COMPLETE NOTES

11TH PHYSICS FULL BOOK NOTES - SHORT QS & NUMERICALS

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SHORT QUESTIONS

1.1 Name several repetitive phenomenon occurring in nature which could serve as reasonable time standard.

Ans. Any natural phenomenon that repeats itself after exactly same time interval can be used as a measure of time. The repetitive phenomenon could serve as reasonable time standard, occurring in nature are as follows:

1. **Sun:** Sun served as reasonable time standard because sunset and sunrises gives the information of time.
2. **Moon:** Moon is also reasonable time standard because it gives the information of time.
3. **Weather:** Changing of weather can also give information about time.
4. Rotation of Earth on its axis.
5. Rotation of Earth around the sun.
6. Oscillation of a simple pendulum.

1.2 Give draw backs to use the period of a pendulum as a time standard.

Ans. As we know that the time period of a simple pendulum depends upon the length and value of g at any place. Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(i) It is clear that time period of a simple pendulum depends upon the value of g which is different at different places. So a pendulum of same length may have different time period at difference places. So period of pendulum cannot be taken as standard for measuring time.

(ii) **Friction:** Time period of a simple pendulum changes due to air resistance.

(iii) **Temperature:** In summer due to increase in temperature, length of simple pendulum changes so time period changes.

1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

Ans. It is very useful to have two units for the amount of substance i.e., kilogram and mole. If we want to consider a specific amounts of mass without considering number of microscopic atoms present in it, it is useful to use **kilogram**. Because one kilogram of different substances contains different number of molecules. While if we want to consider a fixed number of atoms present in it then it is useful to use **mole**. Because one mole of any substance contains the same number of atoms or molecules.

1.4 Three student's measured the length of a needle with a scale on which minimum division is 1 mm and recorded as (i) 0.2145m (ii) 0.21m (iii) 0.214m. Which record is correct and why?

Ans. In these records (iii) 0.214 m is more correct than the other records because the least count of a scale is 1 mm which can be written as 0.001 m. So according to this figure, the student measure that type of record is correct.

1.5 An old saying is that "A chain is only as strong as its weakest link". What analogue statement can you make regarding experimental data used in computation?

Ans. The analogous statement regarding experimental data used in computation is "A result obtained from an experimental data used in computation is only as accurate as its least accurate reading".

1.6 The period of simple pendulum is measured by a stopwatch. What type of errors are possible in the time period?

Ans. When the period of a simple pendulum is measured by a stopwatch, the following types of errors are possible:

1. **Systematic Error:** The error due to the fault in the measuring instrument is called systematic error i.e., zero error.
2. **Personal Error:** The error due to the faulty procedure of an observer is called personal error.

1.7 Does dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.

Ans. Dimensional analysis does not give any information about the constant of proportionality or dimensionless constant. For example

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

The numerical value of this constant cannot determined by dimensional analysis.

1.8 Write the dimension of:

(i) Pressure (ii) Density

Ans. (i) Dimensions of Pressure:

$$\text{As} \quad P = \frac{F}{A} = \frac{ma}{A}$$

$$\text{Unit of P} = \frac{\text{kg ms}^{-2}}{\text{m}^2} = \text{kg m}^{-1}\text{s}^{-2}$$

$$\Rightarrow \quad [P] = [ML^{-1}T^{-2}]$$

(ii) Dimensions of Density:

$$\text{As} \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Unit of density} = \frac{\text{kg}}{\text{m}^3} = \text{kg m}^{-3}$$

$$[\text{Density}] = [ML^{-3}]$$

1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that:

$$[\lambda] = [L], [v] = [LT^{-1}] \text{ and } [f] = [T^{-1}]$$

Decide which of the following is correct, $f = v\lambda$ or $f = \frac{v}{\lambda}$.

Ans. In 1st case if $f = v\lambda$ where f is frequency. Its dimension is $[T^{-1}]$, v is speed, its dimensions are $[LT^{-1}]$.

λ is the wavelength, its dimension is [L].

So, $[T^{-1}] = [LT^{-1}][L]$

$[T^{-1}] = [L^2T^{-1}]$

Hence the equation $f = v\lambda$ is not dimensionally correct because left hand side dimension is not equal to right hand side dimension.

In second case

$$f = \frac{v}{\lambda}$$

So $$[T^{-1}] = \frac{[LT^{-1}]}{[L]}$$

$$[T^{-1}] = [T^{-1}]$$

Hence the equation $f = \frac{v}{\lambda}$ is dimensionally correct because left hand side dimensions is equal to right hand side dimension.

PROBLEMS WITH SOLUTIONS

PROBLEMS 1.1

A light year is the distance light travels in one year. How many metres are there in one light year? (Speed of light = 3.0×10^8 ms^{-1})

Data

Time $= t = 1$ year

$= 365$ days

$= 365 \times 24 \times 3600$

$= 31536000$ sec.

Speed of light $= C = 3 \times 10^8$ m/s

To Find

Distance covered by light $= d = ?$

SOLUTION

As we know that

$$v = \frac{d}{t}$$

$$d = v \times t$$

$$= C \times t$$

$$= 3 \times 10^8 \times 31536000$$

$$= 94608000 \times 10^8$$

$$= 9.46 \times 10^{15} \text{ m}$$

Result

Distance covered by light $= d = 9.46 \times 10^{15}$ m

PROBLEM 1.2

(a) How many seconds are there in 1 year?

(b) How many nanoseconds in 1 year?

(c) How many years in 1 second?

Data

One year $= 365$ days

$= 365 \times 24 \times 3600$

$= 3.1536000$

$= 3.15 \times 10^7$ sec.

To Find

(a) Seconds in one year = ?

(b) Nanosecond in one year = ?

(c) Years in one second = ?

SOLUTION

(a) As we know that

$$1 \text{ year} = 365 \text{ days}$$
$$= 365 \times 24 \times 3600$$
$$= 3.15 \times 10^{7} \text{ sec.}$$

Seconds in one year $= 3.15 \times 10^{7}$ sec.

(b) As 1 year $= 3.15 \times 10^{7} \times 10^{9}$ n sec. Since 1 ns $= 10^{-9}$ s

$= 3.15 \times 10^{16}$ nanosecond $\therefore$ 1 s $= 10^{9}$ ns

(c) As 1 year $= 3.15 \times 10^{7}$ sec.

$$\frac{1}{3.15 \times 10^{7}} \text{ year} = 1 \text{ second}$$

$$1 \text{ second} = 0.317 \times 10^{-7} \text{ years}$$
$$= 3.17 \times 10^{-8} \text{ years}$$

Result

(a) Number of seconds in one year $= 3.15 \times 10^{7}$ seconds

(b) Number of nanoseconds in one year $= 3.15 \times 10^{16}$ nanosecond

(c) Number of years in one second $= 3.17 \times 10^{-8}$ years

PROBLEM 1.3

The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

Data

Length of rectangular plate = L = 15.3 cm

Width of rectangular plate = W = 12.80 cm

To Find

Area of the plate = A = ?

SOLUTION

As we know that

$$\text{Area} = \text{Length} \times \text{Width}$$
$$= 15.3 \times 12.80$$
$$= 195.84 \text{ cm}^2$$

Result

Area of rectangular plate = A = 196 cm^2

PROBLEM 1.4

Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

Data

The given masses are 2.189, 0.089, 11.8 and 5.32

To Find

Sum of masses upto appropriate precision = ?

SOLUTION

$$\text{Sum of masses} = 2.189 + 0.089 \;\; 11.8 + 5.32$$
$$= 19.398$$
$$= 19.4 \text{ kg}$$

Result

Sum of masses upto appropriate precision = 19.4 kg

PROBLEM 1.5

Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment.

Length of simple pendulum l = 100 cm

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

Data

Length of simple pendulum = l = 100 cm

= 1 m

Time for 20 vibration = t = 40.2 s

Time period = $T = \frac{t}{20} = \frac{40.2}{20} = 2.01$ sec.

Least count of metre scale = 1 mm = 0.1 cm

To Find

Acceleration due to gravity = g = ?

SOLUTION

As we know that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring

$$T^2 = 4\pi^2 \times \frac{l}{g}$$

$$\boxed{g = \frac{4\pi^2 l}{T^2}}$$

Putting the values

$$g = \frac{4(3.14)^2 \times 1}{(2.01)^2}$$

$$= \frac{39.4384}{4.04}$$

$$g = 9.76\ m/s^2$$

Since Uncertainty in length = 0.1 cm

% uncertainty in length = 0.1%

% uncertainty in time = $\frac{0.005}{2.01} \times 100$ Since $\frac{0.1}{20}$ = 0.005 sec.

= 0.25%

Thus Total uncertainty in "g" = % uncertainty in time + 2(% uncertainty in time)

= 0.1 + 2(0.25)

= 0.1 + 0.5

= 0.6%

Thus Uncertainty in calculated value of g = $\frac{0.6}{100} \times 0.97$

= $0.06\ m/s^2$

Hence g = $(9.76 \pm 0.06)\ m/s^2$

Result

Acceleration due to gravity = g = $(9.76 \pm 0.06)\ m/s^2$

PROBLEM 1.6

What are the dimensions and units of gravitational constant G in the formula?

$$\mathbf{F = G\frac{m_1 m_2}{r^2}}$$

Data

The given formula is

$$F = G\frac{m_1 m_2}{r^2}$$

To Find

Dimensions of G = ?

Unit of G = ?

SOLUTION

Now for dimensions

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = \frac{F \times r^2}{m_1 m_2} \qquad \text{Since} \quad F = ma$$

$$= \frac{ma \times r^2}{m_1 \times m_2}$$

$$\text{Unit of } G = \frac{kg \,.\, m/s^2 \times m^2}{kg \,.\, kg}$$

$$= \frac{m^3}{kg\, s^2}$$

$$\text{Dimensions of } G = [M^{-1}L^3T^{-2}]$$

For unit of G

$$G = \frac{F \times r_2}{m_1 \times m_1} = \frac{N\text{-}m^2}{kg \,.\, kg}$$

$$G = Nm^2/kg^2$$

Result

$$\text{Dimensions of } G = [M^{-1}L^3T^{-2}]$$

$$\text{Unit of } G = N\text{-}m^2/kg^2$$

PROBLEM 1.7

Show that the expression $v_f = v_i + at$ dimensionally correct, where v_1 is the velocity at t = 0, a is acceleration and v_f is the velocity at time t.

Data

The given equation is

$$v_f = v_i + at$$

To Find

Is the equation dimensionally correct = ?

SOLUTION

$$\text{Now} \quad v_f = v_i + at$$

In unit form

$$\frac{m}{s} = \frac{m}{s} + \frac{m}{s^2} \times s$$

$$\frac{m}{s} = \frac{m}{s} + \frac{m}{s}$$

$$\frac{L}{T} = \frac{L}{T} + \frac{L}{T}$$

Where 2 is constant so it is dimensionless

$$\frac{L}{T} = \frac{L}{T}$$

$$[LT^{-1}] = [LT^{-1}]$$

$$[LT^{-1}] = [LT^{-1}]$$

Result

Hence the equation $v_f = v_i + at$ is dimensionally correct.

PROBLEM 1.8

The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.

SOLUTION

As we know that the speed of sound depends upon the following two factors

(i) Density ρ^a and (ii) Elasticity E^b

Since $v \propto \rho^a E^b$

$$v = \text{Constant} \times \rho^a E^b \quad \ldots\ldots \text{(i)}$$

Writing dimensions of quantities on both the sides.

Dimensions of velocity $v = \left[\dfrac{S}{T}\right] = [LT^{-1}]$

The dimensions of density $\rho = \dfrac{\text{Mass}}{\text{Volume}}$

$$\text{Unit of } \rho = \frac{kg}{m^3}$$

$$[\,\rho\,] = [ML^{-3}]$$

and Dimensions of elasticity $E = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F}{A}$

Where strain has no dimensions

$$E = \frac{ma}{A}$$

$$\text{Unit of E} = \frac{kg\ m/s^2}{m^2}$$

$$[\,E\,] = \frac{kg}{m.s^2} = [ML^{-1}T^{-2}]$$

Putting in equation (i)

$$[LT^{-1}] = \text{Constant } [ML^{-3}]^a\,[ML^{-1}T^{-2}]^b$$

$$[LT^{-1}] = \text{Constant} \times [M^aL^{-3a}][M^bL^{-b}T^{-2b}]$$

$$= \text{Constant} \times [M^{a+b}\,L^{-3a-b}\,T^{-2b}]$$

Comparing the exponents

For L $\quad -3a - b = 1$

For T $\quad -2b = -1$

For M $\quad a + b = 0$

As $\quad -2b = -1$

$$b = \frac{1}{2}$$

and $\quad a + b = 0$

$$a + \frac{1}{2} = 0$$

$$\boxed{a = -\frac{1}{2}}$$

Putting the values in eq. (i)

$$v = \text{Constant} \times \rho^{-1/2} E^{1/2}$$

$$v = \text{Constant} \times \frac{E^{1/2}}{\rho^{1/2}}$$

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

Result

The formula for the speed of sound is

$$v = \text{Constant} \times \sqrt{\frac{E}{\rho}}$$

PROBLEM 1.9

Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

SOLUTION

The given equation is

$$E = mc^2$$

Writing the dimension of both sides

$$\text{Dimension of energy (E)} = \text{Work} = F \,.\, d$$

$$= ma \,.\, d$$

$$\text{Unit of work} = kg \frac{m}{s^2} \,.\, m$$

$$= kg \frac{m^2}{s^2}$$

$$[W] = [ML^2T^{-2}] \quad \ldots\ldots (i)$$

$$\text{Unit of } mc^2 = kg \left(\frac{m}{s}\right)^2$$

$$= kg \frac{m^2}{s^2}$$

$$[mc^2] = [ML^2T^{-2}] \quad \ldots\ldots (ii)$$

From eq. (i) and (ii)

$$[ML^2T^{-2}] = [ML^2T^{-2}]$$

Result

Hence the Einstein's equation $E = mc^2$ is dimensionally consistent.

PROBLEM 1.10

Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v?

SOLUTION

According to statement, the acceleration of particle moving in a circle can be written as

$$a \propto r^n v^m$$

$$a = \text{Constant} \times r^n v^m \qquad \ldots\ldots \text{(i)}$$

Writing the dimension of both sides

Dimensions of acceleration $= [\,a\,] = [LT^{-2}]$

Dimensions of radius $= [\,r\,] = m = [\,L\,]$

Dimensions of velocity $= [\,v\,] = [LT^{-1}]$

Putting in eq. (i)

$$[LT^{-2}] = \text{Constant} \times [\,L\,]^n\,[LT^{-1}]^m$$

$$[LT^{-2}] = \text{Constant} \times [\,L^n\,]\,[L^m T^{-m}]$$

$$[LT^{-2}] = \text{Constant} \times [L^{n+m}\,T^{-m}]$$

Comparing the exponents

$$n + m = 1$$

$$-m = -2$$

$$\boxed{m = 2}$$

Putting in above

$$n + 2 = 1$$

$$n = 1 - 2$$

$$\boxed{n = -1}$$

Putting in eq. (i)

$$a = \text{Constant} \times r^{-1} v^2$$

$$a = \text{Constant} \times \frac{v^2}{r}$$

Result

The acceleration of a particle moves with velocity in a circle of radius r is

$$a = \text{Constant} \times \frac{v^2}{r}$$

SHORT QUESTIONS

2.1 Define the terms (i) unit vector (ii) Position vector (iii) Components of a vector.

Ans. **(i) Unit Vector:** A vector whose magnitude is one called unit vector. It is used to find the direction of a vector. The formula for the unit vector is

$$\hat{A} = \frac{\vec{A}}{|A|}$$

(ii) Position Vector: It is a vector that describe the location of a particle with respect to origin.

The position vector $\vec{r}$ of point P(a, b) in x-y plane is given by

$$\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{k}$ is the unit vector along z-axis. In three dimension, the position and $\vec{r}$ from origin will

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z-axis respectively.

(iii) Component of a Vector: The part of a vector effective in a particular direction is called the components of a vector. Usually a vector has two or more components, one along x-axis is called horizontal component and other along y-axis is called vertical component.

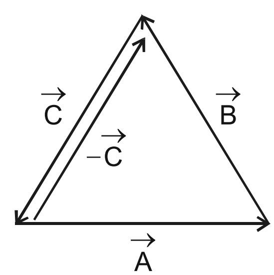
2.2 The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

Ans. The resultant of three vectors of equal magnitudes is equal to zero if they are represented by the three adjacent sides of a triangle as shown. If we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. By using head to tail rule where $-\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$.

Hence

$$-\vec{C} = \vec{A} + \vec{B}$$

$$\vec{A} + \vec{B} + \vec{C} = 0$$

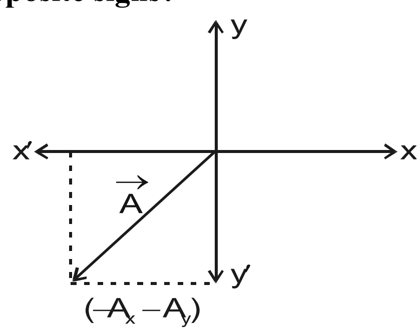


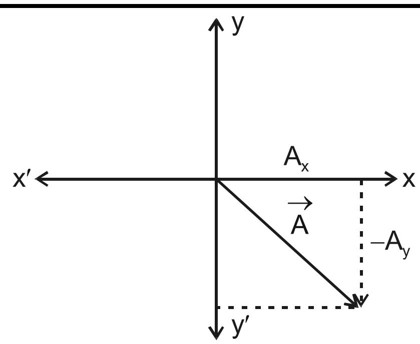
Thus the vector sum of three vectors is zero.

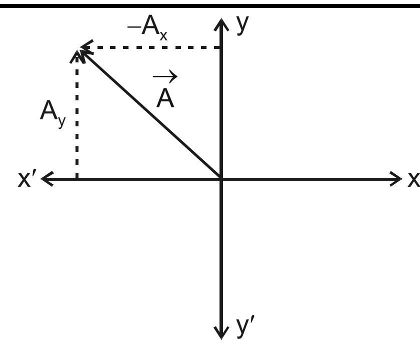
2.3 Vector A lies in the xy-plane. For what orientation will both of its rectangular components be negative. For what orientation will its components have opposite signs?

Ans. ***Case-I:*** If a vector $\vec{A}$ lies in third quadrant then both of its rectangular components A_x and A_y will be negative as shown.

Case-II: If a vector $\vec{A}$ lies in second and fourth quadrant then both of it, rectangular components A_x and A_y have in opposite sign as shown.







2.4 If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explains.

Ans. No, its magnitude cannot be zero because, the magnitude of vector contains the sum of square of its components. So if one of the components of a vector is not zero and even if they have the opposite signs then the magnitude of a vector cannot be zero. According to formula

$$A = \sqrt{A_x^2 + A_y^2}$$

If $A_y = 0$

Then $A = \sqrt{A_x^2 + 0^2}$

$$A = \sqrt{A_x^2}$$

$$A = A_x$$

$\therefore A \neq 0$

So if one of the rectangular components of a vector is not zero then its magnitude cannot be zero.

2.5 Can a vector have a component greater than the vector's magnitude?

Ans. (i) No, the magnitude of a vector cannot have a component greater than its magnitude because the components of a vector is always less in magnitude of resultant vector. Only in case of equilateral triangle, they are equal.

(ii) Yes, the statement is correct if we do not take the case of rectangular component. So a vector has a component greater than vector magnitude.

2.6 Can the magnitude of a vector have a negative value?

Ans. No, the magnitude of a vector cannot have a negative value. The magnitude of a vector always has a positive value. For example, if we have a vector $-3\vec{A}$, where 3 is the magnitude of a vector and the negative sign shows its direction.

(OR)

As magnitude of $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2}$$

Hence magnitude of a vector cannot have a negative value. e.g.,

If $A_x = -5$ and $A_y = 2$

then $A = \sqrt{(-5)^2 + (2)^2}$

$$= \sqrt{25 + 4}$$

$$= \sqrt{29}$$

2.7 If $\vec{A} + \vec{B} = \vec{0}$, what can you say about the components of the two vectors?

Ans. If
$$\vec{A} + \vec{B} = \vec{0}$$
$$\vec{A} = -\vec{B}$$

In case of rectangular components

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$
$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -B_x\hat{i} - B_y\hat{j} - B_z\hat{k}$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$A_x = -B_x$$
$$A_y = -B_y$$
$$A_z = -B_z$$

So it means that if the sum of the two vectors is zero then their rectangular components will be of the same magnitude but in opposite direction.

2.8 Under what circumstances would a vector have components that are equal in magnitude?

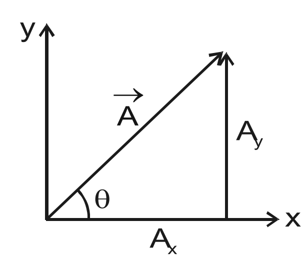
Ans. If θ be the angle which vector $\vec{A}$ makes with horizontal line having components A_x and A_y then

$$\tan\theta = \frac{A_y}{A_x}$$
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

According to question

$$A_x = A_y$$

then
$$\theta = \tan^{-1}\left(\frac{A_y}{A_y}\right)$$
$$\theta = \tan^{-1}(1)$$
$$\theta = 45°$$



So, if $\vec{A}$ makes an angle of 45° with x-axis then its both components will be equal in magnitude.

2.9 Is it possible to add a vector quantity to a scalar quantity? Explain.

Ans. No, a vector quantity cannot be added to a scalar quantity because scalar has only magnitude while vector has both the magnitude and direction. So they cannot be added to each other.

2.10 Can you add zero to a null vector?

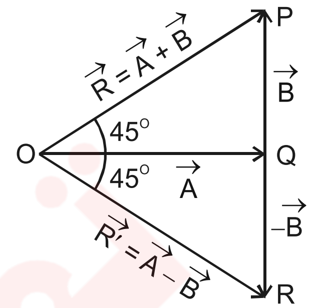
Ans. No, zero is not added to a null vector because zero is a scalar and null vector is a vector quantity.

2.11 Two vectors have unequal magnitudes. Can their sum be zero? Explain.

Ans. No, the sum of two vectors of unequal magnitude cannot be zero. It is only possible when two vectors have same magnitude and in opposite direction.

2.12 Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ having equal magnitudes as shown. From head to tail rule, $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are the resultant having same magnitude because they are the hypotenuse of right angled triangle.



Since in ΔOPQ

$$\angle POQ = \angle QPO$$
$$= 45°$$

and magnitude of $\vec{R}$ will be

$$|\vec{R}| = \sqrt{A^2 + B^2}$$

and in ΔOQR

$$\angle ROQ = \angle ORQ = 45°$$

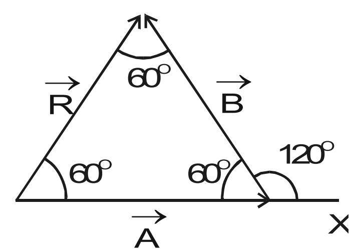
So $\angle POQ + \angle ROQ = 45° + 45°$

$$= 90°$$

Hence the resultants $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are perpendicular to each other and they are in equal length.

2.13 How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude.

Ans. If two vectors $\vec{A}$ and $\vec{B}$ make an angle of 120°. Then their resultant would have the same magnitude as that of $\vec{A}$ and $\vec{B}$.



We know that:

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB + \cos\theta}$$

According to given condition A = B = R.

$$\because \quad R = \sqrt{R^2 + R^2 + 2R^2 \cos\theta}$$

$$R^2 = 2R^2 + 2R^2 \cos\theta$$

$$R^2 - 2R^2 = 2R^2 \cos\theta$$

$$\frac{-R^2}{2R^2} = \cos\theta \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120°$$

2.14 The two vectors to be combined have magnitudes 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct.

(i) 100N (ii) 70N (iii) 20N

Ans. When 60 N and 35 N forces are in same direction then maximum resultant is

$$= 60 + 35 = 95 \text{ N}$$

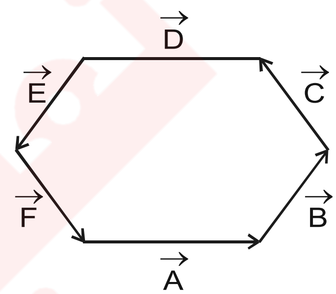
When 60 N and 35 N forces are in opposite direction then minimum resultant is

$$= 60 - 35 = 25 \text{ N}$$

Hence the range of the force is 25 N – 95 N then (ii) 70 N is correct.

2.15 Suppose the sides of a closed polygon represent vector arranged head to tail? What is the sum of these vectors?

Ans. We know that if the vectors are arranged by head to tail rule, which makes a closed polygon then its resultant is zero because there is no place to draw resultant. Consider $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, $\vec{E}$ and $\vec{F}$ are the vectors which are arranged by head to tail rule then their resultant is zero i.e.,



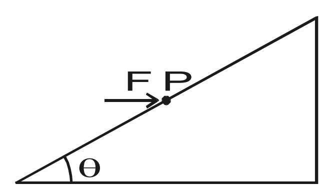
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = 0$$

2.16 Identify the correct answer:

(i) Two ships X and Y are traveling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be.

(a) East (b) West (c) South-east (d) South-west

(ii) A horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at an angle θ to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is:



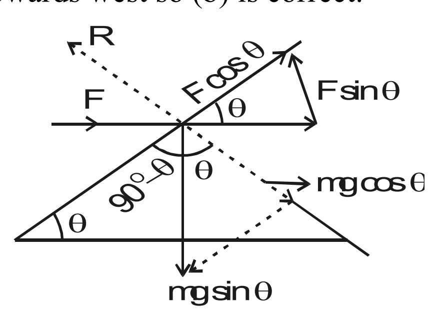
(a) $F\cos\theta - mg\sin\theta$ (b) $F\sin\theta - mg\cos\theta$

(c) $F\cos\theta + mg\cos\theta$ (d) $F\sin\theta + mg\sin\theta$

(e) $mg\tan\theta$

Ans. (i) We know that the ship x is moving towards North from shore and according to observer on ship y, the ship x is moving towards north-east direction so ship y is approaching towards the line of motion of ship x. Thus the motion of ship y is towards west so (b) is correct.

(ii) Now the horizontal force F and weight mg of the body can be resolved into its rectangular components as shown. The force acting up along the plane of the surface is



$$= F\cos\theta - mg\sin\theta$$

So (a) is correct.

2.17 If all the components of the vectors, $\vec{A}_1$ and $\vec{A}_2$ were reversed, how would this alter $\vec{A}_1 \times \vec{A}_2$?

Ans. The vectors $\vec{A}_1$ and $\vec{A}_2$ can be resolved into its rectangular components i.e.,

$$\vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} + A_{1z}\hat{k}$$

$$\vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{j} + A_{2z}\hat{k}$$

$$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

On reversing the components

$$\vec{A}_1 = -A_{1x}\hat{i} - A_{1y}\hat{j} - A_{1z}\hat{k}$$

$$\vec{A}_2 = -A_{2x}\hat{i} - A_{2y}\hat{j} - A_{2z}\hat{k}$$

So $$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -A_{1x} & -A_{1y} & -A_{1z} \\ -A_{2x} & -A_{2y} & -A_{2z} \end{vmatrix}$$

Taking (–1) as common from R_1 and R_2

$$\vec{A}_1 \times \vec{A}_2 = (-)(-)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

$$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

Hence if the components of the vectors are reversed then there is no effect on $\vec{A}_1 \times \vec{A}_2$.

2.18 Name the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = 0$.

Ans. We know that

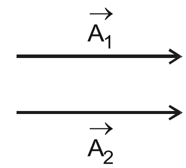
$$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta\, \hat{n}$$

Therefore the required three conditions are

(i) Either $\vec{A}_1$ or $\vec{A}_2$ is null vector i.e., $|\vec{A}_1| = |\vec{A}_2| = 0$.

$$\vec{A}_1 \times \vec{A}_2 = 0$$

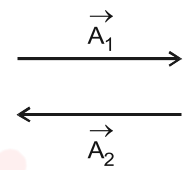
(ii) $\vec{A}_1$ and $\vec{A}_2$ are parallel i.e., $\theta = 0°$.



$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin 0° \, \hat{n} \qquad \because \sin 0° = 0$$

$$\vec{A}_1 \times \vec{A}_2 = 0$$

(iii) $\vec{A}_1$ and $\vec{A}_2$ are antiparallel i.e., $\theta = 180°$



$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin 180° \, \hat{n}$$

$$\Rightarrow \quad \vec{A}_1 \times \vec{A}_2 = 0 \qquad \because \sin 180° = 0$$

2.19 Identify true or false statements and explain the reason:

(a) A body in equilibrium implies that it is not moving nor rotating.

(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

Ans. (a) This statement is false because the body is said to be in equilibrium if it is moving with constant velocity or rotating with constant angular velocity.

(OR)

This statement is true because when a body is not moving nor rotating then it is in static equilibrium.

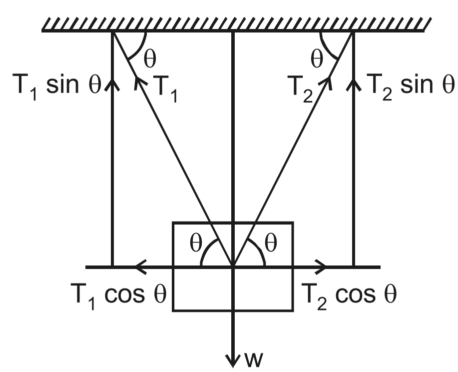
(b) This statement is true because when coplanar forces (vectors) acting on a body in the form of a closed polygon then $\sum \vec{F} = \vec{0}$ i.e., 1^{st} condition of equilibrium, is satisfied so the body is in translational equilibrium.

(OR)

This statement is false because there may be any torque due to these forces i.e., 2^{nd} condition of equilibrium is not satisfied so the body is not in complete equilibrium.

2.20 A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the settings will be minimum.

Ans. Let we suspend the picture from the wall by two strings as shown in figure.



Let T_1 and T_2 be the tension in string from figure.

Along x-axis

$$T_1 \cos \theta = T_2 \cos \theta$$

$$\Rightarrow \quad T_1 = T_2 = T$$

Along y-axis

$$T_1 \sin \theta + T_2 \sin \theta = w$$

$$T \sin \theta + T \sin \theta = w \qquad \because T_1 = T_2 = T_3$$

$$T = \frac{w}{2 \sin \theta}$$

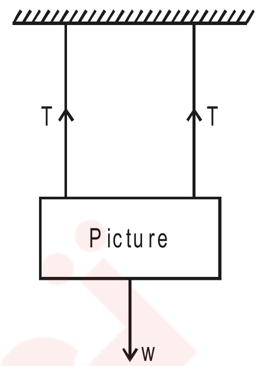
For minimum tension sin θ should have max. value and max. value of sin θ = 1.

$$\Rightarrow \quad \sin \theta = 1$$

$$\sin \theta = \sin 90°$$

$$\theta = 90°$$

So for minimum tension θ should be 90° as shown in figure.



From figure

$$2T = w$$

$$T = \frac{w}{2}$$

So only in this case, the tension in both the string will be minimum.

2.21 Can a body rotate about its centre of gravity under the action of its weight?

Ans. No, a body cannot rotate about its centre of gravity under the action of weight because the line of action of force (weight) passes through its center of gravity (pivot) i.e., movement arm r = 0.

So
$$\tau = rF$$

$$\tau = (0) F$$

$$\tau = 0$$

So the torque is zero.

PROBLEMS WITH SOLUTIONS

PROBLEM 2.1

Suppose, in a rectangular coordinates system, a vector $\vec{A}$ has its tail at the point P(–2, –3) and its tip at Q(3, 9). Determine the distance between these two points?

Data

The given points are

P(–2, –3) and Q(3, 9)

To Find

Distance between P and Q = r = ?

SOLUTION

Position vector of a $= \vec{r}_1 = -2\hat{i} - 3\hat{j}$

Position vector of b $= \vec{r}_2 = 3\hat{i} + 9\hat{j}$

$$\Rightarrow \quad \vec{A} = \vec{r}_2 - \vec{r}_1$$

$$= 3\hat{i} + 9\hat{j} - (-2\hat{i} - 3\hat{j})$$

$$= 3\hat{i} + 9\hat{j} + 2\hat{i} + 3\hat{j}$$

$$\vec{A} = 5\hat{i} + 12\hat{j} \qquad \ldots\ldots \text{(i)}$$

The magnitude of $\vec{A}$ is:

$$A = \sqrt{(5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

Result

Distance between P and Q = r = 13 units

PROBLEM 2.2

A certain corner of a room is selected as the origin of the rectangular coordinate system. If an insect is crawling on an adjacent wall at a point having coordinates (2, 1), where the units are in meters? What is the distance of the insect from this corner of the room?

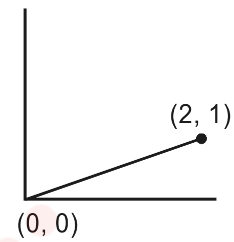
Data

In a coordinate system, the given point is (2, 1)

To Find

Distance of insect from the corner of room = r = ?

SOLUTION



Let the corner of the room whose coordinate are (0, 0)

The position vector is

$$\vec{r} = 2\hat{i} + 1\hat{j}$$

The magnitude of position vector $\vec{r}$ is

$$|\vec{r}| = \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{4 + 1}$$

$$= \sqrt{5}$$

$$r = 2.24 \text{ m}$$

Result

Distance of insect = r = 2.24 m

PROBLEM 2.3

What is the unit vector in the vector $\vec{A} = 4\hat{i} + 3\hat{j}$?

Data

The given vector is

$$\vec{A} = 4\hat{i} + 3\hat{j}$$

To Find

Unit vector = $\hat{A}$ = ?

SOLUTION

By formula

$$\hat{A} = \frac{\vec{A}}{|A|}$$

$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{4^2 + 3^2}}$$

$$= \frac{4\hat{i} + 3\hat{j}}{\sqrt{16 + 9}}$$

$$\hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}$$

Result

$$\text{Unit vector} = \hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}$$

PROBLEM 2.4

Two particles are located at $\vec{r}_1 = 3\hat{i} + 7\hat{j}$ and $\vec{r}_2 = -2\hat{i} + 3\hat{j}$ respectively. Find both the magnitude of vector $(\vec{r}_2 - \vec{r}_1)$ and its orientation with respect to the x-axis.

Data

The position vectors are

$$\vec{r}_1 = 3\hat{i} + 7\hat{j}$$

$$\vec{r}_2 = -2\hat{i} + 3\hat{j}$$

To Find

$$\text{Magnitude of vector} = \vec{r}_2 - \vec{r}_1 = ?$$

$$\text{Direction} = \theta = ?$$

SOLUTION

As we know that

$$\vec{r}_2 - \vec{r}_1 = -2\hat{i} + 3\hat{j} - (3\hat{i} + 7\hat{j})$$

$$= -2\hat{i} + 3\hat{j} - 3\hat{i} - 7\hat{j}$$

$$= -5\hat{i} - 4\hat{j}$$

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{(-5)^2 + (-4)^2}$$

$$= \sqrt{25 + 16} = \sqrt{41}$$

$$|\vec{r}_2 - \vec{r}_1| = 6.4$$

Orientation w.r.t. x-axis:

$$\phi = \tan^{-1}\left(\frac{\text{y-component}}{\text{x-component}}\right)$$

$$= \tan^{-1}\left(\frac{4}{5}\right)$$

$$\phi = 39°$$

As both components are –ive so $(\vec{r}_2 - \vec{r}_1)$ lies in 3rd quadrant.

$$\Rightarrow \quad \theta = 180 + \phi$$
$$= 180 + 39$$
$$\theta = 219°$$

Result

Magnitude of vector $\vec{r}_2 - \vec{r}_1 = 6.4$

Orientation $= \theta = 219°$

PROBLEM 2.5

If a $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i} + \hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4\hat{i} + 7\hat{j}$. What is the magnitude of vector $\vec{A}$?

Data

If $\vec{B}$ is added to $\vec{A}$

$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j}$$

If $\vec{B}$ is subtracted from $\vec{A}$

$$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}$$

To Find

Magnitude of vector $\vec{A} = ?$

SOLUTION

Then
$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j}$$
$$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j}$$
$$2\vec{A} = 2\hat{i} + 8\hat{j}$$
$$2\vec{A} = 2(\hat{i} + 4\hat{j})$$
$$\vec{A} = \hat{i} + 4\hat{j}$$

Magnitude of vector $\vec{A} = \sqrt{(1)^2 + (4)^2}$
$$= \sqrt{1 + 16}$$
$$= \sqrt{17}$$
$$= 4.12$$

Result

Magnitude of vector $\vec{A} = 4.12$

PROBLEM 2.6

Given that $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} - 4\hat{j}$, find the magnitude and direction of

(a) $\vec{C} = \vec{A} + \vec{B}$ (b) $\vec{D} = 3\vec{A} - 2\vec{B}$

Data

The given vectors are

$$\vec{A} = 2\hat{i} + 3\hat{j}$$

$$\vec{B} = 3\hat{i} - 4\hat{j}$$

To Find

(a) Magnitude of $\vec{C} = \vec{A} + \vec{B} = ?$

Direction of $\vec{C} = ?$

(b) Magnitude of $\vec{D} = 3\vec{A} - 2\vec{B} = ?$

Direction of $\vec{D} = ?$

SOLUTION

(a) Now $\vec{C} = \vec{A} + \vec{B}$

Putting the values

$$\vec{C} = 2\hat{i} + 3\hat{j} + 3\hat{j} - 4\hat{j}$$

$$\vec{C} = 5\hat{i} - \hat{j}$$

Magnitude of $\vec{C} = \sqrt{(5)^2 + (-1)^2}$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$= 5.1$$

For direction, $\tan\phi = \dfrac{C_y}{C_x}$

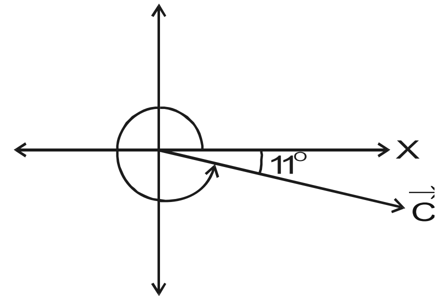
$$\phi = \tan^{-1}\left(\frac{C_y}{C_x}\right)$$ where $C_y = -1$, $C_x = 5$

So $\phi = \tan^{-1}\left(\dfrac{1}{5}\right)$

$$\phi = 11°$$

Since C_x is positive and C_y is negative.

So $\vec{C}$ lies in 4th quadrant.



$$\theta = 360° - 11°$$
$$\theta = 349°$$

(b) Now
$$\vec{D} = 3\vec{A} - 2\vec{B}$$
$$= 3(2\hat{i} + 3\hat{j}) - 2(3\hat{i} - 4\hat{j})$$
$$= 5\hat{i} + 9\hat{j} - 6\hat{i} + 8\hat{j}$$
$$= 17\hat{j}$$

So the magnitude of $\vec{D} = \sqrt{(17)^2}$
$$= 17$$

And the direction of $\vec{D}$ is

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right)$$ where $D_y = 17$, $D_x = 0$

$$\theta = \tan^{-1}\left(\frac{17}{0}\right)$$
$$= \tan^{-1}(\infty)$$
$$\theta = 90°$$

Result

(a) Magnitude of vector $\vec{C} = 5.0$

Direction of vector $\vec{C} = \theta = 349°$

(b) Magnitude of vector $\vec{D} = 17$

Direction of vector $\vec{D} = \theta = 90°$

PROBLEM 2.7

Find the angle between two vectors $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$.

Data

The given vectors are

$$\vec{A} = 5\hat{i} + \hat{j}$$
$$\vec{B} = 2\hat{i} + 4\hat{j}$$

To Find

Angle between the vectors $= \theta = ?$

SOLUTION

As we know that

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\boxed{\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}} \qquad \ldots\ldots \text{(i)}$$

where $\vec{A} \cdot \vec{B} = (5\hat{i} + \hat{j}) \cdot (2\hat{i} + 4\hat{j})$

$$= 10(\hat{i} \cdot \hat{i}) + 20(\hat{i} \cdot \hat{j}) + 2(\hat{j} \cdot \hat{i}) + 4(\hat{j} \cdot \hat{j})$$

Since $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

So $\vec{A} \cdot \vec{B} = 10 + 4 = 14$

The magnitude of vector $\vec{A} = \sqrt{5^2 + 1^2}$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$$A = 5.1$$

The magnitude of vector $\vec{B} = \sqrt{2^2 + 4^2}$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$B = 4.5$$

Putting the value in eq. (i)

$$\cos\theta = \frac{14}{(5.1)(4.5)}$$

$$\cos\theta = 0.61$$

$$\theta = \cos^{-1}(0.61)$$

$$\theta = 52°$$

Result

Angle between vectors $= \theta = 52°$

PROBLEM 2.8

Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line from the point (2, –1) to the point (6, 4).

Data

$$\text{Force} = \vec{F} = 3\hat{i} + 2\hat{j}$$

and the given points are A(2, –1), B(6, 4).

To Find

Work done = W = ?

SOLUTION

By formula

$$W = \vec{F} \cdot \vec{d}$$

Where $\vec{d}$ is distance between two given points

$$\text{So Coordinates } \vec{d} = (6, 4) - (2, -1)$$
$$= (6 - 2, 4 + 1)$$
$$= (4, 5)$$

$$\text{Now } \vec{d} = 4\hat{i} + 5\hat{j}$$

$$\text{Therefore } W = (3\hat{i} + 2\hat{j}) \cdot (4\hat{i} + 5\hat{j})$$
$$= 12(\hat{i} \cdot \hat{i}) + 10(\hat{j} \cdot \hat{j})$$
$$= 12 + 10$$
$$= 22 \text{ Joule}$$

Result

Work done = W = 22 J

PROBLEM 2.9

Show that the three vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Data

The given vectors are

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$$

To Find

Are the vectors mutually perpendicular = ?

SOLUTION

If $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C} = \vec{C} \cdot \vec{A} = 0$ so the vectors are mutual perpendicular.

Thus
$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$$
$$= 2(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) + 1(\hat{k} \cdot \hat{k})$$
$$= 2 - 3 + 1$$
$$\vec{A} \cdot \vec{B} = 0$$

and
$$\vec{B} \cdot \vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} - 5\hat{k})$$
$$= 8(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$$
$$= 8 - 3 - 5$$
$$\vec{B} \cdot \vec{C} = 0$$
$$\vec{C} \cdot \vec{A} = (4\hat{i} + \hat{j} - 5\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$
$$= 4(\hat{i} \cdot \hat{i}) + 1(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$$
$$= 4 + 1 - 5$$
$$\vec{C} \cdot \vec{A} = 0$$

Hence
$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = \vec{C} \cdot \vec{A} = 0$$

Result

So the given three vectors are mutually perpendicular.

PROBLEM 2.10

Given that $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{k}$, find the length of the projection of $\vec{A}$ on $\vec{B}$.

Data

The given vectors are

$$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$$
$$\vec{B} = 3\hat{i} - 4\hat{k}$$

To Find

Projection of $\vec{A}$ on $\vec{B}$ = $A \cos \theta$ = ?

SOLUTION

By formula

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\boxed{A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}}$$

Now
$$\vec{A} \cdot \vec{B} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 4\hat{k})$$
$$= 3(\hat{i} \cdot \hat{i}) - 12(\hat{k} \cdot \hat{k})$$
$$= 3 - 12$$
$$= -9$$

And magnitude of vector $\vec{B}$

$$B = \sqrt{(3)^2 + (-4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

Putting in above equation

$$A\cos\theta = \frac{-9}{5}$$

Result

Projection of $\vec{A}$ on $\vec{B}$ $= A\cos\theta = -\frac{9}{5}$

PROBLEM 2.11

Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 unit north, 3 unit west and 8 unit east, respectively. Describe carefully (a) $\vec{A} \times \vec{B}$ (b) $\vec{A} \times \vec{C}$ (c) $\vec{B} \times \vec{C}$.

Data

The given vectors are

$$\vec{A} = 4 \text{ units north}$$

$$\vec{B} = 3 \text{ units west}$$

$$\vec{C} = 8 \text{ units east}$$

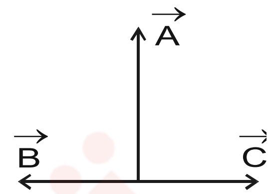
To Find

(a) $\vec{A} \times \vec{B} = ?$

(b) $\vec{A} \times \vec{C} = ?$

(c) $\vec{B} \times \vec{C} = ?$

SOLUTION



(a) As $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

$\vec{A} \times \vec{B} = (4)(3) \sin 90^\circ\, \hat{n}$

$=$ 12 units vertically upward according to right hand rule

(b) $\vec{A} \times \vec{C} = AC \sin\theta\, \hat{n}$

$= (4)(8) \sin 90^\circ\, \hat{n}$

$=$ 32 units vertically downward

(c) $\vec{B} \times \vec{C} = BC \sin\theta\, \hat{n}$

$= (3)(8) \sin 180^\circ\, \hat{n}$

$= 24(0)\, \hat{n}$

$\vec{B} \times \vec{C} = \vec{0}$

Result

(a) $\vec{A} \times \vec{B} =$ 12 units vertically upward

(b) $\vec{A} \times \vec{C} =$ 32 units vertically downward

(c) $\vec{B} \times \vec{C} = \vec{0}$ (Null vector)

PROBLEM 2.12

The torque or turning effect of force about a given point is given by $\vec{r} \times \vec{F}$ where $\vec{r}$ is the vector from the given point to the point of application of $\vec{F}$. Consider a force $\vec{F} = -3\hat{i} + \hat{j} + \hat{k}$ acting on the point $7\hat{i} + 3\hat{j} + \hat{k}$ (m). What is the torque in Nm about the origin?

Data

Force $= \vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$

Position vector $= \vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

To Find

Torque acting $= \vec{\tau} = \vec{r} \times \vec{F} = ?$

SOLUTION

As $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} 7 & 1 \\ -3 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} 7 & 3 \\ -3 & 1 \end{vmatrix}$$

$$= \hat{i}\,(15 - 1) - \hat{j}\,(35 + 3) + \hat{k}\,(7 + 9)$$

$$= 14\hat{i} - 38\hat{j} + 16\hat{k}$$

Result

Torque acting $= \vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$ (Nm)

PROBLEM 2.13

The line of action of force $\vec{F} = \hat{i} - 2\hat{j}$, passes through the point whose position vector is $(-\hat{j} + \hat{k})$. Find (a) the moment of F about the origin (b) the moment of $\vec{F}$ about the point of which the position vector is $\hat{i} + \hat{k}$.

Data

Force $= \vec{F} = \hat{i} - 2\hat{j}$

Position vector $= \vec{r} = -\hat{j} + \hat{k}$

To Find

(a) Moment of force about origin $= \vec{\tau} = ?$

(b) Moment of force about $\hat{i} + \hat{k} = \vec{\tau}' = ?$

SOLUTION

(a) First for the moment of force about origin is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 1 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \hat{i}\,(0 + 2) - \hat{j}\,(0 - 1) + \hat{k}\,(0 + 1)$$

$$\vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$$

(b) The moment of force about $\hat{i} + \hat{k}$ is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

So $$\vec{r}' = (-\hat{j} + \hat{k}) - (\hat{i} + \hat{k})$$

$$= -\hat{j} + \hat{k} - \hat{i} - \hat{k}$$

$$= -\hat{i} - \hat{j}$$

So $$\vec{\tau}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 0 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= \hat{i}\,(0) - \hat{j}\,(0) + \hat{k}\,(2 + 1)$$

$$= 0\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\vec{\tau}' = 3\hat{k}$$

Result

(a) Moment of force about origin $= \vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$

(b) Moment of force about $\hat{i} + \hat{k} = \vec{\tau}' = 3\hat{k}$

PROBLEM 2.14

The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vector.

Data

Magnitude of dot product $= \vec{A}\cdot\vec{B} = 6\sqrt{3}$

Magnitude of vector product $= |\vec{A}\times\vec{B}| = 6$

To Find

Angle between the vectors $= \theta = ?$

SOLUTION

As $\quad \vec{A}\cdot\vec{B} = AB\cos\theta$ ...... (i)

$|\vec{A}\times\vec{B}| = AB\sin\theta$ ...... (ii)

Divide eq. (ii) by (i)

$$\frac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}} = \frac{AB\sin\theta}{AB\cos\theta}$$

$$\frac{6}{6\sqrt{3}} = \tan\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

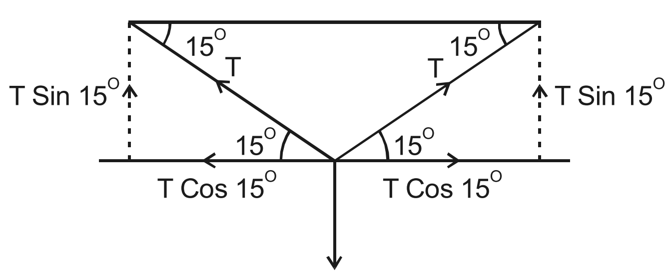
$$\theta = 30^\circ$$

Result

Angle between vectors $= \theta = 30^\circ$

PROBLEM 2.15

A load of 10N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.



Data

Load $= W = 10\text{ N}$

Angle with horizontal $= \theta = 15^\circ$

To Find

Tension in the string $= T = ?$

SOLUTION

According to 1st condition of equilibrium.

Now using $\sum F_y = 0$ and $\sum F_x = 0$

So $T_1 \sin\theta + T_2 \sin\theta - \text{Load} = 0$ $\qquad T_2 \cos 15° - T_1 \cos 15° = 0$

$2T \sin\theta = \text{Load}$ $\qquad T_2 \cos 15° = T_1 \cos 15°$

$T = \dfrac{\text{Load}}{2\sin\theta}$ $\qquad T_1 = T_2$

$= \dfrac{5}{0.2588}$ $\qquad \therefore \quad T_1 = T_2 = T$

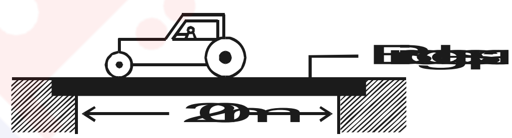
$T = 19.3\ \text{N}$

Result

Tension in string $= T = 19.3\ \text{N}$

PROBLEM 2.16

A tractor of weight 15000N crosses a single span bridge of weight 8000 N and of length 21m. The span bridge is supported half a metre from either end. The bridge is supported half a metre from either end. The tractors front wheels take 1/3 of the total weight of the tractor, and the rear wheels are 3m behind the front wheels. Calculate the force on the bridge supports when the rear wheels are at the middle of the bridge span.



Data

Weight of tractor $= W_1 = 15000\ \text{N}$

Weight of bridge $= W_2 = 8000\ \text{N}$

Length of bridge $= l_1 = 21\ \text{m}$

Length of tractor $= l_2 = 3.0\ \text{m}$

Weight of front wheels $= W_3 = \dfrac{1}{3} \times 15000$

$= 5000\ \text{N}$

Weight of rear wheels $= W_4 = 15000 - 5000$

$= 10000\ \text{N}$

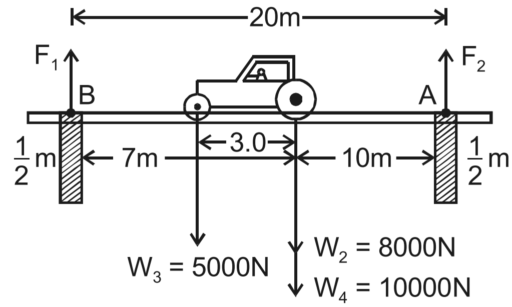
To Find

Forces on the bridge supports

$F_1 = ?$

and $F_2 = ?$

SOLUTION



By using 1st condition of equilibrium

$$\Sigma F_y = 0$$

$$F_1 + F_2 - 5000 - 8000 - 10000 = 0$$

$$F_1 + F_2 = 23000 \text{ N} \qquad \ldots\ldots \text{(i)}$$

According to 2nd condition of equilibrium

$$\Sigma \tau = 0$$

Suppose the point B is the point of rotation therefore

Torque of Force $F_1 = \tau_1 = F_1 \times 0$

$= 0$

Torque of Force $F_2 = \tau_2 = F_2 \times 20$

Torque of Force W_2 and $W_4 = \tau_3 = -18000 \times 10$

Torque of Force $W_3 = \tau_4 = -5000 \times 7$

Then $\tau_1 + \tau_2 + \tau_3 + \tau_4 = 0$

$$0 + F_2 \times 20 - 180000 - 35000 = 0$$

$$20F_2 = 215000$$

$$F_2 = \frac{215000}{20}$$

$$F_2 = 10750 \text{ N}$$

$$F_2 = 10.750 \times 10^3 \text{ N}$$

$$F_2 = 10.75 \text{ KN}$$

Putting in eq. (i)

$$F_1 + 10750 = 23000$$

$$F_1 = 23000 - 10750$$

$$= 12250 \text{ N}$$

$$F_1 = 12.25 \times 10^3 \text{ N}$$

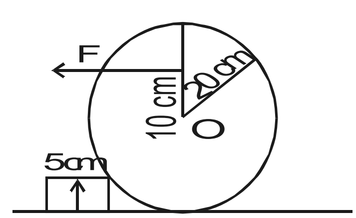
$$= 12.25 \text{ KN}$$

Result

Force on the bridge supports

$F_1 = 12.25$ KN and $F_2 = 10.75$ KN

PROBLEM 2.17



A spherical ball of weight 50N is to be lifted over the step as shown in the figure.

Calculate the minimum force needed just to lift it above the floor.

Data

Weight of the spherical ball = W = 50 N

Radius of the spherical ball = r = 20 cm

= 0.2 m

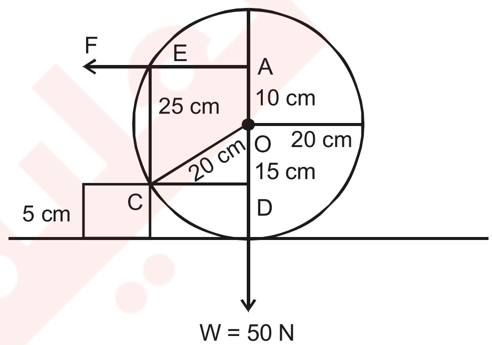
Height of the step = h = 5 cm

= 0.05 m

To Find

Minimum force required to lift the ball = F = ?

SOLUTION



Now consider the triangle ΔOCD is right angled triangle, by Pythagorean theorem

$$(OC)^2 = (OD)^2 + (CD)^2$$

$$(CD)^2 = (OC)^2 - (OD)^2$$

$$= (20)^2 - (15)^2$$

$$\sqrt{(CD)^2} = \sqrt{1750}$$

$$= 13.2 \text{ cm}$$

$$CD = 0.13 \text{ m}$$

By using 2nd condition of equilibrium

$$\sum\tau = 0$$

So $$-W \times CD + F \times CB = 0$$

$$-50 \times 0.13 + F \times 0.25 = 0$$

$$0.25\ F = 6.5$$

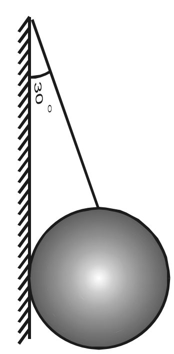
$$F = \frac{6.5}{0.25}$$

$$F = 26 \text{ N}$$

Result

Minimum force required to left ball = F = 26 N

PROBLEM 2.18



A uniform sphere of weight 10 N is held by a string attached to a frictionless wall so that the string makes an angle of 30° with the wall. Find the tension in the string and the force exerted on the sphere by the wall.

Data

Weight of the sphere = W = 10.0 N

Angle of string with wall = θ = 30°

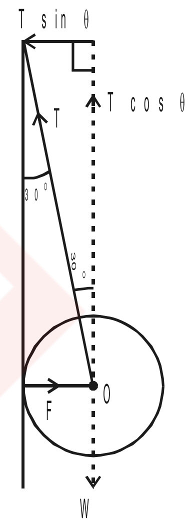
To Find

(a) Tension in the string = T = ?

(b) Force exerted by the wall = F = ?

SOLUTION

(a) Let F be the force exerted on sphere by wall, T is the tension in string and W is the weight of the sphere.



By using 1st condition of equilibrium

$$\Sigma F_y = 0 \quad \text{and} \quad \Sigma F_x = 0$$

So $F - T\sin\theta = 0$

$$F = T\sin\theta$$

and $T\cos\theta - W = 0$

$$T\cos\theta = W$$

$$T = \frac{W}{\cos\theta}$$

$$= \frac{10.0}{\cos 30} = \frac{10.0}{0.866}$$

$$T = 11.55\ \text{N}$$

and $F = T\sin\theta$

$$= 11.55 \sin 30^\circ$$

$$F = 5.8\ \text{N}$$

Result

(a) Tension in the string = T = 11.55 N

(b) Force exerted by the wall = F = 5.8 N

SHORT QUESTIONS

3.1 What is the difference between uniform and variable velocity. From the explanation of variable velocity, define acceleration. Give SI units of velocity and acceleration.

Ans. **Uniform Velocity:** The velocity of a body is said to be uniform if it covers equal displacement in equal interval of time.

Variable Velocity: The velocity of a body is said to be variable if it covers unequal displacement in unequal interval of time.

Acceleration: From the variable velocity, the rate of change of velocity is called acceleration.

Let a body is moving with velocity $\vec{v}_i$. After small time Δt its velocity changes from $\vec{v}_i$ to $\vec{v}_f$ then the change in velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. So

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

SI Unit of Velocity: The SI unit of velocity is m/s or km/hr.

SI Unit of Acceleration: The SI unit of acceleration is m/s^2.

3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

Ans. When an object is thrown vertically upward, the sign of acceleration due to gravity is negative relative to velocity. But when the object is thrown downward, the sign of acceleration due to gravity is taken as positive because velocity and acceleration are in same direction.

3.3 Can the velocity of an object reverse direction when acceleration is constant? If so, give an example.

Ans. Yes, the velocity of an object can reverse its direction when acceleration is constant.

Example: When an object is thrown vertically upward then during upward motion its velocity decreases, the direction of velocity will be in upward while direction of acceleration due to gravity will be in downward and when it reach at the highest point its velocity become zero but during downward of object the direction of velocity will be in downward while direction of acceleration due to gravity will again in downward thus we see that in this case the velocity reverse the direction while acceleration is constant.

3.4 Specify the correct statements:

(a) An object can have a constant velocity even its speed is changing.

(b) An object can have a constant speed even its velocity is changing.

(c) An object can have a zero velocity even its acceleration is not zero.

(d) An object subjected to a constant acceleration can reverse its velocity.

Ans. (a) It is false statement because an object cannot have a constant velocity even its speed is changing.

(b) It is true when the object is moving along a circular path.

(c) It is true because when an object is thrown vertically upward, at maximum height, velocity is zero but acceleration is not zero, it is a = g.

(d) It is true. Yes an object subjected to a constant acceleration can reverse its velocity.

3.5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have larger speed when it strikes the ground? Ignore air friction.

Ans. Both the balls have the same speed on striking the ground but time is different. When the velocity of the ball thrown upward with initial velocity v_i, it will has same velocity v_i when it return back and passes the man so as the initial velocities of a ball is same for both cases, therefore the final velocities will also be same.

3.6 Explain the circumstances in which the velocity "v" and acceleration "a" of a car are:

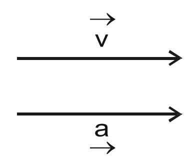
(i) Parallel **(ii) Anti-parallel**

(iii) Perpendicular to one another **(iv) "v" is zero but "a" is not**

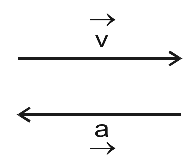
(v) "a" is zero but "v" is not zero

Ans. Following are the circumstances when velocity and acceleration of car:

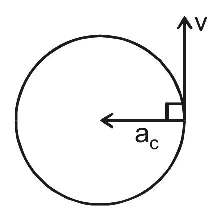
(i) Parallel: When the velocity of a car is increasing along a straight path then velocity and acceleration are parallel to each other.



(ii) Anti-parallel: When the velocity of car is decreasing along the straight line then velocity and acceleration are anti-parallel to each other.



(iii) Perpendicular to one another: The velocity and acceleration of a car are perpendicular to each other when the car is moving along a circular path.



(iv) v is zero but a is not zero: The velocity of a car becomes to zero when the brakes are applied and the car comes to rest due to acceleration in opposite direction.

(v) a is zero but v is not zero: Acceleration is zero when the car is moving with uniform acceleration.

3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

Ans. Yes, the motion with constant velocity is a special case of motion with constant acceleration. This statement is true.

Explanation: we know that when a body moves with constant velocity then its acceleration will be zero i.e., there is no rate of change of velocity so whenever it moves with constant velocity its acceleration will remain zero that is constant here zero is also a constant quantity. Therefore motion with constant velocity is a special case of motion with constant acceleration.

3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

Ans. Consider a body of mass "m" moving with velocity v_i. Let a force F is applied on the body which changes the velocity from v_i to v_f then according to 1st equation of motion.

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

But from Newton's second law of motion

$$F = ma$$

$$F = m\left(\frac{v_f - v_i}{t}\right)$$

$$F = \frac{mv_f - mv_i}{t}$$

Where mv_f is the final momentum and mv_i is the initial momentum so,

$$\frac{mv_f - mv_i}{t} = \text{Rate of change of momentum}$$

$$F = \text{Rate of changed momentum}$$

Newton's Second Law of Motion in Terms of Momentum: Newton's second law of motion in terms of momentum states, "the rate of change of momentum is equal to applied force".

3.9 Define impulse and show that how it is related to linear momentum?

Ans. **Impulse:** When a very large force acts on a body for a very short interval of time then the product of such a force and time is called impulse. It is a vector quantity

$$\text{Impulse} = I = \text{Force} \times \text{Time}$$

$$I = F \times \Delta t$$

As we know that

$$F \times \Delta t = mv_f - mv_i$$

So $$I = mv_f - mv_i$$

$$\vec{I} = m\Delta\vec{v}$$

$$\vec{I} = \Delta\vec{P} = \text{Change in momentum}$$

This shows that impulse is equal to change in momentum.

3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

Ans. **Law of Conservation of Linear Momentum:** This law states that the total linear momentum of an isolated system remains constant.

Importance of an Isolated System: This law holds good only for isolated system. An isolated system is one at which there is no external force acting. If the system is not isolated but the external forces are very small as compared to interacting forces so this law can also be applied on such a system.

3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

Ans. **Elastic Collision:** These collision in which kinetic energy remains constant is called elastic collisions.

Inelastic Collision: These collision in which kinetic energy does not remain constant is called inelastic collisions.

In Case of Bouncing Ball: If the ideal bouncing ball returns to the same height where it is dropped then the collision is elastic collision. If the bouncing ball will not returned to the same height then the collision is inelastic. So due to change of energy, kinetic energy does not remain constant.

For example; when a heavy ball is dropped on to the surface of earth, it rebounds upto very little height because maximum K.E is lost due to friction and also changes into heat and sound energies. So in most cases, the K.E is not conserved. Thus momentum and K.E are conserved in all types of collisions. However the K.E is conserved only in elastic collision.

3.12 Explain what is meant by projectile motion. Derive expressions for

(a) The time of flight (b) The range of projectile.

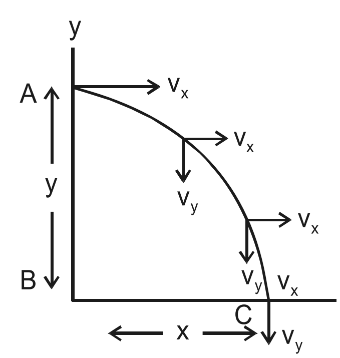
Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

Ans. **Projectile Motion:** When an object is thrown in air making a certain angle with horizontal, so that object moves under the action of gravity and moves along a curved path, is called as "projectile". Its motion is called "projectile motion". Its path is called trajectory. Its path is parabolic. **(OR)** Projectile motion is two dimensional motion under constant acceleration due to gravity.

The body thrown is called projectile and the curved path followed by it is called trajectory.

Examples:

1. Motion of football kicked off by a player.
2. A ball thrown by a cricketer.
3. Missile fired from launching pad.



Consider a body thrown in horizontal direction with horizontal velocity v_x from point A having vertical height 'y'. In the absence of horizontal force, the horizontal components v_x remain constant all along the motion. If the body hits at point 'C' the horizontal distance 'x' covered by the body is given by

$$x = v_x t$$

Where 't' is the time taken by body to move from A to C.

The body not only covers distance in forward direction but also moves down under the action of gravity. The downward vertical velocity of body under the action of gravity goes on increasing continuously. This vertical motion is same as for freely falling body. The distance covered by body in downward direction is AB = y and is given by

$$S = y = v_{iy}t + \frac{1}{2}at^2$$

As the ball at 'A' has only the horizontal velocity so

v_{iy} (initial vertical velocity) $= 0$ and $a = g$

So $y = \frac{1}{2}gt^2$

$$\boxed{y = \frac{1}{2}gt^2}$$

Time of Flight of Projectile: The time taken by body to cover the distance from place of projection to the place where it hits the ground, is called time of flight of projectile. The time of flight can be calculated by using 2nd equation of motion:

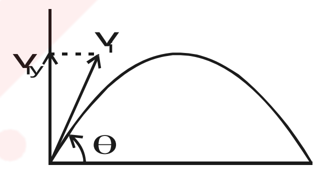
$$S = v_i t + \frac{1}{2}gt^2$$

As the ball returns to ground, so net vertical distance is zero. i.e.,

$$S = 0 \quad \text{and} \quad v_i = v_{iy} = v_i \sin\theta$$

The above equation becomes

$$0 = v_i \sin\theta t - \frac{1}{2}gt^2$$



$$\frac{1}{2}gt^2 = v_i \sin\theta t \quad \text{or} \quad \frac{1}{2}gt = v_i \sin\theta$$

or $$t = \frac{2v_i \sin\theta}{g}$$

Where 't' is the time of flight of projectile.

Range of Projectile: Max. distance which a projectile conversion the horizontal direction is called the range of projectile. In order to find R

$$R = v_{ix} \times t$$

$$= \frac{v_i \cos\theta \times 2v_i \sin\theta}{g}$$

$$= \frac{v_i^2}{g} 2\sin\theta\cos\theta$$

$$R = \frac{v_i^2}{g}\sin 2\theta$$

The formula for the **range of projectile** is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

For maximum horizontal range sin 2θ must have maximum value and we know that the maximum value of sin 2θ is 1.

$\because$ $\sin 2\theta = 1$

$$\sin 2\theta = \sin 90^\circ$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

So above equation becomes

$$R_{max.} = \frac{v_i^2 \sin 2(45°)}{g}$$

$$R_{max.} = \frac{v_i^2 \sin 90°}{g}$$

$$R_{max.} = \frac{v_i^2}{g}$$

So the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

3.13 At what point or points in its path does a projectile have its minimum speed, its maximum speed?

Ans. The speed of the projectile is maximum at the point of projection and also at the point where it hits the ground. While the speed of projectile is minimum when it reaches the maximum height.

3.14 Each of the following questions is followed by four answers, one of which is correct answer. Identified that answer.

(i) What is meant by a ballistic trajectory?

(a) The paths followed by an un-powered and unguided projectile is called ballistic trajectory.

(b) The path followed by the powered and unguided projectile is called ballistic trajectory.

(c) The path followed by un-powered but guided projectile.

(d) The path followed by powered and guided projectile.

(ii) What happens when two-body system undergoes elastic collision?

(a) The momentum of the system changes.

(b) The momentum of the system does not change.

(c) The bodies come to rest after collision.

(d) The energy conservation law is violated.

Ans. **(i)** (a) is correct.

(ii) (b) is correct.

PROBLEMS WITH SOLUTIONS

PROBLEM 3.1

A helicopter is ascending vertically at the rate of 19.6 ms^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

METHOD-I

Data

Initial vertical velocity of helicopter = v_i = 19.6 m/s

Since initial velocity of the stone is upward and stone moves downward.

∴ Vertical distance travelled by stone = S = –156.8 m

To Find

Time taken by stone to reach the ground = t = ?

SOLUTION

By using 2^{nd} equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$

$$-156.8 = 19.6\,t + \frac{1}{2} \times -9.8\,t^2$$

$$-156.8 = 19.6\,t - 4.9\,t^2$$

$$4.9\,t^2 = -19.6\,t - 156.8 = 0$$

Dividing by 4.9

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t - 8) + 4(t - 8) = 0$$

$$(t - 8)(t + 4) = 0$$

$$t - 8 = 0 \quad , \quad t + 4 = 0$$

$$t = 8 \text{ sec.} \quad , \quad t = -4 \text{ sec.}$$

Since time is always positive so ignoring the negative time hence.

Result

Time taken by stone = t = 8 sec.

METHOD-II

Data

Initial vertical velocity of helicopter = v_i = -19.6 m/s

Vertical distance = S = 156.8 m

To Find

Time taken by stone to reach the ground = t = ?

SOLUTION

By using the 2nd equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$

$$156.8 = -19.6\,t + \frac{1}{2} \times 9.8\,t^2$$

$$156.8 = -19.6\,t + 4.9\,t^2$$

Divide by 4.9

$$\frac{156.8}{4.9} = \frac{-19.6}{4.9} t + \frac{4.9}{4.9} t^2$$

$$32 = -4t + t^2$$

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t - 8) + 4(t - 8) = 0$$

$$(t + 4)(t - 8) = 0$$

$$t + 4 = 0 \quad , \quad t - 8 = 0$$

$t = -4$ sec. (neglected), $t = 8$ sec.

Result

Time taken by stone to reach the ground = t = 8 sec.

PROBLEM 3.2

Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (ms^{-1})	**0**	**10**	**20**	**20**	**20**	**20**	**0**
Time (s)	**0**	**30**	**60**	**90**	**120**	**150**	**180**

Use the graph to calculate

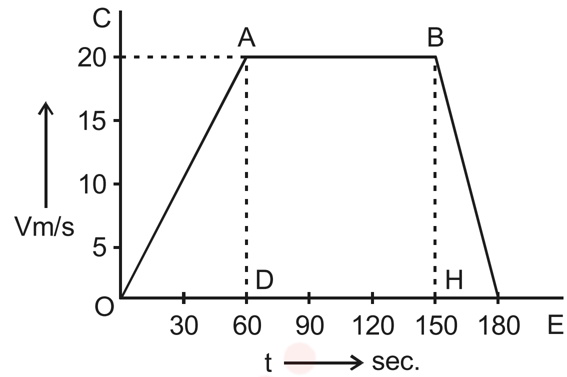
(a) The initial acceleration

(b) The final acceleration

(c) The total distance travelled by the motorcyclist.

SOLUTION

The velocity time graph is as shown.



(a) For initial acceleration

Initial acceleration

$$a_i = \frac{\text{Change in velocity}}{\text{Time}}$$

$$a_i = \frac{\Delta v}{\Delta t}$$

Since $\Delta v = 20$ m/s

$\Delta t = 60$ sec.

So, $$a_i = \frac{20}{60}$$

$$= \frac{1}{3} \text{ m/s}^2$$

$$a_i = 0.33 \text{ m/s}^2$$

(b) For final acceleration

$$a_f = \frac{\text{Change in velocity}}{\text{Time}}$$

Since $\Delta v = v_f - v_i$

$= 0 - 20$

$= -20$ m/s

and $\Delta t = 30$ sec.

$$a_f = \frac{-20}{30}$$

$$a_f = -0.67 \text{ m/s}^2$$

(c) For total distance travelled by motorcyclist

Total distance = Area of ΔOAD + Area of rectangle ABHD + Area of ΔBHE

Thus;

Area of ΔOAD $= \frac{1}{2}$ Base × Height

$= \frac{1}{2} \times 60 \times 20$

$= 600$ m

Area of rectangle ABHD = Length × Breadth

$= 90 \times 20$

$= 1800$ m/s

Area of $\Delta BHE = \frac{1}{2}$ Base × Height

$= \frac{1}{2} \times 30 \times 20$

$= 300$ m

Putting in above equation

Total distance travelled $= 600 + 1800 + 300$

$= 2700$ m

$= 2.7$ km

Result

(a) Initial acceleration $= a_i = 0.33$ m/s^2

(b) Final acceleration $= a_f = -0.67$ m/s^2

(c) Total distance travelled by motorcyclist = 2.7 km

PROBLEM 3.3

A proton moving with speed of 1.0×10^7 ms^{-1} passes through a 0.02 cm thick sheet of paper and emerges with a speed of 2.0×10^6 ms^{-1}. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Data

Initial speed of proton $= v_i = 1.0 \times 10^7$ m/s

Distance covered $= S = 0.02$ cm

$= 2 \times 10^{-4}$ m

Final speed of proton $= v_f = 2.0 \times 10^6$ m/s

To Find

Retardation (negative acceleration) = a = ?

Time taken = t = ?

SOLUTION

For the retardation by using 3rd equation of motion

$$2as = v_f^2 - v_i^2$$

$$2a \times 2 \times 10^{-4} = (2 \times 10^6)^2 - (1.0 \times 10^7)^2$$

$$4 \times 10^{-4}\,a = 4 \times 10^{12} - 1.0 \times 10^{14}$$

$$= 10^{12}\,(4 - 1.0 \times 10^2)$$

$$= 10^{12}\,(4 - 100)$$

$$4 \times 10^{-4}\,a = -96 \times 10^{12}$$

$$a = \frac{-96 \times 10^{12}}{4 \times 10^{-4}}$$

$$a = -24 \times 10^{16}$$

$$a = -2.4 \times 10^{17} \text{ m/s}^2$$

Negative sign shows retardation.

For the time taken, by using 1st equation of motion

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$= \frac{2.0 \times 10^6 - 1.0 \times 10^7}{-2.4 \times 10^{17}}$$

$$= \frac{10^6 (2 - 1.0 \times 10)}{-2.4 \times 10^{17}}$$

$$= \frac{10^6 (2 - 10)}{-2.4 \times 10^{17}}$$

$$= \frac{-8 \times 10^6}{-2.4 \times 10^{17}}$$

$$= 3.3 \times 10^{6-17}$$

$$= 3.3 \times 10^{-11} \text{ sec.}$$

Result

Retardation (negative acceleration) $= a = -2.4 \times 10^7 \text{ m/s}^2$

Time taken $= t = 3.3 \times 10^{-11}$ sec.

PROBLEM 3.4

Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the magnitude of ratio of their velocities after the spring has been released?

Data

1st mass $= m_1$

2nd mass $= m_2$

Initial velocity of mass m_1 $= v_i = 0$

Initial velocity of mass m_2 $= v_i = 0$

To Find

Ratio of their velocities $= \frac{v_1}{v_2} = ?$

SOLUTION

According to law of conservation momentum

$$m_1v_i + m_2v_i = m_1v_1 + m_2v_2$$

Therefore;

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

Result

Hence after releasing the spring, the ratio of magnitude of their velocities is equal to the inverse ratio of their masses.

PROBLEM 3.5

An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of 1.0×10^{-4} ms^{-1} and at a rate of 1.0×10^{-13} kgs^{-1}. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

(a) If there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

(b) If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

Data

Mass of amoeba $= m = 1.0 \times 10^{-12}$ kg

Speed of ejected water $= v = 1.0 \times 10^{-4}$ m/s

Rate of water $= \frac{m}{t} = 1.0 \times 10^{-13}$ kg/s

To Find

(a) Acceleration of amoeba $= a = ?$

(b) Force of surrounding water $= F = ?$

SOLUTION

(a) By formula

$$\boxed{F = \frac{m}{t} \times v}$$

$$F = 1.0 \times 10^{-13} \times 1.0 \times 10^{-4}$$

$$= 1.0 \times 10^{-17}\ \text{N}$$

So by second law of motion

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{1.0 \times 10^{-17}}{1.0 \times 10^{-12}}$$

$$= 1.0 \times 10^{-17+12}$$

$$= 1.0 \times 10^{-5}\ m/s^2$$

(b) The force of surrounding water is

$$F = \frac{m}{t} \times v$$

$$= 1.0 \times 10^{-13} \times 1.0 \times 10^{-4}$$

$$= 1.0 \times 10^{-17}\ N$$

Result

(a) Acceleration of amoeba = a = 1.0×10^{-5} m/s^2

(b) Force of surrounding water = F = 1.0×10^{-17} N

PROBLEM 3.6

A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of 3 ms^{-1}, how fast will the can be going?

Data

Mass of can = m_1 = 40 g = 0.04 kg

Mass of wooden block = m_2 = 200 g = 0.2 kg

Speed of wooden block = v_2' = 3 m/s

To Find

Speed of can = v_1' = ?

SOLUTION

According to law of conservation of momentum

Momentum before explosion = Momentum after explosion

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$0 = m_1v_1' + m_2v_2'$$

$$m_1v_1' = -m_2v_2'$$

$$\boxed{v_1' = -\frac{m_2 v_2}{m_1}}$$

$$= -\frac{0.2 \times 3}{0.04}$$

$$= -15 \text{ m/s}$$

Result

Speed of can $= v_1' = 15$ m/s

–ve sign shows that the can and wooden block moves in opposite direction.

PROBLEM 3.7

An electron ($m = 9.1 \times 10^{-31}$ kg) traveling at 2.0×10^7 ms^{-1} undergoes a head on collision with a hydrogen atom ($m = 1.67 \times 10^{-27}$ kg) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of hydrogen atom?

Data

Mass of electron $= m_1 = 9.1 \times 10^{-31}$ kg

Velocity of electron $= v_1 = 2.0 \times 10^7$ m/s

Mass of hydrogen atom $= m_2 = 1.67 \times 10^{-27}$ kg

Velocity of hydrogen atom $= v_2 = 0$

To Find

Velocity of hydrogen atom $= v_2' = ?$

SOLUTION

$$\boxed{v_2' = \frac{2m_1 v_1}{m_1 + m_2}}$$

$$v_2' = \frac{2 \times 9.1 \times 10^{-31} \times 2.0 \times 10^7}{9.1 \times 10^{-31} + 1.67 \times 10^{-27}}$$

$$= \frac{36.4 \times 10^{-31+7}}{10^{-27}(9.1 \times 10^{-4} + 1.67)}$$

$$= \frac{36.4 \times 10^{-24+27}}{0.00091 + 1.67}$$

$$= \frac{36.4 \times 10^3}{1.67091}$$

$$= 21.78 \times 10^3$$

$$= 2.18 \times 10^4 \text{ m/s}$$

Result

Velocity of hydrogen atom $= v_2' = 2.18 \times 10^4$ m/s

PROBLEM 3.8

A truck weighing 2500 kg and moving with a velocity of 21 ms^{-1} collides with a stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

Data

Mass of truck $= m_1 = 2500$ kg

Velocity of truck $= v_1 = 21$ m/s

Mass of car $= m_2 = 1000$ kg

Velocity of car $= v_2 = 0$

To Find

Common velocity after collision $= v = ?$

SOLUTION

According to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Since $v_1' = v_2' = v$

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$\boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

Putting values

$$v = \frac{2500 \times 21 + 1000 \times 0}{2500 + 1000}$$

$$= \frac{52500}{3500}$$

$$v = 15 \text{ m/s}$$

Result

Common velocity of truck and car after collision $= v = 15$ m/s

PROBLEM 3.9

Two blocks of masses 2.0 kg and 0.5 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring in 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

Data

Mass of 1st block $= m_1 = 2.0$ kg

Mass of 2nd block $= m_2 = 0.5$ kg

Elastic potential energy $= P.E = 10$ J

To Find

Velocity of mass $m_1 = v'_1 = ?$

Velocity of mass $m_2 = v'_2 = ?$

SOLUTION

According to law of conservation of energy

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$\frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = 10$$

$$m_1v_1'^2 + m_2v_2'^2 = 20$$

$$2v_1'^2 + 0.5v_2'^2 = 20 \quad \ldots\ldots \text{(i)}$$

And according to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

$$m_1v'_1 + m_2v'_2 = 0$$

$$2v'_1 + 0.5v'_2 = 0$$

$$0.5v'_2 = -2v'_1$$

$$v'_2 = -\frac{2v'_1}{0.5} = -4v'_1$$

$$\boxed{v'_2 = -4v'_1}$$

Putting eq. (i)

$$2v_1'^2 + 0.5(-4v'_1)^2 = 20$$

$$2v_1'^2 + 0.5(16v_1'^2) = 20$$

$$2v_1'^2 + 8v_1'^2 = 20$$

$$10v_1'^2 = 20$$

$$v_1'^2 = \frac{20}{10}$$

$$v'_1 = 1.41 \text{ m/s}$$

and $v'_2 = -4v_1$

$$= -4(1.41)$$

$$v'_2 = -5.65 \text{ m/s}$$

Result

Velocity of mass $m_1 = v'_1 = 1.41$ m/s

Velocity of mass $m_2 = v'_2 = -5.65$ m/s

PROBLEM 3.10

A football is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

Data

Angle with horizontal = θ = 30°

Horizontal distance = R = 40 m

The value of g = 9.8 m/s^2

To Find

Initial speed of ball = v_i = ?

SOLUTION

By formula

$$\boxed{R = \frac{v_i^2 \sin 2\theta}{g}}$$

$$v_i^2 = \frac{R \times g}{\sin 2\theta}$$

$$= \frac{40 \times 9.8}{\sin 2(30)}$$

$$= \frac{392}{0.866}$$

$$\sqrt{v_i^2} = \sqrt{452.64}$$

$$v_i = 21.27$$

$$= 21.3 \text{ m/s}$$

Result

Initial velocity of ball = v_i = 21.3 m/s

PROBLEM 3.11

A ball is thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How far off it hit the ground and with what velocity?

Data

Initial horizontal velocity = v_{ix} = 21 m/s

Initial vertical velocity = v_{iy} = 0

Vertical distance = y = 10 m

To Find

Horizontal distance = R = x = ?

Velocity to hit the ground = v = ?

SOLUTION

Formula for horizontal distance

$$\boxed{x = v_{ix} \times t} \qquad \text{...... (i)}$$

For time

$$y = v_{iy}t + \frac{1}{2}gt^2$$

$$10 = 0 \times t + \frac{1}{2} \times 9.8\,t^2$$

$$10 = 4.9\,t^2$$

$$t^2 = \frac{10}{4.9}$$

$$t^2 = 2.04$$

$$t = 1.42 \text{ sec.}$$

Therefore; putting in eq. (i)

$$x = R = v_{ix} \times t$$

$$= 21 \times 1.42$$

$$= 29.8$$

$$= 30 \text{ m}$$

For velocity

$$v = \sqrt{v_{fx}^2 + v_{fy}^2}$$

As $v_{fx} = v_{ix} = 21$ m/s

and $v_{fy} = v_{iy} + gt$

$$= 0 + 9.8 \times 1.42$$

$$v_{fy} = 13.91 \text{ m/s}$$

So $v = \sqrt{(21)^2 + (13.91)^2}$

$$= \sqrt{441 + 193.48}$$

$$= \sqrt{634.48}$$

$$v = 25.1 \text{ m/s}$$

Result

Horizontal distance = x = R = 30 m

Velocity to hit the ground = v = 25 m/s

PROBLEM 3.12

A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(a) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

(b) How long was it in air?

Data

Height of bomber $= y = 490\text{ m}$

Horizontal velocity $= v_{ix} = 300\text{ km/hr}$

$$= \frac{300 \times 1000}{3600}$$

$$= 83.3\text{ m/s}$$

To Find

(a) Horizontal distance $= x = R = ?$

(b) Time in air $= t = ?$

SOLUTION

(a) For horizontal distance

$$\boxed{x = v_{ix} \times t} \quad \ldots\ldots \text{(i)}$$

For time

$$y = v_{iy}t + \frac{1}{2}gt^2 \qquad \text{Since} \quad v_{iy} = 0$$

$$490 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$$

$$490 = 4.9\, t^2$$

$$t^2 = \frac{490}{4.9}$$

$$t^2 = 100$$

$$t = 10\text{ sec.}$$

So putting in eq. (i), we get

$$x = v_{ix} \times t$$

$$x = 10 \times 83.3$$

$$= 833\text{ m}$$

(b) Time in air $= t = 10\text{ sec.}$

Result

(a) Horizontal distance $= x = 833\text{ m}$

(b) Time in air $= t = 10\text{ sec.}$

PROBLEM 3.13

Find the angle of projection of a projectile for which its maximum height and horizontal range are equal.

Data

The given that

Horizontal range = Maximum height

To Find

Angle of projection = θ = ?

SOLUTION

As we know that the range of projection is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

and maximum height is

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

Therefore; $R = h$

$$\frac{v_i^2 \sin 2\theta}{g} = \frac{v_i^2 \sin^2 \theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2 \theta}{2}$$

Since $\sin 2\theta = 2 \sin \theta \cos \theta$

$$2 \sin \theta \cos \theta = \frac{\sin^2 \theta}{2}$$

$$2 \cos \theta = \frac{\sin \theta}{2}$$

$$2 \times 2 = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = 4$$

$$\theta = \tan^{-1}(4)$$

$$\theta = 75.9$$

$$= 76°$$

Result

Angle of projection = θ = 76°

PROBLEM 3.14

Prove that for angles of projection, which exceed or fall short of 45^o by equal amounts, the ranges are equal.

SOLUTION

Let the two angles 30° and 60° which exceed or fall short of 45° by equal of 15°.

Now we have to find, the ranges at these two angles so the range of projectile is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

When θ = 30°

$$R_1 = \frac{v_i^2 \sin 2(30°)}{g}$$

$$R_1 = \frac{v_i^2 \sin 60°}{g}$$

$$R_1 = \frac{0.866\, v_i^2}{g}$$

And when θ = 60°

$$R_2 = \frac{v_i^2 \sin 2(60°)}{g}$$

$$R_2 = \frac{v_i^2 \times 0.866}{g}$$

$$R_2 = \frac{0.866\, v_i^2}{g}$$

Thus $R_1 = R_2$

Result

Hence for angle of projection which exceed or fall short of 45° by equal amounts, the ranges are equal.

PROBLEM 3.15

A SLBM (submarine launched ballistic missile) is fired from a distance of 3000 km. If the Earth were flat and the angle of launch is 45^o with horizontal, find the time taken by SLBM to hit the target and the velocity with which the missile is fired.

Data

Horizontal distance = x = 3000 km

= 3000 × 1000

= 3×10^6 m

Angle of launch = θ = 45°

To Find

Time taken by SLBM to hit the ground $= t = ?$

Velocity of the missile $= v_i = ?$

SOLUTION

For velocity of the missile

$$R = x = \frac{v_i^2 \sin 2\theta}{g}$$

$$\boxed{v_i^2 = \frac{x \times g}{\sin 2\theta}}$$

$$v_i^2 = \frac{3 \times 10^6 \times 9.8}{\sin 2(45°)}$$

$$v_i^2 = \frac{29.4 \times 10^6}{\sin 90°} \qquad \text{Since} \quad \sin 90° = 1$$

$$v_i^2 = 29.4 \times 10^6$$

$$v_i = 5.42 \times 10^3 \text{ m/s}$$

$$= 5.42 \text{ km/s}$$

For time

$$\boxed{t = \frac{2v_i \sin\theta}{g}}$$

$$t = \frac{2(5.42 \times 10^3) \sin 45°}{9.8}$$

$$= \frac{10.84 \times 10^3 \times 0.707}{9.8}$$

$$t = 0.982 \times 10^3$$

$$t = 782 \text{ sec.}$$

$$t = 13 \text{ min.}$$

Result

Time taken by SLMB to hit the ground $= t = 13$ min.

Velocity of the missile $= v_i = 5.42$ km/s

SHORT QUESTIONS

4.1 A person holds a bag of groceries while standing still, taking to a friend. A car is stationery with its engine running. From the stand point of work, how are these two situations similar?

Ans. In both the situations there is no work done because both the bodies have zero displacement i.e.,

$$W = \vec{F} \cdot \vec{d}$$

$$= Fd \cos \theta \qquad \text{But} \qquad d = 0$$

$$W = F(0) \cos \theta$$

$$W = 0$$

Hence no work is done in both the cases.

4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10m.

Ans. ***Data***

Mass = m = 10 kg

Vertical height = h = 10 m

To Find

Work done in KJ = ?

Solution

By formula:

$$\text{Work done} = W = mgh$$

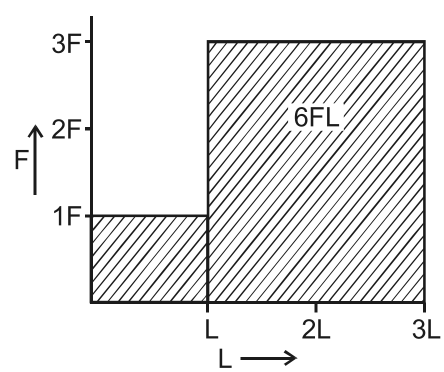
$$= 10 \times 10 \times 9.8$$

$$= 980 \text{ J} = \frac{980}{1000} \text{ KJ}$$

Work done = 0.98 KJ

4.3 A force F acts through a distance L. The force is then increased to 3 F, and then acts through a further distance of 2L. Draw the work diagram to scale.

Ans. Work diagram is a force distance graph as shown in figure. A force F acts through a distance L then work done is FL. If the force is increased to 3F through a distance 2L then



$$\text{Work done} = 3F \cdot 2L$$

$$= 6FL$$

Hence the total:

$$\text{Work done} = FL + 6FL$$

$$= 7FL$$

4.4 In which case is more work done? When a 50 kg bag of books is lifted through 30 cm, or when a 50 kg create is pushed through 2m across the floor with a force of 50 N?

Ans. In 1^{st} case when $m = 50$ kg

$h = 50 \text{ cm} = 0.5 \text{ m}$

Work done in this case is:

$$\text{Work done} = W_1 = mgh$$
$$= 50 \times 0.5 \times 9.8$$
$$W_1 = 245 \text{ J}$$

In 2^{nd} case:

$m = 50$ kg

$d = 2$ m

$F = 50$ N

Work done in this case is:

$$W_2 = F \,.\, d$$
$$= 50 \times 2$$
$$W_2 = 100 \text{ J}$$

It is clear that more work is done in 1^{st} case.

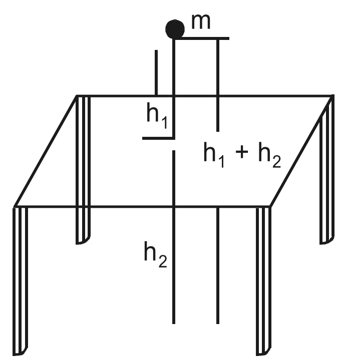
4.5 An object has 1 J of potential energy. Explain what does it mean?

Ans. As $1 \text{ J} = 1 \text{ N} \times 1 \text{ m}$

When one Newton force is applied on a body in lifting it through a height of one metre. Then work done is stored in a body as P.E which is 1 Joule i.e., the body has the ability of doing 1 Joule of work.

4.6 A ball of mass m is held at a height h_1 above a table. The table top is at a height h_2 above the floor. One student says that the ball has potential energy mgh_1 but another says that it is mg $(h_1 + h_2)$. Who is correct?

Ans. The ball is at a height h_1 with respect to the table and the table is at height h_2 with respect to the ground, so the ball is at a height of $h_1 + h_2$ from the ground. Therefore the potential energy stored in the ball with respect to the top is mgh_1 and with respect to ground is $mg(h_1 + h_2)$. Hence both the students are correct because one student is telling with respect to the top and other is telling with respect to the ground.



4.7 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

Ans. When the rocket re-enters into the atmosphere then a part of its kinetic energy is used to do work against friction with air and dust particles present in atmosphere which will be appear in the form of heat therefore due to this reason nose cone of rocket become very hot.

4.8 What sort of energy is in the following:

(a) Compressed spring

(b) Water in a high dam

(c) A moving car

Ans. (a) A compressed spring has elastic potential energy.

(b) A water in a high dam has gravitational potential energy.

(c) A moving car has kinetic energy.

4.9 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

Ans. At a certain height, a cup has gravitational potential energy. When it is dropped its gravitational potential energy decreases and kinetic energy increases. Just before striking the floor, kinetic energy is maximum. On striking, kinetic energy is changed into sound energy, heat energy and energy to break the cup if air friction is ignored. If air friction is present then some part of K.E is also used to overcome this friction.

4.10 A boy uses a catapult to throw a stone which accidentally smashes a green house window. List the possible energy changes.

Ans. Elastic potential energy stored in catapult, which is transferred to the stone as its kinetic energy. When the stone strikes the green house window, its kinetic energy changes into sound, heat and energy to break the glass window.

PROBLEMS WITH SOLUTIONS

PROBLEM 4.1

A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20m long.

Data

Force exerted = F = 40 N

Angle = θ = 20°

Length of strip of grass = d = 20 m

To Find

Work done = W = ?

SOLUTION

By formula

$$W = \vec{F} \cdot \vec{d}$$

$$\boxed{W = Fd\cos\theta}$$

$$W = (40)(20)\cos 20°$$

$$= 751.7 \text{ J}$$

$$= 7.5 \times 10^2 \text{ J}$$

Result

Work done = W = 7.5×10^2 J

PROBLEM 4.2

A rain drop (m = 3.35×10^{-5}kg) falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100 m, how much work is done by.

(a) Gravity and (b) Friction

Data

Mass of raindrop = m = 3.35×10^{-5} kg

Height = h = 100 m

To Find

(a) Work done by gravity = W = ?

(b) Work done by friction = W = ?

SOLUTION

(a) As we know that work done by gravity is

$$W = \vec{F} \cdot \vec{d}$$

$$= Fd \cos 0^\circ$$

$$= Fd\,(1)$$

$$W = mgh$$

Putting values

$$W = 3.35 \times 10^{-5} \times 9.8 \times 100$$

$$W = 0.0328\ J$$

(b) Now work done by friction is

$$W = \vec{F} \cdot \vec{d}$$

$$= Fd \cos 180^\circ$$

$$= Fd\,(-`1)$$

$$= -mgh$$

Putting values

$$W = -3.35 \times 10^{-5} \times 9.8 \times 100$$

$$= -0.0328\ J$$

Result

(a) Work done by gravity $= W = 0.0328\ J$

(b) Work done by friction $= W = -0.0328\ J$

PROBLEM 4.3

Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of another?

Data

Number of bricks $= 10$

Mass of each brick $= m = 1.5\ kg$

Height of each brick $= h = 6\ cm$

$= 0.06\ m$

To Find

Work done by placing one on the top $= W = ?$

SOLUTION

Now the total work done placing one brick on the top of another is

$$W = mgh + 1\ mgh + 2\ mgh + 3\ mgh + 4\ mgh + 5\ mgh + 6\ mgh + 7\ mgh + 8\ mgh + 9\ mgh$$

$$= (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)mgh$$

$$= 45\ mgh$$

$$= 45 \times 1.5 \times 9.8 \times 0.06$$

$$= 39.69\ J$$

$$W = 40\ J$$

Result

Work done by placing one on the top of another $= W = 40\ J$

PROBLEM 4.4

A car mass 800 kg traveling at 54 kmh^{-1} is brought to rest in 60 meters. Find the average retarding force on the car. What has happened to original kinetic energy?

Data

Mass of car $= m = 800\ kg$

Initial velocity of car $= v_i = 54\ km/hr$

$$= \frac{54 \times 1000}{3600}$$

$$= 15\ m/s$$

Distance $= d = 60\ m$

Final velocity of car $= v_f = 0$

To Find

Average force $= F = ?$

SOLUTION

By formula

$$F \times d = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

$$F \times 60 = \frac{1}{2} m(0^2 - 15^2)$$

$$60F = \frac{1}{2} \times 800(-15^2)$$

$$= 400 \times -225$$

$$60F = 400 \times -225$$

$$60F = -90000$$

$$F = \frac{-90000}{60}$$

$$F = -1500 \text{ N} \quad \text{(negative shows force is retarding)}$$

Result

Average force = F = 1500 N

Note: Original K.E. energy has been used in doing work against the friction between tyre and road.

PROBLEM 4.5

A 1000 kg automobile at the top of an incline 10 metre high and 100m long is released and rolls down the hill. What is its speed at the incline if the average retarding force due to friction is 480 N?

Data

Mass of automobile = m = 1000 kg

Height of inclined plane = h = 10 m

Length of plane = S = 100 m

Retarding force = f = 480 N

To Find

Speed of automobile = v = ?

SOLUTION

As Loss of P.E = Gain in K.E + Work done against friction

$$mgh = \frac{1}{2}mv^2 + fS$$

$$\frac{1}{2}mv = mgh - fS$$

$$\frac{1}{2} \times 1000\, v^2 = 1000 \times 9.8 \times 10 - 480 \times 100$$

$$500\, v^2 = 98000 - 48000$$

$$v^2 = 50{,}000$$

$$v^2 = \frac{50{,}000}{500}$$

$$v^2 = 100$$

$$v = 10 \text{ ms}^{-1}$$

Result

Speed of automobile = v = 10 m/s

PROBLEM 4.6

100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water 1000 kg m^{-3}, find.

(a) The increase in P.E.

(b) The power delivered by the pump.

Data

Volume of water $= V = 100\ m^3$

Height of tank $= h = 10\ m$

Time taken $= t = 20$ min.

$= 20 \times 60$

$= 1200$ sec.

To Find

(a) Increase in P.E. $= ?$

(b) Power delivered by pump $= P = ?$

SOLUTION

(a) For increase in P.E.

$$\boxed{P.E = mgh}$$

But Mass = Volume × Density

$= 100 \times 1000$

$m = 10^5$ kg

So $P.E = 10^5 \times 9.8 \times 10$

$P.E = 9.8 \times 10^6$ J

(b) For power delivered by pump

$$\text{Power} = \frac{P.E}{\text{Time}}$$

$$= \frac{9.8 \times 10^6}{1200}$$

$$P = 8.16 \times 10^{-3+6}$$

$$= 8.16 \times 10^3 \text{ watt}$$

$$= 8.2 \text{ k watt}$$

Result

(a) Increase in P.E $= 9.8 \times 10^6$ J

(b) Power delivered $= P = 8.2$ k watt

PROBLEM 4.7

A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 kmh^{-1}. What power (kW) must the engine develop?

Data

Force required = F = 400 N

Velocity of automobile = v = 80 km/h

$$= \frac{80 \times 1000}{3600}$$

$$= 22.22 \text{ m/s}$$

To Find

Power = P = ?

SOLUTION

As we know that

$$P = \vec{F} . \vec{v}$$

$$= Fv \cos\theta \qquad \text{But} \qquad \theta = 0°$$

$$P = Fv \cos 0°$$

$$P = Fv$$

$$= (400)(82.22)$$

$$= 8888 \text{ watt}$$

$$= 8.88 \text{ k watt}$$

$$= 8.9 \text{ k watt}$$

Result

Power = P = 8.9 k watt

PROBLEM 4.8

How large a force is required to accelerate an electron (m = 9.1 × 10^{-31}kg) from rest to a speed of 2.0 × 10^{7} ms^{-1} through a distance of 5.0 cm?

Data

Mass of electron = m = 9.1×10^{-31} kg

Initial velocity = v_i = 0

Final velocity = v_f = 2×10^7 m/s

Distance = d = 5 cm = 0.05 m

To Find

Force required = F = ?

SOLUTION

According to work energy principle

$$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$Fd = \frac{1}{2}mv_f^2 \qquad \text{Since} \quad v_i = 0$$

$$F \times 0.05 = \frac{1}{2} \times 9.1 \times 10^{-31}\,(2 \times 10^7)^2$$

$$0.05\,F = 18.2 \times 10^{-31+14}$$

$$0.05\,F = 18.2 \times 10^{-17}$$

$$F = \frac{18.2 \times 10^{-17}}{0.05}$$

$$F = 364 \times 10^{-17}$$

$$F = 3.64 \times 10^{-15}\ \text{N}$$

Result

Force required $= F = 3.64 \times 10^{-15}$ N

PROBLEM 4.9

A diver weighing 750 N dives from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting air friction.

Data

Weight of diver $= W = 750$ N

Height of board $= h_1 = 10$ m

Height of point from water $= h_2 = 5$ m

To Find

Speed of diver $= v = ?$

SOLUTION

As we know that

Gain in K.E = Loss of P.E

$$\frac{1}{2}mv^2 = mgh$$

$$\sqrt{v^2} = \sqrt{2gh}$$

$$v = \sqrt{2g(h_1 - h_2)}$$

$$v = \sqrt{2 \times 9.8 \times 5} \qquad \text{Since} \quad h = h_1 - h_2$$

$$v = 9.89 \text{ m/s}$$

$$v = 9.9 \text{ m/s}$$

Result

Speed of diver $= v = 9.9$ m/s

PROBLEM 4.10

A child starts from ret at the top of a slide of height 4.0m, (a) what is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of 6 ms^{-1}, what percentage of his total energy at the top of the slide is lost as a result of friction?

Data

Height of slide $= h = 4$ m

To Find

(a) Speed at bottom $= v = ?$

(b) % of total energy lost $= ?$

If $v' = 6$ m/s

SOLUTION

(a) As we know that

Gain in K.E = Loss of P.E

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \times 4}$$

$$= \sqrt{78.4}$$

$$= 8.8 \text{ m/s}$$

K.E when the child is moving with 8.8 m/s

$$\text{K.E} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\,(8.8)^2$$

$$\text{K.E} = 38.72 \text{ mJ}$$

(b) K.E when the child is moving with 6 m/s

$$K.E' = \frac{1}{2} mv'^2$$

$$= \frac{1}{2} m (6)^2$$

$$K.E' = 18 \text{ mJ}$$

Loss of energy $= K.E - K.E'$

$= 38.72 \text{ m} - 18 \text{ m}$

$= 20.72 \text{ mJ}$

% loss of energy $= \dfrac{20.72 \text{ m}}{38.72 \text{ m}} \times 100$

$= 53.5\%$

$= 54\%$

Result

(a) Speed of child at bottom = v = 8.8 m/s

(b) % loss of energy = 54%

SHORT QUESTIONS

5.1 Explain the difference between tangential velocity and the angular velocity, if one of these is given for a wheel of known radius, how will you find the other?

Ans. **Tangential Velocity:** The tangential velocity v_T is the linear velocity of a particles moving along a curve or a circle directed along the tangent at any point on the curve.

Angular Velocity: The angular velocity ω is the rate of change of angular displacement moving along a curved path.

Relation: The tangential velocity v_T, angular velocity ω and the radius r of the wheel are related by the relation

$$v_T = r\omega$$

If one of these is given for a wheel of known radius r then the other one can be find by using the above relation.

5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object to follow a circular path?

Ans. The force which is required to keep the body moving in a circular path and directed towards the centre of the circular path is called centripetal force. It is denoted by F_c and mathematically, it can be written as

$$F_c = \frac{mv^2}{r}$$

For uniform circular motion, it must be under the influence of a force which changes the direction of motion continuously. So the centripetal force is always needed if a body is to maintain its circular motion.

5.3 What is meant by moment of inertia? Explain its significance.

Ans. **Moment of Inertia:** It is defined as the product of mass of particle and square of its perpendicular distance from axis of rotation. It is denoted by I. Mathematically it can be written as

$$I = mr^2$$

Where m is the mass of the particle and r is the perpendicular distance of particle from the axis of rotation.

Significance: The moment of inertia plays the same role in angular motion as the mass in linear motion. As mass of a body is called measure of inertia but moment of inertia is the property of a body which resists to change its uniform circular motion.

5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum.

Ans. **Angular Momentum:** The angular momentum of an object is defined as the cross product of position vector $\vec{r}$ with respect to the axis of rotation and the linear momentum $\vec{P}$. It is denoted by $\vec{L}$. Mathematically

$$\vec{L} = \vec{r} \times \vec{P}$$

$$= rP \sin\theta\, \hat{n}$$

It is a vector quantity so the direction of angular momentum $\vec{L}$ is perpendicular to the plane contained by $\vec{r}$ and $\vec{P}$.

Law of Conservation of Angular Momentum: This law states that if no external torque acts on a system, the total angular momentum of a system remains constant.

Mathematically

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 \;\ldots\ldots\; = \text{Constant}$$

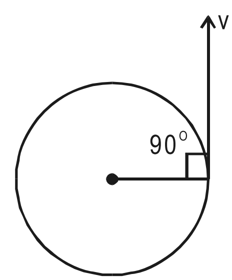
5.5 Show that orbital angular momentum $L_o = mvr$.

Ans. From the definition of angular momentum

$$\vec{L}_o = \vec{r} \times \vec{P}$$

$$L_o = rP \sin\theta = mvr \sin\theta$$

where θ is the angle between position vector $\vec{r}$ and velocity $\vec{v}$. In case of circular orbital motion, the angle between radius and tangential velocity is 90°. Hence



$$L_o = rP \sin\theta = mvr \sin 90° \quad \text{where} \quad \sin 90° = 1$$

$$L_o = mrv \;(1)$$

Hence $\quad L_o = mvr$

5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Ans. When a satellite is moving in a circular orbit, it has an acceleration $a_c = \dfrac{v^2}{r}$ which is supplied by gravity. The low flying earth satellite have an acceleration $a_c = g = 9.8\ m/s^2$. Thus

$$g = \frac{v^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

where r is the radius of earth = R = 6400 km.

So $\quad v = \sqrt{9.8 \times 6400}$

$$= 7.9\ \text{km/s}$$

This is the minimum velocity necessary to put a satellite into orbit around the earth.

5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

Ans. Direction of Angular Momentum: As we know that the angular momentum is always directed perpendicular to the plane containing $\vec{v}$ and $\vec{r}$ according to right hand rule.

$$\vec{L} = \vec{r} \times \vec{P} = (\vec{r} \times \vec{v})m$$

Direction of Angular Velocity: The direction of angular velocity is taken along the axis of rotation given by right hand rule. If rotation is along the curl of the fingers of right hand, then the outward thumb gives the direction of angular velocity. According to this rule, for counter clockwise rotation, the direction of angular velocity is outward along the axis of rotation.

5.8 Explain why an object, orbiting the Earth, is said to be freely falling. Use you explanation to point out why objects appear weightless under certain circumstances.

Ans. An object such as an artificial satellite orbits the Earth due to force of gravity. Its centripetal acceleration is equal to the acceleration due to gravity directed towards the centre of the Earth. Hence, a satellite is always falling towards the centre of Earth with acceleration equal to “g” and is a freely falling object. It does not hit the ground because of its tangential velocity and curvature of the Earth.

In the fame of reference of a freely falling object the bodies appears weightless because both, the body inside and the frame of reference, are falling with the same acceleration. The relative acceleration of inside body with respect to its frame of reference is zero. Hence, it seems that no force is acting on the body and when it is suspended to a spring balance, the spring balance shows its weight as zero.

5.9 When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

Ans. The mud flies off the tyre of a moving bicycle along the tangent as it leaves the path of circular motion. Actually the mud is sticked with the surface of tyre due to adhesive force (the attraction force existing between the molecules of different surfaces). This adhesive force provides the necessary centripetal force to the mud to move in a circular path. If the speed of rotation is increased then at a certain instant this adhesive force becomes unable to meet the demand of centripetal force and hence, it will no longer move in circular path. As, in the circular path mud has linear velocity along the tangent at every point so when it leaves its circular path it moves along tangent at that point.

5.10 A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

Ans. The velocity of the disc on reaching the bottom of the inclined plane is given as

$$v_{Disc} = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

and for a hoop, the velocity of hoop on reaching the bottom of the inclined plane is given as

$$v_{Hoop} = \sqrt{gh}$$

$$v_{Disc} = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

$$v_{Disc} = \sqrt{\frac{4}{3}}\, V_{Hoop}$$

As $\sqrt{\frac{4}{3}} > 1$

$\therefore$ $v_{Disc} > V_{Hoop}$

From these relations, it is clear that the velocity of disc is greater than the velocity of hoop so the disc moves faster as compared to hoop.

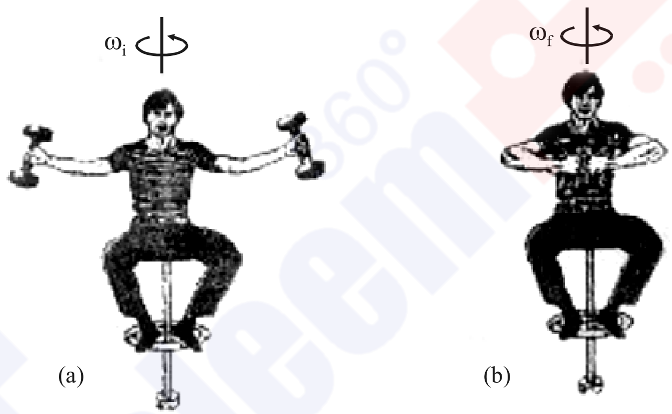
5.11 Why does a diver change his body position before diving in the pool?

Ans. Before lifting off the diving board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about an axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into closed tuck position. As angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

where ω_1 and ω_2 are the angular speeds before and after diving. Hence the diver spins faster when its moment of inertia becomes smaller.

5.12 A student holds two dumb-bells without stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumbbell towards his chest (Fig. 5.25). What will be the effect on rate of rotation?



Ans. By using the law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

With out-stretched arms, the moment of inertia is I_1 and his angular speed is ω_1. Their product $I_1\omega_1$ is equal to angular moment which remains constant. When he pulls the dumbbells towards his chest, his moment of inertia decrease and he spins faster as ω_2 increases to keep the product $I_2\omega_2$ constant.

5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission?

Ans. A satellite communication system can be set up by placing several geo stationary satellites in orbit over different point on the surface of the Earth. One such satellite covers 120^o of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites.

PROBLEMS WITH SOLUTIONS

PROBLEM 5.1

A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8m.

Data

Diameter of beam = Length of arc = S = 2.50 m

Distance of moon from the earth = r = 3.8×10^8 m

To Find

Divergence angle = θ = ?

SOLUTION

By formula

$$S = r\theta$$

$$\boxed{\theta = \frac{S}{r}}$$

$$\theta = \frac{2.50}{3.8 \times 10^8}$$

$$= 0.657 \times 10^{-8}$$

$$= 6.6 \times 10^{-9} \text{ rad}$$

Result

Divergence angle = θ = 6.6×10^{-9} rad

PROBLEM 5.2

A gramophone record turntable accelerate from rest to an angular velocity of 45.0 rev min^{-1} in 1.60s. What is its average angular acceleration?

Data

Initial angular velocity = ω_i = 0

Final angular velocity = ω_f = 45.0 rev/min.

$$= 45 \times \frac{2\pi}{60} \text{ rad/s}$$

$$= 1.5\pi \text{ rad/s}$$

Time interval = Δt = 1.60 sec.

To Find

Angular acceleration $= \alpha = ?$

SOLUTION

By formula

$$\boxed{\alpha = \frac{\omega_f - \omega_i}{\Delta t}}$$

$$= \frac{1.5\pi - 0}{1.6}$$

$$= \frac{1.5 \times 3.14}{1.6}$$

$$\alpha = 2.9 \text{ rad/s}^2$$

Result

Angular acceleration $= \alpha = 2.9 \text{ rad/s}^2$

PROBLEM 5.3

A body of moment of inertia I = 0.80 kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

Data

Moment of inertia $= I = 0.80 \text{ kgm}^2$

Constant angular velocity $= \omega = 100 \text{ rad/s}$

To Find

Angular momentum $= \vec{L} = ?$

Torque $= \vec{\tau} = ?$

SOLUTION

For angular momentum

$$\boxed{L = I\omega}$$

$$L = 0.80 \times 100$$

$$= 80 \text{ kgm}^2/\text{s}^2 \qquad \text{As} \quad \text{kgm}^2/\text{s}^2 = \text{J.s}$$

So $L = 80 \text{ J.s}$

For torque to sustain its motion

$$\boxed{\tau = I \times \alpha}$$

Since the body is moving with constant angular velocity so $\alpha = 0$

$$\tau = I \times 0$$

$$\tau = 0$$

Result

Angular momentum $= \vec{L} = 80$ J.s

Torque $= \vec{\tau} = 0$

PROBLEM 5.4

Consider the rotating cylinder shown in Fig. 5.26. Suppose that m = 5.0 kg, F = 0.60 N and r = 0.20 m. Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder. (Moment of inertia of cylinder = $\frac{1}{2}mr^2$)

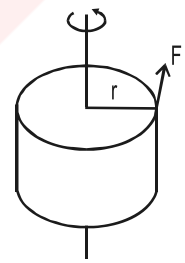


Fig. 5.26

Data

Mass of cylinder $= m = 5.0$ kg

Force acting on cylinder $= F = 0.60$ N

Radius of cylinder $= r = 0.20$ m

To Find

(a) Torque acting on the cylinder $= \tau = ?$

(b) Angular acceleration $= \alpha = ?$

SOLUTION

(a) For torque acting on the cylinder

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin\theta \qquad \text{But} \qquad \theta = 90^\circ$$

$$\boxed{\tau = rF}$$

$$= 0.20 \times 0.60$$

$$= 0.12 \text{ Nm}$$

(b) For angular acceleration

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} \qquad \text{But} \qquad I = \frac{1}{2}mr^2$$

$$\alpha = \frac{\tau}{\frac{1}{2}mr^2}$$

$$\boxed{\alpha = \frac{2\tau}{mr^2}}$$

Putting the values

$$\alpha = \frac{2 \times 0.12}{5.0 \times (0.20)^2}$$

$$= 1.2 \text{ rad/s}^2$$

Result

(a) Torque acting on the cylinder $= \tau = 0.12$ N.m

(b) Angular acceleration $= \alpha = 1.2$ rad/s^2

PROBLEM 5.5

Calculate the angular momentum of a star of mass 2.0×10^{30} kg and radius 7.0×10^5km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

Data

Mass of star $= M = 2.0 \times 10^{30}$ kg

Radius of star $= R = 7.0 \times 10^5$ km

$= 7.0 \times 10^8$ m

Time period of star $= T = 20$ days

$= 20 \times 24 \times 3600$

$= 1728000$ sec.

To Find

Angular momentum $= \vec{L} = ?$

Kinetic energy $= (K.E)_{rot} = ?$

SOLUTION

By using formula

$$L = I\omega \quad \text{But} \quad I = \frac{2}{5}MR^2$$

$$\text{and} \quad \omega = \frac{2\pi}{T}$$

So

$$\boxed{L = \frac{2\pi}{T} \times \frac{2}{5}MR^2}$$

$$= \frac{2(3.14)}{1728000} \times \frac{2 \times 2.0 \times 10^{30} \times (7.0 \times 10^{8})^2}{5}$$

$$= \frac{1230.80 \times 10^{46}}{8640000}$$

$$= 1.42 \times 10^{46-4}$$

$$= 1.42 \times 10^{42} \text{ J.s}$$

For rotational kinetic energy

$$(K.E)_{rot} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5}MR^2 \times \left(\frac{2\pi}{T}\right)^2$$

$$= \frac{1}{5}MR^2 \times \frac{4\pi^2}{T^2}$$

$$= \frac{2.0 \times 10^{30} \times (7.0 \times 10^{8})^2 \times 4(3.14)^2}{5 \times (1728000)^2}$$

$$= \frac{3864.9 \times 10^{30+16}}{1.49 \times 10^{13}}$$

$$= 2.58 \times 10^{46-10}$$

$$(K.E)_{rot} = 2.58 \times 10^{36} \text{ J}$$

Result

Angular momentum $= \vec{L} = 1.42 \times 10^{42}$ J.s

Kinetic energy $= (K.E)_{rot} = 2.58 \times 10^{36}$ J

PROBLEM 5.6

A 1000 kg car traveling with a speed of 144 km h^{-1} rounds a curve of radius 100 m. Find the necessary centripetal force.

Data

Mass of car $= m = 1000$ kg

Speed of car $= v = 144$ km/h

$$= \frac{144 \times 1000}{3600} \text{ m/s}$$

$$= 40 \text{ m/s}$$

Radius of curve $= r = 100$ m

To Find

Centripetal force $= F_c = ?$

SOLUTION

By using formula

$$\boxed{F_c = \frac{mv^2}{r}}$$

$$F_c = \frac{1000 \times (40)^2}{100}$$

$$= 16000 \text{ N}$$

$$= 1.6 \times 10^4 \text{ N}$$

Result

Centripetal force $= F_c = 1.6 \times 10^4$ N

PROBLEM 5.7

What is the least speed at which an aeroplane can execute a vertical loop of 1.0km radius so that there will be not tendency for the pilot to fall down at the highest point?

Data

Radius of loop $= r = 1.0$ km $= 1000$ m

Acceleration due to gravity $= g = 9.8$ m/s^2

To Find

Least speed of aeroplane $= v = ?$

SOLUTION

By formula

$$\boxed{v = \sqrt{gr}}$$

$$= \sqrt{9.8 \times 1000}$$

$$= \sqrt{9800}$$

$$= 98.9 \text{ m/s}$$

$$= 99 \text{ m/s}$$

Result

Least speed of aeroplane $= v = 99$ m/s

PROBLEM 5.8

The Moon orbit the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^8 m. Radius of the Moon is 1.74×10^6 m.

Data

Distance between earth and moon $= r = 3.85 \times 10^8$ m

Radius of moon $= R_m = 1.74 \times 10^6$ m

To Find

$$\text{Ratio} = \frac{L_s}{L_o} = ?$$

SOLUTION

The spin angular momentum of the moon is

$$L_s = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

$$\boxed{L_s = \frac{2}{5}MR_m^2\omega} \qquad \ldots\ldots \text{(i)}$$

The orbital angular momentum is given by

$$\boxed{L_o = Mr^2\omega} \qquad \ldots\ldots \text{(ii)}$$

where $MR^2 = I$ (moment of inertia)

Divide eq. (i) by (ii)

$$\frac{L_s}{L_o} = \frac{\frac{2}{5}MR_m^2\omega}{Mr^2\omega}$$

$$\boxed{\frac{L_s}{L_o} = \frac{2R_m^2}{5r^2}}$$

$$= \frac{2 \times (1.74 \times 10^6)^2}{5 \times (3.85 \times 10^8)^2}$$

$$= \frac{6.05 \times 10^{12}}{74.11 \times 10^{16}}$$

$$= 0.0816 \times 10^{12-16}$$

$$= 0.0816 \times 10^{-4}$$

$$\frac{L_s}{L_o} = 8.2 \times 10^{-6}$$

Result

$$\text{Ratio} = \frac{L_s}{L_o} = 8.2 \times 10^{-6}$$

PROBLEM 5.9

The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then? (For sphere I = 2/5 MR^2).

Data

Time period $= T_1 = 1$ day

$= 24$ hr.

Moment of inertia $= I = \frac{2}{5} MR^2$

To Find

Time period $= T_2 = ?$

SOLUTION

According to law of angular momentum

$$\boxed{I_1\omega_1 = I_2\omega_2} \qquad \ldots\ldots \text{(i)}$$

where $I_1 = \frac{2}{5} MR_1^2$ According to question

$I_2 = \frac{2}{5} MR_2^2$ $R_2 = \frac{R_1}{2}$

$$I_2 = \frac{2}{5} M \left(\frac{R_1}{2}\right)^2$$

$$I_2 = \frac{2}{5}M\frac{R_1^2}{4} = \frac{1}{10}MR_1^2$$

and $\omega_1 = \frac{2\pi}{T_1}$ and $\omega_2 = \frac{2\pi}{T_2}$

Putting in eq. (i)

$$\frac{2}{5}MR_1^2 \times \frac{2\pi}{T_1} = \frac{1}{10}MR_1^2 \times \frac{2\pi}{T_2}$$

$$\frac{2}{5T_1} = \frac{1}{10T_2}$$

$$20T_2 = 5T_1$$

$$\boxed{T_2 = \frac{5T_1}{20}}$$

$$T_2 = \frac{5T_1}{20}$$

$$= \frac{5 \times 24}{20}$$

$$T_2 = 6 \text{ hr.}$$

Result

Earth completes its rotation = T_2 = 6 hr.

PROBLEM 5.10

What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} kg and its radius as 6400 km).

Data

Radius of earth = R = 6400 km

= 6400×1000

= 6400000 m

Height of circular orbit = h = 900 km

= 900×1000

= 900000 m

Mass of earth = M = 6.0×10^{24} kg

To Find

Orbit speed = v = ?

SOLUTION

By formula

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

where $r = h + R$

$= 6400 + 900$

$= 7300 \text{ km}$

$= 7300 \times 1000$

$= 7300000 \text{ m} = 7300 \times 10^{3} \text{ m}$

Then $v = \sqrt{\dfrac{6.63 \times 10^{-11} \times 6.0 \times 10^{24}}{7300 \times 10^{3}}}$

$= \sqrt{5.44 \times 10^{-11-3-3+24}}$

$= \sqrt{5.44 \times 10^{7}}$

$= \sqrt{54.4 \times 10^{6}}$

$= 7.37 \times 10^{3} \text{ m/s}$

$= 7.4 \times 10^{3} \text{ m/s}$

Result

Orbital speed $= v = 7.4 \text{ km/s}$

SHORT QUESTIONS

6.1 Explain what do you understand by the term viscosity?

Ans. **Viscosity:** The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. It is denoted by η. The SI unit of viscosity is Ns/m^2.

6.2 What is meant by drag force? What are the factors upon which drag force acting upon a small sphere of radius r, moving down through a liquid, depend?

Ans. **Drag Force:** An object moving through a fluid experiences a retarding force called drag force. This force increases as the speed of object through the fluid increases. In case of a spherical object moving through a fluid, the expression of drag forces is:

$$F = 6\pi\eta r v$$

This shows that drag force depends on the following factor:

(i) Radius r of the spherical body.

(ii) Speed v of the body.

(iii) Coefficient of viscosity η.

6.3 Why fog droplets appear to be suspended in air?

Ans. As we know that the expression for the terminal velocity is

$$v_t = \frac{mg}{6\pi\eta r}$$

where $\frac{g}{6\pi\eta r}$ is constant so

$$v_t \propto m$$

This shows that terminal velocity is directly proportional to mass.

As the mass of the fog droplet is very small therefore the terminal velocity is very small. So the droplet appears to be suspended in air.

6.4 Explain the difference between laminar flow and turbulent flow.

Ans. **Laminar Flow:** The fluid, flow is said to laminar if every particle of the fluid that passes a point moves along the same path as followed by particles which passed that point earlier.

Turbulent Flow: The irregular or unsteady flow of the fluid is called turbulent flow. In turbulent flow, there is a great disorder and constantly changing flow path.

6.5 State Bernoulli's relation for a liquid in motion and describe some of its applications.

Ans. **Bernoulli's Theorem:** This theorem states that "the sum of pressure, kinetic energy per unit volume and potential energy per unit volume in a steady flow of an incompressible and non-viscous fluid remains constant at any point of its path".

Mathematically it is expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$$

where ρ is the density of the liquid and g is the acceleration due to gravity.

Applications of Bernoulli's Theorem: Following are the applications of Bernoulli's theorem:

(i) Operation of paint sprayer or perfume sprayer.

(ii) Swing of a cricket ball.

(iii) Working of a carburetor of a car.

(iv) Working of a filter pump.

6.6 A person is standing near a fast moving train. Is there any danger that he will fall towards it?

Ans. We know that according to Bernoulli's principle "the pressure will be low where the speed of the fluid is high and vice versa therefore when a person is standing near a fast moving train the speed of air between person and train is very high and pressure will be low while the speed of air behind the person is low so pressure is high, thus a force will act from high pressure to low pressure therefore the person will be in danger.

6.7 Identify the correct answer. What do you infer from Bernoulli's theorem?

(i) Where the speed of the fluid is high the pressure will be low.

(ii) Where the speed of the fluid is high the pressure is also high.

(iii) This theorem is valid only for turbulent flow of the liquid.

Ans. (i) is correct where the speed of the fluid is high, the pressure will be low.

6.8 Two row boats moving parallel in the same direction are pulled towards each other. Explain.

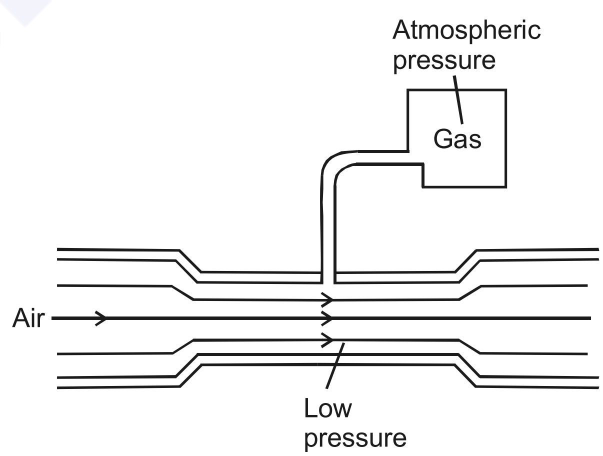
Ans. We know that according to Bernoulli's principle "the pressure will be low where the speed of fluid is high and vice versa therefore when two row boats are moving parallel in the same direction then the speed of water between the boats will be fast and pressure will be low while on the opposite sides of the boats the speed of water will be small so pressure will be high, thus a force will act from high pressure to low pressure therefore two boats are pulled towards each other.

6.9 Explain, how the swing is produced in a fast moving cricket ball.

Ans. We know that according to Bernoulli's principle "the pressure will be low where the speed of the fluid is high and vice versa". Therefore when a cricket ball is thrown by a fast bowler, the speed of air on the shining side will be fast and pressure will be low while on the rough side the speed of air is less and pressure is high. So a face will act from high pressure to the low pressure and the ball moves in a curved path called swing.

6.10 Explain the working of a carburetor of a motorcar using Bernoulli's principle and non viscous liquid is constant; thus.

Ans. Carburetor of car engine uses a venturi duct to feed the correct mixture of air and petrol to the cylinders. Air is drawn through the duct along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating a low pressure in the duct, which draws petrol vapour into air stream as shown in figure.



6.11 For which position will the maximum blood pressure in the body have the smallest value. (a) Standing up right (b) Sitting (c) Lying horizontally (d) Standing on one's head?

Ans. The blood pressure is measured at the level of heart. When lying horizontally heart does not has to work as hard as to pump against gravity because all parts of the body are in level with heart. So (c) is correct.

6.12 In an orbiting space station, would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck?

Ans. No, under weightlessness condition the blood pressure will be equal in major arteries of leg and neck in an orbiting space station.

PROBLEMS WITH SOLUTIONS

PROBLEM 6.1

Certain globular protein particle has a density of 1246 kg m^{-3}. It falls through pure water ($\eta = 8.0 \times 10^{-4}$ $Nm^{-2}s$) with a terminal speed of 3.0 cm h^{-1}. Find the radius of the particle.

Data

Density of protein particle $= \rho = 1246\ kg/m^3$

Viscosity $= \eta = 8.0 \times 10^{-4}\ Ns/m^2$

Terminal speed $= v_t = 3.0\ cm/h$

$= 0.03\ m/h$

$= \dfrac{0.03}{3600}$

$= 8.33 \times 10^{-6}\ m/s$

To Find

Radius of the particle $= r = ?$

SOLUTION

By formula

$$v_t = \frac{2gr^2\rho}{9\eta}$$

$$\boxed{r^2 = \frac{v_t \times 9\eta}{2\rho g}}$$

$$= \frac{8.33 \times 10^{-6} \times 9 \times 8.0 \times 10^{-4}}{2 \times 1246 \times 9.8}$$

$$= \frac{599.77 \times 10^{-4-6}}{24421.6}$$

$$r^2 = 0.024 \times 10^{-10}$$

$$r = 0.156 \times 10^{-5}$$

$$r = 1.56 \times 10^{-6}\ m$$

Result

Radius of the particle $= r = 1.56 \times 10^{-6}\ m$

PROBLEM 6.2

Water flows through a hose, whose internal diameter is 1 cm, at a speed of 1 ms^{-1}. What should be the diameter of the nozzle if the water is to emerge at 21 ms^{-1}?

Data

Internal diameter of pipe $= d_1 = 1$ cm

$= 0.01$ m

Speed of water in the hose $= v_1 = 1$ m/s

Speed of emerging water $= v_2 = 21$ m/s

To Find

Diameter of the nozzle $= d_2 = ?$

SOLUTION

According to equation of continuity

$$\boxed{A_1v_1 = A_2v_2} \quad \ldots\ldots \text{(i)}$$

But $\quad A = \pi r^2$

So $\quad A_1 = \pi\left(\frac{d_1}{2}\right)^2$

$$= \pi\frac{d_1^2}{4}$$

and $\quad A_2 = \pi\left(\frac{d_2}{2}\right)^2$

$$= \pi\frac{d_2^2}{4}$$

Putting in eq. (i)

$$\pi\frac{d_1^2}{4}v_1 = \pi\frac{d_2^2}{4}v_2$$

$$d_1^2v_1 = d_2^2v_2$$

$$\boxed{d_2^2 = \frac{d_1^2v_1}{v_2}}$$

$$= \frac{(0.01)^2 \times 1}{21}$$

$$d_2^2 = 4.76 \times 10^{-6}$$

$$d_2 = \sqrt{4.76 \times 10^{-6}}$$
$$= 2.18 \times 10^{-3}$$
$$= 0.21 \times 10^{-2} \text{ m}$$
$$d_2 = 0.21 \text{ cm}$$

Result

Diameter of nozzle $= d_2 = 0.21$ cm

PROBLEM 6.3

The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15m above the point of leak.

(i) With what speed does the water rush from the hole?

(ii) If the hole has an area of 0.060 cm², how much water flows out in one second?

Data

Height of water $= h_1 - h_2 = 15$ m

Area of hole $= A = 0.060 \text{ cm}^2$

$= 0.060 \times 10^{-4} \text{ m}^2$

To Find

(a) Speed of water from the hole $= v = ?$

(b) Volume per second of water $= V = ?$

SOLUTION

(a) For speed of water, using Torricelli's theorem

$$\boxed{v = \sqrt{2g(h_1 - h_2)}}$$
$$= \sqrt{2 \times 9.8 \times 15}$$
$$= \sqrt{294}$$
$$v = 17 \text{ m/s}$$

(b) For volume per second, by using equation of continuity

$$A_1v_1 = A_2v_2 = \text{Volume per second}$$

or

$$\boxed{\text{Volume per second of water} = Av}$$

So $= 0.060 \times 10^{-4} \times 17$

$= 1.02 \times 10^{-4} \text{ m}^3$

Volume per second of water $= 102 \text{ cm}^3$

Result

(a) Speed of water from hole $= v = 17$ m/s

(b) Volume per second of water $= 102 \text{ cm}^3$

PROBLEM 6.4

Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1}, while at another point 3m higher, the speed is 4.0 ms^{-1}. If the pressure is 80 KPa at the lower point, what is pressure at the upper point?

Data

Speed of water at one point = v_1 = 3 m/s

Height of upper point = h_2 = 3 m

Speed of water at lower point = v_2 = 4 m/s

Height of lower point = h_1 = 0

Pressure at lower point = P_1 = 80 KPa

= 80×1000

= 80,000 Pa

Density of water = ρ = 1000 kg/m^3

To Find

Pressure at upper point = P_2 = ?

SOLUTION

By using Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

When $h_1 = 0$

So $$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\boxed{P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 - \rho g h_2}$$

$$= 80000 + \frac{1}{2} \times 1000 \times (3)^2 - \frac{1}{2} \times 1000 \times (4)^2 - 1000 \times 9.8 \times 3$$

$$= 80000 + 4500 - 8000 - 29400$$

$$P_2 = 47100 \text{ Pa}$$

$$P_2 = 47.1 \text{ KPa}$$

Result

Pressure at upper end = P_2 = 47 KPa

PROBLEM 6.5

An airplane wing is designed so that when the speed of the air across the top of the wing is 450 ms^{-1}, the speed of air below the wing is 410 ms^{-1}. What is the pressure difference between the top and bottom of the wings? (Density of air = 1.29 kgm^{-3}).

Data

Speed of air above the wing $= v_1 = 450$ m/s

Speed of air below the wing $= v_2 = 410$ m/s

Air density $= \rho = 1.29$ kg/m^3

To Find

Pressure difference $= P_2 - P_1 = ?$

SOLUTION

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since $\rho g h_1 = \rho g h_2$

So $\boxed{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2}$

$$P_2 - P_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

$$= \frac{1}{2}\rho (v_1^2 - v_2^2)$$

$$= \frac{1}{2} \times 1.29\,(450^2 - 410^2)$$

$$= \frac{1}{2} \times 1.29 \times 34400$$

$$P_2 - P_1 = 22188 \text{ Pa}$$

$$P_2 - P_1 = 22.1 \text{ KPa}$$

Result

Pressure difference $= P_2 - P_1 = 22.1$ KPa

PROBLEM 6.6

The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about 30 cms^{-1}. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8×10^{-4} cm, there are literally millions of them so that their total cross section is about 2000 cm^2.

Data

Radius of aorta $= r_1 = 1.0$ cm

$= 0.01$ m

Speed of blood $= v_1 = 30$ cm/s

$= 0.3$ m/s

Area of one capillary $= A = 8 \times 10^{-4}$ cm

$= 8 \times 10^{-6}$ m

Total area of cross-section of capillaries $= A = 2000\ \text{cm}^2$

$= 0.2\ \text{m}^2$

To Find

Average speed of blood $= v_2 = ?$

SOLUTION

By using equation of continuity

$$A_1 v_1 = A_2 v_2$$

But $A_1 = \pi r_1^2$

So $\pi r_1^2 v_1 = A_2 v_2$

$$\boxed{v_2 = \frac{\pi r_1^2 v_1}{A_2}}$$

$$= \frac{3.14 \times (0.01)^2 \times 0.3}{0.2}$$

$$v_2 = 4.71 \times 10^{-4}\ \text{m/s}$$

$$v_2 = 5.0 \times 10^{-4}\ \text{m/s}$$

Result

Average speed of blood $= v_2 = 5.0 \times 10^{-4}$ m/s

PROBLEM 6.7

How large must a heating duct be if air moving 3.0ms^{-1} along it can replenish the air in a room of 300m^3 volume every 15 min? Assume the air's density remains constant.

Data

Speed of air in duct $= v_1 = 3.0$ m/s

Volume of air $= V = 300\ \text{m}^3$

Time $= t = 15$ min.

$= 15 \times 60$

$= 900$ sec.

To Find

Size / Length of duct $= r = ?$

SOLUTION

As we know that

Volume per second $= Av$

$$\frac{V}{t} = Av \qquad \text{As} \quad A = \pi r^2$$

$$\boxed{\frac{V}{t} = \pi r^2 v}$$

$$\boxed{r^2 = \frac{V}{t \times \pi v}}$$

$$= \frac{300}{900 \times 3.14 \times 3.0}$$

$$r^2 = 0.035$$

$$r = 0.188$$

$$r = 0.19 \text{ m}$$

$$r = 19 \text{ cm}$$

Result

Size / Length of duct $= r = 19$ cm

PROBLEM 6.8

An airplane design calls for a "lift" due to the net force of the moving air on the wing of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is 160 ms^{-1}? The density of air is 1.29 kgm^{-3} and assume maximum thickness of wing be one meter.

Data

Pressure difference $= P_1 - P_2 = 1000$ N/m^2

Speed of air past the lower surface $= v_1 = 160$ m/s

Density of air $= \rho = 1.29$ kg/m^3

To Find

Speed of air over the upper surface $= v_2 = ?$

SOLUTION

By using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since $\rho g h_1 = \rho g h_2$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\boxed{P_1 - P_2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2}$$

$$1000 + \frac{1}{2} \times 1.29 \times (160)^2 = \frac{1}{2} \times 1.29\, v_2^2$$

$$1000 + 16512 = 0.645\, v_2^2$$

$$v_2^2 = \frac{17512}{0.645}$$

$$v_2^2 = 27150.3$$

$$v_2 = 164.7$$

$$v_2 = 165 \text{ m/s}$$

Result

Speed of air over upper surface $= v_2 = 165$ m/s

PROBLEM 6.9

What gauge pressure is required in the city mains for a stream flow a fire hose connected to the city mains to reach a vertical height of 15 m?

Data

Vertical height $= h_1 - h_2 = 15$ m

Density of water $= \rho = 1000 \text{ kg/m}^3$

To Find

Pressure difference $= P_2 - P_1 = ?$

SOLUTION

By using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since $\frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$

So $P_1 + \rho g h_1 = P_2 + \rho g h_2$

$$\boxed{P_2 - P_1 = \rho g h_1 - \rho g h_2}$$

$$= \rho g(h_1 - h_2)$$

$$= 1000 \times 9.8 \times 15$$

$$P_2 - P_1 = 147000 \text{ Pa}$$

$$P_2 - P_1 = 147 \text{ KPa}$$

Result

Pressure difference $= P_2 - P_1 = 147$ KPa

SHORT QUESTIONS

7.1 Name two characteristics of simple harmonic motion.

Ans. Two characteristics of simple Harmonic Motion are given as:

(i) Acceleration of a vibrating body is directly proportional to the displacement and is always directed towards the mean position i.e.,

$$a \propto -x$$

(ii) Total energy of the particle executing simple harmonic motion remains conserved.

$$E_{total} = K.E + P.E = \text{Constant}$$

(iii) Simple harmonic motion can be represented by function of sine or cosine in the form of equation i.e.,

$$x = x_o \sin(\omega t + \phi)$$

and $$x = x_o \cos(\omega t + \phi)$$

where ϕ is a measure of phase.

7.2 Does frequency depend on amplitude for harmonic oscillators?

Ans. No, the frequency of oscillator is independent of the amplitude of oscillator.

(i) In case of mass-spring system, the frequency of mass is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

According to this relation, the frequency of oscillator depends upon mass and spring constant but it does not depend upon the amplitude of oscillator.

(ii) In case of simple pendulum, the frequency of the harmonic oscillator is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

This relation shows that the frequency does not depend upon the amplitude but it depends upon the length of pendulum and acceleration due to gravity.

7.3 Can we realize an ideal simple pendulum?

Ans. No we cannot realize an ideal simple pendulum because ideal simple pendulum consists of heavy but small mass suspended from a frictionless support by means of an inextensible string. As there conditions are impossible to attain therefore we cannot realize an ideal simple pendulum.

7.4 What is the total distance traveled by an object moving with SHM in a time equal to its period, if its amplitude is A?

Ans. As we know that time period of a simple harmonic motion is the time required to complete one vibration. If A is the amplitude of vibration then the distance travelled by an object in a time equal to its period is 4A.

7.5 What happens to the period of simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

Ans. We know that the time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Let T′ be the time period of simple pendulum when length becomes double.

i.e., $l' = 2l$

Then
$$T' = 2\pi\sqrt{\frac{2l}{g}}$$
$$= 2\pi\sqrt{\frac{l}{g}} \times \sqrt{2}$$
$$T' = \sqrt{2}\,T$$
$$= 1.41\,T$$

If the length of pendulum is doubled then its time period increases by 1.41 times the original time period.

Mass: If mass of pendulum is doubled, there is no change in time period because it is independent of mass.

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

Ans. No, the acceleration of a simple harmonic oscillator does not remain constant. The acceleration of harmonic oscillator varies with displacement because:

$$a = -\omega^2 x$$

where ω^2 is constant

So
$$a = -\text{Constant} \times x$$
$$a \propto -x$$

This shows that acceleration is directly proportional to displacement.

As displacement is changing during motion, therefore acceleration is also changing.

The acceleration will be zero at the mean position i.e., $x = 0$.

So,
$$a = -\omega^2(0)$$
$$a = 0$$

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

Ans. **Phase Angle:** The angle which specifies the displacement as well as the direction of motion of the point executing SHM is called phase angle.

The phase gives the information about the state of motion of the vibrating point. We can get the waveform of SHM by applying the concept of phase.

The phase angle does not define angle between maximum displacement and the driving force.

7.8 Under what conditions does the addition of two simple harmonic motions produce resultant, which is also simple harmonic?

Ans. In order to produce resultant SHM by the addition of two simple harmonic motions following conditions must be required:

(i) Two SHMs are parallel i.e., in same direction.

(ii) Two SHMs are in phase.

(iii) Two SHMs vibrate with same frequency.

7.9 Show that in SHM, the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

Ans. As the velocity and acceleration of a SHM are

$$a = -\omega^2 x \quad \text{and} \quad v = \omega\sqrt{x_o^2 - x^2}$$

At the mean position x = 0

Therefore;

$$a = -\omega^2(0) \quad \text{and} \quad v = \omega x_o = \text{Maximum value}$$

$$a = 0$$

At the extreme position $x = x_o$. Therefore

$$a = -\omega^2(x_o) \quad \text{and} \quad v = \omega\sqrt{x_o^2 - x_o^2}$$

$$a = -\omega^2 x_o \quad \text{and} \quad v = 0 = \text{Minimum value}$$

Thus it is clear that in SHM, acceleration is zero when then velocity is greatest and the velocity is zero when the acceleration is greatest.

7.10 In relation to SHM, explain the equations:

(i) $y = A \sin(\omega t + \varphi)$ (ii) $a = -\omega^2 x$

Ans. (i) $y = A \sin(\omega t + \phi)$

(ii) $a = -\omega^2 x$

Here, y = Instantaneous displacement

A = Amplitude

ϕ = Initial phase

ωt = Angle subtended in time 't'

This equation represents instantaneous the acceleration of an object executing SHM in which "a" represents acceleration, "ω" is the angular frequency and x represents its instantaneous displacement.

7.11 Explain the relation between total energy, potential energy and kinetic energy for body oscillating with SHM.

Ans. According to law of conservation of energy, the total energy of a body executing SHM remains constant. The K.E is maximum at the mean position and zero at the extreme position while the potential energy is maximum at the extreme position and zero at the mean position.

7.12 Describe some common phenomena in which resonance plays an important role.

Ans. The phenomenon resonance plays a very important role in:

(i) Musical instrument.

(ii) Producing electrical resonance in radio set with transmission of a particular radio frequency.

(iii) **In microwave oven:** The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz.

7.13 If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Ans. If a mass spring system is hung vertically and set into oscillations, the motion eventually stops due to air resistance and friction. Because of these frictional forces, energy is dissipated into heat and the system does not oscillate.

PROBLEMS WITH SOLUTIONS

PROBLEM 7.1

A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

Data

Mass of body	= m_1	=	100 g
		=	0.1 kg
Extension in the string	= x	=	4.0 cm
		=	0.04 m
Time period	= T	=	0.568 sec.
Amplitude	= x_o	=	10 cm
		=	0.1 m

To Find

Mass of object pulling the spring = m_2 = ?

SOLUTION

By using the formula

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

But for the value of K

$$F = Kx \quad \text{and} \quad F = m_1 g$$

$$K = \frac{F}{x} = \frac{m_1 g}{x}$$

$$K = \frac{0.1 \times 9.8}{0.04}$$

$$= 24.5 \text{ N/m}$$

Therefore; $T = 2\pi\sqrt{\frac{m_2}{K}}$

Squaring

$$T^2 = 4\pi^2 \frac{m_2}{K}$$

$$\boxed{m_2 = \frac{T^2 \times K}{4\pi^2}}$$

Putting the values

$$m_2 = \frac{(0.568)^2 \times 24.5}{4(3.14)^2}$$

$$m_2 = 0.200 \text{ kg}$$

$$m_2 = 200 \text{ g}$$

Result

Mass of object pulling the spring = m_2 = 200 g

PROBLEM 7.2

A load of 15.0 g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

Data

Mass elongates = m_1 = 15.0 g = 0.015 kg

Extension in the spring = x = 2.00 cm = 0.02 m

Mass attached with spring = m_2 = 294 g = 0.294 kg

To Find

(i) Time period = T = ?

(ii) Spring constant = K = ?

(iii) Maximum speed = v_o = ?

SOLUTION

(i) For time period

$$T = 2\pi\sqrt{\frac{m_2}{K}}$$

But

$$K = \frac{m_1 g}{x}$$

$$= \frac{0.015 \times 9.8}{0.02}$$

$$= 7.35 \text{ N/m}$$

Therefore;

$$T = 2(3.14)\sqrt{\frac{0.294}{7.35}}$$

$$T = 1.256 \text{ sec.}$$

(ii) Spring constant $= K = \frac{F}{x} = \frac{m_1 g}{x}$

$$K = \frac{0.015 \times 9.8}{0.02}$$

$$= 7.35 \text{ N/m}$$

(iii) For maximum speed

$$v_o = x_o \sqrt{\frac{K}{m_2}}$$

$$= 0.1 \sqrt{\frac{7.35}{0.294}}$$

$$= 0.5 \text{ m/s}$$

$$v_o = 50 \text{ cm/s}$$

Result

(i) Time period $= T = 1.256$ sec.

(ii) Spring constant $= K = 7.35$ N/m

(iii) Maximum speed $= v_o = 50$ cm/s

PROBLEM 7.3

An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find

(i) Period

(ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

Data

Mass of body $= m = 8.0$ kg

Amplitude $= x_o = 30$ cm $= 0.3$ m

Restoring force $= F = 60$ N

Displacement $= x = 30$ cm $= 0.3$ m

To Find

(i) Time period $= T = ?$

(ii) Acceleration $= a = ?$

Speed $= v = ?$

Kinetic energy $= K.E = ?$

Potential energy $= P.E = ?$

When

Displacement $= x_1 = 12$ cm $= 0.12$ m

SOLUTION

(i) For time period

$$T = 2\pi\sqrt{\frac{m}{K}}$$

As $$K = \frac{F}{x}$$

$$K = \frac{60}{0.3}$$

$$= 200 \text{ N/m}$$

So $$T = 2(3.14)\sqrt{\frac{8.0}{200}}$$

$$= 1.256 \text{ sec.}$$

$$T = 1.3 \text{ sec.}$$

(ii) For acceleration

$$a = -\frac{K}{m}x$$

$$= -\frac{200}{8.0} \times 0.12$$

$$a = 3 \text{ m/s}^2$$

For speed

$$v = \omega\sqrt{x_o^2 - x_1^2}$$

But $$\omega = \frac{2\pi}{T}$$

$$= \frac{2(3.14)}{1.3}$$

$$= 4.83 \text{ rad/s}$$

Therefore; $$v = 4.83\sqrt{(0.3)^2 - (0.12)^2}$$

$$v = 1.33 \text{ m/s}$$

For kinetic energy

$$K.E = \frac{1}{2}K(x_o^2 - x_1^2)$$

$$= \frac{1}{2} \times 200\,(0.3^2 - 0.12^2)$$

$$= 100(0.0756)$$

$$= 7.56 \text{ J}$$

For potential energy

$$P.E = \frac{1}{2}Kx_1^2$$

$$= \frac{1}{2} \times 200(0.12)^2$$

$$= 1.44 \text{ J}$$

Result

(i)	Time period	=	T	=	1.3 sec.
(ii)	Acceleration	=	a	=	3 m/s^2
	Speed	=	v	=	1.33 m/s
	Kinetic energy	=	K.E	=	7.56 J
	Potential energy	=	P.E	=	1.44 J

PROBLEM 7.4

A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant K = 1960 Nm^{-1}. Find the maximum distance through which the spring will be compressed.

Data

Mass of block	=	m	=	4.0 kg
Height	=	h	=	0.80 m
Spring constant	=	K	=	1960 N/m

To Find

Maximum distance = x_o = ?

SOLUTION

By formula

$$P.E = \frac{1}{2}Kx_o^2$$

$$\boxed{x_o^2 = \frac{2P.E}{K}} \quad \ldots\ldots (i)$$

But

$$P.E = mgh$$

$$= 4.0 \times 9.8 \times 0.80$$

$$= 31.36 \text{ J}$$

Putting in eq. (i)

So

$$x_o^2 = \frac{2 \times 31.36}{1960}$$

$$x_o = 0.178$$

$$= 0.18 \text{ m}$$

Result

Maximum distance = x_o = 0.18 m

= 18 cm

PROBLEM 7.5

A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where g = 9.8 ms^{-2}?

Data

Length of simple pendulum = l = 50.0 cm

= 0.50 m

Value of g = 9.8 m/s^2

To Find

Frequency of vibration = f = ?

SOLUTION

By formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2(3.14)\sqrt{\frac{0.5}{9.8}}$$

$$T = 1.41 \text{ sec.}$$

But $$f = \frac{1}{T}$$

$$= \frac{1}{1.41}$$

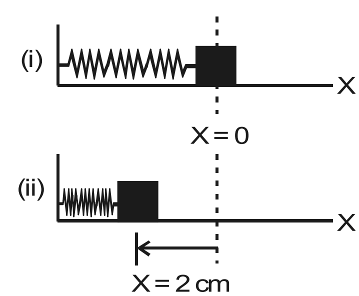
$$f = 0.70 \text{ Hz}$$

Result

Frequency of vibration = f = 0.70 Hz

PROBLEM 7.6

A block of mass 1.6 kg is attached to a spring with spring constant 1000 Nm^{-1}, as shown in Fig. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x = 0, if the surface is frictionless.



Data

Mass of block = m = 1.6 kg

Spring constant = K = 1000 N/m

Maximum displacement = x_o = 2.0 cm

= 0.02 m

To Find

Velocity of the block = v_o = ?

SOLUTION

By formula

$$v_o = x_o\sqrt{\frac{K}{m}}$$

$$= 0.02\sqrt{\frac{100}{1.6}}$$

$$v_o = 0.50 \text{ m/s}$$

Result

Velocity of block $= v_o = 0.50$ m/s

PROBLEM 7.7

A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

Data

Mass of car $= m_1 = 1300$ kg

Spring constant for each $= K = 20{,}000$ N/m

Mass of persons $= m_2 = 160$ kg

Spring constant for 4 springs $= 4 \times 20{,}000$

$= 80{,}000$ N/m

To Find

Frequency of vibration $= f = ?$

SOLUTION

By formula

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

But $m = m_1 + m_2$

$= 1300 + 160$

$= 1460$ kg

Therefore

$$f = \frac{1}{2(3.14)}\sqrt{\frac{80000}{1460}}$$

$$f = 1.18 \text{ Hz}$$

Result

Frequency of vibration $= f = 1.18$ Hz

PROBLEM 7.8

Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t$$

What is the displacement of the object after 2.0 s?

Data

The given equation is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t$$

Time $= t = 2.0$ sec.

To Find

Amplitude $= x_o = ?$

Frequency $= f = ?$

Time period $= T = ?$

SOLUTION

As we know that the equation for SHM is

$$x = x_o \cos \omega t \quad \text{...... (i)}$$

and the given equation is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t \quad \text{...... (ii)}$$

Comparing the eq. (i) and (ii)

$$x_o = 0.25$$

So

Amplitude $= x_o = 0.25$ m

$$\omega = \frac{\pi}{8}$$

But $T = \dfrac{2\pi}{\omega}$

$$T = \frac{2\pi}{\frac{\pi}{8}}$$

$$T = 2\pi \times \frac{8}{\pi}$$

Time period $= T = 16$ sec.

$$\text{Frequency} = f = \frac{1}{T}$$

$$f = \frac{1}{16}\ \text{Hz}$$

For displacement after 2 sec. is

$$x = 0.25\cos\frac{\pi}{8}t$$

$$= 0.25\cos\left(\frac{\pi}{8}\times 2\right)$$

$$= 0.25\cos\left(\frac{\pi}{4}\right)$$

$$= 0.25 \times 0.707$$

$$= 0.18\ \text{m}$$

Result

Amplitude $= x_o = 0.25$ m

Time period $= T = 16$ sec.

Frequency $= f = \frac{1}{16}$ Hz

Displacement $= x = 0.18$ m

SHORT QUESTIONS

8.1 What feature do longitudinal waves have in common with transverse waves?

Ans. The following features are common in transverse wave and longitudinal waves:

(i) Both are mechanical waves.

(ii) Both waves transfer energy from one point to another point but not matter.

(iii) Both produce disturbance in the medium through which they travel.

(iv) The relation $v = f\lambda$ holds for both the waves here v is speed of wave, "f", frequency and λ is wavelength.

8.2 The five possible waveforms obtained, when the output from a microphone is fed into the y-input of cathode ray oscilloscope, with the time base on, are shown in Fig. 8.23. These waveform are obtained under the same adjustment of the cathode ray oscilloscope controls, indicate the waveform:

(a) which trace represents the loudest note?

(b) which trace represents the highest frequency?

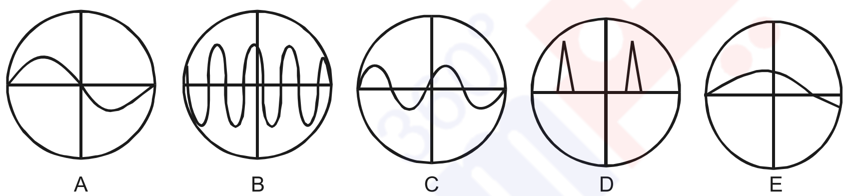


Fig. 8.23

Ans. (a) The trace (B) represents the loudest sound because the loudness of sound depends upon the amplitude of vibration.

(b) The trace (B) represents the highest frequency because the number of waves per second are maximum.

8.3 Is it possible for two identical waves traveling in the same direction along a string to give rise to a stationary wave?

Ans. No, the stationary waves are not produced when the two identical waves are travelling the same direction. Because in order to produce stationary waves, two identical waves should travel in opposite direction.

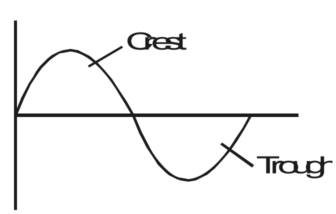
8.4 A wave is produced along a stretched string but some of its particle permanently shows zero displacement. What types of wave is it?

Ans. It will be a stationary wave because in stationary wave, some points will remain permanently at rest i.e., zero displacement called node.

8.5 Explain the terms crest, through, node, and anti-node.

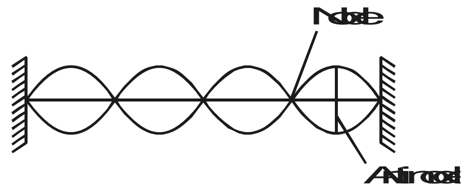
Ans. **Crest:** The upper portion of the transverse wave from the mean position is called crest.

Trough: The lower portion of the transverse wave from the mean position is called trough.



Node: The point at which the displacement of the stationary wave is zero is called node.

Anti-node: The point at which the displacement of the stationary wave is maximum is called anti-node.



8.6 Why does sound travel faster in solid than in gases?

Ans. As we know that the speed of sound is given by the expression

$$v = \sqrt{\frac{E}{\rho}}$$

From this equation we see that speed of sound is directly proportional to square root of modulus of elasticity. As the elasticity of medium is greater for solid as compared to gases and density is also greater for solids. So the effect of elasticity is greater than density thus the sound travels faster in solids than in gases.

8.7 How are beats useful in tuning musical instruments?

Ans. In musical instruments, various notes can be produced by changing the length of air column. To get a particular note, a standard instrument is taken and is sounded together with musical instrument which is to be tuned. The number of beats produced per second are recorded. The frequency of the instrument to be tuned is so adjusted that it gives no beat with standard instrument. So the musical instrument is tuned to a particular frequency with the help of phenomenon of beats.

8.8 When two notes of frequency f_1 and f_2 are sounded together, beats are formed. If $f_1 > f_2$, what will be frequency of beats?

(i) $f_1 + f_2$ **(ii)** $\frac{1}{2}(f_1 + f_2)$ **(iii)** $f_1 - f_2$ **(iv)** $\frac{1}{2}(f_2 - f_1)$

Ans. The number of beats produced in one second is equal to the difference in the frequencies of two notes. So:

Number of beats per second $= f_1 - f_2$

Hence the correct answer is (iii).

8.9 As a result of a distant explosion, an observer senses a ground tremor and then hears the explosion. Explain the time difference.

Ans. We know that sound waves travel faster through solid as compared to gases. So the time difference is due to that waves produced by explosions reach through solid ground much faster than the f sound waves travelling through air.

8.10 Explain why sound travels faster in warm air than in cold air.

Ans. We know that the expression for speed of sound in air is

$$v = \sqrt{\frac{rP}{\rho}}$$

From this equation we see that speed of sound is inversely proportional to $\sqrt{\rho}$. Which means that if density of air is greater, speed of sound will be small and vice versa. As warm air has smaller density than that of cold air therefore sound travel faster in warm air than in cold air.

8.11 How should a sound source move with respect to an observer so that frequency of its sound does not change?

Ans. If both sound waves and observer are moving in same direction with same velocity then their relative velocity is equal to zero. So the frequency of its sound does not change.

PROBLEMS WITH SOLUTIONS

PROBLEM 8.1

The wavelength of the signals from a radio transmitter is 1500 m and the frequency is 200 KHz. What is the wavelength for a transmitter operating at 1000 KHz and with what speed the radio waves travel?

Data

Wavelength of signals = λ_1 = 1500 m

Frequency of signals = f_1 = 200 KHz

= 200×1000 Hz

= 2×10^5 Hz

Frequency for transmitter = f_2 = 1000 KHz

= 1000×1000 Hz

= 10^6 Hz

To Find

Wavelength for transmitter = λ_2 = ?

Speed of radio waves = v = ?

SOLUTION

By formula

$$v = f\lambda$$

For speed of radio waves

$$v = f_1\lambda_1$$

$$= 2 \times 10^5 \times 1500$$

$$= 3000 \times 10^5$$

$$= 3 \times 10^8 \text{ m/s}$$

And for wavelength

$$v = f_2\lambda_2$$

$$\lambda_2 = \frac{v}{f_2}$$

$$= \frac{3 \times 10^8}{10^6}$$

$$= 3 \times 10^{8-6}$$

$$= 3 \times 10^2$$

$$\lambda_2 = 300 \text{ m}$$

Result

Wavelength for transmitter $= \lambda_2 = 300$ m

Speed of radio waves $= v = 3 \times 10^8$ m/s

PROBLEM 8.2

Two speakers are arranged as shown in Fig. 8.24. The distance between them is 3 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.00 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the centre of the line and directly opposite each speaker. Calculate the speed of sound.

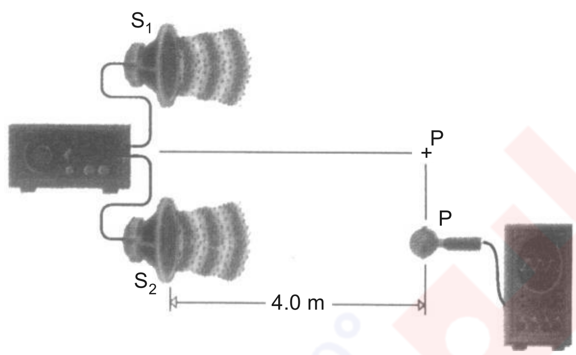


Fig. 8.24

Data

Distance between speakers $= d = 3.0$ m

Frequency of sound $= f = 344$ Hz

Distance between microphone and speakers $= L = 4.0$ m

To Find

Speed of sound $= v = ?$

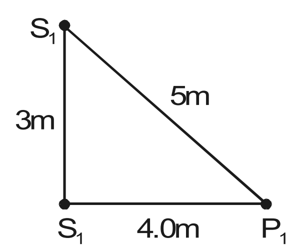
SOLUTION

By formula

$$\boxed{v = f\lambda}$$

Where λ = Path difference

$= S_1P_1 - S_2P_1$



Consider the $\Delta S_1P_1S_2$ is right angle triangle then

$$(S_1P_1)^2 = (S_1S_2)^2 + (S_2P_1)^2$$

$$= (3)^2 + (4)^2$$

$$(S_1P_1)^2 = 9 + 16$$

$$S_1P_1 = \sqrt{25}$$

$$= 5$$

So $\lambda = S_1P_1 - S_2P_1$

$= 5 - 4$

$\lambda = 1 \text{ m}$

Then $v = f\lambda$

$= 1 \times 344$

$v = 344 \text{ m/s}$

Result

Speed of sound $= v = 344 \text{ m/s}$

PROBLEM 8.3

A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments, at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency?

Data

Length of string $= l = 120 \text{ cm}$

$= 1.20 \text{ m}$

Number of segments $= N = 4$

Frequency of four segments $= f_4 = 120 \text{ Hz}$

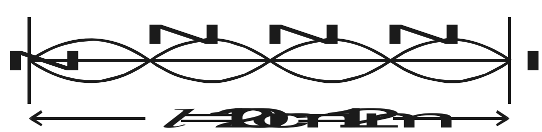
To Find

Fundamental frequency $= f_1 = ?$

Wavelength $= \lambda = ?$

SOLUTION

For the wavelength



$$l = 4 \times \frac{\lambda}{2}$$

$$\lambda = \frac{2l}{4} = \frac{2 \times 1.2}{4}$$

$$\lambda = 0.6 \text{ m}$$

and for fundamental frequency

$$f_4 = 4f_1$$

$$\boxed{f_1 = \frac{f_4}{4}}$$

$$f_1 = \frac{120}{4}$$

$$f_1 = 30 \text{ Hz}$$

Result

Fundamental frequency $= f_1 = 30$ Hz

Wavelength $= \lambda = 0.6$ m

PROBLEM 8.4

The frequency of the note emitted by a stretched string is 300 Hz. What will be the frequency of this note when

(a) the length of the wave is reduced by one-third without changing the tension.

(b) the tension is increased by one-third without changing the length of the wire.

Data

Frequency of the note $= f = 300$ Hz

To Find

(a) Frequency $= f_1 = ?$ (When the length of the wave reduced by one-third)

(b) Frequency $= f_2 = ?$ (When the tension is increased by one-third)

SOLUTION

(a) Suppose the length of wave $= \lambda$

One-third of that length $= \frac{1}{3}\lambda$

After reducing the length $= \lambda - \frac{1}{3}\lambda$

$$\lambda' = \frac{2}{3}\lambda$$

So according to relation

$$\lambda f = \frac{2}{3}\lambda f_1$$

$$\frac{2}{3} f_1 = f$$

$$f_1 = \frac{3}{2} f$$

$$f_1 = \frac{3}{2} \times 300$$

$$f_1 = 450 \text{ Hz}$$

(b) As we know that

$$f_2 = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

Suppose the tension $= F$

One-third of that tension $= \frac{1}{3}F$

After increasing the tension, the new tension becomes $= F' = F + \frac{1}{3}F$

$$= \frac{4}{3}F$$

Therefore the frequency at that tension is

$$f_2 = \frac{1}{2l}\sqrt{\frac{F'}{m}}$$

$$f_2 = \frac{1}{2l}\sqrt{\frac{\frac{4}{3}F}{m}}$$

$$f_2 = \frac{1}{2l}\sqrt{\frac{F}{m}} \times \sqrt{\frac{4}{3}}$$

Therefore; $f_2 = f \times \sqrt{\frac{4}{3}}$

$$f_2 = 300 \times 1.15$$

$$= 346.4 \text{ Hz}$$

Result

(a) Frequency when length of the wave is reduced $= f_1 = 450$ Hz

(b) Frequency when tension is increased $= f_2 = 346$ Hz

PROBLEM 8.5

An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is

(a) open at both ends.

(b) closed at one end.

(Speed of sound = 350 ms^{-1})

Data

Length of organ pipe $= l = 50$ cm

$= 0.5$ m

Speed of sound $= v = 350$ m/s

To Find

(a) Frequency $= f_1 = ?$

Frequency $= f_2 = ?$ (When open at both ends)

(b) Frequency $= f_1 = ?$

Frequency $= f_3 = ?$ (When closed at one end)

SOLUTION

(a) When pipe is opened at both ends

$$v = f\lambda$$

$$f_1 = \frac{v}{\lambda}$$

Since $l = \frac{\lambda}{2}$

$$\lambda = 2l$$

$$= 2 \times 0.5$$

$$= 1.0$$

So $f_1 = \frac{350}{1.0} = 350 \text{ Hz}$

Next harmonic frequency $= f_2 = 2f_1$

$$= 2 \times 350$$

$$= 700 \text{ Hz}$$

(b) When pipe is closed at one-end

$f_1 = \frac{v}{\lambda}$ But $l = \frac{\lambda}{4}$

$$\lambda = 4l$$

$$= 4 \times 0.5$$

$$= 2.0$$

So $f_1 = \frac{350}{2.0} = 175 \text{ Hz}$

For next harmonic $= f_3 = 175 \times 3$

$$= 525 \text{ Hz}$$

Result

(a) Frequency $= f_1 = 350$ Hz

Next harmonic frequency $= f_2 = 700$ Hz (When pipe is opened at both ends)

(b) Frequency $= f_2 = 175$ Hz

Next harmonic frequency $= f_3 = 525$ Hz (When pipe is closed at one end)

PROBLEM 8.6

A church organ consists of pipes, each open at one end of different lengths. The minimum length is 30 mm and the longest is 4m. Calculate the frequency range of the fundamental notes.

(Speed of sound = 340 ms^{-1})

Data

Minimum length of pipe $= l_1 = 30$ mm

$= 30 \times 10^{-3}$ m

$= 0.03$ m

Maximum length of pipe $= l_2 = 4$ m

Speed of sound $= v = 340$ m/s

To Find

Frequency for minimum length $= f_1 = ?$

Frequency for maximum length $= f_2 = ?$

SOLUTION

By formula

$$f_1 = \frac{v}{4l_1} \quad \text{(When the pipes are opened at one-end)}$$

For minimum length Since $\lambda = 4l_1$

$$f_1 = \frac{340}{4(0.03)}$$

$$= \frac{340}{0.12}$$

$$f_1 = 2833 \text{ Hz}$$

For maximum length

$$f_2 = \frac{v}{4l_2}$$

$$= \frac{340}{4 \times 4} = \frac{340}{16} = 21.2 \text{ Hz}$$

Result

Frequency for minimum length $= f_1 = 2833$ Hz

Frequency for maximum length $= f_2 = 21$ Hz

PROBLEM 8.7

Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prong. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

Data

Number of beats per second before loadings $= n = 3$

Number of beats after loading $= n = 1$

Frequency of 1st tuning fork $= f_A = 256$ Hz

To Find

Frequency of 2nd tuning fork $= f_B = ?$

SOLUTION

By formula

$$f_A - f_B = \pm n$$

$$f_B = f_A \pm n$$

$$= 256 \pm 3$$

$$f_A = 256 + 3 \quad \text{or} \quad f_B = 256 - 3$$

$$f_B = 259 \text{ Hz} \quad \text{or} \quad f_B = 253 \text{ Hz}$$

As after loading number of beats reduces so.

Result

Frequency of 2nd tuning fork $= f_B = 253$ Hz

PROBLEM 8.8

Two cars P and Q are traveling along a motorway in the same direction. The leading car P travels at a steady speed of 12 ms^{-1}; the other car Q, traveling at a steady speed of 20 ms^{-1}, sound its horn to emit a steady note which P's driver estimates, has a frequency of 830 Hz. What frequency does Q's own driver hear?

(Speed of sound = 340 ms^{-1})

Data

Speed of car P $=$ 12 m/s

Speed of car Q $=$ 20 m/s

Relative speed $= u_s = 20 - 12$

$= 8$ m/s

Speed of sound $= v = 340$ m/s

Apparent frequency $= f_A = 830$ Hz

To Find

Frequency heard by Q's driver $= f = ?$

SOLUTION

By using formula, when the source moves towards the observer

$$f_A = \left(\frac{v}{v - u_s}\right) f$$

$$\boxed{f = \frac{(v - u_s)\, f_A}{v}}$$

$$= \frac{340 - 8}{340} \times 830$$

$$= \frac{332 \times 830}{340}$$

$$f = 810 \text{ Hz}$$

Result

Frequency heard by Q's driver = f = 810 Hz

PROBLEM 8.9

A train sounds its horn before it sets off from the station and an observer waiting on the plateform estimates its frequency at 1200 Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the plateform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50 s?

(Speed of sound = 340 ms^{-1})

Data

Fundamental frequency = f = 1200 Hz

Apparent frequency = f_A = 1140 Hz

Speed of sound = v = 340 m/s

Time taken = t = 50 sec.

To Find

Speed of train = u_s = ?

Distance covered by train = S = ?

SOLUTION

By formula, when the source moves away from the observer

$$f_A = \frac{v}{v + u_s} f$$

$$v + u_s = \frac{v}{f_A} f$$

$$\boxed{u_s = \frac{vf}{f_A} - v}$$

$$u_s = \frac{340 \times 1200}{1140} - 340$$

$$= 357.894 - 340$$

$$u_s = 17.894 \text{ m/s}$$

For the distance covered by the train

$$S = u_{ave} \times t$$

$$u_{ave} = \frac{0 + 17.9}{2} = \frac{17.9}{2} = 8.95 \text{ m/s}$$

So, $S = 8.95 \times 50$

$= 448$ m

Result

Speed of train $= u_s = 17.89$ m/s

Distance covered by train $= S = 448$ m

PROBLEM 8.10

The absorption spectrum of faint galaxy is measured and the wavelength of one of the lines identified as the Calcium α line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in a laboratory.

(a) Is the galaxy moving towards or away from the Earth?

(b) Calculated the speed of the galaxy relative to Earth.

(Speed of light = 3.0×10^8 ms^{-1})

Data

Apparent wavelength $= \lambda_A = 478$ nm

$= 478 \times 10^{-9}$ m

Fundamental wavelength $= \lambda = 397$ nm

$= 397 \times 10^{-9}$ m

Speed of light $= C = 3 \times 10^8$ m/s

To Find

(a) Is the galaxy moving towards or away from the earth?

(b) Speed of galaxy relative to earth $= u_s = ?$

SOLUTION

(a) As we know that

$$f = \frac{C}{\lambda}$$

For apparent wavelength

$$f_A = \frac{C}{\lambda_A}$$

$$= \frac{3 \times 10^8}{478 \times 10^{-9}}$$

$$= 6.27 \times 10^{8+9-3}$$

$$= 6.27 \times 10^{14} \text{ Hz}$$

For fundamental wavelength

$$f = \frac{C}{\lambda}$$

$$= \frac{3 \times 10^8}{397 \times 10^{-9}}$$

$$= 7.55 \times 10^{14} \text{ Hz}$$

So $f_A < f$

The galaxy is moving away from the earth.

(b) For the speed of galaxy, when the galaxy is moving away from the earth

$$f_A = \frac{v}{v + u_s} f$$

$$v + u_s = \frac{v_f}{f_A}$$

$$\boxed{u_s = \frac{v_f}{f_A} - v}$$

$$= \frac{3 \times 10^8 \times 7.55 \times 10^{14}}{6.27 \times 10^{14}} - 3 \times 10^8$$

$$u_s = 3.612 \times 10^8 - 3 \times 10^8$$

$$= 0.612 \times 10^8$$

$$= 6.12 \times 10^7 \text{ m/s}$$

Result

(a) Galaxy is moving away from the earth.

(b) Speed of galaxy $= u_s = 6.12 \times 10^7$ m/s

SHORT QUESTIONS

9.1 Under what conditions two or more sources of light behave as coherent sources?

Ans. Two or more sources of light behaves as coherent sources if they have no phase difference or have a constant phase difference between the waves emitted by them.

A common method for producing two coherent light sources is to use single source to illuminate a screen containing two narrow slits. Hence two or more sources derived from a single source of light behaves as coherent source because they are in the same phase.

9.2 How is the distance between interference fringes affected by the separation between the slits of Young's experiment? Can fringes disappear?

Ans. The fringe spacing or distance between two consecutive bright or dark fringes in young's experiment is given by

$$\Delta y = \frac{\lambda L}{d}$$

where λ is the wavelength of light used, L is the distance of screen and source and d is the separation of slits. This relation shows that the fringe spacing is inversely proportional to the separation of slits. It means that greater the separation between the slits, the smaller will be fringe spacing. If by increasing the separation, the fringe spacing decreases and the bright fringes becomes so close that uniform intensity of light is seen and fringes disappear.

9.3 Can visible light produce interference fringes? Explain.

Ans. Yes, the white light or visible light can produce the interference fringes. Since the white light is the mixture of seven colours so each colour will produce interference fringes corresponding to its own wavelength. Hence the fringe pattern will be coloured but the fringes will be so closed that it would be difficult to observe the interference fringes of visible or white light.

9.4 In the Young's experiment, one of the slits is covered with blue filter and other with red filter. What would be the pattern of light intensity on the screen?

Ans. Since red and blue light have different wavelengths and will not be in phase coherence. Therefore there will be no dark and bright bands on the screen. So the interference of light cannot be observed properly. For better interference pattern, the monochromatic light should be used.

9.5 Explain whether the Young's experiment is an experiment for studying interference or diffraction effects of light.

Ans. Young's double slit experiment is basically used to study the interference of light. However spreading of light around the corners of the slits also produce diffraction of light. However interference on the screen takes place only when the light coming out of the narrow slits suffers from diffraction first.

9.6 An oil film spreading over a wet footpath shows colours. Explain how does it happen?

Ans. The colours are seen on the oil film spreading on the wet foot path due to interference of light waves. When a light beam is incident, a part of it is reflected from the upper surface of the then oil film and a part of it is reflected from the lower surface of the thin film. The two reflected beams are coherent. When oil film is very thin, these coherent beams overlap. Hence constructive and destructive interference exhibit colours.

9.7 Could you obtain Newton's rings with transmitted light? If yes, would the pattern be different from that obtained with reflected light?

Ans. Yes, Newton rings can be obtained with transmitted light. However no phase is changed in transmitted light, so bright fringes can be replaced by dark fringes and so on. In case of transmitted light, the central point is bright.

9.8 In the white light spectrum obtained with a diffraction grating, the third order image of a wavelength coincides with the fourth order image of a second wavelength. Calculate the ratio of the two wavelengths.

Ans. For diffraction grating, the equation is

$$d \sin \theta = n\lambda$$

where d sin θ is the phase difference, and n is the number of order and λ is the wavelength of light used.

For first wavelength λ_1 and 3rd order, n = 3

$$d \sin \theta = 3\lambda_1 \qquad \ldots\ldots \text{(i)}$$

and for second wavelength λ_2 and fourth order, n = 4

$$d \sin \theta = 4\lambda_2 \qquad \ldots\ldots \text{(i)}$$

Equating the eq. (i) and (ii)

$$3\lambda_1 = 4\lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

$$\lambda_1 : \lambda_2 = 4 : 3$$

9.9 How would you manage to get more orders of spectra using a diffraction grating?

Ans. For diffraction grating, the equation is given by

$$d \sin \theta = n\lambda$$

where d is the grating element, λ is the wavelength. In order to get more orders of spectra from θ = 0° to θ = 90° for a given wavelength, the grating element d must be increased i.e. Less number of lines per unit length be ruled on diffraction grating.

9.10 Why the polaroid sunglasses are better than ordinary sunglasses?

Ans. The sunlight reflected from smooth surfaces such as water, wet roads, lakes and glass is horizontally polarized and produces glare. This glare can be reduced by using polariod sunglasses because they can decrease the intensity of light passing through them. Hence the polariod sunglasses are better than ordinary sunglasses. Since polariod sunglass reduce the glare of light entering into the eye.

9.11 How would you distinguish between un-polarized and plane-polarized lights?

Ans. Un-polarized and plane polarized light can be distinguished from each other by using polarizer. When light is viewed through polarizer and it can be seen continuously even if the polarizer is rotated, the light seen is unpolarized. However if on rotating the polarizer, the light becomes dim and cuts off by rotating the polarizer through 90°, then the light observed is plane polarized light.

9.12 Fill in the Blanks:

(i) **According to ———— principle, each point on a wavefront acts as a source of secondary ————.**

(ii) **In Young's experiment, the distance between two adjacent bright fringes for violet light is ———— than that for green light.**

(iii) **The distance between bright fringes in the interference pattern ———— as the wavelength of light used increase.**

(iv) **A diffraction grating is used to make a diffraction pattern for yellow light and then for red light. The distances between the red spots will be ———— than that for yellow light.**

(v) **The phenomenon of polarization of light reveals that light waves are ———— waves.**

(vi) **A polaroid is a commercial ————.**

(vii) **A polaroid glass ———— glare of light produced at a road surface.**

Ans.

(i) Huygen's, wavelets	(ii) less	(iii) increases	(iv) more
(v) transverse	(vi) polarizer	(vii) eliminates (reduces)	

PROBLEMS WITH SOLUTIONS

PROBLEM 9.1

Light of wavelength 546 nm is allowed to illuminate the slits of Young's experiment. The separation between the slits is 0.10 mm and the distance of the screen from the slits where interference effects are observed is 20 cm. At what angle the first minimum will fall? What will be the linear distance on the screen between adjacent maxima?

Data

Wavelength of light $= \lambda = 546\ \text{nm}$

$= 546 \times 10^{-9}\ \text{m}$

Separation between the slits $= d = 0.10\ \text{mm}$

$= 0.10 \times 10^{-3}\ \text{m}$

Distance of screen from the slit $= L = 20\ \text{cm}$

$= 20 \times 10^{-2}\ \text{m}$

To Find

Angle for minimum fall $= \theta = ?$

Fringe spacing $= \Delta y = ?$

SOLUTION

Angle for minimum fall is

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

$$\sin\theta = \frac{\left(m + \frac{1}{2}\right)\lambda}{d}$$

For 1st minimum m = 0

$$\sin\theta = \frac{\lambda}{2d}$$

$$\sin\theta = \frac{546 \times 10^{-9}}{2 \times 0.10 \times 10^{-3}}$$

$$\sin\theta = 2730 \times 10^{-9+3}$$

$$\sin\theta = 2730 \times 10^{-6}$$

$$\theta = \sin^{-1}(0.002730)$$

$$\theta = 0.156$$

$$= 0.16^\circ$$

For Fringe spacing

$$\boxed{\Delta y = \frac{L\lambda}{d}}$$

$$\Delta y = \frac{20 \times 10^{-2} \times 546 \times 10^{-9}}{0.10 \times 10^{-3}}$$

$$= 109200 \times 10^{-2-9+3}$$

$$= 109200 \times 10^{-8}$$

$$= 1.09 \times 10^{-3}\ \text{m}$$

$$\Delta y = 1.09\ \text{mm}$$

Result

Angle for minimum fall = θ = $0.16°$

Fringe spacing = Δy = 1.09 mm

PROBLEM 9.2

Calculate the wavelength of light, which illuminates two slits 0.5 mm apart and produces interference pattern on a screen placed 200 cm away from the slits. The first bright fringe is observed at a distance of 2.40 mm from the central bright image.

Data

Distance between slits = d = 0.5 mm

= 0.5×10^{-3} m

Distance of screen from slits = L = 200 cm

= 2 m

For 1st bright fringe = m = 1

Distance of 1st bright fringe = y = 2.40 mm

= 2.40×10^{-3} m

To Find

Wavelength of light = λ = ?

SOLUTION

By formula

$$y = \frac{m\lambda L}{d}$$

$$\boxed{\lambda = \frac{y \times d}{mL}}$$

Putting the value, for 1st bright $m = 1$

$$\lambda = \frac{2.40 \times 10^{-3} \times 0.5 \times 10^{-3}}{1 \times 2}$$

$$= 0.6 \times 10^{-6}$$

$$= 6 \times 10^{-7}\ \text{m}$$

$$= 600 \times 10^{-9}\ \text{m}$$

$$\lambda = 600\ \text{nm}$$

Result

Wavelength of light $= \lambda = 600$ nm

or $= 6 \times 10^{-7}$ m

PROBLEM 9.3

In a double slit experiment the second order maximum occurs at θ = 0.25°. The wavelength is 650nm. Determine the slit separation.

Data

Second order maxima $= m = 2$

Angle $= \theta = 0.25°$

Wavelength of light $= \lambda = 650$ nm

$= 650 \times 10^{-9}$ m

To Find

Slit separation $= d = ?$

SOLUTION

By formula

$$d \sin\theta = m\lambda$$

$$\boxed{d = \frac{m\lambda}{\sin\theta}}$$

$$d = \frac{2 \times 650 \times 10^{-9}}{\sin 0.25°}$$

$$= \frac{1300 \times 10^{-9}}{4.36 \times 10^{-3}}$$

$$= \frac{1300 \times 10^{-9}}{0.00430}$$

$$= 298165.1 \times 10^{-9}$$

$$= 0.298 \times 10^{-3}$$

$$= 0.30 \times 10^{-3}\ \text{m}$$

$$d = 0.30\ \text{mm}$$

Result

Slit separation $= d = 0.30$ mm

PROBLEM 9.4

A monochromatic light of λ = 588 nm is allowed to fall on the half silvered glass plate G_1, in the Michelson interferometer. If mirror M_1 is moved through 0.233 mm, how many fringes will be observed to shift?

Data

Wavelength of light $= \lambda = 588$ nm

$= 588 \times 10^{-9}$ m

Distance moved by mirror M_1 $= L = 0.233$ mm

$= 0.233 \times 10^{-3}$ m

To Find

Number of fringes $= m = ?$

SOLUTION

By formula

$$L = m\frac{\lambda}{2}$$

$$\boxed{m = \frac{2L}{\lambda}}$$

$$m = \frac{2 \times 0.233 \times 10^{-3}}{588 \times 10^{-9}}$$

$$= \frac{0.466 \times 10^{-3}}{588 \times 10^{-9}}$$

$$= 7.92 \times 10^{-3-4+9}$$

$$= 7.92 \times 10^{2}$$

$$m = 792$$

Result

Number of fringes $= m = 792$

PROBLEM 9.5

A second order spectrum is formed at an angle of 38.0° when light falls normally on a diffraction grating having 5400 lines per centimetre. Determine wavelength of the light used.

Data

For second order spectrum $= n = 2$

Angle of diffraction $= \theta = 38.0°$

Number of lines on grating $= N = 5400$ lines/cm

$= 540000$ lines/m

To Find

Wavelength of light $= \lambda = ?$

SOLUTION

By formula

$$d \sin\theta = n\lambda$$

$$\boxed{\lambda = \frac{d \sin\theta}{n}} \qquad \text{But} \qquad d = \frac{1}{N} = \frac{1}{540000}$$

$$\lambda = \frac{\frac{1}{540000} \times \sin 38.0}{2}$$

$$= \frac{0.615}{540000 \times 2}$$

$$= \frac{0.615}{1080000}$$

$$= 5.69 \times 10^{-7}$$

$$= 5.70 \times 10^{-7} \text{ m}$$

$$= 570 \times 10^{-9} \text{ m}$$

$$\lambda = 570 \text{ nm}$$

Result

Wavelength of light $= \lambda = 570$ nm

PROBLEM 9.6

A light is incident normally on a grating which has 2500 lines per centimetre. Compute the wavelength of a spectral line for which the deviation in second order is 15.0°.

Data

Number of lines on grating $= N = 2500$ /cm

$= 2500 \times 100$ /m

$= 250000$ /m

Angle of diffraction $= \theta = 15°$

For second order $= n = 2$

To Find

Wavelength of light $= \lambda = ?$

SOLUTION

By formula

$$d \sin\theta = n\lambda$$

$$\lambda = \frac{d \sin\theta}{n}$$

As $$d = \frac{1}{N}$$

$$= \frac{1}{250000}$$

$$\text{So} \qquad \lambda = \frac{\frac{1}{250000} \times \sin 15^\circ}{2}$$

$$= \frac{0.2588}{500000}$$

$$= 5.176 \times 10^{-7}$$

$$= 517.6 \times 10^{-9} \text{ m}$$

$$\lambda = 518 \text{ nm}$$

Result

Wavelength of light = λ = 518 nm

PROBLEM 9.7

Sodium light (= 589 nm) is incident normally on a grating having 3000 lines per centimetre. What is the highest order of the spectrum obtained with this grating?

Data

Wavelength of sodium light = λ = 589 nm

= 589×10^{-9} m

Number of lines on grating = N = 3000 /cm

= 300000 /m

Angle of grating (for highest order) = θ = 90°

To Find

Highest order of spectrum = n = ?

SOLUTION

By formula

$$d \sin \theta = n\lambda$$

$$\boxed{n = \frac{d \sin \theta}{\lambda}} \qquad \text{But} \qquad d = \frac{1}{N} = \frac{1}{300000}$$

$$\text{So} \qquad n = \frac{\frac{1}{300000} \times \sin 90^\circ}{589 \times 10^{-9}}$$

$$= 5.66 \times 10^{-9} \times 10^{9}$$

$$n = 5.66$$

Result

Highest order of spectrum = n = 5^{th}

PROBLEM 9.8

Blue light of wavelength 480 nm illuminates a diffraction grating. The second order image is formed at an angle of 30° from the central image. How many lines in a centimeter of the grating have been ruled?

Data

Wavelength of light $= \lambda = 480 \text{ nm}$

$= 480 \times 10^{-9} \text{ m}$

Angle $= \theta = 30°$

For second order image $= n = 2$

To Find

Number of lines per centimeter $= N = ?$

SOLUTION

By using the formula

$$d \sin\theta = n\lambda$$

$$\boxed{d = \frac{n\lambda}{\sin\theta}}$$

$$= \frac{2 \times 480 \times 10^{-9}}{\sin 30°}$$

$$= \frac{960 \times 10^{-9}}{0.5}$$

$$d = 1920 \times 10^{-9}$$

But $N = \dfrac{1}{d}$

$$= \frac{1}{1920 \times 10^{-9}}$$

$$= 5.20 \times 10^{-4+9}$$

$$= 5.20 \times 10^{5} \text{ /m}$$

$$N = 5.20 \times 10^{3} \text{ /cm}$$

Result

Number of lines per centimeter $= N = 5.2 \times 10^{3}$

PROBLEM 9.9

X-rays of wavelength 0.150 nm are observed to undergo a first order reflection at a Bragg angle of 13.3° from a quartz (SiO_2) crystal. What is the interplaner spacing of the reflecting planes in the crystal?

Data

Wavelength of light $= \lambda = 0.150 \text{ nm}$

$= 0.150 \times 10^{-9} \text{ m}$

For 1^{st} order reflection $= n = 1$

Bragg's angle $= \theta = 13.3°$

To Find

Interplaner spacing $= d = ?$

SOLUTION

According to Bragg's law

$$2d \sin\theta = n\lambda$$

$$\boxed{d = \frac{n\lambda}{2\sin\theta}}$$

Putting the values

$$d = \frac{1 \times 0.150 \times 10^{-9}}{2 \sin 13.3°}$$

$$= \frac{0.150 \times 10^{-9}}{0.230 \times 2}$$

$$= \frac{0.150 \times 10^{-9}}{0.460}$$

$$= 0.326 \times 10^{-9}\ \text{m}$$

$$d = 0.326\ \text{nm}$$

Result

Interplaner spacing of the reflecting planes in the crystal $= d = 0.326$ nm

or $d = 0.326 \times 10^{-9}$ m

PROBLEM 9.10

An X-ray beam of wavelength λ undergoes a first order reflection from a crystal when its angle of incidence to a crystal face is 26.5°, and an X-ray beam of wavelength 0.097 nm undergoes a third order reflection when its angle of incidence to that face is 60.0°. Assuming that the two beams reflect from the same family of planes, calculate (a) the interplanar spacing of the planes and (b) the wavelength λ.

Data

For 1^{st} wavelength λ_1

Angle of incidence $= \theta_1 = 26.5°$

First order reflection $= n_1 = 1$

For second wavelength λ_2

Angle of incidence $= \theta_2 = 60°$

Third order reflection $= n_2 = 3$

Wavelength of second beam $= \lambda_2 = 0.097$ nm

$= 0.097 \times 10^{-9}$ m

To Find

(a) Interplaner spacing of the planes $= d = ?$

(b) Wavelength of 1^{st} beam $= \lambda_1 = ?$

SOLUTION

(a) For interplaner spacing, by using Bragg's law

$$2d \sin\theta = n\lambda$$

For 1st beam

$$2d \sin\theta_1 = n_1\lambda_1$$

$$\boxed{\lambda_1 = \frac{2d \sin\theta_1}{n_1}}$$

Putting the values

$$\lambda_1 = \frac{2 \times d \sin 26.5°}{1}$$

$$\lambda_1 = 0.892\ d \qquad \ldots\ldots \text{(i)}$$

For 2nd beam

$$\lambda_2 = \frac{2d \sin\theta_2}{n_2}$$

$$\boxed{d = \frac{\lambda_2 n_2}{2 \sin\theta_2}}$$

$$d = \frac{0.097 \times 10^{-9} \times 3}{2 \sin 60°}$$

$$= \frac{0.291 \times 10^{-9}}{2 \times 0.866}$$

$$= 0.168 \times 10^{-9}\ \text{m}$$

$$d = 0.168\ \text{nm}$$

For 1st wavelength, putting in eq. (i)

$$\lambda_1 = 0.892 \times d$$

$$= 0.892 \times 0.168$$

$$\lambda_1 = 0.150\ \text{nm}$$

Result

(a) Interplaner spacing $= d = 0.168$ nm

(b) Wavelength of 1st beam $= \lambda_1 = 0.150$ nm

SHORT QUESTIONS

10.1 What do you understand by linear magnification and angular magnification? Explain how a convex lens is used as a magnifier?

Ans. **Linear Magnification:** It is defined as the ratio of the size of image to the size of object. Mathematically

$$M = \frac{I}{O} = \frac{q}{p}$$

Angular Magnification: It is the ratio of the angle subtended by the image as seen through the optical instrument to the angle subtended by the object at the unaided eye.

$$M = \frac{\beta}{\alpha}$$

Both linear and angular magnification has no unit.

A Convex lens as a Magnifier: A convex lens of short focal length can be used as magnifying glass because when an object is placed between lens and focus then the image formed is erect, virtual and very much magnified.

10.2 Explain the difference between angular magnification and resolving power of an optical instrument. What limits the magnification of an optical instrument?

Ans. Angular magnification means how large or magnified image is formed by the instrument but resolving power is its ability to provide the minor details of an object under examination. The magnification of an optical instrument is limited due to defects in the lenses. Such as chromatic and spherical aberrations.

10.3 Why would it be advantageous to use blue light with a compound microscope?

Ans. We know that the expression for resolving power is:

$$R \propto \frac{1}{\alpha_{min.}}$$

Here, $$\alpha_{min.} = 1.22\frac{\lambda}{D}$$

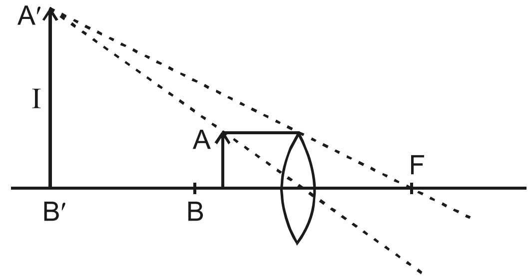
From this equation we see that resolving power is inversely proportional to $\alpha_{min.}$ and $\alpha_{min.}$ depends upon wavelength and diameter of lens. As blue light has short wavelength therefore it will produce less diffraction and resolving power of compound microscope will increase.

10.4 One can buy a cheap microscope for use by the children. The images seen in such a microscope have coloured edges. Why is this so?

Ans. It is due to the defect of lenses known as chromatic aberrations. This is because of the prism like formation of the lens in which all rays of white light cannot meet at a single point therefore image is not sharp and has coloured edges.

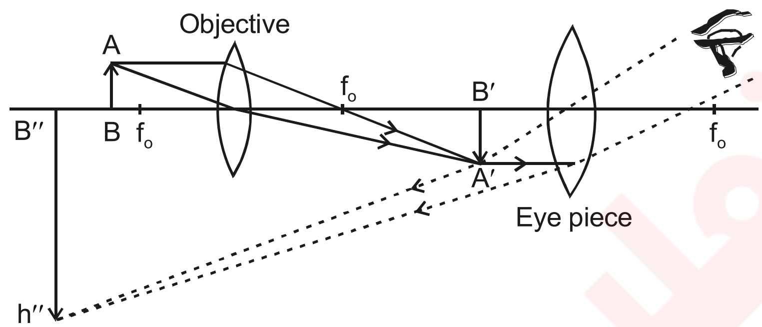
10.5 Describe with the help of diagram, how (a) a single biconvex lens can be used as a magnifying glass. (b) biconvex lenses can be arranged to form a microscope.

Ans. (a) Ray diagram of a biconvex lens used as magnifying glass:



Simple microscope or magnifying glass

(b) Ray diagram of two biconvex lens arranged to make microscope.

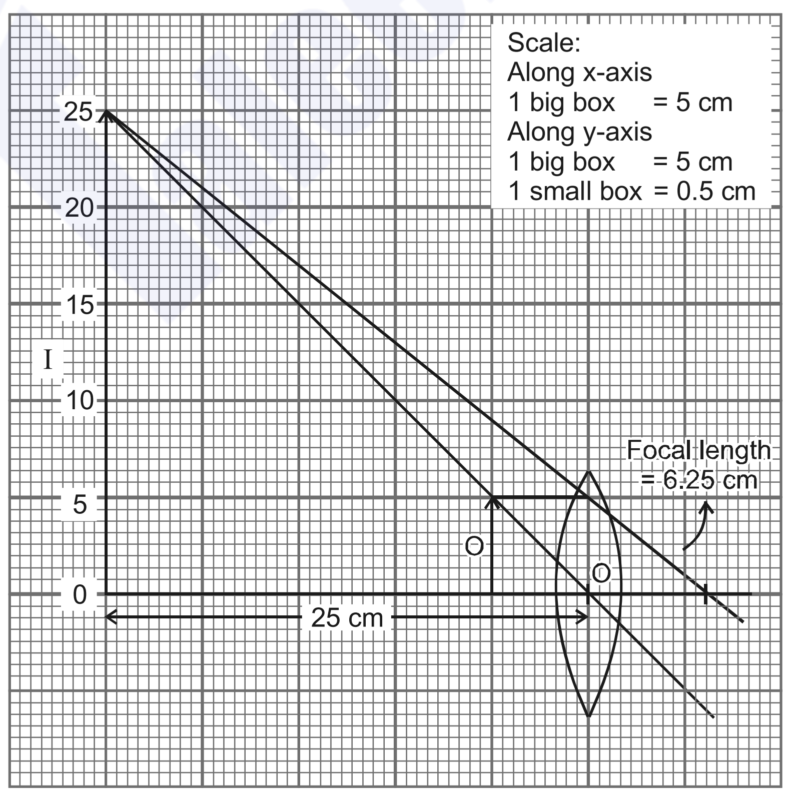


Compound microscope

10.6 If a person were looking through a telescope at the full moon, how would the appearance of the moon be changed by covering half of the objective lens?

Ans. The person will see the full image of the moon if half of the objective lens of a telescope is covered but its brightness is reduced because less light is transmitted through the lens.

10.7 A magnifying glass gives a five times enlarged image at a distance of 25 cm from the lens. Find, by ray diagram, the focal length of the lens.



Data

Magnification	=	M	=	5
Image distance	=	q	=	25 cm
Focal length of lens	=	f	=	?

Solution

By formula

$$M = 1 + \frac{d}{f}$$

$$5 = 1 + \frac{25}{f}$$

$$5 - 1 = \frac{25}{f}$$

$$4f = 25$$

$$f = \frac{25}{4}$$

$$f = 6.2 \text{ cm}$$

10.8 Identify the correct answer:

(i) The resolving power of a compound microscope depends on;

(a) The refrective index of the medium in which the object is placed.

(b) The diameter of the objective lens.

(c) The angle subtended by the objective lens at the object.

(d) The position of an observer's eye with regard to the eye lens.

(ii) The resolving power of an astronomical telescope depends on:

(a) The focal length of the objective lens.

(b) The least distance of distinct vision of the observer.

(c) The focal length of the eye lens.

(d) The diameter of the objective lens.

Ans. (i) The formula for the resolving power of a lens of diameter D is given by

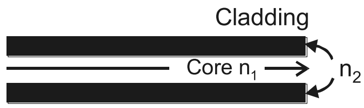
$$R = \frac{D}{1.22\lambda}$$

Hence the resolving power of a lens depends upon the diameter of objective so (b) is correct.

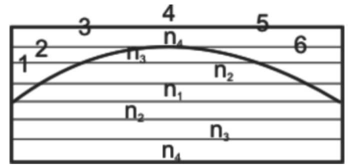
(ii) As we know that the resolving power of an astronomical telescope depends upon the diameter of objective lens so (d) is correct.

10.9 Draw sketches showing the different light paths through a single-mode and a multimode fibre. Why is the single-mode fibre preferred in telecommunications?

Ans.



(a)



(b)

It has a very thin core of about 5 μm diameter and has a relatively larger cladding. It can carry more than 14 TV channels or 14000 phone calls so it is preferred in telecommunication.

10.10 How the light signal is transmitted through the optical fibre?

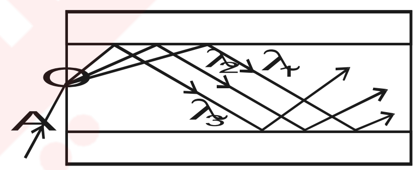
Ans. The light signals are transmitted through the optical fibre on the principle of:

(i) Total internal reflection.

(ii) Continuous refraction.

10.11 How the power is lost in optical fibre through dispersion? Explain.

Ans. If the source of light signals is not monochromatic then the light will disperse while propagating through the core of the optical fibre into different wavelength so the light of different wavelengths reaches the other end of the fibre at different times and the signal received is distorted. So the power is lost in optical fibre through dispersion.



PROBLEMS WITH SOLUTIONS

PROBLEM 10.1

A converging lens of focal length 5.0 cm is used as a magnifying glass. If the near point of the observer is 25 cm and the lens is held close to the eye, calculate (i) the distance of the object from the lens (ii) the angular magnification. What is the angular magnification when the final image is formed at infinity?

Data

Focal length of the lens = f = 5.0 cm

Distance of near point = q = d = 25 cm

To Find

(i) Distance of object from lens = p = ?

(ii) Angular magnification = M = ?

Angular magnification when the image is at infinity = M′ = ?

SOLUTION

(i) By lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

As q = –d, the image is virtual so

$$\frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

$$\frac{1}{5} = \frac{1}{p} - \frac{1}{25}$$

$$\frac{1}{p} = \frac{1}{5} + \frac{1}{25}$$

$$= \frac{5+1}{25}$$

$$\frac{1}{p} = \frac{6}{25}$$

$$p = \frac{25}{6} = 4.16 \text{ cm}$$

(ii) As we know that

$$\text{Angular magnification} = M = 1 + \frac{d}{f}$$

$$= 1 + \frac{25}{5}$$

$$= 1 + 5$$

$$M = 6$$

For angular magnification when the image is at infinity then the object must be at focus. So

$$M' = \frac{d}{f} \qquad \text{Since} \quad p = f$$

$$M' = \frac{d}{f} = \frac{25}{5}$$

$$M' = 5$$

Result

(i) Distance of object from the lens $= p = 4.16$ cm

(ii) Angular magnification $= M = 6$

Angular magnification when the image is at infinity $= M' = 5$

PROBLEM 10.2

A telescope objective has focal length 96 cm and diameter 12 cm. Calculate the focal length and minimum diameter of a simple eye piece lens for use with the telescope, if the linear magnification required is 24 times and all the light transmitted by the objective from a distant point on the telescope axis is to fall on the eye piece.

Data

Focal length of objective $= f_o = 96$ cm

Diameter of objective $= d_o = 12$ cm

Linear magnification $= M = 24$

To Find

Focal length of eye-piece $= f_e = ?$

Diameter of eye-piece $= d_e = ?$

SOLUTION

By using the formula for linear magnification in case of telescope

$$M = \frac{f_o}{f_e}$$

$$\boxed{f_e = \frac{f_o}{M}}$$

$$f_e = \frac{96}{24}$$

$$f_e = 4 \text{ cm}$$

For the diameter of eye-piece

$$\boxed{\frac{f_o}{f_e} = \frac{d_o}{d_e}}$$

$$d_e = \frac{f_e \times d_o}{f_o}$$

$$= \frac{4 \times 12}{96}$$

$$d_e = 0.5 \text{ cm}$$

Result

Focal length of eye-piece $= f_e = 4$ cm

Diameter of eye-piece $= d_e = 0.5$ cm

PROBLEM 10.3

A telescope is made of an objective of focal length 20 cm and an eye piece of 5.0 cm, both convex lenses. Find the angular magnification.

Data

Focal length of objective $= f_o = 20$ cm

Focal length of eye-piece $= f_e = 5.0$ cm

To Find

Angular magnification $= M = ?$

SOLUTION

By formula

$$\boxed{M = \frac{f_o}{f_e}}$$

$$M = \frac{20}{5.0}$$

$$M = 4$$

Result

Angular magnification $= M = 4$

PROBLEM 10.4

A simple astronomical telescope in normal adjustment has an objective of focal length 100 cm and an eye piece of focal length 5.0 cm. (i) Where is the final image formed (ii) Calculate the angular magnification.

Data

Focal length of objective $= f_o = 100$ cm

Focal length of eye-piece $= f_e = 5.0$ cm

To Find

(i) Distance of final image $= q = ?$

(ii) Angular magnification $= M = ?$

SOLUTION

(i) For distance of final image

$$\frac{1}{f_e} = \frac{1}{p} + \frac{1}{q}$$

The final image is virtual so

$$\frac{1}{f_e} = \frac{1}{p} - \frac{1}{q} \qquad \text{Since} \quad p = f_e = 5.0\text{ cm}$$

So
$$\frac{1}{5.0} = \frac{1}{5.0} - \frac{1}{q}$$

$$\frac{1}{q} = \frac{0}{5.0}$$

$$q = \frac{5.0}{0}$$

$$q = \infty \text{ (infinity)}$$

(ii) For angular magnification

$$M = \frac{f_o}{f_e} = \frac{100}{5.0}$$

$$M = 20$$

Result

(i) Distance of final image = $q = \infty$

(ii) Angular magnification = $M = 20$

PROBLEM 10.5

Point object is placed on the axis of and 3.6 cm from a thin convex lens of focal length 3.0 cm. A second thin convex lens of focal length 16.0 cm is placed coaxial the first and 26.0 cm from it on the side away from the object. Find the position of the final image produced by the two lenses.

Data

Distance of object = p_1 = 3.6 cm

Distance between lenses = L = 26 cm

Focal length of 1st lens = f_1 = 3.0 cm

Focal length of 2nd lens = f_2 = 16.0 cm

To Find

Position of the final image = q_2 = ?

SOLUTION

For 1st law

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{q_1}$$

$$\frac{1}{3.0} = \frac{1}{3.6} + \frac{1}{q_1}$$

$$\frac{1}{3} - \frac{1}{3.6} = \frac{1}{q_1}$$

$$\frac{1}{3} - \frac{10}{36} = \frac{1}{q_1}$$

$$\frac{12-10}{36} = \frac{1}{q_1}$$

$$q_1 = \frac{36}{2}$$

$$q_1 = 18 \text{ cm}$$

Distance of object from the 2nd lens $= p_2 = L - q_1$

$$= 26 - 18$$

$$p_2 = 8 \text{ cm}$$

So for 2nd lens

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q_2}$$

So $$\frac{1}{16} = \frac{1}{8} + \frac{1}{q_2}$$

$$\frac{1}{q_2} = \frac{1}{10} - \frac{1}{8}$$

$$= \frac{1-2}{16}$$

$$\frac{1}{q_2} = -\frac{1}{16}$$

$$q_2 = -16 \text{ cm}$$

Result

Position of final image $= q_2 = -16$ cm

PROBLEM 10.6

A compound microscope has lenses of focal length 1.0 cm and 3.0 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed, 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument.

Data

Focal length of objective = f_o = 1.0 cm

Focal length of eye-piece = f_e = 3.0 cm

Distance of object from objective = p_1 = 1.2 cm

Distance of final image = q_2 = 25 cm

To Find

Separation between the lenses = L = ?

Magnification of the instrument = M = ?

SOLUTION

For objective

$$\frac{1}{f_o} = \frac{1}{p_1} + \frac{1}{q_1}$$

$$\frac{1}{10} = \frac{1}{1.2} + \frac{1}{q_1}$$

$$\frac{1}{1} - \frac{1}{1.2} = \frac{1}{q_1}$$

$$1 - \frac{10}{12} = \frac{1}{q_1}$$

$$\frac{12-10}{12} = \frac{1}{q_1}$$

$$\frac{2}{12} = \frac{1}{q_1}$$

$$q_1 = \frac{12}{2}$$

$$q_1 = 6 \text{ cm}$$

For eye-piece of the compound microscope

$$\frac{1}{f_e} = \frac{1}{p_2} + \frac{1}{q_2}$$

Since the final image is virtual

So $$\frac{1}{f_e} = \frac{1}{p_2} - \frac{1}{q_2}$$

$$\frac{1}{3.0} = \frac{1}{p_2} - \frac{1}{25}$$

$$\frac{1}{3} + \frac{1}{25} = \frac{1}{p_2}$$

$$\frac{25+3}{75} = \frac{1}{p_2}$$

$$p_2 = \frac{75}{28}$$

$$p_2 = 2.67 \text{ cm}$$

So the separation between the lens $= L = q_1 + p_2$

$= 6 + 2.67$

$= 8.67$ cm

For magnification of the instrument

$$M = \frac{q_1}{p_1}\left(1 + \frac{d}{f_e}\right)$$

$$= \frac{6}{1.2}\left(1 + \frac{25}{3.0}\right)$$

$$= 5(1 + 8.33)$$

$$= 46.7$$

$$M = 47$$

Result

Separation between the lenses $= L = 8.67$ cm

Magnification of the instrument $= M = 47$

PROBLEM 10.7

Sodium light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective is 0.90 cm, (i) find the limiting angle of resolution (ii) using visible light of any wavelength, what is the maximum limit of resolution for this microscope.

Data

Wavelength of sodium light $= \lambda = 589$ nm

$= 589 \times 10^{-9}$ m

Aperture of lens $= D = 0.90$ cm

$= 0.90 \times 10^{-2}$ m

To Find

(i) Limiting angle of resolution $= \alpha_{min} = ?$

(ii) For any visible light, limiting angle of resolution $= \alpha'_{min} = ?$

SOLUTION

(i) By formula, for the limiting angle of resolution

$$\alpha_{min} = \frac{1.22\lambda}{D}$$

$$= \frac{1.22 \times 589 \times 10^{-9}}{6.90 \times 10^{-2}}$$

$$= 794.3 \times 10^{-9+2}$$
$$= 794.3 \times 10^{-7}$$
$$= 7.9 \times 10^{-5} \text{ rad.}$$

(ii) For any visible light

Wavelength of violet light $= \lambda = 400$ nm

$= 400 \times 10^{-9}$ m

Therefore;

$$\boxed{\alpha'_{min} = \frac{1.22\lambda}{D}}$$

$$\alpha'_{min} = \frac{1.22 \times 400 \times 10^{-9}}{0.90 \times 10^{-2}}$$
$$= 542.22 \times 10^{-9+2}$$
$$= 542.2 \times 10^{-7}$$
$$= 5.4 \times 10^{-5} \text{ rad.}$$

Result

(i) Limiting angle of resolution $= \alpha_{min} = 7.9 \times 10^{-5}$ rad.

(ii) For any visible light, limiting angle of resolution $= \alpha'_{min} = 5.4 \times 10^{-5}$ rad.

PROBLEM 10.8

An astronomical telescope having magnifying power of 5 consist of two thin lenses 24 cm apart. Find the focal lengths of the lenses.

Data

Magnification of the lenses $= M = 5$

Distance between lenses $= L = 24$ cm

To Find

Focal length of objective $= f_o = ?$

Focal length of eye-piece $= f_e = ?$

SOLUTION

As we know that the magnification of the astronomical telescope is

$$M = \frac{f_o}{f_e}$$

$$5 = \frac{f_o}{f_e}$$

$$\boxed{f_o = 5f_e}$$

and the length of telescope is

$$L = f_o + f_e$$

$$f_o + f_e = 24$$

$$f_e + 5f_e = 24$$

$$6f_e = 24$$

$$f_e = \frac{24}{6}$$

$$f_e = 4 \text{ cm}$$

and $$f_o = 5f_e$$

$$f_o = 5(4)$$

$$= 20 \text{ cm}$$

Result

Focal length of objective $= f_o = 20$ cm

Focal length of eye-piece $= f_e = 4$ cm

PROBLEM 10.9

Glass light pipe in air will totally internally reflect a light ray if its angle of incidence is atleast 39°. What is the minimum angle for total internal reflection if pipe is in water? (Refractive Index of water = 1.33).

Data

Angle of incidence $= \theta_c = 39°$

Refractive index for water $= n_2 = 1.33$

To Find

Minimum angle for total internal reflection for water $= \theta_1 = ?$

SOLUTION

According to Snell's law

$$n_1 \sin \angle i = n_2 \sin \angle r$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\boxed{\sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1}} \qquad \ldots\ldots \text{(i)}$$

But $$n_1 = \frac{1}{\sin \theta_c}$$

$$= \frac{1}{\sin 39°}$$

$$n_1 = 1.59$$

Putting in eq. (i)

So $\sin\theta_1 = \dfrac{1.33 \sin 90°}{1.59}$

$\sin\theta_1 = 0.83$

$\theta_1 = \sin^{-1}(0.83)$

$\theta_1 = 57°$

Result

Minimum angle for total internal reflection for water $= \theta_1 = 57°$

PROBLEM 10.10

The refractive index of the core and cladding of an optical fibre are 1.6 and 1.4 respectively. Calculate (i) the critical angle for the interface (ii) the maximum angle of incidence in the air of a ray which enters the fibre and is incident at the critical angle on the interface.

Data

Refractive index of core $= n_1 = 1.6$

Refractive index of cladding $= n_2 = 1.4$

To Find

(i) Critical angle $= \theta_c = ?$

(ii) Maximum angle of incidence for air $= \theta_1' = ?$

SOLUTION

(i) By using Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

But $\theta_1 = \theta_c$

$\theta_2 = 90°$

So $n_1 \sin\theta_c = n_2 \sin 90°$

$\sin\theta_c = \dfrac{n_2}{n_1}$

$\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

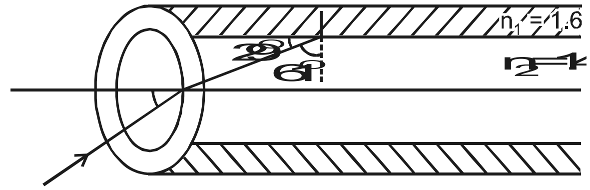
$= \sin^{-1}\left(\dfrac{1.4}{1.6}\right)$

$\theta_c = 61°$

(ii) For maximum angle of incidence $\theta_c = 61°$ and the angle of refraction is

$\theta'_c = 90° - 61°$

$\theta'_c = 29°$



Therefore;

$$n_1 \sin\theta'_1 = n_2 \sin\theta'_2$$

For air $n_1 = 1$

For core $n_2 = 1.6$

So $1 \times \sin\theta'_1 = 1.6 \times \sin 29°$

$$\sin\theta'_1 = 0.77$$

$$\theta'_1 = \sin^{-1}(0.77)$$

$$\theta'_1 = 50°.8$$

$$= 51°$$

Result

(i) Critical angle $= \theta_c = 61°$

(ii) Maximum angle of incidence for air $= \theta'_1 = 51°$

SHORT QUESTIONS

11.1 Why is average velocity of the molecules in a gas zero but the average of the square of velocities is not zero?

Ans. According to kinetic molecular theory of gasses, there are a large number of molecules which are in random motion. Due to random motion of molecules, the number of molecules on the average moving in any direction with certain velocity is equal to number of molecules moving on opposite direction with the same velocity. So their average velocity will be zero because their vector sum will be zero i.e.,

$$v + (-v) = 0$$

but we know that square of negative quantity is positive therefore when we take average of the square of velocities it will not be zero, i.e., $V^2 + (-V)^2$ is not zero.

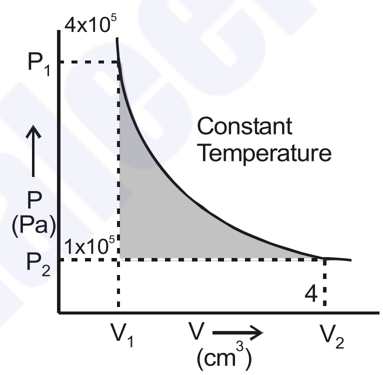
11.2 Why does the pressure of a gas in a car tyre increase when it is driven through some distance?

Ans. When a car is driven on the road through some distance. There is force of friction between the tyre and road. Due to this force of friction, the tyre heats up and the gas inside the tyre. Work done by the car is converted into heat which raises the temperature of the gas in a tyre. This increases the kinetic energy of the molecules. Since pressure is directly proportional to the average kinetic energy. i.e.,

$$P \propto \langle K.E \rangle$$

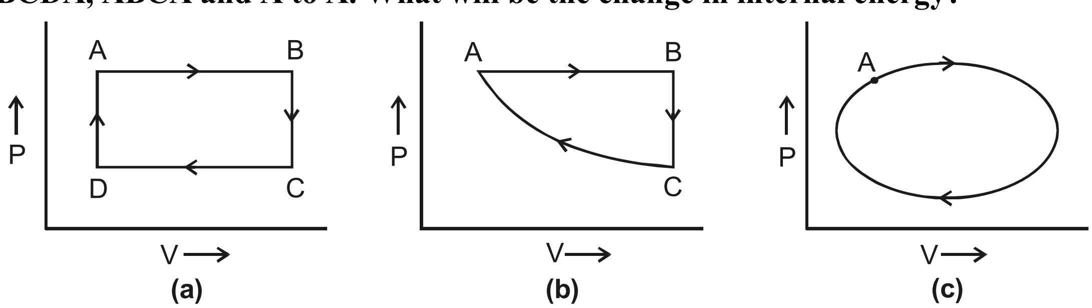
Hence pressure of a gas in a car tyre increases.

11.3 A system undergoes from state P_1V_1 to state P_2V_2 as shown in figure 11.12. What will be the change in internal energy?



Ans. As internal energy depends on temperature. In this case temperature is constant. So internal energy is also constant. Hence there will be no change in internal energy.

11.4 Variation in volume by pressure is given in Fig. 11.13. A gas is taken along the paths ABCDA, ABCA and A to A. What will be the change in internal energy?



Ans. There will be no change in the internal energy in all three cases because the system returns to its initial state.

11.5 Specific heat of a gas at constant pressure is greater than specific heat at constant volume. Why?

Ans. When a gas is heated at constant volume, heat supplied only increase the internal energy i.e., temperature because the piston is fixed so no external work is done to expand the gas i.e., W = 0, the total heat supplied is used to increase the internal energy and temperature of the gas. But when a gas is heated at constant pressure, the heat supplied is used in two ways i.e., some heat is used to do external work to expand the gas and the remaining heat is used to increase the internal energy.

Thus more heat is required at constant pressure than at constant volume. So the specific heat at constant pressure is greater than specific heat at constant volume. i.e.,

$$C_p > C_v$$

11.6 Give an example of a process in which no heat is transferred to or from the system but the temperature of the system changes.

Ans. In adiabatic expansion or compression, no heat is transferred to or from the system but the temperature of the system changes. During adiabatic expansion temperature of the system falls while during adiabatic compression temperature of the system rises.

11.7 Is it possible to convert internal energy into mechanical energy? Explain with an example.

Ans. Yes, in adiabatic expansion, internal energy is changed into mechanical energy.

According to 1st law of thermodynamics

$$Q = \Delta U + W$$

In adiabatic process $Q = 0$

Then $$0 = \Delta U + W$$

$$W = -\Delta U$$

Thus the internal energy decreases because some internal energy is converted into mechanical energy.

Example: In petrol engine, hot gasses expand and the piston moves so internal energy is converted into work.

11.8 Is it possible to construct a heat engine that will not expel heat into the atmosphere?

Ans. No it is impossible to construct a heat engine that will not expect heat into the atmosphere. According to second law of thermodynamics (Kelvin's statement).

"No heat engine operating continuously in a cycle can convert all the heat supplied into work".

11.9 A thermos flask containing milk as a system is shaken rapidly. Does the temperature of milk rise?

Ans. When the milk is shaken rapidly, kinetic energy of the milk molecules increases which causes the increase in temperature and internal energy of the molecules of milk. When we are shaking, some work is done on it which converts into K.E of the molecules of milk so the temperature of milk increases.

11.10 What happens to the temperature of the room, when a air conditioner is left running on a table in the middle of the room?

Ans. Temperature of the room increases, as heat absorbed from the room is expelled in the same room. Also work done by the compressor is changed into heat which is expelled in the same room.

11.11 Can the mechanical energy be converted completely into heat energy? If so give an example.

Ans. Yes, mechanical energy can be converted into heat energy. When work is done in compressing the gas by adiabatic process, the increase in internal energy of the gas is equal to the work done according to 1^{st} law of thermodynamics.

$$Q = \Delta U + W$$

In adiabatic process Q = 0.

$$0 = \Delta U + W$$

As work is done on gas show work will be negative:

$$0 = \Delta U - W$$

$$\Rightarrow \quad \Delta U = W$$

This shows that in adiabatic process mechanical work is converted into increase in internal energy.

Example: If we rub our hands, the whole mechanical energy is converted into heat energy.

11.12 Does entropy of a system increases or decreases due to friction?

Ans. Entropy of a system increases, as work done due to friction is changed into heat and this heat goes into surrounding and becomes useless. According to law of increase of entropy, entropy increases for irreversible process.

11.13 Give an example of a natural process that involves an increases in entropy.

Ans. When ice is melted due to high temperature of surroundings. The heat transferred to ice from surroundings is positive. Since $\Delta S = \frac{\Delta Q}{T}$. As ΔS is positive thus the entropy of this natural process increases.

11.14 An adiabatic change is the one in which.

(a) No heat is added to or taken out of system.

(b) No change of temperature takes place.

(c) Boyle's law is applicable.

(d) Pressure and volume remains constant.

Ans. (a) is correct because in an adiabatic process, no heat enters or leaves the system.

11.15 Which one of the following process is irreversible?

(a) Slow compressions of an elastic spring.

(b) Slow evaporation of a substance in an isolated vessel.

(c) Slow compression of a gas.

(d) A chemical explosion.

Ans. (d) is correct because a chemical explosion cannot be reversed. It is a irreversible process.

11.16 An ideal reversible heat engine has.

(a) 100% efficiency.

(b) Highest efficiency.

(d) An efficiency which depends on the nature of working substance.

(d) None of these.

Ans. (b) is correct because according to 2^{nd} law of thermodynamics, the efficiency of an ideal heat engine cannot be 100%. It has highest efficiency.

PROBLEMS WITH SOLUTIONS

PROBLEM 11.1

Estimate the average speed of nitrogen molecules in air under standard conditions of pressure and temperature.

Data

At S.T.P

Temperature $= T = 0°C = 0 + 273 = 273\ K$

Pressure $= P = 1$ atm.

To Find

Average speed of nitrogen molecules $= <v> = ?$

SOLUTION

As we know that

$$\boxed{T = \frac{2}{3K} < \frac{1}{2} mv^2 >}$$

$$3KT = <mv^2>$$

$$<v^2> = \frac{3KT}{m} \qquad \ldots\ldots (i)$$

where K = Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

m = Mass of nitrogen molecules

Since Molecular mass of nitrogen $= m = 28$ g

$= 0.028$ kg

$$m = \frac{\text{Molecular mass of nitrogen}}{\text{Avogadro number } (N_A)}$$

$$m = \frac{0.028}{6.022 \times 10^{23}}$$

$$= 4.64 \times 10^{-3-23} \text{ kg}$$

$$= 4.64 \times 10^{-26} \text{ kg}$$

Putting in eq. (i)

$$<v^2> = \frac{3 \times 1.38 \times 10^{-23} \times 273}{4.64 \times 10^{-26}}$$

$$= 243.58 \times 10^{-23+26}$$

$$= 243.58 \times 10^3$$

$$\sqrt{<v^2>} = \sqrt{243580}$$

$$<v> = 493 \text{ m/s}$$

Result

Average speed of nitrogen molecule = $<v>$ = 493 m/s

PROBLEM 11.2

Show that ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the square root of the inverse ratio of their masses.

Data

Mass of 1^{st} gas = m_1

Mass of 2^{nd} gas = m_2

Velocity of molecules of 1^{st} gas = v_1

Velocity of molecules of 2^{nd} gas = v_2

To Find

Ratio = ?

SOLUTION

As we know that

$$T = \frac{2}{3K} < \frac{1}{2} mv^2 >$$

For 1^{st} gas

$$T = \frac{1}{3K} < m_1 v_1^2 > \qquad \text{...... (i)}$$

For 2^{nd} gas

$$T = \frac{1}{3K} < m_2 v_2^2 > \qquad \text{...... (ii)}$$

Divide equation (i) by (ii)

$$\frac{T}{T} = \frac{\frac{1}{3K} < m_1 v_1^2 >}{\frac{1}{3K} < m_2 v_2^2 >}$$

$$1 = \frac{< m_1 v_1^2 >}{< m_2 v_2^2 >}$$

$$\frac{m_2}{m_1} = \frac{< v_1^2 >}{< v_2^2 >}$$

$$\sqrt{<\frac{v_1^2}{v_2^2}>} = \sqrt{\frac{m_2}{m_1}}$$

$$<\frac{v_1}{v_2}> = \sqrt{\frac{m_2}{m_1}}$$

Result

Thus the ratio of the root mean square speeds of molecules of two different gases at certain temperature is equal to the square root of the inverse ratio of their mass.

PROBLEM 11.3

A sample of gas is compressed to one half of its initial volume at constant pressure of 1.25×10^5 Nm^{-2}. During the compression, 100 J of work is done on the gas. Determine the final volume of the gas.

Data

Pressure = P = 1.25×10^5 N/m^2

Work done = ΔW = 100 J

To Find

Final volume of the gas = V_2 = ?

SOLUTION

According to relation

$$\Delta W = P\Delta V$$

where ΔV = Change in volume

$$= V_1 - V_2 = V - \frac{V}{2}$$

$$\Delta V = \frac{V}{2} = V_2$$

So $\Delta W = P\left(\frac{V}{2}\right)$

$$\Delta W = PV_2$$

$$\boxed{V_2 = \frac{\Delta W}{P}}$$

$$= \frac{100}{1.25 \times 10^5}$$

$$= 80 \times 10^{-5}$$

$$= 8 \times 10^{-4}\ \text{m}^3$$

Result

Final volume of the gas = V_2 = 8×10^{-4} m^3

PROBLEM 11.4

A thermodynamic system undergoes a process in which its internal energy decreases by 300 J. If at the same time 120J of work is done on the system, find the heat lost by the system.

Data

Decrease in internal energy = ΔU = -300 J

Work done on the system = ΔW = -120 J

To Find

Heat lost by the system = ΔQ = ?

SOLUTION

By using 1st law of thermodynamic

$$\Delta Q = \Delta U + \Delta W$$

$$= -300 + (-120)$$

$$= -300 - 120$$

$$\Delta Q = -420 \text{ J}$$

Result

Heat lost by the system = ΔQ = -420 J

PROBLEM 11.5

A carnot engine utilises an ideal gas. The source temperature is 227°C and the sink temperature is 127°C. Find the efficiency of the engine. Also find the heat input from the source and heat rejected to the sink when 10000J of work is done.

Data

Source temperature = T_1 = $227°C + 273$

= 500 K

Sink temperature = T_2 = $127°C + 273$

= 400 K

Work done = W = $10{,}000$ J

To Find

Efficiency of the engine = η = ?

Heat absorbed = Q_1 = ?

Heat rejected = Q_2 = ?

SOLUTION

For the efficiency of the engine

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$= \left(1 - \frac{400}{500}\right) \times 100$$

$$= \left(\frac{500 - 400}{500}\right) \times 100$$

$$= \frac{100}{500} \times 100$$

$$\eta = 20\%$$

For heat, as we know that

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

$$\boxed{\eta = \frac{W}{Q_1}}$$

$$\frac{20}{100} = \frac{10000}{Q_1}$$

$$\frac{1}{5} = \frac{10000}{Q_1}$$

$$Q_1 = 50000 \text{ J}$$

$$= 5 \times 10^4 \text{ J}$$

For heat rejected

$$W = Q_1 - Q_2$$

$$\boxed{W = Q_1 - W}$$

$$= 50000 - 10000$$

$$= 40000 \text{ J}$$

$$Q_2 = 4 \times 10^4 \text{ J}$$

Result

Efficiency of engine $= \eta = 20\%$

Heat absorbed $= Q_1 = 5 \times 10^5$ J

Heat rejected $= Q_2 = 4 \times 10^4$ J

PROBLEM 11.6

A reversible engine works between two temperatures whose difference is 100°C. If it absorbs 746J of heat from the source and rejects 546J to the sink, calculate the temperature of the source and the sink.

Data

Temperature difference $= T_1 - T_2 = 100°C = 100\text{ K}$

Heat absorbed $= Q_1 = 746\text{ J}$

Heat rejected $= Q_2 = 546\text{ J}$

To Find

Temperature of source $= T_1 = ?$

Temperature of sink $= T_2 = ?$

SOLUTION

By formula

$$\text{Efficiency} = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$$

$$= \left(1 - \frac{546}{746}\right) \times 100$$

$$= \frac{746 - 546}{746} \times 100$$

$$= \frac{200}{746} \times 100$$

$$\eta = 26.8\%$$

As $T_1 - T_2 = 100°C = 100\text{ K}$

$$T_1 - T_2 = 100$$

Therefore;

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\frac{26.8}{100} = \frac{100}{T_1}$$

$$26.8\, T_1 = 100 \times 100$$

$$T_1 = \frac{100 \times 100}{26.8}$$

$$T_1 = 373\text{ K} - 273$$

$$= 100°C$$

So $T_2 = 0°C$

Result

Temperature of source $= T_1 = 100°C$

Temperature of sink $= T_2 = 0°C$

PROBLEM 11.7

A mechanical engineer develops an engine, working between 327°C and 27°C and claim to have an efficiency of 52%. Does he claim correctly? Explain.

Data

Temperature of source $= T_1 = 327°C + 273$

$= 600\ K$

Temperature of sink $= T_2 = 27°C + 273$

$= 300\ K$

Given efficiency $= \eta = 52\%$

To Find

Is his claim correct = ?

SOLUTION

By formula

$$\boxed{\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100}$$

$$= \left(1 - \frac{300}{600}\right) \times 100$$

$$= \frac{300}{600} \times 100$$

$$= 50\%$$

Result

As the efficiency of engine is 52% but calculated efficiency of the engine is 50%. So Mechanical Engineer's claim is not correct.

PROBLEM 11.8

A heat engine performs 100J of work and at the same time rejects 400J of heat energy to the cold reservoirs. What is the efficiency of the engine?

Data

Work done $= W = 100\ J$

Heat rejected $= Q_2 = 400\ J$

To Find

Efficiency of the engine $= \eta = ?$

SOLUTION

By formula

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} \times 100$$

Since $Q_1 - Q_2 = W$

$$Q_1 = W + Q_2$$

$$= 100 + 400$$

$$= 500 \text{ J}$$

$$\boxed{\eta = \frac{W}{Q_1} \times 100}$$

$$\eta = \frac{100}{500} \times 100$$

$$= 20\%$$

Result

Efficiency of the engine $= \eta = 20\%$

PROBLEM 11.9

A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees the temperature of the source be increased?

Data

Temperature of sink $= T_2 = 7°C + 273$

$= 280$ K

Efficiency $= \eta = 50\% = \frac{50}{100} = \frac{1}{2}$

Efficiency $= \eta' = 70\% = \frac{70}{100} = \frac{7}{10}$

To Find

Increase in temperature of source $= \Delta T = ?$

SOLUTION

By formula

$$\eta = 1 - \frac{T_2}{T_1}$$

when $\eta = \frac{1}{2}$

$$\frac{1}{2} = 1 - \frac{280}{T_1}$$

$$\frac{1}{2} = \frac{T_1 - 280}{T_1}$$

$$2(T_1 - 280) = T_1$$

$$2T_1 - 560 = T_1$$

$$2T_1 - T_1 = 560$$

$$\boxed{T_1 = 560\ K}$$

when $\eta' = \frac{7}{10}$

$$\eta' = 1 - \frac{T_2}{T_1'}$$

$$\frac{7}{10} = \frac{T_1' - 280}{T_1'}$$

$$10(T_1' - 280) = 7T_1'$$

$$10T_1' - 2800 = 7T_1'$$

$$10T_1' - 7T_1' = 2800$$

$$3T_1' = 2800$$

$$T_1' = \frac{2800}{3}$$

$$T_1' = 933.3\ K$$

Increase in temperature of source

$$\Delta T = T_1' - T_1$$

$$= 933 - 560$$

$$= 373\ K$$

Result

Increase in temperature of source = 373 K or 373°C

PROBLEM 11.10

A steam engine has boiler that operates at 450K. The heat changes water to steam, which derives the piston. The exhaust temperature of the outside air is about 300K. What is maximum efficiency of this steam engine?

Data

Temperature of source = T_1 = 450 K

Temperature of sink = T_2 = 300 K

To Find

Maximum efficiency of steam engine = η = ?

SOLUTION

By formula

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$= \left(1 - \frac{300}{450}\right) \times 100$$

$$= \frac{450 - 300}{450} \times 100$$

$$= \frac{150}{450} \times 100$$

$$\eta = 33.3\%$$

Result

Efficiency of steam engine $= \eta = 33.3\%$

PROBLEM 11.11

336J of energy is required to melt 1g of ice at 0°C what is the change in entropy of 30g of water at 0°C as it changed to ice at 0°C by a refrigerator?

Data

Heat of fusion of ice $= H_f = 336\ J/g$

Mass of water $= m = 30\ g$

Temperature $= T = 0°C + 273 = 273\ K$

To Find

Change in entropy $= \Delta S = ?$

SOLUTION

By formula

$$\boxed{\Delta S = -\frac{\Delta Q}{T}}$$

But $\Delta Q = mH_f$

$$= 30 \times 336\ J$$

So

$$\Delta S = -\frac{30 \times 336}{273}$$

$$= -36.9\ J/K$$

Result

Change in entropy $= \Delta S = -36.9\ J/K$