COMPLETE NOTES

11TH PHYSICS FULL BOOK NOTES - LONG QS & EXAMPLES

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Chapter 1

MEASUREMENTS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand what is Physics.
- Understand that all physical quantities consist of a numerical magnitude and a unit.
- Describe and use base units, supplementary units, and derived units.
- Understand and use the scientific notation.
- Use the standard prefixes and their symbols to indicate decimal sub-multiplies or multiples to both base and derived units.
- Understand and use the conventions for indicating units.
- Understand the distinction between systematic errors and random errors.
- Understand and use the significant figures.
- Understand the distinction between precision and accuracy.
- Use dimensionality to check the homogeneity of physical equations.
- Derive formulae in simple cases using dimensions.

INTRODUCTION TO PHYSICS

Man has always been curious to know about things. He started to observe, think and wondering about the world around him. He tried to find ways to organize the disorder in observed facts about natural phenomena and material objects things in orderly manner. His attempts resulted in the birth of a single discipline (Branch) of science, called natural philosophy.

There was a huge increase in the volume of scientific knowledge up till the beginning of nineteenth century and it was found necessary to classify the study of nature into two branch.

(i) Biological Sciences.

(ii) Physical Science.

Biological Sciences

The science which deals with living things such as botany, zoology etc are called biological sciences.

Physical Sciences

The science which deals with non living things such as chemistry, astronomy, geology etc are called physical sciences.

Physics is important and basic part of physical science besides its other disciplines such as chemistry, astronomy geology etc. Physics is an experimental science and scientific method emphasizes the need of accurate measurement of different phenomena or of man made objects.

Areas of Physics
Mechanics
Heat & thermodynamics
Electromagnetism
Optics
Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic physics
Molecular physics
Nuclear physics
Solid-state physics
Particle physics
Superconductivity
Super fluidity
Plasma physics
Magneto hydrodynamics
Space physics

Frontiers of Fundamental Science

At the present time there are three main frontiers of fundamental science.

(1) The world of the extremely large, the universe itself, Radio telescopes now gather information from the far side of the universe and have recently detected, as radio waves, the "fire light" of the big bang which probably started off the expanding universe nearly 20 billion years ago.

(2) The world of extremely small, that of the particles such as, electron, protons, neutrons, mesons and others.

(3) The world of complex matter, it is also the world of "middle sized" things, from molecules at one extreme to the earth at the other. This is all fundamental physics, which is heart of science.

Q.1 Define physics. Give its main branches.

Ans. **PHYSICS**

"The branch of science which deals with the study of matter and energy and the relationship between them is called physics."

The study of physics involves investigating of such things as the laws of motion, the structure of space and time, the nature and type of forces that hold different materials together, the interaction between different particles, the interaction of electromagnetic radiation with matter and so on.

Branches of Physics

By the end of 19^{th} century many physicists started believing that everything about physics has been discovered. However, about the beginning of the 20^{th} century many new experimental facts showed that the laws formulated by the previous scientists need modifications. Further researches gave birth to many new disciplines (branches).

Interdisciplinary Areas of Physics
Astrophysics
Biophysics
Chemical physics
Engineering physics
Geophysics
Medical physics
Physical oceanography
Physics of music

(1) Nuclear Physics

The branch of physics which deals with atomic nuclei, is called nuclear physics.

(2) Particle Physics

The branch of physics which is concerned with the ultimate particles of which matter is composed of is called particle physics.

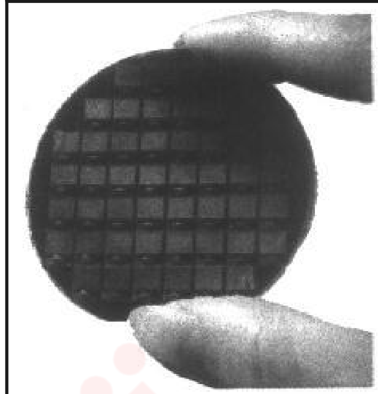
(3) Relativistic Mechanics

The branch of physics which deals with velocities approaching that of light is called relativistic mechanics.

(4) Solid State Physics

The branch of physics which is concerned with the structure and properties of solid materials is called solid state physics.

Do You Know?



Computer chips are made from wafers of the metalloid silicon, a semiconductor.

Other Branches of Science

Physics is the most fundamental of all sciences and provides other branches of science, basic principles and fundamental laws. This overlapping of physics and other fields gave birth to new branches such as physical Chemistry, biophysics, astrophysics, health physics etc.

Q.2 What is the role of physics in technology?

Ans. ROLE OF PHYSICS IN TECHNOLOGY

Physics also plays an important role in development of technology and engineering.

Science and technology are potent force for the change in the outlook of mankind. The information media and fast means of communications have brought all parts of the world in close contact with one another. Events in one part of the world are immediately reverberate round the globe.

We are living in the age of information technology. The computer networks are products of chips developed from basic ideas of physics. The chips are made of silicon. Silicon can be obtained from sand. It is upto us whether we make a sand castle or computer out of it.

Q.3 What do you mean by physical quantities? Also describe its types.

Ans. PHYSICAL QUANTITIES

The foundation of physics rests upon physical quantities in terms of which the laws of physics one expressed. Therefore the qualities have to be measured accurately.

All those quantities which can be measured are called physical quantities.

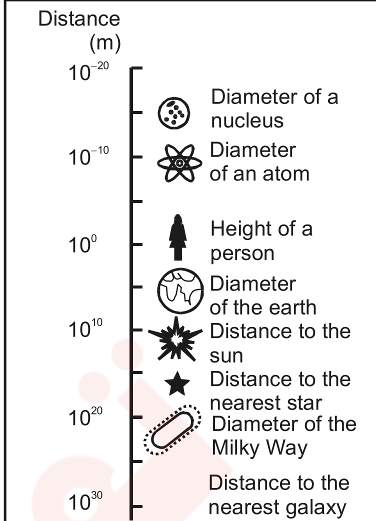
e.g., mass length, time, velocity, force, density temperature, electric current, volume, acceleration etc.

Physical quantities have been divided into two categories.

(i) Base quantities.

(ii) Derived quantities.

Do You Know?



Order of magnitude of some distances

(i) Base Quantities

Base quantities are those quantities which cannot be defined in terms of other physical quantities. These are the minimum number of physical quantities in term of which other physical quantities can be defined. The typical examples of base quantities are length, mass and time.

(ii) Derived Quantities

Those physical quantities whose definitions are expressed in terms of other physical quantities are called derived quantities.

The examples of derived quantities are velocity, acceleration, force, momentum etc.

Measurement of Base Quantities

The measurement of base quantity is based on two steps.

(i) The choice of standard.

(ii) The establishment of a procedure for comparing the quantity to be measured with the standard so that number and a unit are determined as the measure of that quantity.

Characteristics of an Ideal Standard

An ideal standard has two principal characteristics: It is accessible and it is invariable. These two requirements are often incompatible and a compromise has to be made between them.

Unit

To measure a physical quantity, a standard size of that quantity is required. This standard size is known as unit for that particular physical quantity.

Q.4 What is international system of units?

Ans. **INTERNATIONAL SYSTEM OF UNITS**

In 1960 an international committee agreed on a set of definitions and standard to describe the physical quantities. The system that was established is called system international (SI). Since then SI units are being used by world's scientific community in all scientific works. The international system of units (SI) is built up from three kinds of units.

(1) Base units (2) Supplementary units (3) Derived units

Q.5 What are base units? Define the base units of SI.

Ans. **BASE UNITS**

There are seven base units for various physical quantities namely; length, mass, time, temperature, electric current, luminous intensity and amount of substance (with special reference to the number of particles).

The name of base units for these physical quantities together with symbols are listed in table:

Physical Quantity	SI Unit	Symbol
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Thermodynamic temperature	Kelvin	K
Intensity of light	Candela	cd
Amount of substance	Mole	mol

Q.6 What are supplementary units? Define them.

Ans. **SUPPLEMENTARY UNITS**

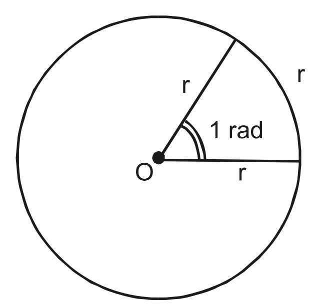
The general conference on weights and measures has not yet classified certain units of the SI under either base units or derived units. These SI units are called supplementary units. This class contains two units, which are:

(1) Radian (Plane angle)

(2) Steradian (Solid angle)

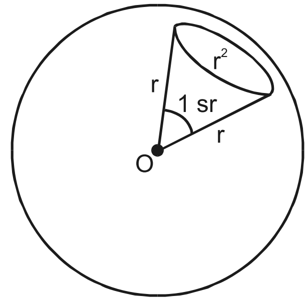
Radian

The radian is the plane angle between two radii of a circle which cut off on the circumference an arc, equal in length to the radius, as shown in figure.



Steradian

The steradian is the solid angle (three dimensional angle) subtended at the centre of the sphere by an area of its surface equal to the square of radius of the sphere, as shown in figure.



Q.7 What are derived units? Give some examples.

Ans. **DERIVED UNITS**

SI units for measuring all other physical quantities are derived from the base and supplementary units. Some of the derived units are given below:

Physical Quantity	Unit	Symbol	In terms of base units
Force	Newton	N	kg ms^{-2}
Work	Joule	J	N m = kg m^2 s^{-2}
Power	Watt	W	Js^{-1} = kg m^2 s^{-3}
Pressure	Pascal	Pa	Nm^{-2} = kg m^{-1} s^{-2}
Electric charge	Coulomb	C	A s

Q.8 What do you understand by term?

Ans. **SCIENTIFIC NOTATION**

Numbers are expressed in standard form called scientific notation, which employs power of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal.

Scientific Notation Explain the Uses of Prefix

Example

The number 134.7 should be written as

$$134.7 = 1.374 \times 10^2$$

Similarly, the number 0.0023 can be written as

$$0.0023 = 2.3 \times 10^{-3}$$

Some Prefixes for Powers of Ten		
Factor	**Prefix**	**Symbol**
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	mili	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deca	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

Conventions for Indicating Units

Use of SI units requires special care, more particularly in writing prefixes.

Following points should be kept in mind while using units:

(i) Full name of the unit does not begin with a capital even if named after a scientist e.g. newton.

(ii) The symbol of unit named after a scientist has initial capital letters such as N for newton.

(iii) The prefix should be written before the unit without any space, such as 1×10^{-3}, m is written as 1 mm. Standard prefixes are given in table 1.4.

(iv) A combination of base units is written each with one space apart. For example, newton metre is written as N m.

(v) Compound prefixes are not allowed. For example, 1μμF may be written as 1pF.

(vi) A number such as 5.4×10^4 cm may be expressed in scientific notation as 5.0×10^2m.

(vii) When a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone. Thus, $1\text{km}^2 = 1\ (\text{km})^2 = 1 \times 10^6\text{m}^2$.

(viii) Measurement in practical work should be recorded immediately in the most convenient unit, e.g., micrometer screw guage measurement in mm, and the mass of calorimeter in grams (g). But before calculation for the result, all measurements must be converted to the appropriate SI base units.

Q.9 Explain the phenomenon of errors and uncertainties.

Ans. **ERRORS AND UNCERTAINTIES**

All physical measurements are uncertain or imprecise to some extent. It is very difficult to eliminate all possible errors or uncertainties in a measurement. The errors in a measurement may occur due to.

(1) Negligence or inexperience of a person.

(2) The faulty apparatus.

(3) Inappropriate method or technique.

The uncertainty may occur due to inadequacy or limitations of an instrument, natural variations of the object being measured or natural imperfection of a person's senses. However the uncertainty is also usually described as an error in a measurement.

Types of Errors

There are two types of errors.

(1) Random error.

(2) Systematic error.

(1) Random Error

Random error is said to occur when repeated measurements of a quantity, give different values under the same conditions. It is due to some unknown causes.

Repeating the measurements several times and taking an average can reduce the effect of random errors.

(2) Systematic Errors

Systematic error refers to an effect that influences all measurements of a particular quantity equally. It produces consistent difference in reading.

It occurs due to:

(i) Zero error of instrument.

(ii) Poor calibration of instruments, or incorrect marking etc.

Systematic error can be reduced by comparing the instrument with an other which is known to be more accurate. Thus for systematic error, a correction factor can be applied to reduce error.

Q.10 Describe the significant figures. Also discuss its rules.

Ans. **SIGNIFICANT FIGURES**

We know that physics is based on measurements whenever a physical quantity is measured; there is some uncertainty about its determined value. This uncertainty may be due to a number of reasons. One reason is the type of instrument, being used. We know that every measuring instrument is calibrated to a certain smallest division and this fact put a limit to the degree of accuracy while measuring with it.

Suppose that we want to measure the length of a straight line with the help of a meter rod calibrated in millimeters. Let the end point of the line lies between 10.3 and 10.4 cm marks. By convention if the end of line does not touch or cross the midpoint of the smallest division, the reading is confined to the previous division. In case the end of line seems to be touching or have crossed the midpoint, the reading is extended to the next division.

For Your Information

	Interval (s)
Age of the universe	5×10^{17}
Age of the Earth	1.4×10^{17}
One year	3.2×10^{7}
One day	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radio waves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

Approximate Values of Some Time Intervals

By applying the above rule the position of the edge of line recorded as 12.7cm with the help of a meter rod calibrated in millimeters may lie between 12.65cm and 12.75cm. Thus in this example the maximum uncertainty is ±0.05 cm. It is, infact, equivalent to an uncertainty of 0.1cm equal to the least count of the instrument divided into two parts, half above and half below the recorded reading.

Thus the correct way of recording the above reading is

$$12.7 \pm 0.05 \text{ cm}$$

The recorded value of the length of the straight line i.e. 12.7 cm contains three digits (1, 2, 7) out of which two digits 1 and 2 are accurately known while the third digit i.e. 7 is a doubtful one. Thus significant figures may be stated as:

"In any measurement, the accurately known digits and the first doubtful digit are called significant figures." **(OR)** *"A significant figure is the one which is known to be reasonably reliable".*

If the above mentioned measurement is taken by a better measuring instrument which is exact up to hundredth of a centimeter, it would have been recorded as 12.70cm. In this case number of significant figures is four (1, 2, 7 and 0).

Thus, we can say that as we improve the quality of our measuring instrument and techniques, we extend the measured result to more and more significant figures and correspondingly improve the experimental accuracy of the result.

General Rules

There are some general rules in order to find the number of significant figures in final result.

(1) All digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are significant. However zeros may or may not be significant. In case of zeros, the following rules may be adopted.

(a) A zero between two significant figures is itself significant.

(b) Zeros to the left of significant figures are not significant. For example, none of the zero in 0.00467 or 02.59 is significant.

(c) Zeros to the right of significant figure may or may not be significant.

   (i) In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant.

   (ii) In integers, such as 8,000 kg, the number of significant zeros is determined by the accuracy of the measuring instrument. If the measuring scale has a least count of 1kg then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3kg and so on.

(2) Significant Figures in Multiplication and Division of Numbers

In multiplying or dividing numbers i.e.,

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.4576898 \times 10^{3}$$

Let us see, how many numbers should be retained in the answer. As the factor 3.64×10^4, the least accurate in the above calculations has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off.

Rules for Rounding Off Numbers

Following are the rules for rounding off numbers.

(a) If the first digit to be dropped is less than 5, the last digit to be retained should remain unchanged. e.g. 12.4 is rounded off as 12.

(b) If the first digit to be dropped is more than 5, the last digit to be retained is increased by one. e.g. 12.6 is rounded off as 13.

(c) If the digit to be droped is 5, and the number following 5 is not zero then the last digit to be retained is increased by one e.g. 12.51 is rounded off as 13.

(d) If the digit to be droped is 5, and the number following 5 is zero then the last digit to be retained follows odd even rule. i.e., if the digit to be retained is odd it is increased by one but left as it is if it is even.

e.g.,

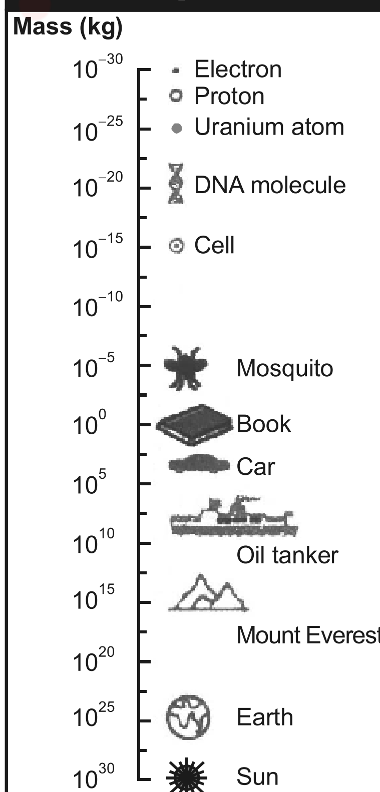
13.50	is rounded off as	14
14.50	is rounded off as	14
15.5	is rounded off as	15
16.5	is rounded off as	16

Examples of Numbers to be Rounded Off

The following numbers are rounded off to three significant figures as follows.

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.9
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Interesting Information



Order of magnitude of some masses

Following this rule, the correct answer of the computation given in section (2) is 1.46×10^3.

(3) Addition or Subtraction of Numbers

In adding or subtracting numbers, the number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters.

For example, we wish to add the following quantities expressed in meters.

(i)		(ii)	
	72.1		2.7543
	3.42		4.10
	0.003		1.273
	75.523m		8.1273
	75.5m		8.13m

In Case (i) the number 72.1 has the smallest number of decimal places, thus answer is rounded off to the same decimal position which is then 75.5m.

In case (ii) the number 4.10 has the smallest number of decimal places, and hence answer is rounded off to the same decimal position which is then 8.13m.

Do You Know?

Mass can be thought of as a form of energy. In fact the mass is highly concentrated form of energy. Einstein's famous equation, $E=mc^2$ means
Energy = Mass x Speed of light^2
According to this equation 1 kg mass is actually 9×10^{16} J energy.

Q.11 Explain the term precision and accuracy.

Ans. **PRECISION AND ACCURACY**

In measurements made in physics, the term precision and accuracy are frequently used. The precision of a measurement is determined by the instrument or device being used and accuracy of measurement depends on the fractional or percentage uncertainty in that measurement. Let us make it clear by examples.

Example-I

Let the length of an object is recorded as 25.5cm by using meter rod having smallest division in millimeter. This measurement is difference of two readings that is initial and final positions. In case of single reading uncertainty is taken as ± 0.05cm. But in present reading uncertainty is taken double due to the reading of initial and final position i.e.

Uncertainty = ±0.05 ±0.05 = ±0.1cm

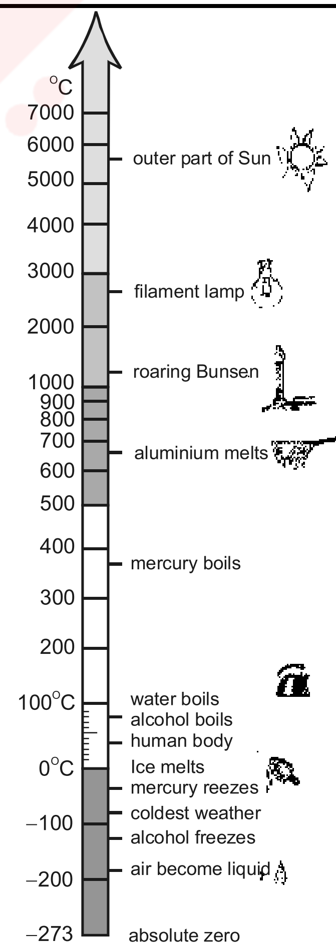
This uncertainty is called absolute uncertainty and absolute uncertainty in effect is equal to least count of the measuring instrument.

Thus,

Precision or absolute uncertainty (least count) = ±0.1cm

$$\text{Fractional uncertainty} = \frac{0.1\text{cm}}{25.5\text{cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1}{25.5} \times \frac{100}{100} = \frac{0.4}{100} = 0.4\%$$



Some Specific Temperatures

Example-II

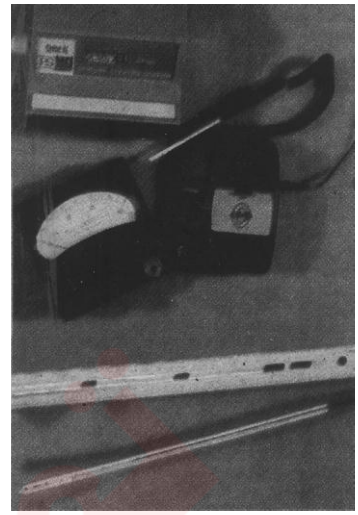
Another measurement is recorded as 0.45cm. It is taken by vernier callipers with least count as 0.01cm.

Now, Precision or absolute uncertainty (least count) = ±0.01cm

$$\text{Fractional uncertainty} = \frac{0.01\text{cm}}{0.45\text{cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.01\text{cm}}{0.45\text{cm}} \times \frac{100}{100} = \frac{2.0}{100} = 2.0\%$$

For Your Information



We use many devices to measure physical quantities, such as length, time and temperature. They all have some limit of precision.

Conclusion

The reading 25.5cm taken by meter rule is although less precise but is more accurate having less percentage uncertainty or error where as the reading 0.45cm taken by vernier callipers is more precise but less accurate.

In fact, it is relative measurement which is important. The smaller a physical quantity, a more precise instrument should be used Here the measurement 0.45cm demands that a more precise instrument, such as micrometer screw guage with least count 0.001cm should have been used. Thus we can define precision and accuracy.

Precision

A precise measurement is the one which has less absolute uncertainty.

Accuracy

An accurate measurement is the one which has less fractional or percentage uncertainty or error.

Q.12 How will you assess the total uncertainty in the final result? Explain in different cases.

Ans. **ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT**

To assess the total uncertainty or error, it is necessary to evaluate the uncertainties in all the factors involved in that calculation. The maximum possible uncertainty or error in the final result can be found as follows.

(1) For Addition and Subtraction

For addition and subtraction absolute uncertainties are added.

For example, the distance x determined by the difference between two separate position measurements.

$$x_1 = 10.5 \pm 0.1\text{cm}$$

and $$x_2 = 26.8 \pm 0.1\text{cm}$$

The difference between them is recorded as

$$x = x_2 - x_1$$

$$x = 26.8 \pm 0.1 - 10.5 \pm 0.1$$

$$x = 16.3 \pm 0.2\text{cm}$$

(2) For Multiplication and Division

For multiplication and division percentage uncertainties are added. For example, we determine maximum uncertainty in the value of resistance R of a conductor determined by formula.

$$R = \frac{V}{I}$$

Where V = Potential difference

and I = Current

The given values of V and I are.

$$V = 5.2 \pm 0.1V$$

and $I = 0.84 \pm 0.05A$

The %age uncertainty for V is $= \frac{0.1V}{5.2V} \times \frac{100}{100} = \frac{2}{100} = 2\%.$

The %age uncertainty for I is $= \frac{0.05A}{0.84A} \times \frac{100}{100} = \frac{6}{100} = 6\%$

Hence total uncertainty in the value of R is.

= % age uncertainty for V + % age uncertainty for I.

= 2% + 6%

= 8%

Thus,

$$R = \frac{5.2V}{0.84A} = 6.19\ VA^{-1} = 6.19$$

Ohms with % age uncertainty of 8%.

Now,

$$8\%\text{ of }6.2 = 6.2 \times \frac{8}{100} = 0.5$$

Thus,

$$R = 6.2 \pm 0.5\text{ ohms}$$

The result is rounded off to two significant digits because both V and I have two significant figures. Uncertainty being an estimate only, is recorded by one significant figure.

(3) For Power Factor

For power factor multiply the percentage uncertainty by that factor i.e.

Total % age uncertainty = Power factor × % age uncertainy.

Example

Let us calculate the volume of a sphere given by formula.

$$V = \frac{4}{3}\pi r^3$$

Now,

% age uncertainty in V = power factor × % age uncertainty in r.

∴ % age uncertainty in V = 3 × % age uncertainty in r.

Let the radius of sphere is measured as 2.25 cm by a vernier calliper with least count 0.01 cm, then.

Radius r is recorded as

$$r = 2.25 \pm 0.01\text{cm}$$

Now,

Absolute uncertainty = least count = 0.01cm

$$\%\text{ age uncertainty in } r = \frac{0.01\text{cm}}{2.25\text{cm}} \times \frac{100}{100} = \frac{0.4}{100} = 0.4\%$$

$\therefore$ Total % age uncertainty in V = $3 \times 0.4\%$ = 1.2%

Now,

$$\text{Volume} = V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}(3.14)(2.25\text{cm})^3$$

$$= 47.689\text{ cm}^3$$

Thus,

Volume V = 47.689 cm^3 with 1.2% of uncertainty.

As, 1.2% of 47.689 = 0.6

$$\therefore \quad V = 47.7 \pm 0.6\text{ cm}^3$$

For Your Information

Colour printing uses just four colours-cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

(4) For Uncertainty in the Average Value of Many Measurements

(i) Find the average value of measured values.

(ii) Find the deviation of each measured value from the average value.

(iii) The mean deviation is the uncertainty in the average value.

Example

There are six readings of the micrometer screw guage to measure the diameter of a wire in mm.

The readings are.

1.20, 1.22, 1.23, 1.19, 1.22, and 1.21.

Then,

$$\text{Average} = \frac{1.20 + 1.22 + 1.23 + 1.19 + 1.22 + 1.21}{6}$$

$$= 1.21 \text{ mm}$$

The deviation of each value ie the difference between each recording and average value, without regard of sign, are.

0.01, 0.01, 0.02, 0.02, 0.01, and 0.

$$\text{Mean of deviations} = \frac{0.01 + 0.01 + 0.02 + 0.02 + 0.01 + 0}{6}$$

$$= 0.01 \text{ mm}$$

Thus uncertainty in mean diameter i.e., 1.21 mm is 0.01 mm recorded as

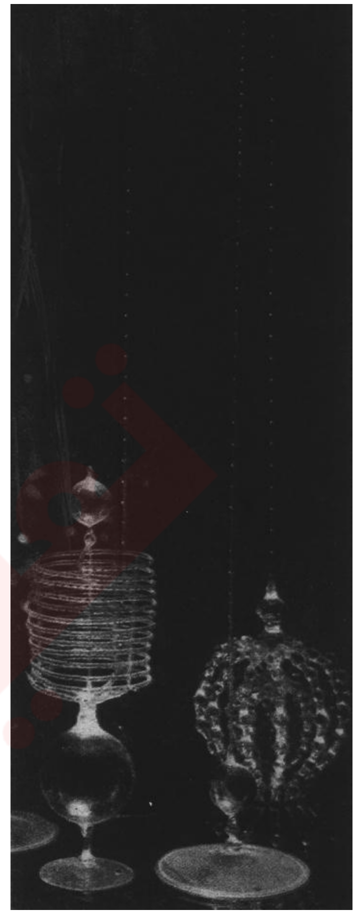
$$\boxed{\text{Diameter} = 1.21 \pm 0.01 \text{ mm}}$$

(5) For the Uncertainty in Timing Experiment

The uncertainty in the time period of a vibrating body is found by dividing the least count of timing device by the number of vibrations ie.

$$\text{Uncertainty in time period} = \frac{\text{Least count}}{\text{No. of vibrations}}$$

For Your Information



These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Academia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.

Example

The time of 30 vibrations of a simple pendulum recorded by a stopwatch accurate up to one tenth of a second (least count) is 54.6 s.

Now, Time period $= T = \frac{54.6\text{ s}}{30} = 1.82$ Sec

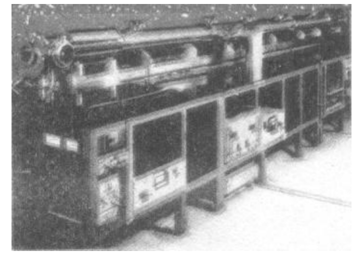
Uncertainty $= \frac{\text{Least count}}{\text{No. of vibrations}} = \frac{0.1\text{ s}}{30} = 0.003\text{S}$

Thus time period T is

$$\boxed{T = 1.82 \pm 0.003\text{ S}}$$

Note: *It is advisable to count large number of swings to reduce timing uncertainty.*

For Your Information



Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

Q.13 What do you understand from the dimensions of physical quantities?

Ans. DIMENSIONS OF PHYSICAL QUANTITIES

"The qualitative nature of the physical quantity is considered a dimensions. It is denoted by a specific symbol written within square brackets."

(OR)

Dimensions of physical quantity is a relationship between derived physical quantity and the base quantity.

Different quantities such as length, breadth, diameter, light year which are measured in meter denote the same dimensions and has the dimensions of length [L]. Similarly, the dimensions of mass and time are denoted by [M] and [T] respectively.

Other quantities that we measure have dimensions which are combinations of these dimensions.

(1) Dimensions of Speed

$$\text{Speed} = \frac{\text{length}}{\text{time}} = \frac{L}{T}$$

$$\text{Dimensions of speed} = \frac{\text{dimension of length}}{\text{dimension of time}}$$

or $$[V] = \frac{[L]}{[T]}$$

$\Rightarrow$ $$[V] = [L][T^{-1}] = [LT^{-1}]$$

(2) Dimensions of Acceleration

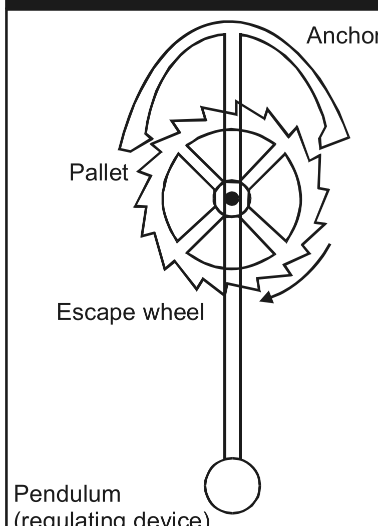
$$\text{Acceleration} = \frac{\text{velocity}}{\text{time}}$$

$$\text{Dimensions of acceleration} = \frac{\text{dimension of V}}{\text{dimension of I}}$$

$$[a] = \frac{[V]}{[T]}$$

$$[a] = \frac{[LT^{-1}]}{[T]} = [LT^{-1}][T^{-1}] = [LT^{-2}]$$

Do You Know?



The device which made the pendulum clock practical.

(3) Dimensions of Force

Since, $F = ma$

$\therefore$ $[F] = [m]\,[a]$

$= [M]\,[LT^{-2}]$ $\quad \therefore [a] = [LT^{-2}]$

$= [MLT^{-2}]$

Using the method of dimensions called the dimensional analysis, we can check the correctness of a given formula or an equation and can also derive it. Dimensional analysis makes use of the fact that expression of the dimensions can be manipulated as algebraic quantities.

(i) Checking the Homogeneity of Physical Equation

In order to check the correctness of an equation, we are to show that the dimensions of the quantities on both sides of the equation are the same, irrespective of the form of the formula. This is called the principle of homogeneity of dimensions.

(ii) Derivation of a Possible Formula

The success of this method for deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends.

SOLVED EXAMPLES

EXAMPLE 1.1

The length, breadth and thickness of a sheet are 3.233m, 2.105 m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.

Data

Length of sheet $= l = 3.233\text{m}$

Breadth of sheet $= \text{b} = 2.105\text{m}$

Thickness of sheet $= \text{h} = 1.05\text{cm} = 1.05 \times 10^{-2}\text{m}$

To Find

Volume of sheet $= \text{V} = ?$

SOLUTION

Volume is given by

$$V = l \times b \times h$$

Putting values, we get

$$V = 3.233 \times 2.105 \times 1.05 \times 10^{-2}$$

$$= 7.14573825 \times 10^{-2}\ \text{m}^3$$

As the factor 1.05 cm has minimum number of significant figures equal to three, therefore, volume is recorded up to 3 significant figures.

Hence, $\boxed{V = 7.15\ \text{m}^3}$

Result

Volume of sheet $= \text{V} = 7.15\ \text{m}^3$

EXAMPLE 1.2

The mass of a metal box measured by a lever balance is 2.2 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision?

Data

Mass of box $= \text{m} = 2.2\ \text{kg}$

Mass of 1st coin $= \text{m}_1 = 10.01\text{g} = 0.01001\text{kg}$

Mass of 2nd coin $= \text{m}_2 = 10.02\text{g} = 0.01002\text{kg}$

To Find

Total mass of box $= \text{M} = ?$

SOLUTION

Total mass when silver coins are added.

$$M = m + m_1 + m_2$$
$$= 2.2 + 0.01001 + 0.01002$$
$$= 2.22003 \text{ kg}$$

Since least precise is 2.2 kg having one decimal Place, hence total mass should be to one decimal place which is the appropriate precision. Thus,

Total mass = 2.2 kg

Result

Total mass = 2.2 kg

EXAMPLE 1.3

The diameter and length of a metal cylinder measured with the help of vernier calipers of least count 0.01 cm are 1.22 cm and 5.35 cm. Calculate the volume V of the cylinder and uncertainty in it.

Data

Least count of vernier callipers = 0.01cm

Diameter of cylinder = D = 1.22 cm

Length of cylinder = l = 5.35 cm

To Find

Volume of cylinder = V = ?

Uncertainty in volume = ?

SOLUTION

Absolute uncertainty in length = 0.01 cm

% age uncertainty in length $= \dfrac{0.01}{5.33} \times \dfrac{100}{100} = \dfrac{0.2}{100} = 0.2\%$

Absolute uncertainty in diameter = 0.01 cm

% uncertainty in diameter $= \dfrac{0.01}{1.22} \times \dfrac{100}{100} = \dfrac{0.8}{100} = 0.8\%$

As volume is given by

$$V = \pi r^2 l$$
$$V = \frac{\pi d^2 l}{4}$$

$\therefore$ Total uncertainty in V = 2 (% uncertainty in d) + (% uncertainty in l)

$= 2\,(.8) + .2$

$= 1.6 + 2 = 1.8\%$

Now, $V = \dfrac{(3.14)\,(1.22)^2\,(5.33)}{4} = 6.2509079 \text{ cm}^3$

Thus, $V = 6.2509079\ cm^3$ with uncertainty 1.8%.

As, 1.8% of 6.2509079 = 0.1

Thus, $V = 6.2 \pm 0.1\ cm^3$

Where $6.2\ cm^3$ is calculated volume and $0.1\ cm^3$ is the uncertainty in it.

Result

Volume of cylinder = $6.2509079\ cm^3$

Uncertainty in volume = $6.2 \pm 0.1\ cm^3$

EXAMPLE 1.4

Check the correctness of the relation $v = \sqrt{\frac{F \times l}{m}}$ where V is the speed of transverse wave on a stretched string of tension F, length l and mass m.

SOLUTION

The given formula is

$$V = \sqrt{\frac{F \times l}{m}}$$

Dimensions of L.H.S = $[V] = [LT^{-1}]$

Dimensions of R.H.S = $\left[\frac{F \times l}{m}\right]^{1/2}$

$$= \left[\frac{[F]\,[l]}{[m]}\right]^{1/2} = \left[\frac{[MLT^{-2}]\,[L]}{[M]}\right]^{1/2}$$

$$= [L^2T^{-2}]^{1/2}$$

$$= [LT^{-1}]$$

As Dimensions of LHS = Dimensions of RHS.

Hence formula is dimensionally correct.

EXAMPLE 1.5

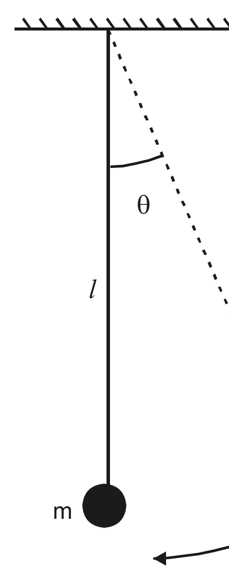
Derive a relation for the time period of a simple pendulum (Fig. 1.2) using dimensionsal analysis. The various possible factors on which the time period T may depend are

(i) Length of the pendulum (l).

(ii) Mass of the bob (m).

(iii) Angle θ which the thread makes with the vertical.

(iv) Acceleration due to gravity (g).



Data

Length of the pendulum = l

Mass of bob = m

Angle which the thread makes with vertical = θ

Acceleration due to gravity = g

To Find

Relation for time period = T = ?

SOLUTION

The relation for time period T will be of the form

$$T \propto m^a \times l^b \times \theta^c \times g^d$$

or $$T = \text{Constant } m^a\, l^b\, \theta^c\, g^d \qquad \ldots\ldots (1)$$

Now we find the values of powers a, b, c and d.

Now, Dimensions of $\theta = [LL^{-1}] = 1$ $\qquad \therefore s = r\theta$

And, Dimensions of $g = [LT^{-2}]$

$$\theta = \frac{S}{r} = \frac{L}{L}$$

$$= LL^{-1} = 1$$

Writing dimensions on both sides, we get.

$$[T] = \text{Constant } [M]^a [L]^b [1]^c [LT^{-2}]^d$$

$$= \text{Constant } [M]^a [L]^b [1]^c [L]^d [T]^{-2d}$$

$$[T] = \text{Constant } [M]^a [L]^{b+d} [T]^{-2d} \qquad ([1]^c = 1)$$

Comparing the exponents of M, L and T on both sides

$$[M]^o = [M]^a \Rightarrow a = 0$$

$$[L]^o = [L]^{b+d} \Rightarrow b + d = 0 \text{ or } b = -d = (-1/2) = ½$$

$$[T]^1 = [T]^{-2d} \Rightarrow -2d = 1 \text{ or } d = -1/2$$

Thus, $a = 0$

$b = 1/2$

$d = -1/2$

Putting values of a, b, d and θ in eq. (1), we get.

$$T = \text{Constant } m^o \times l^{1/2} \times 1 \times g^{-1/2}$$

or $$T = \text{Constant}\frac{l^{1/2}}{g^{1/2}}$$

or $$T = \text{Constant}\sqrt{\frac{l}{g}}$$

$$\boxed{T = \text{Constant}\sqrt{\frac{l}{g}}}$$

Result

Relation for time period = $T = \text{Constant}\sqrt{\frac{l}{g}}$

EXAMPLE 1.6

Find the dimensions and hence, the SI units of coefficient of viscosity η in the relation of Stokes law for the drag force F for a spherical object of radius r moving with velocity v given as F = 6 πηrv.

Data

$$F = 6\pi\eta rv$$

To Find

Dimensions of η = ?

Units of η = ?

SOLUTION

We are given

$$F = 6\pi\eta rv \qquad \ldots\ldots (1)$$

Now, 6π is a number having no dimensions. It is not accounted in dimensional analysis, then.

$$[F] = [\eta\, r\, v]$$

$$\Rightarrow \qquad [\eta] = \frac{[F]}{[r]\,[v]}$$

Substituting (putting) the dimensions of F, r and v in R.H.S, we get

$$[\eta] = \frac{[MLT^{-2}]}{[L]\,[LT^{-1}]}$$

$$[\eta] = [ML^{-1}T^{-1}]$$

Units of η

SI unit of η are kg $m^{-1}s^{-1}$.

Result

Dimension of η = $[ML^{-1}T^{-1}]$

Unit of η = kg $m^{-1}s^{-1}$

Chapter 2

VECTORS AND EQUILIBRIUM

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand the definition of scalars and vectors.
- Understand and use rectangular coordinate system.
- Understand the idea of unit vector, null vector and position vector.
- Represent a vector as two perpendicular components (rectangular components).
- Understand multiplication of vectors and solve problems.
- Define the moment of force or torque.
- Appreciate the use of the torque due to a force.
- Appreciate the applications of the principle of moments.

Q.1 Define scalars and vectors. Give examples.

Ans. **SCALAR QUANTITIES**

Those physical quantities which are completely described by magnitude with proper units are called scalar quantities. e.g. time, current, speed etc. Scalars are added, subtracted, divided and multiplied by ordinary arithmetic rules.

VECTOR QUANTITIES

Those physical quantities which are completely described by magnitude with proper units as well as direction are called vector quantities. e.g. force, torque etc. Vectors are not added, subtracted, divided and multiplied by ordinary arithmetic rules but it can be used as vector addition, vector multiplication and vector subtraction.

Q.2 Describe how a vector quantity is represented?

Ans. **REPRESENTATION OF A VECTOR**

(i) By Letter

A vector is usually represented by a bold face letter that is **A** or by a letter with an arrow drawn above or below it that is $\vec{A}$ or $\underline{A}$.

The magnitude of a vector is denoted by $|\vec{A}|$ (modulus) or A.

(ii) By Graphically

A vector is represented graphically by a directed line segment with an arrow-head in the direction of the vector. The length of the line segment, according to the suitable scale, corresponds to the magnitude of the vector. The length of the line is called its magnitude and arrow head indicates the direction.

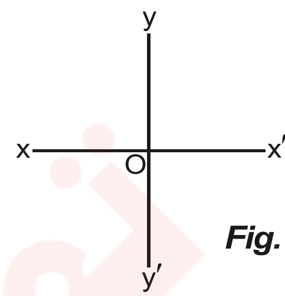


Fig. (i)

Q.3 Describe the rectangular coordinate system.

Ans. **RECTANGULAR COORDINATE SYSTEM**

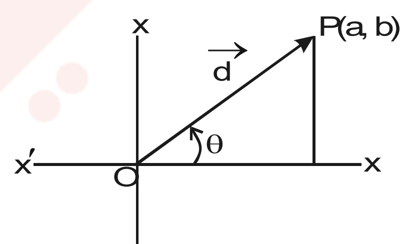
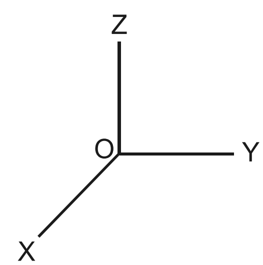


Fig. (ii)

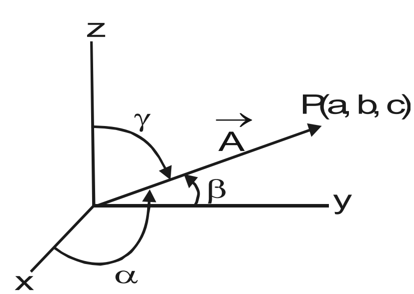
Two reference lines drawn at right angles as shown in figure. They are known as coordinate axes and their point of intersection is known as origin. This system of coordinate axes is called Cartesian or rectangular coordinate system.

One of the lines is named as x-axis, and the other the y-axis. Usually the x-axis is taken as the horizontal axis, with the positive direction to the right, and the y-axis as the vertical axis with the positive direction upward.



The direction of a vector in a plane is denoted by the angle which representative line of the vector makes with positive x-axis in the anti-clockwise direction as shown in Fig. (ii).

The direction of a vector in space requires another axis which is at right angle to both x and y axes, as shown in figure which is called z-axis.



The direction of a vector in space is specified by the three angles which the representative line of the vector makes with x, y and z axes respectively as shown in figure. The point P of a vector A is thus denoted by three coordinates (a, b, c).

Q.4 Explain the following terms:

(i) Addition of vector ***(ii) Resultant vector***

(iii) Vector subtraction ***(iv) Multiplication of a vector***

(v) Unit vector ***(vi) Null vector***

(vii) Equal vector

Ans. **ADDITION OF VECTORS**

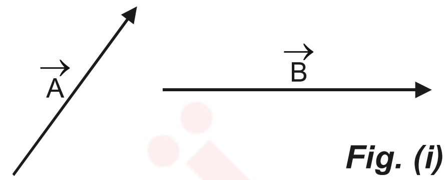


Fig. (i)

Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. (i).

Their sum is obtained by drawing, their representative lines in such a way that tail of vector $\vec{B}$ coincides with the head of vector $\vec{A}$. Now if we join the tail of $\vec{A}$ to the head of $\vec{B}$ as shown in Fig. (ii).

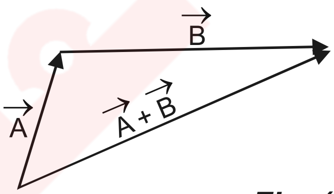


Fig. (ii)

If we join the tail of first vector with the head of last vector, it will represent the vector sum $\vec{A}+\vec{B}$ in magnitude and direction. This is known as head to tail rule of vector addition. Similarly the sum $\vec{B}+\vec{A}$ is represented by doted lines as shown in Fig. (iii). It is clear from Fig. (iii) that

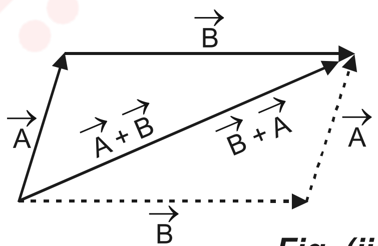


Fig. (iii)

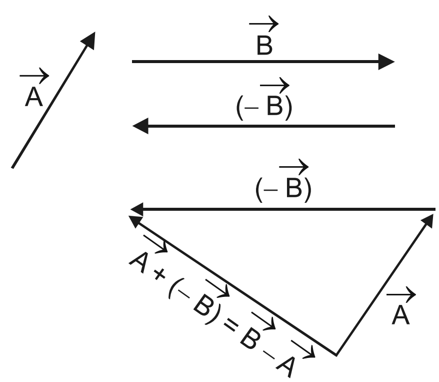
$$\vec{A}+\vec{B} = \vec{B}+\vec{A}$$

So the vector addition is said to be commutative. It means that when vectors are added, the result is same for any order of addition.

RESULTANT VECTOR

The resultant of number of vectors is that single vector which would have the same effect as all the original vectors taken together.

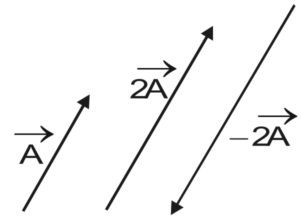
VECTOR SUBTRACTION



The subtraction of a vector is equal to the addition of the same vector with its direction reversed. To subtract vector $\vec{B}$ from vector $\vec{A}$ reverse the direction of $\vec{B}$ and add it to $\vec{A}$ as shown in figure.

MULTIPLICATION OF A VECTOR BY A SCALAR

The product of a vector $\vec{A}$ and a number $n > 0$ is defined to be a new vector $n\vec{A}$ having the same direction as $\vec{A}$ but a magnitude n times the magnitude of $\vec{A}$ as shown in figure.



If the vector is multiplied by a negative number, then its direction is reversed.

For Example

When velocity (vector) is multiplied by scalar i.e. mass m, the product is a new vector quantity called momentum having same dimensions as product of mass and velocity.

UNIT VECTOR

For information

$$\left|\hat{A}\right| = \frac{\left|\vec{A}\right|}{A} = \frac{A}{A} = 1$$

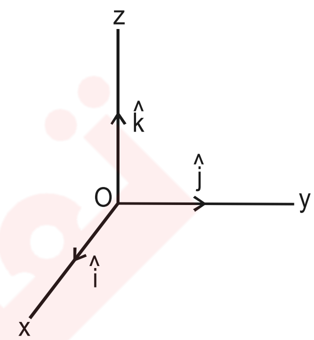
A unit vector in a given direction is a vector with magnitude one in that direction.

It is used to represent the direction of a vector.

A unit vector in the direction of $\vec{A}$ is written as $\hat{A}$.

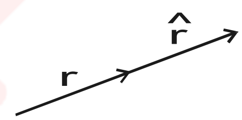
As $\vec{A} = A\,\hat{A}$

$$\hat{A} = \frac{\vec{A}}{A}$$

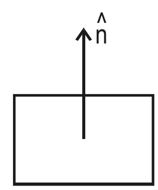


The direction along x, y and z axes is represented by unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. Two of the more frequently used unit vectors are:

(i) The vector $\hat{r}$ which represents the direction of the vector r as shown.



(ii) The vector $\hat{n}$ which represents the direction of a normal drawn on a surface as shown.



NULL VECTORS

It is a vector of zero magnitude and arbitrary direction.

For example, sum of a vector and its negative vector is a null vector.

$$\vec{A} + \left(-\vec{A}\right) = \vec{0}$$

EQUAL VECTORS

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal if they have the same magnitude and direction, regardless of the position of their initial points.

This means that parallel vectors of the same magnitude are equal to each other.

Q.5 Define rectangular components of a vector. How will you find resultant vector of the rectangular components are given?

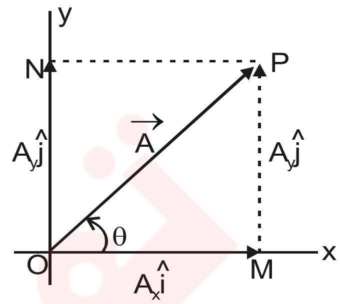
Ans.

RECTANGULAR COMPONENTS OF A VECTOR

The splitting up of a vector into its parts is called the resolution of a vector. Usually a vector can be resolved into two parts called the components of a vector. **(OR)** A component of a vector is its effective value in a given direction. The component along x-axis is called horizontal component and the component along y-axis is called the vertical component. If these components are right angle to each other then they are called rectangular components of a vector.

Explanation

Consider a vector $\vec{A}$ represented by $\vec{OP}$ making an angle θ with x-axis. Draw projection OM of vector $\vec{OP}$ on x-axis and projection ON of vector OP on y-axis as shown in figure Projection OM being along x-direction is represented by $A_x \hat{i}$ and projection ON = MP along y-direction is represented by $A_y \hat{j}$.



By head to tail rule

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

Here $A_x\hat{i}$ and $A_y\hat{j}$ are the components of $\vec{A}$. Since these components are at right angle to each other, hence, are called rectangular components.

Considering the right angle triangle OPM

$$\frac{\text{Base}}{\text{Hypotenuse}} = \cos\theta \quad \Rightarrow \quad \cos\theta = \frac{OM}{OP}$$

$$\frac{A_x}{A} = \cos\theta$$

$$A_x = A\cos\theta$$

It is the magnitude of x-component of vector $\vec{A}$.

Now $$\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \sin\theta$$

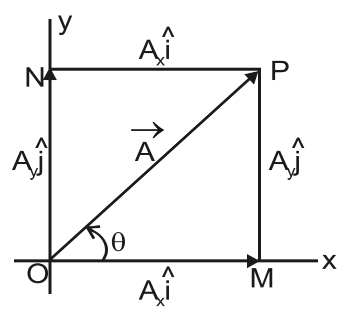
$$\frac{A_y}{A} = \sin\theta$$

$$A_y = A\sin\theta$$

It is the magnitude of y-component of $\vec{A}$.

Determination of a Vector from its Rectangular Components

If rectangular components $A_x \hat{i}$ and $A_y \hat{j}$ vectors are given. Then we can find the magnitude and direction of the vector.



Using Pythagorean theorem.

$$(H)^2 = (B)^2 + (P)^2$$

$$A^2 = A_x^2 + A_y^2$$

The magnitude of resultant

$$A = \sqrt{A_x^2 + A_y^2}$$

The direction of resultant

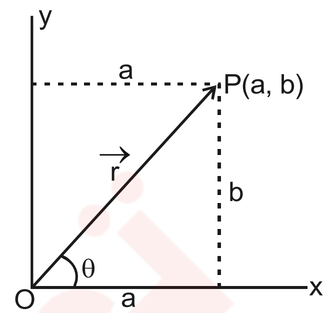
For direction, $\tan\theta = \dfrac{A_y}{A_x}$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Q.6 Define position vector.

Ans. **POSITION VECTOR**

It is a vector that describes the location of a particle with respect to the origin. It is represented by a straight line drawn in such a way that its tail coincides with the origin and the head with point P (a, b) as shown in figure. The projection of position vector r on the x and y-axes are the coordinates a and b and they are rectangular components of the vector r.



$$\vec{r} = a\,\hat{i} + b\,\hat{j}$$

and $$r = \sqrt{a^2 + b^2}$$

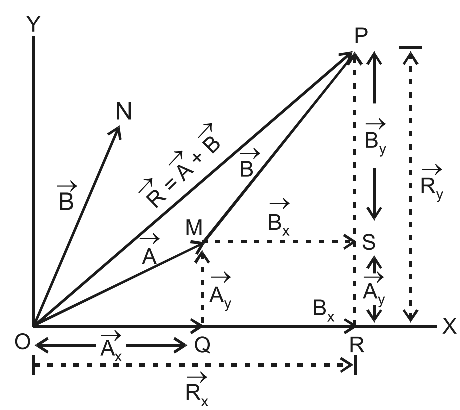
In three dimension $\vec{r} = a\,\hat{i} + b\,\hat{j} + c\,\hat{k}$

$$r = \sqrt{a^2 + b^2 + c^2}$$

Q.7 Explain the addition of vectors by rectangular components.

Ans. **VECTOR ADDITION BY RECTANGULAR COMPONENTS**

Consider two vectors $\vec{A}$ and $\vec{B}$ which are represented by two directed lines OM and ON respectively. The $\vec{B}$ is added to $\vec{A}$ by head to tail rule as shown in figure. The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is given in direction and magnitude by the $\overrightarrow{OP}$.



Resolving $\vec{A}$, $\vec{B}$ and $\vec{R}$ vector into their rectangular components.

In the figure, $\vec{A}_x$, $\vec{B}_x$ and $\vec{R}_x$ are the x–components of the vectors $\vec{A}$, $\vec{B}$, $\vec{R}$ and their magnitude are given by the lines OQ, MS, OR respectively.

From figure:

$$OR = OQ + QR$$

$$\therefore \quad QR = MS$$

$$OR = OQ + MS$$

$$\therefore \quad R_x = A_x + B_x \quad \text{......... (i)}$$

$$\vec{R}_x = R_x\,\hat{i} = A_x\,\hat{i} + B_x\,\hat{i}$$

$$R_x\,\hat{i} = (A_x + B_x)\,\hat{i}$$

Equation (i) means that the sum of the magnitude of x–components of two vectors, which are to be added is equal to the x–component of the resultant.

Similarly $RP = RS + SP$

$\therefore$ $RS = QM$

$\therefore$ $RP = QM + SP$

$$R_y = A_y + B_y \qquad \ldots\ldots\ldots \text{(ii)}$$

$$\vec{R}_y = R_y\,\hat{j} = (A_y + B_y)\,\hat{j}$$

Equation (ii) means that the sum of the magnitudes of y-components of two vectors is equal to the magnitude of y-component of the resultant.

Since $R_x\,\hat{i}$ and $R_y\,\hat{j}$ are rectangular components of the $\vec{R}$.

$$\vec{R} = R_x\,\hat{i} + R_y\,\hat{j} \qquad \ldots\ldots\ldots \text{(iii)}$$

Putting the values of R_x and R_y.

$$\therefore \quad \vec{R} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j}$$

Its magnitude is

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

For direction

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

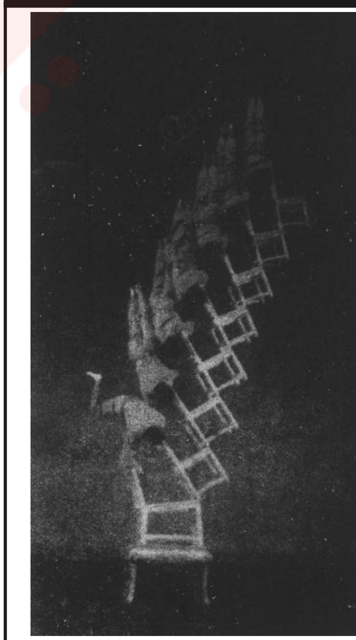
$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

Similarly for any number of coplanar vectors $\vec{A}, \vec{B}, \vec{C}$ ....

$$R = \sqrt{(A_x + B_x + C_x + \ldots)^2 + (A_y + B_y + C_y + \ldots)^2}$$

$$\theta = \tan^{-1}\left(\frac{A_y + B_y + C_y + \ldots}{A_x + B_x + C_x + \ldots}\right)$$

Do You Know?



The Chinese acrobats in this incredible balancing act are in equilibrium.

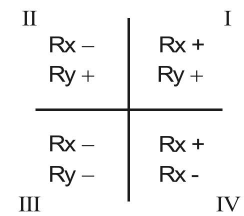
Important Steps for Vector Addition by Rectangular Components

Following are the important steps for the vector addition by rectangular components:

(i) Find x and y components of all given vectors.

(ii) Find x-components , R_x of the resultant vector by adding the x–components of all the vectors.

(iii) Find y-components , R_y of the resultant vector by adding the y–components of all the vectors.



(iv) Find magnitude of resultant vector $\vec{R}$ using

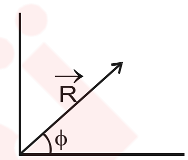
$$R = \sqrt{R_x^2 + R_y^2}$$

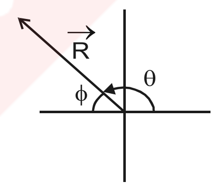
(v) Find the direction of resultant vector by using

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Where θ is the angle, which resultant vector makes with positive x-axis. Irrespective of sign of R_x and R_y determine the value of

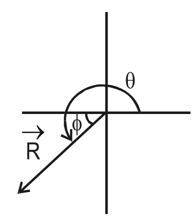
$$\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$





(i) If both R_x, R_y are + ve then $\vec{R}$ lies in first quadrant.

$\therefore \quad \theta = \phi$

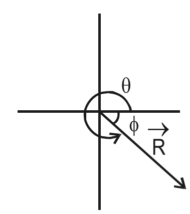


(ii) If R_x is –ve and R_y is +ve then $\vec{R}$ lies in second quadrant.

$\therefore \quad \theta = 180^o - \phi$

(iii) If R_x, R_y both are –ve then $\vec{R}$ lies in third quadrant.

$\therefore \quad \theta = 180^o + \phi$



(iv) If R_x is +ve and R_y is –ve then $\vec{R}$ lies in fourth quadrant.

$\therefore \quad \theta = 360^o - \phi$

Point to Ponder

Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?

Ans. **We should keep our legs apart so that when we bend, our centre of gravity will remain within live of action and we will remain in stable equilibrium.**

Note: If θ is angle between $\vec{A}$ and $\vec{B}$, then their resultant is:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

PRODUCT OF TWO VECTORS

There are two types of vector multiplication.

(i) Scalar Product

(ii) Vector Product

Q.8 Define scalar product. Also write at least five characteristics of scalar product.

Ans. SCALAR OR DOT-PRODUCT

When the product of two vectors results into a scalar quantity then the product is called scalar product. The scalar product between two vectors can be expressed as by putting a dot (•) between the vectors and can be written as $\vec{A} \cdot \vec{B}$. So this product is also called dot product.

Examples

(i) $W = \vec{F} \cdot \vec{d}$, where work is a scalar quantity because it is the dot product of two vectors force and displacement.

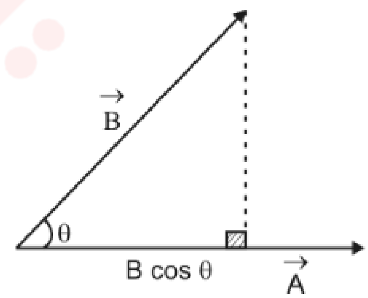
(ii) $P = \vec{F} \cdot \vec{V}$, where power is a vector because it is the dot product of two vectors force and velocity.

Explanation

Consider $\vec{A}$ and $\vec{B}$ are the two vectors having θ angle between them, then the scalar product can be written as:

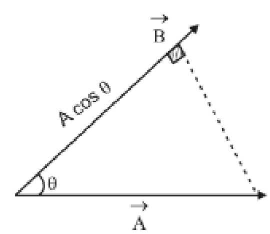
$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

Now draw perpendicular from head of $\vec{B}$ on $\vec{A}$. Also draw a perpendicular from head of $\vec{A}$ on $\vec{B}$.



$$\vec{A} \cdot \vec{B} = AB\cos\theta$$
$$= A(B\cos\theta)$$
$$= \text{Magnitude of } \vec{A} \text{ times } \left(\text{Projection of } \vec{B} \text{ on } \vec{A}\right)$$

Similarly



$$\vec{B} \cdot \vec{A} = BA\cos\theta$$
$$= B(A\cos\theta)$$
$$= \text{Magnitude of } \vec{B} \text{ times } \left(\text{Projection of } \vec{A} \text{ on } \vec{B}\right)$$
$$= \text{Magnitude of } \vec{B}\left(\text{Magnitude of component of } \vec{A} \text{ in direction of } \vec{B}\right)$$

Characteristics of Scalar Product

Following are the characteristics of scalar product:

1. Scalar product is commutative (change of order of vectors has no effect) i.e., $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

Proof

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta \qquad \ldots\ldots\ldots (1)$$

$$\vec{B} \cdot \vec{A} = BA \cos\theta$$

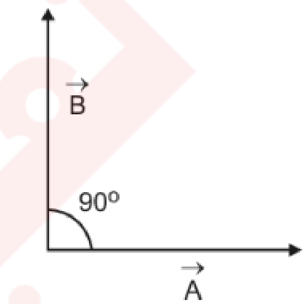
Since AB = BA

$$\therefore \quad \vec{B} \cdot \vec{A} = AB \cos\theta \qquad \ldots\ldots\ldots (2)$$

From equations (1) and (2).

$$\therefore \quad \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

2. The scalar product of two mutually perpendicular vector is zero i.e., $\vec{A} \cdot \vec{B} = 0$.



Now $\quad \vec{A} \cdot \vec{B} = AB \cos\theta$

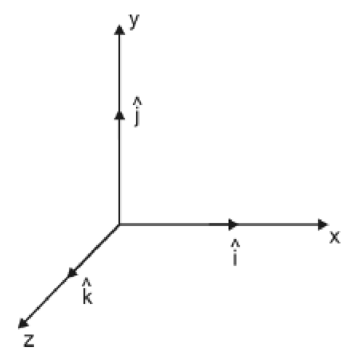
But θ = 90°

$$\therefore \quad \vec{A} \cdot \vec{B} = AB \cos 90^o$$
$$= AB\,(0)$$
$$\vec{A} \cdot \vec{B} = 0$$

In case of unit vector

$$\hat{i} \cdot \hat{j} = 0$$
$$\hat{j} \cdot \hat{k} = 0$$
$$\hat{k} \cdot \hat{i} = 0$$

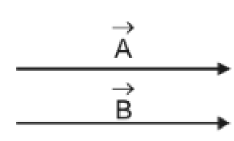
$$\boxed{\begin{aligned} \hat{i} \cdot \hat{j} &= |\hat{i}|\,|\hat{j}| \cos 90^o \\ &= 1 \times 1 \times 0 \\ &= 0 \end{aligned}}$$



3. The scalar product of two parallel vectors is equal to the product of their magnitudes i.e., $\vec{A} \cdot \vec{B} = AB$.

Now $\quad \vec{A} \cdot \vec{B} = AB \cos\theta$

But θ = 0° because the vectors are parallel



As $\quad \vec{A} \cdot \vec{B} = AB \cos\theta$

$$\therefore \quad \vec{A} \cdot \vec{B} = AB \cos 0^o$$
$$= AB \times 1$$
$$= AB$$

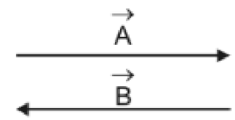
In the case of unit vectors

$$\hat{i}\cdot\hat{i} = 1,\ \hat{j}\cdot\hat{j} = 1,\ \hat{k}\cdot\hat{k} = 1.$$

$$\hat{i}\cdot\hat{i} = |\hat{i}|\,|\hat{i}|\cos 0^\circ = 1 \times 1 \times 1 = 1$$

For anti–parallel vectors

i.e. $\theta = 180^\circ$



As $\quad \vec{A}\cdot\vec{B} = AB\cos 180^\circ$

$\qquad\qquad = AB(-1)$

$\therefore \quad \vec{A}\cdot\vec{B} = -AB$

4. The self scalar product of a $\vec{A}$ is equal to square of its magnitude i.e. $\vec{A}\cdot\vec{A} = A^2$.

As $\quad \vec{A}\cdot\vec{B} = AB\cos\theta$

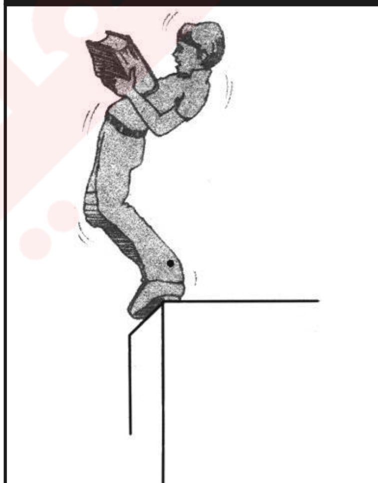
Put $\quad \vec{B} = \vec{A}$

$\therefore \quad \vec{A}\cdot\vec{A} = AA\cos 0^\circ$

$\qquad\qquad = A^2 \times 1$

$\qquad\qquad = A^2$

What Should You Do?



You are falling off the edge. What should you do to avoid falling?

Ans. **In order to avoid falling we should bend in backward direction so that our centre of gravity will remain within line of action and we will remain In stable equilibrium.**

5. Scalar product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

Let $\quad \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Their scalar product can be written as

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= A_xB_x\,\hat{i}\cdot\hat{i} + A_xB_y\,\hat{i}\cdot\hat{j} + A_xB_z\,\hat{i}\cdot\hat{k}$$
$$+ A_yB_x\,\hat{j}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_yB_z\,\hat{j}\cdot\hat{k}$$
$$+ A_zB_x\,\hat{k}\cdot\hat{i} + A_zB_y\,\hat{k}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}$$

$$= A_xB_x(1) + A_xB_y(0) + A_xB_z(0)$$
$$+ A_yB_x(0) + A_yB_y(1) + A_yB_z(0)$$
$$+ A_zB_x(0) + A_zB_y(0) + A_zB_z(1)$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

6. Scalar product holds distributive law, i.e.,

$$\vec{A}\cdot(\vec{B} + \vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

7. Scalar product is associative, i.e.,

$$mn(\vec{A}\cdot\vec{B}) = mn\,\vec{A}\cdot\vec{B}$$

$$= \vec{A} \cdot mn\,\vec{B}$$

$$= m\vec{A} \cdot n\vec{B}$$

Q.9 Define vector product. Also explain the characteristic of vector product.

Ans. **VECTOR OR CROSS PRODUCT**

When the product of two vectors results into a vector quantity, then the product is called vector product. The vector product between two vectors can be expressed as by putting a cross (×) and can be written as $\vec{A} \times \vec{B}$. So this product is also called cross product.

Examples

(i) $\vec{\tau} = \vec{r} \times \vec{F}$ where torque is a vector because it is a cross product of moment arm and force.

(ii) $\vec{L} = \vec{r} \times \vec{p}$ where angular momentum is a vector because it is the cross-product of perpendicular distance and momentum.

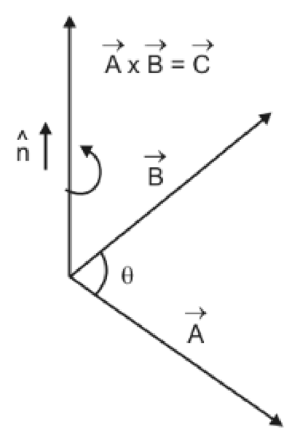
(iii) $\vec{F} = q\left(\vec{V} \times \vec{B}\right)$ where force is a vector because it is the cross-product of velocity and magnetic field.

Explanation

Consider $\vec{A}$ and $\vec{B}$ are the two vectors making an angle θ with each other, then their vector product is

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.



$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

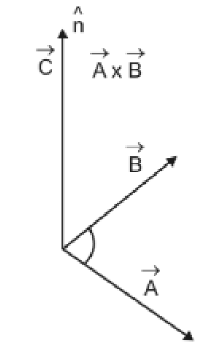
$$\left|\vec{A} \times \vec{B}\right| = AB \sin\theta \left|\hat{n}\right|$$

$$= AB \sin\theta \times 1$$

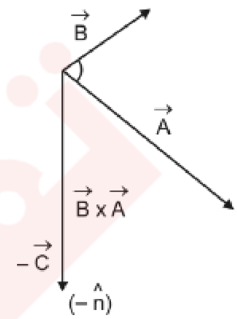
$$= AB \sin\theta$$

The direction of $\vec{A} \times \vec{B}$ is obtained by the right hand rule.

Right Hand Rule



Join the tails of two vectors $\vec{A}$ and $\vec{B}$ which determine the plane containing them. Rotate first vector in the plane, $(\vec{A})$ towards the second vector $(\vec{B})$ through the smaller of the two possible angles. Hold the right hand in such a way that thumb should be erect and fingers should be curl along the direction of rotation of $\vec{A}$, then thumb points toward the direction of $\vec{A} \times \vec{B}$.



Characteristics of Cross Product

Following are the characteristics of cross product:

1. Vector product is non-commutative i.e.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

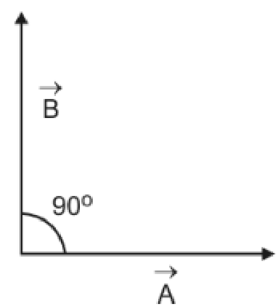
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

As by right hand-rule $\vec{A} \times \vec{B}$ is out of the paper and $\vec{B} \times \vec{A}$ is into the paper i.e., they have same magnitude but opposite in direction, therefore

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

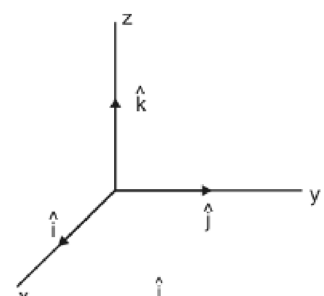
2. Cross product of two perpendicular vectors has maximum magnitude i.e.

$$\vec{A} \times \vec{B} = AB\,\hat{n}$$

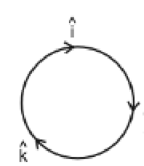


As
$$\vec{A} \times \vec{B} = AB \sin \hat{n}$$
$$= AB \sin 90^\circ\,\hat{n}$$
$$= AB\,\hat{n}$$

In case of unit vector



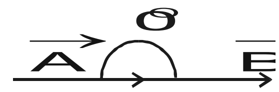
$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$
$$\hat{j} \times \hat{i} = -\hat{k}$$
$$\hat{k} \times \hat{j} = -\hat{i}$$
$$\hat{i} \times \hat{k} = -\hat{j}$$



3. The cross product of two parallel vector is null vector i.e. $\vec{A} \times \vec{B} = \vec{0}$

As $\vec{A} \times \vec{B} = AB \sin\theta \hat{n}$

But $\theta = 0°$ because the vectors are parallel



As $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

$\therefore \quad \vec{A} \times \vec{B} = AB \sin 0^o\, \hat{n}$

$= AB\,(0)\, \hat{n} = 0\, \hat{n}$

$\vec{A} \times \vec{B} = \vec{0}$

In case of unit vector

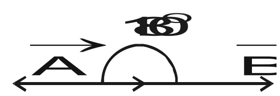
$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

For anti-parallel vectors

$\theta = 180^o$



$\therefore \quad \vec{A} \times \vec{B} = AB \sin 180^o\, \hat{n}$

$= AB\,(0)\, \hat{n}$

$= 0\, \hat{n}$

$\vec{A} \times \vec{B} = \vec{0}$

4. Self vector product is equal to a null vector.

$\vec{A} \times \vec{A} = AA \sin 0^o\, \hat{n}$

$= A^2\,(0)\, \hat{n}$

$= 0\, \hat{n}$

$\vec{A} \times \vec{A} = \vec{0}$

5. Cross product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \left(A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}\right) \times \left(B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}\right)$$

$$= A_x B_x\,\hat{i} \times \hat{i} + A_x B_y\,\hat{i} \times \hat{j} + A_x B_z\,\hat{i} \times \hat{k}$$

$$+ A_y B_x\,\hat{j} \times \hat{i} + A_y B_y\,\hat{j} \times \hat{j} + A_y B_z\,\hat{j} \times \hat{k}$$

$$+ A_z B_x\,\hat{k} \times \hat{i} + A_z B_y\,\hat{k} \times \hat{j} + A_z B_z\,\hat{k} \times \hat{k}$$

$$= A_x B_x\,(0) + A_x B_y\left(\hat{k}\right) + A_x B_z\left(-\hat{j}\right)$$

$$+ A_y B_x\left(-\hat{k}\right) + A_y B_y\,(0) + A_y B_z\left(\hat{i}\right)$$

$$+ A_z B_x\left(\hat{j}\right) + A_z B_y\left(-\hat{i}\right) + A_z B_z\,(0)$$

$$= A_x B_y\,\hat{k} - A_x B_z\,\hat{j} - A_y B_x\,\hat{k} + A_y B_z\,\hat{i} + A_z B_x\,\hat{j} - A_z B_y\,\hat{i}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\,\hat{i} + (A_z B_x - A_x B_z)\,\hat{j} + (A_x B_y - A_y B_x)\,\hat{k}$$

or
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}\,(A_yB_z - A_zB_y) - \hat{j}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

$$= \hat{i}\,(A_yB_z - A_zB_y) - \hat{j}\,[-1(A_zB_x - A_xB_z)] + \hat{k}\,(A_xB_y - A_yB_x)$$

$$= \hat{i}\,(A_yB_z - A_zB_y) + \hat{j}\,(A_zB_x - A_xB_z) + \hat{k}\,(A_xB_y - A_yB_x)$$

6. The magnitude of $\vec{A} \times \vec{B}$ is equal to the area of the parallelogram formed with $\vec{A}$ and $\vec{B}$ as two adjacent sides i.e.

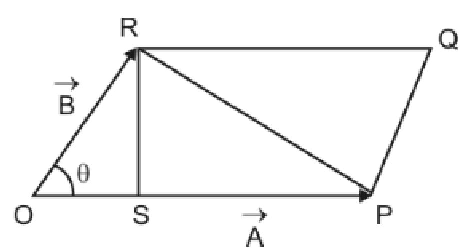
$$\left|\vec{A} \times \vec{B}\right| = \textbf{Area of parallelogram}$$

Solution

As $\quad \vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta \quad \text{.........} \;(i)$$

Consider ΔORS



$$\frac{RS}{OR} = \sin\theta$$

$$RS = OR\sin\theta$$

$$RS = B\sin\theta$$

Equation (i) becomes

$$\left|\vec{A} \times \vec{B}\right| = (OP)\,(RS)$$

Multiply and divide by 2

$$\left|\vec{A} \times \vec{B}\right| = \frac{2}{2}(OP)\,(RS)$$

$$= 2\left(\frac{1}{2}(\text{Base})\,(\text{Height})\right)$$

$$= 2\,[\text{Area of } \Delta OPR]$$

$$= \text{Area of } \Delta OPR + \text{Area of } \Delta PQR$$

$$\therefore \quad \left|\vec{A} \times \vec{B}\right| = \text{Area of parallelogram}$$

7. Vector product is distributive, i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

8. Vector product is associative, i.e.,

$$mn(\vec{A} \times \vec{B}) = mn\,\vec{A} \times \vec{B}$$

$$= m\vec{A} \times n\vec{B}$$

$$= \vec{A} \times mn\,\vec{B}$$

***Q.10 Explain torque. Also calculate the torque due to a free $\vec{F}$* acting on the rigid body.**

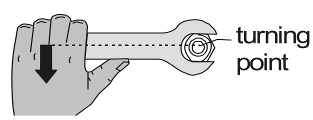
Ans.

TORQUE (MOMENT OF FORCE)

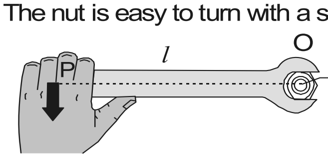
The turning effect of a force is called its torque and is equal to the product of Force and the perpendicular distance from its line of action to the pivot which is the point around which the body rotates. This perpendicular distance between line of action of force and pivot is called moment arm.

Magnitude of the Torque is represented by "τ" is given by

$$\tau = l\,F$$



The nut is easy to turn with a spanner.



It is easier still if the spanner has a long handle.

Torque depends upon two factors:

(i) Force

(ii) Moment arm

When the line of action of the applied force passed through the pivot, then

Moment arm $l = 0$

$$\therefore \quad \tau = 0\,(F)$$

$$\tau = 0$$

By convention, anti–clockwise torque is taken as positive and clock–wise torque is taken as negative.

Unit

SI unit of torque is "Nm".

Dimensions

$$[\tau] = Nm$$
$$= kg\ m / s^2 \times m$$
$$= kg\ m^2 / s^2$$
$$= ML^2 / T^2$$
$$= [ML^2\ T^{-2}]$$

$F = ma$
$N = kg\ m/s^2$

Torque Due to a Force $\vec{F}$ Acting on a Rigid Body

Under the action of a force, if distance between the points of the body remains same such a body is called rigid body.

Let the force $\vec{F}$ acts on a rigid body at point P whose position vector relative to pivot O is $\vec{r}$. The force $\vec{F}$ can be resolved into two rectangular components i.e., $F \sin\theta$ perpendicular to $\vec{r}$ and $F \cos\theta$ in the direction of $\vec{r}$ as shown in Fig. (i). The torque due to $F \cos\theta$ is zero as its line of action of force passes through pivot O. Therefore torque due to $\vec{F}$ is equal to the torque due to $F \sin\theta$. As

$$\tau = \text{Moment arm} \times F$$
$$\tau = r F \sin\theta$$

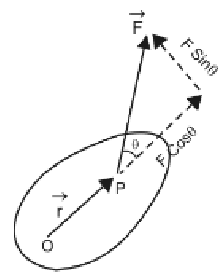


Fig. (i)

Alternatively the moment arm "l" is equal to the magnitude of the component of $\vec{r}$ perpendicular to the line of action $\vec{F}$ as shown in Fig. (ii).

As $\tau = l F$

$$\tau = r \sin\theta\, F$$
$$\tau = r F \sin\theta$$

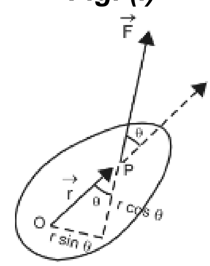


Fig. (ii)

Where θ is the angle between $\vec{r}$ and $\vec{F}$. From equations (1) and (2) torque can also be define as

Definition

The vector product of position vector $\vec{r}$ and the force $\vec{F}$ is also called torque.

$$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = r F \sin\theta \, \hat{n}$$

where $r F \sin\theta$ is the magnitude of torque. The direction of torque represented by $\hat{n}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$ given by right hand rule.

Note: *Torque is a vector quantity.*

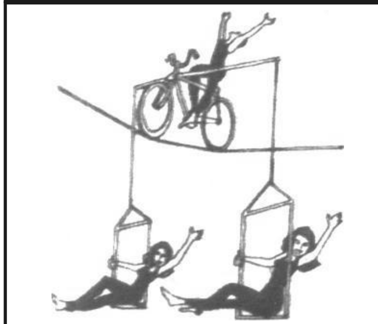
If $\vec{r}$ and $\vec{F}$ are in same direction, then torque will be minimum (0).

If $\vec{r}$ and $\vec{F}$ are in opposite direction, then torque will be minimum (0).

If $\vec{r}$ and $\vec{F}$ are perpendicular to each other, then torque will be maximum (rF).

Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration. Torque is the analogous of force for rotational motion. If the body is at rest or rotating with uniform angular velocity, the angular acceleration will be zero. In this case the torque acting on the body will be zero.

Point to Ponder



Do you think the rider in the above figure is really in danger? What if people below were removed?

Ans. **There is no danger for the rider in the above figure because he is in stable equilibrium and no change if the people below were removed.**

Can You Do?

Stand with one arm and the side of one foot pressed against a wall. Can you raise the other leg side ways? If not, then why not?

Ans. **Yes.**

Q.11 Define equilibrium. Also state the conditions of equilibrium.

Ans. **EQUILIBRIUM**

If a body, under the action of a number of forces, is at rest or moving with uniform velocity, it is said to be in equilibrium.

1. Static Equilibrium

If a body is at rest, it is said to be in Static Equilibrium. For example book lying on the table.

2. Dynamic Equilibrium

If a body is moving with uniform velocity, or rotating with uniform angular velocity, it is said to be in Dynamic Equilibrium. For example jumping of paratrooper.

First Condition of Equilibrium

The vector sum of all the forces acting on a body must be equal to zero.

$$\Sigma \vec{F} = \vec{0}$$

In case of coplanar forces, the resultant force $\vec{F}_x$ is equal to sum of x–directed forces acting on the body. Therefore

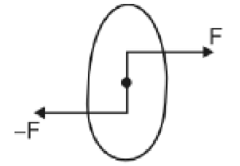
$$\therefore \qquad \Sigma \vec{F}_x = \vec{0}$$

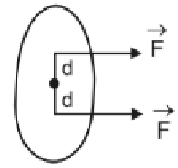
Similarly for y–directed forces, the resultant $\vec{F}_y$ should be zero. Therefore

$$\therefore \qquad \Sigma \vec{F}_y = \vec{0}$$

Second Condition of Equilibrium

Let two equal and opposite forces are acting on body as shown in figure.





Although, the first condition of equilibrium is satisfied, yet it may rotate having clockwise turning effect.

Thus for a body in equilibrium, the vector sum of all the torques acting on it about arbitrary axis should be zero. This is known as second condition of equilibrium. Mathematically

$$\Sigma \vec{\tau} = \vec{0}$$

Requirements for a body to be in complete equilibrium are

(i) $\Sigma \vec{F} = \vec{0}$

i.e. $\Sigma \vec{F}_x = \vec{0}$

$\Sigma \vec{F}_y = \vec{0}$

and $\Sigma \vec{\tau} = \vec{0}$

When first condition is satisfied, there is no linear acceleration and body will be in translation equilibrium (linear equilibrium).

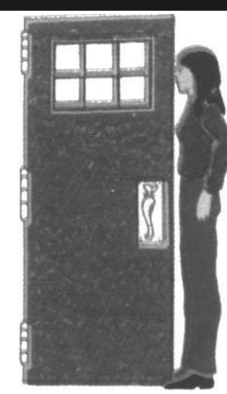
When second condition is satisfied that is no angular acceleration and the body will be in rotational a equilibrium.

For a body to be in complete equilibrium, both conditions should be satisfied, i.e., both linear acceleration and angular acceleration should be zero.

Interesting Application

A concurrent force system in equilibrium. The tension applied can be adjusted as desired.

Can You Do?



With your nose touching the end of the door, put your feet astride the door and try to rise up on your toes.

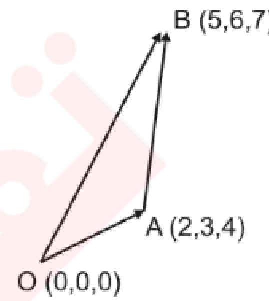
SOLVED EXAMPLES

EXAMPLE 2.1

The position of two aeroplanes at any instant are represented by points A (2, 3, 4) and B (5, 6, 7) from an origin 'O' in km as shown in the figure.

(a) What are their position vectors.

(b) Calculate the distance between two aeroplanes.



Data

Position of two acroplanes are

A(2, 3, 4) and B(5, 6, 7)

To Find

Distance between two aeroplanes AB = ?

SOLUTION

Position vector of aeroplane A is

$$\overrightarrow{OA} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

and that of B is

$$\overrightarrow{OB} = (5, 6, 7) - (0, 0, 0)$$

$$= 5\hat{i} + 6\hat{j} + 7\hat{k}$$

By head to tail rule

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\therefore \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= 5\hat{i} + 6\hat{j} + 7\hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$$

$$= 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$\therefore$ Distance between two aeroplanes is

$$AB = \sqrt{(3)^2 + (3)^2 + (3)^2}$$

$$= \sqrt{9 + 9 + 9}$$

$$= \sqrt{27}$$

$$= 5.2 \text{ km}$$

Result

Distance between two aeroplanes = AB = 5.2 km

EXAMPLE 2.2

Two forces of magnitudes 10N and 20N act on a body in directions making angles 30^o and 60^o with x-axis respectively. Find the resultant force.

Data

First force = F_1 = 10 N

Second force = F_2 = 20 N

First angle = $\theta_1 = 30^o$

Second angle = $\theta_2 = 60^o$

To Find

Resultant force = $\vec{F}$ = ?

SOLUTION

Resolving F_1 and F_2 into its components

$$F_{1x} = F_1 \cos 30^o = 10 \times 0.866 = 8.66 \text{ N}$$

$$F_{1y} = F_1 \sin 30^o = 10 \times 0.5 = 5 \text{ N}$$

$$F_{2x} = F_2 \cos 60^o = 20 \times 0.5 = 10 \text{ N}$$

$$F_{2y} = F_2 \sin 60^o = 20 \times 0.866 = 17.32 \text{ N}$$

Now $F_x = F_{1x} + F_{2x} = 8.66 + 10 = 18.66$ N

Also $F_y = F_{1y} + F_{2y} = 5 + 17.32 = 22.32$ N

Magnitude of resultant is

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(18.66)^2 + (22.32)^2}$$

$$= \sqrt{348.19 + 498.18}$$

$$= \sqrt{84637}$$

$$F = 29 \text{ N}$$

For direction using

$$\phi = \tan^{-1}\frac{F_y}{F_x}$$

Putting values

$$\phi = \tan^{-1}\frac{22.32}{18.66}$$

$$= \tan^{-1} 1.196$$

$$\phi = 50^o$$

Since F_x and F_y are positive hence resultant lies in first quadrant.

$\therefore \quad \theta = \phi$

$\therefore \quad \theta = 50^o$

Result

Resultant force $= F = 29$ N

EXAMPLE 2.3

Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Data

Angle between two forces F_1 and F_2 $\quad \theta = ?$

Such that

$$F_1 = F_2 = F$$

Also $\quad R = F$

SOLUTION

Using $\quad R = \sqrt{F_1^2 + F_2^2 + 2\, F_1\, F_2 \cos\theta}$

$\because \quad F_1 = F_2 = R = F$

$\therefore \quad F = \sqrt{F^2 + F^2 + 2\, F\, F \cos\theta}$

$$F = \sqrt{2\,F^2 + 2\,F^2\cos\theta}$$

Squaring both sides

$$F^2 = 2\,F^2 + 2\,F^2\cos\theta$$

$$F^2 = 2F^2(1 + \cos\theta)$$

$$1 = 2(1 + \cos\theta)$$

$$\frac{1}{2} = 1 + \cos\theta$$

$$\cos\theta = \frac{1}{2} - 1$$

$$= -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120^o$$

EXAMPLE 2.4

A force $F = 2\hat{i} + 3\hat{j}$ units, has its point of application moved from point A(1, 3) to the point B(5, 7). Find the work done.

Data

$$\vec{F} = 2\hat{i} + 3\hat{j}$$

Distance covered from

A (1, 3) to B (5, 7)

To Find

Work done = W = ?

SOLUTION

Using $\vec{d} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$$= (5 - 1)\hat{i} + (7 - 3)\hat{j}$$

$$\vec{d} = 4\hat{i} + 4\hat{j}$$

Now using

$$W = \vec{F} \cdot \vec{d}$$

$$\therefore \quad W = \left(2\hat{i} + 3\hat{j}\right) \cdot \left(4\hat{i} + 4\hat{j}\right)$$

$$= 8 + 12$$

$$= 20 \text{ unit}$$

Result

Work done = W = 20 Joule

EXAMPLE 2.5

Find the projection of vector $\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$ in the direction of vector $\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$.

Data

$$\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$$

$$\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$$

To Find

Projection of $\vec{A}$ along $\vec{B}$ = $A\cos\theta$ = ?

SOLUTION

Using $\vec{A} \cdot \vec{B} = AB\cos\theta$

$$\therefore \quad A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}$$

$$= \frac{(2\hat{i} - 8\hat{j} + \hat{k}) \cdot (3\hat{i} - 4\hat{j} - 12\hat{k})}{\sqrt{(3)^2 + (-4)^2 + (-12)^2}}$$

$$= \frac{6 + 32 - 12}{\sqrt{9 + 16 + 144}}$$

$$= \frac{26}{\sqrt{169}}$$

$$= \frac{26}{13}$$

$$= 2$$

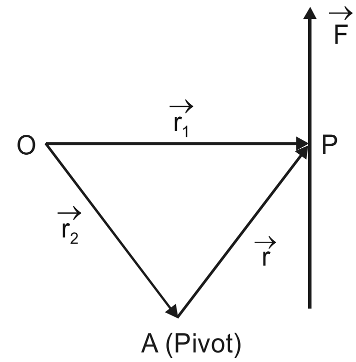
Result

Projection of $\vec{A}$ along $\vec{B}$ = $A\cos\theta$ = 2

EXAMPLE 2.6

The line of action of a force $\vec{F}$ passes through a point P of a body whose position vector in metre is $\hat{i} - 2\hat{j} + \hat{k}$. If $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ (in Newton) determine the torque about the point 'A' whose position vector (in metre) is $2\hat{i} + \hat{j} + \hat{k}$.

Data



$$\vec{r}_1 = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k} \text{ (N)}$$

To Find

$$\vec{\tau} \text{ about A} = ?$$

SOLUTION

$$\vec{r}_2 = 2\hat{i} + \hat{j} + \hat{k}$$

By head to tail rule

$$\vec{r}_1 = \vec{r}_2 + \vec{r}$$

$$\therefore \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

Putting values

$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} + \hat{k})$$

$$= \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - \hat{j} - \hat{k}$$

$$= -\hat{i} - 3\hat{j}$$

Now using

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 0 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12 + 0) - \hat{j}(-4 - 0) + \hat{k}(3 + 6)$$

$$= -12\hat{i} + 4\hat{j} + 9\hat{k} \text{ (Nm)}$$

Result

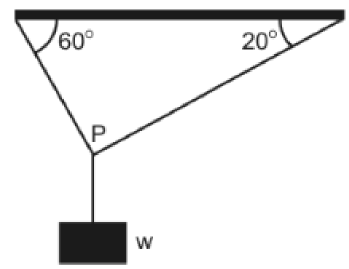
$$\vec{\tau} \text{ about A} = -12\hat{i} + 4\hat{j} + 9\hat{k} \text{ (Nm)}$$

EXAMPLE 2.7

A load is suspended by two cords as shown in the figure. Determine the maximum load that can be suspended at P if maximum breaking stress of the cord used is 50N.

Data

Force = F = 50 N



To Find

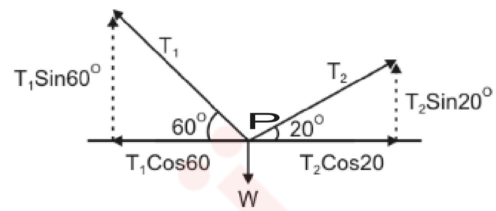
Maximum load = W = ?

SOLUTION

Resolving T_1 and T_2 into its components

Using 1st condition of equilibrium

Now using



$\Sigma F_y = 0$ and $\Sigma F_x = 0$

$T_1 \sin 60^o + T_2 \sin 20^o - W = 0$ $\quad$ $T_2 \cos 20° - T_1 \cos 60° = 0$

$50\,(0.866) + 26.6\,(0.34) = W$ $\quad$ $T_1 = 1.88\,T_2$

$W = 52\text{ N}$ $\quad$ Since $T_1 > T_2$

$\therefore T_1$ has maximum stress.

$T_1 = 50\text{ N}$

then $T_2 = 26.6\text{ N}$

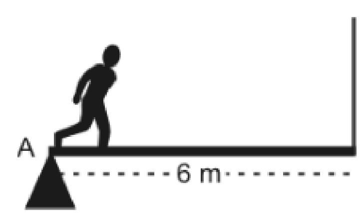
Result

Maximum load = W = 52 N

EXAMPLE 2.8

A uniform beam of 200N is supported horizontally as shown. If the breaking stress of the rope is 400N how far can the man of weight 400N walk from point A on the beam as shown in figure?

Data



Weight of beam = 200 N

Breaking stress of rope = 400N

Distance of man from point A = d = ?

Weight of man = 400N

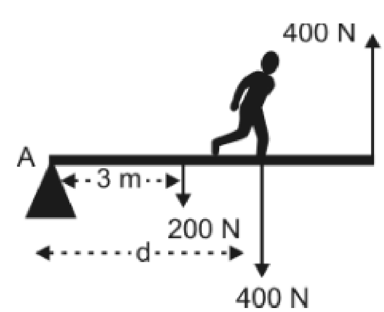
To Find

Distance = d = ?

SOLUTION

Taking point A as pivot

Using $\sum \tau = 0$



$400 \times 6 - 400 \times d - 200 \times 3 = 0$

$2400 - 400\,d - 600 = 0$

$400\,d = 1800$

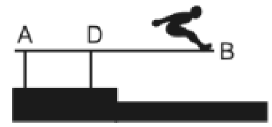
$d = 4.5\text{ m}$

Result

Distance $= d = 4.5 \text{ m}$

EXAMPLE 2.9

A body weighing 300N is standing at the edge of a uniform diving board 4.0m in length. The weight of the board is 200N. Find the forces exerted by pedestals on the board.



Data

Weight of boy $= 300\text{N}$

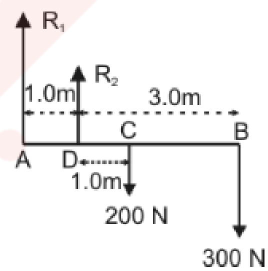
Length of board $= 4\text{m}$

Weight of board $= 200\text{N}$

To Find

Forces exerted by pedestals on board $= ?$

SOLUTION



Let R_1 and R_2 are the reaction forces exerted by the pedestals on the board. A little consideration will show that R_1 is in the wrong direction because the board must be actually pressed down in order to keep it in equilibrium. We will see that this assumption will be automatically corrected by calculations.

Now applying condition of equilibrium

$$\Sigma F_y = 0$$

$$R_1 + R_2 = 200 + 300$$

$$R_1 + R_2 = 500\text{N} \quad \ldots\ldots\ldots (1)$$

Now applying $\Sigma \tau = 0$ about point D.

$$-R_1 \text{ x } AD - 300 \text{ x } DB - 200 \text{ x } DC = 0$$

$$-R_1 \text{ x } 1 - 300 \text{ x } 3 - 200 \text{ x } 1 = 6$$

$$-R_1 - 900 - 200 = 0$$

$$-R_1 = 1100$$

$$R_1 = -1100 \text{ N}$$

Negative sign shows R_1 is directed downward.

⇒ Putting value of R_1 in equation (1).

$$-1100 + R_2 = 500$$

$$R_2 = 500 + 1100$$

$$= 1600 \text{ N}$$

R_1	=	–1100 N	=	1.1 KN
R_2	=	1600 N	=	1.6 KN

Result

Force exerted by pedestals on board $= R_1 = 1.1$ KN

$= R_2 = 1.6$ KN

The negative sign tR_1 shows that it is directed downward.

Chapter 3

MOTION AND FORCE

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

Understand displacement from its definition and illustration.

Understand velocity, average velocity and instantaneous velocity.

Understand acceleration, average acceleration & instantaneous acceleration.

Understand the significance of area under velocity-time graph.

Recall Newton's Laws of motion.

Describe Newton's second law of motion as rate of change of momentum.

Define impulse as a product of impulsive force and time.

Describe law of conservation of momentum.

Describe the force produced due to flow of water.

Understand the process of rocket propulsion (simple treatment).

Understand projectile motion in a non-resistive medium.

Derive time of flight, maximum height and horizontal range of projectile motion.

Q.1 Define motion and rest.

Ans. **MOTION**

If a body is changing its position with respect to its surroundings then the body is said to be in motion.

REST

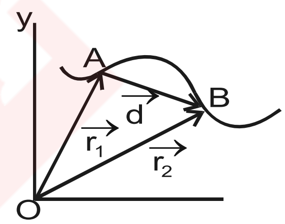
If a body is not changing its position with respect to some observer then the body is said to be at rest.

Q.2 Define displacement and distance.

Ans. **DISPLACEMENT**

The displacement is a change in the position of body from its initial position to its final position, or the shortest distance between two points is called displacement.

The displacement can be represented as a vector that describes how far and in what direction the body has been displaced from its initial position. The tail of displacement vector is located at the position where the displacement started and its tip is located at final position where displacement ended. If a body is moving along a curve as shown with A as its initial position and B as its final position then the displacement $\vec{d}$ of the body is represented by AB.



If $\vec{r}_1$ is position vector of A and $\vec{r}_2$ that of B then by head to tail rule

$$\vec{r}_1 + \vec{d} = \vec{r}_2$$

$$\therefore \quad \vec{d} = \vec{r}_2 - \vec{r}_1$$

It is a vector quantity and its SI unit is metre (m).

Distance

It is the separation between the two points. It is a scalar quantity and its SI unit is metre (m).

Q.3 Define velocity and types of velocity.

Ans. **VELOCITY**

The rate of change of displacement is known as velocity. Its direction is along the direction of displacement. So if $\vec{d}$ is the total displacement of the body in time t, then its average velocity during the interval t is defines as

$$\vec{V}_{av} = \frac{\vec{d}}{t}$$

It is a vector quantity and SI unit is m/s.

Dimensions

$$[\vec{V}] = m/s$$
$$= L/T$$
$$= [LT^{-1}]$$

Types of Velocity

There are three types of velocity:

(i) Uniform velocity (ii) Variable velocity (iii) Instantaneous velocity

(i) Uniform Velocity

If a body covers equal displacements in equal interval of times, however small may be interval the velocity is said to be uniform velocity.

(ii) Variable Velocity

If a body covers equal displacement in unequal interval of times however small may be the interval then it is said to be variable velocity. And its motion is non-uniform.

(iii) Instantaneous Velocity

Velocity of a body at any instant is called instantaneous velocity. **(OR)** The instantaneous velocity is also defined as the limiting value of $\frac{\Delta \vec{d}}{\Delta t}$ as the time interval Δt following the time Δt approaches to zero. Mathematically

$$\vec{V}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

Note: *If the instantaneous velocity does not change the body is said to be moving with uniform velocity.*

Q.4 Define acceleration with its units.

Ans. **ACCELERATION**

The time rate of change of velocity of a body is called acceleration. As velocity is a vector so any change in velocity may be due to change in its magnitude or change in its direction or both. Consider a body whose velocity $\vec{V}_1$ at any time t changes to $\vec{V}_2$ in small time interval Δt, therefore the change in velocity $\Delta \vec{V}$ is

$$\Delta \vec{V} = \vec{V}_2 - \vec{V}_1$$

The average acceleration during time interval Δt is given by

$$\vec{a}_{ave} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\Delta \vec{V}}{\Delta t}$$

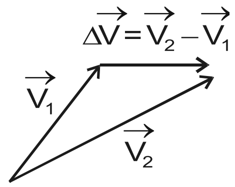
If the velocity of the body is increasing then its acceleration is positive while if the velocity of the body is decreasing then its acceleration is negative. The SI unit of acceleration is m/s^2.

For Your Information

Typical Speeds

Speed, ms^{-1}	Motion
300 000 000	Light, radio waves, x-rays, microwaves (in vacuum)
210 000	Earth-Sun travel around the galaxy
29 600	Earth around the Sun
1 000	Moon around the Earth
980	SR-71 reconnaissance jet
333	Sound (in air)
267	Commercial jet airliner
62	Commercial automobile (max.)
37	Falcon in a dive
29	Running cheetah
10	100-metres dash (max.)
9	Porpoise swimming
5	Flying bee
4	Human running
2	Human swimming

0.01	Walking ant



Dimensions

$$[\vec{a}] = m/s^2$$

$$= L/T^2 = [LT^{-2}]$$

Instantaneous Acceleration

Acceleration of a body at a particular instant is known as instantaneous acceleration.

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

Note: *For a body moving with uniform acceleration, its average acceleration is equal to the instantaneously accelerated.*

Q.5 Explain velocity-time graph.

Ans. **VELOCITY-TIME GRAPH**

Graphs which show the variation of velocity of an object with time are called velocity-time graphs. In such graphs, the time is taken along positive x-axis because it is the independent quantity.

When velocity of car is constant

When velocity of car is constant, its velocity–time graph is a horizontal straight line as shown in Fig. (i).

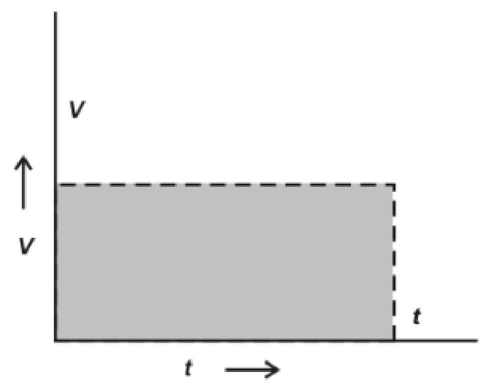


Fig. (i)

As the distance covered by the object is

$$S = vt$$

This distance moved by an object can also be found by using its velocity-time graph by calculating area under this graph. This area is shown shaded in Fig. (i).

As it is a rectangle

$\therefore$ Area under the graph = Height x Width

H x W = vt

A = vt

Hence distance covered = Area under V – t graph.

When car moves with constant acceleration

When the car moves with constant acceleration, the velocity-time graph is a straight line which rises the same height for equal intervals of time as shown in Fig. (ii).

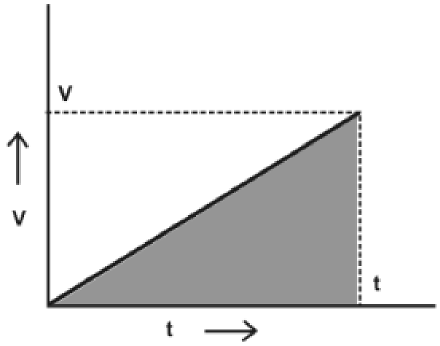


Fig. (ii)

Here the velocity of the object increased uniformly from O to V in time "t". Therefore

$$\therefore \quad V_{av} = \frac{0 + V}{2}$$

$$= \frac{1}{2} V$$

$$\therefore \quad S = V_{av}\, t$$

$$S = \frac{1}{2} V\, t$$

Now we calculate area under velocity-time graph which is equal to the area of the triangle shaded as shown in Fig. (ii).

$$\text{Area of } \Delta = \frac{1}{2} \text{(Base) (Height)}$$

$$= \frac{1}{2} (t)\,(v)$$

$$= \frac{1}{2} v\, t$$

Hence distance covered = area under V–t graph

"In this case average acceleration of an object can be found by finding the slope of its velocity–time graph".

Note: *The area between the velocity–time graph and the time–axis is numerically equal to the distance covered by the object.*

When the car moves with increasing acceleration

When the car moves with increasing acceleration (non–uniform velocity) the velocity-time graph is a curve as shown in Fig. (iii).

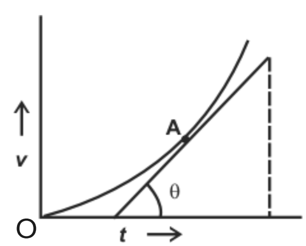


Fig. (iii)

The point A on the graph corresponds to time t. The magnitude of the Instantaneous acceleration at this instant is numerically equal to the slope of the tangent at point A on the velocity-time graph of the object as shown in Fig. (iii).

REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

Suppose an object is moving with uniform acceleration 'a' along a straight line. If initial velocity of the object is 'V_1' and final velocity 'V_f' after a time interval t. And 'S' is distance covered then we have

$$V_f = V_i + at \qquad \ldots\ldots (1)$$

$$S = \left(\frac{V_f + V_i}{2}\right) x\, t \quad \ldots\ldots (2)$$

$$S = V_i\, t + \frac{1}{2} at^2 \quad \ldots\ldots (3)$$

$$V_f^2 = V_i^2 + 2a\, S \quad \ldots\ldots (4)$$

These equations are useful only for linear motion with uniform acceleration.

When the object moves along the straight line, the direction of motion does not change. In this case all the vector can be manipulated like scalars. In such problems the direction of initial is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity.

In the absence of air resistance, all objects near the surface of earth, moves towards the earth with a uniform acceleration. This acceleration, is known as acceleration due to gravity. It is denoted by 'g'. Its average value near the earth surface is taken as 9.8 ms^{-2} in the down ward direction.

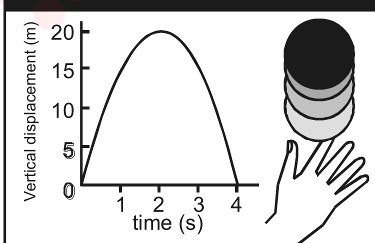
Note: *The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing 'a' by 'g'.*

Q.6 State and explain Newton's laws of motion.

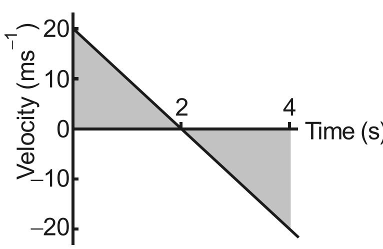
Ans. **NEWTON'S LAWS OF MOTION**

Newton's laws are empirical laws deduced from experiments. They were clearly stated for the 1st time by Sir Isaac Newton who published them in 1687 in his famous book called "Principia". Newton's laws are applicable only for speed which is negligible compared to speed of light. For very fast moving objects, such as atomic particle in an accelerator, relativistic mechanics developed by Einstein is applicable.

Do You Know?



How the displacement of a vertically thrown ball varies with time.



How the velocity of a vertically thrown ball varies with time? Velocity is upwards positive.

NEWTON'S FIRST LAW OF MOTION

A body at rest will remain at rest and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force. This is also known as Law of "Inertia".

Inertia

The property of an object tending to maintain to the state of rest or state of uniform motion is known as object's inertia. The mass of the object is a quantitative measure of its inertia.

Frame of Reference

The space bounded by three mutually perpendicular lines is known as frame of reference. There are two types:

(i) Inertial frame of reference

The frame of reference in which Newton's laws of motions holds is known as inertial frame of reference. It is non–accelerated frame of reference.

(ii) Non-inertial frame of reference

A frame of reference in which Newton's laws of motions does not hold is known as non-inertial frame of reference. It is accelerated frame of reference.

e.g., A frame of reference stationed on earth is approximately an inertial frame of reference.

NEWTON'S SECOND LAW OF MOTION

A force applied on a body produces acceleration in its own direction. The acceleration produced is directly proportional with the applied force and inversely proportional with the mass of the body.

Mathematically, it is expressed as

$$\therefore \quad \vec{a} \propto \vec{F} \qquad \text{.........(i)}$$

$$\vec{a} \propto \frac{1}{m} \qquad \text{.........(ii)}$$

Combining (i) and (ii)

$$\vec{a} \propto \frac{\vec{F}}{m}$$

$$\vec{a} = k\frac{\vec{F}}{m}$$

where K = constant of proportionality.

If F = 1 N , m = 1 kg

a = 1 m/s^2

then K = 1

If S.I. units are used then

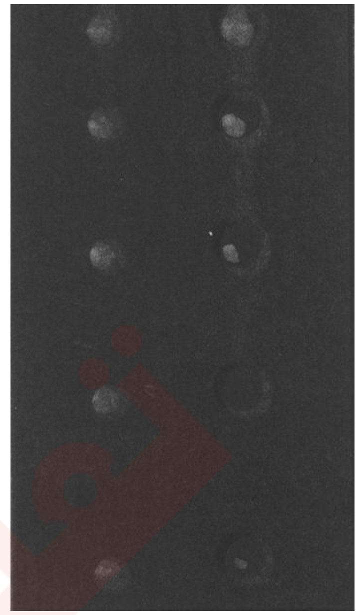
$$\therefore \quad \vec{F} = m\vec{a}$$

NEWTON'S THIRD LAW OF MOTION

Action and reaction are equal but in opposite direction.

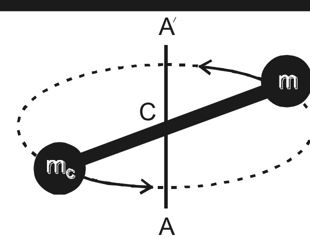
For example, whenever an interaction occurs between two objects, each object exerts the same force on the other, but in the opposite direction and for the same length of time. Each force in action-reaction pair acts only on one of the two bodies, the action and reaction forces never act on the same body.

Do You Know?



At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.

For Your Information



A measurement of mass independent of gravity. The unknown mass m and a calibrated mass m_c are mounted on a light weight rod. If the masses are equal, the rod will rotate without wobble about its centre.

Point to Ponder

A car accelerates along a road. Which force actually moves the car?
Ans: Force of friction

Q.7 What is the linear momentum? Also define its units.

Ans. MOMENTUM (LINEAR MOMENTUM)

It is defined as the "the product of mass and velocity of the object."

It is denoted by "P".

Mathematically

$$\vec{P} = m\vec{V}$$

or The quantity of motion in a moving body is called linear momentum.

Linear momentum is a vector quantity and has the direction in direction of velocity.

The magnitude of momentum depends upon the mass of body and velocity of the body.

Unit

The SI unit of momentum is kg m/s. It is also Ns.

Dimensions

$$[P] = \text{Kg m/s}$$

$$= \text{ML/T}$$

$$= [\text{MLT}^{-1}]$$

Q. Show that kg m/s is equal to Ns?

Ans. Kg m/s = Ns

As L.H.S. = Kg m/s

Multiple and divide by s

$$= \text{Kg m/s} \times \frac{s}{s}$$

$$= [\text{Kg m/s}^2] \times s$$

$$= \text{Ns}$$

$$= \text{R.H.S}$$

Q.8 How force and linear momentum are related? (OR) State Newton's second law of motion in terms of momentum.

Ans. MOMENTUM AND NEWTON'S SECOND LAW OF MOTION

Consider a body of mass 'm' moving with an initial velocity $\vec{V}_i$. Suppose an external force $\vec{F}$ acts upon if for time 't' after which velocity becomes $\vec{V}_f$.

Interesting Information

Throwing a package onto shore from a boat that was previously at rest causes the boat to move outward from shore (Newton's third law).

As, $$\vec{V}_f = \vec{V}_i + \vec{a}\,t$$

$$\vec{a}\,t = \vec{V}_f - \vec{V}_i$$

$$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{t} \quad \ldots\ldots\ldots (1)$$

From Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}}{m} \qquad \ldots\ldots\ldots (2)$$

Point to Ponder

Which will be more effective in knocking a bear down.
i. a rubber bullet or
Ii. a lead bullet of the same Momentum

Ans. **Rubber bullet will be more effective in knocking a bear down because its rate of change of momentum will be greater than that of lead bullet.**

From equation (1) and (2)

$$\frac{\vec{F}}{m} = \frac{\vec{V}_f - \vec{V}_i}{t}$$

$$\vec{F} = \frac{m\vec{V}_f - \vec{V}_i}{t}$$

$$\vec{F} = \frac{\vec{P}_f - \vec{P}_i}{t}$$

$$\vec{F} = \frac{\Delta\vec{P}}{t}$$

Hence second law of motion in term of momentum can also be stated as "the time rate of change of momentum of a body equals the applied force."

Q.9 Define impulse and show that it is change in momentum.

Ans. IMPULSE

When a very large force acts on a body for a very short interval of time the momentum of the body changes. The product of such a force and time is called the impulse. It is denoted by I and it is a vector quantity.

$$\therefore \text{ Impulse} = \vec{I} = \vec{F} \times t$$

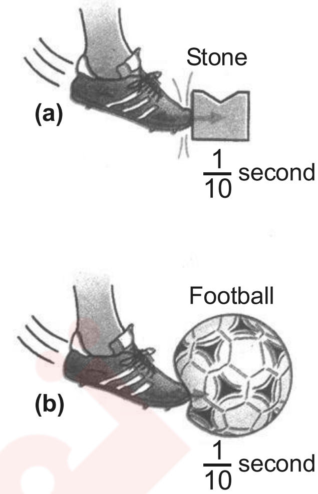
$$\text{As} \quad \vec{F} = \frac{m\vec{V}_f - m\vec{V}_i}{t}$$

$$\vec{F} \times t = m\vec{V}_f - m\vec{V}_i$$

$$\therefore \quad \text{Impulse} = \text{change in momentum } (\Delta\vec{P})$$

Unit: Its unit is Kg ms^{-1} or Ns.

Point to Ponder



Which hurt you in the above situations (a) or (b) and think why?

Point to Ponder

Does a moving object have impulse?

Ans. There are two possibilities:

i. If a body is moving with constant velocity then change in momentum will be zero therefore impulse will be zero but if a body moves with variable velocity then there will be change in momentum and then the moving body will have impulse.

Do You Know?

Your hair acts like a crumple zone on your skull. A force of 5 N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Q.10 State and explain law of conservation of linear momentum.

Ans. LAW OF CONSERVATION OF MOMENTUM

Isolated System

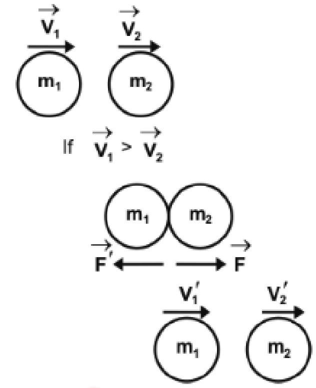
It is a system on which no external agency exerts any force. e.g., The molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion but, no external force can exert on them.

Statement

This law states that the total linear momentum of an isolated system remains constant.

Explanation

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively. Both the balls collide and after collision, the ball of mass m_1 moves with velocity $\vec{V}_1'$ and m_2 moves with velocity $\vec{V}_2'$ in the same direction as shown in figure.



To find the change in momentum we use

$$\vec{F} \times t = m\vec{V}_f - m\vec{V}_1$$

For mass m_1

$$\vec{F} \times t = m_1\vec{V}_1' - m_1\vec{V}_1 \qquad \text{......... (1)}$$

Similarly for mass m_2

$$\vec{F}' \times t = m_2\vec{V}_2' - m_2\vec{V}_2 \qquad \text{......... (2)}$$

Adding (1) and (2)

$$\vec{F} \times t + \vec{F}' \times t = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

$$(\vec{F} + \vec{F}')\,t = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

Since the action F is equal and opposite to the reaction force F′

i.e., $$\vec{F}' = -\vec{F}$$

$$\therefore \quad t(\vec{F} - \vec{F}) = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

$$0 = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$$

which means that total initial momentum of the system before collision is equal to the final momentum of the system after collision Consequently the total change in momentum of the isolated two ball system is zero.

Point to Ponder

What is the effect on the speed of a fighter plane chasing another when it opens fire? What happens to the speed of pursued plane when it returns the fire?

Ans. **The speed of fighter plane chasing another will decrease due to law of conservation of momentum. While the speed of pursued plane will increase.**

Do You Wear Seat Belts?

When a moving car stops quickly, the passengers move forward towards the windshield. Seat belts change the forces of motion and prevent the passengers from moving. Thus the chance of injury is greatly reduced.

Do You Know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

Q.11 Define elastic and inelastic collision.

Ans. ELASTIC AND INELASTIC COLLISIONS

Collision

When two or more object come close enough so that there is some sort of interaction between them, with or without the presence of external force, we say a collision has been taken place between the objects.

There are two types of collision:

1. Head-on collision, such a collision in which after collision balls move in same direction as they move before collision.
2. Oblique collision (direction of balls changes after collision).

Elastic Collision

In the ideal case when no K.E is lost, the collision is said to be perfectly elastic.

For example, when a hard ball is dropped on to a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor.

Inelastic Collision

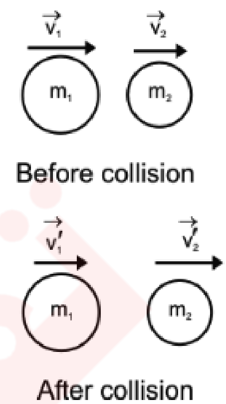
A collision in which the Kinetic Energy of the system is not conserved is called Inelastic Collision.

When two tennis balls collide then after collision, they will rebound with velocities less than the velocities before the impact. During this process, a portion of K.E. is lost, partly due to friction as the molecules in the ball move past one another when the balls distort and partly due to its change into heat and sound energies.

Note: Momentum and total energy are conserved in all types of collisions.

Q.12 Discuss elastic collision in one dimension and prove that speed of approach speed of released. (OR) Derive the formula of final velocities of two balls after an elastic collision in one dimensions.

Ans. ELASTIC COLLISION IN ONE DIMENSION



Consider two smooth, non–rotating balls of masses m_1 and m_2 moving initially with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after collision be $\vec{V}_1'$ and $\vec{V}_2'$ respectively, as shown in figure.

Consider direction of the velocity and momentum to the right.

Since the collision is elastic therefore both momentum and K.E. are conserved.

By Appling Law of conservation of momentum

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2' \quad \ldots\ldots\ldots (1)$$

$$m_1\vec{V}_1 - m_1\vec{V}_1' = m_2\vec{V}_2' - m_2\vec{V}_2$$

$$m_1(\vec{V}_1 - \vec{V}_1') = m_2(\vec{V}_2' - \vec{V}_2) \quad \ldots\ldots\ldots (2)$$

Using law of conservation of K.E.

$$\frac{1}{2}m_1\vec{V}_1^2 + \frac{1}{2}m_2\vec{V}_2^2 = \frac{1}{2}m_1\vec{V}_1'^2 + \frac{1}{2}m_2\vec{V}_2'^2$$

$$\frac{1}{2}\left(m_1\vec{V}_1^2 + m_2\vec{V}_2^2\right) = \frac{1}{2}\left(m_1\vec{V}_1'^2 + m_2\vec{V}_2'^2\right)$$

$$m_1\vec{V}_1^2 + m_2\vec{V}_2^2 = m_1\vec{V}_1'^2 + m_2\vec{V}_2'^2$$

$$m_1\vec{V}_1^2 - m_1\vec{V}_1'^2 = m_2\vec{V}_2'^2 - m_2\vec{V}_2^2$$

$$m_1\left(\vec{V}_1^2 - \vec{V}_1'^2\right) = m_2\left(\vec{V}_2'^2 - \vec{V}_2^2\right)$$

$$m_1(\vec{V}_1 - \vec{V}_1')(\vec{V}_1 + \vec{V}_2') = m_2(\vec{V}_2' - \vec{V}_2)(\vec{V}_2' + \vec{V}_2) \quad \ldots\ldots\ldots (3)$$

Dividing equation (3) by equation (2)

$$\frac{m_1(\vec{V}_1 - \vec{V}_1')(\vec{V}_1 + \vec{V}_1')}{m_1(\vec{V}_1 - \vec{V}_1')} = \frac{m_2(\vec{V}_2' - \vec{V}_2)(\vec{V}_2' + \vec{V}_2)}{m_2(\vec{V}_2' - \vec{V}_2)}$$

$$\vec{V}_1 + \vec{V}'_1 = \vec{V}'_2 + \vec{V}_2 \qquad \text{......... (4)}$$

$$\vec{V}_1 - \vec{V}_2 = \vec{V}'_2 - \vec{V}'_1$$

$$\vec{V}_1 - \vec{V}_2 = -(\vec{V}'_1 - \vec{V}'_2)$$

$$\vec{V}_{rel} = -\vec{V}'_{rel}$$

Before collision $(\vec{V}_1 - \vec{V}_2)$ is the velocity of first ball relative to second ball. Similarly $(\vec{V}'_2 - \vec{V}'_1)$ is the velocity of second ball relative to first ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation. i.e.,

$$\left\{\begin{matrix}\text{Magnitude of relative}\\ \text{velocity of approach}\end{matrix}\right\} = \left\{\begin{matrix}\text{Magnitude of relative}\\ \text{velocity of separation}\end{matrix}\right\}$$

Calculation of Velocity $\vec{V}_1'$ and $\vec{V}_2'$:

From equation (4)

$$\vec{V}_1 + \vec{V}'_1 = \vec{V}'_2 + \vec{V}_2$$

$$\vec{V}'_2 = \vec{V}_1 + \vec{V}'_1 - \vec{V}_2$$

Put this value in equation (1)

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}'_1 + m_2\left(\vec{V}_1 + \vec{V}'_1 - \vec{V}_2\right)$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}'_1 + m_2\vec{V}_1 + m_2\vec{V}'_1 - m_2\vec{V}_2$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 + m_2\vec{V}_2 - m_2\vec{V}_1 = (m_1 + m_2)\,\vec{V}'_1$$

Dividing both sides by $(m_1 + m_2)$

$$\frac{(m_1 - m_2)\,\vec{V}_1 + 2m_2\vec{V}_2}{(m_1 + m_2)} = \frac{(m_1 + m_2)\,\vec{V}'_1}{(m_1 + m_2)}$$

$$\vec{V}'_1 = \frac{(m_1 - m_2)\,\vec{V}_1}{(m_1 + m_2)} + \frac{2m_2\vec{V}_2}{(m_1 + m_2)} \qquad \text{......... (5)}$$

From equation (4)

$$\vec{V}'_1 = \vec{V}'_2 + \vec{V}_2 - \vec{V}_1$$

Do You Know?

If another car crashes into back of yours, the head-rest of the car seat can save you from serious neck injury. It helps to accelerate your head forward with the same rate as the rest of your body.

Put this value in equation (1)

$$\therefore \quad m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\left(\vec{V}_2' + \vec{V}_2 - \vec{V}_1\right) + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_2' + m_1\vec{V}_2 - m_1\vec{V}_2 + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 + m_1\vec{V}_1 - m_1\vec{V}_1 = (m_1 + m_2)\vec{V}_2'$$

$$2m_1\vec{V}_1 + (m_2 - m_1)\vec{V}_2 = (m_1 + m_2)\,\vec{V}_2'$$

Dividing both sides by $(m_1 + m_2)$

$$\therefore \quad \vec{V}_2' = \frac{2m_1\vec{V}_1}{m_1 + m_2} + \frac{(m_2 - m_1)\,\vec{V}_2}{m_1 + m_2} \qquad \ldots\ldots\ldots (6)$$

Point to Ponder

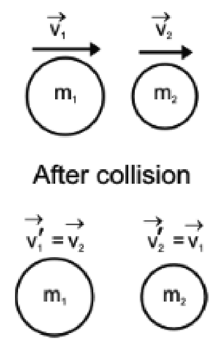
In thrill machine rides at amusement parks, there can be an acceleration of 3g or more. But without head rests, acceleration like this would not be safe. Think why not?

Q.13 Discuss the various cases of elastic collision in dimensions.

Ans. **SPECIAL CASES:**

Case-I: When $m_1 = m_2 = m$

Putting this in equation (5) and equation (6)



$$\therefore \quad \vec{V}_1' = \frac{(m - m)\vec{V}_1}{m + m} + \frac{2m_2\vec{V}_2}{m + m}$$

$$= 0 + \frac{2m\vec{V}_2}{2m} = \frac{2m\vec{V}_2}{2m}$$

$$\vec{V}_1' = \vec{V}_2$$

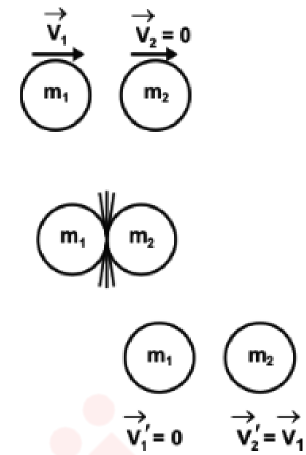
Now $$\vec{V}_2' = \frac{2m\vec{V}_1}{m + m} + \frac{(m - m)\vec{V}_2}{m + m}$$

$$= \frac{2m\vec{V}_1}{2m} + 0$$

$$\vec{V}_2' = \vec{V}_1$$

It means that when two balls of equal mass collide elastically, they simply exchange their velocities.

Case-II: When $m_1 = m_2 = m$ and $\vec{V}_2 = 0$ i.e., target ball at rest



Put this value in eq. (5) and (6)

$$\therefore \quad \vec{V}_1' = \frac{(m-m)\vec{V}_1}{m+m} + \frac{2m\,(0)}{m+m}$$

$$= 0 + 0$$

$$\vec{V}_1' = 0$$

$$\text{Now,} \quad \vec{V}_2' = \frac{2m\vec{V}_1}{m+m} + \frac{(m-m)\,(0)}{m+m}$$

$$= \frac{2m\vec{V}_1}{2m} + 0$$

$$\vec{V}_2' = \vec{V}$$

In this case the ball m_1 comes to rest after collision while ball m_2 that was at rest began to move with $\vec{V}_1$.

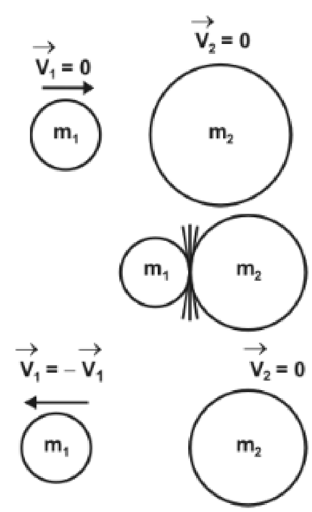
Case-III: When a light body collides with a massive body which is at rest.

i.e., $\vec{V}_2 = 0$

also $m_2 >> m_1$

i.e., $m_1 \simeq 0$

Putting this value in equation (5) and equation (6).



$$\therefore \quad \vec{V}_1' = \frac{0-m_2}{0+m_2}\vec{V}_1 + \frac{2m_2}{0+m_2}(0)$$

$$= -\frac{m_2}{m_2}\vec{V}_1$$

$$\vec{V}_1' = -\vec{V}_1$$

$$\text{Also} \quad \vec{V}_2' = \frac{m_2-m_1}{m_1+m_2}\vec{V}_2 + \frac{2m_1\vec{V}_1}{m_1+m_2}$$

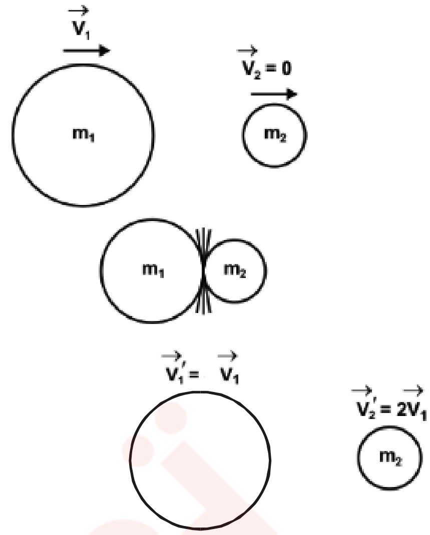
$$= 0 + \frac{2\,(0)\,\vec{V}_1}{0+m_2}$$

$$= 0 + 0$$

$$\vec{V}_2' = 0$$

This means that m_1 will bounce back with same velocity while m_2 remains stationary.

Case-IV: When a massive body collides with a lighter body at rest.



i.e., $\vec{V}_2 = 0$

As $m_1 >> m_2$

$\therefore$ $m_2 \simeq 0$

Putting this value in equation (5) and equation (6).

$$\vec{V}_1' = \frac{m_1 - 0}{m_1 + 0}\vec{V}_1 + \frac{2\,(0)\,(0)}{m_1 + 0}$$

$$= \frac{m_1\vec{V}_1}{m_1} + 0$$

$$\vec{V}_1' = \vec{V}_1$$

Also

$$\vec{V}_2' = \frac{0 - m}{m_1 - 0}(0) + \frac{2m_1\vec{V}_1}{m_1 + 0}$$

$$= 0 + \frac{2m_1\vec{V}_1}{m_1}$$

$$\vec{V}_2' = 2\vec{V}_1$$

Hence after the collision there is practically no change in the velocity of massive body but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body.

Q.14 Derive a relation for the force due to water flow on the wall.

Ans. **FORCE DUE TO WATER FLOW**

When water from a horizontal pipe strikes a wall normally, a force is exerted on the wall. Suppose the water strikes the wall normally with velocity V and comes to rest on striking the wall, then

Change in velocity = Final velocity – Initial velocity

= 0 – V

= – V

If 'm' is the mass of water that strikes the wall, then

Change in momentum = m (– V) = – m V

According to Newton's second law, time rate of change of momentum is equal to force applied i.e.,

$$F = \frac{\text{Change in momentum}}{\text{time}}$$

or

$$F = -\frac{m\,v}{t} = -\left(\frac{m}{t}\right)v \qquad \ldots\ldots\ldots (1)$$

or Force = –(Mass per second) x (Change in velocity)

This is the force exerted by the wall on water.

From third law of motion the reaction force exerted by water upon the wall is equal but opposite. Therefore, equation (1) becomes

$$F = -\frac{(-m\,v)}{t} = \frac{m\,v}{t}$$

Thus, force can be calculated from the product of the mass of water striking normally per second and change in velocity.

For Example

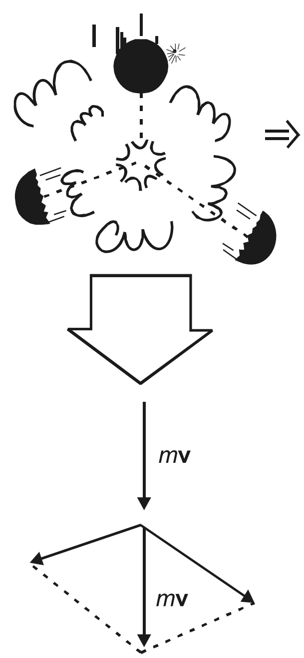
Suppose the water flows out from a pipe at 3 kgs^{-1} and its velocity changes from 5 ms^{-1} to zero on striking the ball, then,

$$\text{Force} = 3\ \text{kgs}-1 \times (5\ \text{ms}^{-1} - 0) = 15\ \text{kgms}^{-2} = 15\ \text{N}$$

Q.15 Write a note on momentum and explosive forces.

Ans. MOMENTUM AND EXPLOSIVE FORCES

There are many examples where momentum changes are produced by explosive forces within an isolated system for example, when a shell explodes in mid-air, fragments fly off in different directions. The total momentum of all its fragments equals the initial momentum of the shell. Suppose a falling bomb explodes into pieces as shown in figure. The momenta of the bomb fragments combine by vector addition to equal to original momentum of the falling bomb.



Consider another example of bullet of mass m fired from the rifle of mass M with a velocity v. Initially, the total momentum of the bullet and rifle is zero. From the principle of conservation of linear momentum, when the bullet fired, the total momentum of bullet and rifle still remain zero, since no external force has acted on them. Thus if V′ is the velocity of the rifle then.

$$\therefore \quad 0 + 0 = m\,V + MV'$$

$$0 = m\,V + MV'$$

$$MV' = -m\,V$$

$$V' = -\frac{m\,V}{M}$$

The momentum of the rifle is thus equal and opposite to that of the bullet. Since mass of rifle is much greater than bullet, it follows that the rifle moves back or rifle with only a fraction of velocity of the bullet.

Q.16 What do you know about rocket propulsion?

Ans. ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ingited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines. The rocket gains momentum to the momentum of the gas expelled from the engine it in opposite direction. The rocket engines continue to gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So instead of traveling at steady speed the rocket gets faster and faster sol long the engines are operating.

Rocket carries its own fuel in the form of a liquid or solid and oxygen. It can, therefore, work at great heights where little or no air is present. In order to provide enough inward thrust to overcome gravity, a typical rocket resumes about 10000 kgs^{-1} of fuel and eject the burnt at speed of over 4000 ms^{-1}. In effect, more than 10% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to take the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving to carry the space craft further up at every greater speed.

If m is the mass of the gases ejected per second with velocity relative to the rocket, the change in momentum per second of ejecting gases is mv. This equals the thrust produced in by the engine on the body of the rocket. So, the acceleration, $\vec{a}$ of the rocket is.

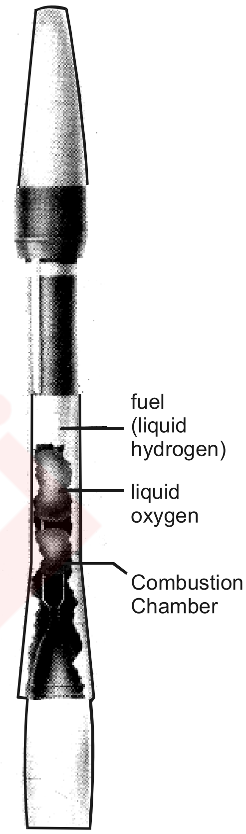


Fig. Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals. The gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

$$\vec{F} = \frac{m\vec{V}}{t}$$

As $t = 1$ sec

$$\vec{F} = \frac{m\vec{V}}{1}$$

$$\vec{F} = m\vec{V}$$

$$M\vec{a} = m\vec{V}$$

$$\vec{a} = \frac{m\vec{V}}{M}$$

Q.17 Define projectile motion. Also calculate the velocity of the projectile of any instant. (OR) Define projectile motion. Also derive the formulae for the time of flight and height of projectile. (OR) Explain projectile motion. Also derive the formula for the range of projectile.

Ans. PROJECTILE MOTION

When an object is thrown in air making a certain angle with horizontal, so that object moves under the action of gravity and moves along a curved path, is called as "projectile". Its motion is called "projectile motion". Its path is called trajectory. Its path is parabolic. **(OR)** Projectile motion is two dimensional motion under constant acceleration due to gravity.

For Example:

(i) A ball thrown by a cricketer

(ii) A foot-ball kicked by a player.

(iii) A missile fired from a launching pad.

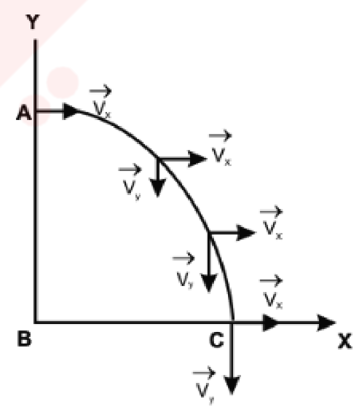
Explanation

Consider the motion of a ball when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downwards, until it strikes something.

Suppose that the ball leaves the hand of the thrower at point A, as shown in figure. And its velocity at that instant is completely horizontal, i.e., $\vec{V}_x$.

according to Newton's 1st law of motion, there will be no acceleration in horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight, is the force of gravity. There is no horizontal force acting on it, so its horizontal velocity will remain unchanged and will be $\vec{V}_x$, until the ball hits something.

Hence, the horizontal distance is



$$S = V_i\,t + \frac{1}{2}a\,t^2$$

$$x = V_{ix}\,t + \frac{1}{2}a_x\,t^2$$

$\because \quad V_x = \text{Constant}$

$\therefore \quad a_x = 0$

$\therefore \quad X = V_{ix}\,t$

This is the horizontal distance.

The ball will accelerate downward under the force of gravity and hence $\vec{a} = g$. This vertical motion is the same as for a freely falling body.

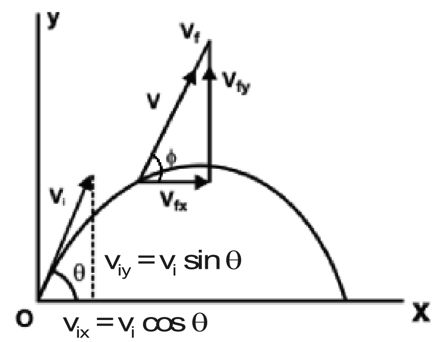
As, $\quad y = V_{iy}\,t + \frac{1}{2}a_y\,t^2$

$\therefore \quad V_{iy} = 0$

$a_y = g$

$\therefore \quad y = \frac{1}{2}gf^2$

Velocity of the Projectile at any Instant



Suppose that a projectile is fired in a direction making an angle θ with the horizontal by velocity V_i as shown in Fig.

At any instant the velocity of the projectile has two components (1) horizontal component, (2) vertical component. These two components are independent to each other. During the motion of the projectile horizontal component of the velocity remains same, so

$$a_x = 0$$

because we have neglected air resistance and no other force is acting along this direction.

As the projectile moves up under the action of gravity, so

$$a_y = -g$$

As $\quad V_f = V_i + a\,t$

For two dimensional – motion

$$V_{fx} = V_{ix} + a_x\,t$$

$\because \quad a_x = 0$

$\therefore \quad V_{fx} = V_{ix} + 0\,(t)$

$$V_{fx} = V_{ix} = V_i \cos\theta \qquad \ldots\ldots\ldots (1)$$

Also, $\quad V_f = V_i + a\,t$

$$V_{fy} = V_{iy} + a_y\,t$$

$$V_{iy} = V_i \sin\theta,\ a_y = -g$$

$\therefore \quad V_{fy} = V_i \sin\theta - g\,t \qquad \ldots\ldots\ldots (2)$

Velocity of the projectile at any instant is

$$\vec{V} = \vec{V}_f = V_{fx}\,\hat{i} + V_{fy}\,\hat{j}$$

Putting the values of V_{fx} and V_{fy}

$$\vec{V} = V_i \cos\theta\,\hat{i} + (V_i \sin\theta - g\,t)\,\hat{j}$$

Its magnitude is

$$V = \sqrt{V_{fx}^2 + V_{fy}^2}$$

or $\quad V = \sqrt{(V_i \cos\theta)^2 + (V_i \sin\theta - g\,t)^2}$

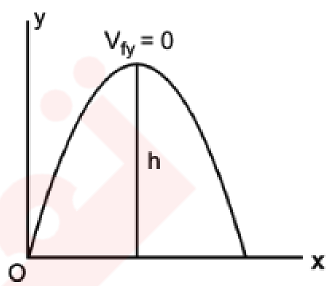
For direction

$$\tan\phi = \frac{V_{fy}}{V_{fx}}$$

$$\phi = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right)$$

Height of the Projectile

The maximum vertical distance which a projectile covers is called height of projectile. In order to determine the maximum height of projectile attains we used the third equation of motion.



Using $2aS = V_f^2 - V_i^2$

or $2a_yy = V_{fy}^2 - V_{iy}^2$ ……… (1)

$\because$ $V_{fy} = 0$

As the body comes to rest after reaching highest point

Also $a_y = -g$

$V_{iy} = V_i \sin\theta$

Putting these values in equation (1)

$\therefore$ $2(-g)y = (0)^2 - (Vi\sin\theta)^2$

$$-2gy = -V_i^2\sin^2\theta$$

$$2gy = V_i^2\sin^2\theta$$

$\because$ $S = y = h$

$$2gh = V_i^2\sin^2\theta$$

$$\boxed{\therefore\ h = \frac{V_i^2\sin^2\theta}{2g}}$$

Time of Flight

"The time taken by the body to cover the distance from the place of its projection to the place where it hits the ground at the same level is called the time of flight."

As the body goes up and comes back to same level, thus covering no vertical displacement.

$\therefore \quad S = y = h = 0$

Also $\quad V_{iy} = V_i \sin\theta$

$a_y = -g$

As $\quad S = V_{iy}\, t + \frac{1}{2} a_y t^2$

$$0 = V_i \sin\theta\, t + \frac{1}{2}(-g)\, t^2$$

$$0 = V_i \sin\theta\, t - \frac{1}{2} g\, t^2$$

$$\frac{1}{2} g\, t^2 = V_i \sin\theta\, t$$

$$\frac{1}{2} g\, t = V_i \sin\theta$$

$$\boxed{t = \frac{2 V_i \sin\theta}{g}}$$

Time to reach maximum Height

As, $\quad V_f = V_i + a\,t$

$V_{fy} = V_{iy} + a_y\, t$

As, Motion is upward

$\therefore \quad a_y = -g$

$V_{fy} = 0$

$$0 = V_i \sin\theta - g\,t$$

$$gt = V_i \sin\theta$$

$$t = \frac{V_i \sin\theta}{g}$$

Range of the Projectile

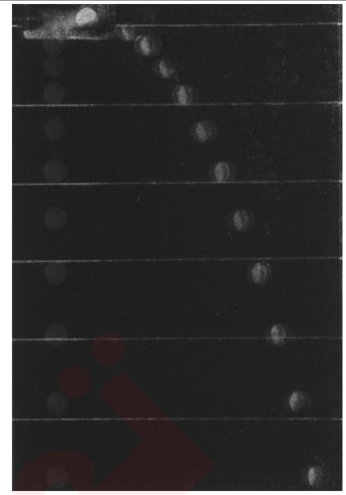
"The maximum distance, which a projectile covers in the horizontal direction is called the range of the projectile. It is denoted by "R".

In order to find "R", we multiply the horizontal component of the velocity with time of flight.

Hence, $\quad R = V_{ix} \times t$

$$= V_i \cos\theta \times \frac{2 V_i \sin\theta}{g}$$

Interesting Information

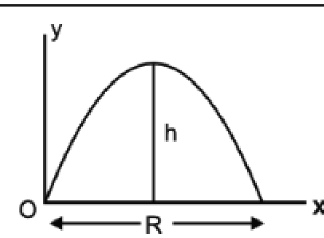


A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

Point to Ponder

Water is projected from two rubber pipes at the same speed-from one at an angle of 30° and from the other at 60°. Why are the ranges equal?

Ans. **Because the range will be maximum at 45°. But if the angles of projection which exceed or fall short of 45° by equal amounts, the ranges are equal.**



$$= \frac{V_i^2 (2 \sin\theta \cos\theta)}{g}$$

$$R = \frac{V_i^2 \sin 2\theta}{g} \quad (\because\ 2\sin\theta\cos\theta = \sin 2\theta)$$

Thus the range of projectile depends upon the velocity of projection and the angle of projection.

Maximum Range

R will be maximum

when, $\sin 2\theta = 1$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^{o}$$

$$\theta = \frac{90^{o}}{2}$$

$$\theta = 45^{o}$$

$$\therefore \quad R_{max} = \frac{V_i^2}{g}$$

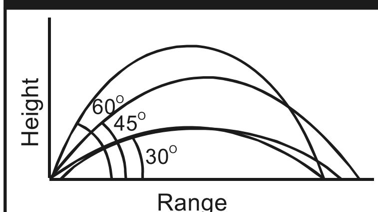
Application to Ballistic Missiles

A ballistic flight is that in which a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity. An un–powered and un-guided missile is called a ballistic missile and the path followed by it is called ballistic trajectory.

As discussed before, a ballistic missile moves in a way that is the result of the superposition of two independent motions: a straight line inertial flight in the direction of the launch and a vertical gravity fall. By law of inertia, an object should sail straight off in the direction thrown, at constant speed equal to its initial speed particularly in empty space. But the downward force of gravity will alter straight path into a curved trajectory. For short ranges and flat Earth approximation, the trajectory is parabolic but the dragless ballistic trajectory for spherical Earth should actually be elliptical. At high speed and for long trajectories the air friction is not negligible and some times the force of air friction is more than gravity. It affects both horizontal as well as vertical motions. Therefore, it is completely unrealistic to neglect the aerodynamic forces.

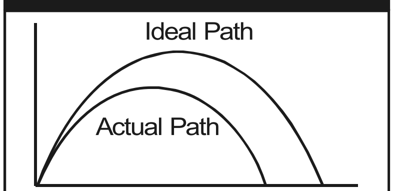
The shooting of a missile on a selected distant spot is a major element of warfare. It undergoes complicated motions due to air friction and wind etc. Consequently the angle of projection cannot be found by the geometry of the situation at the moment of launching. The actual flights of missiles are worked out to high degrees of precision and the result were contained in tabular form. The modified equation of trajectory is too complicated to be discussed here. The ballistic missiles are useful precision, powered and remote control guided missiles are used.

Do You Know?



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

Do You Know?



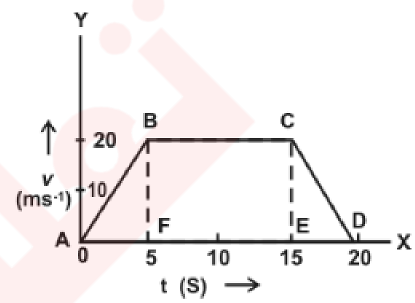
In the presence of air friction the trajectory of a high speed projectile fall short of a parabolic path.

SOLVED EXAMPLES

EXAMPLE 3.1

The velocity time graph of a car moving on a straight road is shown in figure. Describe the motion of the car and find the distance covered.

SOLUTION



The graph shows that car starts from rest i.e., $V_i = 0$ and its velocity becomes 20 m/s in time 5 sec. Then

Average acceleration is given by

$$a = \frac{\Delta V}{\Delta t} = \frac{20}{5} = 4 \text{ m/s}^2$$

Motion of car from B to C

The graph shows that car moves with uniform velocity of 20 m/s^2. Here acceleration is zero.

Motion of Car From C to D

The graph shows that the acceleration decreases during last four seconds

and $$a = \frac{\Delta V}{\Delta t} = \frac{-20}{4} = -5 \text{ m/s}^2$$

Negative Sign Shows that Velocity is Decreasing

Distance Covered by Car

Distance covered = Area of trapezium OABCO

Now

Area of trapezium $= \frac{1}{2}$(sum of parallel sides) x (distance between parallel sides)

$$= \frac{1}{2}(10 + 19) \times 20$$

$$= \frac{1}{2}(29)(20) = 290$$

Thus

Distance covered = 290 m

Note: The distance covered in above example can also be determined by applying.

Distance covered = Area of ΔABF + Area of rectangle BCEF + Area of ΔCDE.

EXAMPLE 3.2

A 1500 kg car has its velocity reduced from 20 ms^{-1} to 15 ms^{-1} in 3.S. How large was the average retarding force.

Data

Mass = m = 1500 kg

Initial velocity = V_i = 20 m s^{-1}

Final velocity = V_f = 15 m s^{-1}

Time taken = t = 3.0 s

To Find

Force = F = ?

SOLUTION

We know that

$$F = \frac{m V_f - m V_i}{t}$$

$$= \frac{m (V_f - V_i)}{t}$$

Putting values, we get

$$F = \frac{1500 (15 - 20)}{3}$$

$$= \frac{1500 \times (-5)}{3}$$

$$= -2500 \text{ N}$$

$$\boxed{F = -2500 \text{ N}}$$

The negative sign indicates the force is retarding.

Result

Force = F = –2500 N

= 2.5 KN

EXAMPLE 3.3

Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 ms^{-1} and 4 ms^{-1} respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 ms^{-1}?

Data

Mass of 1st ball = m_1 = 2 kg

Mass of 2nd ball = m_2 = 3 kg

Velocity of 1st ball = V_1 = 6 m s^{-1}

Velocity of 2nd ball = V_2 = -4 m s^{-1}

Velocity after collision = V_2' = -3 m s^{-1}

To Find

Velocity of 1st ball after collision = V_1' = ?

SOLUTION

Using law of conservation of momentum.

Momentum of the system before collision = Momentum of the system after collision

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

Putting values

$$2(6) + 3(-4) = 2V_1' + 3(-3)$$

$$12 - 12 = 2V_1' - 9$$

$$0 = 2V_1' - 9$$

$$2V_1' = 9$$

$$V_1' = 4.5 \text{ m/s}$$

Result

Velocity of 1st ball after collision = V_1' = 4.5 m/s

EXAMPLE 3.4

A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 ms^{-1} to the right while the second ball is at rest. If the collisions were perfectly elastic what would be the velocity of the two balls after the collision?

Data

Mass of 1st ball = m_1 = 70 g = 0.07 kg

Mass of 2nd ball = m_2 = 140 g = 0.14 kg

Velocity of 1st ball = V_1 = 9 m s^{-1}

Velocity of 2nd ball = V_2 = 0

To Find

Velocity of 1st ball after collision = V_1' = ?

Velocity of 2nd ball after collision = V_2' = ?

SOLUTION

Using $V_1' = \frac{m_1 - m_2}{m_1 + m_2} V_1 + \frac{2 m_2 V_2}{m_1 + m_2}$

$V_1' = \frac{0.07 - 0.14}{0.07 + 0.14} \times 9$

$V_1' = -3$ m/s

Now, $V_2' = \frac{2m}{m_1 + m_2} V_1 + \frac{m_2 - m_1}{m_1 + m_2} V_2$

$V_2' = \frac{2 \times 0.14}{0.07 + 0.14} \times 9 + 0$

$V_2' = 6\ ms^{-1}$

Result

Velocity of 1st ball after collision = V_1' = -3 m/s

Velocity of 2nd ball after collision = V_2' = 6 m/s

EXAMPLE 3.5

A 100 g golf ball is moving to the right with a velocity of 20 ms^{-1}. It makes a head on collision with a 8 kg steel ball, initially at rest. Compute velocities of the ball after collision.

Data

Mass of 1st ball = m_1 = 100 g

$= \frac{100}{1000}$ kg

= 0.1 kg

Mass of 2nd ball = m_2 = 8 kg

Velocity of 1st ball = V_1 = 20 m s^{-1}

Velocity of 2nd ball = V_2 = 0

To Find

Velocity of 1st ball after collision = V_1' = ?

Velocity of 2nd ball after collision = V_2' = ?

SOLUTION

Using $V_1' = \frac{m_1 - m_2}{m_1 + m_2} V_1 + \frac{2m_2 V_2}{m_1 + m_2}$

$V_1' = \frac{0.1 - 8}{0.1 + 8} \times 20 + 0$

$$V_1' = -19.5 \text{ m s}^{-1}$$

Now, $$V_2' = \frac{2m}{m_1 + m_2} V_1 + \frac{m_2 - m_1}{m_1 + m_2} V_2$$

$$V_2' = \frac{2 \times 0.1}{0.1 + 8} \times 20 + 0$$

$$V_2' = 0.49 \text{ m s}^{-1}$$

$$V_2' = 0.5 \text{ m s}^{-1}$$

Result

Velocity of 1st ball after collision = V_1' = -19.5 m/s

Velocity of 2nd ball after collision = V_2' = 0.5 m/s

EXAMPLE 3.6

A hose pipe ejects water at a speed of 0.3 ms^{-1} through a hole of area 50 cm^2. If the water strikes a wall normally, calculate the force on the wall, assuming the velocity of the water normal to the wall is zero after striking.

Data

$$V = 0.3 \text{ m s}^{-1}$$

$$A = 50 \text{ cm}^2$$

$$= 50 \times 10^{-4} \text{ m}^2$$

To Find

Force on the ball = F = ?

SOLUTION

Using $$F = \frac{m\,V}{t} \quad \text{......... (1)}$$

As $$\rho = \frac{m}{V}$$

$$m = \rho V$$

$$\therefore \frac{V}{t} \text{ (volume of water per second)} = AV$$

$$= 50 \times 10^{-4} \times .3$$

$$= 0.0015 \text{ m}^3$$

Since $$\frac{m}{t} = \rho \frac{V}{t}$$

$$\frac{m}{t} = 1000 \times 0.0015$$

$$= 1.5 \text{ kg/s}$$

Putting equation (1)

$$\therefore \quad F = 1.5\ (.3)$$

$$= 0.45 \text{ N}$$

Result

Force on the ball = F = 0.45 N

EXAMPLE 3.7

A ball is thrown with a speed of 30 ms^{-1} in a direction 30^o above the horizontal. Determine the height to which it rises, the time of flight and horizontal range.

Data

Angle with horizontal = θ = 30^o

Initial speed = V_i = 30 m s^{-1}

To Find

Vertical height = h = ?

Time of flight = t = ?

Horizontal range = R = ?

SOLUTION

For Vertical Height

$$h = \frac{V_i^2 \sin^2\theta}{2g}$$

$$h = \frac{(30 \sin 30)^2}{2 \times 9.8}$$

$$= 11.5 \text{ m}$$

For Range of Projectile

$$R = \frac{V_i^2 \sin^2\theta}{g}$$

$$R = \frac{(30)^2 \sin 2 \times 30}{9.8}$$

$$R = \frac{(30)^2 \sin 60}{9.8}$$

$$R = \frac{(30)^2 (0.866)}{9.8}$$

$$R = 79.5 \text{ m}$$

For Time of Flight

$$t = \frac{2 V_i \sin\theta}{g}$$

$$t = \frac{2 \times 30 \times \sin 30}{9.8}$$

$$t = 3.1 \text{ sec}$$

Result

Vertical height = h = 11.5 m

Horizontal range = R = 79.5 m

Time of flight = t = 3.1 sec.

EXAMPLE 3.8

In example 3.7 calculate the maximum range and the height reached by the ball if the angles of projection are (i) 45^o (ii) 60^o.

Data

Initial speed = V_i = 30 m/s

Angle with horizontal = θ = 30^o

To Find

Maximum range = R = ?

Vertical height = h = ?

When: (i) $\theta = 45°$

(ii) $\theta = 60°$

SOLUTION

As we know that

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

(i) When $\theta = 45°$

$$R = \frac{(30)^2 \sin 2 \times 45^o}{9.8}$$

$$= \frac{900 \times \sin 90}{9.8}$$

$$= \frac{900 \times 1}{9.8}$$

$$\boxed{R = 91.8 \text{ m}}$$

and vertical height

Using formula

$$h = \frac{V_i^2 \sin^2\theta}{2g}$$

$$= \frac{(30)^2 (\sin 45)^2}{2\,(9.8)}$$

$$= \frac{900\,(0.707)^2}{19.6}$$

$$= 23 \text{ m}$$

$$\boxed{h = 23 \text{ m}}$$

(ii) When θ = 60°

We know that

$$R = \frac{V_i^2 \sin 2\,\theta}{g}$$

$$= \frac{(30)^2 \sin 2 \times 60^o}{9.8}$$

$$= \frac{900 \times \sin 120}{9.8}$$

$$= \frac{900 \times 0.866}{9.8}$$

$$\boxed{R = 80 \text{ m}}$$

and vertical height is

Using formula

$$h = \frac{V_i^2 \sin^2 \theta}{2\,g}$$

$$= \frac{(30)^2 (\sin 60)^2}{2\,(9.8)}$$

$$= \frac{900\,(0.866)^2}{19.6}$$

$$= 34.4 \text{ m}$$

$$\boxed{h = 34.4 \text{ m}}$$

Result

(i) When θ = 45°

Maximum range = R = 91.8 m

Vertical height = h = 23 m

(ii) When θ = 60°

Maximum range = R = 80 m

Vertical height = h = 34.4 m

Chapter 4

WORK AND ENERGY

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand the concept of work in terms of the product of a force and displacement in the direction of the force.
- Understand and derive the formula Work = wd = mgh for work done in a gravitational field near Earth's surface.
- Relate power to work done.
- Define power as the product of force and velocity.
- Quote examples of power from everyday life.
- Explain the two types of mechanical energy.
- Understand the work-energy principle.
- Derive an expression for absolute potential energy.
- Define escape velocity.
- Give examples of conservation of energies from everyday life.
- Describe some non-conventional sources of energy.

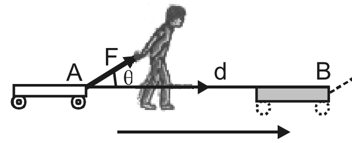
Q.1 Define the term work done. Describe the special cases when the work done is positive, negative and zero.

Ans. **WORK DONE BY A CONSTANT FORCE**

Work done on a body by a constant force is defined as;

"The product of the magnitude of the displacement and the component of the force in the direction of the displacement."

Consider, an object which is being pulled by a constant force F, at an angle θ to the direction of motion. The force moves the object from A to B through a displacement $\vec{d}$, as shown in figure.



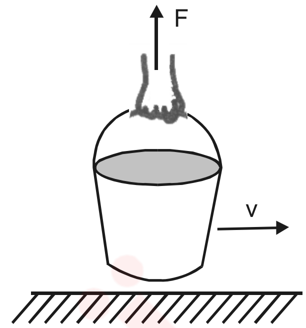
Since $\quad W = Fd$

$$W = (F \cos\theta)\, d$$

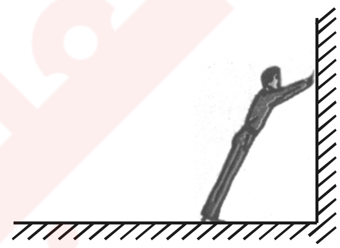
$$W = F d \cos\theta$$

Where $F \cos\theta$ is the component of the force in the direction of $\vec{d}$. So,

$$W = \vec{F} \cdot \vec{d}$$



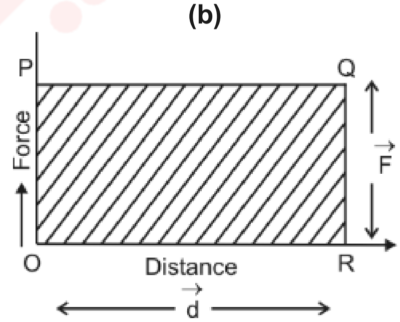
(a)


(b)

So work is also defined as; "The dot product of force $\vec{F}$ and displacement $\vec{d}$." When a constant force acts through a distance $\vec{d}$, the event can be plotted on a simple graph as show in figure. The displacement is plotted along the x-axis and the force along y-axis.

As the force is constant so the graph will be a horizontal straight line. Clearly, the shaded area in figure is also Fd.

Hence area under a force-displacement graph can be taken to represent the work done by the force. If $\vec{F}$ is not in the direction of the displacement the graph is plotted between $F \cos\theta$ and d.



From the definition of work we find the following important results

(i) Work is a scalar quantity.

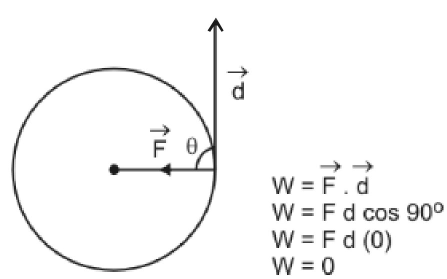
(ii) If $\theta < 90^o$, work is done and it is said to be positive work e.g., work is maximum when F is parallel to the direction of motion of the body i.e., $\theta = 0^o$.

$$W = F d \cos 0^o$$

$$W = Fd \times 1$$

$$W = F d$$

(iii) If $\theta = 90^o$, no work is done



Example

Work done by the centripetal force is 0 and when a person holding a pail by force F is moving forward then $\theta = 90^o$.

$$\because \quad W = F d \cos 90^o$$

$$W = F d\, (0)$$

$$W = 0$$

(iv) If $\theta > 90^o$, the work done is said to be negative.

Example

When a body moves against the force of friction on a horizontal plane i.e., $\theta = 180^o$.

$$\therefore \quad W = F\,d\cos 180^o$$

$$W = F\,d\,(-1)$$

$$W = -F\,d$$

(v) SI unit of work is Nm known as joule (J).

$$\boxed{1J = 1N \times 1m}$$

Joule

The SI unit of work is Joule.

The amount of work done is 1 J when a force of 1N is acting on a body displaces it, through a distance of 1m.

$$\therefore \quad 1J = Nm$$

Dimensions

$$[W] = J$$
$$= Nm$$
$$= kg\,m/s^2 \times m$$
$$= kg\,m^2/s^2 \qquad (\because\ F = ma,\quad N = kg\,m/s^2)$$
$$= [ML^2T^{-2}]$$

Q.2 Explain how can you calculate the work done by a variable force?

Ans. **WORK DONE BY A VARIABLE FORCE**

Variable Force

"If magnitude or direction of force changes during the process of the doing work, is called variable force."

Example

(i) As a rocket moves away from the earth, work is done against the force of gravity, which varies as the inverse square of the distance from

Earth's centre $\left(F = \dfrac{G\,Mm}{r^2}\right)$

(ii) Force exerted by a spring increases with the amount of stretch $(F \propto x)$.

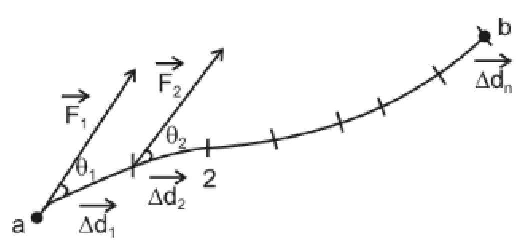


Fig. A particle acted upon by variable force, moves along the path shown from point a to point b.

Consider the path of a particle in the xy-plane as it moves from point a to point b. The path has been divided into n short intervals of displacement $\Delta d_1, \Delta d_2, \ldots\ldots \Delta d_n$ and $F_1, F_2, \ldots\ldots F_n$ are the forces acting during these intervals.

During each small interval, the force is supposed to be approximately constant. So the work done by the first interval can then be written as;

$$\Delta W_1 = F_1 . \Delta d_1 = F_1 \cos\theta_1 \Delta d_1$$

and in the second interval;

$$\Delta W_2 = F_2 . \Delta d_2 = F_2 \cos\theta_2 \Delta d_2$$

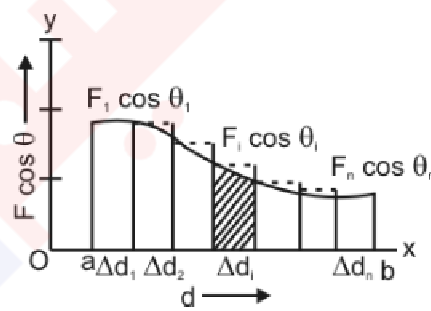
and so on. The total work done in moving the object can be calculated by adding all these terms

$$W_{total} = \Delta W_1 + \Delta W_2 + \ldots\ldots\ldots + \Delta W_n \qquad \ldots\ldots\ldots \text{(i)}$$

$$= F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + \ldots\ldots\ldots + F_n \cos\theta_n d_n$$

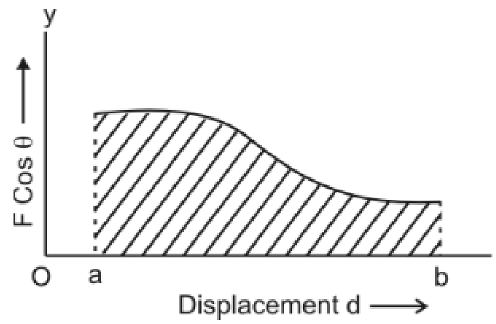
$$W_{total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \qquad \ldots\ldots\ldots \text{(ii)}$$

We can examine this graphically by plotting $F \cos\theta$ verses d as shown in Fig. The displacement d has been subdivided into n equal intervals. The value of $F \cos\theta$ at the beginning of each interval is indicated in the figure by horizontal lines.



Now the ith shaded rectangle has an area $F_i \cos\theta_i \Delta d_i$ which is the work done during the ith interval. Thus, the work done by given equation (ii) equals the sum of the areas of all the rectangles. If we subdivide the distance into a large number of intervals so that each Δd becomes very small, the work done given by equation (ii) becomes more accurate. If we let each Δd to approach zero then we obtain an exact result for the work done, such as

$$W_{total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i$$



In this limit Δd approaches to zero, the total area of the rectangles approaches the area between the $F \cos\theta$ curve and d-axis from a to b as show in figure shaded.

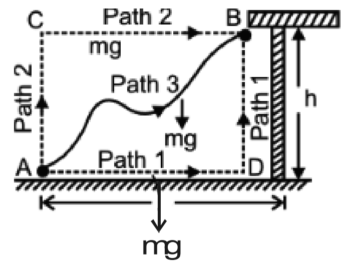
Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos\theta$ verses d curve between the two points a and b as shown in figure.

Q.3 Prove that the work done by gravitational field is independent of the path followed by the body.

Ans. WORK DONE BY GRAVITATIONAL FIELD

"The space around the Earth in which its gravitational force acts on a body, is called the gravitational field."

When an object is moved in the gravitational field, the work is done by the gravitational force. if displacement is in the direction of the gravitational force, the work is positive. If the displacement is against the gravitational force, the work is negative.



Consider an object of mass "m" being displaced with constant velocity from point A to B along various paths in the presence of a gravitational force, as shown in figure.

In this case gravitational force is equal to the weight "mg" of the object.

Work Done Along Path ADB

The work done by the gravitational field along the path ADB can be divided into two steps i.e., the 1st step from A to D, the work done along this path is zero because the weight mg is perpendicular to this path i.e.,

$$\begin{aligned} W_{AD} &= \vec{F} \cdot \vec{d} \\ &= Fd \cos 90^\circ \\ &= mgd\,(0) \\ &= 0 \end{aligned}$$

and the 2nd step is from D to B, the work done along this path is

$$\begin{aligned} W_{DB} &= \vec{F} \cdot \vec{d} \\ &= Fd \cos 180^\circ \\ &= mgh\,(-1) \\ &= -mgh \end{aligned}$$

The total work done along ADB is

$$\begin{aligned} W_{ADB} &= W_{AD} + W_{DB} \\ &= 0 + (-mgh) \\ &= -mgh \qquad \text{...... (i)} \end{aligned}$$

Work Done Along Path ACB

The work done by the gravitational field along the path ACB can be divided into two steps i.e., one from A to C, the work done along AC is

$$\begin{aligned} W_{AC} &= \vec{F} \cdot \vec{d} \\ &= Fd \cos 180^\circ \\ &= mgh\,(-1) \\ &= -mgh \end{aligned}$$

and the second step from C to A the work done along CA is zero because

$$\begin{aligned} W_{CA} &= \vec{F} \cdot \vec{d} \\ &= Fd \cos 90^\circ \\ &= Fd\,(0) \\ &= 0 \end{aligned}$$

Thus the total work done along ACB is

$$W_{ACB} = W_{AC} + W_{CB}$$
$$= -mgh + 0$$
$$W_{ACD} = -mgh \quad \ldots\ldots \text{(ii)}$$

Work Done Along the Curved Path

Imagine the curved path, to be divided into a series of horizontal and vertical steps as shown in figure. There is no work done along the horizontal steps, because mg is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacement.

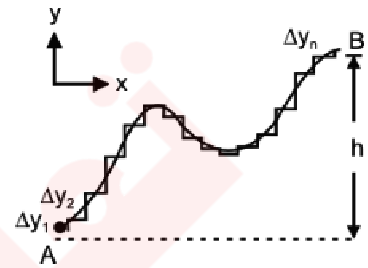
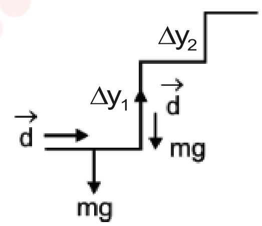


Fig. A smooth path may be replaced by a series of infinitesimal x and y displacements. Work is done only during they displacements.

$$W_1 = \vec{F} \cdot \vec{d}$$
$$= mg\,\Delta y_1 \cos 180^o$$
$$= -mg\,\Delta y_1$$

Similarly

$$W_2 = -mg\,\Delta y_2$$
$$\vdots \qquad \vdots \quad \vdots$$
$$\vdots \qquad \vdots \quad \vdots$$
$$W_n = -mg\,\Delta y_n$$
$$W_{AB} = W_1 + W_2 + \ldots\ldots + W_n$$
$$= -\,mg\,\Delta y_1 - mg\,\Delta y_2 \ldots\ldots mg\,\Delta y_n$$
$$= -\,mg\,(\Delta y_1 + \Delta y_2 + \ldots\ldots + \Delta y_n)$$



As $\quad \Delta y_1 + \Delta y_2 + \ldots\ldots + \Delta y_n = h$

$\therefore \quad W_{AB} = -mg\,h \quad \ldots\ldots$ (iii)

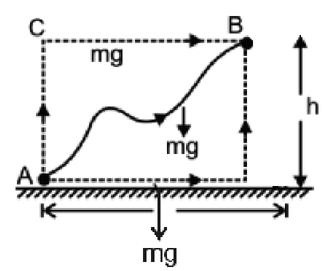
We conclude that

From eq. (i), (ii) and (iii)

"Work done in the Earth's gravitational field is independent of the path followed by the body".

Q.4 Prove that work done along the closed path in a gravitational field is zero.

Ans. Consider a closed path ABCA in a gravitational field. In order to calculate the work done along a closed path, we proceed as.



The total work done along this path is

$$W_{total} = W_{A\rightarrow B} + W_{B\rightarrow C} + W_{C\rightarrow A}$$

Since $\quad W_{A\rightarrow B} = -mgh$

and $\quad W_{B\rightarrow C} = \vec{F} \cdot \vec{d}$

$$= Fd\cos 90°$$
$$= Fd\,(0)$$
$$= 0$$

and $W_{C\to A} = \vec{F} \cdot \vec{d}$

$= Fd \cos 0°$

$= mgh\,(0)$

$= mgh$

Thus $W_{total} = -mgh + 0 + mgh$

$= 0$

Thus the work done along a closed path is zero.

Conservative Field

"The field in which the work done is independent of the path followed and work done in a closed path be zero, is called a conservative field."

For Example:

Gravitational field and electrostatic field.

Note: *The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.*

Q.5 Define power and give the values of average power and instantaneous power. Also give the unit of power.

Ans. **POWER**

"The rate of doing work is known as power." **(OR)** Power is the measure of the rate at which work is being done. If work ΔW is done in the time interval Δt then average power P_{ave} during the interval Δt is defined as

$$P_{ave} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power

If work is expressed as a function of time, the instantaneous power P at any instant is defined as:

$$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Average power is equal to instantaneous power if work is done at constant rate.

Power and Velocity

To Show that $P = \vec{F} \cdot \vec{V}$.

Proof: When the propeller of a motor boat causes the water to

For Your Information

Conservative Forces

Gravitational force

Elastic spring force

Electric force

Non-Conservative Forces

Frictional force

Air resistance

Tension in a string

Normal force

Propulsion force of a rocket

Propulsion force of a motor

For Your Information

Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	1.3×10^8
Car at 90 km h^{-1}	1.1×10^5
Electric heater	2×10^3

Colour TV	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

exert, a constant force $\vec{F}$ on the boat, it moves with constant velocity $\vec{V}$. The power delivered by the motor at any instant is given by;

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Since, $$\Delta W = \vec{F} \cdot \Delta \vec{d}$$

$$\therefore \quad P = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \vec{F} \cdot \frac{\Delta \vec{d}}{\Delta t}$$

$$= \vec{F} \cdot \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} \quad \left(\lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} = \vec{V} \right)$$

$$P = \vec{F} \cdot \vec{V}$$

Hence power is also defined as: "The dot product of force and velocity."

Power is a scalar quantity.

Unit

SI unit of power is watt.

Watt

"Rate of doing 1J of work in one second is called watt."

$$1\ \text{W} = 1\ \text{J}/1\ \text{sec}$$

Do You Know?

It takes about 9×10^9 J to make a car and the car then uses about 1×10^{12} J of energy from petrol in its lifetime

Dimensions

$$P = \text{Watt}$$

As $$P = \text{J/s}$$

$$[P] = \frac{\text{Nm}}{\text{Sec}} \quad (\because\ 1\text{J} = \text{Nm})$$

$$= \frac{\text{kg m}/\text{s}^2 \times \text{m}}{\text{s}}$$

$$= \text{kg m}^2/\text{s}^3$$

$$= [ML^2/T^3]$$

$$= [ML^2 T^{-3}]$$

In electrical measurements, the unit of work is watt x second

$$\left(1\text{W} = \frac{1\text{J}}{1\text{S}} \text{ and } 1\text{W} \times 1\text{S} = 1\text{J} \right).$$

Commercial unit of electrical energy is kilowatt hour (k w h).

Kilowatt Hour

One kilowatt hour is the work done in one hour by an agency whose power is one kilowatt.

$$\therefore \quad 1\ \text{k w h} = 1000\ \text{W} \times 3600\ \text{sec}$$
$$= 36 \times 10^5\ \text{W sec}$$
$$= 36 \times 10^5\ \text{J} \qquad (\because\ \text{W sec} = \text{J})$$
$$= 3.6 \times 10^6\ \text{J}$$
$$= 3.6\ \text{M J}$$
$$1\ \text{h p} = 746\ \text{watt}$$

Q.6 Define energy. What do you mean by K.E and P.E with its formula?

Ans. **ENERGY**

"Energy of a body is its capacity to do work."

For example; mechanical, heat, sound energies etc.

There are two basic forms of mechanical energy

1. Kinetic energy
2. Potential energy

Kinetic Energy

The energy possessed by a body due to its motion is called kinetic energy.

Mathematically:

$$\text{K.E.} = \frac{1}{2} m v^2$$

Potential Energy

The energy possessed by a body due to its position is called potential energy. There are two types of potential energy:

(i) Gravitational potential energy.

(ii) Elastic potential energy.

Gravitational Potential Energy

The energy possessed by a body because of its position in a force field. e.g. gravitational field. The potential energy due to gravitational field near the surface of the Earth at a height h is given by the formula.

$$\text{P.E.} = mgh$$

This is called gravitational potential energy. The gravitational P.E. is always determined relative to some

For Your Information

Approximate Energy Values Source

arbitrary position which is assigned the value of zero P.E. In the present case, this reference level is the surface of the Earth as position of zero P.E. In some cases a point at infinity from the Earth can also be chosen as zero reference level.

Source	Energy (J)
Burning 1 ton coal	30×10^9
Burning 1 litre petrol	5×10^7
K.E. of car at 90 km h^{-1}	1×10^6
Running person at 10 km h^{-1}	3×10^2
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

Elastic Potential Energy

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Q.7 Derive work energy relation or principle.

Ans. **WORK-ENERGY PRINCIPLE**

This principle states that, work done on the body equals change in its K.E.

Consider a body of mass "m" moving with velocity "V_i". A force F acting through a distance d increases the velocity to V_f, then by using third equation of motion.

$$2ad = V_f^2 - V_i^2$$

$$d = \frac{1}{2a}\left(V_f^2 - V_i^2\right) \quad \ldots\ldots\ldots (1)$$

Tid-bits

[illegible]

As $F = m\,a \quad \ldots\ldots\ldots (2)$

Multiply equation (1) with equation (2)

$$Fd = \frac{m\,a}{2a}\left(V_f^2 - V_i^2\right)$$

$$W = \frac{1}{2}m\left(V_f^2 - V_i^2\right) \qquad (\because Fd = W)$$

$$= \frac{1}{2}m\,V_f^2 - \frac{1}{2}m\,V_i^2$$

$$= K.E_f - K.E_i$$

$$\therefore \quad W = \text{Change in K.E.}$$

This is known as work–energy principle.

Note: *If a body is raised up from the Earth's surface, the work done changes the gravitational potential energy and if a spring is compressed, the work done on it equals the increase in its elastic potential energy.*

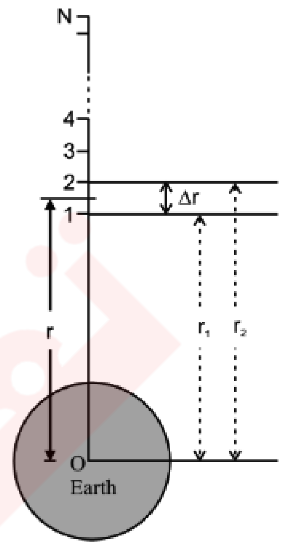
Q.8 Define absolute potential energy. Calculate the value of absolute potential in the gravitational field.

Ans. **ABSOLUTE POTENTIAL ENERGY**

"The work done by gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero" is called absolute P.E.

Work done by the gravitational force is true only near the surface of the earth where the gravitational force is nearly constant. But if the body is displaced through a large distance from point 1 to N as shown in figure. Then the gravitational force will not remain same, since it varies inversely to the square of the distance $\left(F_g = \frac{G M m}{r^2}\right)\left(\therefore \; F_g \propto \frac{1}{r^2}\right)$.

Thus we divide the distance between point 1 and N into small steps each of length Δr so that the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps.



If r_1 and r_2 are distances of points 1 and 2 respectively from the centre O of the Earth as shown in Fig.

The distance, between the centre of 1st step and the centre of Earth will be

$$r = \frac{r_1 + r_2}{2} \;\ldots\ldots (1) \text{ (average distance)}$$

As $\Delta r = r_2 - r_1$

$$r_2 = r_1 + \Delta r$$

Putting this value of r_2 in equation (1)

$$\therefore \quad r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

$$r = \frac{2r_1}{2} + \frac{\Delta r}{2}$$

$$\therefore \quad r = r_1 + \frac{\Delta r}{2}$$

Squaring both sides

$$r^2 = r_1^2 + \left(\frac{\Delta r}{2}\right)^2 + 2r_1 \frac{\Delta r}{2}$$

$$r^2 = r_1^2 + \frac{(\Delta r)^2}{4} + r_1 \Delta r$$

Since Δr is very small so $\dfrac{\Delta r^2}{4}$ is neglected.

$$\therefore \quad r^2 = r_1^2 + r_1\,\Delta r$$

Putting value of Δr

$$\therefore \quad r^2 = r_1^2 + r_1\,(r_2 - r_1)$$

$$\therefore \quad r^2 = r_1^2 + r_1\,r_2 - r_1^2$$

$$r^2 = r_1\,r_2$$

The gravitational force F at the centre of this step is

$$F = \frac{G\,Mm}{r^2}$$

Putting value of r^2

$$\therefore \quad F = \frac{G\,Mm}{r_1\,r_2}$$

As this force is assumed to be constant during the interval Δr, so the work done is

$$W_{1\to 2} = \vec{F}\,.\,\Delta\vec{r}$$

$$= F\,\Delta r \cos 180^\circ$$

$$= F\,\Delta r\,(-1)$$

$$= -F\,\Delta r$$

Putting the value of F and Δr

$$\therefore \quad W_{1\to 2} = -\frac{G\,Mm}{r_1\,r_2}(r_2 - r_1)$$

$$= -G\,Mm\left(\frac{r_2}{r_1\,r_2} - \frac{r_1}{r_1\,r_2}\right)$$

$$= -G\,Mm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly,

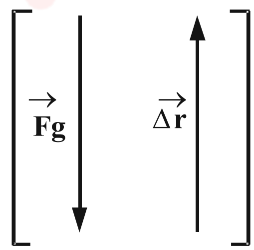
$$W_{2\to 3} = -G\,Mm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$W_{N-1\to N} = -G\,Mm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Do You Know?

There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.



Tid-bits

[illegible]

Hence, $W_{total} = W_{1\to 2} + W_{2\to 3} + \ldots\ldots\ldots + W_{N-1\to N}$

$$\therefore \quad W_{total} = -G\,Mm\left(\frac{1}{r_1}-\frac{1}{r_2}\right) - G\,Mm\left(\frac{1}{r_2}-\frac{1}{r_3}\right)\ldots\ldots\ldots - G\,Mm\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)$$

$$W_{total} = -G\,Mm\left[\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\ldots\ldots\ldots+\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)\right]$$

$$= -G\,Mm\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{1}{r_2}-\frac{1}{r_3}+\ldots\ldots\ldots+\frac{1}{r_{N-1}}-\frac{1}{r_N}\right]$$

$$= -G\,Mm\left(\frac{1}{r_1}-\frac{1}{r_N}\right)$$

If the point N is situated at an infinity distance from the centre of the earth then

$$r_N = \infty$$

$$\frac{1}{r_N} = \frac{1}{\infty} = 0$$

$$\therefore \quad W_{total} = -G\,Mm\left(\frac{1}{r_1}-0\right)$$

$$W_{total} = -\frac{G\,Mm}{r_1}$$

Therefore, gravitational potential energy of a body at a distance 'r' from the center of Earth is;

$$U = -\frac{G\,Mm}{r}$$

This is also known as the absolute value of potential energy.

(i) When r increases, the gravitational force does negative work and U increases i.e., becomes less negative.

(ii) When r decreases, the body falls towards the Earth, the work, is positive and potential energy decreases i.e., becomes more negative.

(iii) The absolute P.E. on the surface of Earth is found by putting r = R (Radius of Earth).

$$\therefore \quad U = -\frac{G\,Mm}{R}$$

Negative sign shows that Earth's gravitational field for mass 'm' is attractive.

Q.9 Define escape velocity. Also derive the relation for the escape velocity.

Ans. **ESCAPE VELOCITY**

"The minimum initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity."

The escape velocity corresponds to initial K.E. gained by the body which carries it to an infinite distance, from the surface of Earth.

$$\therefore \quad \text{Initial K.E.} = \frac{1}{2}\,m\,V_{esc}^2$$

We know that work done in lifting a body from Earth's surface to an infinite distance is equal to increase (change) in its potential energy. i.e.,

$$\therefore \quad \text{Increase in P.E.} = 0 - \left(-\frac{G\,Mm}{R}\right)$$

$$= G\frac{Mm}{R}$$

The body will escape out of the gravitational field if the initial K.E. of the body is equal to absolute P.E.

$$\frac{1}{2} m V_{esc}^2 = \frac{G\,Mm}{R}$$

$$V_{esc}^2 = \frac{2\,G\,M}{R}$$

Taking square root

$$V_{esc} = \sqrt{\frac{2\,G\,M}{R}} \quad \ldots\ldots\ldots (1)$$

As mass of earth is

$$M = \frac{g\,R^2}{G}$$

$$\therefore \quad G\,M = g\,R^2$$

Putting this value in equation (1)

$$\therefore \quad V_{esc} = \sqrt{\frac{2\,g\,R^2}{R}}$$

$$V_{esc} = \sqrt{2\,g\,R}$$

As $R = 6.4 \times 10^6$ m

$g = 9.8\ \text{m/s}^2$

$$\therefore \quad V_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$V_{esc} = 11.2 \times 10^3\ \text{m/s}$$

or $V_{esc} = 11.2$ km / s

The value of V_{esc} comes at to be approximately 11 km s^{-1}.

Note: *Escape velocity does not depend on mass of the object.*

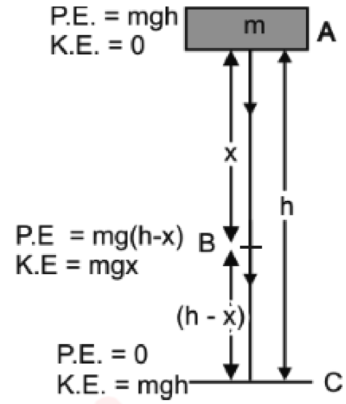
For Your Information	
Some Escape Speeds (kms^{-1})	
Heavenly body	**Escape speed**
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Naptuna	25.4
Satum	37.0
Jupiter	61

Q.10 Explain the phenomena inter conversion of potential energy and kinetic energy.

Ans. **INTER CONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY**

Consider a body of mass 'm' at rest, at a height 'h' above the surface of the Earth as shown in figure. At the position A, the body has P.E. = mg h with respect to the earth and K.E. = 0. We release the body and as it falls, we can examine how K.E. and P.E. associated with it interchange.

Let calculate P.E. and K.E. at the position B when the body has fallen a distance x, ignoring air friction.



P.E. = mg (h – x)

$$\text{K.E.} = \frac{1}{2} m V_B^2$$

V_B can be calculated by

Using $V_f^2 = V_i^2 + 2 g S$ ……… (1)

Here $V_f = V_B$

$V_i = 0$

$S = x$

Putting these values in equation (1)

$\therefore$ $V_B^2 = 0 + 2 g x$

$= 2 g x$

$\therefore$ K.E. at point B is

$$\therefore \quad \text{K.E.} = \frac{1}{2} m V_B^2$$

$$\text{K.E.} = \frac{1}{2} m (2 g x)$$

$= mg x$

Total energy at B = P.E. + K.E.

= mg (h – x) + mg x

= mg h – mg x + mg x

= mgh

At position 'C', just before the body strikes the Earth, P.E. = 0 and K.E. = $\frac{1}{2} m V_C^2$ where V_C can be found out by the following expression.

$$V_C^2 = V_i^2 + 2 g h = 2 g h$$

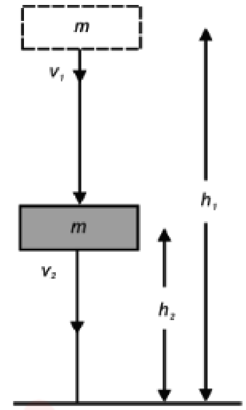
As $V_i = 0$

$$\text{i.e.,} \quad \text{K.E.} = \frac{1}{2} m V_C^2$$

$$= \frac{1}{2} m \times 2 g h$$

$= mg h$

Thus at point C, K.E. is equal to the original value of the P.E. of the body. Actually when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its K.E. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus, from figure



$$\text{Loss in P.E.} = \text{Gain in K.E.}$$

$$mg\,(h_1 - h_2) = \frac{1}{2}\,m\left(V_2^2 - V_1^2\right)$$

Where V_1 and V_2 are velocities of the body at the height h_1 and h_2 respectively. This result is truly only when frictional force is not considered.

If we assume that a frictional force 'f' is present during the downward motion, then a part of P.E. is used in doing work against friction equal to $f\,h$. The remaining P.E. $= mg\,h - f\,h$ is converted into K.E.

Hence, $$mg\,h - f\,h = \frac{1}{2}\,m\,V^2$$

or $$mg\,h = \frac{1}{2}\,m\,V^2 + f\,h$$

Thus, Loss in P.E. = Gain in K.E. + Work done against friction.

CONSERVATION OF ENERGY

The kinetic and potential energies are both different forms of the same basic quantity, i.e., mechanical energy. Total mechanical energy of a body is the sum of the kinetic energy and potential energy. In our previous discussion of a falling body, potential energy may change into kinetic energy and vice versa, but the total energy remains constant. Mathematically,

$$\text{Total energy} = \text{P.E.} + \text{K.E.} = \text{Constant}$$

This is a special case of the law of conservation of energy which states that:

"Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant".

This is one of the basic laws of physics. We daily observe many energy transformations from one form to another. Some forms, such as electrical and chemical energy, are more easily transferred than others, such as heat. Ultimately all energy transfers result in heating of the environment and energy is wasted. For example, the P.E. of the falling object changes to K.E., but on striking the ground, the K.E. changes into heat and sound. If it seems is an energy transfer that some energy has disappeared, the lost energy is often converted into heat. This appears to be the fate of all available energies and is one reason why new sources of useful energy have to be developed.

For Your Information

Source of energy	Original source
Solar	Sun
Bio mass	Sun
Fossil fuels	Sun
Wind	Sun
Waves	Sun
Hydro electric	Sun
Tides	Moon
Geothermal	Earth

Energy Sources

Renewable	Non-renewable
Hydroelectric	Coal
Wind	Natural gas
Tides	Oil
Geothermal*	Uranium
Biomass	Oil shale
Sunlight	Tar sands
Ethanol/Methanol**	

* Individual fields may run off

** Renewable when made from bio mass

Q.11 Describe briefly the non-conventional energy sources.

Ans. NON-CONVENTIONAL ENERGY SOURCES

These are the energy sources not very common these days. However, it is expected, that these sources will contribute substantially to the world energy demand of the future. Some of these are introduced briefly here.

Energy From Tides

One very novel example of obtaining energy from gravitational field is the energy obtained from tides. Gravitational force of the moon gives rise to tides in the sea. The tides raise the water in the sea roughly twice a day. If the water at the high tide is trapped in a basin by constructing a dam, then it is possible to use this as a source of energy. The dam is filled at high tide and water is released in a controlled way at low tide to drive the turbines. At the next high tide the dam is filled again and the in rushing water also drives turbines and generates electricity as shown systematically in the figure.

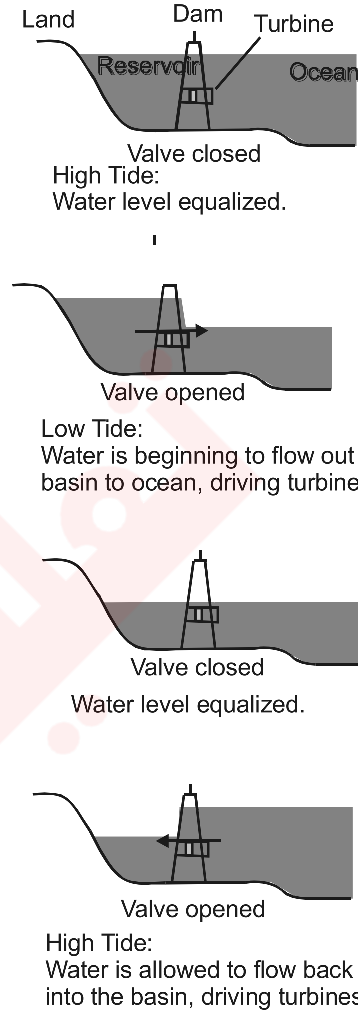
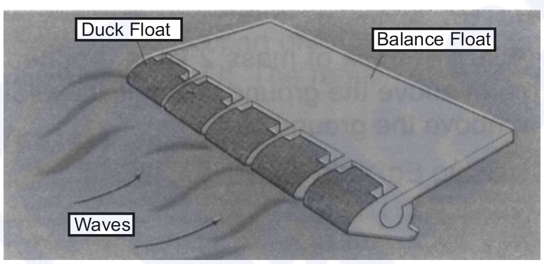


Fig. Tidal power plant. Turbine are located inside the dam.

Energy From Waves

The tidal movement and the winds blowing across the surface of the ocean produce strong water waves. Their energy can be utilized to generate electricity. A method of harnessing wave energy is to use large floats which move up and down with the waves. One such device invented by Professor Salter is known Salter's duck (figure). It consists of two parts (i) Duck float (ii) Balance float.



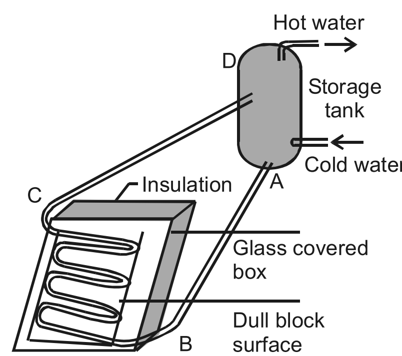
The wave energy makes duck float move relative to the balance float. The relative motion of the duck float is then used to run electricity generators.

Do You Know?

The pull of the Moon does not only pull the sea up and down. This tidal effect can also distort the continents pulling land up and down by as much as 25 cm.

Solar Energy

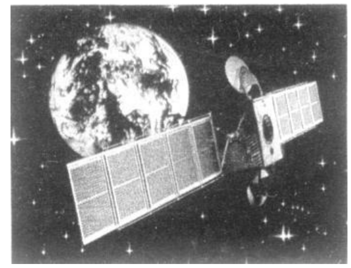
The Earth receives huge amount of energy directly from the Sun each day. Solar energy at normal incidence outside the Earth's atmosphere is about 1.4 kWm^{-2} which is referred as solar constant. While passing through the atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the intensity of the solar energy reaching the Earth's surface is about $1kWm^{-2}$. This energy can be used directly to heat water using large solar reflectors and thermal absorbers or be converted to electricity. In one method the flat plate collectors are used for heating water. A typical collector is shown in figure (a). It has a blackened surface which absorbs energy directly from solar radiation. Cold water passes over the surface and is heated upto about 70°C.



(a)

Much higher temperature can be achieved by concentrating solar radiation on to a small surface area by using huge reflectors (mirrors) or lenses to produced steam for running a turbine.

The other method is the direct conversion of sunlight into electricity through the use of semi conductor devices called solar cells also known as photo voltaic cells. Solar cells are thin wafers made from silicon. Electrons in the silicon gain energy from sunlight to create a voltage. The voltage produced by a single voltaic cell is very low. In order to get sufficient high voltage for practical use, a large number of such cells are connected in series forming a solar cell panel.



(b)

For cloudy days or nights, electric energy can be stored during the Sun light in Nickle cadmium batteries by connecting them to solar panels. These batteries can then provide power to electrical appliances at nights or on cloudy days.

Solar cells, although, are expensive but last a long time and have low running cost. Solar cells are used to power satellites having large solar panels which are kept facing the Sun figure (b). Other examples of the use of solar cells are remote ground based weather stations and rain forecast communication systems. Solar calculators are also in use now a days.

Q.12 Explain the energy from biomass.

Ans. **ENERGY FROM BIOMASS**

Biomass is a potential source of renewable energy. This includes all the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy or bio conversion refers to the use of this material as fuel or its conversion into fuels.

There are many methods used for the conversion of biomass into fuels. But the most common are:

(1) Direct combustion.

(2) Fermentation.

Direct combustion method is usually applied to get energy from waste products commonly known as solid waste. It will be discussed in the next section.

Biofuel such as ethanol (alcohol) is a replacement of gasoline. It is obtained by fermentation of biomass using enzymes and by decomposition through bacterial action in the absence of air (oxygen).

The rotting of biomass in a closed tank called a digester produces Biogas which can be piped out to use for cooking and heating (figure).

The waste material of the process is a good organic fertilizer. Thus, production of biogas provides us energy source and also solves the problem of organic waste disposal.

For Your Information

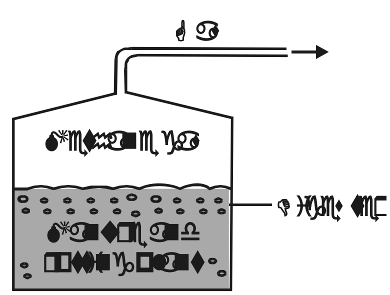
The rapid growth of human population has put a strain on our natural resources. A sustainable society minimizes waste and maximizes the benefit from each resource. Minimizing the use of energy is an other method of conservation we can save energy by,

(i) turning off lights and electrical appliances when not in use

(ii) using fluorescent bulbs instead of incandescent bulbs

(iii) using sunlight in offices, commercial centers and houses during daylight hours

(iv) taking short hot shawers.



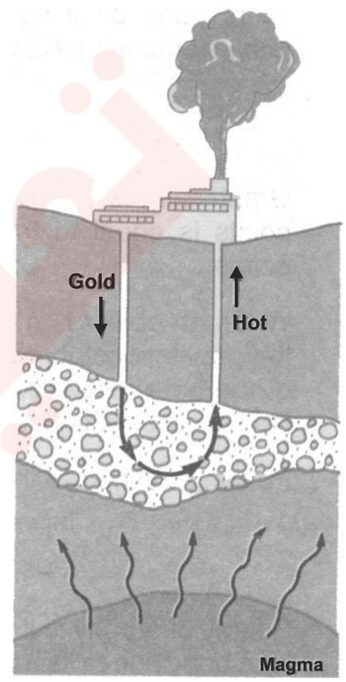
Energy from Waste Products

Waste products like wood waste, crop residue, and particularly municipal solid waste can be used to get energy by direct conversion. It is probably the most commonly used conversion process in which waste material is burnt in a confined container. Heat produced in this way is directly utilized in the boiler to produce steam that can run turbine generator.

Q.13 Explain geothermal energy.

Ans. GEOTHERMAL ENERGY

This is the heat energy extracted from inside the Earth in the form of hot water or steam. Heat within the Earth is generated by the following processes.



1. Radioactive Decay

The energy, heating the rocks, is constantly being released by the decay of radioactive elements.

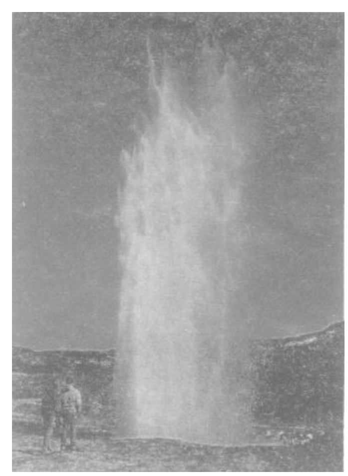
2. Residual Heat of the Earth

At some places hot igneous rocks, usually within 10 km of the Earth's surface, are in a molten and partly molten state. They conduct heat energy from the Earth's interior which is still very hot. The temperature of these rocks is about 200°C or more.

3. Compression of Material

The compression of material deep inside the Earth also causes generation of heat energy.

In some place water beneath the ground is in contact with hot rocks and is raised to high temperature and pressure. It comes to the surface as hot springs, geysers, or steam vents. The steam can be directed to turn turbines of electric generators.



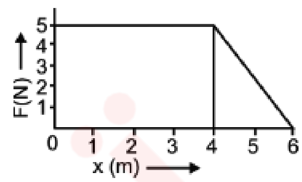
Where water is not present and hot rocks are not very deep, the water is pumped down through them which returns as steam (figure). The steam then can be used to drive turbines or for direct heating.

An interesting phenomenon of geothermal energy is a geyser. It is a hot spring that discharges steam and hot water, intermittently releasing an explosive column into the air (figure). Most geysers erupt at irregular intervals. They usually occur in volcanic regions. Extraction of geothermal heat energy often occurs closer to geyser sights. This extraction seriously disturbs geyser system by reducing heat flow and aquifer pressure. Aquifer is a layer of rock holding water that allows water to percolate through it with pressure.

SOLVED EXAMPLES

EXAMPLE 4.1

A force F acting on an object varies with distance x as shown in the figure. Calculate the work done by the force as the object moves from x = 0 to x = 6 m.



Data

Force = F

Distance = x

To Find

Work done = W = ?

SOLUTION

[Total area under the graph] = (Area of rectangle) + (Area of triangle)

$$= (4 \times 5) + \frac{1}{2}(6-4)(5)$$

$$= 20 + \frac{1}{2}(2)(5)$$

$$= 20 + 5 = 25 \text{ J}$$

Since area under F–d graph is equal to work done

Hence, W = 25 J

Result

Work done by the force = W = 25 J

EXAMPLE 4.2

A 70 kg man runs up a long flight of stairs in 4.05 s. The vertical height to the stairs is 4.5m calculate his power out put in watts.

Data

Mass of man = m = 70 kg

= t = 4 sec

Height of stairs = h = 4.5 m

To Find

Power output = P = ?

SOLUTION

Using

$$P = \frac{W}{t}$$

Since $W = P.E. = mgh$

$$\therefore \quad P = \frac{mgh}{t}$$

$$= \frac{70 \times 9.8 \times 4.5}{4}$$

$$= 7.7 \times 10^2 \text{ watt}$$

Result

Power output $= P = 7.7 \times 10^2$ watt

EXAMPLE 4.3

A brick of mass 2.0 kg is dropped from a rest position 5.0 m above the ground. What is its velocity at a height of 3.0 m above the ground?

Data

Mass of brick $= m = 2$ kg

rest position $= h_1 = 5$ m

At a height $= h_2 = 3$ m

To Find

Velocity $= V = ?$

SOLUTION

Using Loss of P.E. = Gain in K.E.

$$mg(h_1 - h_2) = \frac{1}{2} m (V_2^2 - V_1^2)$$

$$\therefore \quad V_1 = 0 \quad \text{and} \quad V_2 = V$$

$$\therefore \quad mg(h_1 - h_2) = \frac{1}{2} m (V^2 - 0^2)$$

$$g(h_1 - h_2) = \frac{1}{2} V^2$$

$$V^2 = 2g(h_1 - h_2)$$

Taking square root

$$V = \sqrt{2\, g\, (h_1 - h_2)}$$

Putting values

$$V = \sqrt{2 \times 9.8\, (5 - 3)}$$

$$= \sqrt{2 \times 9.8 \times 2}$$

$$= \sqrt{39.2}$$

$$V = 6.26 \text{ m/s}$$

$$= 6.3 \text{ m/s}$$

Result

Velocity at a height 3.0 m $= V = 6.3$ m/s

Chapter 5

CIRCULAR MOTION

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Describe angular motion.
- Define angular displacement, angular velocity and angular acceleration.
- Define radian and convert an angle from radian measure to degree and vice versa.
- Derive the equation $a_c = r\omega^2 = v^2/r$ and $F_c = m\omega^2\ r = mv^2/r$.
- Understand and describe moment of inertia of a body.
- Understand the concept of angular momentum.
- Describe examples of conservation of angular momentum.
- Describe the motion of artificial satellites.
- Understand that the objects in satellites appear to be weightless.
- Understand that how and why artificial gravity is produced.
- Calculate the radius of geo-stationary orbits and orbital velocity of satellites.
- Describe Newton's and Einstein's views of gravitation.

Q.1 Define circular motion.

Ans. **CIRCULAR MOTION**

When a body moves in such a way that its distance from the fixed point remains constant then such a motion is called circular motion.

For Example:

(i) Motion of earth around the sun.

(ii) Motion of moon around the earth.

(iii) Motion of electron around the nucleus.

Q.2 Explain angular displacement with its units.

Ans. **ANGULAR DISPLACEMENT**

The angle through which a particle moves in an interval of time while moving along a circle is called angular displacement.

Consider the motion of a single particle P of mass m in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of a mass less rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. (i). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. (ii). The z–axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis Fig. (iii).

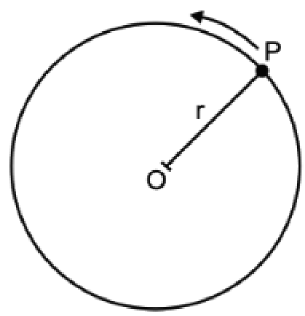


Fig. (i)

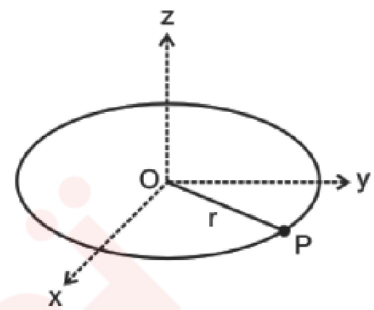


Fig. (ii)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt.

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clockwise.

For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

The direction associated with $\Delta\theta$ is along the axis rotation and is given by right hand rule which states that

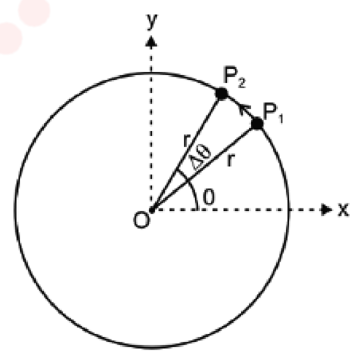


Fig. (iii)

Right Hand Rule

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement, as shown in Fig. (iv).

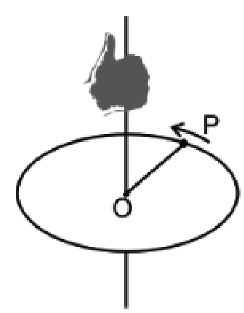


Fig. (iv)

Unit

There are three units of angular displacement

(i) degree

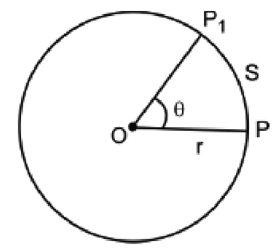
(ii) revolution

(iii) radian

But radian is SI unit. The angular displacement is one radian if the angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.

Q.3 Show that $S = r\theta$.

Ans. Consider an arc of length S of a circle of radius r which subtends an angle θ at the centre of the circle. Its value in radian is given by



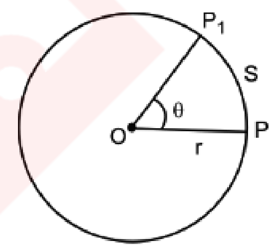
$$\theta = \frac{\text{Arc length}}{\text{Radius}} \text{ (rad)}$$

$$\theta = \frac{S}{r}$$

$$S = r\theta$$

Q.4 Show that 1 radian = 57.3°.

Ans. If 'OP' is rotating, the point 'P' covers a distance S = 2πr in one revolution of P. In radian, it is



$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

$$1 \text{ revolution} = 2\pi \text{ rad.} = 360°$$

$$1 \text{ rad.} = \frac{360°}{2\pi}$$

$$1 \text{ rad.} = 57.3°$$

$$\therefore \quad 2\pi \text{ radian} = 360^o$$

$$1 \text{ radian} = \frac{360^o}{2\pi}$$

$$= \frac{180^o}{3.14}$$

$$1 \text{ radian} = 57.3^o$$

Q.5 Define angular velocity and instantaneous angular velocity with its units.

Ans. **ANGULAR VELOCITY**

"The rate of change of angular displacement is called angular velocity". **(OR)** The angular velocity is also defined as the rate at which the angular displacement is changing with time. It is denoted by ω. If Δθ is the angular displacement during the time interval Δt, the average angular velocity during the interval is

$$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$$

It is a vector quantity. Its direction is along the axis of rotation and given by right hand rule.

Dimensions

$$[\omega] = [T^{-1}]$$

Instantaneous Angular Velocity

The instantaneous angular velocity ω is the limit of the ratio $\frac{\Delta\theta}{\Delta t}$ as Δt following instant t, approaches to zero. Thus

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Units

Angular velocity measured in radians per second which is SI units. Sometime it is also measures in terms of **revolution per minute**.

Q.6 Define angular acceleration and instantaneous angular acceleration with its units.

Ans. **ANGULAR ACCELERATION**

The rate of change of angular velocity is called angular acceleration.

It is denoted by α.

It is a vector quantity. Its direction is along axis of rotation and given by right hand rule. If ω_i and ω_f are the value of instantaneous velocity of a rotating body at time t_i and t_f then, the average angular acceleration during the intervals $t_f - t_i$ is given by

$$\alpha_{ave} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$\therefore \quad \alpha = \frac{\Delta\omega}{\Delta t}$$

Unit

Its SI unit is radian per sec^2.

Dimensions

$$[\alpha] = [T^{-2}]$$

Instantaneous Angular Acceleration

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches to zero then mathematically

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

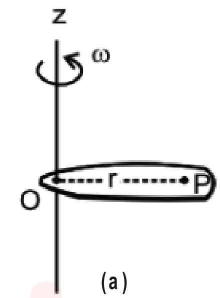
Explanation

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now we consider the rotation of a rigid body as shown in figure. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation. It is usually referred as reference line. As the body rotates, line OP also rotates with it with the same angular velocity and angular acceleration. Thus the rotation of a rigid body can be described by the rotation of the reference line OP and all the terms that we defined with the help of rotating line OP are also valid for the rotational motion of a rigid body. In future while dealing with rotation of rigid body, we will replace it by its reference line OP.

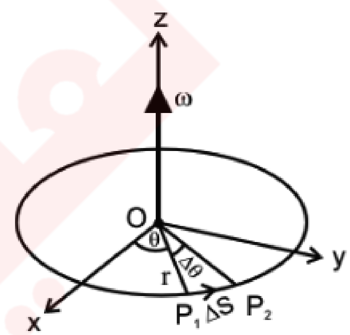
Q.7 Show that $V = r\omega$ (relation between linear and angular velocity).

Ans. Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. (a).

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity v whereas the line OP rotates with angular velocity ω as shown in Fig. (b). We are interested in finding out the relation between ω and v. As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, we consider its magnitude only which can also be treated as a scalar.



(a)



(b)

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP has an angular displacement $\Delta\theta$ radian during this interval. ΔS and $\Delta\theta$ are related by equation.

$$\Delta S = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \qquad \text{...... (1)}$$

In the limit when $\Delta t \to 0$ the ratio $\frac{\Delta S}{\Delta t}$ represents v, the magnitude of the velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So Eq. (1) becomes

$$v = r\omega \qquad \text{...... (2)}$$

In Fig. (b), it can be seen that the point P is moving along the arc P_1P_2. In the limit when $\Delta t \to 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why the linear velocity of the point P is also known as tangential velocity.

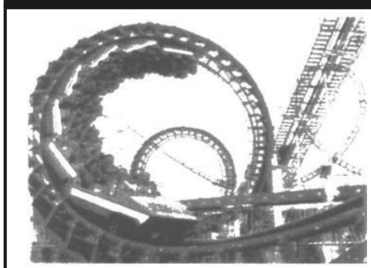
Q.8 Show that $a_t = r\alpha$ (relation between linear and angular acceleration).

Ans. If the reference line OP is rotating with an angular acceleration α, the point P will also have a linear or tangential acceleration a_t. Using Eq. (1) it can be shown that the two accelerations are related by

$$a_t = r\alpha \qquad \text{...... (1)}$$

Eqs. (1) and (2) show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body displacement, angular speed and angular acceleration at any instant. Thus by the use of angular variables we can describe the motion of the entire body in a simple way.

Point to Ponder



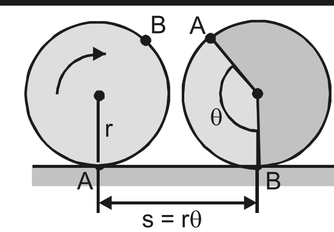
You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

Equations of Angular Motion

The equation of angular motion are exactly analogous to those in linear motion except that θ, ω and α have replaced S, v and a, respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear	Angular	
$v_f = v_i + at$	$\omega_f = \omega_1 + \alpha t$	…… (1)
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$	…… (2)
$S = v_i t + \frac{1}{2} at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	…… (3)

Do You Know?



As the wheel turns through an angle, it lays out a tangential distance S = rθ.

The angular equations (1) to (3) hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence they can be manipulated as scalars.

Q.9 What is centripetal force? Also derive the expression for centripetal force.

Ans. CENTRIPETAL FORCE

The force needed to bend the normally straight path of the particle into a circular path is called the centripetal force.

It is denoted by F_C and mathematically

$$F_C = \frac{m V^2}{r}$$

Explanation

If the particle moves from A to B with uniform speed V as shown in Fig. (i). The velocity of the particle changes its direction but not its magnitude. The change in velocity is shown in Fig. (ii). Hence acceleration of the particle is

$$a = \frac{\Delta V}{\Delta t} \quad \text{……… (1)}$$

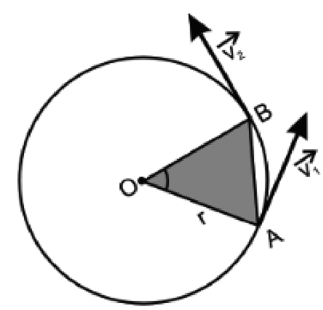


Fig. (i)

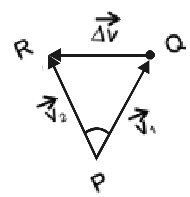


Fig. (ii)

Where Δt is the time taken by the particle to travel from A to B. Let the velocities at A and B are $\vec{V}_1$ and $\vec{V}_2$. Since speed of the particle is V, so the time taken to travel a distance S, as shown in Fig. (i) is

$$\Delta t = \frac{S}{V} \qquad (\because\ S = V\,t)$$

Put in equation (1)

So
$$a = V\frac{\Delta V}{S} \quad \text{……… (2)}$$

Let us now draw a triangle PQR such that PQ is parallel and equal to $\vec{V}_1$ and PR is parallel and equal to $\vec{V}_2$, as shown in Fig. (ii). We know that the radius of a circle is perpendicular to its tangent, so OA is perpendicular to $\vec{V}_1$ and OB is perpendicular to $\vec{V}_2$ Fig. (i). Therefore, angle AOB equals the angle QPR between $\vec{V}_1$ and $\vec{V}_2$. Further, as $\vec{V}_1 = \vec{V}_2 = \vec{V}$ and OA = OB, both triangles are isosceles. From geometry, we know "two isosceles triangles are similar, if the angles between their equal arms are equal. Hence, the triangle OAB Fig. (i) is similar to triangle PQR Fig. (ii). Hence, we can write

$$\frac{\Delta V}{AB} = \frac{V}{r}$$

$$\frac{\Delta V}{V} = \frac{AB}{r}$$

If the point B is close to the point A on the circle, as will be the case when $\Delta t \to 0$, the arc AB is of nearly the same length as the line AB. To that approximation,

$$AB = S$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{S}{r}$$

$$\therefore \quad \Delta V = S\frac{V}{r}$$

Putting this value in the equation (2), we get

$$a = \frac{V}{S}\, S\frac{V}{r}$$

$$\therefore \quad a = \frac{V^2}{r} \qquad \ldots\ldots\ldots (3)$$

Where a is the instantaneous acceleration. As this acceleration is caused by the centripetal force, it is called **centripetal acceleration** denoted by a_c. This acceleration is directed along the radius towards the centre of the circle. In Fig. (i) and (ii), since PQ is perpendicular to OA and PR is perpendicular to OB, so QR is perpendicular to AB. It may be noted that QR is parallel to the perpendicular bisector of AB. As the acceleration of the object moving in the circle parallel to ΔV when $AB \to 0$, so centripetal acceleration is directed along radius towards the centre of the circle. It can, concluded that

The instantaneous acceleration of an object traveling with uniform speed in a circle is directed towards the centre of the circle and is called centripetal acceleration.

The centripetal force has the same direction as the centripetal acceleration and its value is given by

$$F_C = m\,a_c = \frac{m\,V^2}{r}$$

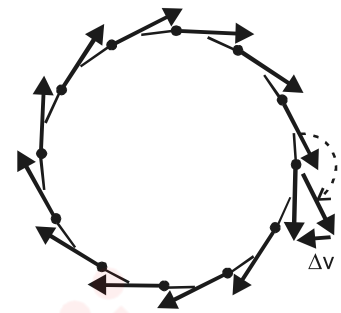
In angular motion.

$$\text{Put} \quad V = r\,\omega$$

$$\text{then} \quad F_C = \frac{mr\omega^2}{r}$$

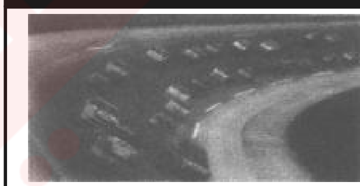
$$F_C = mr\omega^2$$

Do You Know?



Direction of motion changes continuously in circular motion.

Tid-bits



Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

Do You Know?

Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

Q.10 What is moment of inertia? Find an expression for inertia of mass m rotating about the point O.

Ans. MOMENT OF INERTIA

Consider a mass m attached to a massless rod at O as shown in Fig. (i). Let us assume that the bearing at the pivot O is frictionless and that the mass of the rod is negligible. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence this will accelerate the mass according to

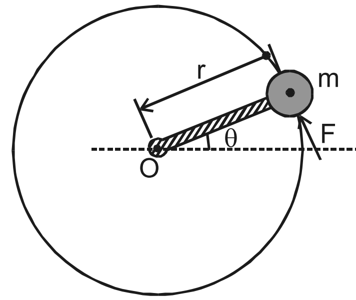


Fig. The force F causes a torque about the axis O and gives the mass m an angular acceleration about the pivot point.

$$F = ma$$

In doing so the force will cause the mass to rotate about O. Since tangential acceleration a_T is related to angular acceleration α by the equation.

$$a_T = r\alpha$$

So, $$F = m r \alpha$$

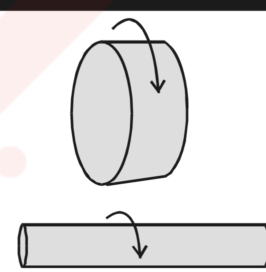
Multiply both sides by r

$$\therefore \quad rF = \tau = \text{torque} = m r^2 \alpha$$

which is rotational analogue of the Newton's second law of motion, F = ma.

Here F is replaced by τ, a by α and m by mr^2. The quantity $m r^2$ is known as the moment of inertia and is represented by I. It may be defined as the product of mass and square of distance from the point of rotation. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

Do You Know?

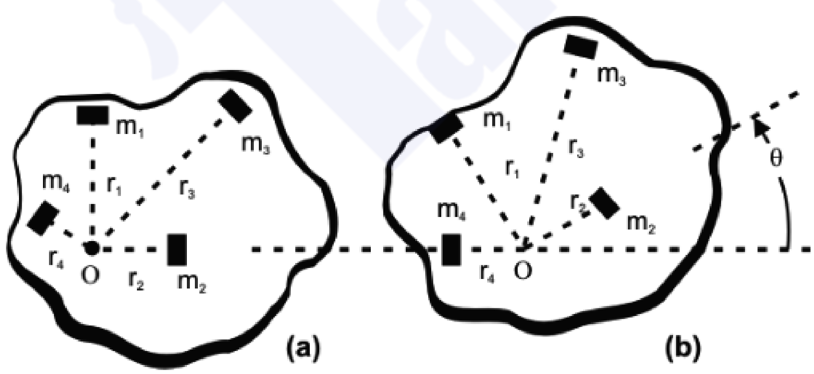


Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

Moment of Inertia of a Rigid Body

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means mass distribution is not uniform. As shown in figure, rigid body is made up of n small pieces of masses.

For Your Information



$m_1, m_2, \ldots\ldots, m_n$ at distance $r_1, r_2, \ldots\ldots, r_n$ from the axis of rotation O. Let the body be rotating with the angular acceleration α, so the magnitude of the torque acting on m_1 is

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is

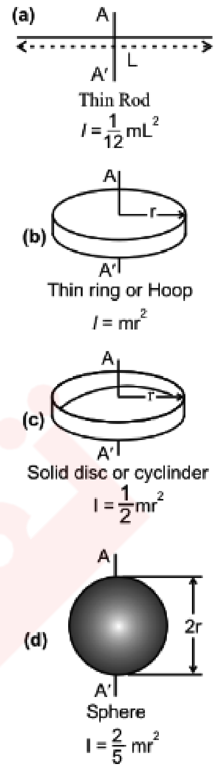
$$\tau_2 = m_2 r_2^2 \alpha_2$$

and so on.

Hence the body is rigid, so all the masses are rotating with same angular acceleration α,

Total torque τ_{total} is then given by

$$\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2)\, \alpha$$



$$= \left(\sum_{i=1}^{n} m_i \, r_i^2 \right) \alpha$$

$$\tau = I \alpha$$

where $\quad I = \sum_{i=1}^{n} m_i \, r_i^2$

Q.11 Explain angular momentum. Derive a relation between angular momentum and moment of inertia.

Ans. **ANGULAR MOMENTUM**

Angular momentum is the product of linear momentum and moment arm for momentum. **(OR)** A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum L of a particle of mass m moving with velocity V and momentum p relative to origin O is defined as

$$\vec{L} = \vec{r} \times \vec{P}$$

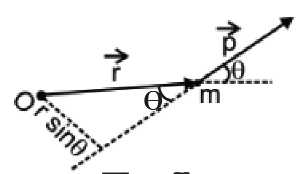


Fig. (i)

where $\vec{r}$ is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude

$$L = rp \sin\theta = m\, r\, V \sin\theta \quad (\because \; P = mV)$$

where θ is the angle between r and p. The direction of L is perpendicular to the plane formed by r and p and is given by the right hand rule of vector product. SI unit of angular momentum is $kg\, m^2 s^{-1}$ or $J\, s$.

If the particle is moving in a circle of radius r with uniform angular velocity ω, then angle between r and tangent velocity is 90^o. Hence

$$L = m\,r\,V \sin 90^o = m\,r\,V$$

But $V = r\,\omega$

Hence $L = m\,r^2\,\omega$

$$L = I\omega \qquad (\because I = mr^2)$$

Now consider a symmetric rigid rotating about a fixed axis through the centre of mass as shown in Fig. (ii). Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω. The magnitude of the angular momentum of the particle of mass m_i is $m_i\,V_i\,r_i$ about the origin O. The direction of L_i is the same as that of ω. Since $V_i = r_i\,\omega$, the angular momentum of the ith particle is $m_i\,r_i^2\,\omega$. Summing this over all particles gives the total angular momentum of the rigid body.

Q. Show that $Kgm^2/s = Js$.

Ans. Taking:

L.H.S $= Kg\,m^2/s$

Multiply and divide by s

$$= Kgm^2/s \times \frac{s}{s}$$

$$= Kgm^2/s^2 \times s$$

$$= Kgm/s^2\ ms$$

$$= Nms \quad (\because Nm = J)$$

$$= Js$$

$$= R.H.S$$

$$L = \left(\sum_{i=1}^{n} m_i\,r_i^2\right) = I\,\omega$$

Where I is the moment of inertia of the rigid body about the axis of rotation.

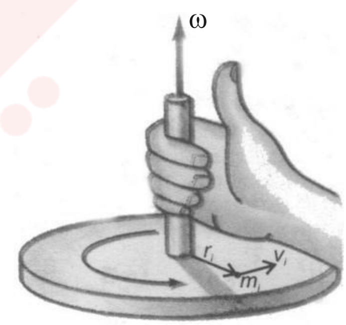


Fig. (ii)

Physicists usually make a distinction between spin angular momentum (L_s) and orbital angular momentum (L_c). The spin angular momentum is the angular momentum of spinning body, while orbital angular momentum is associated with the motion of a body along a circular path.

The difference is illustrated in Fig. (iii). In the usual circumstances concerning orbital angular momentum, the orbital radius is large as compared to the size of the body, hence, the body may be considered to be a point object.

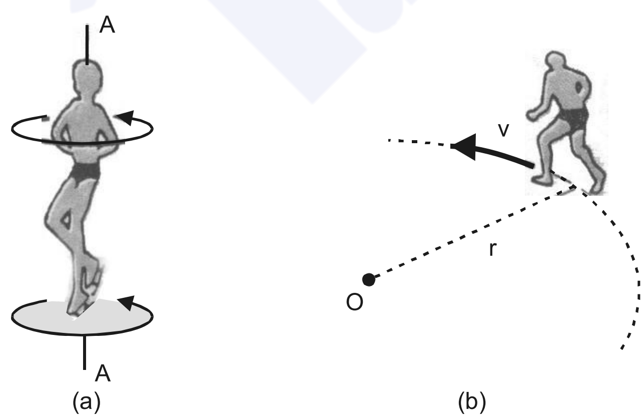
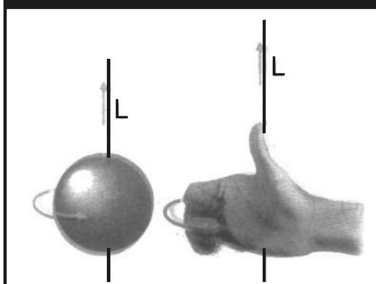


Fig. (iii)

For Your Information



(a) (b)

The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Q.12 State and explain law of conservation of angular momentum.

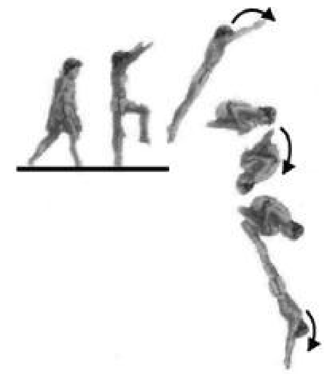
Ans. LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots\ldots = \text{constant}$$

or

$$I\,\omega = \text{Constant}$$



A man diving from a diving board.

Fig. (i)

This is illustrated by the diver in Fig. (i). The diver pushes off the board with a small angular velocity about a horizontal axis through his centre gravity G. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about this axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed position. As the angular momentum is conserved, so

$$I_1\,\omega_1 = I_2\,\omega_2$$

Hence, the diver must spin faster when moment of inertia becomes smaller to conserve angular momentum. This enables the diver to take extra somersaults.

The angular momentum is a vector quantity with direction along axis of rotation.

The direction of angular momentum along axis of rotation also remain fixed. This is illustrated by the fact given below.

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

This fact is of great importance for the Earth as it moves around the Sun. no other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

Point to Ponder

Why does the coasting rotating system slow down as water drips into the beaker?

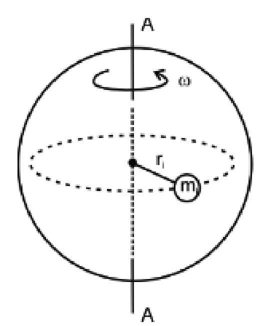
Do You Know?

The law of conservation of angular momentum is important in many sports, particularly in diving gymnastics and ice skating

Q.13 Define rotational kinetic energy. Also calculate the expression for rotational K.E.

Ans. ROTATIONAL KINETIC ENERGY

If a body is spinning about an axis with constant angular velocity ω, each point of the body is moving in a circular path and, therefore, has some K.E. To determine the total K.E. of a spinning body, we imagine it to be composed of tiny pieces of mass m_1, m_2, …… If a piece of mass m_1 is at a distance r_i from the axis of rotation, as shown in Fig. it is moving in a circle with speed



$$V_i = r_i\,\omega$$

Thus the K.E. of this piece is

$$K.E_i = \frac{1}{2} m_i V_i^2 = \frac{1}{2} m_i (r_i \omega)^2$$

$$K.E_i = \frac{1}{2} m_i r_i^2 \omega^2$$

The rotational K.E. of the whole body is the sum of the kinetic energies of all the parts. So we have

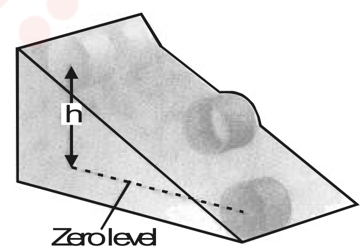
$$K.E_{rot} = \frac{1}{2}(m_1 r_1^2 \omega^2 + m_2 r_2^2 \omega^2 + \ldots\ldots)$$

$$K.E_{rot} = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots)\,\omega^2$$

$$\therefore \quad K.E_{rot} = \frac{1}{2} I \omega^2$$

Q.14 Calculate the rotational K.E of a disc and a hoop with its velocities.

Ans. **ROTATIONAL KINETIC ENERGY OF A DISC AND A HOOP**



As, $K.E_{rot} = \frac{1}{2} I \omega^2$ .... (1)

For disc

$$I = \frac{1}{2} m r^2$$

Putting in eq (1)

$$\therefore \quad K.E_{rot} = \frac{1}{2}\left(\frac{1}{2} m r^2\right)\omega^2$$

$$K.E_{rot} = \frac{1}{4} m r^2 \omega^2$$

As, $V = r\omega$

Squaring

$$V^2 = r^2 \omega^2$$

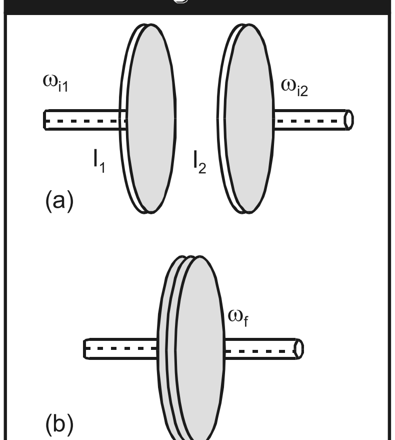
$$\therefore \quad K.E_{rot} = \frac{1}{4} m V^2$$

For Hoop

$$I = m r^2$$

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

Interesting Information



Rotational collision - the clutch

$$\therefore \quad K.E_{rot} = \frac{1}{2} m r^2 \omega^2$$

$$K.E_{rot} = \frac{1}{2} m V^2$$

When both starts moving down an inclined plane of height h, their motion consists of both rotational and transnational motions. If no energy is lost against friction, the total K.E. of disc or hoop on reaching the bottom of incline must be equal to its P.E. at top.

$$P.E = K.E_{rot} + K.E_{trans}$$

Velocity of a Disc

$$mgh = \frac{1}{4} m V^2 + \frac{1}{2} m V^2$$

$$mgh = m V^2 \left(\frac{1}{4} + \frac{1}{2}\right)$$

$$mgh = m V^2 \left(\frac{1+2}{4}\right)$$

$$mgh = \frac{3}{4} m V^2$$

$$gh = \frac{3}{4} V^2$$

$$V^2 = \frac{4}{3} g h$$

$$V = \sqrt{\frac{4gh}{3}}$$

For Your Information

As the sphere rolls to the bottom of the incline, its gravitational potential energy is changed to kinetic energy of rotation and translation.

Velocity of a Hoop

$$P.E. = K.E_{rot} + K.E_{trans}$$

$$mgh = \frac{1}{2} m V^2 + \frac{1}{2} m V^2$$

$$mgh = m V^2 \left(\frac{1}{2} + \frac{1}{2}\right)$$

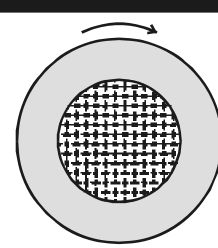
$$mgh = m V^2 \left(\frac{1+1}{2}\right)$$

$$mgh = m V^2$$

$$gh = V^2$$

$$V = \sqrt{g h}$$

Do You Know?



As the wheel rolls, it has both rotational and translational kinetic energy.

Q.15 What are artificial satellite? Find the expression for minimum velocity and period to put a satellite into the orbit.

Ans. ARTIFICIAL SATELLITES

"Satellites are objects that orbit around the Earth." They are put into orbit by rockets and are held in orbit by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 ms^{-2} towards the centre of the Earth. If they do not, they would fly off in a straight line tangent to the Earth.

When the satellite is moving in a circle, it has an acceleration

$$a_c = \frac{V^2}{r}$$

In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity and we have;

$$g = \frac{V^2}{R} \qquad \ldots\ldots\ldots (1)$$

where V is the orbital velocity and R is the radius of earth, which is 6400 km.

From equation (1)

$$V^2 = g\,h$$

$$V = \sqrt{g\,R}$$

$$= \sqrt{9.8 \times 6.4 \times 10^6}$$

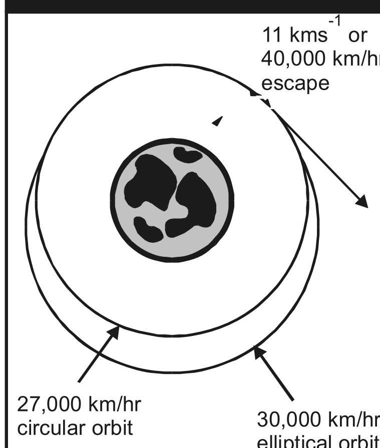
$$= \sqrt{62.72 \times 10^6}$$

$$= 7.9 \times 10^3 \text{ m/s}$$

$$V = 7.9 \text{ km/s}$$

This is the minimum velocity necessary to put a satellite into orbit and called critical velocity.

For Your Information



Satellites Orbits

Time Period of a Satellite

The period T is given by

$$T = \frac{2\pi R}{V} \qquad \left(\because \begin{array}{l} S = V\,T \\ S = 2\pi R \end{array} \right)$$

$$T = \frac{2\,(3.14)\,6400}{7.9}$$

$$= 5060 \text{ sec}$$

$$= \frac{5060}{60} \text{ min}$$

$$T = 84 \text{ min}$$

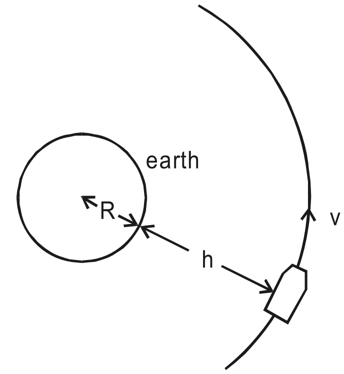
As $$F_g = \frac{GMm}{r^2}$$

$$ma = \frac{GMm}{r^2}$$

$$a \propto \frac{1}{r^2}$$

Higher the satellite the slower will the required speed and longer it will take to complete one revolution around the earth.

Close orbiting satellites orbit the Earth at a height of about 400 km. Twenty four such satellites form the Global positioning system. An airline pilot, sailor or any other person can now use a pocket size instrument or mobile phone to find his position on the Earth's surface to within 10 m accuracy.



Q.16 What do you understand by real and apparent weight?

Ans. **REAL AND APPARENT WEIGHT**

On the Earth the weight of an object is the gravitational pull of the Earth on the object. The weight of an object is measured by a spring balance. The force exerted by the object on the scale is equal to the pull due to gravity on the object i.e., weight of the object.

Consider an object of mass 'm' suspended by a string and spring balance in a lift as shown in fig. The reading of spring balance indicates tension in string and it shows apparent weight of object. Its value depends upon acceleration of lift.

> **Tidbits**
>
> The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

Case I

When lift is at rest or moving with uniform velocity.

In this case

$$a = 0$$

As $F = ma$

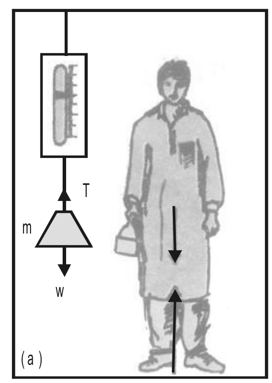
$\therefore$ $F_{net} = m(0) = 0$

As $F_{net} = T - w$

$$0 = T - w$$

$$T = w$$

The scale shows real weight of the object.



at rest
a = 0
T = w

Case II

When lift is moving upward with acceleration 'a'.

In this case

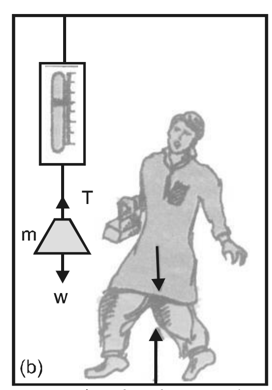
$$F_{net} = T - w$$

$$T - w = ma \qquad (\because F_{net} = ma)$$

$$T = w + ma$$

$$T = mg + ma$$

$$T = m(g + a)$$



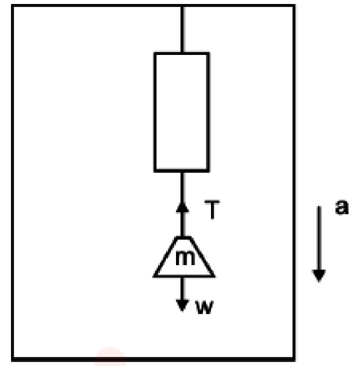
acceleration downward
w - T = ma
T = w - ma

The object will have weight more than its real weight by an amount ma.

Case III

When lift is moving downward with acceleration 'a'.

In this case



$$F_{net} = w - T$$

$$ma = w - T$$

$$T = w - ma \quad \ldots\ldots\ldots (1)$$

The tension is less than w by an amount ma. To a person in the accelerated lift the object appears to weight less than w.

Do You Know?

Your apparent weight differs from your true weight when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ridge when that velocity is constant.

Case IV

When lift is falling freely under gravity

i.e., $a = g$

From equation (1)

$$T = mg - ma$$

$$T = m(g - a)$$

$$\because \quad a = g$$

$$\therefore \quad T = m(g - g)$$

$$T = m(0)$$

$$T = 0$$

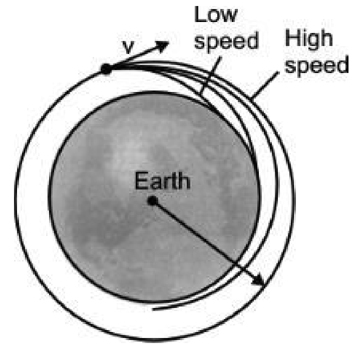
The apparent weight of the object will be shown by the scale to be zero. It is understood from these considerations that apparent weight of the object is not equal to its true weight in an accelerating system. It is equal and opposite to the force required to stop it from falling in that frame of reference.

Q.17 Explain the phenomena of weightless in a satellite.

Ans. WEIGHTLESSNESS IN A SATELLITE AND GRAVITY FREE SYSTEM

When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless. It does not matter where the object is whether it is falling under force of attraction of the Earth, the Sun, or some distant star.

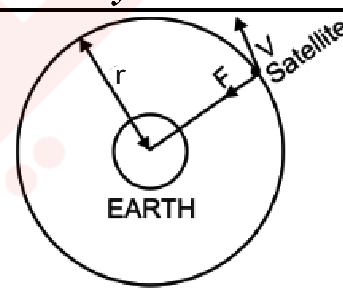
An Earth's satellite is freely falling object. The statement may be surprising at first, but it is easily seen to be correct. Consider the behaviour of a projectile shot parallel to horizontal surface of the Earth in the absence of air friction. If the projectile is thrown at successively larger speed then during its free fall to the Earth, the curvature of path decreases with increasing horizontal speeds. If object is thrown fast enough parallel to the Earth, curvature of its path will match the curvature of the Earth as shown in figure. In this case the space ship will simply circle the Earth.



The space ship is accelerating towards the centre of the Earth at all times since it circles round the Earth. Its radial acceleration is simply g, the free fall acceleration. In fact the space ship is falling towards the centre of the Earth at all the times, but the curvature of the Earth prevents space ship from hitting. Since the space ship is in free fall all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellites. Such a system is called gravity free system.

Q.18 What is orbital velocity? Define an expression for orbital velocity.

Ans. **ORBITAL VELOCITY**



The earth and some other planets revolve round the Sun nearly circular paths. The artificial satellites launched by men also adopt nearly circular course around the Earth. This type of motion is called orbital motion.

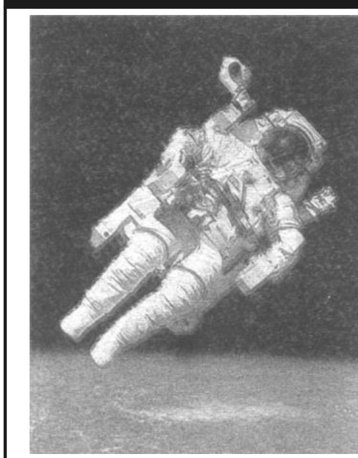
Figure shows a satellite going round the Earth in a circular path. The mass of the satellite is m_s and V is its orbital speed. The mass of the Earth is M and r represents radius of the orbit. This force $\frac{m_s V^2}{r}$ is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equation the gravitational force to the required centripetal force, gives

$$\frac{G\, m_s M}{r^2} = \frac{m_s V^2}{r}$$

$$V = \sqrt{\frac{GM}{r}} \qquad \text{.........} (1)$$

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus any satellite orbiting at distance r from Earth's centre must have the orbital speed given by equation (1). Any speed less than this will bring the satellite tumbling back to the Earth.

Tid-bits



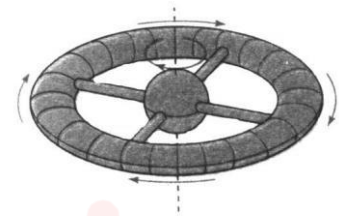
In 1984, at a height of 100 km above Hawaii island with a speed of 29000 kmh^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Q.19 What do you understand by artificial gravity? Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.

Ans. **ARTIFICIAL GRAVITY**

In a gravity free space satellite there will be no force that will force any body to any side of the spacecraft. If this satellite is to stay in orbit over an extended period of time this weightlessness may affect the performance of astronauts present in that spacecraft. To over come this difficulty, an artificial gravity is created in the spacecraft. This could enable the crew of the spaceships to function in an almost normal manner. For

this the spaceship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.



Consider a spacecraft of the shape as shown in Fig. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed ω.

Then its angular acceleration is given by

$$a_c = R\omega^2$$

$$\therefore \quad a_c = \frac{V^2}{R}$$
$$V = R\omega$$
$$\therefore \quad a_c = \frac{V^2}{R}$$
$$a_c = R\omega^2$$

But $$\omega = \frac{2\pi}{t}$$

Where t is period of revolution of spaceship.

Hence, $$a_c = R\left(\frac{2\pi}{t}\right)^2 = \frac{4\pi^2 R}{t^2}$$

As frequency

$$f = \frac{1}{t}$$

$$a_c = R\,4\pi^2 f^2$$

or $$f^2 = \frac{a_c}{4\pi^2 R}$$

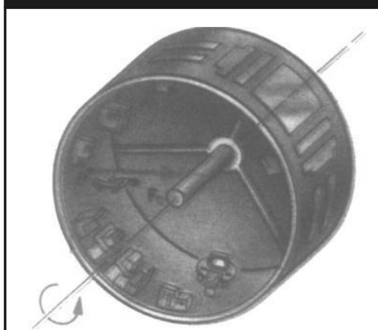
or $$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

Here the force of gravity provides required centripetal acceleration, therefore,

$$a_c = g$$

and $$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

Do You Know?



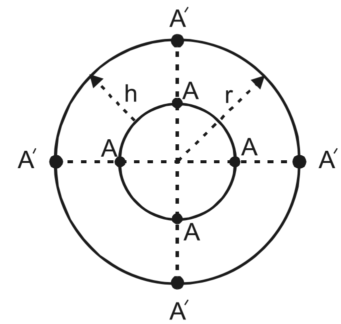
The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

When the spaceship rotates with this frequency, artificial gravity like Earth is provided to the inhabitants of the spaceship.

Q.20 What are geo stationary satellites? Derive an expression for the orbital radius of geo stationary satellites.

Ans. **GEO STATIONARY ORBITS**

"The satellite whose orbital motion is synchronized with the rotation of Earth is called geo stationary satellite and the orbit in which geo stationary satellite revolves around the Earth is called geo stationary orbit."



In this way the synchronous satellite remains always over same point on equator as Earth spins on its axis.

Such a satellite is very useful for worldwide communication, weather observations, navigation and other military uses.

Radius of Geo Stationary Orbit

As, $V = \sqrt{\dfrac{GM}{r}}$ ……… (1)

But this speed must be equal to the average speed of the satellite in one day.

i.e. $V = \dfrac{S}{T}$

$$V = \frac{2\pi r}{T} \quad \text{……… (2)}$$

Where T is the revolution of satellite, that is equal to one day. This means that the satellite must move in one complete orbit in a time of exactly one day. As the Earth rotates in one day and the satellite will revolve around. Earth is one day, the satellite at A will always stay over same point A on Earth as shown in figure.

Equating above equation (1) and (2),

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring both sides;

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

or $$r^3 = \frac{GM\,T^2}{4\pi^2}$$

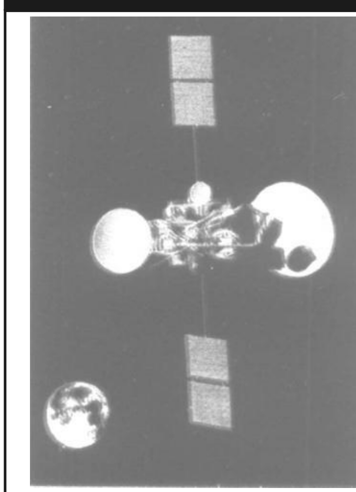
or $$r = \left(\frac{GM\,T^2}{4\pi^2}\right)^{1/3}$$

Putting the value we get

$$r = 4.23 \times 10^4 \text{ km}$$

which is orbital radius measured from centre of the Earth, for a geostationary satellite. A satellite at this height will always stay directly above a particular point on the surface of the Earth. The height of such satellite above equator is 36000 km.

Do You Know?



A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationery. It is used now for international communications.

Q.21 Write a note on communication satellites.

Ans. COMMUNICATION SATELLITES

A satellite communication system can be set up by placing several geo stationary satellites in orbit over different point on the surface of the Earth. One such satellite covers 120^{o} of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites shown in figure.

The whole Earth can be covered by just there geostationary satellite.

Since these geo stationary satellites seem to hover over one place on the Earth, continuous communication with any place on the surface of the Earth can be made. Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the Earth. The energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries can also pick up the signals from the satellite using a dish antenna on your house.

The largest satellite system is managed by 126 countries, international Telecommunication satellite organization (INTELSAT). An INTELSAT VI satellite is shown in Fig. It operates at microwave frequency of 4, 6, 11 and 14 GHz and has capacity of 30,000 two way telephone circuits plus three T.V. channels.

Do You Know?

$1\,GHz = 10^{9}\,Hz$

Q.22 Discuss Newton's and Einstein's views of gravitation.

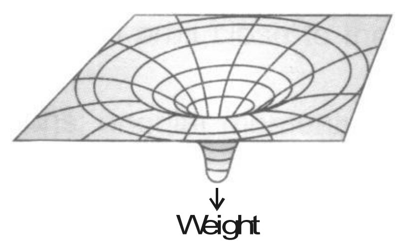
Ans. NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION

According to Newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

Do You Know?

The gravity can bend light. The gravity of a star can also be used to focus light from stars

According to Einstein's theory, space time is curved, especially locally near massive bodies. To visualize this, we might think of space as a thin rubber sheet; if a heavy weight is hung from it, curves as shown in Fig. The weight corresponds to a huge mass that causes space itself to curve. Thus, in Einstein's theory we do not speak of the force of gravity acting on bodies; instead we say that bodies and light rays move along geodesics (equivalent to straight lines in plane geometry) in curved space time. Thus a body at rest or moving slowly near the great mass would follow a geodesic towards that body.

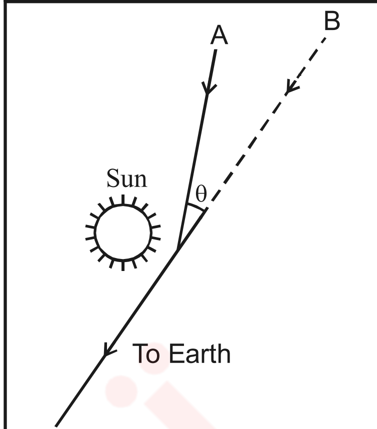


Rubber sheet analogy for curved space-time.

Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity; but explicitly said that he offered no explanation of why gravity should follow an inverse square law. Einstein's theory also says that gravity follow inverse square law (Except in strong gravitational fields), but it tells us why this should be so that is why Einstein's theory is better than Newton's even thought it includes Newton's theory with in itself and gives us same answer as Newton's theory everywhere except where the gravitational field is very strong.

Einstein inferred that if gravity and acceleration are precisely equivalent, gravity must bend light, by a precisely amount that could be calculated. This was not entirely startling suggestions Newton's theory, based on the idea light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exact twice as great as it is according to Newton's theory. Why the bending to straight caused by the gravity of the Sun was measured during a solar eclipse in 1919, and found, match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

Interesting Information



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ = 1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

SOLVED EXAMPLES

EXAMPLE 5.1

An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming declaration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Data

Initial angular velocity $= \omega_i = 3$ rev / s

Final angular velocity $= \omega_f = 0$

Time $= \Delta t = 18$ sec

To Find

Angular declaration $= \alpha = ?$

Number of revolutions $= \theta = ?$

SOLUTION

Using $\alpha = \dfrac{\Delta\omega}{\Delta t}$

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

$$\alpha = \frac{0-3}{18}$$

$$\alpha = -0.167 \text{ rev/s}^2$$

For angular motion

$$\theta = \omega_i\, t + \frac{1}{2}\alpha\, t^2$$

$$\theta = 3 \times 18 + \frac{1}{2}(-0.167)\,(18)^2$$

$$\theta = 27 \text{ rev}$$

Result

Angular declaration $= \alpha = 0.167$ rev/s^2

Number of revolutions $= \theta = 27$ rev

EXAMPLE 5.2

A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path?

Data

Mass of car = m = 1000 kg

Radius of circular path = r = 10 m

Speed of car = v = 10 ms^{-1}

To Find

Force required = F_C = ?

SOLUTION

Using $F_C = \dfrac{m\,V^2}{r}$

$$F_C = \frac{1000\,(10)^2}{10}$$

$$F_C = 100 \times 100$$

$$F_C = 10^4\ \text{N}$$

Result

Force required = $F_C = 1.0 \times 10^4$ N

This force must be supplied by the frictional force of the pavement on the wheels.

EXAMPLE 5.3

A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. What will be the tension in the spring when the ball is at the point A of the path and its speed is v at this point?

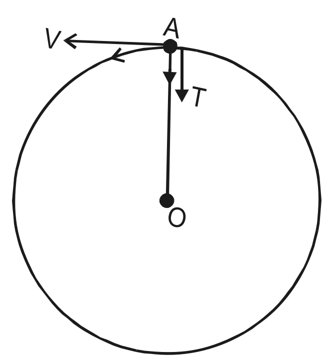
SOLUTION

At point A two forces are acting on the ball.

(1) Tension in string.

(2) Weight of ball w.

As these force act along the radius at point A, therefore,



$$T + w = \frac{m\,V^2}{r} \qquad \ldots\ldots\ldots (1)$$

$$T = \frac{m\,V^2}{r} - w$$

$$T = \frac{m\,V^2}{r} - mg$$

$$T = m\left(\frac{V^2}{r} - g\right)$$

If $\frac{V^2}{r} = g$

Then $T = m(g - g) = 0$

$T = 0$ and the centripetal force just equal to weight.

EXAMPLE 5.4

The mass of Earth is 6.00 x 10^{24} kg. The distance r from Earth to the Sun is 1.50 x 10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16 x 10^7 s).

Data

Mass of earth $= M_e = 6 \times 10^{24}$ kg

Distance between Earth and Sun $= r = 1.50 \times 10^{11}$ m

Time period $= T = 3.16 \times 10^7$ sec

To Find

Orbital angular momentum $= L_o = ?$

SOLUTION

Using $L_o = I\omega$ .... (1)

Here $I = Mer^2$

As $T = \frac{2\pi}{T}$

$\therefore \quad \omega = \frac{2\pi}{T}$

$\therefore$ equation (1) becomes

$$L_o = M_e r^2 \frac{2\pi}{T}$$

Putting values

$$L_o = 6 \times 10^{24} (1.50 \times 10^{11})^2 \frac{2(3.14)}{3.16 \times 10^7}$$

$$= \frac{37.68 \times 2.25 \times 10^{24} \times 10^{22}}{3.16 \times 10^7}$$

$$= \frac{84.78}{3.16} \times 10^{46-7}$$

$$= 26.83 \times 10^{39}$$

$$L_o = 2.68 \times 10^{40} \text{ Js}$$

Result

Orbital angular momentum $= L_o = 2.68 \times 10^{40}$ Js

EXAMPLE 5.5

A disc without slipping rolls down a hill of height 10.0 m. If the disc stars from rest at the top of the hill, what is its speed at the bottom?

Data

Height of hill $= h = 10$ m

To Find

Speed of disc at bottom $= V = ?$

SOLUTION

Using
$$V = \sqrt{\frac{4\,gh}{3}}$$

$$V = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$$

$$V = \sqrt{\frac{392}{3}}$$

$$V = \sqrt{130.66}$$

$$V = 11.4 \text{ m/s}$$

Result

Speed of disc at bottom $= V = 11.4$ m/s

EXAMPLE 5.6

An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Data

Distance of satellite from surface of Earth $= h = 384000$ km

Mass of earth $= M_e = 6 \times 10^{24}$ kg

Radius $= R = 6400$ km

To Find

Period of one revolution $= T = ?$

SOLUTION

As distance of satellite from centre of Earth is

$$r = h + R$$

$\therefore$ Using $V = \sqrt{\frac{GM}{r}}$

$$= \sqrt{\frac{40.02 \times 10^{13}}{390400000}}$$

$$= \sqrt{0.000001025 \times 10^{12}}$$

$$= 0.00101 \times 10^{6}$$

$$= 1.01 \times 10^{3} \text{ m/s}$$

$$= 1.01 \text{ km/s}$$

Now using:

$$T = \frac{2 \times 3.14 \times 290400000}{1.01 \times 10^{3}}$$

$$= \frac{1 \text{ day}}{60 \times 60 \times 24}$$

$$T = 27.5 \text{ day}$$

Result

Period of one revolution = T = 27.5 days

EXAMPLE 5.7

Radio and T.V. signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate.

(a) What is the orbital radius for a synchronous satellite?

(b) What is its speed?

Data

$$T = 24 \text{ h}$$

$$T = 24 \times 60 \times 60 \text{ sec}$$

$$T = 86400 \text{ sec}$$

To Find

(a) Orbital radius = r = ?

(b) Speed of satellite = V = ?

SOLUTION

(a) Using $r = \left(\frac{GM\,T^2}{4\pi^2}\right)^{1/3}$

Putting values

$$r = \left[\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} (86400)^2}{4 (3.14)^2}\right]^{1/3}$$

$$= \left[\frac{40.02 \times 10^{13} \times 7.46 \times 10^{9}}{4 \times 9.86}\right]^{1/3}$$

$$= \left[\frac{298.55 \times 10^{22}}{39.44}\right]^{1/3}$$

$$= [7.57 \times 10^{22}]^{1/3}$$

$$= (75.7 \times 10^{21})^{1/3}$$

$$= (75.7)^{1/3} \; 10^{21 \times 1/3}$$

$$= [(4.23)^3]^{1/3} \; 10^7$$

$$r = 4.23 \times 10^7 \text{ m}$$

(b) Now using

$$V = \frac{2\pi r}{T}$$

Putting values

$$= \frac{2 (3.14) (4.23 \times 10^7)}{86400}$$

$$= \frac{26.56 \times 10^7}{86400}$$

$$= 3.07 \times 10^{-4} \times 10^7$$

$$= 3.07 \times 10^3 \text{ m/s}$$

$$V = 3.1 \text{ km/s}$$

Result

(a) Orbital radius = r = 4.23×10^7 m

(b) Speed of satellite = V = 3.1 km/s

Chapter 6

FLUID DYNAMICS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand that viscous forces in a fluid cause a retarding force on an object moving through it.
- Use Stokes' law to derive an expression for terminal velocity of a spherical body falling through a viscous fluid under laminar conditions.
- Understand the terms steady (laminar, streamline) flow, incompressible flow, non-viscous flow as applied to the motion of an ideal fluid.
- Appreciate the equation of continuity Av = Constant for the flow of an ideal and incompressible fluid.
- Appreciate that the equation of continuity is a form of the principle of conservation of mass.
- Derive Bernoulli's equation in form $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$.
- Explain how Bernoulli effect is applied in the filter pump, atomizers, in the flow of air over an aerofoil, venturimeter and in blood physics.
- Give qualitative explanations for the swing of a spinning ball.

INTRODUCTION

The study of fluids in motion is relatively complicated, but analysis can be simplified by making a few assumptions. The analysis is further simplified by the use of two important conservation principles; the conservation of mass and the conservation of energy. The law of conservation of mass gives us the equation of continuity while the law of conservation of energy is the basis of Bernoulli's equation. The equation of continuity and the Bernoulli's equation along with their applications in aeroplane and blood circulation are discussed in this chapter.

Q.1 Define fluid dynamics.

Ans. **FLUID DYNAMICS**

"The branch of physics which deals with the fluid in motion" is called fluid dynamics.

Fluid

"Those substances which can flow from one point to the other are called fluid."

For examples; liquids and gases.

Q.2 Explain what do you understand by viscosity.

Ans. **VISCOSITY**

"The frictional effect between different layers of the flowing fluid is described in terms of viscosity of fluid." **(OR)** "The property of fluid due to which they resist their flow is also known as viscosity." **(OR)** Fraction in fluids is known as viscosity.

Viscosity measures how much force is required to slide one layer of liquid over another layer. Substances that do not flow easily, such as thick tar and honey etc; have large co–efficient of viscosities, (η). Substances which flow easily like water have small coefficient of viscosities. Since, liquids and gases have non zero viscosity, a force is required if an object is to be moved through them.

Unit

As, $F = 6\pi\eta r v$

$$\eta = \frac{F}{6\pi r v}$$

As $= \dfrac{N}{m\ m/s}$

$$= \frac{Ns}{m^2}$$

$$= Nm^{-2}\ s$$

or $\eta = kg\ m / s^2\ m^{-2}\ s$

$$= kg\ m^{-1}\ s^{-1}$$

Dimensions

$$[\eta] = [ML^{-1}\ T^{-1}]$$

Q.3 Define drag force and Stoke's law.

Ans. **DRAG FORCE (VISCOUS DRAG)**

"An object moving through a fluid experiences a retarding force is called a drag force." The drag force

For Your Information

Viscosities of Liquids and Gases at 30°C	
Material	**Viscosity 10^{-3} (Nsm^{-2})**

increases as the speed of the object increases.

Stoke's Law

The drag force 'F' on a sphere of radius 'r' moving slowly with speed 'v' through a fluid of viscosity 'η' is given by Stoke's law as under

$$F_d = 6\pi\eta r v_t$$

At high speeds the force is no longer proportional to the speed.

Note: *With rise in temperature, viscosity of liquid decreases and viscosity of gases increases.*

Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.000
Plasma	1.6
Glycerin	6.29

Q.4 Define terminal velocity. Also derive the expression for terminal velocity. (OR) Define terminal velocity. Prove that terminal velocity is directly proportional to the square of the radius.

Ans. **TERMINAL VELOCITY**

Maximum constant velocity of an object falling vertically downward when the weight of the object is equal to drag force is called terminal velocity.

Explanation

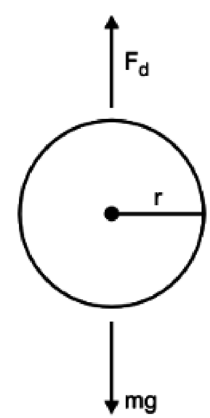
Consider a water droplet such as that of fog falling vertically, the air drag on the water droplet increases with speed. The droplet accelerates rapidly under the force of gravity which pulls the droplet downward. However, the upward drag force on it increases as the speed of the droplet increases.

Since, the droplet is moving downward.

$$\therefore \quad mg > F_d$$

The net force on droplet is

$$F_{net} = mg - F_d$$



As the speed of droplet continues to increase, the drag force eventually approaches the weight in the magnitude. Finally when the magnitude of drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with Constant speed (Maximum speed) is called Terminal velocity.

$$\therefore \quad F_{net} = 0$$

$$\therefore \quad 0 = mg - F_d$$

$$mg = F_d$$

$$mg = 6\pi\eta r V_t$$

$$V_t = \frac{mg}{6\pi \eta r} \quad \text{.........} (1)$$

or $V_t \propto m$

Where r, g, η are constant then

The more massive an object, faster it falls through a fluid.

As, ρ (density) = m / V

$$m = \rho V$$

As droplet is of spherical shape; so

Volume of sphere is

$$V = \frac{4}{3}\pi r^3$$

Then $m = \rho \times \frac{4}{3}\pi r^3$

Putting this value in equation (1);

$$\therefore \quad V_t = \frac{\rho \, 4/3 \, \pi r^3 g}{6\pi \eta r}$$

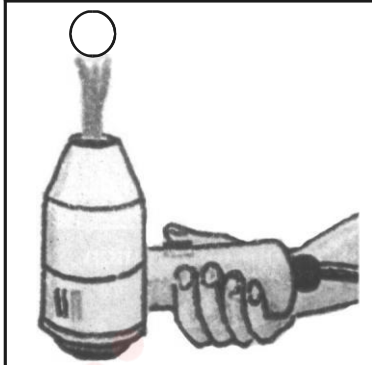
$$= \frac{4r^2 \rho g}{18 \eta}$$

$$V_t = \frac{2 \rho g r^2}{9 \eta}$$ which is the expression for the terminal velocity

or $V_t \propto r^2$ where $\frac{2\rho g}{9\eta}$ is constant

Hence, terminal velocity is directly proportional to the square of radius of droplet.

Can You Do That?



A table tennis ball can be made suspended in the stream of air coming from the nozzle of chair dryer.

Q.5 What is fluid flow? What is the difference between stream line and turbulent flow?

Ans. **FLUID FLOW**

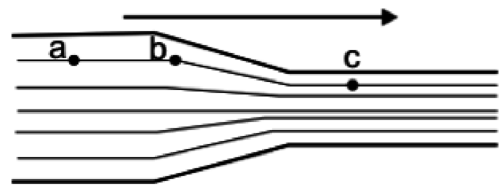
When a fluid is in motion, its flow can be:

(i) Streamline flow

(ii) Turbulent flow

(i) Streamline Flow

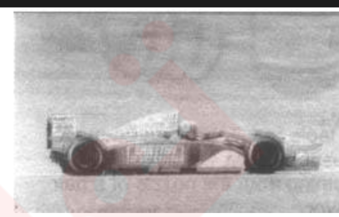
"The flow is said to be streamline or laminar if every particle that passes a particular point, moves along exactly the same path as followed by particles which passed that points earlier." **(OR)** "If velocity of the particles at different points does not change with time, the flow is called streamline flow."



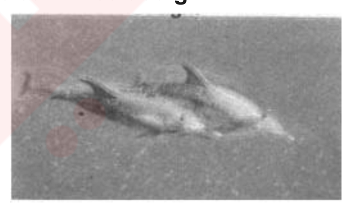
Stream Line

In this case each particle of fluid moves along a smooth path called a streamline as shown in figure. The different streamline cannot cross each other. This condition is called steady flow condition. If the streamlines cross each other the particle will go in one direction or other and flow will not be steady flow, it will be a turbulent flow.

Do You Know?



Formula one racing cars have a streamlined design.



Dolphins have streamlined bodies to assist their movement in water.

(ii) Turbulent Flow

"The irregular or unsteady flow of the fluid is called turbulent flow."

Ideal Fluid

Such a fluid which satisfy the following conditions is called an ideal fluid.

(i) The fluid is non–viscous i.e. there is no internal frictional force between adjacent layers of fluid.

(ii) The fluid is incompressible i.e. its density is constant.

(iii) The fluid motion is steady.

Q.6 State and explain equation of continuity.

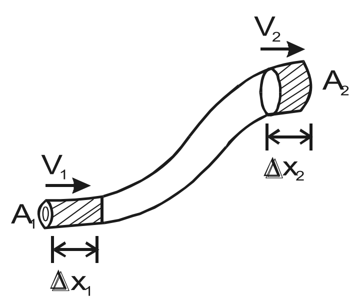
Ans. EQUATION OF CONTINUITY

Statement

The product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate. **(OR)** If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved i.e., the mass that flows into the bottom of the pipe through A_1 in a time Δt must be equal to mass of the liquid that flows out A_2 in the same time.

Explanation

Consider a fluid flowing through a pipe of non–uniform size. The particles in the fluid move along the streamlines in a steady state flow as shown in Fig.



In a small time Δt, the fluid at the lower end of tube moves a distance Δx_1, with a velocity V_1. If A_1 is area of cross-section of this end, then the mass of the fluid contained in the shaded region is

As, $\rho = m / V$

$m = \rho V$

or $\Delta m_1 = \rho V$

Since Volume V = Length x Area of the cross section of pipe

$$= \Delta x_1 \times A_1$$

$$\therefore \quad \Delta m_1 = \rho_1 A_1 \Delta x_1 \qquad \ldots\ldots\ldots (1)$$

As, $\quad S = V\,t$

$$\therefore \quad \Delta x_1 = V_1 \Delta t$$

Putting in equation (1)

$$\therefore \quad \Delta m_1 = \rho_1 A_1 V_1 \Delta t \qquad \ldots\ldots\ldots (2)$$

Tidbits

As the water falls, its speed increases and so its cross sectional area decreases as mandated by the continuity equation.

Where ρ_1 is the density of fluid. Similarly the fluid that moves with velocity V_2 through the upper end of pipe of area A_2 in the same time Δt has a mass

$$\Delta m_2 = \rho_2 A_2 V_2 \Delta t \qquad \ldots\ldots\ldots (3)$$

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved.

$$\therefore \quad \Delta m_1 = \Delta m_2$$

Putting values from equation (2) and equation (3)

$$\rho_1 A_1 V_1 \Delta t = \rho_2 A_2 V_2 \Delta t$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This equation is known as the equation of continuity. Since density is constant for the steady flow of incompressible fluid.

$$\therefore \quad \rho_1 = \rho_2 = \rho$$

$$\therefore \quad \rho A_1 V_1 = \rho A_2 V_2$$

$$A_1 V_1 = A_2 V_2$$

or $\quad AV = \text{Constant}$

or $\quad V = \dfrac{1}{A}\,\text{Constant}$

$$V \propto \frac{1}{A}$$

Note: *Equation of continuity obeys law of conservation of mass.*

Q.7 State and explain Bernoulli's equation.

Ans. BERNOULLI'S EQUATION (OR THEOREM)

Introduction

A fundamental equation in fluid dynamics which deals with the steady flow of an incompressible and non-viscous fluid is called Bernoulli's equation.

Statement

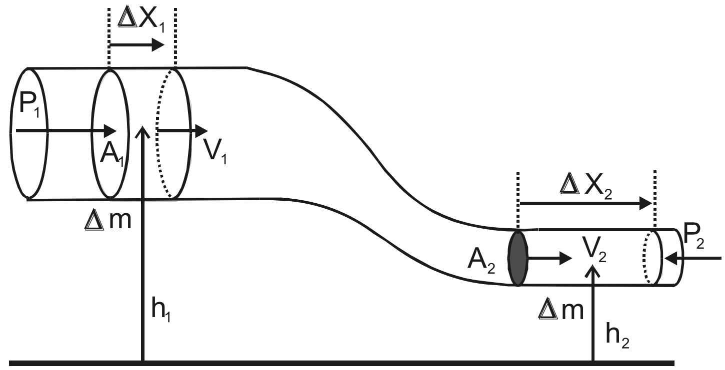
Bernoulli's theorem states that the sum of pressure, K.E. per unit volume and P.E. per unit volume, in a steady flow of an incompressible and non–viscous liquid has the same value."

Mathematically,

$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{Constant}$$

Explanation

Consider the flow of fluid which is incompressible, non-viscous and flows in a steady state manner through the pipe in time 't' as shown in figure.



The force on the upper end of the fluid is P_1A_1 (since $P = F/A \therefore F = PA$) where P_1 is the pressure and A_1 is the area of cross section of pipe at the upper end. The work done on the fluid, in moving it through a distance Δx_1 will be

$$W_1 = \vec{F} \cdot \vec{\Delta x_1}$$
$$= F \Delta x_1 \cos 0^o$$
$$= P_1 A_1 \Delta x_1 \times 1$$
$$W_1 = P_1 A_1 \Delta x_1 \qquad \text{......... (1)}$$

If V_1 is velocity of fluid at this end then,

As $S = V t$

$$\Delta x_1 = V_1 t$$

$\therefore$ equation (1) becomes

$$W_1 = P_1 A_1 V_1 t$$

Similarly work done on the fluid at lower end is,

$$W_2 = \vec{F}_2 \cdot \vec{\Delta x_2}$$
$$= F_2 \Delta x_2 \cos 180^o$$
$$= P_2 A_2 \Delta x_2 (-1) \qquad (\because F_2 = P_2 A_2)$$
$$= -P_2 A_2 \Delta x_2$$
$$W_2 = -P_2 A_2 V_2 t \qquad (\because \Delta x_2 = V_2 t)$$

Where V_2 is the velocity of the fluid at lower end. P_2 is the pressure, A_2 is the area of cross section of lower end and Δx_2 is the distance moved by the fluid in the same time interval 't'. The work W_2 is taken to be negative because this work is done against the fluid force.

Total work done

$$W = W_1 + W_2$$
$$\therefore \quad W = p_1 A_1 V_1 t + (-p_2 V_2 A_2 t)$$
$$W = p_1 A_1 V_1 t - p_2 A_2 V_2 t \qquad \text{......... (2)}$$

From equation of continuity

$$A_1 V_1 = A_2 V_2$$

Multiply both sides by t

$$A_1 V_1 t = A_2 V_2 t = V \text{ (Volume)}$$

$$\left[\begin{array}{l} \Delta m = \rho AV t \\ \dfrac{\Delta m}{\rho} = AV t \\ V = AV t \end{array} \right]$$

$\therefore$ equation (2) becomes

$$\therefore \quad W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V$$

Since $\rho = \dfrac{m}{V}$

$$\therefore \quad V = \frac{m}{\rho}$$

$$\therefore \quad W = \frac{m}{\rho}(P_1 - P_2) \qquad \ldots\ldots\ldots (3)$$

Part of this work is utilized by the fluid in changing its K.E. and a part of it is used in changing its gravitational P.E.

$$\text{Change in K.E.} = \Delta \text{K.E} = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$$

$$\text{Change in P.E.} = \Delta \text{P.E.} = mg h_2 - mg h_1$$

Where h_1 and h_2 are the heights of upper and lower ends of pipe respectively.

Applying law of conservation of energy to this volume of the fluid.

$$W = \Delta\text{K.E.} + \Delta\text{P.E.}$$

$$\frac{m}{\rho}(P_1 - P_2) = m\left(\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2 + g h_2 - g h_1\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 + \rho g h_2 - \rho g h_1$$

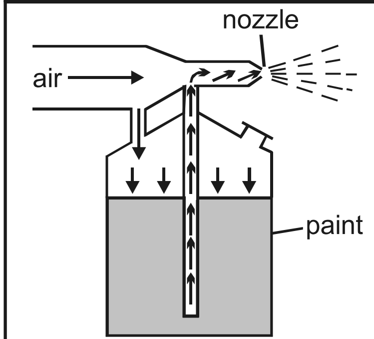
$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

This is Bernoulli's equation and is often expressed

As $P + \dfrac{1}{2}\rho V^2 + \rho g h = \text{Constant}$

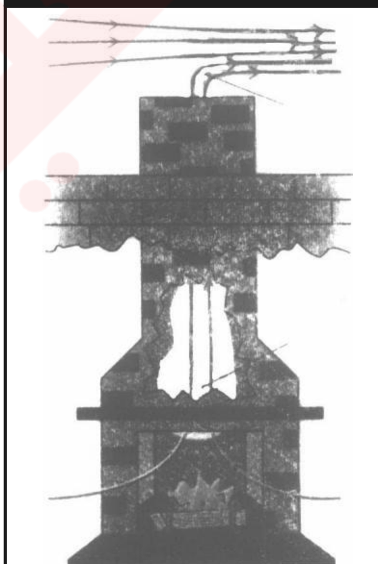
Note: Law of conservation of energy is the basis of Bernoulli's theorem.

Interesting Information



A stream of air passing over a tube dipped in a liquid will cause the liquid to rise in the tube as shown. This effect is used in perfume bottles and paint sprayers.

Do You Know?



A chimney works best when it is tall and exposed to air currents, which reduces the pressure at the top and force the upward flow of smoke.

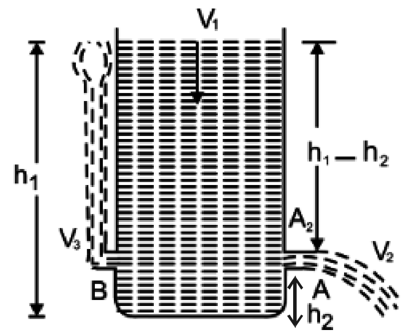
APPLICATIONS OF BERNOULLI'S EQUATION

There are two applications of Bernoulli's theorem:

Q.8 State and explain Torricelli's theorem.

Ans. **TORRICELLI'S THEOREM**

Statement



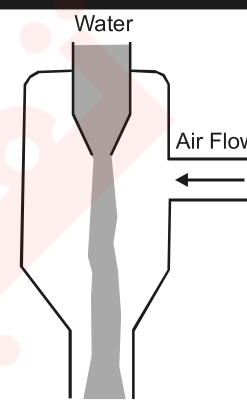
"The speed of efflux (outward flow of gas or liquid) is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity."

Explanation

Suppose a large tank of fluid has two small orifices A and B on it as shown in figure. Now we find speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds V_2 and V_3 will be much larger than the speed V_1 of the top of surface of water. Therefore, $V_1 \approx 0$.

For Your Information



A filter pump has a constriction in the centre, so that a jet of water from the tap flows faster here. This cause a drop in pressure near it and air, therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump.

As Bernoulli's equation is

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

Putting $V_1 = 0$

$$\therefore \quad P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

Since $P_1 = P_2 = P$ (Atmospheric pressure)

$$\therefore \quad P + \rho g h_1 = P + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\rho g h_1 = \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\frac{1}{2}\rho V_2^2 = \rho g h_1 - \rho g h_2$$

$$\frac{1}{2}\rho V_2^2 = \rho g (h_1 - h_2)$$

$$\frac{1}{2} V_2^2 = g (h_1 - h_2)$$

$$V_2^2 = 2g (h_1 - h_2)$$

Taking square root on both sides.

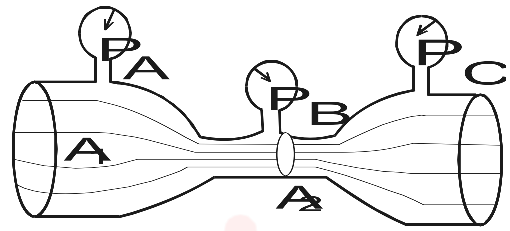
$$V_2 = \sqrt{2g (h_1 - h_2)}$$

Which is the Torricelli's theorem.

Note: Notice that the speed of the efflux of liquid is the same as the speed of a ball that falls through a height $(h_1 - h_2)$. The top level of the tank has moved down a little and the P.E. has been transferred into K.E. of the efflux of fluid. If the orifice had been pointed upward as at B shown in figure, this K.E. would allow the liquid to rise to the level of water tank. In practice, viscous-energy losses would alter the result to some extent.

Q.9 What is the relation between fluid speed and pressure? (OR) Prove that where the pressure is low, the speed will be high.

Ans. RELATION BETWEEN SPEED AND PRESSURE OF THE FLUID



Suppose that water flows through a pipe system as shown in figure.

Clearly, the water will flow faster at B than it does at A or C. $\left(\because V \propto \frac{1}{A}\right)$.

Now we compare the pressure at B with that at A. As the Bernoulli's equation is

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_B$$

Since the average height at both places is same.

$\therefore$ P.E. is same at both places.

$$\therefore \quad P_A + \frac{1}{2}\rho V_A^2 + \rho g h = P_B + \frac{1}{2}\rho V_B^2 + \rho g h$$

$$\therefore \quad P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

Let $V_A = 0.20$ m/s, $V_B = 2$m/s

For water;

$$\rho = 1000 \text{ kg/m}^3$$

$$\therefore \quad P_A + \frac{1}{2}(1000)(2)^2 = P_B + \frac{1}{2}(1000)(2)^2$$

$$P_A + 500\,(0.04) = P_B + 500 \times 4$$

$$P_A + 20 = P_B + 2000$$

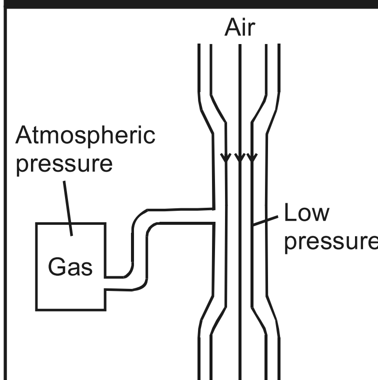
$$P_A - P_B = 2000 - 20$$

$$P_A - P_B = 1980 \text{ Nm}^{-2} \text{ (Pa)}$$

$$\therefore \quad P_A > P_B$$

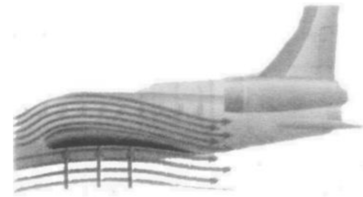
This shows that the pressure in the narrow pipe where the streamlines are closer together is much smaller than in the wider pipe. Thus, where the speed is high, the pressure will be low.

Interesting Information



The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

Example



The lift on an aeroplane is due to this effect. The flow of air around an aeroplane wing is illustrated in figure. The wing is designed to deflect the air so that streamlines are closer together above the wing than below it. We have seen in figure that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward.

Similarly, when a tennis ball is hit by a racket in such a way that spins as well as moves forward, the velocity of the air on one side of the ball increases (figure) due to spin and air speed in the same direction as at B and hence, the pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

Q.10 Explain venturi relation.

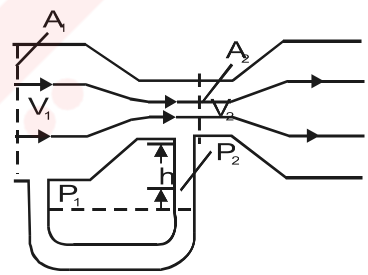
Ans. VENTURI RELATION

If one of the pipe has a much smaller diameter than the other, as shown in Fig. It is assumed that the pipes are horizontal so that $\rho g h$ terms become equal and can, therefore, be dropped then

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - \frac{1}{2}\rho V_1^2\right)$$

$$= \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$



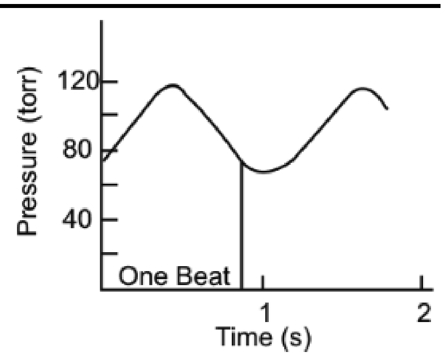
As the cross sectional area A_2 is small as compared to the area A_1, then from equation of continuity $V_1 = (A_2/A_1)\,V_2$, will be small as compared to V_2. Thus for flow from a large pipe to a small pipe we can neglect V_1 on the right hand side of equation, hence,

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2$$

This is known as Venturi relation, which is used in Venturi–meter, a device used to measure speed of liquid flow.

Q.11 Explain the flow of blood inside the human body. How the blood pressure is measured?

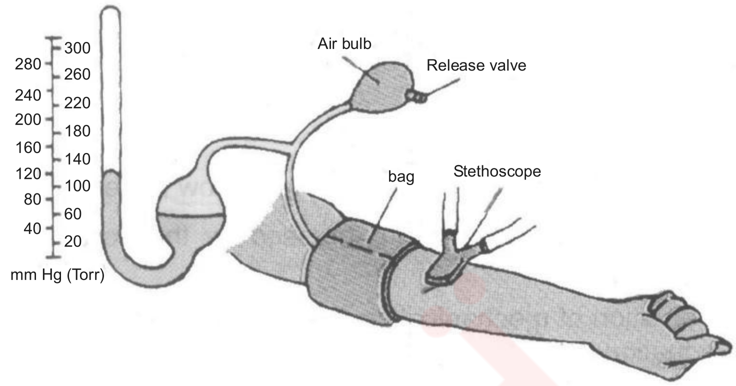
Ans. BLOOD FLOW



Blood is incompressible fluid with density nearly equal to that of water. High concentration of red blood cells (about 50%) increases its viscosity from three to five times that of water. Blood vessels are not rigid. They stretch like a rubber hose. Under normal circumstances the volume of blood is sufficient to keep the vessels inflated at all times, even in the relaxed state between heart beats. This means that there is tension in the walls of blood vessels and so pressure of blood inside is greater than external atmospheric pressure.

Figure shows the variation in blood pressure as the heart beats. The pressure varies from high (systolic) pressure of 120 torr to a low (diastolic) pressure of about 70 – 80 torr between beats in normal healthy person.

The instrument which is used to measure the blood pressure dynamically is called sphygmomanometer. It is shown in figure.



An inflatable bag is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. The effect is to squeeze the arm and compress the blood vessels inside. When the external pressure applied becomes larger than systolic pressure, the vessels collapse cutting off the flow of blood. Opening the release valve on the bag gradually decreases the external pressure.

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point the first surges of blood flow through the narrow stricture which produces a high flow speed. As a result, the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow changes from turbulent to laminar and the gurgle in stethoscope disappear. This is the signal to record diastolic pressure.

Note: (i) 1 Torr = 133.3 Nm^{-2}.

(ii) For blood pressure Torr or mm of Hg is used instead of SI unit of pressure i.e. Nm^{-2}.

SOLVED EXAMPLES

EXAMPLE 6.1

A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that η for air = 19×10^{-6} kg $m^{-1}s^{-1}$ and density of water ρ = 1000 kgm^{-3}.

Data

Radius of water droplet $= r = 0.010$ cm

$= 0.010 \times 10^{-2}$ m

$= 1 \times 10^{-4}$ m

For air $= \eta = 19 \times 10^{-6}$ kg m^{-1} s^{-1}

Density of water $= \rho = 1000$ kg m^{-3}

To Find

Terminal velocity $= V_t = ?$

SOLUTION

Using: $V_t = \dfrac{2\, g\, r^2\, \rho}{9\, \eta}$

$$= \frac{2\,(9.8)\,(1 \times 10^{-4})^2\, 1000}{9 \times 19 \times 10^{-6}}$$

$$= \frac{2000 \times 9.8 \times 10^{-8}}{9 \times 19 \times 10^{-6}}$$

$$= \frac{9.8 \times 2 \times 10^{-5} \times 10^{6}}{9 \times 019}$$

$$= \frac{2 \times 9.8 \times 10}{9 \times 019}$$

$$= \frac{196}{171}$$

$= 1.1$ m/s

Result

Terminal velocity $= V_t = 1.1$ m/s

EXAMPLE 16.2

A water hose with an internal diameter of 20mm at the outlet discharges 30 kg of water in 60s. Calculate the water speed at the outlet. Assume the density of water is 1000 kgm^{-3} and its flow is steady.

Data

Diameter of hose $= d = 20\text{mm}$
$= 20 \times 10^{-3}$ m
$= 2 \times 10^{-2}$ m

Mass of water $= m = 30$ kg

Time $= t = 60$ sec

Density of water $= \rho = 1000 \text{ kg m}^{-3}$

To Find

Speed of water $= V = ?$

SOLUTION

From equation of continuity

$$m = \rho A V \Delta t$$

Also

$$V = \frac{m}{\rho A \Delta t}$$

Since area of cross section $A = \pi r^2$

$$\therefore \quad V = \frac{m}{\rho \pi r^2 \Delta t}$$

$$= \frac{30}{1000 \times 3.14 \left(\frac{d}{2}\right)^2 60} \qquad \left(\therefore r = \frac{d}{2}\right)$$

$$= \frac{30}{1000 \times 3.14 \frac{d^2}{4} \times 60}$$

$$= \frac{30 \times 4}{1000 \times 3.14 \times 60 \, (2 \times 10^{-2})^2}$$

$$= \frac{120}{1000 \times 3.14 \times 60 \times 4 \times 10^{-4}}$$

$$= \frac{2}{3.14 \times 4 \times 10^{-1}}$$

$$= \frac{10}{6.28}$$

$$= 1.6 \text{ m/s}$$

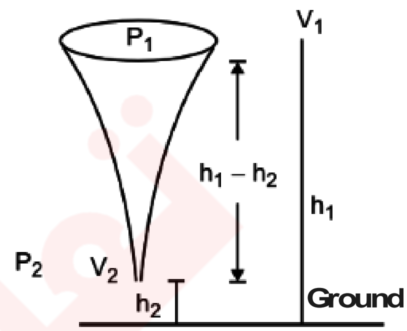
Result

Speed of water = V = 1.6 m/s

EXAMPLE 6.3

Water flows down hill through a closed vertical funnel. The flow speed at the top is 12.0 cms^{-1}. The flow speed at the bottom is twice the speed at the top. If the funnel is 40 cm long and the pressure at the top is 1.103×10^5 Nm^{-2}, what is the pressure at the bottom?

Data



Flow speed at top = V_1 = 12 cm/s

= 0.12 m/s

Flow speed at bottom = V_2 = 24 cm/s

= 0.24 m/s

Length of funnel = h = $h_1 - h_2$

= 40 cm

= 0.4 m

Pressure at top = P_1 = 1.013×10^5 Nm^{-2}

To Find

Pressure at bottom = P_2 = ?

SOLUTION

Using Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 + \rho g h_1 - \rho g h_2$$

$$= P_1 + \tfrac{1}{2}\rho (V_1^2 - V_2^2) + \rho g (h_1 - h_2)$$

$$= 1.013 \times 10^5 + \frac{1}{2}(1000)\,[(.12)^2 - (.24)^2] + 1000 \times 9.8 \times 0.4$$

$$= 1.013 \times 10^5 + 500\,(.0144 - 0.0576) + 980 \times 4$$

$$= 1.013 \times 10^5 + 500\,(-\,0.0432) + 3920$$

$$= 1.013 \times 10^5 - 21.6 + 3920$$

$$= 1.013 \times 10^5 + 3898.4$$

$$= 1.013 \times 10^5 + 0.039984 \times 10^5$$

$$= 10^5\,(1.013 + 0.03898)$$

$$= 1.05 \times 10^5 \text{ Nm}^{-2}$$

Result

Pressure at bottom = P_2 = 1.05×10^5 Nm^{-2}

Chapter 7

OSCILLATIONS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

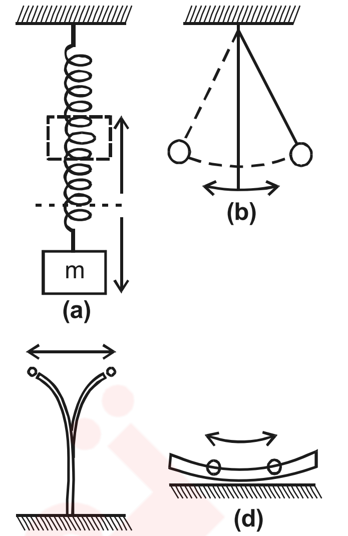
- Investigate the motion of an oscillator using experimental, analytical and graphical methods.
- Show that the motion of mass attached to a spring is simple harmonic.
- Understand that the motion of simple pendulum is simple harmonic and to calculate its time period.
- Understand and use the terms amplitude, time period, frequency, angular frequency and phase difference.
- Know and use of solutions in the form of $x = x_o \cos \omega t$ or $y = y_o \sin \omega t$.
- Describe the interchange between kinetic and potential energies during SHM.
- Describe practical examples of free and forced oscillations.

Q.1 Define oscillatory motion.

Ans. OSCILLATORY MOTION

"Such a motion in which a body moves to and fro about a mean position, is called oscillatory or vibratory motion."

The oscillatory motion is called periodic motion when it repeats itself after regular intervals of time.

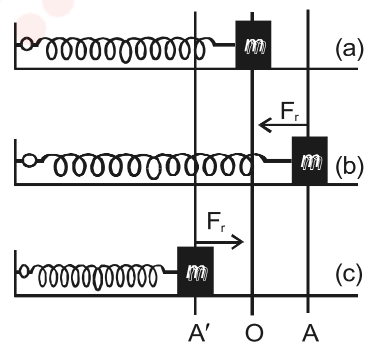


(Vibrating objects)

Examples

1. A mass, suspended from a spring, when pulled down and then released starts oscillating.
2. The bob of a simple pendulum, when displaced from its rest position and released vibrates.
3. A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.
4. A steel ball rolling in a curved dish.

Thus to get oscillations, a body is pulled away from its rest or equilibrium position and then released. The body oscillates due to a restoring force. Under the action of this restoring force, the body accelerates and it overshoots the rest position due to inertia. The restoring force then pulls it back. The restoring force is always directed towards the rest position and so the acceleration is also directed towards the rest or mean position.

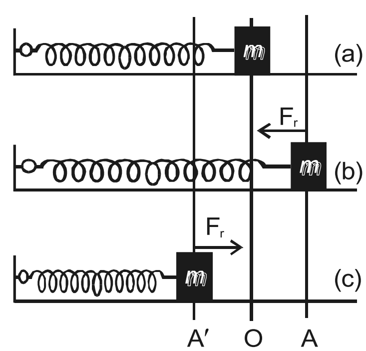


It is observed that the vibrating bodies produce waves. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges, appear to be rigid, they actually vibrate. The architects and the engineers who design and build them, take this fact into account.

Q.2 Explain simple harmonic motion. (OR) Derive an expression for the acceleration of a body vibrating under elastic restoring force.

Ans. SIMPLE HARMONIC MOTION

Consider a body of mass 'm' attached with a spring of spring constant (K). The other end of the spring attached with the fixed support and the spring is placed on the smooth horizontal surface. When the body is pulled towards right from the mean position through a displacement 'x', a deforming force $F = Kx$ is needed. When the body is released, an elastic restoring force which is equal and opposite to the deforming force comes into play and restore the position of the body. But due to inertia, the body over shoot the mean position and goes towards left from the mean position. Once again an elastic restoring force brings the body back towards the mean position but due to inertia, the body moves towards right from the mean position.



Thus under the action of restoring force and inertia, the body continue its vibratory motion for a long time between these two positions A and A′. The body speeds up while moving towards mean position and slows down while moving away from mean position. This means that the acceleration of the body is always directed towards the mean position. This acceleration can be calculated as follows:

$$F = -Kx \quad \text{(Elastic restoring force)}$$

$$F = ma \quad \text{(Newton's IInd law of motion)}$$

Comparing

$$ma = -Kx$$

$$a = -\frac{Kx}{m}$$

Where $\frac{K}{m}$ is constant

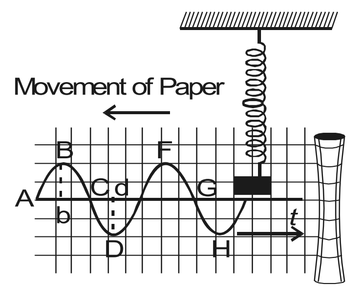
then, $$a = \text{Constant}\,(-x)$$

$$a \propto -x$$

Thus such a motion in which acceleration of the body is always directed towards the mean position and is directly proportional to the displacement called simple harmonic motion.

Q.3 Define instantaneous displacement and amplitude.

Ans. INSTANTANEOUS DISPLACEMENT AND AMPLITUDE OF VIBRATION



"When a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement." It is zero at the instant when the body is at the mean position and is maximum at the extreme positions.

It is zero when the body is at mean position and is maximum at the extreme positions. It is denoted by x.

Amplitude

"The maximum value of displacement (where Hook's law is valid), on either side of mean position is called amplitude. It is denoted by x_0.

Vibrations

"A complete round trip of body in motion vibrating motion is called vibration."

Time Period

"It is the time required to complete one vibration. It is denoted by 'T'."

Frequency

"Number of vibrations completed in one second is called frequency. It is denoted by 'f'."

$$f = \frac{1}{T}$$

Unit

Cycles per-second or Hertz (Hz) or vibrations per-second.

Angular Frequency

If 'T' is the time period of a body executing SHM, its angular frequency is

$$\omega = \frac{2\pi}{T}$$

Since $\frac{1}{T} = f$

$\therefore \quad \omega = 2\pi f$

ω = Angular frequency

$\theta = \omega t$

$s = Vt$

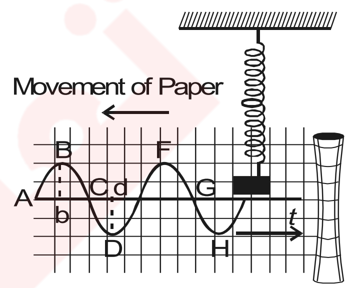
$2\pi r = r\omega T$

$2\pi = \omega T$

$\omega = \frac{2\pi}{T}$

Wave Form of SHM

The arrangement as shown in figure can be used to record the vibration in displacement with time for a mass-spring system. The strip of paper is moving at a constant speed from right to left, thus providing a time scale on the strip. A pen is attached with the vibrating mass, which records its displacement against time as shown in figure.



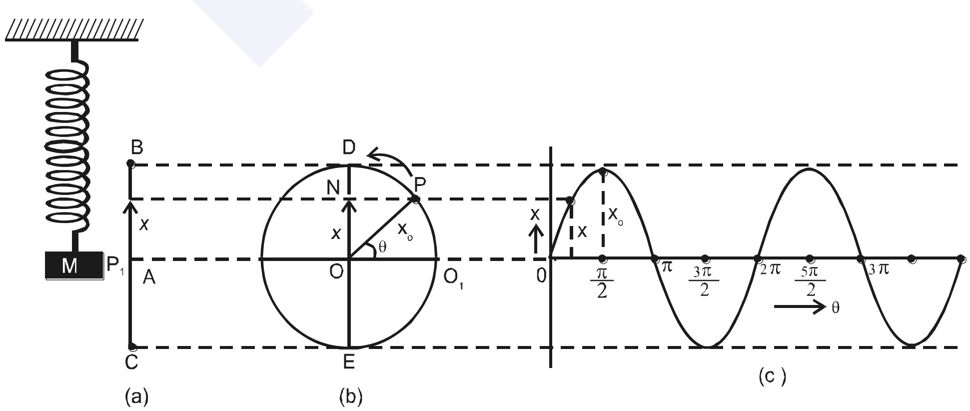
It can be seen that the curve showing the variation of displacement with time is a sine curve. It is usually known as wave-form of SHM.

Q.4 Discuss simple harmonic motion on the bases of uniform circular motion. Also calculate displacement and instantaneous velocity.

Ans. SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

Consider a mass 'm', attached with the end of a vertically suspended spring, vibrate simple harmonically with period 'T', frequency 'f' and amplitude 'x_0'. The motion of mass is displayed by the 'P_1' on the line 'BC' with 'A' as mean position, and 'B', 'C' as extreme position, as shown in figure.

Assuming 'A' as the position of the pointer at $t = 0$, it will move so that it is at 'B', 'A', 'C' and back to 'A' at instants T/4, T/2, 3T/4 and T respectively. This will complete one cycle of vibration with amplitude of vibration being $x_0 = AB = AC$.



(a) (b) (c)

Consider another point 'P' moving on a circle of radius 'x_0' with a uniform angular frequency $\omega = 2\pi/T$, where 'T' is the time period of the vibration of the pointer. It may be noted that the radius of the circle is equal to the amplitude of the pointer's motion.

Consider the motion of the point 'N', the projection of 'P' on the diameter 'DE' drawn parallel to the line of vibration of the pointer as shown in figure. Note that the level of points 'D' and 'E' is the same as the points 'B' and 'C'. As 'P' describes uniform circular motion with a constant angular speed 'ω', 'N' oscillates to and fro on the diameter 'DE' with time period T. Assuming O_1, to be the position of P at t = 0, the position of the point N at the instants 0, T/4, T/2, 3T/4 and T will be at the points O, D, O, E and O respectively. A comparison of the motion of N with that of the pointer P_1 shows that it is a replica of the pointer's motion. Thus the expressions of the displacement, velocity and acceleration for the motion of N also hold good for the pointer P_1, executing SHM.

Displacement

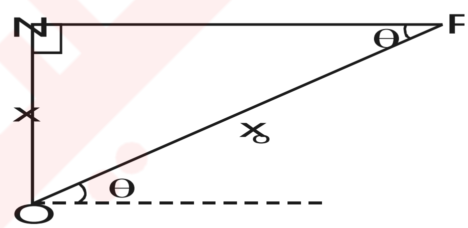
If we count the time t = 0 from the instant when P is passing through O_1, the angle, which the radius 'OP' sweeps out in time 't', is $\theta = \omega t$. The displacement x of the point N at instant will be

$$x = ON = OP \sin \angle QOP$$

Since $OP = x_o$

$$\angle O_1OP = \theta = \omega t$$

$$x = x_o \sin \omega t$$



This will be also the displacement of the pointer at the instant 't'.

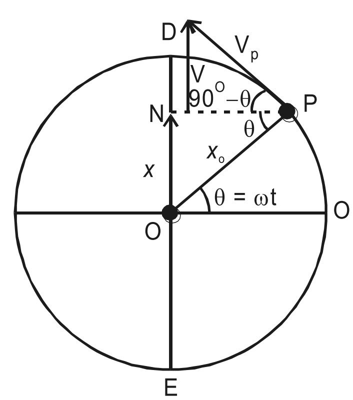
Instantaneous Velocity

The velocity of point 'P' at the instant 't' will be directed along the tangent to the circle at 'P' and its magnitude will be

$$V = r\,\omega$$

$$V_P = x_0\,\omega \quad (\because\ r = x_0) \qquad \text{......... (i)}$$

As the motion of 'N' on the diameter 'DE' is due to the motion of 'P' on the circle, the velocity of 'N' is actually the component of 'V_P' in a direction parallel to the diameter 'DE'.



From figure this component is

$$\frac{V}{V_P} = \sin(90^o - \theta)$$

$$V = V_P \cos\theta \quad \because \sin(90^o - \theta) = \cos\theta$$

Putting the value of V_P.

$$\therefore \quad V = x_0\,\omega\cos\theta \qquad \text{......... (ii)}$$

The direction of velocity of 'N' depends upon the value of phase angle θ. When 'θ' is between 0^o to 90^o the direction is from 'O' to 'D', for 'θ' between 90^o to 270^o, its direction is from 'D' to 'E'. When θ is between 270^o to 360^o, the direction of motion is from 'E' to 'O'.

Consider ΔONP

$$\frac{\text{Base}}{\text{Hypt}} = \cos\theta$$

$$\frac{NP}{OP} = \cos\theta$$

$$\frac{NP}{x_0} = \cos\theta \qquad \ldots\ldots\ldots \text{(iii)}$$

By using Pythagorean theorem,

$$(H)^2 = (P)^2 + (B)^2$$

$$x_0^2 = x^2 + (NP)^2$$

$$(NP)^2 = x_0^2 - x^2$$

$$NP = \sqrt{x_0^2 - x^2}$$

Putting this value in equation (iii).

$$\therefore \qquad \cos\theta = \frac{\sqrt{x_0^2 - x^2}}{x_0}$$

Putting this value in equation (ii).

$$\therefore \qquad V = x_0\,\omega\,\frac{\sqrt{x_0^2 - x^2}}{x_0}$$

$$\boxed{V = \omega\sqrt{x_0^2 - x^2}}$$

This equation shows that at mean position, where $x = 0$, the velocity is maximum and at the extreme positions where $x = x_0$, the velocity is zero.

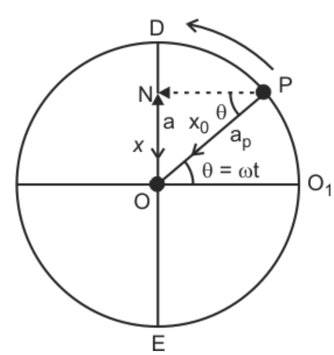
Q.5 Derive the relation for acceleration in terms of ω.

Ans. ACCELERATION IN TERMS OF ω

When the point P is moving on a circle, it has an acceleration

$$a_p = x_o\omega^2 \qquad \text{because} \qquad \boxed{a_p = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2}$$

$$a_p = \frac{V_P^2}{r} \qquad \ldots \text{(i)} \qquad (\because a_c = a_p)$$



At instant t, its direction will be along PO. The acceleration of the point N will be component of the acceleration a_p along the diameter DE on which N moves due to motion of P. The value of this component is

$$a_p \sin\theta = x_o\omega^2 \sin\theta$$

Thus the acceleration of N is

$$a = x_o\omega^2 \sin\theta$$

Considering ΔONP

$$\frac{a}{a_p} = \sin\theta$$

$$a = a_p \sin\theta$$

Putting value of a_p

$$\therefore \quad a = x_0\,\omega^2 \sin\theta \qquad \ldots\ldots\ldots \text{(ii)}$$

Considering ΔONP

$$\frac{ON}{OP} = \sin\theta$$

$$\therefore \quad \sin\theta = \frac{x}{x_0}$$

Putting this value in equation (ii)

$$\therefore \quad a = x_0\,\omega^2\left(\frac{x}{x_0}\right)$$

$$\vec{a} = \omega^2\,\vec{x}$$

Since acceleration of N is directed towards mean position. So

$$\therefore \quad \vec{a} = -\,\omega^2\,\vec{x}$$

$$\text{or} \quad \vec{a} \propto -\,\vec{x}$$

This shows that the acceleration is proportional to displacement and is directed towards the mean position, which is the characteristics of SHM. Thus N is executing SHM with same amplitude, period and instantaneous displacement as the pointer P_1. This conforms that the motion of N is just a replica, of the pointers motion.

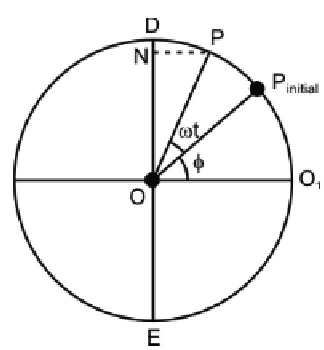
Q.6 What is meant by phase angle?

Ans. **PHASE**

"The angle $\theta = \omega t$, which specifies the displacement as well as the direction of motion of the point, executing SHM is known as phase."

It is the angle, which the rotating radius 'OP' makes with reference direction 'OO_1' at any instant 't'.

At 't' = 0, the position of rotating radius 'OP' is along 'OO_1' so that 'N' is at its mean position and the displacement is 'O'. In general at t = 0, 'OP' can make any angle 'ϕ' with the reference 'OO_1' as shown in Fig. 1.



In time 't' the radius will rotate by 'ωt'. So, now 'OP' would make an angle $(\omega t + \phi)$ with 'OO_1' as the instant 't' and displacement 'x' at that instant is given by

$$x = x_0 \sin(\omega t + \phi)$$

Now the phase angle is

$$\theta = \omega t + \phi$$

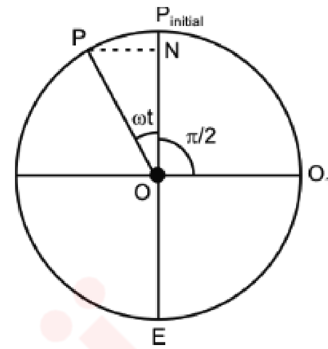
at $t = 0$

$$\therefore \quad \theta = \phi$$

So ϕ is the initial phase. If we take initial phase as 90^o then,

$$x = x_0 \sin(\omega t + 90^o)$$

$$x = x_0 \cos \omega t$$



This also gives the "displacement" of SHM, but in this case the 'N' is starting its motion from extreme position instead of mean position as shown in figure. Now shadow 'N' is moving along horizontal diameter.

Q.7 Discuss the motion of mass attach with one end of spring placed on horizontal surface. A horizontal mass spring system also derive the expressions for time period, instantaneous displacement and velocity.

Ans. A HORIZONTAL MASS-SPRING SYSTEM

Consider the vibrating mass attached to a spring as shown whose acceleration at any instant is given by

$$a = -\frac{k}{m}x \qquad \text{.........} (1)$$

As k and m are constant. We see that the acceleration is directly proportional to the displacement x and its direction is towards the mean position. In case of circular motion, the acceleration is

$$a = -\omega^2 x \qquad \text{.........} (2)$$

Comparing (1) and (2)

$$-\omega^2 x = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

Taking square root

$$\omega = \sqrt{\frac{k}{m}}$$

As $T = \frac{2\pi}{\omega}$

Time Period

As the time period of the mass is

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Instantaneous Displacement

The instantaneous displacement of mass is given by

$$x = x_0 \sin \omega t$$

Putting value of ω

$$x = x_0 \sin \sqrt{\frac{k}{m}}\ t$$

Instantaneous Velocity

The instantaneous velocity V of mass m is given by

$$V = \omega\sqrt{x_0^2 - x^2}$$

Putting value of ω

$$V = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$$

$$V = \sqrt{\frac{k}{m}}\sqrt{x_0^2\left(1 - \frac{x^2}{x_0^2}\right)}$$

$$V = x_0\sqrt{\frac{k}{m}}\ \sqrt{1 - \frac{x^2}{x_0^2}} \qquad \text{.........} (3)$$

This equation shows that velocity of mass gets maximum equal to V_0 when $x = 0$ i.e. at mean position.

$$V_0 = \sqrt{\frac{k}{m}}\sqrt{x_0^2(1-0)}$$

$$V_0 = \sqrt{\frac{k}{m}}\sqrt{x_0^2}$$

$$V_0 = \sqrt{\frac{k}{m}}\ (x_0)$$

$$V_0 = x_0\sqrt{\frac{k}{m}}$$

Putting $V_0 = x_0\sqrt{\frac{k}{m}}$ in equation (3)

$$\therefore \qquad V = V_0\sqrt{1 - \frac{x^2}{x_0^2}}$$

Note: The formula derived for displacement and velocity are also valid for vertically suspended mass-spring system provided air friction is not considered.

Q.8 What is simple pendulum. Show that its motion is S.H.M. Also derive the expression for time period and frequency.

Ans. **SIMPLE PENDULUM**

It is an arrangement in which a heavy bob is suspended with light and inextensible string. "A simple pendulum consists of a small heavy mass 'm', suspended by a light inextensible string of length 'l', fixed at its upper end," by a frictionless support as shown in the figure.

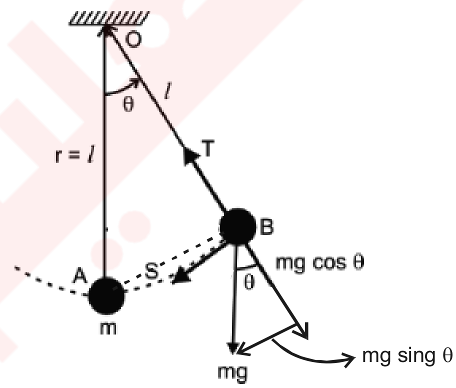
Explanation

When such a pendulum is displaced from its mean position through a small angle 'θ' to position 'B' and released. It starts oscillating to and fro.

Two forces are acting on bob

(i) tension 'T' along the string.

(ii) Its weight 'mg' downwards. The weight 'mg' can be resolved into two components.



(i) mg sin θ along the tangent at B.

(ii) mg cos θ along the string to balance the tension in the string.

i.e. $T = mg\cos\theta$

Therefore 'mg sin θ' is responsible for the motion of the bob. The restoring force at 'B' will be,

$$F = -mg\sin\theta$$

As $F = ma$

$$\therefore \quad ma = -mg\sin\theta$$

$$a = -g\sin\theta \quad \text{......... (1)}$$

If θ is very small then $\sin\theta \approx \theta$ (in radian)

Equation (1) becomes

$$\therefore \quad a = -g\theta \quad \text{......... (2)}$$

As θ is very small.

$\therefore \quad \text{Arc } x = \text{chord } x$

As $\quad s = r\theta$

$\text{Arc length} = s$

$\because \quad r = l \quad \text{and} \quad s = x$

$\therefore \quad x = l\theta$

$$\theta = \frac{x}{l}$$

Putting this value in equation (2).

$$\therefore \quad a = -g\frac{x}{l}$$

$$a = -\frac{g}{l}x \qquad \text{.........} (3)$$

Since, $\frac{g}{l}$ is constant

So, $\quad a = -\text{Constant } x$

$\therefore \quad a \propto -x$

which shows that the acceleration of a simple pendulum is directly proportional towards the displacement and is always directed toward the mean position. So the motion of simple pendulum is a simple harmonic motion.

But for simple harmonic motion

$$a = -\omega^2 x \qquad \text{.........} (4)$$

From (3) and (4)

$$-\omega^2 x = -\frac{g}{l} x$$

$$\omega^2 = \frac{g}{l}$$

Taking square root

$$\therefore \quad \omega = \sqrt{\frac{g}{l}}$$

Time Period

$\therefore$ time period of simple pendulum is

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{g/l}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

"This equation shows that the time period depends only on the length of the pendulum and the acceleration due to gravity. It is independent of the mass of the bob."

It is also independent of amplitude.

Q. What is the length of the second's pendulum?

Ans. **As, $T = 2$ sec**

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring the both sides

$$T^2 = 4\pi^2\left(\sqrt{\frac{l}{g}}\right)$$

$$l = \frac{T^2 g}{4\pi^2}$$

$$l = \frac{(2)^2 \times 9.8}{4\,(3.14)^2}$$

$$l = \frac{4 \times 9.8}{4 \times 9.86}$$

$$l = \frac{9.8}{9.86}$$

$$= 0.9939 \text{ m}$$

$$l = 99.39 \text{ cm}$$

Second's Pendulum

"A second's pendulum is a pendulum which completes one vibration in two seconds."

Hence time period of such a pendulum is 2 second.

As, $T = 2$ sec

$$\therefore \quad f = \frac{1}{T}$$

$$f = \frac{1}{2}$$

$$f = 0.5 \text{ Hz}$$

Q.9 Explain the energy conservation in S.H.M.

Ans. **ENERGY CONSERVATION IN SHM**

When the mass 'm' is pulled slowly, the spring is stretched by an amount 'x_0' against the elastic restoring force 'F'. It is assumed that the stretching is done slowly, so that acceleration is zero.

According to Hook's law:

$$F = k x_0$$

If displacement $= 0$ then $F = 0$

If displacement $= x_0$ then $F = k x_0$

$$F = \text{Average force}$$

$$= \frac{0 + k\,x_0}{2}$$

$$= \frac{1}{2} k\,x_0$$

Work done in moving the mass ‘m’ through ‘x_0’ is

$$W = F\,d$$

$$W = \frac{1}{2} k\,x_0 \times x_0$$

$$W = \frac{1}{2} k\,x_0^2$$

This work appears as elastic P.E. of spring.

Hence, Elastic P.E. $= \frac{1}{2} k\,x_0^2$

This gives maximum P.E. at the extreme position.

$$P.E_{max} = \frac{1}{2} k\,x_0^2$$

Energy at Extreme Position

At extreme position,

$$P.E. = \frac{1}{2} k\,x_0^2$$

$$K.E. = 0$$

$\therefore$ Total energy $= E = P.E. + K.E.$

$$E = \frac{1}{2} k\,x_0^2 + 0$$

So, $E = \frac{1}{2} k\,x_0^2$

Energy at any Instant ‘t’

At any instant ‘t’, if the displacement is ‘x’, then P.E. at that instant is

$$P.E. = \frac{1}{2} k\,x^2$$

the velocity at that instant is;

$$V = x_0 \sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_0^2}\right)}$$

K.E. at that instant is

$$\text{K.E.} = \frac{1}{2} m V^2$$

Putting the value of 'V'.

$$\text{K.E.} = \frac{1}{2} m x_0^2 \left(\frac{k}{m} \left(1 - \frac{x^2}{x_0^2} \right) \right)$$

$$\text{K.E.} = \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right)$$

$$\therefore \quad E = \text{P.E.} + \text{K.E.}$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 \left(1 - \frac{x^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 - \frac{1}{2} k x_0^2 \left(\frac{x^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2$$

$$E_{total} = \frac{1}{2} k x_0^2$$

Energy at Mean Position

K.E. is maximum at mean position.

$$\text{i.e.,} \quad x = 0$$

$$\text{K.E.}_{max} = \frac{1}{2} k x_0^2 \left(1 - \frac{(0)^2}{x_0^2} \right)$$

$$= \frac{1}{2} k x_0^2 \left(1 - \frac{0}{x_0^2} \right)$$

$$= \frac{1}{2} k x_0^2 (1 - 0)$$

$$\text{K.E.}_{max} = \frac{1}{2} k x_0^2$$

$$\text{Also} \quad \text{P.E.} = 0$$

$$\therefore \quad E = \text{P.E.} + \text{K.E.}$$

$$E = 0 + \frac{1}{2} k x_0^2$$

$$E = \frac{1}{2} k x_0^2$$

"Hence the total energy of the vibrating mass and spring system is constant." When the K.E of the mass is maximum, the P.E of the spring is zero. On the other hand, when the P.E of the spring is maximum, the K.E of the mean is zero.

Q.10 Define free and forced oscillations.

Ans. FREE AND FORCED OSCILLATIONS

Free Oscillations

"A body is said to be executing free vibrations, when it oscillates without the interference of an external force. the frequency of these free vibrations is known as its natural frequency."

Example

A simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

"If a freely oscillating system is subjected to an external force, then forced vibrations will take place."

Example

When the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

The vibrations of a factory floor caused by running of heavy machinery.

DRIVEN HARMONIC OSCILLATOR

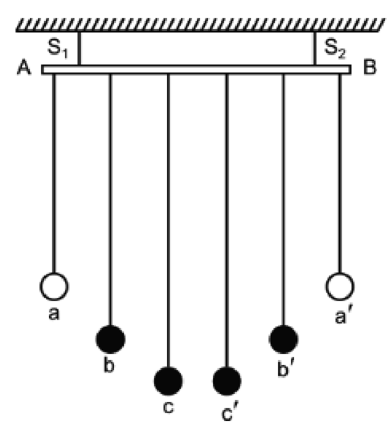
"A physical system under going forced vibrations is known as driven harmonic oscillator."

Q.11 Explain the phenomenon resonance with example.

Ans. RESONANCE

Definition

"The resonance occurs, when the frequency of the applied force is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator."



To demonstrate the resonance effect, an apparatus is shown in the Fig. A horizontal rod 'AB' is supported by two strings S_1 and S_2. Three pairs of pendulums aa', bb' and cc' are suspended to this rod. The length of each pair is the same but is different for different pairs. If one of these pendulums, says c, is displaced in a direction perpendicular to the plane of the paper, then its resultant oscillatory motion causes in rod AB a very slight disturbing motion, whose period is the same as that of c'.

Due to this slight motion of rod, each of remaining pendulums (aa', bb' and cc') under go a slight periodic motion. This causes the pendulum c', whose length and, hence, period is exactly the same as that of c, to oscillate back and forth with steadily increasing amplitude. However, the amplitude of the other pendulums remain small through out the subsequent motions of c and c′, because their natural periods are not the same as that of the disturbing force due to rod AB.

Example 1

A swing is a good example of mechanical resonance.

Example 2

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Example 3

Tuning a radio is the best example of electrical resonance.

Example 4

The heating and cooking of food very efficiently and evenly by microwave oven.

Do You Know?

All structures are likely to resonate at one or more frequencies. This can cause problem. It is especially important to test all the components in helicopters and aeroplanes; resonance in an aeroplane's wing or a helicopter rotor could be very dangerous.

Interesting Information

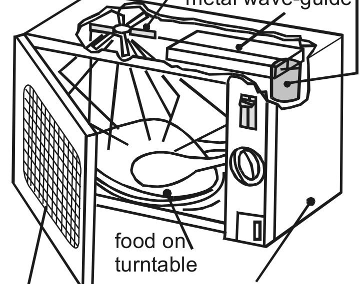
The collapse of Tacoma Narrow Bridge (USA) is suspected to be due to violent resonance oscillations.

Advantages and Disadvantages of Resonance

We come across many examples of resonance in every day life. A swing is a good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

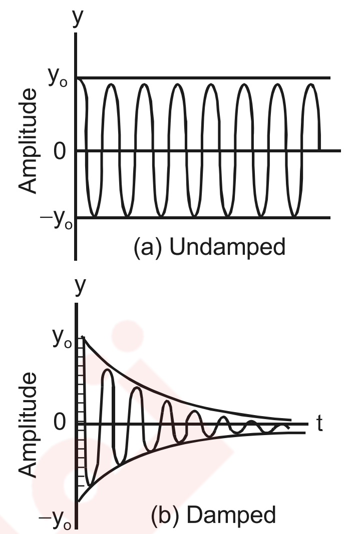


Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven figure. The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency, the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

Q.12 What are the damped oscillations and sharpness of resonance?

Ans. **DAMPED OSCILLATIONS**

This is a common observation that the amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.



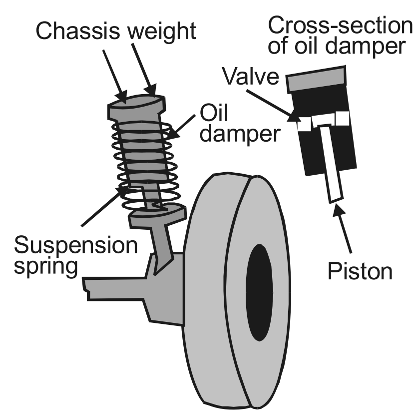
We know from our everyday experience that the motion of any macroscopic system is accompanied by frictional effects. While describing the motion of a simple pendulum, this effects was completely ignored. As the bob of the pendulum moves to and fro, then in addition to the weight of the bob and the tension in the string, bob experiences viscous drag due to its motion through the air. Thus simple harmonic motion is an idealization (Fig. a). In practice, the amplitude of this motion gradually becomes smaller and smaller because of friction and air resistance because the energy of the oscillator is used up in doing work against the resistive forces. Fig. b shows how the amplitude of a damped simple harmonic wave changes, with time as compared with an ideal un-damped harmonic wave. Thus we see that

Damping is the process whereby energy is dissipated from the oscillating system.

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillation.

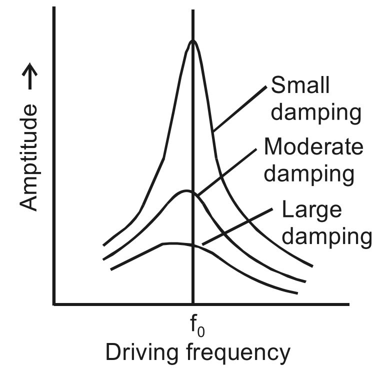
Q.13 What is sharpness of resonance?

Ans. SHARPNESS OF RESONANCE



At resonance, the amplitude of vibration becomes very large when damping is small. Thus, damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency slightly different from the resonant frequency. Where as a heavily damped system has a fairly flat resonance curve as is shown in an amplitude frequency graph in figure.

The effect of damping can be observed by attaching a very light mass such as pith ball, and another of the same length carrying a heavy mass such as lead bob of equal size, to a rod as shown in figure (1). They are set into motion (vibrations) by a third pendulum of equal length attached to the same rod. It is observed that the amplitude of lead bob is much greater than that of pith ball. The damping effect for the pith ball due to air resistance is much greater than that for lead bob. Thus, the sharpness of the resonance curve of the resonating system depends on the frictional loss of energy.


Amptitude →
Small damping
Moderate damping
Large damping
f_0
Driving frequency

SOLVED EXAMPLES

EXAMPLE 7.1

A block weighing 4.0 kg extends a spring by 0.16 m from its un–stretched position. The block is removed and a 0.5 kg body is hung from the same spring. If the spring is now stretched and then released, what is the period of vibration?

Data

Mass of block = m = 4 kg

Extension in spring = x = 0.16 m

Mass of new body = m' = 0.50 kg

To Find

Time period = T = ?

SOLUTION

Using: $F = k\,x$

$$\frac{F}{x} = K$$

Putting the values

$$K = \frac{4 \times 9.8}{0.16}$$

$$K = 245\ \text{N/m}$$

Using, $T = 2\pi\sqrt{\frac{m'}{k}}$

Putting the values

$$T = 2\pi\sqrt{\frac{0.5}{245}}$$

$$T = 0.285\ \text{sec}$$

Result

Time period = T = 0.285 sec

EXAMPLE 7.2

What should be the length of a simple pendulum whose period is 1.0 second at a place where $g = 9.8\ \text{ms}^{-2}$? What is the frequency of such a pendulum?

Data

Length of simple pendulum = l = ?

Time period = T = 1 sec

Value of g $= 9.8\ m/s^2$

To Find

Frequency $= f = ?$

SOLUTION

Using, $T = 2\pi\sqrt{\frac{l}{g}}$

Squaring both sides

$$T^2 = 4\pi^2\left(\frac{l}{g}\right)$$

$$\frac{T^2\,g}{4\pi^2} = l$$

Putting the values,

$$l = \frac{(1)^2\,(9.8)}{4 \times \pi^2}$$

$$l = 0.25\ m$$

and $f = \frac{1}{T}$

$$f = \frac{1}{1}$$

$$f = 1\ Hz$$

Result

Frequency of vibration $= f = 1\ Hz$

EXAMPLE 7.3

A spring whose spring constant is 80.8 Nm^{-1} vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down so that on being released, it may pass the mean position with a velocity of 1.0 ms^{-1}.

Data

Spring constant $= K = 80.0\ Nm^{-1}$

Mass $= m = 1.0\ kg$

Maximum velocity $= V_{max} = 1.0\ ms^{-1}$

To Find

Distance $= x_0 = ?$

SOLUTION

At mean position

$$K.E._{max} = \frac{1}{2} k x_0^2$$

$$\frac{1}{2} m V_{max}^2 = \frac{1}{2} k x_0^2$$

$$m V_{max}^2 = k x_0^2$$

Putting the values;

$$(1)(1)^2 = (80)(x_0)^2$$

$$\frac{1}{80} = x_0^2$$

Taking square root

$$x_o = \frac{1}{\sqrt{80}}$$

$$x_o = 0.11 \text{ m}$$

Result

Distance = x_o = 0.11 m

Chapter 8

WAVES

LEARNING OBJECTIVE

At the end of this chapter the students will be able to:

- Recall the generation and propagation of waves.
- Describe the nature of the motions in transverse and longitudinal waves.
- Understand and use the terms wavelength, frequency and speed of wave.
- Understand and use the equation $v = f\lambda$.
- Understand and describe Newton's formula of speed of sound.
- Derive Laplace correction in Newton's formula of speed of sound.
- Derive the formula $v = v_o + 0.61t$.
- Explain and use the principle of superposition.
- Understand the terms interference and beats.
- Understand and describe reflection of waves.
- Explain the formation of a stationary wave using graphical method.
- Understand the terms node and anti-node.
- Understand and describe modes of vibration of string.
- Understand and describe Doppler's effect and its causes.

Waves transport energy without transporting matter. The energy transportation is carried by a disturbance, which spreads out from a source. We are well familiar with different types of waves such as water waves in the ocean, or gently formed ripples on a still pond due to rain drop. When a musician plucks a guitar-string, sound waves are generated which on reaching our ear, produce the sensation of music. Wave disturbances may also come in a concentrated bundle like the shock waves from an aeroplane flying at supersonic speed. Whatever may be the nature of waves, the mechanism by which it transports energy is the same. A succession of oscillatory motions are always involved. The wave is generated by an oscillation in the vibrating body and propagation of wave through space is by means of oscillations. The waves which propagate by the oscillation of material particles are known as mechanical waves.

There is another class of waves which, instead of material particles, propagate out in space due to oscillations of electric and magnetic fields. Such waves are known as electromagnetic waves. We will undertake the study of electromagnetic waves at a later stage. Here we will consider the mechanical waves only. The waves generated in ropes, strings, coil of springs, water and air are all mechanical waves.

So far we have been considering motion of individual particles but in case of mechanical waves, we study the collective motion of particles. An example will help us here. If you look at a black and white picture in a newspaper with a magnifying glass, you will discover that the picture is made up of many closely spaced dots. If you do not use the magnifier, you do not see the dots. What you see is the collective effect of dots in the form of a picture. Thus what we see as mechanical wave is actually the effect of oscillations of a very large number of particles of the medium through which the wave is passing.

Do You Know?

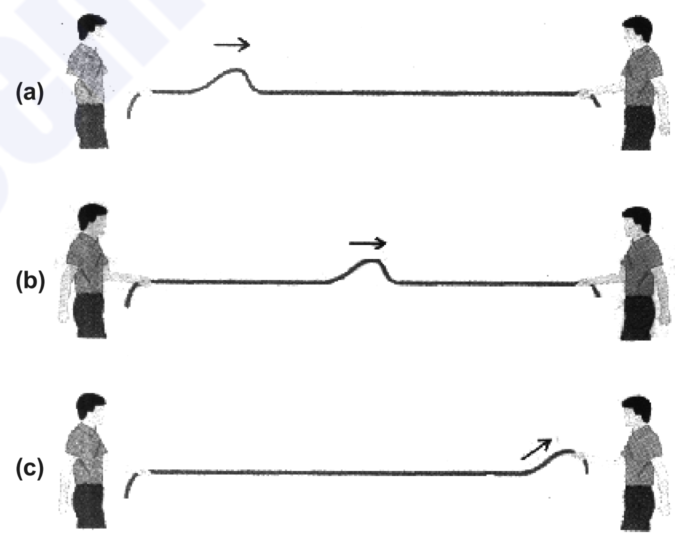
Ultrasonic waves are particularly useful for undersea communication and detection systems. High frequency radio waves, used in radars travel just a few centimetres in water, whereas highly directional beams of ultrasonic waves can be made to travel many kilometres.

PROGRESSIVE WAVES

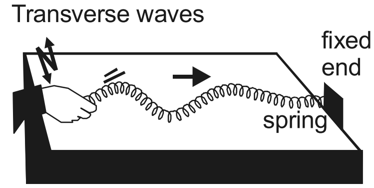
Drop a pebble into water. Ripples will be produced and spread out across the water. The ripples are the examples of progressive waves because they carry energy across the water surface. A wave, which transfers energy by moving away from the source of disturbance, is called a progressive or travelling wave. There are two kinds of progressive waves – transverse waves and longitudinal waves.

Transverse and Longitudinal Waves

Consider two persons holding opposite ends of a rope or a hosepipe. Suddenly one person gives one up and down jerk to the rope. This disturbs the rope and creates a hump in it which travels along the rope towards the other person (Figure a and b).



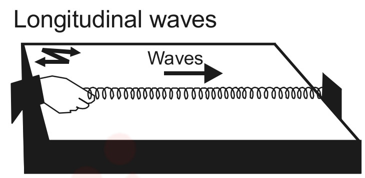
When this hump reaches the other person, it causes his hand to move up (Figure c). Thus the energy and momentum imparted to the end of the rope by the first person has reached the other end of the rope by travelling through the rope i.e., a wave has been set up on the rope in the form of a moving hump. We call this type of wave a pulse. The forward motion of the pulse from one end of the rope to the other is an example of progressive wave. The hand jerking the end of the rope is the source of the wave. The rope is the medium in which the wave moves.



(a)

A large and loose spring coil (slinky spring) can be used to demonstrate the effect of the motion of the source in generating waves in a medium. It is better that the spring is laid on a smooth table with its one end fixed so that the spring does not sag under gravity.

If the free end of the spring is vibrated from side to side, a pulse of wave having a displacement pattern shown in figure (a) will be generated which will move along the spring.



(b)

If the end of the spring is moved back and forth, along the direction of the spring itself as shown in figure (b), a wave with back and forth displacement will travel along the spring. Waves like those in figure (a) in which displacement of the spring is perpendicular to the direction of the waves are called transverse waves. Waves like those in figure (b) in which displacements are in the direction of propagation of the waves are called longitudinal waves. In this example the coil of spring is the medium, so in general we can say that

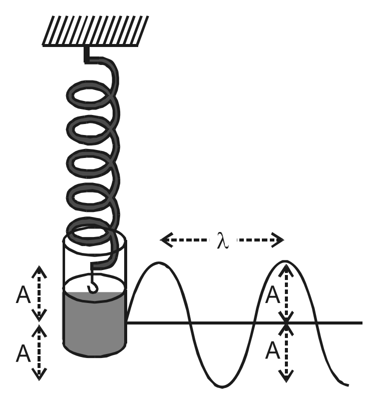
Transverse waves are those in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves and longitudinal waves are those in which the particles of the medium have displacements along the direction of propagation of waves.

Both types of waves can be set up in solids. In fluids, however, transverse waves die out very quickly and usually cannot be produced at all. That is why, sound waves in air are longitudinal in nature.

PERIODIC WAVES

Upto now we have considered wave in the form of a pulse which is set up by a single disturbance in a medium like the snapping of one end of a rope or a coil spring. Continuous, regular and rhythmic disturbances in a medium result from periodic vibrations of a source which cause periodic waves in that medium. A good example of a periodic vibrator is an oscillating mass-spring system (figure a). We have already studied in the previous chapter that the mass of such a system executes SHM.

Transverse Periodic Waves

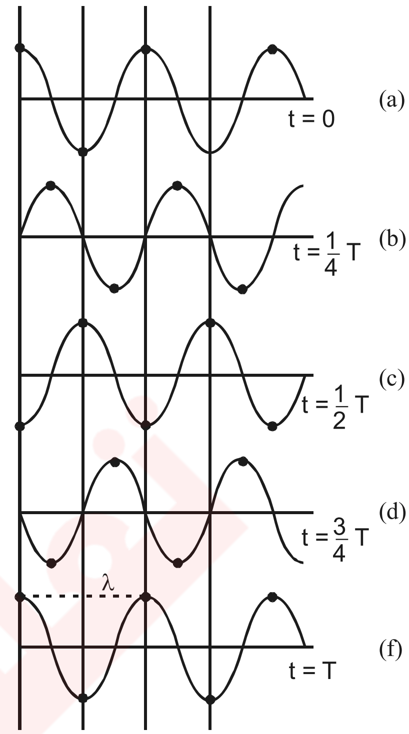


Imagine an experiment where one end of a rope is fastened to a mass spring vibrator. As the mass vibrates up and down, we observe a transverse periodic wave travelling along the length of rope (figure b). The wave consists of crests and troughs. The crest is a pattern in which the rope is displaced above its equilibrium position, and in troughs, it has a displacement below its equilibrium position.

As the source executes harmonic motion up and down with amplitude A and frequency f, ideally every point along the length of the rope executes SHM in turn, with the same amplitude and frequency. The wave travels towards right as crests and troughs in turn, replace one another, but the points on the rope simply oscillates up and down. The amplitude of the wave is the maximum value of the displacement in a crest or trough and it is equal to the amplitude of the vibrator. The distance between any two consecutive crests or troughs is the same all along the length of the rope. This distance is called the wavelength of the periodic wave and is usually denoted by the Greek letter lambda λ (figure b).

In principle, the speed of the wave can be measured by timing the motion of a wave crest over a measured distance. But it is not always convenient to observe the motion of the crest. As discussed below, however, the speed of a periodic wave can be found indirectly from its frequency and wavelength.

As a wave progresses, each point in the medium oscillates periodically with the frequency and period of the source. Figure illustrates a periodic wave moving to the right, as it might look in photographic snapshots taken every 1/4 period. Follow the progress of the crest that started out from the extreme left at t = 0. The time that this crest takes to move a distance of one wavelength is equal to the time required for a point in the medium to go through one complete oscillation. That is the crest moves one wavelength λ in one period of oscillation T. The speed v of the crest is therefore,



$$v = \frac{\text{Distance moved}}{\text{Corresponding time interval}} = \frac{\lambda}{T}$$

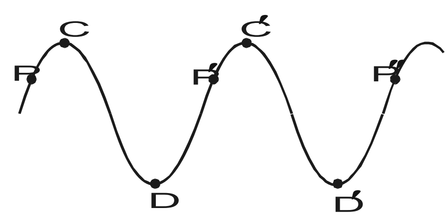
All parts of the wave pattern move with the same speed, so the speed of any one crest is just the speed of the wave. We can therefore, say that the speed v of the waves is

$$v = \frac{\lambda}{T} \qquad \text{...... (1)}$$

but $\frac{1}{T}$ = f, where f is the frequency of the wave. It is the same as the frequency of the vibrator, generating the waves. Thus eq. (1) becomes

$$v = f\lambda \qquad \text{...... (2)}$$

Phase Relationship between Two Points on a Wave



The profile of periodic waves generated by a source executing SHM is represented by a sine curve. Figure shows the snapshot of a periodic wave passing through a medium. In this figure, set of points are shown which are moving in unison as the periodic wave passes. The points C and C′, as they move up and down, are always in the same state of vibration i.e., they always have identical displacements and velocities. Alternatively, we can say that as the wave passes, the points C and C′ move in phase. We may also say that C′ leads C by one time period of 2π radian. Any point at a distance x, C lags behind by phase angle.

$$\phi = \frac{2\pi x}{\lambda}$$

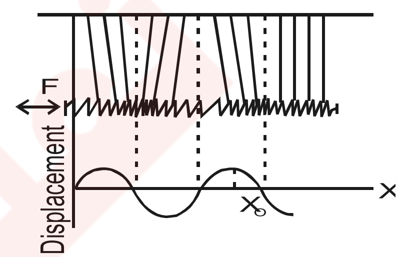
So is the case with points D and D′. Indeed there are infinitely many such points along the medium which are vibrating in phase. Points separated from one another through distances of λ, 2λ, 3λ,

…… are all in phase with each other. These points can be anywhere along the wave and need not correspond with only the highest and lowest points. For example, points such as P, P′, P″, …… are all in phase. Each is separated from the next by a distance λ.

Some of the points are exactly out of step. For example, when point C reaches its maximum upward displacement, at the same time D reaches its maximum downward displacement. At the instant that C begins to go down, D begins to move up. Points such as these are called one half period out of phase. Any two points separated from one another by $\frac{\lambda}{2}$, $3\frac{\lambda}{2}$, $5\frac{\lambda}{2}$, …… are out of phase.

Longitudinal Periodic Waves

In the previous section we have considered the generation of transverse periodic waves. Now we will see how the longitudinal periodic waves can be generated.



Consider a coil of spring as shown in figure. It is suspended by threads so that it can vibrate horizontally. Suppose an oscillating force F is applied to its end as indicated. The force will alternately stretch and compress the spring, thereby sending a series of stretched regions (called rarefaction) and compressions down the spring. We will see the oscillating force causes a longitudinal wave to move down the spring. This type of wave generated in springs is also called a compressional wave. Clearly in a compressional wave, the particles in the path of wave move back and forth along the line of propagation of the wave.

Notice in figure, the supporting threads would be exactly vertical if the spring were undisturbed. The disturbance passing down the spring causes displacements of the elements of the spring from their equilibrium positions. In figure, the displacements of the threads from the vertical are a direct measure of the displacements of the spring elements. It is, therefore, an easy way to graph the displacements of the spring elements from their equilibrium positions and this is done in the lower part of the figure.

Q.1 Define progressive waves with its types.

Ans. **PROGRESSIVE WAVES (TRAVELLING WAVE)**

A wave, which transfers energy in moving away from the source of disturbance, is called a progressive or travelling waves

There are two kinds of traveling waves

(i) Transverse waves.

(ii) Longitudinal waves.

(i) Transverse Waves

"Transverse waves are those in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves."

e.g., water waves, light waves.

(ii) Longitudinal Waves (Compressional Waves)

"Longitudinal waves are those in which particles of the medium have displacement along the direction of propagation of waves."

e.g., sound waves.

Both types of waves can be setup in solids in liquids however, transverse waves die out very quickly and usually cannot be produced at all that is why sound waves in air one longitudinal in nature.

Periodic Waves

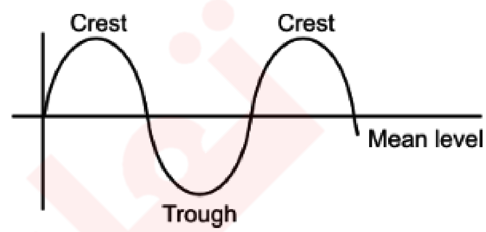
"Continuous, regular and rhythmic disturbances in a medium result from periodic vibrations of a source which cause periodic waves in that medium."

e.g., Oscillating mass–spring system.

Transverse Periodic Waves

Crest

"The portion of transverse wave above its mean position, is called crest."



Trough

"The potion of transverse wave below its mean position, is called trough."

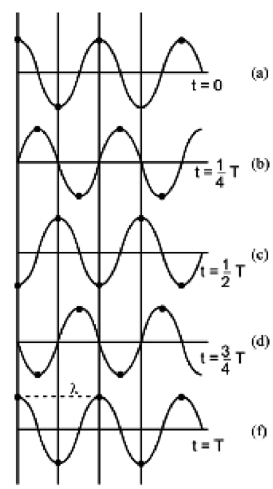
Wavelength

"The distance between any two consecutive crests or troughs, is called wave length."

It is denoted by a Greek letter Lambda (λ).

Q.2 Show that $V = \lambda f$.

Ans. The time that the crest takes to move a distance of one wave length is equal to the time required for a point in the medium to complete one oscillation i.e., crest moves one wave length 'λ' in one period of oscillation 'T', the speed 'V of the crest (wave) is



As, $V = \dfrac{\text{Distance moved}}{\text{Corresponding time interval}}$

$\therefore \quad V = \dfrac{\lambda}{T} \quad \left(\begin{array}{l} S = V\,t \\ V = \dfrac{S}{t} \end{array} \right)$

Since, $\dfrac{1}{T} = f$

$\therefore \quad V = \lambda f$

Which is the relation between speed, frequency and wavelength.

Longitudinal Periodic Waves

Compression

"The portion of longitudinal wave where particles of medium are very close to each other is called compression."

Rarefaction

"The portion of longitudinal wave where particles of medium are far apart from each other is called rarefaction."

Q.3 Explain Newton's formula for the speed of sound in air.

Ans. SPEED OF SOUND IN AIR

(i) Newton's Formula for Speed of Sound in Air

When one particle of the medium is disturbed, the disturbance in the form of wave travel in all directions in the medium. The velocity of disturbance depends upon the density and the elasticity of the medium. The lighter the density of the medium, more quickly the disturbance moves from point to point and similarly greater the elasticity of the medium, more quickly disturbance will be propogated from point to point in the medium. Newton proposed the following formula for the velocity of sound through the materials which is as follows:

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\text{Elasticity}}{\text{Density}}}$$

Where $\in$ is the elasticity of the medium and ρ is the density.

Newton assumed that when a sound wave travels through air, the temperature of the air during compression remains constant and pressure changes from P to $(P + \Delta P)$ and therefore, the volume changes from V to $(V - \Delta V)$. According to Boyle's law

$$PV = (P + \Delta P)(V - \Delta V)$$

or $$PV = PV - P\Delta V + V\Delta P - \Delta P\Delta V$$

The product $\Delta P\Delta V$ is very small and can be neglected. So, the above equation becomes:

$$0 = -P\Delta V + V\Delta P$$

$$P = \frac{V\Delta P}{\Delta V}$$

$$P = \frac{\Delta P}{\frac{\Delta V}{V}}$$

Where ΔP = Stress

Speed of sound in different media	
Medium	**Speed ms^{-1}**
Solids at 20°C	
Lead	1320
Copper	3600
Aluminium	5100
Iron	5130
Glass	5500
Liquids at 20°C	
Methanol	1120
Water	1483
Gasses of S.T.P.	
Carbon dioxide	258
Oxygen	315
Air	332
Helium	972
Hydrogen	1286

$$\frac{\Delta V}{V} = \text{Volume strain}$$

$$P = \frac{\text{Stress}}{\text{Volume strain}} = \text{Elasticity}$$

The above equation becomes:

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}}$$

On substituting the values of atmospheric pressure and density of air at S.T.P. in above equation, we find that the speed of sound waves in air comes out to be 280 ms^{-1}, whereas its experimental value is 332 ms^{-1}.

Q.4 How laplace correct the speed of sound in air?

Ans. LAPLACE CORRECTION FOR VELOCITY OF SOUND IN AIR

The sound waves travel in the form of compressions and rarefactions. The compressions and rarefactions are so rapid, the temperature of air does not remain constant. The temperature increases due to compressions and the temperature decreases due to the rarefactions. Therefore during compression the air does not lose heat due to conduction and during rarefaction it does not gain heat. Thus the temperature throughout the medium does not remain constant. The relation between volume and pressure (PV = Constant) is not true but it is given as

$$PV^{\gamma} = \text{Constant}$$

Where γ is constant and its value depends upon the nature of the gas where

$$\gamma = \frac{\text{Molar specific heat at constant pressure}}{\text{Molar specific heat at constant volume}}$$

If 'P' be the pressure then the change in pressure is very small which is $P + \Delta P$ therefore volume decreases from V to $V - \Delta V$ then

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma}$$

$$PV^{\gamma} = (P + \Delta P)V^{\gamma}\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

Applying Binomial theorem:

$$P = (P + \Delta P)\left(1 - \gamma\frac{\Delta V}{V}\right)$$

or

$$P = P - \gamma P\frac{\Delta V}{V} + \Delta P - \gamma \Delta P\frac{\Delta V}{V}$$

Where $\left(\gamma \Delta P\frac{\Delta V}{V}\right)$ is negligible. Hence, we have

$$0 = -\gamma P\frac{\Delta V}{V} + \Delta P$$

For Your Information

Values of Constant

Types of gas	γ
Monoatomic	1.67
Diatomic	1.40
Polyatomic	1.29

For Your Information

Ranges of Hearing

Organisms	Frequencies (Hz)
Dolphin	150 – 150,000
Bat	1000 – 120,000

Cat	60 – 70,000
Dog	15 – 50,000
Human	20 – 20,000

or $$\frac{\Delta P}{\frac{\Delta V}{V}} = \gamma P = E$$

Therefore

$$V = \sqrt{\frac{E}{\rho}}$$

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

On substituting the value of atmospheric pressure and γ then the speed of the sound is 333 m/s. This value of speed of sound is very close to the experimental value. Thus the laplace correction must therefore be correct.

Q.5 What is effect of variation of pressure on the speed of sound?

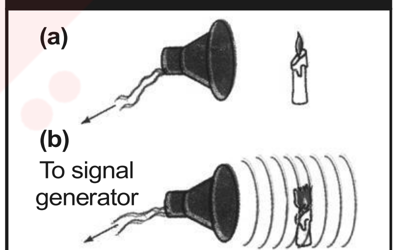
Ans. EFFECT OF PRESSURE ON SPEED OF SOUND

As, $$V = \sqrt{\frac{\gamma P}{\rho}}$$

$$\left(\begin{array}{l} \rho = \frac{m}{V} \\ P \rightarrow \text{increase} \\ V \rightarrow \text{decreases} \\ \rho \rightarrow \text{increase} \\ \therefore\ P \propto \rho \end{array}\right)$$

Since density is proportional to the pressure so the speed of sound is not affected by the variation in pressure of the gas.

Tidbits



Sound waves cause the candle flame to flicker.

Q.6 What is effect of density on the speed of sound?

Ans. EFFECT OF DENSITY ON SPEED OF SOUND

As, $$V = \sqrt{\frac{\gamma P}{\rho}}$$

At the same temperature and pressure for the gases having the same value of γ, the velocity is inversely proportional to the square root of their density.

i.e., $$V \propto \frac{1}{\sqrt{\rho}}$$

Note: *Speed of sound in hydrogen is four times its speed in oxygen as density of the oxygen is sixteen times that of the hydrogen.*

Q.7 What is the effect of temperature on the speed of sound in air?

Ans. EFFECT OF TEMPERATURE ON SPEED OF SOUND

When a gas is heated at constant pressure, its volume is increased and hence, its density is decreased.

As, $$V = \sqrt{\frac{\gamma P}{\rho}}$$

So, the speed is increased with the rise of temperature.

Let, V_0 = Speed of sound at 0^oC , ρ_0 = Density of gas at 0^oC

V_t = Speed of sound at t^oC , ρ_t = Density of gas at t^oC

then, $$V_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad \ldots\ldots\ldots (1)$$

and $$V_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad \ldots\ldots\ldots (2)$$

Dividing equation (2) by (1)

$$\frac{V_t}{V_0} = \sqrt{\frac{\gamma P/\rho_t}{\gamma P/\rho_0}}$$

$$\frac{V_t}{V_0} = \frac{\sqrt{\gamma P}}{\sqrt{\rho_t}} \times \frac{\sqrt{\rho_0}}{\sqrt{\gamma P}}$$

$$\frac{V_t}{V_0} = \frac{\sqrt{\rho_0}}{\sqrt{\rho_t}}$$

$$\frac{V_t}{V_0} = \frac{\sqrt{\rho_0}}{\sqrt{\rho_t}} \quad \ldots\ldots\ldots (3)$$

If V_0 is the volume of a gas at temperature 0^oC and V_t is volume at t^oC, then by using volume expansion.

$$V_t = V_0 (1 + \beta t)$$

Where β is the coefficient of volume expansion of the gas. For all gases, its value is about $\frac{1}{273}$.

Hence, $$V_t = V_0 \left(1 + \frac{t}{273}\right)$$

As $$\rho = \frac{m}{V}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$V = \frac{m}{\rho}$$

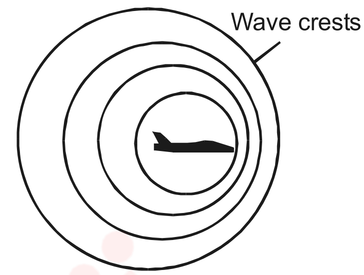
$$\frac{m}{\rho_t} = \frac{m}{\rho_0}\left(1 + \frac{t}{273}\right)$$

$$\frac{1}{\rho_t} = \frac{1}{\rho_0}\left(1 + \frac{t}{273}\right)$$

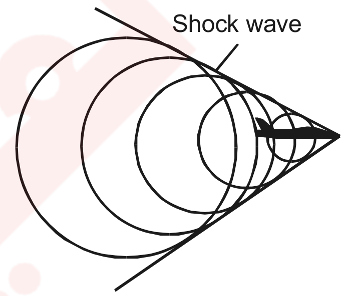
$$\rho_0 = \rho_t\left(1 + \frac{t}{273}\right)$$

Putting this value in equation (3).

Do You Know?



Slower than the speed of sound.



Faster than the speed of sound.

What happens when a jet plane like Concorde flies faster than the speed of sound?
A conical surface of concentrated sound energy sweeps over the ground as a supersonic plane passes overhead. It is known as sonic boom.

$$V_t - V_o \propto V, t$$
$$V_t - V_o = \beta V_o t$$
$$V_t = V_o + \beta V_o t$$
$$= V_o(1 + \beta t)$$

$$\frac{V_t}{V_0} = \sqrt{\frac{\rho_t\left(1+\frac{t}{273}\right)}{\rho_t}}$$

$$\frac{V_t}{V_0} = \sqrt{1+\frac{t}{273}} \qquad \text{.........} (4)$$

$$\frac{V_t}{V_0} = \sqrt{\frac{273+t}{273}}$$

$$\frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}}$$

Where T and T_0 are the absolute temperature. Corresponding to 5°C and 0°C respectively. Thus the speed of sound is directly proportional to the square root of the absolute temperature.

Now, using Binomial theorem and neglecting high power, we have, eq. (4) as:

$$\frac{V_t}{V_0} = \left(1+\frac{t}{273}\right)^{1/2}$$

$$\frac{V_t}{V_0} = 1+\frac{1}{2}\left(\frac{t}{273}\right)$$

$$\frac{V_t}{V_0} = \left(1+\frac{t}{546}\right)$$

$$V_t = V_0 + \frac{V_0\, t}{546}$$

As, $V_0 = 332$ m/s

Putting the value in the 2nd factor,

$$V_t = V_0 + \frac{332}{546}\, t$$

$$V_t = V_0 + 0.61\, t$$

This shows that one degree Celsius rise in temperature produces approximately 0.61 m/s (61 cm/s) increase in the speed of sound.

Q.8 State the principle of superposition.

Ans. PRINCIPLE OF SUPERPOSITION

So far, we have considered single waves. What happens when two waves encounter each other in the same medium? Suppose two waves approach each other on a coil of spring, one travelling towards the right and the other travelling towards left. Figure shows that you would see happening on the spring. The waves pass through each other without being modified. After the encounter, each wave shape looks just as it did before and is travelling along just as it was before.

This phenomenon of passing through each other unchanged can be observed with all types of waves. You can easily see that it is true for surface ripples.

But what is going on during the time when the two waves overlap? Figure (c) shows that the displacements they produce just add up. At each instant, the spring's displacement at any point in the overlap region is just the sum of the displacements that would be caused by each of the two waves separately.

Thus, if a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves be y_1, y_2, ……, y_n, then the resultant displacement of the particle, under the simultaneous action of these n waves is the algebraic sum of all the displacements i.e.,

$$Y = y_1 + y_2 + \ldots\ldots + y_n$$

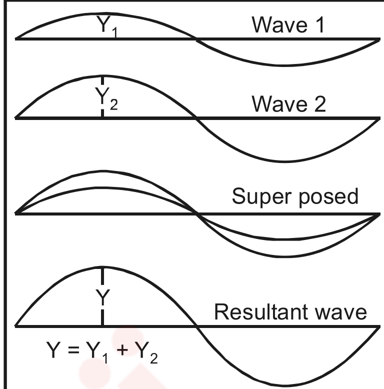
This is called principle of superposition.

Again, if two waves which cross each other have opposite phase, their resultant displacement will be

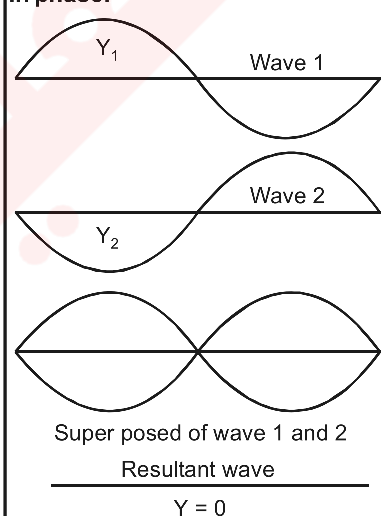
$$Y = y_1 - y_2$$

Particularly if $y_1 = y_2$ then result displacement Y = 0. Principle of superposition leads to many interesting phenomena with waves.

For Your Information



Superposition of two waves of the same frequency which are exactly in phase.



Superposition of two waves of the same frequency which are exactly out of phase.

(i) Two waves having same frequency and travelling in the same direction (Interference).

(ii) Two waves of slightly different frequencies and travelling in the same direction (Beats).

(iii) Two waves of equal frequency travelling in opposite direction (Stationary waves).

Q.9 State and explain the phenomenon of interference of sound.

Ans. INTERFERENCE

"Superposition of two waves having the same frequency and traveling in the same direction results in a phenomenon, called interference." There are two types:

(i) Constructive interference

(ii) Destructive interference

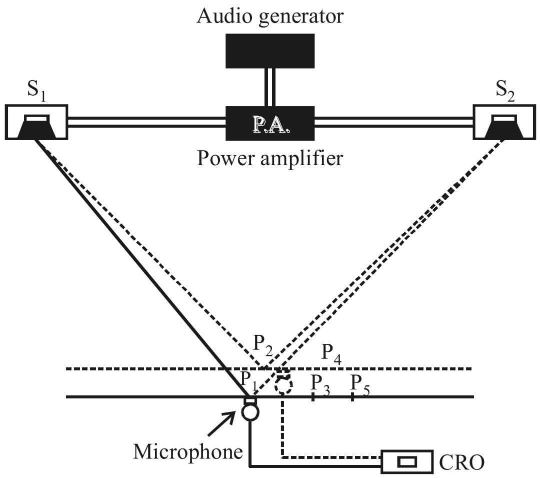


Fig. (a)

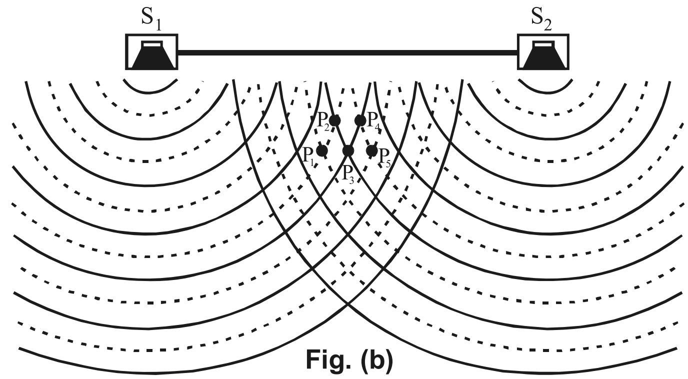


Fig. (b)

(i) Constructive Interference

"If two waves arrive at a point in phase i.e., compression of one wave falls on compression on other wave and rarefaction of one wave falls on the rarefaction of other wave, then resultant sound is loudest."

or Whenever path difference is an integral multiple of wavelength the two waves are added up. This effect is called constructive interference.

$\therefore$ Condition for constructive interference can be written as

$$\Delta S = n\lambda$$

where $n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

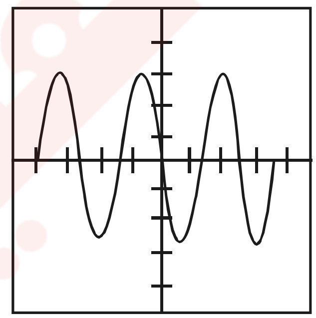


Fig. (c)
Constructive interference
Large displacement is displayed on the CRO screen

(ii) Destructive Interference

"If two waves reach a point out of phase i.e., compression of one wave falls on the rarefaction of other wave and rarefaction of one wave falls on the compression of other wave, then resultant sound will be minimum."

or At points where displacements of two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength. This effect is called destructive interference.

Condition for destructive interference is

$$\Delta S = (2n+1)\frac{\lambda}{2}$$

where $n = 0, \pm 1, \pm 2, \ldots\ldots$

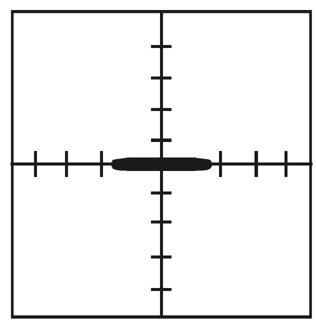


Fig. (d)
Destructive interference
Zero displacement is displayed on the CRO screen

Explanation

An experimental set up to observe interference effect in sound waves as shown in figure.

Two loud speakers S_1 and S_2 act as two sources of harmonic sound waves of a fixed frequency produced by an audio generator. A microphone attached to a sensitive cathode ray oscilloscope (CRO) act as a detector of sound waves. The CRO is a device to display the input signal into waveform on its screen. The microphone is placed at various points, turn by turn, infront of the loud speakers as shown in the figure.

At points P_1, P_3 and P_5, we find that compressions met with compressions and rarefactions. So, the displacement of the two waves are added up at these points and large resultant displacement is produced. At points P_2 and P_4, compressions met with rarefactions so they cancel each other effect. The resultant displacement becomes zero. Now we have to find the path difference between the waves at point P_1 is

$$\Delta S = S_2P_1 - S_1P_1$$

$$\Delta S = 4\frac{1}{2}\lambda - 3\frac{1}{2}\lambda = \lambda$$

But $\Delta S = n\lambda$

For constructive interference.

Where n = 0, 1, 2, 3, ……

For destructive interference

$$\Delta S = S_2P_2 - S_1P_2$$

$$\Delta S = 4\lambda - 3\frac{1}{2}\lambda = \frac{1}{2}\lambda$$

So, $\Delta S = \left(n + \frac{1}{2}\right)\lambda$

Where n = 0, 1, 2, 3, ……

Q.10 What are beats? How they are produced? Show that the number of beats is equal to the difference between the frequencies of the tuning forks.

Ans. **BEATS**

Beat is the combined effect of two sound waves having frequencies slightly different from each other.

Consider two tuning forks each having frequency 32 cps. Slightly load (with wax or ring) one tuning fork so that frequency decreases a little. Let the frequency becomes 30 cps. The two tuning forks are sounded together and held at equal distance from the ear. Let at t = 0, the two forks are in phase. i.e., their right prongs moves towards right producing compressions at the same time and louder sound is heard by the listener.

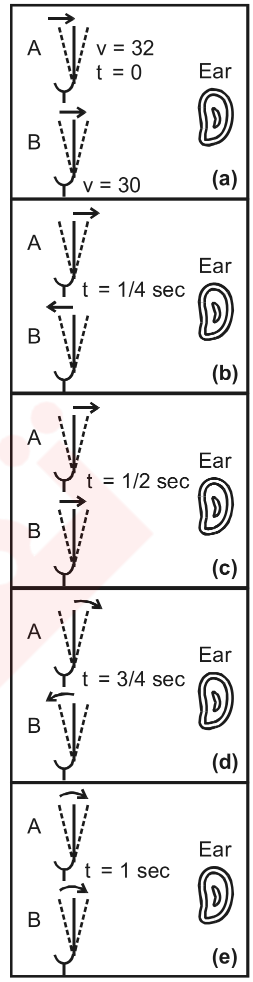
With passage of time, the tuning fork B (30 cps) begins to fall behind ‘A’.

At $t = \frac{1}{4}$ sec, 'A' has completed 8 vib and 'B' has completed $7\frac{1}{2}$ vib. The prongs are now out of phase and no sound is heard due to destructive interference.

At $t = \frac{1}{2}$ sec, 'A' has completed 16 vib and 'B' completes 15 vib. The prongs of forks become in phase and louder sound is heard. At $t = \frac{3}{4}$ sec, 'A' has completed 24 vib while 'B' completes $22\frac{1}{2}$ vib. The prongs are now in opposite phase and once again no sound is heard.

At t = 1 sec, both forks have completed 32 vib and 30 vib. The prongs become in phase and max or louder sound is heard.

It is observed that in 1 sec, the sound falls in intensity twice. The sudden fall of sound in intensity is known as beat. Thus two beats are produced/sec which is equal to the difference in frequencies of two forks (32 – 30 = 2).



Definition of Beats

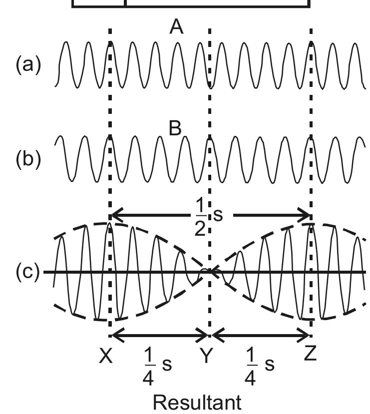
The periodic alteration of sound between maximum and minimum loudness as many times a second as the difference in frequencies is called phenomenon of beats.

$$\boxed{\text{No of beats/sec} = \text{Difference in frequencies}}$$

$$\pm n = f_A - f_B$$

Graphical Explanation of Beats

The displacements of the particles of the medium due to two waves are plotted separately as function of time. The resultant displacement of any particle will be the sum of the displacement due to each of the two waves. The resultant wave which is produced in shown in figure (c). It is seen that amplitude of resultant waves changes with time. The change in amplitude gives rise to production of beat.



Formation of Beats

Uses of Phenomenon of Beats

The phenomenon of beats is used to find out:

(i) Unknown frequency

(ii) To tune a musical instrument

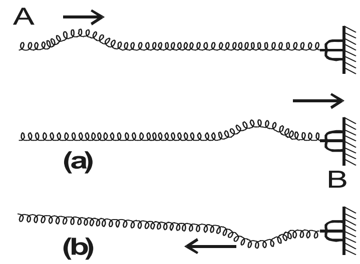
Q.11 Write a note on reflection of waves.

Ans. REFLECTION OF WAVES

In an extensivemedium, a wave travels in all direction from its source with a velocity depending upon the properties of the medium. However, when the wave comes across the boundary of two media, a part of it is reflected back. The reflected wave has the same wavelength and frequency but its phase may change depending upon the nature of the boundary.

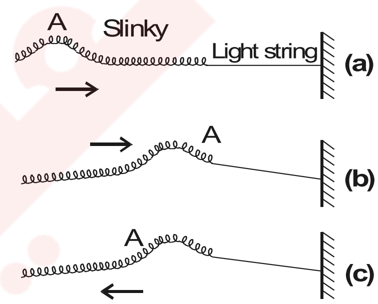
Now we will discuss two most common cases of reflection at the boundary. These cases will be explained with the help of waves travelling in slinky spring. (A slinky spring is a loose spring which has small initial length but a relatively large extended length).

One end of the slinky spring is tied to a rigid support on a smooth horizontal table. When a sharp jerk is given up to the free end of the slinky spring towards the side A, a displacement or a crest will travel from free end to the boundary (Figure a). It will exert a force on bound end towards the side A. Since this end is rigidly bound and acts as a denser medium, It will exert a reaction force on the spring in opposite direction. This force will produce displacement towards B and a trough will travel backwards along the spring (Figure b).



From the above discussion it can be concluded that whenever a transverse wave, travelling in a rarer medium, encounters a denser medium, it bounces back such that the direction of its displacement is reversed. An incident crest on reflection becomes a trough.

This experiment is repeated with a little variation by attaching one end of a light string to a slinky spring and the other end to the rigid support as shown in figure. If now the spring is given a sharp jerk towards A, a crest travels along the spring as shown in figure. When this crest reaches the spring-string boundary, it exert a force on the string towards the side it does not oppose the motion of the spring. The end of the spring, therefore, continues its displacement towards A. The spring behaves as if it has been plucked up. In other words a spring crest is again created at the boundary of the spring-string system, which travels backwards along the spring. From this it can be concluded that when a transverse wave travelling in a denser medium, is reflected from the **boundary of a rarer** medium, the direction of its displacement remains the same. An incident crest is reflected as a crest. We are already familiar with the fact that the direction of displacement is reversed when there is change of 180° in the phase of vibration. So, the above conclusion can be written as follows:



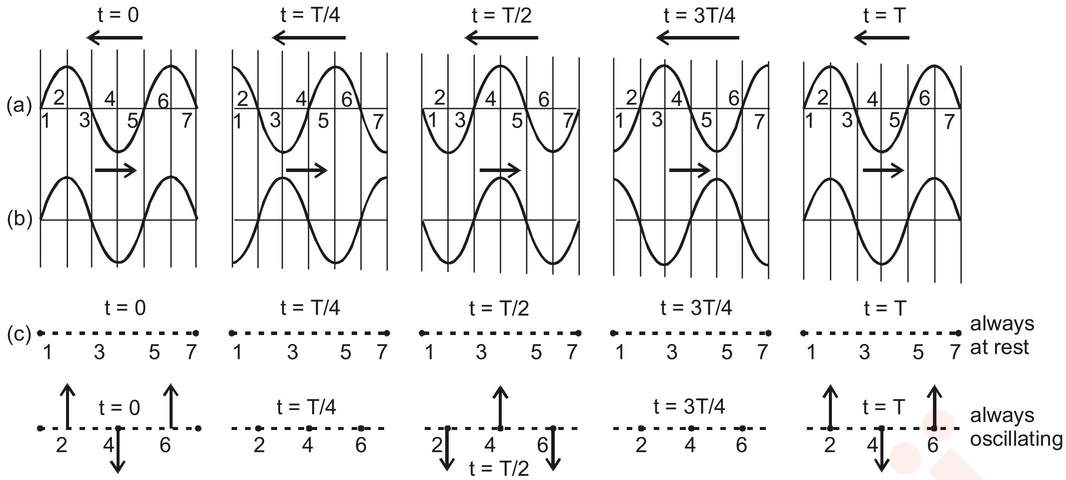
(i) If a transverse wave travelling in a rarer medium is incident on a denser medium, it is reflected such that it undergoes a phase change of 180°.

(ii) If a transverse wave travelling in a denser medium is incident on a rarer medium, it is reflected without any change in phase.

Q.12 What are stationary waves? How they one produced? Define node and anti-node.

Ans. STATIONARY WAVES

Now let us consider the superposition of two waves moving along a string in opposite directions. Figure (a, b) shows the profile of two such waves at instants = 0, T/4, 3/4 T and T, where T is the time period of the wave. We are interested in finding out the displacements of the points 1, 2, 3, 4, 5, 6 and 7 at these instants as the waves superpose. From the figure (a, b), it is obvious that the points 1, 2, 3, etc.,



are distant λ/4 apart, λ being the wavelength of the waves. We can determine the resultant displacement of these points by applying the principle of superposition. Figure (c) shows the resultant displacement of the points 1, 3, 5 and 7 at the instants t = 0, T/4, T/2, 3T/4 and T. It can be seen that the resultant displacement of these points is always zero. These points of the medium are known as nodes. Figure (c) shows that the distance between two consecutive nodes is λ/2. Figure (d) shows the resultant displacement of the points 2, 4 and 6 at the instant t = 0, T/4, T/2, 3T/4 and T. The figure shows that these points are moving with an amplitude which is the sum of the amplitudes of the component waves. These points are known as antinodes. They are situated midway between the nodes and are also λ/2 apart. The distance between a node and the next antinode is λ/4. Such a pattern of nodes and antinodes is known as a stationary or standing wave.

Energy in a wave moves because of the motion of the particles of the medium. The nodes always remain at rest, so energy cannot flow past these points. Hence energy remains "standing" in the medium between nodes, although it alternates between potential and kinetic forms. When the antinodes are all at their extreme displacements, the energy stored is wholly potential and when they are simultaneously passing through their equilibrium positions, the energy is wholly kinetic.

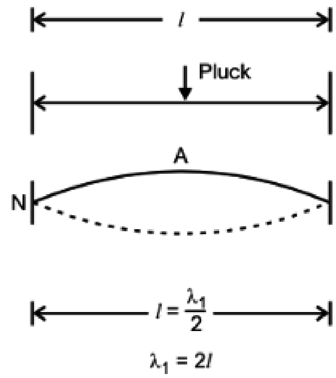
An easy way to generate a stationary wave is to superpose a wave travelling down a string with its reflection travelling in opposite direction as explained in the next section.

Q.13 Explain the stationary waves in a stretched string. Also calculate the frequencies.

Ans. STATIONARY WAVES IN A STRETCHED STRING

Consider a string of length 'l' which is kept stretched by clamping its ends so that the tension in the string is F. If the string is plucked at its middle point, two transverse waves will originate from this point. One of them will move towards the left end of the string and the other towards the right end. When these waves reach the two clamped ends, they are reflected back, thus giving rise to stationary waves. The string will vibrate with such a frequency f_1, so that nodes are formed at two fixed ends (clamped ends) and anti-nodes between them. Thus the string vibrates in one loop as shown in Fig.

If λ_1 is the wavelength of this mode of vibration (1st mode of vibration) so,



As $l = \frac{\lambda_1}{2}$

$$\lambda_1 = 2l \quad \dots\dots\dots \text{(i)}$$

$$V = \lambda f$$

$$f_1 = \frac{V}{\lambda_1}$$

Putting value of λ_1.

$$\therefore \quad f_1 = \frac{V}{2l} \quad \dots\dots\dots \text{(ii)}$$

The speed 'V' of the waves in the string depends upon the tension F of the string and mass per unit length of the string. It is given by

$$V = \sqrt{\frac{F}{m}}$$

$$\therefore \quad f_1 = \frac{\sqrt{F/m}}{2l}$$

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}} \quad \dots\dots\dots \text{(iii)}$$

If the string is plucked from $\frac{1}{4}$ of its length, then again stationary waves will be set up, but now the string vibrate in two loops with f_2. If λ_2 is wave length in this case then,

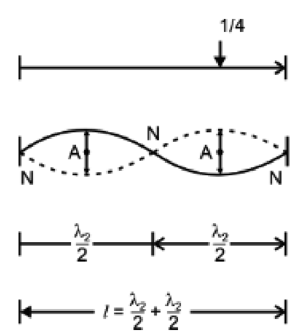
As, $l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$

$$l = \frac{\lambda_2 + \lambda_2}{2}$$

$$l = \frac{2\lambda_2}{2}$$

$$l = \lambda_2$$

$$\therefore \quad f_2 = \frac{V}{\lambda_2}$$



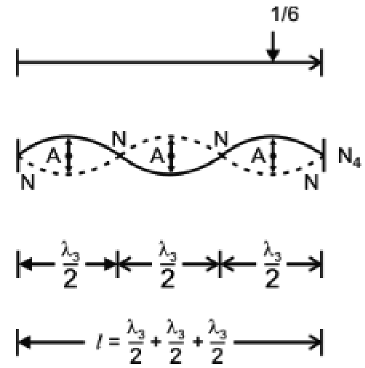
Putting value of λ_2

$$\therefore \quad f_2 = 2\left(\frac{V}{2l}\right)$$

$$f_2 = 2f_1$$

This shows that when string vibrates in two loops, its frequency is doubled and wave length becomes half than it vibrates in one loop.

Similarly if the string is plucked from $\frac{1}{6}$ th of its length, it vibrates in three loops as shown in figure.



As $\quad l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$

$$l = 3\frac{\lambda_3}{2}$$

$$\lambda_3 = \frac{2l}{3}$$

$$\therefore \quad f_3 = \frac{V}{\lambda_3}$$

Putting value of λ_3

$$f_3 = \frac{V}{2l/3}$$

$$f_3 = 3\left(\frac{V}{2l}\right)$$

$$f_3 = 3f_1$$

It means that when string vibrates in three loops, its frequency is three times the frequency when it vibrates in one loop.

Thus we can generalize that if the string is made to vibrate in 'n' loop, then

$$f_n = nf_1$$

and wave length is;

$$\lambda_n = \frac{2l}{n}$$

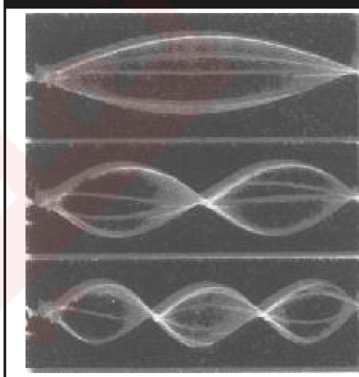
where n = 1, 2, 3, ……

It is clear that as the string vibrates in more and more loops, its frequency goes on increasing and the wave length gets shorter. However the product of frequency and wave length is always equal to V, (speed of the wave).

The above discussion clearly establishes that the stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ………, n f_1 which is known as harmonic series. The fundamental frequency 'f_1' is called first harmonic (over tone), 'f_2' is called second harmonic (over tone), and so on.

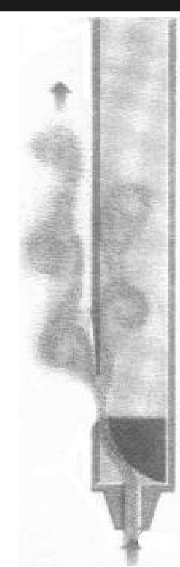
Note: *The frequency of a string on a musical instrument can be changed either by varying the tension or by changing the length. For example the tension in guitar and violin strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck, thereby changing the length of the vibrating portion of the string.*

Do You Know?



A standing-wave pattern is formed when the length of the string is an integral multiple of half wavelength; otherwise no standing wave is formed.

For Your Information



In an organ pipe, the primary driving mechanism is wavering, sheet like jet of air from flute-slit, which interacts with the upper lip and the air column in the pipe to maintain a steady oscillation.

Q.14 Describe the stationary waves in air column.

Ans. **STATIONARY WAVES IN AIR COLUMNS**

Stationary waves can be set in other media also, such as air column. A common example of vibrating air column is in the organ pipe. The relationship between the incident wave and the reflected wave depends on whether the reflecting end of the pipe is open or closed. If the reflecting end is open, the air molecules have complete freedom of motion and this behaves as an antinode. If the reflecting end is closed, then it behaves as a node because the movement of the molecules is restricted. The modes of vibration of an air column in a pipe open at both ends are shown in figure.

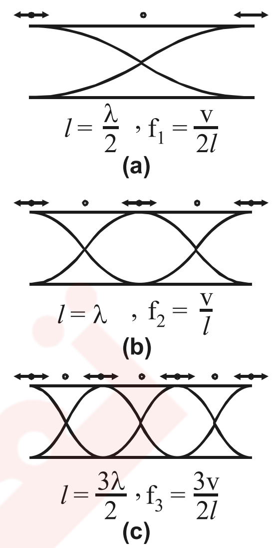


Fig. Stationary longitudinal waves in a pipe open at both ends.

In figure, the longitudinal waves set up in the pipe have been represented by transverse curved lines indicating the varying amplitude of vibration of the air particles at points along the axis of the pipe. However, it must be kept in mind that air vibrations are longitudinal along the length of the pipe. The wavelength 'λ_n' of nth harmonic and its frequency 'f_n' of any harmonic is given by

$$\lambda_n = \frac{2l}{n} \quad , \quad f_n = \frac{v}{\lambda_n} = \frac{nv}{2l} \quad \ldots\ldots (1)$$

$$n = 1, 2, 3, 4, \ldots\ldots$$

where 'v' is the speed of sound in air and 'l' is the length of the pipe. The equation (10 can also be written as

$$f_n = nf_1 \quad \ldots\ldots (2)$$

If a pipe is closed at one end and open at the other, the closed end is a node. The modes of vibration in this case are shown in figure.

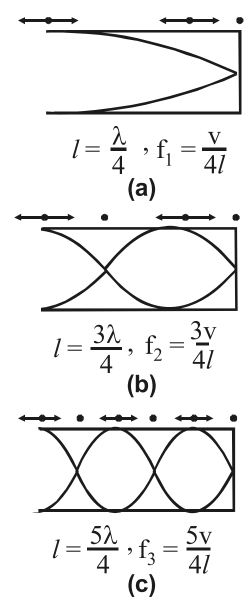


Fig. Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

In case of fundamental note, the distance between a node and antinode is one fourth of the wavelength,

Hence, $l = \frac{\lambda_1}{4}$ or $\lambda_1 = 4l$

Since $v = f\lambda$

Hence, $f_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$

It can be proved that in a pipe closed at one end, only odd harmonics are generated, which are given by the equation.

$$f_n = \frac{nv}{4l} \quad \ldots\ldots (3)$$

where $n = 1, 3, 5, \ldots\ldots$

This shows that the pipe, which is open at both ends, is richer in harmonics.

Case I

When pipe is open at both ends

In 1st mode

$$l = \frac{2\lambda_1}{4}$$

$$l = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2l$$

As, $V = \lambda f$

$$f_1 = \frac{V}{\lambda_1}$$

Putting value of 'λ_1',

$$\therefore \quad f_1 = \frac{V}{2l}$$

In 2nd mode

$$l = \frac{\lambda_2 + 2\lambda_2 + \lambda_2}{4}$$

$$l = \frac{4\lambda_2}{4}$$

$$l = \lambda_2$$

$$\therefore \quad f_2 = \frac{V}{l}$$

Multiply and divided by (2).

$$f_2 = 2\left(\frac{V}{2l}\right)$$

In 3rd mode

$$l = \frac{\lambda_3 + 2\lambda_3 + 2\lambda_3 + \lambda_3}{4}$$

$$l = \frac{6\lambda_3}{4}$$

$$l = \frac{3\lambda_3}{2}$$

$$l = \frac{2l}{3}$$

As, $f_3 = \dfrac{V}{\lambda_3}$

Putting value of λ_3

$$\therefore \quad f_3 = \frac{V}{2l/3}$$

$$f_3 = 3\left(\frac{V}{2l}\right)$$

Now, $f_n = n\left(\dfrac{V}{2l}\right)$

where, n = 1, 2, 3, ……

Case II

When pipe is open at one end and closed at other end.

In 1st mode

$$\lambda_1 = 4l$$

$$f_1 = \frac{V}{\lambda_1}$$

Putting value of λ_1

$$f_1 = \frac{V}{4l}$$

In 2nd mode

$$l = \frac{\lambda_2}{4} + \frac{\lambda_2}{2}$$

$$l = \frac{\lambda_2 + 2\lambda_2}{4}$$

$$l = \frac{3\lambda_2}{4}$$

$$\lambda_2 = \frac{4l}{3}$$

$$f = \frac{V}{\lambda_2}$$

Putting value of λ_2

$$f_3 = \frac{V}{4l/3}$$

$$f_3 = \frac{3V}{4l}$$

$$f_3 = 3\,(f_1)$$

We can generalize,

$$\therefore \quad f_n = \frac{n\,V}{4l}$$

where $n = 1, 3, 5, \ldots\ldots$

Note: *The pipe which is open at both ends is richer in harmonics.*

Interesting Information

Echolocation allows dolphins to detect small differences in the shape, size and thickness of objects.

Q.15 What is Doppler effect? Describe expression for apparently changed frequency when the observer is at rest while the source is in motion. (OR) What is Doppler effect? Describe the expression for apparently changed frequency when the observer is in motion while source is at rest.

Ans. DOPPLER EFFECT

Introduction

This effect was observed by Johann Doppler while he was observing the frequency of light emitted from distant stars. In some cases the frequency of light emitted from a star was found to be slightly different from that emitted from a similar source on the earth. He found that the change in frequency of light depends on the motion of star relative to the earth.

Definition

"The apparent change in the pitch OR frequency of a source of sound, when there is a relative motion between the source of sound and the observer, is called Doppler effect."

For example, when an observer is standing on a railway platform, the pitch of the whistle of a approaching train is heard to be higher. But when the same train moves away, the pitch of the whistle becomes lower.

Explanation

Suppose 'V' is the velocity of the sound in a medium and a source emits a sound of frequency 'f' and wave length 'λ'. If both the source and the observer are stationary, then the waves received by the observer in one second are;

$$f = \frac{V}{\lambda}$$

Case-A

When observer 'O' is moving towards a stationary source 'S'

If an observer ('O') moves towards, a source 'S' with velocity U_0, the relative velocity of the waves and the observer is increased to $(V + U_0)$, then number of waves received in one second apparent frequency increases

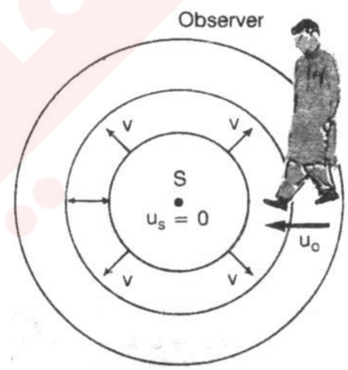


Fig. An observer moving with velocity u_o towards a stationary source hears a frequency f_A that is greater than the source frequency.

$$f_A = \frac{V}{\lambda}$$

Putting the value

$$f_A = \frac{V + U_0}{V/f} \quad \left(\because \lambda = \frac{V}{f}\right)$$

$$f_A = \left(\frac{V + U_0}{V}\right) f$$

Hence, $\dfrac{V + U_0}{V} > 1$

$\therefore \quad f_A > f$

This means that when an observer 'O' is moving towards a stationary source S, the frequency of sound increases.

Case-B

When observer 'O' moves away from a stationary source S

When observer 'O' moves away from stationary source 'S' with velocity 'U_0,' then relative velocity of sound waves and the observer is, $(V - U_0)$, hence, number of waves received by the observer, per–second are reduced and is given by

$$f_B = \frac{V - U_0}{\lambda}$$

Putting value of

$$\lambda = \frac{V}{f}$$

$$\therefore \quad f_B = \frac{V - U_0}{V/f}$$

$$f_B = \left(\frac{V - U_0}{V}\right) f$$

As, $\dfrac{V - U_0}{V} < 1$

$$\therefore \quad f_B < f$$

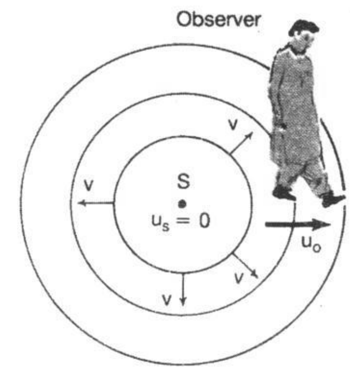


Fig. An observer moving with velocity u_o away from stationary source hears a frequency f_B that is smaller than the source frequency.

This means that when observer 'O' moves away from stationary source 'S,' the apparent frequency decreases.

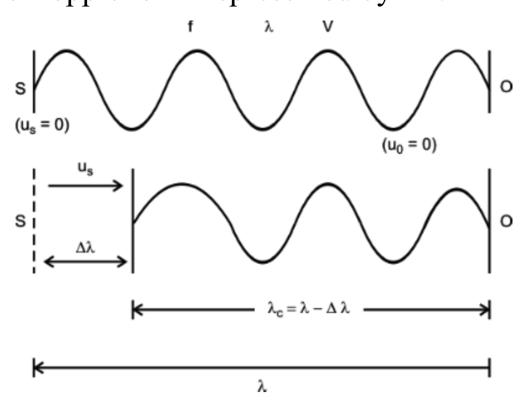
Case-C

When source 'S' is moving towards the stationary observer 'O'

If the source 'S' is moving towards the observer with velocity 'U_s' then in one second, the waves are compressed, (Wave length decreases), by an amount known as Doppler shift represented by $\Delta\lambda$.

As, $V = f\lambda$

$$\Delta\lambda = \frac{U_s}{f}$$



The compression of waves is due to the fact that same number of waves are contained in a shorter space depending upon the velocity of the source.

The wave length for observer 'C' is then, $\lambda_c = \lambda - \Delta\lambda$.

Putting the values of λ and $\Delta\lambda$.

$$\therefore \quad \lambda_c = \frac{V}{f} - \frac{U_s}{f}$$

$$\lambda_c = \frac{V - U_s}{f}$$

The modified frequency for observer 'C' is

$$f_c = \frac{V}{\lambda_c}$$

Putting value of λ_c

$$\therefore \quad f_c = \frac{V}{\frac{V - U_s}{f}}$$

$$f_c = \left(\frac{V}{V - U_s}\right) f$$

$$\text{As,} \quad \frac{V}{V - U_s} > 1$$

$$\therefore \quad f_c > f$$

This means that the observed frequency increases when the source is moving towards the observer.

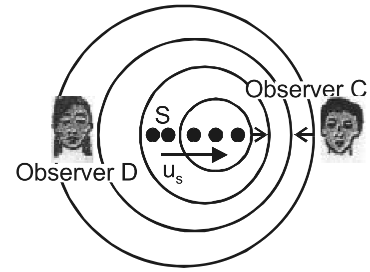
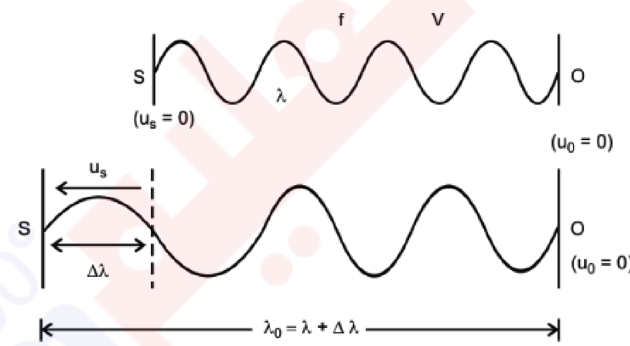


Fig. A source moving with velocity u_s towards a stationary observer C and away from stationary observer D. Observer C hears an increased and observer D hears a decreased frequency.

Case-D

When source 'S' is moving away from stationary observer 'O'

If the source is moving away from the observer 'O' with velocity 'U_s,' then in one second the waves are stretched (wave length increase) by an amount $\Delta\lambda$.



$$\Delta\lambda = \frac{U_s}{f}$$

The stretched of the waves is due to the fact that same number of wave are contained in larger (longer) space, depending upon the velocity of the source.

The wave length for observer 'D' is, $\lambda_D = \lambda + \Delta\lambda$.

Putting values of λ and $\Delta\lambda$.

$$\therefore \quad \lambda_D = \frac{V}{f} + \frac{U_s}{f}$$

$$\lambda_D = \frac{V + U_s}{f}$$

The modified frequency for observer 'D' will be

$$f_D = \frac{V}{\lambda_D}$$

Putting value of λ_D

$$\therefore \quad f_D = \frac{V}{\frac{V + U_s}{f}}$$

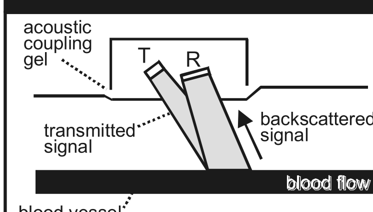
$$f_D = \left(\frac{V}{V + U_s}\right) f$$

$$\text{As,} \quad \frac{V}{V + U_s} < 1$$

$$\therefore \quad f_D < f$$

This means that the observed frequency decreases, when the source is moving away from the observer.

Do You Know?

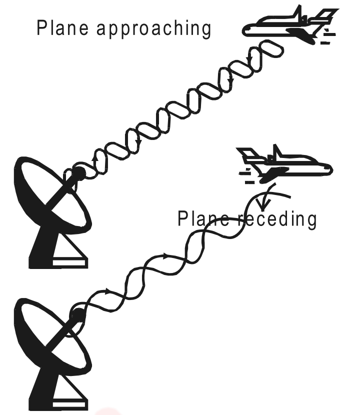


The Doppler effect can be used to monitor blood flow through major arteries. Ultrasound waves of frequencies 5 MHz to 10 MHz are directed towards the artery and a receiver detects the back scattered signal. The apparent frequency depends on the velocity of flow of the blood.

Q.16 What are the applications of Doppler effect?

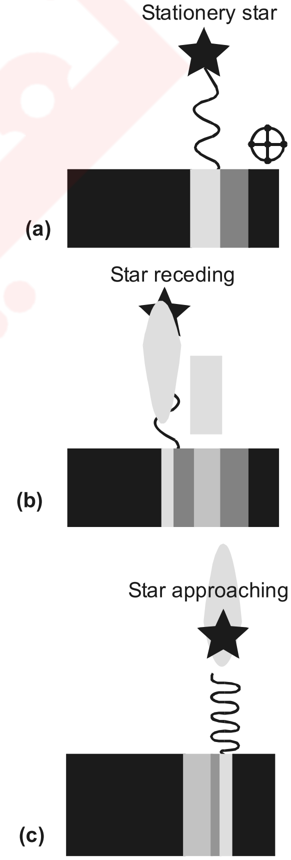
Ans. APPLICATIONS OF DOPPLER EFFECT

Doppler effect is applied in working of radar system. Radar uses radio waves to find the elevation and speed of an aeroplane. Radar is a device which transmits and receives radio waves. The radio waves transmitted from radar are reflected back from aeroplane and are received by radar. If the aeroplane is moving towards the radar then the wavelength of reflected wave is shorter. If the plane is moving away from the radar then the wavelength of reflected wave is longer as shown in figure.



A frequency shift is used in a radar to detect the motion of an aeroplane.

The difference of wavelength of transmitted and reflected waves is used to determine the speed of aeroplane. Term SONAR (is acronym) stands for sound navigation and ranging. Sonar is the name of the technique used for detecting the presence of objects under water by acoustical echo. In Sonar "Doppler detection" depends upon relative speed of the target and the detector. It employs. The Doppler effect in which an apparent change in frequency occurs when source and observer are in relative motion with respect to one another.



In military it is used for detection and location of submarine antisubmarine weapons and depth measurement of sea. Astronomers use Doppler effect to calculate the speeds of distant stars and galaxies. By comparing the line spectrum of light from the star with light from a laboratory source, the Doppler shift of stars light can be measured. Then speed of star can be calculated.

Doppler effect is used to determine whether a particular star or galaxy is approaching the earth or moving away from the earth. Light from the star is measured with the help of spectrometer. It has been found that stars moving towards the earth show a blue shift. Thus is because the emitted waves by the star have shorter wavelength than of the star had been at rest. So the spectrum is shifted towards shorter wavelength i.e., to the blue end of the spectrum.

It has been found that stars moving away from the earth show a red shift. This is because the emitted waves by the star have longer wavelength than of the star had been at rest. So the spectrum is shifted towards longer wavelength i.e., to the red end of the spectrum. Astronomers have discovered that all the distant galaxies are moving away from us. They have also measured their speed by measuring their red shift. Another important application of the Doppler shift using electromagnetic waves is radar speed trap. Microwaves are emitted from a transmitter in the form of short bursts. Each burst is reflected back by any moving car in the path of microwaves. The reflected microwaves are received back as Doppler's shift. By measuring Doppler shift the speed of the car can be calculated by computer programme.

Do You Know?

Bats navigate and find food by echo location.

SOLVED EXAMPLES

EXAMPLE 8.1

Find the temperature at which the velocity of sound in air is two times its velocity at 10^oC.

Data

Velocity of sound = $V_t = 2\,V_{10}$

$T_{10} = 10 + 273 = 283$ K

To Find

Temperature = T = ?

SOLUTION

Using $\dfrac{V_t}{V_{10}} = \sqrt{\dfrac{T}{T_{10}}}$

Since $V_t = 2\,V_{10}$

$\therefore \quad \dfrac{2\,V_{10}}{V_{10}} = \sqrt{\dfrac{T}{283}}$

$$\sqrt{\frac{T}{283}} = 2$$

Square both sides

$$\frac{T}{283} = 4$$

$$T = 4 \times 283$$

$$T = 1132 \text{ K}$$

or $T = 1132 - 273$

$$T = 859^oC$$

Result

Temperature = T = 859°C or 1132 K

EXAMPLE 8.2

A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

Data

Number of beats = n = 4

After loading B,

Number of beats = $n' = 6$

Frequency of tuning fork A = f_A = 320 Hz

To Find

Frequency of tuning fork B = f_B = ? when loaded

SOLUTION

Using $f_A - f_B = \pm n$

$320 - f_B = \pm 4$

$f_B = 320 + 4$, $f_B = 320 - 4$

$f_B = 324$ Hz , $f_B = 316$ Hz

By loading B, its frequency will decrease. Thus if 324 Hz is original frequency the beat frequency will reduce. But in this case beat frequency increases.

$\therefore$ $f_B = 316$ Hz

After loading B,

$f_B = 316 - 2$

$f_B = 314$ Hz

Result

Frequency of fork B = 314 Hz

EXAMPLE 8.3

A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked?

Data

Weight of wire = W = 80 N

$\therefore$ Tension in wire = F = 80 N

Diameter of wire = d = 0.50 mm

= 0.50×10^{-3} m

Length of wire = l = 1.5 m

To Find

Fundamental frequency = f_1 = ?

SOLUTION

Using $f_1 = \dfrac{1}{2l}\sqrt{\dfrac{F}{m}}$

Where m = Mass per unit length

Since $\rho = \dfrac{m}{V}$ ……… (1)

Since Volume = Area x Length

$\therefore \quad V = \pi r^2 l$

Put in (1)

$\therefore \quad m = \rho \pi r^2 l$

$$m = \rho \pi \left(\frac{d}{2}\right)^2 l$$

$\therefore$ Mass per unit length is

$$m = 7.8 \times 10^3 (3.14) \frac{(.50 \times 10^{-3})^2}{4} \times 1$$

$$= 1.53 \times 10^{-3} \text{ kg/m}$$

$$\therefore \quad f_1 = \frac{1}{2(1.5)} \sqrt{\frac{80}{1.53 \times 10^{-3}}}$$

$$f_1 = \frac{1}{3} \sqrt{\frac{80 \times 10^3}{1.53}}$$

$$f_1 = \frac{1}{3} \sqrt{\frac{80000}{1.53}}$$

$$f_1 = \frac{1}{3} \sqrt{52287.58}$$

$$f_1 = 76.22 \text{ Hz}$$

Result

Fundamental frequency = f_1 = 76.22 Hz

EXAMPLE 8.4

A pipe has a length of 1 m. determine the frequencies of the fundamental and the first two harmonics

(a) If the pipe is open at both ends.

(b) If the pipe is closed at one end.

(Speed of sound in air = 340 ms^{-1})

Data

$l = 1$ m

$V = 340$ ms^{-1}

To Find

(a) When the pipe is open at both ends

Fundamental frequency = f_1 = ?

Next 1st harmonics = f_2 = ?

Next 2nd harmonics = f_3 = ?

(b) When the pipe is closed at one end

Fundamental frequency = f_1 = ?

Next 3^{rd} harmonics = f_3 = ?

Next 5^{th} harmonics = f_5 = ?

SOLUTION

(a) When pipe is open at both end,

Using $f_n = \dfrac{nV}{2l}$

$n = 1, 2, 3, 4, \ldots\ldots$

Here $f_1 = \dfrac{V}{2l}$

$f_1 = \dfrac{340}{2 \times 1}$

$f_1 = 170 \text{ Hz}$

So, $f_2 = 2f_1$

$f_2 = 340 \text{ Hz}$

$f_3 = 3f_1$

$f_3 = 510 \text{ Hz}$

(b) When pipe is closed at one end and is open at the other

Using, $f_n = \dfrac{nV}{4l}$

$n = 1, 3, 5, 7, \ldots\ldots$

Here, $f_1 = \dfrac{V}{4l}$

$f_1 = \dfrac{340}{4 \times 1}$

$f_1 = 85 \text{ Hz}$

and, $f_3 = 3f_1 = 3 \times 85$

$f_3 = 3f_1 = 255 \text{ Hz}$

$f_5 = 5f_1 = 5 \times 85$

$f_5 = 5f_1 = 425 \text{ Hz}$

Result

(a) When the pipe is open at both ends

Fundamental frequency = f_1 = 170 Hz

Next 1^{st} harmonics = f_2 = 340 Hz

Next 2^{nd} harmonics = f_3 = 510 Hz

(b) When the pipe is closed at one end

Fundamental frequency $= f_1 = 85$ Hz

Next 3rd harmonics $= f_3 = 255$ Hz

Next 5th harmonics $= f_5 = 425$ Hz

EXAMPLE 8.5

A train is approaching a station at $90kmh^{-1}$ sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed?

(Speed of sound $= 340ms^{-1}$)

Data

Speed of train $= U_s = 90$ km/h

$= 90 \times 1000$ m/3600s

$= 25$ m/s

Frequency of source $= f_0 = 1000$ Hz

Speed of sound $= V = 340$ m/s

To Find

Apparent frequency $= f' = ?$

Apparent frequency when train moves away from station with same speed $f'' = ?$

SOLUTION

When train is moving towards listener, then by using

$$f' = \left(\frac{V}{V - u_s}\right) f$$

Putting values.

$$f' = \left(\frac{340}{340 - 25}\right) \times 1000$$

$$= 1079.4 \text{ Hz.}$$

When train is moving away from listener then by using.

$$f'' = \left(\frac{V}{V - u_s}\right) f$$

Putting values.

$$f'' = \left(\frac{340}{340 + 25}\right) \times 1000$$

$$= 931.5 \text{ Hz}$$

Result

Apparent frequency $= f' = 1079.4$ Hz

Apparent frequency when train moves away from station with same speed

$f'' = 931.5$ Hz

Chapter 9

PHYSICAL OPTICS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

Understand the concept of wavefront.

State Huygen's principle.

Use Huygen's principle to explain linear superposition of light.

Understand interference of flight.

Recognize and express colour patterns in thin films.

Describe the formation of Newton's rings.

Understand the working of Michelson's interferometer and its uses.

Explain the meaning of the term diffraction.

Describe diffraction at a single slit.

Derive the equation for angular position of first minimum.

Derive the equation d sin θ = mλ.

Carry out calculations using the diffraction grating formula.

Describe the phenomenon of diffraction of X-rays by crystals.

Understand the effect of rotation of Polaroid on polarization.

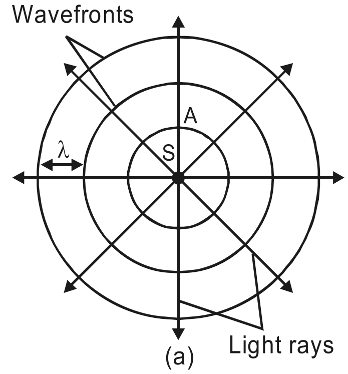
Understand how plane polarized light is produced and detected.

INTRODUCTION

Light is a type of energy which produces sensation of vision. But how does this energy propagate? In 1678, Huygen's, an element Dutch scientist, proposed that light energy from a luminous source travels in space as waves. The experimental evidence in support of wave theory in Huygen's time was not convincing. However, Young's interference experiment performed for the first time in 1801 proved wave nature of light and thus established the Huygen's wave theory. In this chapter you will study the properties of light, associated with its wave nature.

Q.1 Define wave front with its types.

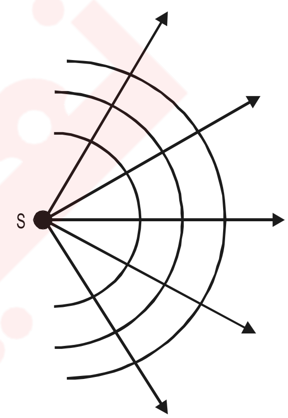
Ans. WAVE FRONTS



"Such a surface on which all the points have the same phase of vibration is known as wave front."

Explanation

Consider a point source of light 'S'. The waves emitted from the source propagated in all direction with a speed 'c'. After time 't' these waves will reach the surface of sphere with centre as a source of light and radius as ct.



Every point on the surface of sphere will be set into vibration by the waves. As the distance of all these points from the source is the same, as their state of vibration will be identical. With the time passing the waves moves farther giving rise to new wave fronts. All these wave fronts will be concentric spheres of increasing radius. Thus we can say that the waves propagated in sphere by the motion of wave fronts. There are two types.

(1) Spherical wave fronts.

(2) Plane wave fronts.

Spherical Wave Fronts

The wave fronts in which the electromagnetic waves are propagated in spherical form with the source is called spherical wave front.

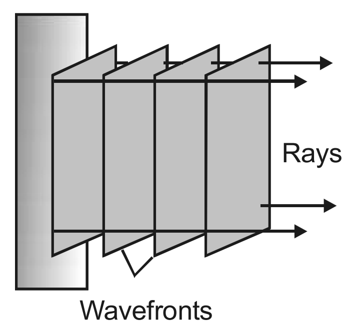


Fig. Spherical wave fronts (a) and plane wavefronts (b) spaced a wavelength apart. The arrows represent rays.

The spherical wave fronts transmitted energy equally in all directions and the direction in which the energy is travel is called a ray.

Plane Wave Fronts

A small portion of spherical wave fronts which is far away from the source is called the plane wave fronts.

The plane wave fronts are parallel to each other and the rays are perpendicular to each plane surface.

Do You Know?

[illegible]

Q.2 State and explain Huygen's principle.

Ans. HUYGEN'S PRINCIPLE

This principle used to determine the shape and location of the new wave fronts. Huygen's principle enables us to determine the shape and location of the new wavefronts. This principle consist of two parts.

(1) Every point of a wave front considered as a source of secondary wavelets which spread out in forward direction with a speed equal to the speed of propagation of the waves.

(2) The new position of the wavefront after a certain time can be found by constructing a surface that touches all the secondary wavelets.

AB represents the wavefront at any instant. In order to determine the wavefront at time t + Δt, draw secondary wavelets with centre at various points on the wavefront AB and radius as cΔt where c is the speed of the propagation of the waves as shown in fig. The new wavefront at time t + Δt is A′B′ which is a tangent to all the secondary wavelets.

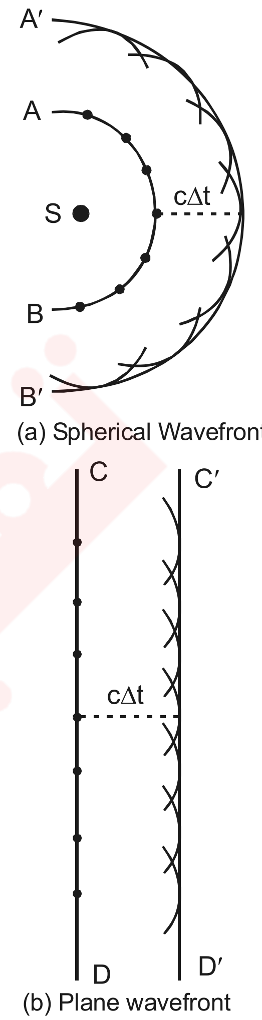


Fig. Huygens' construction for determining the position of the wavefronts AB and CD after a time interval Δt. A′B′ and C′D′ are the new positions of the wavefronts.

Q.3 What is the interference of light? Write down the condition of interference.

Ans. INTERFERENCE OF LIGHT WAVES

Definition

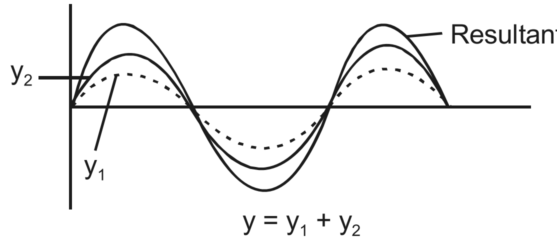
"When two light waves of same frequency and amplitude traveling in the same direction are superposed in such a way that they reinforce at some points while they cancel at the other points, they are set to produce interference of light waves."

There are two types of interference of light waves.

(1) Constructive Interference.

(2) Destructive Interference.

(1) Constructive Interference

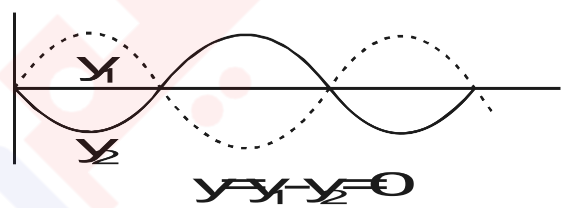


That kind of interference in which the two waves reinforce each other i.e. crest of one wave falls on the crest of other wave and similarly trough of one wave falls on the trough of other wave is called constructive interference. In this case the amplitude of the resultant will be greater than either of the individual wave. The constructive interference take place when the path difference between the two waves is an integral multiple of the wavelength i.e.

$$S = m\lambda$$

where $m = 0, 1, 2, 3, \ldots\ldots\ldots$

(2) Destructive Interference



That kind of interference in which the two waves cancel each other crest of one wave falls on the trough of other wave is called destructive interference.

In this case the amplitude of the resultant wave will be less than either of the individual waves. The path difference between the two waves is given as

$$S = \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, 3, \ldots\ldots\ldots$

Conditions for Detectable Interference

When two waves travel in the same medium, they would interfere constructively or destructively. The amplitude of the resultant wave will be greater than either of the individual waves, if they interfere constructively. In the case of destructive interference, the amplitude of the resultant wave will be less than either of the individual waves.

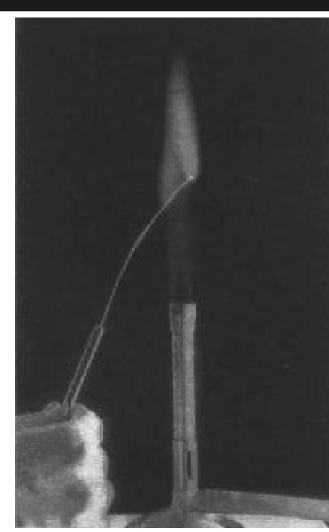
Interference of light waves is not easy to observe because of the random emission of light from a source. The following conditions must be met, in order to observe the phenomenon.

1. The interfering beams must be monochromatic, i.e., of a single wavelength.
2. The interfering beams of light must be coherent.

Explanation

Consider two or more sources of light waves of the same wavelength. If the sources send out crests or troughs at the same instant, the individual waves maintain a constant phase difference with

For Your Information



Monochromatic Light

Sodium chloride in a flame gives out pure yellow light. This lights is not a mixture of red and green.

one another. The monochromatic sources of light which emit waves, having a constant phase difference, are called coherent sources.

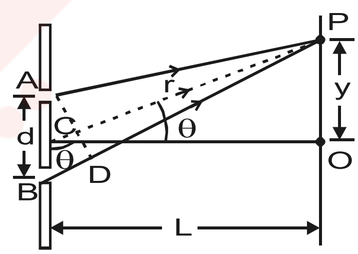
A common method of producing two coherent light beams is to use a monochromatic source to illuminate a screen containing two small holes, usually in the shape of slits. The light emerging from the two slits is coherent because a single source produces the original beam and two slits serve only to split it into two parts. The points on a Huygen's wavefront which send out secondary wavelets are also coherent sources of light.

Q.4 Explain Young's double slit experiment for interference of light waves. Also derive an expression for fringe spacing.

Ans. **YOUNG'S DOUBLE SLIT EXPERIMENT FOR INTERFERENCE OF LIGHT WAVES**

A single narrow slit is illuminated by monochromatic light of wavelength λ. Light from this slit is made to pass through two narrow slits which are closely spaced. When light falls on screen from these slits, interference is produced. The screen is covered with alternate bright and dark bands called interference fringes. The distance between two consecutive dark or bright fringes is called fringe spacing.

A and B are two narrow slits separated by a very small distance d. They are illuminated by a monochromatic light of wavelength λ. A screen is placed at a distance L from the slits. O is the central point of the screen which is at equal distance from the slits. O is therefore always a bright point since light waves from slits A and B reach in phase.



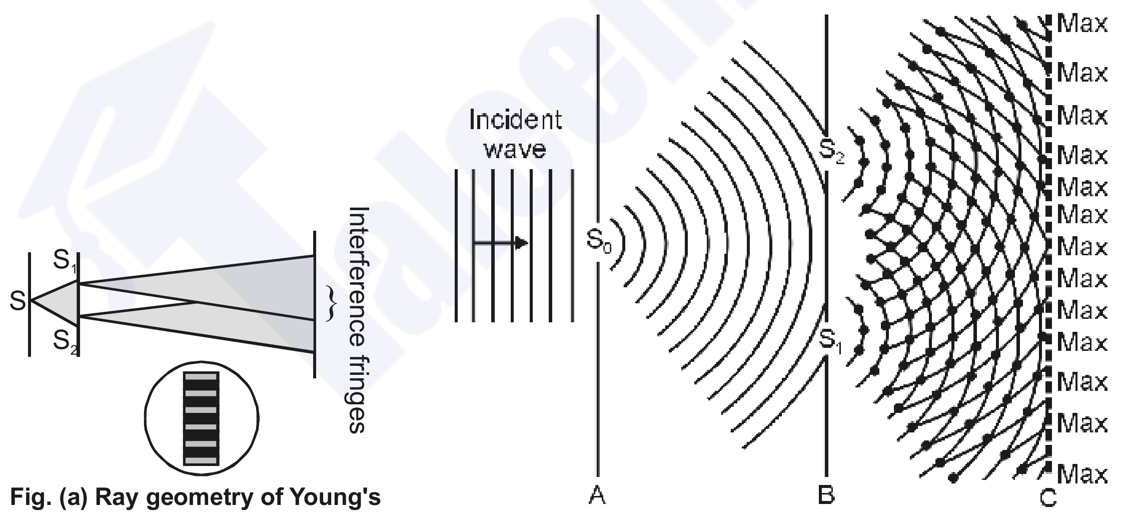


Fig. (a) Ray geometry of Young's double slit experiment.

Fig. (b) Young's double slit experiment for interference of light.

'P' is any point on the screen at a distance 'y' from the centre 'O'. Light reaching at point P from the slit B has to cover a longer distance as compared to the light from the 'A'. Draw perpendicular from slit 'A' on BP which is the path difference between the light waves reaching at point P, from the slits A and B.

If S = $m\lambda$

where m = 0, 1, 2, 3, ...

The interference produced is constructive and the point P is a bright point. However if.

$$S = \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, 3, \ldots\ldots\ldots$

The interference produced is destructive and the point P is a dark point. To find out the path difference between the waves, we proceed as follows. Join C with point P them from the figure.

$$\frac{BD}{AB} = \sin\theta$$

$$BD = AB\sin\theta$$

$$BD = d\sin\theta$$

The point P is to be a bright point

$$d\sin\theta = m\pi$$

where $m = 0, 1, 2, 3, \ldots\ldots\ldots$

the point P to be a dark point

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, 3, \ldots\ldots\ldots$

Calculation of Y

To calculate the distance between the point P and the centre of the screen.

Therefore $\tan\theta = \dfrac{PO}{OC}$

$$\tan\theta = \frac{Y}{L}$$

$$L\tan\theta = y$$

usually the point P is very close to the point C, so the angle θ is very small then

$$\tan\theta \simeq \sin\theta$$

$$L\sin\theta = y$$

But $d\sin\theta = m\lambda$

$$\sin\theta = \frac{m\lambda}{d}$$

$$L \times \frac{m\lambda}{d} = y$$

$$m\lambda = \frac{dy}{L}$$

$$\lambda = \frac{dy}{mL}$$

Knowing the values of d and L, measuring the distance y and counting, the number of fringes, Then the wavelength λ can be calculated.

Fringe Width or Fringe Spacing

It is the distance between two consecutive bright and dark fringes which is calculated as

For Your Information		
θ°	**sin θ**	**tan θ**
2	0.035	0.035
4	0.070	0.070
6	0.104	0.105
8	0.139	0.140
10	0.174	0.176

Position of mth dark fringe $= \left(m + \frac{1}{2}\right)\frac{\lambda L}{d}$

Position of (m – 1)th dark fringe $= \left(m - 1 + \frac{1}{2}\right)\frac{\lambda L}{d}$

Fringe spacing $= \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} - \left(m - 1 + \frac{1}{2}\right)\frac{\lambda L}{d}$

$= \left(m + \frac{1}{2} - m + 1 - \frac{1}{2}\right)\frac{\lambda L}{d}$

For dark fringes $= \frac{\lambda L}{d}$

For bright fringes, position of mth bright fringe $= \frac{m\lambda L}{d}$

Position of (m – 1) th bright fringe $= (m-1)\frac{\lambda L}{d}$

Fringe width $= y_{m+1} - y_m$

$= (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$

$= (m+1-m)\frac{\lambda L}{d}$

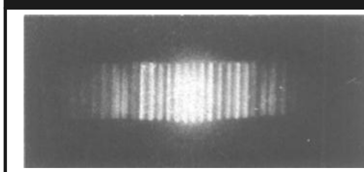
$= \frac{\lambda L}{d}$

If the distance b/w the adjacent bright fringes is ΔY, then.

$$\Delta Y = \frac{\lambda L}{d}$$

Similarly, the distance between two adjacent dark fringes can be proved to be $\frac{\lambda L}{d}$. It is found that the bright and dark fringes are of equal width and are equally spaced. The fringe spacing increases if red light is used as compared to blue light. The fringe spacing varies directly with distance L between the slits and screen and inversely with the separation d of the slits. If the separation d between the two slits, the order m of the bright or dark fringe and fringe spacing ΔY are known, the wavelength λ, of the light used for interference effect can be determined by using above equation.

Tidbits



An interference pattern formed with white light.

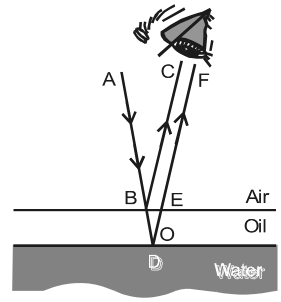
Interesting Information

Colours seen on oily water surface are due to interference of incident white light.

Q.5 Explain the phenomenon of interference in thin films.

Ans. INTERFERENCE IN THIN FILMS

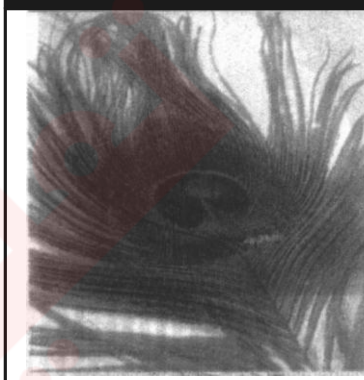
A thin film is a transparent medium whose thickness is comparable with the wavelength of light. Brilliant and beautiful colours in soap bubbles and oil film on the surface of water are due to interference of light reflected from the two surfaces of the film as explained below.



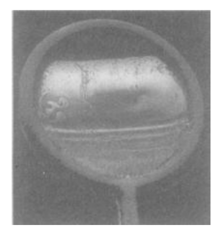
Geometrical construction of interference of light due to a thin oil film.

Consider a thin film of a refracting medium. A beam AB of monochromatic light of wavelength λ is incident on its upper surface. It is partly reflected along BC and partly refracted into the medium along BD. At D it is again partly reflected inside the medium along DE and then at E refracted along EF as shown in figure. The beams BC and EF, being the parts of the same primary beam have a phase coherence. As the film is thin, so the separation between the beam BC and EF will be very small, and they will superpose and the result of their interference will be detected by the eye. It can be seen in figure that the original beam splits into two parts BC and EF due to the thin film which enter the eye after covering different lengths of paths. Their path difference depends upon (i) thickness and nature of the film and (ii) angle of incidence. If the two reflected waves reinforce each other, then the film as seem with the help of a parallel beam of monochromatic light will look bright. However, if the thickness of the film and angle of incidence are such that the two reflected waves cancel each other, the film will look dark.

Do You Know?


The vivid iridescence of peacock feathers due to interference of the light reflected from its complex layered surface.

If white light is incident on a film of irregular thickness at all possible angles, we should consider the interference pattern due to each spectral colour separately. It is quite possible that at a certain place on the film its thickness and the angle of incidence of light are such that the condition of destructive interference of one colour is being satisfied. Hence, that portion of the film will exhibit the remaining constituent colours of the white light as shown in figure.


Interference pattern produced by a thin soap film illuminated by white light.

Q.6 What are Newton's rings? Describe the experiment arrangement of producing the Newton rings. Why does central spot of Newton rings look dark?

Ans. NEWTON'S RINGS

When a plano-convex lens of long focal length is placed in contact with a plane glass plate (Fig. a) a thin air firm is enclosed between the upper surface of the glass plate and the lower surface of the lens. The thickness of the air film is almost zero at the point of contact O and it gradually increases as one proceeds towards the periphery of the lens. Thus, the points where the thickness of air film is constant, will lie on a circle with O as centre.

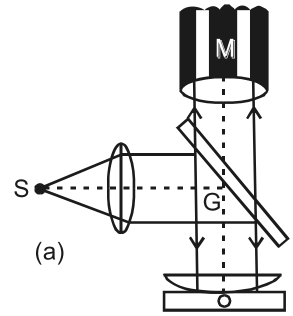


Fig. (a) Experimental arrangement for observing Newton's rings.

By means of a sheet of glass G, a parallel beam of monochromatic light is reflected towards the plano-convex lens L. Any ray of monochromatic light that strikes the upper surface of the air film nearly along normal is partly reflected and partly refracted. The ray refracted in the air film is also reflected partly at the lower surface of the film. The two reflected rays, i.e., produced at the upper and lower surfaces of the film, are coherent and interfere constructively or destructively.

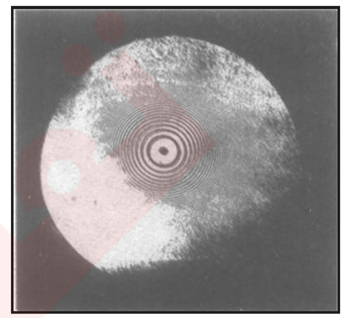


Fig. (b) A pattern of Newton's rings due to interference of monochromatic light.

At the point of contact of the lens and the glass plate, the thickness of the film is effectively zero but due to reflection at the lower surface of air film from denser medium, an additional path difference of $\lambda/2$ is introduced. Consequently, the centre of Newton rings is dark due to destructive interference.

Q.7 Describe principle and working of Michelson's interferometer. How can you find the wavelength of light used?

Ans. MICHELSON'S INTERFEROMETER

Michelson's interferometer is an instrument that can be used to measure distance with extremely high precision. **Albert A. Michelson** devised this instrument in **1881** using the idea of interference of light rays. The essential features of a Michelson's interferometer are shown schematically in figure.

Monochromatic light from an extended source falls on a half silvered glass plate G_1 that partially reflects it and partially transmits it. The reflected portion labelled as I in the figure travels a distance L_1 to mirror M_1, which reflects the beam back towards G_1. The half silvered plate G_1 partially transmits this portion that finally arrives at the observer's eye. The transmitted portion of the original beam labelled as II, travels a distance L_2 to mirror M_2 which reflects the beam back toward G_1. The beam II partially reflected by G_1 also arrives the observer's eye finally. The plate G_2, cut from the same piece of glass as G_1, is introduced in the path of beam II as a compensator plate. G_2, therefore, equalizes the path length of the beams I and II in glass. The two beams having their different paths are coherent. They produce interference effects when they arrive at observer's eyes. The observer then sees a series of parallel interference fringes.

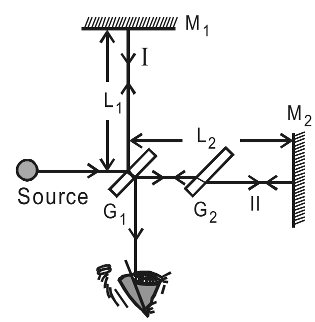


Fig. Schematic diagram of a Michelson's interferometer.

In a practical interferometer, the mirror M_1 can be moved along the direction perpendicular to its surface by means of a precision screw. As the length L_1 is changed, the pattern of interference fringes is observed to shift. If M_1 is displaced through a distance equal to $\lambda/2$, a path difference of double of this displacement is produced, i.e., equal to λ. Thus a fringe is seen shifted forward across the line of reference of cross wire in the eye piece of the telescope used to vie the fringes.

A fringe is shifted, each time the mirror is displaced through $\lambda/2$. Hence, by counting the number m of the fringes which are shifted by the displacement L of the mirror, we can write the equation.

$$L = m\frac{\lambda}{2} \qquad \text{......... (1)}$$

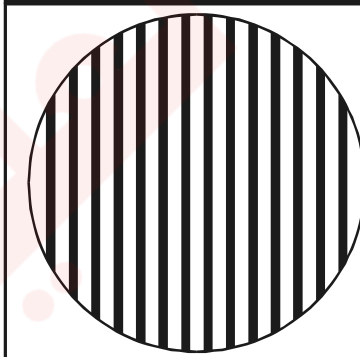
Very precise length measurements can be made with an interferometer. The motion of mirror M_1 by only $\lambda/4$ produces a clear difference between brightness and darkness. For λ = 400 nm, this means a high precision of 100 nm or 10^{-4} mm.

Michelson measured the length of standard metre in terms of the wavelength of red cadmium light and showed that the standard metre was equivalent to 1, 553, 163.5 wavelengths of this light.

For Your Information

A photograph of Michelson Interferometer.

For Your Information



Interference fringes in the Michelson Interformeter.

Q.8 Explain the phenomenon of diffraction of light.

Ans. **DIFFRACTION OF LIGHT**

In Young's double slit experiment for the interference of light, the central region of the fringe system is bright. If light travels in a straight path, the central region should appear dark i.e., the shadow of the screen between the two slits. Another simple experiment can be performed for exhibiting the same effect.

Consider that a small and smooth steel ball of about 3mm in diameter is illuminated by a point source of light. The shadow of the object is received on a screen as shown in fig. The shadow of the spherical object is not completely dark but has a bright spot at its centre. According to Huygen's principle, each point on the rim of the sphere behaves as a source of secondary wavelets which illuminate the central region of the shadow.

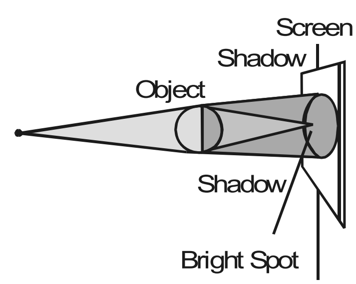


Fig. Bending of light caused by its passage past spherical object.

These two experiments clearly show that when light travels past an obstacle, it does not proceed exactly along a straight path, but bends around the obstacle.

The property of bending of light around obstacles and spreading of light wave into the geometrical shadow of an obstacle is called diffraction.

The phenomenon is found to be prominent when the wavelength of light is large as compared to the size of the obstacle or aperture of light. The diffraction of light occurs, due to the interference

Point to Ponder

Hold two fingers close together to form a slit. Look at a light bulb through the slit. Observe the pattern of light being seen and think why it is so.

between rays coming from different parts of the same wavefront.

Q.9 Explain the diffraction of due to light passing through a narrow slit.

Ans. DIFFRACTION DUE TO A NARROW SLIT

Fig. shows the experimental arrangement for studying diffraction of light due to a narrow slit. The slit AB of width d is illuminated by a parallel beam of monochromatic light of wavelength λ. The screen S is placed parallel to the slit for observing the effects of the diffraction of light. A small portion of the incident wavefront passes through the narrow slit. Each point of this section of the wavefront sends out secondary wavelets to the screen. These wavelets then interfere to produce the diffraction pattern. It becomes simple to deal with rays instead of wavefronts as shown in the figure. In this figure, only nine rays have been drawn whereas actually there are a large number of them. Let us consider rays 1 and 5 which are in phase on in the wavefront AB. When these reach the wavefront AC, ray 5 would have a path difference ab say equal to $\lambda/2$. Thus, when these two rays reach point P on the screen; they will interfere destructively. Similarly, each pair 2 and 6, 3 and 7, 4 and 8 differ in path by $\lambda/2$ and will do the same. But the path difference ab = d/2 sin θ.

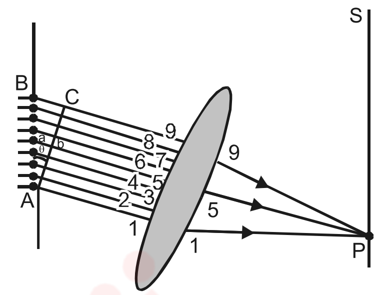


Fig. Diffraction of light due to a narrow slit AB. The dots represent the sources of secondary wavelets.

The equation for the first minimum is, then

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

or

$$d\sin\theta = \lambda \qquad \ldots\ldots (2)$$

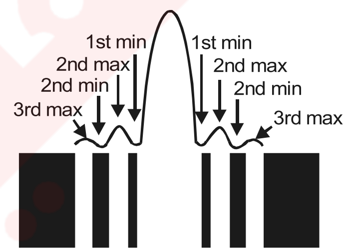


Fig. (a) Diffraction pattern of mono chromatic light produced due to a single slit; graphical representation and photograph of the pattern.

In general, the conditions for different orders of minima on either side of centre are given by

$$d\sin\theta = m\lambda \quad \text{where}$$

$$m = \pm(1, 2, 3, \ldots\ldots) \qquad \ldots\ldots (3)$$

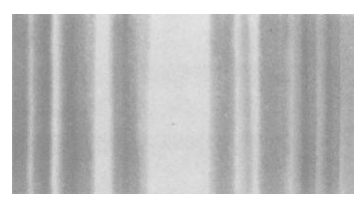


Fig.(b) Diffraction pattern produced by white light through a single still.

The region between any two consecutive minima both above and below O will be bright. A narrow slit, therefore, produces a series of bright and dark regions with the first bright region at the cerntre of the pattern. Such a diffraction pattern is shown in Fig. (a) and (b).

Q.10 Describe the diffraction grating and obtained the grating equation to find the wavelength of light used.

Ans. DIFFRACTION GRATING

A diffraction grating consists of a glass plate on which very fine equidistant parallel lines (scratches) are drawn by means of a fine diamond point. The lines (Scratches) acts as opaque, while the spacing between the lines on glass plate act as slits. A typical diffraction grating has about 400 to 5000 lines per centimeter.

In order to understand how a grating diffracts light, consider a parallel beam of monochromatic light illuminating the grating at normal incidence. A few of the equally spaced narrow slits are shown in the figure. The distance between two adjacent slits is d, called grating element. Its value is obtained by dividing the length L of the grating by the total number N of the lines ruled on it. The sections of wavefront that pass through the slits behaves as sources of secondary wavelets according to Huygen's principle.

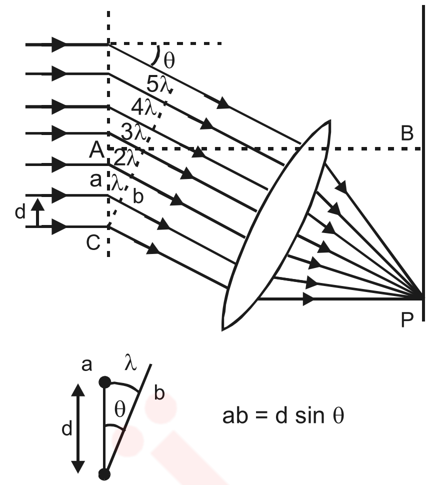


Fig. Diffraction of light due to grating.

In figure consider the parallel rays which after diffraction through the grating make an angle θ with AB, the normal to grating. They are then brought to focus on the screen at P by a convex lens. If the path difference between rays No. 1 and 2 is one wavelength λ, they will reinforce each other at P. As the incident beam consists of parallel rays, the rays from any two consecutives slits will differ in path by λ when they arrive at P. They will, therefore, interfere constructively. Hence, the condition for constructive interference is that ab, the path difference between two consecutive rays, should be equal to λ i.e.,

$$ab = \lambda \quad \ldots\ldots\ldots (4)$$

$$ab = d \sin\theta \quad \ldots\ldots\ldots (5)$$

d being the grating element. Substituting the value of ab in Eq. (4).

$$d \sin\theta = \lambda \quad \ldots\ldots\ldots (6)$$

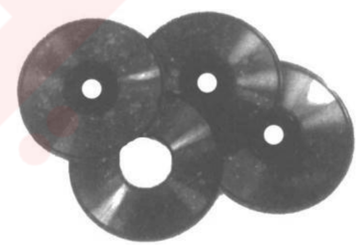
According to Eq. (5), when θ = 0 i.e., along the direction of normal to the grating, the path difference between the rays coming out from the slits of the grating will be zero. So we will get a bright image in this direction. This is known as zero order image formed by the grating. If we increase θ on either side of this direction, a value of θ will be arrived at which d sin θ will become λ and according to Eq. (6), we will again get a bright image. This is known as first order image of the grating. In this way if we continue increasing θ, we will get the second, third, etc. images on either side of the zero order image with dark regions in between. The second, third order bright images would occur accordingly as d sin θ becoming equal to 2 λ, 3 λ, etc. Thus the Eq. (6) can be written in more general form as.

$$d \sin\theta = n\lambda \quad \ldots\ldots\ldots (7)$$

Where, $n = 0 \pm 1 \pm 2 \pm 3$ etc.

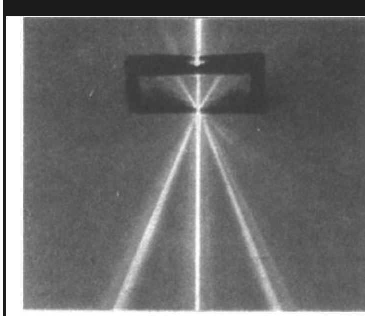
However, if the incident light contains different wavelengths, the image of each wavelength for a certain value of n is diffracted in a different direction. Thus, separate images are obtained corresponding to each wavelength or colour. Eq. (7) shows that the value of θ depends upon n, so the images of different colours are much separated in higher orders.

Interesting Information



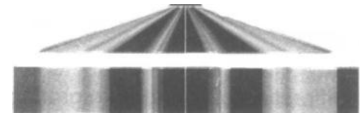
The fine rulings, each 0.5μm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are composite of the diffraction patterns from the ruling.

Can You Tell?



Light waves projected through this diffraction grating produce an interference pattern. What colours are between the bands of interference?

For Your Information



Diffraction of white light by a fine diffraction grating.

Q.11 Explain diffraction of X-rays by crystals.

Ans. **DIFFRACTION OF X-RAYS BY CRYSTALS**

X-rays are the electromagnetic waves of much shorter wavelength, about 10^{-10} m.

In order to observe the effects of diffraction, the grating spacing must be of the order of the wavelength of the radiation used. The regular array of atoms in a crystal forms a natural diffraction grating with spacing that is typically $\approx 10^{-10}$ m. The scattering of X-rays from the atoms in a crystalline lattice gives rise to diffraction effects very similar to those observed with visible light incident on ordinary grating.

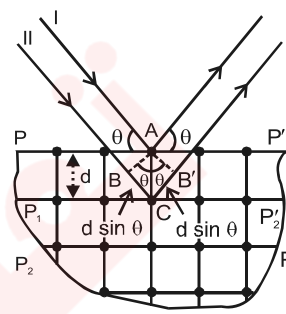
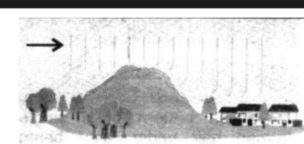


Fig. Diffraction of X-rays from the lattice plane of crystal.

The study of atomic structure of crystals by X-rays was initiated in 1914 by W.H. Bragg and W.L Bragg with remarkable achievements. They found that a monochromatic beam of X-rays was reflected from a crystal plane as if it acted like mirror. To understand this effect, a series of atomic planes of constant interplanar spacing d parallel to a crystal face are shown by lines PP′ $P_1P'_1$, $P_2P'_2$ and so on, in figure.

Interesting Application



Diffraction of radio waves

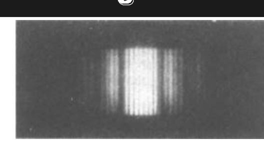
Suppose an x-ray beam is incident at an angle ‘θ’ on one of the planes as shown in figure. The beam can be reflected from both upper and lower planes of atoms. The beam reflected from lower plane travels some extra distance as compared to the beam reflected from the upper plane. The effective path difference between the two reflected beams is 2d sinθ. Therefore, for reinforcement, the path difference should be an integral multiple of the wavelength. Thus

$$2d \sin \theta = n\lambda \qquad \text{......... (8)}$$

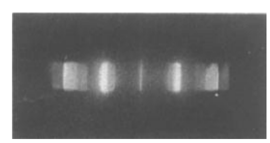
The value of n is referred to as the order of reflection. The equation (8) is known as the Bragg equation. It can be used to determine interplanar spacing between similar parallel planes of a crystal if X-rays of known wavelength are allowed to diffract from the crystal.

X-ray diffraction has been very useful in determining the structure of biologically important molecules such as hemoglobin, which is an important constituent of blood, and double helix structure of DNA.

Interesting Information

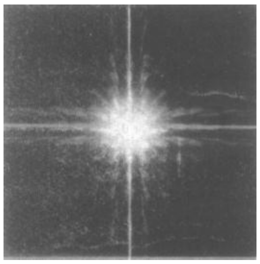


The spectrum of white light due to diffraction gratting of 100 silts.



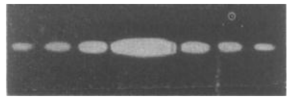
The spectrum of white light due to diffraction grating of 2000 slits.

Interesting Illustration



A multi-aperture diffraction pattern. This is a picture of a white-light point source shot through a place of tightly woven cloth.

Tidbits



Diffraction pattern of a single human hair under laser beam illumination.

For Your Information

Looking through two polarizes. When they are "crossed", very little light passes through.

Q.12 Explain the phenomenon polarization.

Ans. **POLARIZATION**

In transverse mechanical waves, such as produced in a stretched string, the vibrations of the particles of the medium are perpendicular to the direction of propagation of the waves. The vibration can be oriented along vertical, horizontal or any other direction. In each of these cases, the transverse mechanical wave is said to be polarized. The plane of polarization is the plane containing the direction of vibration of the particles of the medium and the direction of propagation of the wave.

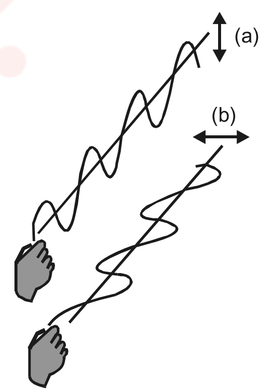


Fig. Transverse waves on a string polarized.
(a) in a vertical plane and
(b) in a horizontal plane

A light wave produced by oscillating charge consists of a periodic variation of electric field vector accompanied by the magnetic field vector at right angle to each other. Ordinary light has components of vibration in all possible planes. Such a light is unpolarized. On the other hand, if the vibrations are confined only in one plane, the light is said to be polarized.

Production and Detection of Plane Polarized Light

The light emitted by an ordinary incandescent bulb (and also by the Sun) is unpolarized, because its (electrical) vibrations are randomly oriented in space. It is possible to obtain plane polarized beam of light from un-polarized light by removing all waves from the beam except those having vibrations along one particular direction. This can be achieved by various processes such as selective absorption, reflection from different surfaces, refraction through crystals and scattering by small particles.

The selective absorption method is the most common method to obtain plane polarized light by using certain types of materials called dichroic substances. These materials transmit only those waves, whose vibrations are parallel to a particular direction and will absorb those waves whose vibrations are in other directions. One such commercial polarizing material is a polaroid.

If un-polazied light is made incident on a sheet of polaroid, the transmitted light will be plane polarized. If a second sheet of polaroid is placed in such a way that the axes of the polaroids, shown by straight lines drawn on them, are parallel, the light is transmitted through the second polaroid also. If the second polaroid is slowly rotated about the beam of light, as axis of rotation, the light emerging out of the second polaroid gets dimmer and dimmer and disappears when the axes become mutually perpendicular. The light reappears on further rotation and becomes brightest when the axes are again parallel to each other.

This experiment proves that light waves are transverse waves. If the light waves were longitudinal, they would never disappear even if the two polaroids were mutually perpendicular.

Reflection of light from water, glass, snow and rough road surfaces, for larger angles of incidences, produces glare. Since the reflected light is partially polarized, glare can considerably by reduced by using polaroid sunglasses.

Sunlight also becomes partially polarized because of scattering by air molecules of the Earth's atmosphere. This effect can be observed by looking directly up through a pair of sunglasses made of polarizing glass. AT certain orientations of the lenses, less light passes through than at others.

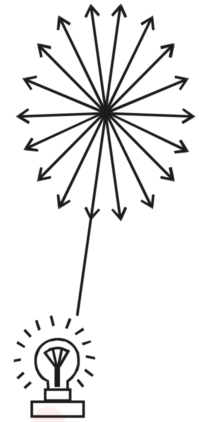


Fig. An unpolarized light, due to incandescent bulb, has vibrations in all directions.

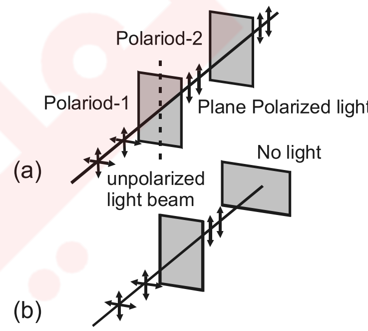
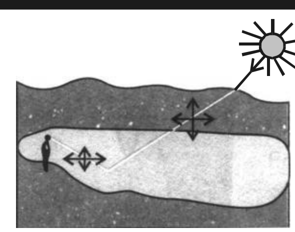


Fig. Experimental arrangement to show that light waves are transverse. The lines with arrows indicates electric vibrations of light waves.

Do You Know?



Light reflected from smooth surface of water is partially polarized parallel to the surface.

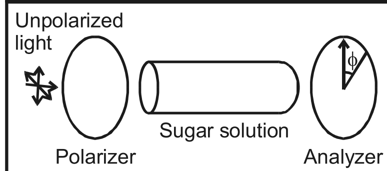
Q.13 Define optical rotation.

Ans. OPTICAL ROTATION

When a plane polarized light is passed through certain crystals, they rotate the plane of polarization. Quartz and sodium chlorate crystals are typical examples, which are termed as optically active crystals.

A few millimeter thickness of such crystals will rotate the plane of polarization by many degrees. Certain organic substances, such as sugar and tartaric acid, show optical rotation when they are in solution. This property of optically active substances can be used to determine their concentration in the solutions.

Interesting Information



Sugar solution rotates the plane of polarization of incident light so that it is no longer horizontal but at an angle. The analyzer thus stops the light when rotated from the vertical (crossed) positions.

SOLVED EXAMPLES

EXAMPLE 9.1

The distance between the slits in Young's double slit experiment is 0.25 cm. Interference fringes are formed on a screen placed at a distance of 100cm from the slits. The distance of the third dark fringe from the central bright fringe is 0.059 cm. Find the wavelength of the incident light.

Data

Distance between slits = d = 0.25cm

= 0.25×10^{-2} m

Distance between slits and screen = L = 100 cm

= 1 m

Distance of third dark fringe = Y = 0.059 cm

= 0.059×10^{-2} m

To Find

Wavelength of light = λ = ?

SOLUTION

Using for dark fringe

$$Y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d}$$

where $m = 0, 1, 2, \ldots\ldots$

For 3rd dark fringe

$$m = 2$$

Putting values in above equation

$$0.059 \times 10^{-2} = \left(2 + \frac{1}{2}\right)\lambda \frac{1}{0.25 \times 10^{-2}}$$

$$\therefore \quad \lambda\left(\frac{4+1}{2}\right) = 0.059 \times 10^{-2} \times 0.25 \times 10^{-2}$$

$$\frac{5}{2}\lambda = 0.01475 \times 10^{-4}$$

$$\lambda = \frac{0.0295}{5} \times 10^{-4}$$

$$= 5.9 \times 10^{-3} \times 10^{-4}$$

$$= 5.9 \times 10^{-7}\ \text{m}$$

$$= 590 \times 10^{-9}\ \text{m}$$

$$= 590\ \text{nm}$$

Result

Wavelength of light $= \lambda = 590$ nm

EXAMPLE 9.2

Yellow sodium light of wavelength 589 nm, emitted by a single source passes through two narrow slits 1.00 mm apart. The interference pattern is observed on a screen 225 cm away. How far apart are two adjacent bright fringes?

Data

$$\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$$

$$d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$L = 225 \text{ cm} = 225 \times 10^{-2} \text{ m}$$

To Find

Distance between two adjacent bright fringes $= \Delta y = ?$

SOLUTION

Using

$$\Delta Y = \frac{\lambda L}{d}$$

Putting values

$$\Delta Y = \frac{589 \times 10^{-9} \times 225 \times 10^{-2}}{1 \times 10^{-3}}$$

$$= 132525 \times 10^{-11+3}$$

$$= 132525 \times 10^{-8}$$

$$= 1.325 \times 10^{-3}$$

$$= 1.33 \times 10^{-3} \text{ m}$$

$$= 1.33 \text{ mm}$$

Result

Distance between two adjacent bright fringes $= \Delta Y = 1.33$ mm

EXAMPLE 9.3

Light of wavelength 450nm is included on a diffraction grating on which 5000 lines/cm have been ruled.

(i) How many orders of spectrum can be observed on either side of the direct beam?

(ii) Determine the angle corresponding to each other.

Data

Wavelength of light $= \lambda = 450\text{nm}$

$= 450 \times 10^{-9}$ m

Grating element $= d = \frac{1}{5000}$ cm

$= \frac{1}{500000}$ m

To Find

(i) Orders of spectra $= n = ?$

(ii) Angle $= \theta = ?$

SOLUTION

Using formula

$$d \sin\theta = n\lambda$$

or
$$n = \frac{d \sin\theta}{\lambda} \quad \ldots\ldots\ldots (1)$$

For maximum number of order of spectra, $\sin\theta = 1$ i.e., $\theta = 90°$

Putting values in (1), we get

$$n = \left(\frac{1}{500000}\right)\left(\frac{\sin 90^o}{450 \times 159}\right)$$

$$= \frac{1}{500000 \times 450 \times 10^{-9}}$$

$$= \frac{10^{-9}}{500000 \times 450} = 4.4$$

Hence max order of spectrum $= 4$

(ii) For First Order Spectrum.

$$n = 1$$

Using the formula

$$d \sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d} = \frac{1 \times 450 \times 10^{-9}}{\frac{1}{500000}}$$

$$= \frac{1 \times 4450 \times 10^{-9} \times 500000}{1}$$

$$\sin\theta = 0.225$$

$$\Rightarrow \quad \theta = \sin^{-1}(0.225)$$
$$= 13^{o}$$

For 2nd Order Spectrum

Putting values, we get n = 2

$$\sin\theta = \frac{2 \times 450 \times 10^{-9}}{\frac{1}{500000}}$$
$$= 2 \times 450 \times 10^{-9} \times 500000$$
$$\sin\theta = 0.45$$
$$\theta = \sin^{-1}(0.45)$$
$$\theta = 26.7^{o}$$

For 3rd Order Spectrum

$$\sin\theta = \frac{n\lambda}{d}$$
$$= \frac{3 \times 450 \times 10^{-9}}{\frac{1}{500000}}$$
$$= 3 \times 450 \times 10^{-9} \times 500000$$
$$\sin\theta = 0.675$$
$$\theta = \sin^{-1}(0.675)$$
$$\theta = 42.50^{\circ}$$

For 4th Order Spectrum

$$\sin\theta = \frac{n\lambda}{d} \quad n = 4$$
$$= \frac{4 \times 450 \times 10^{-9}}{\frac{1}{500000}}$$
$$= 4 \times 450 \times 10^{-9} \times 500000$$
$$\sin\theta = (0.9)$$
$$\theta = \sin^{-1}(0.9)$$
$$\theta = 62.2^{o}$$

Result

(i) Order of spectra = n = 4

(ii) Angle = θ = 13°

Chapter 10

OPTICAL INSTRUMENT

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Recognize the term of least distance of distinct vision.
- Understand the terms magnifying power and resolving power.
- Understand the working of spectrometer.
- Describe Michelson rotating mirror method to find the speed of light.
- Understand the principles of optical fibre.
- Identify the types of optical fibres.
- Appreciate the applications of optical fibres.

INTRODUCTION

In this chapter, some optical instruments that are based on the principles of reflection and refraction, will be discussed. The most common of these instruments are the magnifying glass, compound microscope and telescopes. We shall also study magnification and resolving powers of these optical instruments. The spectrometer and an arrangement for measurement of speed of light are also described. An introduction to optical fibres, which has developed a great importance in medical diagnostics, telecommunication and computer networking, is also included.

Q.1 Define least distance of distinct vision.

Ans. **LEAST DISTANCE OF DISTINCT VISION**

"The minimum distance from the eye at which an object appears to be distinct is called least distance of distinct vision OR near point."

The distance is about 25 cm from the eye. It is denoted by 'd'. If the object is held closer to the eye than this distance the image formed will be blurred and fuzzy. The location of the near point however changes with age.

Q.2 Define linear magnification.

Ans. **LINEAR MAGNIFICATION**

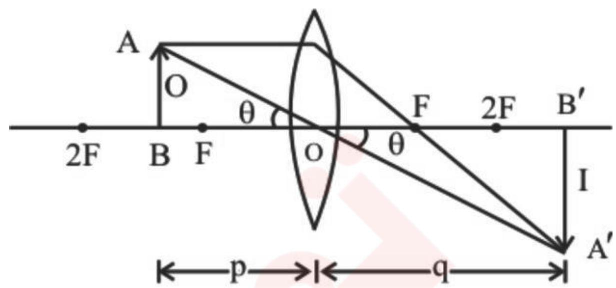
"The ratio of the size of the image to the size of the object is called magnification."

It is denoted by 'M'.

$$\therefore \quad M = \frac{I}{O}$$

Q. Show that $M = \frac{I}{O} = \frac{q}{p}\left(OR\ M = \frac{q}{p}\right)$

Ans. When an object is placed in front of a convex lens at a point beyond its focus, a real and inverted image of the object is formed as shown in figure.



ΔABO and $\Delta A'B'O$ are similar triangles.

For similar triangles the ratio of the length of the corresponding sides is equal i.e.,

$$\frac{I}{O} = \frac{OB'}{OB}$$

$$\frac{I}{O} = \frac{q}{p} \quad \left\{ \begin{array}{l} \because OB' = q = \text{image distance} \\ OB = p = \text{object distance} \end{array} \right.$$

As, $$M = \frac{I}{O}$$

$$\therefore \quad M = \frac{I}{O} = \frac{q}{p}$$

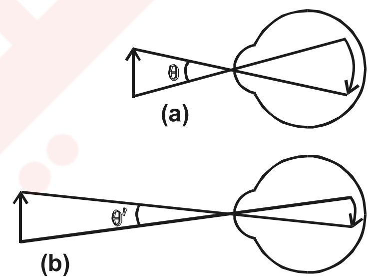


Fig. When the same object is viewed at a shorter distance, the image on the retina of the eye is greater; so the object appears larger and more details can be seen. The angle θ the object subtends in (a) is greater than θ' in (b).

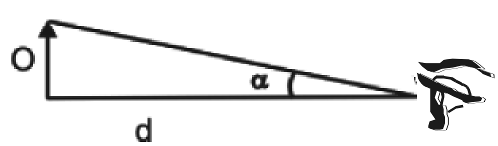
1. Visual Angle

The angle subtended by an object at the eye, is called visual angle.

The apparent size of an object depends upon its actual size and on the angle subtended by it at the eye. Thus the closer the object is to the eye, the greater is the angle subtended and larger appears the size of the object.

2. Object Angle

"When the object is placed at 'd', then the angle subtended at a naked (unaided) eye, by the object is called object angle".

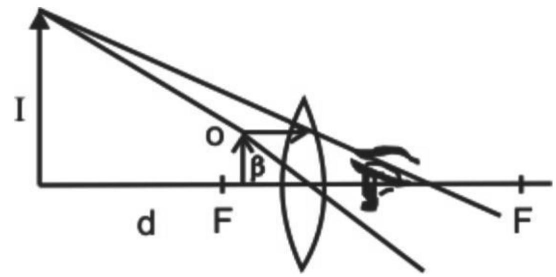


It is denoted by α.

3. Image Angle

"When the angle subtended by the image, formed at 'd', on the eye, is called image angle".

It is denoted by β.



Q.3 Define angular magnification.

Ans. **ANGULAR MAGNIFICATION**

"The ratio of the angles subtended by the image as seen through the optical device (aided eye) to that subtended by the object at the unaided eye, is called angular magnification".

It is also called magnifying power.

$$\therefore \quad M = \frac{\beta}{\alpha}$$

Tid-bits

If you find it difficult to read small print, make a pinhole in a piece of paper and hold it in front of your eye close to the page. You will see the print clearly.

Q.4 Define resolving power.

Ans. **RESOLVING POWER**

"The resolving power of an instrument is its ability to reveal the minor details of the object under examination."

Resolving power is expressed as the reciprocal of minimum angle which two point sources subtends at the instrument so that their images are seen as two distinct spots of light rather than one Raleigh showed that for light of wavelength 'λ', through a lens of diameter D, the resolving power

$$R = \frac{1}{\alpha_{min}} = \frac{D}{1.22\,\lambda}$$

where $\alpha_{min} = 1.22\,\dfrac{\lambda}{D}$

$$\therefore \quad R = \frac{1}{1.22\,\lambda/D} = \frac{D}{1.22\,\lambda}$$

The smaller the value of α_{min} greater is the resolving power because two distant objects which one close together can be seen separated through the instrument.

In case of a grating spectrometer, the resolving power 'R' of the grating is defined as;

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

Thus we see that a grating with high resolving power can distinguished small difference in wavelength. If 'N' is the number on rulings of the gratings, it can be shown that the resolving power in the mth order (m) diffraction equals the product i.e., $N \times m$.

$$R = N \times m$$

Q.5 What is simple microscope? Derive an expression for the magnifying power of simple microscope.

Ans. SIMPLE MICROSCOPE OR (MAGNIFYING GLASS) OR (MAGNIFIER)

Definition

"An optical instrument, which is used to see small objects distinctly, is called simple microscope."

Construction

It consist of a biconvex lens.

Principle

When the object is placed inside the focal point of the lens, then erect, magnified and virtual image is formed at 'd' (near point or 25 cm).

Magnification

The image formed by the object, when placed at 'd' on the eye as shown in Fig. 2 A lens is placed just in front of the eye and the object is placed in front of the lens, in such a way that a virtual image is formed at 'd', from the eye. The size of the image is now much larger than without the lens.

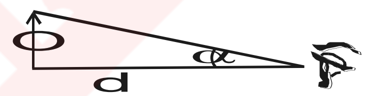


Fig1

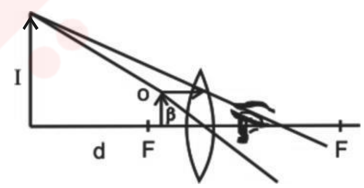


Fig 2

If 'β' and 'α' are the respective angles subtended by the object when seen through the lens and when viewed directly, then,

$$M = \frac{\beta}{\alpha} \quad \text{......... (1)}$$

From Fig. 1

$$\tan\alpha = \frac{o}{d}$$

and from Fig. 2

$$\tan\beta = \frac{I}{d}$$

If α and β are very small then,

$$\tan\alpha \approx \alpha$$

$$\tan\beta \approx \beta$$

$$\therefore \quad \alpha = \frac{o}{d}$$

$$\beta = \frac{I}{d}$$

Putting the values of 'α' and 'β' in equation (1).

$$\therefore \quad M = \frac{I/d}{o/d}$$

$$M = \frac{I}{d} \times \frac{d}{o}$$

$$M = \frac{I}{o}$$

As, $$\frac{I}{o} = \frac{q}{p}$$

$$\therefore \quad M = \frac{q}{p}$$

Since the image is at the least distance of distinct vision i.e., q = d.

$$\therefore \quad M = \frac{d}{p} \quad \ldots\ldots\ldots (2)$$

The lens formula is,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

For virtual image

$$q = -q$$

$$\therefore \quad \frac{1}{f} = \frac{1}{p} - \frac{1}{q}$$

Since $$q = d$$

So, $$\frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

Multiply by 'd' both sides

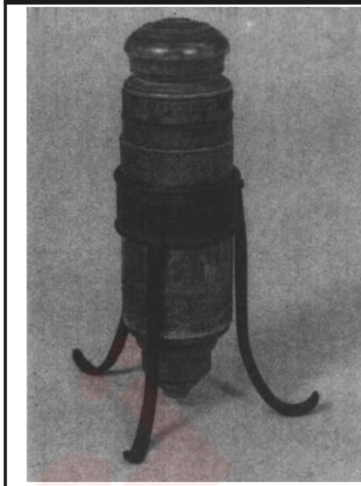
$$\frac{d}{f} = \frac{d}{p} - \frac{d}{d}$$

$$\frac{d}{f} = M - 1 \qquad \left(\because \text{ from eq. (2) } M = \frac{d}{p}\right)$$

$$\therefore \quad M = 1 + \frac{d}{f}$$

It is clear for a lens of high angular magnification, the focal length will be small.

For Your Information



A seventeenth century microscope which could be moved up and down in its support ring (courtesy of the Museum of the History of Science Florence).

Q.6 Describe the construction of compound microscope. Also derive the expression for its magnifying power.

Ans. COMPOUND MICROSCOPE

Whenever high magnification is desired, a compound microscope is used.

Construction

It consists of two convex lenses, an objective lens (lens towards the object) of very short focal length and an eye piece (lens towards the eye) of comparatively longer focal length. The ray diagram is shown.

The object of height 'h' is placed just beyond the principal focus of the objective. This produces a real, magnified image of height 'h_1' at a place situated with in the focal point of the eye piece. The eye piece is used as a magnifying glass to see the image formed by the objective, which becomes object for eye piece. The final image seen by the eye through microscope is virtual and very much enlarged. In normal adjustment, the eye piece is positioned so that the final image is formed at the near point of the eye i.e., at 'd'.

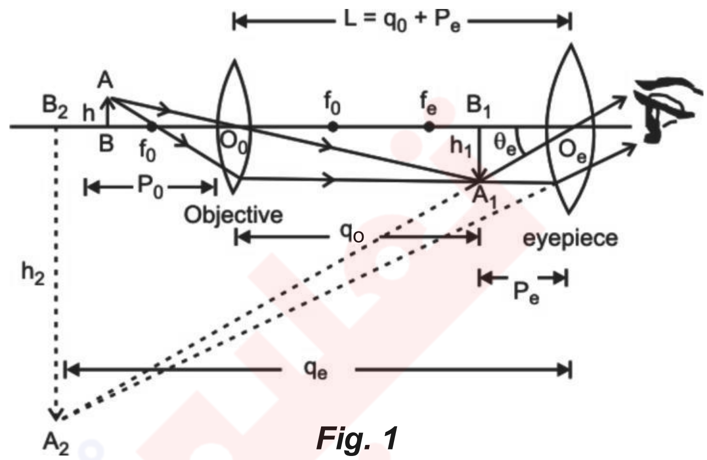


Fig. 1

Magnification

Here, $$M = \frac{\tan \theta_e}{\tan \theta}$$

Where 'θ_e' is the angle subtended by the final image 'h_2' and 'θ' is the angle subtended by the object at eye h, if placed at 'd'.

From Fig. '1' and '2'.

$$\tan \theta_e = \frac{h_2}{d} \quad \text{and} \quad \tan \theta = \frac{h}{d}$$

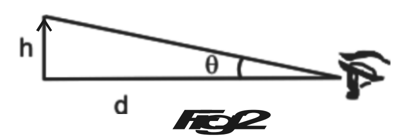


Fig2

$$\therefore \quad M = \frac{h_2 / d}{h / d}$$

$$M = \frac{h_2}{d} \times \frac{h}{d}$$

$$M = \frac{h_2}{h}$$

or $$M = \frac{h_1}{h} \times \frac{h_2}{h_1}$$

Where '$\frac{h_1}{h}$' is the linear magnification 'M_1' of the objective and '$\frac{h_2}{h_1}$' is the magnification 'M_2' of the eye piece.

Hence total magnification is;

$$M = M_1 \times M_2$$

Since $M_1 = \frac{q_0}{p_0}$

and $M_2 = 1 + \frac{d}{f_e}$ ($\because$ eye piece is working as a simple microscope)

$\therefore$ $M = \frac{q_0}{p_0}\left(1 + \frac{d}{f_e}\right)$

Note Since the object is placed just beyond the principle focus of the objective.

$\therefore$ $P_0 \approx f_0$

As, $L = q_0 + p_e$

As the image h_1 is placed very closed to eye piece

$\therefore$ $p_e \approx 0$

$\therefore$ $L = q_0$

$\therefore$ $M = \frac{L}{f_0}\left(1 + \frac{d}{f_e}\right)$

The limit to which a microscope can be used to resolve details, depends on the width of the objective. A wider objective and use of blue light of short wavelength produces less diffraction and allows more details to be seen.

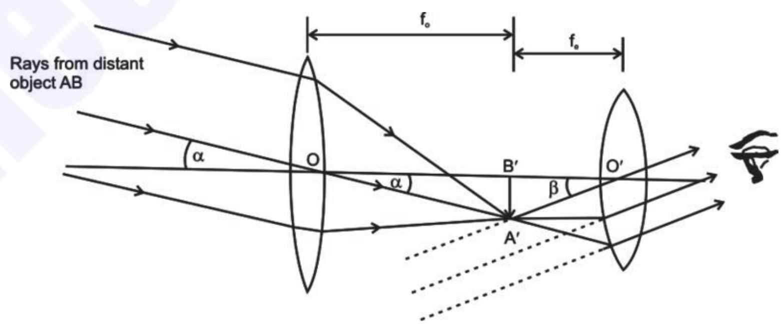
Q.7 What is astronomical telescope? Describe the construction and magnifying power of astronomical telescope.

Ans. **ASTRONOMICAL TELESCOPE**

"An optical device, which is used to see the heavenly objects like moon and stars."

It is a refracting telescope.

A simple astronomical telescope consists of two convex lenses, an **objective** of long focal length 'f_0' and an **eye piece** of short focal length f_e. The objective forms a real, inverted and diminished image $A'B'$ of a distant object AB. This real image $A'B'$ acts as object for the eye piece which is used as a magnifying glass.



The final image seen through the eye-piece is virtual, enlarged and inverted, as shown in the figure.

When a very distant object is viewed, the rays of light coming from any of its point (say its top) are considered parallel and these parallel rays are converged by the objective to form a real image $A'B'$ at its focus. If it is desired to see the final image through the eye–piece without any strain on the eye, the eye-piece must be placed so that the image $A'B'$ lies at its focus. The rays after refraction through the eye-piece will becomes parallel and the final image appears to be formed at infinity. In this condition the image $A'B'$ formed by the objective lies at the focus of both the objective and eye–piece and the telescope is said to be in normal adjustment.

Magnification

The angle ‘α’ subtended at the unaided eye is practically same as subtended at the objective.

Consider $\Delta OA'B'$

$$\alpha = \tan\alpha = \frac{A'B'}{OB'}$$

$$\therefore \quad \alpha = \frac{A'B'}{f_0}$$

Considering $\Delta O'A'B'$

$$\beta = \tan\beta = \frac{A'B'}{O'B'}$$

$$\therefore \quad \beta = \frac{A'B'}{f_e}$$

Hence magnifying power of telescope is

$$M = \frac{\beta}{\alpha}$$

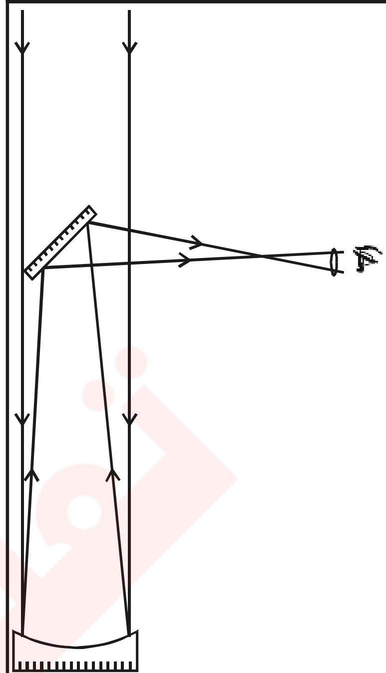
Putting values of ‘α’ and ‘β’.

$$\therefore \quad M = \frac{A'B'/f_e}{A'B'/f_0}$$

$$M = \frac{A'B'}{f_e} \times \frac{f_0}{A'B'}$$

$$M = \frac{f_0}{f_e}$$

For Your Information



Reflecting Telescope

Large astronomical telescopes are reflecting type made from specially shaped very large mirrors used as objectives. With such telescopes, astronomers can study stars which are millions light year away.

The length of telescope is the distance between objective and eye–piece of a telescope in normal adjustment.

$$\therefore \quad L = f_0 + f_e$$

Note: Besides having a high magnifying power another problem which confronts the astronomers while designing a telescope to see the distant planets and stars is that they would like to gather as much light form the object as possible. This difficulty is overcome by using the objective of large aperture so that it collects a great amount of light from the astronomical objects. Thus a good telescope has an objective of long focal length and large aperture.

Q.8 What is spectrometer? Also discuss its three parts.

Ans. **SPECTROMETER**

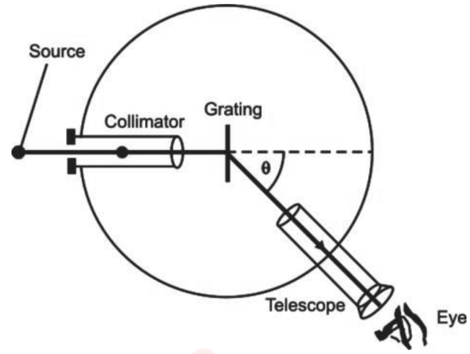
Definition

“A spectrometer is an optical device used to study spectra from different sources of light.”

Explanation

With the help of a spectrometer, the deviation of light by a glass prism and the refractive index of the material of the prism can be measured quite accurately. Spectrometer can be used to measure the wavelength of the light.

The essential components of a spectrometer are shown in the figure.



Collimator

It consists of a fixed metallic tube with a convex lens at one end and an adjustable slit at the other end.

The width of the slit can be adjusted. When the slit is just at the focus of the convex lens then the rays coming out of the lens become parallel.

Turn Table

A prism or a grating is placed on a turn table which is capable of rotating about a fixed vertical axis.

A circular scale graduated in half degrees is attached with it.

Telescope

A telescope is attached with a venire scale and is rotate-able about the same vertical axis as the turn table. A circular scale graduated in half degrees is attached with it.

The telescope is adjusted in such a way that the rays of light entering it, are focused at the cross wires near the eye–piece.

Note: *Before using a spectrometer, one should be sure that the collimator is so adjusted that parallel rays of light emerge out of its convex lens. The telescope is adjusted in such a way that the rays of light entering it are focused at the cross wires near the eye-piece. Finally, the refracting edge of the prism must be parallel to the axis of rotation of the telescope so that the turn table is levelled. This can be done by using the levelling screws.*

Q.9 Describe Michelson's experiment to determine the speed of light.

Ans. **SPEED OF LIGHT**

Light travels very fast and rapidly. Therefore, it is very difficult to measure its speed. Galileo was the first man who made an attempt to measure the speed of light but he got a failure in his attempt to measure the speed of light. Yet he agreed with the fact that light does take sometime to travel from one point to another. Michelson performed an experiment for the determination of the speed of light.

Michelson's Experiment

In this experiment, the speed of light was measured by measuring the time taken by light to cover a round trip between two mountains. The distance between two mountains was measure accurately. The experimental setup is shown in figure.

An eight sided polished mirror 'M' is mounted on the shaft of a motor whose velocity can be varied. Suppose the mirror is stationary in the position, shown in figure.

A beam of light from the face '1' of the mirror 'M' falls at the plane mirror 'm' placed at a distance 'd', from 'M'. The light is reflected back from the mirror 'm' and falls on the face '3' of mirror 'M'. On reflection from face '3', it enters the telescope.

If the mirror 'M' is rotated clockwise initially the source will not be visible through the telescope. When 'M' gains a certain speed the source 'S' becomes visible. This happens when the time taken by the light in moving from 'M' to 'm' and back to 'M' is equal to the time taken by face '2' to move to the position of face '1'.

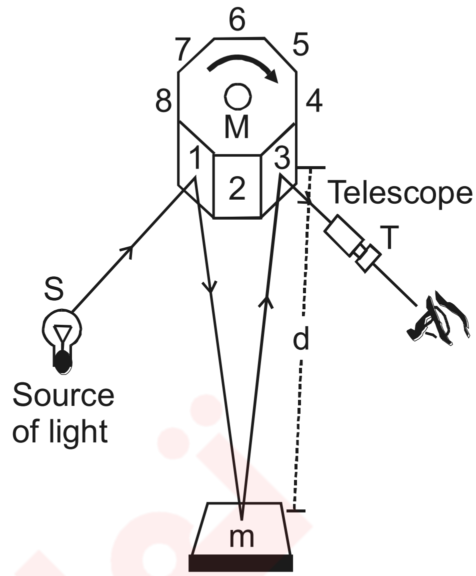


Fig. Michelson's method for measurement of speed of light.

Angle subtended by any side of eight sided mirror at the centre is $\frac{2\pi}{8}$. If 'f' is the frequency of the 'M' when the source 'S' is visible through the telescope, then the time taken by the mirror to complete 1 revolution $= \frac{1}{f}$.

Time to complete $\frac{1}{8}$ vibration $= \frac{1}{8f}$.

$$\therefore \quad t = \frac{1}{8f}$$

As	θ	=	wt
	t	=	$\frac{\theta}{w}$
$\therefore$	w	=	$2\pi f$
$\therefore$	t	=	$\frac{\theta}{2\pi f}$
	t	=	$\frac{2\frac{\pi}{8}}{2\pi f}$
	t	=	$\frac{1}{8f}$

The time taken by the light from 'M' to 'm' and back to 'M' is;

Using $\quad S = V t$

$$S = Ct \;(\because V = C)$$

$$2d = Ct$$

$$\therefore \quad t = \frac{2d}{C}$$

These two times are equal.

$$\therefore \quad \frac{2d}{C} = \frac{1}{8f}$$

$$C = 2d \times 8f$$

$$C = 16\,d\,f$$

$$C = 2.99792458 \times 10^8\ ms^{-1}$$

$$C = 3 \times 10^8\ ms^{-1}$$

The speed of light in other materials is always less than C. In media other than vacuum, it depends upon the nature of the medium. However, the speed of light in air is approximately equal to that in vacuum.

Q.10 Write a note on the fibre optics.

Ans. **INTRODUCTION TO FIBRE OPTICS**

For hundred of years man has communicated using flashes of reflected sunlight by day and lanterns by night. Navy signalmen still use powerful blinder lights to transmit coded messages to other ships during periods of radio-silence. Light communication has not been confined to simple dots and dashes. It is an interesting but little known fact that Alexander Graham Bell invented a device known as "photo phone" shortly after his invention of telephone. Bell's photo phone used a modulated beam of reflected sunlight focused upon a Selenium detector several hundred metres away. With a device, bell was able to transmit a voice message via a beam of light. The idea remained dormant for many years. During the recent past the idea of transmission of light through thin optical fibres has been revived and is now being used in communication technology.

The use of light as transmission carries wave in fiber optics has several advantages over radio wave carries these advantages are (a) wide band-width capability and (b) immunity from electromagnetic interference.

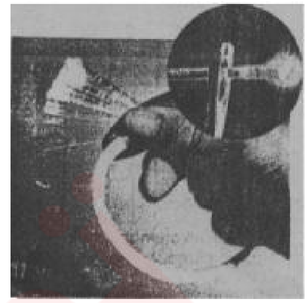
It is also used to transmit light around corners and into inaccessible places so that the formerly unobservable could be viewed. The use of fibre optic tools in industry is now very common, and their importance as diagnostic tools in medicine has been proved.

Recently the fibre optic technology has evolved into something much more important and useful and communication system of enormous capabilities.

One feature of such a system is its ability to transmit thousands of telephone conversations, several television programs and numerous data signals between stations through one or two flexible, hair thin threads of optical fibre. With the tremendous information carrying capacity called the bandwidth, fibre optic systems have undoubtedly made practical such services as two ways television which was too costly before the development of fibre optics. These systems also allow word processing, image transmitting and receiving equipment to operate efficiently.

In addition to give an extremely wide bandwidth, the fibre optic system has much thinner and light weight cables. An optical fibre with its protective case may be typically 6.0 mm in diameter, and yet it can replace a 7.62 cm diameter bundle of copper wire now used to carry the same amount of signals.

Point to Ponder



Each of the thin optical fibres is small enough to fit through the eye of a needle. Why is the size of the fibre important?

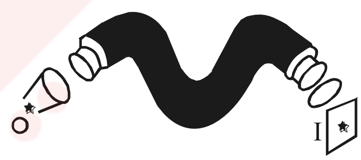


Fig. (a) Optical fibre image

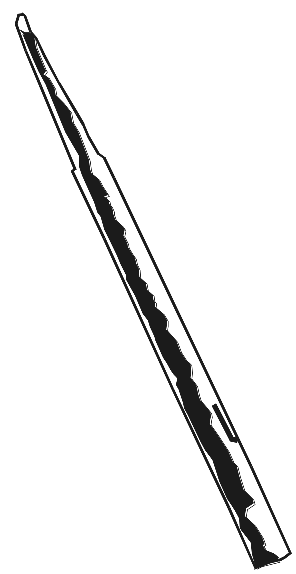


Fig. (b) A precision diamond scalpel for use in eye surgery. The illumination is obtained by light passing through a fibre optic light guide.

Q.11 What are the basic principles of optical fibre? Explain then.

Ans. **FIBRE OPTIC PRINCIPLES**

Propagation of light in an optical fibre requires the light should be confined within the fibre. This may be done by total internal reflection and continuous refraction.

Total Internal Reflection

One of the qualities of any optically transparent material is the speed at which light travels within the materials, i.e., it depends upon the refractive index n of the material. The index of refraction is merely the ratio of the speed of light c in vacuum to the speed of light v in that material.

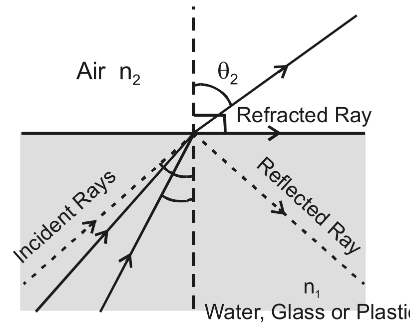


Fig. (a) If the angle of refraction in the air is 90° the angle of incidence is called the critical angle.

Expressed mathematically,

$$n = \frac{c}{v}$$

The boundary between two optical media, e.g., glass and air having different refractive indices can reflect or refract light rays. The amount and direction of reflection or refraction is determined by the amount of difference in refractive indices as well as the angle at which the rays strike the boundary. At some angle of incidence, the angle of refraction is equal to 90^o when a ray of light is passing through glass to air. This angle of incidence is called the angle θ_c, shown in figure.

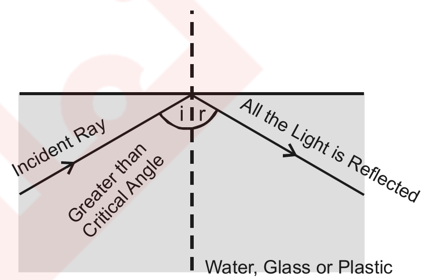


Fig. (b) For angles of incidence greater than the critical angle, all the light is reflected; none is refracted into the air.

We are already familiar with Snell's law;

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

From Fig.

when $\theta_1 = \theta_c$, $\theta_2 = 90^o$

thus, $n_1 \sin\theta_c = n_2$ or $\sin\theta_c = \dfrac{n_2}{n_1}$

For incident angles equal to or greater than the critical angle, the glass–air boundary will act as a mirror and no light escapes from the glass Fig. For glass–air boundary, we have

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.0}{1.5} \quad \text{or} \quad \theta_c = 41.8^o$$

Let us now assume that the glass is formed into a long, round rod. We know that all the light rays striking the internal surface of the glass at angles of incidence greater than 41.8^o (critical angle) will be reflected back into the glass while those with angle less than 41.8^o will escape from the glass Fig. Ray 1 is injected into the rod so that it strikes the glass air boundary at an angle of incidence about 30^o.

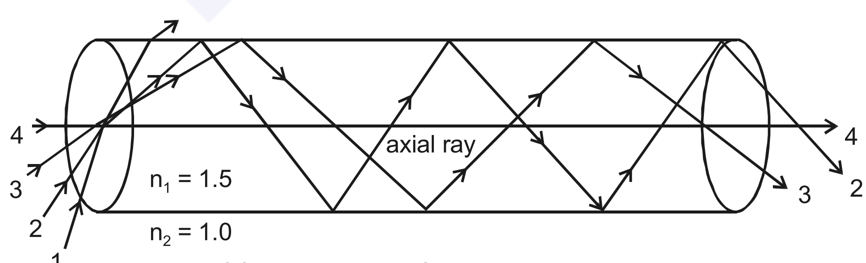


Fig. (a) Propagation of light within a glass rod.

Since this is less than the critical angle, it will escape from the rod and be lost. Ray 2 at 42^o will be reflected back into the rod, as will ray 3 at 60^o. Since the angle of reflection equals the angle of incidence, these two rays will continue to propagate down the rod, along paths determined by the original angles of incidence. Ray 4 is called an axial ray since its path is parallel to the axis of the rod. Axial rays will travel directly down this straight and rigid rod. However, in a flexible glass fibre they will be subjected to the laws of reflection figure.

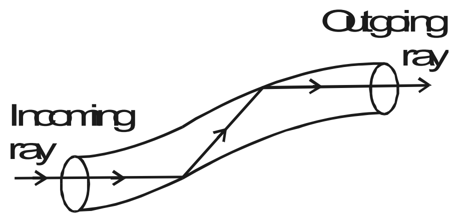


Fig. (b) Light propagation within a flexible glass fibre.

Optical fibres that propagate light by total internal reflection are the most wisely used.

Continuous Refraction

There is another mode of propagation of light through optical fibres in which light is continuously refracted within the fibre. For this purpose central core has high refractive index (high density). This layer is called cladding. Such a type of fibre is called multi-mode step index fibre whose cross sectional view is shown in figure.

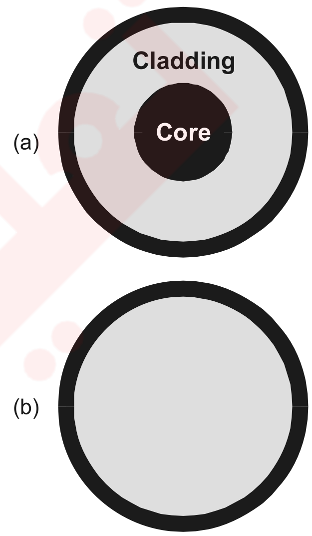


Fig. Cross sectional view of
(a) Multi-mode step index fibre
(b) Multi-mode graded index fibre

Now a days, a new type of optical fibre is used in which the central core has high refractive index (high density) and its density gradually decreases towards its periphery. This type of optical fibre is called a multi mode graded index fibre. Its cross sectional view is shown in Fig.

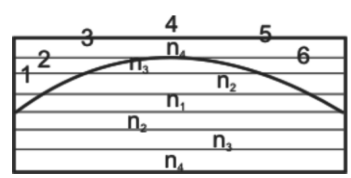


Fig. Light propagation within a hypothetical multi layer fibre.

In both these fibres, the propagation of light signal is through continuous refraction. We already know that a ray passing a denser medium to a rarer medium bends away from the normal and vice versa. In step index or graded index fibre, a ray of light entering the optical fibre, as shown in Fig. is continuously refracted through these steps and is reflected from the surface of the outer layer. Hence light is transmitted by continuous refraction and total internal refraction.

Q.12 What are different types of optical fibre?

Ans. TYPES OF OPTICAL FIBRES

There are three types of optical fibres which are classified on the basis of the mode by which they propagate light. These are (i) single mode step index (ii) multi mode step index and (iii) multi mode graded index. The term 'mode' is described as the method by which light is propagated within the fibre, i.e., various path the light can take in traveling down the fibre. The optical fibre is also covered by a plastic jacket for protection.

(i) Single Mode Step Index Fibre

Single mode or mono mode step index fibre has a very thin core of about 5 μm diameter and has a relatively larger cladding (of glass or plastic) as shown in the figure. Since it has a very thin core, a strong monochromatic light source i.e., a laser source has to be used to send light signals through it. It can carry more than 14 TV channels or 14000 phone calls.

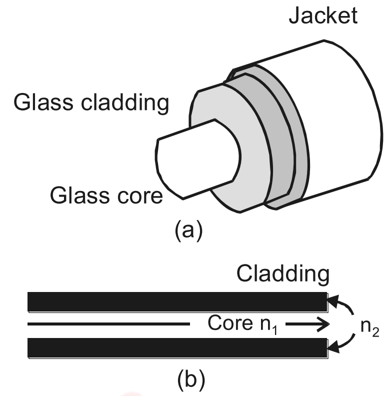


Fig. Single-mode step-index fibre.

(ii) Multi Mode Step Index Fibre

This type of fiber has a core of relatively larger diameter such as 50 μm. It is mostly used for carrying white light but due to dispersion effects, it is useful for a short distance only. The fibre core has a constant refractive index n_1; such as 1.52, from its centre of boundary with the cladding as shown in Fig. The refractive index then changes to a lower value n_2, such as 1.48, which remains constant through out the cladding.

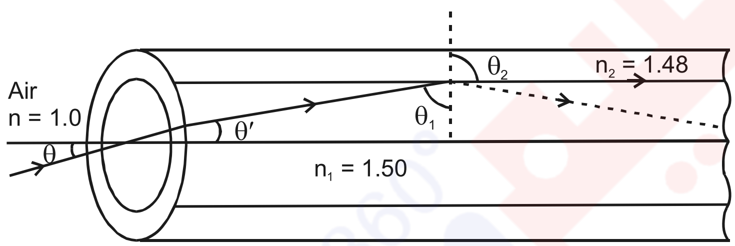


Fig. Light propagation through multi-mode step-index fibre.

This is called a step–index multimode fibre, because the refractive index steps down from 1.52 to 1.48 at boundary with cladding.

(iii) Multimode Graded Index Fibre

Multi mode graded index fibre has core which ranges in diameter from 50 to 1000 μm. It has a core of relatively high refractive index and the refractive index decreases gradually from the middle to the outer surface of the fibre. There is no noticeable boundary between core and cladding.

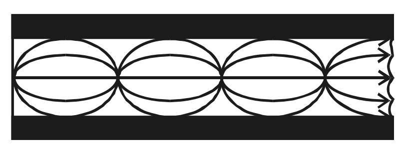


Fig. Light propagation through multi-mode graded-index fibre.

This type of fibre is called multi mode graded–index fibre (Fig.) and is useful for long distance application in which white light is used. The mode of transmission of light through this type of fibre is also the same, i.e., continuous refraction from the surfaces of smoothly decreasing refractive index and total internal reflection from the boundary of the outer surfaces.

Q.13 Explain the signal transmission and conversion to sound in fibre optics.

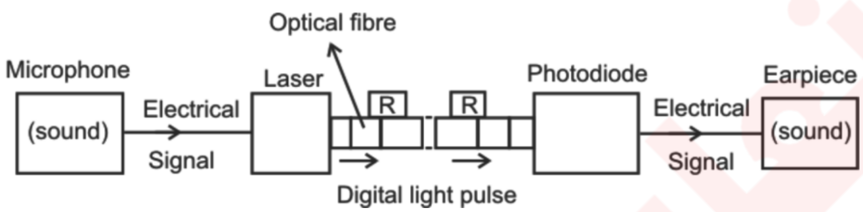
Ans. SIGNAL TRANSMISSION AND CONVERSION TO SOUND

A fibre optic communication system consists of three major components (i) a transmitter that converts electrical signals to light signals, (ii) an optical fibre for guiding the signals and (iii) a receiver that captures the light signals at the other end of the fibre and reconverts them to electric signals.

A light source in the transmitter can be either a semiconductor laser or a light emitting diode (LED). With either device, the light emitted is an invisible infrared signals. The typical wavelength is 1.3 μm.

Such a light will travel much faster through optical fibres than will either visible ultra–violet light. The lasers and LEDs used in this application are tiny units (less than half the size of the thumbnail) in order to match the size of fibres. To transmit information by light waves, whether it is an audio signal, a television signal or a computer data signal, it is necessary to modulate the light waves.

The most common method of modulation is called digital modulation in which the laser or LED is flashed on and off at an extremely fast rate. A pulse of light represents the number 1 and the absence of light represents zero. In a sense, instead of flashes of light traveling down the fibre, ones (1s) and zeros (0s) are moving down the path.



With computer type equipment, any communication can be represented by a particular pattern or code of these 1s and 0s. The receiver is programmed to decode the 1s and 0s, thus it receives, the sound, pictures or data as required. Digital modulation is expressed in bits (binary digit) or megabits (10^6 bits) per second, where a bit is a 1 or a 0.

Despite the ultra-purity (99.99 % glass) of the optical fibre, the light signals eventually become dim and must be regenerated by devices called repeaters. Repeaters are typically placed about 30 km apart, but in the newer systems they may be separated by as much as 100 km.

At the end of the fibre, a photodiode converts the light signals, which are then amplified and decoded, if necessary, to reconstruct the signals originally transmitted (figure).

Q.14 Explain how the power of light signals is lost by scattering and absorption during propagation?

Ans. LOSSES OF POWER

When a light signals travel along fibre by multiple reflection, some light is absorbed due to impurities in the glass. Some of it is scattered by groups of atoms which are formed at places such as joints when fibres are joined together. Careful manufacturing can reduce the power loss by scattering and absorption.

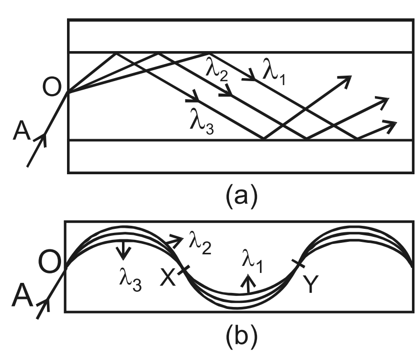


Fig. Light paths in (a) step-index and (b) graded-index fibre.

The disadvantage of the step-index fibre Fig. (a) can considerably be reduced by using a graded index fibre. As shown in the Fig. (b), the different wavelengths still take different paths and are totally internally reflected at different layers, but still they are focused at the same points like X and Y etc. It is possible because the speed is inversely-proportional to the refractive index. So the wavelength λ_1 travels a longer path than λ_2 or λ_3 but at a greater speed.

Inspite of the different dispersion, all the wavelengths arrive at the other end of the fibre at the same time. With a step-index fibre, the overall time of difference may be about $33ns$ per km length of fibre. Using in a graded index fiber, the time difference is reduced to about $1ns$ per km.

SOLVED EXAMPLES

EXAMPLE 10.1

A microscope has an objective lens of focal length 10 mm. And an eye piece of 25 mm focal length. What is the distance between lenses and its magnification, if the object is in sharp focus when it is 10.5 mm from objective?

Data

$$f_0 = 10 \text{ mm}$$
$$f_e = 25 \text{ mm}$$
$$p_0 = 10.5 \text{ mm}$$

To Find

Distance between lenses = L = ?

Magnification = M = ?

SOLUTION

Using

$$\frac{1}{p_0}+\frac{1}{q_0} = \frac{1}{f_0}$$

$$\frac{1}{10.5}+\frac{1}{q_0} = \frac{1}{10}$$

$$\frac{1}{q_0} = \frac{1}{10}-\frac{1}{10.5}$$

$$\frac{1}{q_0} = \frac{1}{10}-\frac{10}{105}$$

$$\frac{1}{q_0} = \frac{1}{10}-\frac{2}{21}$$

$$\frac{1}{q_0} = \frac{21-20}{210}$$

$$\frac{1}{q_0} = \frac{1}{210}$$

So, $q_0 = 210 \text{ mm}$

Now using

$$\frac{1}{f_e} = \frac{1}{p_e}+\frac{1}{q_e}$$

$$\frac{1}{p_e} = \frac{1}{f_e} - \frac{1}{q_e}$$

For eye piece $q_e = d$ and image is virtual

$$\therefore \quad q_e = -d$$
$$= -25 \text{ cm}$$
$$q_e = -250 \text{ mm}$$
$$\therefore \quad \frac{1}{p_e} = \frac{1}{25} - \frac{1}{-250}$$
$$= \frac{1}{25} + \frac{1}{250}$$
$$= \frac{10+1}{250}$$
$$\frac{1}{p_e} = \frac{11}{250}$$
$$p_e = \frac{250}{11}$$
$$= 22.7 \text{ mm}$$

Now using

$$L = q_0 + p_e$$
$$= 210 + 22.7$$
$$= 232.7 \text{ mm}$$

Magnification by objective $= M_1 = \frac{q_o}{p_o}$

$$= \frac{210}{10.5}$$
$$= 20$$

Magnification by eye-piece $= M_2 = \frac{-250}{22.7}$

$$= -11.0$$

Total magnification $= M = M_1 \times M_2$

$$= 20(-11)$$
$$= -220$$

Result

Distance between lenses $= L = 232.7$ mm

Magnification $= M = 220$

EXAMPLE 10.2

Calculate the critical angle and angle of entry for an optical fibre having core of refractive index 1.50 and cladding of refractive index 1.48.

Data

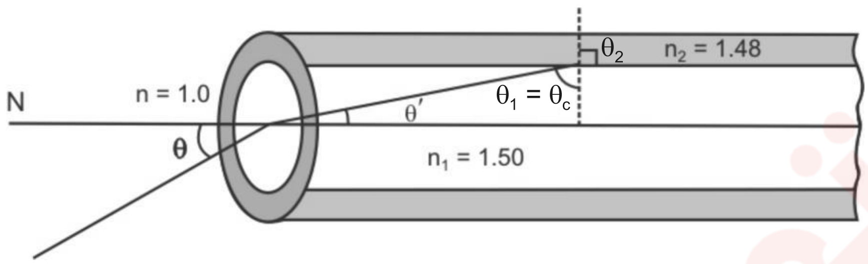
Refractive index of core $= n_1 = 1.50$

Refractive index of cladding $= n_2 = 1.48$

To Find

Critical angle $= \theta_c = ?$

Angle of entry $= \theta = ?$



SOLUTION

For core-cladding interface.

From Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

When, $\theta_1 = \theta_c$, $\theta_2 = 90^o$

So, $$1.5 \sin\theta_c = 1.48 \sin 90^o$$

$$\sin\theta_c = \frac{1.48 \times 1}{1.5}$$

$$= 0.986$$

$$\theta_c = \sin^{-1}(0.986)$$

$$= 80.6^o$$

Now using Snell's law for air-core interface.

$$n \sin\theta = n_1 \sin\theta'$$

$$1 \sin\theta = 1.5 \sin\theta' \quad \ldots\ldots\ldots (1)$$

From figure

$$\theta' + \theta_c + 90^o = 180^o$$

$$\theta' + 80.6 + 90^o = 180^o$$

$$\theta' = 180^o - 170.6$$

$$= 9.4^o$$

Put in equation (1)

$\therefore$ $$\sin\theta = 1.5 \sin 9.4^o$$

$$\theta = \sin^{-1}(0.245)$$

$$= 14.2^o$$

Result

Critical angle $= \theta_c = 80.6°$

Angle of entry $= \theta = 14.2°$

Note: *If light beam is incident at the end of the optical fibre at an angle greater than 14.20, the total internal reflection would not take plane.*

Chapter 11

HEAT AND THERMODYNAMICS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

State the basic postulates of kinetic theory of gases.

Derive gas laws on the basis of kinetic theory.

Describe that the internal energy of an ideal gas is due to kinetic energy of its molecules.

Understand and use the terms work and heat in thermodynamics.

Differentiate between isothermal and adiabatic processes.

Explain the molar specific heats of a gas.

Apply first law of thermodynamics to derive $C_p - C_v = R$.

Explain the second law of thermodynamics and its meaning in terms of entropy.

Understand the concept of reversible and irreversible processes.

Define the term heat engine.

Understand and describe Carnot theorem.

Describe the thermodynamic scale of temperature.

Explain the term entropy.

INTRODUCTION

Thermodynamics deals with various phenomena of energy and related properties of matter, especially the transformation of heat into other forms of energy. An example of such transformation is the process converting heat into mechanical work. Thermodynamics thus plays central role in technology, since almost all the raw energy available for our use is liberated in the form of heat. In this chapter we shall study the behaviour of gases and laws of thermodynamics, their significance and applications.

Q.1 Define thermodynamics.

Ans. **THERMODYNAMICS**

"Thermodynamics is the branch of physics, which deals with transformation of heat energy into mechanical energy (work)."

Q.2 What are the basic assumption of kinetic molecular theory of gases?

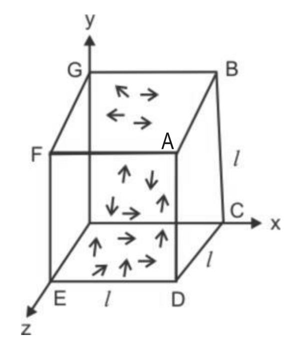
Ans. **KINETIC THEORY OF GASES**

Matter is composed of tiny molecules which are not in the state of rest. These molecules move in different direction and make collision with each other as well as with the walls of the container. The molecular kinetic theory of gases based on the following postulates.

1. A finite volume of gas consists of a very large number of molecules.
2. The size of the molecule is much smaller than the separation between molecules.
3. The gas molecules are in random motion and may change their direction of motion after every collision.
4. Collision between gas molecules themselves and with walls of container are assumed to be perfectly elastic.
5. Molecules do not exert force on each other except during a collision.

Q.3 Define pressure. Also derive the expression for the pressure of a gas.

Ans. **PRESSURE OF GAS**



According to kinetic theory of gas,

Molecules collide with the walls of container and exert pressure on the walls. An expression for this can be obtained as follow

Consider a cubical container of side of length 'l', which has 'N' number of molecules moving in random motion as shown in figure.

Area of each face $= A = l \times l = l^2$

Volume of container $= V = A \times l$

$= l^2 \times l = l^3$

Consider a signal molecule of mass 'm' moving with velocity '$\vec{V}_1$', which has three components $\vec{V}_{1x}, \vec{V}_{1y}, \vec{V}_{1z}$ along x, y and z axes.

Momentum of molecule before striking the face

ABCDA $= P_i = m V_{1x}$

If collision is assumed to be perfectly elastic, the molecule will rebound from this face with same velocity.

$\therefore$ Momentum after collision $= P_f = -m V_{1x}$

$\therefore$ Change in momentum $= \Delta P = P_f - P_i$

$$\Delta P = -m V_{1x} - m V_{1x}$$

$$\Delta P = -2m V_{1x} \qquad \ldots\ldots\ldots (1)$$

After recoil the molecule travels to the opposite face EFGHE and collides with it, rebounds and travels back to the face 'ABCDA' after covering a distance $2l$. The time 'Δt', between two successive collision with face 'ABCDA' is

As, $S = V t$

$$2l = V_{1x} \Delta t$$

$$\Delta t = \frac{2l}{V_{1x}} \qquad \ldots\ldots\ldots (2)$$

So, the number of collisions per–second that the molecule will make with this face is;

$$\Delta t = \frac{\Delta V_{1x}}{2l} \qquad \left(\because\ f = \frac{1}{\Delta t}\right)$$

$$\left(f = \frac{1}{\frac{2l}{V_{1x}}}\right)$$

$$\left(f = \frac{V_{1x}}{2l}\right)$$

Now according to Newton's 2nd law, in term of momentum, the force 'F_m' exerted on the molecule by the wall, is equal to rate of change of momentum i.e.,

$$F_m = \frac{\Delta P}{\Delta t}$$

Putting value of ΔP.

$$\therefore \quad F_m = -\frac{2m V_{1x}}{\Delta t} \qquad \ldots\ldots\ldots (3)$$

According to Newton's 3rd law of motion force F_{1x} exerted by the molecule on the face ABCDA is equal but opposite so,

$$F_{1x} = -F_m$$

Putting value of F_m from equation (3)

$$\therefore \quad F_{1x} = -\left(\frac{-2m V_{1x}}{\Delta t}\right)$$

$$F_{1x} = \frac{2m V_{1x}}{\Delta t}$$

Putting value of Δt.

$$\therefore \quad F_{1x} = \frac{2m V_{1x}}{2l/V_{1x}}$$

$$F_{1x} = \frac{m}{l} V_{1x}^2$$

Similarly the forces due to all other molecule can be determined. Thus the total x–directed force F_x due to 'N' number of molecules moving with velocities $V_1, V_2, \ldots\ldots, V_N$ is

$$F_x = F_{1x} + F_{2x} + \ldots\ldots + F_{Nx}$$

$$\therefore \quad F_x = \frac{m}{l} V_{1x}^2 + \frac{m}{l} V_{2x}^2 + \ldots\ldots + \frac{m}{l} V_{Nx}^2$$

$$F_x = \frac{m}{l} \left(V_{1x}^2 + V_{2x}^2 + \ldots\ldots + V_{Nx}^2 \right)$$

As pressure is normal force per unit area, hence pressure P_x, on the face perpendicular to x-axis is

$$P_x = \frac{F_x}{A} = \frac{F_x}{l^2}$$

$$P_x = \frac{1}{l^2} \left(\frac{m V_{1x}^2}{l} + \frac{m V_{2x}^2}{l} + \frac{m V_{3x}^2}{l} + \ldots\ldots + \frac{m V_{Nx}^2}{l} \right)$$

$$P_x = \frac{m}{l^3} \left(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \ldots\ldots + V_{Nx}^2 \right) \quad \ldots\ldots\ldots (4)$$

As the mass of single molecule is m, the mass of N molecule will be $m N$

Since density;

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m N}{l^3}$$

Hence, $$\frac{m}{l^3} = \frac{\rho}{N}$$

Substituting the value of $\frac{m}{l^3}$ in equation (4).

$$\therefore \quad P_x = \frac{\rho}{N} \left(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \ldots\ldots + V_{Nx}^2 \right)$$

or $$P_x = \rho \left(\frac{V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \ldots\ldots + V_{Nx}^2}{N} \right) \quad \ldots\ldots\ldots (5)$$

where $$= \left(\frac{V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \ldots\ldots + V_{Nx}^2}{N} \right)$$

is called the mean of squared velocities of the molecules moving along 'x' direction, known as mean square velocity, represented by $< V_x^2 >$.

$\therefore$ equation (5) becomes

$$P_x = \rho < V_x^2 > \quad \ldots\ldots\ldots (6)$$

Similarly pressure on the faces perpendicular to 'y' and 'z' axes will be

$$P_y = \rho < V_y^2 > \quad \text{and} \quad P_z = \rho < V_z^2 >$$

The mean square of all the component velocities will be equal. Hence,

$$< V_x^2 > = < V_y^2 > = < V_z^2 >$$

and from vector addition

$$< V^2 > = < V_x^2 > + < V_y^2 > + < V_z^2 >$$

$$< V^2 > \;=\; < V_x^2 > + < V_x^2 > + < V_x^2 >$$

thus, $$< V^2 > \;=\; 3 < V_x^2 >$$

or $$< V_x^2 > \;=\; \frac{1}{3} < V^2 >$$

Putting this value of $< V_x^2 >$ in equation (6).

$$\therefore \qquad P_x \;=\; \frac{\rho}{3} < V^2 >$$

By Pascal's law the pressure on the other sides and everywhere inside the vessel will be the same provided the gas is of uniform density. So,

$$P_x \;=\; P_y \;=\; P_z \;=\; \frac{\rho}{3} < V^2 >$$

Thus in general,

$$P \;=\; \frac{1}{3}\rho < V^2 >$$

Since $$\text{Density} \;=\; \rho \;=\; \frac{m\,N}{V}$$

Hence, $$P \;=\; \frac{m\,N}{V} < V^2 >$$

Multiply and divide by (2)

$$P \;=\; \frac{2}{3}\frac{N}{V} < \frac{1}{2} m\,V^2 >$$

$$P \;=\; \frac{2}{3} N_0 < \frac{1}{2} m\,V^2 >$$

Where 'N_0' is the number of molecules per unit volume.

Thus, $$P \;=\; \text{Constant} \; < \text{K.E.} >$$

$$\therefore \qquad P \;\propto\; < \text{K.E.} >$$

Hence pressure exerted by the gas is directly proportional to the average translational K.E. of the gas molecules.

INTERPRETATION OF TEMPERATURE

Q.4 Show that absolute temperature is directly proportional to average translational K.E.

Ans. From ideal gas law;

$$PV \;=\; n\,RT \qquad \ldots\ldots\ldots (1)$$

Where 'n' is the number of moles of the gas contained in volume 'V' at absolute temperature T and R is called universal gas constant. Its value is $8.314 \text{ J mole}^{-1} \text{ K}^{-1}$.

If N_A is Avogadro number, then

$$n = \frac{N}{N_A}$$

$\therefore$ equation (1) becomes

$$PV = \frac{N}{N_A}RT$$

$$PV = N\frac{R}{N_A}T$$

$$PV = NkT \qquad \ldots\ldots\ldots (2)$$

Where $k = \frac{R}{N_A}$ is the Boltzman constant and has the value $= 1.38 \times 10^{-23}\ J\ K^{-1}$

As, $$P = \frac{2}{3}\frac{N}{V}<\frac{1}{2}mV^2>$$

$$PV = \frac{2}{3}N<\frac{1}{2}mV^2> \qquad \ldots\ldots\ldots (3)$$

Comparing (2) and (3)

$$NkT = \frac{2}{3}N<\frac{1}{2}mV^2>$$

$$kT = \frac{2}{3}<\frac{1}{2}mV^2>$$

$$T = \frac{2}{3k}<\frac{1}{2}mV^2>$$

$$T = \text{Constant}<\frac{1}{2}mV^2>$$

or $$T \propto <K.E.>$$

DERIVATION OF GAS LAWS

Q.5 Derive Boyle's law from the kinetic molecules theory of gases.

Ans. **BOYLE'S LAW**

As, $$P = \frac{2}{3}\frac{N}{V}<\frac{1}{2}mV^2>$$

$$PV = \frac{2}{3}N<\frac{1}{2}mV^2>$$

If we keep the temperature constant.

$\therefore <\frac{1}{2}mV^2>$ is constant $\quad (\because\ T \propto K.E.)$

$$PV = \text{Constant}$$

or $$P \propto \frac{1}{V}$$

Thus pressure 'P' is inversely proportional to volume V at constant temperature of gas, which is Boyle's law.

Q.6 Derive Charles's from the kinetic molecular theory of gases.

Ans. **CHARLES'S LAW**

As, $P = \frac{2}{3}\frac{N}{V} < \frac{1}{2} m V^2 >$

$V = \frac{2}{3}\frac{N}{P} < \frac{1}{2} m V^2 >$

If pressure is kept constant.

$\therefore \quad V = \text{Constant} < \frac{1}{2} m V^2 >$

$\therefore \quad V \propto < \frac{1}{2} m V^2 >$

or $\quad V \propto T \quad \left(\because T \propto < \frac{1}{2} m V^2 >\right)$

Thus volume is directly proportional to absolute temperature of gas, provided pressure is kept constant.

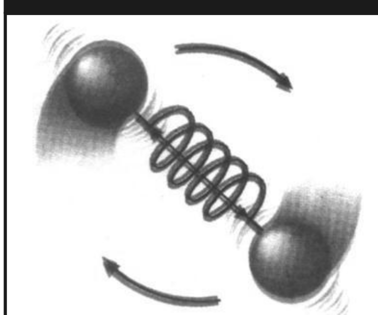
Q.7 What is internal energy?

Ans. **INTERNAL ENERGY**

"The sum of all forms of molecular energies (K.E. and P.E.) of a substance is termed as its internal energy."

In the study of thermodynamics, usually ideal gas is considered as a working substances. The molecules of an ideal gas are mere mass points, which exert forces on one another. So the internal energy of an ideal gas system is generally the translational K.E. of its molecules. Since the temperature of a system is defined as the average K.E. of its molecules, thus for an ideal gas system, the internal energy is directly proportional to its temperature.

Do You Know?



A diatomic gas molecule has both transnational and rotational energy. It also has vibrational energy associated with the spring like bond between its atoms.

When we heat a substance, energy associated with its atoms or molecules is increased i.e., heat is converted to internal energy.

It is important to note that energy can be added to a system even though no heat transfer takes place. For example, when two objects are rubbed together, their internal energy increases because of mechanical work. The increase in temperature of the object is an indication of increase in the internal energy. Similarly, when an object slides over any surface and comes to rest because of frictional forces, the mechanical work done on or by the system is partially converted into internal energy.

In thermodynamics, internal energy is a function of state. Consequently, it does not depend on path but depends on initial and final states of the system. consider a system, which undergoes a pressure and volume change from P_a and V_a to P_b and V_b respectively, regardless of the process by which

System

It is the collection of matter, which has distinct boundaries.

For example a gas enclosed in a cylinder.

Q.8 What is the relation between work and heat?

Ans. **WORK AND HEAT**

Heat is the form of energy, which can travel from one body to another.

Its unit is joule (J).

Consider the gas enclosed in a cylinder with a movable frictionless piston of cross-sectional area A. In equilibrium the system occupies volume 'V', and exerts a pressure 'P' on the walls of the cylinder and its piston. The force 'F' exerted by the gas on the piston is PA.

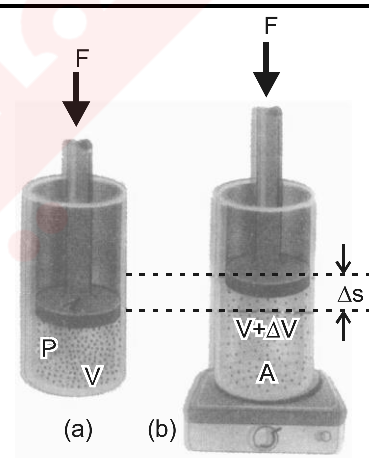


Fig. A gas sealed in a cylinder by a weightless, frictionaless piston. The constant downward applied force F equals PA, and when the poiston is displaced, downward work is done on the gas.

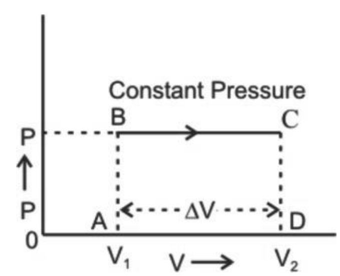
We assume that the gas expands through ΔV very slowly, so that it remains in equilibrium. As the piston moves up through a small distance ΔY, the work (W) done by the gas is;

$$W = F\,\Delta Y = PA\,\Delta Y$$

Since $A\,\Delta Y = \Delta V$ (Change in volume)

Hence $W = P\,\Delta V$

The work done can also be calculated by area of the curve under $P - V$ graph, as shown in figure.



Q.9 State and explain first law of thermodynamics.

Ans. **FIRST LAW OF THERMODYNAMIC**

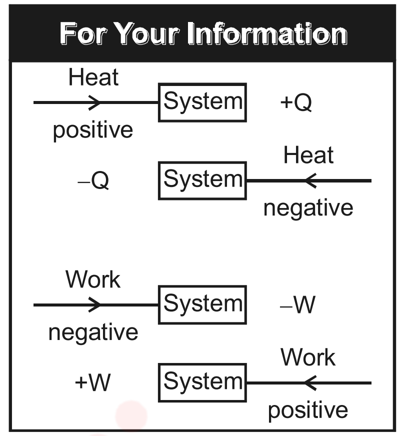
"In any thermodynamic process, when heat 'Q' is added to the system, this energy appears as an increase in the internal energy 'ΔU' stored in the system plus the work 'W' done by the system on its surroundings."

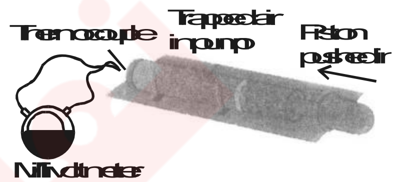
Let 'Q' be the heat supplied to the system, so that the change in internal energy is ΔU and total work done by the system is 'W', so

$$Q = \Delta U + W \qquad \ldots\ldots\ldots (1)$$

$$(\Delta U = U_2 - U_1)$$

When heat supplied to the system then K.E. of molecules increases, so internal energy increase. Also molecules push the piston upward and work is done by the system.





From equation (1).

$$\Delta U = Q - W \qquad \ldots\ldots\ldots (2)$$

So, change in internal energy is defined as;

"The heat supplied to the system–work done by the system."

This law is the extended form of the law of conservation of energy.

Examples

(1) Heat engine (2) A bicycle pump (3) Human metabolism

Q.10 Describe the isothermal process and also draw P-V diagram.

Ans. **ISOTHERMAL PROCESS**

"The process, in which temperature of the system remains constant is called an isothermal process."

Consider a system consisting of an ideal gas, in a cylinder having conducting base and non-conducting walls and piston. The piston is frictionless and moveable.

The cylinder is placed on heat reservoir. In order to keep the temperature of the gas constant, the expansion or compression must take place slowly because transfer of heat requires time. In case of an ideal gas potential energy associated with its molecules is zero. Hence the internal energy of an ideal gas depends only on its temperature, which is constant

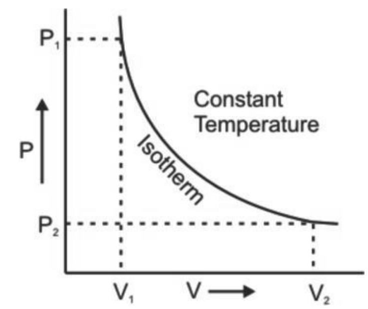
$$\therefore \quad \Delta U = 0$$

Hence 1st law of thermodynamics becomes;

$$Q = \Delta U + W$$

$$Q = 0 + W$$

$$Q = W$$



This shows that if the gas expands and does an external work 'W', then an amount of heat 'Q', has to be supplied to keep the temperature constant. Conversely, if the gas is compressed i.e., work is done on the system, then heat 'Q' should leave the system to keep the temperature constant.

"The curve representing an isothermal process is called an isotherm."

The relation for isothermal process is;

$$PV = \text{Constant}$$

Q.11 Describe adiabatic process and also draw P-V diagram.

Ans. ADIABATIC PROCESS

"An adiabatic process is the one, in which no heat enters or leaves, the system."

i.e., $Q = 0$

This prevention of heat may be achieved either by surrounding the system with a layer of insulting material or by performing expansion or compression process quickly, because heat flow require finite time and any process performed quickly is practically adiabatic.

e.g.,
1. The rapid escape of air from a burst tyre.
2. The rapid expansion and compression of air through, which a sound wave is passing.
3. Cloud formation in the atmosphere.

Consider a system consisting of a gas enclosed in a heat insulator. For adiabatic process;

$$Q = 0$$

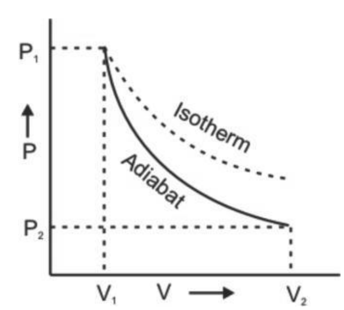
$\therefore$ 1st law of thermodynamics becomes;

$$Q = \Delta U + W$$

$$O = \Delta U + W$$

$$W = -\Delta U$$

In this case work is done by the system and is equal to decrease in internal energy i.e., work is done at the cost of internal energy.



or $\Delta U = -W$

In this case work is done on the system and is equal to increase in internal energy.

The curve is called adiabatic and is obtained by

$$PV^{\gamma} = \text{Constant}$$

where, $\gamma = \dfrac{C_P}{C_V}$

Isochoric Process

Such a process in which volume is constant.

Isobaric Process

Such a process in which pressure is constant.

Q.12 Define molar specific heat of a gas.

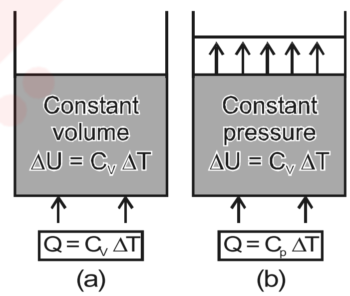
Ans. MOLAR SPECIFIC HEAT OF A GAS

"The amount of heat required to raise the temperature of one mole of the substance through 1K, is called molar specific heat of a gas."

Consider 'n' number of moles of a substance. The amount of heat 'Q', absorbed by it depends on;

1. Number of moles i.e. $Q \propto n$ .... (1)
2. Temperature difference i.e., $Q \propto \Delta T$ .... (2)

Combining (1) and (2).



$$Q \propto n\,\Delta T$$

$$Q = C_m\, n\,\Delta T$$

$$\therefore \quad C_m = \frac{Q}{n\,\Delta T}$$

where C_m is the molar specific heat.

Unit

J/mole k or J $mole^{-1}$ k^{-1}

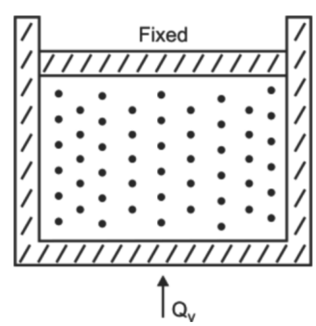
Q.13 Define molar specific heat at constant volume.

Ans. MOLAR SPECIFIC HEAT AT CONSTANT VOLUME

"The amount of heat required to raise the temperature of one mole of the gas through 1K, at constant volume, is called molar specific heat at constant volume."

It is represented by C_V.

For 'n' moles



$$C_V = \frac{Q_V}{n\,\Delta T}$$

For 1 mole

$$C_V = \frac{Q_V}{\Delta T}$$

$$Q_V = C_V\,\Delta T$$

As, $W = P\,\Delta V$

Since piston is fixed

$\therefore \quad \Delta V = 0$

$\therefore \quad W = P\,(0)$

$W = 0$

Now applying 1st law of thermodynamics, at constant volume;

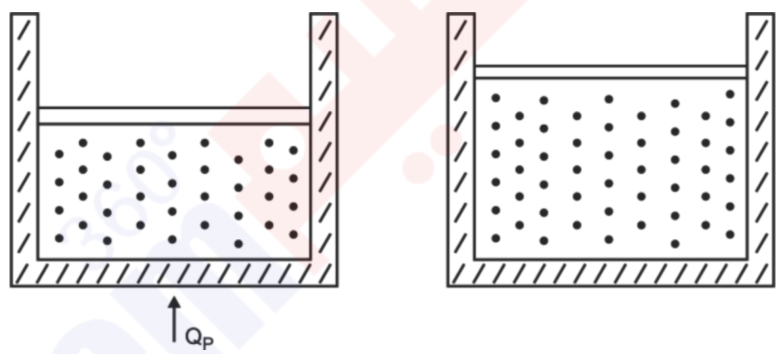
$\therefore \quad Q_V = \Delta U + W$

$C_V\,\Delta T = \Delta U + 0$

$\therefore \quad \Delta U = C_V\,\Delta T$

Q.14 Define molar specific heat at constant pressure.

Ans. MOLAR SPECIFIC HEAT AT CONSTANT PRESSURE



"The amount of heat required to raise the temperature of one mole of a gas, through 1K at constant pressure, is called molar specific heat at constant pressure."

For 'n' moles;

$$C_P = \frac{Q_P}{n\,\Delta T}$$

For 1 mole

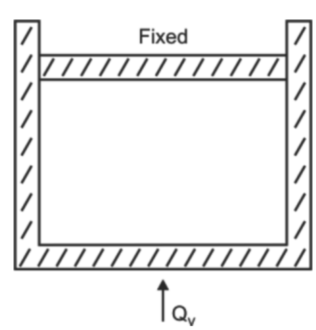
$$C_P = \frac{Q_P}{\Delta T}$$

$$Q_P = C_P\,\Delta T$$

Q.15 Prove that molar heat capacity at constant pressure is greater than molar heat capacity at constant volume. Show that $C_P - C_V = R$.

Ans. Consider one mole of an ideal gas, contained in a cylinder, fitted with a fixed piston. As the piston is fixed and the gas is heated, its volume remains constant and all heat supplied goes to increase K.E. of gas molecules due to which temperature of gas increase. If 'Q_V' is heat supplied and 'Δt' is rise in temperature, then by the definition of 'C_V' we have,

$$Q_V = C_V\,\Delta T \qquad \ldots\ldots\ldots (1)$$



As piston is fixed

$\therefore \quad \Delta V = 0$

As $W = P\,\Delta V$

$W = P\,(0)$

$W = 0$

Now applying 1st law of thermodynamics at constant volume;

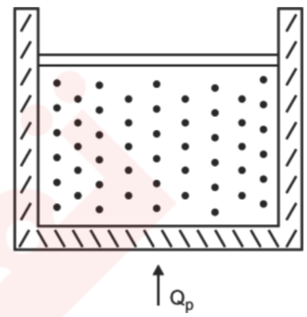
$$Q_V = \Delta U + W$$

$$C_V\,\Delta T = \Delta U + 0$$

$$\Delta U = C_V\,\Delta T \qquad \ldots\ldots (2)$$

Now if the piston is free to move through small distance, when 'Q_P' is supplied, from definition of 'C_P'.

$$Q_P = C_P\,\Delta T \qquad \ldots\ldots (3)$$



When gas is heated, its temperature rises and doing some work in pushing the piston upward at constant external pressure.

As $$W = P\,\Delta V \qquad \ldots\ldots (4)$$

Using ideal gas equation for one mole.

$$\therefore \qquad PV = RT \left(\begin{array}{c} PV = n\,RT \\ n = 1 \end{array}\right) \ldots\ldots (5)$$

At constant pressure 'P', amount of work done by one mole of a gas due to expansion 'ΔV' caused by the rise in temperature ΔT so equation (5) becomes.

$$P\Delta V = R\,\Delta T \qquad \ldots\ldots\ldots (6)$$

Comparing (4) and (6).

$$W = R\,\Delta T \qquad \ldots\ldots\ldots (7)$$

Now using 1st law of thermodynamics at constant pressure.

$$Q_P = \Delta U + W \qquad \ldots\ldots\ldots (8)$$

When one mole of a gas is heated at constant pressure, the internal energy increases, by the same amount, as at constant volume for the same rise in temperature 'ΔT'.

i.e., $(\Delta U = \Delta U_P = \Delta U_V = C_V\,\Delta T)$

Putting values of Q, ΔU, W from equation (3), (2) and equation (7) in equation (8).

$$\therefore \qquad C_P\,\Delta T = C_V\,\Delta T + R\,\Delta T$$

$$C_P\,\Delta T = \Delta T\,(C_V + R)$$

$$C_P = C_V + R$$

$$C_P - C_V = R$$

As R is positive

$$\therefore \qquad C_P > C_V$$

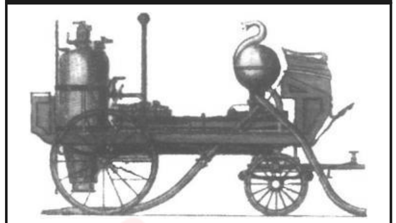
REVERSIBLE AND IRREVERSIBLE PROCESSES

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings. In the reverse process, the working substance passes through the same stages as in the direct process but thermal and mechanical effects at each stage are exactly reversed. If heat is absorbed in the direct process, it will be given out in the reverse process and if work is done by the substance in the direct process, work will be done on the substance in the reverse process. Hence, the working substance is restored to its original conditions.

A succession of events which bring the system back to its initial condition is called a cycle. A reversible cycle is the one in which all the changes are reversible.

Although no actual change is completely reversible but the processes of liquefaction and evaporation of a substance, performed slowly, are practically reversible. Similarly the slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by slowly decreasing the pressure on the piston to reverse the operation.

Do You Know?



The steam engine is a thermodynamics system.

If a process cannot be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.

All changes which occur suddenly or which involve friction or dissipation of energy through condition, convection or radiation are irreversible. An example of highly irreversible process is an explosion.

Q.16 What is heat engine?

Ans. **HEAT ENGINE**

A heat engine converts some thermal energy to mechanical work. Usually the heat comes from the burning of a fuel. The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off showing the ability to do work. This observation helped to develop a steam engine.

Basically a heat engine (figure) consists of hot reservoir or source which can supply heat at high temperature and a cold reservoir or sink into which heat is rejected at a lower temperature. A working substance is needed which can absorb heat Q_1 from source, converts some of it into work W by its expansion and rejects the rest heat Q_2 to the cold reservoir or sink. A heat engine is made cyclic to provide a continuous supply of work.

Q.17 State second law of thermodynamics.

Ans. SECOND LAW OF THERMODYNAMICS

First law of thermodynamics tells us that heat energy can be converted into equivalent amount of work, but it is silent about the conditions under which this conversion takes place. The second law is concerned with the circumstances in which heat can be converted into work and direction of flow of heat.

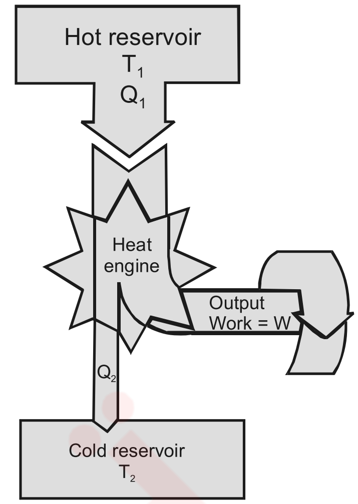


Fig. Schematic representation of a heat engine. The engine absorbs heat Q_1 from the hot reservoir, expels heat Q_2 to the cold reservoir and does work W.

Before initiating the discussion on formal statement of the second law of thermodynamics, let us analyze briefly the factual operation of an engine. The engine or the system represented by the block diagram absorbs a quantity of heat Q_1 from the heat source at temperature T_1. It does work W and expels heat Q_2 to low temperature reservoir at temperature T_2. As the working substance goes through a cyclic process, in which the substance eventually returns to its initial state, the change in internal energy is zero. Hence from the first law of thermodynamics, net work done should be equal to the net heat absorbed.

$$W = Q_1 - Q_2$$

In practice, the petrol engine of a motor car extracts heat from the burning fuel and converts a fraction of this energy to mechanical energy or work and expels the rest to atmosphere. It has been observed that petrol engines convert roughly 25% and diesel engines 35 to 40% available heat energy into work.

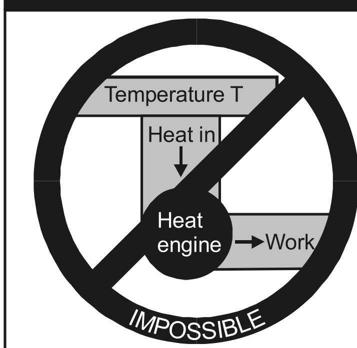
The second law of thermodynamics is a format statement based on these observations. It can be stated in a number of different ways.

According to Lord Kelvin's statement based on the work of a heat engine.

It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

This means that a single heat reservoir, no matter how much energy it contains, cannot be made to perform any work. This is true for oceans and our atmosphere which contain a large amount of heat energy but cannot be converted into useful mechanical work. As a consequence of second law of thermodynamics, two bodies at different temperatures are essential for the conversion of heat into work. Hence for the working of heat engine there must be a source of heat at a high temperature and a sink at low temperature to which heat may be expelled. The reason for our inability to utilize the heat contents of oceans and atmosphere is that there is no reservoir at a temperature lower than any one of the two.

Tidbits



According to the Kelvin statement of the second law of thermodynamics, the process pictured here is impossible. Heat from a source at a single temperature cannot be converted entirely into work.

Q.18 What is Carnot engine? Describe the four processes (steps) of the operating cycle of the Carnot engine.

Ans. CARNOT ENGINE

It is an ideal heat engine, whose efficiency is nearly 100%.

In 1840 CARNOT described an ideal engine using isothermal and adiabatic processes, he showed that a heat engine operating in an ideal reversible cycle, between to heat reservoirs, of different temperatures, would be the most efficiency engine.

Working Principle

It absorbs some heat from H.T.R., convert a part of it into work and reject the remaining part to L.T.R.

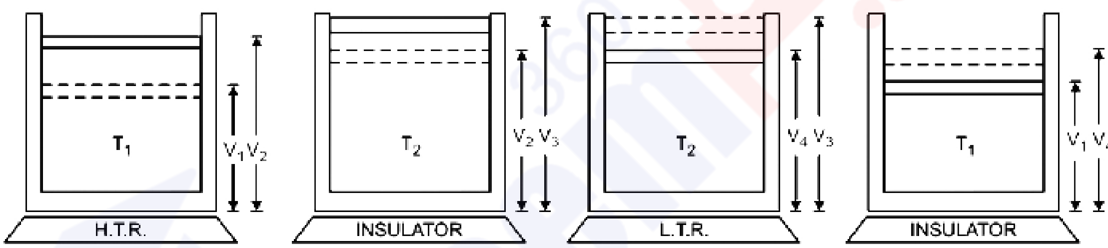
Construction

It consists of an ideal gas, enclosed in a cylinder, provided by a frictionless, movable and non-conducting piston. The walls of the cylinder are non-conducting, while the base is conducting to heat.

The operating cycle of CARNOT engine is called CARNOT cycle.

Carnot Cycle

It consist of 4 processes, which are shown on PV diagram, as shown in Fig.



Process 1

The cylinder is placed on H.T.R. and the gas is allowed to expand isothermally of temperature T_1.

Heat Q_1 flows into the cylinder due to conduction through base and the gas does some work on piston. The process is represented by curve AB.

Process 2

The cylinder is placed on an insulator and the gas is allowed to expand.

As no heat can enter or leave the system, the expansion is adiabatic and the temperature of gas fall from T_1 to T_2, the gas does work on the piston. The process is represented by curve BC.

Process 3

The cylinder is placed on a L.T.R. and the gas is compressed slowly.

During this compression temperature of gas rises, but heat Q_2 is rejected to L.T.R. making temperature constant, some work is done on the gas. The process is represented by curve CD.

Process 4

Finally the cylinder is once again placed on an insulator and the gas is compressed adiabatically.

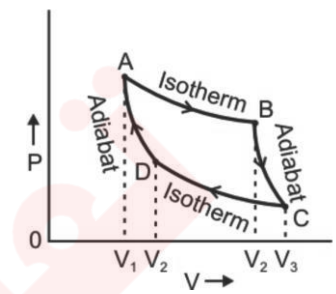
During this compression, temperature of the gas increases from T_2 to T_1. The process is represented by curve DA.

Q.19 What is Carnot's theorem? Also calculate the efficiency of such a engine in terms of temperature of source and sink.

Ans. CARNOT'S THEOREM

"No heat engine can be more efficient than a CARNOT engine operating between the same two temperature."

The Carnot's theorem can be extended to state that all Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.



All real heat engines are less efficient than Carnot engine due to friction and other heat losses.

As the working substance returns to the initial state, there is no change in its internal energy i.e., $\Delta U = 0$.

The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram. It can also be estimated from net heat 'ΔQ', absorbed in one cycle.

$$\Delta Q = Q_1 - Q_2$$

From 1st law of thermodynamics;

$$Q = \Delta U + \Delta W$$

$$\Delta W = Q_1 - Q_2 \qquad (\because \ \Delta U = 0)$$

The efficiency η of the heat engine is defined as;

$$\eta = \frac{(\text{Output})}{(\text{Input})}$$

thus, $$\eta = \frac{Q_1 - Q_2}{Q_1}$$

or, $$\eta = 1 - \frac{Q_2}{Q_1}$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to Kelvin temperature. So Q_1 and Q_2 are proportional to Kelvin temperature, T_1 and T_2 respectively and hence

$$\eta = 1 - \frac{T_2}{T_1}$$

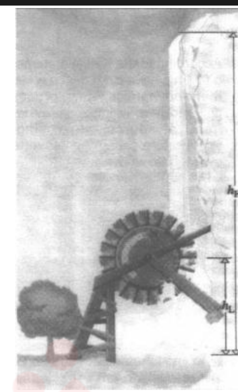
The efficiency in % age.

$$\%\text{ age efficiency} = \left(1 - \frac{T_2}{T_1}\right) 100\ \%$$

Thus the efficiency of CARNOT engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. The larger the temperature difference of two reservoirs, the greater is the efficiency. But it can never be one or 100% unless cold reservoir is at absolute zero temperature.

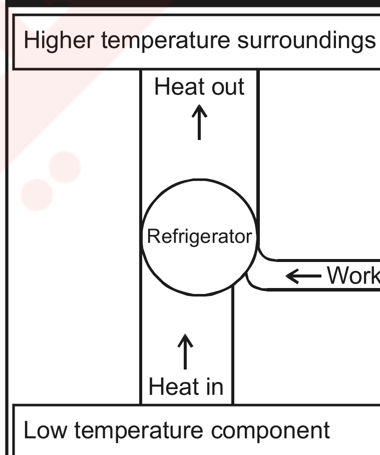
i.e., $T_2 = 0K$

Interesting Information



A waterfall analogy for the heat engine.

Do You Know?

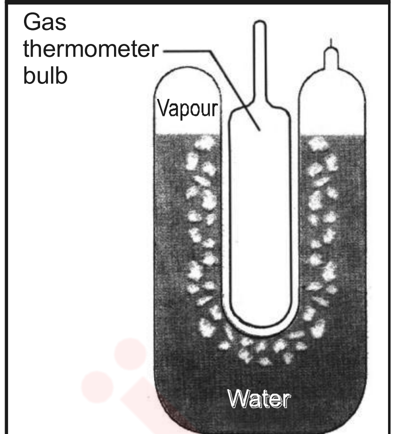


A refrigerator transfers heat from a low-temperature compartment to higher-temperature surroundings with the help of external work. It is a heat engine operating in reverse order.

Q.20 What is thermodynamic scale of temperature?

Ans. **THERMODYNAMIC SCALE OF TEMPERATURE**

For Your Information



A triple-point cell, in which solid ice, liquid water, and water vapour coexist in thermal equilibrium. By international agreement, the temperature of the mixture has been defined to be 273.16 K. The bulb of a constant-volume gas thermometer is shown inserted into the well of the cell.

The CARNOT cycle provides us the basis to define a temperature scale which is independent of material properties of the working substance.

The thermodynamic scale of temperature is defined by choosing 273.16 K as absolute temperature of the triple point of water as one fixed point and absolute zero as the other.

Unit

The unit of thermodynamic scale is Kelvin.

Kelvin

As, 273.16 K = Thermodynamic temperature of the triple point of water.

$\therefore$ $1K = \frac{1}{273.16} \times$ Thermodynamic temperature of the triple point of water.

Triple Point of Water

"It is a state in which ice, water and vapour co–exists in equilibrium and it occurs uniquely, at one particular pressure (0.61 K pa) and temperature (273.16 K)."

If heat Q is absorbed or rejected by system at T and Q_s is heat absorbed or rejected by system when it is at temperature of triple point of water then unknown temperature T is given by

$$\frac{Q}{Q_3} = \frac{T}{273.16}$$

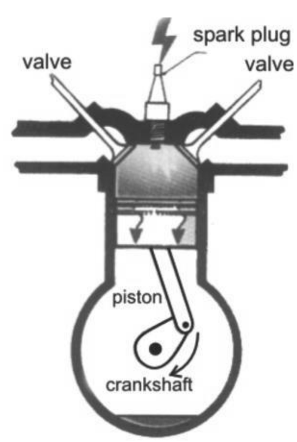
$$\therefore \quad T = \frac{Q}{Q_3}(273.16)$$

Since this scale is independent of working substance hence can be applied at very low temperature.

Q.21 What is the principle, construction and working of a petrol engine?

Ans. **PETROL ENGINE**

Although different engines may differ in their construction technology but they are based on principle of a Carnot cycle. A typical four stroke petrol engine also under goes four successive processes in each cycle.



Construction

It consists of pistons, crankshaft, sparking plug and valves, as shown in figure.

(1) Intake Stroke

The cycle starts on intake stroke in which piston moves outward and petrol air mixture is drawn through an inlet valve into the cylinder from carburetor at atmospheric pressure.

(2) Compression Stroke

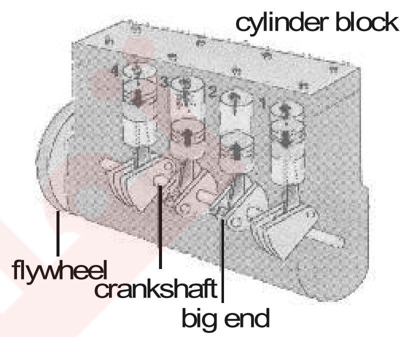
On the compression stroke, the inlet valve is closed and the mixture is compressed adiabatically by inward movement of the piston.

(3) Power Stroke

An electric spark is produced by the sparking plug. On the power stroke a spark fires the mixture causing a rapid increase in pressure and temperature. The burning mixture expands adiabatically and forces the piston to move outward. This is the only working stroke in the cycle which delivers power to crank shaft to derive the fly wheel.

(4) Exhaust Stroke

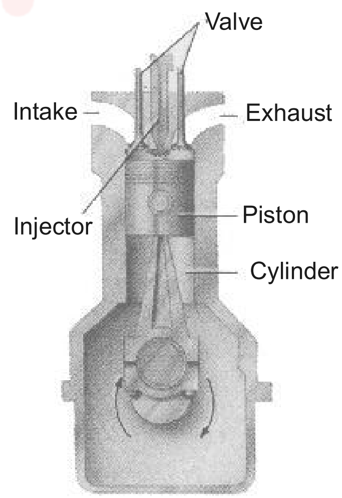
On the exhaust stroke, the outlet valve opens. The residual gases are expelled and piston moves inward.



The cycle then begins again. Most motor bikes have one cylinder engine but cars usually have four cylinders on the same crankshaft. The cylinders are timed to fire turn by turn in succession for smooth running of the car. The actual efficiency of properly tuned engine is not more than 25% to 30% due to friction and other heat losses.

Q.22 What is diesel engine?

Ans. DIESEL ENGINE



No spark plug is needed in diesel engine. Diesel is sprayed into the cylinder but maximum compression by a strong compression pump.

Because air is at very high temperature immediately after compression, the fuel mixture ignites on contact with the air in the cylinder and pushed the piston upward.

The operation of diesel engine is very easy and its efficiency is greater than that of petrol engine which is about 35% to 40%.

Q.23 What is entropy? Explain it with examples.

Ans. ENTROPY

The concept of entropy was introduced into the study of thermodynamics by **Rudolph Clausius** in 1856 to give a quantitative basis for the second law. It provides another variable to describe the state of a system to go along with pressure, volume, temperature and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of heat ΔQ at absolute temperature T, then the increase in the state variable called entropy ΔS of the system is given by

$$\Delta S = \frac{\Delta Q}{T}$$

Like potential energy or internal energy, it is the change in entropy of the system which is important. Change in entropy is positive when heat is added and negative when heat is removed from the system. Suppose, an amount of heat Q flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 when $T_1 > T_2$. The change in entropy of the reservoir, at temperature T_1, which loses heat, decreases by Q/T_1 and of the reservoir at temperature T_2, which gains heat, increases by Q/T_2. As $T_1 > T_2$ so Q/T_2 will be greater than Q/T_1 i.e., $Q/T_2 > Q/T_1$.

Hence, net change in entropy $= \dfrac{Q}{T_2} - \dfrac{Q}{T_1}$ is positive.

It follows that in all natural processes where heat flows from one system to another, there is always a net increase in entropy. This is another statement of 2nd law of thermodynamics. According to this law

If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

It is observed that a natural process tends to proceed towards a state of greater disorder. Thus, there is a relation between entropy and molecular disorder. For example an irreversible heat flow from a hot to a cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of increase in average molecular speeds and therefore, the randomness of molecular motion. Similarly, free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus in both examples, entropy is said to be increased.

We can conclude that only those processes are probable for which entropy of the system increases or remains constant. The process for which entropy remains constant is a reversible process; whereas for all irreversible processes, entropy of the system increases.

Every time entropy increases, the opportunity to convert some heat into work is lost. For example there is an increase in entropy when hot and cold waters are mixed. Then warm water which results cannot be separated into a hot layer and a cold layer. There has been no loss of energy but some of the energy is no longer available for conversion into work. Therefore, increase in entropy means degradation of energy from a higher level where more work can be extracted to a lower level at which less or no useful work can be done. The energy in a sense is degraded, going from more orderly form to less orderly form, eventually ending up as thermal energy.

In all real processes where heat transfer occurs, the energy available for doing useful work decreases. In other words the entropy increases, thereby decreasing the entropy, it is at the expense of net increase in entropy for some other system. When all the systems are taken together as the universe, the entropy of the universe always increases.

Do You Know?

Approximate efficiencies of various devices

Device	Efficiency (%)
Electric generator	70-99
Electric motor	50-93
Dry cell battery	90
Domestic gas furnace	70-85
Storage battery	72
Hydrogen-oxygen fuel cell	60
Liquid fuel rocket	47
Steam turbine	35-46
Fossil-fuel power plant	30-40
Nuclear reactor	39
Aircraft gas turbine engine	36
Solid-state laser	30
Internal combustion gasoline engine	20-30
Gallium arsenide solar cells	>20
Fluorescent lamp	20
Silicon solar cell	12-15
Steam locomotive	8
Incandescent lamp	5
Watt's steam engine	1

ENVIRONMENTAL CRISIS AS ENTROPY CRISIS

The second law of thermodynamics provides us the key for both understanding our environmental crisis, and for understanding how we must deal with this crisis.

From a human standpoint the environmental crisis results from our attempts to order nature for our comforts and greed. From a physical standpoint, however, the environmental crisis is an entropy or disorder crisis resulting from our futile efforts to ignore the second law of thermodynamics. According to which, any increase in the order in a system will produce an even greater increase in entropy or disorder in the environment. An individual impact may not have a major consequence but an impact of large number of all individuals disorder producing activities can affect the overall life support system.

The energy processes we use are not very efficient. As a result most of the energy is lost as heat to the environment. Although we can improve the efficiency but 2nd law eventually imposes an upper limit on efficiency improvement. Thermal pollution is an inevitable consequence of 2nd law of thermodynamics and the heat is the ultimate death of any form of energy. The increase in thermal pollution of the environment means increase in the entropy and that causes great concern. Even small temperature changes in the environment can have significant effects on metabolic rates in plants and animals. This can cause serious disruption of the overall ecological balance.

For Your Information

The jet engines on this aircraft convert thermal energy to work, but the visible exhaust clearly shows that a considerable amount of thermal energy is lost as waste heat.

In addition to thermal pollution, the most energy transformation processes such as heat engines used for transportation and for power generation cause air pollution. In effect, all forms of energy production have some undesirable effects and in some cases all problems can not be anticipated in advance.

The imperative from thermodynamics is that whenever you do anything, be sure to take into account its present and possible future impact on your environment. This is an ecological imperative that we must consider now if we are to prevent a drastic degradation of life on our beautiful but fragile Earth.

SOLVED EXAMPLES

EXAMPLE 11.1

What is the average transnational kinetic energy of molecules in a gas at temperature 27 ^{o}C?

Data

Temperature = T = 27 ^{o}C

= 27 + 273

= 300 K

To Find

Average translational kinetic energy = <K.E.> = ?

SOLUTION

Using

$$T = \frac{2}{3K} \langle K.E. \rangle$$

$$\therefore \quad \langle K.E. \rangle = \frac{3\,K\,T}{2}$$

Putting values

$$\langle K.E. \rangle = \frac{3 \times 1.38 \times 10^{-23} \times 300}{2}$$

$$= \frac{1242 \times 10^{-23}}{2}$$

$$= 621 \times 10^{-23}\ J$$

$$= 6.21 \times 10^{-21}\ J$$

Result

Average translation kinetic energy = <K.E.> = 6.21×10^{-21} J

EXAMPLE 11.2

Find the average speed of oxygen molecule in the air at S.T.P.

To Find

Average speed of oxygen molecule = < V > = ?

SOLUTION

At S.T.P.

T = 0 ^{o}C

= 0 + 273

$$T = 273\text{ K}$$

Using

$$T = \frac{2}{3K}\left< \frac{1}{2} m V^2 \right>$$

$$T = \frac{m}{3\text{ K}} <V^2>$$

or $$<V^2> = \frac{3\text{ K T}}{m} \quad \ldots\ldots\ldots (1)$$

As mass of one molecule of O_2 is

$$m = \frac{\text{Molecular mass of } O_2}{N_A}$$

$$= \frac{32\text{ g}}{6.02 \times 10^{23}}$$

$$= \frac{32}{6.02 \times 10^{26}}$$

$$= 5.31 \times 10^{-26}\text{ kg}$$

Putting values in equation (1)

$$\therefore \quad <V^2> = \frac{3 \times 1.38 \times 10^{-23} \times 273}{5.31 \times 10^{-26}}$$

$$= \frac{1130.22}{5.31} \times 10^{-23+26}$$

$$= 212.85 \times 10^3$$

$$<V^2> = 21.29 \times 10^4$$

Taking square root

$$<V> = 4.61 \times 10^2\text{ m/s}$$

$$= 461\text{ m/s}$$

Result

Average speed of oxygen molecule = $<V>$ = 461 m/s

EXAMPLE 11.3

A gas is enclosed in a container fitted with a piston of cross–sectional area 0.10m^2. The pressure of the gas is maintained at 8000 Nm^{-2}. When heat is slowly transferred, the piston is pushed up through a distance of 4.0cm. If 42J heat is transferred to the system during the expansion, what is the change in internal energy of the system?

Data

Area of piston = A = 0.10 m^2

Pressure of gas = P = 8000 Nm^{-2}

Distance moved by piston = ΔY = 4 cm

$$= \frac{4}{100}$$

$= 0.04$ m

Heat transferred $= Q = 42J$

To Find

Change in internal energy $= \Delta U = ?$

SOLUTION

Using

$$W = P \Delta V$$

or

$$W = PA \Delta Y$$

$$= 8000 \times 0.10 \times 0.04 \qquad (\because \Delta V = A\Delta y)$$

$$= 32 \text{ J} \qquad (\text{Volume} = \text{Area} \times \text{Height})$$

Using 1st law of thermodynamics

$$Q = \Delta U + W$$

$$\therefore \quad \Delta U = Q - W$$

$$= 42 - 32$$

$$= 10 \text{ J}$$

Result

Change in internal energy $= \Delta U = 10$ J

EXAMPLE 11.4

The turbine in a steam power plant takes steam from a boiler at 427 ^{o}C and exhausts into a low temperature reservoir at 77 ^{o}C. What is the maximum possible efficiency.

Data

Temperature of source $= T_1 = 427\,^{o}C$

$= 427 + 273$

$= 700$ K

Temperature of sink $= T_2 = 77\,^{o}C$

$= 77 + 273$

$= 350$ K

To Find

Efficiency $= \eta = ?$

SOLUTION

Using

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{350}{700}$$

$$= \frac{700 - 350}{700} = \frac{350}{700}$$

$$\eta = \frac{1}{2}$$

$$\eta\% = \frac{1}{2} \times 100$$

$$= 50\%$$

Result

Efficiency $= \eta = 50\%$

EXAMPLE 11.5

Calculate the entropy change when 1.0kg ice at $0^{o}C$ melts into water at $0^{o}C$. Latent heat of fusion of ice $L_f = 3.36 \times 10^5$ J kg^{-1}.

Data

Temperature $= T = 0^{o}C$

$= 0 + 273$

$= 273$ K

Mass of ice $= m = 1$ kg

Latent heat of fusion $= L_f = 3.36 \times 10^5$ J/kg

To Find

Entropy change $= \Delta S = ?$

SOLUTION

Using

$$\Delta Q = m L_f$$

$$= 1 \times 3.36 \times 10^5$$

$$= 3.36 \times 10^5 \text{ J}$$

Now using

$$\Delta S = \frac{\Delta Q}{T}$$

Putting values

$$\Delta S = \frac{3.36 \times 10^5}{273}$$

$$= 0.012 \times 10^5$$

$$= 1.2 \times 10^3 \text{ J k}^{-1}$$

Positive sign shows entropy increases because ice changes into water. Because the increase in entropy in this case is a measure of increase in the disorder of water molecules that change from solid to liquid state.

Result

Entropy change $= \Delta S = 1.2 \times 10^3 \text{ Jk}^{-1}$