ax)(1-cx) + C(1-ax)(1-bx) \quad (2)$$

Putting $x = \frac{1}{a}$ in eq. (2) we have

$$1 = A\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right) + B\left(1-\frac{a}{a}\right)\left(1-\frac{c}{a}\right) + C\left(1-\frac{a}{a}\right)\left(1-\frac{b}{a}\right)$$

$$\Rightarrow 1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right) + B(1-1)\left(1-\frac{c}{a}\right) + C(1-1)\left(1-\frac{b}{a}\right)$$

$$\Rightarrow 1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right) + B(0)\left(1-\frac{c}{a}\right) + C(0)\left(1-\frac{b}{a}\right)$$

$$\Rightarrow 1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right) + 0 + 0$$

$$\Rightarrow 1 = \frac{A(a-b)(a-c)}{a^2} \quad \Rightarrow \quad \boxed{A = \frac{a^2}{(a-b)(a-c)}}$$

Putting $x = \frac{1}{b}$ in eq. (2), we have

$$1 = A\left(1-\frac{b}{b}\right)\left(1-\frac{c}{b}\right) + B\left(1-\frac{a}{b}\right)\left(1-\frac{c}{b}\right) + C\left(1-\frac{a}{b}\right)\left(1-\frac{b}{b}\right)$$

$$\Rightarrow 1 = A(1-1)\left(1-\frac{c}{b}\right) + B\left(\frac{b-a}{b}\right)\left(\frac{b-c}{b}\right) + C\left(1-\frac{a}{b}\right)(1-1)$$

$$\Rightarrow 1 = A(0)\left(1-\frac{c}{b}\right) + B\left(\frac{b-a}{b}\right)\left(\frac{b-c}{b}\right) + C\left(1-\frac{a}{b}\right)(0)$$

$$\Rightarrow 1 = 0 + B\,\frac{(b-a)(b-c)}{b^2} + 0 \Rightarrow 1 = B\,\frac{(b-a)(b-c)}{b^2} \Rightarrow$$

$$\boxed{B = \frac{b^2}{(b-a)(b-c)}}$$

Putting $x = \frac{1}{c}$ in eq. (2), we have

$$1 = A\left(1-\frac{b}{c}\right)\left(1-\frac{c}{c}\right) + B\left(1-\frac{a}{c}\right)\left(1-\frac{c}{c}\right) + C\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)$$

$$\Rightarrow 1 = A\left(1-\frac{b}{c}\right)(1-1) + B\left(1-\frac{a}{c}\right)(1-1) + C\left(\frac{c-a}{c}\right)\left(\frac{c-b}{c}\right)$$

$$\Rightarrow 1 = A\left(1-\frac{b}{c}\right)(0) + B\left(1-\frac{a}{c}\right)(0) + C\left(\frac{c-a}{c}\right)\left(\frac{c-b}{c}\right)$$

$$\Rightarrow 1 = 0 + 0 + \frac{C(c-a)(c-b)}{c^2} \quad \Rightarrow 1 = \frac{C(c-a)(c-b)}{c^2}$$

$$\Rightarrow \boxed{C = \frac{c^2}{(c-a)(c-b)}}$$

Putting the values of A, B and C in eq. (1), we have

$$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{a^2}{(a-b)(a-c)(1-ax)} + \frac{b^2}{(b-a)(b-c)(1-bx)} + \frac{c^2}{(c-a)(c-b)(1-cx)}$$

Hence required partial fractions are

$$\frac{a^2}{(a-b)(a-c)(1-ax)} + \frac{b^2}{(b-a)(b-c)(1-bx)} + \frac{c^2}{(c-a)(c-b)(1-cx)}$$

Q.2 Resolve $\frac{1}{(x-1)(2x-1)(3x-1)}$ **into partial fraction.** **(Mtn 17 G-I)**

Ans. Given that: $\frac{1}{(x-1)(2x-1)(3x-1)}$

Consider: $\frac{1}{(x-1)(2x-1)(3x-1)}$

$$= \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{3x-1} \quad \ldots\ldots (1)$$

Multiplying by $(x-1)(2x-1)(3x-1)$:

$$1 = A(2x-1)(3x-1) + B(x-1)(3x-1) + C(x-1)(2x-1) \quad \ldots\ldots (2)$$

Put $x-1 = 0$, $x = 1$

$$1 = A(2(1)-1)(3(1)-1)$$
$$= A(2-1)(3-1)$$
$$1 = A(1)(2) = 2A$$
$$\Rightarrow \boxed{A = \frac{1}{2}}$$

Put $2x-1 = 0$, $2x = 1$, $x = \frac{1}{2}$

$$1 = B\left(\frac{1}{2}-1\right)\left(3\left(\frac{1}{2}\right)-1\right)$$
$$= B\left(\frac{1}{2}-1\right)\left(\frac{3}{2}-1\right)$$
$$1 = B\left(\frac{1-2}{2}\right)\left(\frac{3-2}{2}\right)$$
$$= B\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)$$
$$1 = -\frac{1}{4}B \Rightarrow \boxed{B = -4}$$

Put $3x - 1 = 0$

$3x = 1$

$x = \frac{1}{3}$

$$1 = C\left(\frac{1}{3} - 1\right)\left(2\left(\frac{1}{3}\right) - 1\right)$$
$$= C\left(\frac{1}{3} - 1\right)\left(\frac{2}{3} - 1\right)$$
$$1 = C\left(\frac{1-3}{3}\right)\left(\frac{2-3}{3}\right)$$
$$= C\left(\frac{-2}{3}\right)\left(\frac{-1}{3}\right)$$
$$1 = \frac{2}{9}C \Rightarrow \boxed{C = \frac{9}{2}}$$

So, from eq. (1):

$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{\frac{1}{2}}{x-1} + \frac{-4}{2x-1} + \frac{\frac{9}{2}}{3x-1}$$

$$\boxed{\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{1}{2(x-1)} - \frac{4}{2x-1} + \frac{9}{2(3x-1)}}$$

Q.3 Resolve $\frac{3x^2 - 4x - 5}{(x-2)(x^2+7x+10)}$ into partial fractions. (D.G.K 11 G-I)(Lhr 16 G-I)(Sgd 18)

Ans. Given that: $\frac{3x^2 - 4x - 5}{(x-2)(x^2+7x+10)}$

$$= \frac{3x^2 - 4x - 5}{(x-2)(x^2+5x+2x+10)}$$
$$= \frac{3x^2 - 4x - 5}{(x-2)(x(x+5)+2(x+5))}$$
$$= \frac{3x^2 - 4x - 5}{(x-2)(x+5)(x+2)}$$

Consider:

$$\frac{3x^2 - 4x - 5}{(x-2)(x+5)(x+2)} = \frac{A}{x-2} + \frac{B}{x+5} + \frac{C}{x+2} \quad \ldots\ldots (1)$$

Multiplying by $(x-2)(x+5)(x+2)$:

$3x^2 - 4x - 5 = A(x+5)(x+2) + B(x-2)(x+2) + C(x-2)(x+5) \quad \ldots\ldots (2)$

Put $x - 2 = 0$

$x = 2$

$$3(2)^2 - 4(2) - 5 = A(2+5)(2+2)$$
$$3(4) - 8 - 5 = A(7)(4)$$
$$\Rightarrow 12 - 8 - 5 = 28A$$
$$1 = 28A \Rightarrow \boxed{A = \frac{-1}{28}}$$

Put $x + 5 = 0$

$x = -5$

$$3(-5)^2 - 4(-5) - 5 = B(-5-2)(-5+2)$$
$$3(25) + 20 - 5 = B(-7)(-3)$$
$$75 + 20 - 5 = 21B \Rightarrow 90 = 21B$$
$$B = \frac{90}{21} \Rightarrow \boxed{B = \frac{30}{7}}$$

Put $x + 2 = 0$

$x = -2$

$$3(-2)^2 - 4(-2) - 5 = C(-2-2)(-2+5)$$
$$3(4) + 8 - 5 = C(-4)(3)$$
$$\Rightarrow 12 + 8 - 5 = -12C$$
$$15 = -12C \Rightarrow C = \frac{15}{-12}$$
$$\Rightarrow \boxed{C = -\frac{5}{4}}$$

So, from eq. (1):

$$\frac{3x^2 - 4x - 5}{(x-2)(x+5)(x+2)} = \frac{\frac{-1}{28}}{x-2} + \frac{\frac{30}{7}}{x+5} + \frac{\frac{-5}{4}}{x+2}$$

$$\boxed{\frac{3x^2 - 4x - 5}{(x-2)(x+5)(x+2)} = -\frac{1}{28(x-2)} + \frac{30}{7(x+5)} - \frac{5}{4(x+2)}}$$

Q.4 Resolve into partial fractions $\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1}$.

(Fbd 14)(Lhr 15 G-II)(Gjw 16)

Ans. Given that: $\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1}$

$$\begin{array}{r|l} 2x^2 - x - 1 \;) & 6x^3 + 5x^2 - 7 \quad (3x + 4 \\ & \pm 6x^3 \mp 3x^2 \mp 3x \\ \hline & 8x^2 + 3x - 7 \\ & \pm 8x^2 \mp 4x \mp 4 \\ \hline & 7x - 3 \end{array}$$

$$= 3x + 4 + \frac{7x-3}{2x^2 - x - 1} = 3x + 4 + \frac{7x-3}{2x^2 - 2x + x - 1}$$
$$= 3x + 4 + \frac{7x-3}{2x(x-1) + 1(x-1)}$$
$$= 3x + 4 + \frac{7x-3}{(x-1)(2x+1)} \quad \ldots\ldots (1)$$

Consider: $\frac{7x-3}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1} \quad \ldots\ldots (2)$

Multiplying by $(x-1)(2x+1)$:

$7x - 3 = A(2x+1) + B(x-1) \quad \ldots\ldots (3)$

Put

$x - 1 = 0$

$x = 1$

$$7(1) - 3 = A(2(1)+1)$$
$$\Rightarrow 7 - 3 = A(2+1)$$
$$4 = 3A \Rightarrow \boxed{A = \frac{4}{3}}$$

Put

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$$7\left(-\frac{1}{2}\right) - 3 = B\left(-\frac{1}{2} + 1\right)$$
$$-\frac{7}{2} - 3 = B\left(\frac{-1+2}{2}\right)$$
$$\Rightarrow \frac{-7-6}{2} = B\left(\frac{1}{2}\right)$$
$$-7 - 6 = B \Rightarrow \boxed{B = -13}$$

So, from eq. (1):

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{\frac{4}{3}}{x-1} + \frac{-13}{2x+1} = \frac{4}{3(x-1)} - \frac{13}{2x+1}$$

Now, from eq. (1):

$$\boxed{\frac{6x^3+5x^2-7}{2x^2-x-1} = 3x+4+\frac{4}{3(x-1)}-\frac{13}{2x+1}}$$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The conditional equation $3x = 8$ is true only if x is equal to: (Mtn 08 G-II)

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) 2

Q.2 Partial fraction of $\frac{x^2}{(x-a)(x-b)}$ will be of the form: (Mtn 10) (DGK 11)

(a) $\frac{A}{x-a}+\frac{B}{x-b}$ (b) $\frac{A}{x-a}-\frac{B}{x-b}$

(c) $1+\frac{A}{x-a}+\frac{B}{x-b}$ (d) $\frac{A}{x-a}+\frac{Bx-C}{x-b}$

Q.3 $\frac{x^3+1}{Q(x)}$ will be proper fraction if degree of Q(x) < (Mtn 11 G-II)

(a) 4 (b) 3

(c) 2 (d) 1

Q.4 Partial fraction of $\frac{1}{(x+1)(x-2)}$ are of the form: (Mtn 12 G-II)

(a) $\frac{A}{x+1}$ (b) $\frac{B}{x-1}$

(c) $\frac{A}{x+1}+\frac{B}{x-2}$ (d) $\frac{A}{x+1}+\frac{Bx+C}{x-2}$

Q.5 Partial fraction of $\frac{1}{x^2-1}$ are the form: (Mtn 12 G-II) (Mtn 15 G-II)(Rwp 13) (Swl 17)(Lhr 17 G-I)

(a) $\frac{Ax+B}{x^2-1}$ (b) $\frac{A}{x-1}+\frac{B}{x+1}$

(c) $\frac{A}{x-1}$ (d) $\frac{B}{x+1}$

Q.6 Types of rational fraction are: (Lhr 15 G-II)(Mtn 18 G-II)(Fbd 19)

(a) 3 (b) 2

(c) 4 (d) 1

Q.7 Partial fraction of $\frac{x^2+1}{(x-1)(x+1)}$ is of the form: (Mtn 15, 17 G-I)(Rwp 17 G-I)(Sgd, Gjw 16)

(a) $1+\frac{A}{x-1}+\frac{B}{x+1}$ (b) $\frac{A}{x-1}+\frac{B}{x+1}$

(c) $1+\frac{Ax+B}{x^2-1}+\frac{Ax+B}{x-1}$ (d) $\frac{Ax+B}{x-1}+\frac{C}{x+1}$

Q.8 The fraction $\frac{2x^2+5}{x-3}$ is: (Rwp 15)

(a) Proper (b) Rational

(c) Polynomial (d) Improper

Q.9 $\frac{x^3+1}{(x-1)(x+2)}$ is: (DGK 13 G-I)

(a) Proper fraction (b) Improper fraction

(c) Identity (d) Mixed fraction

Q.10 $\frac{p(x)}{x^2+1}$ is proper fraction if degree of polynomial p(x) is: (DGK 13 G-II)

(a) 2 (b) 11

(c) Less than 2 (d) Greater than 2

Q.11 Partial fraction of $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$ will be of the form: (Fbd 12)(Bpr 11 G-I)

(a) $\frac{A}{x-2}+\frac{B}{x-4}+\frac{C}{x-6}$

(b) $\frac{A}{(x-2)(x-4)}+\frac{B}{(x-4)(x-4)}$

(c) $1+\frac{A}{x-4}+\frac{B}{x-4}+\frac{C}{x-2}$

(d) $\frac{A}{x-2}+\frac{Bx+C}{(x-4)(x-6)}$

Q.12 The relation of the form $\frac{P(x)}{Q(x)}$, $Q(x) \neq 0$ is called a: (Bpr 12)

(a) Number (b) Integer

(c) Equation (d) Fraction

Q.13 $\frac{x^2+x-1}{q(x)}$ will be improper fraction: (Rwp 11)(Bpr 13)

(a) Degree of $q(x) = 2$ (b) Degree of $q(x) = 3$

(c) Degree of $q(x) = 4$ (d) Degree of $q(x) = 5$

Q.14 The fraction $\frac{3}{x+1}$ is: (Sgd 13 G-I)

(a) Proper fraction (b) Improper fraction

(c) Common fraction (d) Irrational fraction

Q.15 $(a+b)x = ax+bx$ is called: (Sgd 13 G-II)

(a) Identity (b) Equation

(c) Conditional (d) Fraction

Q.16 $\frac{x^2-3}{3x+1}$ **is a:** (Mtn 14 G-I)

(a) Polynomial (b) Equation
(c) Proper fraction (d) Improper fraction

Q.17 The improper fraction can be changed into proper fraction by: (Lhr 15 G-II) (Mtn 14 G-II)

(a) Addition (b) Subtraction
(c) Multiplication (d) Division

Q.18 $\frac{x}{2x+3}$ **is** ———— (D.G.K 14 G-I)

(a) Proper fraction (b) Improper fraction
(c) Identity (d) Mixed fraction

Q.19 The rational fraction $\frac{(x-1)(x-3)}{(x-2)(x-5)}$ **is:** (Swl 14)

(a) Proper fraction (b) Improper fraction
(c) Both (d) None of these

Q.20 The rational fraction $\frac{x^2-1}{x^3-2}$ **is:** (Swl 14)

(a) Proper fraction (b) Improper fraction
(c) Both (d) None of these

Q.21 A relation in which the equality is true for only finite values of unknowns is called an: (Fbd 14)

(a) Identity (b) Equation
(c) Trigonometric equation
(d) Algebraic relation

Q.22 Partial fraction of $\frac{x}{(x-1)(x-2)}$ **will be:** (Rwp 14)(DGK 19)

(a) $\frac{A}{x-1}+\frac{B}{x-2}$ (b) $\frac{1}{x-1}$
(c) $\frac{1}{x+1}$ (d) $1+\frac{A}{x-1}+\frac{B}{x-2}$

Q.23 Partial fraction of $\frac{1}{x(x+1)}$ **are of the form:** (Lhr 14 G-II)

(a) $\frac{1}{x-1}+\frac{1}{x+1}$ (b) $\frac{1}{x-1}-\frac{1}{x+1}$
(c) $\frac{1}{x}+\frac{1}{x+1}$ (d) $\frac{1}{x}-\frac{1}{x+1}$

Q.24 Partial fraction of $\frac{7x+25}{(x+3)(x+4)}$ **is of the form:** (Lhr 14 G-I)

(a) $\frac{A}{x-3}+\frac{B}{x+4}$ (b) $\frac{A}{x+3}+\frac{B}{x+4}$
(c) $\frac{Ax+B}{x+3}+\frac{C}{x+4}$ (d) $\frac{Ax+B}{(x+3)(x+4)}$

Q.25 Partial fractions of $\frac{2}{x^2-1}$ **are:** (Lhr 13 G-I , G-II)

(a) $\frac{1}{x-1}+\frac{1}{x+1}$ (b) $\frac{1}{x-1}-\frac{1}{x+1}$
(c) $\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$ (d) $\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$

Q.26 $\frac{x^3+1}{(x-1)(x+2)}$ **is a** (Gjw 15)

(a) Proper fraction (b) Improper fraction
(c) Relation (d) Equation

Q.27 Partial fraction of $\frac{7x+25}{(x+3)(x+4)}$ **will be:**

(a) $\frac{A}{x+3}-\frac{Bx}{x+4}$ (b) $\frac{A}{x+3}+\frac{Bx}{x+4}$
(c) $\frac{A}{x+3}+\frac{B}{x+4}$ (d) $\frac{Ax}{x+3}+\frac{B}{x+4}$

Q.28 Conditional equation $3x-1=0$, is true only if: (Lhr 12 G-II)

(a) $x=3$ (b) $x=-3$
(c) $x=\frac{1}{3}$ (d) $x=-\frac{1}{3}$

Q.29 A fraction $\frac{P(x)}{Q(x)}$ **in which degree of P(x) = degree of Q(x) is called:** (Gjw 11)

(a) Proper fraction (b) Improper fraction
(c) Common fraction (d) None of these

Q.30 A fraction in which the degree of numerator is greater than the degree of denominator is called: (Gjw 11)

(a) Proper fraction (b) An improper fraction
(c) Equation (d) Algebraic relation

Q.31 The quotient of two polynomials $\frac{P(x)}{Q(x)}$ **where** $Q(x)\neq 0$ **, with no common factors, is called a**

(a) Irrational function (b) Polynomial
(c) Rational function (d) None of these.

Q.32 When a rational function is separated into partial fractions, the result is:

(a) An equation (b) An identity
(c) An inequation (d) None of these.

Q.33 The conditional equation $\frac{1}{x}=3$ **holds if x = ----**

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) None of these.

Q.34 The rational function $\frac{P(x)}{Q(x)}$ **where** $Q(x)\neq 0$ **is proper rational function if**

(a) Degree of P(x) = Degree of Q(x)
(b) Degree of P(x) < Degree of Q(x)
(c) Degree of P(x) > Degree of Q(x)
(d) None of these.

Q.35 The rational function $\frac{2x-1}{x(x-1)(x-3)}$ is

(a) Proper (b) Improper
(c) both (d) None of these

Q.36 $\frac{2x-1}{x(x-1)(x-3)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-3)}$, then C is

(a) $\frac{1}{6}$ (b) $-\frac{1}{2}$
(c) $\frac{5}{6}$ (d) None of these.

Q.37 The rational function $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$ is

(a) Proper (b) Improper
(c) both (d) None of these.

Q.38 If $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$, then A is

(a) $\frac{b}{(b-a)(b-c)}$ (b) $\frac{c}{(c-a)(c-b)}$
(c) $\frac{a}{(a-b)(a-c)}$ (d) None of these.

Q.39 If $\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$, then B is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{1}{3}$ (d) None of these.

Q.40 $\frac{3x^2-4x-5}{(x-2)(x+2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+5}$, then C is

(a) $-\frac{1}{28}$ (b) $\frac{30}{7}$
(c) $-\frac{5}{4}$ (d) None of these.

Q.41 An importnt rational fraction can be reduced to a mixed form by. (Mtn 16 G-II)

(a) Addition (b) Multiplication
(c) Factorization (d) Division

Q.42 $(x-1)^2 = x^2 - 2x + 1$ is called. (Rwp 16)

(a) Equation (b) Conditional
(c) Identity (d) Fraction

Q.43 $\frac{x^2}{1-x^2}$ is. (Fbd Board 2016)

(a) Proper fraction (b) Improper fraction
(c) Equation (d) Decimal

Q.44 The conditional equation $\frac{x-1}{3} = 2$ holds if x is equal to. (Bpr 16)

(a) 8 (b) 7
(c) 6 (d) 5

Q.45 Partial fraction of $\frac{2x}{(x^2-1)(x+2)}$ are of the type. **(A.J.K 17)**

(a) $\frac{Ax+B}{x^2-1}$ (b) $\frac{A}{x-1} + \frac{B}{x+2}$
(c) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$
(d) $\frac{A}{x^2-1} + \frac{B}{x+2}$

Q.46 $\frac{x^2+2x-6}{Q(x)}$ is improper if. **(D.G.K 17 G-I)**

(a) Degree of Q(x) = 2
(b) Degree of Q(x) = 3
(c) Degree of Q(x) = 4
(d) Degree of Q(x) = 5

Q.47 $\frac{x^2+1}{Q(x)}$ will be proper fraction of degree of q(x) equals. **(D.G.K 17 G-II)(Rwp 18)**

(a) 1 (b) 2
(c) 3 (d) 0

Q.48 $(x+3)(x+y) = x^2 + 7x + 12$ is. **(Fbd 17)**

(a) Equation (b) Function
(c) Identity (d) Conditional equation

Q.49 The fraction $\frac{x^2+7x+3}{x+1}$ is. **(Lhr 17 G-II)**

(a) Improper (b) Proper
(c) Equivalent (d) Identity

Q.50 The fraction $\frac{x-3}{x+1}$ is. **(Mtn 17 G-II)**

(a) Improper (b) Proper
(c) Identity (d) Equivalent

Q.51 The conditional equation 5x = 4 is true if x=1. **(Fbd 18)**

(a) 4 (b) 5
(c) $\frac{5}{4}$ (d) $\frac{4}{5}$

Q.52 The fraction $\frac{3x^2+5}{x+1}$ is. **(Lhr 18 G-I)**

(a) Proper fraction (b) Polynomial
(c) Partial fraction (d) Improper fraction

Q.53 The type of rational fraction $\frac{3x^2-1}{x-2}$ is. **(Mtn 18 G-I)**

(a) Proper (b) Improper
(c) Polynomial (d) Identity

Q.54 The given form $(x-4)^2 = x^2 - 8x + 16$ is. **(Sgd 18)**

(a) A transcendental (b) Cubic equation
(c) An identity (d) An equation

Q.55 $(x-1)^2 = x^2 - 2x + 1$ is called: (Gjw 19 G-II)

(a) equation (b) inequality

(c) identity (d) polynomial

Q.56 In $\frac{P(x)}{Q(x)}$, if degree of P(x) ≥ degree of Q(x), then fraction is. (Bpr 19)

(a) Proper (b) Improper

(c) Irrational (d) Identity

Q.57 If $\frac{3}{(x-1)(x+2)} = \frac{1}{x-1} + \frac{A}{x+2}$ then "A" is equal to:

(a) −1 (b) 3

(c) 2 (d) 1

Q.58 If $(2x+1) = A(x+1) + B(x+2)$, then A=_______ (Sgd 19)

(a) 3 (b) 4

(c) 5 (d) 1

Q.59 $\frac{p(x)}{x^2+1}$ will be proper fraction if degree of p (x) is (Swl 19)

(a) 1 (b) 2

(c) 3 (d) 4

EXERCISE 5.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Resolve $\frac{1}{(x-1)^2(x+1)}$ into Partial Fractions.

Ans. $\frac{1}{(x-1)^2(x+1)} = \frac{1}{(x+1)(x-1)^2}$

Let $\frac{1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ (1)

Multiplying both sides by $(x+1)(x-1)^2$, we have

$1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$ (2)

Putting $x = -1$ in eq. (2), we have

$1 = A(-1-1)^2 + B(-1-1)(-1+1) + C(-1+1)$

$\Rightarrow 1 = A(-2)^2 + B(-2)(0) + C(0) \Rightarrow 1 = A(4) + 0 + 0 \Rightarrow 1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$

Putting $x = 1$ in eq. (2) we have

$1 = A(1-1)^2 + B(1-1)(1+1) + C(1+1) \Rightarrow 1 = A(0)^2 + B(0)(2) + C(2)$

$\Rightarrow 1 = 0 + 0 + 2C \Rightarrow 1 = 2C \Rightarrow \boxed{C = \frac{1}{2}}$

Again from eq. (2)

$1 = A(x^2 - 2x + 1) + B(x^2 - 1) + C(x + 1)$

$0 \cdot x^2 + 0 \cdot x + 1 = (A+B)x^2 + (-2A + C)x + (A - B + C)$

Equating the coefficients of x^2, we have

$A + B = 0 \Rightarrow B = -A \Rightarrow \boxed{B = -\frac{1}{4}}$

Thus $A = \frac{1}{4},\ B = -\frac{1}{4},\ C = \frac{1}{2}$

Putting the values of A, B and C in eq. (1), we have

$$\frac{1}{(x+1)(x-1)^2} = \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

Hence required partial fractions are

$$\frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

Q.2 Resolve $\frac{1}{(x+1)^2(x^2-1)}$ into Partial Fractions.

Ans. Here denominator $= (x+1)^2(x^2-1) = (x+1)^2(x+1)(x-1) = (x+1)^3(x-1)$

$\therefore \frac{1}{(x+1)^2(x^2-1)} = \frac{1}{(x+1)^3(x-1)}$

Suppose

$$\frac{1}{(x-1)(x+1)^3} \equiv \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

$\Rightarrow 1 \equiv A(x+1)^3 + B(x+1)^2(x-1) + C(x-1)(x+1) + D(x-1)$ ... (i)

$\Rightarrow 1 \equiv A(x^3 + 3x^2 + 3x + 1) + B(x^3 + x^2 - x - 1) + C(x^2 - 1) + D(x - 1)$

$\Rightarrow 1 \equiv (A+B)x^3 + (3A + B + C)x^2 + (3A - B + D)x + (A - B - C - D)$ ... (ii)

Putting $x - 1 = 0 \Rightarrow x = 1$ in (i), we get $1 = A(2)^3$

$\Rightarrow \boxed{A = \frac{1}{8}}$

Putting $x + 1 = 0 \Rightarrow x = -1$ in (i), we get $1 = D(-1-1)$

$\Rightarrow \boxed{D = -\frac{1}{2}}$

Equating the coefficients of x^3 and x^2 in (ii), we get

$0 = A + B \Rightarrow B = -A \Rightarrow \boxed{B = -\frac{1}{8}}$

and $0 = 3A + B + C \Rightarrow 0 = \frac{3}{8} - \frac{1}{8} + C \Rightarrow \boxed{C = -\frac{1}{4}}$

Hence the partial fractions are

$$\frac{\frac{1}{8}}{x+1} + \frac{-\frac{1}{8}}{x+1} + \frac{-\frac{1}{4}}{(x+1)^2} + \frac{-\frac{1}{2}}{(x+1)^3}$$

or $\frac{1}{8(x+1)} - \frac{1}{8(x+1)} - \frac{1}{4(x+1)^2} - \frac{1}{2(x+1)^2}$

Q.3 Resolve $\dfrac{x^2+x-1}{(x+2)^3}$ into partial fractions.

(Rwp 12 G-I)

Ans. Suppose $\dfrac{x^2+x-1}{(x+2)^3} \equiv \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{(x+2)^3}$

$\Rightarrow \quad x^2+x-1 \equiv A(x+2)^2 + B(x+2) + C \quad \ldots\ldots \text{(i)}$

$\Rightarrow \quad x^2+x-1 = A(x^2+4x+4)+B(x+2)+C \quad \ldots\ldots \text{(ii)}$

Putting $x+2=0$ in (i), we get

$(-2)^2+(-2)-1 = A(0)+B(0)+C \quad \Rightarrow \quad \boxed{C = 1}$

Equating the coefficients of x^2 and x in (ii), we get $\boxed{A = 1}$

and $\quad 1 = 4A + B \Rightarrow 1 = 4 + B \Rightarrow \boxed{B = -3}$

Hence the partial fractions are

$\dfrac{1}{x+2} - \dfrac{3}{(x+2)^2} + \dfrac{1}{(x+2)^3}$.

Q.4 Resolve $\dfrac{9}{(x+2)^2\ (x-1)}$ into Partial Fractions.

(Sgd 13 G-I)

Ans. Let $\dfrac{9}{(x-1)(x+2)^2} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$ (1)

Multiplying both sides of eq. (1) by $(x-1)(x+2)^2$, we have

$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$ (2)

Putting $x = 1$ is eq. (2) we have

$9 = A(1+2)^2 + B(1-1)(1+2) + C(1-1)$

$\Rightarrow \quad 9 = A(3)^2 + B(0)(3) + C(1-1)$

$\Rightarrow \quad 9 = 9A + 0 + 0 \Rightarrow 9 = 9A \Rightarrow \boxed{A = 1}$

Putting $x = -2$ in eq. (2) we have

$9 = A(0)^2 + B(-3)(0) + C(-3) \Rightarrow 9 = 0 + 0 + (-3)(C)$

$\Rightarrow 9 = -3C$

$-3C = 9 \Rightarrow \boxed{C = -3}$

From eq. (2)

$9 = A(x^2+4x+4) + B(x^2+x-2) + C(x-1)$

$9 = (A+B)x^2 + (4A+B+C)x + (4A-2B-C)$

Equating the coefficient of x^2, we have

$A + B = 0 \Rightarrow B = -A \Rightarrow \boxed{B = -1}$

Thus $A = 1$, $B = -1$, and $C = -3$

Putting the values of A, B and C in eq. (1), we have

$\dfrac{9}{(x-1)(x+2)^2} = \dfrac{1}{x-1} - \dfrac{1}{x+2} - \dfrac{3}{(x+2)^2}$

Hence required partial fractions are $\dfrac{1}{x-1} - \dfrac{1}{(x+2)} - \dfrac{3}{x+3}$

LONG QUESTIONS

Q.1 Resolve into partial fractions $\dfrac{9}{(x+2)^2(x-1)}$.

(Lhr 13 G-I)(D.G.K 14 G-I)

Ans. See Short Question 5

Q.2 Resolve $\dfrac{x^2}{(x-2)(x-1)^2}$ into partial fractions.

(Gjw 13 G-I)(Lhr 17 G-II)(Fbd 18)
(Bpr 14, 16)(Rwp 17 G-I)

Ans. Given that: $\dfrac{x^2}{(x-2)(x-1)^2}$

Consider: $\dfrac{x^2}{(x-2)(x-1)^2}$

$= \dfrac{A}{x-2} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2} \quad \ldots\ldots (1)$

Multiplying by $(x-2)(x-1)^2$:

$x^2 = A(x-1)^2 + B(x-1)(x-2) + C(x-2) \quad \ldots\ldots (2)$

$x^2 = A(x^2-2x+1) + B(x^2-2x-x+2) + Cx - 2C$

$x^2 = Ax^2 - 2Ax + A + Bx^2 - 3Bx + 2B + Cx - 2C$

$x^2 = (A+B)x^2 + (-2A-3B+C)x + (A+2B-2C) \ldots (3)$

Put $x - 2 = 0$, $x = 2$: $\quad (2)^2 = A(2-1)^2 \Rightarrow 4 = A(1)^2 \Rightarrow \boxed{A = 4}$

Put $x - 1 = 0$, $x = 1$: $\quad (1)^2 = C(1-2) \Rightarrow 1 = C(-1) = -C$, $\boxed{C = -1}$

Comparing coefficients of x^2, x, x^0 in eq. (3):

$A + B = 1 \Rightarrow 4 + B = 1 \Rightarrow B = 1 - 4$

$\Rightarrow \boxed{B = -3}$

So, from eq. (1):

$$\boxed{\dfrac{x^2}{(x-2)(x-1)^2} = \dfrac{4}{x-2} - \dfrac{3}{x-1} - \dfrac{1}{(x-1)^2}}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The partial fraction of $\dfrac{1}{(x+1)(x^2-1)}$ will be of the form: (Rwp 12, 17 G-II)
(Bpr 14)(Mtn 09)(Lhr 12 G-I)(Sgd 17)

(a) $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-1}$

(b) $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-1}$

(c) $\dfrac{A}{x+1} + \dfrac{B}{x+1} + \dfrac{C}{x-1}$

(d) $\dfrac{A}{x+1} + \dfrac{B}{x+1} + \dfrac{C}{x^2-1}$

Q.2 **The partial fraction of $\dfrac{1}{(x-1)^2(x+1)}$ is of the form:** (Lhr 10)

(a) $\dfrac{A}{x+1}+\dfrac{B}{(x-1)^2}$ (b) $\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$

(c) $\dfrac{A}{x-1}+\dfrac{Bx+C}{(x-1)^2}$ (d) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x-1}+\dfrac{Dx+E}{(x-1)^2}$

Q.3 **If $\dfrac{1}{x(x+1)^3}=\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}+\dfrac{D}{(x+1)^3}$, then D is**

(a) 1 (b) 2

(c) -1 (d) None of these.

Q.4 **If $\dfrac{x-1}{(x-2)(x+1)^3}=\dfrac{A}{x-2}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}+\dfrac{D}{(x+1)^3}$, then D is**

(a) $\dfrac{1}{27}$ (b) $\dfrac{2}{3}$

(c) $-\dfrac{1}{9}$ (d) None of these.

Q.5 $\dfrac{x^2}{(x^2-1)(x+1)}=\dfrac{A}{x-1}+\dfrac{B}{x+1}+$ ______ (Lhr 16 G-I)

(a) $\dfrac{C}{x+1}$ (b) $\dfrac{C}{x-1}$

(c) $\dfrac{C}{(x+1)^2}$ (d) $\dfrac{x+1}{x^2+1}$

Q.6 $\dfrac{2x^2}{(x-3)(x+2)}$ **is a fraction:** (Mtn 19 G-I)

(a) Proper (b) Improper

(c) Identity (d) Irrational

EXERCISE 5.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 **Resolve $\dfrac{3x-11}{(x^2+1)(x+3)}$ into Partial Fractions.** **(Sgd 13 G-I)(Gjw 18)**

Ans. Suppose $\dfrac{3x-11}{(x^2+1)(x+3)}\equiv\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+3}$

$\Rightarrow$ $3x-11=(Ax+B)(x+3)+C(x^2+1)$ …… (i)

$\Rightarrow$ $3x-11=(A+C)x^2+(3A+B)x+(3B+C)$ …… (ii)

Putting $x+3=0 \Rightarrow x=-3$ in (i), we get

$-9-11=C(9+1)\Rightarrow \boxed{C=-2}$

Equating the coefficients of x^2 and x in (ii), we get

$0=A+C\Rightarrow A=-C\Rightarrow \boxed{A=2}$

and $3=3A+B\Rightarrow B=3-3A\Rightarrow B=3-6\Rightarrow \boxed{B=-3}$

Hence the partial fractions are $\dfrac{2x-3}{x^2+1}-\dfrac{2}{x+3}$.

Q.2 **Resolve $\dfrac{9x-7}{(x^2+1)(x+3)}$ into Partial Fractions.** **(Fbd 12)**

Ans. Let $\dfrac{9x-7}{(x+3)(x^2+1)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+1}$ (1)

Multiplying both sides by $(x+3)(x^2+1)$, we have

$9x-7=A(x^2+1)+(Bx+C)(x+3)$ (2)

Putting $x=-3$, we have

$9(-3)-7=A((-3)^2+1)+(B(-3)+C)(-3+3)$

$-27-7=A(9+1)+(-3B+C)(0)$

$-34=A(10)+0\Rightarrow -34=10A\Rightarrow \boxed{A=-\dfrac{17}{5}}$

From eq. (2)

$9x-7=A(x^2+1)+Bx^2+3Bx+Cx+3C$

$9x-7=(A+B)x^2+(3B+C)x+(A+3C)$

Equaitng the coefficients of x^2 and x, we have

For x^2, $A+B=0$ (3)

For x, $3B+C=9$ (4)

From eq. (3) $B=-A=-(-\dfrac{17}{5})\Rightarrow \boxed{B=\dfrac{17}{5}}$

From eq. (4) $C=9-3B=9-3\times\dfrac{17}{5}=\dfrac{45-51}{5}=-\dfrac{6}{5}$

$\Rightarrow \boxed{C=-\dfrac{6}{5}}$

Thus $A=-\dfrac{17}{5}$, $B=\dfrac{17}{5}$, $C=-\dfrac{6}{5}$

Hence $\dfrac{9x-7}{(x+3)(x^2+1)}=\dfrac{-17}{5(x+3)}+\dfrac{17x-6}{5(x^2+1)}$

Hence required partial fractions are $\dfrac{-17}{5(x+3)}+\dfrac{17x-6}{5(x^2+1)}$

Q.3 **Resolve into Partial Fractions: $\dfrac{3x+7}{(x^2+4)(x+3)}$** **(Fbd 12 G-I)**

Ans. Let $\dfrac{3x+7}{(x+3)(x^2+4)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4}$ (1)

Multiplying both sides by $(x+3)(x^2+4)$, we have

$3x+7=A(x^2+4)+(Bx+C)(x+3)$ (2)

Putting $x=-3$ in eq. (2), we have

$-9+7=A[9+4]+[B(-3)+C](0)$

$-2=A[13]+0\Rightarrow -2=13A\Rightarrow \boxed{A=-\dfrac{2}{13}}$

Again from eq. (2)

$$0\cdot x^2 + 3x + 7 = (A+B)x^2 + (3B + C)x + (4A + 3C)$$

Equating the coefficients of x^2, x and constant, we have

For x^2, $\quad A + B = 0 \qquad (3)$

For x, $\quad 3B + C = 3 \qquad (4)$

From eq. (3) $B = -A \Rightarrow \boxed{B = \frac{2}{13}}$

From eq. (4) $C = 3 - 3B = 3 - 3\times\frac{2}{13} \Rightarrow \boxed{C = \frac{33}{13}}$

Thus $A = -\frac{2}{13}, B = \frac{2}{13}, C = \frac{33}{13}$

Putting the values of A, B, C in eq. (1)

Hence $\frac{3x+7}{(x+3)(x^2+4)} = -\frac{2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$

Hence required partial fractions are

$$-\frac{2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$$

LONG QUESTIONS

Q.1 Resolve into partial fraction $\frac{3x+7}{(x^2+4)(x+3)}$.

(Rwp 13 G-I)

Ans. See Short Question 3

Q.2 Resolve into partial fraction $\frac{x^2+1}{x^3+1}$.

(Mtn 11 G-II)(Sgd 16)(Mtn 18 G-II)

Ans. Given that: $\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)}$

Consider: $\frac{x^2+1}{(x+1)(x^2-x+1)}$

$$= \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad \ldots\ldots (1)$$

Multiplying by $(x+1)(x^2-x+1)$:

$$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1) \quad \ldots\ldots (2)$$

$$x^2+1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$$

$$x^2+1 = (A+B)x^2 + (-A+B+C)x + (A+C) \quad \ldots (3)$$

Put $x + 1 = 0$, $x = -1$:

$$(-1)^2 + 1 = A((-1)^2 - (-1) + 1)$$

$$1 + 1 = A(1+1+1) \Rightarrow 2 = 3A$$

$$\Rightarrow \boxed{A = \frac{2}{3}}$$

Comparing coefficients of x^2, x, x^0 in eq. (3):

$$A+B = 1 \Rightarrow \frac{2}{3} + B = 1 \Rightarrow B = 1 - \frac{2}{3}$$

$$= \frac{3-2}{3} \Rightarrow \boxed{B = \frac{1}{3}}$$

$$-A + B + C = 0 \Rightarrow -\frac{2}{3} + \frac{1}{3} + C = 0$$

$$\Rightarrow \frac{-2+1}{3} + C = 0$$

$$-\frac{1}{3} + C = 0 \Rightarrow \boxed{C = \frac{1}{3}}$$

So, from eq. (1):

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{\frac{2}{3}}{x+1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2-x+1}$$

$$\Rightarrow \frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{\frac{x+1}{3}}{x^2-x+1}$$

$$\boxed{\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}}$$

Q.3 Resolve into partial fraction $\frac{1}{(x-1)^2(x^2+2)}$.

(Sgd 17)

Ans. Given that: $\frac{1}{(x-1)^2(x^2+2)}$

Consider: $\frac{1}{(x-1)^2(x^2+2)}$

$$= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2} \quad \ldots\ldots (1)$$

Multiplying by $(x-1)^2(x^2+2)$:

$$1 = A(x-1)(x^2+2) + B(x^2+2) + (Cx+D)(x-1)^2 \ldots (2)$$

$$1 = A(x^3 - x^2 + 2x - 2) + Bx^2 + 2B + (Cx + D)(x^2 - 2x + 1)$$

$$1 = Ax^3 - Ax^2 + 2Ax - 2A + Bx^2 + 2B + Cx^3 - 2Cx^2 + Cx + Dx^2 - 2Dx + D$$

$$1 = (A+C)x^3 + (-A+B-2C+D)x^2 + (2A + C - 2D)x + (-2A + 2B + D) \quad \ldots (3)$$

Put $x - 1 = 0$, $x = 1$:

$$1 = B((1)^2 + 2) = B(1+2)$$

$$= 3B$$

$$\boxed{B = \frac{1}{3}}$$

Comparing coefficients of x^3, x^2, x, x^0 in eq. (3):

$$A + C = 0 \Rightarrow A = -C \quad \ldots\ldots (i)$$

$$-A + B - 2C + D = 0 \Rightarrow -A + B - 2C + D = 0$$

Putting value of eq. (i):

$$-(-C) + \frac{1}{3} - 2C + D = 0 \Rightarrow \frac{1}{3} - C + D = 0$$

$$\Rightarrow D = C - \frac{1}{3} \quad \ldots\ldots (ii)$$

$$2A + C - 2D = 0$$

Putting value of eq. (i) and eq. (ii):

$$2(-C) + C - 2\left(C - \frac{1}{3}\right) \equiv 0 \Rightarrow -2C + C - 2C + \frac{1}{3} \equiv 0$$

$$-3C + \frac{1}{3} = 0 \Rightarrow 3C = \frac{1}{3} \Rightarrow \boxed{C = \frac{1}{9}}$$

Putting value of C in eq. (i) and eq. (ii):

$$\boxed{A = -\frac{1}{9}} \Rightarrow D = \frac{1}{9} - \frac{1}{3} = \frac{1-3}{9}$$

$$\Rightarrow \boxed{D = -\frac{2}{9}}$$

So, from eq. (1):

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{\frac{-1}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{\frac{1}{9}x - \frac{2}{9}}{x^2+2}$$

$$= -\frac{1}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{\frac{x-2}{9}}{x^2+2}$$

$$\boxed{\frac{1}{(x-1)^2(x^2+2)} = -\frac{1}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{x-2}{9(x^2+2)}}$$

Q.3 Resolve into partial fractions $\frac{x^2-2x+3}{x^4+x^2+1}$.

(Sgd 13 G-II)(A.J.K 17)

Ans. Given that: $\frac{x^2-2x+3}{x^4+x^2+1} = \frac{x^2-2x+3}{x^4+2x^2+1-x^2}$

$$= \frac{x^2-2x+3}{(x^2+1)^2 - x^2}$$

$$= \frac{x^2-2x+3}{(x^2+1-x)(x^2+1+x)}$$

Consider: $\frac{x^2-2x+3}{(x^2+1-x)(x^2+1+x)}$

$$= \frac{Ax+B}{x^2+1-x} + \frac{Cx+D}{x^2+1+x} \quad \ldots\ldots (1)$$

Multiplying by $(x^2+1-x)(x^2+1+x)$:

$$x^2 - 2x + 3 = (Ax+B)(x^2+1+x) + (Cx+D)(x^2+1-x) \quad \ldots\ldots (2)$$

$$x^2 - 2x + 3 = Ax^3 + Ax + Ax^2 + Bx^2 + B + Bx + Cx^3 + Cx - Cx^2 + Dx^2 + D - Dx$$

$$x^2 - 2x + 3 = (A+C)x^3 + (A+B-C+D)x^2 + (A+B+C-D)x + (B+D) \quad \ldots\ldots (3)$$

Comparing coefficients of x^3, x^2, x, x^0 in eq. (3):

$$A + C = 0 \Rightarrow A = -C \quad \ldots\ldots (i)$$

$$A + B - C + D = 1$$

Putting value of eq. (i):

$$-C + B - C + D = 1 \Rightarrow B - 2C + D = 1$$

$$B \equiv 1 + 2C - D \quad \ldots\ldots (ii)$$

$$A + B + C - D = -2$$

Putting value of eq. (i) and eq. (ii):

$$-C+1+2C-D+C-D = -2 \Rightarrow 1+2C-2D = -2$$

$$2D = 1+2C+2 = 3+2C \Rightarrow D = \frac{3+2C}{2} \quad \ldots\ldots (iii)$$

$$B + D = 3$$

Putting value of eq. (ii):

$$1 + 2C - D + D = 3 \Rightarrow 1 + 2C = 3$$

$$\Rightarrow 2C = 3 - 1 = 2 \Rightarrow \boxed{C = 1}$$

Putting value of C in eq. (i) and eq. (iii):

$$\boxed{A = -1} \Rightarrow D = \frac{3+2(1)}{2} = \frac{3+2}{2}$$

$$\Rightarrow \boxed{D = \frac{5}{2}}$$

Putting values of C and D in eq. (ii):

$$B = 1 + 2(1) - \frac{5}{2} = 1 + 2 - \frac{5}{2} = \frac{2+4-5}{2}$$

$$\Rightarrow \boxed{B = \frac{1}{2}}$$

So, from eq. (1):

$$\frac{x^2-2x+3}{(x^2+1-x)(x^2+1+x)} = \frac{-x+\frac{1}{2}}{x^2+1-x} + \frac{x+\frac{5}{2}}{x^2+1+x}$$

$$\frac{x^2-2x+3}{x^4+x^2+1} = \frac{\frac{-2x+1}{2}}{x^2+1-x} + \frac{\frac{2x+5}{2}}{x^2+1+x}$$

$$\boxed{\frac{x^2-2x+3}{x^4+x^2+1} = \frac{-2x+1}{2(x^2+1-x)} + \frac{2x+5}{2(x^2+1+x)}}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Partial fraction $\frac{1}{x^3-1}$ are of the form:

(Mtn 08)(Lhr 19 G-II)

(a) $\frac{A}{x-1} + \frac{B}{x^2-x+1}$ (b) $\frac{A}{x-1} + \frac{Bx+C}{x^2-x+1}$

(c) $\frac{A}{x-1} - \frac{Bx+C}{x^2+x+1}$ (d) $\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$

Q.2 Partial fraction of $\frac{1}{x^3+1}$ will be of the form:

(Mtn 13 G-II) (D.G.K 12, 15)

(a) $\frac{A}{x-1} + \frac{B}{x^2-x+1}$ (b) $\frac{A}{x+1} - \frac{B}{x^2-x+1}$

(c) $\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ (d) $\frac{Ax+B}{x^2+1} + \frac{B}{x^2-x+1}$

Q.3 $\dfrac{x^4}{1-x^4}$ **is:** (DGK 14) (Mtn 13 G-I) (Fbd 11)

(a) Equation (b) Identity
(c) Proper fraction (d) Improper fraction

Q.4 Partial fraction of $\dfrac{3x-11}{(x+3)(x^2+1)}$ **are of the form:** (Gjw 14)

(a) $\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+1}$ (b) $\dfrac{A}{x+3}+\dfrac{B}{x^2+1}$
(c) $\dfrac{Ax+B}{x+3}+\dfrac{C}{x^2+1}$ (d) $\dfrac{A}{x+3}+\dfrac{B+C}{x^2+1}$

Q.5 $\dfrac{1}{(x^2+1)(x+1)}=\dfrac{A}{x+1}+\ldots\ldots\ldots$ (Gjw 13)

(a) $\dfrac{B}{x^2+1}$ (b) $\dfrac{Bx}{x^2+1}$
(c) $\dfrac{Bx+C}{x^2+1}$ (d) $\dfrac{B+C}{x^2+1}$

Q.6 The partial fraction of $\dfrac{x+5}{(x-1)(x^2+1)}$ **are of the form:** (DGK 15 G-II)(Fbd 19 G-II)

(a) $\dfrac{Ax}{x-1}+\dfrac{B}{x^2+1}$ (b) $\dfrac{A}{x-1}+\dfrac{B}{x^2+1}$
(c) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$ (d) None

Q.7 The rational function $\dfrac{x^3+1}{x^2+1}$ **is**

(a) Proper (b) Improper
(c) both (d) None of these.

Q.8 If $\dfrac{9x-7}{(x^2+1)(x+3)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+1}$**, then B is**

(a) $\dfrac{17}{5}$ (b) $-\dfrac{6}{5}$
(c) $-\dfrac{17}{5}$ (d) None of these.

Q.9 If $\dfrac{x^2+15}{(x^2+2x+5)(x-1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+2x+5}$, **then B is**

(a) -5 (b) -1
(c) 2 (d) None of these.

Q.10 The partial fraction of $\dfrac{9x-7}{(x^2+1)(x-3)}$ **are of the form:** (Lhr 11 G-I)

(a) $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+3}$ (b) $\dfrac{A}{x^2+1}+\dfrac{B}{x+3}$
(c) $\dfrac{A}{x^2+1}$ (d) $\dfrac{B}{x+3}$

Q.11 If $\dfrac{3x-11}{(x^2+1)(x+3)}=\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+3}$ **, then C is**

(a) 2 (b) -2
(c) 3 (d) None of these

Q.12 Partial fraction of $\dfrac{x+4}{(x-1)(x+2)}$ **will be.** (Bpr 18)

(a) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+2}$ (b) $\dfrac{A}{x-1}+\dfrac{B}{x^2+2}$
(c) $\dfrac{Ax}{x-1}+\dfrac{Bx+C}{x^2+2}$ (d) $\dfrac{A}{x-1}+\dfrac{Bx}{x^2+2}$

Q.13 The fraction $\dfrac{x+1}{x^2+2}$ **is.** (Gjw 18)

(a) Proper fraction (b) Improper fraction
(c) Identity (d) Mixed

Q.14 Partial fraction of $\dfrac{1}{(x^2+1)(x-1)}$ **are of the form.** (Lhr 18 G-II)

(a) $\dfrac{A}{x^2+1}+\dfrac{B}{x-1}$ (b) $\dfrac{A}{x+1}+\dfrac{B}{x^2+1}+\dfrac{c}{x-1}$
(c) $\dfrac{A}{x^2+1}+\dfrac{Bx+C}{x-1}$ (d) $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-1}$

Q.15 The partial fractions of $\dfrac{x+5}{(x+1)(x^2+1)}$ **will be of the form:** (Fbd 19 G-II)

(a) $\dfrac{A}{x+1}+\dfrac{B}{x^2+1}$ (b) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}$
(c) $\dfrac{Ax+B}{x+1}+\dfrac{C}{x^2+1}$ (d) $\dfrac{A}{x+1}+\dfrac{Bx}{x^2+1}$

Q.16 Partial fractions of $\dfrac{1}{x^3-1}$ **will be of the form:** (Lhr 19 G-II)

(a) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+x+1}$ (b) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+x+1}$
(c) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2-x+1}$ (d) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2-x+1}$

Chapter 6

SEQUENCE AND SERIES

EXERCISE 6.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define sequence. **(Gjw 19 G-I)**

Ans. A function whose domain is set of natural numbers is called sequence.

Q.2 Write the 1st three terms of the sequence of $a_n = \dfrac{n}{2n+1}$ **(Mtn 16 G-I)(Gjw 19 G-I)**

Ans. $a_n = \dfrac{n}{2n+1}$

Put $n = 1, 2, 3$

$$a_1 = \frac{1}{2(1)+1} = \frac{1}{3}$$

$$a_2 = \frac{2}{2(2)+1} = \frac{2}{5}$$

$$a_3 = \frac{3}{2(3)+1} = \frac{3}{7}$$

$$a_4 = \frac{4}{2(4)+1} = \frac{4}{9}$$

Q.3 Write the first four terms of $a_n = \dfrac{1}{2^n}$. **(A.J.K 17)**

Ans. $a_n = \dfrac{1}{2^n}$

Putting $n = 1, 2, 3, 4, \ldots\ldots\ldots$

$$a_1 = \frac{1}{2^1} = \frac{1}{2}\ ,\quad a_2 = \frac{1}{2^2} = \frac{1}{4}$$

$$a_3 = \frac{1}{2^3} = \frac{1}{8}\ ,\quad a_4 = \frac{1}{2^4} = \frac{1}{16}$$

Q.4 Write the first four terms of the sequence of $a_n = (-1)^n n^2$ **(Mtn 18 G-I)**

Ans. Putting, $n = 1, 2, 3, 4$

$a_1 = (-1)^1 1^2 = (-1)(1) = -1$

$a_2 = (-1)^2 2^2 = (+1)(4) = 4$

$a_3 = (-1)^3 3^2 = (-1)(9) = -9$

$a_4 = (-1)^4 4^2 = (+1)(16) = +16$

Q.5 Write the next two terms of the sequence.
$1, -3, 5, -7, 9, -11$ ——— **(Rwp 16)(DGK-19)**

Ans. $a_1 = 1$

$a_2 = -3$

$a_3 = 5$

$a_4 = -7$

$a_5 = 9$

$a_6 = -11$

$a_7 = 13$

$a_8 = -15$

Q.6 Find the next two terms of 1, 3, 7, 15, 31, ... **(Fbd 18)(Mtn 19 G-I)**

Ans. $a = 1$

$a_2 = 3 = 2(1) + 1 = 2a_1 + 1$

$a_3 = 7 = 2(3) + 1 = 2a_2 + 1$

$a_4 = 15 = 2(7) + 1 = 2a_3 + 1$

$a_5 = 31 = 2(15) + 1 = 2a_4 + 1$

$a_6 = 2a_5 + 1 = 2(31) + 1 = 63$

$a_7 = 2a_6 + 1 = 2(63) + 1 = 127$

Q.7 Find a_7 of 2, 6, 11, 17, ... **(Swl 17)(Lhr 18 G-I)(Bpr 19)**

Ans. $a_1 = 2$

$a_2 = 6 = 2 + 4 = a_1 + 4$

$a_3 = 11 = 6 + 5 = a_2 + 5$

$a_4 = 17 = 11 + 6 = a_3 + 6$

$a_5 = a_4 + 7 = 17 + 7 = 24$

$a_6 = a_5 + 8 = 24 + 8 = 32$

$a_7 = a_6 + 9 = 32 + 9 = 41$

Q.8 Write the first three terms of sequence if $a_n - a_{n-1} = n + 2$, $a_1 = 2$. **(Gjw 17)(Sgd 18)**

Ans. $a_n = a_{n-1} + n + 2$

Putting $n = 2, 3, 4$

$a_2 = a_1 + 2 + 2 = 2 + 4 = 6$

$a_3 = a_2 + 3 + 2 = 6 + 5 = 11$

$a_4 = a_3 + 4 + 2 = 11 + 6 = 17$

Q.9 Find the sequence if $a_n - a_{n-1} = n + 1$ and $a_4 = 14$. **(Gjw 10)**

Ans. Putting $n = 2, 3, 4$ in $a_n - a_{n-1} = n + 1$, we have

$a_2 - a_1 = 3$ ...... (i)

$a_3 - a_2 = 4$ ...... (ii)

$a_4 - a_3 = 5$ ...... (iii)

From (iii), $a_3 = a_4 - 5 = 14 - 5 = 9$ ($\because$ $a_4 = 14$)

From (ii), $a_2 = a_3 - 4 = 9 - 4 = 5$ ($\because$ $a_3 = 9$)

And from (i), $a_1 = a_2 - 3 = 5 - 3 = 2$

Thus the sequence is 2, 5, 9, 14, 20, ...

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If the nth term of an A.P is $\frac{1}{2}(3-n)$, the first three terms are: (Mtn 13 G-II) (D.G.K 15 G-I)(Lhr 17 G-II)

(a) 3, 2, 1 (b) 1, 2, 3
(c) 1, 2, 1 (d) $1, \frac{1}{2}, 0$

Q.2 The nth term of sequence $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$, ——— is: (D.G.K 13)

(a) $\frac{n}{2n-1}$ (b) $\frac{n}{2n+1}$
(c) $\frac{n}{3n-1}$ (d) $\frac{n}{3n+1}$

Q.3 If $a_n - a_{n-1} = n + 1$ and $a_4 = 14$, then a_5 is equal to: (Lhr 15 G-I)

(a) 9 (b) 14
(c) 20 (d) 5

Q.4 Next term of the sequence −1, 2, 12, 40, ……… is: (Rwp 11)

(a) 52 (b) 64
(c) 80 (d) 112

Q.5 If $a_{n-3} = 2n - 5$, then the nth term is: (Lhr 11 G-II)(Rwp 12)

(a) $2n+1$ (b) $2n-1$
(c) $3n+1$ (d) $3n-1$

Q.6 If $a_n = (-1)^{n+1}$, then 26th term of the sequence is: (Lhr 14 G-II)(Mtn 15 G-II) (Rwp 13)

(a) 1 (b) −1
(c) 26 (d) 2

Q.7 The next term in 7, 9, 12, ……… is: (Fbd 14)(Mtn 14 G-II)

(a) 16 (b) 14
(c) 15 (d) 18

Q.8 The fifth term of $a_n = 2n + 3$ is: (Mtn 14 G-II)

(a) −13 (b) −7
(c) 7 (d) 13

Q.9 The 4th term of sequence $a_n = \frac{n}{2n+1}$ is: (D.G.K 14)

(a) 4 (b) 3
(c) $\frac{4}{3}$ (d) $\frac{4}{9}$

Q.10 If $a_{n-2} = 3n - 11$, then 5th term is: (Lhr 10 G-I)

(a) 4 (b) 7
(c) 10 (d) 13

Q.11 Next term of the sequence 1, 3, 7, 15, 31 …… (Lhr 13)

(a) 63 (b) 62
(c) 80 (d) 81

Q.12 a_7 of the sequence 2, 6, 11, 17, ……… (Lhr 13 G-I, G-II)

(a) 41 (b) 51
(c) 32 (d) 40

Q.13 The 3rd term of sequence $a_n = (-1)^n (n-7)$ is: **(Lhr 13 G-I)**

(a) 8 (b) 4
(c) −4 (d) −8

Q.14 If $a_n = 3n + 1$, then the series is

(a) 4, 3, 2, 1, 0 (b) 4, 5, 6, 7
(c) 4, 7, 10, 13 (d) 4, 8, 9, 13

Q.15 The 3rd term of sequence $a_n = (-1)^n (n-7)$ is: **(Lhr 13 G-I)**

(a) 8 (b) 4
(c) −4 (d) −8

Q.16 Next term of the sequence 1, 3, 7, 15 …… is: **(Gjw 11)**

(a) 23 (b) 27
(c) 31 (d) 33

Q.17 If $a_n = (n+1)\, a_{n-1}$, $a_1 = 1$, third term of the sequence is

(a) 3 (b) 60
(c) 12 (d) None of these.

Q.18 What is the next term in the sequence 8, 3, − 2, ……..?

(a) − 7 (b) − 9
(c) − 1 (d) None of these.

Q.19 nth term of the sequence 1, 4, 9, 16, 25, ……. is

(a) $n + 3$ (b) n
(c) n^2 (d) $n^2 + 1$

Q.20 The next term of the sequence 1,3,6,10, —— is.

(a) 13 (b) 15
(c) 17 (d) 19

Q.21 The third term of sequence $a_n = (-1)^n (n-7)$ is. (Sgd 18)

(a) 8 (b) 4
(c) −8 (d) −4

Q.22 If in an Sequence, $a_n = \frac{(-1)^{n-1}n}{2n+1}$, then a_4 equals: (Fbd 19 G-I)

(a) 4 (b) 3
(c) $\frac{4}{3}$ (d) $\frac{-4}{9}$

Q.25 If $a_{n-2} = 3n - 11$, then 6th term is: (Fbd 19 G-II)

(a) 13 (b) 7
(c) 15 (d) 11

Q.26 The 8th term of sequence 1, −3, 5, −7 is:

(a) 15 (b) − 15
(c) 14 (d) −14

Q.27 If $a_n = \frac{(-1)^{n+1}}{2^n}$, then $a_5 =$: (Bpr 19)

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$
(c) $\frac{1}{32}$ (d) $\frac{1}{64}$

Q.26 If $a_n = (-1)^{n+1}$, then $a_{26} =$: (Mtn 19 G-I)

(a) 1 (b) −1
(c) i (d) $-i$

Q.27 If in A.P; $a_{n-3} = 2n - 5$, then a_n will be equal to: (Rwp 19)

(a) 2 n + 1 (b) 2 n − 1
(c) n + 1 (d) n − 1

EXERCISE 6.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define arithmetic sequence.

Ans. A sequence $\{a_n\}$ is an arithmetic sequence or progression (A.P.) if $a_n - a_{n-1} = d$ for all $n \in N$, $n > 1$, and d is called common difference and we can get general term $= a_n = a + \overline{n-1}\ d$.

Q.2 Find the 13th term of the sequence x, 1, 2 – x, ... (Lhr 10 G-II)

Ans. The given sequence is x, 1, 2 – x, ...

Here $a = x$

Common difference $= d = 1 - x$

Now $a_{13} = a + 12d$
$= x + 12(1 - x)$
$= x + 12 - 12x$
$= 12 - 11x$

Q.3 Find the nth term of the sequence $\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots$ (Lhr 08 G-I)

Ans. Given sequence is $\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots$ is not an A.P. but 4, 7, 10,... is an A.P. with a = 4 , d = 3

∴ $a_n = a + (n-1)d = 4 + (n-1)(3) = 4 + 3n - 3 = 3n + 1$

Thus nth term of the A.P. is $\left(\frac{3n+1}{3}\right)^2$

Q.4 Which term of the A.P. 5, 2, − 1, ... is − 85? (Gjw 12)(Lhr 13 G-I, 17 G-II)(Rwp 16)(Swl 19)

Ans. Given A.P. is 5, 2, − 1, ..., − 85

Here a = 5, Common difference = d = 2 − 5 = −3

$a_n = -85$ and n = ?

Now $a_n = a + (n-1)d$

$\Rightarrow -85 = 5 + (n-1)(-3)$

$\Rightarrow -85 = 5 - 3n + 3$

$\Rightarrow -85 = 8 - 3n$

$\Rightarrow 3n = 8 + 85$

$\Rightarrow 3n = 93$

$\Rightarrow n = 31$

Thus $a_{31} = -85$

Q.5 Which term of the A.P. −2, 4, 10 , ... is 148? (Lhr 12 G-I)(Gjw 16, 19 G-I)

Ans. Given A.P. − 2, 4, 10, ... , 148

Here a = −2

d = 4 − (−2) = 4 + 2 = 6

$a_n = 148$ and n = ?

Now $a_n = a + (n-1)d$

$\Rightarrow 148 = -2 + (n-1)(6)$

$\Rightarrow 148 = -2 + 6n - 6$

$\Rightarrow 148 = 6n - 8$

$\Rightarrow 6n = 148 + 8$

$\Rightarrow 6n = 156$

$\Rightarrow n = \frac{156}{6} \Rightarrow n = 26$

Thus $a_{26} = 148$

Q.6 If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P. , Show that $b = \frac{2ac}{a+c}$. (Gjw 13, 18)(Mtn 16, 19 G-II)(Fbd, Lhr 19 G-II)

Ans. Given that

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Therefore

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{2}{b} = \frac{c+a}{ac}$

$\Rightarrow \frac{1}{b} = \frac{a+c}{2ac}$

$\Rightarrow b = \frac{2ac}{a+c}$

Q.7 Find the number of terms in the A.P if $a_1 = 3$, $d = 7$ and $a_n = 59$. **(Fbd 19)**

Ans. $a_1 = 3$, $d = 7$, $a_n = 59$ using

$an = a + \overline{n-1}\, d$

$59 = 3 + (n-1)7 = 3 + 7n - 7 = 7n - 4$

$4 + 59 = 7$

$63 = 7n$

$n = \frac{63}{7} = 9$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 21st term of the sequence 2, 4, 6, ……… (Mtn 11 G-I , 13 G-I)

(a) 40 (b) 42
(c) 44 (d) 46

Q.2 If $a_1 = 5$ and other consecutive terms are 23, 26, 29, then 3rd term is: (D.G.K 11)

(a) 8 (b) 11
(c) 14 (d) 17

Q.3 2, 4, 6, 8, 10, 12, ………… (D.G.K 13 G-I)

(a) G.P (b) A.P
(c) Geometric series (d) None

Q.4 The nth term of an A.P is $\frac{1}{2}(3n)$, the first three terms are: (Bpr 2011 G-I)

(a) 3, 2, 1 (b) 1, 2, 3
(c) $\frac{3}{2}, 3, \frac{9}{2}$ (d) $\frac{3}{2}, 6, \frac{9}{2}$

Q.5 (n + 1)th term of an A.P is: (Rwp 15)(Sgd 13 G-II)

(a) $a_1 + \overline{n-1}\, d$ (b) $a_1 - (n-1)d$
(c) $a_1 + nd$ (d) $a_1 - nd$

Q.6 Common difference of arithmetic sequence 3, 5, 7, ……… is (Gjw 15)

(a) $\frac{-2}{15}$ (b) $\frac{2}{15}$
(c) 2 (d) $\frac{1}{2}$

Q.7 Common difference of A.P 17, 13, 9 , ……… (Rwp 14)

(a) 4 (b) –4
(c) 30 (d) 15

Q.8 If a, A, b are in A.P, the 2A = (Lhr 11 G-II)(Lhr 12 G-I)

(a) $\frac{a+b}{2}$ (b) $a + b$
(c) $a - b$ (d) $\frac{a-b}{2}$

Q.9 What is the next term in the sequence 10, 7, 4, 1 ……..?

(a) 2 (b) – 2
(c) – 3 (d) None of these.

Q.10 What is the common difference of the sequence 11, 5, – 1 , ……?

(a) 6 (b) – 6
(c) 1/6 (d) None of these.

Q.11 The n th term of a arithmetic sequence (A.P) with usual notation is given by **(Mtn Board 2017 G-II)**

(a) $a + (n-1)d$ (b) $a \times (n+1)d$
(c) $a + (n+1)d$ (d) $a - (n+1)d$

Q.12 What is the general term of the sequence given below ?
2, 4, 6, 8, ……….

(a) 2n (b) n + 1
(c) $2n^2$ (d) None of these.

Q.13 If $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P., then

(a) $b = \frac{a+c}{2ac}$ (b) $b = \frac{2ac}{a+c}$
(c) $b = \frac{2ac}{a-c}$ (d) None of these.

Q.14 23rd term of the A.P. 7, 5, 3, 1 …… is:

(a) 57 (b) 47
(c) – 37 (d) None of these.

Q.15 If (k + 1), 3k and (4k + 2) be any three consecutive terms of an A.P., then the value of k is:

(a) 3 (b) 0
(c) 1 (d) None of these.

Q.16 The fifth term of the sequence $a_n = 2n + 1$.

(a) 7 (b) 9
(c) 11 (d) 13

EXERCISE 6.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define arithmetic mean.

Ans. The terms in the middle of first and last term of an A.P. is called arithmetic mean (A.M.).

Le A be A.M between a and b then a, A, b are in A.P.

$\therefore$ $A - a = b - a$

$2A = a + b$

$A = \frac{a+b}{2}$

Q.2 Find A.M between x – 3 and x + 5. **(Rwp 17 G-I)**

Ans. $a = x - 3$, $b = x + 5$

$A = \frac{a+b}{2} = \frac{x-3+x+5}{2} = \frac{2x+2}{2} = \frac{x(x+1)}{2} = x+1$

Q.3 Find A.M between $3\sqrt{5}$ and $5\sqrt{5}$.
(Mtn 16 G-I)(A.J.K 17)(Mtn 18 G-II)

Ans. $A = \frac{a+b}{2}$

$= \frac{3\sqrt{5}+5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$

Q.4 Find A.M between $1 - x + x^2$ and $1 + x + x^2$. (Lhr 16 G-I) (Mtn 17 G-I,II)

Ans. Here $a = 1 - x + x^2$, $b = 1 + x + x^2$

$A = \frac{a+b}{2} = \frac{1 - x + x^2 + 1 + x + x^2}{2}$

$= \frac{2+2x^2}{2} = \frac{2(1+x^2)^2}{2}$

$A = 1 + x^2$

Q.5 Find two A.Ms between 5 and 8. (Swl 17)

Ans. Let A_1 and A_2 be two A.Ms between 5 and 8, then

$5, A_1, A_2, 8$

$a_n = a + n - 1d$

$8 = 5 + (4-1)d$

$8 = 5 + 3d$

$3 = 3d \Rightarrow \boxed{d = 1}$

$A_1 = a + d = 5 + 1 = 6$

$A_2 = A_1 + d = 6 + 1 = 7$

Q.6 Find three A.M's between 3 and 11.
(Mtn 08 G-I)(sgd 16)

Ans. Let A_1, A_2, A_3 be three A.M's between 3 and 11. Then

$3, A_1, A_2, A_3, 11$ are in A.P.

Here $a = 3, n = 5, a_5 = 11, d = ?$

Using the formula

$a_n = a + (n-1)d$ for $n = 5$, we have

$a_5 = a + (5-1)d$

$\Rightarrow 11 = 3 + 4d$

$\Rightarrow 4d = 11 - 3$

$\Rightarrow 4d = 8$

$\Rightarrow d = 2$

Hence $A_1 = a + d = 3 + 2 = 5$

$A_2 = A_1 + d = 5 + 2 = 7$

$A_3 = A_2 + d = 7 + 2 = 9$

Thus three A.Ms between 3 and 11 are 5,7,9

Q.7 Find two A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.
(D.G. K 12, 17 G-I)(Lhr 13 G-I)

Ans. Let A_1, A_2 be two A.Ms between $\sqrt{2}$ and $3\sqrt{2}$, then

$\sqrt{2}, A_1, A_2, 3\sqrt{2}$ are in A.P.

Here $a_1 = \sqrt{2}$, $a_4 = 3\sqrt{2}$

Using $a_n = a_1 + (n-1)d$, we get

$a_4 = a_1 + (4-1)d$

or $3\sqrt{2} = \sqrt{2} + 3d$

$\Rightarrow 3\sqrt{2} - \sqrt{2} = 3d$

$\Rightarrow d = \frac{2\sqrt{2}}{3}$

Now $A_1 = a_1 + d = \sqrt{2} + \frac{2\sqrt{2}}{3} = \frac{3\sqrt{2} + 2\sqrt{2}}{3} = \frac{5\sqrt{2}}{3}$

$A_2 = A_1 + d = \frac{5\sqrt{2}}{3} + \frac{2\sqrt{2}}{3} = \frac{5\sqrt{2} + 2\sqrt{2}}{3} = \frac{7\sqrt{2}}{3}$

Therefore, $\frac{5\sqrt{2}}{3}, \frac{7\sqrt{2}}{3}$ are two A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.

Q.8 If 5, 8 are two A.Ms between a and b, find a and b. (Sgd 17)(Rwp 18, 16)(Lhr 11, 19 G-II)
(Fbd 19)(Rwp 19)

Ans. a, 5, 8, b are in A.P.

$d = 8 - 5 = 3$

$5 - a = 3 \Rightarrow a = 5 - 3 = 2$

$b - 8 = 3 \Rightarrow b = 11$

LONG QUESTIONS

Q.1 Find n so that $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be the A.M between a and b. (Mtn 13 G-II, 19 G-I)
(D.G.K 13, 15 G-I)(Bpr 18)(Mtn 18 G-II)(Srd 18)

Ans. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is A.M. a and b. Then

$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$

$\Rightarrow 2[a^n + b^n] = (a+b)[a^{n-1} + b^{n-1}]$

$2a^n + 2b^n = a^n + a \cdot b^{n-1} + b \cdot a^{n-1} + b^n$

$2a^n - a^n - b \cdot a^{n-1} = a \cdot b^{n-1} + b^n - 2b^n$

$a^n - b \cdot a^{n-1} = a \cdot b^{n-1} - b^n \Rightarrow a \cdot a^{n-1} - b \cdot a^{n-1}$

$= a \cdot b^{n-1} - b \cdot b^{n-1}$

$[a-b]a^{n-1} = (a-b)b^{n-1} \Rightarrow a^{n-1} = b^{n-1}$

This is possible only if: $n - 1 = 0$

$\Rightarrow \boxed{n = 1}$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Arithmetic mean between 2 and 6 is:
(Mtn 08 G-I)

(a) 2 (b) 3

(c) 4 (d) $\sqrt{12}$

Q.2 Arithmetic mean between $\sqrt{2}$ and $3\sqrt{2}$ is: (Mtn 08)(Sgd 16)(Bpr 14)

(a) $2\sqrt{2}$ (b) $\sqrt{2}$
(c) 6 (d) 3

Q.3 A.M between two numbers $\frac{1}{a}$ and $\frac{1}{b}$ is: (Bpr 15)(Mtn 10 G-II)(Sgd, Rwp 17) (Lhr 17 G-I)(Gjw 18)

(a) $\frac{a+b}{ab}$ (b) $\frac{a+b}{2ab}$
(c) $\frac{2ab}{a+b}$ (d) $\frac{a-b}{ab}$

Q.4 A.M between two members x – 1 and x + 1 is: (Mtn 11 G-II)(Fbd 16)

(a) 0 (b) 2
(c) 2x (d) x

Q.5 A.M between a and c is: (Mtn 12 G-II)

(a) $\frac{a-c}{2}$ (b) $\sqrt{ac}$
(c) $\frac{a+c}{d}$ (d) $\frac{a+c}{2}$

Q.6 The A.M between $2+\sqrt{2}$ and $2-\sqrt{2}$ is: (Mtn 13, 17 G-II)(Fbd 19)

(a) 2 (b) $\sqrt{2}$
(c) 0 (d) 4

Q.7 A.M between two numbers –2 and –6 is: (Rwp 11)

(a) 4 (b) –4
(c) $\pm\sqrt{12}$ (d) –3

Q.8 The A.M between two numbers a and b is: (Bpr 14) (Fbd 11)

(a) $\frac{a+b}{2ab}$ (b) $\frac{2ab}{a+b}$
(c) $\frac{a+b}{2}$ (d) $\frac{2}{a+b}$

Q.9 If a, A, b are in A.P, then A = (Sgd 13)

(a) $\frac{a-b}{2}$ (b) $\frac{a+b}{2}$
(c) $\sqrt{ab}$ (d) $\frac{2ab}{a+b}$

Q.10 If a, A, b are in A.P, then 2A = ____________ (Fbd 15)

(a) $\frac{a+b}{2}$ (b) a + b
(c) 2 (a + b) (d) a – b

Q.11 The A.M between x – 3 and x + 5 is: (Lhr 13 G-I, G-II)

(a) x + 1 (b) x – 1
(c) x – 3 (d) x + 5

Q.12 If 5 is A.M between –5 and b then b is equal to: (Gjw 12)

(a) 0 (b) 15
(c) –15 (d) 10

Q.13 20th term of 1 + 3 + 5 + ... is: (Lhr 12 G-II)

(a) 38 (b) 39
(c) 40 (d) 41

Q.14 A.M between $3\sqrt{5}$ and $5\sqrt{5}$ is: (Rwp 16)

(a) $4\sqrt{5}$ (b) $5\sqrt{5}$
(c) 10 (d) $2\sqrt{5}$

Q.15 The arithmetic mean between $\frac{1}{2}$ and $\frac{1}{4}$ is (Gjw 19 G-II)

(a) $\frac{3}{8}$ (b) $\frac{3}{4}$
(c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

EXERCISE 6.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define arithmetic series.

Ans. Sum of n terms of an A.P is called arithmetic series (A.S).

Let $S_n = a + (a + d) + (a + 2d) + ... (a_n - d) + a_n$

Also $S_n = a_n + (a_n - d) + ... + (a + d) + a$

Adding above equations

$2S_n = (a + a_n) + (a + a_n) + ... + (a + a_n) + (a + a_n)$

$2S_n = n(a + a_n)$

$$S_n = \frac{n}{2}[a + a_n] = \frac{n}{2}[a + a + \overline{n-1}\,d]$$

$$S_n = \frac{n}{2}[2a + \overline{n-1}\,d]$$

Q.2 Sum the series: $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + ... + a_{13}$. (Mtn 08 G-II)

Ans. $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \;... + a_{13}$

Here $a = \frac{3}{\sqrt{2}}$, $d = 2\sqrt{2} - \frac{3}{\sqrt{2}} = \frac{2(2)-3}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

n = 13

Now using the sum formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}\left[2\left(\frac{3}{\sqrt{2}}\right) + (13-1)\frac{1}{\sqrt{2}}\right]$$

$$= \frac{13}{2}\left[\frac{6+12}{\sqrt{2}}\right] = \frac{13}{2}\left(\frac{18}{\sqrt{2}}\right) = \frac{13}{1}\left\{\frac{9}{\sqrt{2}}\right\} = \frac{117}{\sqrt{2}}$$

Q.3 How many terms of $(-7) + (-5) + (-3) + \text{------}$ amount to 65?

(Sgd 16)(Swl 17)(Mtn 18 G-I)(Bpr 18)

Ans. $(-7) + (-5) + (-3) + \text{--------}$

$a = -7\ ,\ d = -5 - (-7) = -5 + 7 = 2$

$S_n = 65\ ,\ n = ?$

$S_n = \frac{n}{2}\ [2a + \overline{n-1}\ d]$

$65 = \frac{n}{2}[(2(-7) + (n-1)2]$

$130 = n[-14 + 2n - 2]$

$130 = n(2n - 16)$

$130 = 2n^2 - 16n$

$0 = 2n^2 - 16n - 130$

$0 = 2(n^2 - 8n - 65)$

$\Rightarrow\quad n^2 - 8n - 65 = 0$

$n^2 - 13n + 5n - 65 = 0$

$n(n - 13) + 5(n - 13) = 0$

$(n - 13)(n + 5) = 0$

Either $n - 13 = 0$, $n = 13$ | or $n + 5 = 0$, $n = -5$ it is rejected.

Q.4 Sum the series upto 10th term. (Sgd 18, 19)

$1.11 + 1.41 + 1.71 + \ldots\ldots\ldots$

Ans. $1.11 + 1.41 + 1.71 + \ldots\ldots\ldots$

$a = 1.11\ ,\ d = 1.41 - 1.11 = 0.30$

$n = 10$

Using $S_n = \frac{n}{2}[2a + \overline{n-1}\ d]$

$S_n = \frac{10}{2}\ [2(1.11) + 9(0.3)] = 5[2.22 + 2.7] = 24.6$

Q.5 Sum the series $(x - a) + (x + a) + (x + 3a) + \ldots..$

(A.J.K 17)

Ans. $d = (x + a) - (x - a) = x + a - x + a = 2a$

$S_n = \frac{n}{2}\ [2a + \overline{n-1}\ d]$

$= \frac{n}{2}\ [2(x - a) + (n - 1)2a] = \frac{n}{2}\ [2x - 2a + 2na - 2a]$

$= \frac{n}{2}\ [2x + 2na - 4a]$

$= \frac{2n}{2}\ [x + na - 2a] = n[x + (n - 1)a]$

Q.6 Sum the series

$1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \text{--------} +$ 3n term (D.G.K. 16 G-I)

Ans.

$1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \text{--------} +$ 3n term

$(1 + 4 - 7) + (10 + 13 - 16) + (19 + 22 - 25) + \text{------} +$ n term

$-2 + 7 + 16 + \text{--------}$

$a = -2\ ,\ d = 7 - (-2) = 9$

Using

$S_n = \frac{n}{2}\ [2a + \overline{n-1}\ d]$

$S_n = \frac{n}{2}\ [2(-2) + (n - 1)9]$

$= \frac{n}{2}\ [-4 + 9n - 9] = \frac{n}{2}\ (9n - 13)$

Q.7 If $S_n = n(2n - 1)$, then find the series.

(Rwp 17 G-II)(Fbd 18)

Ans. $S_n = n(2n - 1)$

Putting $n = 1, 2$

$S_1 = 1(2(1) - 1) = 1(2 - 1) = 1(1) = 1 = a_1$

$S_2 = 2(2(2) - 1) = 2(4 - 1) = 2(3) = 6 = a_1 + a_2$

$a_2 = 6 - 1 = 5$

$d = a_2 - a_1 = 5 - 1 = 4$

$a_3 = a_2 + d = 5 + 4 = 9$

$\therefore$ Series is $1 + 5 + 9 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Q.8 The sum of three numbers in an A.P. is 24 and their product is 440. Find the numbers.

(Lhr 10 G-I)

Ans. Let the three numbers in A.P be $a - d, a, a + d$

By given condition: $(a - d) + a + (a + d) = 24$

$\Rightarrow\quad 3a = 24 \quad\Rightarrow\quad a = 8$

Again by given condition

$(a - d)\ (a)\ (a + d) = 440$

$a(a^2 - d^2) = 440$

Putting $a = 8$

$8\ (8^2 - d^2) = 440$

or $\quad 64 - d^2 = \frac{440}{8}$

or $\quad d^2 = 64 - 55$

or $\quad d^2 = 9 \Rightarrow\quad d = \pm 3$

Thus

$a = 8, d = 3$	$a = 8, d = -3$
$a - d = 8 - 3 = 5$	$a - d = 8 - (-3) = 11$
$a = 8$	$a = 8$
$a + d = 8 + 3 = 11$	$a + d = 8 + (-3) = 5$

Thus the required numbers in A.P are 5, 8, 11 or 11, 8, 5.

LONG QUESTIONS

Q.1 The sum of 9 terms of an A.P is 171 and its eighth term is 31. Find the series.

(Lhr 16 G-I)(Rwp 18)

Ans. Let a, ar, ar^2, ar^3 be the four terms of G.P. Then

$a + ar + ar^2 + ar^3 = 80 \Rightarrow a(1 + r + r^2 + r^3) = 80$

$a[(1 + r) + r^2(1+r)] = 80 \Rightarrow a(1+r)(1+r^2)$

$= 80 \quad \ldots\ldots (1)$

and $\dfrac{ar + ar^3}{2} = 30 \Rightarrow ar(1 + r^2) = 30 \times 2$

$\Rightarrow \quad ar(1 + r^2) = 60 \quad \ldots\ldots (2)$

Dividing eq. (1) and eq. (2):

$\dfrac{a(1 + r)(1 + r^2)}{ar(1 + r^2)} = \dfrac{80}{60} \Rightarrow \dfrac{1 + r}{r} = \dfrac{4}{3}$

$\Rightarrow \quad 3(1 + r) = 4r \Rightarrow 3 + 3r = 4r$

$3 = 4r - 3r \Rightarrow r = 3$

Putting value of r in eq. (2):

$a(3)(1 + (3)^2) = 60 \Rightarrow 3a(1 + 9) = 60$

$\Rightarrow \quad 3a(10) = 60$

$3a = 60 \Rightarrow a = \dfrac{60}{30} = 2$

$ar = 2(3) = 6\ ;\ ar^2 = 2(3)^2 = 2(9) = 18$

$ar^3 = 2(3)^3 = 2(27) = 54$

$\Rightarrow \quad \boxed{2, 6, 18, 54}$

Q.2 The sum of three numbers in A.P is 24 and their product is 440. Find the numbers.

(Lhr 10 G-I, 15 G-II)(Fbd 18)

Solution: Let a – d, a, a + d be the three numbers in A.P. Then

$a - d + a + a + d = 24 \Rightarrow 3a = 24$

$\Rightarrow \quad a = \dfrac{24}{3} = 8$

and $a(a - d)(a + d) = 440 \Rightarrow 8(8 - d)(8+d) = 440$

$\Rightarrow \quad 8(64 - d_2) = 440$

$64 - d^2 = \dfrac{440}{8} = 55 \Rightarrow 64 - 55 = d^2$

$\Rightarrow \quad 9 = d^2 \Rightarrow d = \pm 3$

When a = 8 and d = 3, then:

$a - d = 8 - 3 = 5$

$a = 8$

$a + d = 8 + 3 = 11$

$\therefore \boxed{5, 8, 11}$

When a = 8 and d = –3, then:

$a - d = 8 - (-3) = 8 + 3$

$= 11$

$a = 8$

$a + d = 8 - 3 = 5$

$\therefore \boxed{11, 8, 5}$

Q.3 Find four terms of A.P whose sum is 32 and sum of whose squares is 276. (Mtn 16 G-I)

Ans. Let a – 3d, a – d, a + d, a + 3d be the four numbers in A.P. Then

$a - 3d + a - d + a + d + a + 3d = 32$

$\Rightarrow \quad 4a = 32 \Rightarrow a = \dfrac{32}{4} = 8$

and $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 276$

$(8 - 3d)^2 + (8 - d)^2 + (8 + d)^2 + (8 + 3d)^2 = 276$

$64 + 9d^2 - 48d + 64 + d^2 - 16d + 64 + d^2 + 16d$

$+ 64 + 9d^2 + 48 = 0$

$256 + 20d^2 = 276 \Rightarrow 20d^2 = 276 - 256 = 20$

$\Rightarrow \quad d^2 = 1 \Rightarrow d = \pm 1$

When a = 8 and d = 1, then:

$a - 3d = 8 - 3(1)$

$= 8 - 3 = 5$

$a - d = 8 - 1 = 7$

$a + d = 8 + 1 = 9$

$a + 3d = 8 + 3(1)$

$= 8 + 3 = 11$

$\therefore \boxed{5, 7, 9, 11}$

When a = 8 and d = –1, then:

$a - 3d = 8 - 3(-1)$

$= 8 + 3 = 11$

$a - d = 8 - (-1)$

$= 8 + 1 = 9$

$a + d = 8 - 1 = 7$

$a + 3d = 8 + 3(-1)$

$= 8 - 3 = 5$

$\therefore \boxed{11, 9, 7, 5}$

Q.4 Find five numbers in A.P. whose sum is 25 and sum of whose square is 135. (Mtn 12 G-I)

Ans. Let a – 2d, a – d, a, a + d, a + 2d be the five numbers in A.P. Then

$a - 2d + a - d + a + a + d + a + 2d = 25$

$\Rightarrow \quad 5a = 25 \Rightarrow a = \dfrac{25}{5} = 5$

and $(a - 2d)^2 + (a - d)^2 + a^2 + (a+d)^2 + (a+2d)^2 = 135$

$(5 - 2d)^2 + (5 - d)^2 + (5)^2 + (5+d)^2 + (5 + 2d)^2 = 135$

$25 + 4d^2 - 20d + 25 + d^2 - 10d + 25 + 25 + d^2$

$+ 10d + 25 + 4d^2 + 20d = 135$

$125 + 10d^2 = 135 \Rightarrow 10d^2 = 135 - 125 \Rightarrow 10d^2$

$= 10 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

When a = 5 and d = 1, then:

$a - 3d = 5 - 2(1)$

$= 5 - 2 = 3$

$a - d = 5 - 1 = 4$

$a = 5$

$a + d = 5 + 1 = 6$

$a + 2d = 5 + 2(1)$

$= 5 + 2 = 7$

$\therefore \boxed{3, 4, 5, 6, 7}$

When a = 5 and d = –1, then:

$a - 2d = 5 - 2(-1)$

$= 5 + 2 = 7$

$a - d = 5 - (-1)$

$= 5 + 1 = 6$

$a = 5$

$a + d = 5 - 1 = 4$

$a + 2d = 5 + 2(-1)$

$= 5 - 2 = 3$

$\therefore \boxed{7, 6, 5, 4, 3}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Sum of term of an A.P is called: (Mtn 13 G-I)

(a) Geometric series
(b) Arithmetic series
(c) Arithmetic progression
(d) Harmonic progression

Q.2 The sum of four numbers in an A.P. is 24, and the sum of whose squares is 164, then common difference is

(a) 1 (b) −1
(c) ±1 (d) None of these.

Q.3 Sum the A.P −7 + (−5) + (−3) ... upto 6 terms is. (A.J.K 17 G-I)

(a) −12 (b) 12
(c) 9 (d) −9

Q.4 Formula for the sum of n terms of A.P (Arithmetic progression). (Lhr 18 G-II)

(a) $a_n = a_1 + (n-1)d$ (b) $S_n = \frac{n}{2}(a_1 + a_n)$

(c) $S_n = \frac{a_1(1-r^n)}{1-r}$ (d) $S = \frac{a_1}{1-r}$

EXERCISE 6.6

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define geometric progression. (Lhr 11 G-II)(Sgd 19)

Ans. A sequence $\{a_n\}$ is a geometric sequence or geometric progression if $\frac{a_n}{a_{n-1}}$ is the same non-zero number for all $n \in N$ and $n > 1$. The quotient $\frac{a_n}{a_{n-1}}$ is usually denoted by r and is called common ratio of the G.P.

Q.2 Find the 5th term of the G.P. 3, 6, 12, ... (D.G.K 12 G-I)(Gjw 18)

Ans. Given G.P. is 3, 6, 12, ...

Here $a = 3$, $r = \frac{\text{second term}}{\text{first term}} = \frac{6}{3} = 2$

We know that the nth term of a G.P. is given by $a_n = ar^{n-1}$

For $n = 5$, $a_5 = ar^{5-1} = 3(2)^4 = 48$

Q.3 Find the 11th term of G.P, $1 + i, 2, \frac{4}{1+i}$, ...

Ans. $1 + i, 2, \frac{4}{1+i}$, ...

$$a = \frac{2}{1+i} = \frac{2}{1+i} \times \frac{1-i}{1-i} = \frac{2(1-i)}{1^2+1^2} = \frac{2(1-i)}{2} = 1 - i$$

$a_n = a\,r^{n-1}$

$= (1+i)(1-i)^{11-1} = (1+i)(1-i)^{10}$

$= (1+i)[(1-i)^2]^5 = (1+i)(1-1-2i)^5$

$= (1+i)(-2i)^5 = (1+i)(-32i^5)$

$= (1+i)(-32i^4.i) = (1+i)(-32(1)\,i)$

$= 32(-i-i^2) = 32(1-i)$

Q.4 Find the 12th term of G.P if (Gjw 19 G-II)
$1 + i, 2i, -2 + 2i$, ——— (Mtn 16 G-I)(Bpr 19)

Ans. $a = 1 + i$

$$r = \frac{2i}{1+i} = \frac{2i}{1+i} \times \frac{1-i}{1-i} = \frac{2i(1-i)}{1^2-i^2}$$

$$= \frac{2i(1-i)}{1-(-1)} = \frac{1i(1-i)}{2} = i(1-i) = i + 1$$

$a_n = a\,r^{n-1}$

$a_{12} = a\,r^{11}$

$= (1+i)(1+i)^{11} = (1+i)^{12}$

$= [(1+i)^2]^6 = [1-1+2i]^6$

$= [2i]^6 = 2^6 . i^6 = 64\, i^4 . i^2$

$= 64(1)(-1) = -64$

Q.5 Show that the reciprocals of the terms of the geometric sequence a_1, a_1r^2, a_1r^4, ... form another geometric sequence. (Mtn 08 G-I)

Ans. Given geometric sequence is

a_1, a_1r^2, a_1r^4, ...

The reciprocals of the terms are

$\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}$, ...

Here $\frac{a_2}{a_1} = \frac{\text{second term}}{\text{first term}} = \frac{\frac{1}{a_1 r^2}}{\frac{1}{a_1}}$

$= \frac{1}{a_1r^2} \times \frac{a_1}{1} = \frac{1}{r^2} \longrightarrow$ (i)

also $\frac{a_3}{a_2} = \frac{\text{third term}}{\text{second term}} = \frac{\frac{1}{a_1 r^4}}{\frac{1}{a_1 r^2}}$

$= \frac{1}{a_1r^4} \times \frac{a_1r^2}{1} = \frac{1}{r^2} \longrightarrow$ (ii)

From (i) and (ii)

$\frac{a_2}{a_1} = \frac{a_3}{a_2}$ i.e., the common ratio is same

Hence the sequence $\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}$, ... is a G.P.

Q.6 If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in G.P. Show that the common ratio is $\pm\sqrt{\frac{a}{c}}$. **(Lhr 10 G-II, 2016, 2017 G-I) (Fbd 16)(Sgd 17)(Gjw 17)(Sgd 18)**

Ans. Given $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P. Let r be the common ratio.

$\therefore \quad r = \frac{1/b}{1/a} = \frac{a}{b} \quad$ (i) $\quad$ also $r = \frac{1/c}{1/b} = \frac{b}{c} \quad$ (ii)

Multiplying (i) and (ii)

$$r^2 = \frac{a}{b} \times \frac{b}{c} \Rightarrow r^2 = \frac{a}{c} \Rightarrow r = \pm\sqrt{\frac{a}{c}}$$

LONG QUESTIONS

Q.1 Find 11th term of G.P $1 + i, 2, \frac{4}{1+i}$ .... **(Rwp 13 G-I)(Mtn 19 G-I)**

Ans. See Short Question 3

Q.2 If a, b, c, d are in G.P. prove that $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P. **(Lhr 17 G-I)**

Ans. Since a, b, c, d are in G.P., so

$$\frac{b}{c} = \frac{c}{b} = \frac{d}{c} \quad (= r)$$

$$\therefore \frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac \quad \ldots\ldots (1)$$

$$\frac{b}{a} = \frac{d}{c} \Rightarrow bc = ad \quad \ldots\ldots (2)$$

$$\frac{c}{b} = \frac{d}{c} \Rightarrow c^2 = bd \quad \ldots\ldots (3)$$

Now, $(b^2 - c^2)^2 = (b^2)^2 - 2b^2c^2 + (c^2)^2 = (b^2)^2 - b^2c^2 - b^2c^2 + (c^2)^2$

$= a^2c^2 - b^2c^2 - a^2d^2 + b^2d^2 = c^2(a^2 - b^2) - d^2(a^2 - b^2)$

$= (a^2 - b^2)(c^2 - d^2) \Rightarrow \frac{b^2 - c^2}{a^2 - b^2} = \frac{c^2 - d^2}{b^2 - c^2}$

(Common ratio)

This shows that $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Number of geometric sequence can not be: **(Mtn 11 G-II)**

(a) Zero (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) 1

Q.2 Fifth term of G.P 3, 6, 12, ________ is: (Rwp 15 G-II)(D.G.K 12) (D.G.K 13 G-II)

(a) 15 (b) 48
(c) 2 (d) 36

Q.3 $(-1)^n$ is: (D.G.K 13 G-II)

(a) A.P (b) G.P
(c) H.P (d) Not a sequence

Q.4 The general term of the G.P is: (Lhr 15 G-I)(Bpr 12)

(a) $a_n = a_1 + \overline{n-1}\,d$ (b) $a_1 = a_n + (n-1)d$
(c) $a_n = a_1 r^{n-1}$ (d) $a_n = d + (n-1)a_1$

Q.5 No term of geometric sequence can be: (Mtn 15 G-I)(D.G.K 17 G-I)(Rwp 17 G-I)(Lhr 19 G-II)

(a) 0 (b) 1
(c) 2 (d) 3

Q.6 The next term of G.P 1, 2, 4, 8, 16, ... is: (Swl 14)

(a) 15 (b) 19
(c) 21 (d) 32

Q.7 $5, \frac{5}{2}, \frac{5}{4}$, ... (Lhr Board 2014 G-II)

(a) Series (b) A.P
(c) G.P (d) H.P

Q.8 What is the general term of the geometric sequence $-1, 1, -1, 1$, ...?

(a) $(-1)^n$ (b) $(1)^n$
(c) $(-1)^{n-1}$ (d) None of these.

Q.9 What is the nth term of the G.P. for which a = 8, r = 3/2 and n = 5 ?

(a) 27/2 (b) 57/2
(c) 81/2 (d) None of these.

Q.10 n^{th} term of G.P is. (Rwp 16)

(a) $a_1 r^n$ (b) $a_1 r^{n-1}$
(c) $\frac{a}{r^n}$ (d) $\frac{r^n}{a}$

Q.11 In geometric sequence nth term is (Mtn 18 G-I)

(a) $a_1 + (n-1)d$ (b) $\frac{n}{2}[2a_1 + (n-1)d]$
(c) $\frac{a_1}{1-r}$ (d) $a_1 r^{n-1}$

Q.12 No term of geometric sequence can be: (Lhr 19 G-II)

(a) 0 (b) 1
(c) 2 (d) 3

EXERCISE 6.7

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define geometric mean. **(Gjw 12)(Mtn 17 G-II)**

Ans. A number G is a said to be a geometric mean (G.M.) between two numbers a and b if a, G, b are in G.P.

$$\therefore \quad \frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab \Rightarrow G = \pm\sqrt{ab}$$

Q.2 Insert two G.Ms. between 2 and 16.
(Mtn 08 G-II)(Sgd 17)(D.G.K 17 G-I)(Lhr 11 G-I)

Ans. Let G_1 , G_2 be the two G.Ms. between 2 and 16

Then 2, G_1 , G_2 , 16 are in G.P.

Here $a = 2$, $n = 4$, $a_4 = 16$, $r = ?$

Using $a_n = a\,r^{n-1}$

For $n = 4$, we have

$$a_4 = a\,r^{4-1}$$
$$a_4 = a\,r^3$$
$$16 = (2)\,r^3$$
$$r^3 = \frac{16}{2}$$
$$r^3 = 8$$
$$r^3 = 2^3$$
$$\Rightarrow \quad r = 2$$
$$r^3 - 2^3 = 0$$
$$(r-2)(r^2 + 2r + 2^2) = 0$$

Either

$$r - 2 = 0$$
$$r = 2$$

or $\quad r^2 + 2r + 4 = 0$

$$r = \frac{-2 \pm \sqrt{2^2 - 4.1.4}}{2.1} = \frac{-2 \pm \sqrt{4-16}}{2}$$
$$= \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2\sqrt{3}\,i}{2} = -1 \pm \sqrt{3}\,i$$

When $r = 2$

$$G_1 = ar = 2(2) = 4$$
$$G_2 = ar^2 = 2(2)^2 = 8$$

When $\;r = -1 + \sqrt{3}\,i$

$$G_1 = ar = 2(-2 + \sqrt{3}\,i)$$
$$G_2 = ar = 2(-1 + \sqrt{3}\,i)^2 = 2(1 - 2 - 2\sqrt{3}\,i)$$
$$= 2(-2 - 2\sqrt{3}\,i) = -4(1 + \sqrt{3}\,i)$$

When $\;r = -1 - \sqrt{3}\,i$

$$G_1 = ar = 2(-1 - \sqrt{3}\,i) = -2(1 + \sqrt{3}\,i)$$
$$G_2 = ar^2 = 2(-1 - \sqrt{3}\,i) = 2(1 - 3 + 2\sqrt{3}\,i)$$
$$= 2(-2 + 2\sqrt{3}\,i) + 4(-1 + \sqrt{3}\,i)$$

Q.3 Find G.M. between –2i and 8i
(Gjw 11, 2016)(Lhr 08 G-I)(A.J.K 17)
(Mtn 18 G-I)(Fbd 18)(Sgd 19)

Ans. Here $\;a = -2i\,,\; b = 8i$

$$G = \pm\sqrt{ab}$$
$$G = \pm\sqrt{(-2i)\,(8i)} = \pm\sqrt{-16 \cdot i^2} = \pm\sqrt{-16\,(-1)}$$
$$= \pm\sqrt{16} = \pm\,4$$

Q.4 Insert two G.Ms between 1 and 8. (Mtn 19 G-I)
(Fbd 17)(Lhr 09 G-II)(Rwp 17 G-II)(Lhr 18 G-II)

Ans. Let G_1 and G_2 be two G.Ms between 1 and 8, then 1, G_1, G_2, 8 are in G.P

Using $\quad a_n = a\,r^{n-1}$

$$8 = (1)r^{4-1}$$
$$8 = r^3$$
$$8 - r^3 = 0$$
$$2^3 - r^3 = 0$$
$$(2 - r)(2^2 + 2r + r^2) = 0$$

Either $\quad 2 - r = 0$

$$r = 0$$

or $\quad 4 + 2r + r^2 = 0$

$$r = \frac{-2 \pm \sqrt{2^2 - 4.1.4}}{2.1} = \frac{-2 \pm \sqrt{4-16}}{2}$$
$$= \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2\sqrt{3}\,i}{2} = -1 \pm \sqrt{3}\,i$$

When $\;r = 2$

$$G_1 = ar = 1(2) = 2$$
$$G_2 = ar_2 = 1(2)^2 = 4$$

When $\;r = -1 + \sqrt{3}\,i$

$$G_1 = ar = 1\,(-1 + \sqrt{3}\,i) = -1 + \sqrt{3}\,i$$
$$G_2 = ar^2 = 1(-1 - \sqrt{3}\,i) = 1 - 3 + 2\sqrt{3}\,i$$
$$= -2 + 2\sqrt{3}\,i$$

When $\;r = -1 - \sqrt{3}\,i$

$$G_1 = ar = 1(-1 - \sqrt{3}\,i)$$
$$G_2 = ar^2 = 1(-1 - \sqrt{3}\,i)^2 = 1(1 - 3 + 2\sqrt{3}\,i)$$
$$= -2 + 2\sqrt{3}\,i$$

LONG QUESTIONS

Q.1 For what value of 'n' $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the positive geometric mean between a and b.
(D.G.K 11 G-I, 2013 G-II)
(Lhr 13 G-II, 15 G-I)(Rwp 16)
(Mtn 16 G-II)(Lhr 18-GII)(Gjw 19 G-II)

Ans. If $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is G.M. between a and b. Then

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \sqrt{ab} \;\Rightarrow\; \frac{a^n + b^n}{a^{n-1} + b^{n-1}} = a^{\frac{1}{2}}b^{\frac{1}{2}}$$
$$a^n + b^n = a^{\frac{1}{2}}b^{\frac{1}{2}}\,[a^{n-1} + b^{n-1}]$$
$$\Rightarrow \quad a^n + b^n = a^{n-\frac{1}{2}}.\,b^{\frac{1}{2}} + a^{\frac{1}{2}}.\,b^{n-\frac{1}{2}}$$
$$a^n - a^{n-\frac{1}{2}}.\,b^{\frac{1}{2}} = a^{\frac{1}{2}}.\,b^{n-\frac{1}{2}} - b^n$$
$$a^{\frac{1}{2}}.\,a^{n-\frac{1}{2}} - a^{n-\frac{1}{2}}.\,b^{\frac{1}{2}} = a^{\frac{1}{2}}.\,b^{n-\frac{1}{2}} - b^{1/2} - b^{n-\frac{1}{2}}$$
$$a^{n-\frac{1}{2}}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right] = b^{n-\frac{1}{2}}\left[a^{\frac{1}{2}} - b^{\frac{1}{2}}\right] \Rightarrow a^{n-\frac{1}{2}} = b^{n-\frac{1}{2}}$$

This is possible only if: $n - \frac{1}{2} = 0 \Rightarrow \boxed{n = \frac{1}{2}}$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ **is a geometric mean between a and b if n =** (Bpr 13) (Mtn 10 G-I)

(a) 0 (b) -1

(c) $-\dfrac{1}{2}$ (d) $\dfrac{1}{2}$

Q.2 G.M between 2 and 8 is equal to: (Mtn 12 G-I)

(a) 2 (b) 8

(c) 16 (d) ± 4

Q.3 The geometric mean between $-2i$ and $8i$ are
(Lhr 15 G-II)(Rwp, Mtn 17 G-I)(Bpr 18)(Rwp 18)

(a) ± 4 (b) ± 3

(c) ± 2 (d) ± 1

Q.4 The geometric mean between 1 and 16 is
(Lhr 15 G-I)(Gjw 15)

(a) 4 (b) -4

(c) ± 4 (d) $\pm \dfrac{1}{4}$

Q.5 The geometric mean between 4 and 16 is:
(D.G.K 14)(Lhr 17 G-II)(Lhr 18 G-II)

(a) ± 4 (b) ± 6

(c) ± 8 (d) ± 16

Q.6 G.M between -2 and 8 is:
(D.G.K 14 G-II) (Gjw 12)(Lhr 18 G-I)

(a) 16 (b) 3

(c) -4 (d) $\pm 4i$

Q.7 If $\dfrac{1}{a}, \dfrac{1}{b}$ and $\dfrac{1}{c}$ are in G.P. , then common ratio is

(a) $\pm\sqrt{\dfrac{a}{c}}$ (b) $\pm\sqrt{\dfrac{c}{a}}$

(c) $\pm\dfrac{a}{c}$ (d) None of these.

Q.8 With usual notation, the product of n geometric means between a and b is.
(Mtn 16 G-II)

(a) G^n (b) nG

(c) $(A)^n$ (d) $(G)^{\frac{n}{2}}$

Q.9 Geometric mean between -2 and 9. (Fbd 17)

(a) $\pm 3\sqrt{2}\,i$ (b) $\dfrac{-16}{3}$

(c) ± 4 (d) $\pm 4i$

Q.10 The geometric means between 2 and 4 all.
(Sgd 17)

(a) $\pm 8\sqrt{2}$ (b) $\pm 4\sqrt{2}$

(c) $\pm 2\sqrt{2}$ (d) $\pm 3\sqrt{2}$

Q.11 Which of the following cannot be term of G.P.
(Bpr 16)

(a) -1 (b) 0

(c) 1 (d) 5

Q.12 If $a = 2i$, $b = 4i$, then G = ______ (Lhr 12 G-II)

(a) $\pm 2\sqrt{2}\,i$ (b) $\sqrt{6}\,i$

(c) $\pm 2i$ (d) $4i$

Q.13 Geometric Mean between $4i$ and $-16i$ is:
(Mtn 19 G-I)

(a) 8 (b) -8

(c) ± 8 (d) ± 64

Q.14 The nth root of product of n Geometric Means between a and b is equal to: (Rwp 19)

(a) $(ab)^{1/n}$ (b) $a^n b^n$

(c) $n\sqrt{ab}$ (d) $\sqrt{ab}$

EXERCISE 6.8

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define geometric series. **(Lhr 19 G-II)**

Ans. Sum of n terms of a G.P is called geometric series (G.S).

Let $Sn = a + ar + ar^2 + \dots + ar^{n-1}$ ... (i)

Multiplying with r

$r\,S_n = ar + ar^2 + ar^3 + \dots + ar^n$ ... (ii)

Subtracting equation (ii) from equation (i), we get

$S_n - r\,S_n = a - ar^n$

$(1 - r)\,S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ if $|r| < 1$

$= \dfrac{a(r^n - 1)}{r - 1}$ if $|r| > 1$

Q.2 Define infinite geometric series.

Ans. If number of terms approaches to infinity, then such a geometric series is called infinite geometric series. It is usually denoted by S and is given by.

$S = a + ar + ar + \dots$

If $|r| < 1$, then $r^n \to 0$ when

$n \to \infty$, therefore

$S = \dfrac{a(1 - 0)}{1 - r} = \dfrac{a}{1 - r}$

Such a series is called convergent.

If $|r| > 1$, then $r^n \to \infty$

When $n \to \infty$, therefore,

$S = \infty$, such a series is called divergent.

Q.3 Find sum of first 15 terms of G.P.

$1, \frac{1}{3}, \frac{1}{9}, ...$ **(D.G.K 17 G-I)**

Ans. $a = 1$

$r = \frac{\frac{1}{3}}{1} = \frac{1}{3}$

$n = 15$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{1\left(1-\left(\frac{1}{3}\right)^{15}\right)}{1-\frac{1}{3}} = \frac{1-\frac{1}{14348907}}{\frac{2}{3}}$$

$$= \frac{3}{2}\left[\frac{14348907-1}{14348907}\right]$$

$$= \frac{3}{2}\left[\frac{14348906}{143489077}\right] = \frac{7174453}{4782969}$$

Q.4 Find the sum of the infinite geometric series: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$ **(Lhr 08 G-I)(Mtn 16 G-II)**

Ans. Given $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$

Here $a = \frac{1}{2}, r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times 2 = \frac{1}{2} < 1$

Using sum formula for infinite geometric series

$$S_\infty = \frac{a}{1-r} = \frac{\left(\frac{1}{2}\right)}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{2-1}{2}} = \frac{1}{2} \times \frac{2}{1} = 1$$

Q.5 Find the sum of $\frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3} +$ ———— **(Sgd 16)**

Ans. $\frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3} +$ ————

$a = \frac{9}{4}$

$r = \frac{\frac{3}{2}}{\frac{9}{4}} = \frac{3}{2} \times \frac{4}{9} = \frac{2}{3}$

$$S = \frac{a}{1-r} = \frac{\frac{9}{4}}{1-\frac{2}{3}} = \frac{\frac{9}{4}}{\frac{3-2}{3}} = \frac{\frac{9}{4}}{\frac{1}{3}} = \frac{9}{4} \times \frac{3}{1} = \frac{27}{4}$$

Q.6 Find the sum of the infinite G.P. **(Gjw 18)**

$2, \sqrt{2}, 1, ...$

Ans. $2, \sqrt{2}, 1, ...$

$a = 2, \; r = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}} < 1$

$$S = \frac{a}{1-r} = \frac{2}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{2}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{2\sqrt{2}(\sqrt{2}+1)}{2-1} = \frac{2\sqrt{2}(\sqrt{2}+1)}{1}$$

$= 2\sqrt{2}(\sqrt{2}+1)$ Ans.

Q.7 Convert recurring decimal $0.\dot{7}$ into vulgar fraction. **(Lhr 17 G-I)(Rwp 17 G-I)**

Ans. $0.\dot{7} = 0.7777$ ....

$= 0.7 + 0.07 + 0.007 +$ ....

$= 7[0.1 + 0.01 + 0.001 +$ ....$]$

$= 7\left[\frac{0.1}{1-0.1}\right] = 7\left[\frac{0.1}{0.9}\right] = 7\left(\frac{1}{9}\right) = \frac{7}{9}$

Q.8 Convert the recurring decimal $2.\dot{2}\dot{3}$ into the equivalent common fraction. **(Mtn 16 G-I)(Bpr 16)**

Ans. $2.\dot{2}\dot{3} = 2.23232323$ ————

$= 2 + 0.23 + 0.0023 + 0.000023 +$ ————

$= 2 + 0.23[0.01 + 0.001 + 0.000001 +$ ——$]$

$= 2 + \frac{23}{100}\left[\frac{0.01}{1-0.01}\right] = 2 + \frac{23}{100}\left[\frac{0.01}{0.99}\right]$

$= 2 + \frac{23}{100}\left(\frac{1}{99}\right) = 2 + \frac{23}{9900} = 2\frac{23}{9900}$

Q.9 Find vulgar fraction equivalent to $1.\dot{5}\dot{3}$ **(Fbd 16)**

Ans. $1.\dot{5}\dot{3} = 1.53535353$ ————

$= 1 + 0.53 + 0.0053 + 0.000053 +$ ————

$= 1 + 53[0.01 + 0.0001 + 0.000001 +$ ——$]$

$= 1 + 53\left[\frac{0.01}{1-0.01}\right] = 1 + 53\left[\frac{0.01}{0.99}\right] = 1 + 53\left(\frac{1}{99}\right) = 1\frac{53}{99}$

Q.10 Find vulgar fraction equivalent to $1.\dot{3}\dot{4}$. **(Bpr 18)(Sgd 18)**

Ans. $1.\dot{3}\dot{4} = 1.343434\ \ldots\ldots$

$= 1 + 0.34 + 0.0034 + 0.000034 + \ldots\ldots$

$= 1 + 34[0.01 + 0.0001 + 0.000001 + \ldots]$

$$= 1 + 34\left[\frac{0.01}{1-0.01}\right] = 1 + 34\left[\frac{0.01}{0.99}\right]$$

$$= 1 + 34\left[\frac{1}{99}\right] = 1 + \frac{34}{99} = 1\frac{34}{99} \quad \text{Ans.}$$

Q.11 If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \cdots$ and if $0 < x < 2$, then prove that $x = \frac{2y}{1+y}$.

(Lhr 13 G-I, 17 G-II)(Mtn 18 G-II)(Fbd 19 G-II)

Ans. $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \cdots\infty$

Here $a = \frac{x}{2}$, $r = \dfrac{\left(\frac{1}{4}x^2\right)}{\left(\frac{x}{2}\right)} = \frac{x^2}{4}\times\frac{2}{x} = \frac{x}{2}$

Using sum formula for infinite geometric series

$$S_\infty = \frac{a}{1-r}$$

$$y = \frac{\frac{x}{2}}{1-\frac{x}{2}} = \frac{\frac{x}{2}}{\frac{2-x}{2}} = \frac{x}{2-x}$$

$y(2-x) = x \Rightarrow 2y - xy = x$

$x + xy = 2y \Rightarrow x(1+y) = 2y$

$$x = \frac{2y}{1+y}$$

Q.12 If $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \cdots$ Show that $x = 2\left(\frac{y-1}{y}\right)$. **(Lhr 11 G-I)(Gjw 16)(Rwp 18)**

Ans. $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \cdots$

Here $a = 1$, $r = \dfrac{\frac{x}{2}}{1} = \frac{x}{2}$

Using the sum formula for infinite geometric series

$$S_\infty = \frac{a}{1-r}$$

$$y = \frac{1}{1-\frac{x}{2}} = \frac{1}{\left(\frac{2-x}{2}\right)} = \frac{2}{2-x}$$

$\Rightarrow \quad y(2-x) = 2$

$\Rightarrow \quad 2y - xy = 2$

$\Rightarrow \quad xy = 2y - 2$

$\Rightarrow \quad x = \frac{2(y-1)}{y}$

Q.13 If $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \cdots$ and if $0 < x < \frac{3}{2}$, then show that $x = \frac{3y}{2(1+y)}$ **(Lhr 10 G-II)**

Ans. $y = \frac{2}{3}x + \frac{4}{9}x^2 + \cdots\infty$

Here $a = \frac{2}{3}x$, $r = \dfrac{\frac{4}{9}x^2}{\frac{2}{3}x} = \frac{4x^2}{9}\times\frac{3}{2x} = \frac{2x}{3}$

Using the sum formula for infinite geometric series

$$S_\infty = \frac{a}{1-r}$$

$$\text{So } y = \frac{\frac{2}{3}x}{1-\frac{2x}{3}} = \frac{\frac{2x}{3}}{\frac{3-2x}{3}} = \frac{2x}{3-2x}$$

$(3-2x)y = 2x$

$3y - 2xy = 2x$

$2x + 2xy = 3y$

$2x(1+y) = 3y$

$$x = \frac{3y}{2(1+y)}$$

Q.14 Sum the series $2 + (1-i) + \frac{1}{i} + \ldots +$ to 8 terms. **(DGK 19)**

Ans. See Long Question 1

LONG QUESTIONS

Q.1 Sum the series $2 + (1-i) + \frac{1}{i} + \ldots + \ldots$ to 8 terms. **(D.G.K 17 G-I)(Mtn 17 G-II)**

Ans. Given that: $2 + (1-i) + \frac{1}{i} + \ldots\ldots$ to 8 terms

Here, we can take: $a = 2$, $r = \frac{1-i}{2}$, $n = 8$

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r} \Rightarrow S_8 = \frac{2\left[1-\left(\frac{1-i}{2}\right)^8\right]}{1-\frac{1-i}{2}}$$

$$= \frac{2\left[1-\left(\frac{1-i}{2}\right)^8\right]}{\frac{2-1+i}{2}} = \frac{2\left[1-\left(\frac{1-i}{2}\right)^8\right]}{\frac{1+i}{2}} = \frac{4}{1+i}\left[1-\frac{(1-i)^8}{2^8}\right]$$

$$= \frac{4(1-i)}{1-i^2}\left[1-\frac{((1-i)^2)^4}{256}\right]$$

$$= \frac{4(1-i)}{1-(-1)}\left[1-\frac{(1+i^2-2i)^4}{256}\right]$$

$$= \frac{4(1-i)}{1+1}\left[1-\frac{(1-1-2i)^4}{256}\right]$$

$$= \frac{4(1-i)}{2}\left[1-\frac{(-2i)^4}{256}\right] = 2(1-i)\left[1-\frac{16i^4}{256}\right]$$

$$= 2(1-i)\left[1-\frac{16(i^2)^2}{256}\right]$$

$$= 2(1-i)\left[1-\frac{16(-1)^2}{256}\right] = 2(1-i)$$

$$\left[1-\frac{16}{256}\right] = 2(1-i)\left[1-\frac{1}{16}\right]$$

$$= 2(1-i)\left[\frac{16-1}{16}\right] = 2(1-i)\left[\frac{15}{16}\right] = \frac{15}{8}(1-i)$$

Q.2 Find the vulgar fraction of $1.\dot{5}\dot{3}$. (Sgd 13 G-I)

Ans. See Short Question 11

Q.3 If $y = 1+\frac{x}{2}+\frac{x^2}{4}+ \ldots$ show that $x = 2\left(\frac{y-1}{y}\right)$. (Gjw 17)

Ans. See Short Question 14

Q.4 If $y = \frac{x}{2}+\frac{1}{4}x^2+\frac{1}{8}x^3+\ldots\ldots$ and if $0 < x < 2$, then prove that $x = \frac{2y}{1+y}$ (D.G.K 14 G-I)

Ans. See Short Question 13

Q.5 If $y = \frac{2}{3}x+\frac{4}{9}x^3+\ldots$ and if $a < x < \frac{3}{2}$ then show that $x = \frac{2y}{2(1+y)}$. (Bpr 12, 13 G-I)(Sgd 16)

Ans. See Short Question 15

Q.6 Sum of an infinite geometric series is 9 and the sum of squares of its terms is $\frac{81}{5}$. Find the series. (Sgd 13 G-II)(D.G.K 16 G-I)(Rwp 19)

Ans. Given that: Sum of infinite geometric series = 9

$$\frac{a}{1-r} = 9 \Rightarrow a = 9(1-r) \quad \ldots\ldots (*)$$

$$\& \quad a^2 = 81(1-r)^2 \quad \ldots\ldots (1)$$

Sum of the squares of its terms $= \frac{81}{5}$

$$\Rightarrow \frac{a^2}{1-r^2} = \frac{81}{5} \quad \ldots\ldots (2)$$

Putting value of eq. (1) in eq. (2):

$$\frac{81(1-r)^2}{1-r^2} = \frac{81}{5} \Rightarrow \frac{(1-r)^2}{(1-r)(1+r)} = \frac{1}{5}$$

$$\Rightarrow \frac{1-r}{1+r} = \frac{1}{5}$$

$$5(1-r) = 1+r \Rightarrow 5-5r = 1+r$$

$$\Rightarrow 5-1 = 5r+r \Rightarrow 4 = 6r$$

$$r = \frac{4}{6} = \frac{2}{3}$$

Putting value of r in eq. (*): $a = 9\left(1-\frac{2}{3}\right) = 9$

$$\left(\frac{3-2}{3}\right) = 9\left(\frac{1}{3}\right) = 3$$

The required series is: $3+3\left(\frac{2}{3}\right)+3\left(\frac{2}{3}\right)^2$

$+ \ldots\ldots \Rightarrow \boxed{3+2+\frac{4}{3}+\ldots\ldots}$

Q.7 The sum of an infinite geometric series is half the sum of the squares of its terms. If the sum of its first two terms is $\frac{9}{2}$. Find the series.(A.J.K 17)

Ans. The infinite geometric series is: $a+ar+ar^2+\ldots$

According to given condition, we take:

$$\frac{a}{1-r} = \frac{1}{2}\cdot\frac{a^2}{1-r^2} \Rightarrow \frac{a}{1-r} = \frac{a^2}{2(1-r^2)}$$

$$\Rightarrow \frac{a}{1-r} = \frac{a^2}{2(1-r)(1+r)}$$

$$1 = \frac{a}{2(1+r)} \Rightarrow 2(1+r) = a \Rightarrow a = 2+2r \quad \ldots (1)$$

and $\quad a+ar = \frac{9}{2} \Rightarrow a(1+r) = \frac{9}{2} \quad \ldots\ldots (2)$

Putting value of eq. (1) in eq. (2):

$$(2+2r)(1+r) = \frac{9}{2} \Rightarrow 2+2r+2r+2r^2 = \frac{9}{2}$$

$$2+4r+2r^2 = \frac{9}{2} \Rightarrow 2(2+4r+2r^2) = 9$$

$$4+8r+4r^2 = 9 \Rightarrow 4r^2+8r+4-9 = 0$$

$$4r^2+8r-5 = 0 \Rightarrow 4r^2+10r-2r-5 = 0$$

$$2r(2r+5)-1(2r+5) = 0 \Rightarrow (2r+5)(2r-1) = 0$$

Either $2r-1 = 0$ or $2r+5 = 0$

$2r = 1 \qquad 2r = -5$

$r = \frac{1}{2} \qquad r = -\frac{5}{2}$

When $r = \frac{1}{2}$, then from eq. (1):

$$a = 2+2\left(\frac{1}{2}\right) \Rightarrow \boxed{a = 2+1 = 3}$$

The series is: $\boxed{3+\frac{3}{2}+\frac{3}{4}+\ldots\ldots}$

When $r = -\frac{5}{2}$, then from eq. (1):

$$a = 2 + 2\left(-\frac{5}{2}\right) \Rightarrow \boxed{a = 2 - 5 = -3}$$

The series is: $\boxed{-3 + \frac{15}{2} - \frac{75}{4} + \ldots\ldots}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 An infinite geometric series is divergent. if: (Mtn 12 G-II) (Mtn 08 G-I)

(a) $|r| < 1$ (b) $|r| > 1$
(c) $r < 1$ (d) $r > 1$

Q.2 For an infinite geometric series for which $|r| < 1$, S_∞ is: (Mtn 15 G-II) (Mtn 09 G-I)

(a) $\frac{a(1+r)}{1-r}$ (b) $\frac{a_1}{1+r}$
(c) $\frac{a_1+1}{2r}$ (d) $\frac{a_1}{1-r}$

Q.3 Sum of series is $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots$ is: (Mtn 11 G-I)

(a) 1 (b) -1
(c) 2 (d) -2

Q.4 The sum of an infinite geometric series is equal to: (Rwp 13) (Mtn 12 G-I)

(a) $\frac{a}{1-r}$ (b) $\frac{a}{1+r}$
(c) $\frac{a}{1-r^2}$ (d) $\frac{a}{1+r^2}$

Q.5 $S_n = \frac{a_1(1-r^n)}{1-r}$ is convergent if:

(a) $|r| < 1$ (b) $|r| > 1$
(c) $r = 1$ (d) $r = -1$

Q.6 If $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} +$ ——— an infinite geometric series, then sum is: (D.G.K 11)

(a) $\frac{1}{4}$ (b) $\frac{5}{4}$
(c) $\frac{4}{5}$ (d) $\frac{3}{4}$

Q.7 If $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \ldots\ldots\ldots$, then interval of convergence is: (Lhr 12 G-I)

(a) $0 < x < \frac{3}{2}$ (b) $\frac{-3}{2} < x < 0$
(c) $\frac{1}{2} < x < \frac{-1}{2}$ (d) $\frac{-3}{2} < x < \frac{3}{2}$

Q.8 The series 3 + 33 + 333 + ... is

(a) A.P. (b) G.P.
(c) H.P. (d) None of these.

Q.9 The series $r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots$ is

(a) A.P. (b) G.P.
(c) H.P. (d) None of these.

Q.10 The series $2 + (1-i) + \left(\frac{1}{i}\right) + \ldots$ is

(a) A.P. (b) G.P.
(c) H.P. (d) None of these.

Q.11 The series $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \cdots$ is convergent in the interval

(a) $-2 < x < 2$ (b) $-3 < x < 3$
(c) $-4 < x < 4$ (d) None of these.

Q.12 The series $1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \ldots\ldots\ldots$ and $|x| < 2$: (Lhr 13 G-II)

(a) Arithmetic (b) Harmonic
(c) Convergent (d) Divergent

Q.13 The series $a + ar + ar^2 + \ldots \infty$ is convergent if. (D.G.K 17 G-I)

(a) $|r| > 1$ (b) $|r| < 1$
(c) $|r| = 1$ (d) $|r| \neq 1$

Q.14 An infinite geometric series in convergent if. (Fbd 18)

(a) $|r| \leq 1$ (b) $|r| < 1$
(c) $|r| < 2$ (d) $|r| > 2$

Q.15 The series $a + ar + ar^2 + — \infty$ converges, if (Swl 19)

(a) $|r| > 1$ (b) $|r| \geq 1$
(c) $|r \leq 1|$ (d) $|r| < 1$

EXERCISE 6.10

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define Harmonic progression. **(Sgd 17)**

Ans. Harmonic Progression (H.P)

A sequence of numbers is called a harmonic sequence or progression if the reciprocal of its terms are in arithmetic progression.

Q.2 Define harmonic mean.

Ans. The terms in the middle of first and last term of an H.P is called harmonic means.

Q.3 Find the 9th term of the H.P. $-\frac{1}{5}, -\frac{1}{3}, -1, \ldots\ldots$

(Mtn 08 G-II)

Ans. As $-\frac{1}{5}, -\frac{1}{3}, -1, \ldots\ldots$ are in H.P.

$\Rightarrow$ **$-5, -3, -1, \ldots..$ are in A.P.**

Here $a = -5, d = -3 - (-5) = -3 + 5 = 2, n = 9$

Now $a_9 = a + (9-1)d$

$= -5 + 8(2) = -5 + 16 = 11$ (In A.P.)

Hence $a_9 = \frac{1}{11}$ (In H.P.)

$= G^2$

Q.4 Find the 9th term of the sequence $-\frac{1}{5}, -\frac{1}{3}, -1, \ldots\ldots$ **(Lhr 10 G-II)(Fbd 19)**

Ans. As $-\frac{1}{5}, -\frac{1}{3}, -1, \ldots\ldots$ are in H.P.

$\Rightarrow$ $-5, -3, -1, \ldots..$ are in A.P.

Here $a = -5, d = -3 - (-5) = -3 + 5 = 2, n = 9$

$a_9 = a + (9-1)d$

$= -5 + 8(2) = -5 + 16 = 11$ (In A.P.)

Hence $a_9 = \frac{1}{11}$ (In H.P.)

Q.5 Find 12th terms of the harmonic sequence. $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, $ ———.

(Lhr 16 G-I, 18 G-II)(Gjw, Mtn 19 G-I)(Fbd 19 G-I)

Ans. $\frac{1}{3}, \frac{2}{9}, \frac{1}{6},$ ——— All in H.P

$3, \frac{9}{2}, 6,$ ——— All in A.P

$a = 3, d = \frac{9}{2} - 3 = \frac{9-6}{2} = \frac{3}{2}, n = 12$

Using $a_n = a + \overline{n-1}\, d^2$

$a_{12} = a + 11d$

$a_{12} = 3 + 11\left(\frac{3}{2}\right) = \frac{6+33}{2} = \frac{39}{2}$

12 term in H.P. $= \frac{2}{39}$

Q.6 In H.P, $a_1 = \frac{-1}{3}$, $a_5 = \frac{1}{5}$ and find a_9.

(Bpr 16)(Lhr 17 G-II)

Ans. First term in H.P $= a_1 = \frac{-1}{3}$

First term in A.P $= -3 = a'$

Fifth term in H.P $= a_5 = \frac{1}{5}$

Fifth term in A.P $= 5 = a'_5$

Using $a'_n = a' + \overline{n-1}\, d$

$5 = a + 4d$

$5 = -3 + 4d$

$8 = 4d$

$\boxed{d = 2}$

$a'_9 = a' + 8d = -3 + 8(2) = -3 + 16 = 13$

9^{th} term in H.P $= \frac{1}{13}$

Q.7 Find H.M. between -2 and -8. **(Lhr 09 G-I)**

Ans. Here $a = -2, b = -8$

We know H.M. $= \frac{2ab}{a+b}$

Putting values a and b, we have

H.M. $= \frac{2(-2)(-8)}{(-2)+(-8)} = \frac{32}{-10} = -\frac{16}{5}$

Q.8 Find H if $a = -2$, $b = -6$. **(Gjw 14)(Sgd 18)**

Ans. $H = \frac{2ab}{a+b} = \frac{2(-2)(-6)}{(-2)+(-6)} = \frac{24}{-8} = -3$

Q.9 Prove that A<G<H $(G < 0)$, if $a = -2$, $b = -8$

(Lhr 13 G-II)

Ans. $A = \frac{a+b}{2} = \frac{-2-8}{2} = -\frac{10}{2} = -5$ (i)

$G = -\sqrt{ab} = -\sqrt{(-2)(-8)} = -\sqrt{16} = -4$ (ii)

$(\because G < 0)$

$H = \frac{2ab}{a+b} = \frac{2(-2)(-8)}{-2-8} = -\frac{32}{10} = -\frac{16}{5} = -3{\cdot}2$ (iii)

From (i), (ii) and (iii)

clearly $A < G < H$

Q.10 The first term of an H.P. is $-\frac{1}{3}$ and the fifth term is $\frac{1}{5}$. Find its 9th term.

(Mtn 08 G-I)(Lhr 13 G-I)

Ans. In H.P. $a_1 = -\frac{1}{3}, a_5 = \frac{1}{5}, a_9 = ?$

In A.P. $a_1 = -3$, $a_5 = 5$

Now in A.P.

$a_1 = -3$ ....(i)

$a_5 = 5 \Rightarrow a_1 + 4d = 5$ ....(ii)

Using equation (i) in equation (ii), we have

$-3 + 4d = 5$

$4d = 5 + 3$

$4d = 8$

$d = \frac{8}{4} = 2$

Thus $a_1 = -3, \; d = 2$

Now $a_9 = a + (9-1)d$ $\quad \because a_n = a + (n-1)d$

$= -3 + 8(2) = -3 + 16 = 13$

$a_9 = 13$ in A.P.

$a_9 = \dfrac{1}{13}$ in H.P.

Q.11 If $a = 2i$, $b = 4i$, Show that $AH = G^2$ **(Sgd 16)(Gjw 18)**

Ans. $A = \dfrac{a+b}{2} = \dfrac{2i+4i}{2} = \dfrac{6i}{2} = 3i$

$H = \dfrac{2ab}{a+b} = \dfrac{2(2i)(4i)}{2i+4i} = \dfrac{16i^2}{6i} = \dfrac{-16}{6i} = -\dfrac{8}{3i}$

$G = \sqrt{ab} = \sqrt{(2i)(4i)} = \sqrt{8(-1)} = 2\sqrt{2}\,i$

$AH = (3i)\left(-\dfrac{8}{3i}\right) = -8 = (2\sqrt{2}\,i)^2 = G^2$

LONG QUESTIONS

Q.1 Find n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be H.M. between a and b. **(Lhr 13 G-I)(Swl 14)(DGK 19)**

Ans. If $\dfrac{a^{n+1}+b^{n+1}}{a^{n-1}+b^{n-1}}$ is H.M. between a and b. Then

$\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \dfrac{2ab}{a+b} \Rightarrow (a+b)[a^{n+1}+b^{n+1}]$

$= 2ab[a^n+b^n]$

$a^{n+2} + a\,.\,b^n + a^n\,.\,b + b^{n+2} = 2a^{n+1}\,.\,b + 2a\,.\,b^{n+1}$

$a^{n+2} + a^{n+1}\,.\,b - 2a^{n+1}\,.\,b = 2a\,.\,b^{n+1} - a.b^{n+1} - b^{n+2}$

$a^{n+2} - a^{n+1}\,.\,b = ab^{n+1} - b^{n+2} \Rightarrow a^{n+1}\,.\,a - a^{n+1}\,.\,b$

$= a\,.\,b^{n+1} - b^{n+1}\,.\,b$

$a^{n+1}[a-b] = b^{n+1}[a-b] \Rightarrow a^{n+1} = b^{n+1}$

This is possible only if: $n+1 = 0 \Rightarrow \boxed{n = -1}$

Q.2 If the H.M and A.M between two numbers and 4 and 9/2 respectively find the numbers. **(D.G.K 14 G-II)**

Ans. Let x and y be the numbers. Then

H.M. $= 4 \Rightarrow \dfrac{2xy}{x+y} = 4 \Rightarrow xy$

$= 2(x+y)$ ....(1)

and A.M. $= \dfrac{9}{2} \Rightarrow x+y = 9 \Rightarrow y = 9 - x$ ...... (2)

Putting value of eq. (2) in eq. (1):

$x(9-x) = 2(x+9-x) \Rightarrow 9x - x^2 = 18$

$\Rightarrow x^2 - 9x + 18 = 0$

$x^2 - 6x - 3x + 18 = 0 \Rightarrow x(x-6) - 3(x-6) = 0$

$(x-6)(x-3) = 0 \Rightarrow x = 3, 6$

When x = 3, then from eq. (2):

$y = 9 - 3 = 6$

$\therefore \boxed{3, 6}$

When x = 6, then from eq. (2):

$y = 9 - 6 = 3$

$\therefore \boxed{6, 3}$

Q.3 If the (positive) G.M and H.M. between two numbers are 4 and $\dfrac{16}{5}$, find the numbers. **(Rwp 17 G-II)**

Ans. Let x and y be the two numbers. Then

G.M. $= 4 \Rightarrow \sqrt{xy} = 4 \Rightarrow xy = 16$ ...... (1)

and H.M. $= \dfrac{16}{5} \Rightarrow \dfrac{2xy}{x+y} = \dfrac{16}{5} \Rightarrow \dfrac{2(16)}{x+y} = \dfrac{16}{5}$

$\Rightarrow \dfrac{2}{x+y} = \dfrac{1}{5}$

$x+y = 10 \Rightarrow y = 10 - x$ ...... (2)

Putting value of eq. (2) in eq. (1):

$x(10-x) = 16 \Rightarrow 10x - x^2 = 16$

$\Rightarrow x^2 - 10x + 16 = 0$

$x^2 - 8x - 2x + 16 = 0 \Rightarrow x(x-8) - 2(x-8) = 0$

$(x-8)(x-2) = 0 \Rightarrow x = 2, 8$

When x = 2, then from eq. (2):

$y = 10 - 2 = 8$

$\therefore \boxed{2, 8}$

When x = 8, then from eq. (2):

$y = 10 - 8 = 2$

$\therefore \boxed{8, 2}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Reciprocal of terms of geometric sequence: (Mtn 08)

(a) Arithmetic sequence
(b) Harmonic sequence
(c) Geometric sequence
(d) None of these

Q.2 If $\dfrac{1}{5}, \dfrac{1}{8}$ are two harmonic means between x and y, the y is equal to: (Mtn 09 G-I)

(a) $\dfrac{1}{10}$ (b) $\dfrac{1}{11}$

(c) $\dfrac{1}{12}$ (d) $\dfrac{1}{13}$

Q.3 12th term of the sequence $\dfrac{1}{3}, \dfrac{2}{9}, \dfrac{1}{6}$, ... is: (Mtn 09 G-II)

(a) $\dfrac{39}{2}$ (b) 39

(c) $\dfrac{2}{39}$ (d) $\dfrac{1}{39}$

Q.4 If A, G, H denote the A.M, +ve G.M and H.M between two +ve real numbers, then A, G, H form: (Mtn 09 G-II)(Rwp 17 G-II)

(a) $A > G > H$ (b) $A < H < G$
(c) $G < H < A$ (d) $H < A < G$

Q.5 If a, b distinct +ve real numbers, and $G = \sqrt{ab}$, then: (Gjw 11)(Mtn 10 G-II)

(a) $A < G < H$ (b) $-A < -G < -H$
(c) $A < G > H$ (d) $H < G < A$

Q.6 If a and b are two positive distinct real numbers then: (Bpr 11 G-I)(Fbd 12)(gjw 18)
(a) $A > G$ (b) $A < G$
(c) $A = G$ (d) $A \geq G$

Q.7 If $a = -1$, $b = 5$, then $A \times H$ is equal to: (Bpr 2013)
(a) -5 (b) $\frac{-5}{2}$
(c) 5 (d) $\frac{2}{5}$

Q.8 Reciprocal of A.P is. (Mtn 16 G-I)(DGK 19)
(a) A.P (b) G.P
(c) H.P (d) None of these

Q.9 With usual notation AH equal: (Mtn 14 G-I)
(a) A^2 (b) H^2
(c) G^2 (d) $-G^2$

Q.10 If a, b, G are all negative, then which is true: (D.G.K 14 G-II)
(a) $A < G < H$ (b) $A > G > H$
(c) $A = G = H$ (d) $H > A > G$

Q.11 A, G, H are arithmetic, geometric, harmonic means between a and b respectively then A, G, H are in: (Swl 19)
(a) G.P (b) A.P
(c) H.P (d) All of these

Q.12 Let $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ be H.M between a and b, then: (Lhr 10 G-I)
(a) $n = 0$ (b) $n = 1$
(c) $n = \frac{1}{2}$ (d) $n = -1$

Q.13 For any two numbers a and b, $\frac{2ab}{a+b}$ is: (Lhr 14 G-II)
(a) A.M (b) G.M
(c) H.M (d) None

Q.14 If A, G and H are arithmetic geometric and harmonic means, then $\frac{G}{H}$ = ________
(a) $\frac{A}{G}$ (b) $\frac{G}{A}$
(c) $\frac{H}{G}$ (d) G^2

Q.15 If the numbers $\frac{1}{k}$, $\frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, the value of k is
(a) 1 (b) 2
(c) 3 (d) 4

Q.16 A sequence of numbers whose reciprocal form an arithmetic sequence, is known as
(a) arithmetic sequence (b)geometric sequence
(c) harmonic sequence (d) None of these.

Q.17 $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, —. is equal to. (Sgd 16)
(a) H.P (b) A.P
(c) G.P (d) Arithmetic series

Q.18 If $a = 1$, $b = 5$, then $A \times H$ is equal to. (Mtn 16 G-I)(Mtn 18 G-II)
(a) 5 (b) -5
(c) $-\frac{5}{2}$ (d) $\frac{2}{5}$

Q.19 H.M between 3 and 7 is. (Gjw 16)(Lhr 17 G-I)
(a) $\frac{5}{21}$ (b) $\frac{21}{5}$
(c) 5 (d) 21

Q.20 For any two members a and b, G^2 is equal to. (with usual notation) (Bpr 16)(Sgd 18)
(a) $A \times H$ (b) $A \div H$
(c) $A - H$ (d) $A + H$

Q.21 The harmonic mean between a and b is: (Lhr 15 G-II, 16 G-I)(Swl 14) (Bpr 13, 19) (DGK 11)(Mtn 18 G-II)
(a) $\frac{a+b}{2}$ (b) $\frac{2ab}{a+b}$ (c) $\pm\sqrt{ab}$ (d) $\frac{a+b}{2ab}$

Q.22 If $\frac{4}{7}$ be the third term of H.P, then third term of A.P is. (A.J.K 17)
(a) $\frac{7}{4}$ (b) $\frac{4}{7}$ (c) 4 (d) 7

Q.23 Reciprocal terms of harmonic sequence equals. (D.G.K 17 G-II)
(a) A.P (b) H.P
(c) G.P (d) Arithmetic series

Q.24 If $a = 1$, $b = 5$, then $A \times H =$ (Fbd 17)
(a) 5 (b) -5
(c) $\frac{-5}{2}$ (d) $\frac{2}{5}$

Q.25 A, G, H are in. (Swl 17)
(a) A.P (b) G.P
(c) H.P (d) Series

Q.26 For two +ve real numbers, with usual notation. (Gjw 18)
(a) $A > G$ (b) $A = G$
(c) $A \geq G$ (d) $A < G$

Q.27 The 10th term of $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$, ... is. (Lhr 18 G-I)
(a) 30 (b) 28 (c) $\frac{1}{29}$ (d) $\frac{1}{32}$

Q.28 If A ,G ,H have their usual meaning, G^2= (Sgd 19)
(a) H (b) A (c) $A \times H$ (d) A/H

Q.29 A , G , H , are in (Swl 19)
(a) A . P (b) G . P
(c) H . P (d) series

Chapter 7

PERMUTATIONS, COMBINATION AND PROBABILITY

EXERCISE 7.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define factorial.

Ans. Let n be a positive integer then the product $n(n-1)(n-2)\ \dots\ 3.2.1$ is denoted by $n!$ or $\underline{|n}$ and is called n factorial.

Q.2 Write 8.7.6.5 in the factorial form.
(Rwp 17 G-I)

Ans. 8.7.6.5

$$= \frac{8.7.6.5\ \underline{|4}}{\underline{|4}} = \frac{\underline{|8}}{\underline{|4}}$$

Q.3 Write in the factorial form $n(n-1)(n-2)\ \dots\dots\ (n-r+1)$ **(Mtn 08 G-II)**
(Fbd, Sgd, A.J.K 17)(Rwp 16)

Ans. $n(n-1)(n-2)\ \dots\dots\ (n-r+1)$

$$= n(n-1)(n-2)\ \dots\dots\ [n-(r-1)]$$

$$= \frac{n(n-1)(n-2)\ \dots\dots \{n-(r-1)\}\{n-(r)\}!}{(n-r)!}$$

$$= \frac{n(n-1)(n-2)\ \dots\dots\ (n-r+1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

Q.4 Write $\frac{(n+1)(n)(n-1)}{3.2.1}$ in the factorial form
(Mtn 11, 16, 17 G-I)(Fbd 18, 19 G-II)

Ans. $\frac{(n+1)(n)(n-1)}{3.2.1}$

$$= \frac{(n+1)(n)(n-1)(n-2)!}{3.2.1.(n-2)!} = \frac{(n+1)!}{3!(n-2)!}$$

Q.5 Write $(n+2)(n+1)(n)$ in factorial form.
(Gjw 16)(Lhr 16 G-I)(Mtn 18)

Ans. $(n+2)(n+1)(n) = \frac{(n+2)(n+1)(n)(n-1)!}{(n-1)!}$

$$= \frac{(n+2)!}{(n-1)!}$$

Q.6 Write $\frac{8\cdot7\cdot6}{3\cdot2\cdot1}$ in factorial form. **(Lhr 14 G-II)**

Ans. $\frac{8\cdot7\cdot6}{3\cdot2\cdot1} = \frac{8\cdot7\cdot6\cdot5!}{3\cdot2\cdot1\cdot5!} = \frac{8!}{3!5!}$

Q.7 Evaluate $\frac{8!}{4!.2!}$ **(Lhr 14 G-I)(Mtn 16 G-II)**

Ans. $\frac{8!}{4!.2!} = \frac{8.7.6.5.4!}{4!.2.1} = 8.7.3.5 = 840$

Q.8 Evaluate $\frac{\underline{|9}}{\underline{|6}\ \underline{|3}}$. **(D.G.K 17 G-I)**

Ans. $\frac{\underline{|9}}{\underline{|6}\ \underline{|3}} = \frac{9.8.7\ \underline{|6}}{\underline{|6}} = 84$

Q.9 Evaluate $\frac{15!}{15!(15-15)!}$. **(Lhr 18 G-I)**

Ans. $\frac{15!}{15!(15-15)!} = \frac{15!}{15!\ 0!} = \frac{15!}{(15!)(1)} = 1$ Ans.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Value of $\frac{9!}{6!\ 3!}$ is: (Mtn 08 G-II)

(a) 84 (b) 48
(c) 24 (d) 42

Q.2 $(3!)(0!) =$

(a) 3! (b) 0
(c) 3 (d) ∞

Q.3 The factorial form of 6.5.4 is equal to:
(Mtn 13 G-II)(Rwp 17 G-I)(Fbd 18)

(a) $\frac{6!}{3!}$ (b) $\frac{3!}{6!}$
(c) $\frac{1}{6!}$ (d) 6!

Q.4 Factorial form of $n(n-1)(n-2)\ \dots\ (n-r+1)$ is:
(DGK 11)

(a) $\frac{(n+1)!}{(n-r)!}$ (b) $\frac{n!}{(n+r)!}$
(c) $\frac{n!}{(n-r)!}$ (d) $\frac{(n+1)!}{(n+r)!}$

Q.5 0! (DGK 13 G-I)

(a) 0 (b) 1
(c) −1 (d) 2

Q.6 Factorial form of $\frac{(n+1)\ n(n-1)}{3\ 2\ 1}$ is: (DGK 13 G-II)

(a) $\frac{(n+1)!}{3!}$ (b) $\frac{(n-2)!}{3!}$

(c) $\frac{(n+1)!}{3!\,(n-2)!}$ (d) $\frac{(n+1)(n-2)!}{3!}$

Q.7 9.8.7.6 = (Bpr 12)

(a) $\frac{9!}{5!}$ (b) $\frac{5!}{9!}$

(c) 5! (d) 3!

Q.8 The value of $\frac{\underline{|3}}{\underline{|0}}$ is equal to:

(Rwp 13, 15)(Mtn 15 G-II)(Swl 17)(Lhr 17 G-I)

(a) 0 (b) $\propto$

(c) 3 (d) 6

Q.9 5! = (Mtn 14 G-II)(Gjw 015)

(a) 140 (b) 120

(c) 5 (d) 0

Q.10 $\frac{6!}{8!}$ = (DGK 14 G-I & G-II)

(a) $\frac{1}{65}$ (b) 65

(c) 56 (d) $\frac{1}{56}$

Q.11 The factorial form of $\frac{10.9}{2.1}$ is: (Lhr 14 G-I)

(a) $\frac{\underline{|10}}{\underline{|2}}$ (b) $\frac{\underline{|10}}{\underline{|8}\ \underline{|2}}$

(c) $\frac{\underline{|10}\ \underline{|8}}{\underline{|2}}$ (d) $\frac{\underline{|8}}{\underline{|4}}$

Q.12 Factorial form of (n + 2)(n + 1)n, is:

(Lhr 13 G-II) (Gjw 12)

(a) $\frac{(n+2)!}{(n-1)!}$ (b) $\frac{(n+2)!}{n!}$

(c) $\frac{n!}{(n+2)!}$ (d) $(n+2)!$

Q.13 n! stands for

(a) product of first n natural numbers

(b) sum of first n natural numbers

(c) product of first n integers

(d) None of these.

Q.14 Factorial form n(n − 1) (n − 2) = (Lhr 11 G-I)

(a) $\frac{n!}{(n-1)!}$ (b) $\frac{n!}{(n-1)!}$

(c) $\frac{n!}{(n-3)!}$ (d) $\frac{n!}{(n+3)!}$

Q.15 $(n-1)(n-2)(n-3)$ —— $(n-r+1)$ = (Mtn 16 G-II)

(a) $\frac{(n-1)!}{(n-r)!}$ (b) $\frac{n!}{(n-r)!}$

(c) $\frac{(n-1)!}{n(n-r+2)!}$ (d) $\frac{n!}{(n-r+1)!}$

Q.16 If n = 1, then n $\underline{|n-1}$ is (Rwp 16)

(a) Zero (b) 1

(c) 2 (d) −1

Q.17 6! + 0! is equal to (Bpr 16)

(a) 719 (b) 720

(c) 721 (d) 722

Q.18 $\frac{6!}{3!\,3!}$ = ... (A.J.K 17)

(a) 6 (b) 8

(c) 10 (d) 20

Q.19 The value of $\frac{4!}{0!}$ is. (Lhr 18 G-I)

(a) 24 (b) 4

(c) 0 (d) Infinity

Q.20 Simplify form of $\frac{10!}{7!}$ is equal to: (Lhr 18 G-II)

(a) 720 (b) 620

(c) 520 (d) 420

Q.21 $9 \times 8 \times 7$ is equal to: (Sgd 18)

(a) 9! (b) $\frac{9!}{7!}$

(c) $\frac{3!}{2!}$ (d) $\frac{9!}{6!}$

Q.22 0! = (Fbd 19 G-I)

(a) 0 (b) 1

(c) −1 (d) 2

Q.23 $\frac{8!}{7!}$ = (Bpr 19)

(a) 7! (b) 7

(c) 8 (d) 8!

Q.24 Factorial form of n (n-1) (n-2) is: (DGK 19)

(a) $\frac{n!}{(n-1)!}$ (b) $\frac{n!}{(n-2)}!$

(c) $\frac{n!}{(n-3)}!$ (d) $\frac{n!}{(n+3)}!$

Q.25 The value of 4! . 0!. 1! is: (Lhr 19 G-II)

(a) 0 (b) 1

(c) 4 (d) 24

Q.26 The factorial form of (Mtn 19 G-I)

$n(n-1)\ (n-2)$ - - - $(n-r+1)$ is:

(a) $\frac{n!}{(n-r)!}$ (b) $(n-1)!$

(c) n! (d) $\frac{n!}{(n-r+1)!}$

EXERCISE 7.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 State fundamental principle of counting. **(Mtn 17 G-II)**

Ans. Fundamental Principle of Counting:

Suppose A and B are two events. The first event A can occur in b different ways. After A has occurred, B can occur in q different ways. The number of ways that the two events can occur is the product p.q.

Q.2 Evaluate $^{12}P_5$ **(Lhr 06 G-I)(Rwp 17 G-I)**

Ans. $^{12}P_5 = \frac{12!}{(12-5)!} = \frac{12.11.10.9.8.7!}{7!} = 95040$

Q.3 Evaluate $^{10}P_7$ **(Gjw 10)**

Ans. $^{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!}$

$= \frac{10.9.8.7.6.5.4.3!}{3!}$

$= 10.9.8.7.6.5.4 = 604800$

Q.4 Evaluate $^{20}P_3$ without calculator. **(Gjw 16)**

Ans. $^{20}P_3 = \frac{\underline{|20}}{\underline{|20-3}} = \frac{20.19.18.\underline{|17}}{\underline{|17}} = 6840$

Q.5 Find the value of n when $^{11}P_n = 11 \times 10 \times 9$
(Lhr 08, 16 G-I, 17 G-II, 12 G-II)(Sgd 16)
(Swl, Fbd 17)(Gjw 14, 18)(Mtn 18 G-II)

Ans. $\frac{11!}{(11-n)!} = 11 \times 10 \times 9 \Rightarrow \frac{11!}{(11-n)!} = 990$

$\Rightarrow \frac{(11-n)!}{11!} = \frac{1}{990} \quad \Rightarrow \quad (11-n)! = \frac{11!}{990}$

$\Rightarrow (11-n)! = \frac{11 \times 10 \times 9 \times \underline{|8}}{11 \times 9 \times 10} \Rightarrow (11-n)! = 8!$

$\Rightarrow 11 - n = 8 \Rightarrow n = 11 - 8 \Rightarrow n = 3$

Q.6 Find n if $^nP_2 = 30$. **(Lhr 09 G-II)(Bpr 16)**
(Rwp 18)(Sgd 18)

Ans. $^nP_2 = 30 \quad \Rightarrow \frac{n!}{(n-2)!} = 30$

$\Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 30 \Rightarrow n(n-1) = 30$

$\Rightarrow n^2 - n - 30 = 0 \quad \Rightarrow n^2 - 6n + 5n - 30 = 0$

$\Rightarrow n(n-6) + 5(n-6) = 0 \Rightarrow (n-6)(n+5) = 0$

$\Rightarrow n - 6 = 0, n + 5 = 0 \quad \Rightarrow n = 6, n = -5$

Q.7 Prove that $^nP_r = n \times {}^{n-1}P_{r-1}$
(Lhr 13 G-I)(Mtn 16, 17 G-I)

Ans. R.H.S. $= n\,{}^{n-1}P_{r-1}$

$= n\frac{(n-1)!}{(n-1-r+1)!} = \frac{n(n-1)!}{(n-r)!}$

$= \frac{n!}{(n-r)!} = {}^nP_r = $ L.H.S.

Q.8 Find the value of n if $^nP_4 : {}^{n-1}P_3 = 9 : 1$
(D.G.K 16 G-I)(Rwp 16)(A.J.K 17)
(Bpr 18)(Mtn 19 G-I)

Ans. $^nP_4 : {}^{n-1}P_3 = 9 : 1$

$9\,{}^{n-1}P_3 = 1 \,.\, {}^nP_4$

$9 \,.\, \frac{\underline{|n-1}}{\underline{|n-1-3}} = \frac{\underline{|n}}{\underline{|n-4}}$

$9\frac{\underline{|n-1}}{\underline{|n-4}} = \frac{n\underline{|n-1}}{\underline{|n-4}}$

$\boxed{9 = n}$

Q.9 How many words can be formed from the letters of the following word 'PLANE' using all letters when no letter is to be repeated.
(Lhr 10 G-I)(Gjw 17)(Rwp 19)

Ans. Here we have to form permutations of 5 taken 5 letters at a time i.e.

$^5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{(0)!} = \frac{5!}{1} = 5! = 5.4.3.2.1 = 120$

Q.10 How many words can be formed from the letters of OBJECT using all letters, no letter is to be repeated. **(Lhr 11 G-II)(Fbd 16)(Bpr 19)**

Ans. Here we have to form permutations of 6 letters taken 6 at a time i.e.

$^6P_6 = \frac{6!}{(6-6)!} = 6! = 6.5.4.3.2.1 = 720$

Q.11 How many 3–digit numbers can be formed by using each one of the digits 2, 3, 5, 7, 9
(Mtn 16 G-II)(Rwp 17 G-II)

Ans. Number of 3–digit numbers

$= P_3 = \frac{\underline{|5}}{\underline{|5-3}} = \frac{120}{2} = 60$

Q.12 How many signals can be given by 5 flags of different colours, using 3 flags at a time?
(Gjw 13),(Lhr 13 G-II)

Ans. Total number of flags $= n = 5$

Number of signals using 3 flags

$= {}^5P_3 \quad = \frac{5!}{(5-3)!} = \frac{5.4.3.2!}{2!}$

$= 5.4.3 \quad = 60$

Q.13 How many 5 digits multiples of "5" can be formed from the digits 2, 3, 5, 7, 9 when no digit is repeated. **(Mtn 18 G-I)**

Ans. Number of 5-digits multiples of 5.

= Number of numbers when 5 is at units place

= 4!

= 24

LONG QUESTIONS

Q.1 How many 6-digit numbers can be formed without repeating any digit from the digits 0,1,2,3,4,5,? In how many of them will 0 be at the tens place? (Lhr 15 G-I)

Ans. Digits = 0, 1, 2, 3, 4, 5

Number of total digits = 6 ; Using digits at a time = 6

Here, we can take: $n = 6$

Numbers of the form 1⊗⊗⊗⊗⊗ $= {}^5P_5 = \frac{5!}{0!}$

$= 120$

Numbers of the form 2⊗⊗⊗⊗⊗ $= {}^5P_5 = \frac{5!}{0!}$

$= 120$

Numbers of the form 3⊗⊗⊗⊗⊗ $= {}^5P_5 = \frac{5!}{0!}$

$= 120$

Numbers of the form 4⊗⊗⊗⊗⊗ $= {}^5P_5 = \frac{5!}{0!}$

$= 120$

Numbers of the form 5⊗⊗⊗⊗⊗ $= {}^5P_5 = \frac{5!}{0!}$

$= 120$

Total numbers $= 5 \times 120 = 600$

Numbers with "0" at tens place $= {}^5P_5 = \frac{5!}{0!}$

$= 120$

Q.2 Prove that ${}^nP_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$ (D.G.K 14 G-II)

Ans. L.H.S.

$= {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

$= \frac{\underline{|n-1}}{\underline{|n-1-r}} + r.\frac{\underline{|n-1}}{\underline{|n-1-r+1}}$

$= \frac{\underline{|n-1}}{\underline{|n-1-r}} + r\frac{\underline{|n-1}}{(n-r)\underline{|n-1-r}}$

$= \frac{\underline{|n-1}}{\underline{|n-1-r}}\left[1+\frac{r}{n-r}\right]$

$= \frac{\underline{|n-1}}{\underline{|n-1-r}}\left[\frac{n-r+r}{n-r}\right]$

$= \frac{n\,\underline{|n-1}}{(n-r)\underline{|n-r-1}} = \frac{\underline{|n}}{\underline{|n-r}} = {}^nP_r = \text{L.H.S}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The value of n when ${}^{11}P_n = 11.10.9$ is:

(a) 1 (b) 3
(c) 2 (d) 0

Q.2 ${}^nP_n$ equals:
(D.G.K 17 G-I)(Lhr 12 G-I)(Mtn 13 G-I)(Sgd 19)

(a) n! (b) (n + 1)!
(c) 1 (d) n

Q.3 ${}^nP_2 = 30 \Rightarrow n =$ (DGK 15 G-I) (Mtn 13 G-II)

(a) 6 (b) 4
(c) 5 (d) 720

Q.4 n different objects can be arranged taken all at a time: (DGK 15 G-II) (Gjw 13) (DGK 12)

(a) (n − 1)! ways (b) (n + 1) ways
(c) n! ways (d) (2n)! ways

Q.5 With usual notation ${}^6P_4$ equals:
(Lhr 11 G-II) (Rwp 14)(DGK 14) (Mtn 14 G-I)

(a) 160 (b) 260
(c) 360 (d) 340

Q.6 The value of ${}^{16}P_1$ is: (Mtn 14 G-II)

(a) 16 (b) 20
(c) 80 (d) 90

Q.7 ${}^{20}P_3$ = (DGK 14 G-II)(Mtn 11 G-II)(Gjw 18)

(a) 4050 (b) 5040
(c) 6840 (d) 4068

Q.8 The value of ${}^6P_1$ is: (Rwp 14)

(a) 18 (b) 12
(c) 6 (d) 0

Q.9 How many arrangements can be made of the letters a, b, c, d taken three at a time?

(a) 20 (b) 22
(c) 24 (d) 26

Q.10 The number of signals that can be made by hoisting 6 flags of different colors one above the other, when any of them may be hoisted at once is

(a) 720 (b) 1950
(c) 1956 (d) None of these.

Q.11 ${}^4P_3$ is equal to. (Sgd 16)(D.G.K 17 G-II)

(a) ${}^4P_1$ (b) ${}^4P_2$
(c) ${}^4P_4$ (d) ${}^5P_4$

Q.12 If $^nP_2 = 30$, then n is equal to. (Lhr 16 G-I)

(a) 6 (b) 5
(c) 4 (d) 10

Q.13 ^nP_r = ? (Where n > 0 , r > 0) (Fbd 16, 2017)(Bpr 14)(Gjw 16)

(a) $\frac{n!}{r!}$ (b) $\frac{n!}{(n-r)!}$
(c) $\frac{(n-r)!}{r!}$ (d) $\frac{n!}{r!\,(n-r)!}$

Q.14 The value of $^5P_2$ is. (Mtn 17 G-I)

(a) 5 (b) 10
(c) 15 (d) 20

Q.15 $\frac{n!}{(n-r)!}$ = (Mtn 11 G-I)

(a) ^nC_r (b) $^{n-1}C_r$
(c) $^{n+1}C_{r+1}$ (d) ^nP_r

Q.16 $^6P_3$ = : (Fbd 19 G-II)

(a) 36 (b) 6
(c) 18 (d) 120

Q.17 Number of signals given by 5 flags of different colours using 3 flags at a time equals. (Rwp 19)

(a) 30 (b) 40
(c) 50 (d) 60

Q.18 ^nP_n = (Sgd 19)

(a) n (b) 0
(c) 1 (d) n!

EXERCISE 7.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define circular permutation. **(Lhr 09 G-I)(Rwp 19)**

Ans. The permutations of things which can be represented by the points on a circle are called circular permutations.

Q.2 How many arrangements of the letters of the word 'PAKISTAN' taken all together, can be made? **(Gjw 13)(Gjw 18)**

Ans. The word " PAKISTAN " contains 8 letters of which
A is repeated 2 times
P comes only once
T comes only once
N comes only once
K comes only once
S comes only once
and I comes only once
Therefore the total number of words that can be formed from the word "PAKISTAN" are

$$= \frac{8!}{2!\,1!\,1!\,1!\,1!\,1!\,1!} = \frac{8.7.6.5.4.3.2.1}{2} = 20160$$

Q.3 How many arrangement of the word "Mathematics" taken all together. **(D.G K 17 G-I)(A.J.K 17)**

Ans. Mathematics
Number of letters = n = 11
No of Ms = n_1 = 2
Number of Ts = n_2 = 2
Number of As = n_3 = 2

Number pf arrangements $= \frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2}$

$= \frac{11.10.9.8.7.\lfloor 6}{2\times 2\times 2}$

$= 11\times 10\times 9\times 7\times 720$

$= 4989600$

Q.4 How many numbers can be formed from the digits 2, 2, 3, 3, 4, 4? **(Bpr 16)**

Ans. 2,2, 3, 3, 4, 4
No. of digits = n = 6
No. of 2s = n_1 = 2
No. of 3s = n_2 = 2
No. of 4s = n_3 = 2

No. of numbers $= \frac{6!}{2!\,2!\,2!} = \frac{720}{2\times2\times2} = 90$

Q.5 How many numbers greater than 1000,000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4? **(Mtn 11 G-I)(Rwp 16)**

Ans. See Long Question 5

Q.6 In how many ways can 4 keys be arranged on a circular key ring? **(Gjw 12)(Rwp 18) (Sgd 17)(Lhr 17 G-I)**

Ans. 4 keys can be arranged on a circular key ring in $\frac{1}{2}$ (3!) or 3 ways. (Taking clockwise or anti-clockwise only)

Q.7 In how many ways a necklace of 6 beads of different colours be made? **(Lhr 13 G-II)(Sgd 19)**

Ans. Number of necklaces $\frac{1}{2}(5!) = \frac{1}{2}(120) = 60$

Q.8 How many arrangements of the letters of the word, taken all together, can be made 'PAKISTAN'. **(Fbd 19)**

Ans. Total no. of letters = 9
P is repeated 2 times
A is repeated 3 times
T is repeated 2 times
K and N come once

∴ Total Number of words $= \binom{9}{2,3,2,1,1}$

$= \frac{9!}{2!\,3!\,2!\,1!\,1!} = 15120$

Q.9 How many arrangements of the letters of the word 'MATHEMATICS', taken all together, can be made? (Gjw 19 G-II)

Ans. Total no. of letters = 11

M is repeated 2 times

A is repeated 2 times

T is repeated 2 times

H, E, I, C and S come once

$\therefore$ Total Number of words = $\binom{11}{2,2,2,1,1,1,1,1}$

$= \dfrac{11!}{2!\,2!\,2!\,1!\,1!1!\,1!1!} = 4989600$

LONG QUESTIONS

Q.1 How many numbers greater than 1000,000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4? (Mtn 15 G-I)

Ans. Given digits = 0, 2, 2, 2, 3, 4, 4

Number of total digits = 7 ; Digit 0 is repeated = 1

Digit 2 is repeated = 3 ; Digit 3 is repeated = 1

Digit 4 is repeated = 2

Numbers greater than 2000000 = $\binom{6}{1,2,1,2}$

$= \dfrac{6!}{1!\,2!\,1!\,2!} = \dfrac{720}{2\times 2} = \dfrac{720}{4} = 180$

Numbers greater than 3000000 = $\binom{6}{1,3,2}$

$= \dfrac{6!}{1!\,3!\,2!} = \dfrac{720}{6\times 2} = \dfrac{720}{12} = 60$

Numbers greater than 4000000 = $\binom{6}{1,3,1,1}$

$= \dfrac{6!}{1!\,3!\,1!\,1!} = \dfrac{720}{6} = 120$

Total numbers greater than 100000

= 180+60+120 = 360

MULTIPLE CHOICE QUESTIONS

❑ *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 4 keys can be arranged on a circular key ring in: (Mtn 09 G-I)

(a) 5 ways (b) 4 ways

(c) 3 ways (d) 2 ways

Q.2 The number of ways in which 5 persons can be seated at a round table are: (Bpr 11 G-I)

(a) 2! (b) 3!

(c) 4! (d) 5!

Q.3 Circular permutation of n non-living things is given by: (Fbd 11)

(a) n! (b) $\dfrac{n!}{2}$

(c) (n − 1)! (d) $\dfrac{(n-1)!}{2}$

Q.4 5 persons can be seated at a round table is: (Sgd 13 G-I) (Fbd 12) (Lhr 10 G-II)(Fbd 17)

(a) 24 ways (b) 25 ways

(c) 20 ways (d) 120 days

Q.5 5 keys can be arranged in a circular ring in number of ways: (Lhr 14, 15 G-II, 10 G-I)

(a) 24 (b) 12

(c) 6 (d) 5

Q.6 How many arrangements of the letters of the word PAKPATTAN can be made, taken all together.

(a) 15130 (b) 1512

(c) 15120 (d) None of these.

Q.7 How many necklaces can be made from 6 beads of different colours?

(a) 120 (b) 60

(c) 36 (d) None of these.

Q.8 How many arrangements of the letters of the word MATHEMATICS can be made, taken all together.

(a) 4989000 (b) 4989600

(c) 4989600 (d) None of these.

Q.9 Number of permutations of n different things taken r at a time is denoted by

(a) ${}^{n}C_{r}$ (b) ${}^{n}P_{r}$

(c) n! (d) None of these.

Q.10 How many arrangements of the letters of the word PAKISTAN can be made, taken all together.

(a) 21160 (b) 20160

(c) 20170 (d) None of these.

EXERCISE 7.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define combination.

Ans. Number of selections of n objects taking r at a time is called combination. It is denoted by ${}^{n}C_{r}$ or $\binom{n}{r}$ and is given by $\dfrac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$.

Q.2 Evaluate ${}^{20}C_{17}$ (D.G K 17)

Ans. ${}^{20}C_{17} = \dfrac{20!}{(20-17)!(17)!}$

$= \dfrac{20.19.18.17!}{3!\,17!} = \dfrac{20.19.18}{1.2.3} = 1140$

Q.3 Evaluate $^{12}C_3$ **(Lhr 08 G-I)**

Ans. $^{12}C_3 = \dfrac{12!}{3!(12-3)!}$ Using $^nC_r = \dfrac{n!}{r!\,(n-r)!}$

$= \dfrac{12.11.10.9!}{3.2.1.9!} = \dfrac{12.11.10}{3.2.1} = 220$

Q.4 Prove that $^nC_r = {}^nC_{n-r}$
(Gjw 14)(Lhr 11, 19 G-II)(Fbd 16)
(D.G.K 16 G-I)(Gjw, Sgd 17)(Rwp 17 G-II)

Ans. $^nC_{n-r} = \dfrac{n!}{(n-r)!\,(n-r+r)!} = \dfrac{n!}{(n-r)!\,r!}$

$\Rightarrow$ $^nC_{n-r} = {}^nC_r$

Q.5 Find the value of n, when $^nC_{12} = {}^nC_6$
(Gjw 13, 16)(Lhr 12 G-I)(Lhr 11 G-I)
(Fbd 16)(Swl 17, 19)(A.J.K 17)(Sgd 18)(Mtn 18)

Ans. $^nC_{12} = {}^nC_6$

$\Rightarrow$ $^nC_{n-12} = {}^nC_6$

$\Rightarrow$ $n-12 = 6 \Rightarrow n = 18$

Q.6 Find the value of n , when $^nC_5 = {}^nC_4$
(Lhr 14 G-I)(Mtn 16 G-I-II)(Mtn 18 G-I)(Rwp 19)

Ans. $^nC_5 = {}^nC_4$

$\therefore$ $^nC_{n-5} = {}^nC_4$ Q $^nC_r = {}^nC_{n-r}$

$\Rightarrow$ $n-5 = 4 \Rightarrow n = 9$

Q.7 How many diagonals can be formed by joining the vertices of the polygon having 5 sides?(Gjw 10)(Lhr 09 G-II)

Ans. See Long Question 3

Q.8 Find the value of n, when $^nC_{10} = \dfrac{12\times 11}{2!}$
(Gjw 11)(Lhr 16 G-I)(Lhr 13 G-I)(DGK 19)
(Mtn 17 G-I)(Rwp 18)(Fbd 18)(Lhr 18 G-I)

Ans. $^nC_{10} = \dfrac{12\times 11}{2!} = \dfrac{12\times 11\times 10!}{2!\times 10!} = \dfrac{12!}{2!\,(12-2)!}$

i.e. $^nC_{10} = {}^{12}C_2$

$\Rightarrow$ $^nC_{n-10} = {}^{12}C_2$ Q $^nC_r = {}^nC_{n-r}$

$\Rightarrow$ $n-10 = 2$

$\Rightarrow$ $n = 12$

Q.9 If $C_8^n = C_{12}^n$, find n.(Rwp 17 G-I)(Gjw 19 G-I)

Ans. As $C_r^n = C_{n-r}^n$

$\therefore$ $n-12 = 8$

$\boxed{n = 20}$

Q.10 Find the number of the diagonals of a 6-sided figure. **(Lhr 10, 13 G-I)(Sgd, Gjw 17)**
(Bpr 18)(Lhr 18 G-I)

Ans. A 6-sided figure has 6 vertices.
Joining any two vertices we get a line segment

$\therefore$ Number of line segments $= {}^6C_2 = \dfrac{6!}{2!\,4!} = 15$

But these line segments include 6 sides of the figure.

$\therefore$ Number of diagonals $= 15 - 6 = 9$

Q.11 How many diagonals of 8 side figure?(Bpr 16)

Ans. No of diagonals $= C_2^8 - 8 = 28 - 8 = 20$

Q.12 Find the number of diagonals of 12-sided figure. (Rwp 16)

Ans. Number of sides $= n = 12$

Number of diagonals $= C_2^{12}$ — no. of sides

$= \dfrac{12\,.\,11}{1\,.\,2} - 12 = 66 - 12 = 54$

Q.13 There are 8 men and 10 women member of a club. How many committees of seven can be formed having 4 women? **(D.G.K 16 G-I)**

Ans. No. of committees of seven member with 4 women

$= C_4^{10} \times C_3^8$

$= \dfrac{10.9.8.7}{1.2.3.4} \times \dfrac{8.7.6}{1.2.3} = 11760$

LONG QUESTIONS

Q.1 Find the values of 'n' are 'r', when $^nC_r = 35$ and $^nP_r = 210$ **(D.G.K 11 G-I) (D.G.K 14 G-I)**

Ans. See Short Question 10

Q.2 How many diagonals and triangles can be formed by joining the vertices of polygon having 5 sides. **(D.G.K 16 G-I)**

Ans. Number of sides of polygon $= 5$

Number of vertices of 5 sided polygon $= 5$

Number of vertices using for a line $= 2$

Here, we can take: $n = 5,\ r = 2$

Number of line segments $= {}^5C_2 = \dfrac{5!}{2!\,3!}$

$= \dfrac{120}{2\times 6} = \dfrac{120}{12} = 10$

Number of diagonals $= 10 - 5 = 5$

Number of vertices using for a triangle $= 3$

Here, we can take: $n = 5,\ r = 3$

Number of triangles formed $= {}^5C_3 = \dfrac{5!}{3!\,2!}$

$= \dfrac{120}{6\times 2} = \dfrac{120}{12} = 10$

Q.3 Prove that $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
(Bpr 11 G-I, 16)(Swl 14)(Rwp 11 G-I, 13 G-II)

Ans. L.H.S.$= {}^nC_r + {}^nC_{r-1}$

$= \dfrac{n!}{r!\,(n-r)!} + \dfrac{n!}{(r-1)!\,(n-(r-1))!}$

$= \dfrac{n!}{r\,.\,(r-1)!\,(n-r)!} + \dfrac{n!}{(r-1)!\,(n-(r-1))(n-r)!}$

$= \dfrac{n!}{(r-1)!\,(n-r)!}\left[\dfrac{1}{r} + \dfrac{1}{n-(r-1)}\right]$

$= \dfrac{n!}{(r-1)!\,(n-r)!}\left[\dfrac{1}{r} + \dfrac{1}{n-r+1}\right]$

$$= \frac{n!}{(r-1)!\,(n-r)!}\cdot\frac{n-r+1+r}{r(n-r+1)}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\cdot\frac{n+1}{r(n-(r-1))}$$

$$= \frac{(n+1)n!}{r(r-1)!\,(n-(r-1))(n-r)!} = \frac{(n+1)!}{r!\,(n-(r-1))!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!} = \frac{(n+1)!}{r!\,((n+1)-r)!} = {}^{n+1}C_r = \text{R.H.S.}$$

Q.4 Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$

(Bpr 12 G-I, 14)(Mtn 16 G-II)(Sgd 16)
(Fbd 12 G-I, 16)(D.G.K 15 G-I)(Lhr 19 G-II)

Ans. L.H.S.$= {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$$= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-1-(r-1))!}$$

$$= \frac{(n-r)!}{r(r-1)!(n-r-1)!} + \frac{(n-1)!}{(r-1)!(n-1-r+1)!}$$

$$= \frac{(n-1)!}{r(r-1)!(n-r-1)!} + \frac{(n-1)!}{(r-1)!(n-r)!}$$

$$= \frac{(n-1)!}{r(r-1)!(n-r-1)!} + \frac{(n-1)!}{(r-1)!(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!}\left[\frac{1}{r}+\frac{1}{n-r}\right]$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!}\left[\frac{n-r+r}{r(n-r)}\right]$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!}\cdot\frac{n}{r(n-r)}$$

$$= \frac{n(n-1)!}{r(r-1)!(n-r)(n-r-1)!}$$

$$= \frac{n!}{r!(n-r)!} = {}^nC_r = \text{R.H.S.}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 ${}^5C_2$ is equal to: (Mtn 08)
(a) Zero (b) 10
(c) n! (d) n

Q.2 ${}^{n-1}C_r + {}^{n-1}C_{r-1}$ equals: (Mtn 09 G-II)
(a) ${}^nC_r$ (b) ${}^{n-1}C_{r+1}$
(c) ${}^{n-1}C_r$ (d) ${}^nC_{r+1}$

Q.3 ${}^nC_r\, r! =$ (Fbd 14) (Mtn 10 G-I)(Gjw 16)
(a) ${}^{n+1}P_r$ (b) ${}^nP_{r+1}$
(c) ${}^{n-1}P_r$ (d) ${}^nP_r$

Q.4 ${}^nC_n =$
(Mtn 11 G-I)(Lhr 13 G-II)(Fbd 15)(Lhr 17 G-II)
(a) 0 (b) n
(c) n! (d) 1

Q.5 The value of n when ${}^nC_2 = {}^nC_7$ is equal to:
(a) 5 (b) 4
(c) 8 (d) 9

Q.6 ${}^nC_n$ has the value equal to: (Rwp 12)(Bpr 13)
(a) 1 (b) n
(c) 0 (d) 2n

Q.7 ${}^8C_3 =$ (Rwp 12)(Bpr 13)
(a) ${}^8P_3$ (b) ${}^8C_5$
(c) ${}^3C_8$ (d) ${}^5C_8$

Q.8 If ${}^nC_8 = {}^nC_{12}$, then n is equal to:
(Lhr 14 G-I)(Rwp 13)(Rwp 18)
(Mtn 17 G-I)(Mtn 18 G-I)
(a) 4 (b) 8
(c) 20 (d) 12

Q.9 ${}^6C_2$ (Sgd 13 G-I)
(a) 5 (b) 10
(c) 15 (d) 20

Q.10 ${}^nC_r$ also can be written as: (Gjw 11)
(a) P(n, r) (b) C(n, r)
(c) P(r, n) (d) C(r, n)

Q.11 How many diagonals can be formed by joining the vertices of the polygon having 5 sides.
(a) 10 (b) 15
(c) 5 (d) None of these

Q.12 How many triangles can be formed by joining the vertices of the polygon having 5 sides.
(a) 20 (b) 15
(c) 10 (d) None of these.

Q.13 How many triangles can be formed by joining the vertices of the polygon having 12 sides.
(a) 202 (b) 220
(c) 110 (d) None of these.

Q.14 The total number of combinations of n different things taken 1 , 2 , 3 .....n at a time is
(a) $2^n - 1$ (b) $2^n + 1$
(c) 2^{n+1} (d) 2^{n-1}

Q.15 If $C_r^n = {}^nC_q$, which of the following rust be true (Mtn 16 G-I)
(a) $r \neq q$ (b) $r + q = n$
(c) $r - q = n$ (d) $q = 0$

Q.16 The number of handshakes that can be exchanged among a party of 10 students if every student shakes hands once with every student is
(a) ${}^{10}P_2$ (b) ${}^{10}P_{10}$
(c) ${}^{10}C_2 - 10$ (d) ${}^{10}C_2$

Q.17 C_r^n is equal to. (Lhr 17 G-I)
(a) $\frac{n!}{r!}$ (b) $\frac{n!}{(n-r)!}$
(c) $n(n-r)!$ (d) $\frac{n!}{n!\,(n-r)!}$

Q.18 C_0^n is equal to. (Lhr 17 G-I, 18 G-II)

(a) ${}^nP_2$ (b) ${}^nC_n$

(c) ${}^nC_2$ (d) ${}^nC_{n+1}$

Q.19 $\frac{{}^nP_r}{r!}$ is equal to. (Lhr 17 G-II)

(a) ${}^nC_r$ (b) ${}^nC_{r-1}$

(c) ${}^{n+1}C_r$ (d) ${}^{n-1}C_r$

Q.20 ${}^nC_r + {}^nC_{r-1} =$ (Fbd 18)

(a) ${}^nC_r$ (b) ${}^{n+1}C_r$

(c) ${}^{n+1}C_{r+1}$ (d) ${}^{n-1}C_{r-1}$

Q.21 ${}^nC_{r-1} + {}^nC_{r-2}$ is equal to: (Mtn 18 G-II)

(a) ${}^nC_{r-1}$ (b) ${}^{n+1}C_{r-1}$

(c) ${}^{n+1}C_{r-2}$ (d) ${}^nC_{r-2}$

Q.22 With usual notation notation ${}^nC_r$ equals. (Fbd 19 G-I)

(a) ${}^nC_{r-n}$ (b) ${}^nC_n$

(c) ${}^nC_{n-r}$ (d) ${}^{n-r}C_n$

Q.23 ${}^{n-1}C_r + {}^{n-1}C_{r-1}$ equals (Gjw 19 G-II)

(a) ${}^{n+1}C_r$ (b) ${}^{n+1}C_{r+1}$

(c) ${}^nC_r$ (d) ${}^{n-1}C_r$

Q.24 $\frac{n!}{(n-r)!r!}$ is equal to: (Rwp 19)

(a) ${}^rC_n$ (b) ${}^rP_n$

(c) ${}^nC_r$ (d) ${}^nP_r$

Q.25 If ${}^nC_{10} = {}^nC_{14}$ then n= (Sgd 19)

(a) 24 (b) 8

(c) 20 (d) 18

Q.26 If ${}^nc_4 - {}^nc_5 = 0$, then n = (Swl 19)

(a) 0 (b) 4

(c) 6 (d) 9

EXERCISE 7.5

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define Probability. **(Gjw 14)(Rwp 18)**

Ans. Probability is the branch of mathematics in which the chance of an event happening is predicted before it occurs.

Q.2 Define the Sample Space and give a simple example. **(Swl 17)(Gjw 11)(Fbd 16)**

Ans. The set S of all possible outcomes of a given experiment is called the sample space. Each element of a sample space is called a sample point.

Example:

In tossing a fair coin, the possible outcomes are a Head (H) or a Tail (T)

$\therefore$ The sample space $S = \{H, T\}$.

Q.3 Define an Event. **(Gjw 10)(Sgd 16)**

Ans. A particular outcome E of an experiment is called an event. Thus an event is a subset of the sample space S.

Q.4 Define mutually exclusive events. **(Gjw 16)**

Ans. If two events A and B of a sample space S are disjoint then A and B are said to be mutually exclusive events.

Q.5 A die is rolled. What is the probability that the dots on the top are greater than 4?

(Lhr 12 G-I)(Gjw 13)(Mtn 16 G-II)

Ans. Sample space $S = \{1, 2, 3, 4, 5, 6\}$

$\Rightarrow \quad n(S) = 6$

The event E that the dots on the top are greater than 4 $= \{5, 6\}$

$\Rightarrow \quad n(E) = 2$

$\therefore \quad \text{Probability} = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Q.6 There are 5 green and 3 red balls in a box, one ball is taken out, find the probability that the ball drawn is green. **(Lhr 09 G-I)(Lhr 18 G-I)**

Ans. S = {5 green, 3 red balls}

A = {5 green balls}

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{8}$

MULTIPLE CHOICE QUESTIONS

- *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 If P(E) = 1, then event is known is: (Mtn 08 G-II)

(a) Known as mutually exclusive

(b) Equally likely

(c) Certain (d) Not possible

Q.2 From a box containing slips numbered 1, 2, 3, 4, 5. One slip is picked up, the probability that the slip bears a prime number is: (Mtn 09 G-II)

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) 1

Q.3 If S = {1, 2, 3, 4}, A = {1, 2}, the P(A) = (Mtn 10, 17 G-II)(Rwp 12, 17 G-I)

(a) 2 (b) 4

(c) 1 (d) $\frac{1}{2}$

Q.4 A die is rolled then n(S) is: (Rwp 12) (Mtn 11 G-II)(D.G.K 17 G-II)

(a) 36 (b) 6

(c) 1 (d) 9

Q.5 When one die is rolled, the probability of even number:(DGK 12)(Mtn 11 G-I, 12 G-II, 13 G-I)

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Q.6 If S is sample space, A is event, then probability P(A) is: (DGK 13 G-II)(Mtn 15 G-I)

(a) $P(A) = 1$ (b) $0 < P(A) < 1$

(c) $0 \le P(A) \le 1$ (d) $P(A) \le 1$

Q.7 The probability that an event E does not occur is:

(Lhr 10 G-II)(Bpr 11 G-I) (Fbd 12)(Bpr 18)

(a) $P(E) = \frac{n(E)}{n(S)}$ (b) $P(\bar{E}) = \frac{n(E)}{n(\bar{S})}$

(c) $P(E) - P(E) - 1$ (d) $P(\bar{E}) = 1 - P(E)$

Q.8 A fair coin is tossed then probability of head is: (Lhr 15 G-II)

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) $\frac{1}{3}$

Q.9 The probability to get an odd number is a dice is thrown once is ________

(Lhr 12 G-II) (Sgd 13)

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) 2 (d) $\frac{1}{3}$

Q.10 For two mutually exclusive events A and B: (Sgd 13)

(a) $A \cup B = \phi$ (b) $A \cap B = \phi$

(c) $A - B = \phi$ (d) $A \cup B = A \cap B$

Q.11 If A = { }, then P(A) is equal to: (Rwp 14)

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) None of these

Q.12 Probability of an impossible event is: (Gjw 12)

(a) $P(E) = 1$ (b) $P(E) = 2$

(c) $P(E) = 0$ (d) $P(E) = \frac{1}{2}$

Q.13 There are 5 red and 3 black balls in a box one ball is taken out, the probability that ball is blue: (Lhr 11 G-II)

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$

(c) 1 (d) Zero

Q.14 If a coin is tossed, the probability of head is: (Lhr 11 G-I)

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$

Q.15 If n(S) = 20, n(B) = 2, the P(B) (DGK 15 G-I)

(a) $\frac{1}{10}$ (b) $\frac{9}{10}$

(c) 10 (d) $\frac{10}{9}$

Q.16 For two non-mutually exclusive events A and B

(a) $P(A \cup B) = P(A) + P(B) + P(A \cap B)$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cup B)$

(d) $P(A \cup B) = P(A) + P(B)$

Q.17 A die is rolled, the probability that the dots on top are less than 4 is equal to:

(DGK 15 G-II) (Fbd 11)(Gjw 18)

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1

Q.18 If a fair coin is tossed, then probability of head appears. (Fbd 16)

(a) $\frac{1}{2}$ (b) 2

(c) 0 (d) 1

Q.19 If S is sample space, E is an event, then P(E) is: (Bpr 12) (DGK 11)

(a) $\frac{n(E)}{n(S)}$ (b) $\frac{n(S)}{n(E)}$

(c) $n(E) - n(S)$ (d) $n(E) + n(S)$

Q.20 When a fair coin is tossed two times, then sample space is. (A.J.K 17)

(a) {H, T} (b) {H, HH, T}

(c) {HH, HT, TH, TT}

(d) {HH, TT}

Q.21 For an event A, the range of its probability equals. (Sgd 17)(D.G.K 17 G-II)(Swl 19)

(a) $0 \le P(A) \le 1$ (b) $0 < P(-A) < 1$

(c) $-1 < P(A) < 1$ (d) $-1 \le (P(A) \le 1$

Q.22 An event is said to be sure if P(A) = (Swl 17)

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Q.23 If S is a sample space and E = S is an event, then P(E) is equal to. (Mtn 16 G-I)

(a) [0,1] (b) 0

(c) 1 (d) (0, 2)

Q.24 A die is thrown, what is the probability to get 3 dots. (Bpr 19)

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

Q.25 If n(S) = 20, n(B) =2, then P(B) equals: (DGK 19)

(a) 10 (b) $\frac{1}{10}$

(c) $-\frac{1}{10}$ (d) 1

Q.26 Probability of impossible event is: (Lhr 19 G-II)

(a) $\frac{1}{2}$ (b) 1

(c) 0 (d) 2

EXERCISE 7.7

SHORT ANSWERS TO THE QUESTIONS

Q.1 A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.
(Lhr 10 G-II)(Mtn 11 G-I, 18)(Fbd 14)

Ans. Number of red marbles = 10 , Number of white marbles = 30

Number of black marbles = 20

Total marbles = 60

The total possible out comes = ${}^{60}C_1 = 60$

Favourable out comes {i.e. red or white} = ${}^{10}C_1 + {}^{30}C_1 = 10 + 30 = 40$

$\therefore$ P(Red or White marble)

$$= \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{40}{60} = \frac{2}{3}$$

LONG QUESTIONS

Q.1 A die is thrown twice. What is the probability that the sum of the number of dots shown in 3 or 11? **(Mtn 12, 19 G-I)**

Ans. Given that:

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$
$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$
$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$
$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$
$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$
$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$\Rightarrow$ n = 36

A = Sum is 3 = {(1, 2), (2, 1)}

$m = 2 \Rightarrow P(A) = \frac{m}{n} = \frac{2}{36}$

B = Sum is 11 = {(5, 6), (6, 5)}

$m = 2 \Rightarrow P(B) = \frac{m}{n} = \frac{2}{36}$

Now, $A \cap B = \{\ \}$

So, $P(A \cup B) = P(A) + P(B) = \frac{2}{36} + \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$

Q.2 A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.
(Mtn 11 G-I)(Fbd 14)(Mtn 18)

Ans. See Short Question 2

Q.3 Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6? **(Fbd 19 G-II)**

Ans.

$$S = \left\{\begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix}\right\}$$

$\Rightarrow$ n(S) = 36

A = Event sum is 4 = {(1,3),(2,2),(3,1)} $\Rightarrow$ n(A) = 3

B = Event sum is 6

= {(1,5),(2,4),(3,3),(4,2),(5,1)} $\Rightarrow$ n(B) = 5

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$ $P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

As A and B are disjoint, so

$P(A \cup B) = P(A) + P(B) = \frac{3}{36} + \frac{5}{36} = \frac{8}{36} = \frac{2}{9}$

MULTIPLE CHOICE QUESTIONS

❑ *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 If A and B are overlapping set the P(A ∪ B) is equal to: (Mtn 08)(Rwp 18)(Lhr 18 G-I)

(a) $P(A) + P(B)$ (b) $P(A)+P(B) - P(A \cap B)$

(c) $P(A) - P(B)$ (d) $P(A) - P(A \cap B)$

Q.2 If A and B are disjoint, the P(A ∪ B) equals:
(Mtn 09, 12 G-I)(Bpr 15)(Rwp 17 G-II)

(a) $P(A) - P(B)$ (b) $P(A) . P(B)$

(c) $\frac{P(A)}{P(B)}$ (d) $P(A) + P(B)$

Q.3 A die is rolled once the probability that the dots on the top are greater than four is:
(Lhr 10, 12 G-I)

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Q.4 What is the probability that a number selected from the numbers 1 , 2 , 3 , 4 , 5 , ... , 16 is a prime number is ?

(a) $\frac{1}{16}$ (b) $\frac{5}{8}$

(c) $\frac{3}{8}$ (d) None of the above

Q.5 In a siMtneous throw of two dice, what is the probability of getting a total of 7?

(a) $\frac{1}{6}$ (b) $\frac{7}{12}$

(c) $\frac{7}{36}$ (d) None of the above

Q.6 If E and F be mutually exclusive events such that P (E) = 0.4 and P (F) = 0.5 , then P(E∪F) is :

(a) 0.2 (b) 0.1

(c) 0.9 (d) None of the above

Q.7 In a single throw of two dice, the probability of getting a total of 6 is

(a) $\frac{6}{36}$ (b) $\frac{5}{36}$

(c) $\frac{1}{36}$ (d) None of these.

Q.8 If A and B are mutually exclusive events, then P(A ∪ B) equal to: (Mtn 15 G-II)(Bpr 14)

(a) P(A) + P(B) (b) P(A) − P(B)

(c) P(AB) (d) P(A) + P(B) − P(A∩B)

Q.9 If E and F be events in a sample space such that P (E∪F) = 0.8 , P (E∩F) = 0.3 and P(E) = 0.5, then P(F) is

(a) 0.6 (b) 1

(c) 0.8 (d) None

Q.10 If A and B are disjoint then P(A∪B) equals (Gjw 19 G-II)

(a) P(A) − P(B) (b) P(A) P(B)

(c) $\frac{P(A)}{P(B)}$ (d) P(A) + P(B)

Q.11 When *A* and *B* are two disjoint events then *P (A ∪ B)* =: (Mtn 19 G-I)

(a) P (A) − P (B)

(b) P (A) + P (B) − P (A ∩ B)

(c) P (A) − P (A ∩ B)

(d) P (A) + P (B)

EXERCISE 7.8

SHORT ANSWERS TO THE QUESTIONS

Q.1 Determine the probability of getting 2 heads in two successive tosses of a balanced coin.

(Lhr 13 G-II)(Gjw 11)

Ans. When two coins are tossed then the possible out comes are.

S = {HH , HT , TH , TT}

n(S) = 4

H = Event that 2 heads getting

n(H) = 1

Hence required probability is

$$P(H) = \frac{n(H)}{n(S)} = \frac{1}{4}$$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 For independent events, then P(A ∩ B) is:

(Mtn 15 G-I) (Mtn 14 G-I)(Sgd 17)

(a) P(A) + P(B) (b) P(A) − P(B)

(c) P(A) . P(B) (d) $\frac{P(A)}{P(B)}$

Q.2 For two independent events A and B if P(A) = P(B) = $\frac{1}{2}$, then P(A∩B) is: (Rwp 15)

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) Zero

Q.3 If A and B are independent events and P(A) = $\frac{5}{7}$ and P(B) = $\frac{7}{9}$ then P(A ∩ B) is: (Gjw 14)

(a) $\frac{5}{7}$ (b) $\frac{7}{9}$

(c) $\frac{9}{7}$ (d) $\frac{5}{9}$

Q.4 When P(A∩B) = P(A) P(B), then A and B are:

(a) Mutually exclusive (b) Equally likely

(c) Independent (d) Dependent

Q.5 One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a king?

(a) $\frac{1}{13}$ (b) $\frac{1}{52}$

(c) $\frac{3}{13}$ (d) None of the above

Q.6 What is the probability of getting a king or a queen in a single draw from a pack of 52 cards?

(a) $\frac{1}{26}$ (b) $\frac{1}{13}$

(c) $\frac{2}{13}$ (d) None of the above

Q.7 In a siMtneous throw of two dice , what is the probability of getting a total of 10 or 11?

(a) $\frac{7}{12}$ (b) $\frac{5}{36}$

(c) $\frac{1}{6}$ (d) None of the above

Q.8 if A and B are independent events and $P(A) = 0.8$, $P(B) = 0.7$ then $P(A \cap B)$ equals (Lhr 16 G-I)

(a) 0.56 (b) $\frac{8}{7}$

(c) $\frac{7}{8}$ (d) 0.1

Q.9 If two dice are thrown siMtneously, then the number of elements in the sample space are. (Rwp 17 G-II)

(a) 6 (b) 12

(c) 24 (d) 36

Q.10 If A and B are independent events, probability $P(A \cup B) =$ (Mtn 16 G-II)

(a) $P(A) \cap P(B)$ (b) $P(A) . P(B)$

(c) $P(A) + P(B)$

(d) $P(A) + P(B) - P(A) . P(B)$

Q.11 F or two events A and B if $P(A) = P(B) = \frac{1}{3}$ then probability $P(A \cap B) =$ (Mtn 18 G-I)

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) 1

Q.12 A fair coin is tossed twice then probability of getting one head and one tail is: (Fbd 19 G-II)

(a) 1 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Chapter 8

MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

EXERCISE 8.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 State principle of mathematical induction. **(Lhr 09 G-I)(Sgd 17)(Fbd 18)**

Ans. If a proposition or statement S(n) for each positive integer n is such that

(1) S(1) is true i.e., S(n) is true for n = 1 and

(2) S(k + 1) is true whenever S(k) is true for any positive integer k, then S(n) is true for all positive integers.

Q.2 Prove that $1 + 3 + 5 + \cdots + (2n-1) = n^2$ for n = 1, 2. **(Lhr 08 G-I)(Rwp 18)(Fbd 19)**

Ans. $1 + 3 + 5 + \cdots + (2n-1) = n^2$

For n = 1	For n = 2
L.H.S. = 1	L.H.S. = 1 + 3 = 4
R.H.S. = $(1)^2 = 1$	R.H.S. = $(2)^2 = 4$
L.H.S. = R.H.S.	L.H.S. = R.H.S.
Statement is true for n = 1	Statement is true for n = 2

Q.3 Prove that:

$$\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \text{——} + \binom{n+2}{3} = \binom{n+3}{4}$$

is true for n = 1, 2. **(Sgd 16)**

Ans. For n = 1

$\binom{3}{3} = \binom{1+3}{4}$

$\binom{3}{3} = \binom{4}{4}$

$1 = 1$

For n = 2

$\binom{3}{3} + \binom{4}{3} = \binom{5}{4}$

$1 + 4 = 5$ it is true.

Q.4 Show that inequality $4^n > 3^n + 4$ is true for n = 2, 3 **(Rwp 16)**

Ans. Put n = 2

$4^2 > 3^2 + 4$

$16 > 9 + 4$

$16 > 13$ it is true

Put n = 3

$4^3 > 3^3 + 4$

$64 > 27 + 4$

$64 > 31$ it is true

Q.5 Show that $2 + 6 + 8 + \text{——} + 2 \times 3^{n-1} = 3^n - 1$ is true for n = 1, n = 2. **(Fbd 16)**

Ans. Put n = 1

$2 = 3^1 - 1$

n = 2 it is true

Put n = 2

$2 + 6 = 3^2 - 1$

$8 = 9 - 1$

$8 = 8$ it is true

Q.6 $1 + 5 + 9 + \ldots + (4n-3) = n(2n-1)$.
Prove for n = 1, 2
(D.G.K 17 G-I)(Rwp 17 G-II)(Lhr 17 G-I)

Ans. Put n = 1

$1 = 1(2(1) - 1)$

$= 1(2-1) = 1(1) = 1$

Put n = 2

$1 + 5 = 2(2(2) - 1)$

$6 = 2(4-1) = 2(3) = 6$

Q.7 Show that $4^n > 3^n + 4$ is not true for n = 1. **(Rwp 17 G-I)**

Ans. Put = n

$4^n > 3^n + 4$

$4^1 > 3^1 + 4$

$4 > 7$ It is not true.

Q.8 Prove that $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = 2\left[1 - \frac{1}{2^n}\right]$
for n = 1, 2 (Gjw 12)(Sgd 17)(Lhr 11, 17 G-II)
(Bpr 18)(Mtn 18 G-I)

Ans.

For n = 1	For n = 2
L.H.S. = 1	L.H.S. = $1 + \frac{1}{2} = \frac{3}{2}$
R.H.S. $= 2\left[1 - \frac{1}{2^1}\right]$	R.H.S. $= 2\left[1 - \frac{1}{2^2}\right]$
$= 2\left[\frac{2-1}{2}\right] = 1$	$= 2\left[\frac{4-1}{4}\right] = \frac{3}{2}$
L.H.S. = R.H.S.	L.H.S. = R.H.S.
Statement is true for n = 1	Statement is true for n = 2

Q.9 Use mathematical induction to prove for every positive integer n

$$\mathbf{1 + 4 + 7 + \ldots\ldots + (3n - 2) = \frac{n(3n-1)}{2}}$$

(Lhr 12 G-I)(A.J.K 17)

Ans. $1 + 4 + 7 + \ldots\ldots + (3n - 2) = \dfrac{n(3n-1)}{2}$ …… (1)

C-1 For $n = 1$

L.H.S. of (1) $= 1$

R.H.S. of (1) $= \dfrac{1(3(1)-1)}{2} = \dfrac{3-1}{2} = 1$

L.H.S. of (1) = R.H.S. of (1)

Statement is true for $n = 1$, so C-1 is satisfied.

C-2 Suppose statement is true for $n = k$

i.e. $1 + 4 + 7 + \ldots\ldots + (3k - 2) = \dfrac{k(3k-1)}{2}$

C-3 Now we want to prove for $n = k + 1$

i.e. $1 + 4 + 7 + \cdots + (3k - 2) + (3k + 1) = \dfrac{(k+1)(3k+2)}{2}$

$$\begin{aligned}
\text{L.H.S.} &= \{1 + 4 + 7 + \cdots + (3k - 2)\} + (3k + 1) \\
&= \frac{k(3k-1)}{2} + (3k + 1) \\
&= \frac{k(3k-1) + 2(3k+1)}{2} \\
&= \frac{3k^2 - k + 6k + 2}{2} \\
&= \frac{3k^2 + 3k + 2k + 2}{2} \\
&= \frac{3k(k+1) + 2(k+1)}{2} \\
&= \frac{(k+1)(3k+2)}{2} = \text{R.H.S.}
\end{aligned}$$

Truth for $n = k$, Implies truth for $n = k + 1$

Hence by the principle of mathematical induction the statement is true for all positive integral values of n.

Q.10 Show that $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$ for integral values of $n > 0$ (Mtn 12, 16 G-II)

Ans. $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$ …… (1)

C-1 For $n = 1$

L.H.S. of (1)$= 4(1) - 3 = 4 - 3 = 1$

R.H.S. of (1)$= 1\,(2(1) - 1) = 1\,(2 - 1) = 1$

L.H.S. of (1) = R.H.S. of (1)

Statement is true for $n = 1$, so C-1 is satisfied.

C-2 Suppose statement is true for $n = k$

i.e. $1 + 5 + 9 + \ldots\ldots + (4k - 3) = k\,(2k - 1)$

C-3 Now we want to prove for $n = k + 1$

i.e. $1+5+9+\ldots\ldots+(4k - 3)+(4k + 1) = (k+1)\,(2k + 1)$

$$\begin{aligned}
\text{L.H.S.} &= 1 + 5 + 9 + \ldots\ldots + (4k - 3) + (4k + 1) \\
&= \{1 + 5 + 9 + \ldots\ldots + (4k - 3)\} + (4k + 1) \\
&= k\,(2k - 1) + 4k + 1 \\
&= 2k^2 - k + 4k + 1 \\
&= 2k^2 + 3k + 1 \\
&= 2k^2 + 2k + k + 1 \\
&= 2k\,(k + 1) + 1(k + 1) \\
&= (k+1)(2k + 1) = \text{R.H.S.}
\end{aligned}$$

Truth for $n = k$, Implies truth for $n = k + 1$

Hence by the principle of mathematical induction the statement is true for all positive integral values of n.

Q.11 Prove the formula for n = 1, 2

$$\mathbf{1 + 2 + 4 + \ldots + 2^{n-1} = 2^n - 1}$$

(Gjw 19 G-II)(Sgd, Swl 19)

Ans. For $n = 1$, $1 = 2^1 - 1$ or $1 = 1$ True

For $n = 2$, $1 + 2 = 2^2 - 1 \Rightarrow 3 = 4 - 1$ True

For $n = 3$, $1 + 2 + 4 = 2^3 - 1 \Rightarrow 7 = 8 - 1$ True

Q.12 Prove the formula for n = 1 and n = 2:

$$\mathbf{1 + 4 + 7 + \ldots + (3n - 2) = \frac{n(3n-1)}{2}.}$$

(Mtn 19 G-I)

Ans. Put $n = 1$

$1 = \dfrac{1(3(1)-1)}{2} \Rightarrow 1 = \dfrac{1(3-1)}{2}$

$\Rightarrow$ $1 = 1$ True

Put $n = 2$

$1 + 4 = \dfrac{2(3(2)-1)}{2} \Rightarrow 5 = \dfrac{2(6-1)}{2}$

$\Rightarrow$ $5 = 5$ True

LONG QUESTIONS

Q.1 Use the mathematical induction to prove that

$$1 + 4 + 7 + \ldots + (3n - 2) = \frac{n(3n-1)}{2}.$$

(Lhr 17 G-II)

Ans. See Short Question 17

Q.2 Use mathematical induction to prove the formula for every positive integer n.

$$1 + \frac{1}{2} + \frac{1}{4} + \text{———} + \frac{1}{2^{n-1}} = 2\left[1 - \frac{1}{2^n}\right]$$

(Mtn 13 G-I)

Ans. Given that: $\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \ldots + \dfrac{1}{3^n} = \dfrac{1}{2}\left[1 - \dfrac{1}{3^n}\right]$

Case-1: For $n = 1$, so $\dfrac{1}{3} = \dfrac{1}{2}\left[1 - \dfrac{1}{3}\right] = \dfrac{1}{2}\left[\dfrac{3-1}{3}\right]$

$= \dfrac{1}{2} \cdot \dfrac{2}{3} \Rightarrow \dfrac{1}{3} = \dfrac{1}{3}$ (True)

Case-2: Suppose that it is true for n = k.

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots+\frac{1}{3^k} = \frac{1}{2}\left[1-\frac{1}{3^k}\right] \qquad \ldots\ldots (1)$$

Adding $\frac{1}{3^{k+1}}$ on both sides of eq. (1):

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots+\frac{1}{3^k}+\frac{1}{3^{k+1}} = \frac{1}{2}\left[1-\frac{1}{3^k}\right]+\frac{1}{3^{k+1}}$$

$$= \frac{1}{2}-\frac{1}{2.3^k}+\frac{1}{3.3^k} = \frac{1}{2}+\frac{1}{3.3^k}-\frac{1}{2.3^k} = \frac{1}{2}+\frac{2-3}{6.3^k}$$

$$= \frac{1}{2}-\frac{1}{6.3^k} = \frac{1}{2}\left[1-\frac{1}{3.3^k}\right] = \frac{1}{2}\left[1-\frac{1}{3^{k+1}}\right]$$

Which is true for n = k + 1, so C – 2 is satisfied. Hence given statement is true for all positive integers.

Q.3 Use mathematical induction to prove that the following formula holds for every positive integer n. (Mtn 18 G-II)

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\ldots+\frac{1}{(3n-1)(3n+2)} = \frac{n}{2(3n+2)}$$

Ans. Given that:

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\ldots+\frac{1}{(3n-1)(3n+2)}$$

$$= \frac{n}{2(3n+2)}$$

Case-1: For n = 1, so $\frac{1}{2\times5} = \frac{1}{2(3(1)+2)}$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{2(3+2)} = \frac{1}{2\times5} \Rightarrow \frac{1}{10} = \frac{1}{10} \text{ (True)}$$

Case-2: Suppose that it is true for n = k.

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\ldots+\frac{1}{(3k-1)(3k+2)}$$

$$= \frac{k}{2(3k+2)} \qquad \ldots\ldots (1)$$

Adding $\frac{1}{(3k+2)(3k+5)}$ on both sides of eq. (1):

$$\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\ldots+\frac{1}{(3k-1)(3k+2)}+\frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{2(3k+2)}+\frac{1}{(3k+2)(3k+3)} = \frac{1}{3k+2}\left[\frac{k}{2}+\frac{1}{3k+5}\right]$$

$$= \frac{1}{3k+2}\left[\frac{3k^2+5k+2}{2(3k+5)}\right]$$

$$= \frac{1}{3k+2}\cdot\frac{3k^2+3k+2k+2}{2(3k+3+2)}$$

$$= \frac{1}{3k+2}\cdot\frac{3k(k+1)+2(k+1)}{2[3(k+1)+2]}$$

$$= \frac{1}{3k+2}\cdot\frac{(k+1)(3k+2)}{2[3(k+1)+2]} = \frac{k+1}{2[3(k+1)+2]}$$

Which is true for n = k + 1, so C – 2 is satisfied. Hence given statement is true for all positive integers.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 **$3^n < n!$ for integral values:** (Mtn 08)
(Lhr 10 G-I, 12 G-II)

(a) $n > 6$ (b) $n \geq 5$
(c) $n < 4$ (d) $n \leq 3$

Q.2 $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\ldots\ldots+\binom{n+2}{3} =$
(Mtn 15 G-I) (DGK 12)(Mtn 09 G-I)

(a) $\binom{n+3}{3}$ (b) $\binom{n+4}{3}$
(c) $\binom{n+4}{2}$ (d) $\binom{n+3}{4}$

Q.3 **$n! > 2^n - 1$ is true for integral values of n >:**
(Mtn 10 G-I)

(a) 2 (b) 3
(c) 4 (d) 1

Q.4 **If n is integer, then $n^2 > n + 3$ is true for:**
(Mtn 13, 17 G-I, 10 G-II)(Fbd 12) (Swl 13)
(Gjw 13)(Rwp 17 G-II)(Mtn 18 G-II)(Sgd 18)

(a) $n \geq 3$ (b) $n \geq 2$
(c) $n \geq 1$ (d) $n < 3$

Q.5 **If n is any +ve integer then $4^n > 3^n + 4$ is true for all values if n:** (Mtn 18 G-II)
(Lhr 14, 18 G-I)(DGK 14) (Bpr 12 G-I, 19)

(a) $n > 2$ (b) $n \geq 2$
(c) $n < 2$ (d) $n \leq 2$

Q.6 **If n is positive integer then $n! > 3^{n-1}$ is true when:** (Rwp 11)

(a) $n \leq 3$ (b) $n \geq 3$
(c) $n \geq 5$ (d) $n \geq 1$

Q.7 **If n is +ve integer, then $n! > n^2$ is true when:**
(Rwp 13)(Lhr 11 G-II)(Sgd 13)(Gjw 11)(Fbd 19)

(a) $n \geq 4$ (b) $n \geq 3$
(c) $n < 4$ (d) $n = 2$

Q.8 **$n! > n^2$ is true for:** (Sgd 13) (Gjw 11)(Gjw 18)

(a) $n \geq 1$ (b) $n \geq 3$
(c) $n \geq 2$ (d) $n \geq 4$

Q.9 **$n! > 2^n - 1$ is true if:** (Rwp 14)
(Lhr 13 G-II)(D.G.K 17 G-I)(Gjw 12)
(Lhr 12 G-I)(DGK 14 G-II)(Fbd 18)

(a) $n \geq 4$ (b) $n < 4$
(c) $n = 2$ (d) $n = 1$

Q.10 **If n a positive integer, then $5^n - 2^n$ is divisible by:** (Fbd 11) (Mtn 09 G-II)

(a) 2 (b) 5
(c) 3 (d) 6

Q.11 The inequalities $4^n > 3^n + 4$ is true for n equal to: (Lhr 11 G-II)
(a) 2 (b) 1
(c) -1 (d) -2

Q.12 If n is any positive integer, then
$$a + (a + d) + (a + 2d) + \ldots + [a + (n-1)d] =$$
(a) $[2a + (n-1)d]$ (b) $\frac{n}{2}[2a + (n+1)d]$
(c) $\frac{n}{2}[a + (n-1)d]$ (d) $\frac{n}{2}[2a + (n-1)d]$

Q.13 If n is any positive integer, then
$$\frac{1}{3}+\frac{1}{9}+\ldots+\frac{1}{3^n} =$$
(a) $\frac{1}{3}\left(1-\frac{1}{3^n}\right)$ (b) $\frac{1}{4}\left(1-\frac{1}{3^n}\right)$
(c) $\frac{1}{3}\left(1-\frac{1}{2^n}\right)$ (d) $\frac{1}{2}\left(1-\frac{1}{3^n}\right)$

Q.14 If n is any positive integer, then $r + r^2 + r^3 + \ldots + r^n$ = -----------------
(a) $\frac{(1-r^n)}{1-r}$; $(r \neq 1)$ (b) $\frac{r(1-r^n)}{1-r}$; $(r \neq 1)$
(c) $\frac{r(1+r^n)}{1-r}$; $(r \neq 1)$ (d) $\frac{r(1-r^n)}{1-r^n}$; $(r \neq 1)$

Q.15 If n is a positive integer, then
$$2^1 + 2^2 + 2^3 + \ldots + 2^n =$$
(a) $2(2^{n+1} - 1)$ (b) $2(2^n + 1)$
(c) $2(2^n - 1)$ (d) $2(3^n - 1)$

Q.16 If n is a positive integer, then
$$\binom{5}{5} + \binom{6}{5} + \binom{7}{5} + \ldots\ldots + \binom{n+4}{5} =$$
(a) $\binom{n+5}{4}$ (b) $\binom{n+5}{5}$
(c) $\binom{n+4}{4}$ (d) $\binom{n+5}{6}$

Q.17 If n is a positive integer, then $n! > 3^{n-1}$ is true for all
(a) $n \geq 5$ (b) $n > 3$
(c) $n \geq 3$ (d) $n > 5$

Q.18 The inequality $n! > 2^n + 1$ is valid of. (D.G.K 17 G-II)
(a) $n < 4$ (b) $n \geq 4$
(c) $n \leq 3$ (d) $n = 3$

Q.19 $n^2 - n + 41$ represents a prime number for all $n \in N$, where (Mtn 17 G-II)
(a) $n \geq 100$ (b) $n \geq 200$
(c) $n \leq 40$ (d) $n \leq 50$

Q.20 $x - y$ is a factor of: (Mtn 09 G-I)
(a) $x^n + y^n$ (b) $x^{2n} + y^{2n}$
(c) $x^{n+1} + y^{n+1}$ (d) $x^n - y^n$

Q.21 Which one is not divisible by 2 for all the integral values of n. (Lhr 18 G-II)
(a) $n^3 - n$ (b) $5^n - 1$
(c) $5^n - 2^n$ (d) $n^2 + n$

Q.22 If n is any positive integer then $2^n > 2(n+1)$ is true for all: (DGK 19)
(a) $n \leq 3$ (b) $n < 3$
(c) $n \geq 3$ (d) $n > 3$

EXERCISE 8.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 State the Binomial theorem. (D.G.K 16 G-I)

Ans. Let n be a positive integer and
$$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \text{——} + \binom{n}{r}a^{n-r}x^r + \text{——} + \binom{n}{n}x^n$$
Where a and x are real numbers.

Q.2 Calculate by means of binomial theorem $(2.02)^4$. **(Mtn 09 G-I)(Lhr 11 G-I) (Fbd 17)(Rwp 17 G-I)(Gjw 19 G-II)**

Ans. $(2.02)^4 = (2 + 0.02)^4$
$$= \binom{4}{0}(2)^4(0.02)^0 + \binom{4}{1}(2)^3(0.02)^1 + \binom{4}{2}(2)^2(0.02)^2 + \binom{4}{3}(2)^1(0.02)^3 + \binom{4}{4}(2)^0(0.02)^4$$
$= 1 \times 16 + 4\,(8)\,(0.02) + 6\,(4)\,(0.0004) + 4\,(2)\,(0.000008) + 1(0.00000016)$
$= 16 + 0.64 + 0.0096 + 0.000064 + 0.00000016$
$= 16.64966416$

Q.3 Calculate $(0.97)^3$ by means of binomial theorem. **(Lhr 10, 16, 17 G-I)(A.J.K 17)(Fbd 18)(Mtn 19 G-I)**

Ans. $(0.97)^3 = (1 - 0.03)^3$
$$= \binom{3}{0}(1)^3(-0.03)^0 + \binom{3}{1}(1)^2(-0.03)^1 + \binom{3}{2}(1)^1(-0.03)^2 + \binom{3}{3}(1)^0(-0.03)^3$$
$= 1 + 3(-0.03) + 3(0.0009) - 0.000027$
$= 1 - 0.09 + 0.0027 - 0.000027$
$= 1.0027 - 0.090027 = 0.912673$

Q.4 Calculate $(9.98)^4$ by means of binomial theorem.
(Gjw 16)(Mtn 18 G-I)

Ans. $(9.98)^4 = (10-0.02)^4$

$$= \binom{4}{0}10^4 + \binom{4}{1}10^3(-0.02) + \binom{4}{2}(10)^2(-0.02)^2$$
$$+ \binom{4}{3}(10)(-0.02)^3 + \binom{4}{4}(-0.02)^4$$
$$= 1.(10000)+4.(1000)(-0.02)+\frac{4.3}{1.2}(100)(0.0004)$$
$$+ 4.10(-0.000008) + 1.(0.00000016)$$
$$= 10000 - 890 + 0.24 - 0.00032 + 0.00000016$$
$$= 9920.23968$$

Q.5 Using binomial theorem expand $(9.9)^5$.
(Sgd 18, 19)

Ans. Given that: $(9.9)^5 = (10-0.1)^5$

$$= [10 + (-0.1)]^5$$
$$= \binom{5}{0}(10)^5(-0.1)^0 + \binom{5}{1}(10)^4(-0.1)^1$$
$$+ \binom{5}{2}(10)^3(-0.1)^2 + \binom{5}{3}(10)^2(-0.1)^3$$
$$+ \binom{5}{3}(10)^1(-0.1)^4 + \binom{5}{5}(10)^0(-0.1)^5$$
$$= 1 \times 100000 \times 1 + 5 \times 10000 \times (-0.1)$$
$$+ 10 \times 1000 \times 0.01 + 10 \times 100 \times (-0.001)$$
$$+ 5 \times 10 \times 0.0001 + 1 \times 1 \times (-0.00001)$$
$$= 100000 - 5000 + 100 - 1 + 0.005 - 0.0001$$
$$= 95099.00499$$

Q.6 Expand $(a + 2b)^4$ using binomial theorem.
(Swl 17)

Ans. $(a + 2b)^3$

$$= \binom{3}{0}a^3 + \binom{3}{1}a^2(2b) + \binom{3}{2}a^1(2b^2) + \binom{3}{3}(2b)^3$$
$$= a^3 + 3.a^2(2b) + \frac{3.2}{1.2}a(4b^2) + 1.(8b^3)$$
$$= a^3 + 6a^2b + 12ab^2 + 8b^3$$

Q.7 Find the general term of $(3-2x)^7$
(Gjw 10)(Lhr 18 G-I)

Ans. Let T_{r+1} be the general term. Then

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$
$$= \binom{7}{r}(3)^{7-r}(-2x)^r$$
$$= \binom{7}{r}3^{7-r}.(-2)^r(x)^r$$

Q.8 Calculate r for finding the term independent of x in the expansion of $\left(\frac{x}{2}+\frac{2}{x^2}\right)^{12}$ **(Mtn 12 G-I)**

Ans. Let T_{r+1} be the term independent of x in the expansion of $\left(\frac{x}{2}+\frac{2}{x^2}\right)^{12}$, then

$$T_{r+1} = \binom{12}{r}\left(\frac{x}{2}\right)^{12-r}\left(\frac{2}{x^2}\right)^r$$
$$= \binom{12}{r}\frac{x^{12-r}}{2^{12-r}}.2^r.x^{-2r} = \binom{12}{r}2^{2r-12}x^{12-3r}$$

As the term is independent of x, so exponent of x, will be zero.

That is, $12 - 3r = 0 \Rightarrow r = 4$

Q.9 Find the fifth term in the expansion of $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$. **(Lhr 10 G-I)**
(Rwp 17 G-II)(D.G.K 17 G-I)(Swl 19)

Ans. See Long Question 6

Q.10 Find 6^{th} term in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$
(Bpr 11 G-I, 16, 18, 19)(Rwp 16)(DGK 19)

Ans. We know that the rth term in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$ is

$$T_r = \binom{n}{r-1}a^{n-(r-1)}b^{r-1}$$

For 6^{th} term, put $r = 6$, we get

$$T_6 = \binom{10}{5}(x^2)^{10-5}\left(-\frac{3}{2x}\right)^5$$
$$T_6 = \frac{10.9.8.7.6}{5.4.3.2.1}x^{2(5)}\left(-\frac{3}{2}\right)^5\frac{1}{x^5}$$
$$T_6 = 252\,x^{10-5}\left(-\frac{243}{32}\right)$$
$$T_6 = -1913.625\,x^5$$

Q.11 Find the term involving x^4 in the expansion of $(3-2x)^7$. **(Rwp 11 G-I)(Rwp 18)**

Ans. See Long Question 7

Q.12 Find the 6^{th} term in the expansion of $\left(x-\frac{2}{x}\right)^{10}$
(Fbd 16)

Ans. See Long Question 4

Q.13 Find the coefficient of x^5 in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$ **(Lhr 13 G-I)(Mtn 16 G-II)**

Ans. See Long Question 3

Q.14 Show that $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + ... + n\binom{n}{n} = n2^{n-1}$. **(Lhr 17 G-II)**

Ans. L.H.S

$$= \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + ... + n\binom{n}{n}$$

$$= n + 2 . \frac{n(n-1)}{2!} + 3 . \frac{n(n-1)(n-2)}{3!} + ... + n.1$$

$$= n\left[1 + n - 1 + \frac{(n-1)(n-2)}{2!} + ... + 1\right]$$

$$= n\left[\binom{n-1}{0} + \binom{n-1}{1} + \binom{n-1}{2} + ... + \binom{n-1}{n-1}\right]$$

$$= n\,2^{n-1} = \text{R.H.S}$$

Q.15 Find the fifth term of $\left(\frac{3x}{2} - \frac{1}{3x}\right)^{11}$. (Gjw 19 G-I)

Ans. $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{11} = \left(\frac{3}{2}x + \frac{-1}{3x}\right)^{11}$

Here $n = 11$, $a = \frac{3}{2}x$, $b = \frac{-1}{3x}$, $r + 1 = 5 \Rightarrow r = 4$

$$\therefore \quad T_5 = \binom{n}{4} a^{n-4} x^4 = \binom{11}{4}\left(\frac{3}{2}x\right)^{11-4}\left(\frac{-1}{3x}\right)^4$$

$$= 330\left(\frac{3}{2}x\right)^7 \frac{(-1)^4}{3^4x^4}$$

$$= 330\frac{3^7}{2^7}x^7 . \frac{1}{3^4x^4} = 330 \times \frac{3^3}{2^7}x^3$$

$$= 330 \times \frac{27}{128}x^3 = 165 \times \frac{27}{64}x^3$$

$$\boxed{T_5 = \frac{4455}{64}x^3}$$

Q.16 Find the fifth term in the expansion of $\left(\frac{3}{2}x - \frac{1}{3x}\right)$. **(Swl 19)**

Ans. Here $n = 11$, $a = \frac{3}{2}x$

$b = \frac{-1}{3x}$, $r+1=5 \Rightarrow r=4$

$$\therefore \quad T_5 = \binom{n}{4} a^{n-4} x^4 = \binom{11}{4}\left(\frac{3}{2}x\right)^{11-4}\left(\frac{-1}{3x}\right)^4$$

$$= 330\left(\frac{3}{2}x\right)^7 \frac{(-1)^4}{3^4x^4}$$

$$= 330\frac{3^7}{2^7}x^7 . \frac{1}{3^4x^4} = 330 \times \frac{3^3}{2^7}x^3$$

$$= 330 \times \frac{27}{128}x^3 = 165 \times \frac{27}{64}x^3$$

$$\boxed{T_5 = \frac{4455}{64}x^3}$$

LONG QUESTIONS

Q.1 Calculate by binomial theorem $(0.97)^3$. **(Lhr 13 G-II)**

Ans. See Short Question 3.

Q.2 Find the coefficient of x^5 in the expansion of $\left(x^2 - \frac{3}{2x}\right)$. (D.G.K 14 G-I, II)(Rwp 16)(Sgd 13)

Ans. Given that: $\left(x^2 - \frac{3}{2x}\right)^{10}$

Here, we can take: $a = x^2$, $x = \frac{-3}{2x}$, $n = 10$

$$\therefore \quad T_{r+1} = \binom{n}{r} a^{n-r} x^r = \binom{10}{r}(x^2)^{10-r}\left(\frac{-3}{2x}\right)^r$$

$$= \binom{10}{r} x^{20-2r} \frac{(-3)^r}{(2)^r x^r}$$

$$= \binom{10}{r} \frac{(-3)^r}{(2)^r} x^{20-3r} \qquad ...... (1)$$

For the coefficient of the term involving x^5, so

$20 - 3r = 5 \Rightarrow 20 - 5 = 3r \Rightarrow 15 = 3r$

$\Rightarrow r = \frac{15}{3} = 5$

Putting value of r in eq. (1):

$$T_6 = \binom{10}{5} \frac{(-3)^5}{(2)^5} x^5 = 252 . \frac{-243}{32}$$

$$x^5 = 63 . \frac{-243}{8}x^5 = \frac{-15309}{8}x^5$$

$$\boxed{\text{Coefficient} = \frac{-15309}{8}}$$

Q.3 Find the term independent of x in $\left(x - \frac{2}{x}\right)^{10}$. **(Fbd 16, 18)(D.G.K 15 G-II)**

Ans. (i) Given that: $\left(x - \frac{2}{x}\right)^{10}$

Here, we can take: $a = x$, $x = \frac{-2}{x}$, $n = 10$

$$\therefore \quad T_{r+1} = \binom{n}{r} a^{n-r} x^r = \binom{10}{r} x^{10-r}\left(\frac{-2}{x}\right)^r$$

$$= \binom{10}{r} x^{10-r} \frac{(-2)^r}{x^r}$$

$$= \binom{10}{r} (-2)^r x^{10-2r} \qquad ...... (1)$$

For the term independent of x, so

$10 - 2r = 0 \Rightarrow 10 = 2r \Rightarrow r = \frac{10}{2} = 5$

Putting value of r in eq. (1):

$$T_6 = \binom{10}{5} (-2)^5 x^0 = 252(-32)(1) = -8064$$

Q.4 Determine the middle term in $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$.

(D.G.K 17 G-I)(Mtn 18 G-I)(Rwp 17 G-II)(Bpr 19)

Ans. Given that: $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$

Here, we can take: $a = \frac{3}{2}x,\ x = \frac{-1}{3x},\ n = 11$

$$\text{Middle term} = \left(\frac{n+1}{2}\right)^{th}\text{term} + \left(\frac{n+3}{2}\right)^{th}\text{term}$$

$$= \left(\frac{11+1}{2}\right)^{th}\text{term} + \left(\frac{11+3}{2}\right)^{th}\text{term}$$

$$= 6^{th}\text{ term} + 7^{th}\text{ term} = \binom{n}{5}a^{n-5}x^5 + \binom{n}{6}a^{n-6}x^6$$

$$= \binom{11}{5}\left(\frac{3x}{2}\right)^{11-5}\left(\frac{-1}{3x}\right)^5 + \binom{11}{6}\left(\frac{3x}{2}\right)^{11-6}\left(\frac{-1}{3x}\right)^6$$

$$= \binom{11}{5}\left(\frac{3x}{2}\right)^6\frac{(-1)^5}{(3x)^5} + \binom{11}{6}\left(\frac{3x}{2}\right)^5\frac{(-1)}{(3x)^6}$$

$$= \binom{11}{5}\frac{(3x)^6}{(2)^6}\cdot\frac{-1}{(3x)^5} + \binom{11}{6}\frac{(3x)^5}{(2)^5}\cdot\frac{1}{(3x)^6}$$

$$= \binom{11}{5}\frac{-(3x)^1}{64} + \binom{11}{6}\frac{1}{32(3x)!}$$

$$= 462\times\frac{-3x}{64} + 462\times\frac{1}{96x}$$

$$= 231\cdot\frac{-3x}{32} + 231\cdot\frac{1}{48x} = -\frac{693}{32}x + \frac{231}{48x}$$

$$= -\frac{693}{32}x + \frac{77}{16x}$$

Q.5 Find the term involving x^4 in the expansion of $(3-2x)^7$. **(Lhr 17 G-I)(Rwp 18)**

Ans. Given that: $(3-2x)^7$

Here, we can take: $a = 3,\ x = -2x,\ n = 7$

$\therefore\ T_{r+1} = \binom{n}{r}a^{n-r}x^r = \binom{7}{4}(3)^{7-r}(-2x)^r = \binom{7}{r}(3)^{7-r}(-2)^r x^r \ldots\ldots (1)$

For the term involving x^4, so $r = 4$

Putting the value of r in eq. (1):

$$T_5 = \binom{7}{4}(3)^{7-4}(-2)^4x^4 = 35(3)^3(16)x^4$$

$$= 35(27)(16)x^4 = 15120x^4$$

Q.6 Find the coefficient of term involving x^{-1} in the expansion of $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$. **(Bpr 19)**

Ans. $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11} = \left(\frac{3}{2}x+\frac{-1}{3x}\right)^{11}$

Here $n = 11,\ a = \frac{3}{2}x,\ b = \frac{-1}{3x}$

$$\therefore\ T_{r+1} = \binom{n}{r}a^{n-r}b^r = \binom{11}{r}\left(\frac{3}{2}x\right)^{11-r}\left(\frac{-1}{3x}\right)^r$$

$$= \binom{11}{r}\frac{3^{11-r}}{2^{11-r}}x^{11-r}\cdot\frac{(-1)^r}{3^r\cdot x^r}$$

$$= \binom{11}{r}\frac{3^{11-2r}}{2^{11-r}}(-1)^r x^{11-2r} \ldots\ldots (1)$$

To find term involving x^{-1}

put $11-2r = -1 \Rightarrow 2r = 11+1 = 12$

$\Rightarrow\ r = \frac{12}{2} = 6$

Putting value of r in eq. (1):

$$T_{6+1} = \binom{11}{6}\frac{3^{11-12}}{2^{11-6}}(-1)^6x^{-1}$$

$$\Rightarrow\ T_7 = \binom{11}{6}\frac{3^{-1}}{2^5}(1)x^{-1} = 462\times\frac{1/3}{32}x^{-1}$$

$$= 231\times\frac{1}{3\times16}x^{-1} = 77\times\frac{1}{16}x^{-1} = \frac{77}{16}x^{-1}$$

$$\boxed{\text{Coefficient of term involving } x^{-1} = \frac{77}{16}}$$

Q.7 Find the term in the expansion of $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$ involving x^5. **(Fbd 19 G-II)**

Ans. $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11} = \left(\frac{3}{2}x+\frac{-1}{3x}\right)^{11}$

Here $n = 11,\ a = \frac{3}{2}x,\ b = \frac{-1}{3x}$

$$\therefore\ T_{r+1} = \binom{n}{r}a^{n-r}b^r = \binom{11}{r}\left(\frac{3}{2}x\right)^{11-r}\cdot\left(\frac{-1}{3x}\right)^r$$

$$= \binom{11}{r}\frac{3^{11-r}}{2^{11-r}}\cdot x^{11-r}\cdot\frac{(-1)^r}{3^r\cdot x^r}$$

$$= \binom{11}{r}\frac{3^{11-2r}}{2^{11-r}}\cdot(-1)^r x^{11-2r} \ldots\ldots (1)$$

To find term involving x^5

put $11-2r = 5 \Rightarrow 2r = 11-5 = 6$

$\Rightarrow\ r = \frac{6}{2} = 3$

$$T_{3+1} = \binom{11}{3}\frac{3^{11-6}}{2^{11-3}}(-1)^3x^5$$

$$\Rightarrow\ T_4 = 165\frac{3^5}{2^8}(-1)x^5 = 165\times\frac{243}{256}(-1)x^5$$

$$\boxed{T_4 = -\frac{40095}{256}x^5}$$

Q.8 Find the coefficient of x^5 in the expansion of $\left(x^2 - \dfrac{3}{2x}\right)^{10}$. **(Gjw 19 G-I)(Sgd 19)**

Ans. Here $a = x^2$, $b = \dfrac{-3}{2x}$, $n = 10$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{10}{r} (x^2)^{10-r} \left(\frac{-3}{2x}\right)^r = \binom{10}{r} x^{20-2r} \left(\frac{-3}{2}\right)^r . x^{-r}$$

$$= \binom{10}{r} x^{20-2r-r} \left(\frac{-3}{2}\right)^r = \binom{10}{r} x^{20-3r} \left(\frac{-3}{2}\right)^r \quad \text{— (1)}$$

To find term involving x^5

Put $20 - 3r = 5 \Rightarrow 20 - 5 = 3r \Rightarrow 3r = 15$

$\Rightarrow$ $r = 5$ in eq. (1)

$$T_6 = \binom{10}{5} x^5 \left(\frac{-3}{2}\right)^5 = 252 \frac{-243}{32} x^5 = \frac{-15309}{8} x^5$$

$$\boxed{\text{Thus co-efficient of } x^5 \text{ is } \frac{-15309}{8}}$$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If n is even then the middle term in expansion of $(a + x)^n$ is: (Rwp 15) (Mtn 08) (D.G.K 15 G-II)(Sgd 17)

(a) $\left(\frac{n}{2} + 1\right)$th term (b) $\left(\frac{n}{2} - 1\right)$th term
(c) $\frac{n}{2}$ th term (d) $\frac{n+1}{2}$ th term

Q.2 The sum of exponent of a, b in every term in the expansion of $(a + b)^n$ is: (Gjw 14)(Mtn 10, 16 G-I)

(a) Zero (b) 1
(c) n (d) 2n

Q.3 $\binom{n}{r} a^{n-r} b^r$ is ______ term of $(a + b)^n$:

(a) $(r + 1)^{th}$ (b) $(r + 2)^{th}$
(c) $(r + 3)^{th}$ (d) rth

Q.4 The middle term in the expansion of $(1 + 2x)^6$ is: (Lhr 15 G-I)

(a) Third (b) Fourth
(c) Fifth (d) Sixth

Q.5 In expansion $(a + b)^{16}$, middle term will be:

(a) 11 (b) 12
(c) 8 (d) 9

Q.6 The number of terms in the expansion of $(1 - x)^7$ is: (Mtn 12 G-I)

(a) 8 (b) 7
(c) 9 (d) 10

Q.7 General term of the expansion $(a + b)^n$ is: (Mtn 13 G-I)(Gjw 12) (Bpr 11 G-I)

(a) $\binom{n}{r} a^{n-r} b^{n-r}$ (b) $\binom{n}{r} a^{n-r} b^r$
(c) $\binom{n}{r} a^n b^{n-r}$ (d) $a^n b^r$

Q.8 If n is even then the middle term in the expansion of $(a + b)^n$ is: (Lhr 12 G-II)(Mtn 13 G-II)(Fbd 16)

(a) $\left(\frac{n}{2}\right)$ th term (b) $\left(\frac{n+2}{2}\right)$ th term
(c) $\left(\frac{n+1}{2}\right)$ th term (d) $\left(\frac{n+3}{2}\right)$ th term

Q.9 The co-efficient of last term in expansion of $(a-b)^5$ is: (Mtn 14 G-II) (DGK 13)

(a) 1 (b) −1
(c) 0 (d) 2

Q.10 In the expansion of $(a + b)^7$, then second term is: (D.G.K 13, 14 G-I)(Gjw 15)

(a) a^7 (b) $7a^6b$
(c) $7ab^6$ (d) 8

Q.11 In the expansion of $(a + b)^n$ by binomial theorem n must by: (D.G.K 13 G-II)

(a) Real no. (b) Positive integer
(c) Integer (d) Complex no.

Q.12 The general term of the binomial expansion $(a+x)^n$, $n \in N$ is: (Lhr 13 G-II)(Bpr 11 G-I)(Rwp 16)

(a) ${}^nC_1\, a^n x^r$ (b) ${}^nC_2\, a^r x^r$
(c) ${}^nC_r\, (ax)^{n-r}$ (d) ${}^nC_r\, a^{n-r} x^r$

Q.13 In the expansion of $\left(\frac{x}{2} - \frac{2}{x^2}\right)^6$, the co-efficient of x^3 is: (Bpr 12)

(a) $\frac{-3}{8}$ (b) $\frac{-8}{3}$
(c) −3 (d) −8

Q.14 Sum of odd co-efficient in the expansion of $(1+x)^7$ is. (Gjw 16)(Sgd 17) (Lhr 15 G-II, 16 G-I)(Mtn 15 G-II)(Rwp 13)

(a) $1+\frac{1}{2}x$ (b) $1-\frac{1}{2}x$
(c) $1-x$ (d) $1-2x$

Q.15 The sum of even co-efficients in the expansion of $(a+x)^n$ is ———— when a = 1 (Gjw 18) (Bpr 13) (Rwp 11)

(a) 2^n (b) 2^{n+1}
(c) 2^{n-1} (d) 2^{2n}

Q.16 General term of the expansion $(a+b)^n$ is: (Fbd 12) (Swl 13)

(a) $\binom{n}{0}a^{n-r}b^{n-r}$ (b) $\binom{n}{r}a^{n-r}b^r$
(c) $\binom{n}{r}a^n b^{n-r}$ (d) $a^n b^r$

Q.17 $(a+b)^6$ contains: (Rwp 11)

(a) 5 terms (b) 6 terms
(c) 7 terms (d) 4 terms

Q.18 In expansion $(x+y)^{10}$, middle term is: (Rwp 12)

(a) T_6 (b) T_5
(c) T_7 (d) T_4

Q.19 The index of $\left(2a-\frac{x}{a}\right)^4$ is: (Fbd 11)

(a) 2 (b) 3
(c) 4 (d) 5

Q.20 The number of terms in the expansion of $(a+b)^{20}$ is: (Mtn 14 G-I)

(a) 18 (b) 19
(c) 20 (d) 21

Q.21 The sum of binomial co-efficients in th expansion of $(1+x)^4$:(Rwp 12, 15, 16) (Bpr 14)

(a) 8 (b) 10
(c) 16 (d) 32

Q.22 The number of terms in the binomial expansion of $(a+b)^{15}$ are: (Swl 14)

(a) 15 (b) 14
(c) 16 (d) 13

Q.23 The number of terms in expansion of $(a+b)^8$: (Bpr 14)

(a) 8 (b) 9
(c) 10 (d) 11

Q.24 The number of terms in the expansion of $(2a+b)^{13}$ are: (Lhr 10 G-I)(Fbd 18)

(a) 12 (b) 13
(c) 14 (d) 15

Q.25 The co-efficient of last term in expansion of $(a-b)^5$ or $(x-y)^5$: (Gjw 15)(Mtn 18 G-II)

(a) 1 (b) −1
(c) 0 (d) 2

Q.26 The number of term in the expansion of $(a+b)^7$ is: (Lhr 13 G-I)

(a) 4 (b) 6
(c) 7 (d) 8

Q.27 The middle term in the expansion of $(3-2x)^{10}$ is: (Lhr 12 G-I)

(a) T_4 (b) T_5
(c) T_6 (d) T_7

Q.28 Sum of coefficient in the expansion of $(1+x)^n$: (DGK 15 G-II) (Gjw 11)(Mtn 09 G-II)(Lhr 14 G-I)

(a) 2^{n-1} (b) 2^n
(c) 2^{n+1} (d) n^2

Q.29 Total terms in the expansion of $(a+x)^n$ are: (Gjw 11)

(a) n (b) n + 1
(c) n! (d) (n + 1)!

Q.30 The number of terms in the expansion of $(x-3)^{10}$ is: (Lhr 11 G-I)

(a) 10 (b) 11
(c) 12 (d) 13

Q.31 Middle term in expansion of $(x+y)^{21}$ is:

(a) 10^{th}, 11^{th} (b) 9^{th}, 10^{th}
(c) 11^{th}, 12^{th} (d) 12^{th}, 13^{th}

Q.32 Number of terms in the expansion of $(1+x)^{2n+1}$ is: (Lhr 17 G-I)

(a) 2n + 1 (b) 2n
(c) 2n + 2 (d) 3n + 1

Q.33 The co-efficient of last term in the expression of $(a+b)^5$ is (DGK 15 G-I)

(a) 1 (b) −1
(c) 0 (d) 5

Q.34 In the expansion of $(a+2b)^5$ the coefficient of b^3 is

(a) $10a^4$ (b) $40a^3$
(c) $80a$ (d) $80a^2$

Q.35 In the expansion of $\left(3a-\frac{x}{3a}\right)^4$ the coefficient of x^3 is

(a) 81 (b) 36
(c) 6 (d) $\frac{4}{9a^2}$

Q.36 In the expansion of $\left(2x^3 - \dfrac{1}{2a}\right)^6$ the coefficient of x^{12} is

(a) 64 (b) $\dfrac{-96}{a}$

(c) $\dfrac{60}{a^2}$ (d) $\dfrac{-20}{a^5}$

Q.37 In the expansion of $(x + 2y)^5$, the last term is:

(a) $64y^6$ (b) $32y^6$

(c) $32y^5$ (d) None of these

Q.38 The middle term in the expansion of $\left(x + \dfrac{1}{x}\right)^{10}$ is

(a) ${}^{10}C_6$ (b) ${}^{10}C_5$

(c) ${}^{10}C_7$ (d) None of these.

Q.39 The general term in the expansion of $\left(x + \dfrac{1}{x}\right)^n$ is:

(a) ${}^nC_r\, x^{n-2r}$ (b) ${}^nC_r\, x^{n-r}$

(c) ${}^nC_r\, x^{n+2r}$ (d) None of these.

Q.40 Total number of terms in the expansion of $(a + b)^5 + (a - b)^5$ after simplification are:

(a) 3 (b) 6

(c) 9 (d) 5

Q.41 In the expansion of $(a + b)^n$, the sum of two indices of a and b is equal to

(a) $n + 1$ (b) $n - 1$

(c) n (d) None of these

Q.42 The middle term in the expansion of $(a + b)^{20}$ is

(a) 10^{th} term (b) 11^{th} term

(c) 12^{th} term (d) 13^{th} term

Q.43 The middle term of $(x - y)^{18}$ is :

(a) 9th (b) 10th

(c) 11th (d) None of these

Q.44 The middle terms of $(x + y)^{23}$ are:

(a) 10 and 11 (b) 11 and 12

(c) 12 and 13 (d) None of these

Q.45 There is no integer n for which 3^n is. (Mtn 16 G-II)

(a) Odd (b) Even

(c) Prime (d) Complex

Q.46 An algebraic expression consisting of two terms is called. (Sgd 16)

(a) Monomial (b) Trinomial

(c) Polynomial (d) Binomial

Q.47 The rth term in the expansion of $(a + x)^n$ is. (Lhr 16 G-I)

(a) $\binom{n}{r-1} a^{n-r+1} x^{r-1}$ (b) $\binom{n}{r} a^n x^{n-1}$

(c) $\binom{n}{r+1} a^r x^{n-r}$ (d) $\binom{n}{r} a^{n-r} x^r$

Q.48 $(r + 1)$th term in the expansion of $(a + b)^n$ is. (Gjw 16)

(a) $\binom{n}{r} a^{n-r} b^r$ (b) $\binom{n}{r} a^{n-r} b^{r-1}$

(c) $\binom{n}{r} a^{n+r} b^r$ (d) $\binom{n}{r} a^{n-r} b^{r+1}$

Q.49 Number of terms in the expansion of $(2x - y)^9$ is. (Bpr 16)

(a) 8 (b) 9

(c) 10 (d) 11

Q.50 The coefficient of the last term in the expansion of $(a + b)^n$ is: (Mtn 13 G-II)

(a) 1 (b) -1

(c) 0 (d) 2

Q.51 Middle term of $\left(x - \dfrac{1}{x}\right)^{10}$ is: (D.G.K 14 G-II)

(a) 5th term (b) 6th term

(c) 7th term (d) 11th term

Q.52 Middle term in the expansion of $(a + x)^2$ is. (A.J.K 17)

(a) 6^{th} (b) 7^{th}

(c) 8^{th} (d) 5^{th}

Q.53 The number of terms in the expansion of $(x^2 - 1)$ is. (Lhr 17 G-II)

(a) 2 (b) 7

(c) 8 (d) 12

Q.54 The number of terms in the expansion of $(a + b)^5$ are. (Mtn 17 G-II)

(a) 6 (b) 5

(c) 2 (d) 7

Q.55 In the expansion of $(3 + x)^4$, middle term will be. (Rwp 17 G-I)

(a) 81 (b) $54x^2$

(c) $26x^2$ (d) x^2

Q.56 The sum of odd co-efficient in the expansion of $(1 + x)^5$ is. (Rwp 17 G-I)

(a) 16 (b) 32

(c) 25 (d) 5

Q.57 The number of terms in the expansion of $(a + x)^{10}$ is. (Sgd 17)

(a) 10 (b) 11

(c) 12 (d) 9

Q.58 Sum of odd co-efficient in the expansion of $(1 + x)^4$ is. (Bpr 14)
(a) 8 (b) 10
(c) 12 (d) 16

Q.59 The number of terms in the expansion of $(a + b)^8$ is. (Fbd 17)
(a) 8 (b) 9
(c) 10 (d) 11

Q.60 Sum of even co-efficients, the expansion $(1 + x)^3$ is: (Mtn 11 G-II)
(a) 2 (b) 4
(c) 6 (d) 8

Q.61 The sum of odd co-efficients in the expansion of $(1 + x)^5$ is: (Lhr 10 G-II)
(a) 16 (b) 32
(c) 25 (d) 5

Q.62 Sum of odd co-efficient in the expansion (Fbd 19)
$(1 + x)^n$ is. (Gjw 16)
(a) n^2 (b) 2^{n-2}
(c) 2^{n-1} (d) 2^n

Q.63 In the expansion of $(3 + x)^4$, middle term will be. (Mtn 18 G-II)
(a) 81 (b) $54x^2$
(c) $26x^2$ (d) x^4

Q.64 In the expansion of $(x + y)^8$, middle term is. (Rwp 18)
(a) T_4 (b) T_6
(c) T_3 (d) T_5

Q.65 If n is a positive integers $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots\dots + \binom{n}{n-1}$ is equal to. (Rwp 18)
(a) 2^n (b) 2^{n+1}
(c) 2^{n-1} (d) 13^n

Q.66 The number of terms in the expansion of $(x + y)^9$ is: (Fbd 19 G-II)
(a) 9 (b) 8
(c) 10 (d) 11

Q.67 Sum of binomial coefficients in the expansion of $(1 + x)^5$ is: (Fbd 19 G-II)
(a) 32 (b) 16
(c) 10 (d) 8

Q.68 The middle term in expansion of $(a + x)^n$ when n is even is (Gjw 19 G-II)(Swl 19)
(a) $\left(\frac{n}{2}+1\right)$ th term (b) $\left(\frac{n}{2}-1\right)$ th term
(c) $\left(\frac{n}{2}\right)$ th term (d) $\left(\frac{n+1}{2}\right)$ th term

Q.69 Total number of terms in expansion of $\left(\frac{x}{2}-\frac{2}{x^2}\right)^{16}$ are: (Bpr 19)
(a) 17 (b) 16
(c) 15 (d) 14

Q.70 Number of terms in the expansion of $(1+x)^{2n+1}$ is: (DGK 19)
(a) $2n + 1$ (b) $2n$
(c) $2n + 2$ (d) $3n + 1$

Q.71 Middle terms in the expansion of $(x + y)^{11}$ are:
(a) T_6, T_7 (b) T_5, T_6
(c) T_7, T_8 (d) T_8, T_9

Q.72 In the expansion of $(3 - 2x)^8$, 5^{th} term will be its: (Mtn 19 G-I)
(a) Last term (b) 2^{nd} last term
(c) 3^{rd} last term (d) Middle term

Q.73 Sum of even co-efficient in the expansion of $(1 + x)^n$ equals: (Rwp 19)
(a) 2^{n+1} (b) 2^{n-1}
(c) 2^n (d) 2^{1-n}

Q.74 The sum of coefficients in the expension of $(1 + x)^5$ is (Sgd 19)
(a) 8 (b) 16
(c) 32 (d) 64

Q.75 2^{nd} term of $(a + b)^7$ is (Swl 19)
(a) a^7 (b) $7ab^6$
(c) $7a^6b$ (d) $7ab$

EXERCISE 8.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 State binomial series.
(Lhr 09 G-I)(Mtn 17 G-I)(D.G.K 16 G-I)

Ans. If n is not a positive integer i.e., if it is negative or fraction, the expansion.

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}x^r + \cdots$$

is used for solving the problem. The expression on R.H.S is called binomial series. This expansion is valid for $|x| < 1$.

Q.2 Write the formula for the expansion of $(1+x)^n$, when n is a negative integer or fraction.
(Lhr 11 G-I)

Ans. When n is a negative integer or fraction, then

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)\dots\dots(n-r+1)}{r!}x^r + \dots\dots\dots$$

provided $|x| < 1$

The above series is called binomial series.

Q.3 Expand $(8-2x)^{-1}$ up to four terms.
(Lhr 08 G-I)(Fbd 16)(Mtn 17, 18 G-II)(D.G.K 17 G-I)

Ans. $(8-2x)^{-1} = 8^{-1}\left(1-\frac{2x}{8}\right)^{-1} = \frac{1}{8}\left(1-\frac{x}{4}\right)^{-1}$

$$= \frac{1}{8}\left(1-\frac{x}{4}\right)^{-1}$$

$$= \frac{1}{8}\left\{1 + (-1)\left(-\frac{x}{4}\right) + \frac{(-1)(-1-1)}{2!}\left(-\frac{x}{4}\right)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{x}{4}\right)^3 + \ldots\ldots\right\}$$

$$= \frac{1}{8}\left[1 + \frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{64} + \ldots\ldots\right]$$

$$= \frac{1}{8} + \frac{x}{32} + \frac{x^2}{128} + \frac{x^3}{512} + \ldots\ldots \text{ valid if } |x| < 4$$

Q.4 Expand $(1-x)^{\frac{1}{2}}$ upto 3 terms.
(Lhr 11 G-II, 17 G-I)(Swl 19)

Ans. $(1-x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-x)^2$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \ldots\ldots\ldots \text{ valid if } |x| < 1$$

Q.5 Expand $(4-3x)^{\frac{1}{2}}$ up to 4 terms if $|x| < \frac{4}{3}$
(Lhr 14 G-I) (Gjw 11)

Ans. $(4-3x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{\frac{1}{2}} = 2\left(1-\frac{3x}{4}\right)^{\frac{1}{2}}$

$$= 2\left\{1 + \left(\frac{1}{2}\right)\left(-\frac{3x}{4}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{3x}{4}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(-\frac{3x}{4}\right)^3 + \ldots\right\}$$

$$= 2 - \frac{3}{4}x - \frac{9}{64}x^2 - \frac{27}{512}x^3 - \ldots\ldots\ldots \text{ valid if } |x| < \frac{4}{3}$$

Q.6 Find the first four terms of $(1+2x)^{-1}$.
(Lhr 16 G-I)(Rwp 16,17 G-I)(Mtn 16 G-I)(Fbd 18)

Ans. $(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-1-1)}{2!}(2x)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}(2x)^3 + \text{——}$

$$= 1 - 2x + \frac{(-1)(-2)}{2}(4x^2) + \frac{(-1)(-2)(-3)}{6}(8x^3) + \text{——}$$

$$= 1 - 2x + 4x^2 - 8x^3 + \text{——}$$

Q.7 Expand $(1-2x)^{\frac{1}{3}}$ upto three terms.
(Lhr 17 G-II)

Ans. $(1-2x)^{\frac{1}{3}} = 1 + \frac{1}{3}(-2x) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}(-2x)^2 + \ldots$

$$= 1 - \frac{2}{3}x + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2!}(4x^2) + \ldots$$

$$= 1 - \frac{2}{3}x - \frac{4}{9}x^2 + \ldots$$

Q.8 Expand the series $(1+x)^{\frac{-1}{3}}$ upto 3 terms.
(A.J.K, Swl 17)(Mtn 17 G-I)(Lhr, Gjw 19 G-II)(Sgd 19)

Ans. $(1+x)^{\frac{-1}{3}}$

$$= 1 + \left(\frac{-1}{3}\right)x + \frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)}{2!}x^2 + \ldots \frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)\left(\frac{-1}{3}-2\right)x^3}{3!}$$

$$= 1 - \frac{1}{3}x + \frac{\left(\frac{-1}{3}\right)\left(\frac{-4}{3}\right)}{2}x^2 + \ldots \frac{\left(\frac{-1}{3}\right)\left(\frac{-4}{3}-1\right)\left(\frac{-7}{3}\right)x^3}{6}$$

$$= 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \ldots$$

Q.9 Expand upto 4 terms $(2-3x)^{-2}$ (Lhr 12 G-I)

Ans. $(2-3x)^{-2} = 2^{-2}\left(1-\frac{3x}{2}\right)^{-2} = \frac{1}{4}\left(1-\frac{3x}{2}\right)^{-2}$

$$= \frac{1}{4}\left(1-\frac{3x}{2}\right)^{-2}$$

$$= \frac{1}{4}\left\{1 + (-2)\left(-\frac{3x}{2}\right) + \frac{(-2)(-2-1)}{2!}\left(-\frac{3x}{2}\right)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}\left(-\frac{3x}{2}\right)^3 + \ldots\right\}$$

$$= \frac{1}{4}\left\{1 + 3x + \frac{27x^2}{4} + \frac{27x^3}{2} + \ldots\ldots\right\}$$

$$= \frac{1}{4} + \frac{3x}{4} + \frac{27x^2}{16} + \frac{27x^3}{8} + \ldots\ldots$$

valid if $|x| < \frac{2}{3}$

Q.10 Prove the binomial series $\sqrt[3]{30} \approx 3.107$
(Lhr 11 G-I)

Ans. $\sqrt[3]{30} = (30)^{1/3} = (27+3)^{1/3}$

$= \left[27\left(1+\frac{3}{27}\right)\right]^{1/3} = (27)^{1/3}\left(1+\frac{1}{9}\right)^{1/3}$

$= 3\left(1+\frac{1}{9}\right)^{1/3}$

$= 3\left\{1+\frac{1}{3}\cdot\frac{1}{9}+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{1}{9}\right)^2+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}\left(\frac{1}{9}\right)^3+\ldots\ldots\right\}$

$= 3\left[1+\frac{1}{3}\cdot\frac{1}{9}-\frac{1}{9}\left(\frac{1}{9}\right)^2+\frac{5}{81}\left(\frac{1}{9}\right)^3+\ldots\ldots\right]$

$= 3\left[1+\frac{1}{27}-\left(\frac{1}{27}\right)^2+\Lambda\right]$

$\approx 3[1+.03704-.001372] = 3[1.035668] = 3.107004$

Thus $\sqrt[3]{30} \approx 3.107$

Q.11 Evaluate $\sqrt[5]{31}$ correct to three places of decimal. **(Lhr 16)(DGK 19)**

Ans. $\sqrt[5]{31} = (31)^{\frac{1}{5}}$

$= (32-1)^{\frac{1}{5}}$

$= \left[32\left(1-\frac{1}{32}\right)\right]^{\frac{1}{5}} = \left[2^5\left(1-\frac{1}{32}\right)\right]^{\frac{1}{5}}$

$= (2^5)^{\frac{1}{5}}\left(1-\frac{1}{32}\right)^{\frac{1}{5}} = 2^1\left(1-\frac{1}{32}\right)^{\frac{1}{5}}$

$= 2\left[1-\frac{1}{5}\left(\frac{1}{32}\right)+\frac{\left(\frac{1}{5}\right)\left(\frac{1}{5}-1\right)}{2!}\left(\frac{1}{32}\right)^2+\text{——}\right]$

$\cong 2[1-0.00625] \cong 1.9875$

Q.12 Use binomial theorem, find the value of $\frac{\sqrt{7}}{\sqrt{8}}$ nearest hunderth. **(D.G.K 16 G-I)**

Ans. $\frac{\sqrt{7}}{\sqrt{8}} = \left(\frac{7}{8}\right)^{\frac{1}{2}} = \left(1-\frac{1}{8}\right)^{\frac{1}{2}}$

$= 1+\frac{1}{2}\left(-\frac{1}{8}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}+\text{——}$

$= 1-\frac{1}{16}-\frac{1}{8}\left(\frac{1}{64}\right)+\text{——}$

$= 1-0.0625-0.001953125 = 0.9355$

Q.13 Evaluate $(1.03)^{\frac{1}{3}}$ by binomial theorem upto three places of decimals.
(Gjw 17)(Lhr 18 G-I)(Rwp 18)(Fbd 19 G-I, II)

Ans. $(1.03)^{\frac{1}{3}} = (1+0.03)^{\frac{1}{3}}$

$= 1+\frac{1}{3}(0.03)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}(0.03)^2$

$= 1+0.01+\frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}(0.0009)+\ldots$

$= 1+0.01-0.0009 = 1.0091$

Q.14 If x is so small that its square and higher powers can be neglected, then show that $\frac{1-x}{\sqrt{1+x}} \approx 1-\frac{3}{2}x.$ **(Mtn 16 G-II, 08 G-I)**
(Lhr 09 G-I)(Gjw 13, 16)(Rwp 19)

Ans. L.H.S. $= \frac{1-x}{\sqrt{1+x}} = (1-x)(1+x)^{-\frac{1}{2}}$

$= (1-x)\left\{1+\left(-\frac{1}{2}\right)(x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}x^2+\ldots\right\}$

$\approx (1-x)\left\{1-\frac{x}{2}\right\}$

Neglecting the terms with x^2 and higher powers of x

$\approx 1-\frac{x}{2}-x+\frac{x^2}{2} = 1-\frac{x+2x}{2}$ Neglecting the term with x^2

$\approx 1-\frac{3x}{2} = \text{R.H.S.}$

Q.15 If x is so small that its square and higher powers can be neglected, then show that $\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1+\frac{3}{2}x.$
(Mtn 17 G-I)(Mtn 18 G-II)(Gjw 18)

Ans. L.H.S

$= \frac{\sqrt{1+2x}}{\sqrt{1-x}}$

$= (1+2x)^{\frac{1}{2}}(1-x)^{\frac{-1}{2}}$

$= \left[1+\frac{1}{2}(2x)+\text{Neglecting higher powers of x}\right]$

$\left[1+\left(\frac{-1}{2}\right)(-x)+\text{Neglecting higher powers of x}\right]$

$$\approx (1+x)\left(1+\frac{1}{2}x\right)$$

$$\approx 1+\frac{1}{2}x + x + \text{neglecting higher power of x}$$

$$= 1+\frac{3}{2}x$$

Q.16 If x is so small that its square and higher powers can be neglected, then show that $\dfrac{\sqrt{4+x}}{(1-x)^3} \approx 2+\dfrac{25}{4}x$. **(Gjw 10) (Fbd 17)**

Ans. L.H.S. $= \dfrac{\sqrt{4+x}}{(1-x)^3}$

$$= (4+x)^{\frac{1}{2}}(1-x)^{-3} \quad \text{.........(i)}$$

Take, $(4+x)^{\frac{1}{2}} = \left[4\left(1+\frac{x}{4}\right)\right]^{\frac{1}{2}} = \left(2^2\right)^{\frac{1}{2}}\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$

$$= 2\left\{1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2+\ldots\right\}$$

$$= 2\left\{1+\left(\frac{x}{8}\right)\right\}$$; Neglecting the terms with x^2 and higher power of x

$$= 2+\frac{x}{4}$$

Also

$$(1-x)^{-3} = \left\{1+(-3)(-x)+\frac{(-3)(-3-1)}{2!}(-x)^2+\cdots\right\}$$

$= \{1+3x\}$ Neglecting x^2 and higher powers of x

Putting all these in equation (i), we get

L.H.S. $= \left\{2+\frac{x}{4}\right\}\{1+3x\}$

$$= 2+6x+\frac{x}{4}+\frac{3}{4}x^2$$

$= 2+\dfrac{24x+x}{4}$ Neglecting the term with x^2

$$= 2+\frac{25}{4}x = \text{R.H.S.}$$

Q.17 Expand $\dfrac{\sqrt{1+2x}}{1-x}$ upto 4-terms, taking the value of x such that the expansion in each case if valid. **(Mtn 18 G-I)**

Ans. Given that: $\dfrac{\sqrt{1+2x}}{1-x} = (1+2x)^{1/2}(1-x)^{-1}$

$$=\left[1+\left(\frac{1}{2}\right)(2x)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(2x)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(2x)^3+\ldots\right]$$

$$.\left[1+(-1)(-x)+\frac{-1(-1-1)}{2!}(-x)^2+\frac{-1(-1-1)(-1-2)}{3!}(-x)^3+\ldots\right]$$

$$=\left[1+x+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}.4x^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}8x^3+\ldots\right].$$

$$\left[1+x+\frac{(-1)(-2)}{2}x^2+\frac{(-1)(-2)(-3)}{6}(-x^3)+\ldots\right]$$

$$=\left[1+x+\frac{-\frac{1}{4}}{2}.4x^2+\frac{\frac{3}{8}}{6}.8x^3+\ldots\right]\left[1+x+\frac{2}{2}x^2+\frac{-6}{-6}(-x^3)+\ldots\right]$$

$$=\left[1+x-\frac{1}{8}.4x^2+\frac{3}{48}.8x^3+\ldots\right][1+x+x^2+x^3+\ldots]$$

$$= 1+x+x^2+x^3+x+x^2+x^3-\frac{1}{2}x^2-\frac{1}{2}x^3+\frac{1}{2}x^3+\ldots$$

$$= 1+2x+\frac{3}{2}x^2+2x^3+\ldots$$

The expansion is valid if:

$|2x| < 1$

$\Rightarrow -1 < 2x < 1$

$-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow |x| < \frac{1}{2}$

and

$|-x| < 1 \Rightarrow |x| < 1$

Q.18 Expand $\sqrt{99}$ by using binomial expansion to find its value upto three places of decimals. **(Bpr 19)**

Ans. $\sqrt{(99)} = (99)^{\frac{1}{2}} = (100-1)^{\frac{1}{2}}$

$$=\left(100\left(1-\frac{1}{100}\right)\right)^{\frac{1}{2}} = 100^{\frac{1}{2}}\left(1-\frac{1}{100}\right)^{\frac{1}{2}}$$

$$= 10(1-0.01)^{\frac{1}{2}}$$

$$= 10\left(1+\left(\frac{1}{2}\right)(-0.01)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(-0.01)^2+---\right)$$

$$\approx 10\left[1-0.005+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(0.0001)\right]$$

$$= 10\left[1-0.005-\frac{1}{8}(0.00001)\right]$$

$$\approx 10(1-0.01-0.0000125) = 10(0.995) = 9.95$$

Thus $\boxed{\sqrt{(99)} \approx 9.95}$

LONG QUESTIONS

Q.1 Expand the following up to 1st 4 terms $(1-x)^{1/2}$.

(D.G.K 13 G-I)

Ans. Given that: $(1-x)^{1/2} = [1+(-x)]^{1/2}$

Here, we can take: $x = -x,\ n = \frac{1}{2}$

$$= 1 + \left(\frac{1}{2}\right)(-x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-x)^3 + \ldots$$

$$= 1 - \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(-x^3) + \ldots$$

$$= 1 - \frac{1}{2}x + \frac{-\frac{1}{4}}{2}x^2 + \frac{\left(\frac{3}{8}\right)}{6}(-x^3) + \ldots$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{3}{48}x^3 + \ldots$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$$

The expansion is valid if: $|-x| < 1 \Rightarrow \boxed{|x| < 1}$

Q.2 If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots\ldots\ldots$ then prove that $y^2 + 2y - 4 = 0$.

(Mtn 11 G-I)(Gjw 16)(Rwp, Fbd 19)

Ans. Given that: $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots$

$$1 + y = 1 + \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots \qquad \ldots\ldots (1)$$

We know that:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \qquad \ldots\ldots (2)$$

Comparing eq. (1) and eq. (2):

$$nx = \frac{2}{5} \Rightarrow x = \frac{2}{5n} \qquad \ldots\ldots (3)$$

and $\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}\left(\frac{2}{5}\right)^2$

Putting value of eq. (3), so

$$\frac{n(n-1)}{2!}\left(\frac{2}{5n}\right)^2 = \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 \Rightarrow \frac{n(n-1)}{2}\cdot\frac{4}{25n^2} = \frac{3}{2}\cdot\frac{4}{25}$$

$$\frac{4(n-1)}{50n} = \frac{12}{50} \Rightarrow 4n - 4 = \frac{12}{50}\times 50n = 12n$$

$$4n - 12n = 4 \Rightarrow -8n = 4 \Rightarrow n = \frac{4}{-8}$$

$$\Rightarrow \boxed{n = -\frac{1}{2}}$$

Putting value of n in eq. (3), so

$$x = \frac{2}{5\left(-\frac{1}{2}\right)} \Rightarrow \boxed{x = -\frac{4}{5}}$$

So, Sum $= (1+x)^n \Rightarrow 1 + y = \left(1 - \frac{4}{5}\right)^{-1/2}$

$$= \left(\frac{5-4}{5}\right)^{-1/2} = \left(\frac{1}{5}\right)^{-1/2} = (5)^{1/2}$$

$$(1+y)^2 = 5 \Rightarrow 1 + 2y + y^2 = 5$$

$$\Rightarrow y^2 + 2y + 1 - 5 = 0$$

$$\boxed{y^2 + 2y - 4 = 0}$$

Q.3 If $y = \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \ldots\ldots\ldots$ then prove that $y^2 + 2y - 2 = 0$.

(Mtn 11 G-II, 15 G-I)(Lhr 13 G-I)(Bpr 16)
(Rwp 11 G-I, 13 G-II)

Ans. Given that:

$$y = \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \ldots$$

$$1 + y = 1 + \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \ldots \qquad \ldots (1)$$

We know that:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \qquad \ldots\ldots (2)$$

Comparing eq. (1) and eq. (2):

$$nx = \frac{1}{3} \Rightarrow x = \frac{1}{3n} \qquad \ldots\ldots (3)$$

and $\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}\left(\frac{1}{3}\right)^2$

Putting value of eq. (3), so

$$\frac{n(n-1)}{2!}\left(\frac{1}{3n}\right)^2 = \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 \Rightarrow \frac{n(n-1)}{2}\cdot\frac{1}{9n^2} = \frac{3}{2}\left(\frac{1}{9}\right)$$

$$\frac{n-1}{18n} = \frac{3}{18} \Rightarrow n - 1 = \frac{3}{18}\times 18n = 3n$$

$$\Rightarrow n - 3n = 1$$

$$-2n = 1 \Rightarrow \boxed{n = -\frac{1}{2}}$$

Putting value of n in eq. (3):

$$x = \frac{1}{3\left(-\frac{1}{2}\right)} \Rightarrow \boxed{x = -\frac{2}{3}}$$

So, $\quad \text{Sum} = (1+x)^n \Rightarrow 1+y = \left(1-\frac{2}{3}\right)^{-1/2}$

$$= \left(\frac{3-2}{3}\right)^{-1/2} = \left(\frac{1}{3}\right)^{-1/2} = (3)^{1/2}$$

$$(1+y)^2 = 3 \Rightarrow 1+2y+y^2 = 3$$

$\Rightarrow \quad y^2+2y+1-3 = 0$

$$\boxed{y^2+2y-2 = 0}$$

Q.4 If $2y = \frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\frac{1.3.5}{3!}\cdot\frac{1}{2^6}+\ldots\ldots\ldots$ then prove that $4y^2+4y-1=0$.

(Mtn 12, 16 G-II)(D.G.K 16 G-I)(A.J.K 17)

Ans. Given that: $2y = \frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\frac{1.3.5}{3!}\cdot\frac{1}{2^6}+\ldots$

$$1+2y = 1+\frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\frac{1.3.5}{3!}\cdot\frac{1}{2^6}+\ldots \quad \ldots\ldots (1)$$

We know that:

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots \quad \ldots\ldots (2)$$

Comparing eq. (1) and eq. (2):

$$nx = \frac{1}{2^2} = \frac{1}{4} \Rightarrow x = \frac{1}{4n} \quad \ldots\ldots (3)$$

and $\quad \frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}\cdot\frac{1}{2^4}$

Putting value of eq. (3), so

$$\frac{n(n-1)}{2!}\left(\frac{1}{4n}\right)^2 = \frac{1.3}{2!}\cdot\frac{1}{2^4} \Rightarrow \frac{n(n-1)}{2}\cdot\frac{1}{16n^2}$$

$$= \frac{3}{2}\cdot\frac{1}{16}$$

$$\frac{n-1}{32n} = \frac{3}{32} \Rightarrow n-1 = \frac{3}{32}\times 32n = 3n$$

$\Rightarrow \quad n-3n = 1$

$$-2n = 1 \Rightarrow \boxed{n = -\frac{1}{2}}$$

Putting the value of n in eq. (3), so

$$x = \frac{1}{4\left(-\frac{1}{2}\right)} = -\frac{1}{2}$$

So, $\quad \text{Sum} = (1+x)^n \Rightarrow 1+2y = \left(1-\frac{1}{2}\right)^{-1/2}$

$$= \left(\frac{2-1}{2}\right)^{-1/2} = \left(\frac{1}{2}\right)^{-1/2} = (2)^{1/2}$$

$$(1+2y)^2 = 2 \Rightarrow 1+4y+4y^2 = 2$$

$\Rightarrow \quad 4y^2+4y+1-2 = 0$

$$\boxed{4y^2+4y-1 = 0}$$

Q.5 If x is so small that its square and higher power can be neglected then show that

$$\sqrt{\frac{1+2x}{1-x}} \approx 1+\frac{3}{2}x$$

(Sgd 13 G-II)

Ans. See Short Question 19.

Q.6 If $p-q$ is small when compared with p or q, show that $\frac{(2n+1)p+(2n-1)q}{(2n-1)p+(2n+1)q} \approx \left(\frac{p+q}{2q}\right)^{\frac{1}{n}}$

(Mtn 10 G-I)

Ans. Since $p-q$ is small, take $p-q = h$; where h is so small that its square and higher powers can be neglected.

$$p-q = h \Rightarrow p = q+h$$

L.H.S. $= \frac{(2n+1)p+(2n-1)q}{(2n-1)p+(2n+1)q}$

$$= \frac{(2n+1)(q+h)+(2n-1)q}{(2n-1)(q+h)+(2n+1)q}$$

$$= \frac{2nq+2nh+q+h+2nq-q}{2nq+2nh-q-h+2nq+q}$$

$$= \frac{4nq+2nh+h}{4nq+2nh-h} = \frac{4nq\left\{1+\left(\frac{2n+1}{4nq}\right)h\right\}}{4nq\left\{1+\left(\frac{2n-1}{4nq}\right)h\right\}}$$

$$= \left\{1+\left(\frac{2n+1}{4nq}\right)h\right\}\left\{1+\left(\frac{2n-1}{4nq}\right)h\right\}^{-1}$$

$$= \left\{1+\left(\frac{2n+1}{4nq}\right)h\right\}$$

$$\left\{1+(-1)\left(\frac{2n-1}{4nq}\right)h+\frac{(-1)(-1-1)}{2!}\left(\frac{2n-1}{4nq}\right)^2h^2+\cdots\right\}$$

$$= \left\{1+\left(\frac{2n+1}{4nq}\right)h\right\}\left\{1-\left(\frac{2n-1}{4nq}\right)h\right\}$$

Neglecting the terms with h^2 and higher powers of h

$$= 1-\left(\frac{2n-1}{4nq}\right)h+\left(\frac{2n+1}{4nq}\right)h-\left(\frac{2n+1}{4nq}\right)\left(\frac{2n-1}{4nq}\right)h^2$$

$= 1-\frac{1}{4nq}h\,(2n-1-2n-1)$ Neglecting the term with h^2

$$= 1-\frac{1}{4nq}h\,(-2) = 1+\frac{h}{2nq}$$

R.H.S. $= \left(\frac{p+q}{2q}\right)^{\frac{1}{n}} = \left(\frac{q+h+q}{2q}\right)^{\frac{1}{n}}$

$$= \left(\frac{2q+h}{2q}\right)^{\frac{1}{n}} = \left(1+\frac{h}{2q}\right)^{\frac{1}{n}}$$

$$= 1 + \frac{1}{n}\left(\frac{h}{2q}\right) + \frac{\frac{1}{n}\left(\frac{1}{n} - 1\right)}{2!}\left(\frac{h}{2q}\right)^2 + \cdots$$

$= 1 + \frac{h}{2nq}$ Neglecting the terms with h^2 and higher powers of h

$\Rightarrow$ L.H.S. = R.H.S.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 The expansion of $(4 + 3x)^{1/2}$ is valid if: (Mtn 09 G-II)

(a) $|x| < 2$ (b) $|x| < 3$
(c) $|x| < 4$ (d) $|x| < 4/3$

Q.2 $1 + 2x + 3x^2 + \ldots\ldots\ldots \infty$ is expansion: (Mtn 10 G-I)

(a) $1 + x$ (b) $(1 + x)^{-1}$
(c) $(1 + x)^{-2}$ (d) $(1 - x)^{-2}$

Q.3 The number of terms in the expansion of $(1+x)^{1/2}$ are: (Mtn 10 G-II)(Rwp 17 G-II)

(a) n (b) $\frac{n}{2}$
(c) r (d) ∞

Q.4 Second term in the expansion of $(1 + 2x)^{1/3}$ is: (Mtn 10 G-II)

(a) x (b) $\frac{2}{3}x$
(c) 3x (d) $-3x$

Q.5 $1 - x + x^2 + \ldots\ldots\ldots$ is explanation of: (Mtn 11 G-I)

(a) $(1 + x)^{-2}$ (b) $(1 + x)^{-3}$
(c) $(1 + x)^{-1}$ (d) $(1 - x)^2$

Q.6 Expansion $(1 + x)^{5/2}$ hold when: (Mtn 11 G-II)

(a) $|x| < 1$ (b) $|x| > 1$
(c) $|x| = 1$ (d) $|x| > 1$

Q.7 The expansion of $(1 + 2x)^{-1}$ is valid if:
(Lhr 05 G-II)(Lhr 14 G-II)(Lhr 11,15 G-II)
(Mtn 12 G-II) (Lhr 17 G-II)(A.J.K 17)

(a) $|x| < 1$ (b) $|x| < \frac{1}{2}$
(c) $|x| = 1$ (d) $|x| < 2$

Q.8 The expansion of $(1 + 2x)^{-2}$ is valid if (Mtn 15 G-II)

(a) $|x| < \frac{1}{2}$ (b) $|x| < 1$
(c) $|x| < 2$ (d) $|x| < 3$

Q.9 The expansion of $(1 - 2x)^{-1}$ is valid if: (Fbd 11)

(a) $|x| > 2$ (b) $|x| < \frac{1}{2}$
(c) $|x| = 2$ (d) $|x| = 0$

Q.10 The second term in the expansion of $\left(1 + \frac{1}{2}x\right)^{-1}$ is: (Fbd 12)

(a) $\frac{-x}{3}$ (b) 2x
(c) $\frac{-x}{2}$ (d) $\frac{x}{2}$

Q.11 The second term in the expansion of $(1 - 2x)^{1/2}$ is: (Mtn 14 G-I)(Lhr 12 G-I)

(a) x (b) $-x$
(c) x^2 (d) 2x

Q.12 2^{nd} term in $(1 - x)^{-1}$ is: (Fbd 15)(Mtn 17 G-I)

(a) 1 (b) x
(c) 2x (d) 3x

Q.13 The second term in the expansion of $(1 - 2x)^{-1/2}$ is: (Fbd 14)(Lhr 13 G-II)

(a) x (b) 2x
(c) 3x (d) 4x

Q.14 The expansion of $(1 + 2x)^{-3}$ is valid only if: (Lhr 10 G-I)

(a) $|x| < 2$ (b) $|x| < \frac{1}{2}$
(c) $|x| < \frac{1}{3}$ (d) $|x| < \frac{1}{6}$

Q.15 The expansion of $(3 - 5x)^{-1/2}$ is valid only if: (Lhr 10 G-II)

(a) $|x| < 5$, (b) $|x| < \frac{5}{3}$
(c) $|x| < \frac{3}{5}$ (d) $|x| < \frac{1}{2}$

Q.16 In the expansion of $\left(\frac{x}{2} + \frac{2}{x}\right)^{12}$, middle term is: (Gjw 14)

(a) T_6 (b) T_7
(c) T_8 (d) T_5

Q.17 Second term of $(1 - 2x)^{1/3}$ is: (Lhr 12 G-I)

(a) $-\frac{2x}{3}$ (b) $\frac{2x}{3}$
(c) $\frac{x}{2}$ (d) $\frac{x}{3}$

Q.18 Expression of $(3-5x)^{1/2}$ is valid if: (DGK 15 G-I)

(a) $|x| > 5$ (b) $|x| < \frac{5}{3}$

(c) $|x| < \frac{3}{5}$ (d) $|x| < \frac{1}{2}$

Q.19 The expansion of $(1-5x)^{-1/3}$ is valid if

(a) $|x| < 1$ (b) $|x| > 1$

(c) $|x| > \frac{1}{5}$ (d) $|x| < \frac{1}{5}$

Q.20 Expansion of $(1-2x)^{\frac{1}{3}}$ is valid if. (D.G.K 17 G-II))

(a) $|x| < 1$ (b) $< \frac{1}{3}$

(c) $|x| < 2$ (d) $|x| < \frac{1}{2}$

Q.21 $1 + 2x + 3x^2 + 4x^3 +$ ——— is the expansion of. (Mtn 16 G-I)

(a) $(1+x)^{-2}$ (b) $(1+x)^2$

(c) $(1-x)^2$ (d) $(1-x)^{-2}$

Q.22 The expansion of $(1+3x)^{\frac{1}{3}}$ is valid only when. (Bpr 16)

(a) $|x| < \frac{1}{2}$ (b) $|x| < \frac{1}{3}$

(c) $|x| < 1$ (d) $|x| = 1$

Q.23 Expansion of $(1+x)^{-1/4}$ is valid only if (Lhr 15 G-I)

(a) $|x| > 1$ (b) $|x| < 1$

(c) $|x| < -1$ (d) $|x| > -1$

Q.24 Number of terms in expansion of $(1+2x)^{1/2}$ are: (Rwp 11)

(a) n (b) n!

(c) 1 (d) ∞

Q.25 The expansion of series $(1-2x)^{1/3}$ is valid only if: (Lhr 11 G-I, 12 G-II)

(a) $|x| > \frac{1}{2}$ (b) $|x| < \frac{1}{2}$

(c) $|x| > \frac{1}{3}$ (d) $|x| < \frac{1}{3}$

Q.26 If $|x| < 1$, then the expansion of $(1-x)^{1/2}$ upto two terms is: (Bpr 12) (Bpr 13)

(a) $1 + \frac{1}{2}x$ (b) $1 - \frac{1}{2}x$

(c) $1 - x$ (d) $1 - 2x$

Q.27 The second term in the expansion of $(1+2x)^{\frac{1}{2}}$ is. (Fbd 17)

(a) x (b) $\frac{2}{15}$

(c) $\frac{1}{2}$ (d) 4x

Q.28 Number of term in the expansion of $(1+x)^{\frac{1}{3}}$ is. (Swl 17)(Sgd 19)

(a) 6 (b) 7

(c) $\frac{1}{3}$ (d) Infinite

Q.29 If $n \in Z^+$ and $|x| < 1$, then the expansion $1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots$ is. (Bpr 18)

(a) Arithmetic series (b) Geometric series

(c) Harmonic series (d) Binomial series

Q.30 The 2nd term in expansion of $\left(1 - \frac{1}{3}x\right)$ is. (Lhr 18 G-I)

(a) $\frac{1}{3}x$ (b) $-\frac{1}{3}x$

(c) 3x (d) 2x

Q.31 The expansion of $(8-2x)^{-1}$ is valid if.

(a) $x > 4$ (b) $|x| < 4$

(c) $|x| = 0$ (d) $|x| = 4$

Q.32 Expansion of $(1+x)^n$ is valid.

(a) $|x| > 1$ (b) $|x| < 1$

(c) $|x| < -$ (d) $|x| > -1$

Q.33 Exp of $(3-5x)^{1/2}$ is valid if: (Lhr 19 G-II)

(a) $|x| < \frac{3}{5}$ (b) $|x| < \frac{5}{3}$

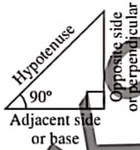
(c) $|x| < 5$ (d) $|x| < 3$

Q.34 Third term in the expansion of $(1-2x)^{1/3}$ is equal to: (Rwp 19)

(a) $-9x^2/4$ (b) $9x^2/4$

(c) $4x^2/9$ (d) $-4x^2/9$

TRIGONOMETRIC FORMULAE



1. $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$

2. $\cos\theta = \dfrac{\text{adjacent side}}{\text{hyp.}} = \dfrac{\text{base}}{\text{hyp.}}$

3. $\tan\theta = \dfrac{\text{per.}}{\text{base}} = \dfrac{\text{opp. side}}{\text{adj. side}}$

4. $\cot\theta = \dfrac{\text{base}}{\text{per.}} = \dfrac{\text{adj. side}}{\text{opp. side}} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

5. $\sec\theta = \dfrac{\text{hyp.}}{\text{adj. side}} = \dfrac{\text{hyp.}}{\text{base}} = \dfrac{1}{\cos\theta}$

6. $\text{cosec}\,\theta = \dfrac{\text{hyp.}}{\text{opp.side}} = \dfrac{\text{hyp.}}{\text{per.}} = \dfrac{1}{\sin\theta}$

7. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

8. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

9. $\sec\theta = \dfrac{1}{\cos\theta}$

10. $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

11. $\sin^2\theta + \cos^2\theta = 1$

or $\sin^2\theta = 1 - \cos^2\theta$

or $\cos^2\theta = 1 - \sin^2\theta$

12. $1 + \tan^2\theta = \sec^2\theta$

or $\tan^2\theta = \sec^2\theta - 1$

13. $1 + \cot^2\theta = \text{cosec}^2\,\theta$

or $\cot^2\theta = \text{cosec}^2\,\theta - 1$

θ	0 $\frac{2\pi}{360^\circ}$	30° $\frac{\pi}{6}$	45° $\frac{\pi}{4}$	60° $\frac{\pi}{3}$	90° $\frac{\pi}{2}$	180° π	270° $\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$

$\sin\theta$ $\text{cosec}\,\theta$	All
$\tan\theta$ $\cot\theta$	$\cos\theta$ $\sec\theta$

Chapter 9

FUNDAMENTALS OF TRIGONOMETRY

EXERCISE 9.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define trigonometry?

Ans. Trigonometry has been derived from three Greek words Tri (three), Goni (angle) and metron (measurement). Literally, it means measurement of triangle.

Q.2 Define angle.

Ans. Two rays with a common starting point forms an angle.

Q.3 Define degree. **(Bpr 19)**

Ans. If the initial ray rotates in anticlockwise direction in such a way that it coincides with itself, then it is said that an angle of 360 degree (360°) is formed.

OR

If we divide circumference of a circle into 360 equal parts, then the angle subtended by one part at the centre of circle is called one **degree**.

Q.4 Define one Radian angle.
(Mtn 13 G-II)(Gjw 16)(Fbd 16, 19)

Ans. One radian is the angle subtended at the center of the circle by an arc of length equal to the radius.

Q.5 Convert 54°45′ into radians. **(Fbd 19 G-II)**
(Lhr 16 G-I)(Rwp 17 G-II)(Mtn 18 G-I)

Ans. $54°45' = 45° + \frac{45°}{60} = 54\frac{3}{4} = \frac{219°}{4}$

$$= \frac{219}{4} \times \frac{\pi}{180} = \frac{219\,\pi}{720}$$

Q.6 Express in radian 120′ 40″.
(Fbd 17)(Gjw 17)

Ans. $120' \, 40''$

$$= 2° \, 40'' = 2° + \frac{40}{360} \text{ degrees} = \frac{181}{90} \times \frac{\pi}{180} \text{ rad.}$$

Q.7 Convert 3″ into radians. **(Mtn 10 G-II)**

Ans. $3'' = 3 \times 1''$

$$= 3 \times \frac{1°}{3600} = \frac{1°}{1200} = \frac{1}{1200} \times 1°$$

$$= \frac{1}{1200} \times \frac{\pi}{180} \text{ radians} = \frac{\pi}{216000} \text{ radians}$$

Q.8 Convert the angle $\frac{2\pi}{3}$ radian into degree.
(Gjw 13)

Ans. $\frac{2\pi}{3} = \frac{2 \times 180°}{3} = 2 \times 60° = 120°$

Q.9 Express $\frac{19\pi}{32}$ into the measure sexagesimal system. **(Lhr 10 G-II)**

Ans.
$$\begin{aligned}
\frac{19\pi}{32} &= \frac{19 \times 180°}{32} \\
&= \frac{3420°}{32} = 106.875° \\
&= 106° + 0.875° \\
&= 106° + 0.875 \times 1° \\
&= 106° + 0.875 \times 60' \\
&= 106° + 52.5' \\
&= 106° + 52' + 0.5' \\
&= 106° + 52' + 0.5 \times 1' \\
&= 106° + 52' + 0.5 \times 60'' \\
&= 106° + 52' + 30'' \\
&= 106° \, 52' \, 30''
\end{aligned}$$

Q.10 Using usual notations find 'r' when $l = 5$ cm, $\theta = \frac{1}{2}$ radian **(Lhr 10 G-I)(Mtn 16 G-I)**

Ans: $l = 5\text{cm}, \theta = \frac{1}{2}$ radian

We know that: $l = r\theta$

$$\therefore r = \frac{l}{\theta} = \frac{5}{\frac{1}{2}} = 5 \times 2 = 10 \text{ cm}$$

Q.11 What is the length of the arc intercepted on a circle of radius 14 cm by the arms of a central angle of 45°?(Lhr 14 G-II)(Gjw 19 G-I)(Sgd 19)

Ans. $l = ?$

$r = 14$ cm

$$\begin{aligned}
\theta &= 45° = 45 \times \frac{\pi}{180} \\
&= 45 \times \frac{22}{7} \times \frac{1}{180} \\
&= \frac{22}{28} = \frac{11}{14} \text{ radians}
\end{aligned}$$

$$l = r\,\theta = 14 \times \frac{11}{14} = 11 \text{ cm.}$$

Q.12 Find λ when $\theta = 65°20'$, r = 18mm

(Bpr 16)(Mtn, Lhr 17 G-II)(Mtn 18 G-II)

Ans. $\theta = 65°20'$

$$= 65° + \frac{20}{60} = 65\frac{1}{3}^{\circ} = \frac{196°}{3}$$

$$= \frac{196}{3} \times \frac{\pi}{180} \text{ rad} = \frac{98\pi}{270}$$

$$\lambda = r\,\theta = 18 \times \frac{98\pi}{270} = 20.525$$

Q.13 Find θ, when l = 1.5 cm, r = 2.5 cm

(Lhr 14 G-I)(Swl 17)(Mtn 19 G-I)

Ans. $\theta = \frac{l}{r} = \frac{1.5}{2.5} = \frac{3}{5}$ radians $=$ 0.6 radians

Q.14 Find l, when $\theta = \pi$ radians, r = 6 cm

(Gjw 14)(Lhr 17 G-I, II)(D.G.K 17 G-I)

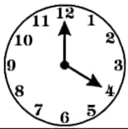
Ans. $\theta = \pi$ radians, r = 6 cm

$l = r\,\theta = 6\pi = 18.86$ cm

Q.15 What is the circular measure of the angle between the hands of a watch at 4 O'clock?

(Sgd 16)(A.J.K 17)(Fbd 18)

Ans. At 4 O'clock, the hands of a watch makes $\frac{4}{12}$th part of angle of whole rotation $= \frac{4}{12}(2\pi) = \frac{2\pi}{3}$ rad.



LONG QUESTIONS

Q.1 A circular wire of radius 6cm is cut straightened and then bent so as to lie along the circumference of a hoop of radius 24cm. Find the measure of the angle which it subtends at the center of hoop. (A.J.K 17)

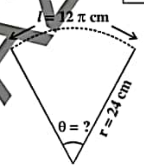
Ans. Given that: Radius of circular wire = 6 cm

$\therefore$ Length of circular wire $= 2\pi \times 6 = 12\pi$ cm

Radius of a hoop = 24 cm

Here, we can take: $l = 12\pi$ cm, r = 24 cm

$$\therefore \quad \theta = \frac{l}{r} = \frac{12\pi}{24} \Rightarrow \boxed{\theta = \frac{\pi}{2}\text{ rad}}$$



MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Relation between arc length of a circular measure of its central angle θ is:

(Mtn 2008 G-II)

(a) $l = r\theta$ (b) $v = l\theta$

(c) $\theta = lr$ (d) $l = \frac{1}{2}r^2\theta$

Q.2 The area of the sector of circular region of radius r is given by:

(Mtn 15 G-I)(Swl 13)(Mtn 09-13 G-I)(D.G.K 17 G-II)

(a) $r^2\theta$ (b) $2r^2\theta$

(c) $\frac{1}{3}r^2\theta$ (d) $\frac{1}{2}r^2\theta$

Q.3 In one hour, the angle (in radian) traversed through by the minute hand of a clock is:

(Bpr 13) (Mtn 2009 G-II) (Mtn 12 G-I) (Gjw 14 G-I)

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 2π

Q.4 If $\tan\theta = \frac{1}{\sqrt{5}}$ and terminal arm of an angle is in its first quadrant, then values of cosec θ:

(Mtn 10 G-I)

(a) $\sqrt{5}$ (b) $\sqrt{6}$

(c) $\sqrt{7}$ (d) $\sqrt{8}$

Q.5 With usual notation l equal to: (Mtn 12 G-II)

(a) r (b) θ

(c) $r\theta$ (d) None of these

Q.6 $\frac{5\pi}{4}$ radians =

(Lhr 13 G-I) (D.G.K. 15 G-I) (Mtn 13)(Fbd 17)

(a) 360° (b) 335°

(c) 270° (d) 225°

Q.7 1 radian =

(Rwp 17 G-I)(Fbd 15 G-II) (DGK 11)(Fbd 18)

(a) 45° (b) 50

(c) 57.29° (d) 60°

Q.8 $\frac{1}{4}$ rotation (anticlockwise) =

(Mtn 14 G-II) (DGK 13)(Lhr 17 G-I)

(a) 45° (b) 90°

(c) 180° (d) 360°

Q.9 Angle between hands of a clock at 2 O-clock:

(Lhr 14 G-II) (DGK 13 G-II)

(a) 30° (b) 60°

(c) 90° (d) 45°

Q.10 If θ = 150°, then its reference angle is: (Bpr 11)
(a) 15° (b) 30°
(c) 45° (d) 60°

Q.11 120° = ________ (Sgd 13)
(a) $\frac{5\pi}{6}$ rad (b) $\frac{2\pi}{3}$ rad
(c) $\frac{3\pi}{4}$ rad (d) $\frac{\pi}{2}$ rad

Q.12 1° or one degree is equal to: (Mtn 14 G-I)(Lhr 16 G-I)
(a) $\frac{\pi}{180}$ rad (b) $\frac{180}{\pi}$ rad
(c) $\frac{\pi}{90}$ rad (d) $\frac{\pi}{360}$ rad

Q.13 A right angle is equal to: (Mtn 14 G-II)
(a) 90° (b) 80°
(c) 60° (d) 45°

Q.14 60th part of 1 degree is called: (Fbd 14)(Mtn 16 G-I)(DGK 19)
(a) Second (b) Radian
(c) Degree (d) Minute

Q.15 If length of arc is equal to radius of circle, the angle subtended at the centre of circle is equal to: (Lhr 12 G-I)(Gjw 10)(Sgd 18)
(a) One degree (b) One radian
(c) 180° (d) π radian

Q.16 π radians = (Lhr 11 G-I)
(a) 180° (b) 180′
(c) 360° (d) 360′

Q.17 Arc length of a circle of radius r, central angle θ rad is: (Lhr 12 G-II)
(a) $\frac{1}{2}r^2\theta$ (b) πr^2
(c) $2\pi r$ (d) $r\theta$

Q.18 $\frac{2\pi}{3}$ radian equal: (Gjw 13) (D.G.K 17 G-I)
(a) 120° (b) 150°
(c) 270° (d) 190°

Q.19 Angle of 30° degrees is equal to: (Lhr 13 G-II)
(a) $\frac{\pi}{3}$ rad (b) $\frac{\pi}{30}$ rad
(c) $\frac{\pi}{6}$ rad (d) $\frac{\pi}{4}$ rad

Q.20 The value of $\frac{7\pi}{9}$ in terms of degree in equal to: (Mtn 15 G-II)
(a) 150° (b) 140°
(c) 130° (d) 120°

Q.21 420^0 = ------------
(a) $\frac{420}{\pi}$ radians (b) $\frac{7\pi}{3}$ radians
(c) $\frac{3\pi}{7}$ radians (d) None of these.

Q.22 $-\frac{\pi}{4}$ = ------------
(a) $\frac{1}{8}$ of a counterclockwise revolution.
(b) $\frac{1}{8}$ of a clockwise revolution.
(c) $\frac{1}{4}$ of a clockwise revolution.
(d) None of these.

Q.23 The 60^{th} part of 1-degree is called one. (Mtn 16 G-I)
(a) Second (b) Minute
(c) Degree (d) Radian

Q.24 $\frac{3\pi}{10}$ radian equals to. (Mtn 17 G-I)
(a) 110° (b) 130°
(c) 45° (d) 54°

Q.25 The angle $\frac{\pi}{12}$ is degree measure. (Mtn 18 G-II)
(a) 30° (b) 20°
(c) 45° (d) 15°

Q.26 The vertex of an angle in standard form is at: (Fbd 19 G-I)
(a) (1 , 0) (b) (0 , 1)
(c) (1 , 1) (d) (0 , 0)

Q.27 $\frac{9\pi}{5}$ rad in degree measure is : (Bpr 19)
(a) 321° (b) 322°
(c) 323° (d) 324°

Q.28 The 60^{th} part of 1-degree is called: (DGK 19)
(a) second (b) minute
(c) degree (d) Radian

Q.29 The measure of angle between hands of a watch at 3 0'clock is: (Mtn 19 G-I)
(a) 30° (b) 60°
(c) 90° (d) 120°°

Q.30 The area of a sector of circular region of radius r and angle θ is equal to: (Rwp 19)
(a) $\frac{1}{2}r\theta^2$ (b) $\frac{1}{2}r^2\theta$
(c) $r\theta^2$ (d) $r^2\theta$

EXERCISE 9.2

SHORT ANSWERS TO THE QUESTIONS

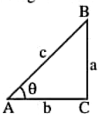
Q.1 Define angle and angle in standard position. **(Lhr 11 G-I)(Lhr 18 G-I)**

Ans. The amount of rotation of a ray from its initial position is known as measure of the angle.

If the initial side of the angle is the positive x-axis and the vertex is at the origin of a rectangular coordinate system, the angle is said to be in standard position.

Q.2 Prove the fundamental identity $\cos^2\theta + \sin^2\theta = 1$. **(Swl 17)(Rwp 19)**

Ans. Consider a right angled ΔABC, as shown in fig.



By the Pythagoras theorem

$c^2 = a^2 + b^2$

Dividing equation by c^2

$\frac{c^2}{c^2} = \frac{a^2}{c^2} + \frac{b^2}{c^2}$

$1 = \sin^2\theta + \cos^2\theta$

i.e. $\sin^2\theta + \cos^2\theta = 1$

Q.3 In which quadrant are the terminal arms of angle lie when $\sin\theta < 0$ and $\cos\theta > 0$. **(D.G.K 17 G-I)**

Ans. In 4^{th} quadrant

Q.4 Find the values of the remaining trigonometric function if $\sin\theta = \frac{12}{13}$ and the terminal arm of the angle is in 1st quadrant. **(Mtn 12 G-I)(A.J.K 17)**

Ans. $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169}$

$\cos^2\theta = \frac{25}{169}$

$\cos\theta = \pm\frac{5}{13}$

As terminal side of the angle is in the I quadrant where cos is positive

$\therefore \quad \cos\theta = \frac{5}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{13}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{5}{12}$

Q.5 If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quad I. Find the values of $\cos\theta$ and $\text{cosec}\,\theta$.(Guj 18)(Sgd 18)

Ans. $\text{Cot}\,\theta = \frac{15}{8}$

Then terminal arm lies in 3^{rd} quadrant where $\sin\theta$ and $\cos\theta$ are –ve using.

$\text{cosec}\,\theta = \pm\sqrt{1+\cot^2\theta}$

taking – value

$\text{cosec}\,\theta = -\sqrt{1+\cot^2\theta} = -\sqrt{1+\left(\frac{15}{8}\right)^2}$

$= -\sqrt{1+\frac{225}{64}} = -\sqrt{\frac{64+225}{64}}$

$= -\sqrt{\frac{289}{64}} = -\frac{17}{8}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cos\theta = \cot\theta \, . \sin\theta$

$\cos\theta = \left(\frac{15}{8}\right)\left(-\frac{8}{17}\right)$

$\cos\theta = \frac{-15}{17}$

Q.6 If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quadrant, find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$ **(Mtn 09 G-II, 19 G-I)**

Ans. $\text{cosec}^2\theta = 1 + \cot^2\theta$

$\text{cosec}^2\theta = 1 + \left(\frac{5}{2}\right)^2 = 1 + \frac{25}{4}$

$= \frac{4+25}{4} = \frac{29}{4}$

$\Rightarrow \quad \text{cosec}\,\theta = \pm\frac{\sqrt{29}}{2}$

Since the terminal arm of the angle is in the I quadrant where $\text{cosec}\,\theta$ is positive.

$\therefore \quad \text{cosec}\,\theta = \frac{\sqrt{29}}{2}$

$\sin\theta = \frac{1}{\text{cosec}\,\theta} = \frac{2}{\sqrt{29}}$

$\because \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\therefore \quad \cos\theta = \cot\theta\sin\theta = \dfrac{5}{2}\times\dfrac{2}{\sqrt{29}} = \dfrac{5}{\sqrt{29}}$

Now $\dfrac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$

$$= \frac{3\times\frac{2}{\sqrt{29}} + 4\times\frac{5}{\sqrt{29}}}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} = \frac{\frac{6+20}{\sqrt{29}}}{\frac{5-2}{\sqrt{29}}} = \frac{26}{3}$$

Q.7 If Sin $\theta = -\dfrac{1}{2}$, terminal arm of θ is not in III Quadrant, find Tan θ. **(Bpr 19)**

Ans. As terminal arm is not in quadrant III, then terminal arm is in quadrant IV, where $\cos\theta$ is +ve

$$\text{Cos}\,\theta = +\sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{1}{2}\right)^2}$$

$$= \sqrt{1-\frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\text{Tan}\,\theta = \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

LONG QUESTIONS

Q.1 If cot $\theta = \dfrac{5}{2}$ and terminal arm of angle is in 1st Quadrant find the value of: $\dfrac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$

(Mtn 11 G-I)(D.G.K 13 G-I)(Fbd 16)

Ans. See Short Question 6

Q.2 Find the value of the remaining trigonometric function when $\cos\phi = \dfrac{9}{41}$ and the terminal arm of the angle is in Quad-IV. (Bpr 14)(DGK 19)

Ans. Given that: $\cos\theta = \dfrac{9}{41} = \dfrac{x}{r}$

Here, θ lies in IV-quadrant, so $x = 9$, $r = 41$

$y = -\sqrt{r^2 - x^2} = -\sqrt{(41)^2 - (9)^2} = -\sqrt{1681-81}$

$= -\sqrt{1600} = -40$

The values of the remaining trigonometric functions are:

$\sin\theta = \dfrac{y}{r} = -\dfrac{40}{41}$, $\quad \tan\theta = \dfrac{y}{x} = -\dfrac{40}{9}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{9}{40}$, $\quad \sec\theta = \dfrac{r}{x} = \dfrac{41}{9}$

$\text{cosec}\,\theta = \dfrac{r}{y} = -\dfrac{41}{40}$

Q.3 Find the values of remaining trigonometric functions $\sin\theta = -\dfrac{1}{\sqrt{2}}$ and the terminal arm of the angle is not in quadrant III.

(Swl 14)(Rwp 13 G-I)(Rwp 18)

Ans. Given that: $\sin\theta = -\dfrac{1}{\sqrt{2}} = \dfrac{y}{r} = \dfrac{-1}{\sqrt{2}}$

Here, θ lies in IV-quadrant, so $y = -1$, $r = \sqrt{2}$

$\therefore \quad x = +\sqrt{r^2 - y^2} = +\sqrt{(\sqrt{2})^2 - (-1)^2} = +\sqrt{2-1}$

$= \sqrt{1} = 1$

The values of the remaining trigonometric functions are:

$\cos\theta = \dfrac{x}{r} = \dfrac{1}{\sqrt{2}}$, $\tan\theta = \dfrac{y}{x} = \dfrac{-1}{1} = -1$

$\cot\theta = \dfrac{x}{y} = \dfrac{1}{-1} = -1$, $\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

$\text{cosec}\,\theta = \dfrac{r}{y} = \dfrac{\sqrt{2}}{-1} = -\sqrt{2}$

Q.4 If cosec $\theta = \dfrac{m^2+1}{2m}$ $\left(0 < \theta < \dfrac{\pi}{2}\right)$ find the value of remaining trigonometric ratio.

(Bpr 11 G-I)(Sgd 16)(D.G.K 14 G-II, 15 G-I)

(Gjw 16)(Rwp 17 G-I)(Lhr 18 G-II)

Ans. Given that: $\text{cosec}\,\theta = \dfrac{m^2+1}{2m} = \dfrac{r}{y}$

Here, θ lies in I-quadrant, so $r = m^2+1$, $y = 2m$

$\therefore \quad x = \sqrt{r^2 - y^2} = \sqrt{(m^2+1)^2 - (2m)^2}$

$= \sqrt{m^4 + 2m^2 + 1 - 4m^2}$

$= \sqrt{m^4 + 1 - 2m^2} = \sqrt{(m^2-1)^2} = |m^2 - 1|$

The values of the remaining trigonometric functions are:

$\sin\theta = \dfrac{y}{r} = \dfrac{2m}{m^2+1}$, $\cos\theta = \dfrac{x}{r} = \dfrac{|m^2-1|}{m^2+1}$

$\tan\theta = \dfrac{y}{x} = \dfrac{2m}{|m^2-1|}$, $\cot\theta = \dfrac{x}{y} = \dfrac{|m^2-1|}{2m}$

$\sec\theta = \dfrac{r}{x} = \dfrac{m^2+1}{|m^2-1|}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If $\cos\theta > 0$ and $\tan\theta < 0$, then terminal arm of angle θ lies in: (Mtn 08 G-I) (D.G.K 11)

(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

Q.2 $\cos^2 4\theta + \sin^2 4\theta =$ (Mtn 10 G-II)
(a) 4 (b) 0
(c) 16 (d) 1

Q.3 $\cos^2 3\theta + \sin^2 3\theta =$ (Fbd 15)
(a) 1 (b) 2
(c) 3 (d) 4

Q.4 $1 + \cot^2\theta =$ (Mtn 11 G-I)
(a) $\sin^2\theta$ (b) $\cos^2\theta$
(c) $\sec^2\theta$ (d) $\text{cosec}^2\theta$

Q.5 **If $\tan\theta > 0$, $\sin\theta < 0$, then θ lies in:** (Rwp 12-13)(Mtn 11 G-I)
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

Q.6 **$\cot^2\theta - \text{cosec}^2\theta$ is equal to:** (Fbd 12) (Bpr 11)
(a) −1 (b) 0
(c) 1 (d) 2

Q.7 **Angle 275° lies in:** (Bpr 12)
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

Q.8 $\sin^2 4A + \cos^2 4A$**:** (Rwp 12)
(a) 4 (b) 3
(c) 2 (d) 1

Q.9 **An angle is said to be in standard position if its vertex is at.** (Mtn 16 G-II)
(a) (1, 1) (b) (−1, 0)
(c) (2, 0) (d) (0, 0)

Q.10 **If $\sin\theta < 0$ and $\cot\theta > 0$, then in which quadrant θ lies:** (Sgd 17)(Lhr 18 G-I) (Sgd 13 G-II) (DGK 14 G-II)(Gjw 10, 11, 15)
(a) 1st (b) 2nd
(c) 3rd (d) 4th

Q.11 **If $\sin\theta < 0$, $\cos\theta < 0$, then θ lies in quadrant:** (Swl 14)
(a) I (b) II
(c) III (d) IV

Q.12 $\sec^2\theta - \tan^2\theta =$ (Bpr 14)
(a) 0 (b) 1
(c) −1 (d) $\frac{1}{2}$

Q.13 **$1 + \cot^2\theta$ is equal:** (Rwp 14) (Lhr 11 G-II)
(a) $\sec^2\theta$ (b) $\frac{1}{\sin^2\theta}$
(c) $\tan^2\theta$ (d) $\frac{1}{\sec^2\theta}$

Q.14 **3 radians is equal to in degrees:** (Lhr 10 G-I)
(a) 169.78° (b) 171.888°
(c) 170.889° (d) 171.5°

Q.15 $1 - \sec^2\theta =$ (Lhr 10 G-II)
(a) $\tan^2\theta$ (b) $-\tan^2\theta$
(c) $\tan^2\theta - 1$ (d) $1 - \tan^2\theta$

Q.16 **If $\tan\theta > 0$ and $\sin\theta > 0$ then, the terminal side lies in:** (Gjw 11)
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

Q.17 **If $\sin\theta > 0$ and $\sec\theta > 0$ the terminal arm of the angle θ lies in:** (Gjw 12)
(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

Q.18 **A right triangle is in which one angle =** (Lhr 13 G-I)
(a) 45° (b) 90°
(c) 270° (d) 360°

Q.19 **$1 + \tan^2\theta$ is equal to:** (Rwp 15)
(a) $\cot\theta$ (b) $\text{cosec}\,\theta$
(c) $\sec^2\theta$ (d) $-\sec\theta$

Q.20 **In first quadrant cosec θ is always ------------, where θ is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.21 **In second quadrant sin θ is always ------------, where α is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.22 **In second quadrant cos θ is always ------------, where θ is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.23 **In fourth quadrant tan θ is always ------------, where α is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.24 **In fourth quadrant cosec θ is always ------------, where α is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.25 **In fourth quadrant sin θ is always ------------, where α is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.26 **In first quadrant sec θ is always ------------, where α is in standard position.**
(a) Positive (b) Negative
(c) Zero (d) Positive and Negative

Q.27 **If $\tan\alpha > 0$ and $\cos\alpha < 0$, then α lies in**
(a) First quadrant (b) Second quadrant
(c) Third quadrant (d) Fourth quadrant

Q.28 If $\cos\alpha < 0$ and $\operatorname{cosec}\alpha < 0$, then α lies in
(a) First quadrant (b) Second quadrant
(c) Third quadrant (d) Fourth quadrant

Q.29 If $\operatorname{cosec}\alpha > 0$ and $\cot\alpha < 0$, then α lies in
(a) First quadrant (b) Second quadrant
(c) Third quadrant (d) Fourth quadrant

Q.30 In a triangle, the side opposite to 90° is called
(a) Base (b) Perpendicular
(c) Hypotenuse (d) None of these.

Q.31 In a right angle triangle, the side opposite to angle is called
(a) Base (b) Perpendicular
(c) Hypotenuse (d) None of these.

Q.32 In a right angle triangle, the side adjacent to angle is called
(a) Base (b) Perpendicular
(c) Hypotenuse (d) None of these.

Q.33 If α lies in third quadrant, then $\sin\alpha + \cos\alpha$ is
(a) Negative (b) Positive
(c) Zero (d) Negative or positive

Q.34 $\sin^2\theta + \cos^2\theta$ equals. (Lhr 14 G-I)
(a) -1 (b) 1
(c) 2 (d) -2

Q.35 The vertex of an angle in standrd form is at.
(Sgd 16)(Fbd 11, 16)
(a) (0, 1) (b) (1, 0)
(c) (1, 1) (d) (0, 0)

Q.36 $\cos^2 2\theta + \sin^2 2\theta$ is equal to. (Rwp 16)
(a) 1 (b) Zero
(c) $\sec^2\theta$ (d) 2

Q.37 $\sin(-\alpha) =$ __________
(Bpr 12)(Gjw 11)(Lhr 11 G-I)
(a) $\sec\alpha$ (b) $-\sin\alpha$
(c) $\sin\alpha$ (d) $\cos\alpha$

Q.38 $\cos(-\theta)$ is. (Mtn 14 G-I)
(a) $-\cos\theta$ (b) $\sin\theta$
(c) $\sec\theta$ (d) $\cos\theta$

Q.39 An angle in standard position whose terminal arm lies on the x-axis or on the y-axis is called.
(Bwp 18)
(a) Obtuse angle (b) Acute angle
(c) Right angle (d) Quadrantal angle

Q.40 An angle is in standard position whose terminal side falls of x-axis or y-axis. (Rwp 18)
(a) General angle (b) Conterminal angle
(c) Quadrantal angle (d) Acute angle

Q.41 $\sin^2(5\theta) + \cos^2(5\theta) =$ (Mtn 18 G-I)
(a) 5 (b) 2
(c) 1 (d) 10

Q.42 If $\sin\theta = \frac{1}{2}$ and $\frac{\pi}{2} < \theta < \pi$ then $\cos\theta =$:
(Fbd 19 G-II)
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{-\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

Q.43 $\frac{\pi}{3}$ rad is an angle. (Swl 19)
(a) acute (b) obtuse
(c) straight (d) reflexive

EXERCISE 9.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Verify $2\sin 45^{\circ} + \frac{1}{2}\operatorname{cosec} 45^{\circ} = \frac{3}{\sqrt{2}}$

(D.G.K 11 G-I)(Sgd 16)(Bpr 16)(Rwp 16)(Sgd 17)
(D.G.K 17 G-I)(Mtn 17, 19 G-I)(Gjw 17)(Rwp 18)

Ans. L.H.S. $= 2\sin 45^{\circ} + \frac{1}{2}\operatorname{cosec} 45^{\circ}$

$$= 2\times\frac{1}{\sqrt{2}} + \frac{1}{2\times\frac{1}{\sqrt{2}}}$$

$$= \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}+\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$$

$$= \frac{3}{\sqrt{2}} = \text{R.H.S.}$$

Hence $2\sin 45^{\circ} + \frac{1}{2}\operatorname{cosec} 45^{\circ} = \frac{3}{\sqrt{2}}$

Q.2 Verify that $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

(Lhr 10 G-I)(Mtn 16 G-I-II)(Rwp 17 G-I)(Swl 19)
(Fbd, Swl, A.J.K 17)(Mtn 18 G-II)(Gjw 19 G-I)

Ans: L.H.S. $= \sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4}$

$$= \sin^2 30^{\circ} + \sin^2 60^{\circ} + \tan^2 45^{\circ}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$$

$$= \frac{1}{4} + \frac{3}{4} + 1 = \frac{1+3+4}{4}$$

$$= \frac{8}{4} = 2 = \text{R.H.S.}$$

Hence $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

Q.3 Evaluate: $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

(Mtn 11 G-II)(Gjw 16)(Fbd 19 G-II)

Ans. $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \dfrac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}} = \dfrac{\frac{3-1}{\sqrt{3}}}{1+1}$

$= \dfrac{2}{\sqrt{3}} \times \dfrac{1}{2} = \dfrac{1}{\sqrt{3}}$

Q.4 Find x, if $\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$ **(Fbd 18)(Guj 18)(Lhr 18 G-II)**
(Mtn 09 G-I)(Fbd 16)(Rwp, Lhr 17 G-II)

Ans. $\tan^2 45° - \cos^2 60° = x \sin 45° \cos 45° \tan 60°$

$\Rightarrow \quad (1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3})$

or $\quad 1 - \frac{1}{4} = x\frac{\sqrt{3}}{2}$

or $\quad \frac{3}{4} = x\frac{\sqrt{3}}{2}$

or $\quad \frac{3}{4}\times\frac{2}{\sqrt{3}} = x$

or $\quad \frac{\sqrt{3}}{2} = x$

or $\quad x = \frac{\sqrt{3}}{2}$

Q.5 Verify $\cos 2\theta = 2\cos^2\theta - 1$ when $\theta = 30°, 45°$
(Mtn 17 G-II)(Lhr 17 G-I)

Ans. $\cos 2\theta = 2\cos^2\theta - 1$

L.H.S
$= \cos 2\theta$
$= \cos(60°)$
$= \frac{1}{2}$

R.H.S
$= 2\cos^2\theta - 1$
$= 2\cos^2 30° - 1$
$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$
$= 2\left(\frac{3}{4}\right) - 1$
$= \frac{3}{2} - 1 = \frac{1}{2}$

$\therefore$ L.H.S = R.H.S

When $\theta = 45°$

L.H.S
$= \cos 2\theta$
$= \cos 90°$
$= 0$

R.H.S
$= 2\cos^2\theta - 1$
$= 2\cos^2 45 - 1$
$= 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1$
$= 2\left(\frac{1}{2}\right) - 1$
$= 1 - 1 = 0$

$\therefore$ L.H.S = R.H.S

Q.5 Evaluate $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$. **(Gjw 19 G-II)**

Ans: $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \dfrac{1-\tan^2 60°}{1+\tan^2 60°}$

$= \dfrac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2} = \dfrac{1-3}{1+3} = \dfrac{-2}{4} = \dfrac{-1}{2}$

LONG QUESTIONS

Q.1 Find the values of all trigonometric function of $-\frac{9}{2}\pi$. **(Fbd 12 G-I)**

Ans. Given that: $-\frac{9\pi}{2} = -810°$

$= -3 \times 360° + 270°$

Now, $\sin\left(\frac{-9\pi}{2}\right) = \sin(-810°) = \sin(-3\times 360° + 270°)$

$= \sin 270° = -1$

and $\cos\left(\frac{-9\pi}{2}\right) = \cos(-810°) = \cos(-3\times360°+270°)$

$= \cos 270° = 0$

$\tan\left(\frac{-9\pi}{2}\right) = \dfrac{\sin\left(-\frac{9\pi}{2}\right)}{\cos\left(-\frac{9\pi}{2}\right)} = \dfrac{-1}{0} = -\infty$

$\cot\left(\frac{-9\pi}{2}\right) = \dfrac{\cos\left(-\frac{9\pi}{2}\right)}{\sin\left(-\frac{9\pi}{2}\right)} = \dfrac{0}{-1} = 0$

$\sec\left(\frac{-9\pi}{2}\right) = \dfrac{1}{\cos\left(-\frac{9\pi}{2}\right)} = \dfrac{1}{0} = \infty$

$\operatorname{cosec}\left(\frac{-9\pi}{2}\right) = \dfrac{1}{\sin\left(-\frac{9\pi}{2}\right)} = \dfrac{1}{-1} = -1$

Q.2 Find the values of the trigonometric functions of the angle $\frac{-17\pi}{3}$. **(Rwp 11 G-I, 13 G-II)**

Ans. Given that: $\frac{-17\pi}{3} = -1020°$

$= -3 \times 360° + 60°$

Now, $\sin\left(\frac{-17\pi}{3}\right) = \sin(-1020°) = \sin(-3\times360°+60°)$

$= \sin 60° = \frac{\sqrt{3}}{2}$

and $\cos\left(\frac{-17\pi}{3}\right) = \cos(-1020°) = \cos(-3\times360°+60°)$

$= \cos 60° = \frac{1}{2}$

$$\tan\left(\frac{-17\pi}{3}\right) = \frac{\sin\left(\frac{-17\pi}{3}\right)}{\cos\left(\frac{-17\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\cot\left(\frac{-17\pi}{3}\right) = \frac{\cos\left(\frac{-17\pi}{3}\right)}{\sin\left(\frac{-17\pi}{3}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\sec\left(\frac{-17\pi}{3}\right) = \frac{1}{\cos\left(\frac{-17\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$$

$$\operatorname{cosec}\left(\frac{-17\pi}{3}\right) = \frac{1}{\sin\left(\frac{-17\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 If $\sin\theta = \frac{\sqrt{3}}{2}$, then θ is equal: (Lhr 15 G-II)

(a) 30° (b) 45°
(c) 60° (d) 90°

Q.2 Value of $\cos 30^0$ = ------------

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}}{2}$
(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{2}}$

Q.3 Value of $\operatorname{cosec} 30^0$ = ------------

(a) 2 (b) $\frac{1}{2}$
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

Q.4 Value of $\operatorname{cosec} 45^0$ = ------------

(a) $\sqrt{3}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

Q.5 Value of $\cos 45^0$ = ------------

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{2}}$
(c) 2 (d) $\frac{1}{\sqrt{3}}$

Q.6 $2\sin 45° + \frac{1}{2}\operatorname{cosec} 45°$ equals

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{\sqrt{2}}{3}$ (d) None of these.

Q.7 Which one is not a quadrantal angle

(a) 0° (b) 90°
(c) 120° (d) 180°

Q.8 Which one is not a quadrantal angle

(a) 0° (b) 90°
(c) 270° (d) 280°

Q.9 Which of the following is not quadrantal angle. (Bpr 16)

(a) $\frac{9}{2}\pi$ (b) 13π
(c) $\frac{4}{3}\pi$ (d) $\frac{\pi}{2}$

Q.10 trigonometric ratios of −330 are same as (A.J.K 17)

(a) 60° (b) 30°
(c) 45° (d) 90°

Q.11 Value of $\cot 60^0$ = ------------

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

Q.12 Value of $\sec 30^0$ = ------------

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{2}{\sqrt{3}}$

Q.13 $\cos(-60°)$ = (Swl 17)

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{\sqrt{3}}{2}$

Q.14 Values of trigonometric functions of quadrantal angle 765° are same as of the angle. (Lhr 18 G-II)

(a) 30° (b) 45°
(c) 60° (d) 90°

Q.15 $\sqrt{2}\ \sin 54° + \dfrac{1}{\sqrt{2}}\ \csc 45° =$: (Fbd 19 G-II)

(a) 1 (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{\sqrt{2}}$ (d) 2

Q.16 If $\tan\theta = \dfrac{1}{\sqrt{3}}$ and θ is in III quadrant then $\cos\theta$ equals (Gjw 19 G-II)

(a) $\sqrt{3}$ (b) $\dfrac{1}{\sqrt{3}}$

(c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

Q.17 Which angle is quadrantal angle: (Lhr 19 G-II)

(a) 45^o (b) 60^o

(c) 270^o (d) 120^o

EXERCISE 9.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 Prove that $\sec\theta\ \text{cosec}\,\theta\ \sin\theta\ \cos\theta = 1$
(Mtn 08 G-I)(Mtn 17 G-I)

Ans. L.H.S. = $\sec\theta\ \text{cosec}\,\theta\ \sin\theta\cos\theta$

$$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}\cdot\sin\theta\cos\theta$$

$$= \left(\frac{1}{\cos\theta}\cos\theta\right)\left(\frac{1}{\sin\theta}\sin\theta\right) = 1 = \text{R.H.S.}$$

Hence $\sec\theta\ \text{cosec}\,\theta\ \sin\theta\cos\theta = 1$

Q.2 Show that $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
(Lhr 12 G-I, 11 G-II)(DGK 19)

Ans. L.H.S. $= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1 + \tan^2\theta - \tan^2\theta \quad (\because 1 + \tan^2\theta = \sec^2\theta)$

$= 1 =$ R.H.S.

Q.3 Prove that $\dfrac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$.
(Mtn 16 G-I)(Rwp 18)

Ans. L.H.S

$$= \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\dfrac{\sin\theta}{\cos\theta}}{\sec^2\theta} = \frac{2\dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos^2\theta}}$$

$$= 2\frac{\sin\theta}{\cos\theta}\times\frac{\cos^2\theta}{1} = 2\sin\theta\cos\theta = \text{R.H.S}$$

Q.4 Prove that $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
(Gjw 11)(Mtn 10 G-II)(Fbd 17, 19 G-II)

Ans. L.H.S. $= 2\cos^2\theta - 1$

$= 2(1 - \sin^2\theta) - 1$

$(\because \cos^2\theta = 1 - \sin^2\theta)$

$= 2 - 2\sin^2\theta - 1$

$= 1 - 2\sin^2\theta$

$=$ R.H.S.

Hence $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Q.5 Prove that $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$.
(Fbd 16)

Ans. L.H.S $= \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 1\,.\,(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta =$ R.H.S

Q.6 Prove $\text{cosec}\,\theta + \tan\theta\sec\theta = \text{cosec}\,\theta\sec^2\theta$
(Gjw 17)

Ans. L.H.S

$= \text{cosec}\,\theta + \tan\theta\sec\theta$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos^2\theta} = \frac{1}{\sin\theta\cos^2\theta} = \frac{1}{\sin\theta}\times\frac{1}{\cos^2\theta}$$

$= \text{cosec}\,\theta\,.\,\sec^2\theta =$ R.H.S

Q.7 Prove that $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$.
(Gjw, Sgd 16)(Bpr 19)

Ans. L.H.S $= (\sec\theta - \tan\theta)^2$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)} = \frac{1-\sin\theta}{1+\sin\theta}$$

$=$ R.H.S

Q.8 Prove that $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \text{cosec}\,\theta$
(Mtn 2008 G-II)(Mtn 10 G-I)
(Lhr 16 G-I)(Bpr 16) (Lhr 17 G-II)

Ans. L.H.S $= \dfrac{\sin\theta}{1+\cos\theta} + \cot\theta$

$$= \frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + \cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+\cos\theta}{\sin\theta(1+\cos\theta)} \left(\because \cos^2\theta + \sin^2\theta = 1\right)$$

$$= \frac{1}{\sin\theta} = \operatorname{cosec}\theta = \textbf{R.H.S}$$

Q.9 Prove that $\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$

(Mtn 12 G-I)(Rwp 16)

Ans. L.H.S. $= \cot^2\theta - \cos^2\theta$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$$

$$= \frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}$$

$$= \frac{\cos^2\theta\left(1-\sin^2\theta\right)}{\sin^2\theta}$$

$$= \frac{\cos^2\theta\cdot\cos^2\theta}{\sin^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}\cdot\cos^2\theta$$

$= \cot^2\theta\cos^2\theta =$ **R.H.S.**

Hence $\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$

Q.10 Prove the identity: $\cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(Lhr 13 G-I) (Mtn 13, 16 G-I)

Ans. R.H.S. $= \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{1} = \cos^2\theta - \sin^2\theta = \text{L.H.S.}$$

Q.11 Prove that $\dfrac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$.

(Gjw 13, 19 G-I)(Rwp 19)

Ans. L.H.S. $= \dfrac{\cot^2\theta - 1}{1+\cot^2\theta} = \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta} - 1}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$

$$= \frac{\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1}$$

$$= \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - \left(1-\cos^2\theta\right)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1 = \text{R.H.S.}$$

Hence $\dfrac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

LONG QUESTIONS

Q.1 Show that $\cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$.

(Mtn 14 G-I)

Ans. See Short Question 13

Q.2 Solve that $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$.

(D.G.K 14 G-I)

Ans. See Short Question 10

Q.3 Show that $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

(Mtn 14 G-II)

Ans. L.H.S. $= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = \text{R.H.S.}$$

Q.4 Prove the result: $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$.

(Sgd 13 G-I)

Domain
$\mathbb{R}$ but $\theta \neq n\pi$

Ans. R.H.S. $= \dfrac{\cot\theta - 1}{\cot\theta + 1}$

$$= \frac{\dfrac{\cos\theta}{\sin\theta} - 1}{\dfrac{\cos\theta}{\sin\theta} + 1}$$

$$= \frac{\dfrac{\cos\theta - \sin\theta}{\sin\theta}}{\dfrac{\cos\theta + \sin\theta}{\sin\theta}} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$= \text{L.H.S.}$

Q.5 Prove that:

$$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$$

(Mtn 15 G-I)

Ans. L.H.S. = $\sin^3\theta - \cos^3\theta$

Domain
$\theta \in \mathbb{R}$

$= (\sin\theta - \cos\theta)[\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta]$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$=$ R.H.S.

Q.6 Prove that

$$\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta.\cos^2\theta)$$

(Mtn 12, 15 G-I)(Rwp 16)(D.G.K 12 G-I)
(Lhr 15 G-I)(Fbd 17)(Mtn 18 G-I)(Fbd 18)

Ans. L.H.S. = $\sin^6\theta - \cos^6\theta$

Domain
$\theta \in \mathbb{R}$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta . \cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(1)^2 - \sin^2\theta\cos^2\theta]$

$= (\sin^2 - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$=$ R.H.S.

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 An equation containing at least one trigonometric function is called (Gjw 15)

(a) Algebraic equation (b) Equation
(c) Linear equation
(d) Trigonometric equation

Q.2 $\mathbf{cosec\,\theta = \frac{1}{\sin\theta}}$**, for all** $\theta \in$ **R but**

(a) $\theta = n\pi, n \in z$ (b) $\theta \neq n\pi, n \in z$
(c) $\theta \neq n\frac{\pi}{2}, n \in z$ (d) $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$

Q.3 $\mathbf{\cot\theta = \frac{\cos\theta}{\sin\theta}}$**, for all** $\theta \in$ **R but**

(a) $\theta = n\pi, n \in z$ (b) $\theta \neq n\pi, n \in z$
(c) $\theta \neq n\frac{\pi}{2}, n \in z$ (d) $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$

Q.4 $\mathbf{\sec\theta = \frac{1}{\cos\theta}}$**, for all** $\theta \in$ **R but**

(a) $\theta \neq n\pi, n \in z$ (b) $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$
(c) $\theta \neq (2n)\frac{\pi}{2}, n \in z$ (d) None of the above

Q.5 $\mathbf{\tan\theta = \frac{\sin\theta}{\cos\theta}}$**, for all** $\theta \in$ **R but**

(a) $\theta \neq n\pi, n \in z$ (b) $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$
(c) $\theta \neq (2n)\frac{\pi}{2}, n \in z$ (d) None of the above

Q.6 $\mathbf{1 + \tan^2\theta = \sec^2\theta}$**, for all** $\theta \in$ **R but**

(a) $\theta \neq n\pi, n \in z$ (b) $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$
(c) $\theta \neq (2n)\frac{\pi}{2}, n \in z$ (d) None of the above

Q.7 $\mathbf{1 + \cot^2\theta = \csc^2\theta}$**, for all** $\theta \in$ **R but**

(a) $\theta = n\pi, n \in z$ (b) $\theta \neq n\pi, n \in z$
(c) $\theta \neq n\frac{\pi}{2}, n \in z$ (d) $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$

Q.8 $\mathbf{(\sec\theta - 1)(\sec\theta + 1) =}$ **-------------**

(a) $\tan^2\theta$ (b) $\sin^2\theta$
(c) $-\tan^2\theta$ (d) $\cot^2\theta$

Q.9 $\mathbf{\frac{\sec\theta}{cosec\,\theta}}$ **is equal to.** (Lhr 17 G-II)

(a) $\cos\theta$ (b) $\tan\theta$
(c) $\cot\theta$ (d) $\sin\theta$

Q.10 If $\mathbf{6\cos^2\theta + 2\sin^2\theta = 5}$**, then** $\mathbf{\tan^2\theta}$ **will be equal to:** (Rwp 19)

(a) $\frac{3}{2}$ (b) 3
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Q.11 $\mathbf{\cot^2\theta - cosec^2\theta =}$ (Sgd 19)

(a) 2 (b) -1
(c) 1 (d) 0

ALLIED ANGLES

$\pi-\theta$ $\frac{\pi}{2}+\theta$	$\frac{\pi}{2}-\theta$ θ
$\pi+\theta$ $\frac{3\pi}{2}-\theta$	$\frac{3\pi}{2}+\theta$ $2\pi-\theta$ θ

1. $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = -\cos\theta$

$\tan(-\theta) = -\tan\theta$

2. $\sin\left(\frac{\pi}{2}-\theta\right) = +\cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right) = +\sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = +\cot\theta$

3. $\sin\left(\frac{\pi}{2}+\theta\right) = +\cos\theta$

$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$

4. $\sin(\pi - 0) = +\sin\theta$

$\cos(\pi - 0) = -\cos\theta$

$\tan(\pi - 0) = -\tan\theta$

w

5. $\sin(\pi + 0) = -\sin\theta$

$\cos(\pi + 0) = -\cos\theta$

$\tan(\pi + 0) = +\tan\theta$

6. $\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right) = +\cot\theta$

7. $\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2}+\theta\right) = +\sin\theta$

$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$

8. $\sin(2\pi - \theta) = -\sin\theta$

$\cos(2\pi - \theta) = +\cos\theta$

$\tan(2\pi - \theta) = -\tan\theta$

9. $\sin(2\pi + \theta) = \sin\theta$

$\cos(2\pi + \theta) = \cos\theta$

$\tan(2\pi + \theta) = \tan\theta$

FUNDAMENTAL LAW OF TRIGONOMETRY

1. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

2. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

3. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

4. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

5. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

6. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

DOUBLE ANGLE

7. $\sin 2\alpha = 2\sin\alpha\cos\alpha$

8. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

9. $\cos 2\alpha = 2\cos^2\alpha - 1$

10. $\cos 2\alpha = 1 - 2\sin^2\alpha$

11. $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

HALF ANGLE

12. $\cos\frac{\alpha}{2} = \sqrt{\dfrac{1+\cos\alpha}{2}}$ or $1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}$

13. $\sin\frac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{2}}$ or $1 - \cos\alpha = 2\sin^2\frac{\alpha}{2}$

14. $\tan\frac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

TRIPLE ANGLE

15. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

16. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

17. $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$

18. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

19. $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

20. $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

21. $-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

22. $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

23. $\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$

24. $\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

25. $\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$

Chapter 10

TRIGONOMETRIC IDENTITIES OF SUM AND DIFFERENCE OF ANGLES

EXERCISE 10.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 State fundamental law of trigonometry.

(Swl 16)

Ans. Let α and β any two angles (real numbers) then $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$ which is called the fundamental law of trigonometry.

Q.2 If α, β, γ are the angles of a ΔABC, then prove that $\sin(\alpha + \beta) = \sin\gamma$.

(A.J.K 17)(Gjw 18)(Mtn 18 G-I)

Ans. $\alpha + \beta + \gamma = 180$

$\alpha + \beta = 180 - \gamma$

L.H.S

$= \sin(\alpha + \beta)$

$= \sin(180 - \gamma)$

$= \sin\gamma = $ R.H.S

Q.3 If α, β, γ are the angles of a triangle ABC, then prove that $\cos(\alpha + \beta) = -\cos\gamma$.

(Sgd 16)(Fbd 18)

Ans. $\alpha + \beta + \gamma = 180$

$\alpha + \beta = 180 - \gamma$

L.H.S $= \cos(\alpha + \beta)$

$= \cos(180 - \gamma) = -\cos\gamma = $ R.H.S

Q.4 If α, β, γ are the angles of a triangle ABC, then prove that $\tan(\alpha + \beta) + \tan\gamma = 0$.

(Lhr 13 G-II, 14 G-I)(Mtn 16 G-II)(Fbd 19 G-II)

Ans. Since α, β and γ are the angles of a triangle

$\therefore \quad \alpha + \beta + \gamma = 180° \Rightarrow \alpha + \beta = 180° - \gamma$

L.H.S. $= \tan(\alpha + \beta) + \tan\gamma = \tan(180° - \gamma) + \tan\gamma$

$= -\tan\gamma + \tan\gamma = 0 = $ R.H.S.

Q.5 Without using the tables and calculator. Find the value of cos 315°. **(Lhr 16 G-I)**

Ans. $\cos(315°) = \cos(3(90°) + 45°)$

$= +\sin 45° = +\dfrac{1}{\sqrt{2}}$

Q.6 Find the value of sec(−300) without table.

(Rwp 17 G-I)

Ans. $\sec(-300)$

$= \sec 300 \qquad (\Theta\ \sec(-\theta) = \sec\theta)$

$= \sec(3(90) + 30°)$

$= +\operatorname{cosec} 30$

$= \dfrac{1}{\sin 30°} = \dfrac{1}{\frac{1}{2}} = 2$

Q.7 Without using the tables, find the value of: tan 1110°. **(Lhr 11 G-I) (Gjw 17)**

Ans. $\tan 1110° = \tan(1080° + 30°) = \tan(3{\times}360° + 30°)$

$= \tan 30° = \dfrac{1}{\sqrt{3}}$

Q.8 Without using the tables, evaluate cot (−855°)

(Lhr 10 G-I)

Ans. $\cot(-855°)$

$= -\cot(855°) = -\cot(855° - 720°) = -\cot(135°)$

$= -\cot(90° + 45°) = -(-\tan 45°)$

$[\because \cot(90° + 45°) = -\tan 45°]$

$= \tan 45° = 1$

Q.9 Prove that: $\sin(180° + \alpha)\sin(90° - \alpha) = -\sin\alpha\cos\alpha$. **(Gjw 10)(Mtn 17 G-I)(Lhr 17 G-II)**

(Mtn 18 G-II)(Sgd 18)

Ans. L.H.S. $= \sin(180° + \alpha)\sin(90° - \alpha)$

$= -\sin\alpha\cos\alpha = $ R.H.S.

Q.10 Without using tables / calculator prove that

$\cos 306° + \cos 234° + \cos 162° + \cos 18° = 0$

(Rwp 16)(Bpr 16)(Sgd 19)

Ans. L.H.S

$= \cos 306° + \cos 234° + \cos 162° + \cos 18°$

$= \cos(360° - 54°) + \cos(180° + 54°) + \cos(180 - 18)$

$+ \cos 18° = \cos 54 - \cos 54° - \cos 18° + \cos 18°$

$= 0 = $ R.H.S

Q.11 Prove that:

$$\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \frac{1}{2}$$

(Mtn 17 G-II)

Ans. L.H.S

$= \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ$

$= \sin(8(90) + 60^\circ)\sin(5(90) + 30^\circ) + \cos(90 + 30^\circ)\sin 30^\circ$

$= (+\sin 60^\circ)(+\cos 30^\circ) + (-\sin 30^\circ)\sin 30^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2} =$ R.H.S

Q.12 Find the value of sin 540 without using calculator.

Ans. sin(540)

$= \sin(6 \times 90 + 0) = \sin 0$

$= \sin(0) = 0$

Q.13 Express sin 319° as a trigonometric function of an angle of positive degree measure of less than 45°. **(Gjw 19 G-I)**

Ans. $\sin(319^\circ) = \sin(4 \times 90^\circ - 41^\circ) = -\sin 41^\circ$

LONG QUESTIONS

Q.1 Prove that $\dfrac{\cos(90^\circ+\theta).\sec(-\theta).\tan(180^\circ-\theta)}{\sec(360^\circ-\theta).\sin(180^\circ-\theta).\cot(90^\circ-\theta)}$

(Mtn 12 G-II)

Ans.

L.H.S. $= \dfrac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$

$= \dfrac{\cos(1\times 90^\circ+\theta).\sec\theta.\tan(2\times 90^\circ-\theta)}{\sec(4\times 90^\circ-\theta)\sin(2\times 90^\circ+\theta)\cot(1\times 90^\circ-\theta)}$

According to allied angle:

$= \dfrac{-\sin\theta.\sec\theta.(-\tan\theta)}{\sec\theta.(-\sin\theta).\tan\theta} = \dfrac{1}{-1} = -1 =$ R.H.S.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $\cos\left(\frac{\pi}{2}-\beta\right)$ **is equal to:** (Mtn 08 G-I)

(a) $\sin\beta$ (b) $-\sin\beta$

(c) $\cos\beta$ (d) $-\cos\beta$

Q.2 $\cos(270+\alpha)$ **equals.** (Mtn 09 G-II)

(a) $\cos\alpha$ (b) $\sin\alpha$

(c) $-\sin\alpha$ (d) $-\cos\alpha$

Q.3 $\sin\left(\frac{3\pi}{2}-\theta\right)$ **equals to:** (Mtn 10 G-I)

(a) $\sin\theta$ (b) $\cos\theta$

(c) $-\sin\theta$ (d) $-\cos\theta$

Q.4 $\sin\left(\alpha-\frac{\pi}{2}\right) =$ ____________

(D.G.K 15 G-I) (Mtn 13 G-II) (Mtn 17 G-I)(Fbd 17) (Swl 17)

(a) $\sec\alpha$ (b) $-\cos\alpha$

(c) $\cos\alpha$ (d) $-\sin\alpha$

Q.5 $\sin 196^\circ =$ ____________ (D.G. Khan 13)

(a) $\sin 16^\circ$ (b) $\cos 33^\circ$

(c) $\cos 16^\circ$ (d) $-\sin 16^\circ$

Q.6 $\tan(270+\theta)$ **equal to.** (Bpr 13)

(a) $\cot\theta$ (b) $-\cot\theta$

(c) $\tan\theta$ (d) $-\tan\theta$

Q.7 $\cos(\pi+\theta) =$ ____________

(Mtn 15 G-II)(Rwp 12)(Rwp 18)

(a) $-\cos\theta$ (b) $\cos\theta$

(c) $\sin\theta$ (d) $-\sin\theta$

Q.8 If $\theta = 150^\circ$, then its reference angle is. (Fbd 12)

(a) 15° (b) 30°

(c) 45° (d) 60°

Q.9 $\cos^2\theta =$ ____________ (Sgd 13)

(a) $\frac{1+\cos\theta}{2}$ (b) $\frac{1+\cos 2\theta}{2}$

(c) $\frac{1-\cos 2\theta}{2}$ (d) $\frac{\cos 2\theta - 1}{2}$

Q.10 $\cos\left(\alpha-\frac{\pi}{2}\right) =$ ____________ (Mtn 14 G-II)

(a) $-\sin\alpha$ (b) $-\cos\alpha$

(c) $\cos\alpha$ (d) $\sin\alpha$

Q.11 $\tan\left(\alpha-\frac{\pi}{2}\right) =$ ____________

(D.G. Khan 14 G-I)

(a) $1\sec\alpha$ (b) $\cot\alpha$

(c) $-\cot\alpha$ (d) $\tan\alpha$

Q.12 $\sin\left(\frac{3\pi}{2}-\theta\right) =$ ____________

(D.G. Khan 14 G-II)

(a) $\cos\theta$ (b) $-\cos\theta$

(c) $\sin\theta$ (d) $-\sin\theta$

Q.13 **sin 390° = ______________** (Swl 14)

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{-1}{\sqrt{2}}$ (d) 1

Q.14 **sin (90 − θ) equal to.** (Bpr 14)

(a) sin θ (b) − sin θ

(c) cos θ (d) − cos θ

Q.15 **Which angle is quadrant angle?** (Bpr 14)

(a) 120° (b) 270°

(c) 60° (d) 45°

Q.16 **$\sin\left(\frac{3\pi}{2}+\theta\right)$ =** (Lhr 10 G-I)

(a) cos θ (b) − cos θ

(c) sin θ (d) − sin θ

Q.17 **cos(π − θ) =** (D.G.K 15 G-II)(Lhr 12 G-I)(Lhr 13 G-II)

(a) sin θ (b) −sin θ

(c) cos θ (d) −cos θ

Q.18 **sin (θ + 270°) = ________** (Gjw 17)

(a) cot θ (b) tan θ

(c) sin θ (d) − cos θ

Q.19 **sin (−300)** (Lhr 13 G-I)

(a) $-\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{2}{\sqrt{3}}$ (d) 0

Q.20 **The value of cos 315° is:** (Lhr 17 G-I)

(a) 0 (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

Q.21 **tan (270 + θ) =** (Lhr 14 G-II)

(a) cot θ (b) tan θ

(c) −cot θ (d) − tan θ

Q.22 **cot (π − α)** (D.G.K 15 G-II)

(a) sin α (b) cot α

(c) − cot α (d) tan α

Q.23 **$\tan\left(\frac{3\pi}{2}-\theta\right)$ is equal to:** (Rwp 15)

(a) tan θ (b) −cot θ

(c) cot θ (d) −tan θ

Q.24 **sin (α + β) = ________**

(a) sin α sin β + csc α cos β

(b) sin α sin β + cos α cos β

(c) sin α cos β − cos α sin β

(d) sin α cos β + cos α sin β

Q.25 **cos (α + β) = ________**

(a) cos α cos β − sin α sin β

(b) cos α sin β + sin α cos β

(c) cos α sin β − sin α cos β

(d) cos α cos β + sin α sin β

Q.26 **sin (α − β) = __________**

(a) sin α cos β + cos α sin β

(b) sin α cos β − cos α sin β

(c) cos α cos β − sin α sin β

(d) cos α cos β + sin α sin β

Q.27 **cos (α − β) = __________**

(a) cos α cos β + sin α sin β

(b) cos α cos β − sin α sin β

(c) sin α cos β + cos α sin β

(d) sin α cos β − cos α sin β

Q.28 **tan (α − β) = __________**

(a) $\frac{\tan\alpha - \tan\beta}{1 - \tan\alpha\tan\beta}$ (b) $\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

(c) $\frac{\tan\alpha + \tan\beta}{1 + \tan\alpha\tan\beta}$ (d) $\frac{\cot\alpha - \cot\beta}{1 + \cot\alpha\cot\beta}$

Q.29 **tan (45° − θ) = __________**

(a) $\frac{\tan\theta + 1}{\tan\theta - 1}$ (b) $\frac{1 + \tan\theta}{1 - \tan\theta}$

(c) $\frac{1 + \tan^2\theta}{1 - \tan^2\theta}$ (d) $\frac{1 - \tan\theta}{1 + \tan\theta}$

Q.30 **$\cos\left(\theta + \frac{3\pi}{2}\right)$** **(Bpr 18)**

(a) −sinθ (b) sinθ

(c) cosθ (d) −cosθ

Q.31 **The angles 90° ± θ, 180° ± θ, 270° ± θ, 360° ± θ are the**

(a) composite angles (b) half angles

(c) quadrantal angles (d) allied angles

Q.32 **A reference angle θ is always**

(a) $0 < \theta < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < \theta < \pi$

(c) $\pi < \theta < \frac{3\pi}{2}$ (d) $\frac{3\pi}{2} < \theta < 2\pi$

Q.33 **If θ lies in first quadrant, then its reference angle is**

(a) θ (b) π − θ

(c) θ − π (d) 2π − θ

Q.34 **If θ lies in second quadrant, then its reference angle is**

(a) θ (b) π − θ

(c) θ − π (d) 2π − θ

Q.35 If θ lies in third quadrant, then its reference angle is

(a) θ (b) $\pi - \theta$

(c) $\theta - \pi$ (d) $2\pi - \theta$

Q.36 If θ lies in fourth quadrant, then its reference angle is

(a) θ (b) $\pi - \theta$

(c) $\theta - \pi$ (d) $2\pi - \theta$

Q.37 If $\theta = 210°$, then its reference angle is

(a) $30°$ (b) $20°$

(c) $60°$ (d) $40°$

Q.38 If $\theta = 300°$, then its reference angle is

(a) $30°$ (b) $20°$

(c) $60°$ (d) $40°$

Q.39 $\tan\left(\frac{3\pi}{2} - \theta\right)$, where θ is a basic angle, will have terminal side in

(a) quad. I (b) quad. II

(c) quad. III (d) quad. IV

Q.40 $\sec(2\pi + \theta)$, where θ is a basic angle, will have terminal side in

(a) quad. I (b) quad. II

(c) quad. III (d) quad. IV

Q.41 $\cos 315^0$ = __________

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) $-\frac{3}{\sqrt{2}}$

Q.42 $\tan(1110°)$ = __________

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

Q.43 $\cot(-855°)$ = __________

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{\sqrt{2}}$

Q.44 $\operatorname{cosec}(2040°)$ = __________

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{2}{\sqrt{3}}$

(c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

Q.45 $\sec(-960°)$ = __________

(a) 0 (b) 2

(c) -2 (d) $\sqrt{2}$

Q.46 $\sin(-780°)$ = __________

(a) $-\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $-\frac{2}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

Q.47 $\cot\left(\frac{3\pi}{2} - \theta\right)$ is equal to. (Sgd 16)

(a) $\tan\theta$ (b) $-\tan\theta$

(c) $\cot\theta$ (d) None of these

Q.48 $\cos\left(\frac{\pi}{2} - \beta\right)$ is equal to.(Rwp 16)

(a) $\sin\beta$ (b) $-\sin\beta$

(c) $\cos\beta$ (d) $-\cos\beta$

Q.49 $\cot(90 - \alpha)$ is equals to. (Lhr 16 G-I)

(a) $-\tan\alpha$ (b) $\tan\alpha$

(c) $\cot\alpha$ (d) $-\cot\alpha$

Q.50 $\sin\left(\frac{3\pi}{2} - \theta\right)$ is equal to. (Bpr 16)

(a) $\sin\theta$ (b) $-\sin\theta$

(c) $\cos\theta$ (d) $-\cos\theta$

Q.51 $\tan\left(\frac{3\pi}{2} + \theta\right)$ (A.J.K 17)

(a) $\cot\theta$ (b) $-\cot\theta$

(c) $\tan\theta$ (d) $\operatorname{cosec}\theta$

Q.52 $\tan(\pi - \alpha)$ is equal to. (Lhr 17 G-II)(Mtn 18 G-I)

(a) $\tan\alpha$ (b) $-\tan$

(c) $-\tan\alpha$ (d) $\cot\alpha$

Q.53 $\frac{3\pi}{2} - \theta$ lies is. (Rwp 17 G-II)

(a) 1st quadrant (b) 2nd quadrant

(c) 3rd quadrant (d) 4th quadrant

Q.54 $\operatorname{cosec}\left(\theta + \frac{\pi}{2}\right)$ = __________ (Mtn 11 G-I)

(a) $\sec\theta$ (b) $-\sec\theta$

(c) $\operatorname{cosec}\theta$ (d) $\operatorname{cosec}\theta$

Q.55 If α, β, γ are the angles of a triangle then $\tan(\alpha+\beta) + \tan\gamma$ is.

(Gjw 12)(Mtn 13 G-II)(Lhr 18 G-I)

(a) 0 (b) 1

(c) 2 (d) -1

Q.56 $\sin\left(\frac{\pi}{2} - \theta\right)$ = (Sgd 18)

(a) $\cos\theta$ (b) $\sin\theta$

(c) $-\cos\theta$ (d) $-\sin\theta$

Q.57 $\cos\left(\frac{3\pi}{2}+\theta\right)$ =: (Fbd 19 G-I)

(a) $\cos\theta$ (b) $-\cos\theta$
(c) $-\sin\theta$ (d) $\sin\theta$

Q.58 cos 315° equals (Gjw 19 G-II)

(a) $\tan(-45°)$ (b) $\tan 45°$
(c) $\sin 45°$ (d) $\operatorname{cosec} 45°$

Q.59 $\cos(\theta - 180°)$ =: (Bpr 19)

(a) $\sin\theta$ (b) $-\cos\theta$
(c) $\cos\theta$ (d) $-\sin\theta$

Q.60 $\sin(-\alpha)$ =: (DGK 19)

(a) $\sec\alpha$ (b) $-\sin\alpha$
(c) $\sin\alpha$ (d) $-\cos\alpha$

Q.61 $\cos\left(\frac{3\pi}{2}-\phi\right)$ **is equal to:** (Lhr 19 G-II)

(a) $-\sin\phi$ (b) $\sin\phi$
(c) $\cos\phi$ (d) $-\cos\phi$

Q.62 The angle $\frac{3\pi}{2}$ - is θ lies in quadrant: (Mtn 19 G-I)

(a) I (b) II
(c) III (d) *IV*

Q.63 $\tan(\alpha - 90)$ = (Swl 19)

(a) $\cot\alpha$ (b) $-\cot\alpha$
(c) $\tan\alpha$ (d) $-\tan\alpha$

EXERCISE 10.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Prove that $\sin(180+\theta) = -\sin\theta$ (A.J.K 17)

Ans. L.H.S

$= \sin(180+\theta)$

$= \sin 180\cos\theta + \cos 180\sin\theta$

$= (0)\sin\theta + (-1)\sin\theta = -\sin\theta$ = R.H.S

Q.2 Prove that: $\cos(\alpha + 45°) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

(Gjw 13)(Mtn 16 G-I-II)

Ans. L.H.S. = $\cos(\alpha + 45°) = \cos\alpha\cos 45° - \sin\alpha\sin 45°$

$$= \cos\alpha \times \frac{1}{\sqrt{2}} - \sin\alpha \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha) = \text{R.H.S.}$$

Q.3 Find the value of cos 105° and sin 105°.

(Fbd 16)(Lhr 14 G-I,19 G-II)(D.G.K 17 G-I)(Rwp 17 G-II)

Ans. $\cos 105° = \cos(45° + 60°)$

$$= \cos 45°\cos 60° - \sin 45°\sin 60°$$

$$= \frac{1}{\sqrt{2}}\times\frac{1}{2} - \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$\sin 105° = \sin(60° + 45°)$

$$= \sin 60°\cos 45° + \cos 60°\sin 45°$$

$$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} + \frac{1}{2}\times\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Q.4 Show that:

$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\beta - \sin^2\alpha$.

(Lhr 12 G-I)(D.G.K 16 G-I)(Sgd 17)(Bpr 18)

Ans. $\cos(\alpha+\beta)\cos(\alpha-\beta)$

$= [\cos\alpha\cos\beta - \sin\alpha\sin\beta]$
$[\cos\alpha\cos\beta + \sin\alpha\sin\beta]$

$= (\cos\alpha\cos\beta)^2 - (\sin\alpha\sin\beta)^2$

$= \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= (1-\sin^2\alpha)\cos^2\beta - \sin^2\alpha(1-\cos^2\beta)$

$= \cos^2\beta - \sin^2\alpha\cos^2\beta - \sin^2\alpha + \sin^2\alpha\cos^2\beta$

$= \cos^2\beta - \sin^2\alpha$

Q.5 Prove that $\tan(45° + A).\tan(45° - A) = 1$.

(Lhr 09, 14 G-II)(Sgd 16)(Bpr 16)(Fbd 17)

Ans. L.H.S. $= \tan(45° + A).\tan(45° - A)$

$$= \frac{\tan 45° + \tan A}{1 - \tan 45°\tan A}\times\frac{\tan 45° - \tan A}{1 + \tan 45°\tan A}$$

$$= \frac{1 + \tan A}{1-(1)\tan A}\times\frac{1-\tan A}{1+(1)\tan A}$$

$$= \frac{1+\tan A}{1-\tan A}\times\frac{1-\tan A}{1+\tan A} = 1 = \text{R.H.S.}$$

Q.6 Prove that $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

(Rwp 17 G-II)(Sgd 18)

Ans. L.H.S

$= \cot(\alpha+\beta)$

$$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$= \frac{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}} = \frac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha}$$

= R.H.S

LONG QUESTIONS

Q.1 Prove that $\dfrac{\cos 80° - \sin 8°}{\cos 8° + \sin 8°} = \tan 37°$

(D.G.K 15 G-I)(Bpr 19)

Ans. R.H.S.= $\tan 37° = \tan(45° - 8°)$

$$= \frac{\tan 45° - \tan 8°}{1 + \tan 45° \,.\, \tan 8°}$$

$$= \frac{1 - \tan 8°}{1 + (1)\,.\,\tan 8°} = \frac{1 - \tan 8°}{1 + \tan 8°} = \frac{1 - \dfrac{\sin 8°}{\cos 8°}}{1 + \dfrac{\sin 8°}{\cos 8°}}$$

$$= \frac{\dfrac{\cos 8° - \sin 8°}{\cos 8°}}{\dfrac{\cos 8° + \sin 8°}{\cos 8°}}$$

$$= \frac{\cos 8° - \sin 8°}{\cos 8° - \sin 8°} = \text{R.H.S.}$$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $\sin\left(\theta + \dfrac{\pi}{6}\right) + \cos\left(\theta + \dfrac{\pi}{3}\right)$ **equals:** (Mtn 09 G-I)

(a) $\sin\theta$ (b) $\cos\theta$

(c) $\sec\theta$ (d) $\tan\theta$

Q.2 $\tan(45° - \theta) =$ ________ (Swl 14)

(a) $\dfrac{\tan\theta + 1}{\tan\theta - 1}$ (b) $\dfrac{1 + \tan\theta}{1 - \tan\theta}$

(c) $\dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$ (d) $\dfrac{1 - \tan\theta}{1 + \tan\theta}$

Q.3 $\sin 319°$ (Rwp 14)

(a) $-\cos 35°$ (b) $\sin 41°$

(c) $-\sin 41°$ (d) $\sin 16°$

Q.4 $\tan(45° + \theta) =$ ________

(a) $\dfrac{\tan\theta + 1}{\tan\theta - 1}$ (b) $\dfrac{1 + \tan\theta}{1 - \tan\theta}$

(c) $\dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$ (d) $\dfrac{1 - \tan\theta}{1 + \tan\theta}$

Q.5 $\sin 15° =$ ________

(a) $\dfrac{\sqrt{3} + 1}{2\sqrt{2}}$ (b) $\dfrac{\sqrt{3} - 1}{2\sqrt{2}}$

(c) $\dfrac{\sqrt{3}}{2\sqrt{2}}$ (d) $\dfrac{2\sqrt{2}}{\sqrt{3} - 1}$

EXERCISE 10.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Find the value of $\cos 2\alpha$ when $\cos\alpha = \dfrac{3}{5}$ where $0 < \alpha < \dfrac{\pi}{2}$ (Bpr 16)(Mtn 08 G-I)(Gjw 19 G-I)

Ans. $\cos 2\alpha = 2\cos^2\alpha - 1$

$$= 2\left(\frac{3}{5}\right)^2 - 1$$

$$= 2\left(\frac{9}{25}\right) - 1 = \frac{18 - 25}{25} = \frac{-7}{25}$$

Q.2 Find the value of $\sin 2\alpha$ if $\sin\alpha = \dfrac{12}{13}$ and $0 < \alpha < \dfrac{\pi}{2}$. (Gjw 10)

Ans. $\cos^2\alpha = 1 - \sin^2\alpha$

$$= 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169}$$

$$= \frac{169 - 144}{169} = \frac{25}{169}$$

$\Rightarrow \cos\alpha = \dfrac{5}{13}$ ($\because$ cos is positive in I quadrant)

Now, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$$

Q.3 Prove that $\dfrac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$.

(Fbd 18)(Lhr 10 G-I)(Mtn 14 G-I)(Rwp 18)

Ans. See Long Question 3

Q.4 $1 + \tan\alpha\tan 2\alpha = \sec 2\alpha$

(Lhr 14 G-II, 12 G-I)(Gjw 16)(Mtn 17 G-II)

Ans. See Long Question 2

Q.5 Prove that: $\sqrt{\dfrac{1 + \sin\alpha}{1 - \sin\alpha}} = \dfrac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}$.

(Rwp 16)(Bpr 19)

Ans. L.H.S

$$= \sqrt{\frac{1 + \sin\alpha}{1 - \sin\alpha}}$$

$$= \sqrt{\frac{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$$

$$= \sqrt{\frac{\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)^2}} = \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}} = \text{R.H.S}$$

Q.6 Prove that $\dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$.

(Gjw 14)

Ans. $\text{L.H.S} = \dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$

$$= \frac{\sin A + 2\sin A\cos A}{1 + \cos A + 2\cos^2 A - 1}$$

$$= \frac{\sin A(1 + 2\cos A)}{\cos A(1 + 2\cos A)}$$

$$= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}$$

Hence $\dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A.$

LONG QUESTIONS

Q.1 Prove that $1 + \tan\alpha\tan 2\alpha = \sec 2\alpha$

(Bpr 11 G-I)

Ans. $\text{L.H.S.} = 1 + \tan\alpha\tan 2\alpha = 1 + \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin 2\alpha}{\cos 2\alpha}$

$$= \frac{\cos 2\alpha\,.\,\cos\alpha + \sin 2\alpha\,.\,\sin\alpha}{\cos 2\alpha\,.\,\cos\alpha}$$

$$= \frac{\cos(2\alpha - \alpha)}{\cos 2\alpha\,.\,\cos\alpha}$$

$$= \frac{\cos\alpha}{\cos 2\alpha\,.\,\cos\alpha} = \frac{1}{\cos 2\alpha} = \sec 2\alpha$$

$= \text{R.H.S.}$

Q.2 Prove that $\dfrac{\sin 2\alpha}{1 + \cos 2\alpha} = \tan\alpha$ **(Mtn 14 G-I)**

Ans. See Short Question 10

Q.3 Show that $\dfrac{\cos 3\theta}{\cos\theta} + \dfrac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$

(D.G.K 14 G-I)(Sgd 17)

Ans. L.H.S

$$= \frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta}$$

$$= \frac{\cos 3\theta\sin\theta + \sin 3\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{\sin 3\theta\cos\theta + \cos 3\theta\sin\theta}{\cos\theta\sin\theta}$$

$$= \frac{\sin(3\theta + \theta)}{\cos\theta\sin\theta} = \frac{\sin 4\theta}{\cos\theta\sin\theta}$$

$$= \frac{2\sin 2\theta\cos 2\theta}{\cos\theta\sin\theta} = \frac{2(2\sin\theta\cos\theta)\cos 2\theta}{\cos\theta\sin\theta}$$

$= 4\cos 2\theta = \text{R.H.S.}$

Q.4 Prove the identity $\sqrt{\dfrac{1 + \sin\alpha}{1 - \sin\alpha}} = \dfrac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}$

(Lhr 15 G-I)

Ans. See Short Question 15

Q.11 Prove that $\dfrac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta} = \tan\theta\tan 2\theta$.

(Gjw 19 G-I)

Ans. $\text{L.H.S} = \dfrac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta}$

$$= \frac{2\sin\theta\sin 2\theta}{\cos\theta + 4\cos^3\theta - 3\cos\theta}$$

$(\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta)$

$$= \frac{2\sin\theta\sin 2\theta}{4\cos^3\theta - 2\cos\theta} = \frac{2\sin\theta\sin 2\theta}{2\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta\sin 2\theta}{\cos\theta\cos 2\theta} = \tan\theta\tan 2\theta = \text{R.H.S}$$

MULTIPLE CHOICE QUESTIONS

- ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $2\sin^2\dfrac{\theta}{2}$ **equals.**

(Mtn 15 G-I, 09 G-II)(Fbd 18)(Guj 18)

(a) $1 + \cos\theta$ (b) $1 + \sin\theta$

(c) $1 - \sin\theta$ (d) $1 - \cos\theta$

Q.2 $\cos^2 A - \sin^2 A =$ ________________

(a) 1 (b) $\sin 2A$

(c) $\cos 2A$ (d) -1

Q.3 $\tan 2\theta =$ ____________ (Mtn 12 G-I)

(a) $\dfrac{2\tan\theta}{1 - \tan^2\theta}$ (b) $\dfrac{2\tan\theta}{1 + \tan^2\theta}$

(c) $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$ (d) $\dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$

Q.4 $\cos 2\theta =$ ______________ (Mtn 12 G-II)

(a) $\dfrac{2\tan\theta}{1 - \tan^2\theta}$ (b) $\dfrac{2\tan\theta}{1 + \tan^2\theta}$

(c) $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$ (d) $\dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$

Q.5 $\cos 2\alpha$ **is equal to.** (D.G. Khan 12)

(a) $2\cos^2\alpha + 1$ (b) $2\cos^2\alpha - 1$

(c) $2\sin^2\alpha + 1$ (d) $2\sin^2\alpha - 1$

Q.6 $2\cos^2\alpha - 1 =$ ____________ (D.G. Khan 13)

(a) $\sin 2\alpha$ (b) $\cos 2\alpha$

(c) $\tan 2\alpha$ (d) $1 - \sin 2\alpha$

Q.7 $\sin 2\theta$ **is equal to.**

(Fbd 15)(Rwp 13) (Mtn 17 G-II)

(a) $\dfrac{2\tan\theta}{1 - \tan^2\theta}$ (b) $\dfrac{2\tan\theta}{1 + \tan^2\theta}$

(c) $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$ (d) $\dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$

Q.8 $\sin 2\theta =$ ________ (Swl 14)

(a) $2\sin\theta\cos\theta$ (b) $2\tan\theta$
(c) $\sec^2\theta$ (d) $2\sin\theta$

Q.9 $\cos\frac{\alpha}{2}$ **is equal to:** (Gjw 10)

(a) $\frac{1+\cos\alpha}{2}$ (b) $\frac{1-\cos\alpha}{2}$
(c) $\frac{1+\sin\alpha}{2}$ (d) $\pm\sqrt{\frac{1+\cos\alpha}{2}}$

Q.10 $\tan 2\alpha =$ (Lhr 12 G-I)

(a) $\frac{2\tan\alpha}{1-\tan^2\alpha}$ (b) $\frac{2\tan\alpha}{1+\tan^2\alpha}$
(c) $\frac{\tan\alpha}{1-\tan^2\alpha}$ (d) $\frac{\tan\alpha}{1+\tan^2\alpha}$

Q.11 $\frac{1-\tan^2\alpha}{1+\tan^2\alpha} =$ (Bpr 15)

(a) $\cos 2\alpha$ (b) $\sin 2\alpha$
(c) $\cos^2 2\alpha$ (d) $\sin^2 2\alpha$

Q.12 $\cos 2\alpha$

(a) $2\cos^2\alpha - 1$ (b) $2\cos^2\alpha + 1$
(c) $2\sin^2\alpha + 1$ (d) $2\sin^2\alpha - 1$

Q.13 $\sin 2\alpha$ **is equal to:** (Lhr 15 G-I)(Mtn 18 G-I)

(a) $1-2\sin^2\alpha$ (b) $2\cos^2\alpha - 1$
(c) $2\sin\alpha\cos\alpha$ (d) $\sin\alpha$

Q.14 $\sin 3\theta =$ ________ **(Gjw 16)**

(a) $3\sin\theta - 4\cos^3\theta$ (b) $4\cos^3\theta - 3\sin\theta$
(c) $4\sin^3\theta - 3\sin\theta$ (d) $3\sin\theta - 4\sin^3\theta$

Q.15 $\cos 3\theta =$ ________

(a) $4\cos^3\theta - 3\cos\theta$ (b) $4\sin^3\theta - 3\cos\theta$
(c) $3\sin\theta - 4\cos^3\theta$ (d) $3\sin\theta - 4\sin^3\theta$

Q.16 $\tan 3\theta =$ ________

(a) $\frac{3\tan\theta + \tan^3\theta}{1-3\tan^2\theta}$ (b) $\frac{3\tan\theta - 3\tan^3\theta}{1+3\tan^2\theta}$
(c) $\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ (d) $\frac{\tan\theta + 3\tan^2\theta}{1+3\tan^2\theta}$

Q.17 $\frac{2\tan\theta}{1+\tan^2\theta}$ **is equal to.**(Mtn 16 G-I)

(a) $\tan 2\theta$ (b) $\cot 2\theta$
(c) $\sin 2\theta$ (d) $\cos 2\theta$

Q.18 $\frac{1+\cos\theta}{\sin\theta} =$ (Fbd 16)

(a) $\sin\theta$ (b) $\cot\frac{\theta}{2}$
(c) $\operatorname{cosec}^2\theta$ (d) $\tan\frac{\theta}{2}$

Q.19 $2\cos^2\frac{\theta}{2}$ **equals.** (D.G.K 17 G-II)

(a) $1+\cos\theta$ (b) $1-\cos\theta$
(c) $1-\sin\theta$ (d) $1+\sin\theta$

Q.20 $\sin\theta$ **equals.** (Rwp 17 G-I)

(a) $2\sin^2\frac{\theta}{2}$ (b) $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
(c) $2\cos^2\frac{\theta}{2}$ (d) $2\tan\frac{\theta}{2}$

Q.21 $\sin\frac{\alpha}{2}$ **is equal to.** (Sgd 17)

(a) $\sqrt{\frac{1-\sin\alpha}{2}}$ (b) $\sqrt{\frac{1-\cos\alpha}{2}}$
(c) $\sqrt{\frac{1+\cos\alpha}{2}}$ (d) $\sqrt{\frac{1-\sin\alpha}{2}}$

EXERCISE 10.4

SHORT ANSWERS TO THE QUESTIONS

Q.1 Express $2\sin 3\theta\cos\theta$ as sum. (Lhr 2006 G-I)

Ans. $2\sin 3\theta\cos\theta = \sin(3\theta+\theta) + \sin(3\theta-\theta)$
$= \sin 4\theta + \sin 2\theta$

Q.2 Express $2\sin 7\theta$ co 3θ as sum or difference. (Lhr 11 G-I)

Ans. $2\sin 7\theta\cos 3\theta$
$= \sin(7\theta+3\theta) + \sin(7\theta-3\theta)$
$= \sin 10\theta + \sin 4\theta$

Q.3 Express $2\sin 7\theta\sin 2\theta$ as a sum or difference. (Lhr 16 G-I)

Ans. $2\sin 7\theta\sin 2\theta$
$= -(-2\sin 7\theta\sin 2\theta)$
$= -[\cos(7\theta+2\theta) - \cos(7\theta-2\theta)]$
$= -[\cos 9\theta - \cos 5\theta] = -\cos 9\theta + \cos 5\theta$

Q.4 Express $\cos(x+y)\sin(x-y)$ as sum or difference. (Rwp 17 G-II)

Ans. $\cos(x+y)\sin(x-y)$
$= \frac{1}{2}[2\cos(x+y)\sin(x-y)]$
$= \frac{1}{2}[\sin(x+y+x-y) - \sin(x+y-x+y)]$
$= \frac{1}{2}[\sin 2x - \sin 2y]$

Q.5 Express $\sin 5\theta\cos 2\theta$ as sum or difference. (Gjw 12)(Gjw 10) (Mtn 16 G-I)

Ans. $2\sin 5\theta\cos 2\theta$
$= \sin(5\theta+2\theta) + \sin(5\theta-2\theta)$
$= \sin 7\theta + \sin 3\theta$

$\Rightarrow \sin 5\theta\cos 2\theta = \frac{1}{2}\{\sin 7\theta + \sin 3\theta\}$

Q.6 Express $\cos 12^\circ + \cos 48^\circ$ as products.

(Lhr 10 G-II)

Ans. $\cos 12^\circ + \cos 48^\circ$

$$= 2\cos\frac{12^\circ + 48^\circ}{2}\cos\frac{12^\circ - 48^\circ}{2}$$

$$= 2\cos\frac{60^\circ}{2}\cos\frac{-36^\circ}{2}$$

$$= 2\cos 30^\circ\cos(-18^\circ)$$

$$= 2\cos 30^\circ\cos 18^\circ \quad \{\because \cos(-\theta) = \cos\theta\}$$

Q.7 Express $\cos 12^\circ + \cos 48^\circ$ as a product.

(Lhr 14 G-II)

Ans. $\cos 12^\circ + \cos 48^\circ = 2\cos\frac{12^\circ + 48^\circ}{2}\cos\frac{12^\circ - 48^\circ}{2}$

$$= 2\cos\left(\frac{60^\circ}{2}\right)\cos\left(\frac{-36^\circ}{2}\right)$$

$$= 2\cos 30^\circ\cos(-18^\circ)$$

$$= 2\cos 30^\circ\cos 18^\circ$$

Q.8 Express $\sin 5x + \sin 7x$ as a product.

(Gjw 14)

Ans. $\sin 5x + \sin 7x$

$$= 2\sin\frac{5x + 7x}{2}\cos\frac{5x - 7x}{2} = 2\sin 6x\cos(-x)$$

$$= 2\sin 6x\cos x \quad (\Theta \cos(-\theta) = \cos\theta)$$

Q.9 Prove that $\sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\cos 2\theta$.

(Rwp 16)(Mtn 17 G-II)

Ans. L.H.S

$$= \sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right)$$

$$= -\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} + \theta\right)\sin\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{4} + \theta + \frac{\pi}{4} - \theta\right) - \cos\left(\frac{\pi}{4} + \theta - \frac{\pi}{4} + \theta\right)\right]$$

$$= -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 2\theta\right]$$

$$= -\frac{1}{2}[0 - \cos 2\theta]$$

$$= \frac{1}{2}\cos 2\theta = \text{R.H.S}$$

Q.10 Prove the identity: $\frac{\sin\alpha - \sin\beta}{\sin\alpha + \sin\beta} = \tan\frac{\alpha - \beta}{2}\cot\frac{\alpha + \beta}{2}$.

(Gjw 13)(Sgd 19)

Ans. L.H.S. $= \frac{\sin\alpha - \sin\beta}{\sin\alpha + \sin\beta}$

$$= \frac{2\cos\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}}{2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}}$$

$$= \cot\frac{\alpha + \beta}{2}\tan\frac{\alpha - \beta}{2} = \text{R.H.S.}$$

Q.11 Prove that $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$

(Lhr 08 G-I)

Ans. L.H.S. $= \cos 20^\circ + \cos 100^\circ + \cos 140^\circ$

$$= 2\cos\frac{20^\circ + 100^\circ}{2}\cos\frac{20^\circ - 100^\circ}{2} + \cos 140^\circ$$

$$= 2\cos 60^\circ\cos(-40^\circ) + \cos 140^\circ$$

$$= 2\times\frac{1}{2}\cos 40^\circ + \cos 140^\circ \quad \{\because \cos(-\theta) = \cos\theta\}$$

$$= \cos 40^\circ + \cos 140^\circ$$

$$= 2\cos\frac{40^\circ + 140^\circ}{2}\cos\frac{40^\circ - 140^\circ}{2}$$

$$= 2\cos 90^\circ\cos(-50^\circ)$$

$$= 0 = \text{R.H.S.} \quad \therefore \cos 90^\circ = 0$$

Hence $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$

LONG QUESTIONS

Q.1 Prove that: $\sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\cos 2\theta$

(Gjw 17)

Ans. See Short Question 16

Q.2 Prove that:

$$\frac{\sin\alpha - \sin\beta}{\sin\alpha + \sin\beta} = \tan\left(\frac{\alpha - \beta}{2}\right)\cot\left(\frac{\alpha + \beta}{2}\right)$$

(Rwp 17 G-II)

Ans. See Short Question 17

Q.3 Prove that $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$

(Bpr 14)

Ans. See Short Question 19

Q.4 Show that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

(Lhr 17 G-II)(Mtn 18 G-I)

(Bpr 12 G-I)(Mtn 15 G-I)(Rwp 11 G-I, 13 G-II)

Ans. L.H.S. $= \frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta}$

$$= \frac{\sin 7\theta + \sin\theta + \sin 5\theta + \sin 3\theta}{\cos 7\theta + \cos\theta + \cos 5\theta + \cos 3\theta}$$

$$= \frac{2\sin\frac{7\theta+\theta}{2}.\cos\frac{7\theta-\theta}{2}+2\sin\frac{5\theta+3\theta}{2}.\cos\frac{5\theta-3\theta}{2}}{2\cos\frac{7\theta+\theta}{2}.\cos\frac{7\theta-\theta}{2}+2\cos\frac{5\theta+3\theta}{2}.\cos\frac{5\theta-3\theta}{2}}$$

$$= \frac{2\sin\frac{8\theta}{2}.\cos\frac{6\theta}{2}+2\sin\frac{8\theta}{2}.\cos\frac{2\theta}{2}}{2\cos\frac{8\theta}{2}.\cos\frac{6\theta}{2}+2\cos\frac{8\theta}{2}.\cos\frac{2\theta}{2}}$$

$$= \frac{2\sin 4\theta.\cos 3\theta+2\sin 4\theta.\cos\theta}{2\cos 4\theta.\cos 3\theta+2\cos 4\theta.\cos\theta}$$

$$= \frac{2\sin 4\theta(\cos 3\theta+\cos\theta)}{2\cos 4\theta(\cos 3\theta+\cos\theta)}$$

$$= \frac{\sin 4\theta}{\cos 4\theta} = \tan 4\theta = \text{R.H.S.}$$

Q.5 Show that (with tables / calculator).

$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$.

(Sgd 16)(Bwp 18)(DGK-19)(Fbd 19 G-II)

Ans. L.H.S. $= \cos 20^\circ \cos 40^\circ \cos 80^\circ$

$$= \frac{1}{2}[2\cos 20^\circ \cos 40^\circ]\cos 80^\circ$$

$$= \frac{1}{2}[\cos(20^\circ+40^\circ)+\cos(20^\circ-40^\circ)]\cos 80^\circ$$

$$= \frac{1}{2}[\cos 60^\circ+\cos(-20^\circ)]\cos 80^\circ$$

$$= \frac{1}{2}\left[\frac{1}{2}+\cos 20^\circ\right]\cos 80^\circ$$

$$= \frac{1}{2}\left[\frac{1+2\cos 20^\circ}{2}\right]\cos 80^\circ$$

$$= \frac{1}{4}[1+2\cos 20^\circ]\cos 80^\circ$$

$$= \frac{1}{4}[\cos 80^\circ+2\cos 80^\circ\cos 20^\circ]$$

$$= \frac{1}{4}[\cos 80^\circ+\cos(80^\circ+20^\circ)$$
$$+\cos(80^\circ-20^\circ)]$$

$$= \frac{1}{4}[\cos 80^\circ+\cos 100^\circ+\cos 60^\circ]$$

$$= \frac{1}{4}\left[2\cos\frac{80^\circ+100^\circ}{2}.\cos\frac{80^\circ-100^\circ}{2}+\frac{1}{2}\right]$$

$$= \frac{1}{4}\left[2\cos\frac{180^\circ}{2}.\cos(-10^\circ)+\frac{1}{2}\right]$$

$$= \frac{1}{4}\left[2\cos 90^\circ\cos 10^\circ+\frac{1}{2}\right]$$

$$= \frac{1}{4}\left[2(0)\cos 10^\circ+\frac{1}{2}\right] = \frac{1}{4}\left[0+\frac{1}{2}\right] = \frac{1}{4}\left[\frac{1}{2}\right]$$

$$= \frac{1}{8} = \text{R.H.S.}$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 **$\sin 8\theta - \sin 4\theta$ equals** (Mtn 09 G-I)

(a) $2\sin 6\theta\cos 2\theta$ (b) $\frac{1}{2}\cos 6\theta\sin 3\theta$

(c) $2\cos 6\theta\cos 2\theta$ (d) $2\cos 6\theta\sin 2\theta$

Q.2 **$\frac{\sin 3x-\sin x}{\cos x-\cos 3x}$ equals** (Mtn 09 G-II)

(a) $\tan 2x$ (b) $\cot 2x$

(c) $\tan x$ (d) $\cot x$

Q.3 **$\cos(\alpha+\beta)-\cos(\alpha-\beta)$**

(a) $2\sin\alpha\cos\beta$ (b) $2\cos\alpha\sin\beta$

(c) $2\cos\alpha\cos\beta$ (d) $-2\sin\alpha\sin\beta$

Q.4 **$\sin(\alpha+\beta)-\sin(\alpha-\beta)$ equal to.** (Bpr 13 G-I)

(a) $2\cos\alpha\sin\beta$ (b) $2\sin\alpha\cos\beta$

(c) $2\cos\alpha\cos\beta$ (d) $-2\sin\alpha\sin\beta$

Q.5 **$\cos\theta + \cos\phi$ = ___________**

(a) $2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

(b) $2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

(c) $-2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

(d) $2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

Q.6 **$\cos\theta - \cos\phi$ = ___________**

(a) $2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

(b) $2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

(c) $2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

(d) $-2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

Q.7 **$\sin\theta + \sin\phi$ = ___________**

(a) $2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

(b) $2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

(c) $2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

(d) $-2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

Q.8 $\sin\theta - \sin\phi =$ ____________

(a) $-2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

(b) $2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$

(c) $2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

(d) $2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}$

Q.9 $2\cos 5\theta \sin 3\theta =$ (Mtn 16 G-II)

(a) $\sin 4\theta - \sin\theta$ (b) $\sin 4\theta + \sin\theta$

(c) $\sin 8\theta - \sin 2\theta$ (d) $\cos 8\theta + \cos 2\theta$

Q.10 $2\sin\alpha\cos\beta =$ ____________

(Fbd, Bpr 14)

(a) $\cos(\alpha+\beta) + \cos(\alpha-\beta)$

(b) $\cos(\alpha+\beta) - \cos(\alpha-\beta)$

(c) $\sin(\alpha+\beta) + \sin(\alpha-\beta)$

(d) $\sin(\alpha+\beta) - \sin(\alpha-\beta)$

Q.11 **$2\sin 7\theta \cos 3\theta$ is equal to.**

(Fbd 11)

(a) $\sin 10\theta + \sin 4\theta$

(b) $\sin 10\theta - \sin 4\theta$

(c) $4\cos^3\theta - 3\cos\theta$

(d) $\cos 10\theta - \cos 4\theta$

Q.12 **$\sin 3\theta + \sin 5\theta$ equals.** (Sgd 13 G-II)

(Gjw 14)

(a) $2\sin 4\theta\cos 2\theta$ (b) $2\cos 4\theta\sin\theta$

(c) $2\cos 4\theta\sin\theta$ (d) $2\sin 4\theta\cos\theta$

Q.13 $2\cos 5\theta\sin 3\theta$. (Lhr 10 G-II)

(a) $\sin 8\theta - \sin 2\theta$ (b) $\sin 8\theta + 2\sin\theta$

(c) $\cos 8\theta + \cos 2\theta$ (d) $\sin 4\theta - \sin\theta$

Q.14 $2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$ (Lhr 18 G-II)

(a) $\sin P + \sin Q$ (b) $\sin P - \sin Q$

(c) $\cos P + \cos Q$ (d) $\cos P - \cos Q$

Q.15 $\cos 48° + \cos 12° =$: (Fbd 19 G-II)

(a) $2\cos 18°$ (b) $3\cos 18°$

(c) $\sqrt{3}\cos 18°$ (d) $\sqrt{2}\cos 18°$

Chapter 11

TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

EXERCISE 11

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define a periodic function and what is period of function. **(DGK 19)**

Ans. A function f is said to be periodic, if there exist a smallest positive constant p such that $f(x + p) = f(x)$ for all x and p is called the period of f.

Q.2 Define the period of a trigonometric function.
(Bpr 16, 18)(Mtn 17 G-II)(Rwp 18)

Ans. Period of a trigonometric function is the smallest +ve number which, when added to the original circular measure of the angle, gives the same value of the function.

Q.3 Find the period of cos 2x **(Mtn 19 G-I)**
(Lhr 10 G-II)(Bpr 14 G-I)(Fbd 14 G-I)(Mtn 17 G-I)

Ans. Since $\cos 2x = \cos(2x + 2\pi) = \cos 2(x + \pi)$

Hence period of $\cos 2x$ is π.

Q.4 Find the period of $\tan\frac{x}{7}$.
(Lhr 14 G-II, 11, 12, 13 G-I)(Gjw 13, 18)
(Mtn 15 G-II)(D.G.K 11, 12 G-I)(Sgd 17)
(D.G.K 12 G-I)(Rwp 11 G-I)

Ans. Since $\tan\frac{x}{7} = \tan\left(\frac{x}{7} + \pi\right) = \tan\frac{1}{7}(x + 7\pi)$

Hence period of $\tan\frac{x}{7}$ is 7π.

Q.5 Find the period of cot 8x
(Bpr 11 G-I)(D.G.K 14 G-I)(Lhr 15 G-II)
(Mtn 16 G-I, II) (Fbd, Swl 19)

Ans. Since $\cot 8x = \cot(8x + \pi) = \cot 8\left(x + \frac{\pi}{8}\right)$

Hence period of $\cot 8x$ is $\frac{\pi}{8}$.

Q.6 Find the period of $3\cos\frac{x}{5}$ **(Fbd 16)**
(Lhr 08 G-I, 15 G-I)(D.G.K 11 G-I, 15 G-II)

Ans. Since $3\cos\frac{x}{5} = 3\cos\left(\frac{x}{5} + 2\pi\right) = 3\cos\frac{1}{5}(x + 10\pi)$

Hence period of $3\cos\frac{x}{5}$ is 10π.

Q.7 Find the period of $\sin\frac{x}{5}$ **(Sgd 16)(Rwp 17 G-II)**
(Gjw 11, 14)(Fbd 18)(Lhr 10 G-I)(Shw 14 G-I)

Ans. Since $\sin\frac{x}{5} = \sin\left(\frac{x}{5} + 2\pi\right) = \sin\frac{1}{5}(x + 10\pi)$

Hence period of $\sin\frac{x}{2}$ is 10π.

Q.8 Find the period of tan 4x.
(Fbd, Gjw 17)(Fbd 19 G-II)

Ans. $\tan 4x$

$= \tan(4x + \pi)$

$= \tan 4\left(x + \frac{\pi}{4}\right)$

Period $= \frac{\pi}{4}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 Period of tan 4x equals: (Mtn 08 G-I)

(a) $\frac{\pi}{4}$ (b) 4π
(c) $\frac{\pi}{2}$ (d) π

Q.2 Period of sin 2x is equal to: (Mtn 08, 17 G-II)

(a) 2π (b) π
(c) 3π (d) $\frac{3\pi}{2}$

Q.3 The period of cot 8x is:
(Mtn 10, 15 G-I, 18 G-II)

(a) $\frac{\pi}{2}$ (b) π
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

Q.4 Period of $\cos\frac{x}{2}$ is: (Fbd 18)(Lhr 18 G-I)
(Mtn 10 G-II)(Lhr 14 G-II)(Rwp 17 G-II)

(a) π (b) 2π
(c) 4π (d) $\frac{\pi}{2}$

Q.5 Domain of y = sec x is: (Mtn 11 G-I)
(a) $0 < x < \infty$ (b) $-\infty \le x \le \infty$
(c) $-\pi \le x \le \pi$ (d) $< x < \pi$

Q.6 The domain of cosine function is: (Mtn 11 G-II)
(a) R (b) Z
(c) C (d) W

Q.7 The range of cot x is: (Bpr 11)(Fbd 12)
(a) r
(b) $[-1, 1]$
(c) $R - \{x / x = n\pi\}$
(d) $R - \left\{x / x = (2x+1)\frac{\pi}{2}\right\}$

Q.8 Period of $\cos\frac{x}{3}$ is: (Mtn 12 G-II)
(a) π (b) 3π
(c) $\frac{2\pi}{3}$ (d) 6π

Q.9 Period of $3\sin\frac{x}{3}$ is: (Shw 13) (Mtn 13 G-I)
(a) π (b) 2π
(c) 3π (d) 6π

Q.10 The range of sin x is:
(D.G.K 15 G-I)(Fbd 15, 16)(Mtn 13 G-II)(Gjw 18)
(a) $[-1,1]$ (b) $[-1,0]$
(c) $[0,2]$ (d) $[-2,2]$

Q.11 Period of $\cos\frac{x}{5}$ is: (D.G.K 12)
(a) $\frac{10}{\pi}$ (b) $\frac{\pi}{5}$
(c) 2π (d) 10π

Q.12 Period of sin x is: (D.G.K 13 G.II)
(a) π (b) 2π
(c) $\frac{\pi}{2}$ (d) 4π

Q.13 The domain of sin x is:
(D.G. Khan 13 G-I, 17 G-II)
(a) $[-1,1]$ (b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
(c) R (d) Q

Q.14 The domain of tan x is:
(Sgd 13) (Bpr 12) (Fbd 11)
(a) $[-1,1]$ (b) R
(c) $R - \left\{x|x = (2n+1)\frac{\pi}{2}\right\}$
(d) $R - \{x|x = n\pi, n \in z\}$

Q.15 The period of tan θ is: (Bpr 13)
(a) π (b) 2π
(c) 3π (d) 4π

Q.16 The smallest positive number p for which f(x+p) = f (x) is called:
(Gjw 12) (D.G.K 15 G-II)
(a) Domain (b) Range
(c) Co-domain (d) Period

Q.17 Range of tangent function is:
(Rwp 11)(Mtn 16 G-I)
(a) R (b) Z
(c) N (d) C

Q.18 Period of tan θ is: (Rwp 12)
(a) $-\pi$ (b) π
(c) 2π (d) -2π

Q.19 Period of $\sin\frac{x}{3}$ is: (Rwp 13)(Lhr 14 G-I)
(a) π (b) 3π
(c) $\frac{2\pi}{3}$ (d) 6π

Q.20 The range of sin x is: (Fbd 15) (D.G.K 15 G-I)
(a) $(-1, 1)$ (b) $[-1, 1)$
(c) $[-1, 1]$ (d) $(-1, 1]$

Q.21 The period of sin x is: (Sgd 13)
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
(c) π (d) 2π

Q.22 Range of y = cos x is equal to:
(Mtn 14 G-I)(Rwp 18)(DGK 19)
(a) $(-1,1)$ (b) $-1 \le y \le 1$
(c) $o < y < 1$ (d) R

Q.23 The period of sec x is: (Mtn 14 G-II)
(a) π (b) 2π
(c) 3π (d) 4π

Q.24 The period of sec is: (D.G.K 14 G-II)(Fbd 17)
(a) π (b) 2π
(c) 3π (d) 4π

Q.25 The range of cos x is: (Lhr 11 G-I)(Shw 14)
(a) R (b) $[-1,1]$
(c) $[-3,3]$ (d) $[0,3]$

Q.26 Period of tan x: (Bpr 14)
(a) 4π (b) 3π
(c) 2π (d) π

Q.27 Period of 3 sin 3x is: (Lhr 15 G-II) (Fbd 14)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) π

Q.28 Period of cot3x: (Rwp 14)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) π

Q.29 Domain of cot θ = (Lhr 10 G-I)

(a) $-\infty < \theta < \infty$ (b) $\theta \neq n\pi$

(c) $-\infty < \theta < \infty$ (d) $\theta = \left(\frac{2n+1}{2}\right)\pi$

Q.30 The range of y = cos x is: (Lhr 10 G-II)(Mtn 15 G-II, 19 G-I)

(a) $-1 \le x \le 1$ (b) $-\infty < x < \infty$

(c) $-1 \le y \le 1$ (d) $-\infty < y < \infty$

Q.31 The period of tan $\frac{x}{2}$ (Rwp 15)(Lhr, D.G.K 17 G-I)

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

Q.32 The domain of y = sin x is: (Lhr 11 G-I)(Shw 17)(Sgd 19)

(a) $[-1, 1]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) R (d) Q

Q.33 Period of cosec θ is: (Gjw 11)(Bpr 19)

(a) 0 (b) π

(c) 2π (d) 4π

Q.34 Period of tan $\frac{x}{7}$ is: (Lhr 12)

(a) π (b) 7π

(c) $\frac{\pi}{7}$ (d) $\pi + 7$

Q.35 Domain of y = cos x is: (Bpr 16)

(a) $[0, 1]$ (b) $\mathbb{R}$

(c) $[-1, 1]$ (d) $[-1, 0]$

Q.36 Cosine is periodic function of period: (Gjw 12)

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

Q.37 The period of tan $\frac{x}{3}$ is: (Lhr 13 G-I)(Rwp 17 G-I)

(a) π (b) 2π

(c) 3π (d) None of these

Q.38 Period of sin 3x is: (Lhr 13 G-II)

(a) 2π (b) $\frac{2\pi}{3}$

(c) $\frac{3\pi}{2}$ (d) 3π

Q.39 Period of sin $\frac{x}{5}$ function is: (Gjw 13)(Rwp 19)

(a) 2π (b) $\frac{2\pi}{5}$

(c) 10π (d) 5π

Q.40 Range of cos x is: (Gjw 14)

(a) $[-1, 1]$ (b) $[0, \infty]$

(c) $[-\infty, 0]$ (d) $[-\infty, +\infty]$

Q.41 The range of sin 2x is: (Bpr 15)

(a) $[-1, 1]$ (b) $[-2, 2]$

(c) $(-1, 1)$ (d) $(-2, 2)$

Q.42 The range of y = sin x is equal to: (Lhr 15 G-I)

(a) $-1 \le y \le 1$ (b) $-1 < y < 1$

(c) $-1 \le x \le 1$ (d) $-1 \le y < 1$

Q.43 Period of tan $\frac{x}{2}$ is: (Rwp 15)

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

Q.44 The period of tan x is (Lhr 16)

(a) π (b) 2π

(c) -3π (d) $\frac{\pi}{2}$

Q.45 The period of sec x is

(a) π (b) 2π

(c) 3π (d) $\frac{\pi}{2}$

Q.46 The period of cot x is

(a) π (b) 2π

(c) 3π (d) $\frac{\pi}{2}$

Q.47 The period of sin 2x is equal to. (Sgd 16)

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) 2π

Q.48 The period of cot 2x is
(a) π (b) 2π
(c) 3π (d) $\frac{\pi}{2}$

Q.49 The period of sec 2x is
(a) π (b) 2π
(c) 3π (d) $\frac{\pi}{2}$

Q.50 The period of csc 2x is
(a) π (b) 2π
(c) 3π (d) $\frac{\pi}{2}$

Q.51 The period of tan 3x is
(a) π (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q.52 The period of cot 3x is
(a) π (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q.53 The period of sec 10x is (Bpr 18)
(a) $\frac{\pi}{2}$ (b) π
(c) $\frac{\pi}{5}$ (d) 2π

Q.54 The period of cot $\frac{x}{3}$ is (Mtn 17 G-II)
(a) π (b) 2π
(c) 3π (d) 4π

Q.55 The period of cosec $\frac{x}{4}$ is
(a) 2π (b) 4π
(c) 8π (d) π

Q.56 Period of cosec 10x is. (Rwp 16)
(a) $\frac{\pi}{10}$ (b) $\frac{2\pi}{5}$
(c) $\frac{\pi}{5}$ (d) $\frac{4\pi}{5}$

Q.57 The period of 2 cos x is. (Gjw 16)(Sgd 18)
(a) 4π (b) 2π
(c) π (d) 3π

Q.58 Period of 3 sin 2x is. (Lhr 18 G-II)
(a) 6π (b) 2π
(c) π (d) $\frac{\pi}{2}$

Q.59 The smallest positive number p for which f(x + p) = f(x) is called. (Mtn 18 G-I)
(a) Index (b) Domain
(c) Co-efficient (d) Period

Q.60 The graph of inverse tangent function $y = \tan^{-1}x$ is obtained by reflecting the the graph of $y = \tan x \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$ about
(a) x-axis (b) y-axis
(c) $y = x$ (d) $y = -x$

Q.61 The period of $\sin\frac{x}{2}$ is: (Fbd 19 G-I)
(a) 2π (b) 4π
(c) π (d) 3π

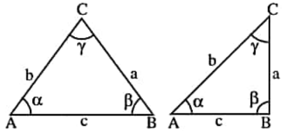
Q.62 The period of $3\cos\frac{x}{5}$ is: (Fbd 19 G-II)
(a) π (b) 10π
(c) $\frac{\pi}{10}$ (d) $\frac{\pi}{5}$

Q.63 Period of cosec 10x is (Gjw 19 G-II)
(a) $\frac{\pi}{10}$ (b) $\frac{2\pi}{5}$
(c) $\frac{\pi}{5}$ (d) $\frac{4\pi}{5}$

Q.64 Range of cotangent function is: (Lhr 19 G-II)
(a) N (b) Z
(c) R (d) C

Q.65 Period of $\frac{1}{2}\sin 2x$ is (Swl 19)
(a) $\frac{\pi}{2}$ (b) π
(c) 2π (d) 4π

THE LAW OF COSINE



1. $a^2 = b^2 + c^2 - 2bc\cos\alpha$

or $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

2. $b^2 = c^2 + a^2 - 2ca\cos\beta$

or $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ca}$

3. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

or $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

LAW OF SINE

4. $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

LAW OF TANGENTS

5. $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\dfrac{\alpha-\beta}{2}\right)}{\tan\left(\dfrac{\alpha+\beta}{2}\right)}$

6. $\dfrac{b-c}{b+c} = \dfrac{\tan\dfrac{\beta-\gamma}{2}}{\tan\dfrac{\beta+\gamma}{2}}$

7. $\dfrac{c-a}{c+a} = \dfrac{\tan\dfrac{\gamma-\alpha}{2}}{\tan\dfrac{\gamma+\alpha}{2}}$

HALF ANGLE FORMULAS

8. $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(S-b)(S-c)}{bc}}$

9. $\sin\dfrac{\beta}{2} = \sqrt{\dfrac{(S-a)(S-c)}{ac}}$

10. $\sin\dfrac{\gamma}{2} = \sqrt{\dfrac{(S-a)(S-b)}{ab}}$

11. $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{S(S-a)}{bc}}$

12. $\cos\dfrac{\beta}{2} = \sqrt{\dfrac{S(S-b)}{bc}}$

13. $\cos\dfrac{\gamma}{2} = \sqrt{\dfrac{S(S-c)}{ab}}$

14. $\tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(S-b)(S-c)}{S(S-a)}}$

15. $\tan\dfrac{\beta}{2} = \sqrt{\dfrac{(S-a)(S-c)}{S(S-b)}}$

16. $\tan\dfrac{\gamma}{2} = \sqrt{\dfrac{(S-a)(S-b)}{S(S-c)}}$

17. Area of triangle $= \Delta = \dfrac{1}{2}\, bc\sin\alpha$

18. Area of triangle $= \Delta = \dfrac{1}{2}\, ac\sin\beta$

19. Area of triangle $= \Delta = \dfrac{1}{2}\, ab\sin\gamma$

20. $\Delta = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$

21. $\Delta = \dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta}$

22. $\Delta = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$

23. Hero's Formula $= \Delta = \sqrt{S(S-a)(S-b)(S-c)}$

(CIRCUM – RADIUS)

24. $R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma} = \dfrac{abc}{4\Delta}$

(Circum – Radius)

25. $r = \dfrac{\Delta}{S}$ (in – radius)

26. $r_1 = \dfrac{\Delta}{S-a}$ (e – radius opposite to vertex A)

27. $r_2 = \dfrac{\Delta}{S-b}$ (e – radius opposite to vertex B)

28. $r_3 = \dfrac{\Delta}{S-c}$ (e – radius opposite to vertex C)

Chapter 12

APPLICATION OF TRIGONOMETRY

EXERCISE 12.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define "right angled triangle". (Sgd 19)

Ans. Right angled triangle: A triangle which has one angle equal to 90° is called right angled triangle.

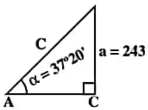
Q.2 Solve the right triangle ABC, in which $\gamma = 90°$, $\alpha = 37°20'$, $a = 243$

(Lhr 13 G-II)(Gjw 18)(Fbd 12 G-I)(D.G.K 14 G-I)

Ans. $\alpha = 37°\,20'$, $\gamma = 90°$, $a = 243$

For β using, $\beta = 90° - \alpha = 90° - 37°\,20' = 52°40'$

Now $\frac{a}{c} = \sin\alpha$



$\Rightarrow \quad c = \frac{a}{\sin 37°20'} = \frac{243}{0.61} = 400.66$

and $\frac{a}{b} = \tan 37°20'$

$\Rightarrow \quad b = \frac{a}{\tan 37°20'} = \frac{243}{0.76} = 318.56$

Thus $\beta = 52°40'$, $b = 318.56$, $c = 400.66$

Q.3 Solve the right triangle ABC, in which $\gamma = 90°$, $\alpha = 62°40'$, $b = 796$

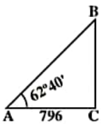
(Lhr 11 G-I)(D.G.K 13 G-I)(Fbd 16)(Gjw 17)

Ans. $\alpha = 62°40'$, $\gamma = 90°$, $b = 796$

$\beta = 90° - \alpha = 90° - 62°40'$

$\Rightarrow \quad \beta = 27°20'$

Now $\frac{b}{c} = \cos\alpha$



$\Rightarrow \quad c = \frac{b}{\cos\alpha} = \frac{796}{\cos 62°40'}$

$= \frac{796}{0.46} = 1733.82$

and $\frac{a}{b} = \tan\alpha \Rightarrow a = b\tan\alpha = 796\,(\tan 62°40')$

$= 796\,(1.9349) = 1540.16$

Thus $\alpha = 62°40'$, $\beta = 27°20'$, $\gamma = 90°$, $a = 1540.16$, $b = 796$, $c = 1733.82$

Q.4 Solve the right triangle ABC, in which $\gamma = 90°$, $a = 3.28$, $b = 5.74$ (Bpr 14 G-I)(D.G.K 14 G-II)

Ans. $a = 3.28$, $b = 5.74$, $\gamma = 90°$

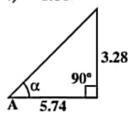
To find c using Pythagoras theorem

$c = \sqrt{a^2 + b^2} = \sqrt{(3.28)^2 + (5.74)^2} = 6.61$

For α using

$\sin\alpha = \frac{a}{c} = \frac{3.28}{6.61} = 0.50$

$\Rightarrow \alpha = \sin^{-1}(0.50) = 29°45'$



For β using,

$\beta = 90° - \alpha = 90° - 29°45' = 60°15'$

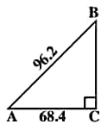
Thus $\alpha = 29°45'$, $\beta = 60°15'$, $c = 6.61$

Q.5 Solve the right triangle ABC, in which $\gamma = 90°$, $b = 68.4$, $c = 96.2$

Ans. $b = 68.4$, $c = 96.2$, $\gamma = 90°$

To find c using Pythagoras theorem

$c^2 = a^2 + b^2$



$\Rightarrow \quad a^2 = c^2 - b^2 \;\Rightarrow a = \sqrt{c^2 - b^2}$

$a = \sqrt{(96.2)^2 - (68.4)^2} = 67.7$

For α $\sin\alpha = \frac{a}{c} = \frac{67.7}{96.2} = 0.70$

$\Rightarrow \quad \alpha = \sin^{-1}(0.70) = 44°\,40'$

For β using

$\Rightarrow \quad \beta = 90° - \alpha = 90° - 44°\,40' = 45°\,20'$

Thus $\alpha = 44°\,40'$, $\beta = 45°\,20'$, $\gamma = 90°$, $a = 67.7$, $b = 68.4$, $c = 96.2$

MULTIPLE CHOICE QUESTIONS

- ❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 In a right triangle, no angle is greater than:

(Lhr 11)(Mtn 17, 18 G-II)

(a) 90°   (b) 80°

(c) 60°   (d) 45°

Q.2 The sum of all the angles of the triangle is:
(Shw 12) (Bpr 12) (Mtn 13 G-II)

(a) 360° (b) 270°
(c) 180° (d) 90°

Q.3 Number of elements of a triangle are.
(Rwp 17 G-I)

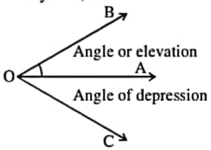
(a) 3 (b) 4
(c) 6 (d) 8

EXERCISE 12.3

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define angles of elevation and depression.
(Sgd, Gjw 16)(D.G.K 16 G-I)(Rwp 17 G-I)(Fbd 19 G-II)

Ans. If $\overrightarrow{OA}$ is the horizontal ray through the eye of observer at point O and there are two objects B and C such that B is above and C is below the horizontal ray $\overrightarrow{OA}$, then

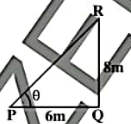


(i) For looking at B above the horizontal ray, we have to raise our eye and $\angle AOB$ is called the angle of elevation and

(ii) For looking at C blow the horizontal ray we have to lower our eye and $\angle AOC$ is called the angle of depression.

Q.2 A vertical pole is 8 m high and the length of its shadow is 6 m. What is the angle of elevation of the Sun at that moment?
(Fbd 14 G-I)(Mtn 17 G-I)

Ans.



Let $|RQ|$ = Length of pole = 8m
$|PQ|$ = Length of shadow of pole = 6m

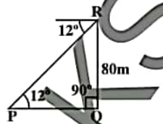
and θ be the angle of elevation of sun at that moment as shown in figure.

$$\text{From } \Delta PQR \ \tan\theta = \frac{|RQ|}{|PQ|} \Rightarrow \tan\theta = \frac{8}{6}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{8}{6}\right) = \tan^{-1}(1.333) = (53.13010)^\circ = 53^\circ 7' 40''$$

Q.3 At the top of a cliff 80 m high, the angle of depression of a boat is 12°. How far is the boat from the cliff?
(Rwp 14 G-I) (Lhr 17 G-II)(Fbd 19 G-II)

Ans.



Let $|RQ|$ = 80 m (Height of cliff)
$|PQ|$ = Distance of boat from cliff

$$\text{From } \Delta PQR \ \tan 12^\circ = \frac{|RQ|}{|PQ|}$$

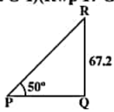
$$|PQ| \tan 12^\circ = |RQ|$$

$$|PQ| = \frac{80}{\tan 12^\circ} = \frac{80}{.21256}$$

$$= 376.37 \text{ m}$$

Q.4 A kite flying at a height of 67.2 m is attached to a fully stretched string inclined at an angle of 55° to the horizontal. Find the length of the string. **(Mtn 12 G-I)(Rwp 17 G-I)**

Ans.



Let $|PR|$ = Length of string

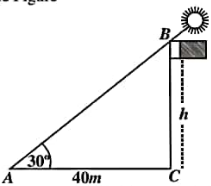
$$\text{From } \Delta PQR, \ \sin 55^\circ = \frac{|RQ|}{|PR|}$$

$$\Rightarrow |PR| \sin 55^\circ = |RQ|$$

$$\Rightarrow |PR| = \frac{|RQ|}{\sin 55^\circ} = \frac{67.2}{0.81915} = 82.036 \text{ m}$$

Q.5 When the angle between the ground and the sun is 30°, flag pole casts a shadow of 40m long. Find the height of the top of the flag. (Sgd 19)

Ans. From the Figure



$|BC|$ = ? (Height of flag pole)
$|AC|$ = 40m (Length of shadow)

In ΔABC

$$\tan 30^\circ = \frac{|BC|}{|AC|}$$

$$0.577 = \frac{|BC|}{40} \Rightarrow |BC| = 23.09\text{m}$$

$$\boxed{|BC| = 23.09 \text{ m}}$$

LONG QUESTIONS

Q.1 A kite flying at a height of 67.2m is attached to a fully stretched string inclined at an angle of 55° to the horizontal. Find the length of the string. (Mtn 12 G-I)

Ans. See Short Question 5.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 A tree of 8m high has the shadow 8m in length, then angle of elevation of sun at that moment is: (Lhr 11 G-II)(Fbd 18)

(a) 15° (b) 30°
(c) 45° (d) 60°

Q.2 Angle below the horizontal line is called: (Lhr 14 G-I)(Mtn 16 G-I)

(a) Right angle (b) Oblique angle
(c) Angle of depression
(d) Angle of electron

Q.3 Angle above the horizontal line is. (Gjw 18)(Bpr 19)

(a) Right angle (b) Oblique angle
(c) Angle of elevation (d) Angle of depression

Q.4 If the shadow of a tree is equal to its height then the angles of elevation of the sun is: (Fbd 19 G-II)

(a) 45° (b) 30°
(c) 60° (d) 90°

EXERCISE 12.4

SHORT ANSWERS TO THE QUESTIONS

Q.5 Solve the triangle ABC, if $\beta = 60°$, $\gamma = 15°$, $b = \sqrt{6}$. (Shw 14 G-I) (Mtn 17 G-I)(A.J.K Board 17)

Ans. $\beta = 60°$, $\gamma = 15°$, $b = \sqrt{6}$

Now $\alpha = 180° - (\beta + \gamma) = 180° - (60° + 15°) = 105°$

Using Law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b\sin\alpha}{\sin\beta} = \frac{\sqrt{6}\sin 105°}{\sin 60°}$$

$$= \frac{\sqrt{6}\,(0.97)}{0.87} = 2.73 = \sqrt{3} + 1$$

also $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} \Rightarrow c = \dfrac{b\sin\gamma}{\sin\beta} = \dfrac{\sqrt{6}\sin 15°}{\sin 60°}$

$$= \frac{\sqrt{6}(0.26)}{0.87} = 0.73 = \sqrt{3} - 1$$

Thus $\alpha = 105°$, $\beta = 60°$, $\gamma = 15°$
$a = \sqrt{3} + 1$, $b = \sqrt{6}$, $c = \sqrt{3} - 1$

Q.6 Solve the triangle ABC, if $b = 125$, $\gamma = 53°$, $\alpha = 47°$ (Bpr 14 G-I)(Gjw 15)

Ans. $b = 125$, $\gamma = 53°$, $\alpha = 47°$

Now $\beta = 180° - (\alpha + \gamma) = 180° - (47° + 53°) = 80°$

Using Law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b\sin\alpha}{\sin\beta} = \frac{125\sin 47°}{\sin 80°} = \frac{125\,(0.73)}{0.985}$$

$= 92.83$

Also $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$$\Rightarrow c = \frac{b\sin\gamma}{\sin\beta} = \frac{125\sin 53°}{\sin 80°} = \frac{125\,(0.7986)}{0.985} = 101.369$$

Thus $\alpha = 47°$, $\beta = 80°$, $\gamma = 53°$, $a = 92.83$,
$b = 125$, $c = 101.369$

Q.8 Solve the triangle ABC, if $a = 53$, $\beta = 88°36'$, $\gamma = 31°54'$. (Lhr 15 G-II)(Sgd 18)

Ans. $a = 53$, $\beta = 88°36'$, $\gamma = 31°54'$

Now $\alpha = 180° - (\beta + \gamma) = 180° - (88°\,36' + 31°\,54') = 59°30'$

Using Law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow b = \frac{a\sin\beta}{\sin\gamma} = \frac{53\sin 88°36'}{\sin 59°30'}$$

$$= \frac{53\,(0.9997)}{0.862} = 61.49$$

Also $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma} \Rightarrow c = \dfrac{a\sin\gamma}{\sin\alpha} = \dfrac{53\sin 31°54'}{\sin 59°30'}$

$$= \frac{53\,(0.53)}{0.862} = 32.5$$

Thus $\alpha = 59°30'$, $b = 61.49$, $c = 32.5$

LONG QUESTIONS

Q.1 Solve the triangle ABC if $a = 53$, $\beta = 88°\,36'$, $\gamma = 31°\,54'$ (Mtn 13 G-I)(Shw 13)(Rwp 14)

Ans. See Short Question 8.

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 A triangle which is not right is known as: (Mtn 08 G-II)

(a) Isosceles triangle (b) Equilateral triangle
(c) Oblique triangle (d) Quadrilateral

Q.2 With usual rotation $a - b + c$ is: (Mtn 10 G-II)

(a) $s + b$ (b) $2(s + b)$
(c) $s - c$ (d) $2(s - b)$

Q.3 If $\alpha = 90°$, then: (Fbd 15)(Mtn 18 G-I)
(a) $c^2 = a^2 + b^2$ (b) $b^2 = a^2 + c^2$
(c) $a^2 = b^2 + c^2$ (d) $a^2 = b^2 - c^2$

Q.4 For a triangle ABC with usual rotation $\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$ equals: (Shw 13)(Mtn 13 G-I)
(a) $\tan\gamma$ (b) $\tan\dfrac{\gamma}{2}$
(c) $\text{Cot}\,\gamma$ (d) $\text{Cot}\dfrac{\gamma}{2}$

Q.5 $\text{Cos}\dfrac{\alpha}{2}$ =
(D.G.K 15 G-II) (Mtn 13 G-II)(Lhr 16 G-I)
(a) $\sqrt{\dfrac{(s-b)(s-c)}{bc}}$ (b) $\sqrt{\dfrac{s(s-a)}{bc}}$
(c) $\sqrt{\dfrac{(s-a)(s-b)}{ab}}$ (d) $\sqrt{\dfrac{s(s-b)}{ac}}$

Q.6 The value of $\dfrac{b^2 + c^2 - a^2}{2bc}$ is: (D.G.K 11)
(a) $\cos\alpha$ (b) $\cos\beta$
(c) $\cos\gamma$ (d) $\sin\alpha$

Q.7 If a,b,c all sides of triangle then $\dfrac{a^2 + c^2 - b^2}{2ac}$
(a) $\cos\alpha$ (b) $\cos\beta$
(c) $\cos\gamma$ (d) $\cos\dfrac{\beta}{2}$

Q.8 Law of cosine $a^2 = b^2 + c^2 - 2bc\cos\alpha$ reduces to Pythagoras theorem if: (D.G.K 13 G-I)
(a) $\alpha = 90°$ (b) $\alpha = 0$
(c) $\alpha = 180°$ (d) $\alpha = 360°$

Q.9 $\sqrt{\dfrac{(s-b)(s-c)}{bc}}$ (D.G.K 13 G-II)
(a) $\cos\dfrac{\alpha}{2}$ (b) $\sin\dfrac{\alpha}{2}$
(c) $\tan\dfrac{\alpha}{2}$ (d) $\sin 2\alpha$

Q.10 In the triangle ABC, the law of tangents is:
(Rwp 13)(Lhr 18 G-II)
(a) $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\dfrac{\alpha-\beta}{2}\right)}{\tan\left(\dfrac{\alpha+\beta}{2}\right)}$ (b) $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\dfrac{\alpha+\beta}{2}\right)}{\tan\left(\dfrac{\alpha-\beta}{2}\right)}$
(c) $\dfrac{a+b}{a-b} = \dfrac{\tan\left(\dfrac{\alpha-\beta}{2}\right)}{\tan\left(\dfrac{\alpha+\beta}{2}\right)}$ (d) $\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

Q.11 $b^2 + c^2 - 2bc\cos\alpha$ equal to: (Bpr 14)
(a) Δ (b) c
(c) a^2 (d) 1

Q.12 In any triangle ABC with usual notation $\dfrac{a^2 + b^2 - c^2}{2ab}$ = (Lhr 12 G-I)
(a) $\cos\alpha$ (b) $\cos\beta$
(c) $\cos\gamma$ (d) $\sin\gamma$

Q.13 If ΔABC is right angle triangle, the law of cosines reduces to:
(a) The law of sines (b) Area of triangle
(c) The law of tangents
(d) The pythagoras theorem

Q.14 In triangle ABC, if $\gamma = 90°$ then
(a) $a^2 = b^2 + c^2$ (b) $b^2 = c^2 + a^2$
(c) $c^2 = a^2 + b^2$ (d) $b = c + a$

Q.15 In law of cosines $a^2 + b^2 - 2ab\cos\gamma$ =
(a) a^2 (b) b^2
(c) c^2 (d) c

Q.16 $\tan\dfrac{\alpha}{2}$ = (D.G.K. Board 15 G-I)
(a) $\sqrt{\dfrac{(S-b)(S-c)}{S(S-a)}}$ (b) $\dfrac{\sqrt{S(S-b)}}{ac}$
(c) $\dfrac{\sqrt{(S-a)(S-b)}}{ab}$ (d) $\dfrac{\sqrt{(S-b)(S-c)}}{bc}$

Q.17 With usual notation $\sqrt{\dfrac{(S-c)(S-b)}{S(S-a)}}$ =
(Mtn 16 G-II)
(a) $\tan\dfrac{\alpha}{2}$ (b) $\tan\dfrac{\beta}{2}$
(c) $\tan\dfrac{\gamma}{2}$ (d) $\cos\dfrac{\alpha}{2}$

Q.18 For any ΔABC, C^2 = (Lhr 16 G-I)
(a) $a^2 + c^2 - 2ac\cos\beta$
(b) $a^2 + b^2 - 2ab\cos\gamma$
(c) $b^2 + c^2 - 2bc\cos\alpha$
(d) $a^2 - c^2 - 2ac\cos\gamma$

Q.19 $\cot\dfrac{\alpha}{2}$ equals to. (Gjw 16)
(a) $\sqrt{\dfrac{S(S-c)}{(S-b)(S-a)}}$ (b) $\sqrt{\dfrac{S(S-a)}{(S-b)(S-c)}}$
(c) $\sqrt{\dfrac{(S-b)(S-a)}{S(S-c)}}$ (d) $\sqrt{\dfrac{(S-b)(S-c)}{S(S-a)}}$

Q.20 $\sqrt{\dfrac{S(S-a)}{bc}}$ =
(Fbd 16)(Mtn 18 G-II, 19 G-I)(Sgd 19)
(a) $\sin\dfrac{\alpha}{2}$ (b) $\sin\dfrac{\beta}{2}$
(c) $\cos\dfrac{\alpha}{2}$ (d) $\cos\dfrac{\beta}{2}$

Q.21 For a triangle with a, b, c and α, β, γ as measures of sides and opposite angles respectively, then $b^2 + c^2 - 2bc\cos\alpha =$: (Fbd 19 G-I)

(a) a^2 (b) b^2
(c) c^2 (d) Δ^2

Q.22 With usual notations, the value of a + b + c is

(a) s (b) 2s
(c) 3s (d) $\frac{s}{2}$

Q.23 In any triangle ABC, with usual notation $b^2 + c^2 - 2bc\cos\alpha =$ (Sgd 19)

(a) Δ (b) 0
(c) a^2 (d) 1

Q.24 $\sqrt{\dfrac{s(s-a)}{bc}} =$ (Sgd 19)

(a) $\sin\alpha/2$ (b) $\sin\beta/2$
(c) $\cos\alpha/2$ (d) $\cos\beta/2$

EXERCISE 12.5

SHORT ANSWERS TO THE QUESTIONS

Q.1 Solve the triangle in which
b = 95, c = 34, α = 52°
(D.G.K 12 GI)(Rwp 14 G-I)

Ans. For a using cosine formula

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$= (95)^2 + (34)^2 - 2(95)(34)\cos 52°$$
$$= 9025 + 1156 - 6460\,(0.61566)$$
$$a^2 = 10181 - 3977.173 = 6203.8269$$
$$\Rightarrow \quad a = 78.7644$$

For β using Law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow \sin\beta = \frac{b\sin\alpha}{a} = \frac{95\sin 52°}{78.7644}$$
$$= \frac{95(0.788)}{78.7644} = 0.9505$$
$$\Rightarrow \quad \beta = \sin^{-1} 0.9505 = 71°53'$$

For γ using

$$\gamma = 180° - (\alpha + \beta)$$
$$= 180° - (52° + 71°53') = 56°07'$$

Thus $\alpha = 52°$, $\beta = 71°53'$, $\gamma = 56°07'$, a = 78.7644, b = 95, c = 34

Q.2 Solve the triangle in which a = 7, b = 3, γ = 38°13′. **(Fbd 14 G-I)**

Ans. For c using cosine formula

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$= (7)^2 + (3)^2 - 2(7)(3)\cos 38°13'$$
$$= 49 + 9 - 42\,(0.7857)$$
$$c^2 = 58 - 32.998$$
$$c^2 = 25.002 \Rightarrow c = 5$$

For β using Law of sines

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$
$$\Rightarrow \quad \sin\alpha = \frac{a\sin\gamma}{c} = \frac{7\sin 38°13'}{5} = \frac{7(0.6186)}{5}$$
$$\sin\alpha = 0.8661 \Rightarrow \alpha = \sin^{-1}(0.8661) = 60°$$

For β using

$$\beta = 180° - (\alpha + \gamma)$$
$$= 180° - (60° + 38°13') = 81°47'$$

Thus $\alpha = 60°$, $\beta = 81°47'$, $\gamma = 38°13'$, a = 7, b = 3, c = 5

Q.3 Solve the triangle DABC in which a = 3, c = 6 and β = 36° 20′ **(Lhr 17 G-II)**

Ans. By law of cosines

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$= 3^2 + 6^2 - 2(3)(6)\cos 36°20'$$
$$= 9 + 36 - 29.001$$
$$= 16$$
$$b = 4$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\sin\alpha = \alpha\frac{\sin\beta}{b} = \frac{3\sin 36°20'}{4}$$
$$= 0.44436$$
$$\alpha = 26°23'$$
$$\alpha + \beta = 62°43'$$
$$r = 180° - (\alpha + \beta)$$
$$= 180 - 62°43'$$
$$= 17°17'$$

Q.4 Solve the triangle AC if
$a = \sqrt{3} - 1$, $b = \sqrt{3} + 1$, $\gamma = 60°$ then find the angles $\alpha_1\beta$. **(Gjw 18)**

Ans. $a = \sqrt{3} - 1 = 0.732$

$$b = \sqrt{3} + 1 = 3732$$
$$c^2 = a^2 + b^2 - 2ab\cos r$$
$$= (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 - 2(\sqrt{3}-1)(\sqrt{3}+1)$$
$$= 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} - 2(3-1)\frac{1}{2}$$
$$= 8 - 2 = 6$$
$$c = \sqrt{6}$$

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(\sqrt{3}+1)^2+(\sqrt{6})^2-(\sqrt{3}-1)^2}{2(\sqrt{3}+1)(\sqrt{6})}$$

$$= \frac{3+1+2\sqrt{3}+6-(3+1-2\sqrt{3})}{2\sqrt{6}(\sqrt{3}+1)}$$

$$= \frac{10+2\sqrt{3}-4+2\sqrt{3}}{2\sqrt{6}(\sqrt{3}+1)} = \frac{6+4\sqrt{3}}{2\sqrt{6}(\sqrt{3}+1)}$$

$$= \frac{12.9282}{13.3840} = 0.965944$$

$$\alpha = 15^\circ$$

$$\alpha + r = 15^\circ + 60^\circ = 75^\circ$$

$$\beta = 180^\circ - (\alpha + r)$$

$$= 180^\circ - 75^\circ = 105^\circ$$

Q.5 Solve the following triangles, using Law of tangent and then Law of sines:
$b = 14.8, c = 16.1, \alpha = 42^\circ 45'$ (Lhr 14 G-I)

Ans. $\beta+\gamma = 180^\circ - \alpha = 180^\circ - 42^\circ 45' = 137^\circ 15' \ldots$(i)

Using Law of tangent

$$\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)} = \frac{b-c}{b+c} \Rightarrow \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{137^\circ 15'}{2}\right)} = \frac{14.8-16.1}{14.8+16.1}$$

$$\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{2.55498} = \frac{-1.3}{30.9}$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = 2.55498\,(-0.04207)$$

$$\tan\left(\frac{\beta-\gamma}{2}\right) = -0.1075 \Rightarrow \left(\frac{\beta-\gamma}{2}\right)$$

$$= \tan^{-1}(-0.1075) = -6^\circ 8'$$

$\Rightarrow \quad \beta - \gamma = -12^\circ 16' \quad \ldots$(ii)

Adding (i) and (ii), we get

$2\beta = 124^\circ 53' \Rightarrow \beta = 62^\circ 26' 30''$

Put in eq.(i), we get

$\gamma = 137^\circ 15' - 62^\circ 26' 30'' = 74^\circ 45' 30''$

For a using Law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b\sin\alpha}{\sin\beta} = \frac{(14.8)\sin 42^\circ 45'}{\sin 62^\circ 26' 30''}$$

$$= \frac{14.8\,(0.6788)}{0.88654} = 11.33$$

Thus $\alpha = 42^\circ 45'$, $\beta = 62^\circ 29' 30''$, $\gamma = 74^\circ 45' 30''$, $a = 11.33$, $b = 14.8$, $c = 16.1$

LONG QUESTIONS

Q.1 Solve the Triangle ABC, if $a = 93$, $c = 101$ and $\beta = 80^\circ$. (Bpr 12 G-I)

Ans. Given that: $a = 93$, $c = 101$, $\beta = 80^\circ$

We know that: $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \quad \alpha + 80^\circ + \gamma = 180^\circ \Rightarrow \alpha + \gamma = 180^\circ - 80^\circ$

$\alpha + \gamma = 100^\circ \quad \ldots\ldots (1)$

$$\frac{\alpha+\gamma}{2} = \frac{100^\circ}{2} = 50^\circ$$

$$\Rightarrow \quad \tan\frac{\alpha+\gamma}{2} = \tan 50^\circ = 1.19^\circ$$

$$\therefore \quad \frac{c-a}{c+a} = \frac{\tan\frac{\gamma-\alpha}{2}}{\tan\frac{\gamma+\alpha}{2}} \Rightarrow \frac{101-93}{101+93} = \frac{\tan\frac{\gamma-\alpha}{2}}{1.19}$$

$$\Rightarrow \quad \frac{8}{194} = \frac{\tan\frac{\gamma-\alpha}{2}}{1.19}$$

$$\frac{8\times 1.19}{194} = \tan\frac{\gamma-\alpha}{2} \Rightarrow \frac{9.52}{194} = \tan\frac{\gamma-\alpha}{2}$$

$$\Rightarrow \quad 0.0491 = \tan\frac{\gamma-\alpha}{2}$$

$$\tan^{-1}(0.0491) = \frac{\gamma-\alpha}{2} \Rightarrow 2.81^\circ = \frac{\gamma-\alpha}{2}$$

$\Rightarrow \quad 5.62^\circ = \gamma - \alpha$

$\gamma - \alpha = 5.62^\circ \quad \ldots\ldots (2)$

Adding eq. (1) and eq. (2):

$$\gamma + \alpha = 100^\circ$$
$$\gamma - \alpha = 5.62^\circ$$

$$2\gamma = 105.62^\circ \Rightarrow \gamma = \frac{105.62^\circ}{2}$$

$$= 52.81^\circ \Rightarrow \boxed{\gamma = 52^\circ 48'}$$

Putting the value of γ in eq. (1), so

$$\alpha + 52.81^\circ = 100^\circ \Rightarrow \alpha = 100^\circ - 52.81^\circ$$

$$= 47.19^\circ \Rightarrow \boxed{\alpha = 47^\circ 11'}$$

and $$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \Rightarrow b = \frac{a\sin\beta}{\sin\alpha}$$

$$= \frac{93\times\sin 80^\circ}{\sin 47.19^\circ} = \frac{93\times 0.9848}{0.7336}$$

$$= \frac{91.5864}{0.7336} \Rightarrow \boxed{b = 124.84}$$

Q.2 Solve the following triangle ABC in which $a = 7$, $b = 3$, $\gamma = 38^\circ 13'$. (Bpr 13 G-I)(Lhr 15 G-I)

Ans. See Short Question 2

Q.3 Solve the ΔABC in which $b = 14.8$, $c = 16.1$ and $\alpha = 42^\circ 45'$. (Mtn 12 G-II)

Ans. See Short Question 5

EXERCISE 12.6

SHORT ANSWERS TO THE QUESTIONS

Q.1 Solve the triangle ABC, by using the law of cosines when a = 7, b = 3, c = 5 (D.G.K 14 G-II)

Ans. We know that: $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\therefore \quad \cos\alpha = \dfrac{9+25-49}{30} = -\dfrac{15}{30} = -\dfrac{1}{2} \Rightarrow \alpha = 120°$

$\cos\beta = \dfrac{c^2+a^2-b^2}{2ca} = \dfrac{25+49-9}{70} = \dfrac{65}{70} = 0.9286$

$\Rightarrow \quad \beta = 21°48'$

and $\gamma = 180° - (\alpha+\beta) = 180° - (120°+21°48') = 38°12'$.

Q.2 Find the measure of the greatest angle, if side of the triangle are 16, 20, 33
(Lhr 10 G-I, 13 G-II)(Rwp 14 G-I)

Ans. Let a = 16, b = 20, c = 32

For the greatest angle of the triangle ABC

Using Consine formula

$\cos\gamma = \dfrac{a^2+b^2-c^2}{2ab}$ (Θc > a , c > b)

$= \dfrac{(16)^2+(20)^2-(33)^2}{2(16)(20)} = \dfrac{256+400-1089}{640}$

$= \dfrac{-433}{640} = -0.68$

$\Rightarrow \gamma = \cos^{-1}(-0.68) = 132°34'$

Q.3 In a triangle ABC, a = 7 , b = 7 , c = 9, then find a, b. (Fbd 17)

Ans. $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{7^2+9^2-7^2}{2(7)(9)}$

$= \dfrac{81}{126} = \dfrac{9}{14}$

$\alpha = \cos^{-1}\left(\dfrac{9}{14}\right) = 50°$

$\cos\beta = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{7^2+9^2-7^2}{2(7)(9)} = \dfrac{81}{126} = \dfrac{9}{14}$

$\beta = \cos^{-1}\left(\dfrac{9}{14}\right) = 50°$

$\alpha + \beta = 50° + 50° = 100$

$r = 180° - (\alpha+\beta)$

$= 180° - 100 = 80°$

Q.4 Solve the following triangles, if a = 32,b = 40,c = 66. (Gjw 13)(Lhr 08 G-I)

Ans. For α using cosine formula

$\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{(40)^2+(66)^2-(32)^2}{2(40)(66)} = \dfrac{1600+4356-1024}{5280}$

$= \dfrac{4932}{5280} = 0.93$

$\alpha = \cos^{-1}(0.93) = 20°56'$

For β again using cosine formula

$\cos\beta = \dfrac{a^2+c^2-b^2}{2ac}$

$= \dfrac{(32)^2+(66)^2-(40)^2}{2(32)(66)} = \dfrac{1024+4356-1600}{4224}$

$= \dfrac{3780}{4224} = 0.89$

$\Rightarrow \beta = \cos^{-1}(0.89) = 26°30'$

For γ using

$\gamma = 180° - (\alpha+\beta)$

$= 180° - (20°56' + 26°30') = 132°34'$

Thus $\alpha = 20°56'$, $\beta = 26°30'$, $\gamma = 132°34'$, a = 32, b = 40 , c = 66

Q.5 Find the smallest angle of the triangle ABC when a = 37.34, b = 3.24 and c = 35.06.
(Rwp 16)(Fbd 16)(Lhr 17 G-II)(Mtn 17 G-I)
(Rwp, Sgd 18)

Ans. The smallest angle is the angle opposite to the smallest side, which is β in this case. So by law of cosines:

$\cos\beta = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{(37.34)^2+(35.05)^2-(3.24)^2}{2(37.34)(35.05)}$

$= 0.998$

$\Rightarrow \beta = \cos^{-1}(0.998) = 3^0\,38'$.

LONG QUESTIONS

Q.1 Solve the triangle ABC in which a = 7, b=7, c = 9.
(Bpr 11 G-I) (Mtn 15 G-I)

Ans. See Short Question 4

Q.2 The sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$ prove that the greatest angle of the triangle is 120°. (Rwp 13)(Gjw 19 G-I, II)
(Mtn 16 G-II)(Fbd 17)(Lahore, Mtn 18 G-II)(Swl 19)

Ans. Let $a = x^2+x+1$, $b = 2x+1$, $c = x^2-1$

Since $a = x^2 + x + 1$ is the largest measure of the side, so α will be the largest angle. Then

$\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{(2x+1)^2+(x^2-1)^2-(x^2+x+1)^2}{2(2x+1)(x^2-1)}$

$= \dfrac{4x^2+4x+1+x^4-2x^2+1-(x^4+x^2+1+2x^3+2x+2x^2)}{2(2x^3-2x+x^2-1)}$

$= \dfrac{4x^2+4x+1+x^4-2x^2+1-x^4-x^2-1-2x^3-2x-2x^2}{2(2x^3-2x+x^2-1)}$

$= \dfrac{-2x^3+2x-x^2+1}{2(2x^3-2x+x^2-1)} = \dfrac{-1(2x^3-2x+x^2-1)}{2(2x^3-2x+x^2-1)}$

$= -\dfrac{1}{2} = -0.5$

$\alpha = \cos^{-1}(-0.5) \Rightarrow \boxed{\alpha = 120°}$

Q.3 Three villages A, B and C are connected by straight roads 6km, 9km and 13km. What angles these roads make with each other? **(Mtn 14 G-II)**

Ans. Let a = 6 km, b = 9 km, c = 13 km

Now, $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{(9)^2+(13)^2-(6)^2}{2(9)(13)}$

$= \dfrac{81+169-36}{234} = \dfrac{214}{234} = 0.9145$

$\alpha = \cos^{-1}(0.9145) = 23.87°$

$\Rightarrow \quad \boxed{\alpha = 23° 52'}$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} \Rightarrow \sin\beta = \dfrac{b\sin\alpha}{a}$

$= \dfrac{9\times\sin 23.87°}{6} = \dfrac{9\times 0.4047}{6}$

$= \dfrac{3.6423}{6} = 0.6071$

$\beta = \sin^{-1}(0.6071) = 37.38°$

$\Rightarrow \quad \boxed{\beta = 37° 22'}$

and $\alpha+\beta+\gamma = 180° \Rightarrow 23.87°+37.38°+\gamma = 180°$

$\Rightarrow \quad 61.25°+\gamma = 180°$

$\gamma = 180° - 61.25° = 118.75°$

$\Rightarrow \quad \boxed{\gamma = 118° 45'}$

EXERCISE 12.7

SHORT ANSWERS TO THE QUESTIONS

Q.1 Find the are of DABC, if a = 18, b = 24, c = 30 **(Gjw 17, 19 G-I))(Rwp 17, 18 G-II)(DGK 19)(Sgd 19)**

Ans. $S = \dfrac{a+b+c}{2} = \dfrac{18+24+30}{2} = \dfrac{72}{2} = 36$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{36(36-18)(36-24)(36-3)}$

$= \sqrt{36\times 18\times 12\times 6}$

$= \sqrt{2\times2\times3\times3\times2\times3\times3\times2\times2\times3\times2\times3}$

$= \sqrt{2^2\times2^2\times2^2\times3^2\times3^2\times3^2}$

$= 2\times2\times2\times3\times3\times3 = 216$

Q.2 Find area of the triangle ABC if a = 32.65, b = 42.81, c = 64.92

Ans. $S = \dfrac{a+b+c}{2}$

$= \dfrac{32.65+42.81+64.92}{2} = 70.19$

Area $= \Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{(70.19)(70.19-32.65)(70.19-42.81)\times(70.19-64.92)}$

Area $= \Delta = 616.60$

Q.3 Find the area of the triangle ABC, if a = 200 , b = 120 , γ = 150° **(Rwp, Swl 19)** **(D.G.K 13 G-II, 14 G-I)(Fbd 12 G-I, 19)**

Ans. Using area formula

$\Delta = \dfrac{1}{2}ab\sin\gamma = \dfrac{1}{2}(200)(120)\sin 150° = 12000$ $(0.50) = 6000$ sq. units

Q.4 Find the area of the triangle ABC, if b = 37, c = 45, α = 30°50′ **(Bpr 12 G-I)(Lhr 15, 16 G-I)(Fbd 17)(Mtn 19 G-I)**

Ans. Using area formula

$\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}(37)(45)\sin 30°50' = \dfrac{1}{2}$ $(37)(45)(0.5126) = 426.69$ sq. units

Q.5 Find the area of the triangle ABC, if b = 25.4 , γ = 36° 41′, α = 45°17′ **(Bpr 14 G-I)(A.J.K 17)**

Ans. $\beta = 180° - (\alpha+\gamma) = 180° - (36° 41' + 45° 17') = 98° 2'$

Using area formula

$\Delta = \dfrac{1}{2}b^2\dfrac{\sin\alpha\sin\gamma}{\sin\beta} = \dfrac{1}{2}(25.4)^2\dfrac{\sin(36°41')\sin(45°17')}{\sin(98° 2')}$

$= \dfrac{322.58\,(0.60)\,(0.71)}{0.99} = \dfrac{136.94}{0.99} = 138.3$ sq. units

Q.6 Find the area of the triangle ABC, if c = 32, α = 47° 24′, γ = 70° 16′ **(Lhr 14 G-I)**

Ans. $\beta = 180° - (\alpha+\gamma) = 180° - (47°24' + 70°16') = 62°20'$

Using area formula

$\Delta = \dfrac{1}{2}c^2\dfrac{\sin\alpha\sin\beta}{\sin\gamma}$

$= \dfrac{1}{2}(32)^2\dfrac{\sin(47°24')\sin(70°16')}{\sin(62°20')}$

$= \dfrac{512\,(0.7361)\,(0.9413)}{0.8857}$

$= \dfrac{709.5203}{1.7714} = 400.54$ sq. Units

Q.7 Find the area of the triangle in which c = 32, $\alpha = 47^0\,24'$ and $\gamma = 70^0\,16'$.

Ans. $\beta = 180^0 - \alpha - \gamma = 180^0 - 47^0\,24' - 70^0\,16' = 62^0\,20'$

$\therefore \Delta = \dfrac{1}{2}c^2\dfrac{\sin\alpha\sin\beta}{\sin\gamma}$

$= \dfrac{1}{2}(32)^2\dfrac{\sin 47^0\,24'\sin 62^0\,20'}{\sin 70^0\,16'}$

$= 354.62(\text{units})^2.$

Q.8 The area of triangle is 2437. If a = 79, and c = 97, then find angle β. (Bpr 19)
(Lhr 13 G-I, II)(Bpr 16)(Rwp 17 G-I)(Fbd, Gjw 18)

Ans. Given that

$\Delta = 2437,\ a = 79,\ c = 97,\ \beta = ?$

But $\Delta = \frac{1}{2}ac\sin\beta$

$\Rightarrow\ 2437 = \frac{1}{2}(79)(97)\sin\beta$

$\Rightarrow\ 4874 = 7663\sin\beta$

$\Rightarrow\ \sin\beta = 0.636$

$\Rightarrow\ \beta = \sin^{-1}(0.636) = 39°30'$

Q.9 The area of triangle is 121.34. If α = 32°15′, β = 65°37′, then find c and angle γ. (Gjw 16)

Ans. Given that: $\Delta = 121.34,\ \alpha = 32°15',\ \beta = 65°37'$

For γ using

$\gamma = 180° - (\alpha+\beta)$

$= 180° - (32°15' + 65°37') = 82°8'$

and $\Delta = \frac{1}{2}c^2\frac{\sin\alpha\sin\beta}{\sin\gamma}$

$121.34 = \frac{1}{2}c^2\frac{\sin(32°15')\sin(65°37')}{\sin(82°8')}$

$121.34 = \frac{1}{2}c^2\frac{(0.5336)(0.9108)}{(0.9906)}$

$c^2 = \frac{2(121.34)(0.9906)}{(0.5336)(0.9108)}$

$\Rightarrow\ c^2 = \sqrt{494.623}\ \Rightarrow c = 22.24$

LONG QUESTIONS

Q.1 Find the area of the triangle ΔABC, in which c = 32, α = 47°24' and β = 70°16'

Ans. See Short Question 11

MULTIPLE CHOICE QUESTIONS

❑ *Each question has four possible answers. Select the correct answer and encircle it.*

Q.1 If length of sides of a Δ are a = 3, b = 4, c = 5, then value of S is: (Mtn 08 G-II)

(a) 9 (b) 6
(c) 12 (d) 7

Q.2 Area of ΔABC:

(a) $\frac{1}{2}ab\sin\alpha$ (b) $ab\sin\gamma$
(c) $\frac{1}{2}ac\sin\beta$ (d) $ab\sin\alpha$

Q.3 In Hero's formula Δ equals to: (Bpr 13)

(a) $\frac{1}{2}bc\sin\alpha$ (b) $\frac{1}{2}ab\sin\alpha$
(c) $\frac{1}{2}ac\sin\beta$ (d) $\sqrt{s(s-a)(s-b)(s-c)}$

Q.4 Area of a triangle in terms of its sides is:

(a) $\sqrt{s(s-a)(s-b)(s-c)}$
(b) $\frac{1}{2}bc\sin\alpha$
(c) $\frac{a-b}{2}$ (d) $\frac{3}{4}ac$

Q.5 If in a triangle ABC, a = 4, b = 10, γ = 30°, then area of triangle is: (Lhr 12 G-II)

(a) 5 (b) 40
(c) 10 (d) 20

Q.6 With usual notation, area of triangle ABC is. (Mtn 16 G-II)

(a) $\sqrt{(S-a)(S-b)(S-c)}$
(b) $\frac{a}{2\sin\beta}$
(c) $\frac{ac\sin\beta}{2}$ (d) $ab\sin\gamma$

Q.7 If Δ is the area of a triangle ABC, then Δ equal: (Rwp 14)(Fbd 16)

(a) $\frac{1}{2}bc\sin\alpha$ (b) $\frac{1}{2}ab\sin\gamma$
(c) $\frac{1}{2}bc\sin\gamma$ (d) $ab\sin\alpha$

Q.8 If Δ is the area of a triangle ABC, then with usual notation Δ =: (Lhr 19 G-II)

(a) $\frac{1}{2}bc\sin B$ (b) $\frac{1}{2}ab\sin a$
(c) $\frac{1}{3}bc\sin a$ (d) $\frac{1}{2}bc\sin a$

Q.9 Area of ΔABC in terms of measure of its all sides is: (Mtn 19 G-I)

(a) $\frac{1}{2}bc\sin a$ (b) $\frac{c^2\sin a\sin\beta}{2\sin y}$
(c) $\frac{1}{2}ca\sin\beta$ (d) $\sqrt{s(s-a)(s-b)(s-c)}$

Q.10 In an oblique triangle, if a = 200 ; b = 120 and included angle y = 150°, then its area will be equal to: (Rwp 19)

(a) 6000 (b) 5000
(c) 2000 (d) 12000

EXERCISE 12.8

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define the term circum-circle. (Lhr 17 G-I)

Ans. Circum-circle: The circle passing through the three vertices of a triangle is called a circum circle. Its center is called the circum center, which is the point of intersection of the right bisectors of the sides of the triangle. Its radius is called circum radius and is denoted by R.

Q.2 Prove that: $R = \dfrac{abc}{4\Delta}$ (Mtn 19 G-I)

(Lhr 14 G-II, 13 G-I)(Bpr 13 G-I)(Fbd 14 G-I, 16) (Rwp 14 G-I) (Mtn 16 G-I)(Gjw 16)(D.G.K 17 G-I)

Proof. We know that $R = \dfrac{a}{2\sin\alpha}$

$$\Rightarrow R = \frac{a}{2 \,.\, 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} \qquad \left(\because \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)$$

$$= \frac{a}{4\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-a)}{bc}}} \text{ (by half angle formulas)}$$

$$= \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

$$\therefore \boxed{R = \frac{abc}{4\Delta}} \qquad \left(\because \Delta = \sqrt{s(s-a)(s-b)(s-c)}\right)$$

Q.3 Define in circle.

(Mtn 16, 18 G-II)(Lhr 19 G-II)(DGK 19)

Ans. In circle:

The circle drawn inside a triangle touching its three sides is called its inscribed circle or in-circle.

Q.4 Define an escribed circle. (Gjw 19 G-I)

Ans. A circle that touches one side of a triangle externally and two sides internally, when extended, is called escribed circle, e-circle or ex-circle.

Q.5 Prove that $\dfrac{1}{r^2} + \dfrac{1}{r_1^2} + \dfrac{1}{r_3^2} + \dfrac{1}{r_3^2} = \dfrac{a^2+b^2+c^2}{\Delta^2}$

(D.G.K 13 G-II)

Ans. $\text{L.H.S} = \dfrac{1}{r^2} + \dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} + \dfrac{1}{r_3^2}$

$$= \frac{s^2}{\Delta^2} + \frac{(s-a)^2}{\Delta^2} + \frac{(s-b)^2}{\Delta^2} + \frac{(s-c)^2}{\Delta^2}$$

$$= \frac{s^2 + (s-a)^2 + (s-b)^2 + (s-c)^2}{\Delta^2}$$

$$= \frac{4s^2 - 2s(a+b+c) + a^2 + b^2 + c^2}{\Delta^2}$$

$$= \frac{4s^2 - 2s\,.\,2s + a^2 + b^2 + c^2}{\Delta^2} = \frac{a^2+b^2+c^2}{\Delta^2} = \text{R.H.S.}$$

$$\text{Hence } \frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$$

Q.6 Prove that: $\Delta = 4\,R\,r\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$

Ans. $\text{R.H.S.} = 4\,R\,r\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$

$$= 4.\frac{abc}{4\Delta}.\frac{\Delta}{s}\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \frac{abc}{s}\sqrt{\frac{s^3(s-a)(s-b)(s-c)}{a^2b^2c^2}}$$

$$= \frac{abc\,s\sqrt{s(s-a)(s-b)(s-c)}}{s.abc}$$

$$= \sqrt{s(s-a)(s-b)(s-c)} = \Delta = \text{L.H.S.}$$

Q.7 Find R, r, if measures of the sides of triangle ABC are a = 13 , b = 14 , c = 15, Find R, r

(Fbd 18)

Ans. $s = \dfrac{a+b+c}{2} = \dfrac{13+14+15}{2} = \dfrac{42}{2} = 21$

$s - a = 21 - 13 = 8,\ s - b = 21 - 14 = 7,\ s - c = 21 - 15 = 6$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21\,(8)\,(7)\,(6)} = \sqrt{7056} = 84$

Now, $R = \dfrac{abc}{4\Delta} = \dfrac{13\,(14)(15)}{4(84)} = \dfrac{2730}{336} = \dfrac{65}{8} = 8.125$

$r = \dfrac{\Delta}{s} = \dfrac{84}{21} = 4$

Q.8 Prove that $\gamma = (S-a)\tan\dfrac{\alpha}{2}$

Ans. $\text{R.H.S.} = (S-a)\tan\dfrac{\alpha}{2}$

$$= (S-a)\sqrt{\frac{(S-b)(S-c)}{S(S-a)}}$$

$$= (S-a)\sqrt{\frac{(S-b)(S-c)}{S(S-a)} \times \frac{S(S-a)}{S(S-a)}}$$

$$= (S-a)\sqrt{\frac{S(S-a)(S-b)(S-c)}{S^2(S-a)^2}}$$

$$= (S-a)\frac{\sqrt{S(S-a)(S-b)(S-c)}}{S(S-a)} = \frac{\Delta}{S} = \gamma$$

Q.9 Show that $r = (s-b)\tan\dfrac{\beta}{2}$ (with usual notation) (D.G.K 16 G-I)

Ans. $\text{R.H.S} = (S-b)\tan\dfrac{\beta}{2}$

$$= (S-b)\sqrt{\frac{(S-a)(S-c)}{S(S-b)}}$$

$$= (S-b)\sqrt{\frac{S(S-a)(S-b)(S-c)}{S^2(S-b)^2}}$$

$$= (S-b)\frac{\sqrt{S(S-a)(S-b)(S-c)}}{S(S-b)} = \frac{\Delta}{S} = r = \text{R.H.S}$$

Q.10 Prove that $(r_3 - r)\cot\frac{r}{2} = c$.

(Sgd 17)(Lhr 18 G-II)(DGK 19)

Ans. L.H.S

$$= (r_3 - r)\cot\frac{r}{2}$$

$$= \left(\frac{\Delta}{S-C} - \frac{\Delta}{S}\right) \cdot \frac{\cos\frac{r}{2}}{\sin\frac{r}{2}}$$

$$= \Delta\left(\frac{S-(S-c)}{S(S-c)}\right) \frac{\sqrt{\frac{S(S-c)}{ab}}}{\sqrt{\frac{(S-a)(S-b)}{ab}}}$$

$$= \Delta\left(\frac{C}{S(S-c)}\right)\sqrt{\frac{S^2(S-c)^2}{(S-a)(S-b)(S)(S-c)}}$$

$$= \Delta\left(\frac{C}{S(S-c)}\right) \cdot \frac{S(S-c)}{\sqrt{S(S-a)(S-b)(S-c)}}$$

$$= \Delta\left(\frac{C}{1}\right) \cdot \frac{1}{\Delta} = c = \text{R.H.S}$$

Q.11 Show that: $a\,b\,c\,(\sin\alpha + \sin\beta + \sin\gamma) = 4\,\Delta s$

(Rwp 16)

Ans. L.H.S. $= a\,b\,c\,(\sin\alpha + \sin\beta + \sin\gamma)$

Putting $\sin\alpha = \frac{a}{2R}$, $\sin\beta = \frac{b}{2R}$, $\sin\gamma = \frac{c}{2R}$

$$\text{L.H.S} = abc\left\{\frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}\right\} = abc\left\{\frac{a+b+c}{2R}\right\}$$

$$= \frac{abc}{R}\left\{\frac{a+b+c}{2}\right\}$$

$$= \frac{abc}{\left(\frac{abc}{4\Delta}\right)}\{s\} = 4\,\Delta s = \text{R.H.S.}$$

LONG QUESTIONS

Q.1 Prove that $r\,r_1\,r_2\,r_3 = \Delta^2$. **(Sgd 13 G-I)**

Ans. See Short Question 11

Q.2 Prove that in equilateral triangle $r:R:r_1 = 1:2:3$ where symbols have their usual meaning.

(D.G.K 11, 13 G-I)(Rwp 11, 13, 17 G-II)

(Lhr, Mtn 17 G-I)

Ans. See Long Question 12

Q.3 Prove that $abc\,(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta S$

(D.G.K 13 G-II, 15 G-I)(Fbd 18)

Ans. See Short Question 25

Q.4 Prove that $(r_1 + r_2)\tan\frac{\gamma}{2} = c$.

(Mtn 11 G-II, 18 G-I)(Fbd 19)

Ans. L.H.S. $= (r_1 + r_2)\tan\frac{\gamma}{2}$

$$= \left[\frac{\Delta}{s-a} + \frac{\Delta}{s-b}\right] \times \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \Delta \times \left[\frac{1}{s-a} + \frac{1}{s-b}\right] \times \sqrt{\frac{(s-a)^2(s-b)^2}{s(s-a)(s-b)(s-c)}}$$

$$= \Delta \times \frac{s-b+s-a}{(s-a)(s-b)} \times \sqrt{\frac{(s-a)^2(s-b)^2}{\Delta^2}}$$

$$= \Delta \times \frac{2s-a-b}{(s-a)(s-b)} \times \frac{(s-a)(s-b)}{\Delta} = 2s-a-b$$

$$= a+b+c-a-b \quad (\because a+b+c = 2s) = c$$

$$= \text{R.H.S.}$$

Q.5 $r_3 = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$

Ans. R.H.S. $= 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$

$$= 4\left(\frac{abc}{4\Delta}\right)\left(\sqrt{\frac{S(S-a)}{bc}}\right)\left(\sqrt{\frac{S(S-b)}{ac}}\right)\left(\sqrt{\frac{(S-a)(S-b)}{ab}}\right)$$

$$= \frac{abc}{\Delta}\sqrt{\frac{S(S-a)}{bc} \times \frac{S(S-b)}{ac} \times \frac{(S-a)(S-b)}{ab}}$$

$$= \frac{abc}{\Delta}\sqrt{\frac{S^2(a-a)^2(S-b)^2}{a^2b^2c^2}}$$

$$= \frac{abc}{\Delta} \times \frac{S(S-a)(S-b)}{abc} = \frac{S(S-a)(S-b)}{\Delta} \times \frac{S-c}{S-c}$$

$$= \frac{S(S-a)(S-b)(S-c)}{\Delta(S-c)} = \frac{\Delta^2}{\Delta(S-c)} = \frac{\Delta}{S-c} = r_3$$

Q.6 Find R, r, r_1, r_2 and r_3 if measures of sides of triangle ABC with usual notations are $a = 13, b = 14, c = 15$ **(Mtn 17 G-II, 18 G-I)**

Ans. Given that: $a = 13$, $b = 14$, $c = 15$

$$s = \frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2} = 21$$

$$s-a = 21-13 = 8\ ,\ s-b = 21-14 = 7\ ,$$

$$s-c = 21-15 = 6$$

$$\therefore \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{7056} = 84$$

Now, $R = \frac{abc}{4\Delta} = \frac{13 \times 14 \times 15}{4 \times 84} = \frac{2730}{336} = \boxed{8.125}$

$$r = \frac{\Delta}{s} = \frac{84}{21} = \boxed{4}\ ;\ r_1 = \frac{\Delta}{s-a} = \frac{84}{8} = \boxed{10.5}$$

$$r_2 = \frac{\Delta}{s-b} = \frac{84}{7} = \boxed{12}\ ;\ r_3 = \frac{\Delta}{s-c} = \frac{84}{6} = \boxed{14}$$

Q.7 Show that r = 4R sin α/2 sin β/2 sin γ/2
(D.G.K 14 G-II)(Mtn 14 G-I)

Ans. R.H.S.= $4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$

$$= 4\times\frac{abc}{4\Delta}\times\sqrt{\frac{(s-b)(s-c)}{bc}}\times\sqrt{\frac{(s-c)(s-a)}{ca}}\times\sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$= \frac{abc}{\Delta}\times\sqrt{\frac{(s-b)(s-c)}{bc}\times\frac{(s-c)(s-a)}{ca}\times\frac{(s-a)(s-b)}{ab}}$$

$$= \frac{abc}{\Delta}\times\sqrt{\frac{(s-a)^2(s-b)^2(s-c)^2}{a^2b^2c^2}}$$

$$= \frac{abc}{\Delta}\times\frac{(s-a)(s-b)(s-c)}{abc}$$

$$= \frac{1}{\Delta}\times\frac{(s-a)(s-b)(s-c)}{1} = \frac{1}{\Delta}\times\frac{s(s-a)(s-b)(s-c)}{s} =$$

$$\frac{1}{\Delta}\times\frac{\Delta^2}{s} = \frac{\Delta}{s} = r = \text{L.H.S.}$$

Q.8 Prove that in equilateral triangle
$r:R:r_1 = 1:2:3$
Where symbols have their usual meaning.
(D.G.K 11, 13 G-I)(Rwp 11, 13, 17 G-II)(Lhr, Mtn 17 G-I)

Ans. See Next Question 15

Q.9 Prove that in an equilateral triangle
$r:R:r_1:r_2:r_3 = 1:2:3:3:3$
(Lhr 13 G-I)(Fbd 14)(Sgd 16)

Ans. As a = b = c for an equilateral triangle

$$S = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)} = \frac{\sqrt{3}\,a^2}{4}$$

$$R = \frac{abc}{4\Delta} = \frac{a.a.a}{r.\frac{\sqrt{3}\,a}{\sqrt{3}\sqrt{3}}}$$

$$= \frac{a}{\sqrt{3}} = \frac{\sqrt{3}\,a}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}\,a}{3}$$

$$r = \frac{\Delta}{S} = \frac{\frac{\sqrt{3}\,a^2}{4}}{\frac{3a}{2}} = \frac{\sqrt{3}\,a^2}{4}\times\frac{2}{3a} = \frac{\sqrt{3}\,a}{6}$$

$$r_1 = \frac{\Delta}{S-a} = \frac{\frac{\sqrt{3}\,a^2}{4}}{\frac{3a}{2}-a}$$

$$r_1 = \frac{\frac{\sqrt{3}\,a^2}{4}}{\frac{a}{2}} = \frac{\sqrt{3}\,a^2}{4}\times\frac{2}{a} = \frac{\sqrt{3}\,a}{2}$$

Similarly

$$r_2 = \frac{\sqrt{3}\,a}{2}$$

$$r_3 = \frac{\sqrt{3}\,a}{2}$$

$\therefore$ $r : R\; r_1 : r_2 : r_3$

$$= \frac{\sqrt{3}\,a}{6} : \frac{\sqrt{3}\,a}{3} : \frac{\sqrt{3}\,a}{2} : \frac{\sqrt{3}\,a}{2} : \frac{\sqrt{3}\,a}{2}$$

$$= \sqrt{3}\,a : 2\sqrt{3}\,a : 3\sqrt{3}\,a : 3\sqrt{3}\,a : 3\sqrt{3}\,a$$

$$= 1:2:3:3:3$$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 In any triangle ABC, with usual rotation $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ is equal to:
(Mtn 08 G-I)(Bpr 18)

(a) $\tan\frac{\alpha}{2}$ (b) $\tan\frac{\beta}{2}$
(c) $\tan\frac{\gamma}{2}$ (d) $\cot\frac{\alpha}{2}$

Q.2 In any triangle ABC, with usual rotation $\frac{\Delta}{s\text{-}a}$ is equal to: (Mtn 08 G-1)

(a) r (b) r_1
(c) r_2 (d) r_3

Q.3 For any equilateral triangle ABC with usual rotation $r:R:r_1$, equals:
(Mtn 09 G-I)(Sgd 17)

(a) 2 : 3 : 1 (b) 3 : 1 : 2
(c) 3 : 2 : 1 (d) 1 : 2 : 3

Q.4 $\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$ equals: (Mtn 09 G-II)

(a) $\tan\frac{\alpha}{2}$ (b) $\tan\frac{\beta}{2}$
(c) $\tan\frac{\gamma}{2}$ (d) None of these

Q.5 In any triangle ABC, the in radius r is given by:
(Mtn 10, 12 G-I, 09, 11 G-II)

(a) $\frac{\Delta}{2S}$ (b) $\frac{\Delta}{3S}$
(c) $\frac{\Delta}{S}$ (d) $\frac{\Delta}{s-a}$

Q.6 In a triangle ABC, Δ = 20 , S = 4, then γ :
(Mtn 11 G-I)

(a) 2 (b) 5
(c) 10 (d) 15

Q.7 $\frac{\Delta}{s} =$ (Mtn 12 G-II , Mtn G-II) (Rwp 12)

(a) R (b) γ_1
(c) γ (d) γ_2

Q.8 In any triangle, with usual notation r_3 is: (Rwp 15)

(a) $\dfrac{\Delta}{S-a}$ (b) $\dfrac{S-b}{\Delta}$

(c) $\dfrac{S-C}{\Delta}$ (d) $\dfrac{\Delta}{S-C}$

Q.9 The circum radius R is equal to: (Mtn 13 G-I, II)(Fbd 11) (Shw 13)(Gjw 13) (Bpr 15, 16)(Multan, D.G.K 15 G-I)(Sgd 16)

(a) $\dfrac{a+b+c}{4\Delta}$ (b) $\dfrac{a+b+c}{\Delta}$

(c) $\dfrac{abc}{\Delta}$ (d) $\dfrac{abc}{4\Delta}$

Q.10 Value of circum radius R is: (Bpr 11)(Fbd 12)(D.G.K 12)(Lhr 10 G-I)(Rwp 19)

(a) $\dfrac{abc}{2\Delta}$ (b) $\dfrac{3\Delta}{abc}$

(c) $\dfrac{\Delta^2}{ac}$ (d) $\dfrac{abc}{4\Delta}$

Q.11 $\dfrac{\Delta}{s-c}$ (D.G.K 13G-II)

(a) γ_1 (b) γ_2

(c) γ_3 (d) γ

Q.12 With usual rotation $\dfrac{abc}{4\Delta}$: (Rwp 11) (Mtn 14)(Fbd 17, 18)

(a) r_1 (b) r_2

(c) r_3 (d) R

Q.13 Radius of ascribed circle opposite to vertex A is: (Lhr 15, 17 G-II) (Gjw 10) (Rwp 13)(Bpr 19)

(a) $\dfrac{\Delta}{s}$ (b) $\dfrac{\Delta}{b}$

(c) $\dfrac{\Delta}{s-a}$ (d) $\dfrac{\Delta}{s-b}$

Q.14 Circum radius of a circle is denoted by:

(a) R (b) r

(c) r_1 (d) r_2

Q.15 In radius r of a triangle is: (Rwp 14)(Gjw 18)

(a) $s\Delta$ (b) $\dfrac{s}{\Delta}$

(c) $\dfrac{\Delta}{S}$ (d) $\dfrac{abc}{4\Delta}$

Q.16 r_2 = with usual notation. (Shw 14)(DGK 19)

(a) $\dfrac{\Delta}{s-a}$ (b) $\dfrac{\Delta}{s-b}$

(c) $\dfrac{\alpha}{\beta}$ (d) $\dfrac{b}{a}$

Q.17 r_1 = (Fbd 14)(Sgd 16)(Lhr 18 G-I)(DGK 19)

(a) $\dfrac{\Delta}{s-a}$ (b) $\dfrac{\Delta}{s-b}$

(c) $\dfrac{\Delta}{s-c}$ (d) $\dfrac{s-a}{\Delta}$

Q.18 The in-radius of the in-circle is: (Lhr 10, 18 G-II)

(a) $\dfrac{abc}{4\Delta}$ (b) $\dfrac{\Delta}{S}$

(c) $\dfrac{\Delta}{S-a}$ (d) $\dfrac{S}{\Delta}$

Q.19 The value of R = (Lhr 11 G-I)(Lhr 09, 18 G-II)

(a) $\dfrac{\Delta}{S}$ (b) $\dfrac{a}{\sin\alpha}$

(c) $\dfrac{b}{\sin\beta}$ (d) $\dfrac{c}{2\sin\gamma}$

Q.20 Radius of escribed circle opposite to vertex B is equal to: (Mtn 15 G-II)

(a) $\dfrac{\Delta}{S}$ (b) $\dfrac{\Delta}{S-a}$

(c) $\dfrac{\Delta}{S-b}$ (d) $\dfrac{\Delta}{S-C}$

Q.21 Radius of e-circle, r_2, is given by: (Gjw 12)(Mtn 18 G-II)

(a) $\dfrac{\Delta}{S-b}$ (b) $\dfrac{\Delta}{S+b}$

(c) $\dfrac{S-b}{\Delta}$ (d) $\dfrac{\Delta}{S+c}$

Q.22 If a, b,c are sides of a triangle then $\dfrac{c^2+a^2-b^2}{2ac}$ = (Lhr 13 G-I)

(a) $\cos\alpha$ (b) $\cos\beta$

(c) $\cos\gamma$ (d) $\sin\beta$

Q.23 A circle passing through three vertices of a triangle is called: (Lhr 13 G-I, G-II, 14 G-II)

(a) Circumcircle (b) In-circle

(c) E-circle (d) Semi-circle

Q.24 Radius of e-circle, r_1, is equal to: (Lhr 13 G-II)(Rwp 17 G-II)

(a) $\dfrac{\Delta}{S-a}$ (b) $\dfrac{\Delta}{\sqrt{S-a}}$

(c) $\dfrac{\Delta}{S-c}$ (d) $\dfrac{\Delta}{S}$

Q.25 For any triangle ABc, with usual notation r_1 is equal to: (Lhr 15 G-I)

(a) $\dfrac{\Delta}{S-b}$ (b) $\dfrac{\Delta}{S-a}$

(c) $\dfrac{S-a}{\Delta}$ (d) $\dfrac{\Delta}{S-C}$

Q.26 $r\,r_1\,r_2\,r_3 =$

(a) Δ^2 (b) Δ

(c) $\frac{abc}{\Delta}$ (d) abc

Q.27 e-radius corresponding to $\angle C$ is (Bpr 18)

(a) $\frac{\Delta}{s-a}$ (b) $\frac{\Delta}{s-b}$

(c) $\frac{\Delta}{s-c}$ (d) $\frac{\Delta}{s}$

Q.28 For any triangle ABC with usual notation $r_2 =$ (Rwp 16)

(a) $\frac{\Delta}{s-a}$ (b) $\frac{\Delta}{s-c}$

(c) $\frac{\Delta}{s-b}$ (d) $\frac{abc}{\Delta}$

Q.29 For any triangle ABC with usual notation $r_3 =$

(a) $\frac{\Delta}{s-a}$ (b) $\frac{\Delta}{s-b}$

(c) $\frac{\Delta}{s-c}$ (d) $\frac{abc}{\Delta}$

Q.30 A circle which touches one side of a triangle externally and the other two sides internally is known as

(a) in-circle (b) escribed-circle

(c) circum-circle (d) None of these.

Q.31 The notation of in-radius is. (Bpr 16)

(a) r (b) R

(c) r_1 (d) Δ

Q.32 With usual notation $\frac{\Delta}{s} =$ (A.J.K 17)

(a) r (b) R

(c) r_1 (d) r_2

Q.33 If in a ΔABC, a = b = c, then. (D.G.K 17 G-I)

(a) $r_1 = r_2$ (b) $r_1 > r_2$

(c) $r_1 < r_2$ (d) $r_1 \neq r_2$

Q.34 e-radius corresponding to $\angle B$ equals. (D.G.K 17 G-II)

(a) $\frac{\Delta}{S-a}$ (b) $\frac{\Delta}{S-b}$

(c) $\frac{\Delta}{S-c}$ (d) $\frac{\Delta}{S}$

Q.35 Notation for radius of in-circle is. (Lhr 17 G-I)

(a) r (b) R

(c) r_1 (d) Δ

Q.36 The point of intersection of the angular bisectors of a triangle is called. (Lhr 17 G-II)

(a) circum-center (b) ortho-center

(c) In-center (d) centroid

Q.37 The radius of inscribed circle. (Rwp 17 G-I)

(a) $\frac{\Delta}{S}$ (b) $\frac{S}{\Delta}$

(c) $\frac{\Delta}{S-c}$ (d) $\frac{4\Delta}{abc}$

Q.38 With usual notation, in any triangle ABC of $\Delta = 20$, a = 4, b = 6, c = 10, then r equals. (Rwp 17 G-II)

(a) 2 (b) 5

(c) 10 (d) 15

Q.39 Radius of escribed circle. (Sgd 17)

(a) $r_1 = \frac{\Delta}{S-a}$ (b) $r_1 = \frac{\Delta}{a}$

(c) $r_1 = \frac{S-a}{\Delta}$ (d) $r_1 = \frac{S}{\Delta}$

Q.40 With usual notation $r^3 =$: (Fbd 19 G-I)

(a) $\frac{\Delta}{s-b}$ (b) $\frac{\Delta}{s-a}$

(c) $\frac{\Delta}{s-c}$ (d) $\Delta^2(s-c)$

Q.41 With usual notations, radius r of inscribed circle is given by (Gjw 19 G-II)

(a) $\frac{\Delta}{s}$ (b) $\frac{s}{\Delta}$

(c) $\frac{\Delta}{s-c}$ (d) $\frac{4\Delta}{abc}$

Q.42 With usual notation R =: (Lhr 19 G-II)

(a) $\frac{b}{2\sin r}$ (b) $\frac{a}{2\sin a}$

(c) $\frac{c}{2\sin a}$ (d) $\frac{a}{2\sin B}$

Q.43 In an equilateral $\Delta\, ABC$ (Swl 19)

(a) $r_1 > r_2$ (b) $r_1 < r_2$

(c) $r_1 = r_2$ (d) $r_1 \neq r_2$

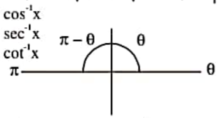
Q.44 With usual notations a + b – c = (Swl 19)

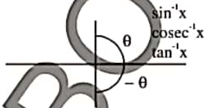
(a) 2s (b) 2s – 2c

(c) 2s – 2b (d) 2s – 2a

FORMULAE

1. $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$ or $\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$
2. $\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$ or $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$
3. $\sin^{-1}x = \frac{\pi}{2} - \text{cosec}^{-1}x$ or $\text{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$
4. $\sin^{-1}A + \sin^{-1}B = \sin^{-1}(A\sqrt{1-B^2} + \sqrt{1-A^2}\,B)$
5. $\sin^{-1}A - \sin^{-1}B = \sin^{-1}(A\sqrt{1-B^2} - B\sqrt{1-A^2})$
6. $\cos^{-1}A + \cos^{-1}B = \cos^{-1}(AB - \sqrt{1-A^2}\sqrt{1-B^2})$
7. $\cos^{-1}A - \cos^{-1}B = \cos^{-1}(AB + \sqrt{1-A^2}\sqrt{1-B^2})$
8. $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$
9. $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$
10. $\sec^{-1}x = \cos^{-1}\frac{1}{x}$ or $\cos^{-1}x = \sec^{-1}\frac{1}{x}$
11. $\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$ or $\sin^{-1}x = \text{cosec}^{-1}\frac{1}{x}$
12. $\cot^{-1}x = \tan^{-1}\frac{1}{x}$ or $\cot^{-1}x = \tan^{-1}x$





PRINCIPAL TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS

Functions	Domain	Range
$y = \sin x$	$\frac{-\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le y \le 1$
$y = \sin^{-1} x$	$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos x$	$0 \le x \le \pi$	$-1 \le y \le 1$
$y \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan x$	$\frac{-\pi}{2} \le x \le \frac{\pi}{2}$	$(-\infty, \infty)$ or **R**
$y = \tan^{-1} x$	$(-\infty, \infty)$ or **R**	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cot x$	$0 < x < \pi$	$(-\infty, \infty)$ or **R**
$y = \cot^{-1} x$	$(-\infty, \infty)$ or **R**	$0 < y < \pi$
$y = \sec x$	$[0, \pi], x \ne \frac{\pi}{2}$	$y \le -1$ or $y \ge 1$
$y \sec^{-1} x$	$x \ge -1$ o $x \le 1$	$[0, \pi], y \ne \frac{\pi}{2}$
$y = \csc x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], x \ne 0$	$y \le -1$ or $y \ge 1$
$y = \csc^{-1} x$	$x \le -1$ or $x \ge 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y \ne 0$

Chapter 13

INVERSE TRIGONOMETRIC FUNCTIONS

EXERCISE 13.1

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define inverse sine function. **(Gjw 19 G-I)**

Ans. The inverse sine function is defined by $y \equiv \sin^{-1} x$ if and only if $x \equiv \sin y$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $-1 \le x \le 1$

Q.2 Devine inverse cosine function.

Ans. The inverse cosine function is defined by $y = \sin^{-1} x \Leftrightarrow x = \sin y$ where $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $-1 \le x \le 1$

Q.3 Define the inverse tangent function.

Ans. The inverse tangent function is defined by:

$y \equiv \tan^{-1} x \Leftrightarrow x = \tan y$ where $-\frac{\pi}{2} < y < \frac{\pi}{2}$

and $-\infty \le x \le +\infty$

Q.4 Evaluate without using tables: $\sin^{-1}(1)$ **(Shw 14 G-I)**

Ans. Let $y = \sin^{-1}(1)$ where $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \quad \sin y = 1 \quad \Rightarrow \quad y = \frac{\pi}{2}$

Thus $\sin^{-1}(1) = \frac{\pi}{2}$

Q.5 Evaluate $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$. **(A.J.K 17)**

Ans. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

Q.6 Evaluate without using tables/calculator:

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Let $y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad \cos y = \left(\frac{\sqrt{3}}{2}\right)$ where $y \in [0, \pi]$

$\Rightarrow \quad y = \frac{\pi}{6} \quad \Rightarrow y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Q.7 Write the domain and range of sin x?

Ans. Domain of $\sin x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range of $\sin x = [-1, 1]$

Q.8 Evaluate without using tables/calculator:

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. **(Bpr 18)**

Ans. Let $y = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad \tan y = -\frac{1}{\sqrt{3}}$ where $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \quad y = -\frac{\pi}{6} \Rightarrow \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

Q.9 Evaluate without using tables/calculator:

$\cos^{-1}\left(\frac{1}{2}\right)$ **(Gjw 18)**

Ans. $y = \cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \cos y = \frac{1}{2}$ where $y \in [0, \pi]$

$\Rightarrow \quad y = \frac{\pi}{3} \Rightarrow y = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Q.10 Evaluate without using tables/calculator:

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Ans. Let $y = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad \tan y = \left(\frac{1}{\sqrt{3}}\right)$ where $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \quad y = \frac{\pi}{6} \quad \Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Q.11 Find the value of $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$. (Rwp 11 G-I)

Ans. Let $y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad \sin y = \frac{1}{\sqrt{2}} \quad ; y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \quad y = \dfrac{\pi}{4}$

$\Rightarrow \quad \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\pi}{4}$

Thus $\cos\left(\sin^{-1}\dfrac{1}{\sqrt{2}}\right) = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$

Q.12 Find the value of $\sec\left(\sin^{-1}\left(-\dfrac{1}{2}\right)\right)$.

(Lhr 15 G-II)(DGK 19)

Ans. Let $y = \sin^{-1}\left(-\dfrac{1}{2}\right)$

$\Rightarrow \sin y = -\dfrac{1}{2} \Rightarrow y = -\dfrac{\pi}{6}$

$\Rightarrow \quad \sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$

Thus $\sec\left(\sin^{-1}\left(-\dfrac{1}{2}\right)\right) = \sec\left(-\dfrac{\pi}{6}\right) = \dfrac{2}{\sqrt{3}}$

Q.13 Find the value of $\sin\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right)$.

(Gjw 14)(Rwp 17 G-II)

Ans. Let $\quad y = \sin^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow \quad \sin y = \dfrac{1}{2}\ ;\ y \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\Rightarrow \quad y = \dfrac{\pi}{6} \Rightarrow \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$

Thus $\quad \sin\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right) = \sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$

Q.14 Find the value of $\sin\left(\tan^{-1}(-1)\right)$

(Lhr 14 G-II)(Bpr 16)

Ans. Let $\quad y = \tan^{-1}(-1)$ where $y \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[$

$\Rightarrow \quad \tan y = -1 \Rightarrow y = -\dfrac{\pi}{4} \Rightarrow \tan^{-1}(-1) = -\dfrac{\pi}{4}$

Thus $\sin\left(\tan^{-1}(-1)\right) = \sin\left(-\dfrac{\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}$

Q.15 Find the value of the expression:

$\tan\left(\cos^{-1}\dfrac{\sqrt{3}}{2}\right)$ **(Sdg 17)**

Ans. Let $y = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow \cos y = \left(\dfrac{\sqrt{3}}{2}\right)$ where $y \in [0, \pi]$

$\Rightarrow y = \dfrac{\pi}{6}$

$y = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{6}$

Thus $\tan\left(\cos^{-1}\dfrac{\sqrt{3}}{2}\right) = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$

Q.16 Find the value of the expression:

$\sin\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right)$

Ans. Let $y = \sin^{-1}\left(\dfrac{1}{2}\right) \Rightarrow \sin y = \dfrac{1}{2}$

where $y \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\Rightarrow y = \dfrac{\pi}{6} \quad \Rightarrow \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$

Thus $\sin\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right) = \sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$

Q.17 Without using calculator, show that

$\cos^{-1}\dfrac{12}{13} = \sin^{-1}\dfrac{5}{13}$ **(Mtn 16 G-I)**

Ans. Let $y = \sin^{-1}\dfrac{5}{13} \Rightarrow y = \dfrac{5}{13}$

Where $y \in \left(0, \dfrac{\pi}{2}\right)$

$\cos y = +\sqrt{1 - \sin^2 y} = \sqrt{1 - \left(\dfrac{5}{13}\right)^2}$

$= \sqrt{\dfrac{169 - 25}{169}} = \sqrt{\dfrac{144}{169}} = \dfrac{12}{13}$

$y = \cos^{-1}\dfrac{12}{13}$

$\sin^{-1}\dfrac{5}{13} = \cos^{-1}\dfrac{12}{13}$

Q.18 Show that: $\cos^{-1}\dfrac{4}{5} = \cot^{-1}\dfrac{4}{3}$

Ans. Let $\cos^{-1}\dfrac{4}{5} = \alpha \quad \Rightarrow \cos\alpha = \dfrac{4}{5}$

Then $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

$= \dfrac{\cos\alpha}{\sqrt{1 - \cos^2\alpha}} = \dfrac{\left(\dfrac{4}{5}\right)}{\sqrt{1 - \left(\dfrac{4}{5}\right)^2}} = \dfrac{\left(\dfrac{4}{5}\right)}{\sqrt{\left(\dfrac{25 - 16}{25}\right)}}$

$= \dfrac{\left(\dfrac{4}{5}\right)}{\sqrt{\dfrac{9}{25}}} = \dfrac{\left(\dfrac{4}{5}\right)}{\left(\dfrac{3}{5}\right)} = \dfrac{4}{3}$

$\Rightarrow \quad \cot\alpha = \dfrac{4}{3}$

$\Rightarrow \quad \alpha = \cot^{-1}\dfrac{4}{3}$

$\Rightarrow \quad \cos^{-1}\dfrac{4}{5} = \cot^{-1}\dfrac{4}{3}$

Q.19 Show that $\tan^{-1}\dfrac{5}{12} = \sin^{-1}\dfrac{5}{13}$

(Mtn 15 G-II)(Fbd 16)(D.G.K 17 G-I)

Ans. Let $\sin^{-1}\dfrac{5}{13} = \alpha \Rightarrow \sin\alpha = \dfrac{5}{13}$

Then $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$$= \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{\frac{5}{13}}{\sqrt{1-\left(\frac{5}{13}\right)^2}}$$

$$= \frac{\frac{5}{13}}{\sqrt{1-\frac{25}{169}}} = \frac{\frac{5}{13}}{\sqrt{\frac{169-25}{169}}} = \frac{\frac{5}{13}}{\sqrt{\frac{144}{169}}}$$

$$= \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{13}\times\frac{13}{12} = \frac{5}{12}$$

$\Rightarrow \quad \tan\alpha = \dfrac{5}{12}$

$\Rightarrow \quad \alpha = \tan^{-1}\dfrac{5}{12}$

Thus $\tan^{-1}\dfrac{5}{12} = \sin^{-1}\dfrac{5}{13}$

Q.20 Show that: $2\cos^{-1}\dfrac{4}{5} = \sin^{-1}\dfrac{24}{25}$

Ans. Let $2\cos^{-1}\dfrac{4}{5} = \alpha \Rightarrow \cos^{-1}\dfrac{4}{5} = \dfrac{\alpha}{2}$

$\Rightarrow \cos\dfrac{\alpha}{2} = \dfrac{4}{5}$

$$\sin\frac{\alpha}{2} = \sqrt{1-\cos^2\frac{\alpha}{2}} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}}$$

$$= \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Now $\sin\alpha = 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} = 2\left(\dfrac{3}{5}\right)\left(\dfrac{4}{5}\right) = \dfrac{24}{25}$

$\Rightarrow \quad \alpha = \sin^{-1}\dfrac{24}{25}$

$\Rightarrow \quad 2\cos^{-1}\dfrac{4}{5} = \sin^{-1}\dfrac{24}{25}$

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $\sin\left(\cos^{-1}\dfrac{1}{2}\right)$ **is equal to:** (Mtn 08 G-I)

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{\sqrt{-3}}{2}$ (d) $\dfrac{-1}{2}$

Q.2 Value of $\cos^{-1}\left(\dfrac{-1}{2}\right)$ **is:** (Mtn 08 G-II)

(a) $\dfrac{\pi}{3}$ (b) $-\dfrac{\pi}{3}$

(c) $\dfrac{5\pi}{3}$ (d) $\dfrac{2\pi}{3}$

Q.3 The value of $\sin\left(\cos^{-1}\dfrac{\sqrt{3}}{2}\right)$ **is:**

(Mtn 10 G-I)(Lhr 11 G-I)(D.G.K 13 G-I)(Rwp 19)

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{\sqrt{3}}{2}$

(c) 1 (d) $\dfrac{1}{2}$

Q.4 $\tan(\tan^{-1}{}_1)$

(a) 1 (b) 45°

(c) 0 (d) 90°

Q.5 The value of $\cos\left(\cos^{-1}\dfrac{1}{2}\right)$ **is equal to:**

(Mtn 12 G-I)

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{\pi}{4}$ (d) $-\dfrac{\pi}{4}$

Q.6 The domain of principal cos function is:

(Mtn 12 G-I)

(a) -1 (b) 1

(c) 0 (d) $[0,\pi]$

Q.7 The value of $\cos(\tan^{-1}0)$ **is: to equal to:**

(Mtn 15 G-II , 12 G-II)(Lhr 18 G-I)

(a) 0 (b) 1

(c) -1 (d) $\dfrac{1}{2}$

Q.8 The value of $\cos\left(\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)\right)$ **is equal to:**

(Mtn 12 G-I)

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{\pi}{4}$ (d) $-\dfrac{\pi}{4}$

Q.9 The domain of principal tan function is: (Mtn 12 G-II)

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $[0, \pi]$

(c) $\frac{\Delta}{b}$ (d) $\frac{\Delta}{s-b}$

Q.10 The domain of principal sinx function: (D.G.K 15 G-II)(Shw 13)(Mtn 13 G-I)

(a) $\left[0\ ,\ \frac{\pi}{2}\right]$ (b) $\left[\frac{-\pi}{2}\ ,\ \frac{\pi}{2}\right]$

(c) $\left[0\ ,\ \frac{3\pi}{2}\right]$ (d) $[0\ ,\ 2\pi]$

Q.11 $\tan^{-1}(-\sqrt{3})$ is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $-\frac{\pi}{3}$ (d) $-\frac{2\pi}{3}$

Q.12 If $\sin^{-1} a = 0$, then value of a is: (Rwp 13)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) 0

Q.13 $\text{Sin}^{-1} x$ is equal to: (Fbd 11)

(a) $\frac{\pi}{2} - \cos^{-1} x$ (b) $\frac{\pi}{2} + \cos^{-1} x$

(c) $\frac{\pi}{2} - \sin^{-1} x$ (d) $\frac{\pi}{2} + \sin^{-1} x$

Q.14 $\cos(\sec^{-1} 1)$ is equal to. (Lhr 17 G-II)

(a) 1 (b) 0

(c) 30° (d) 2

Q.15 $\text{Tan}^{-1}(-1)$ equals: (Fbd 13 G-II)(Mtn 17 G-I)(Shw 17)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{-\pi}{4}$ (d) $-\pi$

Q.16 $\text{Tan}(\tan^{-1}(-1))$ equals: (Fbd 15)(Mtn 14, 19 G-I)(D.G.K 17 G-I)

(a) 1 (b) $\frac{\pi}{4}$

(c) $\frac{-\pi}{4}$ (d) -1

Q.17 $\text{Tan}^{-1}(-1)$ (D.G. 14 G-I)(Shw 17)

(a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $-\pi$

Q.18 $\text{Sin}\left(\cos^{-1}\frac{1}{\sqrt{2}}\right) =$ (D.G.K 14 G-II)

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $-\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2}$

Q.19 $\cos(2\sin^{-1}x)$ equal to: (Bpr 14)

(a) $\sqrt{1+x^2}$ (b) $\sqrt{1-x^2}$

(c) $\sqrt{1+2x^2}$ (d) $1-2x^2$

Q.20 See $\left(\cos^{-1}\frac{1}{2}\right)$ (Fbd 14)(Rwp 14)(Sgd 17)

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Q.21 The value of $\tan^{-1}(-\sqrt{3})$ is: (Lhr 10 G-II)

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

Q.22 $\sec\left(\cos^{-1}\frac{1}{2}\right)$ is equal to: (Gjw 10)

(a) $\frac{1}{2}$ (b) 60°

(c) 30° (d) 2

Q.23 If $\sin 2\theta = 1$, then value of θ is: (Lhr 11 G-II)

(a) 30° (b) 45°

(c) 60° (d) 90°

Q.24 $\text{cosec}^{-1}\sqrt{2} =$ (Gjw 11)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) π

Q.25 The value of $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ is:(Lhr 12)(Gjw 14)

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$

(c) 1 (d) $\frac{1}{2}$

Q.26 Range of function $y = \cos^{-1}x$ is: (Lhr 13 G-I & G-II)

(a) $[-1, 1]$ (b) $(-1, 1)$

(c) $[0, \pi]$ (d) $(0, \pi)$

Q.27 Domain of function $y = \sin^{-1}x$ is:
(Gjw 13)(Lhr 14 G-II)(Mtn 17 G-II)

(a) $0 \le x \le 1$ (b) $-1 \ge x \ge 1$
(c) $-1 \le x \le 1$ (d) $-1 < x < 1$

Q.28 The value of $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is: (Lhr 13 G-I)

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

Q.29 The value of $\sec\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2}\right) =$
(Mtn 15 G-I)(Fbd 17)

(a) $\frac{1}{2}$ (b) 2
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

Q.30 If $x = \text{Sin}^{-1}\frac{\sqrt{3}}{2}$, then x equal to: (Mtn 15 G-II)

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $-\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Q.31 The range of $y = \sin^{-1}x$ is

(a) $[-1, 1]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $[0, \pi]$ (d) $[-\infty, \infty]$

Q.32 The value of $\sin(\tan^{-1}(0)) =$

(a) 0 (b) 1
(c) -1 (d) ∞

Q.33 The domain of principal tangent function is

(a) $]0, \pi[$ (b) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$
(c) $]-1, 1[$ (d) $]-\infty, \infty[$

Q.34 The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{2}$

Q.35 The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{3}$
(c) $\frac{2\pi}{3}$ (d) $-\frac{\pi}{3}$

Q.36 $\sin^{-1}\frac{1}{x} =$

(a) $\sin x$ (b) $\text{cosec}^{-1}\frac{1}{x}$
(c) $\text{cosec}\, x$ (d) $\text{cosec}^{-1}x$

Q.37 $\cos^{-1}\frac{1}{x} =$

(a) $\sec^{-1}x$ (b) $\cos x$
(c) $\sec^{-1}\frac{1}{x}$ (d) $\sec x$

Q.38 The range of principal cosine function is

(a) $[-1, 1]$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $[0, \pi]$ (d) $[-\infty, \infty]$

Q.39 Range of the function $y = \tan^{-1}x$ is

(a) $]0, \pi[$ (b) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$
(c) $]-1, 1[$ (d) $]-\infty, \infty[$

Q.40 The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $\frac{5\pi}{4}$ (d) $\frac{2\pi}{3}$

Q.41 The principal value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$
(c) $-\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

Q.42 The range of principal tangent function is

(a) $]0, \pi[$ (b) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$
(c) $]-1, 1[$ (d) $]-\infty, \infty[$

Q.43 $\tan^{-1}\frac{1}{x} =$

(a) $\cot^{-1}\frac{1}{x}$ (b) $\cot^{-1} x$

(c) $\tan x$ (d) $\cot x$

Q.44 $\tan\left\{\sin^{-1}\left(\frac{1}{2}\right)\right\}$

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 0 (d) $\frac{1}{2}$

Q.45 $\cos(\tan^{-1}\sqrt{3})$ **is equal to.** (Mtn 16 G-I)

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{\sqrt{3}}{2}$

Q.46 $\sin\left(\sin^{-1}\frac{1}{2}\right)$ **is equal to.** (Sgd 16)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

Q.47 $\tan(\sin^{-1}x)$ **is equal to.** (Rwp 16)

(a) $1 + 2x^2$ (b) $1 - x^2$

(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $\frac{2x}{\sqrt{1+x^2}}$

Q.48 **If** $y = \cos^{-1}x$**, then its domain is.** (Lhr 16 G-I)

(a) $-\frac{\pi}{2} < x \le \frac{\pi}{2}$ (b) $-1 \le x \le 1$

(c) $0 \le x \le \pi$ (d) $-\infty < x < +\infty$

Q.49 $\tan^{-1}(-1) =$ (Bpr 16)

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

Q.50 $\cot^{-1}(-1) =$ ______ (A.J.K 17)

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{-3\pi}{4}$ (d) $-\frac{\pi}{4}$

Q.51 $\text{Cos}\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$ **is equal to:** (Gjw 12)(Bpr 12)

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

Q.52 $\cos\left(\sin^{-1}\frac{1}{2}\right)$ **equal.** (D.G.K 17 G-II)

(a) $-\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

Q.53 $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ (Lhr 17 G-I)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

Q.54 $\cos(\tan^{-1}\sqrt{3}) =$: (Fbd 19 G-II)

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

Q.55 $\sin\left(\cos^{-1}\frac{1}{2}\right)$ **equals** (Gjw 19 G-II)

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$

(c) $\frac{-\sqrt{3}}{2}$ (d) $\frac{-1}{2}$

Q.56 $\text{Tan}^{-1}(1) =$ (DGK 19)

(a) $\pi/3$ (b) $\pi/4$

(c) $\pi/6$ (d) π

Q.57 $2\tan^{-1}A$ **equals:** (Lhr 19 G-II)

(a) $\tan^{-1}\left(\frac{A}{1-A^2}\right)$ (b) $\tan^{-1}\left(\frac{2A}{1-A^2}\right)$

(c) $\tan^{-1}\left(\frac{2A}{1+A^2}\right)$ (d) $\tan^{-1}\left(\frac{A}{1+A^2}\right)$

Q.58 $\tan(\tan^{-1}(1)) =$ (Sgd 19)

(a) 1 (b) $\frac{\pi}{4}$

(c) $\pi/3$ (d) 0

Q.29 $\text{Sin}^{-1}(0) + \text{Cos}^{-1}(0) =$ (Swl 19)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{4}$

Q.30 $\sin(\tan^{-1}0^\circ) =$: (Bpr 19)

(a) -1 (b) 1

(c) 0 (d) ∞

EXERCISE 13.2

SHORT ANSWERS TO THE QUESTIONS

Q.1 Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{9}{19}$

(Lhr 14 G-II)(Rwp, Sgd 18)(Fbd 19)

Ans. L.H.S. $=\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}$

$$=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{1}{5}}{1-\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)}\right)=\tan^{-1}\left(\frac{\frac{5+4}{20}}{\frac{20-1}{20}}\right)$$

$$=\tan^{-1}\left(\frac{9}{19}\right)=\text{R.H.S.}$$

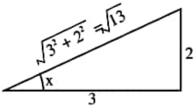
Q.5 Prove that: $2\tan^{-1}\frac{2}{3}=\sin^{-1}\frac{12}{13}$

(Mtn 17 G-II)

Ans. L.H.S. $=2\tan^{-1}\frac{2}{3}=\tan^{-1}\frac{2}{3}+\tan^{-1}\frac{2}{3}$

Let $\tan^{-1}\frac{2}{3}=x$

$\Rightarrow \quad \tan x=\frac{2}{3}\quad ; x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$



$\therefore \quad \sin x=\frac{2}{\sqrt{13}}$

$\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{4}{13}}=\sqrt{\frac{9}{13}}=\frac{3}{\sqrt{13}}$

Now $\sin 2x=2\sin x\cos x=2\frac{2}{\sqrt{13}}\frac{3}{\sqrt{13}}=\frac{12}{13}$

$\Rightarrow \quad 2x=\sin^{-1}\frac{12}{13}\Rightarrow 2\tan^{-1}\left(\frac{2}{3}\right)=\sin^{-1}\frac{12}{13}$

Q.2 Show that: $\sin^{-1}(-x)=-\sin^{-1}x$

(Mtn 17 G-I, II)(Sgd 19)

Ans. $\sin^{-1}(-x)=-\sin^{-1}x$

$\Rightarrow \quad \sin^{-1}(-x)+\sin^{-1}x=0$

Take $\sin^{-1}(-x)+\sin^{-1}x$

$=\sin^{-1}\left\{(-x)\sqrt{1-(x)^2}+x\sqrt{1-(-x)^2}\right\}$

$=\sin^{-1}\left\{-x\sqrt{1-x^2}+x\sqrt{1-x^2}\right\}$

$=\sin^{-1}\{0\}=0$

$\Rightarrow \quad \sin^{-1}(-x)+\sin^{-1}x=0$

$\Rightarrow \quad \sin^{-1}(-x)=-\sin^{-1}x$

Q.3 Show that: $\cos^{-1}(-x)=\pi-\cos^{-1}x$

(Mtn 16 G-II)(Gjw 17)

Ans. Let $y=\cos^{-1}x\Rightarrow \cos y=x$

Consider $\cos(\pi-y)=-\cos y$

$\cos(\pi-\cos^{-1}x)=-x$

$\pi-\cos^{-1}x=\cos^{-1}(-x)$

Q.4 Show that: $\tan^{-1}(-x)=-\tan^{-1}x$

Ans. $\tan^{-1}(-x)=-\tan^{-1}x$

$\Rightarrow \tan^{-1}(-x)+\tan^{-1}(x)=0$

Take $\tan^{-1}(-x)+\tan^{-1}(x)=\tan^{-1}\left\{\frac{(-x)+x}{1-(-x)(x)}\right\}$

$=\tan^{-1}\left\{\frac{-x+x}{1+x^2}\right\}$

$=\tan^{-1}\{0\}=0$

$\Rightarrow \tan^{-1}(-x)+\tan^{-1}(x)=0$

$\Rightarrow \tan^{-1}(-x)=-\tan^{-1}x$

Q.5 Show that: $\sin\left(2\cos^{-1}x\right)=2x\sqrt{1-x^2}$

(Lhr 14 G-I)(Fbd 17, 19 G-II)

Ans. $\cos^{-1}x$ is a real number, say α, then

$\cos^{-1}x=\alpha \Rightarrow \quad x=\cos\alpha \quad ,\alpha\in[0,\pi]$

Now $\sin 2\alpha=2\sin\alpha\cos\alpha=2\cos\alpha\sqrt{\sin^2\alpha}=2\cos\alpha\sqrt{1-\cos^2\alpha}$

$\Rightarrow \quad \sin 2\alpha=2x\sqrt{1-x^2}$

$\Rightarrow \quad \sin\left\{2\cos^{-1}x\right\}=2x\sqrt{1-x^2}$

LONG QUESTIONS

Q.1 Without using calculator, prove that

$\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}$.

(Gjw 13 G-I)(Lhr 13 G-II)
(Sgd 13 G-I-II)(D.G.K 15 G-II)
(Mtn , 13, 16 G-I)(Shw 13, 17)(Rwp 18)

Ans. L.H.S. $=\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}$

$$=\sin^{-1}\left[\frac{3}{5}\cdot\sqrt{1-\left(\frac{8}{17}\right)^2}+\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]$$

$$=\sin^{-1}\left[\frac{3}{5}\cdot\sqrt{1-\frac{64}{289}}+\frac{8}{17}\cdot\sqrt{1-\frac{9}{25}}\right]$$

$$=\sin^{-1}\left[\frac{3}{5}\cdot\sqrt{\frac{289-64}{289}}+\frac{8}{17}\cdot\sqrt{\frac{25-9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\cdot\sqrt{\frac{225}{289}}+\frac{8}{17}\cdot\sqrt{\frac{16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\times\frac{15}{17}+\frac{8}{17}\times\frac{4}{5}\right]$$

$$= \sin^{-1}\left[\frac{45}{85}+\frac{32}{85}\right] = \sin^{-1}\left[\frac{45+32}{85}\right]$$

$$= \sin^{-1}\left(\frac{77}{85}\right) = \text{R.H.S.}$$

Q.2 Prove $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=\cos^{-1}\frac{253}{325}$. (Bpr 19)

(Mtn 15 G-I)(D.G.K 15 G-I)(Fbd 12)(Rwp 17 G-II)

Ans. L.H.S. $= \sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}$

$$= \sin^{-1}\left[\frac{5}{13}\sqrt{1-\left(\frac{7}{25}\right)^2}+\frac{7}{25}\sqrt{1-\left(\frac{5}{13}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\sqrt{1-\frac{49}{625}}+\frac{7}{25}\sqrt{1-\frac{25}{169}}\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\sqrt{\frac{625-49}{625}}+\frac{7}{25}\cdot\sqrt{\frac{169-25}{169}}\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\sqrt{\frac{576}{625}}+\frac{7}{25}\cdot\sqrt{\frac{144}{169}}\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\times\frac{24}{25}+\frac{7}{25}\times\frac{12}{13}\right] = \sin^{-1}\left[\frac{120}{325}+\frac{84}{325}\right]$$

$$= \sin^{-1}\left[\frac{120+84}{325}\right] = \sin^{-1}\left(\frac{204}{325}\right)$$

Let $y = \sin^{-1}\frac{204}{325}$ ...... (1)

$\Rightarrow \sin y = \frac{204}{325}$

$\therefore \cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\left(\frac{204}{325}\right)^2}$

$$= \sqrt{1-\frac{41616}{105625}}$$

$$= \sqrt{\frac{105625-41616}{105625}} = \sqrt{\frac{64009}{105625}} = \frac{253}{325}$$

$y = \cos^{-1}\left(\frac{253}{325}\right)$ ...... (2)

Putting value of eq. (1) in eq. (2):

$$\sin^{-1}\frac{204}{325} = \cos^{-1}\frac{253}{325}$$

Q.3 Prove that: $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{9}{19}$.

(Rwp 13 G-I)(Mtn 14 G-II)(D.G.K 14 G-II, 13 G-I)

Ans. See Short Question 3

Q.4 Prove that $\tan^{-1}\frac{1}{11}+\tan\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

(D.G.K 12 G-I, 13 G-II)(Rwp 16)(Mtn 19 G-I)

Ans. L.H.S. $= \tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{5}{6}\right) = \tan^{-1}\left[\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}\times\frac{5}{6}}\right] = \tan^{-1}\left[\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{5}{66}}\right]$

$$= \tan^{-1}\left[\frac{\frac{6+55}{66}}{\frac{66-5}{66}}\right] = \tan^{-1}\left[\frac{61}{61}\right]$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

R.H.S. $= \tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2} = \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right] = \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{6}}\right]$

$$= \tan^{-1}\left[\frac{\frac{2+3}{6}}{\frac{6-1}{6}}\right] = \tan^{-1}\left[\frac{5}{5}\right]$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

Hence, $\boxed{\text{L.H.S.} = \text{R.H.S.}}$

Q.5 Prove that $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{3}{5}$

(Bpr 14)(Mtn 18 G-I, II)(Fbd 19 G-II)

Ans. Let $y = \cos^{-1}\left(\frac{63}{65}\right) \Rightarrow \cos y = \frac{63}{65}$

$\therefore \sin y = \sqrt{1-\cos^2 y} = \sqrt{1-\left(\frac{63}{65}\right)^2}$

$$= \sqrt{1-\frac{3969}{4225}}$$

$$= \sqrt{\frac{4225-3969}{4225}} = \sqrt{\frac{256}{625}} = \frac{16}{25}$$

$\Rightarrow y = \sin^{-1}\frac{16}{65}$

$$\cos^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{16}{65}\right)$$

and $x = 2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left[\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right]$

$$\left[\because 2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)\right]$$

$$= \tan^{-1}\left[\frac{\frac{2}{5}}{\frac{25-1}{25}}\right] = \tan^{-1}\left[\frac{\frac{2}{5}}{\frac{24}{25}}\right]$$

$$= \tan^{-1}\left[\frac{2}{5}\times\frac{25}{24}\right]$$

$$= \tan^{-1}\left(\frac{1}{1}\times\frac{5}{12}\right) = \tan^{-1}\frac{5}{12} \Rightarrow \tan x = \frac{5}{12}$$

$$\cot x = \frac{12}{5} \Rightarrow \cot^2 x = \left(\frac{12}{5}\right)^2 = \frac{144}{25}$$

$$\operatorname{cosec}^2 x - 1 = \frac{144}{25}$$

$$\Rightarrow \operatorname{cosec}^2 x = \frac{144}{25} + 1$$

$$= \frac{144+25}{25} = \frac{169}{25}$$

$$\operatorname{cosec} x = \sqrt{\frac{169}{25}} = \frac{13}{5} \Rightarrow \sin x = \frac{5}{13}$$

$$\Rightarrow \quad x = \sin^{-1}\left(\frac{5}{13}\right)$$

$$2\tan^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{5}{13}\right)$$

$$\text{L.H.S.} = \cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$$

$$= \sin^{-1}\left(\frac{16}{65}\right) + \sin^{-1}\left(\frac{5}{13}\right)$$

$$= \sin^{-1}\left[\frac{16}{65}\cdot\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\cdot\sqrt{1-\left(\frac{16}{65}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{16}{65}\cdot\sqrt{1-\frac{25}{169}} + \frac{5}{13}\cdot\sqrt{1-\frac{256}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{16}{65}\cdot\sqrt{\frac{169-25}{169}} + \frac{5}{13}\cdot\sqrt{\frac{4225-256}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{16}{65}\cdot\sqrt{\frac{144}{169}} + \frac{5}{13}\sqrt{\frac{3969}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{16}{65}\times\frac{12}{13} + \frac{5}{13}\times\frac{63}{65}\right]$$

$$= \sin^{-1}\left[\frac{192}{845} + \frac{315}{845}\right] = \sin^{-1}\left[\frac{192+315}{845}\right]$$

$$= \sin^{-1}\left[\frac{507}{845}\right]$$

$$= \sin^{-1}\left[\frac{3\times169}{5\times169}\right] = \sin^{-1}\left(\frac{3}{5}\right) = \text{R.H.S.}$$

Q.6 Show that $\tan(\sin^{-1}x) = \dfrac{x}{\sqrt{1-x^2}}$ (Sgd 19)

Ans. See Short Question 11

MULTIPLE CHOICE QUESTIONS

❑ ***Each question has four possible answers. Select the correct answer and encircle it.***

Q.1 $\tan^{-1}A + \tan^{-1}B$ = __________:

(Mtn 10 G-I) (D.G. Khan Board 10)

(a) $\tan^{-1}\left(\frac{A-B}{1+AB}\right)$ (b) $\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

(c) $\tan^{-1}\left(\frac{A+B}{1+AB}\right)$ (d) $\tan^{-1}\left(\frac{A-B}{1-AB}\right)$

Q.2 $\sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

(Sgd 13) (Mtn 10 G-II)

(a) $\sin^{-1}A + \sin^{-1}B$ (b) $\sin^{-1}A - \sin^{-1}B$

(c) $\cos^{-1}A + \cos^{-1}B$ (d) $\cos^{-1}A - \cos^{-1}B$

Q.3 If $\sin^{-1}\frac{1}{2} = \frac{\pi}{2} - x$, then value of x is:

(D.G.K 12)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Q.4 $\tan^{-1}(-x)$ = (D.G.K 13 G-II)

(a) $\pi - \tan^{-1}x$ (b) $-\tan^{-1}x$

(c) $\pi + \tan^{-1}x$ (d) $\cot^{-1}x$

Q.5 $\operatorname{Tan}^{-1}\left(\frac{A-B}{1+AB}\right)$ is equal to: (Bpr 10)

(a) $\tan^{-1}A + \tan^{-1}B$ (b) $\tan^{-1}A - \tan^{-1}B$

(c) $\cot^{-1}A + \cot^{-1}B$ (d) $\cot^{-1}A\cot^{-1}B$

Q.6 $\tan^{-1}A - \tan^{-1}B$ = (Lhr 10 G-II)(Lhr 12 G-II)

(a) $\tan^{-1}\left(\frac{A-B}{1+AB}\right)$ (b) $\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

(c) $\tan^{-1}\left(\frac{2AB}{A-B}\right)$ (d) $\tan^{-1}\left(\frac{1+AB}{A-B}\right)$

Q.7 $\cos^{-1}(-x)$ = (D.G.K 15 G-I)(Fbd 16, 19)

(a) $-\cos^{-1}x$ (b) $\cos^{-1}x$

(c) $\pi - \cos^{-1}x$ (d) $\frac{\pi}{2} - \cos^{-1}x$

Q.8 $\sin^{-1}(-x)$ =

(a) $-\sin^{-1}x$ (b) $\sin^{-1}x$

(c) $\pi + \cos^{-1}x$ (d) $-\cos^{-1}x$

Q.9 $\cos^{-1}(-x) =$

(a) $\pi + \cos^{-1} x$ (b) $\pi - \cos^{-1} x$

(c) $\pi + \sin^{-1} x$ (d) $\pi - \sin^{-1} x$

Q.10 $\tan^{-1} A - \tan^{-1} B =$

(a) $\tan\left(\dfrac{A-B}{1+AB}\right)$ (b) $\tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$

(c) $\tan^{-1}\left(\dfrac{A+B}{1-AB}\right)$ (d) $\tan^{-1}\left(\dfrac{A+B}{1+AB}\right)$

Q.11 $\sin^{-1} A + \sin^{-1} B =$

(a) $\sin^{-1}\left(B\sqrt{1-B^2} + A\sqrt{1-A^2}\right)$

(b) $\sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

(c) $\sin^{-1}\left(A\sqrt{1+B^2} + B\sqrt{1+A^2}\right)$

(d) $\sin^{-1}\left(A^2\sqrt{1-B^2} + B^2\sqrt{1-A^2}\right)$

Q.12 $\sin^{-1} A - \sin^{-1} B =$ (Lhr 18 G-II)

(a) $\sin^{-1}\left(B\sqrt{1-B^2} + A\sqrt{1-A^2}\right)$

(b) $\sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$

(c) $\sin^{-1}\left(A\sqrt{1+B^2} + B\sqrt{1+A^2}\right)$

(d) $\sin^{-1}\left(A^2\sqrt{1-B^2} + B^2\sqrt{1-A^2}\right)$

Q.13 $\cos(2\sin^{-1} x) =$

(a) $1 - 2x^2$ (b) $1 + 2x^2$

(c) $2x^2 - 1$ (d) $1 - x^2$

Q.14 $2\tan^{-1} A$ **equals.** (Bpr 18)(Rwp 17 G-I)(Lhr 19 G-II)

(a) $\tan^{-1}\dfrac{A}{1-A^2}$ (b) $\tan^{-1}\dfrac{2A}{1+A^2}$

(c) $\tan^{-1}\left(\dfrac{-2A}{1+A^2}\right)$ (d) $\tan^{-1}\left(\dfrac{2A}{1-A^2}\right)$

Q.15 $\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{3}\right) =$ (Fbd 18)

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{6}$

Q.16 $\sin^{-1} A + \sin^{-1} B =$ (Mtn 13 G-II)

(a) $\cos^{-1}\left(AB + \sqrt{1-A^2}\sqrt{1-B^2}\right)$

(b) $\sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

(c) $\cos^{-1}(2A-1)$

(d) $\sin{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$

Q.17 $\tan^{-1}\left(\dfrac{A-B}{1+AB}\right)$ **is equal to:** (Fbd 12)(Gjw 16)

(a) $\tan^{-1} A + \tan^{-1} B$ (b) $\tan^{-1} A - \tan^{-1} B$

(c) $\cot^{-1} A + \cot^{-1} B$ (d) $\cot^{-1} A - \cot^{-1} B$

Chapter 14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

SHORT ANSWERS TO THE QUESTIONS

Q.1 Define a trigonometric equation.

(Rwp 16, 17 G-I)(Fbd 18)

Ans. The equations involving trigonometric functions are called trigonometric equations e.g. $\sin^2 x - \cos x = 1$.

Q.2 Solve the equation $\sin x = \frac{1}{2}$ (Bpr 19)

(Rwp 14 G-I)(Gjw 15)(Lhr 18 G-II)(Mtn 18 G-I)

Ans. $\sin x = \frac{1}{2}$

Θ sin x is positive in I and II quadrants with the reference angle $\alpha = \frac{\pi}{6}$

$\Rightarrow x = \frac{\pi}{6}$ and $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ where $x \in [0, 2\pi]$

$\therefore$ General values of x are $\frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$, $n \in Z$

Hence solution set $= \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}$, $n \in Z$

Q.3 Find the solution of the equation $\sin x = -\frac{\sqrt{3}}{2}$ which lie in $[0, 2\pi]$ (Gjw 18, 19 G-I, II)

(Fbd 12 G-I)(D.G.K 14 G-I)(Mtn 16 G-I)(Swl 17)

Ans. $\sin x = -\frac{\sqrt{3}}{2}$

We know that reference angle can be easily calculated by taking positive value of trigonometric function. Let α be the reference angle of angle x, therefore $\sin \alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{3}$

Since sin x is negative (−ve) in III and IV Quadrants with the reference angle $\alpha = \frac{\pi}{3}$

Therefore third quadrant solution is $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ and fourth quadrant solution is $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Thus required solution of equation $\sin x = -\frac{\sqrt{3}}{2}$ is $\frac{4\pi}{3}, \frac{5\pi}{3}$.

Q.4 Solve the equation: $1 + \cos x = 0$ (Swl 19)

(Swl 14 G-I)(Mtn 15 G-II)(Rwp 17 G-II)(Gjw 18)

Ans. $1 + \cos x = 0 \Rightarrow \cos x = -1$

Since cos x is −ve, there is only one solution $x = \pi$ in $[0, 2\pi]$

$\therefore$ General value of x is $\pi + 2n\pi$, $n \in Z$

Hence solution set $= \{\pi + 2n\pi\}$, $n \in Z$

Q.5 Solve the equation: $1 + \cos x = 0$ in $[0, 2\pi]$

(Lhr 16 G-I)(Sgd 17)(Fbd 19 G-II)

Ans. $1 + \cos x = 0 \Rightarrow \cos x = -1$

$\therefore x = \cos^{-1}(-1) = \pi$.

Solution set = { π }

Q.6 Find the solution of the equation $\cot \theta = \frac{1}{\sqrt{3}}$ which lie in $[0, 2\pi]$ (Rwp, Swl 19)

(Lhr 14 G-I, 15 G-I, 18 G-II) (Gjw 17)(Sgd 17, 18, 19)

(D.G.K 14 G-I)(Mtn 16 G-II)(Bpr 16, 18, 19)

Ans. $\cot \theta = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \sqrt{3}$

$\therefore$ tan θ is positive in I and III Quadrants with the reference angle $\alpha = \frac{\pi}{3}$.

Therefore $\theta = \frac{\pi}{3}$ and $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Thus required solution of equation $\cot \theta = \frac{1}{\sqrt{3}}$ is $\frac{\pi}{3}, \frac{4\pi}{3}$.

Q.7 Find the solution of the equation cosec θ = 2 which lie in $[0, 2\pi]$

(Gjw 14)(Lhr 13)(Bpr 12 G-I, 14 G-I)(Sgd 16)(Rwp 19)

(D.G.K 17 G-I)(Lhr 17 G-I)(Mtn 17 G-II)(Fbd 19)

Ans. $\text{cosec}\, \theta = 2 \Rightarrow \sin \theta = \frac{1}{2}$

$\therefore$ sin θ is positive in I and II Quadrants with the reference angle $\alpha = \frac{\pi}{6}$

Therefore $\theta = \frac{\pi}{6}$ and $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Thus required solution of equation cosec θ = 2 is $\frac{\pi}{6}, \frac{5\pi}{6}$

Q.8 Find the solution of the equation sec x = –2 which lie in [0, 2π]
(D.G.K 12 G-I)(Shw 14 G-I)(Rwp 16)(Mtn 19 G-I)
(Rwp 17 G-II)(A.J.K 17) (Fbd 17, 19 G-II)

Ans. $\sec x = -2 \Rightarrow \cos x = -\frac{1}{2}$

We know that reference angle can be easily calculated by taking positive value of trigonometric function. Let α be the reference angle of angle x, therefore $\cos \alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

Since cos x is negative in II and III quadrants with the reference angle $\alpha = \frac{\pi}{3}$

Therefore second quadrant solution is $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and third quadrant solution is

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Thus required solution of equation sec x = –2 is $\frac{2\pi}{3}, \frac{4\pi}{3}$

Q.9 Find the solution set of : $\sin x \cos x = \frac{\sqrt{3}}{4}$.
(Lhr 12 G-II) (Gjw 16)

Ans. $\sin x \cos x = \frac{\sqrt{3}}{4} \Rightarrow \frac{1}{2}(2 \sin x \cos x) = \frac{\sqrt{3}}{4}$

$\Rightarrow \quad \sin 2x = \frac{\sqrt{3}}{2}$

Θ sin 2x is +ve in I and II Quadrants with the reference angle $\alpha = \frac{\pi}{3}$

$\therefore\ 2x = \frac{\pi}{3}$ and $2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ are two solutions in [0, 2π]

As 2π is the period of sin 2x.

$\therefore$ General solution of 2x are $\frac{\pi}{3} + 2n\pi$ and $\frac{2\pi}{3} + 2n\pi$, $n \in Z$

$\Rightarrow$ General solutions of x are $\frac{\pi}{6} + n\pi$ and $\frac{\pi}{3} + n\pi$, $n \in Z$

Hence solution set $= \left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{\pi}{3} + n\pi\right\}$, $n \in Z$

Q.10 Solve the equation $4\cos^2 x - 3 = 0$ when $x \in [0, \pi]$ (Mtn 17 G-I, 18 G-II)(Fbd 19)

Ans. $4\cos^2 x - 3 = 0$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\sqrt{3}}{2}$	$\cos x = -\frac{\sqrt{3}}{2}$
$x = \frac{\pi}{6}$	$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Solution Set $= \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

Q.11 Solve the trigonometric equation $\tan^2 \theta = \frac{1}{3}$ in [0, 2π]
(D.G.K 13 G-II)(Mtn 16 G-II)(Fbd 12 G-I)(Shw 17)

Ans. $\tan^2 \theta = \frac{1}{3}$

$\tan \theta = \pm \frac{1}{\sqrt{3}}$

$\tan \theta = \frac{1}{\sqrt{3}}$	$\tan \theta = -\frac{1}{\sqrt{3}}$
∴ tan θ is positive in I and III quadrants with the reference angle $\alpha = \frac{\pi}{6}$.	∴ tan θ is negative in II and IV quadrants with the reference angle $\alpha = \frac{\pi}{6}$.
Therefore $\theta = \frac{\pi}{6}$ and $\pi + \frac{\pi}{6}$	Therefore $\theta = \pi - \frac{\pi}{6}$ and $2\pi - \frac{\pi}{6}$
$\Rightarrow \theta = \frac{\pi}{6}$ and $\frac{7\pi}{6}$	$\Rightarrow \theta = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$

Hence required solution is $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

Q.12 Find the solution of $\tan^2\theta = \frac{1}{3}$ in [0, π]
(Gjw 16)(Mtn 16 G-I)(DGK 19)

Ans. $\tan^2 \theta = \frac{1}{3}$

$\tan \theta = \pm \frac{1}{\sqrt{3}}$

$\tan \theta = \frac{1}{\sqrt{3}}$	$\tan \theta = \frac{1}{\sqrt{3}}$
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Solution set $= \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

Q.13 Solve the trigonometric equation $\cot^2 \theta = \frac{1}{3}$
Short: (Bpr 3 G-I)(Rwp 13 G-I) (Lhr 19 G-II)

Ans. $\cot^2 \theta = \frac{1}{3}\ \cot \theta = \pm \frac{1}{\sqrt{3}}$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\therefore$ $\tan\theta$ is positive in I and III Quadrants with the reference angle $\alpha = \frac{\pi}{3}$. Therefore $\theta = \frac{\pi}{3}$ and $\pi + \frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ and $\frac{4\pi}{3}$

$\cot\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = -\sqrt{3}$

$\therefore$ $\tan\theta$ is negative in II and IV Quadrants with the reference angle $\alpha = \frac{\pi}{3}$. Therefore $\theta = \pi - \frac{\pi}{3}$ and $2\pi - \frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{2\pi}{3}$ and $\frac{5\pi}{3}$

Hence required solution is $\frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

Q.14 Solve the trigonometric equation $\sec^2\theta = \frac{4}{3}$.

(Lhr 14 G-I), (D.G.K 14 G-II)(Shw 13 G-I)(A.J.K 17) (Rwp 18)(Sgd 19)

Ans. $\sec^2\theta = \frac{4}{3}$

$\sec\theta = \pm\frac{2}{\sqrt{3}}$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$

$\therefore$ $\cos\theta$ is positive in I and IV Quadrants with the reference angle $\alpha = \frac{\pi}{6}$.

Therefore $\theta = \frac{\pi}{6}$ and $2\pi - \frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

$\sec\theta = -\frac{2}{\sqrt{3}}$

$\Rightarrow \quad \cos\theta = -\frac{\sqrt{3}}{2}$

$\therefore$ $\cos\theta$ is negative in II and III quadrants with the reference angle $\alpha = \frac{\pi}{6}$.

Therefore $\theta = \pi - \frac{\pi}{6}$ and $\pi + \frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{5\pi}{6}$ and $\frac{7\pi}{6}$

Hence required solution is $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

Q.15 Solve the trigonometric equation $\text{cosec}^2\theta = \frac{4}{3}$

(Bpr 14 G-I)(Gjw 15) (Sgd 16)(Gjw 17)(D.G.K 17 G-I)

Ans. $\text{cosec}^2\theta = \frac{4}{3} \Rightarrow \text{cosec}\,\theta = \pm\frac{2}{\sqrt{3}}$

$\text{cosec}\,\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore$ $\sin\theta$ is positive in I and II quadrants with the reference angle $\alpha = \frac{\pi}{3}$.

Therefore $\theta = \frac{\pi}{3}$ and $\pi - \frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

$\text{cosec}\,\theta = -\frac{2}{\sqrt{3}}$

$\Rightarrow \quad \sin\theta = -\frac{\sqrt{3}}{2}$

$\therefore$ $\sin\theta$ is negative in III and IV quadrants with the reference angle $\alpha = \frac{\pi}{3}$.

Therefore $\theta = \pi - \frac{\pi}{3}$ and $2\pi - \frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{4\pi}{3}$ and $\frac{5\pi}{3}$

Hence required solution is $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Q.16 Find the solution set of : $\sin x \cos x = \frac{\sqrt{3}}{4}$.

(Lhr 12 G-II) (Gjw 16)(Fbd 18)

Ans. $\sin x \cos x = \frac{\sqrt{3}}{4} \Rightarrow \frac{1}{2}(2\sin x\cos x) = \frac{\sqrt{3}}{4}$

$\Rightarrow \quad \sin 2x = \frac{\sqrt{3}}{2}$

$\because$ $\sin 2x$ is +ve in I and II Quadrants with the reference angle $\alpha = \frac{\pi}{3}$

$\therefore$ $2x = \frac{\pi}{3}$ and $2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ are two solutions in $[0, 2\pi]$

As 2π is the period of $\sin 2x$.

$\therefore$ General solution of $2x$ are $\frac{\pi}{3} + 2n\pi$ and $\frac{2\pi}{3} + 2n\pi$, $n \in Z$

$\Rightarrow$ General solutions of x are $\frac{\pi}{6} + n\pi$ and $\frac{\pi}{3} + n\pi$, $n \in Z$

Hence solution set $= \left\{\frac{\pi}{6} + n\pi\right\} \cup \left\{\frac{\pi}{3} + n\pi\right\}$, $n \in Z$

Q.17 Solve: $\sin x + \cos x = 0$. (DGK 19)

(Lhr 12 G-I, 14 G-II)(Rwp 17 G-I, 18)(Mtn 18 G-I)

Ans. $\sin x + \cos x = 0$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} = 0$ (Dividing by $\cos x \neq 0$)

$\Rightarrow \tan x + 1 = 0 \quad \Rightarrow \tan x = -1$

$\because$ $\tan x$ is −ve in II and IV Quadrants with the reference angle $\alpha = \frac{\pi}{4}$

$\therefore$ $x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ and $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$, where $x \in [0, 2\pi]$

As π is the period of $\tan x$,

$\therefore$ General values of x are $\frac{3\pi}{4}+n\pi$ and $\frac{7\pi}{4}+n\pi$, $n\in Z$

$\therefore$ Solution set $=\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{7\pi}{4}+n\pi\right\}, n\in Z$

$\Theta\left\{\frac{7\pi}{4}+n\pi\right\}\subset\left\{\frac{3\pi}{4}+n\pi\right\}, n\in Z$

Hence the solution set $=\left\{\frac{3\pi}{4}+n\pi\right\}, n\in Z.$

Q.18 Solve $\sin x+\cos x=0$ where $x\in[0,2\pi]$. (Fbd 16)(Mtn 17 G-II)

Ans. $\sin x+\cos x=0$

$\sin x=-\cos x$

$\frac{\sin x}{\cos x}=-\frac{\cos x}{\cos x}$

$\tan x=-1$

$x=\pi-\frac{\pi}{4}, 2\pi-\frac{\pi}{4}$

$x=\frac{3\pi}{4}, \frac{7\pi}{4}$

Q.19 Solve the equation: $2\sin^2\theta-\sin\theta=0$ where $\theta\in[0,2\pi]$ (Lhr 15 G-I)(Fbd 16)

Ans. $2\sin^2\theta-\sin\theta=0 \Rightarrow \sin\theta(2\sin\theta-1)=0$

$\sin\theta=0 \Rightarrow \theta=0,\pi$

$2\sin\theta-1=0$

$\sin\theta=\frac{1}{2}$

$\therefore$ $\sin\theta$ is positive in I and II Quadrants with the reference angle $\alpha=\frac{\pi}{6}$

Therefore $\theta=\frac{\pi}{6}$ and $\theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

Hence required solution is $0,\pi,\frac{\pi}{6},\frac{5\pi}{6}$

Q.20 Solve $\sin^2 x+\cos x=1$ where $x\in[0,2\pi]$. (Bpr 16)(Mtn 17 G-I)(Sgd 18)(Gjw 19 G-II)

Ans. $\sin^2 x+\cos x=1$

$0=1-\sin^2 x-\cos x$

$0=\cos^2 x-\cos x$

$0=\cos x(\cos x-1)$

Either

$\cos x=0$

$x=\frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x-1=0$

$\cos x=1$

$x=0$

Q.21 Solve $2\sin\theta+\cos^2\theta-1=0$ Where $\theta\in[0,2\pi]$ (D.G.K 16 G-I)(Lhr 19 G-II) (Mtn 19 G-I)(Lhr 17 G-I)

Ans. $2\sin\theta+\cos^2\theta-1=0$

$2\sin\theta-(1-\cos^2\theta)=0$

$2\sin\theta-\sin^2\theta=0$

$\sin\theta(2-\sin\theta)=0$

$\sin\theta=0$

$\theta=0,\pi$

$2-\sin\theta=0$

$\sin\theta=2$

Not possible

Solution set $=\{0.\pi\}$

Q.22 Find the solution of the solution. $\tan^2\theta-\sec\theta-1=0$ which be in $[0,2\pi]$ (Lhr 16 G-I)

Ans. $\tan^2\theta-\sec\theta-1=0$

$\sec^2\theta-1-\sec\theta-1=0$

$\sec^2\theta-\sec\theta-2=0$

$\sec^2\theta-2\sec\theta+\sec\theta-2=0$

$(\sec\theta-2)(\sec\theta+1)=0$

$\sec\theta-2=0$

$\sec\theta=2$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}, 2\pi-\frac{\pi}{3}$

$\theta=\frac{\pi}{3}, \frac{5\pi}{3}$

$\sec\theta+1=0$

$\sec\theta=-1$

$\cos\theta=-1$

$\theta=\pi$

Solution set $=\left\{\frac{\pi}{3},\frac{5\pi}{3},\pi\right\}$

Q.23 Find the value of θ satisfying the equation $3\tan^2\theta+2\sqrt{3}\tan\theta+1=0.$ (Rwp 19)

Ans. $3\tan^2\theta+2\sqrt{3}\tan\theta+1=0$

$(\sqrt{3}\tan\theta)^2+2(\sqrt{3}\tan\theta)(1)+(1)^2=0$

$(\sqrt{3}\tan\theta+1)^2=0 \Rightarrow \sqrt{3}\tan\theta+1=0$

$\Rightarrow\sqrt{3}\tan\theta=-1 \Rightarrow \tan\theta=-\frac{1}{\sqrt{3}}$

Since $\tan\theta$ is $-$ve in second and fourth quadrants with the angle $\theta=\frac{\pi}{6}$

In II – Quadrant

$\theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

In IV – Quadrant

$\theta=2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$

As π is period of tangent function so,

General values of θ are $\frac{5\pi}{6}+n\pi$

Solution set $=\left\{\frac{5\pi}{6}+n\pi\right\}, n\in Z$

MULTIPLE CHOICE QUESTIONS

- **Each question has four possible answers. Select the correct answer and encircle it.**

(1) Solution set of $1 + \cos x = 0$ is: (Mtn 08 G-I)

(a) $\left\{\frac{1}{2} + 2n\pi\right\}$ (b) $\{\pi + 2n\pi\}$
(c) $\left\{\frac{3\pi}{2} + 2n\pi\right\}$ (d) $\left\{-\frac{\pi}{2} + 2n\pi\right\}$

(2) If $1 + \cos x = 0$, General solution of x is: (Mtn 08 G-II)(Gjw 18)

(a) $\pi + n\pi$ (b) $\pi + 2n\pi$
(c) $\pi + 3n\pi$ (d) $\pi + 4n\pi$

(3) The solution set of $\tan x = 0$ is given by: (Mtn 08 G-I)

(a) $\{2n\pi\}$ (b) $\{\pi + 2n\pi\}$
(c) $\{n\pi\}$ (d) $\{3n\pi\}$

(4) If $n \in z$, the general solution if $1 + \cos x = 0$ is: (Mtn 09 G-II)

(a) $\{\pi + 2\pi n\}$ (b) $\left\{\frac{\pi}{2} + 2n\pi\right\}$
(c) $\{2n\pi\}$ (d) $\left\{\frac{\pi}{4} + 2n\pi\right\}$

(5) If $\sin x = \frac{1}{2}$, then x (Mtn 15 G-I, 10, 18 G-II)(Lhr 14 G-II)(Sgd 18)

(a) $\frac{\pi}{6}, \frac{5\pi}{6}$ (b) $-\frac{\pi}{6}, \frac{5\pi}{6}$
(c) $\frac{-\pi}{6}, \frac{-5\pi}{6}$ (d) $\frac{2\pi}{3}, \frac{\pi}{3}$

(6) If $\sin x = \frac{1}{2}$, then x = Where $x \in [0, 2\pi]$ (Fbd 15) (Mtn 10 G-II)(D.G.K 17 G-II, 19)

(a) $\frac{\pi}{6}, \frac{7\pi}{6}$ (b) $\frac{\pi}{6}, \frac{5\pi}{6}$
(c) $+\frac{\pi}{3}, \frac{\pi}{6}$ (d) $\frac{\pi}{3}, \frac{2\pi}{3}$

(7) If $\sin x = \frac{\sqrt{3}}{2}$, then reference angle is: (Mtn 10 G-I)(Rwp 11)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $-\frac{\pi}{3}$ (d) $-\frac{\pi}{6}$

(8) If $\cos x = \frac{1}{2}$, then reference angle is: (Mtn 11 G-II)(Lhr 17 G-I)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(9) If $\cos x = -\frac{1}{2}$, then reference angle is: (Mtn 13 G-I)(Lhr 18 G-I)

(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{-\pi}{6}$

(10) Solution of $1 + \cos x = 0$ is: (Lhr 15 G-II) (Fbd 14)(D.G. Khan 13)(Gjw 13)

(a) $\frac{\pi}{2}$ (b) π
(c) 2π (d) $\frac{3\pi}{2}$

(11) The solution of $\operatorname{cosec}\theta = 2$ in the interval $[0, 2\pi]$ (D.G.K 11)

(a) $\frac{\pi}{3}$ and $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$
(c) $\frac{\pi}{2}$ and $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

(12) If $\cos x = -\frac{\sqrt{3}}{2}$, then reference angle is: (D.G.K 12)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

(13) If $\sec x = -\sqrt{2}$, then x = (D.G.K 13 G.II)

(a) $45°, 135°$ (b) $135°, 225°$
(c) $45° - 45°$ (d) $225°, -45°$

(14) If $\sec^2\theta = \frac{4}{3}$, then $\tan^2\theta$: (Bpr 12)

(a) $\frac{1}{3}$ (b) $-\frac{2}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(15) If $\cos x = -\frac{1}{2}$, then the reference angle is: (Shw 13)

(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

(16) $\cot^{-1}(-1) =$ _____ (A.J.K 17)

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $-\frac{3\pi}{4}$ (d) $-\frac{\pi}{2}$

(17) If $\sin x = -\frac{1}{2}$, then $x =$ (Rwp 12)

(a) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

(18) If $\sin x = \frac{-\sqrt{3}}{2}$, then solution is: (Rwp 13, 17 G-II))

(a) $\frac{4\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{4\pi}{3}, \frac{5\pi}{3}$

(c) $\frac{5\pi}{6}, \frac{\pi}{6}$ (d) $\frac{\pi}{3}, \frac{7\pi}{3}$

(19) The solution of the equitation $\tan x = \frac{1}{\sqrt{3}}$ is in: (Fbd 11)

(a) I and II quad (b) I and III quad

(c) II and IV quad (d) I and IV quad

(20) The general solution set of the equation $1 + \cos x = 0$: (Fbd 12)

(a) $2n\pi$, $n \in z$, (b) $\{n\pi\}$, $n \in z$

(c) $\{\pi + 2n\pi\}, n \in z$ (d) $\left\{\frac{\pi}{2} + 2n\pi\right\}$, $n \in z$

(21) If $\sin x = \frac{\sqrt{3}}{2}$ and $x \in [0, 2\pi]$, then $x =$ ______. (Sgd 13 G-I)(Bpr 18)

(a) $\frac{\pi}{3}, \frac{2\pi}{3}$ (b) $\frac{\pi}{4}, \frac{3\pi}{4}$

(c) $\frac{5\pi}{3}, \frac{4\pi}{3}$ (d) $\frac{7\pi}{6}, \frac{5\pi}{6}$

(22) One solution of $\sin x = \frac{-1}{2}$ lies in $[0,2\pi]$ is: (Sgd 13 G-II)

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$ (d) $-\frac{\pi}{3}$

(23) If $\sec x = -2$, the reference angle of $\sec x$ is: (Mtn 14 G-I, 17 G-II)(D.G.K 17 G-I)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $-\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(24) If $\cos 2x = 0$, then solution in 1^{st} quadrant is: (Mtn 14 G-II) (Lhr 11 G-II)(Shw 17)

(a) 30° (b) 45°

(c) 60° (d) 15°

(25) If $\cos x = -\frac{\sqrt{3}}{2}$, then reference angle of $\cos x =$ (D.G.K 14)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

(26) If $\sin x = \frac{1}{2}$, then x is a member of: (D.G.K 14 G-II)

(a) $[0,\pi]$ (b) $[-\pi,0]$

(c) $[\pi, 2\pi]$ (d) $\left[\frac{\pi}{3} + n\pi\right]$

(27) If $\tan x = -1$, then general value of x is: (Bpr 14)(Rwp 18)

(a) $\left\{\frac{3\pi}{4} + n\pi\right\}$ (b) $\left\{\frac{\pi}{6} + n\pi\right\}$

(c) $\left\{\frac{\pi}{4} + n\pi\right\}$ (d) $\left\{\frac{\pi}{3} + n\pi\right\}$

(28) If $\sin\theta = \frac{\sqrt{3}}{2}$, then θ is: (Rwp 14)

(a) 30° (b) 45°

(c) 90° (d) 120°

(29) Solution of the equation $\cos x - 1 = 0$ in $[0, 2\pi]$ is: (D.G.K 15 G-II)

(a) $\{0, \pi\}$ (b) $\{0, 2\pi\}$

(c) $\left\{0, \frac{\pi}{2}\right\}$ (d) $\left\{\frac{\pi}{3}, \frac{3\pi}{2}\right\}$

(30) The equation $\cos^2 x = \frac{3}{4}$ has (Lhr 10 G-II)

(a) One (b) Two

(c) Four (d) Infinite

(31) An equation containing at least one trigonometric function is called: (Gjw 10)(Lhr 17 G-II)

(a) Algebraic equation (b) Quadratic equation

(c) Linear equation

(d) Trigonometric equation

(32) $\cos x = \frac{1}{2}$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(33) If $\cos 2x = 0$, then solution in 1st quadrant is: (Lhr 11 G-II)

(a) 30° (b) 45°

(c) 60° (d) 15°

(34) If $\sin x = \frac{1}{2}$, then reference angle is: (Gjw 11)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(35) Solution of equation $\tan x = \frac{1}{\sqrt{3}}$ lie in:

(Lhr 15 G-I , 12 G-I)

(a) I and II quadrant (b) I and III quadrant

(c) II and IV quadrant (d) I and IV quadrant

(36) Which of the following is the solution of sec x = $\sqrt{2}$, $x \in [0, 2\pi]$. (Lhr 12 G-II)(Gjw 12)

(a) $-\frac{\pi}{4}, \frac{\pi}{4}$ (b) $\frac{\pi}{4}, \frac{3\pi}{4}$

(c) $\frac{\pi}{4}, \frac{5\pi}{4}$ (d) $\frac{\pi}{3}, \frac{\pi}{4}$

(37) If sin x = cos x, then x = (Fbd 17)

(a) 45° (b) 30°

(c) 0° (d) 60°

(38) Solution set of 1 + cos x = 0: (Lhr 13 G-I & G-II)

(a) $\{2n\pi\}$ (b) $\{\pi + 2n\pi\}$

(c) $\{\pi\}$ (d) $\{2\pi + n\pi\}$

(39) If $\cos x = \frac{1}{\sqrt{2}}$, then reference angle is:

(Lhr 14 G-I) (Gjw 14)

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

(40) Reference angle of tan θ = −1 equal:

(D.G.K 15 G-I)

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$

(c) $-\pi$ (d) $\frac{\pi}{2}$

(41) Trigonometric equation has solution. (Bpr 15)

(a) 0 (b) 1

(c) 2 (d) Infinite

(42) The solution of the equation cos x − 1 = 0 in [0,2π] is: (D.G.K 15 G-II)

(a) $\frac{\pi}{2}$ (b) 0, 2π

(c) $-\frac{\pi}{2}, \frac{\pi}{2}$ (d) o, π

(43) Number of solution of trigonometric equation is:

(a) Finite (b) Infinite

(c) Only one (d) All of these

(44) Tan−1 $(-\sqrt{3})$: (Gjw 15)

(a) $\frac{2\pi}{3}$ (b) $-\frac{2\pi}{3}$

(c) $-\frac{\pi}{6}$ (d) $\frac{4\pi}{3}$

(45) The general solution of the equation tan x = − 1 is

(a) $\frac{\pi}{4} + n\pi, n \in Z$ (b) $\frac{\pi}{4} + 2n\pi, n \in Z$

(c) $\frac{3\pi}{4} + n\pi, n \in Z$ (d) $\frac{5\pi}{4} + n\pi, n \in Z$

(46) The general solution of the equation $\tan x = \frac{1}{\sqrt{3}}$ is

(a) $\frac{\pi}{4} + n\pi, n \in Z$ (b) $\frac{\pi}{6} + 2n\pi, n \in Z$

(c) $\frac{5\pi}{6} + n\pi, n \in Z$ (d) $\frac{\pi}{6} + n\pi, n \in Z$

(47) $\cos x = \frac{1}{\sqrt{2}}$, then reference angle is.

(Mtn 17 G-I)(Fbd 18)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(48) If sin x + cos x = 0, then x = ___________.

(a) $\frac{\pi}{4}, -\frac{\pi}{4}$ (b) $-\frac{\pi}{4}, \frac{\pi}{2}$

(c) $-\frac{\pi}{4}, \frac{3\pi}{4}$ (d) none of these

(49) The solutions of $\sin^2 x = \frac{3}{4}$ in the interval [0, π] are ___________.

(a) $\frac{\pi}{3} \& \frac{2\pi}{3}$ (b) $\frac{4\pi}{3} \& \frac{5\pi}{3}$

(c) $-\frac{\pi}{3} \& -\frac{2\pi}{3}$ (d) none of these

(50) Solution of equation 1 + cos θ = 0 are is quadrant. (Mtn 16 G-II)

(a) I and IV (b) II and III

(c) II and IV (d) None of these

(51) If cosx + 1 = 0, then. (Mtn 16 G-I)

(a) $\left\{\frac{\pi}{2} + 2n\pi\right\}$ (b) $\{\pi + 2n\pi\}$

(c) $\{p + n\pi\}$ (d) $\left\{-\frac{\pi}{2} + 2n\pi\right\}$

(52) Solution set of tan 2x = 1 in [0, 2π] is equal to.

(a) $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$ (b) $\left\{\frac{\pi}{8}, \frac{5\pi}{8}\right\}$

(c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (d) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

(53) The solution of equation $\frac{1}{2} + \sin\theta = 0$ are in quadrant. (Rwp 16)
(a) I & IV (b) I & III
(c) III & IV (d) II & IV

(54) $y = \cos x$ is one to one in interval. (Lhr 16 G-I)
(a) $\left[0, \frac{2\pi}{3}\right]$ (b) $[0, 2\pi]$
(c) $[0, \infty[$ (d) $[0, \pi]$

(55) The reference angle of $\tan\theta = -1$ equal (Gjw 16)
(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $-\pi$ (d) π

(56) $\cos x = \frac{1}{2}$ has solution. (Fbd 16)
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(57) Solution of the equation $\cos x = -1$ in $[0, 2\pi]$ is (Bpr 16)
(a) $\{0, \pi\}$ (b) $\{0, 2\pi\}$
(c) $\{\pi\}$ (d) $\{0\}$

(58) If $\cos x = \frac{\sqrt{3}}{2}$, $x \in [0, \pi]$ then x equals. (Rwp 17 G-I)
(a) $-\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$
(c) $\frac{\pi}{6}$ (d) $\frac{7\pi}{6}$

(59) If $n \in Z$, the general solution of the equation $\sin x = 0$ is. (Sgd 17)
(a) $\frac{n\pi}{2}$ (b) $\frac{n\pi}{3}$
(c) $\frac{n\pi}{4}$ (d) $n\pi$

(60) Solution of $\operatorname{cosec}\theta = 2$ in quad I. (A.J.K 17)
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(61) Solution of $\cot\theta = \frac{1}{\sqrt{3}}$ in quadrant III. (Lhr 18 G-II)(Sgd 19)
(a) $\frac{5\pi}{4}$ (b) $\frac{7\pi}{6}$
(c) $\frac{4\pi}{3}$ (d) π

(62) If $\sin x = \frac{1}{2}$, then x = _____________. (Mtn 18 G-I)
(a) $-\frac{\pi}{6}, \frac{5\pi}{6}$ (b) $-\frac{\pi}{6}, -\frac{5\pi}{6}$
(c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$

(63) The solution of $1 + \cos x = 0$ if $0 \le x \le 2\pi$: (Fbd 19 G-II)
(a) $\{0\}$ (b) $\left\{\frac{\pi}{2}\right\}$
(c) $\left\{\frac{\pi}{3}\right\}$ (d) $\{\pi\}$

(64) The solutions of equation $1 + \sin\theta = 0$ are in quadrant (Gjw 19 G-II)
(a) I and IV (b) I and III
(c) II and IV (d) III and IV

(65) The reference angle for $\tan\theta = \sqrt{3}$ is : (Bpr 19)
(a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{-\pi}{3}$

(66) Solution of equation $\tan x = \frac{1}{\sqrt{2}}$ lies in the quadrants: (Lhr 19 G-II)
(a) I and II (b) II and III
(c) I and III (d) I and IV

(67) Solution of $\cot\theta = \frac{1}{\sqrt{3}}$ in IIIrd quadrant is (Sgd 19)
(a) $\frac{5\pi}{4}$ (b) $\frac{7\pi}{4}$
(c) $\frac{4\pi}{3}$ (d) π

(68) If $\sin x = -\frac{\sqrt{3}}{2}$ then x = (Swl 19)
(a) $\frac{\pi}{3}$ (b) $\frac{4\pi}{3}$
(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

FAISALABAD BOARD 2019 (G-I)

MATHEMATICS INTERMEDIATE PART-I

(Objective Type)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) For a , b $\in$ R, either a > b or a = b or a < b is the:

(a) Trichotomy property (b) Left distributive property
(c) Right distributive property (d) Cancellation property

(2) The number of subsets of a set of 4-elements is:

(a) 16 (b) 8 (c) 4 (d) 6

(3) If A is a matrix of order 3×2 then $A^t A$ is of order:

(a) 3×3 (b) 2×3 (c) 2×2 (d) 3×2

(4) If A and B are non-singular matrices then $(AB)^{-1}$

(a) $A^{-1}B^{-1}$ (b) $\frac{1}{AB}$ (c) $B^{-1}A^{-1}$ (d) $(BA)^{-1}$

(5) The polynomial $ax^3 + bx^2 + 8$ has degree:

(a) 8 (b) 3 (c) a + b (d) 5

(6) If α, β are the roots of the equation $x^2 - 4x + 5 = 0$,

(a) 5 (b) - 4 (c) 2 (d) 4

(7) Types of rational fraction are:

(a) 3 (b) 2 (c) 4 (d) 1

(8) If in an Sequence, $a_n = \frac{(-1)^{n-1}n}{2n+1}$, then a_4 equals:

(a) 4 (b) 3 (c) $\frac{4}{3}$ (d) $\frac{-4}{9}$

(9) $\sum_{k=1}^{n} k = 1$

(a) $\frac{n(n+1)}{6}$ (b) $\frac{n(n+1)}{4}$ (c) $\frac{n(n+1)}{2}$ (d) n (n + 1)

(10) 0! =

(a) 0 (b) 1 (c) - 1 (d) 2

(11) **With usual notation notation ${}^{o}C_{r}$ equals.**

(a) ${}^{o}C_{r-n}$ (b) ${}^{n}C_{n}$ (c) ${}^{o}C_{n-r}$ (d) ${}^{n-r}C_{n}$

(12) **$n! > n^2$ is true for:**

(a) $n < 1$ (b) $n < 2$ (c) $n < 3$ (d) $n \geq 4$

(13) **The sum of odd coefficients in the expansion $(1 + x)^n$ is:**

(a) n^2 (b) 2^{n-1} (c) 2^n (d) 2^{n-2}

(14) **The vertex of an angle in standard form is at:**

(a) (1 , 0) (b) (0 , 1) (c) (1 , 1) (d) (0 , 0)

(15) $\cos\left(\dfrac{3\pi}{2} + \theta\right) = :$

(a) $\cos\theta$ (b) $-\cos\theta$ (c) $-\sin\theta$ (d) $\sin\theta$

(16) **The period of $\sin\dfrac{x}{2}$ is:**

(a) 2π (b) 4π (c) π (d) 3π

(17) **For a triangle with a, b, c and α, β, γ as measures of sides and opposite angles respectively, then $b^2 + c^2 - 2bc\cos\alpha = :$**

(a) a^2 (b) b^2 (c) c^2 (d) Δ^2

(18) **With usual notation $r^3 = :$**

(a) (b) (c) (d)

(19) $\cos^{-1}(-x) = :$

(a) $-\cos^{-1}x$ (b) $\cos^{-1}x$ (c) $\pi - \cos^{-1}x$ (d) $\dfrac{\pi}{2} - \cos^{-1}x$

(20) **Solution of $1 + \cos x = 0$ is:**

(a) $\dfrac{\pi}{2}$ (b) π (c) 2π (d) $\dfrac{3\pi}{4}$

FAISALABAD BOARD 2019 (G-I)

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: (16)

(i) Define irrational numbers.

(ii) Name the properties used in these equations: (a) $4 + 9 = 9 + 4$ (b) $1000 \times 1 = 100$

(iii) Prove that $\overline{z} = z$ if z is real.

(iv) Write two proper subsets of $\{a , b , c\}$

(v) Define order of a set.

(vi) Find the inverse of $\{(x , y) \mid y = 2x + 3, x \in R\}$

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(viii) If A and B are square matrices of the same order, then explain why in general $(A + B)(A - B) \neq A^2 - B^2)$

(ix) Define Hermitian matrix.

(x) Prove that $x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$

(xi) If α, β are the roots of $x^2 - px - p - c = 0$, then prove that $(1 + \alpha)(a + \beta) = 1 - c$

(xii) Write two properties of the cube roots of unity.

Q.3 Write answers of any EIGHT questions: (16)

(i) Define conditional equation.

(ii) If $\dfrac{2x+1}{(x-1)(x+2)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{x+3}$ find the value of B.

(iii) Write partial fraction form of $\dfrac{8x^2}{(x^2+1)(1-x^2)}$

(iv) Find the 7^{th} term of $1 , \dfrac{3}{2} , \dfrac{5}{4} , \dfrac{7}{8}$ …………………

(v) Find the number of terms in the A.P if $a_n = 3$, $d = 7$ and $a_n = 59$

(vi) If 5 and 8 are two A.Ms between a and b. Find a and b.

(vii) Find the 9^{th} term of the harmonic sequence $\dfrac{-1}{5} , \dfrac{-1}{3} , -1$,………..

(viii) If the numbers $\dfrac{1}{k} , \dfrac{1}{2k+1}$ and $\dfrac{1}{4k-1}$ are in harmonic sequence, Find k.

(ix) How many arrangements of the letters of the word, taken all together, can be made PAKPATTAN.

(x) Use mathematical induction to prove $1 + 3 + 5 +$ ……….. $+ (2n - 1) = n^2$ is true for $n = 1, n = 2$

(xi) Using binomial theorem find the value of $(1.03)^{\frac{1}{3}}$ upto three decimal places.

(xii) Use binomial theorem to expand $(a - \sqrt{2}\ x)^4$

Q.4 Write answers of any NINE questions: (18)

(i) Define radian.

(ii) Convert $\frac{9\pi}{5}$ to sexagesimal system.

(iii) Prove that $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

(iv) Find the value of tan 15°, without using calculator.

(v) Prove that $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

(vi) Prove that $\cos^2\alpha = \cos^2\alpha - \sin^2\alpha$

(vii) Find the period of cot 8x.

(viii) State the law of sines.

(ix) In the triangle ABC if $\alpha = 35°17'$, $\beta = 45°13'$ and $b = 421$, Find a.

(x) Find the area of the triangle ABC if $a = 200$, $b = 120$, $y = 150°$.

(xi) Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$

(xii) Solve the equation $4\cos^2 x - 3 = 0$ where $x \in [0, 2\pi]$

(xiii) Solve $\text{cosec}\,\theta = 2$, where $\theta \in [0, 2\pi]$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** Prove that the set S = { 1, -1, i, -i } is an abelian group under multiplication. (5)

(b) Obtain the sum of all integers in the first 1000 which are neither divisible by 5 nor by 2. (3)

Q.6 **(a)** Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ (5)

(b) A card is drawn from a deck of 52 playing cards. Find the probability that it is a diamond card or an ace. (3)

Q.7 **(a)** Find the values of a and b if – 2 and 2 are the roots of the polynomial $x^3 - 4x^2 + ax + b$ (5)

(b) If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots\ldots\ldots$, then prove that $y^2 + 2y - 4 = 0$ (3)

Q.8 **(a)** Prove that $\frac{1}{\text{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\theta + \cot\theta}$ (5)

(b) If α, β, y are angles of ΔABC, then prove that $\tan\alpha + \beta + \tan y = \tan\alpha\tan\beta\tan y$ (3)

Q.9 **(a)** Prove that $(r1 + r2)\tan\frac{\gamma}{2} = c$ (5)

(b) Prove that $\sin\text{-}1\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$ (3)

FAISALABAD BOARD 2019 (G-II)

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) **A fair coin is tossed twice then probability of getting one head and one tail is:**

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

(2) **Arithmetic mean between $2+\sqrt{2}$ and $2-\sqrt{2}$ is:**

(a) 0 (b) 2 (c) 4 (d) $2\sqrt{2}$

(3) **If $a_{n-2} = 3n - 11$, then 6th term is:**

(a) 13 (b) 7 (c) 15 (d) 11

(4) **The partial fractions of $\frac{x+5}{(x+1)(x^2+1)}$ will be of the form:**

(a) $\frac{A}{x+1}+\frac{B}{x^2+1}$ (b) $\frac{A}{x+1}+\frac{Bx+C}{x^2+1}$ (c) $\frac{Ax+B}{x+1}+\frac{C}{x^2+1}$ (d) $\frac{A}{x+1}+\frac{Bx}{x^2+1}$

(5) **If ω is cube root of unity then $\omega^{29}+\omega^{28}+1=$:**

(a) 1 (b) 2 (c) 0 (d) -1

(6) **If α, β are the roots of $3x^2 - 2x + 4 = 0$ then $(\alpha+1)(\beta+1) =$:**

(a) $\frac{3}{4}$ (b) $\frac{-4}{3}$ (c) $\frac{4}{3}$ (d) 3

(7) **If** $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ then the value of y will be:

(a) 2 (b) -2 (c) 4 (d) -4

(8) **If A is 4×4 matrix then $|KA| =$:**

(a) $K|A|^4$ (b) $K^2|A|^2$ (c) $K^3|A|$ (d) $K^4|A|$

(9) **If $A \cap B = \varphi$, then n (A - B) is equal to:**

(a) $n(A \cup B)$ (b) $n(A \cap B)$ (c) $n(A)$ (d) $n(B)$

(10) **Multiplicative inverse of (1 , 0) is**

(a) (0 , 1) (b) (1 , 0) (c) (-1 , 0) (d) (0 , -1)

(11) **The solution of $1 + \cos x = 0$ if $0 \le x \le 2\pi$:**

(a) $\{0\}$ (b) $\left\{\frac{\pi}{2}\right\}$ (c) $\left\{\frac{\pi}{3}\right\}$ (d) $\{\pi\}$

(12) $\cos(\tan^{-1}\sqrt{3}) =$:

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

(13) **If the shadow of a tree is equal to its height then the angles of elevation of the sun is:**

(a) 45° (b) 30° (c) 60° (d) 90°

(14) **The period of $3\cos\frac{x}{5}$ is:**

(a) π (b) 10π (c) $\frac{\pi}{10}$ (d) $\frac{\pi}{5}$

(15) $\cos 48° + \cos 12° =$:

(a) $2\cos 18°$ (b) $3\cos 18°$ (c) $\sqrt{3}\cos 18°$ (d) $\sqrt{2}\cos 18°$

(16) $\sqrt{2}\sin 54° + \frac{1}{\sqrt{2}}\csc 45° =$:

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

(17) **The number of terms in the expansion of $(x + y)^9$ is:**

(a) 9 (b) 8 (c) 10 (d) 11

(18) **If $\sin\theta = \frac{1}{2}$ and $\frac{\pi}{2} < \theta < \pi$ then $\cos\theta =$:**

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{-\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(19) **Sum of binomial coefficients in the expansion of $(1 + x)^5$ is:**

(a) 32 (b) 16 (c) 10 (d) 8

(20) ${}^6P_3 =$:

(a) 36 (b) 6 (c) 18 (d) 120

FAISALABAD BOARD 2019 (G-II)

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: **(16)**

(i) Simplify by justifying each step.

(ii) Find the multiplicative inverse of the complex number $(\sqrt{2}, -\sqrt{5})$

(iii) Prove that $\bar{z} = z$ if and only if z is real.

(iv) Write any two proer subsets of the set $\{ x \mid x \in Q \wedge 0 < x \leq 2\}$

(v) Write inverse and contra positive of the conditional $q \rightarrow p$

(vi) Define a semi-group.

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

(viii) If A and B are square matrices of the same order, then explain why in general $(A + B)(A - B) \neq A^2 - B^2$

(ix) Define rank of a matrix.

(x) Solve the equation: $x^3 + x^2 + x + 1 = 0$

(xi) Discuss the nature of the roots of the equation: $2x^2 - 5x + 1 = 0$

(xii) When x4 + 2x3 + kx2 + 3 is divided by x – 2, the remainder is 1. Find the value of K.

Q.3 Write answers of any EIGHT questions: **(16)**

(i) Define an identity equation and give its example.

(ii) Resolve into partial fractions: $\frac{1}{x^2 - 1}$

(iii) Write in mixed form: $\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1}$

(iv) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. show that common difference is $\frac{a - c}{2ac}$

(v) Find the sum of 20 terms of the series, whose rth term is 3r + 1

(vi) If x and y are positive distinct real numbers, show that G.M between x and y is less than A.M.

(vii) If $y = \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \ldots\ldots\ldots\ldots,\ 0 < x < 2$, prove that $x \frac{2y}{1+y}$

(viii) Find the 12^{th} term of harmonic sequence $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \ldots\ldots\ldots\ldots$

(ix) Express in factorial form: $\frac{(n+1)(n)(n-1)}{3.2.1}$

(x) Prove that $n! > 2^n - 1$ is true for n = 5, n = 6

(xi) Using binomial theorem find the value of $(1.03)^{\frac{1}{3}}$ upto three decimal places.

(xii) Use binomial series to find $(1.03)^{\frac{1}{3}}$ upto three places of decimals.

Q.4 Write answers of any NINE questions: (18)

(i) Convert 54°45' into radians.

(ii) Evaluate $\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}}$.

(iii) Prove that $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

(iv) Prove that $\tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right) = 0$

(v) If α, β, γ are angles of a triangle ABC then prove that $\tan(\alpha + \beta) + \tan\gamma = 0$

(vi) Prove that $\dfrac{1 - \cos\alpha}{\sin\alpha} = \tan\dfrac{\alpha}{2}$

(vii) Find the period of tan 4x.

(viii) State the law of cosines (any two).

(ix) At the top of a cliff 80 meters high the angles of depression of a boat is 12°. How far is the boat from the cliff?

(x) Define angles of elevation.

(xi) Show that $\sin(2\cos^{-1}x) = 2x\sqrt{1 - x^2}$

(xii) Find solution of equation $\sec x = -2$ which lie in $[0, 2\pi]$

(xiii) Solve the equation $1 + \cos x = 0$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 (a) If (G , ※) is a group and $a \in G$, then show that inverse of a is unique in G. (5)

(b) If λ, m, n are the pth, qth and rth terms of an A.P. Show that $p(m - n) + q(n - \lambda) + r(\lambda - m) = 0$ (3)

Q.6 (a) Solve the given system of equations by Cramer' rule: (5)

$$\begin{aligned} 2x + 2y + z &= 3 \\ 3x - 2y - 2z &= 1 \\ 5x + y - 3z &= 2 \end{aligned}$$

(b) Two dice are thrown. What is the probability hat the sum of the number of dots appearing on them is 4 or 6? (3)

Q.7 (a) Show that the roots of $x^2 + (mx + c)^2$ will be equal if $c^2 = a^2(1 + m^2)$ (5)

(b) Find the term in the expansion of $\left(\frac{3}{2}x , \frac{1}{3x}\right)^{11}$ involving x^5 (3)

Q.8 (a) If $\tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angles is not in the III quad. Find the value of $\dfrac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta}$ (5)

(b) Without using calculator show that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$ (3)

Q.9 (a) Prove that $\Delta = 4Rr\cos\frac{a}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ (5)

(b) Prove that $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$ (3)

GUJRANWALA BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) $(x-1)^2 = x^2 - 2x + 1$ **is called:**

(a) equation (b) inequality (c) identity (d) polynomial

(2) If ω is complex cube root of unity then ω^{15} equals

(a) 1 (b) zero (c) ω (d) $-\omega$

(3) ${}^{n-1}C_r + {}^{n-1}C_{r-1}$ **equals**

(a) ${}^{n+1}C_r$ (b) ${}^{n+1}C_{r+1}$ (c) ${}^{n}C_r$ (d) ${}^{n-1}C_r$

(4) If $\tan\theta = \frac{1}{\sqrt{3}}$ and θ is in III quadrant then $\cos\theta$ equals

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(5) $\sin(\cos^{-1}\frac{1}{2})$ **equals**

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{-\sqrt{3}}{2}$ (d) $\frac{-1}{2}$

(6) Additive inverse of $a \in R$ is:

(a) 2 (b) 1 (c) $\frac{1}{a}$ (d) $-a$

(7) Period of cosec 10x is

(a) $\frac{\pi}{10}$ (b) $\frac{2\pi}{5}$ (c) $\frac{\pi}{5}$ (d) $\frac{4\pi}{5}$

(8) The middle term in expansion of $(a+x)^n$ when n is even is

(a) $\left(\frac{n}{2}+1\right)$ th term (b) $\left(\frac{n}{2}-1\right)$ th term (c) $\left(\frac{n}{2}\right)$ th term (d) $\left(\frac{n+a}{2}\right)$ th term

(9) With usual notations, radius r of inscribed circle is given by

(a) $\frac{\Delta}{s}$ (b) $\frac{s}{\Delta}$ (c) $\frac{\Delta}{s-c}$ (d) $\frac{4\Delta}{abc}$

(10) cos 315° equals

(a) $\tan(-45°)$ (b) $\tan 45°$ (c) $\sin 45°$ (d) cosec 45°

(11) A reciprocal equation remains unchanged when variable x is replaced by

(a) $-\frac{1}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) $-x$

(12) For any two matrices A and B then $(AB)^t$ equals

(a) AB (b) A^tB^t (c) B^tA^t (d) BA

(13) The solutions of equation $1 + \sin\theta = 0$ are in quadrant

(a) I and IV (b) I and III (c) II and IV (d) III and IV

(14) With usual notations, the value of $a + b + c$ is

(a) s (b) 2s (c) 3s (d) $\frac{s}{2}$

(15) The arithmetic mean between $\frac{1}{2}$ and $\frac{1}{4}$ is

(a) $\frac{3}{8}$ (b) $\frac{3}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

(16) If A and B are disjoint then $P(A \cup B)$ equals

(a) $P(A) - P(B)$ (b) $P(A)\,P(B)$ (c) $\frac{P(A)}{P(B)}$ (d) $P(A) + P(B)$

(17) Expansion of (1 + x)

(a) $|x| > 1$ (b) $|x| < 1$ (c) $|x| < -$ (d) $|x| > -1$

(18) The 8th term of sequence 1, –3, 5, –7 is:

(a) 15 (b) – 15 (c) 14 (d) –14

(19) If $\begin{bmatrix} \gamma & 3 \\ 4 & 2 \end{bmatrix}$ is singular then λ is equal to:

(a) 2 (b) 6 (c) 4 (d) 8

(20) The domain of relation $f = \{(a, 1), (b, 1), (c, 1)\}$ is

(a) $\{a, b, c\}$ (b) $\{a\}$ (c) $\{b\}$ (d) $\{1\}$

GUJRANWALA BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: **(16)**

(i) Separate into real and imaginary parts $\frac{i}{1+i}$

(ii) Simplify $(i)^{101}$

(iii) Show that $\forall z \in c$, $(\bar{z})^2 + z^2$ is a real number.

(iv) For the conditional $p \rightarrow q$. Write its inverse and converse.

(v) Define disjunction of two statements p and q.

(vi) If a , b are elements of a group G, then show that $(ab)^{-1} = b^{-1}a^{-1}$

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(viii) Find the value of λ if $A = \begin{bmatrix} 4 & \lambda \\ 7 & 3 \end{bmatrix}$ is singular.

(ix) Define upper triangular matrix.

(x) Reduce $x^{-2} - 10 = 3x^{-1}$ into quadratic form.

(xi) Show that $(x^3 - y^3) = (x - y)(x - \omega y)$, where ω is a cube root of unity.

(xii) Show that roots of $(p + 1)x^2 - px - q = 0$ are rational.

Q.3 Write answers of any EIGHT questions: **(16)**

(i) Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions.

(ii) Define proper rational fraction.

(iii) For the identity $\frac{2x-3}{x(2x+3)(x-1)} = \frac{A}{x} + \frac{B}{2x+3} + \frac{C}{x-1}$ calculate the value of A and C.

(iv) Write the first four terms of the sequence $a_n = \frac{n}{2n+1}$

(v) How many terms are there in A.P., in which $a_1 = 11$, $a_n = 68$, $d = 3$

(vi) Sum the series $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \ldots\ldots\ldots\ldots$ to n terms.

(vii) Find the 12^{th} term of the G.P $1 + i$, $2i$, $2(1 - i)$, ...

(viii) Find the sum of the following infinite geometric series $4 + 2\sqrt{2} + 2\sqrt{2} + 1 + \ldots\ldots\ldots$

(ix) How many arrangements of the letters of the word 'MATHMATICS', taken all together, can be made?

(x) Prove the formula for a = 1, 2 $\quad 1 + 2 + 4 + \ldots\ldots\ldots + 2^{n-1} = 2^n - 1$

(xi) Calculate (2.02)4 y means of binomial theorem.

(xii) Expand $(1 + x)^{\frac{-1}{3}}$ upto 4-terms, taking the values of x such that the expansion is valid.

Q.4 Write answers of any NINE questions: (18)

(i) What is the length of the arc intercepted on a circle of radius 14 cm by the arms of a central angle of 45°?

(ii) Evaluate: $\dfrac{1 - \tan^2 \frac{\pi}{3}}{1 + \tan^2 \frac{\pi}{3}}$

(iii) Prove that: $\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}$

(iv) Prove that: $\tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right) = 0$

(v) Prove that: $\tan 2a = \dfrac{2 \tan \alpha}{1 - \tan^2 \alpha}$

(vi) Find the value of $\cos 2\alpha$ when $\sin \alpha = \frac{12}{13}$ where $0 < \alpha < \frac{\pi}{2}$

(vii) Find the period of $\tan \frac{x}{3}$

(viii) State law of cosines.

(ix) Find the area of the triangle ABC, given three sides $a = 524$, $b = 276$, $c = 315$

(x) Show that: $r_1 = s \tan\frac{a}{2}$

(xi) Prove that: $\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$

(xii) Find the solution of equation: $\sin x = \frac{-\sqrt{3}}{2}$

(xiii) Solve the equation: $\sin^2 x + \cos x = 1$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** Prove that all 2 x 2 non-singular matrices over the real field form a non-abelian group under multiplication. (5)

(b) For what value of n, $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the positive geometric means between a and b? (3)

Q.6 **(a)** Use Cramer's rule to sole the system: (5)

$$\begin{aligned} 3x_1 + x_2 - x_3 &= 4 \\ x_1 + x_2 - 2x_3 &= -4 \\ -x_1 + 2x_2 - x_3 &= 1 \end{aligned}$$

(b) The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 5 girls be formed? (3)

Q.7 **(a)** Solve $4.2^{2x+1} - 9.2^x + 1 = 0$ (5)

(b) Find the term involving a4 in the expansion of $\left(\frac{2}{x} - a\right)^9$ (3)

Q.8 **(a)** Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ (5)

(b) Reduce $\sin^4\theta$ to an expression involving function of multiple of θ raised to the first power.(3)

Q.9 **(a)** The sides of a triangle are $x^2 + x + 1$, $2x + 1$, $2x + 1$, $x^2 - 1$. Prove that the greatest angle of the triangle is 120°. (5)

(b) Prove that: $\tan\text{-}1 \frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$ (3)

BAHWALPUR BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) The matrix [a b c d] is :

(a) Square (b) Unit (c) Null (d) Row

(2) if A = {a , {a , b }}, then number of elements in P (A) is:

(a) 2 (b) 3 (c) 4 (d) 8

(3) The property used in $(a+1)+\frac{3}{4}=a+(1+\frac{3}{4})$ is:

(a) Closure (b) Associative (c) Commutative (d) Additive

(4) If Order of X = 3×2 and that of A = 2×2 then order of XA =

(a) 3×2 (b) 2×3 (c) 2×2 (d) 3×3

(5) In $\frac{P(x)}{Q(x)}$, if degree of P (x) ≥ degree of Q (x), then fraction is :

(a) Proper (b) Improper (c) Irrational (d) Identity

(6) When x^3-2x^2+3x+3 is divided by $x-3$, the remainder is:

(a) -21 (b) 21 (c) -51 (d) 51

(7) An equation which remains unchanged when x is replaced by $\frac{1}{x}$ is :

(a) Exponential (b) Radical (c) Reducible (d) Reciprocal

(8) If $a_n=\frac{(-1)^{n+1}}{2^n}$, then a_5 = :

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{64}$

(9) A die is thrown, what is the probability to get 3 dots :

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

(10) $\frac{8!}{7!}$ =

(a) 7! (b) 7 (c) 8 (d) 8!

(11) If H is H.M. between "a" and "b" then H = :

(a) $\frac{2ab}{a+b}$ (b) $\frac{a+b}{2ab}$ (c) $\frac{a+b}{2}$ (d) $\pm\sqrt{ab}$

(12) The Statement $4^k > 3^k + 4$ is true for:

(a) $K > 2$ (b) $k \leq 2$ (c) $K \neq 2$ (d) $K \leq 2$

(13) Cos $(\theta - 180^o)$ = :

(a) $\sin\theta$ (b) $-\cos\theta$ (c) $\cos\theta$ (d) $-\sin\theta$

(14) $\frac{9\pi}{5}$ rad in degree measure is :

(a) 321^o (b) 322^o (c) 323^o (d) 324^o

(15) Total number of terms in expansion of $\left(\frac{x}{2}-\frac{2}{x^2}\right)^{16}$ are:

(a) 17 (b) 16 (c) 15 (d) 14

(16) Period of csc θ is :

(a) π (b) $-\pi$ (c) 2π (d) -2π

(17) Sin $(\tan^{-1}0^o)$ = :

(a) -1 (b) 1 (c) 0 (d) ∞

(18) Radius of e – circle opposite to vertex "A" of ΔABC is :

(a) $\frac{\Delta}{s}$ (b) $\frac{\Delta}{s-a}$ (c) $\frac{\Delta}{s-b}$ (d) $\frac{\Delta}{s-c}$

(19) The angle above the Horizontal Line is called and angle of :

(a) Depression (b) Elevation (c) Allied (d) Quadrental

(20) The reference angle for $\tan\theta = \sqrt{3}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{-\pi}{3}$

BAHAWALPUR BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: (16)

(i) If Z_1 and Z_2 are complex numbers then show that $\overline{Z_1 Z_2} = \overline{Z_1}\ \overline{Z_2}$

(ii) If $A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$ then solve the equation $3x - 2A = B$ for X.

(iii) Separate into Real and imaginary Parts $\dfrac{2-7i}{4+5i}$

(iv) If A and B are Overlapping Sets then draw the Venn Diagram of A – B.

(v) Find the Multiplicative inverse of $-3-5i$

(vi) Find Four 4^{th} Roots of 81.

(vii) Define intersection of two sets and give an example.

(viii) Without expansion show that: $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$

(ix) Define Identity Matrix and give an example.

(x) Show that the roots of $px^2 - (p-q)x - q = 0$ are rational.

(xi) If α, β are the roots of $x^2 - px - p - c = 0$ then prove that $(1+\alpha)(1+\beta) = 1 - c$

(xii) Define Monoid.

Q.3 Write answers of any EIGHT questions: (16)

(i) For the Identity $\dfrac{1}{(x-1)(2x-1)(3x-1)} = \dfrac{A}{x-1} + \dfrac{B}{2x-1} + \dfrac{C}{3x-1}$ calculate the value of A

(ii) Find the indicated term of the sequence : 2, 6, 11, 17, ……….. a_7

(iii) Write the first four terms of the A.P. If $a1 = 5$ and other three consecutive terms are 23, 26, 29

(iv) Find the 12^{th} term of the Geometric Sequence: $1 + i$, $2i - 2 + 2i$, ……….

(v) The A.M. between two numbers a and b is 5 and their positive G.M. is 4, find the values of a and b.

(vi) If 5 is the Harmonic Mean between 2 and b, find b.

(vii) How many words an be formed from the letters of the word "OBJECT" using all letters without repeating any letter.

(viii) Prove that $\dfrac{8 \times 10^{n-2}}{6}$ is an integer for n = 1 and n = 2.

(ix) Find 6^{th} term in the expansion of $\left(x^2 - \dfrac{3}{2x}\right)^{10}$

(x) Expand $\sqrt{99}$ by using Binomial Expansion to find its value upto three places of decimals.

(xi) Define Improper Rational Fractions.

(xii) Resolve $\dfrac{1}{x^2-1}$

Q.4 Write answers of any NINE questions: (18)

(i) Define Degree Measure.

(ii) Solve $\text{Sinx} = \frac{1}{2}$

(iii) Find the solutions in $[0, 2\pi]$ $\text{Cot}\theta = \frac{1}{\sqrt{3}}$

(iv) Prove $\frac{\sin 8x + \sin 2x}{\text{Cos } 8x + \text{Cos } 2x} = \text{Tan } 5x$

(v) Prove that $\text{Cos}(\sin^{-1} x) = \sqrt{1 - x^2}$

(vi) Find the period of $\text{Cot}\,\frac{x}{2}$

(vii) If $\text{Sin}\theta = -\frac{1}{2}$, terminal arm of θ is not in III Quadrant, find $\text{Tan}\,\theta$.

(viii) The area of a ΔABC is 2437. If $a = 79$ and $c = 97$, find the angle β.

(ix) Prove that $\Delta = \sqrt{s(s - a)(s - b)(s - c)}$

(x) Prove that $(\text{Sec}\theta - \text{Tan}\theta)^2 = \frac{1 - \text{Sin}\theta}{1 + \text{Sin}\theta}$

(xi) Prove $\text{Sin}(\alpha + \beta).\ \text{Sin}(\alpha - \beta) = \text{Sin}^2\alpha - \text{Sin}^2\beta$

(xii) If $\beta = 52°$, $y = 89°35'$, $a = 89.35$ find the side b of a ΔABC

(xiii) Prove $\sqrt{\frac{1 + \sin\propto}{1 - \propto}} = \frac{\text{Sin}^{\frac{\propto}{2}} + \text{Cos}^{\frac{\propto}{2}}}{\text{Sin}^{\frac{\propto}{2}} - \text{Cos}^{\frac{\propto}{2}}}$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** Convert (AUB) UC = AU(BUC) to logical form and prove by constructing truth table. (5)

(b) Sum to n terms, the series : 3 + 33 + 333 + ………… (3)

Q.6 **(a)** Solve the equation if possible by Cramer's Rule. (5)

$$2x_1 - x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 2x_3 = 6$$
$$-x_1 + 2x_2 - x_3 = 1$$

(b) Find the Probability that sum of dots appearing in two successive throws of two dice is every time 7. (3)

Q.7 **(a)** Find the values of "a" and "b" if "-2" and "2" are the roots of polynomial $x^3 - 4x^2 + ax + b$.(5)

(b) Find the Coefficient of term involving x-1 in the expansion of $(\frac{3}{2}x - \frac{1}{3x})^{11}$ (3)

Q.8 **(a)** Show that the area of a sector of a circular region of radius "r" is $\frac{1}{2}r^2\theta$, where θ is the circular measure of the central angle of the sector. (5)

(b) Prove that $\frac{\text{Cos}8° - \text{Sin}8°}{\text{Cos}8° + \text{Sin}8°} = \tan 37°$ (3)

Q.9 **(a)** Show that $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$ (5)

(b) Prove that $\text{Sin}^{-1}\frac{5}{13} + \text{Sin}^{-1}\frac{7}{25} = \text{Cos}^{-1}\frac{253}{325}$ (3)

D.G. KHAN BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) The multip0licative identity of real numbers is:

(a) 0 (b) 1 (c) 2 (d) 3

(2) The tabular form of the set $\{x \mid x \epsilon Q \Delta x^2=2\}$ is:

(a) $(\sqrt{2}, -\sqrt{2})$ (b) {4} (c) { } (d) {4 , -4}

(3) The additive inverse of a matrix A is:

(a) A (b) -A (c) A^2 (d) $\frac{adj(A)}{|A|}$

(4) If $A = [a_{ij}]_{mxn}$, then cofactor of a_{ij} is:

(a) $(-1)^{ij}M_{ij}$ (b) $(-1)^{i+j}M_{ij}$ (c) $(-1)^{i-j}M_{ij}$ (d) $(1)^{i+j}M_{ij}$

(5) The polynomial $3x^2 + 2x+1$ has degree:

(a) 0 (b) 3 (c) 2 (d) 4

(6) If w is cube root of unity , then $w^3 =$

(a) 1 (b) 0 (c) w^2 (d) 2w

(7) Partial fractions of $\frac{x}{(x-1)(x-2)}$ will be of the form:

(a) $\frac{A}{x-1}+\frac{B}{x+2}$ (b) $\frac{1}{x-1}$ (c) $\frac{1}{x+2}$ (d) $1+\frac{A}{x-1}+\frac{B}{x+2}$

(8) The next term of the sequence 7, 9, 12, ……….. is:

(a) 16 (b) 15 (c) 14 (d) 18

(9) Reciprocal of A.P. is called :

(a) A.P (b) G.P. (c) H.P. (d) H.M

(10) Factorial form of n (n-1) (n-2) is:

(a) $\frac{n!}{(n-1)!}$ (b) $\frac{n!}{(n-2)!}$ (c) $\frac{n!}{(n-3)!}$ (d) $\frac{n!}{(n+3)!}$

(11) If n(S) = 20, n(B) =2, then P(B) equals:

(a) 10 (b) $\frac{1}{10}$ (c) $-\frac{1}{10}$ (d) 1

(12) If n is any positive integer then $2^n > 2\ (n+1)$ us true for all:

(a) $n \leq 3$ (b) $n < 3$ (c) $n \geq 3$ (d) $n > 3$

(13) Number of terms in the expansion of $(1+x)^{2n+1}$ is:

(a) $2n + 1$ (b) $2n$ (c) $2n + 2$ (d) $3n + 1$

(14) The 60^{th} part of 1-degree is called:

(a) second (b) minute (c) degree (d) Radian

(15) $\sin(-\propto) =$:

(a) $\sec \propto$ (b) $-\sin \propto$ (c) $\sin \propto$ (d) $-\cos \propto$

(16) The range of $y = \cos x$ is:

(a) $-1 \leq x \leq 1$ (b) $-\infty < x < \infty$ (c) $-1 \leq y \leq 1$ (d) $-\infty < y < \infty$

(17) Angle below the horizontal ray is called:

(a) Right angle (b) Oblique angle

(c) Angle of depression (d) Angle of elevation

(18) With usual notation, $Y_1 =$

(a) $\frac{\Delta}{s-b}$ (b) $\frac{\Delta}{s-a}$ (c) $\frac{\Delta}{s-c}$ (d) $\frac{s-a}{\Delta}$

(19) $\tan^{-1}(1) =$

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) π

(20) If $\sin x = ½$, then $x =$

(a) $\pi/6, 5\pi/6$ (b) $-\pi/6, 5\pi/6$ (c) $-\pi/6, -5\pi/6$ (d) $\pi/3, 2\pi/3$

D.G. KHAN BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: (16)

(i) Name the property $-3 < -2 \Rightarrow 0 < 1$.

(ii) Simplify $(-i)^{19}$.

(iii) Express the complex number $1 + i\sqrt{3}$ in polar form.

(iv) Define a group.

(v) Differentiate between equal and equivalent sets.

(vi) Define a function. Also give one example of a function.

(vii) Show that $B = \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$ is skew symmetric.

(viii) If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.

(ix) What is the rank of a matrix?

(x) What are the extraneous roots of an equation?

(xi) If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + px^2 + qx + 2$, find the values of P and q.

(xii) Discuss the nature of the roots of equation $x^2 + 2x + 3 = 0$.

Q.3 Write answers of any EIGHT questions: (16)

(i) Define conditional equation.

(ii) If $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$ find value of B.

(iii) Write partial fraction form of $\frac{4x^2+8x}{x^4+2x^2+9}$.

(iv) Find the 8^{th} term of $1, -3, 5, -7, 9, -11, \ldots, a_8$.

(v) If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P, show that the common difference is $\frac{a-c}{2ac}$.

(vi) Which term of the sequence $x^2 - y^2, x + y, \frac{x+y}{x-y}, \ldots$ is $\frac{x+y}{(x-y)^9}$?

(vii) If a^2, b^2 and c^2 are in A.P, show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

(viii) Sum the series $2 + (1 - i) + \frac{1}{i} + \ldots +$ to 8 terms.

(ix) Find the value of n when ${}^nC_{10} = \frac{12 \times 11}{21}$.

(x) Expand $(x + \sqrt{x^2 - 1})^3$.

(xi) Find the 6^{th} term in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$.

(xii) Using binomial theorem find the value of $\sqrt[5]{31}$.

Q.4 Write answers of any NINE questions: (18)

(i) Convert the $35^\circ\, 20'$ to radians.

(ii) Find the value of $\sin\theta$ if $\cos\theta = \frac{9}{41}$ and terminal arm of the angle is in quadrant IV.

(iii) Prove $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$.

(iv) Find the value of $\sin 75^\circ$ without using table / calculator.

(v) Prove that $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$.

(vi) Show that $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$.

(vii) What is period of a function?

(viii) In the right angled triangle ABC if $\gamma = 90^\circ$, $\alpha = 58^\circ\, 13'$ $b = 125.7$. Find a.

(ix) Find area of the triangle ABC, if $a = 18$, $b = 24$, $c = 30$.

(x) Define in-circle of a triangle.

(xi) Find the value of $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$.

(xii) Solve $\sin x + \cos x = 0$ in $[0, \pi]$.

(xiii) Solve $\tan^2\theta = \frac{1}{3}$, $\theta \in [0, \pi]$.

SECTION-II

***Note:* Attempt any THREE questions:** **$10 \times 3 = 30$**

Q.5 **(a)** State and prove the reversal law of inverse. (3)

(b) Find "n" so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the H.M between "a" and "b". (3)

Q.6 **(a)** Solve the system of linear equations (3)

$$x + y = 2$$
$$2x - z = 1$$
$$2y - 3z = -1$$

(b) In how many ways 8 books including 2 on English be arranged on the shelf in such a way that the English books are never together. (3)

Q.7 **(a)** If α, β are the roots of the equation $ax^2 + bx + c = 0$, form the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$. (3)

(b) Identify the following series as binomial expansion and find its sum

$1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \ldots$ (3)

Q.8 **(a)** Prove that: $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1}{1 - 2\sin^2\theta}$. (3)

(b) Prove that: $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$. (3)

Q.9 **(a)** Prove that with usual notations $(\gamma 3 - \gamma)\cot\frac{\gamma}{2} = C$. (3)

(b) Prove that: $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$. (3)

LAHORE BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) $\cos\left(\frac{3\pi}{2}-0\right)$ **is equal to:**

(a) $-\sin\phi$ (b) $\sin\phi$ (c) $\cos\phi$ (d) $-\cos\phi$

(2) Probability of impossible event is:

(a) $\frac{1}{2}$ (b) 1 (c) 0 (d) 2

(3) $2\tan^{-1}A$ **equals:**

(a) $\tan^{-1}\left(\frac{A}{1-A^2}\right)$ (b) $\tan^{-1}\left(\frac{2A}{1-A^2}\right)$ (c) $\tan^{-1}\left(\frac{2A}{1+A^2}\right)$ (d) $\tan^{-1}\left(\frac{A}{1+A^2}\right)$

(4) Which angle is quadrantal angle:

(a) 45° (b) 60° (c) 270° (d) 120°

(5) Solution of equation $\tan x = \frac{1}{\sqrt{2}}$ **lies in the quadrants:**

(a) I and II (b) II and III (c) I and III (d) I and IV

(6) Middle terms in the expansion of $(x+y)^{11}$ **are:**

(a) T_6, T_7 (b) T_5, T_6 (c) T_7, T_8 (d) T_8, T_9

(7) If Δ **is the area of a triangle ABC, then with usual notation** $\Delta =$**:**

(a) $\frac{1}{2}bc\sin B$ (b) $\frac{1}{2}an\sin a$ (c) $\frac{1}{3}bc\sin a$ (d) $\frac{1}{2}bc\sin a$

(8) Range of cotangent function is:

(a) N (b) Z (c) R (d) C

(9) Exp of $(3-5x)^{\frac{1}{2}}$ **is valid if:**

(a) $|x|<\frac{3}{5}$ (b) $|x|<\frac{5}{3}$ (c) $|x|<5$ (d) $|x|<3$

(10) With usual notation R =:

(a) $\frac{b}{2\sin y}$ (b) $\frac{a}{2\sin a}$ (c) $\frac{c}{2\sin a}$ (d) $\frac{a}{2\sin B}$

(11) The sun of the four fourth roots of 81 is:

(a) 0 (b) 81 (c) -81 (d) 3

(12) The property $\forall$ a,b $\in$ 1R, a = b $\Rightarrow$ b = a is called:

(a) Commutative (b) Transitive (c) Symmetric (d) Reflexive

(13) The value of 4! . 0!. 1! is:

(a) 0 (b) 1 (c) 4 (d) 24

(14) A square matrix A= $[a_{ij}]$ in which $a_{ij} = 0$ for $i > j$ is called:

(a) Upper triangular (b) Lower triangular

(c) Symmetric (d) Skew – symmetric

(15) $\sum_{k=1}^{11} (1)^k =$:

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n}{2}$ (c) n (d) $\frac{n(n+1)}{2}$

(16) If $b^2 - 4ac > 0$ but not a perfect square. then roots are:

(a) Equal (b) Complex (c) Rational (d) Irrational

(17) No term of geometric sequence can be:

(a) 0 (b) 1 (c) 2 (d) 3

(18) If A and B are two sets, then A – B =:

(a) $A \cup B^c$ (b) $A \cap B^c$ (c) $(A \cup B)^c$ (d) $(A \cap B)^c$

(19) Partial fractions of $\frac{1}{x^3-1}$ will be of the form:

(a) $\frac{A}{x+1}+\frac{Bx+C}{x^2+x+1}$ (b) $\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$ (c) $\frac{A}{x-1}+\frac{Bx+C}{x^2-x+1}$ (d) $\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$

(20) If A =$[a_{ij}]_{2\times2}$, then $|kA|$ =:

(a) $|A|$ (b) $k^2|A|$ (c) $k|A|$ (d) $k|A|^2$

LAHORE BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: **(16)**

(i) Prove the rule of addition $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

(ii) Find the multiplication inverse of $(\sqrt{2}, -\sqrt{5})$

(iii) Express the complex number $1+i\sqrt{3}$ in polar form.

(iv) Write the power set of $\{a, \{b, c\}\}$

(v) Show that the statement $p \to (p \vee q)$ is tautology.

(vi) Prove that the identity element e in a group G is unique.

(vii) If $A=\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b.

(viii) If $B=\begin{bmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & -2 \end{bmatrix}$, find cofactor B_{21}.

(ix) If A is a skew-symmetric matrix, then show that A^2 is a symmetric matrix.

(x) Solve $x^{-2}-10=3x^{-1}$.

(xi) If α, β are the roots of $x^2-px-p-c=0$ then prove that $(1+\alpha)(1+\beta)=a-c$

(xii) Discuss the nature of roots of the equation $x^2-5x+6=0$

Q.3 Write answers of any EIGHT questions: **(16)**

(i) Define proper fraction.

(ii) If $\frac{x^2-10x+13}{(x-1)(x^2-5x+6)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$, find value of A.

(iii) If $\frac{x}{(x-a)(x-b)(x-c)}=\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$, find value of B.

(iv) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k.

(v) Find sum of infinite geometric series $2+1+0.5+$ ----

(vi) Define geometric mean.

(vii) If 5, 8 are two A.Ms between a and b, find a and b

(viii) If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P, show that $b=\frac{2ac}{a+c}$

(ix) Prove that ${}^nC_r={}^nC_{n-r}$

(x) Expand $(1+x)^{\frac{-1}{3}}$ upto 3 terms.

(xi) Evaluate $\sqrt[3]{30}$ correct to three places of decimal.

(xii) Check whether the statement 5^n-2^n is divisible by 3 for $n=2, 3$ is true or false.

Q.4 Write answers of any NINE questions: **(18)**

(i) Find r, when $\lambda = 56$ cm, $\theta = 45°$

(ii) Find the values of all trigonometric functions for -15π

(iii) Prove that $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

(iv) Express the difference $\cos 7\theta - \cos\theta$ as product.

(v) Prove $\dfrac{1-\cos\alpha}{\sin\alpha} = \tan\dfrac{\alpha}{2}$

(vi) Find the value of $\cos 105°$ without using calculator.

(vii) Find the period of $3\sin\dfrac{2x}{5}$

(viii) With usual notations prove that $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}$

(ix) Define in-circle of the triangle ABC.

(x) State the law of tangent. (any two)

(xi) Show that $\cos(2\sin^{-1}x) = 1 - 2x^2$

(xii) Solve the equation for $\theta \in [0, \pi]$ $\cot^2\theta = \dfrac{4}{3}$

(xiii) Solve the equation for $\theta \in [0, \pi]$ $2\sin\theta + \cos^2 - 1 = 0$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** If G is a group under the operation "$*$" and a, b $\in$ G, find the solutions of the equations : (i) $a * x = b$ (ii) $x * a = b$ (5)

(b) If 7^{th} and 10^{th} terms of an H.P are $\dfrac{1}{3}$ and $\dfrac{5}{21}$ respectively, find its 14^{th} term (3)

Q.6 **(a)** Show that $\begin{vmatrix} a+\lambda & a & a \\ a & a+\lambda & a \\ a & a & a+\lambda \end{vmatrix} = \lambda^2(3a+1)$ (5)

(b) Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$ (3)

Q.7 **(a)** If α, β are the roots of $5x^2 - x - 2 = 0$ form the equation whose roots are $\dfrac{3}{\alpha}$ and $\dfrac{3}{\beta}$ (5)

(b) Use mathematical induction to prove that $n! > n^2$ for integral values of $n \geq 4$. (3)

Q.8 **(a)** A railway train is running on a circular track of radius of radius 500 meters at the rate of 30 km per hour. Through what angle will it turn in 10 sec? (5)

(b) Reduce $\sin 4\,\theta$ to an expression involving only function of multiples of θ raised to the first power. (3)

Q.9 **(a)** Prove that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$ (5)

(b) Prove that $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\dfrac{A+B}{1-AB}$ (3)

MULTAN BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) **Range of function y = cos x is:**

(a) $-\infty < x > \infty$ (b) $-\infty < y < \infty$ (c) $-1 \le y \le 1$ (d) $-1 \le x \le 1$

(2) **In a ΔABC with usual notation $\sqrt{\dfrac{s(s-a)}{bc}}$ =:**

(a) $\sin\frac{a}{2}$ (b) $\cos\frac{a}{2}$ (c) $\cos\frac{\beta}{2}$ (d) $\sin\frac{\beta}{2}$

(3) **Area of ΔABC in terms of measure of its all sides is:**

(a) $\frac{1}{2}bc\sin a$ (b) $\dfrac{c^2 \sin a \sin \beta}{2 \sin y}$ (c) $\frac{1}{2}ca\sin\beta$ (d) $\sqrt{s(s-a)(s-b)(s-c)}$

(4) **Tan (Tan^{-1}(-1) =:**

(a) -1 (b) 1 (c) 2 (d) -2

(5) **Solution set of sin x $=\frac{1}{2}$ is:**

(a) $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ (b) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ (c) $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$ (d) $\{0, \pi$

(6) **If $i = \sqrt{--1}$, then i^{14} =:**

(a) 1 (b) -1 (c) i (d) -i

(7) **The symbol used to denote a biconditional between two propositions is:**

(a) $\rightarrow$ (b) $\wedge$ (c) $\leftrightarrow$ (d) $\vee$

(8) **For a non singular matrix *A*, if *AX* = *B*, then *X* =**

(a) A^{-1B} (b) BA^{-1} (c) $(AB)^{-1}$ (d) $(BA)^{-1}$

(9) **If A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 4 & 5 & 2 \end{bmatrix}$, then M_{13} =:**

(a) 13 (b) 0 (c) 10 (d) 7

(10) **The number of roots of polynomial $8x^6 - 19x^3 - 27 = 0$ are:**

(a) 2 (b) 4 (c) 6 (d) 8

(11) **If s = sum of roots and p = product of roots, then quadratic equation can be written as:**

(a) $x^2 + sx + p = 0$ (b) $s^2 - sx - p = 0$ (c) $x^2 - sx + p = 0$ (d) $sx^2 - sx + p = 0$

(12) $\dfrac{2x^2}{(x - 3)(x + 2)}$ **is a fraction:**

(a) Proper (b) Improper (c) Identity (d) Irrational

(13) **If $a_n = (-1)^{n+1}$, then a_{26} =:**

(a) 1 (b) -1 (c) i (d) -i

(14) **Geometric Mean between 4*i* and – 16*i* is:**

(a) 8 (b) - 8 (c) ± 8 (d) ± 64

(15) **The factorial form of n(n – 1) (n – 2) - - - (n – r + 1) is:**

(a) $\dfrac{n!}{(n - r)!}$ (b) $(n - 1)!$ (c) $n!$ (d) $\dfrac{n!}{(n - r + 1)!}$

(16) **When *A* and *B* are two disjoint events then *P* (*A* ∪ *B*) =:**

(a) P (A) – P (B) (b) P (A) + P (B) – P (A ∩ B)

(c) P (A) – P (A ∩ B) (d) P (A) + P (B)

(17) **The statement 4n” > 3” + 4 is true if:**

(a) $n < 2$ (b) $n \neq 2$ (c) $n \geq 2$ (d) $n \leq 2$

(18) **In the expansion of (3 – 2x)8 , 5th term will be its:**

(a) Last term (b) 2nd last term (c) 3rd last term (d) Middle term

(19) **The measure of angle between hands of a watch at 3 0'clock is:**

(a) 30° (b) 60° (c) 90° (d) 120°°

(20) **The angle $\frac{3\pi}{2}$ - is θ lies in quadrant:**

(a) I (b) II (c) III (d) *IV*

MULTAN BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: **(16)**

(i) Express $(2+\sqrt{-3})\ (3+\sqrt{-3})$ in the form of a + bi and simplify.

(ii) Find the multiplicative inverse of $(-4, 7)$

(iii) Factorize $9a^2 + 16b^2$

(iv) Define union of two sets and give an example.

(v) If A and B are any two sets then prove $(A \cup B)' = A' \cap B'$

(vi) Define tautology and absurdity.

(vii) If A and B are non singular matrices then prove $(AB)^{-1} = B^{-1}A^{-1}$

(viii) Find the inverse of matrix $A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

(ix) If $A = \begin{bmatrix} 0 & 2-3i \\ -2-3i & 0 \end{bmatrix}$ then show that A is skew-hermitian.

(x) Solve the equation $x^{1/2} - x^{1/4} - 6 = 0$

(xi) Using factor theorem show that $(x-1)$ is a factor of $x^2 + 4x - 5$

(xii) The sum of positive number and its reciprocal is $\frac{26}{5}$. Find the number.

Q.3 Write answers of any EIGHT questions: **(16)**

(i) Define “Proper Rational Fraction”.

(ii) Resolve $\frac{x^2+1}{(x+1)(x-1)}$ into Partial Fractions.

(iii) For the identity $\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$ Calculate the value of B.

(iv) Find the next two terms of the sequence: 1, 3, 7, 15, 31, ----

(v) If the nth of the A.P is $3n-1$, find its first three terms.

(vi) Find the 11^{th} term of the geometric sequence: $1+i, 2, \frac{4}{1+i}$, ----

(vii) Insert two G. Ms. between 1 and 8.

(viii) Find the 12^{th} term of the harmonic sequence: $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}$, -----

(ix) Find the value of n when “$P_4 : {}^{n-1}P_3 = 9 : 1$

(x) Prove the formula for n = 1 and n = 2: $1` + 4 + 7 + \text{-----} + (3n-2) = \frac{n(3n-1)}{2}$

(xi) Calculate $(0.97)^3$ by using binomial theorem.

(xii) Expand upto 4 terms: $(2-3x)^{-2}$ taking the values of x such that expansion is valid.

Q.4 Write answers of any NINE questions: **(18)**

(i) Find θ, if $\lambda = 1.5\text{cm}$, $r = 2.5$ cm

(ii) Prove $2\sin 45^\circ + \frac{1}{2}\ \text{cosec}45^\circ = \frac{3}{\sqrt{2}}$

(iii) Prove $(\tan\theta + \cot\theta)^2 = \sec^2\theta \text{cosec}^2\theta$

(iv) Prove $\frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

(v) Prove $\dfrac{\tan\frac{\theta}{2} + \cos\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \sec\theta$

(vi) Prove $\sin\left(\frac{\pi}{4} - \theta\right)\ \sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\ \cos 2\theta$

(vii) Find the period of $\cos 2x$.

(viii) Find the area of a ΔABC, if $b = 37$, $c = 45$, $\alpha = 30^\circ 50'$

(ix) Prove $R = \frac{abc}{4\Delta}$

(x) Prove $r\ r_1 r_2 r_3 = \Delta^2$

(xi) Prove $\cos(\text{Sin}^{-1} x) = \sqrt{1 - x^2}$

(xii) Find the solution of $\sec x = -2$ which lie in $[0, 2\pi]$

(xiii) Find the values of θ satisfying the equation $2\sin\theta + \cos^2\theta - 1 = 0$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** Show that the set $\{1, w, w^2\}$ when $w^3 = 1$ is abelian group w.r.t. ordinary multiplication. (5)

(b) Find n so that $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be A.M between a and b. (3)

Q.6 **(a)** Find the inverse of the matrix $A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \end{bmatrix}$ by using column operation. (5)

(b) A die is thrown twice. What is the probability that the sum of dots shown is 3 or 11. (3)

Q.7 **(a)** Find the condition that $\frac{a}{x-a} + \frac{b}{x-b} = 5$ may have roots equal in magnitude but opposite in signs. (5)

(b) Use binomial theorem to prove that $1 + \frac{1}{4} + \frac{1.3}{4.8} + \frac{1.3.5}{4.8.12} + \text{-------} = \sqrt{2}$ (3)

Q.8 **(a)** If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quadrant, then find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$ (5)

(b) Find the value of $\sin 18^\circ$ without using table or calculator. Hint: $5\theta = 2\theta + 3\theta = 90^\circ$ (3)

Q.9 **(a)** Prove that $\frac{1}{2r\,R} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$ (5)

(b) Prove that $\text{Tan}^{-1}\frac{1}{11}\ \frac{5}{6} = \text{Tan}^{-1}\frac{1}{3} + \text{Tan}^{-1}\frac{1}{2}$ (3)

RAWALPINDI BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) **In an oblique triangle, if a = 200 ; b = 120 and included angle $\gamma = 150^\circ$, then its area will be equal to:**

(a) 6000 **(b)** 5000 **(c)** 2000 **(d)** 12000

(2) **If "R" is the circum-radius, then its value is:**

(a) $\frac{ac}{4\Delta}$ **(b)** $\frac{ab}{4\Delta}$ **(c)** $\frac{abc}{4\Delta}$ **(d)** $\frac{abc}{\Delta}$

(3) **The value of $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ is equal to:**

(a) 1 **(b)** -1 **(c)** $\frac{-1}{2}$ **(d)** $\frac{1}{2}$

(4) **The solution of $\operatorname{cosec}\theta = 2$ in interval $[0,2\pi]$ is equal to:**

(a) $\frac{\pi}{6},\frac{7\pi}{6}$ **(b)** $\frac{\pi}{6},\frac{5\pi}{6}$ **(c)** $\frac{\pi}{3},\frac{5\pi}{6}$ **(d)** $\frac{\pi}{3},\frac{\pi}{6}$

(5) **If $z = \cos\theta + i\sin\theta$, then $|z|$ is equal to:**

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

(6) **For any two subsets A and B of set ∪, then (A∪B) is equal to:**

(a) $A \cup B'$ **(b)** $A \cap B'$ **(c)** $A' \cup B'$ **(d)** $A' \cap B'$

(7) **If "A" is a square matrix and** $(\bar{A})^t = -A$, then "A" is called:

(a) Skew Symmetric **(b)** Symmetric **(c)** Skew Hermitian **(d)** Hermitian

(8) **if** $A=\begin{bmatrix} 4 & x & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ is a singular matrix, then 'x' is equal to:

(a) 3 **(b)** 4 **(c)** 6 **(d)** 7

(9) **If α and β are roots of $ax^2 + bx + c = 0$, then a. β is equal to:**

(a) -b/a **(b)** a/b **(c)** c/u **(d)** a/c

(10) **If "w" is a cube root of unity, then $(1 + w - w^2)(1 - w + w^2)$ will be equal to:**

(a) 3 **(b)** 4 **(c)** 2 **(d)** 1

(11) **If $\frac{3}{(x-1)(x+2)} = \frac{1}{x-1} + \frac{A}{x+2}$ then "A" is equal to:**

(a) -1 (b) 3 (c) 2 (d) 1

(12) **The nth root of product of n Geometric Means between a and b is equal to:**

(a) $(ab)^{1/n}$ (b) $a^n b^n$ (c) $n\sqrt{ab}$ (d) $\sqrt{ab}$

(13) **If in A.P; $a_{n-3} = 2n - 5$, then a_n will be equal to:**

(a) $2n + 1$ (b) $2n - 1$ (c) $n + 1$ (d) $n - 1$

(14) **$\frac{n!}{(n-r)!r!}$ is equal to:**

(a) ${}^rC_n$ (b) ${}^rP_n$ (c) ${}^nC_r$ (d) ${}^nP_r$

(15) **Number of signals given by 5 flags of different colours using 3 flags at a time equals.**

(a) 30 (b) 40 (c) 50 (d) 60

(16) **Sum of even co-efficient in the expansion of $(1 + x)$" equals:**

(a) 2^{n+1} (b) 2^{n-1} (c) 2^n (d) 2^{1-n}

(17) **Third term in the expansion of $(1 - 2x)^{1/3}$ is equal to:**

(a) $-9x^2/4$ (b) $9x^2/4$ (c) $4x^2/9$ (d) $-4x^2/9$

(18) **The area of a sector of circular region of radius r and angle θ is equal to:**

(a) $\frac{1}{2}r\theta^2$ (b) $\frac{1}{2}r^2\theta$ (c) $r\theta^2$ (d) $r^2\theta$

(19) **If $6\cos^2\theta + 2\sin^2\theta = 5$, then $\tan^2\theta$ will be equal to:**

(a) $\frac{3}{2}$ (b) 3 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(20) **Period of $\sin\frac{x}{5}$ is equal to:**

(a) 10π (b) 5π (c) 2π (d) $\frac{2\pi}{5}$

RAWALPINDI BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: (16)

(i) Find the modulus of complex number 3 + 4i.

(ii) Simplify by justifying each step $\dfrac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$ by writing properties.

(iii) Factorize the expression $9a^2 + 16a^2$.

(iv) Define absurdity and give one example.

(v) Solve the system of linear equations. $\begin{matrix} 4x_1 + 3x_2 = 5 \\ 3x_1 - x_2 = 7 \end{matrix}$

(vi) Find the value of x if $\begin{vmatrix} 1 & 2 & 1 \\ 2 & x & 2 \\ 3 & 6 & x \end{vmatrix} = 0$

(vii) Define Row Rank of a matrix.

(viii) Solve the equati8on $x^{-2} - 10 = 3x^{-1}$.

(ix) If A = {1,2,3,4}, B = {3,4,5,6,7,8}, C = {5,6,7,8,10} verify distributivity of union over intersection.

(x) Find the inverse of the relation. {(1, 3), (2,5), (3,7), (4,9), (5,11)}.

(xi) Using remainder theorem to find the remainder when $x^3 - x^2 + 5x + 4$ is divided by $x - 2$.

(xii) Find the roots of the equation $16x^2 + 8x + 1 = 0$ by using quadratic formula.

Q.3 Write answers of any EIGHT questions: (16)

(i) Resolve $\dfrac{1}{x^2 - 1}$ into partial fraction.

(ii) Find 5^{th} of Geometric progression G.P 2,6,12,.......

(iii) Define Circular permutation.

(iv) Expand $(4 - 3x)^{\frac{1}{2}}$ upto three terms.

(v) If $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in Arithmatic progression (A.P) show that common difference is $\dfrac{a-c}{2ac}$.

(vi) If 5,6 are two Arithmatic Means (A.M) between "a" and "b". Find "a" and "b".

(vii) If the numbers $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in (H.P) Hormonic Progression, Find "K".

(viii) How many words can be formed from the letters of "PLAN" using all letters when no letter is to be repeated?

(ix) If ${}^nc_5 = {}^nc_4$, where c stands for combination then find value of n.

(x) Verify the inequality $n > 2^n - 1$ for integral values of $n = 4,5$.

(xi) If x is so small that its square and higher power cab be neglected, show that $\frac{1-x}{\sqrt{1-x}} = 1 - \frac{3}{2}x$.

(xii) Prove that Hormonic Mean (H.M) between two numbers "a" and "b" is $\frac{2ab}{a+b}$.

Q.4 Write answers of any NINE questions: (18)

(i) Prove the fundamental identity $\cos^2\theta + \sin^2\theta = 1$.

(ii) Verify the result $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ for $\theta = 30°$.

(iii) Show that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$.

(iv) Prove that $\cos 330° \sin 600° + \cos 120° \sin 150° = -1$

(v) Find the period of cos ec (10-x).

(vi) Show that $\gamma = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$ with usual notation.

(vii) Find the value of $\cos\left(\sin^{-1}\frac{1}{2}\right)$

(viii) Show that $\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

(ix) Express the following difference as the product of trigonometric functions cosw $7\theta - \cos\theta$.

(x) In any triangle Δ ABC, if c = 16.1, a = 42°45, $\lambda = 74°32'$, then find "β" and "a".

(xi) Find the area of triangle ABC, given two sides and their included angle a = 200, b = 120, γ = 150°.

(xii) Find the solutions of the equation $\cot\theta = \frac{1}{\sqrt{3}}$ in the interval $[0, 2\pi]$.

(xiii) Find the value of θ satisfying the equation $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$.

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** Verify De Morgan's Laws for the given sets: $\cup = \{1, 2, 3, ...20\}$, $A = \{2, 4, 6, ... 20\}$ $B = \{1, 3, 5, ... 19)\}$ (5)

(b) Find the value of λ if A is singular matrix, $A = \begin{bmatrix} 4 & \pi & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$. (3)

Q.6 **(a)** If the roots of $px^2 + qx + q = 0$ are a and b, then prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0.$ (5)

(b) Resolve into partial fraction $\frac{x^4}{1 - x^4}$. (3)

Q.7 **(a)** The sum of an infinite geometric series is 9 and sum of square of its terms is $\frac{81}{5}$. Find the series. (5)

(b) If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots\ldots\ldots\ldots$, then prove that $y^2 + 2y - 4 = 0$. (3)

Q.8 **(a)** A railway train is running on a circular track of radius 500 meters at the rate of 30Km per hour. Through what angle will it turn in 10 sec? (5)

(b) If $\tan\alpha = \frac{-15}{8}$ and $\sin\beta = \frac{-7}{25}$ and neither the terminal side of the angle of measure a nor that of β is in IV quadrant. Find $\sin(\alpha + \beta)$ and co $(\alpha + \beta)$. (3)

Q.9 **(a)** One side of a triangular garden is 30m. If two corner angle are $22°\frac{1}{2}$ and $112°\frac{1}{2}$, find the cost of planting the grass at the rate of Rs.5 per square meter. (5)

(b) Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$. (3)

SARGODHA BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) If A ,G ,H have their usual meaning, G^2=

(a) H (b) A (c) $A \times H$ (d) A/H

(2) ^nP_n =

(a) n (b) 0 (c) 1 (d) n!

(3) The multiplicative inverse of $1 - 2i$ is

(a) $\frac{1+2i}{5}$ (b) $\frac{-1+2i}{5}$ (c) $\frac{1-2i}{5}$ (d) $\frac{1+2i}{3}$

(4) The number of identity elements in a group is

(a) Finite (b) 2 (c) 3 (d) 1

(5) The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is

(a) Null matrix (b) Identity matrix (c) Diagonal matrix (d) Scalar matrix

(6) If $\begin{bmatrix} K & 4 \\ 4 & k \end{bmatrix} = 0$. Then value of K is

(a) ± 16 (b) 0 (c) ± 4 (d) ± 8

(7) The product of roots of the equation $3x^2 + 4x = 0$ is

(a) $\frac{-4}{3}$ (b) $\frac{4}{3}$ (c) 0 (d) 4

(8) When $P(x) = x^3 + 4x^2 - 2x + 5$ is divided by (x - 1), remainder is

(a) 10 (b) -10 (c) 8 (d) -8

(9) If $(2x + 1) = A(x + 1) + B(x + 2)$, then A=_______

(a) 3 (b) 4 (c) 5 (d) 1

(10) The harmonic mean between 3 and 7 is

(a) $\frac{5}{21}$ (b) $\frac{21}{5}$ (c) 5 (d) 21

(11) $\tan(\tan^{-1}(1))$ =

(a) 1 (b) $\frac{\pi}{4}$ (c) $\pi/3$ (d) 0

(12) **Solution of $\cot\theta = \frac{1}{\sqrt{3}}$ in IIIrd quadrant is**

(a) $\frac{5\pi}{4}$ (b) $\frac{7\pi}{4}$ (c) $\frac{4\pi}{3}$ (d) π

(13) **If $^{n}C_{10} = {}^{n}C_{14}$ then n=**

(a) 24 (b) 8 (c) 20 (d) 18

(14) **The number of terms in the expension of $(1 + x)^{1/3}$ is**

(a) 3 (b) 4 (c) Infinite (d) Finite

(15) **The sum of coefficients in the expension of $(1 + x)^5$ is**

(a) 8 (b) 16 (c) 32 (d) 64

(16) **$\cot^2\theta - \operatorname{cosec}^2\theta =$**

(a) 2 (b) -1 (c) 1 (d) 0

(17) **$\tan\left(\frac{\sqrt{3\pi}}{2} + 0\right) =$**

(a) $\cot\theta$ (b) $\tan\theta$ (c) $-\cot\theta$ (d) $-\tan\theta$

(18) **Domain of y = sinx is**

(a) IR (b) [- 1 , 1] (c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) Q

(19) **In any triangle ABC, with usual notation $b^2 + c^2 - 2bc\cos a =$**

(a) Δ (b) 0 (c) a^2 (d) 1

(20) **$\sqrt{\frac{s(s-a)}{bc}} =$**

(a) sin a/2 (b) sin β/2 (c) cos a/2 (d) cos β/2

SARGODHA BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: (16)

(i) Check the closure property w.r.t "x" on $\{-1, 1\}$

(ii) Define modulus of a complex number.

(iii) Find multiplicative inverse of $-3-5i$.

(iv) Write down power set of $\{a\{b, c\}\}$

(v) Construct truth table for an implication.

(vi) Define Semigroup.

(vii) Find x & y if $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$.

(viii) Find A^{-1} if $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$.

(ix) If A is a non-singular matrix, then show that $(a^{-1})^{-1} = A$.

(x) Solve $2x^2 + 12x - 110 = 0$.

(xi) If ω is cube root of unity and $\omega^3 = 1$, then evaluate $\omega 28 + \omega^{29} + 1$

(xii) Discuss the nature of roots of $25x^2 - 30x + 9 = 0$.

Q.3 Write answers of any EIGHT questions: (16)

(i) Define Improper rational fraction and give one example.

(ii) Resolve $\frac{1}{x^2 - 1}$ into partial fractions.

(iii) Convert an improper fraction $\frac{2x^3 + x^2 - x - 3}{x(2x + 3)(x - 1)}$ into mixed form.

(iv) Sum the series $1.11 + 1.41 + 1.71 + \ldots\ldots + a_{10}$

(v) Define a geometric sequence and give an example.

(vi) Insert one real geometric mean between $-2i$ and $8i$.

(vii) Find the sum of infinite geometric series $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1\ldots\ldots$

(viii) If $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in harmonic sequence, find k

(ix) In how many ways the necklaces from 6 beads of different colours can be made.

(x) If $1 + 2 + 4 + \ldots\ldots + 2^{n-1} = 2^n - 1$ then check the statement for $n = 2$ and $n = 3$ is either true or false.

(xi) Evaluate $(9.9)^5$ using binomial theorem upto two decimal places.

(xii) Expand $(1 + x)^{-1/3}$ upto 4 terms.

Q.4 Write answers of any NINE questions: (18)

(i) Define "right angles triangle".

(ii) What is the length of the are intercepted on a circle of radius 14 cms by the arms of a central angle of 45°?

(iii) Find the values of $\sin\theta$ and $\cos\theta$ when $\tan\theta - \frac{1}{3}$ and the terminal arm of the angle is in quad ii.

(iv) Prove that: $\cos 306° + \cos 234° + \cos 162° + \cos 18° = 0$ without using calculator.

(v) Prove that: $\text{sing}\,(45° + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$.

(vi) Prove the identity $\frac{\sin\alpha - \sin\beta}{\sin\alpha + \sin\beta} = \tan\frac{\alpha-\beta}{2}\tan\frac{\alpha+\beta}{2}$.

(vii) Find the period of $\cos\frac{x}{6}$.

(viii) State 'The Law of Sinces'.

(ix) Find the area of the triangle ABC when its sides are $a = 18$, $b = 24$, $c = 30$.

(x) Show that $\sin^{-1}(-x) = -\sin^{-1}x$.

(xi) Find the solutions of the equation $\cot\theta = \frac{1}{\sqrt{3}}$, θ lies in $[0, 2\pi]$

(xii) Solve the equation $\sec^2\theta = \frac{4}{3}$, $\theta\,[0, 2\pi]$

(xiii) When the angle between the ground and the sun is 30°, flag pole casts a shadow of 40m long. Find the height of the top of the flag.

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 **(a)** Show that the set $\{1, \omega, \omega^2\}$, When $\omega^3 = 1$, is an abelian group w.r.t ordinary multiplication. (5)

(b) If $3n^2 + 2n + 1$ be be nth term of the series, find the sum to 2n terms. (3)

Q.6 **(a)** Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$ (5)

(b) Find values of n and r when ${}^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3 : 6 : 11$ (3)

Q.7 **(a)** Solve the euqation $\left(x - \frac{1}{x}\right)^2 + 3\left(x - \frac{1}{x}\right) = 0$ (5)

(b) Find the coefficient of x^5 in the expansion of $\left(x^2 - \frac{3}{3x}\right)^{10}$ (3)

Q.8 **(a)** Prove that identity $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$. (5)

(b) If α, β, γ are the angles of the triangle ABC, show that

$\cot\alpha/2 + \cot\beta/2 + \cot\gamma/2 = \cot\alpha/2\,\cot\beta/w\,\cot{}^{\gamma}/_{w}$ (3)

Q.9 **(a)** Prove that $r = \frac{\Delta}{s}$ with usual notation. (5)

(b) Show that $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$. (3)

SAHIWAL BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

(*Objective Type*)

Total Marks: 20 **Time Allowed: 30 minutes**

Q.1 Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer book. Cutting or filling two or more circles will result in zero mark in the question.

(1) **1. $(1+i)^8 =$**

(a) 2 (b) 4 (c) 8 (d) 16

(2) **The conjunction of two statements p and q is denoted by**

(a) $p \leftrightarrow q$ (b) $p \rightarrow q$ (c) $q \rightarrow p$ (d) $p \wedge q$

(3) **If $|A| = 5$, then $|A^1| =$**

(a) -5 (b) $\frac{1}{5}$ (c) 0 (d) 5

(4) **If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 2$ then $\begin{vmatrix} c & d \\ a & d \end{vmatrix} =$**

(a) 2 (b) -2 (c) ± 2 (d) 0

(5) **No. of roots of the equation $(x-4)^2 = x^2 - 8x + 16$ are**

(a) 2 (b) 4 (c) 8 (d) infinite

(6) **If w is cube root of unity, then $(1 + \omega - \omega 2)^3 =$**

(a) -8ω (b) 8ω (c) 8 (d) -8

(7) **$\frac{p(x)}{x^2+1}$ will be proper fraction if degree of p (x) is**

(a) 1 (b) 2 (c) 3 (d) 4

(8) **The series $a + ar + ar^2 + \cdots \infty$ converges, if**

(a) $|r| > 1$ (b) $|r| \geq 1$ (c) $|r \leq 1|$ (d) $|r| < 1$

(9) **A , G , H , are in**

(a) A . P (b) G . P (c) H . P (d) series

(10) **For an event A, range of its probability P (A) is**

(a) $-1 \leq p(A) \leq 1$ (b) $0 < P(A) < 1$ (c) $0 \leq P(A) \leq 1$ (d) $P(A) = 1$

(11) **If ${}^nc_4 - {}^nc_5 = 0$, then n =**

(a) 0 (b) 4 (c) 6 (d) 9

(12)

(a) $(\frac{n}{2})$ th term (b) $(\frac{n+2}{2})$th term (c) $(\frac{n+1}{2})$th term (d) $(\frac{n+5}{2})$th term

(13) 2^{nd} term of $(a + b)^7$ is

(a) a^7 (b) $7ab^6$ (c) $7a^6b$ (d) $7ab$

(14) $\frac{\pi}{3}$ rad is an angle.

(a) acute (b) obtuse (c) straight (d) reflexive

(15) Tan (a – 90) =

(a) Cot α (b) –Cot α (c) Tan α (d) –Tan α

(16) Period of $\frac{1}{2}\sin 2x$ is

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) 4π

(17) In an equilateral ΔABC

(a) $r_1 > r_2$ (b) $r_1 < r_2$ (c) $r_1 = r_2$ (d) $r_1 \neq r_2$

(18) With usual notations a + b – c =

(a) 2s (b) 2s – 2c (c) 2s – 2b (d) 2s – 2a

(19) $\text{Sin}^{-1}(0) + \text{Cos}^{-1}(0) =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{4}$

(20) If $\text{Sin}x = -\frac{\sqrt{3}}{2}$ then x =

(a) $\frac{\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

SAHIWAL BOARD 2019

MATHEMATICS INTERMEDIATE PART-I

Examination Session 2013-2017

(*Subjective Type*)

Total Marks: 80 **Time Allowed: 2:30 minutes**

SECTION-I

Q.2 Write answers of any EIGHT questions: (16)

(i) Prove that $\overline{z} = z$ iff z is real.

(ii) Separate into real and imaginary parts. $\frac{(-2+3i)^2}{1+i}$

(iii) Does the set $\{0, -1\}$ possess closure property with respect to (i) addition (ii) multiplication?

(iv) Write the set $\{x \mid x \in p \wedge x < 12\}$ id descriptive and tabular form.

(v) Prove that $P \rightarrow p \vee q$ is tautology.

(vi) Define a group.

(vii) If A is any square matrix of order 3, show that $A - A^t$ is skew symmetric.

(viii) Define Skew Symmetric Matrix.

(ix) If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find values of a and b.

(x) Find roots of equation $15x^2 + 2x - a^2 = 0$ by using quadratic formula.

(xi) If α, β are the roots of $3x2 - 2x + 4 = 0$, find value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(xii) Discuss the nature of roots of equation $x^2 + 2x + 3 = 0$.

Q.3 Write answers of any EIGHT questions: (16)

(i) Resolve $\frac{1}{x^2 - 1}$ in to partial fractions.

(ii) Write identity of $\frac{x^2 - 2x + 3}{x^4 + x^2 + 1}$

(iii) Define proper rational fraction.

(iv) If $a_n = \frac{1}{a + (n-1)d}$ m write 1^{st} 4 terms.

(v) Which term of the A.P 5, 2, −1, ---------- is −85?

(vi) Which term of the sequence $x^2 + y^2, x + y, \frac{x+y}{x-y}, ...$ is $\frac{x+y}{(x-y)^9}$

(vii) Sum the series $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + ... \infty$.

(viii) Show $G_2 = AH$, Where A, G and H are arithmetic geometric and harmonic means between a and b.

(ix) Find the value of n, when ${}^nC_{12} = {}^nC_6$

(x) If $1 + 2 + 4 + ... + 2^{n-1}\ 2^n - 1$ then verify for n = 3

(xi) Find the fifth term in the expansion of $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{11}$

(xii) Expand $(1 - x)^{-1/2}$ up to 3 terms.

Q.4 Write answers of any NINE questions: (18)

(i) If α, β, γ are the angles of a triangle ABC then prove that $\text{Cos}\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$

(ii) Prove that $\frac{\sin\theta - \cos\theta \tan\theta/2}{\cos\theta + \sin\theta \tan\theta/2} = \tan\theta/2$.

(iii) Show that $\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$.

(iv) Find the period of Cot8x.

(v) State the "Law of Sines".

(vi) Find the area of the triangle ABC given two sides and their included angle: $a = 200$, $b = 120$, $\gamma = 150°$

(vii) If $\Delta = \frac{1}{2}\,bc\sin\alpha$ then show that $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

(viii) Find the value of $\sec\left(\cos^{-1}\frac{1}{2}\right)$

(ix) Find the solution of $\cot\theta = \frac{1}{\sqrt{3}}$ $\theta \in [0, 2\pi]$

(x) Solve the equation $1 + \cos x = 0$.

(xi) Verify that $\sin^2 \pi/6 + \sin^2 \pi/3 + \tan^2 \pi/4 = 2$.

(xii) $\cos\theta + \tan\theta\sin\theta = \sec\theta$; verify!

(xiii) If $\cos\theta = \frac{9}{41}$ and terminal arm of the angle is in quadrant –IV then find the value of $\tan\theta$

SECTION-II

Note: **Attempt any THREE questions:**

Q.5 (a) Verify De Morgan's Laws for the following sets. (5)

$U = \{1, 2, 3, \ldots\ldots 20\}$

$A = \{2, 4, 6, \ldots\ldots 20\}$

$B = \{1, 3, 5, \ldots\ldots 19\}$

(b) Find n A.Ms between two number a and b (3)

Q.6 (a) Solve the following system by reducing the augmented matrix to echelon form (5)

$x + 2y + z = 2$

$2x + y + 2z = -1$

$2x + 3y - z = 9$

(b) Find the value of n and r when ${}^nC_r = 35$ and ${}^nP_r = 210$ (3)

Q.7 (a) Show that the roots of $x2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2(1 + m^2)$. (5)

(b) Use mathematical induction to prove that $3 + 6 + 9 + \ldots\ldots + 3n = \frac{3n(n+1)}{2}$ for all Positive integers n. (3)

Q.8 (a) Prove that $\frac{\text{Tan}\theta + \sec\theta - 1}{\text{Tan}\theta - \sec\theta + 1} = \text{Tan}\theta + \sec\theta$ (5)

(b) Express $3\sin\theta - 4\cos\theta$ in the form $r\sin(\theta - \phi)$. (3)

Q.9 (a) The sides of a triangle are $x2 + x + 1$, $2x + 1$ and $x2 - 1$. Prove that the greatest angle of triangle is 120°. (5)

(b) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$. (3)

Answers Key

Chapter 1

FUNCTION AND LIMITS

EXERCISE 1.1

1	a	2	c	3	b	4	d	5	a
6	b	7	b	8	a	9	b	10	b
11	c	12	b	13	d	14	a	15	b
16	d	17	d	18	b	19	b	20	a
21	b	22	b	23	b	24	a	25	b
26	c	27	c	28	c	29	d	30	b
31	d	32	d	33	a	34	c	35	d
36	a	37	a	38	a	39	c	40	b
41	a	42	b	43	d	44	a	45	a
46	c	47	c	48	c	49	a	50	c
51	b	52	b	53	a	54	b	55	c
56	d	57	b	58	a	59	d	60	b
61	d								

EXERCISE 1.2

1	d	2	b	3	b	4	b	5	c
6	d	7	d	8	b	9	c	10	d
11	b	12	b	13	d	14	b	15	a
16	c	17	c	18	c	19	c	20	c
21	c	22	a	23	a	24	a	25	c
26	a	27	b	28	c	29	b	30	d
31	c	32	a	33	c	34	a	35	d
36	d	37	b	38	a	39	a	40	c
41	a	42	b	43	d	44	d	45	d
46	a	47	b	48	c	49	c	50	d
51	a	52	b	53	b	54	b		

EXERCISE 1.3

1	a	2	b	3	c	4	a	5	a
6	a	7	b	8	c	9	d	10	a
11	b	12	b	13	b	14	d	15	b
16	c	17	d	18	b	19	d	20	c
21	a	22	d						

Chapter 2

SETS, FUNCTIONS AND GROUPS

EXERCISE 2.1

1	a	2	c	3	c	4	b	5	c
6	b	7	c	8	a	9	d	10	b
11	c	12	b	13	a	14	d	15	b
16	b	17	a	18	a	19	a	20	b
21	d	22	a	23	b	24	b	25	c
26	b	27	a	28	b	29	c	30	d
31	c	32	a	33	b	34	a	35	a
36	a	37	b	38	b	39	c	40	b
41	c	42	a	43	c	44	d	45	a
46	c	47	a	48	c	49	c	50	b
52	c	53	c						

EXERCISE 2.2

1	d	2	b	3	b	4	b	5	a
6	a	7	c	8	a	9	b	10	a
11	c	12	b	13	d	14	a	15	b
16	a	17	b	18	b	19	a	20	d
21	a	22	b	23	b	24	a	25	d
26	c	27	d	28	c	29	c	30	a
31	d	32	d	33	a	34	b	35	b
36	d	37	a	38	c	39	d	40	c
41	b								

EXERCISE 2.3

1	a	2	d	3	a	4	c	5	c
6	b	7	d	8	d				

EXERCISE 2.4

1	d	2	d	3	a	4	c	5	d
6	b	7	b	8	d	9	b	10	a
11	a	12	d	13	d	14	c	15	c
16	b	17	c	18	d	19	b	20	d
21	c	22	d	23	a	24	c	25	a
26	a	27	b	28	b	29	d	30	a
31	b	32	a	33	b	34	b	35	c

EXERCISE 2.5

1	d	2	b	3	a

EXERCISE 2.6

1	b	2	d	3	b	4	b	5	a
6	d	7	b	8	c	9	a	10	d
11	a	12	b	13	a	14	c	15	b
16	b	17	c	18	a				

EXERCISE 2.7

1	b	2	c	3	d

EXERCISE 2.8

1	c	2	a	3	d	4	a	5	d
6	a	7	a	8	c	9	d	10	c
11	a	12	b	13	c	14	c	15	b
16	a	17	b	18	d	19	b		

Chapter 3

MATRICES AND DETERMINANTS

EXERCISE 3.1

1	c	2	d	3	c	4	c	5	b
6	b	7	a	8	b	9	b	10	a
11	c	12	b	13	b	14	b	15	c
16	b	17	a	18	b	19	d	20	b
21	b	22	a	23	b	24	b	25	d
26	a	27	b	28	a	29	a	30	b
31	c	32	c	33	b	34	b	35	a
36	d	37	a	38	c	39	b	40	a
41	a	42	c	43	a	44	d	45	a
46	d	47	c	48	d	49	a	50	c
51	a	52	b						

EXERCISE 3.2

1	b	2	d	3	c	4	a	5	d
6	a	7	d	8	c	9	d	10	c
11	b	12	d	13	c	14	a	15	b
16	d	17	d	18	a	19	a	20	b
21	c	22	c	23	a	24	d	25	c
26	c	27	d	28	d	29	a	30	b
31	a	32	c	b					

EXERCISE 3.3

1	d	2	c	3	d	4	a	5	b
6	d	7	a	8	b	9	c	10	a
11	d	12	a	13	a	14	c	15	c
16	b	17	b	18	b	19	b	20	a
21	a	22	a	23	c	24	b	25	c
26	d	27	a	28	c	29	b	30	b
31	b	32	c	33	c	34	d	35	c
36	b	37	b	38	c	39	d		

EXERCISE 3.4

1	b	2	b	3	d	4	a	5	a
6	a	7	a	8	c	9	d	10	c
11	d	12	c	13	a	14	b	15	b
16	b	17	b	18	c	19	d	20	d
21	c	22	d	23	a	24	c	25	d
26	b	27	a	28	b	29	a	30	a

EXERCISE 3.5

1	d	2	d	3	c

Chapter 4

QUADRATIC EQUATION

EXERCISE 4.1

1	b	2	c	3	b	4	d	5	a
6	c	7	c	8	b	9	c		

EXERCISE 4.2

1	c	2	a	3	d	4	a	5	c
6	b	7	d	8	a	9	c	10	b
11	d	12	c						

EXERCISE 4.4

1	a	2	a	3	a	4	a	5	a
6	b	7	d	8	a	9	c	10	a
11	d	12	d	13	a	14	c	15	d
16	a	17	d	18	c	19	d	20	d
21	a	22	a	23	d	24	c	25	d
26	c	27	c	28	a	29	b	30	c
31	a	32	a	33	b	34	d		

EXERCISE 4.5

1	d	2	a	3	b	4	b	5	a
6	b	7	d	8	c	9	d	10	d
11	c	12	b	13	c	14	d	15	c
16	d	17	c	18	c	19	a	20	c
21	c	22	d	23	d	24	c	25	c
26	d	27	d	28	c	29	b	30	b
31	b	32	b	33	b	34	c	35	c

EXERCISE 4.6

1	b	2	b	3	b	4	a	5	b
6	d	7	d	8	a	9	c	10	d
11	b	12	d	13	d	14	d	15	d
16	d	17	a	18	c	19	b	20	c
21	a	22	d	23	d	24	a	25	c

EXERCISE 4.7

1	a	2	d	3	a	4	a	5	d
6	a	7	a	8	d	9	a	10	d
11	a	12	c	13	d	14	a	15	b
16	a	17	b	18	a	19	c	20	a
21	d	22	d	23	c				

EXERCISE 4.10

1	c	2	b	3	d

Chapter 5

PARTIAL FRACTION

EXERCISE 5.1

1	a	2	c	3	b	4	c	5	b
6	c	7	c	8	d	9	b	10	c
11	c	12	d	13	a	14	a	15	a
16	d	17	d	18	b	19	b	20	a
21	b	22	a	23	c	24	b	25	b
26	b	27	b	28	c	29	b	30	b
31	c	32	a	33	a	34	b	35	a
36	c	37	b	38	c	39	b	40	a
41	d	42	c	43	b	44	b	45	c
46	a	47	c	48	c	49	a	50	a
51	d	52	d	53	b	54	c	55	c
56	b	57	a	58	a	59	a		

EXERCISE 5.2

1	b	2	b	3	c	4	a	5	c
6	b								

EXERCISE 5.3

1	c	2	c	3	d	4	a	5	c
6	c	7	b	8	c	9	c	10	a
11	b	12	a	13	a	14	d	15	b
16	b								

EXERCISE 5.4

1	c

Chapter 6

SEQUENCE AND SERIES

EXERCISE 6.1

1	d	2	b	3	c	4	d	5	a
6	b	7	a	8	d	9	d	10	c
11	a	12	a	13	b	14	c	15	b
16	c	17	c	18	a	19	c	20	b
21	b	22	d	23	a	24	b	25	c
26	b	27	a						

EXERCISE 6.2

1	b	2	a	3	b	4	c	5	c
6	c	7	b	8	b	9	b	10	b
11	a	12	a	13	b	14	c	15	a
16	c								

EXERCISE 6.3

1	c	2	a	3	b	4	d	5	d
6	a	7	b	8	c	9	b	10	b
11	a	12	b	13	b	14	a	15	a

EXERCISE 6.4

1	b	2	c	3	a	4	b

EXERCISE 6.6

1	a	2	b	3	b	4	c	5	a
6	d	7	c	8	c	9	c	10	b
11	d	12	a	11	d				

EXERCISE 6.7

1	b	2	d	3	a	4	c	5	c
6	d	7	a	8	a	9	a	10	c
11	b	12	a	13	c	14	d		

EXERCISE 6.8

1	b	2	d	3	a	4	a	5	a
6	a	7	d	8	d	9	d	10	b
11	a	12	c	13	b	14	b	15	d

EXERCISE 6.10

1	c	2	b	3	c	4	a	5	b
6	a	7	a	8	c	9	c	10	a
11	a	12	d	13	c	14	a	15	c
16	c	17	a	18	a	19	b	20	a
21	a	22	a	23	a	24	b	25	??
26	b	27	c	28	c	29	b		

EXERCISE 6.11

1	b	2	d	3	b	4	b	5	a
6	b	7	c	8	a	9	d	10	c

Chapter 7

PERMUTATIONS, COMBINATION AND PROBABILITY

EXERCISE 7.1

1	a	2	a	3	a	4	c	5	b
6	c	7	a	8	d	9	b	10	d
11	b	12	a	13	a	14	c	15	a
16	b	17	c	18	d	19	a	20	a
21	d	22	b	23	c	24	c	25	d
26	a								

EXERCISE 7.2

1	b	2	a	3	a	4	c	5	c
6	a	7	c	8	c	9	c	10	a
11	c	12	a	13	b	14	d	15	d
16	d	17	d	18	d				

EXERCISE 7.3

1	c	2	c	3	c	4	a	5	b
6	c	7	b	8	c	9	b	10	b

EXERCISE 7.4

1	b	2	a	3	d	4	d	5	d
6	d	7	a	8	b	9	c	10	c
11	b	12	c	13	c	14	b	15	a
16	b	17	d	18	d	19	b	20	a
21	b	22	b	23	c	24	c	25	a
26	d								

EXERCISE 7.5

1	c	2	a	3	d	4	b	5	a
6	c	7	d	8	b	9	a	10	b
11	a	12	c	13	d	14	d	15	a
16	b	17	c	18	a	19	a	20	c
21	a	22	b	23	c	24	b	25	b
26	c								

EXERCISE 7.7

1	b	2	d	3	b	4	c	5	a
6	c	7	b	8	d	9	a	10	d
11	d								

EXERCISE 7.8

1	c	2	a	3	d	4	c	5	a
6	c	7	c	8	a	9	d	10	d
11	a	12	c						

Chapter 8

MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

EXERCISE 8.1

1	a	2	a	3	c	4	a	5	b
6	c	7	d	8	d	9	a	10	c
11	a	12	d	13	d	14	b	15	c
16	b	17	a	18	b	19	c	20	d
21	c	22	d						

EXERCISE 8.2

1	a	2	c	3	a	4	b	5	d
6	a	7	b	8	b	9	a	10	b
11	b	12	d	13	a	14	a	15	c
16	b	17	c	18	a	19	c	20	d
21	c	22	c	23	b	24	c	25	b
26	d	27	c	28	b	29	b	30	b
31	c	32	c	33	a	34	d	35	d
36	c	37	b	38	b	39	a	40	a
41	c	42	b	43	b	44	c	45	b
46	d	47	a	48	a	49	c	50	b
51	c	52	a	53	b	54	a	55	b
56	a	57	b	58	a	59	b	60	b
61	a	62	c	63	b	64	d	65	c
66	c	67	a	68	a	69	a	70	c
71	a	72	d	73	b	74	b	75	c

EXERCISE 8.3

1	d	2	d	3	d	4	b	5	c
6	a	7	b	8	a	9	b	10	c
11	b	12	b	13	a	14	b	15	c
16	b	17	a	18	c	19	d	20	d
21	d	22	b	23	b	24	d	25	b
26	b	27	a	28	d	29	d	30	a
31	b	32	b	33	a	34	d		

Chapter 9

FUNDAMENTALS OF TRIGONOMETRY

EXERCISE 9.1

1	a	2	d	3	d	4	b	5	c
6	d	7	c	8	b	9	b	10	b
11	b	12	a	13	a	14	d	15	b
16	a	17	d	18	a	19	c	20	b
21	b	22	b	23	a	24	d	25	d
26	d	27	d	28	b	29	c	30	b

EXERCISE 9.2

1	d	2	d	3	a	4	d	5	c
6	c	7	d	8	d	9	d	10	c
11	c	12	b	13	b	14	b	15	b
16	c	17	a	18	b	19	c	20	a
21	a	22	b	23	b	24	b	25	b
26	a	27	c	28	c	29	a	30	c
31	b	32	a	33	a	34	b	35	d
36	a	37	c	38	d	39	d	40	c
41	c	42	b	43	a				

EXERCISE 9.3

1	c	2	a	3	a	4	c	5	b
6	a	7	c	8	d	9	c	10	b
11	d	12	d	13	a	14	b	15	d
16	c	17	c						

EXERCISE 9.4

1	d	2	b	3	b	4	b	5	b
6	b	7	b	8	a	9	b	10	c
11	c								

Chapter 10

TRIGONOMETRIC IDENTITIES OF SUM AND DIFFERENCE OF ANGLES

EXERCISE 10.1

1	a	2	a	3	c	4	d	5	d
6	b	7	a	8	b	9	b	10	d
11	c	12	b	13	a	14	c	15	b
16	b	17	d	18	d	19	b	20	d
21	c	22	c	23	c	24	d	25	a
26	b	27	b	28	b	29	d	30	b
31	d	32	a	33	a	34	b	35	c
36	d	37	a	38	c	39	c	40	a
41	a	42	b	43	b	44	b	45	b
46	a	47	a	48	a	49	b	50	d
51	b	52	c	53	c	54	a	55	a
56	a	57	d	58	c	59	b	60	b
61	d	62	c	63	b				

EXERCISE 10.2

1	b	2	d	3	c	4	b	5	b

EXERCISE 10.3

1	d	2	c	3	b	4	c	5	b
6	b	7	b	8	a	9	d	10	a
11	a	12	a	13	c	14	d	15	a
16	c	17	c	18	b	19	a	20	b
21	b								

EXERCISE 10.4

1	d	2	b	3	d	4	a	5	b
6	d	7	b	8	b	9	c	10	c
11	a	12	d	13	a	14	a	15	a

Chapter 11

TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

EXERCISE 11.1

1	a	2	b	3	d	4	c	5	b
6	a	7	c	8	d	9	d	10	a
11	d	12	b	13	c	14	c	15	a
16	d	17	a	18	b	19	d	20	b
21	d	22	b	23	b	24	b	25	b
26	d	27	c	28	a	29	b	30	a
31	b	32	c	33	c	34	b	35	b
36	a	37	c	38	b	39	c	40	a
41	a	42	a	43	b	44	a	45	b
46	b	47	a	48	a	49	a	50	a
51	c	52	c	53	c	54	c	55	c
56	c	57	b	58	c	59	d	60	??
61	b	62	b	63	c	64	c	65	b

Chapter 12

APPLICATION OF TRIGONOMETRY

EXERCISE 12.2

1	a	2	c	3	c	4	a

EXERCISE 12.3

1	c	2	c	3	c

EXERCISE 12.4

1	c	2	d	3	c	4	b	5	b
6	a	7	b	8	a	9	a	10	a
11	c	12	c	13	d	14	c	15	c
16	a	17	a	18	b	19	b	20	c
21	a	22	b	23	c	24	c		

EXERCISE 12.7

1	b	2	c	3	d	4	a	5	c
6	c	7	a	8	d	9	d	10	a

EXERCISE 12.8

1	a	2	b	3	d	4	d	5	c
6	b	7	c	8	d	9	d	10	d
11	c	12	d	13	c	14	a	15	a
16	c	17	b	18	a	19	b	20	d
21	c	22	a	23	c	24	a	25	a
26	d	27	a	28	c	29	c	30	c
31	b	32	a	33	a	34	a	35	b
36	a	37	c	38	a	39	a	40	c
41	a	42	b	43	c	44	b		

Chapter 13

INVERSE TRIGONOMETRIC FUNCTIONS

EXERCISE 13.1

1	a	2	b	3	d	4	a	5	b
6	d	7	b	8	a	9	a	10	b
11	b	12	c	13	a	14	a	15	c
16	d	17	b	18	b	19	d	20	b
21	b	22	d	23	b	24	c	25	d
26	a	27	c	28	b	29	b	30	b
31	d	32	a	33	b	34	c	35	d
36	d	37	a	38	a	39	b	40	a
41	c	42	b	43	b	44	a	45	a
46	d	47	c	48	b	49	b	50	a
51	b	52	b	53	b	54	b	55	a
56	b	57	b	58	a	59	a		

EXERCISE 13.2

1	b	2	a	3	b	4	a	5	b
6	a	7	c	8	a	9	a	10	a
11	b	12	b	13	a	14	b	15	c
16	b	17	b						

Chapter 14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

EXERCISE 14

1	b	2	b	3	c	4	a	5	a
6	b	7	b	8	b	9	b	10	b
11	b	12	c	13	d	14	a	15	a
16	a	17	d	18	b	19	b	20	c
21	a	22	c	23	b	24	b	25	c
26	a	27	a	28	d	29	b	30	d
31	d	32	b	33	b	34	a	35	b
36	c	37	a	38	b	39	b	40	a
41	d	42	b	43	b	44	a	45	c
46	d	47	b	48	c	49	a	50	d
51	b	52	b	53	c	54	d	55	a
56	b	57	c	58	c	59	d	60	a
61	c	62	d	63	d	64	d	65	c
66	c	67	c	68	b				