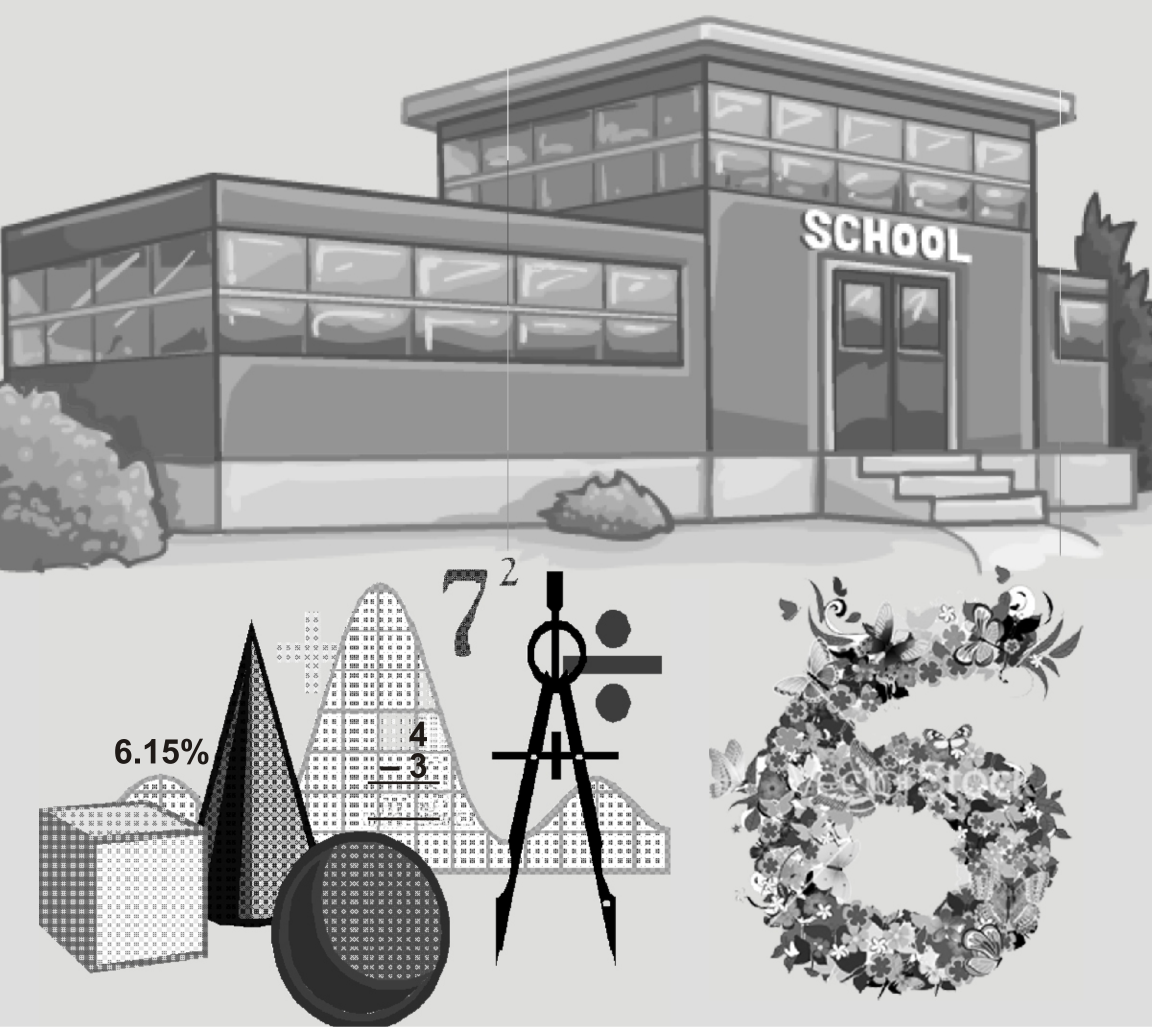
Mathematics 6



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PREFACE

The Sindh Textbook Board is an organization charged with the preparation and publication of textbooks in the province of Sindh. Its prime objective is to develop and produce textbooks which are conductive to equip the new generation with the knowledge and acumen to prepare them to face the challenges of the rapidly changing environment. In this age of knowledge explosion and development of technology not witnessed in the human history, efforts have to be made to ensure that our children do not lag behind. The Board also strives to ensure that Universal Islamic Ideology, culture and traditions are not compromised in developing the textbooks.

To accomplish this noble task, a team of educationists, experts, working teachers and friends endeavor tirelessly to develop text and improve contents, layout and design of the textbooks.

An attempt has made in this textbook to provide horizontal and vertical integration. The efforts of our experts and production personnel can bring about the desired results only if these textbooks are used effectively by teachers and students. Their suggestions will help us in further improving the qualitative contents of textbooks.

Chairman
Sindh Textbook Board

بسم الله الرحمن الرحيم

UNIT 1 SETS

George Cantor

In 19th century, **George Cantor** was the first mathematician who gave the concept of set.

1.1 SET

Define set

We often use some words in our daily life that represent a collection of things like dinner set, sofa set, tea set, group of boys, a team of players, a bunch of keys, crowd of people, a cluster of trees, a flock of sheep. The words set, group, team, bunch, crowd, cluster, flock are used to denote collection of things in daily life.

In mathematics collection of things is called set.

In mathematics we describe a set as:

A set is a collection of **'well defined'** and **'distinct'** objects.

The term **'Well defined'** means a set must have some specific property so, that it can easily be identified whether or not an object belongs to the given set.

Whereas the word **'Distinct'** means different objects.

Let us discuss some examples of collection.

(1) The students of class VI of your school. It is well defined because students of class VI of your school can only belong to it. Each student has unique identity. Thus it is a set.

(2) The collection of favourite books. It is not well defined because a book may be favourite for one person but may not be for another. Thus this collection is not a set.

(3) Name of days of week; Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday. It is a well defined sets because every day is the member of this set, which is distinct.

Recognize notation of a set and its objects/elements

English alphabets A, B, C, . . ., and Z are used to denote sets. The objects of the set are called elements or members of the set.

The elements of the set are listed within brackets { } and separated by commas.

> The symbol ‘ ’ is used to show the membership to a set and read as “belongs to” and ‘ ’ is used for, “does not belong” to a set.

Let us consider the following examples.

Example 1:

(i) B = { 2, 4, 6, 8 }

The members of set B are 2, 4, 6 and 8.

As 2 belongs to set B

So, symbolically we write as 2 B

As 3 does not belong to Set B.

So, symbolically we write as 3 B

Teacher's Note *Teacher should give some more examples of collections from class room and daily life to clarify the concept of set such as name of books in a bag for grade VI and name of instruments in geometry box..*

(ii) D = { x, y, z }

The elements of set D are x, y and z.

Here, x D, but p D

Describe tabular form of a set and demonstrate through examples

As we have already described that all the elements of a set are listed within { } and separated by commas, this form of describing a set is called **tabular form**.

Let us demonstrate this form through examples.

Example 1: Set of the vowels of the English alphabet

In tabular form we write as {a, e, i, o, u} or {i, a, u, e, o}

Example 2: Set of natural numbers upto 100

In tabular form we write as {1, 2, 3, ..., 100}

Example 3: Set of names of three boys whose names start with "A".

In tabular form we write as {Ali, Ahmed, Ahmer}

Example 4: Set of name of days of a week.

In tabular form we write as

{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

Example 5: Explain, why are the following not sets:

(1) A = {5, 6, 6, 7, 8}

(2) B = Collection of intelligent students

Solution:

(1) A = {5, 6, 6, 7, 8}

This is not set because the element 6 is **repeated**.

(2) B = Collection of intelligent students

This is not set because it can not be decided that which students are intelligent.

SETS

Activity 1 For A = {2, 4, 6, 8, 10 }

Fill in the blanks with symbols or

(i)	4 ______ A	(ii)	1 ______ A	(iii)	6 ______ A
(iv)	9 ______ A	(v)	2 ______ A	(vi)	8 ______ A

Activity 2 **Write the given sets in tabular form.**

Set	Tabular Form
(i) Set of first five English alphabets.	{ a, b, c, d, e }
(ii) Set of all factors of 12	
(iii) Set of names of days of a week which start from "F"	
(iv) Set of first five natural numbers	

Points to remember

- The name of a set is denoted by capital letter of English alphabet.
- Every element of a set is separated by a comma ',' in tabular form.
- All the members of set are enclosed in brackets { } in tabular form.
- The order of writing elements does not matter in a set.
- Elements of set do not repeat itself.

Teacher's Note *Teacher should perform the given activities on blackboard with students and also give some more examples.*

EXERCISE 1.1

1. Tick (✓) which of the following are sets?

(i) Collection of the names of Presidents of Pakistan.

(ii) Collection of names of captains of hockey teams of Pakistan.

(iii) Collection of delicious dishes.

(iv) Collection of intelligent students in your class.

(v) Collection of greater numbers.

(vi) Collection of English teachers in your school.

2. If A = {5, 6, 7, 8} and B = {*a, b, c, d*}, fill in the blanks with the symbols or .

(i) 5 ______ A	(ii) 9 ______ A	(iii) m ______ A
(iv) c ______ B	(v) b ______ B	(vi) 7 ______ B

3. Write in tabular form.

(i) Set of name of capitals of the provinces of Pakistan.

(ii) Set of natural numbers from 50 to 70.

(iii) Set of first ten natural numbers exactly divisible by 2.

(iv) Set of letters in the word "pakistan".

(v) Set of mathematics teachers in your school.

(vi) Set of name of colours of national flag.

4. If P = {*a, e, i, o, u*} and Q = {1, 2, 3, ..., 10} then which of the following statements are true or false.

(i) a P ________	(ii) 1 Q ________
(iii) i Q ________	(iv) v P ________
(v) y Q ________	(vi) e P ________
(vii) 3 P ________	(viii) 7 Q ________

Define finite and infinite sets

Consider the sets given in the table 1.

Table 1

A = {2, 4, 6} ,	B = {1, 2, 3, ...}
C = {6, 4, 2} ,	D = {x, y, z}
E = {1, 3, 5, 7, ...} ,	F = {5, 10, 15, 20, 25, ...}

Let us consider set A. It has limited number of elements, so it is **finite set**.

A set which has limited number of elements is called finite set.

In table 1, C and D are also finite sets.
Now consider set B = {1, 2, 3, ...}.
This set has unlimited number of elements. So it is **infinite set**.

A set which has unlimited number of elements is called infinite set.

In table 1, E and F are also infinite sets.

Activity Tick (✓) the finite sets and cross (✗) the infinite sets.

(i) Set of all citizen of Pakistan ☐
(ii) Set of all whole number less than 9999 ☐
(iii) Set of drops of water in river ☐
(iv) The set of all English alphabets ☐
(v) {1, 2, 3, ...} ☐
(vi) {d, o, r} ☐
(vii) {2, 4, 6, ...} ☐

Define empty/void/null sets and singleton

Table 2

A = { 5 } , B = { x }
C = Set of natural numbers less than 1
D = Set of triangles with four sides

In table 2, C has no element because there is no natural number less than 1.

So, C is **an empty set**, which is also called **null or void set**.

A set which has no element is called an empty or null or void set. It is denoted by $\emptyset$ or { }.

In table 2, D is also empty set. Some other examples of empty set are:

Examples:

(i) Set of squares with five sides.
(ii) Set of letters of English alphabet before A.

In table 2, there is a set A = { 5 } which has only one element and is called **singleton set**.

A set having only one element is called singleton.

In table 2, set B is also singleton. Some other examples of singleton are:

Examples:

P = { 8 } Q = { a }, R = {1} and S = {0}

Define equal and equivalent sets

Table 3

A = {1, 2, 3, 4} ,	B = {x, y, z}
C = {4, 3, 2, 1},	D = {a, e, i, o, u}
E = {2, 4, 6, 8, 10},	F = {y, z, x}

Consider sets A and C from the table 3. All the elements of both sets are the same. So they are called **equal sets**.

Two sets A and B are called equal sets if they have all elements same. Symbolically we represent as A = B and read as:
Set A is equal to set B

In table 3, B and F are also equal sets.
When we see D and E, they have same number of elements and these are called **equivalent sets**.

Two sets A and B are called equivalent sets if the number of elements of both sets are same. Symbolically we write as:
A B and read as: Set A is equivalent sets to set B

Also, in the given table 3,

A C and B F

All equal sets are also equivalent sets but all equivalent sets may not be equal sets.

For example in table 3, (i) A = C and also A C

(ii) D E but D E.

Teacher's Note *Teacher should encourage students to make examples of different types of sets themselves.*

Define subset and superset of a set

Consider the sets given in the following table 4.

Table 4

A = { 1 } ,	B = {1, 2, 3}
C = {1, 2, 3, 4},	D = {4, 3, 2, 1}

Let us see sets B and C. All the elements of B are also elements of C. So we say B is **subset** of C.

> If each element of set A is also an element of set B then set A is called subset of set B. Symbolically we write as A B.

In the given table 4,

A B, B C , C D and D C

Remember that

> Every set is subset of itself.

As B B and D D

Again, if we consider B and D.

B is subset of D. So D is called **superset** of B.

> If A is subset of B then the set B is called superset of A. Symbolically we write it as 'B A'

In the given table 4,

B A and D B

Define proper and improper subsets of a set and demonstrate through examples.

Now we consider set B and set D in table 4

Here B D and B D.

So set B is called proper subset of set D.

Proper Subset

If set A is subset of set B and also A is not equal to B then "A is proper subset of set B. Symbolically we write as A B.

In the given table 4, A B and B D

In table 4, we see that set D is subset of set C but set D is also equal to set C. So D is called improper subset of C.

Improper Subset

If set A is subset of set B and A is also equal to B then "A is called improper subset of set B".

In table 4, set C is improper subset of set D.

Activity Identify and write the following sets, from the given table.

(i)	Two finite sets	(ii)	Two infinite sets
(iii)	Two empty sets	(iv)	Two equal sets
(v)	Two equivalent sets	(vi)	Two singleton
(vii)	Two subsets	(viii)	Two super sets

Set of natural numbers between 2 and 3,		Set of factors of 1
Set of triangles with four sides,		D = {2, 4, 6, 8}
A = {1, 3, 5, 7, ...},	B = { 6 },	E = {5, 10, 15, 20, 25, ...}
C = {*a, b, c, d*},		F = {4, 6, 2, 8},
Set of first four letters of English alphabet		

EXERCISE 1.2

1. Give reasons why following collections are not set.

(i) $\{ d, o, o, r \}$ (ii) { ★, O, ★, }

(iii) $\{ a, f, d, \mathrm{a} \}$ (iv) $\{ 2, 2, 3, 3, 4, 4 \}$

(v) Set of beautiful birds (vi) Set of good players

2. Which of the following are finite or infinite sets?

(i) $A = \{ 0, 1, 2, \ldots, 50 \}$.

(ii) $B = \{ 100, 200, 300, 400, \ldots \}$.

(iii) Set of hair on the body of goat.

(iv) Set of legs of a cat.

(v) Set of stars in the sky.

(vi) Set of lines passing through a point.

(vii) Set of natural numbers greater than 20.

(viii) Set of all the cities of Pakistan.

(ix) Set of all the schools in Sindh.

(x) Set of all even numbers.

3. Which of the following are empty sets?

(i) Set of students of your class over 20 years of age.

(ii) Set of letters after Z in English alphabet.

(iii) Set of children whose names start with "K" in your locality.

(iv) Set of name of solar calendar starting with Z.

(v) Set of fishes which live in sand.

(vi) Set of even numbers between 4 and 10.

(vii) $\{ 0 \}$ (viii) $\{ \emptyset \}$

4. Which pairs of the following are equal sets?

(i) $\{ 1, 2, 3 \}$ and $\{ 2, 3, 1 \}$ (ii) $\{ p, a, t \}$ and $\{ t, a, p \}$

(iii) $\{ k, i, t, e \}$ and $\{ b, i, t, e \}$

(iv) $\{ x, y, z \}$ and set of first three letters of English alphabet.

(v) Set of odd numbers less than 2 and $\{ \ \}$

5. Which pairs of the following are equivalent sets?

(i) { *m, i, l, e* } and { *n, i, l, e* } (ii) { *1, 2, 3* } and {3, 2, 1}

(iii) { , ▯, ○, ★, □ } and { 1, 2, 3, 4, 5 }

(iv) { 6, 66, 666, 6666 } and { 666, 7777, 77 }

6. Which of the following statements are true or false?

(i) B = { m, o, n } if B is Set of letters of word "MOON".

(ii) A B if A = { 1, 2 } and B is { 2, 3, 4, 5 }

(iii) If A = { 4, 5, 10 } and B is { 5, 10, 20 } then A B.

(iv) If A = { 0, 1, 2, 3, 5 } and B is Set of natural numbers less than 6, then A = B.

(v) If X is equal to Y, then X is equivalent to Y.

7. Choose appropriate symbol to indicate the relation between sets (=, , __ , , ,)

(i) { 10, 20, 30 } ______ { 5, 10, 15, 20, 25, 30 }

(ii) { 7, 14, 21, 28, 35, 42, 49 } _______ Set of multiples of 7 less than 50.

(iii) { 5, 6, 7, 8, ... } _______ { 7, 5, 8 }

(iv) { 0 } _______ { }

(v) { *t, e, a* } _______ { *a, e, t* }

(vi) { Hyderabad, Karachi, Sukkur }_______Set of all cities of Sindh.

(vii) { *a, e, i, o, u* } _______ Set of letters of English alphabets.

(viii) { 11, 22, 33 } _______ { 1, 2, 3, ... }

REVIEW EXERCISE 1

1. Fill in the blanks.

(i) If A = { a, b, c } then a, b, c are __________ of a set A.

(ii) A set which has no element is called ______________ set.

(iii) A set which has limited number of elements is called a ______ set.

(iv) If two sets A and B have _______________ elements, then they are said to be equal sets.

2. Choose the correct answer.

(i) If B is set of vowels of English alphabet:

(a) p B (b) e B (c) b B (d) a B

(ii) { f } is called:

(a) null set (b) infinite set (c) subset (d) singleton set

(iii) If sets A and B are equal, we use symbol

(a) (b) (c) (d)

(iv) If A = { a, b, c } and B = { a, b, c, d, e } then

(a) A = B (b) A B (c) A B (d) B A

3. Write two examples of set.

4. Write the following sets in tabular form.

(i) A is Set of all even numbers between 2 and 10.

(ii) B is Set of all odd numbers from 1 to 17.

(iii) C is Set of natural numbers divisible by 2 and less than 30.

(iv) D is Set of months of a solar calendar beginning with letter "J".

5. Which of these sets are finite and which of these are infinite.

(i) A is Set of members of your family

(ii) B is Set of all even numbers

(iii) C is Set of prime factors of 60

(iv) D is Set of all multiples of 7

(v) E is Set of one-digit natural numbers

6. Which of the following are empty sets.

(i) L is Set of the names of days of a week which start with “S”.

(ii) F is Name of months of a year having 32 days.

(iii) M is Set of odd numbers exactly divisible by 2.

(iv) N is Set of prime numbers exactly divisible by 2.

7. If A = $\{ d \}$, B = $\{ c, d \}$, C = $\{ a, b, c \}$, D = $\{ a, b \}$ then which of the following statements are true or false.

(i) D C (ii) A __ C (Iii) B D (iv) C __ D

8. Indicate which of the following statements are true and false?

(i) The members of any set are always of same kind.

(ii) The order of the elements of a set does not matter.

(iii) An object can be included in a set repeatedly.

(iv) Any collection of objects is called a set.

(v) The notation “ ” is a symbol to indicate membership of a set.

(vi) A well-defined collection of distinct objects is called a set.

(vii) $\{ a, b, c, d \}$ is not a set.

(viii) The notation “ ” is a symbol to indicate membership of a set.

(ix) 1, 2, 3, 4 is a set.

SUMMARY

- A set is a collection of well defined and distinct objects.
- Each object in a set is an element or member of the set.
- A set is described in two ways.
  - (i) Precisely described in words.
  - (ii) All the elements are listed down separately.
- Finite set has limited number of elements.
- Infinite set has unlimited number of elements.
- A set having no element is an Empty or Null set.
- Two sets are equal if they have all same elements.
- Two sets are equivalent if they have same number of elements.
- B is subset of A if every element of B is also an element of A.
- A set having only one element is called singleton set.

Symbols to Remember

	is member of
	is not member of
Ø or { }	empty set or null set
	is equal to
/	is not equal to
	is equivalent to
	is a subset of
	is a proper subset of
	is a superset of
—	is not a subset of

Unit 2

WHOLE NUMBERS

2.1 NATURAL AND WHOLE NUMBERS

We use numbers '1, 2, 3, 4, 5, ...' for representing the quantity of objects in our daily life.

These numbers 1, 2, 3, 4, ..., are called natural numbers.

When we include '0' in the set of natural numbers, we will get the set of whole numbers.

'0, 1, 2, 3, 4, ...' are called whole numbers.

Here 0 is predecessor of 1 and 1 is successor of 0. Similarly 1 is predecessor of 2 and 2 is successor of 1 and so on.

Differentiate between natural and whole numbers

The only difference between set of natural numbers and set of whole numbers is of number '0', '0' does not belong to the set of Natural numbers. All the other members of both the sets are the same.

Identify natural and whole numbers, and their notations

We denote the set of natural numbers by N.

i.e $\quad N = \{1, 2, 3, 4, 5, 6, ...\}$

The set of whole numbers consist of zero and all natural numbers. It is denoted by W,

i.e. $\quad W = \{0, 1, 2, 3, 4, 5, 6, 7, ...\}$

Look at the following sets:

$N = \{1, 2, 3, ...\}$

$W = \{0, 1, 2, ...\}$

Important facts about natural numbers and whole numbers are:

- 1 is the smallest and the first natural number.
- The first and the smallest whole number is '0'.
- The set of natural numbers is an infinite as we can not count all natural numbers.
- The set of whole numbers is an infinite set.

Represent a given list of whole numbers

Number line helps us to represent given whole numbers. Whole numbers can be represented on a number line by using following steps:

Let us represent numbers 0, 1, 2, 3, 4, 5, ..., on a number line.

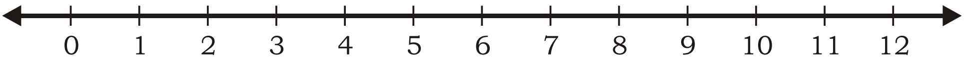
Steps:

(i) Draw a line

(ii) Mark a point to represent the first whole number '0' on the line.

(iii) Mark other points at equal distance and name them 1, 2, 3, 4, 5, ... respectively as shown below:

Thus we get a number line as under:



Note:

(i) On a number line, a number is greater than any number on its left. For example, $1 > 0$, $2 > 1$ and $3 > 2$ etc.

(ii) On a number line a number is less than any number on its right. For example $0 < 2$, $1 < 3$, $5 < 12$.

Represent whole numbers $<$ (or $>$) a given whole number on a number line

Let us represent whole numbers $<$ (or $>$) a given whole number on number line with the help of an example.

Example.

Represent the following on number line.

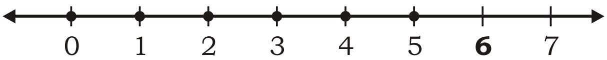
(i) Whole numbers Less than 6

(ii) Whole numbers Greater than 4

Solution:

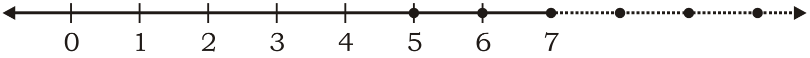
(i) Whole numbers less than 6

Number less than 6 means number to the left side of 6.



Dark shaded dots represent the required whole numbers.
So, 0, 1, 2, 3, 4, 5 are the whole numbers less than 6.

(ii) Whole numbers greater than 4



Dark shaded dots represent the required whole numbers.
So, 5, 6, 7, ... are the required whole numbers.

Represent whole numbers $\geq$ (or $\leq$) a given whole number on a number line

Let us represent whole number $\geq$ (or $\leq$) a given whole number on a number line with the help of an example.

Example. Represent the following whole number on a number line.

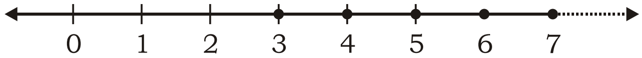
(i) Whole numbers less than or equal to 7
(ii) Whole numbers greater than or equal to 3

Solution: (i) whole number 7

Dark shaded points represent the required whole numbers.

So, the whole numbers less than or equal to 7 are 0, 1, 2, 3, 4, 5, 6, 7.

(ii) Whole number 3



Dark shaded points represent the required whole numbers with continuation.

So, the whole numbers greater than or equal to 3 are 3, 4, 5, 6, 7, ...

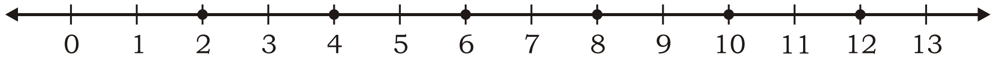
Teacher's Note *Teacher should give more questions to students on blackboard to represent whole numbers on number line.*

Represent whole numbers $>$ but $<$ a given whole number on a number line

Let us represent whole numbers $>$ but $<$ a given whole number on a number line with the help of an example.

Example. Represent the even whole numbers greater than 1 but less than 13 on number line.

Solution: Even whole numbers $>$ 1 but $<$ 13



Dark shaded points represent the required even whole numbers.

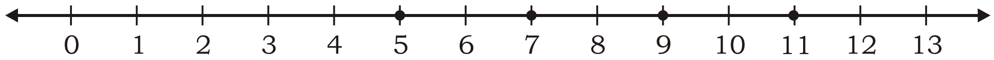
Hence the required even numbers are 2, 4, 6, 8, 10, 12.

Represent whole numbers $\geq$ but $\leq$ a given whole number on a number line

Let us represent whole numbers $\geq$ but $\leq$ a given whole number on a number line with the help of an example.

Example: Represent odd whole numbers greater than or equal to 5 but less than or equal to 11 on a number line.

Solution: Odd whole numbers 5 but 11



Dark shaded points represent the required odd whole numbers.

Hence the required odd whole numbers are 5, 7, 9, 11.

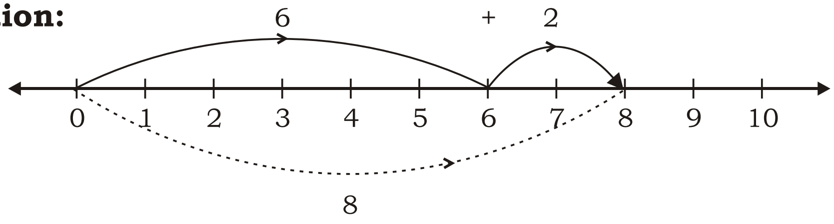
Represent sum of two or more given whole numbers on the number line

The method of adding two or more given whole numbers on the number line is explained with the help of following example.

Teacher's Note *Teacher should draw the number line on blackboard and move a student in front of it to show the sum of numbers on number line.*

Example 1. Find the sum of 6 and 2 by using number line.

Solution:



Its procedure is described below:

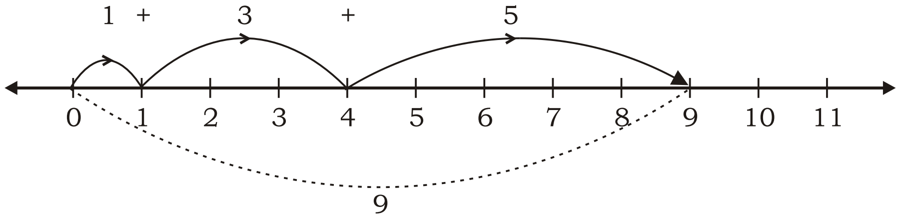
Start from 0, and move 6 units on the right. Again, starting from 6 move 2 units on the right. We reach at number 8.

Hence, $6 + 2 = 8$

Example 2.

With the help of a number line, find the sum of $1 + 3 + 5$

Solution:



Therefore, $1 + 3 + 5 = 9$

EXERCISE 2.1

1. **Write, if possible:**
   - (i) The smallest natural number
   - (ii) The smallest whole number
   - (iii) The largest natural number
   - (iv) The largest whole number
2. **Write first ten natural numbers.**
3. **Write first ten whole numbers.**

4. Represent on number line:

(i) Whole numbers > 8 (ii) Whole numbers < 8

(iii) Whole numbers   8 (iv) Whole numbers   8

(v) Whole numbers > 3 but < 15

(vi) Whole numbers > 5 but   12

(vii) Whole numbers   4 but   11

(viii) Whole numbers   3 but < 9

(ix) Odd whole numbers greater than 3

(x) Even whole numbers greater than or equal to 8 but less than 16.

5. Find sum of the following whole numbers by using number line.

(i) 1 and 5 (ii) 6 and 3

(iii) 10 and 2 (iv) 8 and 4

(v) 2, 3 and 5 (vi) 3, 2 and 4

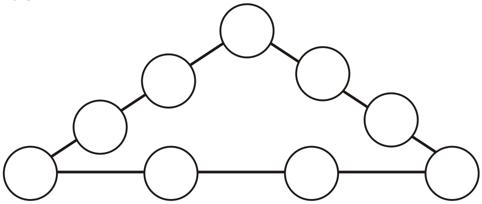
2.2 ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Add and subtract two given whole numbers

We already know how to add and subtract two natural numbers.

Same methods are applied for whole numbers.

How to fill the triangle with 1, 2, 3, 4, 5, 6, 7, 8 and 9 to get the sum of each side equal to 17.



For example, the sum of 5 and 14 is 19 which is again a whole number. Therefore we can say that the sum of whole numbers is always a whole number.

Similarly, 25941 + 58723 = 84664 which is a whole number.

The sum of two whole numbers is always a whole number.

When we subtract 23 from 53, we get 30. Here 23, 53 and 30 are all whole numbers. Hence, the resulting number in subtraction of whole numbers may or may not be a whole number.

Example 1: Add: 389 and 245

$$\begin{array}{r} {}^{1}3\;{}^{1}8\;9 \\ +\;2\;4\;5 \\ \hline \mathbf{6\;3\;4} \\ \hline \end{array}$$

As,
9 + 5 = 14
1 + 8 + 4 = 13
1 + 3 + 2 = 6

Hence 389 + 245 = **634**

Example 2: Subtract 535 from 1000

$$\begin{array}{r} {}^{0}\cancel{1}\;{}^{9}\cancel{0}\;{}^{9}\cancel{0}\;{}^{1}0 \\ -\;5\;3\;5 \\ \hline \mathbf{0\;4\;6\;5} \\ \hline \end{array}$$

As,
10 – 5 = 5
9 – 3 = 6
9 – 5 = 4
0 – 0 = 0

Hence 1000 – 535 = **465**

Example 3: Aslam earns Rs 56835 in a month and his wife earns Rs 35600. Their monthly expenditure is Rs 65000. Find their total monthly income and saving.

Solution: Adding the monthly income of Aslam and his wife.

Monthly income of Aslam	=	56835 rupees
Monthly income of his wife	=	+ 35600 rupees
Total of monthly income	=	92435 rupees

Now, in order to find monthly saving, we subtract monthly expenditure from their total income.

Monthly income	=	92435 rupees
Monthly expenditure	=	65000 rupees
Monthly saving	=	27435 rupees

Hence, their monthly income is Rs 92435 and monthly saving is Rs 27435.

Example 4: Find the sum of smallest five digit number and largest four digit number.

Solution:

Smallest five digit number	10000
Largest four digit number	+ 9999
	19999

Hence, required sum = **19999**

Verify commutative and associative law (under addition) of whole numbers.

(a) Commutative law under addition.

When we add 9 to 30, we get 39 and when we add 30 to 9, then again we get 39. Similarly by adding 100 to 40 and by adding 40 to 100, we get the same whole number 140. In other words, we have

$$9 + 30 = 30 + 9, \text{ and } 100 + 40 = 40 + 100$$

It shows that, for any order of two given whole numbers, their addition give us the same sum. This is known as the *commutative law of whole numbers under addition.*

Thus commutative law under addition is:

> The sum of two whole numbers, in any order, is always same.

Example:

Verify commutative law under addition for whole numbers 85 and 95.

Solution: By Commutative Law under addition, 85 + 95 = 95 + 85

LHS = 85 + 95
= 180

$$\begin{array}{r} 85 \\ +\ 95 \\ \hline \mathbf{180} \end{array}$$

RHS = 95 + 85
= 180

$$\begin{array}{r} 95 \\ +\ 85 \\ \hline \mathbf{180} \end{array}$$

Since, LHS = RHS, therefore 85 + 95 = 95 + 85

Hence, Commutative law under addition is verified.

(b) Associative law under addition.

We can add only two whole numbers at a time. So, we can get the sum of three whole numbers, say 3, 7 and 12 by two ways:

or

$\boxed{3 + 7} + 12$	$3 + \boxed{7 + 12}$
$= 10 + 12$	$= 3 + 19$
$= 22$	$= 22$

Therefore we can say

$(3 + 7) + 12 = 3 + (7 + 12)$

Hence, three whole numbers added in any order give same result.

This is known as associative law under addition.

Thus Associative law under addition is:

The sum of three whole numbers, in any order, is always same.

Example:

Verify Associative law under addition for 23, 59 and 87.

Solution:

By Associative Law under addition, (23 + 59) + 87 = 23 + (59 + 87)

$$\begin{aligned} \text{LHS} &= (23 + 59) + 87 \\ &= 82 + 87 \\ &= 169 \end{aligned} \qquad \begin{array}{r} 23 \\ +\ 59 \\ \hline 82 \\ +\ 87 \\ \hline \mathbf{169} \\ \hline \end{array}$$

$$\begin{aligned} \text{RHS} &= 23 + (59 + 87) \\ &= 23 + 146 \\ &= 169 \end{aligned} \qquad \begin{array}{r} 59 \\ +\ 87 \\ \hline 146 \\ +\ 23 \\ \hline \mathbf{169} \\ \hline \end{array}$$

Since, LHS = RHS, therefore (23 + 59) + 87 = 23 + (59 + 87)

So, Associative law under addition is verified.

Teacher's Note *Teacher should help the students to understand the concepts of laws of whole numbers under addition.*

Activity 1 Fill in the following blanks by using Commutative and Associative laws of addition.

(i) 56123 + 71045 = 71045 + **56123**

(ii) 24125 + (41625 + 7123) = (24125 + ________) + 7123

(iii) 47813 + ________ = 51623 + ________

(iv) 567 + (________ + 1784) = (________ + 962) + ________

Recognize '0' as additive identity

There is a whole number, which has a unique property that no other whole numbers has. The number is 0, and the property is:

When 0 is added to any whole number, the sum is the whole number itself.

For Example: 1 + 0 = 0 + 1 = 1

25 + 0 = 0 + 25 = 25

And the number "0" is called additive identity.

EXERCISE 2.2

1. What is the sum of the largest number of six digits and the smallest number of seven digits?

2. Population of a village is 2700. If 1070 are men and 915 are women, find the number of children.

3. Arif deposited Rs 45800 in his bank account. After a month he withdrew Rs 3500 from it. How much money was left in his account?

4. Kashif had Rs 82000. He gave Rs 6500 to his wife, Rs 10550 to son and Rs 15335 to daughter. How much money was left with him?

5. Verify commutative law under addition for the following.

(i) 7628 and 39780 (ii) 924981 and 228
(iii) 29000 and 10699 (iv) 50102 and 9019854

6. Verify associative law under addition for the following.

(i) 34006, 2389 and 44380 (ii) 583031, 127 and 3405
(iii) 231, 6090 and 25996 (iv) 412, 3007 and 102341

7. Fill in the blanks to make the following statements true.

(i) 5020 + 849 = ________________ + 5020

(ii) 97864 + 0 = ________________

(iii) 749 + 0 = 0 + ______ = ______

(iv) 3971 + (430 + 300) = (3971 + ____________) + 300

(v) 4853 + 93 = 93 + 4853 = _________________

8. Think a number. Double the number and add 9 to it then add the number you started with. Then divide the amount by 3 and also subtract 3 from the quotient. What the number do you get?

2.3 MULTIPLICATION AND DIVISION OF WHOLE NUMBERS

We know that multiplication is repeated addition and division is repeated subtraction.

Multiply and divide two given whole numbers

(a) Multiplication:

The method of multiplying two whole numbers is same as the method of multiplication of natural numbers.

Consider the following multiplications:

5   4 = 20,   10   153 = 1530,   0   2453 = 0

In all these examples, the multiplication of two whole numbers gives a whole number.

> The product of two whole numbers is always a whole number.

Let us consider the following examples.

Example 1:
Multiply 2483 by 253

Solution:

$$\begin{array}{r} 2483 \\ 253 \\ \hline 7449 \\ 124150 \\ 496600 \\ \hline \boxed{\mathbf{628199}} \end{array}$$

Hence

2483   253 = 628199

Example 2: Zahida purchased 20 bangles at the price of Rs 80 each. Find the total amount paid by Zahida.

Solution:

Number of bangles: 20
Price: 80

$$\begin{array}{r} 20 \\ 80 \\ \hline 00 \\ 160 \\ \hline \mathbf{1600} \\ \hline \end{array}$$

Hence the total amount is Rs 1600.

(b) Division:

If we divide 9 by 2, then the quotient is 4 and the remainder is 1.

$$\begin{array}{r} 4 \\ 2\overline{)\,9} \\ -8 \\ \hline 1 \end{array}$$

Let us consider division of some other whole numbers along with their quotients and remainders.

Dividend	Divisor	Quotient	Remainder
9	3	3	0
7	5	1	2
14	8	1	6
56	7	8	0

Note:

- The quotient of a whole number divided by 1 is the number itself.
- The quotient of zero divided by a non-zero whole number is zero.

Example 1: Divide 435 by 3. Write quotient and remainder.

Solution:

```
               145 → quotient
divisor ← 3 ) 435 → dividend
             – 3
             ----
              13
             – 12
             ----
               15
             – 15
             ----
                0 → remainder
```

Hence, 435 3 = 145.

Example 2: The cost of 15 books is Rs 600. Find the cost of one book.

Solution: In order to find the cost of one book we divide the total cost Rs 600 by number of books, i.e. 15.

```
      40
15 ) 600
    – 600
    -----
```

So, the cost of one book is Rs 40.

Example 3:

Find the greatest 4 digit number which is exactly divisible by 23.

Solution:

As greatest 4 digit number is 9999

```
      434
23 ) 9999
    – 92
    -----
       79
     – 69
    -----
      109
     – 92
    -----
       17    = remainder
```

Thus the required number = 9999 – 17

= 9982

EXERCISE 2.3

1. **Find the following products of whole numbers.**

   (i) 854   96   (ii) 736   103

   (iii) 256   1008   (iv) 995   158

2. **Divide and find the quotient and remainder.**

   (i) 7772   58   (ii) 96324   245

   (iii) 16025   1000   (iv) 92845   300

3. A gardener plans to plant 570 trees in 19 rows. Each row should contain equal number of trees. How many trees will be in each row?

4. The shopkeeper purchased 125 television sets. If the cost of each set is Rs 9820, find the cost of all sets.

5. Find the greatest 5 digit number which is exactly divisible by 75.

6. Multiply the greatest number of four digits with the smallest number of three digits.

Verify commutative and associative law (under multiplication) of whole numbers

(a) Commutative law under multiplication

Consider the product of two whole numbers 5 and 6, which is 5   6 = 30. Also 6   5 = 30.

Thus 5 multiplied by 6 is the same as 6 multiplied by 5.

Similarly, 401   98 = 98   401 gives 39298 = 39298, 3756   23 = 23   3756 gives 86388 = 86388, 101   0 = 0   101 gives 0 = 0 and 255   1 = 1   255 gives 255 = 255.

Hence, two whole numbers multiplied in any order gives equal product. This is known as commutative law under multiplication.

> The product of two whole numbers, in any order, is always same.

Associative law under multiplication

Consider the product of three whole numbers 3, 2 and 4. If we first multiply 3 and 2 and multiply the resulting number by 4, we get

$$(3 \quad 2) \quad 4 = 6 \quad 4 = 24.$$

On the other hand, if we multiply 2 with 4 and multiply their product with 3.

we get $3 \quad (2 \quad 4) = 3 \quad 8 = 24$

Thus, $(3 \quad 2) \quad 4 = 3 \quad (2 \quad 4)$

What are the things we can do without any definite order in our daily life?

Hence, three whole numbers multiplied in any order gives equal product. This is the associative law under multiplication.

Thus associative law under multiplication is:

The product of three whole numbers, in any order, is always same.

Recognize '1' as multiplicative identity

The whole number 1 has a special role in multiplication just as the number 0 has in the case of addition. As $3 \quad 1 = 1 \quad 3 = 3$ and $1 \quad 5 = 5 \quad 1 = 5$ etc. Therefore 1 is the only whole number with the property that when 1 is multiplied with any whole number in any order, the result is the same whole number.

So "1" is called multiplicative identity.

Note: The number 0 also has a special property in multiplication. We know that $8 \quad 0 = 0$, $0 \quad 200 = 0$ and $283450 \quad 0 = 0$.

Therefore when 0 is multiplied with any whole number the result is 0.

2.4 MULTIPLICATION AND ADDITION (SUBTRACTION) OF WHOLE NUMBERS

Verify distributive law of multiplication over addition

For the three whole numbers 2, 4 and 7. First we consider 4 + 7 and then 2   (4 + 7) to get 2   11 = 22.

Also (2   4) + (2   7) = 8 + 14 = 22

Therefore 2   (4 + 7) = (2   4) + (2   7)

> What are the things in our daily life must be done in a certain order?

This is called distributive law of multiplication over addition.

Let us demonstrate the distributive law of multiplication over addition with the help of example   5 (10 + 8) = 5   10 + 5   8

Solution:

L.H.S = 5 (10 + 8)	R.H.S = 5 10 + 5 8
= 5 (18)	= 50 + 40
= 90	= 90

$\because$ L.H.S = R.H.S

$\therefore$ 5 (10 + 8) = 5   10 + 5   8

Hence distributive law of multiplication over addition is verified.

Verify distributive law of multiplication over subtraction (with positive difference)

For the above three whole numbers 2, 4 and 7,

2   (7 – 4) = 2   3 = 6 and (2   7) – (2   4) = 14 – 8 = 6

Therefore, 2   (7 – 4) = (2   7) – (2   4)

This is called distributive law of multiplication over subtraction.
Let us demonstrate of Distributive law of multiplication over subtraction with help of example 5 (20 – 2) = 5   20 – 5   2

Solution:

L.H.S = 5 (20 – 2)		R.H.S = 5 20 – 5 2
= 5 (18)		= 100 – 10
= 90		= 90

∵ L.H.S = R.H.S

∴ 5 (20 – 2) = 5   20 – 5   2

Hence distributive law of multiplication under subtraction is verified.

EXERCISE 2.4

1. Fill in the blanks.

(i) 5   0 = __________

(ii) 6   1 = __________

(iii) 2   5 = 5   ______

(iv) 3   (1   4) = (3   1)   _____

(v) 5   (_____   8) = (_____   7)   8

(vi) 541   (645 + ______) = 541   645 + 541   964

(vii) 345   (650 – 125) = 345   650 ______ 345   ______

2. Write True or False

(i) 545   248 = 545 + 248   _________

(ii) 67 – (125 – 12) = (67 – 125) – 12   _________

(iii) 64   (245 – 10) = 64   245 – 64   10   _________

(iv) 95   (25   14) = (95   25) + 14   _________

3. Name and verify the law used in each of the following.

(i) 9 7 = 7 9 (ii) 4 (6 3) = (4 6) 3

(iii) 5 (7 – 1) = 5 7 – 5 1 (iv) 2 (8 + 3) = 2 8 + 2 3

4. Verify commutative law under addition for 530 and 750.

5. Verify Associative Law under multiplication for 240, 425 and 35.

6. Verify distributive law of multiplication over addition for 300, 615 and 975.

REVIEW EXERCISE 2

1. Fill in the blanks by using laws under multiplication of whole numbers.

(i) 2 ______ = 3 ______

(ii) 112 528 = ______ ______

(iii) 5 (3 8) = (5 3) ______

(iv) ______ 7 = 7 ______ = 0

(v) 9 (2 + 16) = ______ 2 + ______ 16

(vi) (20 + 4) 1 = ______ 1 + 4 ______

(vii) 17 (8 – 3) = 17 ______ – ______ 3

(viii) (12 – 6) 32 = 12 ______ – 6 ______

2. Fill in the blanks.

(i) 5 = □ 5

(ii) 15 (10 6) = (□ 10)

(iii) 10 (15 + 6) = (10 15) + (□ 6)

(iv) 10 (100 + 60) = (□ 100) + (10 □)

3. Write True or False.

(i) $5 - 3 = 3 - 5$ (ii) $3 \quad 1 + 7 = 3 \quad 8$

(iii) $2 - (0 - 8) = (2 - 0) + 8$ (iv) $9 + (7 + 5) = (9 + 7) + 5$

(v) $4 \quad (35 \quad 2) = (4 \quad 35) \quad 2$

(vi) $24 - (50 - 6) = (24 - 50) - 6$

(vii) $14 \quad 0 = 14$ (viii) $0 \quad 125 = 0$

(ix) $18 \quad 18 = 0$ (x) $75 \quad 75 = 1$

4. Write the predessor and successor of the following numbers:

(i) 671 (ii) 245

5. Choose the correct answer:

(i) The smallest natural number is _______.
(a) 0 (b) 1 (c) 2 (d) 100

(ii) The predecessor of 1 in the set of whole numbers is ________.
(a) 0 (b) 2 (c) 3 (d) none

(iii) The smallest seven digit number is ________.
(a) 1234567 (b) 9999999
(c) 1111111 (d) 1000000

(iv) The greatest six digit number is ____________
(a) 876543 (b) 999999 (c) 111111 (d) 100000

(v) The numbers divisible by 2 are called_______numbers.
(a) prime (b) even (c) odd (d) whole

6. Draw a number line to represent the following whole numbers.

(i) 0, 1, 3, 9 (ii) Whole numbers > 3

(iii) Whole numbers 8

(iv) Whole numbers > 5 but < 10

(v) Whole numbers 1 but 8

7. Find the sum of 4, 2, 3 and 5 on number line.
8. Find two whole numbers whose sum is:
   (i) 9 (ii) 24 (iii) 31
9. Verify commutative laws of addition and multiplication for 190 and 330.
10. Verify Associative laws of addition and multiplication for 20, 30 and 60.
11. Verify distributive laws of multiplication over addition and subtraction for 700, 500 and 100.
12. What is the cost of 640 photocopies at Rs 1.50 per copy?
13. Find the greatest number of 4 digits which is exactly divisible by 44.

SUMMARY

- Natural numbers are used for counting.
- Some of the set of numbers and their notations are:
  - Set of natural numbers, N = {1, 2, 3, ...}
  - Set of whole numbers, W = {0, 1, 2, ...}
- Sum and product of two whole numbers is also a whole number.
- Addition of whole numbers is commutative and associative.
- Multiplication of whole numbers is commutative and associative.
- Zero is additive identity in the set of whole numbers.
- The multiplicative identity in the set of whole numbers is 1.
- Multiplication is distributive over addition and subtraction (with positive difference) in the set of whole numbers.
- Division of two whole numbers is a whole number only if divisor completely divides the number and remainder is zero.

FACTORS AND MULTIPLES

3.1 FACTORS AND MULTIPLES

Define a factor as a number which divides the dividend completely leaving no remainder.

We know that if a dividend is divided by a divisor, we get quotient and remainder.

Let us consider division of 8 by 2 and 3 separately.

Division by 2

2) 8 (4 or 8   2 = 4
  – 8 remainder = 0
  0

Division by 3

3) 8 (2 or 8   3 = 2
  – 6 remainder = 2
  2

Here 2 is a factor of 8 because remainder is zero or 2 divides 8 exactly. 3 is not a factor of 8 because remainder is not zero or 3 does not divides 8 exactly.

> A factor is a number which divides the dividend completely leaving no remainder.

Example: Which of the following are the factors of 10.

(a) 2 (b) 4 (c) 5 (d) 6

Solutions:

(a) 2) 10 (5
  – 10
  0

10 = 2   5

Since Remainder = 0
So, 2 is a factor of 10

(b) 4) 10 (2
  – 8
  2

Since Remainder is not zero
So, 4 is not a factor of 10

(c) 5) 10 (2
  – 10
  0

10 = 5   2

Since Remainder = 0
So, 5 is a factor of 10

(d) 6) 10 (1
  – 6
  4

Since Remainder is not zero
So, 6 is not a factor of 10

So, 2 and 5 are the factors of 10.

Example 2: Find all the factors of:

(a) 12 (b) 20

Solution:

(a) 12	(b) 20
As $12 = 1 \quad 12$ $12 = 2 \quad 6$ $12 = 3 \quad 4$	As $20 = 1 \quad 20$ $20 = 2 \quad 10$ $20 = 4 \quad 5$
So, 1, 2, 3, 4, 6 and 12 can divide 12 exactly.	So, 1, 2, 4, 5, 10 and 20 can divide 20 exactly.
Hence 1, 2, 3, 4, 6 and 12 are factors of 12.	Hence 1, 2, 4, 5, 10 and 20 are factors of 20.

The number itself and 1 are always the factors of the given number.

Define a multiple as a dividend into which a factor can divide

Let us consider the division of 18 by 2.

$$\begin{array}{r|l} 2) & 18 \;(9 \\ & -18 \\ \hline & 0 \end{array}$$

$\boxed{18 = 2 \quad 9}$

Here the divisor 2 is called factor of 18 because remainder is zero.

The dividend 18 is called multiple of 2 because 2 is factor of 18.

Thus

Multiple of a given number is the dividend, for which the given number is its factor.

So, a number has always unlimited number of multiples.

Teacher's Note *Teacher should help the students to differentiate between factors and multiples by giving more examples.*

Example 1: Is 20 a multiple of 3?

Solution:

$$3\overline{)\,20\,}(6$$
$$\underline{-\,18}$$
$$2$$

Since 3 is not a factor of 20
So, 20 is not a multiple of 3.

Example 2: Find all multiples of 2.

Solution: Multiples of 2 can be found as:

2 1 = 2
2 2 = 4
2 3 = 6
2 4 = 8
and so on

So, the multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, ...

Example 3: Find the multiples of 3 less than 15.

Solution: The multiples of 3 are 3, 6, 9, 12, 15, 18, ...

So, the multiples less than 15 are 3, 6, 9, 12

Example 4: Find the multiples of 4 which are between 9 and 23.

Solution: The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, ...

So, the multiples between 9 and 23 are 12, 16, 20.

Define even numbers as the numbers which are multiples of 2

Any number which is multiple of 2, is called an even number.
As 2, 4, 6, 8, 10, 12, ... are multiples of 2.
So, 2, 4, 6, 8, 10, 12, ... are even numbers.

In other words, an even number is a number which is exactly divisible by 2.

Rule: A number having 0, 2, 4, 6, or 8 at its ones place is an even number.

Example: Which of the following numbers are even:

(a) 257 (b) 7208 (c) 11114 (d) 15683

Solution:

(a) 257 is not even because digit at ones place is 7.

(b) 7208 is even because digit at ones place is 8.

(c) 11114 is even because digit at ones place is 4.

(d) 15683 is not even because digit at ones place is 3.

Define odd numbers as the numbers which are not multiples of 2

A number which is not multiple of 2 is called an odd number. As 1, 3, 5, 7, 9, 11, 13, ... are not multiples of 2.

So, 1, 3, 5, 7, 9, 11, 13, 15, ... are odd numbers.

In other words

An odd number is a number which is not exactly divisible by 2.

Rule: A number having 1, 3, 5, 7 or 9 at its ones place is an odd number.

Example: Identify the odd or even numbers as the multiple of 2 and explain.

(a) 3145 (b) 6784 (c) 9210 (d) 2461

Solution:

	Number	Type	Explanation
(a)	3145	odd	As digit at ones place is 5, which is not even number. Hence, 3145 is odd number.
(b)	6784	even	As digit at ones place is 4, which is even number. So, 6784 is an even number.
(c)	9210	even	As digit at ones place is 0, which is even number. So, 9210 is even number.
(d)	2461	odd	As digit at ones place is 1,which is not even number. So, 2461 is odd number.

Define prime numbers as numbers which have only two factors (i.e., 1 and itself)

A natural number which has only two distinct factors 1 and number itself is called a prime number.

As 2, 3, 5, 7, 11, 13, 17, 19, ... have only two factors.
So, they are prime numbers.

Example: Which of the following numbers is a prime number?

(a) 23 (b) 8

Solution:

(a) 23 has two factors 1 and 23. So, 23 is prime number.

(b) 8 has four factors 1, 2, 4, 8. So, 8 is not a prime number.

Define composite numbers as numbers which have more than two factors

A natural number which has more than two factors is called a composite number.

As 4, 6, 8, 9, 10, 12, 14, 15, ... have more than two factors.
So, they are composite numbers.

Example: Identify the composite or prime numbers with reasons.

(a) 8 (b) 17 (c) 9 (d) 5

Solution:

	Number	Type	Reasons
(a)	8	Composite number	As 1, 2, 4, 8 are the factors of 8, which are more than two. Hence it is composite number.
(b)	17	Prime number	As 1, 17 are the only two distinct factors so, 17 is prime number.
(c)	9	Composite number	As 1, 3, 9 are the factors of 9, which are more than two. Hence it is composite number.
(d)	5	Prime number	As 1, 5 are the only two distinct factors of 5 so, 5 is prime number.

Know that 1 is neither prime nor composite as it has only one factor which is 1 itself

According to the number of factors, there are three types of natural numbers as given below:

Natural Number	Type	Factors
1	Neither prime nor composite	1
2	Prime number	1, 2
3	Prime number	1, 3
4	Composite number	1, 2, 4
5	Prime number	1, 5
6	Composite number	1, 2, 3, 6

So 2, 3, 5, 7, 11, 13, ... are prime numbers.

4, 6, 8, 9, 10, 12, 14, 15, ... are composite numbers and 1 is neither prime nor composite number because it has only one factor which is 1 itself.

Example: Identify prime, composite and the number which is neither prime nor composite from the following:

5, 8, 16, 1, 19

Solution:

Prime numbers are 5 and 19
Composite numbers are 8 and 16
The number which is neither prime nor composite is 1

Know that 1 is a factor of every number.

If we divide any number by 1, the remainder will always be zero. as it is clear from the following examples:

(1) 1 is factor of 5
(2) 1 is factor of 8
(3) 1 is factor of 104

So, 1 is factor of every number because it produces 0 as remainder.

Example: Find factors of 2, 5 and 6 and which number is factor of each of 2, 5 and 6.

Solution:

Factors of 2 are 1, 2
Factors of 5 are 1, 5
Factors of 6 are 1, 2, 3, 6

Here '1' is the only number which is factor of each of 2, 5 and 6.

Know that 2 is the only even prime number whereas all other prime numbers are odd

We know that even natural numbers are 2, 4, 6, 8, 10, ...
From these numbers 2 has only two factors which are 1 and 2.

Whereas all other even numbers have more than two factors.

Hence 2 is the only even number which is also prime.

We know that all prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, ...
From these prime numbers 2 is only even prime number and all other are odd.

EXERCISE 3.1

1. Which of the following are the factors of 20.

(a) 2 (b) 8 (c) 5 (d) 3

2. Write all the factors of each of the following.

(a) 15 (b) 30 (c) 50 (d) 125 (e) 150

3. Which of the following are multiples of 6.

(a) 10 (b) 18 (c) 96 (d) 200

4. Write first seven multiples of:

(a) 4 (b) 7 (c) 12 (d) 15 (e) 20

5. **Write prime numbers between:**

   (a) 6 and 15 (b) 20 and 40 (c) 60 and 95

6. **Write an even prime number.**

7. **Which of the following is even or odd?**

   (a) 10 (b) 47 (c) 75 (d) 91 (e) 100 (f) 117

8. Write all the even numbers between 21 and 51.

9. Write all the odd numbers between 10 and 40.

10. Write all the composite numbers less than 40.

11. Write all the composite numbers between 71 and 101.

12. Write all the prime numbers between 20 and 60.

13. Why 1 is neither prime nor composite?

3.2 TESTS FOR DIVISIBILITY

Test by inspection whether the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15 and 25 can divide a given number.

There are rules by which we can check whether a given number is exactly divisible by 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15 and 25. These rules are called tests for divisibility.

Test of *Divisibility by 2* :

A number is exactly divisible by 2 if the digit at units place of the number is 0, 2, 4, 6 or 8.

Example: 50, 62, 724, 9126, 3338 are exactly divisible by 2.

Test of *Divisibility by 3* :

A number is exactly divisible by 3 if the sum of the digits of the number is divisible by 3.

Example: 24510 is exactly divisible by 3 because $2 + 4 + 5 + 1 + 0 = 12$ and 12 is divisible by 3

Test of *Divisibility by 4* :

A number is divisible by 4 if the number formed by the last two digits of the number is divisible by 4 or the two digits are zeros.

Example: 1002<u>16</u> is divisible by 4 because 16 is divisible by 4.

Test of *Divisibility by 5* :

A number is divisible by 5 if digit at the units place of the number is 0 or 5.

Example: 5210 and 4115 are exactly divisible by 5.

Test of *Divisibility by 6* :

A number is divisible by 6 if the number is exactly divisible by both 2 and 3.

Example: 2142 is divisible by 6 because it is exactly divisible by both 2 and 3.

Test of *Divisibility by 8* :

A number is exactly divisible by 8 if the number formed by the last three digits is exactly divisible by 8 or the last three digits are zero.

Example: 213<u>832</u> is divisible by 8 because 832 is exactly divisible by 8.

Test of *Divisibility by 9* :

A number is exactly divisible by 9 if the sum of the digits of the number is exactly divisible by 9.

Example:

9241011 is exactly divisible by 9 because 9 + 2 + 4 + 1 + 0 + 1 + 1 = 18 and 18 is exactly divisible by 9.

Test of *Divisibility by 10* :

A number is exactly divisible by 10 if the digit at units place of the number is zero.

Example: 25670 is exactly divisible by 10.

Test of *Divisibility by 11* :

A number is exactly divisible by 11 if the difference of the sum of digits at odd places from the sum of its digits at even places is either 0 or exactly divisible by 11.

Example:

7546 is exactly divisible by 11 because (7 + 4) – (5 + 6) = 11 – 11 = 0 and 907665 is divisible by 11 because (9 + 7 + 6) – (0 + 6 + 5) = 22 – 11 = 11 which is exactly divisible by 11.

Test of *Divisibility by 12* :

A number is exactly divisible by 12 if the number is exactly divisible by both 3 and 4.

Example: 234084 is exactly divisible by 12 because it is exactly divisible by 3 and 4 (As 2 + 3 + 4 + 0 + 8 + 4 = 21 is divisible by 3 and 84 is exactly divisible by 4).

Test of *Divisibility by 15* :

A number is exactly divisible by 15 if the number is exactly divisible by both 3 and 5

Example: 26130 is divisible by 15 because it is divisible by both 3 and 5.

Testof *Divisibility by 25* :

A number is exactly divisible by 25 if the number formed by last two digits is exactly divisible by 25 or the last two digits are zeros.

Example: 2341625 is divisible by 25 because 25 is divisible by 25.

Example: Test whether:

(i) 2412006 is exactly divisible by 3

(ii) 6123816 is exactly divisible by 8

(iii) 61710 is exactly divisible by 6

(iv) 36578146 is exactly divisible by 11

(v) 43210284 is exactly divisible by 12

(vi) 32412075 is exactly divisible by 15

Solution:

(i) $2 + 4 + 1 + 2 + 0 + 0 + 6 = 15$. Since 15 is divisible by 3.
So, 2412006 is divisible by 3.

(ii) $6123\underline{816}$ is divisible by 8. Since 816 is divisible by 8.

(iii) $6 + 1 + 7 + 1 + 0 = 15$. Since 15 is divisible by 3 and digit at units place is 0, so it is divisible by 2.
Hence 61710 is divisible by 6.

(iv) Here $(3 + 5 + 8 + 4) – (6 + 7 + 1 + 6) = 20 – 20 = 0$
So, 36578146 is exactly divisible by 11

(v) As $4 + 3 + 2 + 1 + 0 + 2 + 8 + 4 = 24$ is exactly divisible by 3 and 84 is also exactly divisible by 4
So, 43210284 is exactly divisible by 3 and 4
Hence it is exactly divisible by 12

(vi) As $3 + 2 + 4 + 1 + 2 + 0 + 7 + 5 = 24$ is exactly divisible by 3 and there is 5 at ones place
So, 32412075 is exactly divisible by 3 and 5
Hence it is exactly divisible by 15

EXERCISE 3.2

1. Test whether.

(i) 251061 is exactly divisible by 3
(ii) 92348 is exactly divisible by 4
(iii) 49230 is exactly divisible by 9
(iv) 24150 is exactly divisible by 8
(v) 10005 is exactly divisible by 2
(vi) 241566 is exactly divisible by 3
(vii) 34268 is exactly divisible by 2
(viii) 241361 is exactly divisible by 4
(ix) 5123864 is exactly divisible by 8
(x) 4158720 is exactly divisible by 9
(xi) 210006 is exactly divisible by 2
(xii) 1234562 is exactly divisible by 3

2. Which of the following numbers are exactly divisible by 6?

(i) 2456 (ii) 7121700 (iii) 19206

3. Which of the following numbers are exactly divisible by 5?

(i) 24567 (ii) 230590 (iii) 111165

4. Which of the following numbers are exactly divisible by 10?

(i) 25670 (ii) 123600 (iii) 14675

5. Which of the following numbers are exactly divisible by 11?

(i) 215762 (ii) 52958400 (iii) 94857290

6. Which of the following numbers are exactly divisible by 12?

(i) 245168 (ii) 512100 (iii) 2561742

7. Which of the following numbers are exactly divisible by 15?

(i) 51300 (ii) 523449 (iii) 236790

8. Which of the following numbers are exactly divisible by 25?

(i) 2563400 (ii) 341236 (iii) 4102375

9. Which of the following numbers are exactly divisible by 2, 3, and 6?

(i) 14562 (ii) 1101361 (iii) 518214

10. Which of the following numbers are exactly divisible by 4 and 8?

(i) 513864 (ii) 617116 (iii) 2415848

11. Which of the following numbers are exactly divisible by 2, 5 and 10?

(i) 120340 (ii) 210785 (iii) 412360

3.3 FACTORIZATION

The process of factorizing a number into its factors is called factorization.

> **Rule:** Product of factors of a given number is always equal to the number.

For example: (i) 8 = 2   4
Here 2 and 4 are factors of 8

(ii) 12 = 6   2
Here 6 and 2 are factors of 12

Define prime factorization as the process of factorizing a number into its prime factors

The process of factorizing a number into its prime factors is called prime factorization.

For example: (i) 8 = 2   2   2 (ii) 12 = 2   2   3

Prime factorization can be done by two methods as explained in the following example.

Example 1: Find the prime factorization of 60:

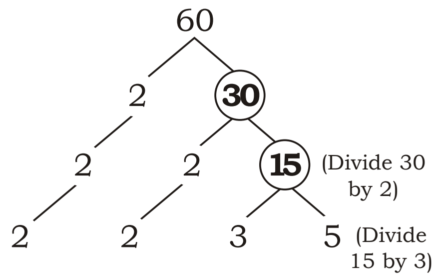
Solution:

Division method

2	60
2	30
3	15
5	5
	1

So, 60 = 2 2 3 5
which is prime factorization of 60

Tree method



So, 60 = 2 2 3 5
which is prime factorization of 60

Example 2: Find the prime factorization of 540:

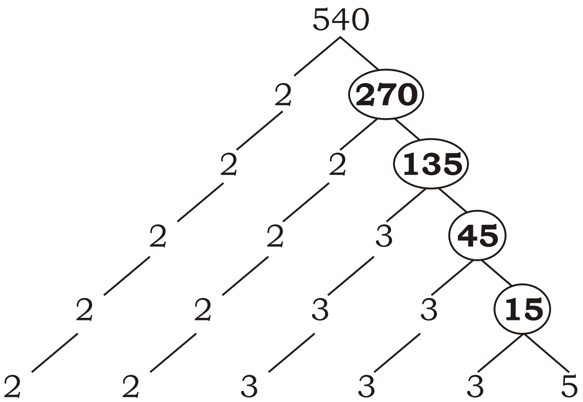
Solution:

Division method

2	540
2	270
3	135
3	45
3	15
5	5
	1

So,
540 = 2 2 3 3 3 5
which is prime factorization of 540.

Tree method



So,
540 = 2 2 3 3 3 5
which is prime factorization of 540.

Teacher's Note *Teacher should help the students to practice both methods of prime factorization.*

Recognize index notation

Consider the factors of 360.

2	360
2	180
2	90
3	45
3	15
5	5
	1

$360 = 2 \quad 2 \quad 2 \quad 3 \quad 3 \quad 5$

Here, factor 2 is 3 times $= 2^3$ (index = 3)
factor 3 is 2 times $= 3^2$ (index = 2)
and factor 5 is 1 time $= 5^1$ (index = 1)

So, prime factorization of 360 can be expressed as

$$\boxed{360 = 2^3 \quad 3^2 \quad 5^1}$$

The prime factorization of 360 in the above form is known as index notation.

Factorize a given number and express its factors in the index notation

Let us factorize a given number and express its factors in the index notation with the help of an example.

Example: Find prime factorization and express it in index notation.

(i) 432 (ii) 1500

Solution:

(i) 432

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Hence,

$432 = 2 \quad 2 \quad 2 \quad 2 \quad 3 \quad 3 \quad 3$
$= 2^4 \quad 3^3$

(ii) 1500

2	1500
2	750
3	375
5	125
5	25
5	5
	1

Hence,

$1500 = 2 \quad 2 \quad 3 \quad 5 \quad 5 \quad 5$
$= 2^2 \quad 3^1 \quad 5^3$

EXERCISE 3.3

1. **Write in index notation.**

   (i) 2 2 2 3 5 3   (ii) 3 5 5 5 7 7

   (iii) 2 3 3 7 7 3 3

2. **Find the prime factorization by division method and express in index notation.**

(i) 24	(ii) 48	(iii) 216	(iv) 250
(v) 468	(vi) 540	(vii) 1024	(viii) 5000

3. **Can you write factorization of a prime number in index notation.**

3.4 HCF (Highest Common Factor)

Define HCF as the greatest number which is a common factor of two or more numbers.

HCF of two or more given numbers is the common factor of the given numbers which is the greatest of all the common factors.

Example: Find HCF of 24 and 40.

Solution:

Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 40 are: 1, 2, 4, 8, 10, 20, 40
Common factors are: 1, 2, 4, 8
The greatest common factor is 8

Hence, HCF = 8

Find HCF of two or more than two numbers by prime factorization and long division method

There are two main methods to find HCF. These methods are explained below:

(i) Prime Factorization Method:

In this method, we use the following rule:

HCF = Product of common factors of two or more numbers.

Example: Find HCF of 48, 36 and 40.

Solution:

2	48
2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

2	40
2	20
2	10
5	5
	1

Here 48 = 2 2 2 2 3
36 = 2 2 3 3
40 = 2 2 2 5

Common factors are 2 and 2

So, HCF = 2 2
= 4

(ii) Long Division Method:

In case of two numbers, first find the HCF by the method as given below:

- Divide greater number by the smaller number.
- Take remainder as divisor and divide the first divisor by it.
- Again remainder will be taken as divisor and 2nd divisor will be treated as dividend.
- Continue this process till remainder becomes zero.
- The last divisor will be HCF.

Note: In case of three numbers, we first find HCF of any two numbers and then continue the process on that HCF and the remaining number till that last divisor, which is HCF of the all three numbers.

Example 1:

Find HCF of 24 and 64 by long division method.

Solution:

$$\begin{array}{l}
24\,\overline{)\ 64\ }(2 \\
\quad -48 \\
\quad \overline{\quad 16\,)\ 24\ }(1 \\
\qquad\quad -16 \\
\text{HCF} \longrightarrow \overline{8\,)\ 16\ }(2 \\
\qquad\qquad\quad -16 \\
\qquad\qquad\quad \overline{\quad 0\quad}
\end{array}$$

So, HCF = 8

Example 2:

Find HCF of 16, 36 and 70.

Solution: First we find HCF of 16 and 36.

$$\begin{array}{l}
16\,\overline{)\ 36\ }(2 \\
\quad -32 \\
\quad \overline{\quad 4\,)\ 16\ }(4 \\
\qquad\quad -16 \\
\qquad\quad \overline{\quad 0\quad}
\end{array}$$

So, HCF of 16 and 36 = 4

Now we find HCF of 4 and 70.

$$\begin{array}{l}
4\,\overline{)\ 70\ }(17 \\
\quad -4 \\
\quad \overline{\quad 30\quad} \\
\quad -28 \\
\text{HCF} \longrightarrow \overline{2\,)\ 4\ }(2 \\
\qquad\qquad -4 \\
\qquad\qquad \overline{\quad 0\quad}
\end{array}$$

Hence, the HCF of 16, 36 and 70 is 2.

EXERCISE 3.4

1. Find the HCF of the following numbers by prime factorization method.

(i) 50, 75 (ii) 98, 196 (iii) 144, 198

(iv) 120, 144, 204 (v) 106, 159, 265

(vi) 12, 48, 36, 24 (vii) 60, 70, 420, 480

2. Find HCF of the following numbers by long division method.

(i) 12, 20 (ii) 81, 117 (iii) 935, 1320

(iv) 252, 576 (v) 2241, 8217, 747

(vi) 30, 120, 90, 210

3. What is the HCF of two prime numbers? (Take any two prime numbers)

3.5 LCM (Least Common Multiple)

Define LCM as the smallest number which is a common multiple of two or more numbers.

LCM of two or more given numbers is the smallest of all their common multiples.

Example: Find the LCM of 3 and 4.

Solution:

Multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ...

Multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

Common multiples of 3 and 4 are: 12, 24, 36, ...
The smallest common multiple = 12
i.e. LCM = 12

Find LCM of two or more number by prime factorization and division method

There are two main methods to find LCM. These are explained below:

(i) Prime Factorization Method:

In this method first we find prime factors of each given number then we find LCM by the following rule.

i.e. LCM = Product of common and non-common factors.

Note: (i) In case of three numbers, we consider a factor as common factor if it is common factor of two numbers or of three numbers.

(ii) Same rule will be applied for more than three numbers.

Example: Find LCM of 12, 18, 24.

Solution:

2	12
2	6
3	3
	1

2	18
3	9
3	3
	1

2	24
2	12
2	6
3	3
	1

Here

12 = 2 2 3

18 = 2 3 3

24 = 2 2 2 3

Here

Common factors are: 2, 2, 3

Non-common factors are: 2, 3

So, LCM = 2 2 3 2 3

= 72

(ii) Division Method:

The main points of this method are given as under.

1. Choose the smallest divisor that can divide at least one of given numbers.
2. Write quotient and the undivided numbers in the second row.
3. Again choose the smallest divisor that can divide at least one of them
4. Continue this process until you get "1" as quotient for each number i.e. (Last row contains all ones)

Then LCM = Product of all divisors.

Example 1:
Find LCM of 3, 4, 5.

Solution:

2	3, 4, 5
2	3, 2, 5
3	3, 1, 5
5	1, 1, 5
	1, 1, 1

Here LCM = 2 2 3 5
= 60

Example 2:
Find LCM of 12, 15, 50.

Solution:

2	12, 15, 50
2	6, 15, 25
3	3, 15, 25
5	1, 5, 25
5	1, 1, 5
	1, 1, 1

Here LCM = 2 2 3 5 5
= 300

Relationship between HCF and LCM

Consider the two number **16** and **24**.

Their product 16 24 = 384

HCF = 8 and LCM = 48

Now multiply HCF and LCM

8 48 = 384

Thus

The product of any two non zero numbers = The product of HCF and LCM

Example: The product of two numbers is 300 and their LCM is 60. What is their HCF?

Solution: The product of two numbers = 300

LCM = 60, HCF = ?

We known that

LCM HCF = Product of two numbers

So, LCM HCF = 300

60 HCF = 300

$$\text{HCF} = \frac{300}{60}$$

$$= 5$$

Thus HCF = 5

EXERCISE 3.5

1. **Using prime factorization, find the LCM of the following numbers.**

   (i) 12, 25 and 40 (ii) 21, 49 and 63

   (iii) 144, 180 and 384 (iv) 108, 135 and 162

   (v) 35, 65 and 75 (vi) 24, 36, 48 and 72

2. **Calculate the LCM of the following numbers by division method.**

   (i) 45 and 55 (ii) 21, 35 and 70

   (iii) 24, 40 and 60 (iv) 72, 108 and 120

3. **The product of two numbers is 360. If their HCF is 6, what is their LCM?**

3.6 Applications of HCF and LCM

Solve real life problems related to HCF and LCM

HCF and LCM are used in daily life as explained in the following examples.

Example 1:

Find the greatest length of a measuring tape which can be used to measure exactly 520 cm and 360 cm.

Solution: The greatest length of the tape is the HCF of 520 cm and 360 cm.

Lets find the required HCF by division method.

```
360) 520 (1
   – 360
   -----------
     160) 360 (2
        – 320
        -----------
           40) 160 (4
             – 160
             -----
```

The HCF of 520 and 360 is 40

Hence, the required length of the tape is 40 cm.

Example 2:

Find the greatest length of wooden scale which can be used to measure 63 cm, 85 cm and 47 cm leaving remainders 3 cm, 5 cm and 7 cm respectively.

Solution:

```
  63, 85, 47
  –3, –5, –7
 ------------
  60, 80, 40
 ------------
```

Here, we have to find HCF of 60, 80 and 40

Teacher's Note *Teacher should give more examples related to daily life to students for their better understanding of applications of HCF and LCM.*

Lets find the HCF by division method.

$$
\begin{array}{l}
60\,\overline{)\;80\;}(1 \\
\quad -60 \\
\hline
\quad 20\,\overline{)\;60\;}(3 \\
\qquad -60 \\
\hline
\qquad\quad 0
\end{array}
$$

As HCF of 60 and 80 is 20
Now we find HCF of 20 and 40

$$
\begin{array}{l}
20\,\overline{)\;40\;}(2 \\
\quad -40 \\
\hline
\qquad 0
\end{array}
$$

As the HCF of 60, 80, 40 is 20
So, the required length of wooden scale is 20 cm.

Example 3: Find the least quantity of sugar which can be exactly measured by 100 gm, 150 gm or 200 gm weights.

Solution: The required least quantity of sugar can be found with the help of LCM.

2	100, 150, 200
2	50, 75, 100
2	25, 75, 50
3	25, 75, 25
5	25, 25, 25
5	5, 5, 5
	1, 1, 1

Here LCM = 2 2 2 3 5 5
= 600

Hence the required quantity of sugar is 600 gm.

Example 4: Find the least number of children which can stand in rows of 10, 30 and 60 children such that each row has 7 children short.

Solution: First of all we find LCM of 10, 30 and 60.

2	10, 30, 60
2	5, 15, 30
3	5, 15, 15
5	5, 5, 5
	1, 1, 1

LCM = 2 2 3 5
= 60

So, the required least number of children = 60 + 7 = 67

Example 5: Three bells ring at intervals of 10, 30 and 40 minutes respectively. At what time will they ring together if they start ringing simultaneously at 9 a.m.

Solution: First of all we find LCM of 10, 30 and 40.

2	10, 30, 40
2	5, 15, 20
2	5, 15, 10
3	5, 15, 5
5	5, 5, 5
	1, 1, 1

Here LCM = 2 2 2 3 5
= 120

They will ring together after 120 minutes together i.e. 120 minutes = 2 hours

Hence the required time = (9 + 2) a.m
= 11 a.m

EXERCISE 3.6

1. Find the greatest length of a wooden scale which can be used to measure 540 cm and 360 cm exactly.
2. Find the greatest number which divides 232 and 305 leaving 7 and 5, respectively as remainder.
3. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.
4. Two tankers contain 600 litres and 570 litres of petrol respectively. Find the maximum capacity of the container which can measure the petrol of either tanker in exact number of times.
5. The length, breadth and height of a room are 8 m, 6 m and 4 m respectively. Determine the longest tape which can measure the three dimensions of the room exactly.
6. Three bells ring at an interval of 5, 10 and 15 minutes respectively. If these started ringing at 8 A.M. Find the time when they will again ring together.

7. Find the least number which when divided by 30, 45 and 60 leaves 9 as remainder.

8. Find the least quantity of milk which can be exactly measured by buckets of capacity 12 litres, 16 litres and 24 litres.

9. The traffic lights at three different road-crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m, after what time will they change again simultaneously?

REVIEW EXERCISE 3

1. Write all the factors of (i) 60 (ii) 250
2. Write first five multiples of (i) 13 (ii) 20
3. Write all prime numbers between 1 and 50.
4. Write all composite numbers between 30 and 60.
5. Which of the following are divisible by 2:
   (i) 31621 (ii) 7008 (iii) 91130 (iv) 5178
6. Which of the following are divisible by 3:
   (i) 51237 (ii) 30001 (iii) 1001001 (iv) 56712
7. Which of the following are divisible by 4:
   (i) 2173 (ii) 41524 (iii) 71611 (iv) 40048
8. Which of the following are divisible by 5:
   (i) 2010 (ii) 31625 (iii) 7128 (iv) 1001
9. Which of the following are divisible by 10:
   (i) 2165 (ii) 71230 (iii) 10000 (iv) 25618
10. Find prime factors of the following by division method and tree method. Also write in index notation.
    (i) 450 (ii) 720

11. Find HCF and LCM by factorization method:

(i) 18, 24 (ii) 12, 15, 40

12. Find HCF and LCM by division method:

(i) 65 and 80 (ii) 26, 65, 169

13. Find the greatest number, that will divide 42, 51 and 67 leaving remainders 2, 1 and 7 respectively.

14. Find the least number which when divided by 20, 30 and 45 leaving 6 as remainder in each case.

SUMMARY

- A factor is a number which divides the dividend exactly.
- A number has limited number of factors.
- Multiple of a given number is the dividend for which the given number is its factor.
- A number has unlimited number of multiples.
- A number which is multiple of 2 is called even number.
- A number which is not multiple of 2 is called an odd number.
- A natural number which has only two distinct factors is called prime number.
- A natural number which has more than two factors is called composite number.
- 1 is neither prime nor composite.
- 1 is factor of every number.
- 2 is the only even number which is also a prime number.

FACTORS AND MULTIPLES

- Divisibility Test a number is divisible by
  - (i) 2 if digit at unit place is 0, 2, 4, 6 or 8.
  - (ii) 3 if the sum of digit is multiple of 3.
  - (iii) 4 if the number formed by last two digit is divisible by 4.
  - (iv) 5 if the digit at unit place is 0 or 5.
  - (v) 6 if the number is divisible by both 2 and 3.
  - (vi) 8 if the number formed by last three digits is divisible by 8.
  - (vii) 9 if sum of digits is divisible by 9.
  - (viii) 10 if digit at units place is zero.
  - (ix) 11 if the difference of sum of digits at odd places from the sum of digits at even places is either 0 or multiple of 11.
  - (x) 12 if the number is divisible by both 3 and 4.
  - (xi) 15 if the number is divisible by both 3 and 5.
  - (xii) 25 if the number formed by last two digits is divisible by 25.
- Product of factors of a given number is always equal to the number.
- HCF = Product of common factors of two or more numbers.
- LCM = Product of common and non-common factors.
- Product of two non-zero number = LCM   HCF

UNIT 4

INTEGERS

4.1 INTEGERS

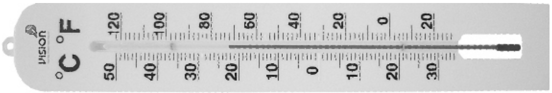
Know that the natural numbers 1, 2, 3, ..., are also called positive integers and the corresponding negative numbers –1, –2, –3, ..., are called negative integers

We are familiar with natural numbers 1, 2, 3, ... These natural numbers are also called positive integers and are used mainly to count and measure the quantities, for example:

- 5 kilograms of apples,
- 15 litres of milk etc.

Similarly there are corresponding negative numbers –1, –2, –3, ... which are called negative integers and are used mainly to measure quantities for example:

- –2 degree centigrade temperature
- –5 metres altitude (5 m below sea level)



The natural numbers 1, 2, 3, ... are also called **positive integers** and the corresponding numbers –1, –2, –3, ... are called **negative integers**.

Know that ‘0’ is an integer which is neither positive nor negative

Besides positive and negative integers, there is an integer which is neither positive nor negative that is ‘0’.

‘0’ is an integer which is neither positive nor negative.

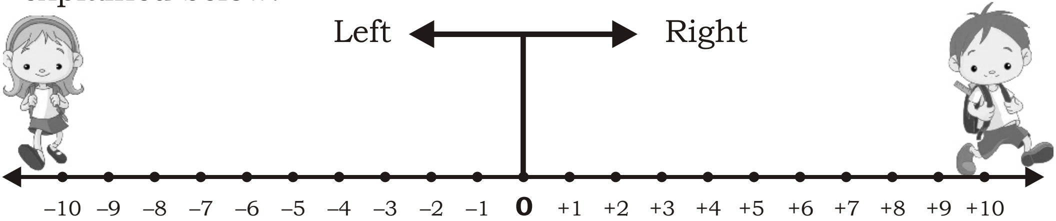
Recognize integers

All the integers are as under:

..., –3, –2, –1, 0, 1, 2, 3, 4, ...

Teacher’s Note *Teacher should give some more examples of directed numbers from daily life.*

These integers are also called **directed numbers** and they are used to represent distance alongwith the direction or position, as explained below:



A boy and girl starting walking from same point 0. One goes on right covers 10 m and other goes on left covers 10 m. The starting point is represented by "0". The distance of 10 m on the right side of starting point is shown by +10 m. The distance of 10 m on the left side of starting point is shown by –10 m.

Example: Fill in the blanks.

(i) If +5 m represents, distance of 5 m towards east then distance of 10 m towards west is **–10 m**.

(ii) If –6 m represents distance of 6 m below sea level then the distance of 8 m above sea level is **+8 m**

Activity **By using integers fill in the blanks.**

(i) If –10 m represents, distance towards west then **+10 m** represents distance towards east.

(ii) If +6 million shows increase in population then __________ shows decrease in population.

(iii) If +20 m shows 20 m distance above sea level then ________ shows 50 m below sea level.

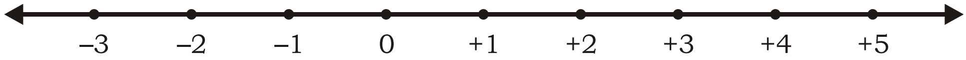
(iv) If –20 rupees represents loss then ______________ represents profit in a business.

(v) If –50 m represents, distance of 50 m towards south then ____________ represents 60 m towards north.

4.2 ORDERING OF INTEGERS

Represent integers on number line

We already know how to represent a natural number on a number line. Let us represent integers on a number line.



This line is called a number line.

Note: The space between any two points or numbers is always same.

Know that on the number line any number lying to the right of zero is positive

Consider a number line.

We observe that all numbers to the right of zero are positive.

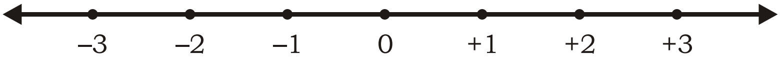
For example: +1 is at right of zero or $+1 > 0$.

+2 is at right of zero which is positive or $+2 > 0$ and so on.

Thus $\boxed{\text{Any number lying to the right of zero is positive.}}$

Know that on the number line any number lying to the left of zero is negative

Let us consider number line.



On this line we observe that all numbers to the left of zero are negative.

For example: –1 is to the left of zero. So –1 is negative or $-1 < 0$.
–2 is to the left of zero so, –2 is negative or $-2 < 0$ and so on.

Hence $\boxed{\text{On a number line, any number to left of zero is negative.}}$

Know that on the number line any number lying to the right of another number is greater

Let us again consider the number line.

We observe that any number to the right of any other number is always greater.

For example +3 is to the right of +1. So, +3 is greater than +1. i.e. +3 > +1

Hence | On a number line, any number to the right of another number is greater. |

Know that on the number line any number lying to the left of another number is smaller

Let us consider the same number line once again.

We observe that any number to the left of any other number is smaller. For example: –2 is to the left of +1 so, –2 is smaller than +1 i.e. – 2 < +1.

Hence | On a number line any number to the left of another number is smaller. |

Example: Decide whether first number is greater or smaller using number line.

(i) +5, –2 (ii) –6, 0 (iii) +4, 0 (iv) –3, –5

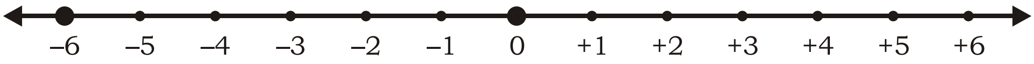
Solution:

(i) Let us represent the numbers on number line.

The number +5 is at the right of –2

Therefore + 5 > –2.

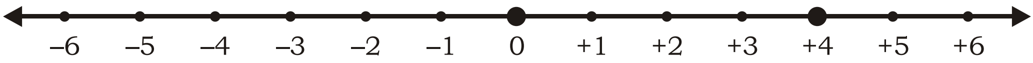
(ii) Let us represent the numbers on number line.



The number –6 is at the left of zero.

Therefore –6 < 0.

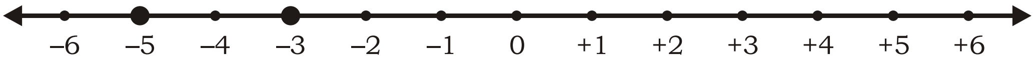
(iii) Let us represent the numbers on number line.



The number +4 is at the right of zero.

Therefore +4 > 0.

(iv) Let us represent the numbers on number line.



The number –3 is at the right of –5.

Therefore –3 > –5.

Activity 1 Fill in the blanks from given options.

(i) –2 is on **left** of +3. (left, right)

(ii) +5 is on ________ of –10. (left, right)

(iii) 0 is on ________ of –6 (left, right)

Activity 2 Fill in the blanks from given options.

(i) +5 **>** –5 (<, >)

(ii) +7 ________ +11 (<, >)

(iii) –9 ________ –4 (<, >)

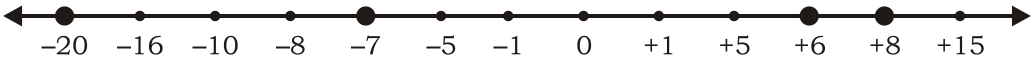
(iv) +2 ________ –20 (<, >)

Know that every positive integer is greater than a negative integer and every negative integer is less than a positive integer

Let us observe the relation between negative and positive numbers from the following examples:

(i) $+6 > -7$ (ii) $+1 > -20$ (iii) $+8 > -1$

Let us represent the number on number line.



It is observed that:

(i) $+6 > -7$ because $+6$ is on right of -7
(ii) $+1 > -20$ because $+1$ is on right of -20
(iii) $+8 > -1$ because $+8$ is on right of -1

From above examples it is clear that all positive numbers are on right of negative numbers.

Hence,

Every positive integer is greater than a negative integer.

From number line it is obvious that all negative numbers are on the left of positive numbers.

Hence,

Every negative integer is less than a positive integer.

For example: (i) $-2 < +6$ (iv) $+7 > -2$
(ii) $-3 < +1$ (v) $+20 > -50$
(iii) $-36 < +2$

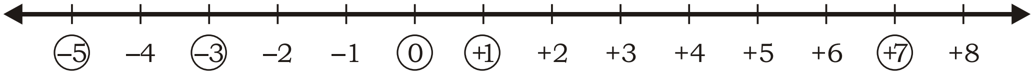
Arrange a given list of integers in ascending and descending order

Given set of integers can be arranged in ascending and descending order with the help of number line.

On a number line the right most number is the greatest and the left most number is the smallest.

Example: Arrange the following numbers –3, 0, 7, 1 –5 in ascending and descending order.

Solution: First of all we indicate the given numbers (–3, 0, 7, 1, –5) on a number line as under.



Here the greatest number is 7 and the smallest number is –5.

So, **Ascending order** (smallest to greatest) is:

–5, –3, 0, 1, 7

Descending order (greatest to smallest) is:

7, 1, 0, –3, –5

4.3 ABSOLUTE OR NUMERICAL VALUE OF AN INTEGER

Define absolute or numerical value of a number as its distance from zero on the number line and is always positive

We know that every integer except zero, on number line represents its distance from zero alongwith its direction as +5 means distance of 5 units on right of zero and –5 means distance of 5 units on the left of zero.

If we ignore direction and just consider the distance then this distance is called absolute or numerical value and we define as:

> Absolute value or numerical value of a number is its distance from zero on the number line and is always positive or zero.

Absolute value of a number is denoted by the symbol "| |"

for example $|-6| = 6$

we read as "Absolute value of –6 as 6"

Similarly

$|+7| = 7$, $|-15| = 15$ and $|0| = 0$

Arrange the absolute or numerical values of the given integers in ascending and descending order

Let us understand the ascending and descending order of absolute or numerical values of given integers with the help of example.

Example:
Write the absolute values of following numbers in ascending and descending orders.
+4, –2, +1, –3, 0, –5, +6

Solution:

Here,

$|+4| = 4$ and $|-3| = 3$

$|-2| = 2$ $| 0 | = 0$

$|+1| = 1$ $|-5| = 5$

$|+6| = 6$

Absolute values of the given integers are:

4, 2, 1, 3, 0, 5, 6

So, ascending order is:

0, 1, 2, 3, 4, 5, 6

And descending order is:

6, 5, 4, 3, 2, 1, 0

EXERCISE 4.1

1. Represent the following integers on a number line.

(i) –2, –1, 0, +1, +2 (ii) –3, –2, –1, 0, +1, +2, +3, +4

(iii) –4, –3, –2, –1, 0, +1, +2 (iv) +5, –5, –4, +3, +1, –2

2. Decide whether first number is greater or smaller.

(i) +15, –6 (ii) –8, 0 (iii) +16, 0

(iv) –2, –8 (v) +7, +9 (vi) –4, –1

3. Fill in the blanks from given options.

(i) –5 is on _______ of + 6 (left, right)
(ii) +6 is on _______ of –7 (left, right)
(iii) 0 is on ________ of –15 (left, right)
(iv) 0 is on ________ of 20 (left, right)

4. Fill in the blanks from given options.

(i) +10 _______ –20 (<, >)
(ii) –16 _______ –4 (<, >)
(iii) +25 _______ –100 (<, >)
(iv) +30 _______ +50 (<, >)
(v) –17 _______ +17 (<, >)
(vi) 0 _________ –5 (<, >)

5. Arrange the given integers in ascending and descending order.

(i) +5, –7, +1, 0, –3, –1 (ii) –3, +4, 0, –1, +2, +5
(iii) 0, –4, +4, –5, +5 (iv) –4, –1, –7, –2, –8

6. Write the absolute values of the following.

(i) –5 (ii) +20 (iii) 0 (iv) –18 (v) +50

7. Arrange the absolute values of given numbers in ascending and descending orders.

(i) –4, +1, –6, +3, 0, +5 (ii) –25, 0, +17, –10, +30, –60
(iii) –20, –10, +11, +7, 0, –4 (iv) +8, –5, +13, –9, –12, +3

8. Write True or False.

(i) Positive integer is always greater than negative integer. ( )
(ii) 0 is a positive integer. ( )
(iii) Every negative integer is less than zero. ( )
(iv) Absolute value of –5 is less than absolute of +4. ( )
(v) –5 is smaller than –10. ( )
(vi) –25 is greater than –100. ( )
(vii) Numerical value of a number can never be negative. ( )

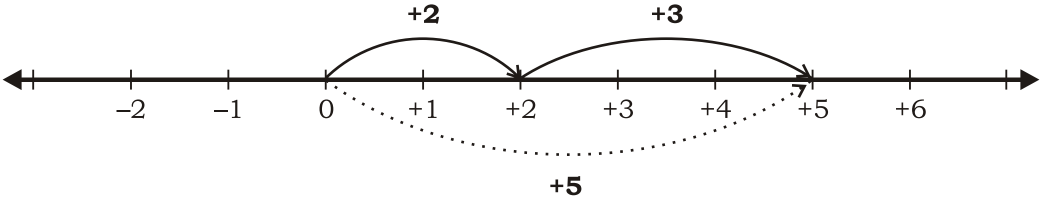
4.4 ADDITION OF INTEGERS

We have already learnt how to find sum of two or more whole numbers on the number line.

Let us revise with the help of following examples.

Example 1: Add +2 and +3 using number line.

Solution:



Here, (+2) + (+3) = +5

Explanation:
Starting from zero, first we move 2 steps on right side of zero reaching at +2.

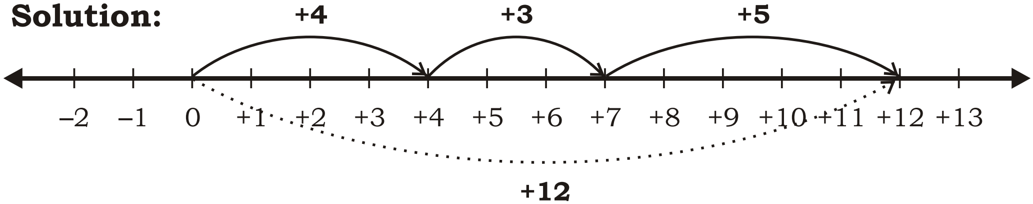
From there move 3 steps on right side reaching at +5

Thus, $\boxed{(+2) + (+3) = (+5)}$

Similarly we can add more than two whole numbers as mentioned in the following example.

Example 2: Find the sum: (+4) + (+3) + (+5) using number line.

Solution:



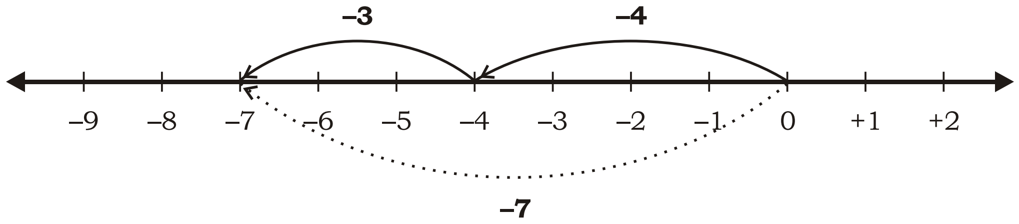
Here, (+4) + (+3) + (+5) = (+12)

Use number line to display sum of two or more given negative integers

Sum of two or more negative integers using number line is explained by the following examples.

Example 1: Find the sum of –4 and –3.

Solution:



Here, (–4) + (–3) = –7

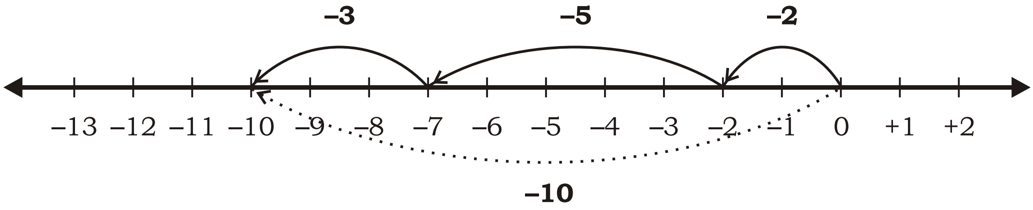
Explanation: Starting from zero, first we move 4 steps from zero on its left reaching at –4

Then from there, we move 3 more steps on left reaching at –7

Thus, $\boxed{(-4) + (-3) = -7}$

Example 2: Find the sum: (–2) + (–5) + (–3).

Solution:



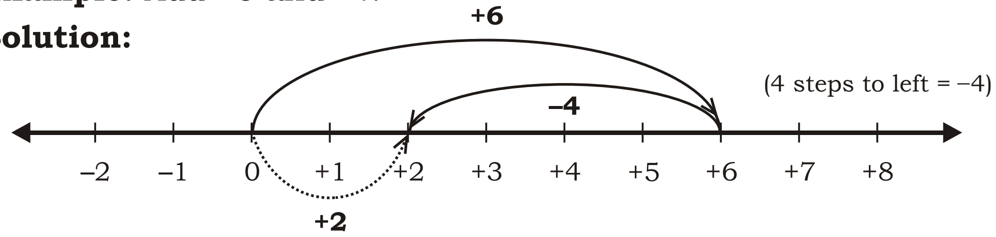
Here, (–2) + (–5) + (–3) = –10

Use number line to display difference of two given positive integers

The sum of a positive and a negative integer is explained with the help of following example.

Example: Add +6 and –4.

Solution:



Here, (+6) + (–4) = +2

Explanation:

First we move 6 steps from zero on its **right** reaching at +6.

Then from there, we move 4 steps on **left** reaching at +2.

Thus, $\boxed{(+6) + (-4) = +2}$

Use number line to display sum of two given integers

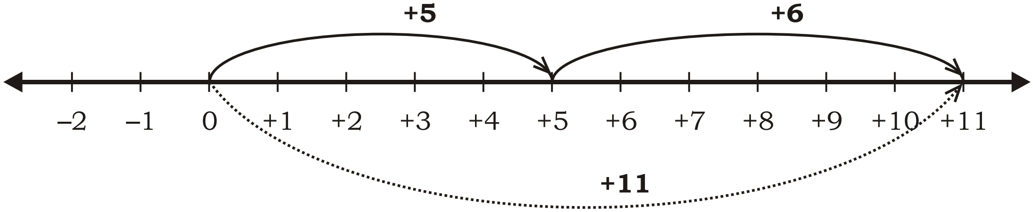
Sum of two given integers is explained with the help of following examples.

Example: Add:

(i) +5 and +6 (ii) –7 and –2

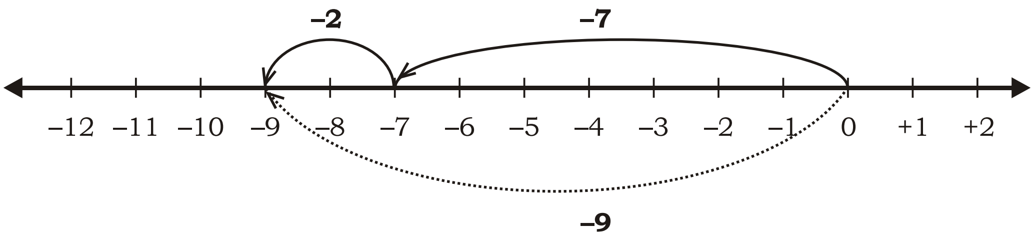
(iii) +8 and –5 (iv) –6 and +2

Solution: (i) +5 and +6



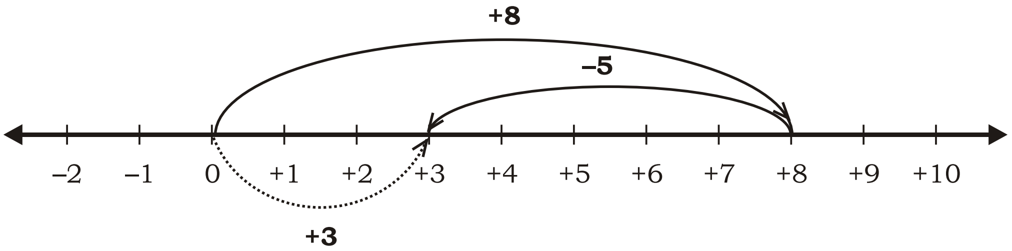
First we move 5 steps on right side of 0 reaching at +5.
Then from there, we move 6 steps on right side reaching at +11.
Here, (+5) + (+6) = +11

Solution: (ii) Add –7 and –2 means to solve (–7) + (–2)



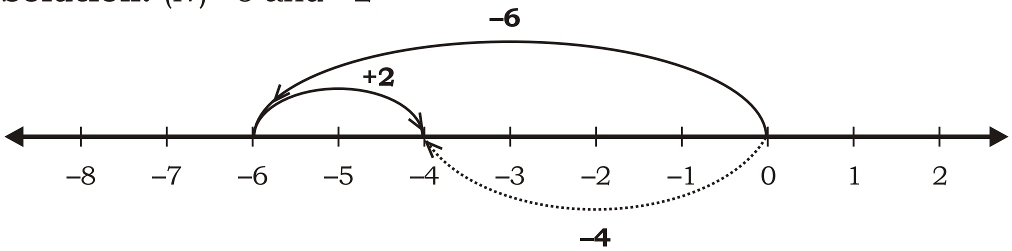
First we move 7 steps from 0 on its left reaching at –7.
Then from there, we move 2 more steps on left reaching at –9.
Here, (–7) + (–2) = –9

Solution: (iii) +8 and –5



First we move 8 steps from zero on its right reaching at (+8).
Then from there we move 5 steps on left; reaching at +3
Here, (+8) + (–5) = +3

Solution: (iv) –6 and +2



Starting from 0, first we move 6 steps from zero on its left, reaching at –6. Then from there we move 2 steps on its right; reaching at point –4.

Here, (–6) + (+2) = –4

Add two integers (with like signs)

Two integers (with like signs) are added using the following three steps.

(i) Take absolute values of given integers.

(ii) Add the absolute values.

(iii) Give the result the common sign.

Examples: Solve: (i) $(+12) + (+13)$ (ii) $(-10) + (-14)$

Solution:

(i) $(+12) + (+13)$

Here, $|+12| = 12$

$|+13| = 13$

Now $(+12) + (+13)$

$= +(12 + 13)$

$= +25$

(ii) $(-10) + (-14)$

Here, $|-10| = 10$

$|-14| = 14$

Now $(-10) + (-14)$

$= -(10 + 14)$

$= -24$

Add two Integers (with unlike signs)

Two integers (with unlike signs) are added using the following three steps.

(i) Take absolute values of given integers.

(ii) Subtract the smaller absolute value from the larger.

(iii) Give the result the sign of integer with the larger absolute value.

Examples: Solve: (i) $(+15) + (-10)$ (ii) $(-20) + (+12)$

Solution:

(i) $(+15) + (-10)$

Here, $|+15| = 15$

$|-10| = 10$

So, $(+15) + (-10)$

$= +(15 - 10)$ (Sign of integer wtih greater absolute value is +)

$= +5$

(ii) $(-20) + (+12)$

Here, $|-20| = 20$

$|+12| = 12$

So, $(-20) + (+12)$

$= -(20 - 12)$ (Sign of integer with greater absolute value is –)

$= -8$

Teacher's Note *Teacher should ask some easy oral questions for good practice of addition of integers.*

EXERCISE 4.2

1. Find the sum using number line.

(i) (+2) + (+7) (ii) (–5) + (–6) (iii) (+7) + (–4)

(iv) (–8) + (+2) (v) (+5) + (– 8) (vi) (+9) + (–9)

2. Find the sum using number line.

(i) (+5) + (+2) + (+3) (ii) (–5) + (–3) + (–4)

(iii) (+4) + (+5) + (+1) (iv) (–2) + (–6) + (–5)

3. Solve the following.

(i) (+10) + (+20) (ii) (–15) + (–25) (iii) (–20) + (–7)

(iv) (+5) + (–14) (v) (–14) + (+8) (vi) (–30) + (+30)

4.5 SUBTRACTION OF INTEGERS

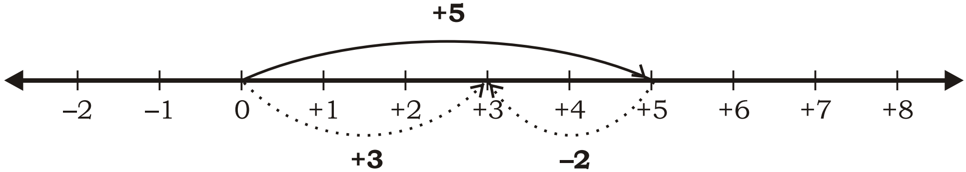
Recognize subtraction as the inverse process of addition

Subtraction is the reverse process of addition because subtracting a subtrahend from a given number means adding the subtrahend with reverse direction in given number.

Let us understand with the following example.

Example 1: Subtract +2 from +5.

Solution: First of all we change the sign of +2 and get –2 then add –2 in +5 on number line.



So, (+5) – (+2)

= (+5) + (–2)

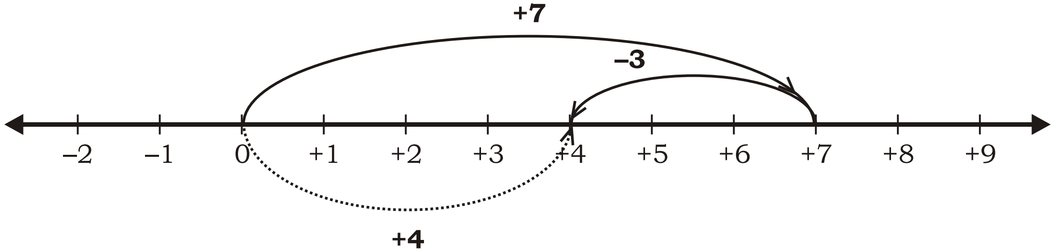
= +3

Hence in order to subtract +2 from +5 we have added +2 with reverse direction (i.e. –2) in +5.

So, we say subtraction is the reverse process of addition.

Example 2: Subtract +3 from +7 using number line.

Solution: To subtract +3 from +7 means;
to solve: (+7) – (+3)



Here, (+7) – (+3) = +4

Subtract one integer from the other by changing the sign of the integer being subtracted and adding according to the rules for addition of integers

It is clear from previous example that if we subtract one integer from other then we have to follow the following rule.

> Subtract one integer from the other by changing the sign of the integer being subtracted and adding according to the rules for addition of integers.

Example 1: Subtract: (i) +2 from +6 (ii) –3 from +8

Solution:

(i) +2 from +6

Subtract +2 from +6 means to find difference between (+6) and (+2)

Difference = (+6) – (+2)
= (+6) + (–2) (changing sign and adding)
= + (6 – 2) ∵ $|+6| = 6$
= + 4 $|-2| = 2$

(ii) –3 from +8

Subtract –3 from +8 means to find difference between (+8) and (–3)

Difference = (+8) – (–3)
= (+8) + (+3) (changing sign and adding)
= + (8 + 3) ∵ $|+8| = 8$
= + 11 $|+3| = 3$

Note: We can solve (+6) + (+2) by adding inversely as under:
(+6) + (+2) = (+6) – (–2) = (+6) + (+2) = +(6+2) = +8

Example 2: Solve: (i) (–8) – (–5) (ii) (+7) – (+10)

Solution:

(i) (–8) – (–5)

= (–8) – (–5)
= (–8) + (+5) (changing sign and adding)
= – (8 – 5) $\because$ |–8| = 8
= – 3 |+5| = 5

(ii) (+7) – (+10)

= (+7) – (+10)
= (+7) + (–10) (changing sign and adding)
= – (10 – 7) $\because$ |–10| = 10
= – 3 |+7| = 7

EXERCISE 4.3

1. Find the difference using number line.

(i) (+6) – (+4) (ii) (–8) – (–3) (iii) (–9) – (–3)
(iv) (–9) – (+5) (v) (+9) – (–2) (vi) (– 7) – (+4)

2. Subtract.

(i) +6 from –10 (ii) –10 from –20 (iii) –8 from –7
(iv) 15 from +20 (v) –6 from + 14 (vi) –9 from –6

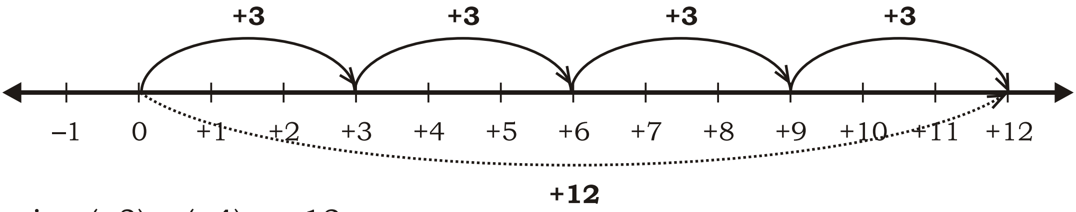
3. Solve the following.

(i) (+25) – (–15) (ii) (–30) – (–25) (iii) (–11) – (+5)
(iv) (+40) – (+30) (v) (–16) – (–18) (vi) (+15) – (–20)
(vii) (–29) – (+17) (viii) (–20) + (–30) (ix) (–27) + (+17)

4.6 MULTIPLICATION OF INTEGERS

We know that multiplication is the simple form of repeated addition for example multiplying +3 by +4 means adding +3 four times i.e. (+3) (+4) = (+3) + (+3) + (+3) + 3

Representation on number line



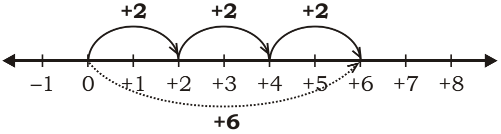
i.e. (+3) (+4) = +12

Recognize that the product of two integers of like signs is a positive integer

Consider the following examples of multiplication using number line.

(i) (+2) (+3)

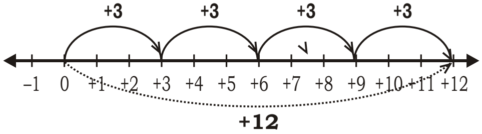
Multiplying +2 with +3 means adding +2 three times in same direction of +2.
i.e.



Here (+2) (+3) = +6

(ii) (–3) (–4)

Multiplying –3 with –4 means adding –3 four times in opposite direction of –3.



(–3) (–4) = (+12)

From these two examples we observe that.

> **Rule 1:**
> The product of two integers of like signs is a positive integer.

For example:

(i) (+3) (+4) = +12 (ii) (–3) (–6) = +18

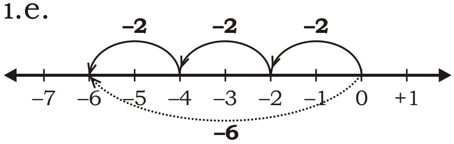
Recognize that the product of two integers of unlike signs is a negative integer

Consider the following examples of multiplication using number line.

(i) (–2)   (+3)

Multiplying –2 with +3 means adding –2 three times in same direction of –2.

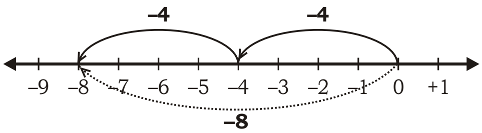
i.e.



So, (–2)   (+3) = –6

(ii) (+4)   (–2)

Multiplying +4 with –2 means adding +4 two times in opposite direction of +4.



So, (+4)   (–2) = –8

From these two examples we observe that.

> **Rule 2:**
> The product of two integers of unlike signs is a negative integer.

For example:

(i) (–2)   (+5) = –10   (ii) (+3)   (–5) = –15

In multiplication of two integers we multiply the absolute values of integers and apply the rules of multiplication of integers.

Examples: Find the product.

(i) (+15)   (–23)   (ii) (–25)   (–14)

Solution:

(i) (+15)   (–23)

= – (15   23) (Applying Rule 2)

= – 345

$$\begin{array}{r} +15 \\ 23 \\ \hline 45 \\ 30\text{x} \\ \hline 345 \\ \hline \end{array}$$

(ii) (–25)   (–14)

= + (25   14) (Applying Rule 1)

= + 350

$$\begin{array}{r} 25 \\ \\ \hline 100 \\ 25\text{x} \\ \hline +350 \\ \hline \end{array}$$

Note: Product of any integer and zero is always zero.

4.7 DIVISION OF INTEGERS

Recognize that division is the inverse process of multiplication

We know that the operation of division is the inverse process of multiplication for example dividing +20 by +5 implies that to find a number which when multiplied by +5 gives +20 as product.

i.e. $\frac{20}{5} = \boxed{} \Rightarrow 20 = \boxed{} \quad 5$

Same number — Guess it?

This number is 4.

because $\frac{20}{5} = \boxed{4} \Rightarrow 20 = \boxed{4} \quad 5$

Recognize that on dividing one integer by another if both the integers have like signs the quotient is positive

Remember that

> Dividing a number with any other number is in fact the product of given number with reciprocal of the divisor.

For example:

(i) $20 \quad (+2) = 20 \quad (+\frac{1}{2})$

$= \frac{20}{2}$

$= 5$

So, $\boxed{20 \quad (+2) = +5}$

(ii) $(-18) \quad (-3) = (-18) \quad (-\frac{1}{3})$

$= +\frac{18}{3}$

$= +6$

So, $\boxed{(-18) \quad (-3) = +6}$

From these two examples we observe that:

> **Rule 1:** For integers of like signs, the quotient is positive.

For example:

(i) $(+10) \quad (+2) = +5$

(ii) $(-20) \quad (-5) = +4$

Recognize that on dividing one integer by another if both the integers have unlike signs the quotient is negative

Let us consider the following examples of division.

(i) $(+8) \quad (-2)$

$= (+8) \quad (-\frac{1}{2})$

$= -\frac{8}{2}$

$= -4$

So, $\boxed{(+8) \quad (-2) = -4}$

(ii) $(-15) \quad (+3)$

$= (-15) \quad (+\frac{1}{3})$

$= -\frac{15}{3}$

$= -5$

So, $\boxed{(-15) \quad (+3) = -5}$

From these two examples we observe that:

> **Rule 2:** For integers of unlike signs, the quotient is negative.

For example:

(i) $(+6) \quad (-2) = -3$

(ii) $(-8) \quad (+4) = -2$

In division of one integer by another, we just perform the division of their absolute values and apply the rules of division.

Examples: Find the quotient.

(i) $(-36) \quad (-4)$

(ii) $(+54) \quad (-2)$

Solution:

(i) $(-36) \quad (-4)$

$= (-36) \quad (-4)$

$= +(36 \quad 4)$ (Applying Rule 1)

$= +9$

$$\begin{array}{r|l} 4\,)\ 36 & 9 \\ -36 & \\ \hline 0 & \end{array}$$

(ii) $(+54) \quad (-2)$

$= (+54) \quad (-2)$

$= -(54 \quad 2)$ (Applying Rule 2)

$= -27$

$$\begin{array}{r|l} 2\,)\ 54 & 27 \\ -4 & \\ \hline 14 & \\ -14 & \\ \hline 0 & \end{array}$$

Teacher's Note *At this level, teacher should only consider division of those integers which are perfectly divisible.*

Know that division of an integer by '0' is not possible

Let us consider division of any integer by 0 for example.

Example: Divide +5 by 0.

Solution:

Dividing +5 by 0 implies to find a number by which when 0 is multiplied the product should be 5

$$0\overline{)\,5\,}(1 \quad , \quad 0\overline{)\,5\,}(2 \quad , \_\_\_\_ \quad , \_\_\_\_ \quad , \_\_\_\_$$
$$\underline{-0} \qquad\qquad \underline{-0}$$
$$5 \qquad\qquad\quad 5$$

This process of division will continue and never finish.

It is impossible as such number does not exist.
Hence division of any integer by 0 is not possible.

EXERCISE 4.4

1. Find the product.

(i) (+15) (–4) (ii) (+20) (+17) (iii) (–16) (–25)

(iv) (–36) (+12) (v) (+35) (–14) (vi) (–43) (–16)

2. Find the quotient.

(i) (+25) (–5) (ii) (–35) (–7) (iii) (+100) (+4)

(iv) (–252) (+4) (v) (–234) (+3) (vi) (–126) (–6)

3. Write true or false.

(i) +5 0 = 0 ( ) (ii) 0 +6 = 0 ( )

(iii) +9 0 = +9 ( ) (iv) +6 0 = 0 ( )

(v) (–10) (–10) = –1 ( ) (vi) (–20) (–1) = +20 ( )

(vii) (+5) (–5) = 1 ( ) (viii) (–18) (–3) = (+6) ( )

REVIEW EXERCISE 4

1. Write all numbers between –5 and + 5.

2. Represent the following integers on a number line.

(i) –5, –2, 0, +3, +5 (ii) –6, –3, –1, +1, +4, +6

3. Write the absolute values of:

(i) –3, +4, –4, +6, 0 (ii) +2, –7, +1, 0, +4, –2

4. Find the sum using number line.

(i) (+6) + (–9) (ii) (–9) + (–6)

(iii) (+9) + (+7) + (–5) (iv) (–1) + (–2) + (3)

5. Find the difference using number line.

(i) (+5) – (+3) (ii) (–7) – (–2) (iii) (+4) – (–3)

6. Solve the following:

(i) (+6) + (–10) (ii) (–10) + (–4)

(iii) (–8) + (+5) (iv) (+20) – (–10)

(v) (–20) – (–15) (vi) (+40) – (+25)

7. Solve the following:

(i) (+10) (–5) (ii) (–6) (–9)

(iii) (+20) (+6) (iv) (–15) (+3)

(v) (+6) 0 (vi) (–10) (–8)

8. Find the quotient.

(i) (+20) (–4) (ii) (–15) (–3)

(iii) (–36) (+6) (iv) (+40) (+10)

SUMMARY

- +1, +2, +3, ... are called positive integers.
- –1, –2, –3, ... are called negative integers.
- 0 is neither positive nor negative.
- Any number to the right of another number is greater.
- Any number to the left of another number is smaller.
- Every positive integer is greater than a negative number.
- Every negative integer is less than a positive integer.
- Absolute value of a number is always non-negative.
- Sum of two positive integers is always positive integer.
- Sum of two negative integers is always negative integer.
- Product of two integers of like sign is always positive integer.
- Product of two integers of unlike signs is always negative integer.
- Product of zero and any other integer is always zero.
- For integers of like signs, the quotient is positive.
- For integers of unlike signs, the quotient is negative.
- Division by zero is not possible.

UNIT 5

SIMPLIFICATIONS

5.1 SIMPLIFICATIONS

In regular routine of our life, we add, multiply, subtract or divide two numbers for different purposes. After each of those cases we get our result as a simplified number. The process of getting a simplified number is known as **simplification**.

Consider this expression.

$$3 + 4 \quad 2$$

We can simplify it by two ways:

(i) If we perform addition of 3 and 4 and then multiply the sum by 2, we get the answer 14.

(ii) If we perform multiplication of 4 and 2 and then add 3 to the product 8, we get the answer 11.

So to get a unique result of our simplification process, we must follow some **rules of preference**; we use brackets to show the preference of operations to be performed.

To know the Kinds of Brackets

We know that for simplification brackets are used to group two or more numbers together with operations.

There are four kinds of brackets.

(i)	———	Vinculum or bar
(ii)	( )	parenthesis or curved brackets or round brackets
(iii)	{ }	braces or curly brackets
(iv)	[ ]	square brackets

SIMPLIFICATIONS

Know the order of preference as, $\overline{\qquad}$, ( ), { } and [ ], to remove (simplify) them from an expression.

In problems involving more than one bracket, the brackets should be removed in the order $\overline{\quad}$, ( ), { } and [ ]. Removing brackets means simplifying expression within the brackets to get the simplified form and then remove the pair of brackets.

We use following rules to remove the brackets.

(i) If there is a plus '+' sign before brackets, the brackets are removed without changing the sign of the number within brackets.

For example, $+ (2 - 5) = + (-3) = -3$

(ii) If there is a minus '–' sign before brackets, the brackets are removed and the sign of the number within brackets is changed.

For example, $- (2 - 5) = - (-3) = 3$

(iii) If there is a number before brackets, the number in the brackets left after simplification is multiplied by this number and the brackets are removed.

For example, $4 (2 - 5) = 4 (-3) = -12$

Also $-5 (3 - 7) = -5 (-4) = + 20$

Example 1: Simplify : $5 + \{16 - 4 \quad 2 \quad 3 - (-6 \quad 2)\}$

Solution: $5 + \{16 - 4 \quad 2 \quad 3 - (-6 \quad 2)\}$

$= 5 + \{16 - 2 \quad 3 - (-3)\}$

$= 5 + \{16 - 6 + 3\}$

$= 5 + (10 + 3)$

$= 5 + 13 = 18$

Example 2: Simplify : $2\,[-2\,\{4\quad 9\quad 8\,(5-\overline{3-4})\}]$

Solution: $2\,[-2\,\{4\quad 9\quad 8\,(5-\overline{3-4})\}]$

$$= 2\,[-2\,\{4\quad 9\quad 8\,(5-(-1)\}]$$

$$= 2\,[-2\,\{4\quad 9\quad 8\,(5+1)\}]$$

$$= 2\,[-2\,\{4\quad 9\quad 8\quad 6\}]$$

$$= 2\,[-2\,[4\quad \frac{9}{8}\quad 6]$$

$$= 2\,[-2\,\{\frac{\overset{1}{\cancel{4}}\quad 9\quad \overset{3}{\cancel{6}}}{\underset{2}{\cancel{8}}}\}]$$

$$= 2\,[-2\,\{9\quad 3\}]$$

$$= 2\,[-2\quad 27]$$

$$= 2\quad -54$$

$$= -108$$

5.2 BODMAS RULE

Recognize BODMAS rule to follow the order in which the operations, to simplify mathematical expressions, are performed.

BODMAS indicates the sequence in which more than one operations are performed in the simplification process of any mathematical problem.

The chain of letters in BODMAS shows meaning of each letter as:

B for "Brackets"
O for "order of operation"
D for "Division"
M for "Multiplication"
A for "Addition"
S for "Subtraction"

Removal of brackets and solving of operation 'of' with operations of DMAS is known as BODMAS rule.

In BODMAS rule, the **of** denotes order of operation.

Then we perform the operations of division, multiplication, addition and subtraction (DMAS) in sequence from left to right. By using BODMAS rule we can simplify any expression involving brackets and arithmetical operations to simplified form.

In short, to follow the BODMAS rule, we first remove **brackets**. Next we perform the operation **'of'**. Then the operation of **division** is performed followed by the operation of **multiplication**. Lastly we perform **addition** followed by the operation of **subtraction**.

After learning about the sequence of operations, we can now apply them to complete the simplification process.

Simplify mathematical expressions involving fractions and decimals grouped with brackets using BODMAS rule

Let us solve some examples using BODMAS rule.

Example 1. Simplify $3\frac{1}{4} \quad \left[\frac{2}{5} + \left\{\left(\frac{7}{8} - \frac{7}{12}\right) + 1\frac{5}{6}\right\} - 2\frac{2}{5}\right]$

Solution: $3\frac{1}{4} \quad \left[\frac{2}{5} + \left\{\left(\frac{7}{8} - \frac{7}{12}\right) + 1\frac{5}{6}\right\} - 2\frac{2}{5}\right]$

$= \frac{13}{4} \quad \left[\frac{2}{5} + \left\{\left(\frac{7}{8} - \frac{7}{12}\right) + \frac{11}{6}\right\} - \frac{12}{5}\right]$ (Converting mixed fractions into improper fractions)

$= \frac{13}{4} \quad \left[\frac{2}{5} + \left\{\left(\frac{21 - 14}{24}\right) + \frac{11}{6}\right\} - \frac{12}{5}\right]$ (Simplifying within parenthesis)

$= \frac{13}{4} \quad \left[\frac{2}{5} + \left\{\frac{7}{24} + \frac{11}{6}\right\} - \frac{12}{5}\right]$ (Removing parenthesis)

$= \frac{13}{4} \quad \left[\frac{2}{5} + \left\{\frac{7 + 44}{24}\right\} - \frac{12}{5}\right]$ (Simplifying within braces)

$$= \frac{13}{4} \quad \left[\frac{2}{5} + \frac{51}{24} - \frac{12}{5}\right]$$ (Removing braces)

$$= \frac{13}{4} \quad \left[\frac{48 + 255 - 288}{120}\right]$$ (Simplifying within square brackets)

$$= \frac{13}{4} \quad \frac{15}{120}$$ (Removing square brackets)

$$= \frac{13}{\cancel{4}_{1}} \quad \frac{\overset{\overset{2}{\cancel{30}}}{\cancel{120}}}{\underset{1}{\cancel{15}}} = 13 \quad 2$$

$$= 26$$

Example 2. Simplify: $1\frac{3}{4} \quad \left[9\frac{5}{8} \quad \left\{\frac{2}{5} + \left(8\frac{2}{5} \quad \overline{4\frac{1}{6} - 3\frac{1}{2}}\right)\right\}\right]$

Solution: $1\frac{3}{4} \quad \left[9\frac{5}{8} \quad \left\{\frac{2}{5} + \left(8\frac{2}{5} \quad \overline{4\frac{1}{6} - 3\frac{1}{2}}\right)\right\}\right]$

$$= \frac{7}{4} \quad \left[\frac{77}{8} \quad \left\{\frac{2}{5} + \left(\frac{42}{5} \quad \overline{\frac{25}{6} - \frac{7}{2}}\right)\right\}\right]$$ (Converting mixed fractions into improper fractions)

$$= \frac{7}{4} \quad \left[\frac{77}{8} \quad \left\{\frac{2}{5} + \left(\frac{42}{5} \quad \overline{\frac{25 - 21}{6}}\right)\right\}\right]$$ (Solving vinculum)

$$= \frac{7}{4} \quad \left[\frac{77}{8} \quad \left\{\frac{2}{5} + \left(\frac{\cancel{42}}{5} \quad \frac{4}{\cancel{6}_{1}}\right)\right\}\right]$$ (Solving within parenthesis)

$$= \frac{7}{4} \quad \left[\frac{77}{8} \quad \left\{\frac{2}{5} + \frac{28}{5}\right\}\right]$$ (Removing parenthesis)

$$= \frac{7}{4} \quad \left[\frac{77}{8} \quad \left\{\frac{\overset{6}{\cancel{30}}}{\underset{1}{\cancel{5}}}\right\}\right]$$ (Solving braces)

$$= \frac{7}{4} \quad \left[\frac{77}{8} \quad \frac{1}{6}\right]$$ (Removing braces)

$$= \frac{7}{4} \quad \frac{77}{48} = \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{4}}} \quad \frac{\overset{12}{\cancel{48}}}{\underset{11}{\cancel{77}}} = \frac{12}{11} = 1\frac{1}{11}$$

Example 3. Simplify: $6.4 - (\overline{3.3 - 1.2} \quad 2.5)$

Solution: $6.4 - (\overline{3.3 - 1.2} \quad 2.5)$ (Solving viculum)

$= 6.4 - (2.1 \quad 2.5)$ (Solving within parenthesis)

$= 6.4 - 5.25$ (Removing parenthesis)

$= 1.15$

Example 4. Simplify: $2.9 \quad \overline{1.8 - 1.3} \quad 3.75$

Solution: $2.9 \quad \overline{1.8 - 1.3} \quad 3.75$

$= 2.9 \quad 0.5 \quad 3.75$

$= 5.4375$

Example 5. Simplify: $3.05 + \left[2.34 \quad \left\{2.4 - \left(1.8 \quad \overline{0.3 + 0.6}\right)\right\}\right]$

Solution: $3.05 + \left[2.34 \quad \left\{2.4 - \left(1.8 \quad \overline{0.3 + 0.6}\right)\right\}\right]$

$= 3.05 + \left[2.34 \quad \left\{2.4 - \left(1.8 \quad 0.9\right)\right\}\right]$

$= 3.05 + \left[2.34 \quad \left\{2.4 - 1.62\right\}\right]$

$= 3.05 + \left[2.34 \quad 0.78\right]$

$= 3.05 + 3$

$= 6.05$

EXERCISE 5.1

Simplify the following:

1. $\frac{1}{5} - \left\{ \frac{2}{5} \quad \frac{4}{15} \quad \frac{1}{2} - \left(\frac{1}{2} + \frac{2}{5} \right) \right\}$

2. $\frac{3}{4} + \left(\frac{4}{3} - \overline{\frac{1}{2} + \frac{2}{3}} + \frac{4}{5} \right)$

3. $\frac{7}{8} + \left\{ \frac{5}{7} + \left(\frac{1}{2} + \frac{1}{3} \right) \quad \frac{5}{6} \right\}$

4. $\frac{5}{2} \quad \left(\frac{5}{6} - \frac{2}{3} \right) \quad \frac{2}{5} \quad \left(\frac{3}{5} + \frac{6}{25} \right)$

5. $\left(\overline{\frac{2}{3} - \frac{3}{4}} \quad \frac{1}{8} \right) \quad \left(\frac{2}{3} - \overline{\frac{3}{4} + \frac{5}{8}} \right)$

6. $1\frac{1}{2} + \left[\frac{1}{3} \quad \left\{ \frac{2}{5} \quad \left(\frac{4}{5} - \overline{\frac{1}{10} + \frac{3}{5}} \right) \right\} \right]$

7. $\left[\frac{1}{2} + \left\{ \left(\frac{3}{4} \quad \frac{9}{16} \right) \quad 1\frac{1}{2} \right\} - 2\frac{1}{4} \right]$

8. $3\frac{1}{2} + \left\{ \left(10\frac{2}{5} - 5\frac{1}{3} \right) \quad 3\frac{2}{3} \right\} - 1\frac{1}{5}$

9. $\left[\left(2\frac{3}{4} + 1\frac{2}{6} \right) \quad \left(1\frac{1}{5} \quad \frac{2}{5} \right) \right]$

10. $\left\{ \left(\frac{1}{2} - \frac{1}{3} \right) + \left(1\frac{2}{3} \quad \frac{1}{9} \right) \right\} - \left\{ \left(\frac{5}{3} - \frac{1}{6} \right) \quad \frac{9}{6} \right\}$

11. $\left[2\frac{1}{4} \quad \left\{ 3\frac{1}{3} - \frac{1}{2} + \left(\frac{5}{3} - \frac{1}{6} \right) \right\} \right] + \frac{5}{6}$

12. $\left(3\frac{1}{2} \quad 5\frac{1}{3} - 10\frac{2}{5}\right) \quad \left(3\frac{1}{8} - 6\frac{1}{2} \quad 2\frac{1}{6}\right)$

13. $\left(2\frac{1}{5} \quad 3\frac{2}{25}\right) \quad 4\frac{2}{5} \quad \left(3 - \frac{1}{5}\right)$

14. $4.02 \quad \left\{1.2 \quad \overline{2.11 + 3.89}\right\}$

15. $4.093 + \left\{5.2 \quad \overline{1.5 + 2.4}\right\}$

16. $5.7 \quad \left\{6.7 + \left(1.3 \quad 7.2 \quad \overline{1.5 \quad 2.4}\right)\right\}$

17. $0.3 \quad 2.6 + \left\{3.7 - \left(1.2 \quad \overline{4.5 \quad 1.5}\right)\right\}$

18. $\left\{\left(3.41 - \overline{1.05 + 1.5}\right) \quad 3.5\right\} \quad 1.25$

19. $30.45 + \left[6.35 \quad \left\{8.95 - \left(3.5 \quad 0.7 \quad 0.01\right)\right\}\right]$

20. $2.297 + \left[0.2 \quad \left\{9 \quad 0.3\right\} - 0.09\right]$

21. $\left\{\left(2.5 - 2\right) + 1.5\right\} \quad 0.5 - 0.05$

22. $2.04 + \left[2.82 \quad \left\{2.4 - \left(1.3 \quad 0.3 + 0.6\right)\right\}\right]$

23. $\left[\left\{7.45 - 1.35 + 1.5\right\} \quad 1.25 \quad 2.5\right]$

Solve real life problems involving fractions and decimals

Fractions and decimals are used in daily life as explained in the following examples.

Example 1.

The maximum marks of Urdu test are 40 and passing marks are $\frac{1}{4}$ of maximum marks. How many marks at least should Haris obtain to pass the test?

Solution: Maximum marks $= 40$

Minimum passing marks $= \frac{1}{4}$ of 40

$= \frac{1}{4} \quad 40$

$= 10$

Therefore, Haris should obtain at least **10 marks** to pass the test.

Example 2.

Fouzia gave $\frac{1}{4}$ of her cake to her sister, another $\frac{2}{3}$ to her two brothers and kept the rest of the cake for herself. What fraction of the cake was left with her.

Solution: Fouzia has 1 cake

She gave $\frac{1}{4}$ and $\frac{2}{3}$ of the cake to her sister and two brothers respectively.

Total cake given $= \frac{1}{4} + \frac{2}{3}$

Cake left with Fouzia $= 1 - \left(\frac{1}{4} + \frac{2}{3}\right) = 1 - \left(\frac{3+8}{12}\right)$

$= 1 - \frac{11}{12} = \frac{12-11}{12} = \frac{1}{12}$

Hence $\frac{1}{12}$ **th** part of the cake was left with Fouzia.

Example 3. There are 5380 suits in a shop. $\frac{3}{5}$ of these are in red colour and $\frac{1}{10}$ these are in green colour and the remaining suits are in multi colours. Find the number of suits in multi colours.

Solution:

The number of suits in red colour $= \frac{3}{5} \quad \overset{1076}{\cancel{5350}} = 3228$

The number of suits in green colour $= \frac{1}{10} \quad 5350 = 538$

The number of suits in multi colours $= 5380 - (3228 + 538)$

$= 5380 - 3766$

$= 1614$

Hence the number of suits in multi colours are 1614.

Example 4.

Najeeb's father bought the following items from market:

(i) 30 eggs at the rate of Rs 10.50 per egg.
(ii) 20 bars of washing soap at the rate of Rs 52.75 per bar.
(iii) 25 pieces of toilet soap at the rate of Rs 45.50 per piece.
(iv) 15 packets washing powder at the rate of Rs 120.80 per packet.

How much money is paid by him?

Solution:

Cost of eggs	= 30	10.50	=	Rs	315.00
Cost of washing soap	= 20	52.75	=	Rs	1055.00
Cost of toilet soap	= 25	45.80	=	Rs	1145.00
Cost of washing powder	= 15	120.80	=	+ Rs	1812.00
		Total		**Rs**	**4327.00**

Total amount spent = 315 + 1055 + 1145 + 1812 = 4327
Hence Najeeb's father paid Rs 4327.

EXERCISE 5.2

1. There are 640 students in total in a school. $\frac{3}{10}$ of the students use own vehicles, $\frac{5}{8}$ use public transport and the remaining students come on foot. How many students of the school come on foot?

2. From a box of 140 oranges, $\frac{3}{7}$ of the oranges were sold and $\frac{1}{14}$ got spoilt. What is the number of the oranges left in the box?

3. A crowd of people was gathered for a cricket match. $\frac{1}{8}$ of crowd had come by train, $\frac{2}{7}$ by bus, $\frac{1}{4}$ by own vehicles and the remaining people came on foot. Find the part of people who came on foot.

4. Maira had Rs 58,000. She gave $\frac{16}{29}$ of it to her mother and $\frac{1}{4}$ of the remainder in Zakat fund. Find the amount left with her?

5. Aamir planted 425 plants of tomatoes. After some days 25 plants died. What fraction of plants was survived?

6. Aijaz gave $\frac{2}{5}$ of his money to his daughter, $\frac{1}{7}$ to his son and the remainder to his wife. What fraction of the amount was given to his wife.

7. Adeel had 26.50 metres of cloth. He used 5.80 metres of cloth for his suit. He gives 11.50 metres of cloth to his brother. How much cloth is left with Adeel in fraction?

8. Anum's family purchased 5 packets of cooking oil at the rate of Rs 121.80 per packet, 3 bags of rice at the rate of Rs 235.50 per bag and 40 kg flour at the rate of Rs 42.70 per kg. Find the total amount paid by her.

9. Umair spent 0.06 of his pocket money on Monday, 0.25 on Tuesday and 0.50 on Wednesday. Remaining money spent on Thursday. What fraction of his pocket money he had on Thursday?

10. Sobia purchased the following items for her home.

(i) Two electric bulbs at the rate of Rs 80.95 each.

(ii) Four small stools at the rate of Rs 105.50 each.

(iii) Three tables at the rate of Rs 530.95 each.

(iv) Six chairs at the rate of Rs 458.30 each.

Find the total amount paid by her.

REVIEW EXERCISE 5

1. Simplify: (i) $1\frac{2}{5} \quad \left\{2\frac{3}{5} - \left(\overline{1\frac{1}{4} - \frac{2}{3}} \quad 1\frac{1}{7}\right)\right\}$

(ii) $2\frac{1}{4} \quad \left[\frac{2}{3} \quad \left\{\frac{1}{3} - \left(\overline{\frac{1}{2} - \frac{1}{5}} \quad 1\frac{1}{2}\right)\right\}\right]$

(iii) $4.3 \quad [3.9 \quad 0.8 - 1.3 + \{4.8 \quad 1.2 - (\overline{1.3 - 1.2} \quad 2.5)\}]$

(iv) $2.9 - [1.05 \quad 1.17 + (1.04 - \overline{2.5 + 1.9})\}]$

2. Solve the following:

(i) Bilal sacrifices a goat and get $13\frac{5}{7}$ kg of meat. He gave one third of meat in charity. How much meat is left with him?

(ii) Azhar solved 15 questions out of 25 questions of mathematics. Find the fractions of solved and unsolved questions.

3. Fareena read $\frac{2}{5}$ of the book in one day. Remaining of the book was read on next day. What part of the book was read on next day? If book contains 150 pages, then find the number of pages she read on first day and the next day.

4. A contractor purchased tiles of different qualities for a building repair as under:

(i) Local Tiles 240.34 square metres at the rate of Rs 327.34 per square metre.

(ii) Imported Tiles 105.28 square metres at the rate of Rs 539.99 per square metre.

Find the total amount paid by the contractor.

5. Write true (T) or false (F) for the following:

(i) The symbol for vinculum is [ ]

(ii) In simplification, if there is a negative sign before a bracket, then the sign inside the bracket remains unchanged, when removing the brackets.

(iii) The expression $35 + 2 \quad 8$ is simplified by first multiplying 2 by 8 and then adding 35 to it.

6. Fill in the blanks:

(i) In BODMAS, M stands for ________.

(ii) The bracket { } is called ___________.

(iii) Brackets are simplified in the order as: vinculum, ____________, ____________ and ___________.

SUMMARY

- BODMAS indicates the sequence in which more than one operations are performed in the simplification process of any mathematical expression.

  Where, **B** stands for **bracket**, **O** for the order of operation **of**, **D** for **division**, **M** for **multiplication**, **A** for **addition** and **S** for **subtraction**.

- While simplifying mathematical expressions involving brackets should be removed in the order first ——, second ( ) then { } and [ ].

- The operations, **division**, **multiplication**, **addition**, and **subtraction** (DMAS) are performed from left to right in order sequence.

- If there is a **plus sign before brackets**, the **brackets** are removed **without changing** the sign of the number within the brackets.

- If there is a **minus sign** before a **pair of brackets**, the sign of the number within the **second brackets is changed** and brackets are removed.

- If there is a number before brackets, the number in the brackets left after simplification is multiplied by this number and the brackets are removed.

UNIT 6 RATIO AND PROPORTION

6.1 RATIO

The word ratio is commonly used in our daily life while distributing the quantities and money and also in household activities. Comparison of two quantities of the same kind is called ratio. This comparison is always in the form of numbers. A ratio is represented by a fraction or the symbol " : ". So, if the cost of two toffees is Rs 2 and Rs 3 then their ratio will be $\frac{2}{3}$ or 2:3, read as the "ratio of 2 is to 3". Similarly a:b read as the "ratio of a is to b". We can also express ratio for more than two quantities as 3:5:4 or a:b:c.

Define ratio as a relation which one quantity bears to another quantity of the same kind with regard to their magnitudes

Let us learn more about ratio from the following Activity.

Activity **Weights of Sahar and Shabana are 45 kg and 15 kg respectively. To relate their weights, consider the following statements.**

- Sahar is 30 kg heavier than Shabana.
- Sahar's weight is 3 times of Shabana's weight.
- Shabana is 30 kg lighter than Sahar.
- Shabana's weight is $\frac{1}{3}$ of Sahar's weight.

Now answer the following:

- Which of the above statements are showing comparison by addition and subtraction?
- Which statements are showing comparison by multiplication and division?

Teacher's Note *Teacher should help the students to enhance the concept of ratio by giving more examples from daily life.*

In the above activity the ratio of magnitudes of Sahar to Shabana's weight is 45:15, where as of Shabana to Sahar's weight is 15:45 which are different ratios.

The order in which a ratio is written is important.

How can we find the ratio between 3 km and 21 m?

We use ratio to compare quantities of the same kind and same units. Such as magnitudes of two lengths, two masses, two volumes etc. Consider the following.

The ratio between quantities of 3 bats and two dolls cannot be written as 3:2 because bat and doll are not of the same kind but we can establish a ratio between 3 bats and 2 bats as 3:2 and 2 dolls and 3 dolls as 2:3

Know that a ratio has no units

When ratio between two quantities is established, no unit is written with the ratio, as ratio represents a number.

For Example:

If Arif has 9 rupees and Saleem has 20 rupees then ratio of their amounts is written as 9:20 (with no unit mentioned)

Example 1.

The age of Amaan is 9 months. The age of his elder sister is 3 years.

The relation between their ages cannot be written as 9:3 because the age of Amaan is given in months and the age of his sister is given in years, these are two different quantities.

So, age of his sister must be converted into months.

As 3 years = 36 months
So, the ratio between the ages of Aman to his sister will be 9:36

To find ratio between two quantities it is necessary that they must be of the same kind.

Hence we can conclude that, ratio is a relation in which one quantity bears to another quantity of the same kind with regard to their magnitudes.

UNIT 6 RATIO AND PROPORTION (Ratio)

Know that of the two quantities forming a ratio, the first one is called antecedent and the second one consequent

If a and b are two quantities of the same kind, the ratio between them will be written as a:b. Here the first element a is called the antecedent and the second element b is called consequent.

For Example:

Ratio	Antecedent	Consequent
2:3	2	3
5:7	5	7
18:23	18	23

Calculate ratio of two numbers

Example: Find the ratio of the following:

(i) Ali has Rs 500 and Seema has Rs 309. Write the ratio of their amounts.

Solution: The required ratio is 500:309

(ii) Nazir get 35 litres of fuel filled in his car and Aijaz filled 48 litres of fuel in his car. Write the ratio of quantities of their fuel.

Solution: The required ratio is 35:48

Reduce given ratio into lowest (equivalent) form

We can find lowest equivalent ratios by dividing each element of the ratio by the same number.

Example 1: Find equivalent ratios of 40:8

Solution:

40 : 8

= 40   2 : 8   2   (Dividing by 2)

= 20 : 4

or 40 : 8

= 40   4 : 8   4 (Dividing by 4)

= 10 : 2

or 40 : 8

= 40   8 : 8   8 (Dividing by 8)

= 5 : 1

So, 20 : 4, 10 : 2 and 5 : 1 are some of the lower equivalent ratios of 40 : 8.

Note: In the above example, 5 : 1 is the **lowest equivalent form** of 40 : 8 as both 5 and 1 are natural numbers having no common factor.

Example 2: Find the ratio between Rs 210 and Rs 105 and write in lowest equivalent form.

Solution: Ratio = 210 : 105

= 70 : 35 (Dividing both by 3)

= 14 : 7 (Dividing both by 5)

= 2 : 1 (Dividing both by 7)

Which is the required ratio.

Example 3: Find the ratios of the following quantities and write in lowest form.

(i) 15 kg and 700 g   (ii) 9 hours and 36 minutes.

Solution:

(i) In the given ratio 15 kg : 700g, the units are different.

As 1 kg = 1000 g

Therefore, 15 kg = (15   1000) = 15000 g

Hence, the ratio in the lowest form

= 15 kg : 700 g

= 15000 g : 700 g

= 15000 : 700

= 150 : 7 (dividing both by 100)

(ii) 9 hours and 36 minutes

As 1 hour = 60 minutes

Therefore, 9 hours = (9   60) minutes = 540 minutes

So, the ratio in the simplest form

= 9 hours : 36 minutes

= 540 minutes : 36 minutes

= 540 : 36

= 15 : 1 (dividing both by 36)

Describe the relationship between ratio and fraction

Ratio is a form of common fraction, where numerator is called antecedent and the denominator is known as consequent.

For example. The fraction $\frac{3}{4}$ is written 3:4 as ratio.

or $\frac{3}{4} = 3 : 4$

Also $\frac{2}{5} = 2 : 5$, $\frac{3}{1} = 3 : 1$, $\frac{25}{39} = 25 : 39$ etc.

Example 1: Two pieces of cloth are of 2 m 50 cm and 75 cm length respectively. Find the ratio between their lengths.

Solution: Let us convert the given lengths in the same units.

As, 1 m = 100 cm,

So, 2 m 50 cm = (2   100)cm + 50 cm

= (200 + 50)cm = 250 cm

Hence, the ratio of their lengths = 2m 50cm : 75cm

= 250cm : 75cm

$= \frac{\cancel{250}^{10}}{\cancel{75}_{3}}$

$= \frac{10}{3} = 10:3$

Example 2: Find ratio of the following and write in lowest form.

(i) $\frac{1}{5}$ and $\frac{2}{3}$ (ii) $1\frac{3}{5}$ m and $\frac{7}{10}$ m

(iii) 1.5 and $7\frac{1}{2}$ (iv) Rs. 3.45 and Rs. 6

Solution:

(i) $\frac{1}{5} : \frac{2}{3} = \frac{1}{5} \quad 15 : \frac{2}{3} \quad 15$

(Multiplyingbothby LCMof5and3)

$= 3 : 10$

(ii) $1\frac{3}{5} : \frac{7}{10} = \frac{8}{5} : \frac{7}{10}$

$= \frac{8}{5} \quad 10 : \frac{7}{10} \quad 10$

(Multiplying both by LCM of 5 and 10)

$= 16 : 7$

(iii) $1.5 : 7\frac{1}{2} = 1.5 : \frac{15}{2}$

$= 1.5 \quad 10 : \frac{15}{\cancel{2}_1} \quad \overset{5}{\cancel{10}}$ (Multiplying both by 10)

$= 15 : 15 \quad 5$

$= 1 : 5$ (dividing both by 15)

(iv) 3.45 : 6 = 345 : 600

(As 3.45 is two degree decimal, so we have convert it in to a whole number by multiply 100. Convert both components of ratio into whole numbers so, we multiply both numbers by 100).

= 69 : 120 (Dividing both by 5)

= 23 : 40 (Dividing both by 3)

EXERCISE 6.1

1. Reduce the following into lowest equivalent form.

(i) $4 : 50$ (ii) $0.8 : 72$ (iii) $3.5 : 4.9$

(iv) $\frac{13}{60} : \frac{7}{15}$ (v) $\frac{5}{6} : \frac{3}{10}$ (vi) $\frac{2}{3} : 5$

(vii) $2\frac{1}{2} : 4$ (viii) $\frac{1}{3} : \frac{1}{6} : \frac{1}{9}$ (ix) $1.5 : 5 : 5.8$

(x) $3\frac{2}{5} : 0.6 : 3.5$ (xi) $\frac{1}{5} : \frac{1}{10} : \frac{1}{15}$ (xii) $\frac{1}{7} : \frac{1}{14} : \frac{1}{21}$

2. Find ratio of the following and write in lowest form.

(i) Rs 150 and Rs 180 (ii) 250 cm and 1 m

(iii) 700 g and 2 kg (iv) 3 hours and 210 minutes

(v) 5 years and 3 months

(vi) 15 days, 2 weeks and 1 month

3. Convert the following fractions into ratio form.

(i) $\frac{2}{9}$ (ii) $\frac{5}{6}$ (iii) $\frac{1}{75}$ (iv) 3 (v) $\frac{p}{q}$

4. Represent the following into fractions.

(i) $1:5$ (ii) $2:19$ (iii) $8:1$ (iv) $75:76$ (v) $x : y$

5. In a Science test, 25 students out of 45 students of class VI were passed. Find the ratio between the passed students and total students.

6. Arshad earns Rs 20,000 per month. Find the ratio of his monthly income and expenditure if he saves Rs. 5000 per month.

7. The measures of sides of two squares are 2 cm and 5 cm. Find the ratio of their perimeters.

8. Weight of a sack of flour is 16 kg and the weight of another sack is 14kg 400g. Find the ratio among their weights.

9. Measures of angles of a triangle are 30° , 60° and 90°. Find the ratio of these angles according to the given order.

6.2 PROPORTION

Two equivalent ratios form a proportion.

For example: 3 : 4 = 6 : 8 is a proportion.
Read as 3 is to 4 equals 6 is to 8.

Know that an equality of two ratios constitutes a proportion, e.g., *a* : *b* :: *c* : *d*, where *a*, *d* are known as extremes and *b*, *c* are called the means

In a proportion, the equality sign can be replaced by "::".
So, for the proportion 2:7 = 6:21, we can write 2:7 :: 6:21

The two middle elements of a proportion are called **means**, where as the elements at both ends of a proportion are known as **extremes**. For example:

$$\underbrace{2 : \overbrace{7 :: 6}^{\textbf{Means}} : 21}_{\textbf{Extremes}}$$

Also, 2,7,6 and 21 are called first, second, third and fourth proportionals respectively of the given proportion.

The equality of two given ratios can be proved by the fact that 2:7::6:21 can also be written as $\frac{2}{7} = \frac{6}{21}$ or 2 21=6 7 or 42=42

This can also be proved with the help of this formula of proportion.

Product of extremes = product of means.

If means are same for example, 1:2 = 2:4 then 2 is called the mean proportional between 1 and 4.

Also 1, 2 and 4 are said to be in continued proportion. In this case the third element 4 is called a third proportional of the first and second. By using these properties of proportions we can find any missing proportional. As shown in the following examples.

Teacher's Note *Teacher should give more examples for practice to clear the concept of students about proportion.*

Example 1: Identify means and extrems in the following:

(i) 2 : 3 = 10 : 15

Solution:

Here

Means are 3 and 10

Extremes are 2 and 15

(ii) 5 : 1 = 10 : 2

Solution:

Here

Means are 1 and 10

Extremes are 5 and 2

Example 2: Decide whether the following numbers are in proportion or not.

(i) 2, 3, 4, 6 (ii) 5, 10, 15, 20

Solution: (i) 2, 3, 4, 6

Here

2 : 3 = 4 : 6

Here Product of means = 3   4 = 12

and product of extremes = 2   6 = 12

Product of means = Product of extremes

2, 3, 4, 6 are in proportion

Solution: (ii) 5 : 10 = 15 : 20

Product of means = 10   15 = 150

Product of extremes = 5   20 = 100

Product of means   Product of extremes

5, 10, 15, 20 are not in proportion

Example 3: Find the value of unknowns in the following proportionals:

(i) 33 : 3 :: x : 27 (ii) 9 : 6 :: 6 : y

Solution:

(i) $33 : 3 : : x : 27$

(product of extremes = product of means)

or $33 \quad 27 = 3 \quad x$

or $\dfrac{33 \quad \cancel{27}^{9}}{\cancel{3}} = x$

or $x = 33 \quad 9$

or $x = 297$

(ii) $9 : 6 : : 6 : y$

or $9 \quad y = 6 \quad 6$

or $y = \dfrac{{}^{2}\cancel{6} \quad \cancel{6}^{2}}{\cancel{9}\,\cancel{3}_{1}}$

or $y = 2 \quad 2$

or $y = 4$

Example 4: Find fourth proportional of 2, 5, 8.

Solution: Let fourth proportional be x.

So, we have

$2 : 5 = 8 : x$

or $2 \quad x = 5 \quad 8$

or $\dfrac{2 \quad x}{2} = \dfrac{5 \quad 8}{2}$

or $x = 20$

So, fourth proportional is 20.

Example 5: Find the mean proportional of 4 and 9.

Solution: Let x be the mean proportional, then

$4 : x : : x : 9$

or $4 \quad 9 = x^2$

or $x^2 = 36$

or $x^2 = 6^2$

or $x = 6$

Hence 6 is the mean proportional of 4 and 9.

EXERCISE 6.2

1. Identify means and extremes in the following:

(i) $2:5 = 8:20$ (ii) $3:4 = 6:8$ (iii) $a:b = c:d$

2. Decide whether the four numbers given in each of the following are in proportion or not:

(i) 18, 24, 30 and 40 (ii) 14, 19, 3 and 4
(iii) 6, 8, 12 and 16 (iv) 15, 20, 16 and 21
(v) 20, 30, 40 and 50 (vi) 21, 57, 28 and 76

3. Find the value of x if the proportion $2 : x = 3 : 7$.

4. Find the fourth proportional in the following:

(i) 2, 3 and 6 (ii) $\frac{11}{24}$, $\frac{7}{15}$ and $\frac{5}{8}$ (iii) 16, 12 and 8
(iv) 76, 28 and 57 (v) 36, 45 and 4 (vi) 40, 30 and 24

5. Find the mean proportion in the following:

(i) 15 and 60 (ii) 18 and 32 (iii) 28 and 63
(iv) 27 and 12 (v) 40 and 90 (vi) 44 and 99

Find proportion (direct and inverse)

(1) Direct proportion:

Consider the following chart.

Quantity of Pencils	Cost
4	Rs 20
3	Rs 15
2	Rs 10
1	Rs 5

What is the proportion of number of pencils and their cost.

In this chart, we can easily observe that the cost of pencils increases or decreases with the corresponding increase or decrease in the quantity of pencils.

Hence we can say that when two quantities are related in such a way that if one quantity increases in a given ratio, the other also increases in the same ratio (and vice versa).

In this situation, there is a **Direct Proportion** between both quantities.

Note: In the above chart, the ratio between 1 pencil and 2 Pencils is same as between the cost Rs 5 and Rs 10.

Examples:

(i) More construction, more materials
(ii) Less money, less shopping
(iii) More students, more teachers
(iv) Less workers, less work

(2) Inverse Proportion:

The following chart is showing number of persons required to complete a work in the given time.

No. of Persons	Time
2	40 hours
4	20 hours
8	10 hours
16	5 hours

What type of proportion is between number of person and time required.

In the above table, we observe that by increasing the number of persons the time required is decreased. (and vice versa).

Hence we can say that when two quantities are related in such a way that if one quantity increases in a given ratio, the other decreases in the same ratio.

In this situation, there is an **inverse proportion** between both quantities.

Note: In the above chart the ratio between 2 persons and 4 persons is same as between 20 and 40.

Examples:

(i) Less speed; more time taken
(ii) More speed; less time taken
(iii) More time; less workers required
(iv) Less time; more workers required

Solve real life problems involving direct and inverse proportion

Consider the following examples.

Example 1: If 20 metre cloth is used to make 4 dresses. How much cloth is required to make such 15 dresses?

Solution: Suppose the required cloth is x metre

Dresses	**Cloth in metres**
4	20
15	x

(More cloth more dresses)

Direct Proportion

$$4 : 15 : : 20 : x$$

As product of means = Product of extremes

or $\quad 4 \quad x = 15 \quad 20$

or $\quad x = \dfrac{15 \quad \cancel{20}^{\,5}}{\cancel{4}_{\,1}}$

or $\quad x = 75$

Hence the required cloth is 75 metre.

Teacher's Note *Teacher should discuss other examples from daily life with the students and help them in solution.*

Example 2: If 3 pipes can fill a tank in 80 minutes. How long will it take to fill the tank if 5 pipes are used?

Solution:

Let x minutes are required.

Pipes	**Time in minutes**
↑ 3	↑ 80
5	x

(More time less pipes) — Inverse Proportion

$$5 : 3 : : 80 : x$$

As product of means = Product of extremes

or $\quad x \quad 5 = 3 \quad 80$

or $\quad x = \dfrac{3 \quad \cancel{80}^{16}}{\cancel{5}_{1}}$

or $\quad x = 48$

Hence 48 minutes are required.

EXERCISE 6.3

1. Ahmed secured 60 marks in a paper of 100 marks. What shall be his score if the paper were of 75 marks?

2. The weight $\frac{5}{9}$ of a piece of metal is 35 kg. What is the weight $\frac{2}{7}$ of the piece?

3. On a map, 80 km are represented by 5 cm. If the distance between two cities on the map is 15 cm. Find the actual distance between them.

4. If a dozen eggs costs Rs 60, find the cost of 32 eggs.

5. Tabassum earns Rs. 42,000 in a month and spends Rs 39,800. Find the ratio of her savings to her income. Also find the ratio of her expenditure to her income.

6. If Naveed can read 45 pages of a book in 75 minutes, find the time he takes to read the book of 876 pages.

7. A book covers 130 pages with 24 lines on every page. If we make the same book of 90 pages with increasing number of lines. How many lines would be there on each page?

8. If 80 workers complete a work in 6 days. Find the number of days required to complete the same by 10 workers.

REVIEW EXERCISE 6

1. Find the ratio in lowest form between the following:

(i) Length and breadth of your copy.

(ii) Number of students present and absent in your class.

(iii) Weight in kg of your two class mates.

(iv) Daily pocket money of your three friends.

(v) Number of pages of the text books of your class of Mathematics, English, and Social Studies.

2. Write equivalent ratio for:

(i) 2 : 3 = ___________ (ii) 10 : 8 = ___________

(iii) 1 : 4 = ___________ (iv) 21 : 35 = ___________

3. Find the value of x in each of the following:

(1) $2 : 5 :: x : 10$ (2) $5 : x :: 10 : 14$

(3) $15 : 30 :: 1 : x$ (4) $x : 12 :: 3 : 4$

(5) $x : 200 :: 18 : 25$ (6) $132 : 216 :: 33 : x$

4. Which of the following are inverse proportions?

(i) The number of copies you buy and their total cost.

(ii) The number of men doing a job and the time taken to finish it.

(iii) The amount of time taken in a journey and speed of the vehicle.

(iv) The number of boxes and the number of Pencils packed in them.

5. Out of 670 students in a school, 350 are boys and rest are girls. Find the ratio of:

(i) Boys to total students. (ii) Girls to total students.

(iii) Girls to boys of the school. (iv) Boys to girls of the school.

6. Rehana can write 26 words in 2 minutes. How much time she will take to write 325 words.

7. 12 kg milk costs Rs 1200. What is the cost of $13\frac{1}{2}$ kg milk?

8. 24 metre high wall is made by 16 workers in a perticular time. How many workers will construct a 18 metre high wall in the same time?

9. Food was stored for 600 persons for 18 days. How many persons will eat the same food in 27 days?

SUMMARY

- Comparison between magnitudes of two quantities of the same kind is called ratio.
- Ratio of two numbers a and b, where b o, is written as

Antecedent —— $a : b$ or $\frac{a}{b}$. —— Numerator

Consequent —— Denominator

- Order of ratio is important i.e. $a : b \quad b : a$ in general
- A ratio remains unchanged if all its elements are multiplied (or divided) by the same number.
- Equality of two ratios is called proportion.
- If a, b, c, d are in proportion, then,

$$a : b : : c : d \text{ or } a : b = c : d$$

Where a, b, c and d are called the First, Second, Third and Fourth proportionals respectively.

- In a, b and c are in continued proportion then,

$$a : b : : b : c$$

- For every $a : b : : c : d$, $ad = bc$
Here a and d are extremes and b and c are means we have,
Product of extremes = Product of means
- When two related quantities, increase or decrease together in the same ratio, they are called directly proportional to each other.
- When in two related quantities, increase in one causes decrease in the other, and decrease in one causes increase in the other, then they called inversely proportional to each other.

UNIT 7

FINANCIAL ARITHMETIC

7.1 PERCENTAGE

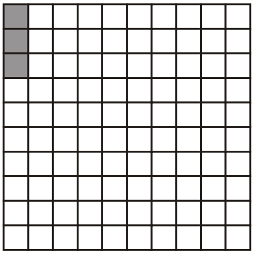
You have often heard the sentences like

"Sale Sale i.e Discount upto 50 percent". Discount 10-60 percent and reduction 10% etc.

In our daily life, we apply percentage on many things like discounts, during shopping etc. To calculate percentage is an important skill during shopping.

The word percent is a short form of the Latin word "per centum". Its meaning is "out of hundred".

Recognize percentage as a fraction with denominator of 100



Look at this figure.
It has been divided into 100 equal parts.
3 out of 100 parts, are coloured.

i.e. $\frac{3}{100}$ of the figure is coloured.

We say that "3 percent" of the figure is coloured.

Percentage is denoted by the symbol %. So, we can say that 3% of the figure is coloured.

Any fraction with 100 as a denominator can be expressed as percentage.

> Percentage is a special type of fraction which indicates parts out of hundred parts.

Teacher's Note *In order to clarify the concept of percentage, teacher should put emphasis on daily life examples and geometrical shapes.*

Convert a percentage to a fraction by expressing it as a fraction with denominator 100 and then simplify

We can convert a percentage to a fraction by expressing it as a fraction with denominator 100 and then simplify.

Consider the following examples.

Example 1: Convert 5% into fraction.

Solution: $5\% = \frac{5}{100}$

$= \frac{\cancel{5}}{\cancel{100}_{20}} = \frac{1}{20}$

Example 2: Convert 72% in fraction.

Solution: $72\% = \frac{72}{100}$

$= \frac{\overset{18}{\cancel{72}}}{\cancel{100}_{25}} = \frac{18}{25}$

Example 3: Convert $7\frac{1}{2}\%$ into fraction.

Solution: $7\frac{1}{2}\% = \frac{15}{2}\% = \frac{15}{2} \quad \frac{1}{100}$

$= \frac{15}{200}$

$= \frac{3}{40}$

Convert a fraction to a percentage by multiplying it with 100%

The fraction having denominators as 100 can easily be converted into percentage.

For example: $\frac{5}{100} = 5\%$ and $\frac{97}{100} = 97\%$

The fractions having denominators other than 100 are converted into percentage by two ways:

(i) First convert the fraction into equivalent fraction having denominator as 100 and then express as percentage.

(ii) Convert any fraction into percentage by multiplying and dividing with 100 or multiplying by 100%.

Example 1: Convert $\frac{1}{2}$ into percentage.

Solution: By converting into equivalent fraction with 100 denominator.

$$\frac{1}{2} = \frac{1 \quad 50}{2 \quad 50} = \frac{50}{100} = 50 \quad \frac{1}{100} = 50\% \; (\because \frac{1}{100} = 1\%)$$

By multiplying with 100%

$$\frac{1}{2} = \frac{1}{2} \quad \frac{100}{100}$$

$$= \frac{100}{2} \quad \frac{1}{100} = 50\% \; (\because \frac{1}{100} = 1\%)$$

Example 2: Convert $\frac{3}{5}$ into percentage.

Solution: By converting into equivalent fraction

$$\frac{3}{5} = \frac{3 \quad 20}{5 \quad 20} = \frac{60}{100} = 60\%$$

By multiplying with 100%

$$\frac{3}{5} = \frac{3}{5} \quad \frac{100}{100}$$

$$= \frac{3 \quad \overset{20}{\cancel{100}}}{\cancel{5}_{1}}$$

$$= 60\% \qquad (\because \frac{1}{100} = 1\%)$$

Example 3: Convert $\frac{5}{8}$ into percentage.

Solution: $\frac{5}{8} = \frac{5}{8} \quad \frac{100}{100}$

$$= (\frac{5}{\cancel{8}_{2}} \quad \frac{\overset{25}{\cancel{100}}}{}) \quad \frac{1}{100} = \frac{125}{2}\%$$

Convert a percentage to a decimal by expressing it as a fraction with denominator 100 and then as a decimal

Method of converting a percentage to a decimal is explained through following examples.

Example 1: Convert 3% into decimal.

Solution: $3\% = \frac{3}{100} = 0.03$

Example 2: Convert $\frac{5}{2}\%$ into decimal.

Solution: $\frac{5}{2}\% = 2.5\%$

$$= \frac{2.5}{100} = 0.025$$

Example 3: Convert 135% into decimal.

Solution: $135\% = \frac{135}{100} = 1.35$

Convert a decimal to percentage by expressing it as a fraction with denominator 100 then as a percentage

Method of converting a decimal fraction into percentage is explained through following examples.

Example 1: Convert 0.5 into percentage.

Solution: $0.5 = \frac{5}{10} = \frac{5 \quad 10}{10 \quad 10} = \frac{50}{100} = 50\%$

Teacher's Note *Teacher should ensure that students can convert simple questions of percentage into fraction and fraction into percentage orally.*

Example 2: Convert 0.25 into percentage.

Solution: $0.25 = \frac{25}{100} = 25\%$

We can also convert decimal into percentage by multiply with 100%.

Example 3: Convert 0.125 into percentage.

Solution:

$$0.125 = 0.125 \quad 100\%$$

$$= \frac{125}{\cancel{1000}_{10}} \quad \overset{1}{\cancel{100}}\%$$

$$= \frac{125}{10}\%$$

$$= 12.5\%$$

Finding percentage of a given quantity

In the following examples, we explain the method of finding the percent of a given quantity.

Example 1: Find 15% of 75.

Solution:

$$15\% \text{ of } 75 = \frac{15}{\cancel{100}_{4}} \quad \overset{3}{\cancel{75}}$$

$$= \frac{45}{4}$$

$$= 11\frac{1}{4}$$

Example 2: Is 20% of 140 greater than 25% of 150.

Solution:

$$20\% \text{ of } 140 = 140 \quad \frac{\overset{1}{\cancel{20}}}{\cancel{100}_{5}} = 28$$

$$25\% \text{ of } 150 = 150 \quad \frac{\overset{1}{\cancel{25}}}{\cancel{100}_{4}} = \frac{75}{2} = 37\frac{1}{2}$$

As $28 < 37\frac{1}{2}$

Therefore, 20% of 140 is not greater than 25% of 150.

Example 3: Write the following fractions in ascending and descending order using percentage.

$$\frac{1}{2}, \frac{7}{5}, \frac{3}{4}$$

Solution:

$$\frac{1}{2} = \frac{1}{\underset{1}{\cancel{2}}}\ \overset{50}{\cancel{100}}\% = 50\%$$

$$\frac{7}{5} = \frac{7}{\underset{1}{\cancel{5}}}\ \overset{20}{\cancel{100}}\% = 140\%$$

$$\frac{3}{4} = \frac{3}{\underset{1}{\cancel{4}}}\ \overset{25}{\cancel{100}}\% = 75\%$$

As $50\% < 75\% < 140\%$

So, Ascending order is:

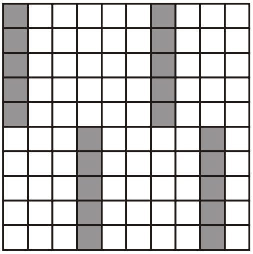
$$\frac{1}{2}, \frac{3}{4}, \frac{7}{5}$$

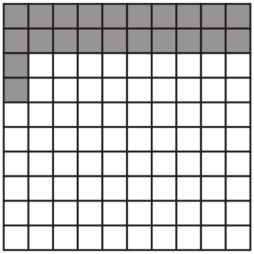
and Descending order is:

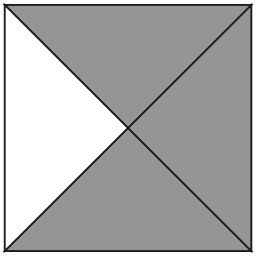
$$\frac{7}{5}, \frac{3}{4}, \frac{1}{2}$$

EXERCISE 7.1

1. What percent of the following figures is shaded.

(i) 

(ii) 

(iii) 

2. **Convert the following into fractions and decimal.**

(i) 25% (ii) 1% (iii) 31% (iv) 15%

(v) $8\frac{1}{2}\%$ (vi) 20.5% (vii) 175% (viii) 115%

(ix) 225% (x) $2\frac{1}{2}\%$

3. **Convert the following fractions into percentage.**

(i) $\frac{1}{4}$ (ii) $\frac{7}{20}$ (iii) $\frac{8}{25}$ (iv) $\frac{5}{4}$

4. **Write the following percentage into decimals.**

(i) 25% (ii) 31% (iii) 1% (iv) $2\frac{1}{2}\%$ (v) 6.5%

5. **Convert the following decimal into percentage.**

(i) 0.025 (ii) 0.4 (iii) 0.85

(iv) 0.105 (v) 12.5 (vi) 125.5

6. **What percentages are the following of 1 metre?**

(i) 70 cm (ii) 50 cm (iii) 75 cm (iv) 125 cm (v) 100 cm

7. **The sale of the books on science on a certain store is 30%, write it in fraction.**

8. **Khalida obtained 96 marks out of 150. What percentage of marks has she obtained?**

9. **Find the following percentages.**

(i) 10 % of 50 (ii) 30 % of 70

(iii) 25% of 250 (iv) 50 % of 50

(v) 75 % of 750 (vi) 125% of 50 cm

(vii) 60 % of 300 litres (viii) 20% of Rs 13.25

(ix) 60.5 % of 8 hours (x) 25% of 172 metres

(xi) 15 % of 210 (xii) 75 % of 1011

(xiii) 65% of 910 (xiv) 35 % of 301

10. Write the following fractions in ascending and descending order using percentage.

$\frac{2}{5}, \frac{1}{4}, \frac{3}{4}$

Solve real life problems involving percentage

Percentages are used in daily life as explained in the following examples.

Example 1: Naheed has Rs 120. She spends 25% of the amount. How much money is left with her?

Solution: Total amount = Rs 120

Rate of expenditure = 25% of the total amount

Amount spends = Rs 120 $\frac{25}{100}$ = 30

Amount left with Naheed = Rs 120 – 30 = 90

Example 2: 75% students out of 40 were present in a test. How many students were absent?

Solution: Total students = 40

percentage of present students = 75%

Number of students present = 75% of 40

= 40 $\frac{75}{100}$ = 30 students

Number of students absent = 40 – 30 = 10 students

Second method.

As, percentage of the students present = 75%

So, percentage of the students absent = 100% – 75% = 25%

So, number of students who were absent = 25% of 40

= 40 $\frac{25}{100}$ = 10 students

Sometimes we need to find total quantity if percentage and given quantity (Percent of quantity) are given. In this case we use the rule

$\boxed{\text{Total quantity} = \text{Given quantity} \quad \text{Percentage}}$

Example 3: Find the quantity if 75% of it is 123.

Solution: By using the formula

Required $\boxed{\textbf{Quantity} = \textbf{Percent of the quantity} \quad \textbf{Rate}}$

Here rate = 75% and percent of the quantity = 123

So, Required quantity = 123   75%

$$= 123 \quad \frac{75}{100}$$

$$= \overset{41}{\cancel{123}} \quad \frac{\overset{4}{\cancel{100}}}{\underset{3}{\cancel{75}}} \qquad \text{Since, } \boxed{\text{Percentage} = \frac{\text{Part of quantity}}{\text{Quantity}} \quad 100\%}$$

$$= 164$$

Hence, the required quantity is 164.

Example 4: 25% of Hassan's salary is Rs 1250. Find his salary.

Solution: Here rate = 25%

Percent of salary = 1250 rupees

Now Salary = percent of salary   rate

$$= 1250 \quad 25\%$$

$$= 1250 \quad \frac{25}{100}$$

$$= 1250 \quad \frac{\overset{4}{\cancel{100}}}{\underset{1}{\cancel{25}}}$$

$$= 5000 \text{ rupees}$$

Hence the salary of Hassan is Rs 5000

Example 5: In an examination Akram obtained 510 marks out of 850. Find the percentage of the marks obtained by him.

Solution: This question can be solved by the following formula.

$$\text{Percentage} = \frac{\text{Part of quantity}}{\text{Quantity}} \quad 100\%$$

Here, Quantity = 850 marks

Part of quantity = 510 marks

So, $\text{Percentage} = \frac{510}{850} \quad 100\% = 60\%$

EXERCISE 7.2

1. I obtained 60 marks out of 75 marks. What is the percentage of my marks?
2. Saima spent Rs 300 out of Rs 500. Find percentage of her expenditure.
3. There were 8500 voters in a village. 34% did not cast their votes. Find the number of voters who cast their votes.
4. The population of a village is 15000. If the population increases by 5% in a year, find the population after one year.
5. Rahila pays 5% of her salary in charity in a month. If she pays Rs 200, find her monthly salary.
6. The 18% of the distance between two cities is 36 km. Find the distance between two cities.
7. Naeem's monthly income is Rs 8000 and he spends Rs 6000 per month. Find the percentage of his expenditure.
8. On a rainy day 600 students out of 750 were present in a school. What percentage of the students were absent.

9. If one angle of a triangle is 90°. Find the other two angles if these angles are 30% and 70% of 90° respectively.

10. Area of square is 100 sq. cm. Find the area of a rectangle whose area is 10% more than the area of given square.

11. In a shop there is a double discount of 10% and 20% on an article having price of Rs 100. Calculate the total discount.

7.2 PROFIT, LOSS AND DISCOUNT

Define selling price and cost price

In every business goods are sold and purchased. Every business is established to earn profit but sometimes a loss to be borne.

The amount which is paid to purchase goods is called **"Cost price"** and the amount which is received by selling goods is called **"Selling price"**.

Define profit, loss and discount

1. Profit Percentage and Loss Percentage

If in a business selling price of an item is more than cost price then there is a *profit* in such transaction. But if selling price of an item is less than cost price then there is a *loss* in such transaction.

Thus

Profit = Selling Price (SP) – Cost Price (CP)

Loss = Cost Price (CP) – Selling Price (SP)

Businessmen generally compare and calculate the profit and loss of goods in percent. For example if a shopkeeper purchases a juice for Rs 50 and sells for Rs 60. Similarly he purchases a milk pack for Rs 100 and sells for Rs 110. In both transactions, he earned Rs 10 as a profit. It seems that in above two cases he earned the same profit which is not fact. So, profit percent or loss percent are useful in such cases.

Now

$$\text{Profit percent} = \frac{\text{Profit}}{\text{CP}} \quad 100\% \quad \text{and} \quad \text{Loss percent} = \frac{\text{Loss}}{\text{CP}} \quad 100\%$$

$$\text{Profit Percent of juice} = \frac{10}{50} \quad 100\% = 20\%$$

$$\text{Profit Percent of milk pack} = \frac{10}{100} \quad 100\% = 10\%$$

This shows that the profit percent is not same in both cases.
Thus profit percent determines the transaction which gives more profit.

2. Discount

The reduction made on the list price or marked price of an article is called **discount** i.e. Net Selling price = Marked Price – Discount.

Remember: The discount percent is calculated only on the marked price and not on selling or cost price.

Example: The marked price of an article is Rs 400 and it is sold at a discount of 10%, find the net selling price of the article.

Solution: Discount = 10% of Rs 400

$$= \frac{10}{100} \quad 400 = \frac{10}{\cancel{100}} \quad \overset{4}{\cancel{400}} = \text{Rs } 40$$

So, Net Selling Price = Marked Price – Discount
= Rs 400 – Rs 40 = Rs 360

Solve real life problems involving profit, loss and discount

Example 1: A school bag is purchased for Rs 180 and is sold out for Rs 225. Find the profit percent.

Solution: Profit = selling price – cost price
= 225 – 180 = 45 rupees

Now Profit Percent $= \dfrac{\text{Profit}}{\text{CP}} \quad 100\%$

$$= \frac{45}{18\cancel{0}} \quad 10\cancel{0}\% = \frac{450}{18}\%$$

$$= 25\%$$

So, Profit = 25%

Example 2: A fruit seller purchased 40 dozens bananas in all and their cost price is Rs 50 per dozen. He sells all the bananas at the rate of Rs 60 per dozen. Find his total profit and profit percent.

Solution: Bananas = 40 dozen

Cost price per dozen = 50 rupees

Total cost price = 50 40 = 2000 rupees

Selling price per dozen = 60 rupees

Total selling price = 60 40 = 2400 rupees

Profit = selling price – cost price

= 2400 – 2000 = 400 rupees

$$\text{Profit Percent} = \frac{\text{Profit}}{\text{CP}} \quad 100\%$$

$$= \frac{40\cancel{0}}{200\cancel{0}} \quad 100\%$$

$$= \frac{1}{5} \quad 100\%$$

$$= 20\%$$

Example 3: Saima purchased a mobile set for Rs 15000. After some time she sold it for Rs 12000. Find her loss percent.

Solution: Here CP = 15000 rupees

SP = 12000 rupees

Since SP < CP, so there is loss

Now Loss = CP – SP

= 15000 – 12000 = 3000 rupees

$$\text{So, Loss Percent} = \frac{\text{Loss}}{\text{CP}} \quad 100\%$$

$$= \frac{{}^{1}\cancel{3000}}{\cancel{15000}_{5}} \quad \cancel{100}\%$$

$$= \frac{1}{5} \quad 100\%$$

$$= 20\%$$

Example 4: Bismah purchased a T.V for Rs 20000 and sold it for Rs 18000 to purchase a new model T.V. Find her profit or loss percent.

Solution: Here CP = 20000 rupees

SP = 18000 rupees

Since SP < CP, so there is loss

Now, Loss = CP – SP

= 20000 – 18000 = 2000 rupees

$$\text{Loss Percent} = \frac{\text{Loss}}{\text{CP}} \quad 100\%$$

$$= \frac{2\cancel{000}}{20\cancel{000}} \quad 100\%$$

$$= \frac{1}{10} \quad 100\%$$

$$= 10\%$$

Example 5: A trader marked his goods at 30% above the cost price. If he allows his customer 20% discount, find his profit percent.

Solution: Let the cost price be Rs 100.

Then marked price = Rs (100 + 30) = Rs 130

Discount = 20% on marked price

$$\frac{20}{100} \quad 130 = \text{Rs } 26$$

So, Net Selling Price = Marked Price – Discount

= 130 – 26 = Rs 104

Thus Profit = Selling Price – Cost Price

= 104 – 100 = Rs 4

Hence, the profit is 4%.

EXERCISE 7.3

1. Find the profit or loss percent in the following.

(i) Eggs are purchased at Rs 120 per dozen and are sold out at Rs 130 per dozen.

(ii) A shopkeeper purchases 25 breads at Rs 10 per bread. He sells breads at Rs 13 per bread as 7 breads are spoiled.

(iii) A man purchases pencils at Rs 60 per dozen and sells at Rs 70 per dozen.

(iv) A man purchases 10 dozen eggs at the rate of Rs 120 per dozen. 3 dozen eggs are broken. Rest of the eggs are sold at the rate of Rs 130 per dozen.

(v) A milkman purchases milk at the rate of Rs 75.50 per litre and sells at the rate of Rs 80 per litre.

2. A book seller purchases 20 books of Mathematics at Rs. 120 per book and sells at Rs 125 per book. Find his profit and profit percent.

3. A grocer purchases 10 kilograms of rice at Rs 120 per kilogram and sells at Rs 125 per kilogram. Find profit percent.

4. A shopkeeper bears 15% loss on the sale of a jacket. If the cost of the jacket is Rs 1000, find his loss.

5. Umaima bought a dinner set for Rs 52000 at 20% discount. Find the actual price of the dinner set.

6. The marked price of ladies purse is Rs 1500 and it is sold at discount of 15%. Find net selling price of the purse.

REVIEW EXERCISE 7

1. Karim bought 50 dozen of pencils at Rs 60 per dozen. He sold them at Rs 13 per pencil. Find his profit or loss percent.

2. A television (T.V) manufacturing company declares that a colour T.V is now available for Rs 13050. Its previous price was Rs 17400. Find the percentage reduction in the price of colour T.V offered by the company.

3. Imran obtained 548 marks out of 800 and his sister obtained 459 out of 600. Whose performance is better?

4. A fruit seller purchases 50 dozen bananas at the rate of Rs 60 per dozen. 10 dozen bananas are spoiled . Rest of the bananas are sold at the rate of Rs 70 per dozen. Find his gain or loss percent.

5. A car was sold at price of Rs 875,000 with a discount of $2\frac{1}{2}$ %. What is actual cost of the car?

6. **Which is greater?**

(i) 0.045 or 5% (ii) 0.64 or 62% (iii) 0.8 or 8%

(iv) 0.15 or 19% (v) $\frac{1}{2}$ or 40% (vi) $\frac{1}{5}$ or 25%

(vii) $\frac{3}{4}$ or 70% (viii) $1\frac{1}{4}$ or 130% (ix) $\frac{1}{10}$ or 9%

7. **(a) Fill in the blanks:**

(i) $\frac{3}{4}$ = ________% (ii) $\frac{5}{8}$ = ________%

(iii) 30% = ________ (fraction)

(iv) 0.125 = ________ %

(v) 38% = ________ (decimal)

(vi) 36% of 120 = ______ (vii) S.P + Loss = ______

(viii) S.P – C.P = ______

(b) Write true (T) or false (F) for the following.

(i) If S.P $<$ C.P so, there is gain.

(ii) If C.P $>$ S.P so, there is loss.

(iii) Loss = C.P – S.P

(iv) S.P + gain = C.P

(v) Loss % = $\frac{\text{Loss}}{\text{S.P}}$ 100%

(vi) Profit = S.P – C.P

(vii) $\frac{\text{Profit} \quad 100\%}{\text{C.P}}$ = Profit %

SUMMARY

- The word **"percent"** means **"out of hundred".** Percentage is denoted by the symbol %
- Percentage can be converted into **common** or **decimal fraction**.
  - (a) To convert a percent into a common fraction, we multiply the number indicating the percent by $\frac{1}{100}$ and simplify the resulting fraction.
  - (b) To convert a percent into a decimal, we remove the % symbol and shift the decimal point two places to the left.
- Fraction or a decimal fraction can be converted into percentage.
  - (a) To convert a decimal into a percent, we shift the decimal point two places to the right and insert percent symbol (%).
  - (b) (i) The fractions having their **denominators** as 100 can easily be converted into percentage.

(ii) The fractions having their denominators other than 100 are converted into percentages in the following ways:

★ First convert the fraction into equivalent fraction having denominator as 100 and then convert it into percentage.

★ First convert the fraction into decimal and then convert it into percentage.

★ Multiply the given fraction by 100%.

- Percent of a quantity = Quantity   rate
- Quantity = Percentage of Quantity   rate
- Percentage = $\dfrac{\text{Part of Quantity}}{\text{Whole Quantity}}$ 100%
- When S.P > C.P, then Profit = S.P – C.P
- When S.P < C.P, then Loss = C.P – S.P
- Profit % = $\dfrac{\text{Profit}}{\text{C.P}}$ 100% and Loss % = $\dfrac{\text{Loss}}{\text{C.P}}$ 100%

UNIT 8 INTRODUCTION TO ALGEBRA

HISTORY OF ALGEBRA

Algebra is the invention of muslims. In 820 AD, a muslim mathematician Muhammad Ibne Mosa Al Khwarzmi wrote a book named Algebra-wal-Almuqabla. The first translation of this book named "Algebra" was published in Europe in Latin which is the base of the subject "Algebra".

8.1 ALGEBRA

Explain the term algebra as an extension of arithmetic in which letter replace the numbers

Man has been using natural numbers 1, 2, 3, ... in his life. As time passed his needs increased and the calculation became more complex. Algebra provides methods to transform and solve difficult and complex mathematical problems in easy way.

In Arithmetic we usually perform calculation of numbers with the help of fundamental operations addition, subtraction, multiplication and division, for example Rafay has 5 apples and Taha has 8 apples. So, the total number of apples is 5 + 8.

The term algebra is an extension of arithmetic in which letters or symbols replace the numbers or quantities. For example Bisma has 5 toffees in a plate and Hifza has some toffees in her purse.

We can represent the total toffees as $5 + x$.

In this example we have shown the unknown quantity of toffees of Hifza by "x".

Teacher's Note *Teacher should give some more examples of unknown quantities from class room or daily life and represent them by x,y or z. Also add or subtract them with known quantities.*

Hence 'the sum of 5 and x' is written in symbols as '$5 + x$'

Similarly other expression can be written in symbols as explained in the example.

Example: Represent into symbols.

In words	In symbols (Algebraic expression)
Sum of 6 and x	$6 + x$
Three times a number y	$3y$
The product of two numbers x and y	xy
Five is greater than four	$5 > 4$
Double of a number e	$2e$

Know that a sentence is a set of words making a complete grammatical structure and conveying full meaning

We speak sentences to convey some ideas in our daily life.

For example: Sum of two and seven is nine.

Symbolically, we write as $\boxed{2 + 7 = 9}$

This sentence conveys full meaning.

So, In languages a sentence is set of words making a complete grammatical structure and conveying full meaning.

Example: Represent the following sentences symbolically.

(1) Double of a number x is 8

Solution: Symbolically we represent it as $2x = 8$.

(2) Five is greater than three

Solution: Symbolically we represent it as $5 > 3$.

Know that Sentences that are either true or false are known as statements and a statement must be either true or false but not both information

We, sometimes speak true sentences or false sentence for example.

(1) Difference of nine and two is seven

i.e. $9 - 2 = 7$

It is a true sentence.

(2) Sum of two and five is four

i.e. $2 + 5 = 4$

It is a false sentence.

Such sentences are known as statements.

Thus

Sentences that are either true or false are known as statements.

A sentence that does not include enough information required to decide whether it is true or false is known as open statement (e.g. $\quad + 2 = 9$) and a number that makes an open statement true is said to satisfy the statement (e.g. $\quad = 7$ makes the statement $\quad + 2 = 9$ true). Also use English alphabet *x* in the open statement $\quad + 2 = 9$ to modify it to $x + 2 = 9$

We know that a statement must be either true or false. But in some cases we have to decide whether a statement is true or false. Such statement is called **open statement**.

For example: $\quad + 5 = 12$ is an open statement.

if we replace $\quad$ with 4 it becomes false i.e. $4 + 5 = 12$ but if we replace $\quad$ with 7 it becomes true. i.e. $7 + 5 = 12$.

If $\quad = 2$, $2 + 5 = 7$, which is not equal to 12. So, it becomes false.

So, we say 7 satisfies the statement

and if we replace $\quad$ with English alphabet *x* it still remains open i.e. $x + 5 = 12$

Example 1: Think and write the value of $\quad$ which makes the following statements true.

(i) $\quad + 6 = 18$ (ii) $\quad - 3 = 7$

Solution: (i) $\quad + 6 = 18$

It will be true if $\quad = 12$ i.e. $\boxed{12 + 6 = 18}$

(ii) $\quad - 3 = 7$

It will be true if $\quad = 10$ i.e. $\boxed{10 - 3 = 7}$

Example 2: Replace the unknowns by the numbers to make statement true.

(i) $2 + \square = 8$ (ii) $17 - \quad = 9$

(iii) $10 \quad = 5$ (iv) $6 \quad x = 42$

Solutions:

(i) $2 + \square = 8$

It will be true if $\square = 6$ i.e. $\boxed{2 + 6 = 8}$

(ii) $17 - \quad = 9$

It will be true if $\quad = 8$ i.e. $\boxed{17 - 8 = 9}$

(iii) $10 \quad = 5$

It will be true if $\quad = 2$ i.e. $\boxed{10 \quad 2 = 5}$

(iv) $6 \quad x = 42$

It will be true if $x = 7$ i.e. $\boxed{6 \quad 7 = 42}$

EXERCISE 8.1

1. **Write the following sentences in symbols.**
   - (i) Sum of a number x and two is eight.
   - (ii) Product of a number y and seven is greater than two.
   - (iii) Sum of six and a number z is less than four.
2. **Which of the following are true or false statements?**
   - (i) $5 + 9 = 18$ (ii) $8 + 3 = 11$ (iii) $20 + 3 = 26$
   - (iv) $21 - 8 = 13$ (v) $18 - 13 = 15$ (vi) $16 - 10 = 6$
3. **Which of the following are open statements?**
   - (i) $\quad + 3 = 9$ (ii) $3a + 4 = 8$ (iii) $9 + 4 = 13$
4. **Find the values of the number or $\square$ which makes the following statements true.**
   - (i) $6 + \quad = 15$ (ii) $\square + 5 = 8$ (iii) $14 - \quad = 8$
   - (iv) $24 - \quad = 20$ (v) $3\square + 1 = 10$ (vi) $35 - \square = 29$

Define variables as letters used to denote numbers in algebra

Consider an open statement $x + 5 > 2$

It will be true for different values of number x.

For example:

for $x = 1$	we get	$1 + 5 > 2$	or	$6 > 2$
for $x = 2$	we get	$2 + 5 > 2$	or	$7 > 2$
for $x = 3$	we get	$3 + 5 > 2$	or	$8 > 2$

and so on

It means the value of number x is not constant, but it is varying.

So, x is called variable.

Hence

> Any of the letter of English alphabet which is used to denote number in algebra is called variable.

Know that any numeral, variable or combination of numerals and variables connected by one or more of the symbols '+' and '–' is known as an algebraic expression (e.g., x + 2y)

In daily life, we often speak set of words which are not sentences.

For example:

(1) Difference of a number x and two
Symbolically we write as $x - 2$

(2) Sum of thrice of a number x and twice of a number y
Symbolically we write as $3x + 2y$

Such expressions like $x - 2$ and $3x + 2y$ are known as algebraic expression. In these expressions we see variables, numerals and fundamental operations.

Thus

> Algebraic expression is the combination of variables, numerals and fundamental operations.

$2 + x$, $5x$, $6x - 3y + 8$ are few examples of algebraic expressions.

Example 1:

Moomal has three brothers. Every brother gives x rupees to Moomal. She already has Rs 7.

(i) Write an algebraic expression for the amount she has.

(ii) If each brother gives Rs 10. Find the amount that Moomal has.

(iii) If each brother gives him Rs 15. Find the amount that she has.

Solution:

(i) Three brothers give her $x + x + x = 3x$
Already Moomal has Rs 7
Total amount that Moomal has $3x + 7$

(ii) If $x = 10$, then Moomal has
$3x + 7 = 3 \quad 10 + 7 = 37$ rupees

(iii) If $x = 15$, then Moomal has
$3x + 7 = 3 \quad 15 + 7 = 52$ rupees

8.2 ALGEBRAIC EXPRESSIONS

Know that x, 2y and 5 are called the terms of the expression x + 2y + 5

The parts of an algebraic expressions are connected to each other by the signs of addition and subtraction, for example in $5x + 4y$, there are two parts, 5x is 1st part and 4y is 2nd part. They are connected by plus sign. The parts $5x$ and 4y are called the 'terms' of the expression $5x + 4y$.

Similarly a+b is an algebraic expression having two terms i.e.a and b.

The expressions $a \quad b$ or $\frac{a}{b}$ and ab, contain only one term.

An algebraic expression consists of terms which are connected by the operations of addition (+) and subtraction (–).

Note: Operations of multiplication ( ) and division ( ) do not connect terms.

Example: Write the number of terms in the following expressions:

(1) $5x + 7y$ (2) $6x \quad 7 + y$ (3) $3x - 4y - 7$ (4) $4 \quad p \quad q$

Solutions:

(1) $5x + 7y$
Number of terms = 2

(2) $6x \quad 7 + y$
Number of terms = 2

(3) $3x - 4y - 7$
Number of terms = 3

(4) $4 \quad p \quad q$
Number of terms = 1

Know that the symbol or number appearing as multiple of a variable used in algebraic term is called its coefficient (e.g. in 2y, 2 is the coefficient of y)

In algebraic expression $3x + 7$, the term $3x$ contains a variable x and a number 3 which is multiplied with x. In this term 3 is the coefficient of x.

> In algebraic expression, the symbol or number appearing as multiple of a variable used in algebraic term is called its coefficient. e.g. in $4x$, 4 is the coefficient of x.

Similarly, in algebraic expression $4a - 3c$, 4 is the coefficient of a and -3 is the coefficient of c.

Similarly, Coefficient of x is 1, which is not written.
and the coefficient of $-x$ is -1, which is also not written.

> *Generally x, y and z are used to write variable and a, b, c are used to denote numbers.*

Know that the number, appearing in algebraic expression, independent of a variable is called a constant term (e.g. in x + 2 y + 5, number 5 is a constant term).

In the algebraic expression $3x + 7$ if $x = 10$, the value of $3x$ is 30 and for $x = 15$, the value of $3x$ is 45. In both cases the second term 7 of the expression $3x + 7$ does not change because 7 is a constant. Similarly in $2x + 9$, 9 is constant.

> In algebraic expression number appearing independent of a variable is called constant term.

Example: Write coefficients, constants and variables of the following expressions.

(1) $6z - 8$ (2) $2x + 3y + 8$

Solution:

(1) $6z - 8$

Here Coefficient is: 6

Constant is: –8

Variables are: z

(2) $2x + 3y + 8$

Here Coefficients are: 2, 3

Constant is: 8

Variable is : x, y

Differentiate between like and unlike terms

The terms having same variable are called 'like terms'. In like terms coefficients and their signs may differ.

For example, $2x$, $-3x$ and $\frac{5}{2}x$ are like terms.

The terms having different variables are called 'unlike terms'.

For example $3x$ and 4y are unlike terms. Similarly -3ℓ and 5m are unlike terms.

Like terms can be combined to give a single term.

As $2x + 3x = 5x$

Here two like terms $2x$ and $3x$ have been combined to give a single term $5x$.

So, Addition or subtraction can not be performed with unlike term

As unlike term 5x and 7y can not be combined to give single term i.e. 5x + 7y can not be further simplified.

EXERCISE 8.2

1. Write the number of terms of the following expressions.

(i) $3 \quad m \quad n$ (ii) p (iii) $2x + 3y$

(iv) $4y + 7$ (v) $4\ell + 3m + 9$ (vi) $7x + 5 + 8$

2. Write the variables of the following expressions.

(i) $8x + 3$ (ii) $-7x + 1$ (iii) $6x + y$ (iv) $\frac{2x}{y}$

(v) $2xy$ (vi) $5 \quad 3yz$ (vii) $3xyz$ (viii) $3xy + 9yz$

3. Write the constants of the following expressions.

(i) $2x + 2$ (ii) $-3 + x$ (iii) $xy + 1$

(iv) $xy + yz + \frac{1}{2}$ (v) $5x + 9$ (vi) $2x + 2y + 10$

4. Write the coefficients of variables of the following.

(i) $-7x$ (ii) $5y$ (iii) $2x + 3y$

(iv) $x + \frac{1}{2}y - \frac{1}{4}z$ (v) $5x + 9$ (vi) $\frac{1}{4}x + 6y + 1$

5. Write like terms in the following.

$2xy$, $4\ell m$, $-7xz$, $140xy$, $13mn$, xz, $\frac{1}{5}xy$, $-9xyz$, $\frac{1}{4}xy$, $-2p$, $\frac{1}{3}xy$, $-2xy$, $5xy$, xyz, $46p$, $5\ell m$

Add and subtract given algebraic expressions

1. Addition of algebraic expressions

We know that $x + x = 2x$ and $x + x + x = 3x$

Here coefficient shows how many times the variable is added.

$$2x + 3x = (x + x) + (x + x + x)$$
$$= x + x + x + x + x = 5x$$

Thus, to add like terms only their coefficients are added and bases and exponents remain the same.

Let us see some examples.

Example 1: Add $7x$ and $5x$

Solution:

Writing the terms vertically:

$$\begin{array}{r} 7x \\ +\ 5x \\ \hline 12x \\ \hline \end{array}$$

Writing the terms horizontally:

$7x + 5x = (7 + 5) = 12x$

It should be noted that in the addition of $7x$ and $5x$ only coefficients are added and variable x remains the same.

Example 2: Find the sum of $3a + 2b + c$ and $8c + 6b + a$

Solution:

Writing the terms vertically:

$$\begin{array}{r} +\ 3a + 2b + c \\ +\ a + 6b + 8c \\ \hline 4a + 8b + 9c \\ \hline \end{array}$$

Writing the terms horizontally:

$$\begin{aligned} \text{Sum} &= (3a + 2b + c) + (a + 6b + 8c) \\ &= 3a + 2b + c + a + 6b + 8c \\ &= 3a + a + 2b + 6b + c + 8c \\ &= 4a + 8b + 9c \end{aligned}$$

Example 3: Add the following:

Solution: $x + 2xy + y,\ 3x + 5xy,\ 5x + xy + 7y$

$$\begin{array}{r} x + 2xy + y \\ 3x + 5xy \quad\ \ \\ 5x + xy + 7y \\ \hline 9x + 8xy + 8y \\ \hline \end{array}$$

- While adding vertically like terms are arranged in same column and then added.
- While adding horizontally like terms are arranged together and added.

2. Subtraction of Algebraic Expressions

We know that subtraction is the inverse operation of addition. For example to subtract 3 from 4 is equivalent to "add – 3 in 4".

$$\begin{aligned}(+4) - (+3) &= (+4) + (-3)\\ &= 1\end{aligned}$$

So to subtract a number from a given number simply change the sign of subtrahend (from – to + and + to –) and add it in the given number.

Example 1: Subtract $6x$ from $9x$.

Solution:

Writing the terms horizontally:

$$\begin{aligned}\text{Difference} &= (+9x) - (+6x) = (+9x) + (-6x)\\ &= 9x - 6x = 3x\end{aligned}$$

Writing the terms vertically:

$$\begin{array}{r} 9x \\ \underline{\underset{-}{+}\, 6x} \\ \underline{3x} \end{array}$$

Example 2: Subtract $-6y$ from $10y$.

Solution:

Writing the terms horizontally:

$$\begin{aligned}\text{Difference} &= (+10y) - (-6y) = (+10y) + (+6y)\\ &= 10y + 6y = 16y\end{aligned}$$

Writing the terms vertically:

$$\begin{array}{r} 10y \\ \underline{\mp\, 6y} \\ \underline{16y} \end{array}$$

Example 3: Subtract $3x - 2y$ from $8x + 4y$

Solution: $(8x + 4y) - (3x - 2y) = 8x + 4y - 3x + 2y$

$$\begin{aligned}&= (8x - 3x) + (4y + 2y)\\ &= 5x + 6y\end{aligned}$$

This question can also be solved as under:

$$\begin{array}{r} +\,8x + 4y \\ \underline{\pm\, 3x \mp 2y} \\ \underline{5x + 6y} \end{array} \quad \text{(by changing signs)}$$

Example 4: Subtract $5x + 2y$ from $8x + 5y$

Solution:

$$\begin{array}{r} +8x+5y \\ \underline{\pm 5x \pm 2y} \\ \underline{3x+3y} \end{array} \quad \text{(by changing signs)}$$

EXERCISE 8.3

1. Add the following expression.

(i) $2x, x, 4x$ (ii) $2a, 3a, 6a, a$

(iii) $8\ell m, 4\ell m, \ell m, 6\ell m$ (iv) $2xy, 4xy, xy, 6xy, 3xy$

2. Simplify:

(i) $2x + 9x$ (ii) $a + 2a + 3a$

(iii) $x + 3x + 6x + 10x$ (iv) $2st + 3st + 5st + 7st$

3. Add:

(i) $3a + 2b, a + b, 4a$ (ii) $6x + 5y + 7z, 2x + 3y + z, y + 2x$

(iii) $pq + qr + pr, qr + 4pr + 2pq, 3pq + 2pr$

(iv) $3c + 4d + 5f, 5c + 7d + 6f$

4. Subtract

(i) $5x$ from $8x$ (ii) $-3y$ from $9y$

(iii) $2x + 3y$ from $6x + 8y$

(iv) Subtract $-20f + 30g + 40$ from $20 + 10f + 20g$.

5. Perform the subtraction.

(i) $\begin{array}{r} 8x \\ \underline{+5x} \\ \underline{\quad\quad} \end{array}$ (ii) $\begin{array}{r} 12ab \\ \underline{-9ab} \\ \underline{\quad\quad} \end{array}$ (iii) $\begin{array}{r} 5z \\ \underline{+3z} \\ \underline{\quad\quad} \end{array}$

(iv) $\begin{array}{r} 10xy \\ \underline{-17xy} \\ \underline{\quad\quad} \end{array}$ (v) $\begin{array}{r} 2x+7y \\ \underline{-3x+5y} \\ \underline{\quad\quad} \end{array}$ (vi) $\begin{array}{r} 2x+15xb+9y \\ \underline{+\ x+20xb+3y} \\ \underline{\quad\quad} \end{array}$

UNIT 8

INTRODUCTION TO ALGEBRA
(Algebraic Expression)

Simplify algebraic expressions grouped with brackets

We know that brackets are used to indicate the order for performing operations. The four kinds of brackets are:

(i) " — " is called a bar or vinculum.

(ii) "( )" is called a round or curved brackets or parentheses.

(iii) "{ }" is called a curly brackets or braces.

(iv) "[ ]" is called box brackets or square brackets.

In algebra, sometimes, we can't simplify an expression into a single term within the brackets. For example, in $2x - (x + y)$ we can't simplify the expression $(x + y)$. For such a situation.

(i) Expand the brackets

(ii) Simplify the whole expression as given below:
$2x - (x + y) = 2x - x - y = x - y$

Hence, $x - y$ is the simplest form of the above mentioned algebraic expression.

–ve sign before the brackets means, change the signs of all the terms within brackets, i.e. $-(a + b) = -a - b$.

Example 1: Simplify the following.

(i) $[5a - \{3b + (6a - 2a + b)\}]$

(ii) $[2a + \{c - a + (a + 2b + c)\}]$

(iii) $xy - [yz - \{zx + xy + 9yz - \overline{zx + xy})\}]$

Solutions:

(i) $[5a - \{3b + (6a - 2a + b)\}]$
$= [5a - \{3b + (4a + b)\}]$
$= [5a - \{3b + 4a + b\}]$
$= [5a - \{4a + 4b\}]$
$= [5a - 4a - 4b]$
$= [a - 4b] = a - 4b$

(ii) $[2a + \{c - a + (a + 2b + c)\}]$
$= [2a + \{c - a + (a + 2b + c)\}]$
$= [2a + \{c - a + a + 2b + c\}]$
$= [2a + \{2b + 2c\}]$
$= [2a + 2b + 2c]$
$= 2a + 2b + 2c$

(iii) $xy - [yz - \{zx + xy + (yz - \overline{zx + xy})\}]$

$= xy - [yz - \{zx + xy + (yz - zx - xy)\}]$

$= xy - [yz - \{\cancel{zx} + \cancel{xy} + yz - \cancel{zx} - \cancel{xy}\}]$

$= xy - [yz - yz] = xy - [\cancel{yz} - \cancel{yz}] = xy - 0 = xy$

Example 2:

Simplify the following expression

$$25 - [-7a - \{-6a + (3 - 5 - 6a)\}]$$

Solution:

$= 25 - [-7a - \{-6a + (3 - 5 - 6a)\}]$

$= 25 - [-7a - \{-6a + (-2 + 6a)\}]$

$= 25 - [-7a - \{-6a - 2 + 6a)\}]$

$= 25 - [-7a - \{-2\}]$

$= 25 - [-7a + 2\}]$

$= 25 + 7a - 2$

$= 7a + 23$

Evaluate and simplify an algebraic expression when the values of variables involved are given

The process of finding the value of an expression by using numbers in place of variables is called evaluation.

Some examples of evaluation are given as under.

Example: If $a = 2$, b = –3 and c = –4, then evaluate the following.

(i) $ab + bc + ca$ (ii) $\dfrac{bc\ \ (b - c)}{a}$

Solution:

(i) $ab + bc + ca$

$= 2 \quad (-3) + (-3) \quad (-4) + (-4) \quad 2$

$= -6 + 12 - 8$

$= -14 + 12 = -2$

(ii) $\dfrac{bc\ \ (b - c)}{a}$

$= \dfrac{(-3) \quad (-4)\ [(-3) - (-4)]}{2}$

$= \dfrac{12\ [-3 + 4]}{2} = \dfrac{12 \quad 1}{2} = 6$

EXERCISE 8.4

1. Simplify the following expressions.

(i) $[x + x + (y + y + 2x)]$

(ii) $[5a - \{3a + (3b - 6b + 4a)\}]$

(iii) $[2a - \{5b - 3\ (2a + \overline{b - 3a})\}]$

(iv) $5\ell - [2 - (3 + 11\ell + 3m + 8\ell)]$

(v) $2\ (x + y) - 3\ [4x + \{7x + 3y + (\overline{5x - 4y})\}]$

2. (i) If $x = 2$, find the values of $3x$ and $-5x$

(ii) If $y = -3$, find the values of

(a) $-3y$ (b) $5y + 7$ (iii) $\frac{5}{3}y$ (iv) $\frac{6}{24}y + 9$

3. If $a = -3$, $b = 5$ and $c = -2$, evaluate the following:

(i) $a + b + c$ (ii) $2a - 3b + c$

(iii) $\frac{a - bc}{c}$ (iv) $\frac{2a + b + c}{abc}$

(v) $\frac{a + b}{c} + \frac{b + c}{a}$ (vi) $\frac{2a + b - c}{a - 3b + c}$

(vii) $\frac{a(2b + 3c)}{3ca}$ (viii) $a + 3abc - b + c$

(ix) $\frac{3a + 4b + 5c}{a + b - c}$ (x) $\frac{a}{b} - \frac{b}{c} + \frac{c}{a} - \frac{a}{c}$

4. If $x = 2$, $y = 3$ then evaluate the following expression.

$8x + [3y - \{6x + (5 - 4x)\}$

5. Abid purchased 5 bags at the rate of Rs x per bag. After paying he has left Rs 3.

Then,

(i) Find the cost of 5 bags.

(ii) Write an algebraic expression for paying the cost of 5 bags and Rs 3 left with him.

(iii) If $x = 30$, find the total amount that Abid had.

REVIEW EXERCISE 8

1. Write in symbols.

(i) Cost of x books, when price of one book is Rs 18.

(ii) 9b plus $5a$ (iii) 3 times x plus 5

(iv) Twice of y minus two third (v) 6 more than p

2. Write the following in words.

(i) $x + 2$ (ii) $3y - 4$ (iii) $x + 5$

(iv) $s + 2t$ (v) $\frac{x - y}{2}$

3. Write the number of terms in each of the following expression.

(i) $2a + 5b$ (ii) $16x$ (iii) $2bc + 2a - c$

(iv) $3xy - 8x + 6y - 2$ (v) $6x \quad 5y + 2z - 6$

4. Write three expressions containing one term, two terms and three terms respectively.

5. Simplify: (i) $4xy + 2xy + \frac{2}{3}xy$ (ii) $9ab + 3ab + \frac{3}{5}ab$

6. Add: (i) $3x + 4y,\ 5x + 9y$

(ii) $2d + 5c + 3b,\ 7c + 5d + 9b$

(iii) $12xy + 3x + 4y,\ 5x + 6y + 8xy$

7. Subtract $x + 3y$ from $2x + 7y$

8. Subtract $x + 3y + 5z$ from $2x - 15y - 9z$

9. Which of the following are true or false statements?

(i) $13 - 5 = 3$ (   )

(ii) $13 - 5 = 5$ (   )

(iii) $13 - 5 = 8$ (   )

10. Find the value of variable which makes the statement true.

(i) $x = 15 - 6$ (ii) $y - 8 = 5$ (iii) $14 = b + 8$

(iv) $4x - 10 = 2$ (v) $12 - 2m = 8$

SUMMARY

- Algebra is generalized arithmetic in which letters (called variables) are also used as numbers.
- A sentence is a set of words making a complete grammatical structure and conveying full meaning.
- A sentence is called TRUE if it satisfies the given condition and FALSE if it does not satisfy the condition.
- A statement is called OPEN if it is either true or false.
- An algebraic expression is the combination of variables, numerals and fundamental operation.
- The number appearing before a variable in a term of algebraic expression is called its **coefficient**.
- The terms having the same variables are called **like terms** otherwise, they are **unlike terms**.
- To add two (or more) algebraic expressions, add their like terms according to the rules for the addition of integers.
- To subtract one expression from the other, change the sign of each term of the expression to be subtracted and add to the expression from which the subtraction is to be performed.
- If an algebraic expression involves two or more than two operations, it is simplified following the rules of BODMAS.
- To evaluate any algebraic expression means to find its value after substituting the given numerical values of each variable in the expression.

UNIT 9

LINEAR EQUATIONS

9.1 ALGEBRAIC EXPRESSIONS

Define an algebraic equation

An algebraic sentence involving the sign of equality "=" is called an algebraic equation or simply an equation e.g.

(i) $6 - 2 = 4$
(ii) $2x + 4 = 8$

are examples of an algebraic equation.

An equation can be compared to the pans of common balance in equilibrium. The two sides of the equation are like the two scale pans and the equality sign shows that scale pans are balanced.

In equation '$2x + 4 = 8$' the left hand side $2x + 4$ is equal to the right hand side 8.

Differentiate between equation and an expression

An algebraic expression is a combination of numbers, variable and arithmetical operations.

Thus $2a$, y, $31 + z$, $2x - 3y$ 4 are all examples of algebraic expressions. Whereas two expressions connected by sign of equality is called an equation. In an equation, the two expressions are equated with the symbol "=".

$2x + 3 = 9$, $x - y = 7$ are examples of equation.

Teacher's Note

Teacher should bring real common balance in class or by any available material make a balance and explain equations by putting different objects or number cards in it.

9.2 LINEAR EQUATIONS

Define linear equation in one variable

An equation in which there is only one variable with degree one, is called linear equation in one variable.

Construct linear expression and linear equation in one variable

We know that an algebraic expression is a combination of numbers, variables and arithmetical operations.

Few examples of expression:

(i) Four is added to twice of a number i.e. twice of a number plus four is: $2x + 4$

(ii) Five is subtracted from thrice of a number i.e. thrice of a number minus five is: $3y - 5$.

(iii) 6 is multiplied by a number i.e. six times of a number is: $6z$

(iv) A number is divided by 8 i.e. eight part of a number is: $\frac{m}{8}$

(v) Two is added to thrice of a number i.e. thrice of a number plus two is: $3n + 2$

(vi) Five times a number is divided by 7 i.e. seventh part of five times of number is: $\frac{5a}{7}$

All the above are algebraic expressions in one variable.

We also know that an algebraic equation is a statement involving the sign of equality. Let us learn how to construct a linear equation.

(i) Four is added to twice of a number the result is 10 i.e. $2x + 4 = 10$

(ii) Three subtracted from thrice of a number the result is 1 i.e. $3y - 3 = 1$

(iii) Three multiplied by a number is 9 i.e. $3z = 9$

(iv) A number divided by 8 is 3 i.e. $\frac{m}{8} = 3$

(v) Two added to thrice of a number is 5 i.e. $3n + 2 = 5$

(vi) Five times of a number divided by 7 is 10 i.e. $\frac{5a}{7} = 10$

All these are linear equations in one variable.

Example 1: Construct the linear equations in one variable of the following.

(i) Four times a number increased by 5 equals 11.

(ii) If a number is subtracted from another number which is 5 more than thrice the first number, the result is 31.

Solution:

(i) Let x be the number. Then four times of x is $4x$ and increasing this by 5 gives $4x + 5$ which is equal to 11. Hence the required equation is:

$$4x + 5 = 11$$

(ii) Let x denote the number.

Then thrice the number is $3x$. Now other number which is 5 more than $3x$ is $3x + 5$. The difference of $3x + 5$ and x is $(3x + 5) - x$ which is equal to 31.

Hence the required equation is:

$$(3x + 5) - x = 31$$

EXERCISE 9.1

Construct the following linear equations.

(1) The difference of a number and twenty is thirty two.

(2) The sum of a number and 8 is 15.

(3) Five multiplied by a number is 35.

(4) Sixteen divided by four is equal to 4.

(5) A number increased by two is 4.

(6) Fifteen decreased from a number, the result is 5.

(7) Twice of a number increased by 4 is 20.

(8) My age is 15 years greater than my brother's age.

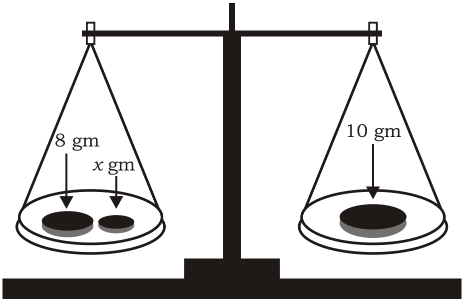
.

Solve simple linear equations involving fractional and decimal coefficient like $\frac{1}{2}x + 5 = x - \frac{1}{3}$.

Consider a problem in which there is weight of 10 gm on one pan of physical balance and there are two weights on other pen one is 8gm and other is unknown.

Let us represent the unknown weight by x. So this problem is written as: $x + 8 = 10$ to keep the balance.

If we put $x = 2$, then $2 + 8 = 10$ is true statement. Thus, a weight of 2 grams is placed with the weight of 8 grams to keep the scale in balance.



Here the sentence $x + 8 = 10$ is called equation and 2 is its solution.

This equation contains one variable and its degree is also one. Such equation is called ***linear equation*** in one variable as we have already studied.

Following are some examples of linear equations.

$x = 4$, $x + 8 = 15$, $5y = 30$ and $5 - 2 = 3t + 8$

> An open sentence which contains the symbol "=" is called equations. The value of the variable which makes the equation true, is called the solution of the equation.

Activity **Let us solve $x + 5 = 8$. Putting different values of x, we get.**

For $x = 1$, we get $1 + 5 = 8$, which is a false statement.

For $x = 2$, we get $2 + 5 = 8$, which is a false statement.

For $x = 3$, we get $3 + 5 = 8$, which is a true statement.

Construct any two linear equation in one variable.

LINEAR EQUATIONS

Example 1: Solve $x + 5 = 11$

Solution:

$$x + 5 = 11$$

Subtracting 5 from both sides, we get.

$$x + 5 - 5 = 11 - 5$$

or $x + 0 = 6$

or $x = 6$

> Same number can be subtracted from both sides of the equation.

Example 2: Solve $x - 8 = 21$

Solution: $x - 8 = 21$

Adding 8 to both sides we get

$$x - 8 + 8 = 21 + 8$$

or $x = 29$

checking $x - 8 = 21$

$29 - 8 = 21$ (by putting $x = 29$)

or $21 = 21$ which is a true sentence.

> Same number can be added to the both sides of the equation.

Example 3: Solve: $7x = 84$

Solution: $7x = 84$

Dividing both sides by 7, we get

$$\frac{7x}{7} = \frac{84}{7}$$

or $x = 12$

checking $7x = 84$

$7 \quad 12 = 84$ (by putting $x = 12$)

or $84 = 84$ which is a true sentence.

> Both sides of equation can be divided by the same non-zero number.

Example 4: Solve $\frac{x}{3} = 7$

Solution: $\frac{x}{3} = 7$

Multiplying both sides by 3, we get

$\frac{x}{3} \quad 3 = 7 \quad 3$

or $x = 21$

checking: $\frac{x}{3} = 7$

$\frac{21}{3} = 7$ (by putting $x = 21$)

or 7 = 7 which is a true sentence

Both sides of equation can be multiplied by the same number.

Both sides of the equation can be interchanged

Let us solve some more equations.

Example 5: Solve: 3t + 5 = 20

Solution: 3t + 5 = 20

3t + 5 – 5 = 20 – 5 (subtracting 5 from both sides)

or 3t = 15

or $\frac{3t}{3} = \frac{15}{3}$ (Dividing both sides by 3)

or t = 5

Verification

3t + 5 = 20

By using t = 5

we get 3 (5) + 5 = 20

or 15 + 5 = 20

or 20 = 20

Which is true sentence hence verified.

Example 6: Find the solution of the following equations:

(i) $\frac{1}{2}a + 5 = a - \frac{1}{3}$ (ii) $\frac{x+1}{2} = \frac{x+4}{3}$ (iii) $\frac{3x+4}{5+2x} = 1$

Solution: (i) $\frac{1}{2}a + 5 = a - \frac{1}{3}$

multiplying both sides by L.C.M 6.

$$6\left(\frac{1}{2}a + 5\right) = 6\left(a - \frac{1}{3}\right)$$

$$\overset{3}{\cancel{6}}\left(\frac{1}{\cancel{2}}a\right) + 6(5) = 6(a) - \overset{2}{\cancel{6}}\left(\frac{1}{\cancel{3}}\right)$$

$$3a + 30 = 6a - 2$$
$$30 + 2 = 6a - 3a$$
$$32 = 3a$$
$$\text{or} \quad 3a = 32$$
$$a = \frac{32}{3}$$

Solution: (ii) $\frac{x+1}{2} = \frac{x+4}{3}$

$6\left(\frac{x+1}{2}\right) = 6\left(\frac{x+4}{3}x + 4\right)$ (Multiplying by LCM 6)

$$3(x+1) = 2(x+4)$$
$$3x + 3 = 2x + 8$$
$$3x - 2x = 8 - 3$$
$$x = 5$$

Solution: (iii) $\frac{3x+4}{5+2x} = 1$

$(\cancel{5+2x})\left(\frac{3x+4}{\cancel{5+2x}}\right) = (5+2x)\ 1$ (Multiplying both sides $(15+2x)$

$$3x + 4 = 1(5+2x)$$
$$3x + 4 = 5 + 2x$$
$$3x - 2x = 5 - 4$$
$$x = 1$$

Teacher's Note *Teacher should help the students in understanding the rules and solving linear equations at every step.*

EXERCISE 9.2

Solve the following equations.

(1)	$x - 7 = 8$	(2)	$y + 5 = 12$	(3)	$x + 2 = 8$
(4)	$7y = 14$	(5)	$3x = 30$	(6)	$y - 16 = 2$
(7)	$\frac{x}{4} = 3$	(8)	$6x + 1 = 49$	(9)	$8x = 32$
(10)	$7y + 2 = 44$	(11)	$6 = x + 9$	(12)	$9x - 21 = 78$
(13)	$\frac{6x-4}{2x+2} = 2$	(14)	$\frac{x}{2} + 4 = x - \frac{1}{3}$	(15)	$\frac{5x-4}{8} = \frac{x+6}{4}$

Solve real life problems involving linear equations

The method of solving word problems is explained through the following examples.

To solve a word problem remember the following steps.

(i) Read the problem carefully. Draw its outline and
(ii) Find what is asked.
(iii) Whatever is asked represent it by a variable x or y.
(iv) Construct an equation according to the given conditions.
(v) Solve the equation to get required answer.
(vi) Check the answer so obtained.

Example 1: Hashim and Qasim both deposited a sum of Rs 47 in the fund of Hilal-e-Ahmer. If the Hashim's share was Rs 25. Find the share of Qasim.

Solution: To solve this problem first read it carefully. In this problem share of Qasim is to be found.

Deposited by both = 47 rupees
Suppose the share of Qasim = x rupees
Share of Hashim = 25 rupees
Share of both = $x + 25$

Teacher's Note
Teacher should give more real life examples for practice to students to clear their concepts more effectively about linear equations.

So, according to the given condition of the problem, we have

$$x + 25 = 47$$

Let us solve it

$$x + 25 = 47$$

or $$x + 25 - 25 = 47 - 25$$

or $$x = 22$$

Hence share of Qasim is Rs 22.

Example 2: Zubeda and Farah jointly planed to go to visit Zoo for entertainment. They both collected Rs 44. Farah paid Rs 6 more than Zubeda. Find the share of Zubeda.

Solution:

Suppose Zubeda paid = x rupees

Farah Paid (Rs 6 more) = $x + 6$ rupees

Total amount = $x + (x + 6)$ rupees

Amount collected = 44 rupees

So, according to the condition of the problem

$$x + (x + 6) = 44$$

$$x + (x + 6) = 44$$

or $$x + x + 6 = 44$$

or $$2x + 6 = 44$$

or $$2x + 6 - 6 = 44 - 6$$

or $$2x = 38$$

or $$\frac{2x}{2} = \frac{38}{2}$$

or $$x = 19$$

Hence, Zubeda paid Rs 19

Example 3: If the sum of a number and 8 is multiplied by 5, then 60 is obtained. Find the number.

Solution: Let the required number be x.

Sum of number and 8 $= x + 8$
Sum multiplied by 5 $= 5(x + 8)$
Given product of sum and 5 $= 60$

So, according to the condition of the problem

$$5(x + 8) = 60$$

or $$5x + 40 = 60$$

or $$5x + 40 - 40 = 60 - 40$$

or $$5x = 20$$

or $$\frac{5x}{5} = \frac{20}{5}$$

or $$x = 4$$

Hence, the required number is 4.

EXERCISE 9.3

1. Sara stitches 44 mirror pieces on a red and green shirt. If 6 more mirror pieces are stitched on red shirt than green shirts, find the mirrors stitched on green shirt.

2. In a cricket match Jamil and Saleem enhanced the score of the team by 84 runs. If Jamil scored 12 runs more than Saleem. Find the score of the Saleem.

3. Sum of a number and its double is 9. Find the number.

4. The product of 7 and the sum of a number and 6 is 77. Find the number.

5. The sum of 6 and four times a number is 42. Find the number.

6. The sum of the ages of mother and her daughter is 22 years. The daughter is 20 years younger than mother. Find the age of her daughter.

7. When Dua opens a book there are two pages in front of her. The sum of the page numbers is 155. If one page number is 50, what is the other page number?

8. The price of a toy was decreased by Rs 7. If new price is 25 rupees. Find the original price.

REVIEW EXERCISE 9

1. Write the algebraic expression of the following:

(i) Sum of x and 4
(ii) A number less than 7
(iii) 9 times a number x
(iv) A number y divided by 6
(v) Sum of two numbers x and y
(vi) Product of a variable x with itself

2. Write an equation for each of the following:

(i) A number x decreased by 3 is 10
(ii) Thrice a number y increased by 5 is 17
(iii) Twice a number z more 4 is 20
(iv) Half of a number m subtracted from 7 is 4
(v) Sum of two consecutive even numbers is 16
(vi) After 8 years I will be thrice as old as I am now

3. Solve the following equations:

(i) $5x - 3 = 18$ (ii) $\frac{y+1}{2} = 5$ (iii) $\frac{m-1}{3} = 1$

(iv) $4y - 10 = 1$ (v) $\frac{p}{7} - 3 = 2$ (vi) $\frac{1}{2}q = 3$

4. A pole x metres long is broken into two parts, one part is 3 metres long and other part is $2x - 17$ metres long. Find the original length of the pole.

5. If a number is tripled and the result is increased by 5, we get 44. Find the number.

6. Twice a number added to half of itself equals 20. Find the number.

7. Thrice a number decreased by 5 exceeds the number by a unit. Find the number.

8. The sum of two consecutive even integers is 34. Find the smallest of them.

SUMMARY

- An algebraic sentence involving the sign of equality "=" is called an **algebraic equation**.
- An algebraic equation in which there is only one variable and degree one is called **Linear Equation in One Variable** or **Simple Linear Equation**.
- 'Solving a **Simple Linear Equation**' means to find that value of **unknown variable** which satisfies the given equation.
- Several real life problems are, first of all, **converted into mathematical form** i.e linear equations and then solved for the unknown by using addition, subtraction, multiplication and division rules.

UNIT 10

GEOMETRY

Geometrical shapes are found every where around us. Geometry is one of the oldest branch of mathematics which links mathematics with practical objects of our lives.

10.1 LINE SEGMENTS

Segment means a part. Line segment is a part of line. It has two distinct end points.

In the following figure we can see a line ℓ with points P and Q marked on it.

The part from P to Q of the above line ℓ is called the line segment PQ (denoted by $\overline{PQ}$ or $\overline{QP}$) and points P and Q are its end points.

How many Points are there on a line?

Line segment can be observed in our environment as edges of door, window, desk, board, table, copy, ruler etc.
Let us revise the differences between line, line segment and Ray with the help of following chart.

With respect to	Line	Line Segment	Ray
End points	No end point	Two end points	One end point
Length	No definite length	Definite length	No definite length
Symbol	Line AB or $\overleftrightarrow{AB}$	Line segment AB or $\overline{AB}$	Ray AB or $\overrightarrow{AB}$

Teacher's Note *Teacher should help the students to build the concept of line segment and differentiate it with line and ray.*

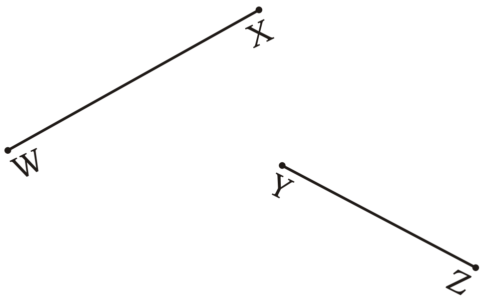
Add measures of two or more line segments

Adding measures of two or more line segments, means to find an another line segment whose length is the sum of measures of two given line segments.

Example: Add the given line segments

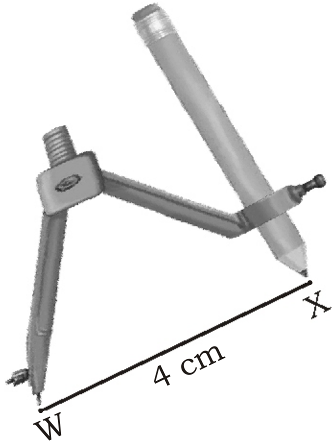
$m\overline{WX} = 4$ cm and $m\overline{YZ} = 3$ cm.

Using pair of compasses, pencil and ruler.



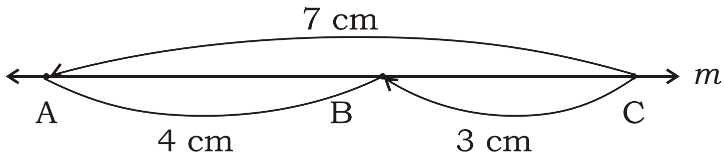
1. Draw a line m of any suitable length and mark a point A on it.

2. Use pair of compasses to measure the length of $\overline{WX} = 4$ cm.



3. Keep the radius equals to 4 cm, with centre at point A, draw an arc to cut line m at point B.

4. Repeat step 2 to get radius equal to 3 cm and with centre B, draw an arc to cut line m in the same direction at point C.



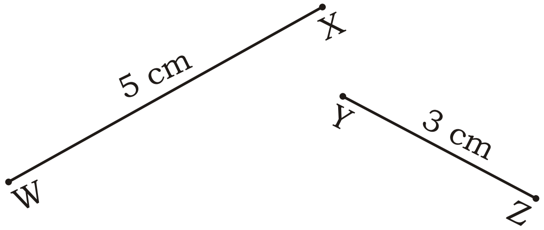
Therefore, $m\overline{AC} = m\overline{WX} + m\overline{YZ} = 4\text{ cm} + 3\text{ cm} = 7\text{ cm}$
Hence $\overline{AC}$ is the required line segment which shows the sum of measures of two line segments.

We can also add measures of more than two line segments in the same way.

Subtract measure of line segment from a longer one

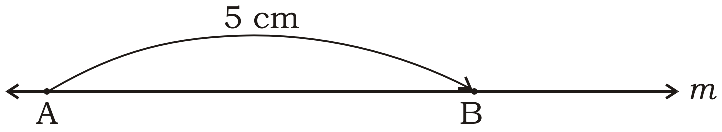
For subtracting measure of one line segment from a longer one, we are going to find another line segment whose measure is the difference of measures of two given line segments.

Example: Subtract the measure of line segment $\overline{YZ}$ from the measure of line segment $\overline{WX}$. Using pair of compasses, pencil and ruler.

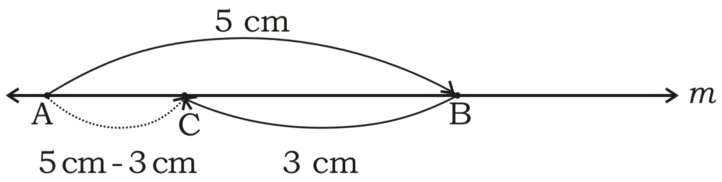


1. Draw a line m of any suitable length and mark a point A on it.

2. Measure the length of $\overline{WX}$ with centre A, draw an arc to cut line m at point B.



3. With centre B and radius equals to $\overline{YZ}$ draw another arc to cut line m at point C in the opposite direction.

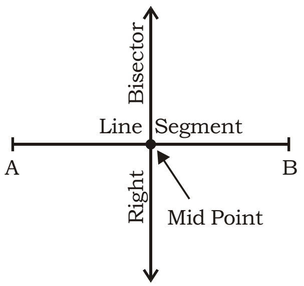


Therefore, $m\overline{AC} = m\overline{WX} - m\overline{YZ} = 5\text{ cm} - 3\text{ cm} = 2\text{ cm}$
Hence $\overline{AC}$ is the required line segment which shows the difference of a line segment from a longer one.

Draw a right bisector of a given line segment using compasses

Right bisector of a given line segment is a line which makes an angle of 90° with the given line segment and divide it into two equal parts.

In the given figure the line ℓ is the right bisector of $\overline{AB}$.



Note: Right bisector is also known as perpendicular bisector.

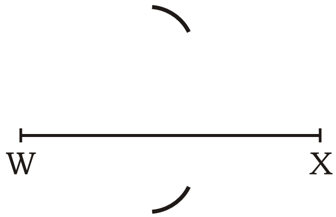
Teacher's Note *Teacher should help the students to understand about right bisector of a line segment.*

Example: Draw a right bisector of $\overline{WX}$ measuring 6 cm by using pair of compasses.

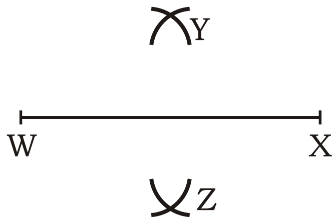
Consider the given line segment WX measuring 6 cm.

Now follow the following steps by using pair of compasses, pencil and ruler.

Steps of construction:



Step I: With centre W and radius more than half of $\overline{WX}$ (say 3.5 cm), draw two arcs on each side of $\overline{WX}$.

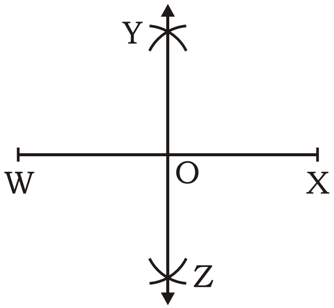


Step II: With centre X draw two arcs of same radius to cut each other at points Y and Z.

Step III: Draw $\overleftrightarrow{YZ}$ which cuts $\overline{WX}$ at point O.

Therefore $\overleftrightarrow{YZ}$ bisects the given line segment WX at point O.

as $m\overline{WO} = m\overline{OX}$.



Hence $\overleftrightarrow{YZ}$ is the required right bisector of $\overline{WX}$.

Note: $\overrightarrow{OY}$ and $\overrightarrow{OZ}$ are also right bisectors of $\overline{WX}$.

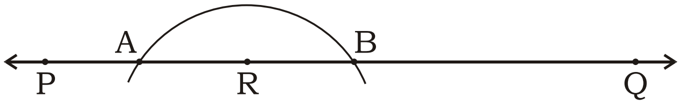
Draw a perpendicular to a given line from a point on it using compasses

Example: Draw a perpendicular to the line PQ at point R on it by using pair of compasses.

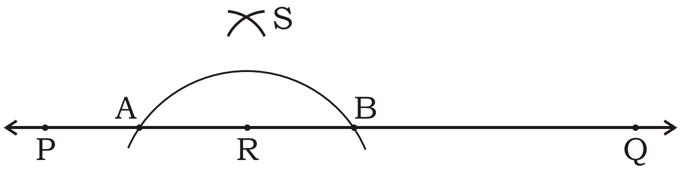
Steps of construction:

Step I: Draw a line PQ, mark point R on it.

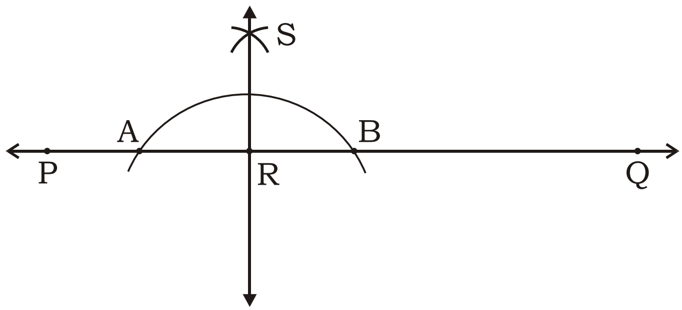
Step II: With centre R draw an arc of suitable radius to cut $\overleftrightarrow{PQ}$ at points A and B, such that $m\overline{RA} = m\overline{RB}$.



Step III: With centre A and B draw two arcs of radius more than $m\overline{RA}$, to cut each other at point S.



Step IV: Join point R and S, and extend it on both sides of line PQ to form $\overleftrightarrow{RS}$.



$\overleftrightarrow{RS}$ is the required perpendicular to $\overleftrightarrow{PQ}$ from a point R on it.

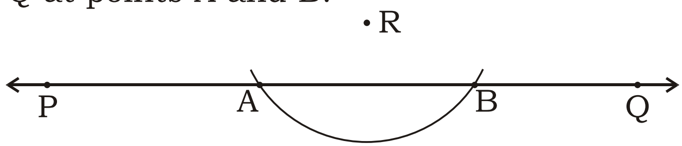
Draw a perpendicular to a given line, from a point outside the line, using compasses

Example: Draw a perpendicular to the line PQ from a point R out side the line by using pair of compasses.

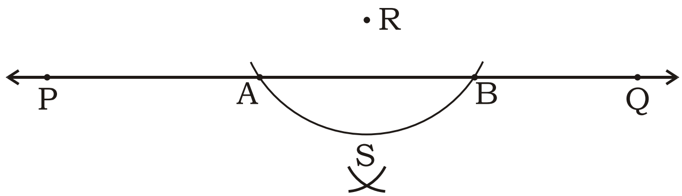
Steps of construction:

Step I: Draw a given line PQ, mark a point R outside it.

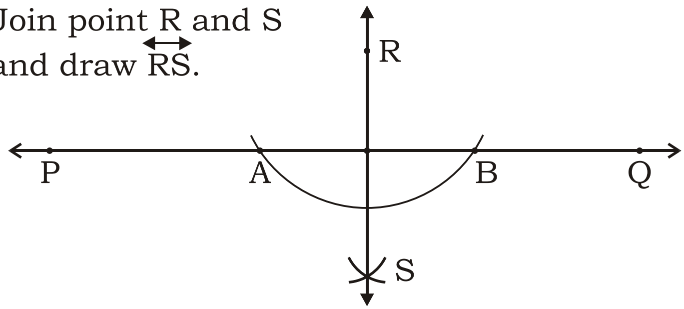
Step II: With centre R draw an arc of suitable radius to cut $\overleftrightarrow{PQ}$ at points A and B.



Step III: With centres A and B, draw two arcs of radius more than half of $\overline{AB}$, to cut each other at point S (in the opposition direction of R).



Step IV: Join point R and S and draw $\overleftrightarrow{RS}$.

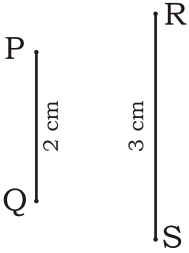


$\overleftrightarrow{RS}$ is the required perpendicular of $\overleftrightarrow{PQ}$ from a point R outside it.

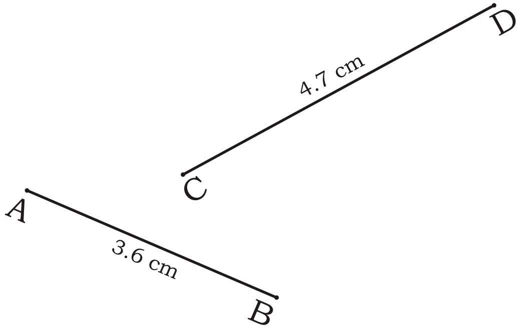
EXERCISE 10.1

1. Find the sum of measures of the following pairs of line segments geometrically.

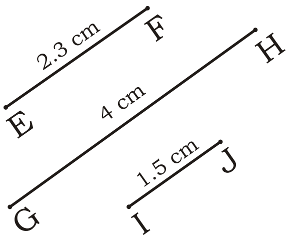
(i)



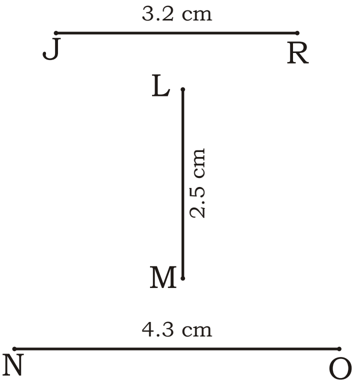
(ii)



(iii)

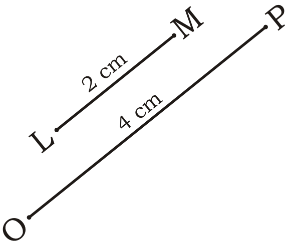


(iv)

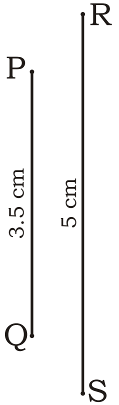


2. Find the difference of measures of the following line segments geometrically.

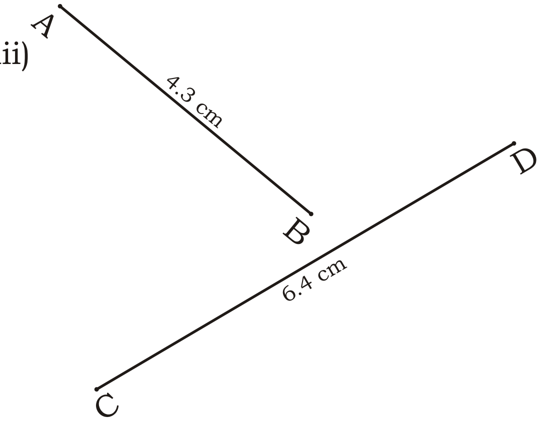
(i)



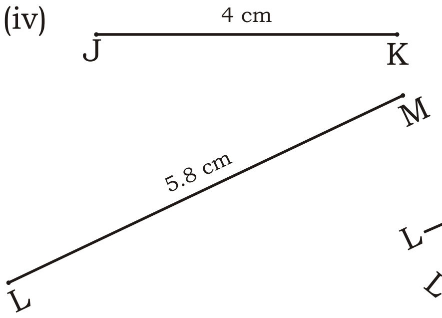
(ii)



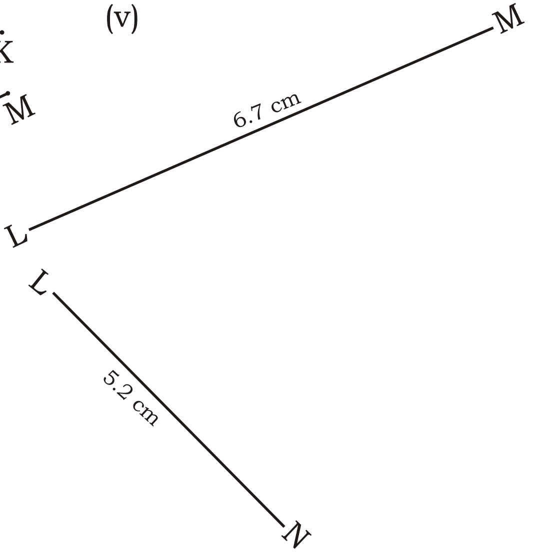
(iii)



(iv)



(v)

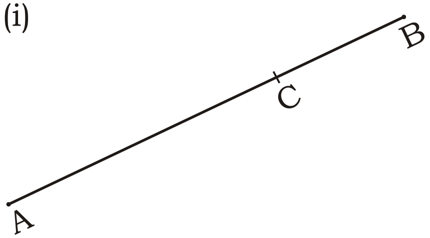


3. Draw right bisectors of the line segments of the following measures.

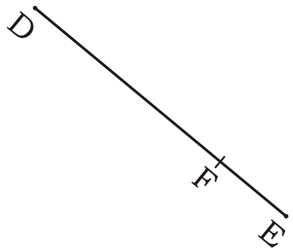
(i) 5 cm (ii) 6.3 cm (iii) 8 cm

4. Draw perpendicular from given points on the following line segments.

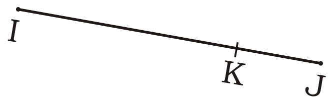
(i)



(ii)

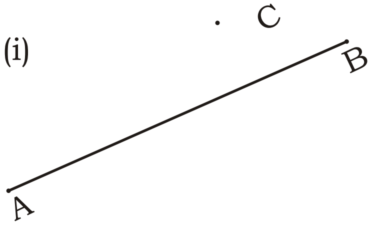


(iii)

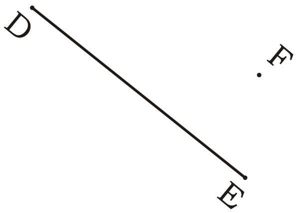


5. Draw perpendiculars from given points outside the following line segments.

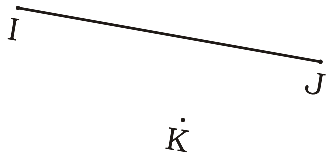
(i)



(ii)



(iii)

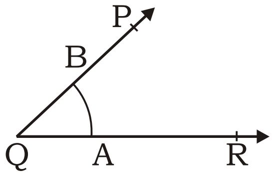


10.2 CONSTRUCTION OF ANGLES

We already know how to construct angles of various measures by using protractor. Let us learn the same with the help of compasses through examples.

Use compasses to construct an angle equal in measure of a given angle

Example: Construct an angle equal in measure of the angle PQR.



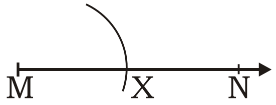
Consider the given angle PQR.

Steps of construction:

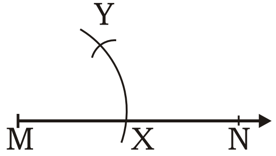
Step I: Draw $\overrightarrow{MN}$

Step II: With center Q draw an arc of any suitable radius, to cut $\overrightarrow{QR}$ and $\overrightarrow{QP}$ at points A and B respectively.

Step III: With centre M, draw an arc of radius equals to $m\overline{QA}$ to cut $\overrightarrow{MN}$ at point X.

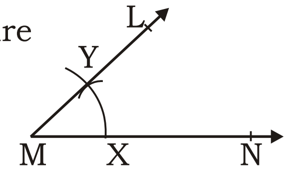


Step IV: With centre X and taking same radius draw an arc to cut the previous arc at point Y.



Step V: Join point M with Y and extend to the point L through point Y. Measure it with protractor as

m LMN = m PQR

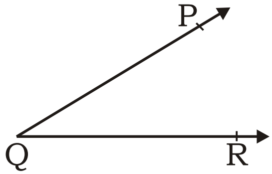


Hence LMN is the required angle.

Use compasses to construct an angle twice in measure of a given angle

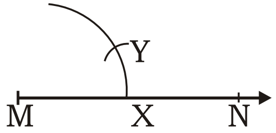
Example: Construct an angle twice in measure of the given angle PQR.

Consider the given angle PQR and measure it.

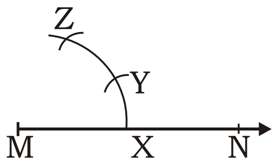


Steps of construction:

Step I: Now follow the steps from I to IV of the previous example and get the arc XY as shown in figure.



Step II: Taking same radius with centre Y, draw another arc to cut the previous arc at point Z.

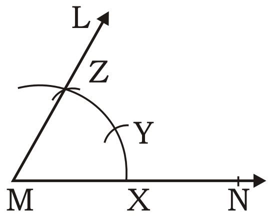


Step III: Draw $\overrightarrow{ML}$ passing through point Z.

Here LMN is the required angle as

m LMN = 2 m PQR.

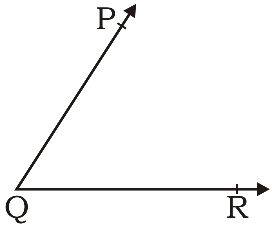
We can verify measure with the help of protractor as the required angle is double of angle PQR



Use compasses to bisect a given angle

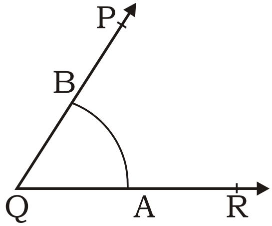
Example: Draw the bisector of the angle PQR.

Consider the given angle PQR.

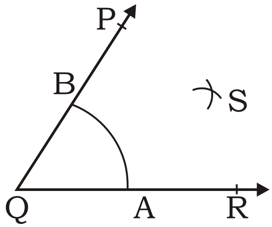


Steps of construction:

Step I: With centre Q draw an arc of any suitable radius to cut $\overrightarrow{QR}$ and $\overrightarrow{QP}$ at points A and B respectively.

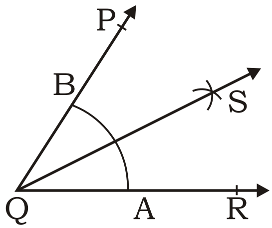


Step II: With centres A and B draw two arcs of suitable radius to cut each other at point S.



Step III: Join point Q with S and draw $\overrightarrow{QS}$.

Here $\overrightarrow{QS}$ bisects PQR in two equal angles i.e m PQS = m RQS



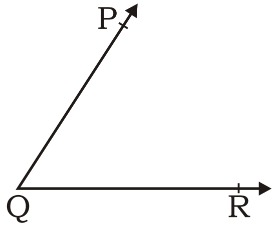
We can verify the result by measuring both the angles with protractor.

Teacher's Note *Teacher should give more examples to students for practice.*

Use compasses to divide a given angle into four equal angles

Example: Divide the angle PQR into four equal angles.

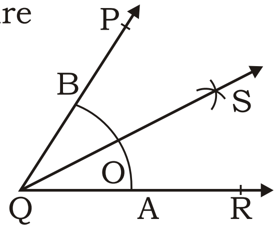
Consider the given angle PQR.



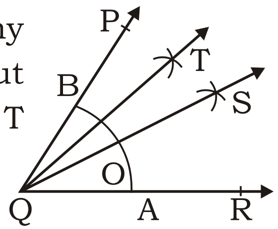
Steps of construction:

Step I: First of all bisect the given angle PQR in two equal angles PQS and RQS using compass as shown in the figure

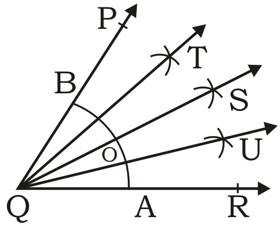
Step II: Name the intersecting point of $\overrightarrow{QS}$ as O.



Step III: With centres O and B and taking any suitable radius, draw two arcs to cut each other at point T. Join Q with T to get $\overrightarrow{QT}$.



Step IV: With centres O and A, repeat the same process to get $\overrightarrow{QU}$.



Now the given PQR has been divided into four equal angles.

i.e m PQT = m TQS = m SQU = m UQR

Note: We can verify measure with the help of protractor.

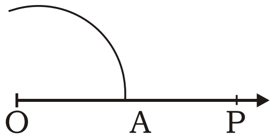
Use compasses to construct the following angles:
60°, 30°, 15°, 90°, 45°, $(22\frac{1}{2})^\circ$, 75°, $(67\frac{1}{2})^\circ$, 120°, 150°, 165°, 135°, 105°

1: Construction of angle of 60°:

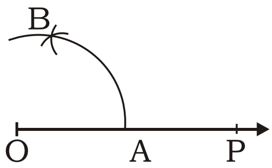
Angle of 60° is the basic angle in construction of angles by using pair of compasses.

Steps of construction:

Step I: Draw $\overrightarrow{OP}$.

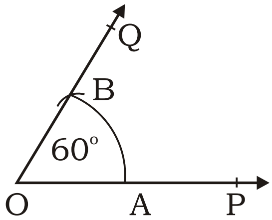


Step II: With centre O draw an arc of any suitable radius to cut $\overrightarrow{OP}$ at point A.



Step III: With centre A, draw another arc of the same radius to cut the previous arc at point B.

Step IV: Join points O and B and extend to the point Q.



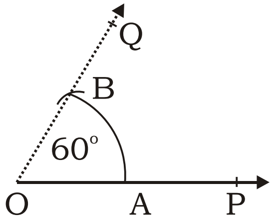
Hence POQ is the required angle of 60°,
i.e *m* POQ = 60°.

Note: We can verify it by using protractor.

2: Construction of angle of 30°:

As $\frac{60^\circ}{2} = 30^\circ$, so we construct an angle of 60° and bisect it in two equal angles of 30°.

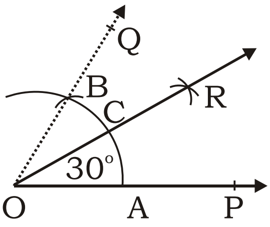
Steps of construction:



Step I: First construct an angle of 60° by using compass as shown in the figure.

Teacher's Note *Teacher should help the students in every step of constructing angles of different measures with the help of geometrical instruments on blackboard as well as in their copies.*

Step II: With centres A and B draw two arcs of any suitable radius to cut each other at point R.



Step III: Join O and R and to draw $\overrightarrow{OR}$ which cuts the arc AB at point C.

Here, *m* POR =30° is the required angle.

We can measure the angles POR and ROQ by using protractor.

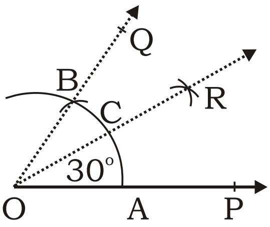
Note: As *m* QOP is bisected, so *m* QOR is also of 30°.

3: Construction of angle of 15°:

As $(\frac{30}{2})^{\circ} = 15^{\circ}$, so we construct an angle of 30°and bisect it to get the angle of 15°.

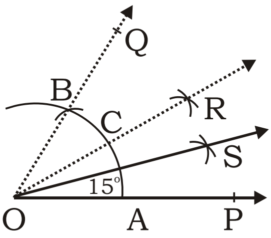
Steps of construction:

Step I: First construct angle of 30° as shown in the figure.



Step II: Name the intersecting point of $\overrightarrow{OR}$ as C.

Step III: With centres C and A and draw two arcs of any convenient radius to cut each other at point S.



Step IV: Join O to S to get $\overrightarrow{OS}$

Here *m* SOP = 15° is the required angle.

Measure the angles SOP and ROS by using protractor.

Note: As ROP is bisected, so *m* ROS is also of 15°.

4: Construction of angle of 90°:

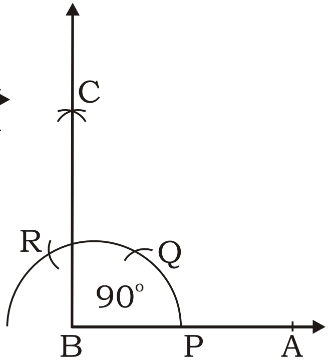
As $60^\circ + 30^\circ = 90^\circ$, so we construct an angle of 60° twice and then bisect its outer angle to get the angle of 90°.

Steps of construction:

Step I: Draw $\overrightarrow{BA}$.

Step II: With centre B and any convenient radius, draw an arc to cut $\overrightarrow{BA}$ at point P.

Step III: Keeping the radius same with centre P, draw another arc to cut the previous arc at point Q. (It is the mark for our first 60° angle).



Step IV: With centre Q and same radius, draw another arc to cut the previous longer arc at point R (It is the mark for 2nd angle of 60°).

Step V: With centres Q and R and same radius, draw two arcs to cut each other at point C (Bisecting the 2nd angle of second 60° to get measure of 30°).

Step VI: Join points B and C and extend to form $\overrightarrow{BC}$.

Here *m* ABC = 90° is the required angle.

Measure the angle ABC by using protractor.

What is the other way to draw a 45° angle?

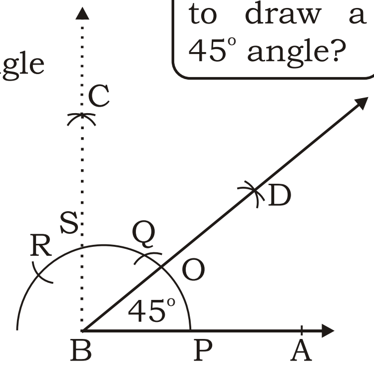
5: Construction of angle of 45°:

As $\left(\frac{90}{2}\right)^\circ = 45^\circ$, so we will construct an angle of 90° and bisect it to get the angle of 45°.

Steps of construction:

Step I: First construct angle of 90° as shown in the figure.

Step II: With the centre B draw an arc PS to cut $\overrightarrow{BA}$ at P and $\overrightarrow{BC}$ at S.



Step 3: With centres S and P and any convenient or similar radii, draw two arcs to cut each other at point D.

Step 4: Join point B to D and extend it to get $\overrightarrow{BD}$.

Measure the angle ABD by using protractor as measurement is 45°.

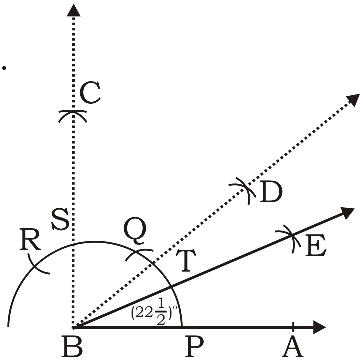
m ABD = 45°. It is the required angle.

Note: As ABC is bisected, so CBD is also of 45°.

6: Construction of angle of $(22\frac{1}{2})^\circ$:

As $(\frac{45}{2})^\circ = (22\frac{1}{2})^\circ$, so we construct an angle of 45°.

and bisect it equally to get the angle of $(22\frac{1}{2})^\circ$.



Steps of construction:

First construct angle of 45° and follow the following steps:

Step I: Name the intersecting point of $\overrightarrow{BD}$ as T.

Step II: With centres T and P and any convenient similar radii, draw two arcs to cut each other at point E.

Step III: Join B to E and extend it to get $\overrightarrow{BE}$.

Hence *m* ABE = $(22\frac{1}{2})^\circ$ is the required angle.

Note: As ABD is bisected, so, DBE is also of $(22\frac{1}{2})^\circ$.

7: Construction of angle of 75°:

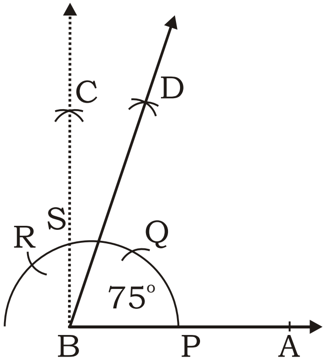
As 90° – 15° = 75°, so we will first construct a 90° angle then subtract 15° from it by bisecting the angle of 30° between 60° and 90° angles to get the angle of 75°.

Steps of construction:

Step I: First construct an angle of 90° as shown in the figure.

Step II: Name the intersecting point of arc PR with $\overrightarrow{BC}$ as S.

Step III: With centres S and Q draw two arcs of equal radius to cut each other at point D.



Step IV: Join point B to D and extend it to get $\overrightarrow{BD}$.

Measure the angle ABD by using protractor as $m\angle ABD = 75°$. Hence it is the required angle.

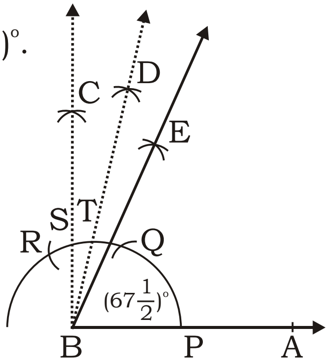
8: Construction of angle of $(67\frac{1}{2})°$:

As $75° - (7\frac{1}{2})° = (67\frac{1}{2})°$, so we will first construct an angle of 75° and then subtract $(7\frac{1}{2})°$ from it by bisecting the angle of 15° between 60° and 75° angles to get the angle of $(67\frac{1}{2})°$.

Steps of construction:

Step I: First construct an angle of 75°. as shown in the figure.

Step II: Name the intersecting point of arc PR with $\overrightarrow{BD}$ as T.



Step III: With centres T and Q draw two arcs of any same radius, to cut each other at point E. (Which gives us $(7\frac{1}{2})^\circ$ measure after 60°)

Step IV: Join point B with E and extend it to get $\overrightarrow{BE}$.

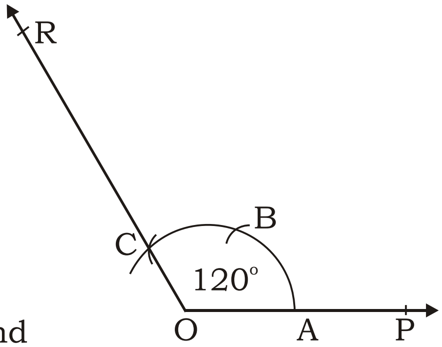
Measure the angle ABE by using protractor m ABE = $(67\frac{1}{2})^\circ$.
Hence it is the required angle.

9: Construction of angle of 120°:

As $60^\circ + 60^\circ = 120^\circ$, so we will construct 60° angle twice to get the angle of 120°.

Steps of construction:

Step I: Follow the process of constructing angle of 60°.



Step II: With cetnre B and keeping the radius same, draw another arc to cut the previous large arc AB at point C.

Step III: Join point O to C and extend it to form $\overrightarrow{OR}$.

Measure the angle POR by using protractor m POR = 120°.
Hence it is the required angle.

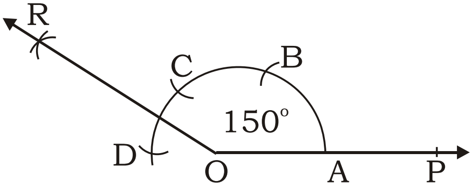
10: Construction of angle of 150°:

As $180^\circ - 30^\circ = 150^\circ$, so we will construct an angle of 60° thrice to get an angle of 180° and then subtract 30° from it by bisecting the last portion of 60° between 120° and 180° to get the angle of 150°.

Steps of construction:

Step I: First construct an angle of 120°.

Step II: With centre C and keeping the radius same, draw another arc to cut the arc AC thrice at point D. (It will give us the angle of 180°).



Step III: With centres C and D and some convenient similar radius, draw two arcs to cut each other at point R.

Step IV: Join point O to R and extend it to form $\overrightarrow{OR}$.

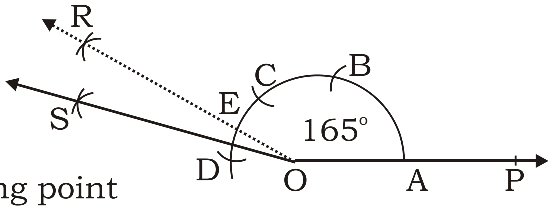
Measure the angle POR by using protractor $m\angle POR = 150°$.
Hence it is the required angle.

11: Construction of angle of 165°:

As 150° + 15° = 165°, so we will first construct an angle of 150° and bisect 30° of its outer part to get the angle of 165°.

Steps of construction:

First construct an angle AOR of 150° as shown in the figure and follow the following steps:



Step I: Name the intersecting point of the arc AD with $\overrightarrow{OR}$ as E.

Step II: With centres E and D draw two arcs of any convenient same radii, to cut each other at point S.

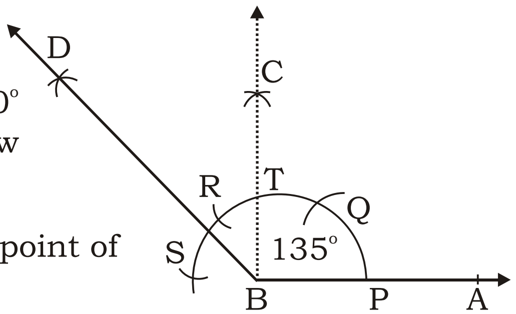
Step III: Join point O with S and extend it to get $\overrightarrow{OS}$.

Measure the angle POS by using protractor $m\angle POS = 165°$.

Hence it is the required angle.

12: Construction of angle of 135°:

As 90° + 45° = 135°, so we will construct an angle of 90° and then an angle of 45° by bisecting its outer part to get the angle of 135°.



Steps of construction:
First construct an angle of 90° as shown in the figure and follow the following steps:

Step I: Name the intersecting point of arc PR wit $\overrightarrow{BC}$ as T.

Step II: With centre R and keeping the radius of arc QR, draw another arc to cut the base arc thrice at point S.

Step III: With centre S and T and any suitable radius, draw two arcs to cut each other at point D.

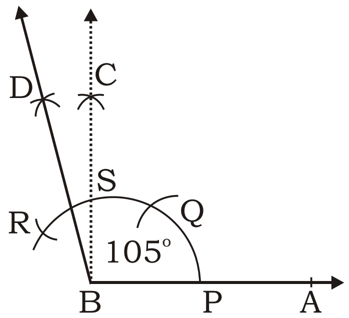
Step IV: Join point B with D to get $\overrightarrow{BD}$.

Measure the angle ABD by using protractor ABD = 135°.
Hence it is the required angle.

13: Construction of angle of 105°.

As 90° + 15° = 105° so, we will construct an angle of 90° and then bisect is outer portion of 30° between 90° and 120° to get the angle of 105°.

Steps of construction:
First construct an angle of 90° as shown in the figure and follow the following steps:



Step I: Name the intersecting point of arc PR with $\overrightarrow{BC}$ as S.

Step II: With centres R and S and any convenient same radii, draw two arcs to cut each other at point D.

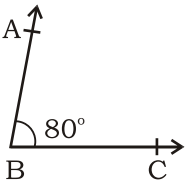
Step III: Join point B with D to get $\overrightarrow{BD}$.

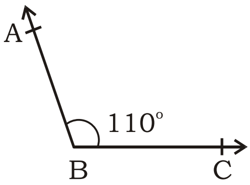
Measure the angle ABD by using protractor m ABD = 105°.

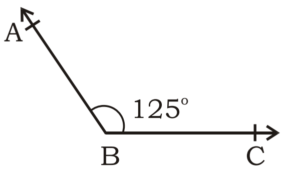
Hence it is the required angle.

EXERCISE 10.2

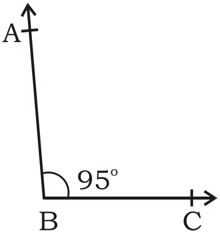
1. Bisect the following angles with the help of compasses.

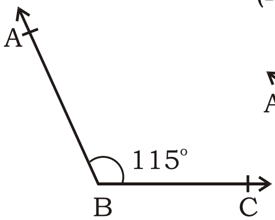
(i)


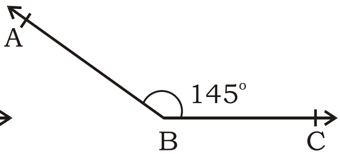
(ii)


(iii)


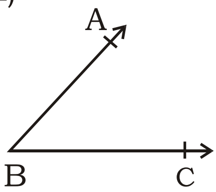
2. Divide the following angles with the help of compasses into four equal angles.

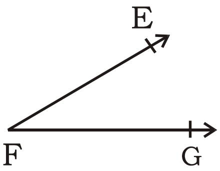
(i)


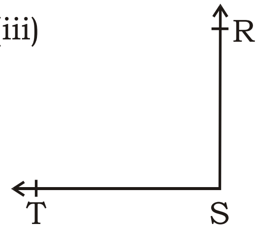
(ii)


(iii)


3. Draw the angle of equal measures and twice of the following angles with the help of pair of compass and ruler. Check your answers with protractor.

(i)


(ii)


(iii)


4. Draw the following angles with the help of pair of compasses and ruler.

(i) 30° (ii) $(22\frac{1}{2})^\circ$ (iii) $(67\frac{1}{2})^\circ$ (iv) 75°

(v) 105° (vi) 120° (vii) 135° (viii) 165°

10.3 CONSTRUCTION OF TRIANGLES

Triangle is a plane closed geometrical figure bounded by three sides. It has three sides and three angles.

We are going to learn different ways of constructing triangles according to the given elements.

Construct a triangle when three sides (SSS) are given.
Caution: Sum of two sides should be greater than the third side

Example: Construct a triangle ABC when

$m\overline{AB} = 4$ cm, $m\overline{BC} = 6$ cm and $m\overline{AC} = 7$ cm.

Steps of construction:

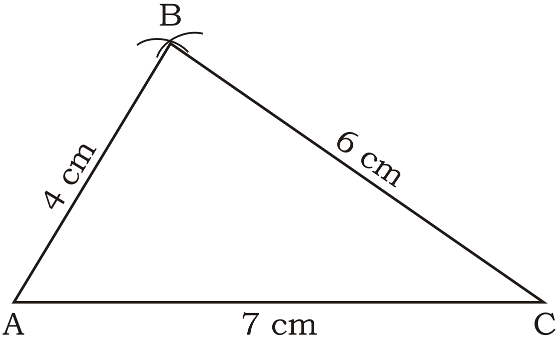
Can we construct a triangle with sides measures 4cm, 3 cm and 2 cm?

Step I: Draw $m\overline{AC} = 7$ cm.

Step II: With centre A and radius equal to 4 cm, draw an arc.

Step III: With centre C and radius equal to 6 cm, draw another arc to cut the previous arc at point B.

Step IV: Join point B to A and C.



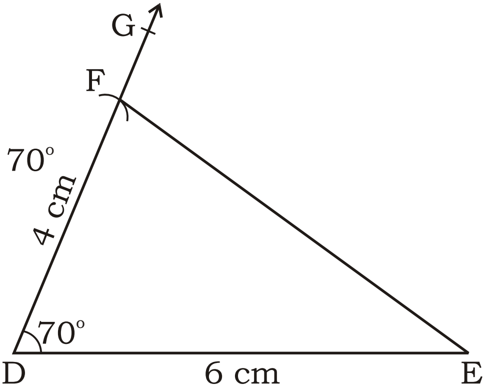
The ABC is constructed.

Construction of triangle is possible only if the sum of any two sides is greater than the third side.

Construct a triangle when two sides and their included angle (SAS) are given

Example: Construct a triangle DEF when

$m\overline{DE} = 6$ cm, $m\overline{DF} = 4$ cm and m D = 70°.



Steps of construction:

Step I: Draw $m\overline{DE} = 6$ cm.

Step II: At point D draw m EDG = 70° by using protractor.

Step III: With centre D and radius equal to 4 cm, draw an arc to cut $\overrightarrow{DG}$ at point F.

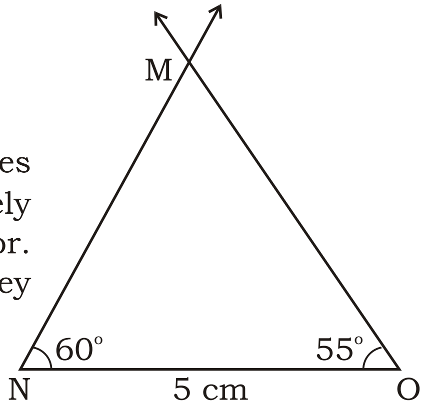
Step IV: Join point F with E.

Hence, the required DEF is constructed.

Construct a triangle when two angles and included side (ASA) are given

Example: Construct a triangle MNO when

$m\overline{NO} = 5$ cm, m N = 60° and m O = 55°.



Steps of construction:

Step I: Draw $m\overline{NO} = 5$ cm.

Step II: At points N and O, draw angles of 60° and 55° respectively with the help of protractor. Name the point where they intersect each other.

Step III: The arms of both angles intersect at point M.

Hence, the required MNO is constructed.

Construct a triangle when hypotenuse and one side (RHS) for a right angled triangle are given

Example: Construct a right angled triangle PQR when hypotenuse $\overline{PQ}$ is 7 cm long and a side $\overline{QR}$ is 5 cm long and m R = 90°.

Steps of construction:

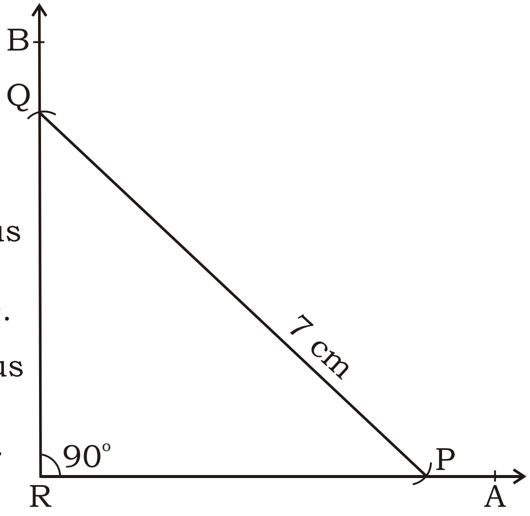
Step I: Draw $\overrightarrow{RA}$ greater than 7 cm.

Step II: At point R draw m ARB = 90°.

Step III: With centre R and radius equal to 5 cm, draw an arc to cut $\overrightarrow{RB}$ at point Q.

Step IV: With centre Q and radius equal to 7 cm, draw an arc to cut $\overrightarrow{RA}$ at point P.

Step V: Join point P and Q.



Hence, the required PQR is constructed and it is the right angled triangle.

EXERCISE 10.3

Construct a triangle ABC when:

1. $m\overline{AB}$ = 3 cm, $m\overline{BC}$ = 5 cm and $m\overline{CA}$ = 7 cm.
2. $m\overline{AB}$ = 2 cm, $m\overline{BC}$ = 5 cm and $m\overline{CA}$ = 5 cm.
3. $m\overline{AB}$ = 5 cm, $m\overline{BC}$ = 7 cm and $m\overline{CA}$ = 9 cm.
4. $m\overline{AB}$ = 8 cm, $m\overline{BC}$ = 6 cm and $m\overline{CA}$ = 10 cm.

Teacher's Note *Teacher should help the students to construct the triangles of different measures on blackboard as well as in their copies by geometrical instruments.*

Construct a triangle MNO when:

5. $m \;\; N = 63^\circ$, $m\overline{NO} = 3$ cm and $m\overline{MN} = 5$ cm.

6. $m \;\; N = 75^\circ$, $m\overline{NO} = 3.4$ cm and $m\overline{MN} = 4$ cm.

7. $m \;\; N = 80^\circ$, $m\overline{NO} = 3.5$ cm and $m\overline{MN} = 4.3$ cm.

8. $m \;\; N = 90^\circ$, $m\overline{NO} = 4$ cm and $m\overline{MN} = 3$ cm.

Construct a triangle PQR when:

9. $m\overline{PQ} = 5$ cm, $m \;\; Q = 40^\circ$, $m \;\; P = 60^\circ$.

10. $m\overline{PQ} = 7$ cm, $m \;\; P = 65^\circ$, $m \;\; Q = 60^\circ$.

11. $m\overline{QR} = 6.5$ cm, $m \;\; Q = 45^\circ$, $m \;\; R = 30^\circ$.

12. $m\overline{QR} = 7.5$ cm, $m \;\; Q = 35^\circ$, $m \;\; R = 40^\circ$.

Construct a right angled triangle XYZ when $m \;\; Z = 90^\circ$.

13. $m\,\overline{XY} = 7$ cm, $m\,\overline{XZ} = 5$ cm.

14. $m\,\overline{XY} = 10$ cm, $m\,\overline{XZ} = 6$ cm.

15. $m\,\overline{XY} = 14$ cm, $m\,\overline{XZ} = 10$ cm.

16. $m\,\overline{XY} = 8$ cm, $m\,\overline{XZ} = 6.5$ cm.

REVIEW EXERCISE 10

1. Give four examples of objects which represent the line segments from your environment.

2. If $m\overline{AB} = 4.3$ cm, $m\overline{CD} = 3.5$ cm and $m\overline{EF} = 2$ cm, construct geometrically a line segment whose length is equal to:

(i) $m\overline{AB} + m\overline{CD} + m\overline{EF}$

(ii) $m\overline{AB} - m\overline{CD}$

(iii) $m\overline{AB} - m\overline{EF}$

3. Draw a line segment measuring 7.6 cm and draw its right bisector. Then bisect its each part into equal parts.

4. Draw angles of the following measures with the help of compasses and bisect them.

(i) 30° (ii) 75° (iii) 105°

5. Construct a triangle ABC according to the following conditions and write steps of construction in each case.

(i) $m\overline{AB} = 8$ cm, $m\overline{AC} = 6$ cm, m A $= 65^\circ$

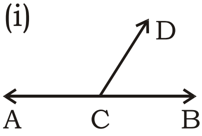
(ii) m A $= 45^\circ$, m C $= 45^\circ$, $m\overline{AC} = 7.5$ cm

(iii) $m\overline{AB} = 5$ cm, $m\overline{AC} = 7$ cm, m B $= 90^\circ$

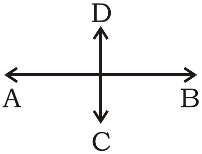
6. Draw is a line segment and name it.

7. Tick the perpendicular bisector of a line AB.

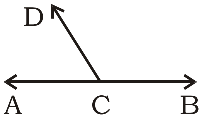
(i)



(ii)



(iii)



8. **Choose the correct answer.**

(i) Line segment AB is denoted by:

(a) $\overleftrightarrow{AB}$ (b) $\overline{AB}$ (c) $\overrightarrow{AB}$ (d) $\overrightarrow{BA}$

(ii) Bisector divides a line segment into ______ equal parts.

(a) two (b) three (c) four (d) five

(iii) In $\overline{PQ}$, the right bisector passes through its:

(a) end point (b) mid point

(c) point P (d) point Q

(iv) An angle is measured by:

(a) Ruler (b) Compasses

(c) Protractor (d) Set squares

9. Write True (T) or False (F) for the following:

(i) A line has a definite length.

(ii) A ray has two end points.

(iii) An angle of 15° is formed by bisecting the angle of 30°.

(iv) A triangle has 6 elements.

(v) A triangle is formed when sum of measures of any two sides is less than the third side.

SUMMARY

- Line segment is a part of line. It has two distinct end points.
- Right bisector is a line, line segment or a ray, which is perpendicular to a given line and also bisects it.
- Line segments and angles can be bisected with the help of ruler and a pair of compasses.
- Angles of measures 60°, 30°, 15°, 90°, 45°, $(22\frac{1}{2})^\circ$, 75°, $(67\frac{1}{2})^\circ$, 120°, 150°, 165°, 135° and 105° can be drawn with the help of ruler and pair of compasses.
- Triangle is a closed geometrical figure bounded by three sides. It has three angles.
- A triangle can be constructed if one of the following set of elements of triangle are known:

  (i) Measures of three sides (SSS).

  (ii) Measures of two sides and included angle (SAS).

  (iii) Measures of two angles and included side (ASA).

  (iv) Measures of hypotenuse and one side (RHS).
- To construct a triangle by SSS, the sum of measures of any two sides must be greater than the third side.

PERIMETER AND AREA

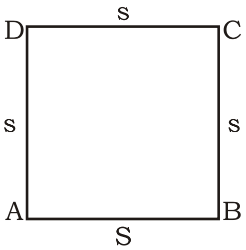
We learnt the concept of perimeter and area in previous class. Let us recall.

> Perimeter is the length of boundary of a plane closed figure and area is the measurement of surface of plane closed figure.

Find perimeter and area of a square and a rectangle

Perimeter of square:

We know that the sum of measures of all the sides of a square is its perimeter.



In the figure, the length of each side of square is S units.

Perimeter = Sum of all sides

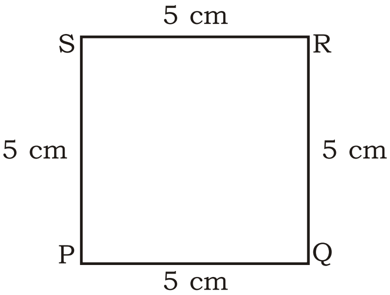
So, Perimeter = S + S + S + S

= 4S

i.e. **Perimeter of square = 4 Side**

Where S denotes the length of a side.

Example 1. Find the perimeter of a square whose each side is 5 cm long.



Solution: Here side = 5 cm

Now Perimeter = 4 side

= 4 5

= 20 cm

Remember that

$$\text{Side of square} = \frac{\text{Perimeter of square}}{4}$$

Teacher's Note *Teacher should revise the concept and formula of perimeter and area of square and rectangle.*

Example 2. Find the length of side of a square whose perimeter is 40 cm.

Solution: Here perimeter = 40 cm

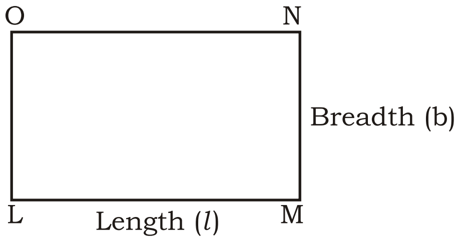
We know that side of square = $\frac{\text{Perimeter of square}}{4}$

$$= \frac{\cancel{40}^{10}}{\cancel{4}_{1}} = \mathbf{10\ cm}$$

So, length of each side is **10 cm**.

Perimeter of rectangle:

We know that the sum of the measures of all sides of a rectangle is called its perimeter. In the figure length and breadth of rectangle are L and B respectively.



Now

Perimeter = Length + Breadth + Length + Breadth

$= l + b + l + b$

$= 2l + 2b = 2(l + b)$

So, $\boxed{\text{Perimeter of rectangle} = 2(l + b)}$

Example 3. Find the perimeter of rectangle whose length and breadth are 12 cm and 8 cm respectively.

Solution: Here l = 12 cm

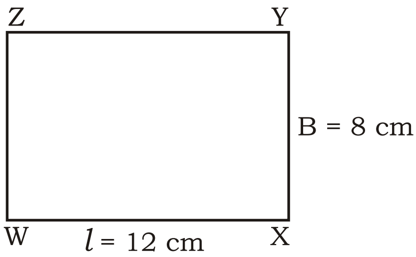
b = 8 cm

Perimeter of rectangle = $2(l + b)$

$= 2(12 + 8)$

$= 2(20)$

$= 40$ cm



So, the required perimeter of given rectangle is 40 cm.

Example 4. Find the length of rectangle whose breadth is 6 cm and its perimeter is 28 cm.

Solution: Here l = ?

b = 6 cm

Perimeter of rectangle = 28 cm

Now Perimeter of rectangle = 2 (l + b)

i.e. $28 = 2(l + 6)$

or $\frac{28}{2} = l + 6$

or $14 = l + 6$

or $14 - 6 = l$

or $8\text{ cm} = l$

or $l = 8\text{ cm}$

So, the required length is **8 cm**.

Area of square:

We know that:

Area of square = side   side

Example 5. Find the area of a square whose each side is 5 cm.

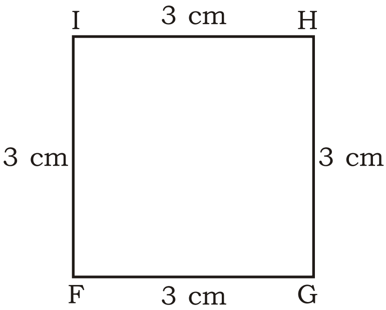
Solution: Here Side = 3 cm

We know that

Area of square = Side   Side

= 3   3

= 9 sq. cm

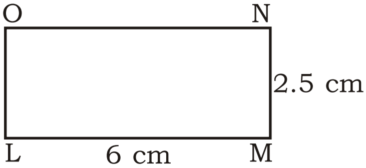


Area of rectangle:

We know that:

Area of rectangle = length   breadth

Example 6. Find the area of rectangle in which length is 6 cm and breadth is 2.5 cm



Solution: Here Length = 6 cm

Breadth = 2.5 cm

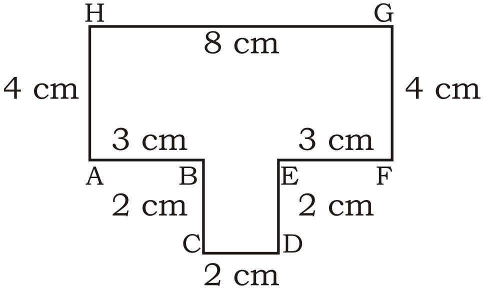
Now Area = length breadth = 6 2.5

= 15.0 sq. cm

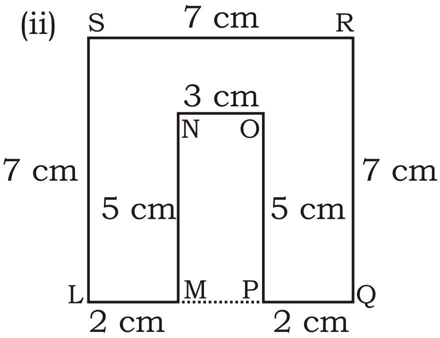
We can also find area and perimeter of complex figures as explained in the following example.

Example 7: Find area and perimeter of the following figures.

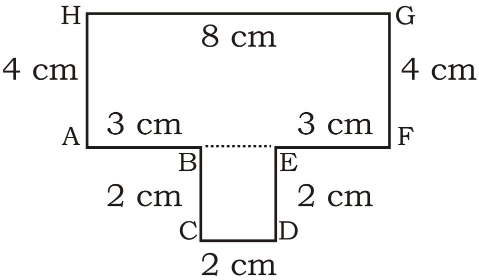
(i)



(ii)



Solution: (i)



Perimeter of figure

According to the figure

Perimeter = 8 + 4 + 3 + 2 + 2
+ 2 + 3 + 4
= 28 cm

Area of figure

In the figure there is a rectangle and square.

According to the figure:

Area of figure = Area of rectangle + Area of square
= 8 4 + 2 2
= 32 + 4
= 36 sq. cm

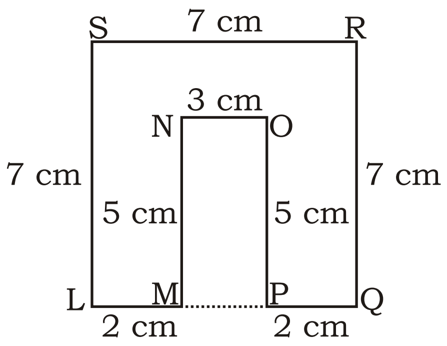
Teacher's Note *Teacher should ensure that the students have done enough practice of problems related to area and perimeter of square and rectangle.*

Solution: (ii)

Perimeter of figure

According to the figure:

Perimeter = 7 + 7 + 7 + 2 + 5
+ 3 + 5 + 2
= 38 cm



Area of figure In the figure there is a square and a rectangle.

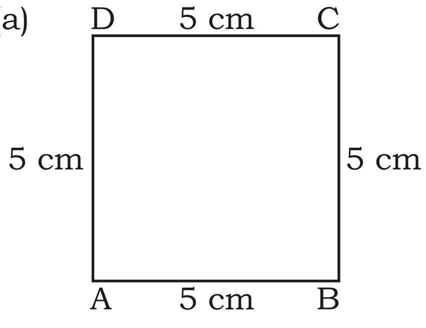
According to th figure

Area of figure = Area of square – Area of rectangle
= 7   7 – 3   5
= 49 – 15
= 34 sq. cm

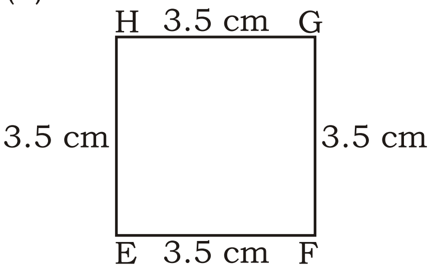
EXERCISE 11.1

1. Find the perimeter and area of each of the following squares.

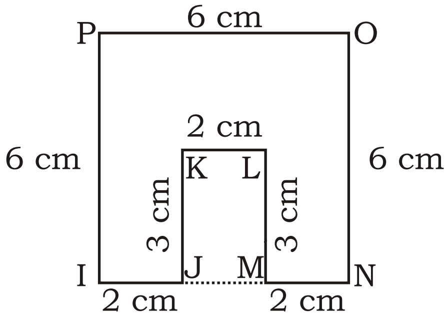
(a)



(b)

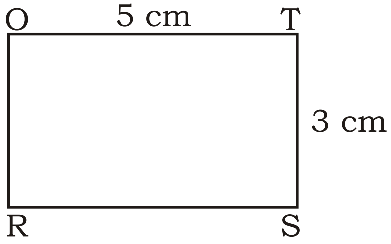


(c)

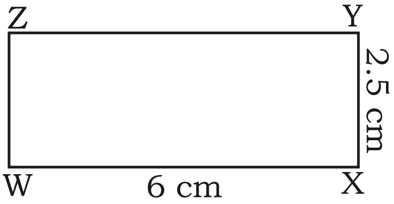


2. Find the perimeter and area of the following rectangles.

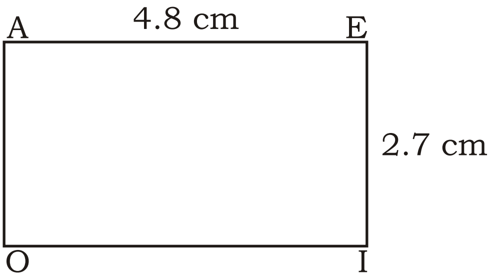
(a)



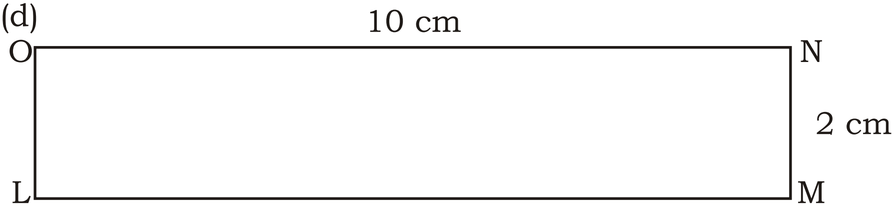
(b)



(c)



(d)



3. Find the perimeter and area of:

(i) Rectangle with sides 5 cm, 2.5 cm, 5 cm and 2.5 cm

(ii) Square with each side 5 cm.

4. Find the side of square whose perimeter is:

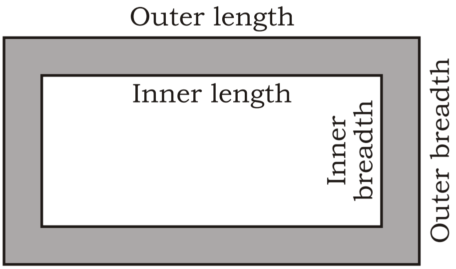
(a) 48 cm (b) 50 cm

5. Find the length of rectangle whose perimeter is 100 m and breadth is 20 m.

6. Find the area of rectangle whose adjacent sides are 6.2 cm and 8.5 cm long.

Find area of path (inside or outside) of a rectangle or square

In the adjacent figure shaded portion represents the path.



Here
Area of the large rectangle (including path)

= Outer length   outer breadth

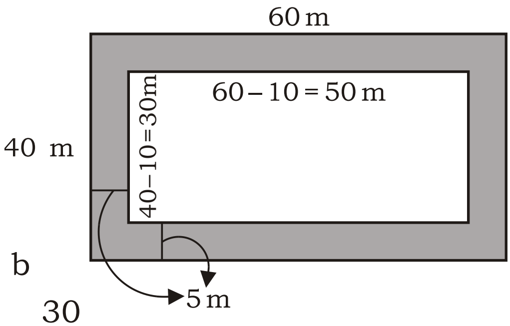
Also area of the small rectangle (excluding path)

= Inner length   Inner breadth

So, Area of path = Area of large rectangle – Area of small rectangle

Example. A field is 60 m long and 40 m wide. A path 5 m wide is constructed all around it inside the field. Find the area of the path.

Solution:



Area of large rectangle

= l b

= 60   40

= 2400 sq. m

Area of small rectangle = l b

= 50   30

= 1500 sq. m

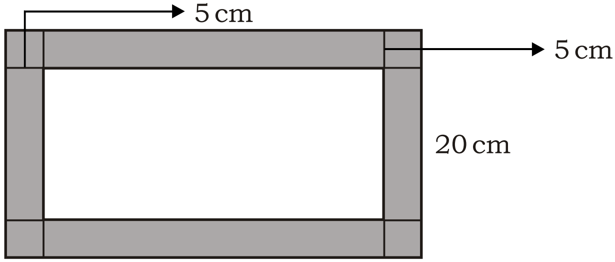
So, Area of path = 2400 – 1500

= 900 sq. m

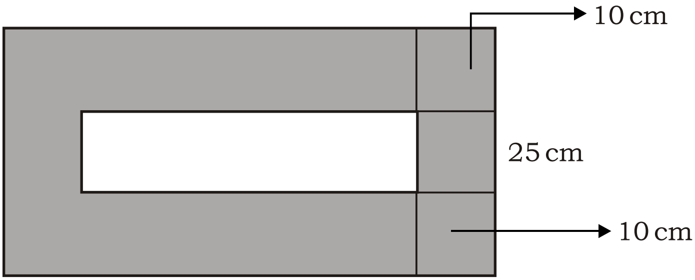
EXERCISE 11.2

1. Find the area of shaded portion in the following figures.

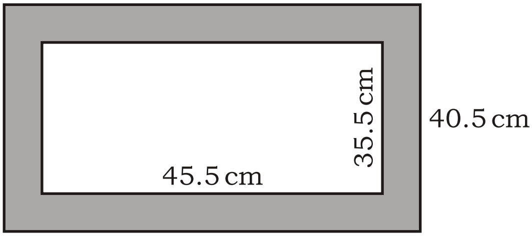
(i)



(ii)



(iii)



2. A field whose length and breadth are 35 cm and 30 cm respectively. A path 5 cm wide constructed all round inside it. Find the area of the path.

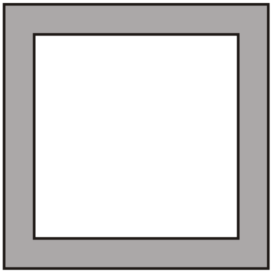
3. A path of 5 cm wide is constructed all round outside a field with dimension 20 cm and 10 cm. Find the area of the path.

Solve real life problems related to perimeter and area of a square and rectangle

Example 1. Danish and Rafay go for jogging every morning. Danish goes around a rectangular field with length 400 m and breadth 300 m. Rafay goes around a square of 300 m length. Who covers more distance.

Solution:

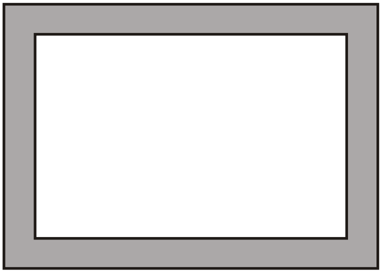
Jogging track for Rafay



300 m

300 m

Jogging track for Danish



300 m

400 m

Distance covered by Rafay = Perimeter of square

= 4 side

= 4 300

= 1200 m

Distance covered by Danish = Perimeter of rectangle

= $2\ (l + b)$

= 2 (400 + 300)

= 2 (700)

= 1400 m

So, Danish covers more distance than Rafay.

Teacher's Note *Teacher should give some more examples from daily life.*

Example 2. Find the cost of ploughing a piece of land 50 m long and 30 m wide at the rate of Rs 50 per sq. m.

Solution:

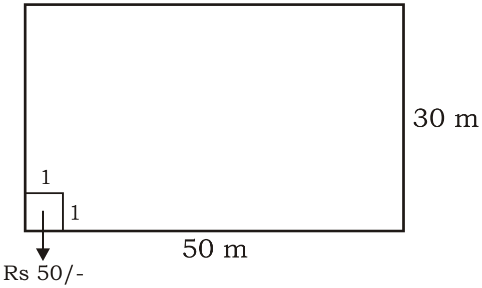
Area of land = Length   breadth

= 50   30

= 1500 sq. m

Cost of ploughing = 1500   50

= **75000 rupees**



EXERCISE 11.3

1. Find the cost of fencing a square shaped park of side 250 m at the rate of Rs 30 per metre.

2. Find the cost of fencing a rectangular park of length 200 m and breadth 150 m at the rate of Rs 30 per metre.

3. Bismah and Umaima goes for jogging every morning. Bismah covers distance around a square of length 200 m and Umaima goes around a rectangular field with length and breadth of 150 m and 100 m respectively. Who covers more distance.

4. Find the length of a plot of width 15 m whose area is 615 sq. m.

5. The length and breadth of a play ground are 35.5 m and 30.5 m respectively. Find the cost of grassing the ground at the rate of 150 sq. m.

6. A garden 120 m long and 8 m wide has a road 3 m wide all around on the garden outside. Find the cost of paving the road at the rate of Rs 200 per sq. m.

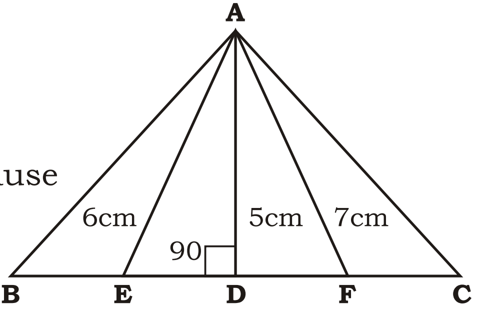
7. Find the area of the floor of a room which is covered by 650 tiles, each tile has area of 0.5 sq. m.

Recognize altitude of a geometric figure as the measure of the shortest distance between the base and its top

Altitude is the shortest distance from the top to the base of a geometrical figure.

In the adjacent figure point A is the top and $\overline{BC}$ is base.

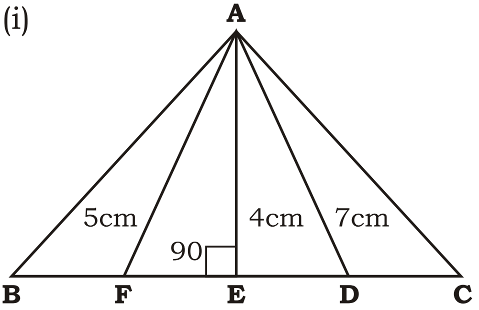
Here $\overline{AD}$ represents altitude because it shows the shortest distance of 5cm from top to the base.



Remember that: Altitude makes right angle with the base

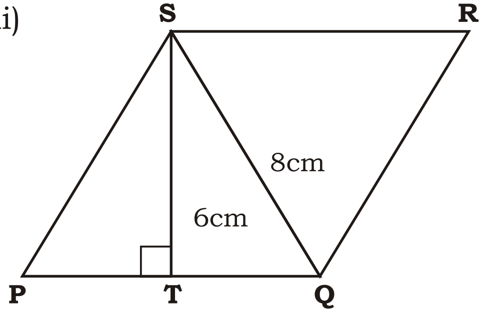
Example. Identify the altitudes in the following figures.

(i)



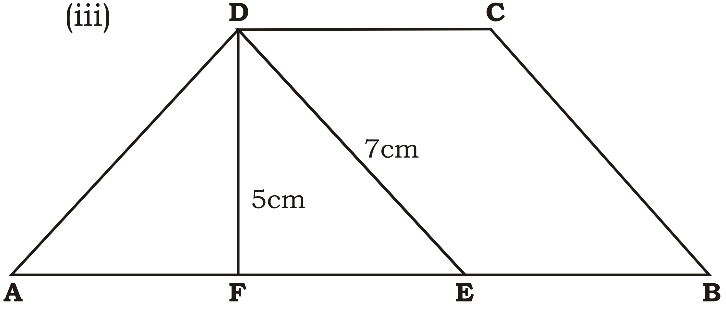
Altitude is $\overline{AE}$ of 4 cm

(ii)



Altitude is $\overline{ST}$ of 6 cm

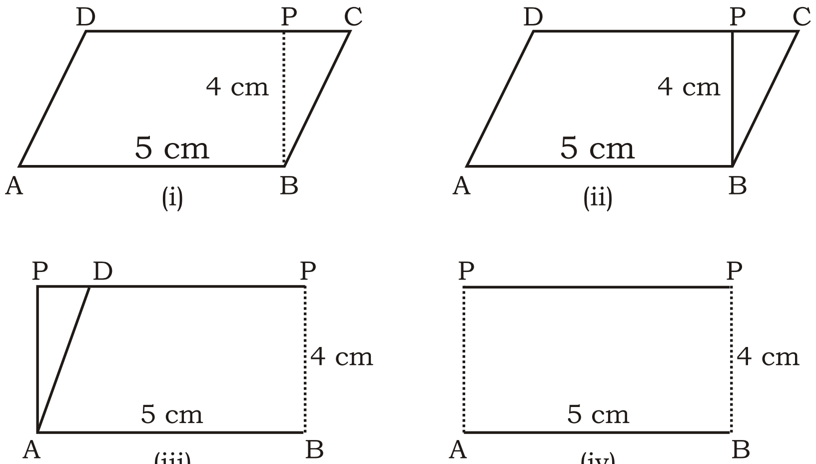
(iii)



Altitude is $\overline{DF}$ of 5 cm

Find area of a parallelogram when altitude and base are given

Consider the following figure of a parallelogram A B C D



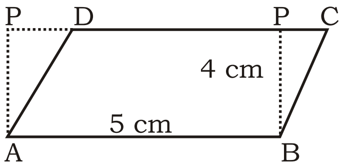
(i) (ii) (iii) (iv)

In figure (i) a perpendicular $\overline{BP}$ is drawn on the base $\overline{AB}$

In figure (ii) the triangular area PBC is being cut and separated.

In figure (iii) this triangular area is placed with $\overline{AD}$. In this way we get a rectangular area ABPP which is shown in figure (iv)

These activities show that a parallelogramic area can be converted into rectangular area. Since we know how to find the area of rectangle, So, it is also easy to find the area of parallelogram.



Area of rectangle = length   breadth

$= m\,\overline{AB} \quad m\,\overline{BP}$

If the value of line segment $\overline{BP}$ is known then the area of parallelogramic region can be found. The line segment BP is called the "altitude" of the parallelogram.

So, area of the parallelogram in above figure is = base   attitude

= 5   4 = 20 sq. cm

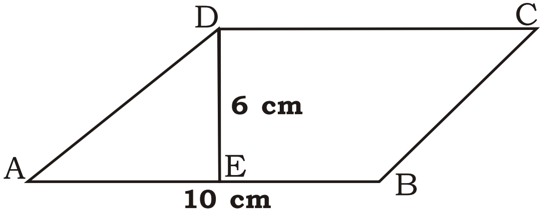
Here $\overline{AB}$ is called the base of the parallelogram.

Hence *area of a parallelogram = base   altitude*

If *h* represents altitude and *b* represents base then

Area of parallelogram = *b*   *h*

Example 1. Find the area of parallelogram whose base is 10 cm and altitude is 6 cm.



Solution: Here

Base = 10 cm

Altitude = 6 cm

Now,

Area of parallelogram = b   h

= 10   6

= 60 sq. cm

Example 2. The area of parallelogram is 60 sq. cm and its base is 5 cm. Find its altitude.

Solution: Here

Base = 5 cm

Area = 60 sq. cm

Altitude = ?

We know that

Area of parallelogram = b   h

i.e. 60 = 5   h

$\frac{\overset{12}{\cancel{60}}}{\cancel{5}} = h$

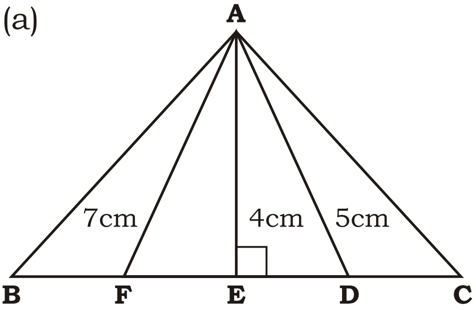
12 = h

So, its altitude is **12 cm** long.

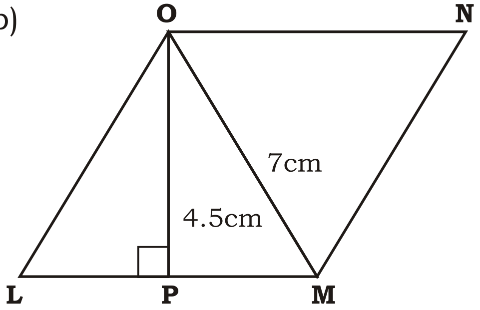
EXERCISE 11.4

1. Identify altitude with its length in the following figures.

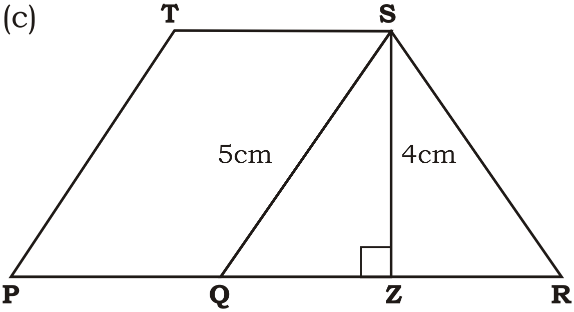
(a)



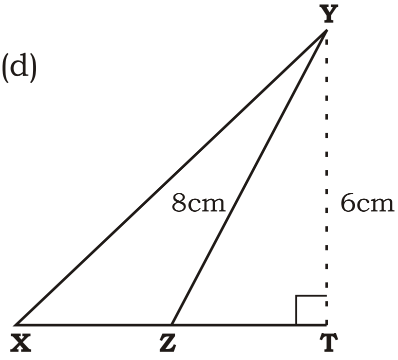
(b)



(c)



(d)



2. Find the area of a parallelogram when its altitude and base are given:

(i) Base = 10 cm, altitude = 6 cm

(ii) Base = 15 cm, altitude = 8 cm

(iii) Base = 4.5 cm, altitude = 5.5 cm

(iv) Base = 6.5 cm, altitude = 4.7 cm

3. The area of a parallelogram is 192 sq. cm and its altitude is 12 cm. Find its base.

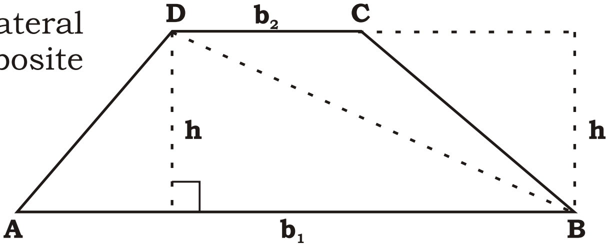
4. The base of parallelogram is 15 m and its area is 180 sq. m. Find its altitude.

5. The altitude of a parallelogram is 1.5 cm and its area is 4.50 sq. cm. Find its base.

Define trapezium and find its area when altitude and measures of the parallel sides are given

A trapezium is a quadrilateral whose only one pair of opposite sides is parallel.

The adjacent trapezium ABCD is divided into two triangles ABD and BCD.



Area of trapezium ABCD = Area of ABD + Area of BCD

$$= \frac{1}{2}h \quad b_1 + \frac{1}{2}h \quad b_2$$

$$= \frac{1}{2}h\,(b_1 + b_2)$$

Where b_1, b_2 are the lengths of parallel sides and

h = altitude or height

So, Area of trapezium = $\frac{1}{2}$ altitude (sum of lengths of parallel sides)

Example. Find the area of trapezium with parallel sides of lengths 6 cm and 10 cm and altitude is 5 cm.

Solution: Here

Altitude = 5 cm

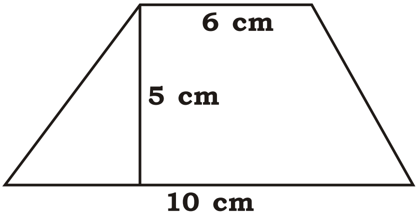
Parallel sides are 6 cm and 10 cm long.

Area of trapezium = $\frac{1}{2}$ altitude (sum of parallel sides)

$= \frac{1}{2} \quad 5\,(6 + 10)$

$= \frac{5}{\cancel{2}} \quad \overset{8}{\cancel{16}} = 5 \quad 8$

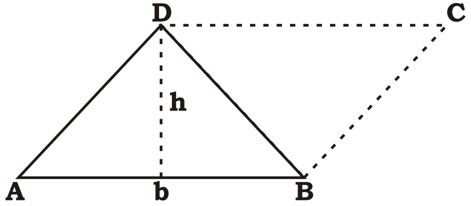
= 40 sq. cm



Teacher's Note *Teacher should ensure that the skill of using formulas have been developed in the students.*

Find area of a triangle when measures of the altitude and base are given.

Look at the parallelogram ABCD which is divided into two equal triangles ABD and BCD.



So,

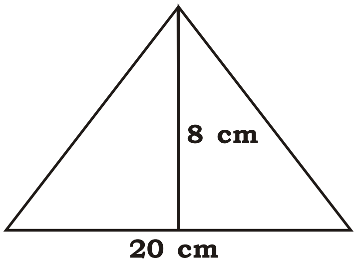
Area of ABD = Half of area of parallelogram

$= \frac{1}{2}$ (Base altitude)

$= \frac{1}{2}$ b h

Where b = base and h = altitude or height

Example 1. Find the area of triangle in which altitude and base are 20 cm and 8 cm respectively.



Solution: Here

Base = 20 cm

Altitude = 8 cm

We know that

Area of triangle $= \frac{1}{2}$ base altitude

$= \frac{1}{\cancel{2}} \quad \overset{10}{\cancel{20}} \quad 8$

= 80 sq. cm

EXERCISE 11.5

1. **Find the area of trapezium when parallel sides and altitude are given as under.**
   (i) Parallel sides are of 5 cm and 6 cm where altitude is 4 cm.
   (ii) Parallel sides are of 3.5 cm and 4.5 cm where altitude is 2 cm.
   (iii) Parallel sides are of 4.5 cm and 6.5 cm where altitude is 5.5 cm.

2. The parallel sides of trapezium are 6 cm and 8 cm, its altitude is 5 cm. Find its area.

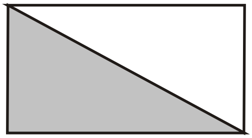
3. If the parallel sides of trapezium are 15 cm and 20 cm and its area is 140 sq. cm. Find the altitude.

4. **Find the area of triangle when its base and altitude are given as under.**
   (i) Base = 8 cm, altitude = 6 cm
   (ii) Base = 9 cm, altitude = 5cm
   (iii) Base = 7 cm, altitude = 2.5 cm
   (iv) Base = 6.5 cm, altitude = 3.5 cm

5. Find the area of triangle whose base and altitude are 6 cm and 5.5 cm respectively.

6. The area of triangle with base 5 cm is 40 sq. cm. Find its height.

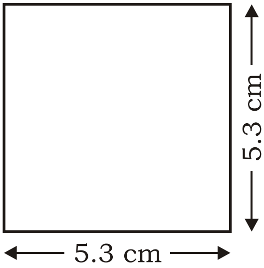
7. Find the area of parallelogram in the figure if area of shaded triangle is 20 sq. m.



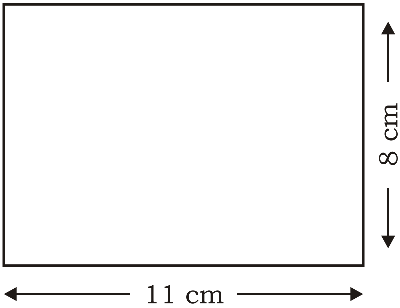
REVIEW EXERCISE 11

1. Find the perimeter and area of the following figures:

(i)

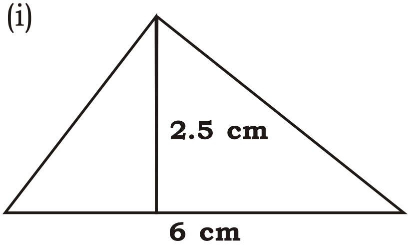


(ii)

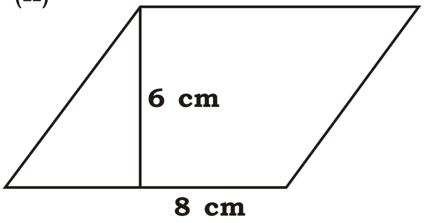


2. Find the area of the following figures:

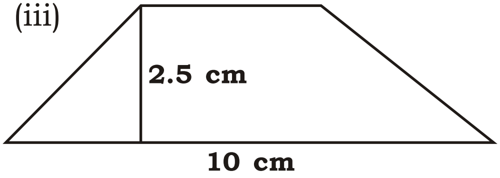
(i)



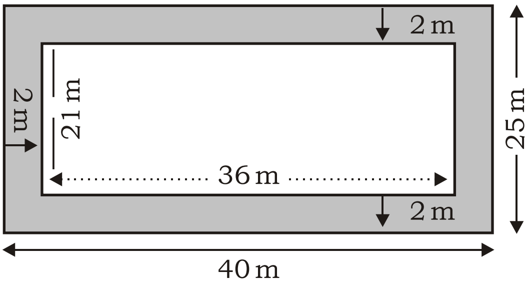
(ii)



(iii)



3. Find area of the shaded portion of the given figure.



4. A tile is a square of side 20 cm. How many such tiles would be required to cover the floor of a square bath-room of side 3 m?

5. The area of a square of side 16 cm is the same as that of a rectangle of length 64 cm. What is the breadth of the rectangle?

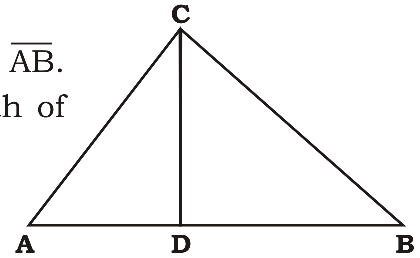
6. The area of a triangle is 72 sq. cm. If its height is 8 cm, find its base.

7. The area of a triangle is 60 sq. cm. If its base is 12 cm, find its altitude.

8. The sides of a right angled triangle forming the right angle are 12 cm and 9 cm respectively. Find the area of that triangle?

9. In ABC, $\overline{CD}$ is perpendicular to $\overline{AB}$. such that m $\overline{AB}$ = 18 cm, length of $\overline{CD}$ = 7.5 cm.

Find area of the ABC.

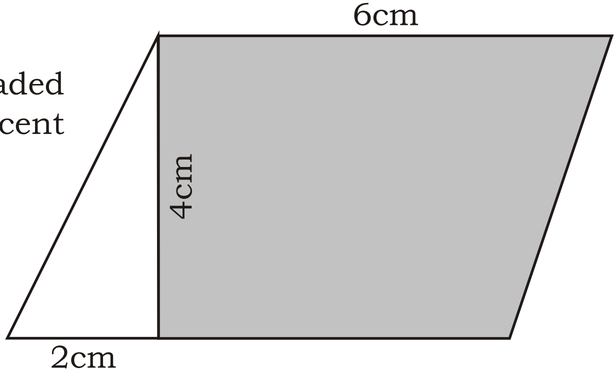


10 The parallel sides of a trapezium are 9 cm and 7 cm respectively. Its height is 5 cm. Find its area?

11. If the parallel sides of a trapezium are 20 cm and 30 cm and its area is 200 sq. cm. Find its height.

12. Area of a trapezium is 120 sq. cm. If its height is 8 cm and one of the parallel sides is 20 cm, find the length of the other side.

13. Find the area of shaded portion in the adjacent figure.



14. Fill in the blanks.

(i) Area is commonly measured in __________.

(ii) Perimeter is commonly measured in ___________.

(iii) Area of a rectangle is the ___________ of the length and breadth.

(iv) Perimeter of a rectangle is the __________ of its 4 sides.

(v) Area of a square is the product of _______________ and _______________.

(vi) Perimeter of a square = ________ ________.

(vii) Formula for finding the perimeter of a rectangle is __________.

(viii) Formula for finding area of a parallelogram is __________.

(ix) Formula for finding area of a triangle is ___________.

(x) Area of a trapezium is _____________.

SUMMARY

- Perimeter of a Rectangle = 2 (L + B).
- Perimeter of a square = 4 side

  and side of a square = $\frac{\text{Perimeter}}{4}$
- Area of a Rectangle = (L B) sq. unit
- Area of a square = (side side) sq. unit
- Area of a parallelogram = (Base Altitude) sq. unit
- Area of a Triangle = $\frac{1}{2}$ (Base Altitude) sq. unit
- Area of a Trapezium = $\frac{1}{2}$ (sum of parallel sides) altitude sq. unit

UNIT 12 THREE DIMENSIONAL SOLIDS

VOLUME AND SURFACE AREA

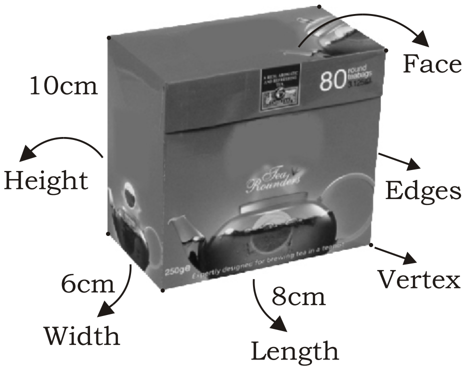
In daily life we come across with solid objects like dice, tea box, tin of ghee, football, funnel etc.

All these objects are three dimensional figures or 3D figures, because they have three dimensions length, width and height.

Let us consider a 3D object tea box as shown here. It has eight vertices, six faces and twelve edges.

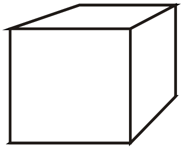
It has length of 8 cm, width of 6 cm and height of 10 cm.

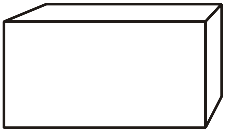
So, it is three dimensional solid figure.

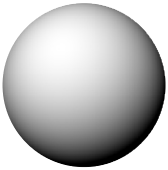


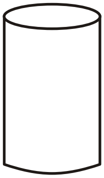
Identify 3D figure (cube, cuboid, sphere, cylinder and cone) with respect to their faces, edges and vertices

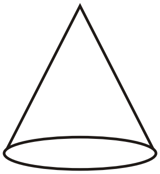
There are many solid figures but we will discuss the properties of five basic 3D figures cube, cuboid, sphere, cylinder and cone. These are shown below:


Cube


Cuboid

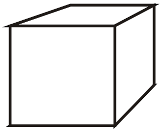
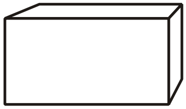
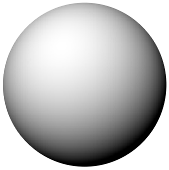
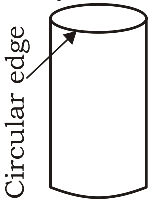
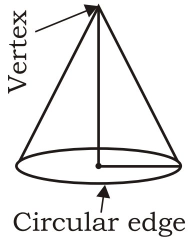

Sphere


Cylinder


Cone

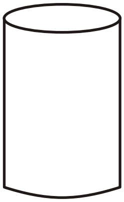
Teacher's Note *Teacher should give some more examples of solid figures from daily life.*

Three Dimensional Figures

3D Figure	Property	No. of Faces	No. of Edges	No. of Vertices
Cube 	It is a solid figure whose length, width and height are equal	6	12	8
Cuboid 	It is a solid figure whose length, width and height are not equal	6	12	8
Sphere 	It is a solid figure with complete round surface.	No	No	No
Cylinder 	It is a solid figure with two opposite circular faces and a curved surface.	2 circular faces	2 Circular edges	No
Cone 	It is a solid figure with one circular face and a curved surface.	1 Circular face	1 Circular edge	1

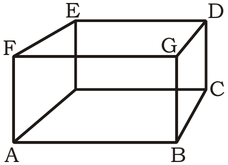
Teacher's Note *In order to teach 3D-figures, teacher should use available 3D-objects.*

Activity **Name the following 3D shapes and fill in the blanks.**

(1) 

Name of figure **Cylinder**

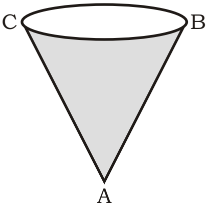
Number of circular faces = _____

(2) 

Name of figure **Cuboid**

$\overline{DE}$ represents _________

Points A, B represent _________

(3) 

Name of figure **Cone**

Point A represents ________

Shaded portion represents ________________

EXERCISE 12.1

1. Identify the objects and write the names of the 3D-figures which represent these objects.

(i) 

(ii) 

(iii) 

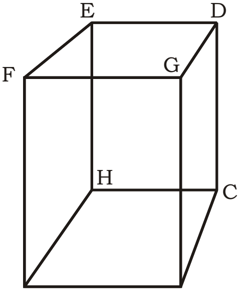
(iv) 

(v) 

(vi) 

2. Fill in the blanks for the following figures.

(i)



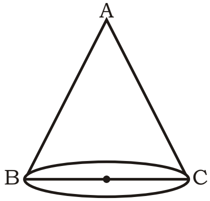
Cuboid

(a) Number of edges = __________

(b) Number of faces = __________

(c) Number of vertices = ________

(ii)

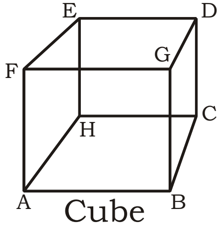


Cone

(a) Point A represents ________________

(b) Number of circular faces __________

(iii)



Cube

(a) $\overline{AB}$ represents ________________

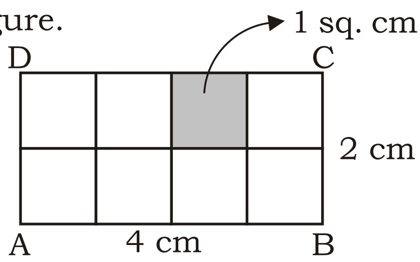
(b) Point A represents ____________

(c) Number of faces ______________

Define and recognize units of surface area and volume

1. Surface area:

We know that area is the measurement of space occupied by a plane closed figure. Area is found by filling the space by square of one unit length as shown in the figure.



Area of square of one unit length is 1 square unit or 1 sq. unit.

There are 8 squares of area 1 sq. cm.

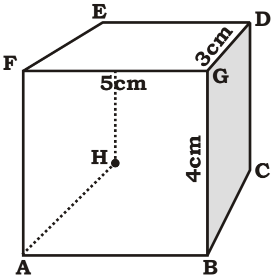
So, area of rectangle = 8 square cm

= 8 sq. cm

Similarly surface area is the measurement of space occupied by the surface of 3D figure which is also measured in square units.

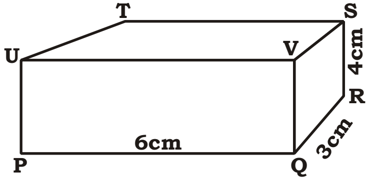
Note: In measurement of surface area of cube and cuboid, we use square of same unit which is used in length, width and height.

Some examples of units of area are sq. cm, sq. m, sq. km etc.



We will use sq. cm to measure surface area of given cube because its length, width and height are given in centimetres.

Similarly, we will use sq. cm for given cuboid to measure the surface area.

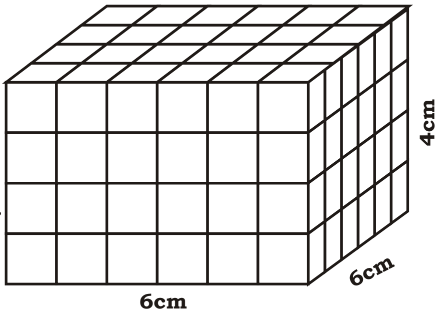


2. Volume:

We know that volume of solid object is the measurement of the space occupied by a 3D-object. Volume is measured by fitting the cubes of one unit length in the space as shown in the figure.

There are 144 cubes of volume of 1 cubic cm each

So,
Volume of cuboid
= 144 cubes of volume of 1 cubic cm.
= 144 cubic cm.

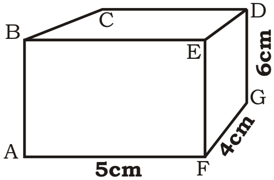


Note:

In measurement of volume of cube and cuboid, we use cubes of same unit which is used in length, width and height.

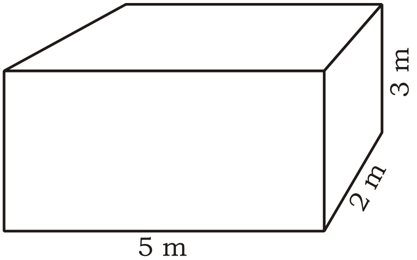
Some units of volume are cubic cm, cubic m, cubic km etc.

For example: In the given cuboid we will use cubic cm to find its volume because its length, width and height are given in centimetres.



Similarly,

The volume of given cuboid will be measured in cubic m.



Activity: Fill in the blanks.

(i) The volume of cuboid of dimensions 5 m, 6 m, 7 m is measured in **cubic m** ________.

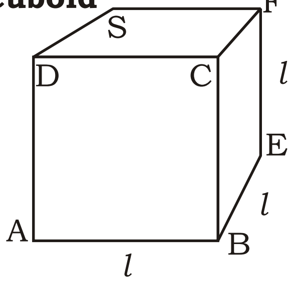
(ii) The volume of cube of each side 6 cm is measured in ________.

(iii) The surface area of cuboid of dimension 6 m, 7 m, 8 m is measured in ________.

(iv) The surface area of cube of length 5 cm is measured in ________.

Find surface area and volume of cube and cuboid

1. Surface area of cube



We know that length, breadth and height are equal in cube.
So, each surface is of same area.

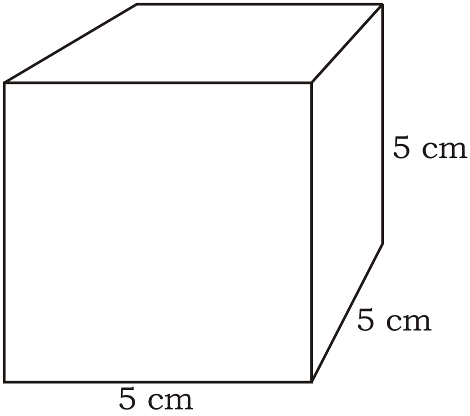
Surface area of cube = Area of 6 equal faces

$= 6 \quad$ Area of face

$= 6 \quad (l \quad l)$

$= 6\,l^2$

Example: Find the surface area of cube with each side of 5 cm.



Solution:

Here

Length = l = 5 cm

We know that

Surface Area $= 6\,l^2$

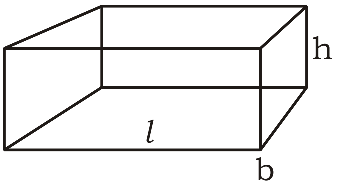
$= 6 \quad (5)^2$

$= 6 \quad 5 \quad 5$

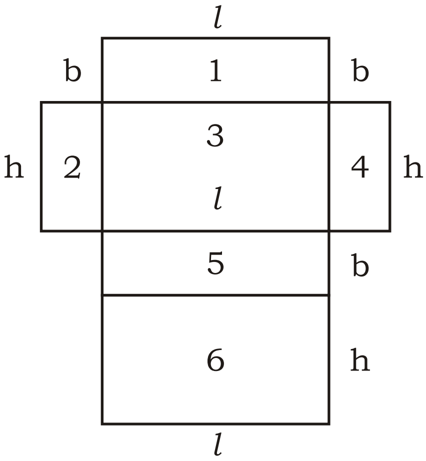
$= 150$ sq. cm

Teacher's Note *Teacher should teach the concept of surface area with the help of available solid objects, for example chalk box and geometry box.*

2. Surface area of cuboid.



In order to find total surface area of cuboid, we unfold all the six faces as shown in the figure and find the total area of its six faces.



Surface area of faces of cuboid

= Area of 1 + Area of 2 + Area of 3

+ Area of 4 + Area of 5 + Area of 6

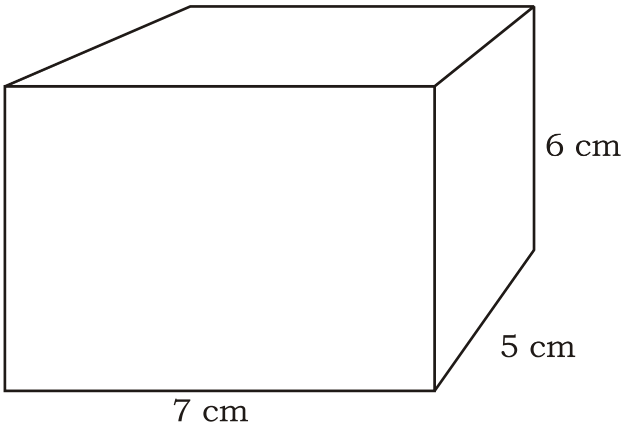
= l b + b   h + h l + b   h

+ l b + h l

= 2 (l b) + 2 (b   h) + 2 (h l)

= 2 (l b + b   h + h l)

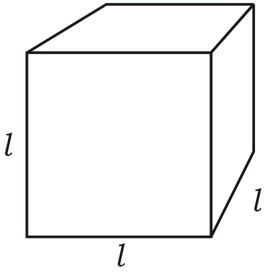
Example: Find the surface area of a cuboid in which
length = 7 cm
breadth = 5 cm
and height = 6 cm.



Solution:
Here
Length = l = 7 cm
Breadth = b = 5 cm
Height = h = 6 cm

Surface area of cuboid = 2 (l b + b   h + h l)
= 2 (7   5 + 5   6 + 6   7)
= 2 (35 + 30 + 42)
= 2 (107)
= 214 sq. cm

3. Volume of Cube

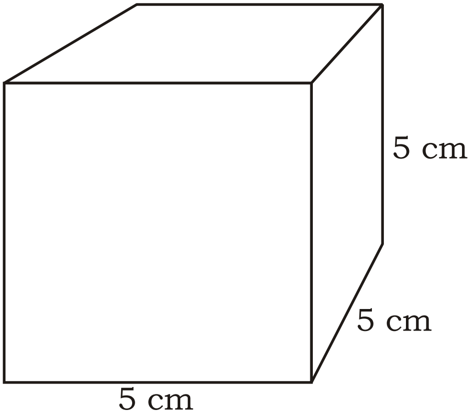


We know that length, breadth and height are equal in any cube.

So, Volume of cube = $l \quad l \quad l$

$= l^3$

Example: Find volume of a cube whose each side is 5 cm long.



Solution:

Here

Length = l = 5 cm

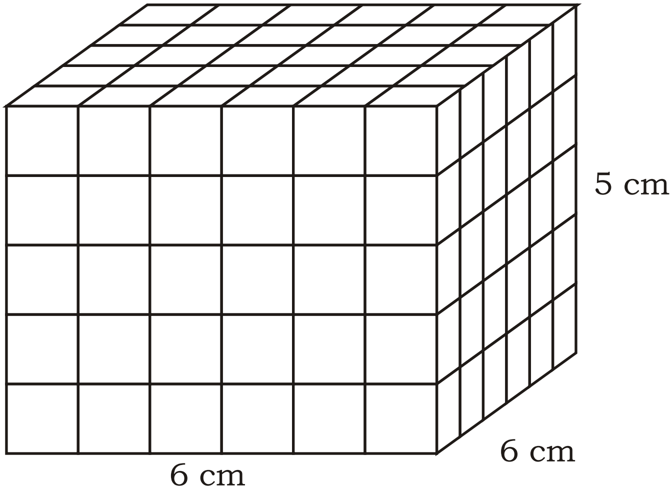
We know that

Volume of Cube = l^3

= 5   5   5

= 125 cubic cm

4. Volume of Cuboid



In order to find formula for volume of cuboid, let us consider a cuboid as shown in the figure.

Here

volume of cuboid

= Area of base   height

= (6   6)   5 cubic cm

= 180 cubic cm

So, $\boxed{\text{Volume of cuboid} = l \quad b \quad h}$

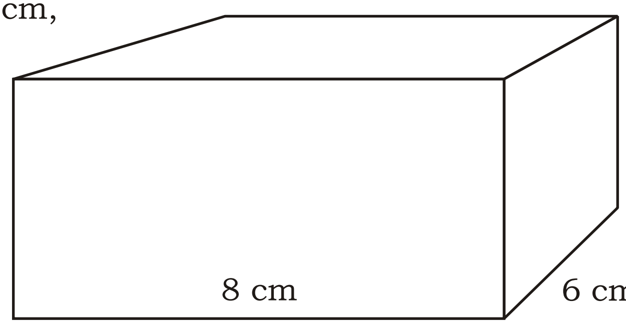
Where

l = Length

b = Breadth

and h = Height

Example: Find the volume of a cuboid in which length = 8 cm, breadth = 6 cm and height = 4 cm.



Solution:

Here

l = 8 cm

b = 6 cm

h = 4 cm

We know that

volume of cuboid = l b h

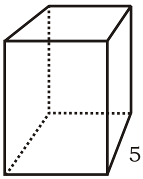
= 8 6 4

= 192 cubic cm

EXERCISE 12.2

1. Write the units of surface area and volume of the given figures.

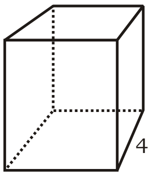
(i)



Unit of surface area = ______________

Unit of volume = ___________________

(ii)



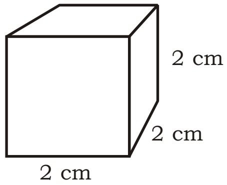
Unit of surface area = ______________

Unit of volume = ___________________

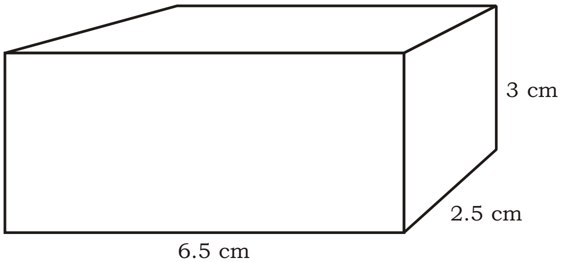
Teacher's Note *Teacher should use solid objects like juice packet and milk packet to teach concept of volume.*

2. Find the surface area and volume of each of the following.

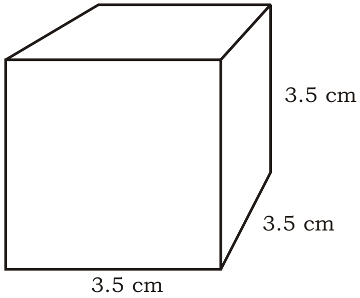
(i)



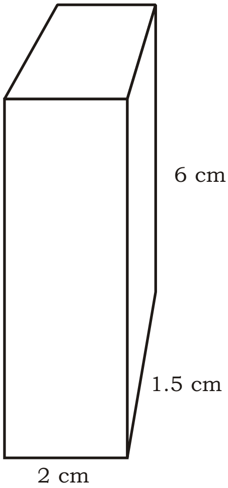
(ii)



(iii)



(iv)



3. Find the volume of the following cubes whose each side is:

(i) 5 cm (ii) 6.2 cm (iii) 2.5 m

4. Find the volume of each of the following cuboids in which:

(i) Length = 9 cm, breadth = 6 cm, height = 4 cm
(ii) Length = 5 cm, breadth = 3.5 cm, height = 4 cm
(iii) Length = 3.5 cm, breadth = 2 cm, height = 2.5 cm

Solve real life problems involving volume and surface area

Example 1: Find the cost of painting a wooden cuboid at the rate of Rs 50 per sq. m if dimensions are 6 m, 5 m and 4 m.

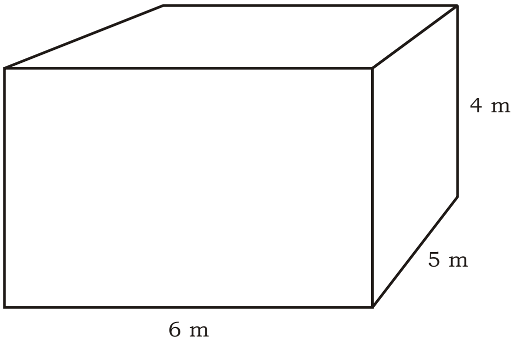
Solution:

Here

$l = 6$ m

b = 5 m

h = 4m



We know that

surface area of cuboid

$= 2\,(l \quad b + b \quad h + h \quad l)$

$= 2\,(6 \quad 5 + 5 \quad 4 + 4 \quad 6)$

$= 2\,(30 + 20 + 24)$

$= 2\,(74)$

= 148 cubic m

Total cost = Surface area x Rate

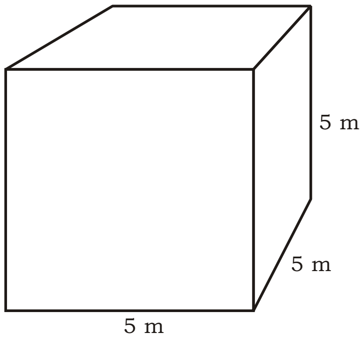
$= 148 \quad 50$

= 7400 rupees

Example 2: How much coloured paper is required to cover a cube of side 5 m completely.

Solution:

Here $l = 5$ m



Now

Surface area of cube $= 6\,(l^2)$

$= 6 \quad l^2$

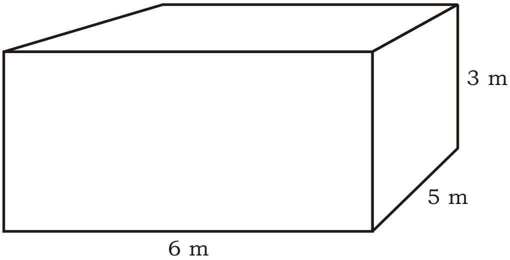
$= 6\,(5 \quad 5)$

$= 6 \quad 25$

= 150 sq. m

Required quantity of paper 150 sq. m

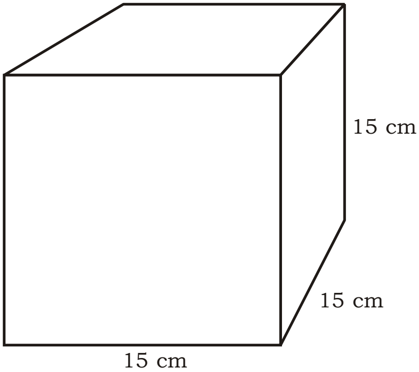
Example 3: Find the capacity of water tank of dimension 6 m, 5 m and 3 m.



Solution:
Here
l = 6 m
b = 5 m
h = 3 m

Now
Volume of cuboid = l b   h
= 6   5   3
= 90 cubic m
So, the required capacity of water tank is 90 cubic m.

Example 4: Find the cost of kerosine oil to fill a cubic tin with each side 15 cm at the rate of Rs 100 per cubic cm.



Solution:
Here l = 15 cm

Volume of cubic tin = l l l or l^3
= 15   15   15
= 5625 cubic cm
Required cost = 5625   100
= 562500 rupees

EXERCISE 12.3

1. Find the cost of polishing cupboard with dimension 3 m, 2 m and 1 m at the rate of Rs 500 per sq. m.
2. A tank is 5 m long, 4 m wide and 2.5 m high. Find the capacity of the tank?
3. The dimension of a brick are 25 cm   10 cm   4.5 cm. How much space will be occupied by 1000 such bricks?

4. Find the cost of painting a wooden cubic box with an edge of 2.5 m long at the rate of Rs 50 per sq. m.

5. Find the capacity of cubical oil tank with each side 5 m long.

6. How many boxes of dimensions 15 cm 10 cm 5 cm are required to fill a box of dimensions 30 cm 20 cm 10 cm.

7. If the cost of cake of dimensions 10 cm 5 cm 5 cm is Rs 100. What is the cost of the cake with dimensions 20 cm 15 cm 15 cm

8. Find the capacity of milk tank of dimensions 3.5 m 3.4 m 5.5 m

9. The dimensions of a tea box are $\frac{20}{3}$ cm $\frac{21}{4}$ cm $\frac{11}{2}$ cm. How much space will be occupied by 100 such boxes.

10. Find the cost of painting steel cube of each side 5.2 m long at the rate of Rs 15.5 per square metre.

REVIEW EXERCISE 12

1. Fill in the blanks.

(i) Cube has ___________ vertices.

(ii) Cone has ___________ circular face.

(iii) Cuboid has__________ faces.

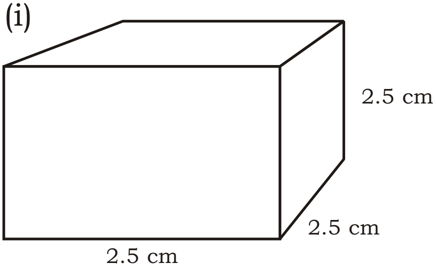
(iv) Cylinder has __________ circular faces.

(v) Cube has ___________ edges.

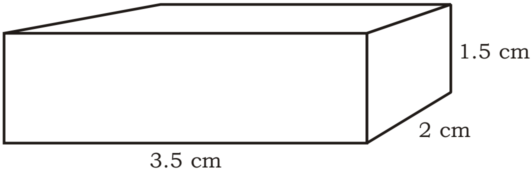
2. Give three examples of 3D figures from daily life.

3. Find the surface area and volume of the following:

(i)



(ii)



4. Find the volume and surface area of a cube whose each side is 1.5 cm long.

5. Find the cost of painting an iron cuboid with dimensions 5 m, 6 m and 7 m at the rate of Rs 100 per sq. m

6. How much milk can be filled in a tank with dimension of 6.5 m 2.7 m and 10 m.

SUMMARY

- Solid objects have three dimensions and are called 3D objects.
- Length, breadth and height of cube are equal.
- Length, breadth and height of cuboid are not equal.
- Cylinder has two circular faces and one curved surface.
- Cone has a circular and a curved surface.
- Sphere has complete round surface.
- Volume of cube = l^3 where l = Length of each side.
- Volume of cuboid = l b h
- Surface area of cuboid = 2 (l b + b h + h l)
- Surface area of cube = 6 l^2

UNIT 13

INFORMATION HANDLING

13.1 TYPES OF DATA

Define data and data collection

We have learnt in previous class that the information collected from any field of study is called data. “A data is set of information and facts which is collected in the form of figures by observation and measurement.

We can collect data by several ways i.e observation, survey, interview, questionnaire etc.

We can analyze and represent the data to derive the result that helps us to examine the given information.

In routine surveys, we collect the information by using a questionnaire like

Tick (✓) your favourite food.

Biryani ☐ Tikka ☐ Fried Fish ☐ Karahi ☐

Few more examples of data:

- Information of favourite game
- Information of favourite subject of students of any school
- Information of most eaten food in any restaurant etc.

We represent the collected data or information in diagram or graphs and interpret it

In previous class we have learnt tally chart which is used for representing simple data.

Teacher's Note *Teacher should develop the questionnaire and complete it with the help of students in the classroom.*

Activity

List the months of the year.
Ask each student in the class which months they were born.
Make a tally chart of this information.

Solution:

Month	Tally marks	No. of students
January		
February		
March		
April		
May		
June		
July		
August		
September		
October		
November		
December		

Teacher's Note *Teacher should complete the table by involving students.*

Distinguish between grouped and ungrouped data

Data which shows the individual information is known ungrouped data.

For example:

Students marks in English test are given in the following table.

Name	Ali	Sana	Umer	Ahmed	Raza	Rahila	Saba	Maria	Faraz	Huma
Marks	**46**	**50**	**30**	**42**	**69**	**59**	**70**	**75**	**40**	**72**

If we arrange or classify the data in groups, it is called grouped data.

Example:

Following are the marks scored out of 100 by 15 students of class VI in test of Science.

76, 51, 65, 62, 70, 63, 69, 62, 75, 61, 80, 55, 65, 59, 60.

The student marks range from 50 to 80.
The marks can be organised into following groups.

51 – 60, 61 – 70, 71 – 80

Marks	Marks obtained	No. of Students
51 – 60	51, 55, 59, 60	4
61 – 70	61, 62, 62, 63, 65, 65, 69, 70	8
71 – 80	75, 76, 80	3

Represent the above data using tally chart.

EXERCISE 13.1

1. Define data.

2. What is difference between grouped and ungrouped data?

3. Which table is showing a grouped data?

(i) Total marks secured by 12 students in an examination.

Names of Student	Ali	Sana	Sara	Adil	Amir	Asma	Huma	Arif	Babar	Anis	Anil	Umar
Marks	581	786	678	725	788	580	690	780	599	509	619	560

(ii) Raza Book Store's sale in a month is shown by the table:

Number of books sold	Days
512 – 611	8
612 – 711	5
712 – 811	4
812 – 911	9

(iii) Visitors in Sindh Museum.

Number of visitors	Days
150 – 199	3
200 – 249	4
250 – 299	1
300 – 349	2
350 – 399	3
400 – 499	2

(iv) Runs scored by players of our school cricket team in one day match.

Player	1	2	3	4	5	6	7	8	9	10	11
Runs	62	41	15	59	22	10	8	2	43	7	21

13.2 BAR GRAPH

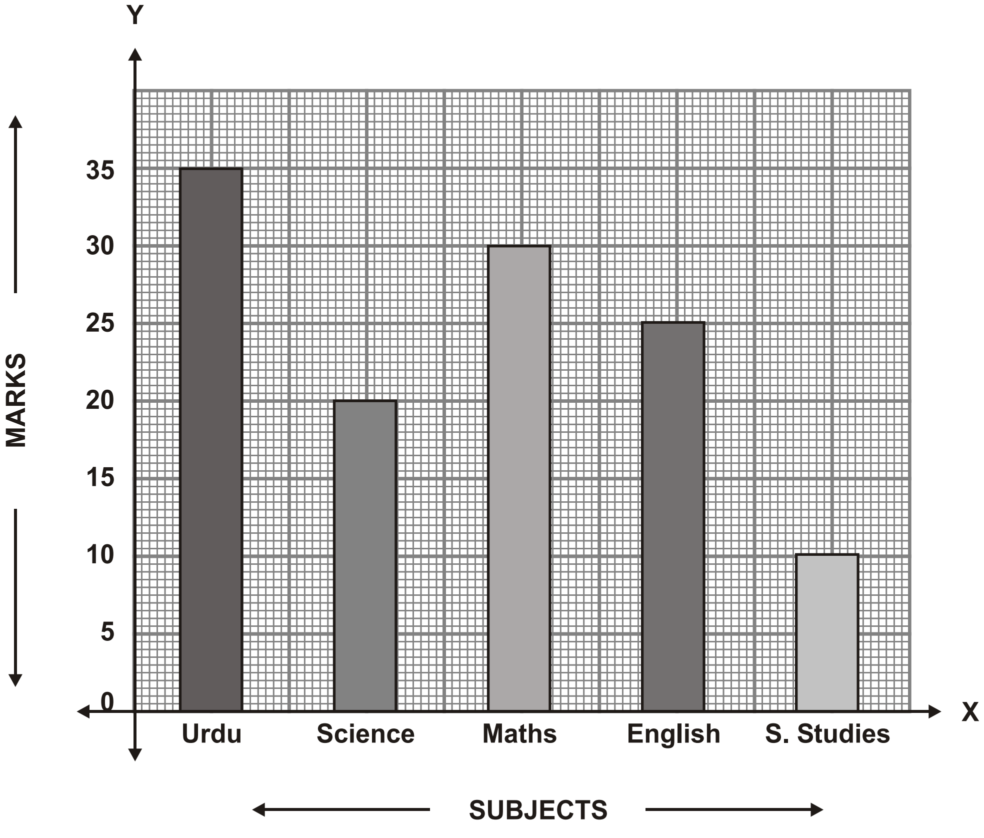
Draw horizontal and vertical bar graphs

We have learnt about Bar Graph in previous classes. A lot of information can be represented by a bar graph. In bar graph data is represented by a number of rectangular bars of the same width with equal spacing. We use two ways to draw bar graph.

1- Vertical Bar Graph
2- Horizontal Bar Graph

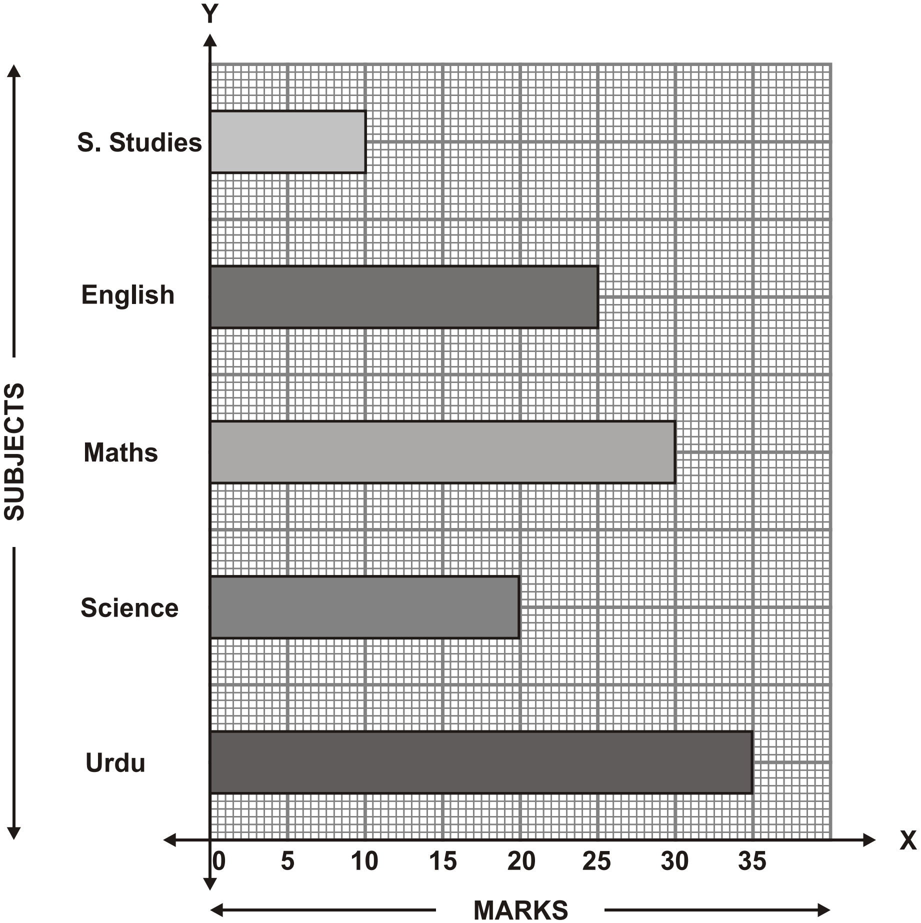
This bar graph represents the test marks of Fozia in different subjects through vertical bars.

Vertical bar graph showing marks of Fozia



This is the horizontal bar graph represents the same test marks of Fozia in different subjects.

Horizontal bar graph showing marks of Fozia



Steps to draw horizontal and vertical bar graphs.

Step 1. Take a graph paper and draw two lines $\overleftrightarrow{OX}$ and $\overleftrightarrow{OY}$, perpendicular to one another. $\overleftrightarrow{\mathbf{OX}}$ is called **X-axis** and $\overleftrightarrow{\mathbf{OY}}$ is called **Y-axis** and **O** is the **origin**. Give the graph a title.

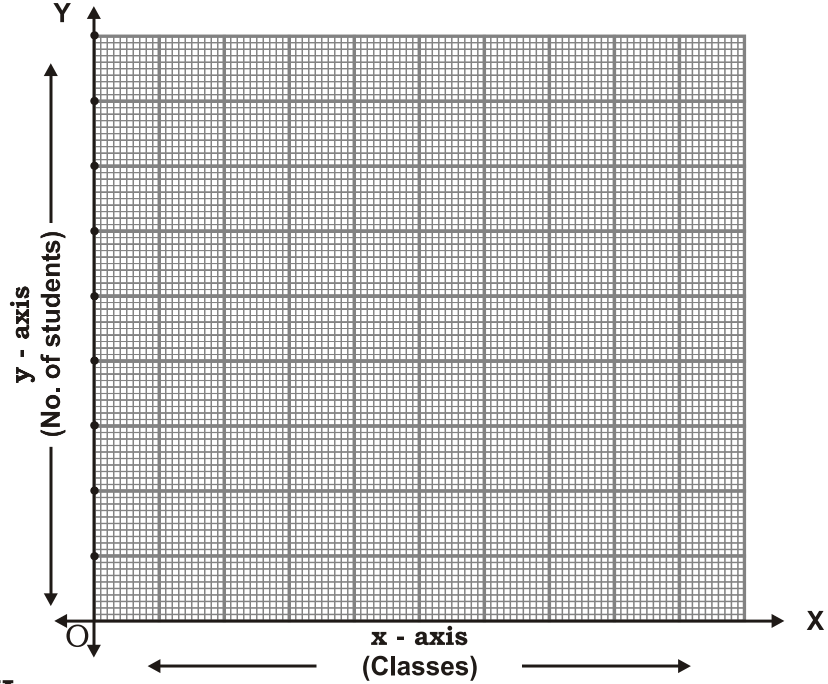
Step 2. Decide a scale for each axis.

Step 3. Make bars of equal breadth at equal distances according to the information.

Example: Draw a vertical bar graph showing the following information.

Class	I	II	III	IV	V
Number of students	45	30	25	36	45

Step I: In this graph the "number of students" are shown on Y-axis and classes are shown on X-axis.



Step II:

Scale: Two small square on Y-axis represents one student.

Length of each bar is taken according to the number of students.

Breadth of each bar is 10 small squares and distance between them is 10 small squares on X-axis.

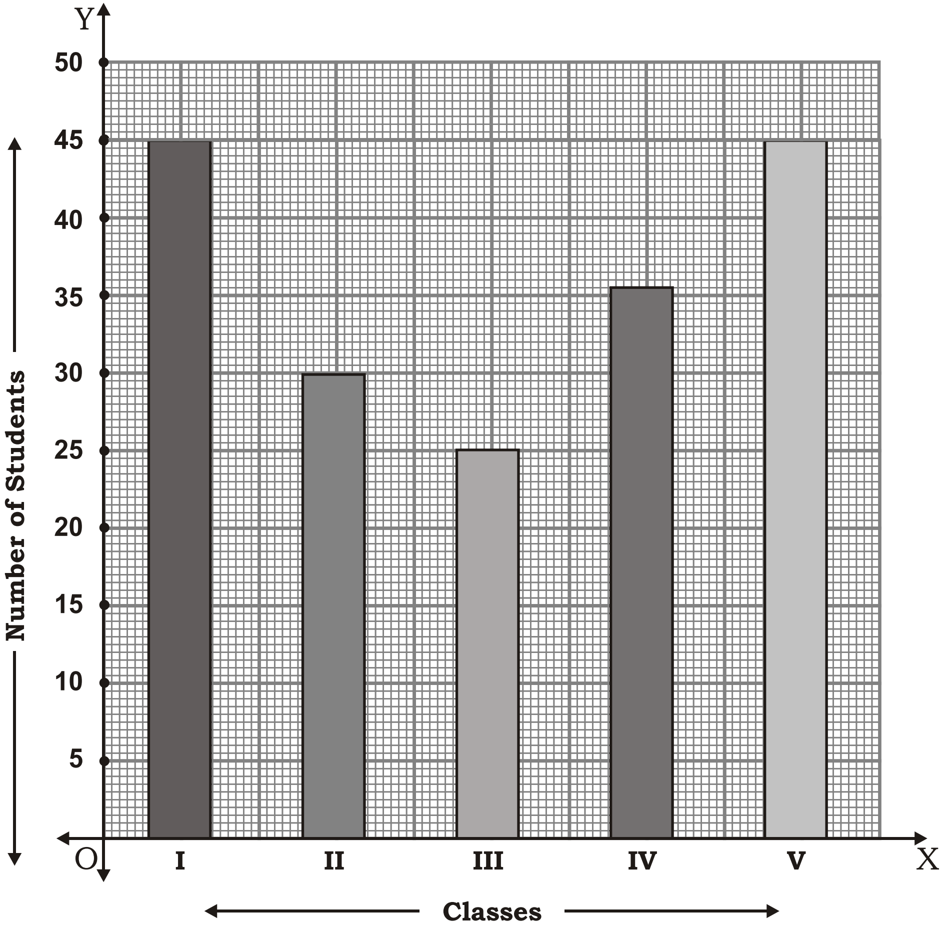
Step III:

In Class I: There are 45 students. Every two small squares on Y-axis represents a student, so we draw the bar graph up to 45 square to represent number of student.

Similarly, draw bars for representing other classes upto the required heights.

Teacher's Note *Teacher should give some more related questions.*

Vertical Bar Graph representing number of students in classes I to V.



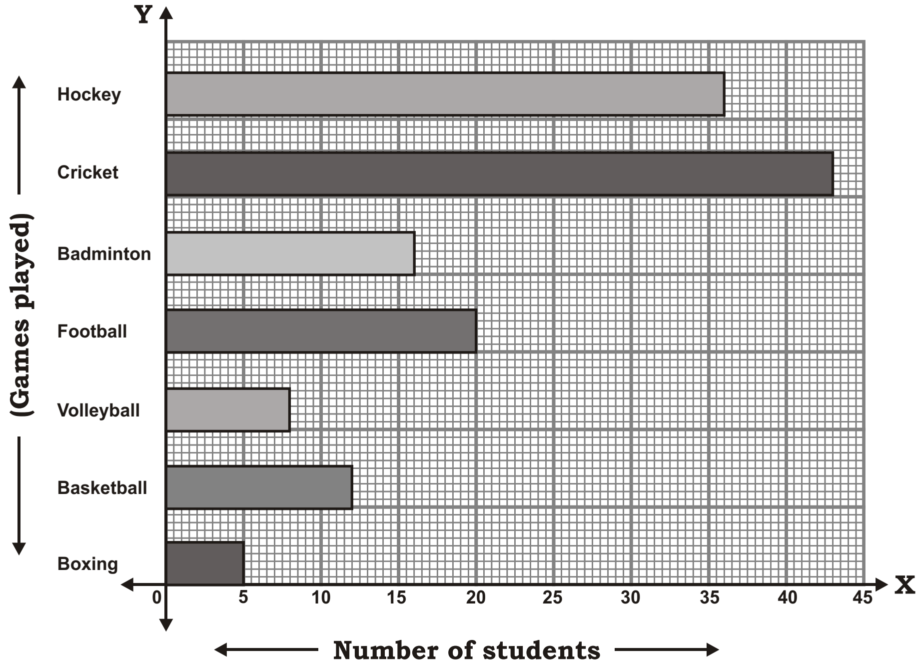
Check list for drawing the bar graph.

1. Read the information carefully.
2. Select proper axes for given information.
3. Make a scale.
4. Draw the bar for each information.
5. Label the graph and give a title.

Example: Games played by students of Class V in a school are given in the table below. Represent data in horizontal bar graph.

Game played	No. of students
Hockey	36
Cricket	43
Badminton	16
Football	20
Volleyball	8
Basketball	12
Boxing	5

1. Read the table, there are seven different types of games played by students.
2. We select x-axis for number of students and y-axis for games played.
3. Make a scale for number of students and played games. 1 unit represents 2 students.
4. Draw horizontal bar for each game.
5. Lable both axis and give the graph a title.



Activity **Write three sentences about the information you get from the previous graph.**

1. Cricket is played by most of the students than any other game. So the longest bar will represent cricket with length of bar at 43 units.

2. ______________________________

3. ______________________________

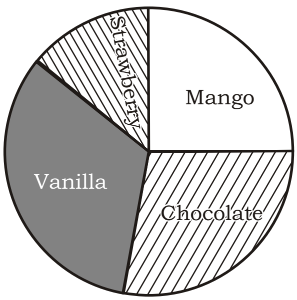
13.3 PIE GRAPH

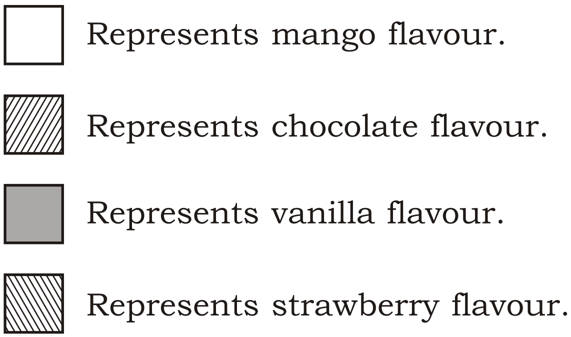
A pie chart is circle graph. When we have many quantities in data, we used a pie graph or pie chart to represent it.

Read a pie graph

In this graph circle is used to represent data. Different quantities are represented by sectors. We know that there are 360° in a circle. The quantities are represented by sectors of a circle . It is useful to show or to compare various quantities at a glance.

Example 1: The data of cone ice cream of different flavours sold at a shop in a day are shown in the pie graph as under.





Study the graph and answer the following questions:

(i) Name the flavour with most sale. Vanilla

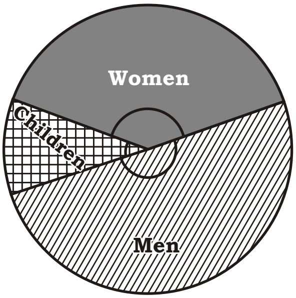
How do you come to know? Because the representing sector is largest in size.

(ii) Name the falvour with least sale. Strawberry

How do you come to know? Because the representing sector is smallest in size.

(iii) Select the flavour which is sold more from mango and vanilla. Chocolate

Activity **The number of 600 pilgrims in a plane is shown by the following Pie graph.**



Here ▨ represents men

■ represents women

▦ represents children

Read the pie graph and answer the following questions:

1. Who are the most on the plane? Men
2. Who are least on the plane?
3. Are there children equal to women passengers?
4. Are men passengers more than women passengers?

EXERCISE 13.2

1. **Draw a vertical bar graph to represent the information given below:**

Monthly Sale of Fans in a Shop

Name of Months	January	February	March	April	May	June
No. of Fans	70	50	35	60	90	95

2. **Represent the following information in a vertical bar graph.**

Monthly Expenses of a Akbar's Family

Items	Food	Education	Rent	Telephone	Electricity	Gas
Expenses (in rupees)	6000	5000	4500	1500	2000	300

3. **Represent the following information in a horizontal bar graph.**

Sale of Variety Show Ticket

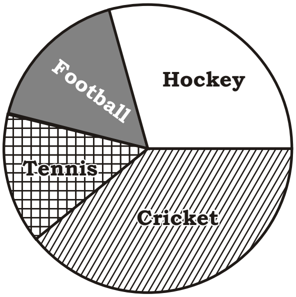
Name of cities	Hyderabad	Sukkur	Larkana	Dadu
Number of Tickets sold	10,000	6,000	3,000	9,000

4. **Draw a horizontal bar graph showing the marks obtained by Amjad during annual examination.**

Subject	Islamiat	English	Urdu	Maths	S.Studies	Science	Sindhi
Marks Obtained	70	60	65	90	45	55	80

5. Following pie graph is representing the games played by the students of class VI.

Study the pie graph and answer the following questions.



(i) Which game is played less?

(ii) Which game is played most?

(iii) Which games are played equally?

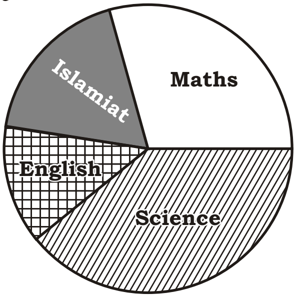
6. The marks obtained in different subjects by Aftab are shown by the following pie graph.

Study the pie graph and answer the following questions.

(i) In which subject, Aftab secured highest marks?

(ii) In which subject, Aftab secured lowest marks?

(iii) In which subject, Aftab secured more marks in science or maths?



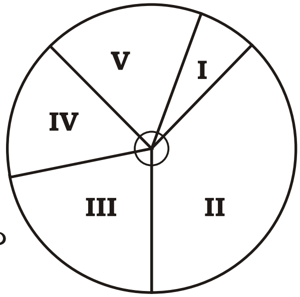
7. The following pie graph shows the attendance of the students of different classes of a school on Saturday.

Study the pie graph and answer the following questions.

(i) In which class, attendance was the highest?

(ii) In which class, attendance was the lowest?

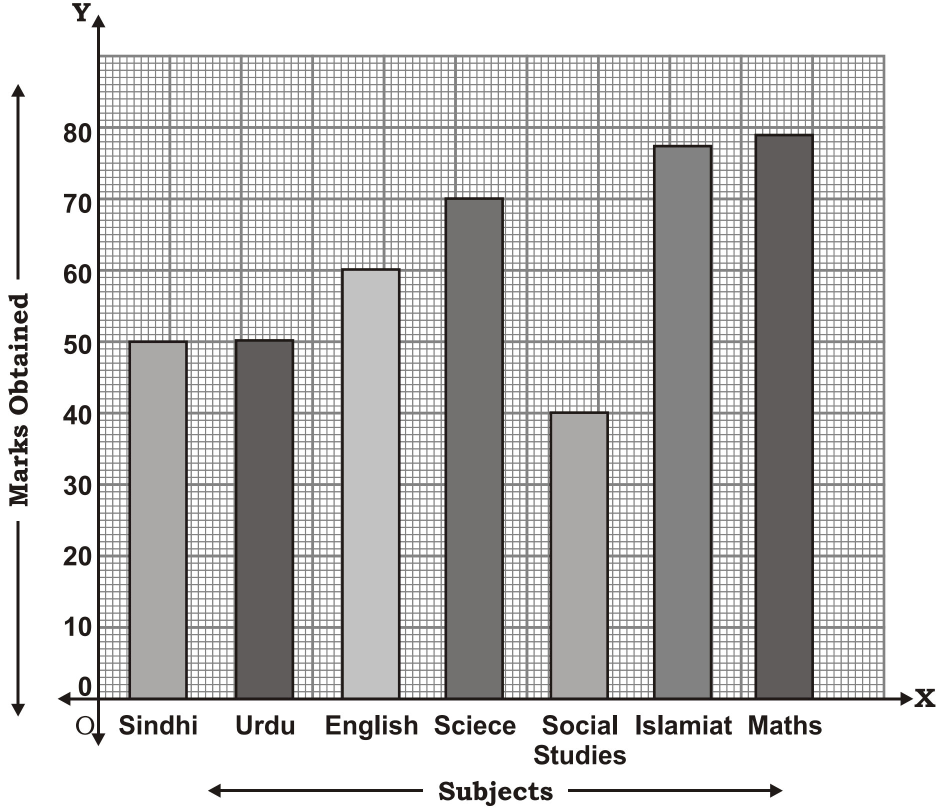
(iii) Which classes have same number of students?



REVIEW EXERCISE 13

1. **The marks obtained by Saira are shown by a vertical bar graph. Answer the given questions by reading the bar graph.**

Graph represents marks obtained by Saira in different subject.



(i) In which subject Saira obtained highest marks and how many?

(ii) In which subject Saira obtained lowest marks and how many?

(iii) In which subjects Saira obtained equal marks and how many?

(iv) Prepare a chart which represents the marks in each subject.

2. Are the following statements true or false?

(i) The reading of numerical data in pictograph is easy.

(ii) In bar graphs numerical data is represented by the rectangular boxes of uniform width.

(iii) To arrange the data in some order is called ungrouped data.

(iv) A rectangular diagram is called pie chart.

3. Fill in the blanks.

(i) To represent numerical data pictorially is called______ graph.

(ii) Representation through_________graphs is easier and better than the pictograph.

(iii) In bar graphs, bars can be drawn either horizontally or ________________.

(iv) _______________ is defined as a set of facts.

(v) Numerical facts which are obtained from field is known as ____________.

SUMMARY

- Data is defined as "a set of observation, measurements or facts".
- Data is called Group Data if classified under certain criteria otherwise it is ungrouped Data.
- In bar graph the quantities are represented by bars of equal width.
- In a bar graph, the bars may be vertical or horizontal.
- In a Pie graph, a circle is divided into a number of sectors, with one sector for each category of Data.

GLOSSARY

Acute angle:	An angle which is less than 90°.
Acute angled triangle:	A triangle which has one of its angle acute angle.
Adjacent angles:	Two angles with a common vertex and a common arm are called adjacent angles.
Angle:	The amount of turning between two arms about a common point.
Antecedent:	The first element of a ratio.
Altitude:	Shortest distance from top to the base of a geometric figure.
Arc:	A part of a circle.
Area:	The space occupied within the boundary of a shape is called an area.
Area of parallelogram:	Area of Parallelogram = Base x Altitude
Area of triangle:	Area of Triangle = $\frac{1}{2}$ x Base x Altitude
Associative property addition:	The property that when any three numbers (fractions) are added in any order, their sum is always the same.
Associative property multiplication:	The property when any three numbers (fractions) are added in any order, their sum is always the same.
Average:	The quantity that represents the given quantities.
Bar graph:	It represents each part of the information in the form of bars (vertical or horizontal).
Block graph:	The graph in which we choose a suitable symbol to represent each part of the information.
Bisecting:	Dividing into two equal parts.
Bracket:	Symbols $\overline{\quad}$, ( ), { } and [ ]
Capacity:	The amount of liquid a container can hold.
Chord:	A line segment joining any two points of a circle.
Commutative property of Multiplication:	The property that any two numbers when multiplied to each other in any order, their product is always same.
Commutative property of addition:	The property that when any two numbers (fractions) are added in any order their sum is always same.

Consequent: The second element of a ratio

Composite number: A natural number having more than two factors.

Cube: A three dimentional solid object having equal length, width and height.

Cuboid: A three dimentional solid object having un equal length, width and height.

Data: Information presented in the form of numbers.

Decimal fraction: A common fraction with a denominator as 10,000, written with a decimal point.

Denominator: Lower number of the common fraction.

Direct proportion: The relationship between two ratios in which increase in one quantity causes a proportional increase in the other quantity and decrease in one quantity causes a improportional decrease in the other quantity.

Directed numbers (Integers): Zero, positive numbers and negative numbers are called directed numbers or integers.

Divisibility: A division in which when a number is divided by another, the remainder is zero.

Edge: A one dimensional line segment joining two vertices.

Equation: A sentence which shows the relation of equality between two algebraic expressions.

Equivalent fraction: The fractions that have the same value.

Equivalent triangle: A triangle in which all the three sides are equal in length.

Even number: A number which is exactly divisible by 2.

Factors: The divisor of a number.

Factorization: A number represented as a product of its factors.

Graph: A pictorial representation of a data.

HCF: Highest Common Factor.

Hypotenuse: Side of a triangle which is opposite to the right angle.

Index notation: A short way of writing a number being multiplied by itself several times.

Integer: A whole number that can be positive, negative or zero.

Inverse proportion: The relationship between the two ratios in which increase in one quantity causes a proportional decrease in the other quantity and a decrease in one quantity causes a proportional increase in the other quantity.

Isosceles triangle:	A triangle with its two sides equal in length.
L.C.M	Least Common Multiple.
Like decimals:	The decimals having same number of decimal places.
Like fractions:	Fraction having same denominator.
Mass:	Quantity of matter present in a body.
Million:	The smallest seven digit number i.e. 1,000,000 (Ten hundred thousand).
Mixed fraction:	A fraction contains both a whole number and a proper common fraction.
Natural number:	A number that occurs commonly and obviously in nature.
Obtuse angle:	An angle which is more than 90°.
Obtuse angled triangle:	A triangle which has one of its angles obtuse angle.
Odd number:	A number which is not exactly divisible by 2.
Parallelogram:	A quadrilateral having two pairs of opposite sides parallel.
Percentage:	The word percent is a short form of the Latin word "Percentum". Percent means out of hundred or per hundred.
Perimeter:	The distance along the sides of a closed shape.
Pie Graph:	A circular graph used to show many data quantities.
Positive integer:	A natural number.
Prime factorization:	A factorization in which every factor is a prime factor.
Prime number:	A natural number having exactly two distinct factors.
Proper fraction:	A fraction whose numerator is less than the denominator.
Proportion:	The quality of two ratios.
Quadrilateral:	A four sided closed figure.
Quotient:	The number shows how many times the divisor has been repeatedly subtracted.
Ratio:	A comparison of two quantities of the same kind.
Radius:	The distance from the centre of the circle to the boundary of the circle.
Rectangle:	A quadrilateral whose opposite sides are equal and have four right angles.
Reflex angle:	An angle of measure greater than 180°.
Right angle:	An angle whose measure is 90°.
Right angled triangle:	A triangle which has one of its angle of the measure 90°.
Right bisector:	A bisector which makes a right angle with the given line segment.

Round off decimals: To round off a decimal nearest to the whole number, check the first decimal place and accordingly round off the number.

Scalene triangle: A triangle whose all sides are of different measures.

Simplification: The process of getting a simplified number.

Sentence: Group of numbers and terms which has complete sense and a relation. There are three types of sentences.
1. True sentences
2. False sentences
3. Open sentences

Square: A quadrilateral whose all four sides are equal and has four right angles.

Straight angle: An angle whose measure equals to 180°.

Subtraction: Symbol (–). The process of finding the difference between two numbers/quantities.

Supplementary angles: Two angles whose sum of the measures is equal to 180°.

Symbol: A sign used to represent an operation, element or relation.

Trapezium: A quadrilateral having one pair of opposite sides parallel.

Unit fraction: Numerator is equal to the denominator.

Unitary method: The process of finding the price of one (unit) item, from which we find the price of a number of similar items.

Unlike decimals: The decimals having different number of decimal places.

Unlike fractions: Fractions whose denominators are not same.

Variable: An alphabetical letter which represents any of the given numbers.

Vertex: An angular point of any shape.

Volume: The measurement of the space occupied by a three dimentional solid object.

Volume of a cube: Volume of a cube = Side x Side x Side

Volume of a cuboid: Volume of a Cuboid = Length x Breadth x Height

Whole number: A number having no fractional or decimal part and no negatives.

ANSWERS

EXERCISE 1.1

1. (i), (ii) and (vi) are sets **2.** (i) (ii) (iii) (iv) (v) (vi)

3. (i) {Karachi, Lahore, Quetta, Peshawar, Gilgit}
(ii) {50, 51, 52, ..., 69, 70} (iii) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
(iv) {p, a, k, i, s, t, n} (vi) {White, Green}

4. (i) True (ii) True (iii) False (iv) False (v) False
(vi) True (vii) False (viii) True

EXERCISE 1.2

1. (i) "0" is repeating (ii) "★" is repeating (iii) "*a*" is repeating
(iv) "2" and "3" and "4" are repeating

2. (i) Finite (ii) Infinite (iii) Infinite (iv) Finite (v) Finite
(vi) Infinite (vii) Infinite (viii) Finite (ix) Finite (x) Infinite

3. (i), (ii), (iv) and (v) are empty sets **4.** (i) and (ii) are equal sets

5. (i), (ii) and (iii) are equivalent sets

6. (i) True (ii) False (iii) True (iv) False (v) True

7. (i) (ii) = (iii) (iv) (v)
(vi) (vii) (viii)

REVIEW EXERCISE 1

1. (i) elements (ii) empty set (iii) all same

2. (i) *d* (ii) *d* (iii) *d* (iv) *c*

4. (i) A = {4, 6, 8} (ii) B = {1, 3, 5, ..., 15, 17}
(iii) C = {2, 4, 6, ..., 26, 28} (vi) D = {January, June, July}

5. (i) A is a finite set (ii) B is an infinite set (iii) C is a finite set
(iv) D is an infinite set (v) E is a finite set

6. F and M are empty sets

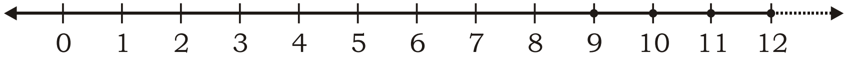
7. (i) True (ii) True (iii) False (iv) True

8. (i) True (ii) True (iii) False (iv) False (v) True
(vi) True (vii) False (viii) False (ix) False

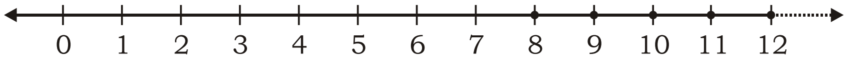
EXERCISE 2.1

1. (i) 1 (ii) 0 (iii) Not possible (iv) Not possible

2. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **3.** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

1. (i)


(ii)


(iii)


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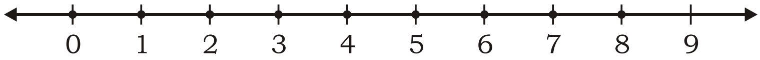

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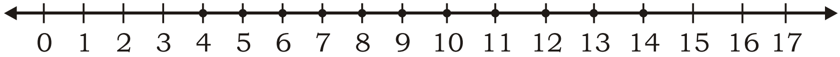

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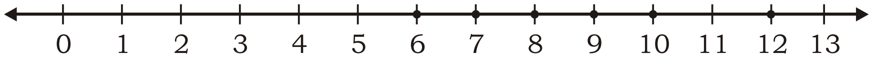

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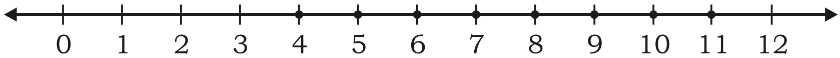

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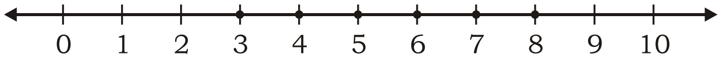

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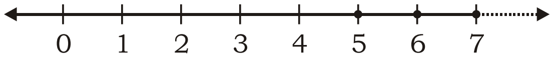
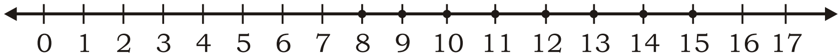
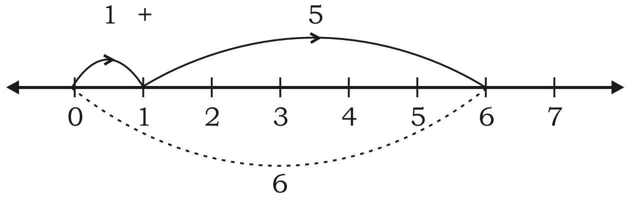


(x)


5. (i)



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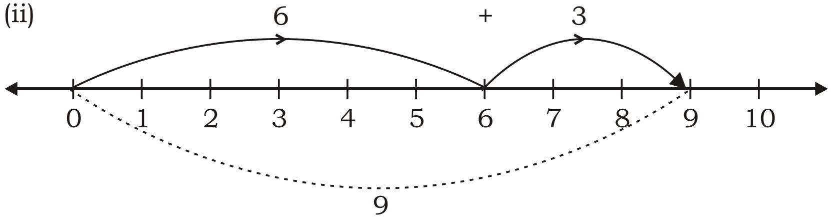

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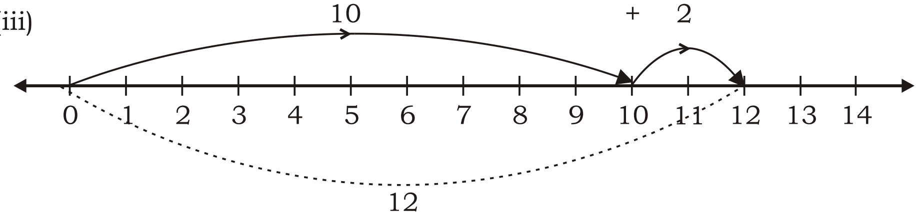

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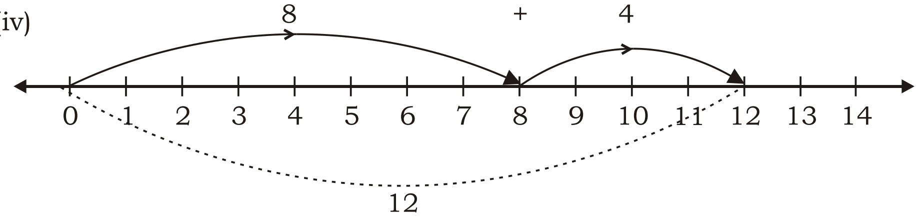
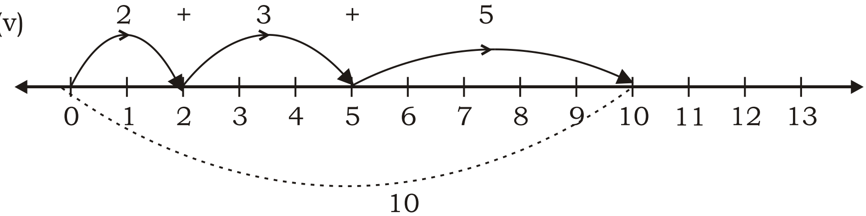
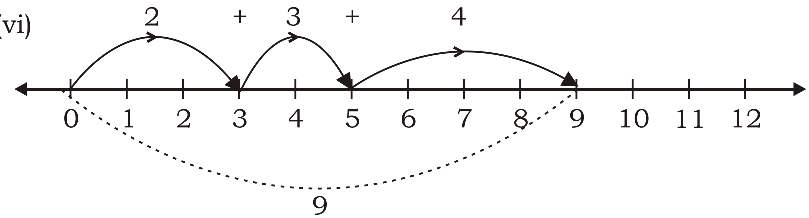


(v)



(vi)



EXERCISE 2.2

1. Required sum is 1,999,999

2. Number of children is 715

3. Rs 42300 left in account

4. Rs 49615 left with him

7. (i) 849 (ii) 97864 (iii) 749,749 (iv) 430 (v) 4946

8. The starting number

EXERCISE 2.3

1. (i) 81984 (ii) 75808 (iii) 258048 (iv) 157210

2. (i) Quotient = 134, Remainder = 0 (ii) Quotient = 393, Remainder = 39
(iii) Quotient = 16, Remainder = 25 (iv) Quotient = 309, Remainder = 145

3. 30 trees in each row

4. Rs 1,227,500

5. Required number = 99975

6. 999900

EXERCISE 2.4

1. (i) 0 (ii) 6 (iii) 2 (iv) 4 (v) 7, 5 (vi) 964 (vii) –, 125

2. (i) False (ii) False (iii) True (iv) True

REVIEW EXERCISE 2

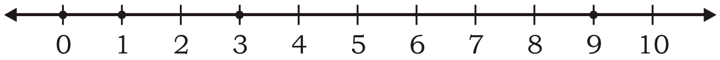
1. (i) 3, 2 (ii) 528, 112 (iii) 8 (iv) 1, 7 (v) 1, 1
(vi) 0, 0 (vii) 9, 9 (viii) 20, 1 (ix) 8, 17 (x) 32, 32

2. (i) False (ii) False (iii) False (iv) True (v) True
(vi) False (vii) False (viii) True (ix) False (x) True

3. (i) Predessor = 670, Successor = 672 (ii) Predessor = 244, Successor = 246

4. (i) b (ii) a (iii) d (iv) b (v) b

5. (i)



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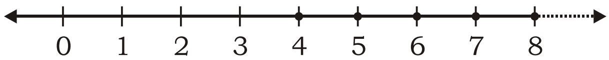
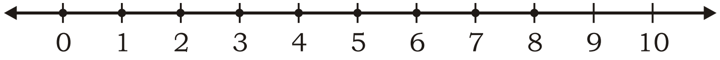
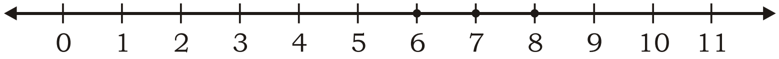
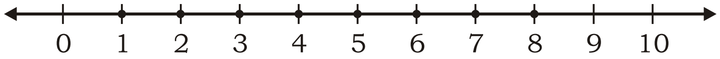
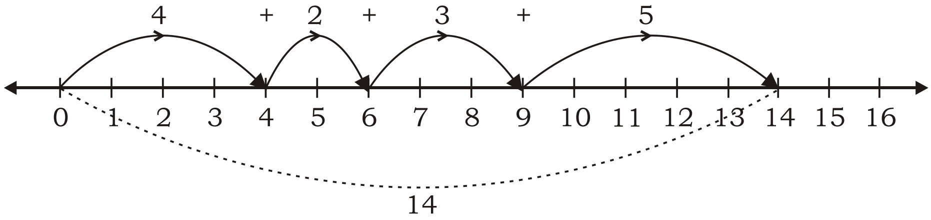


(iii) 

(iv) 

(v) 

6. 

11. Rs 960 **12.** Required number = 9988

EXERCISE 3.1

1. a and c

2. (a) 1, 3, 5, 15 (b) 1, 2, 3, 5, 6, 10, 15, 30 (c) 1, 2, 5, 10, 25, 50
(d) 1, 5, 25, 125 (e) 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150

3. b and c

4. (a) 4, 8, 12, 16, 20, 24, 28 (b) 7, 14, 21, 28, 35, 42, 49
(c) 12, 24, 36, 48, 60, 72, 84 (d) 15, 30, 45, 60, 75, 90, 105

5. (a) 7, 11 and 13 (b) 23, 29, 31 and 37
(c) 61, 67, 71, 73, 79, 83 and 89

6. 2 is an even prime number

7. (a) even (b) odd (c) odd (d) odd (e) even (f) odd

8. 22, 24, 26, 28, 30, 32, 34, ..., 48, 50

9. 11, 13, 15, 17, 19, ..., 37, 39

10. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39.

11. 72, 74, 75, 76, 77, 78, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

12. 23, 29, 31, 37, 41, 43, 47, 51, 53, 59.

13. Because 1 has only one factor.

EXERCISE 3.2

1. (i) Yes (ii) Yes (iii) Yes (iv) No (v) No (vi) Yes
(vii) Yes (viii) No (ix) Yes (x) Yes (xi) Yes (xii) No

2. (ii) and (iii) **3.** (ii) and (iii) **4.** (i) and (ii) **5.** (ii) and (iii)
6. (ii) **7.** (i) and (iii) **8.** (i) and (iii) **9.** (i) and (iii)
10. (i) and (iii) **11.** (i) and (iii)

EXERCISE 3.3

1. (i) 2^3 3^2 5^1 (ii) 3^1 5^3 7^2 (iii) 2^1 3^4 7^2

2. (i) 2^3 3^1 (ii) 2^4 3^1 (iii) 2^3 3^3 (iv) 2^1 5^3
(v) 2^2 3^2 13^1 (vi) 2^2 3^3 5^1 (vii) 2^{10} (vii) 2^3 5^4

3. Yes

EXERCISE 3.4

1. (i) 25 (ii) 98 (iii) 18 (iv) 12 (v) 53 (vi) 12 (vii) 10

2. (i) 4 (ii) 9 (iii) 55 (iv) 36 (v) 747 (vi) 30

EXERCISE 3.5

1. (i) 600 (ii) 441 (iii) 5760 (iv) 1620 (v) 6825 (vi) 144

2. (i) 495 (ii) 210 (iii) 120 (iv) 1080

3. LCM = 6

EXERCISE 3.6

1. Greatest Length = 180 cm
2. Greatest Number = 75
3. Largest Number = 16
4. Maximum Capacity = 30 litres
5. Longest Tape = 2 m
6. Required Time = 8.30 a.m.
7. 189
8. Least Quantity = 48 litres
9. Required Time = 7 minutes 12 seconds

REVIEW EXERCISE 3

1. (i) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60,
(ii) 1, 2, 5, 10, 25, 50, 125, 250

2. (i) 13, 26, 39, 52, 65 (ii) 20, 40, 60, 80, 100

3. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

4. 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50, 52, 54, 55, 56, 57, 58.

5. (ii), (iii) and (iv) **6.** (i), (iii) and (iv) **7.** (ii) and (iv)

8. (i) and (ii) **9.** (ii) and (iii)

10. (i) 2^1 3^2 5^2 (ii) 2^4 3^2 5^1

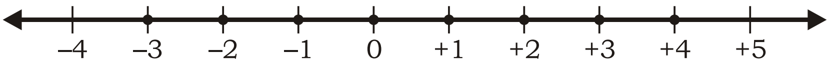
11. (i) HCF = 6, LCM = 72 (ii) HCF = 1, LCM = 120

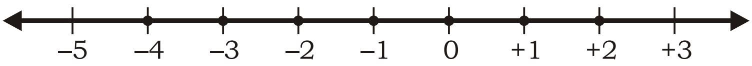
12. (i) HCF = 5, LCM = 1040 (ii) HCF = 13, LCM = 1690

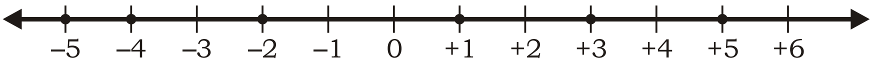
13. Greatest Number = 10 **14.** 186

EXERCISE 4.1

1. (i)


(ii)


(iii)


(iv)


2. (i) +15 > –6 (ii) –8 < 0 (iii) +16 > 0 (iv) –2 > –8 (v) + > < +9 (vi) –4 < –1

3. (i) Left (ii) Right (iii) Right (iv) Left

4. (i) > (ii) < (iii) > (iv) < (v) < (vi) >

5. (i) Ascending order: –7, –3, –1, 0, +1, +5
Descending order: +5, +1, 0, –1, –3, –7

(ii) Ascending order: –3, –1, 0, +2, +4, +5
Descending order: +5, +4, +2, 0, –1, –3

(iii) Ascending order: –5, –4, 0, +4, +5
Descending order: +5, +4, 0, –4, –5

(iv) Ascending order: –8, –7, –4, –2, –1
Descending order: –1, –2, –4, –7, –8

6. (i) 5 (ii) 20 (iii) 0 (iv) 18 (v) 50

7. (i) Ascending order: 0, 1, 3, 4, 5, 6
Descending order: 6, 5, 4, 3, 1, 0

(ii) Ascending order: 0, 10, 17, 25, 30, 60
Descending order: 60, 30, 25, 17, 10, 0

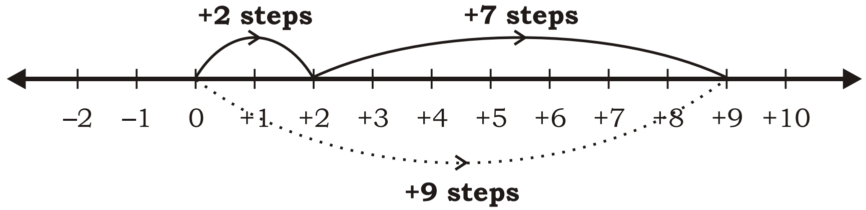
(iii) Ascending order: 0, 4, 7, 10, 11, 20
Descending order: 20, 11, 10, 7, 4, 0

(iv) Ascending order: 3, 5, 8, 9, 12, 13
Descending order: 13, 12, 9, 8, 5, 3

8. (i) True (ii) False (iii) True (iv) False
(v) False (vi) True (vii) True

EXERCISE 4.2

1. (i)



Students may draw number line in their own copies themselves

(ii) –11 steps (iii) +3 steps (iv) – 6 steps
(v) – 3 steps (vi) 0 steps

2. (i) +10 steps (ii) –12 steps (iii) +10 steps (iv) –13 steps

3. (i) +30 (ii) – 40 (iii) –27 (iv) – 9
(v) – 6 (vi) 0

EXERCISE 4.3

1. (i) +2 steps (ii) –5 steps (iii) – 6 steps (iv) –14 steps
(v) +11 steps (vi) –11 steps

2. (i) −16 (ii) −10 (iii) +1 (iv) +5 (v) +20 (vi) +3

3. (i) +40 (ii) −5 (iii) −16 (iv) +10 (v) +2 (vi) +35 (vii) −46 (viii) −50 (ix) −10

EXERCISE 4.4

1. (i) −60 (ii) +340 (iii) +400 (iv) −432 (v) −490 (vi) +688

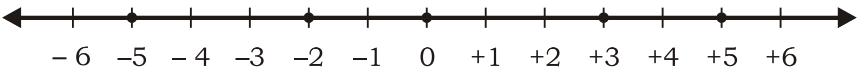
2. (i) −5 (ii) +5 (iii) +25 (iv) −63 (v) −78 (vi) +21

3. (i) False (ii) True (iii) False (iv) True (v) False (vi) True (vii) False (viii) True

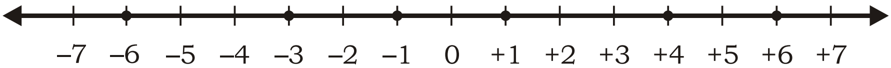
REVIEW EXERCISE 4

1. – 4, –3, –2, –1, 0, +1, +2, +3, +4

2. (i)



(ii)



3. (i) 3, 4, 4, 6, 0 (ii) 2, 7, 1, 0, 4, 2

4. (i) −3 steps (ii) −15 steps (iii) +11 steps (iv) 0 steps

5. (i) +2 steps (ii) −5 steps (iii) +7 steps

6. (i) – 4 (ii) −14 (iii) −3 (iv) +30 (v) −5 (vi) +15

7. (i) −50 (ii) +54 (iii) +120 (iv) – 45 (v) 0 (vi) +80

8. (i) −5 (ii) +5 (iii) – 6 (iv) +4

EXERCISE 5.1

1. $\frac{7}{20}$ 2. $1\frac{43}{60}$ 3. $2\frac{33}{56}$ 4. $\frac{7}{8}$ 5. $\frac{1}{68}$

6. $9\frac{5}{6}$ 7. $\frac{-1}{2}$ 8. $3\frac{15}{22}$ 9. $12\frac{1}{4}$ 10. $14\frac{1}{6}$

11. $10\frac{7}{12}$ **12.** $1\frac{1}{30}$ **13.** $1\frac{6}{49}$ **14.** 28.944 **15.** 24.373

16. 53.01 **17.** 0.88 **18.** 2.408 **19.** 86.965 **20.** 8.207

21. 0.95 **22.** 4.04 **23.** 3.8

EXERCISE 5.2

1. 48 students **2.** 70 oranges left **3.** $\frac{19}{56}$ on foot

4. Rs 19500 left **5.** $\frac{16}{17}$ required fraction **6.** $\frac{16}{35}$ required fraction

7. $\frac{92}{265}$ required fraction **8.** Rs 3023.50 total amount

9. $\frac{19}{100}$ required fraction **10.** Rs 4926.55 required amount

REVIEW EXERCISE 5

1. (i) $\frac{21}{29}$ (ii) $\frac{9}{20}$ (iii) 23.951 (iv) 4.475

2. (i) $9\frac{1}{7}$ kg left (ii) $\frac{3}{5}$ solved and $\frac{2}{5}$ unsolved questions

3. $\frac{3}{5}^{th}$ part on next day, 60 pages on 1^{st} day, 90 pages on next day

4. Rs 135523.03 required amount **5.** (i) True (ii) False (iii) True

6. (i) Multiplication (ii) Braces or curly brackets
(iiii) Parenthesis, braces, square brackets

EXERCISE 6.1

1. (i) 2 : 9 (ii) 5 : 6 (iii) 1 : 75 (iv) 3 : 1 (v) $p : q$

2. (i) $\frac{1}{5}$ (ii) $\frac{2}{19}$ (iii) 8 (iv) $\frac{75}{76}$ (iv) $\frac{x}{y}$

2. (i) 2 : 25 (ii) 1 : 90 (iii) 5 : 7 (iv) 3 : 28 (v) 25 : 9
(vi) 2 : 15 (vii) 5 : 8 (viii) 6 : 3 : 2 (ix) 15 : 50 : 58 (x) 34 : 6 : 35
(xi) 6 : 3 : 2 (xii) 6 : 3 : 21

3. (i) 5 : 6 (ii) 5 : 2 (iii) 7 : 10 (iv) 6 : 7 (v) 20 : 1
(vi) 15 : 14 : 30

4. 5 : 9 **5.** 4 : 3 **6.** 2 : 5 **7.** 10 : 9 **8.** 1 : 2 : 3

EXERCISE 6.2

1. (i) means = 5 and 8, extremes = 2 and 20
(ii) means = 4 and 6, extremes = 3 and 8
(iii) means = b and c, extremes = a and d

2. (i), (ii) and (vi) are in proportion **3.** $x = 4\frac{2}{3}$

4. (i) 9 (ii) $\frac{7}{11}$ (iii) 6 (iv) 21 (v) 5 (vi) 18

5. (i) 30 (ii) 24 (iii) 42 (iv) 18 (v) 60 (vi) 66

EXERCISE 6.3

1. 45 marks **2.** 18 kg **3.** 240 km **4.** Rs 160

5. 11 : 210 , 199 : 210 **6.** 1460 minutes **7.** 39 lines **8.** 48 days

REVIEW EXERCISE 6

2. (i) 4 : 6 (ii) 5 : 4 (iii) 2 : 8 (iv) 3 : 5

3. (i) 4 (ii) 7 (iii) 2 (iv) 9 (v) 144 (vi) 54

4. (ii) and (iii) are inverse proportions

5. (i) 35 : 67 (ii) 32 : 67 (iii) 32 : 35 (iv) 35 : 32

6. 25 minutes **7.** Rs 1350 **8.** 12 workers **9.** 400 persons

EXERCISE 7.1

1. (i) 20% (ii) 22% (iii) 75%

2. (i) $\frac{1}{4}$, 0.25 (ii) $\frac{1}{100}$, 0.01 (iii) $\frac{31}{100}$, 0.31 (iv) $\frac{3}{20}$, 0.15

(v) $\frac{17}{200}$, 0.085 (vi) $\frac{41}{200}$, 0.205 (vii) $\frac{7}{4}$, 1.75 (viii) $\frac{23}{20}$, 1.15

(ix) $\frac{9}{4}$, 2.25 (x) $\frac{1}{40}$, 0.025

3. (i) 25% (ii) 35% (iii) 32% (iv) 125%
4. (i) 0.25 (ii) 0.31 (iii) 0.01 (iv) 0.025 (v) 0.025 (vi) 0.065
5. (i) 2.5% (ii) 40% (iii) 85% (iv) 10.5% (v) 1250% (vi) 12550%
6. (i) 70% (ii) 50% (iii) 75% (iv) 125% (v) 100%
7. $\frac{3}{10}$ **8.** 64%
9. (i) 5 (ii) 21 (iii) $\frac{125}{2}$ (iv) 25 (v) $\frac{1125}{2}$ (vi) 62.5 cm
(vii) 180 litres (viii) Rs 2.65 (ix) 4.84 hours
(x) 43 metres (xi) $\frac{63}{2}$ (xii) $\frac{3033}{4}$ (xiii) $\frac{1183}{2}$ (xiv) $\frac{2107}{20}$
10. Ascending order: $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{4}$, Descending order: $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{4}$

EXERCISE 7.2

1. 80% **2.** 60% **3.** 5610 **4.** 15750 **5.** Rs 4000 **6.** 200 km
7. 75% **8.** 20% **9.** 27°, 63° **10.** 110 sq. m **11.** Rs 28

EXERCISE 7.3

1. (i) Profit: $\frac{25}{3}$% (ii) Loss: $\frac{32}{5}$% (iii) Profit: $\frac{50}{3}$%
(iv) Loss: $\frac{145}{6}$% (v) Profit: 5.96%
2. Profit: Rs 100, $\frac{25}{6}$% **3.** Profit: Rs 50, $\frac{25}{6}$%
4. Loss: Rs 150 **5.** Rs 65000 **6.** Rs 1275

REVIEW EXERCISE 7

1. Profit: 160% **2.** 25%
3. Imran: 68.5%, Sister: 76.5% sister performs better **4.** Loss: $\frac{20}{3}$%
5. Rs 897435.80

6. (i) 5% (ii) 0.64% (iii) 0.8% (iv) 19% (v) $\frac{1}{2}$
(vi) 25% (vii) $\frac{3}{4}$ (viii) 130% (ix) $\frac{1}{10}$

7. **(a)** (i) 75 (ii) 62.5 (iii) $\frac{3}{10}$ (iv) 12.5
(v) 0.38 (vi) 43.2 (vii) C.P (viii) Profit
(b) (i) False (ii) True (iii) True (iv) False
(v) False (vi) True (vii) True

EXERCISE 8.1

1. (i) $x + 2 = 8$ (ii) 7 $y > 2$ (iii) $6 + z < 4$
2. (i) False (ii) True (iii) False (iv) True (v) False (vi) True
3. (i) Open statement (ii) Open statement (iii) Not open statement
4. (i) 9 (ii) 3 (iii) 6 (iv) 4 (v) 3 (vi) 6

EXERCISE 8.2

1. (i) 1 (ii) 1 (iii) 2 (iv) 2 (v) 3 (vi) 2
2. (i) x (ii) x (iii) x, y (iv) x, y (v) x, y (vi) y, z
(vii) x, y, z (viii) x, y, z
3. (i) 2 (ii) –3 (iii) 1 (iv) $\frac{1}{2}$ (v) 9 (vi) 10
4. (i) –7 (ii) 5 (iii) 2, 3 (iv) $1, \frac{1}{2}, -\frac{1}{4}$
(v) 5 (vi) $\frac{1}{4}, 6$
5. $2\,xy$, $140\,xy$, $\frac{1}{5}\,xy$, $\frac{1}{4}\,xy$, $\frac{1}{3}\,xy$, $-2xy$, $5xy$ are like terms,
$4\,\ell m$, $5\,\ell m$ are like terms, $-7xz$, xz are like terms,
$-9xyz$, and xyz are like terms and $-2\,p$ and $46\,p$ are like terms.

EXERCISE 8.3

1. (i) $7\,x$ (ii) $12\,a$ (iii) $19\,\ell m$ (iv) $16\,xy$

2. (i) $11\,x$ (ii) $6\,a$ (iii) $20\,x$ (iv) 17 st

3. (i) $8a + 3b$ (ii) $10x + 9y + 8z$ (iii) $6pq + 2qr + 7pr$
(iv) $8c + 11d + 11f$

4. (i) $3x$ (ii) $12y$ (iii) $4x + 5y$ (iv) $30f - 10g - 20$

5. (i) $3x$ (ii) $21ab$ (iii) $2z$ (iv) $27xy$
(v) $5x + 2y$ (vi) $x - 5xb + 6y$

EXERCISE 8.4

1. (i) $4x + 2y$ (ii) $-2a + 3b$ (iii) $-a - 2b$
(iv) $24\ell + 3m + 1$ (v) $5y - 46x$

2. (i) 6 and -10 (ii) (a) 9 (b) -8 (c) -5 (iv) $\frac{33}{4}$

3. (i) 0 (ii) -23 (iii) $-\frac{7}{2}$ (iv) $-\frac{1}{10}$ (v) -2
(vi) $-\frac{1}{20}$ (vii) $-\frac{2}{3}$ (viii) 80 (ix) $\frac{1}{4}$ (x) $\frac{16}{15}$

4. 16 **5.** (i) Rs $5x$ (ii) $5x + 3$ (iii) Rs 153

REVIEW EXERCISE 8

1. (i) Rs $18x$ (ii) $9b + 5a$ (iii) $3x + 5$ (iv) $2y - \frac{2}{3}$ (v) $6 + p$

2. (i) Sum of x and 2 (ii) Three times y minus four
(iii) Sum of x and 5 (iv) Sum of s and twice of t
(v) Half of difference of x and y

3. (i) 2 (ii) 1 (iii) 3 (iv) 4 (v) 3

4. $5x$, $3x + 4$ and $6x + 5y + 7$

5. (i) $\frac{20}{3}xy$ (ii) $\frac{63}{5}ab$

6. (i) $8x + 13y$ (ii) $12b + 12c + 7d$ (iii) $20xy + 8x + 10y$

7. $x + 4y$ **8.** $x - 18y - 14z$

9. (i) False (ii) False (iii) True

10. (i) 9 (ii) 13 (iii) 6 (iv) 3 (v) 2

EXERCISE 9.1

1. $x - 20 = 32$ **2.** $x + 8 = 15$ **3.** $5x = 35$ **4.** $\frac{16}{4} = 4$

5. $x + 2 = 4$ **6.** $x - 15 = 5$ **7.** $2x + 4 = 20$

8. $a = 15 + b$ where a = my age b = brother's age

EXERCISE 9.2

1. 15 **2.** 7 **3.** 6 **4.** 2 **5.** 10

6. 18 **7.** 12 **8.** 8 **9.** 4 **10.** 6

11. –3 **12.** 11 **13.** 4 **14.** $\frac{26}{3}$ **15.** $\frac{16}{3}$

EXERCISE 9.3

1. 19 **2.** 36 runs **3.** 3 **4.** 5 **5.** 9

6. 1 year **7.** 105 **8.** Rs 32

REVIEW EXERCISE 9

1. (i) $x + 4$ (ii) $x - 7$ (iii) $9x$ (iv) $\frac{4}{6}$
(v) $x + y$ (vi) $8x$

2. (i) $x - 3 = 10$ (ii) $3y + 5 = 17$ (iii) $2z + 4 = 20$ (iv) $7 - \frac{m}{2} = 4$
(v) $2x + 4 = 16$ (vi) $x + 8 = 3x$

3. (i) $\frac{21}{5}$ (ii) 9 (iii) 4 (iv) $\frac{11}{4}$ (v) 35 (iv) 6

4. 14 metres **5.** 13 **6.** 8 **7.** 3 **8.** 16

EXERCISE 10.1

1. (i) 5 cm (ii) 8.3 cm (iii) 7.8 cm (iv) 10.0 cm

2. (i) 2 cm (ii) 1.5 cm (iii) 2.1 cm (iv) 1.8 cm (v) 1.5 cm

REVIEW EXERCISE 10

2. (i) 9.8 cm (ii) 0.8 cm (iii) 2.3 cm

6. PQ **8.** (i) False (ii) False (iii) True (iv) True (v) False

9. (i) b (ii) *a* (iii) b (iv) c

EXERCISE 11.1

1. (a) 20 cm, 25 sq. cm (b) 114 cm, 12.25 sq. cm (c) 30 cm, 30 sq. cm

2. (a) 16 cm, 15 sq. cm (b) 17 cm, 15 sq. cm (c) 15 cm, 12.96 sq. cm
(d) 24 cm, 20 sq. cm

3. (i) 15 cm, 12.5 sq. cm (ii) 20 m, 25 sq. cm

4. (a) 12 cm (b) 12.5 cm

5. 30 cm **6.** 52.7 sq. cm

EXERCISE 11.2

1. (i) 350 sq. cm (ii) 800 sq. cm (iii) 430 sq. cm

2. 550 sq. cm **3.** 400 sq. cm

EXERCISE 11.3

1. 30,000 rupees **2.** 21,000 rupees **3.** Bismah covers more distance

4. 41 m **5.** 162412.5 rupees **6.** 160800 rupees **7.** 325 sq. m

EXERCISE 11.4

1. (a) $\overline{AE}$, 4 cm (b) $\overline{OP}$, 4.5 cm (c) $\overline{ZS}$, 4 cm (d) $\overline{YT}$, 6 cm

2. (i) 60 sq. cm (ii) 120 sq. cm (iii) 24.75 sq. cm (iv) 30.55 sq. cm

3. 16 cm **4.** 12 m **5.** 3 cm

EXERCISE 11.5

1. (i) 22 sq. cm (ii) 8 sq. cm (iii) 30.25 sq. cm

2. 35 sq. cm **3.** 8 cm

4. (i) 24 sq. cm (ii) 22.5 sq. cm (iii) 3.75 sq. cm (iv) 211.375 sq. cm

5. 16.5 sq. cm **6.** 16 cm **7.** 40 sq. cm

REVIEW EXERCISE 11

1. (i) 21.2 cm, 28.09 sq. cm (ii) 38 cm, 88 sq. cm

2. (i) 7.5 sq. cm (ii) 48 sq. cm (iii) 16.25 sq. cm

3. 244 sq. m **4.** 225 **5.** 4 cm **6.** 18 cm **7.** 10 cm

8. 54 sq. cm **9.** 67.5 sq. cm **10.** 40 sq. cm **11.** 8 cm

12. 10 cm **13.** 20 sq. cm

14. (i) sq. units (ii) Units (iii) Product (iv) Sum
(v) Length, breadth (vi) 4, length of side (vii) Perimeter = 2 (ℓ + b)
(viii) Area = Base height (ix) Area = $\frac{1}{2}$ (Base height)
(x) Area = $\frac{h}{2}(b_1 + b_2)$

EXERCISE 12.1

1. (i) sphere (ii) cylinder (iii) cone (iv) cube (v) cuboid (vi) cuboid

2. (i) (a) 12 (b) 6 (c) 8 (ii) (a) vertex (b) 1
(iii) (a) edge (b) vertex (c) 6

EXERCISE 12.2

1. (i) sq. cm, cubic cm (ii) sq. mm, cubic mm

2. (i) 24 sq. cm, 8 cubic cm (ii) 86.5 sq. cm, 48.75 cubic cm
(iii) 73.5 sq. cm, 42.875 cubic cm (iv) 48 sq. cm, 18 cubic cm

3. (i) 125 cubic cm (ii) 238.328 cubic cm
(iii) 15.625 cubic cm

EXERCISE 12.3

1. Rs 11000 **2.** 50 cubic m **3.** 1125000 cubic cm
4. Rs 1875 **5.** 125 cubic m **6.** 8 **7.** Rs 1800
8. 65.45 cubic m **9.** 19250 cubic cm **10.** Rs 2514.72

REVIEW EXERCISE 12

1. (i) 8 (ii) 1 (iii) 6 (iv) 2 (v) 12
2. Tennis ball, Geometry box and Brick
3. (i) 37.5 sq. cm, 15.625 cubic cm (ii) 30.5 sq. cm, 10.5 cubic cm
4. 13.5 sq. cm and 3.375 cubic cm **5.** Rs 21400 **6.** 175.5 cubic m

EXERCISE 13.1

3. (ii), (iii)

EXERCISE 13.2

5. (i) Tennis (ii) Cricket (iii) Hockey and football
6. (i) Science (ii) English (iii) Science
7. (i) Class II (ii) Class V (iii) Class III and IV

REVIEW EXERCISE 13

1. (i) Maths, 79 marks (ii) Social Studies, 40 marks
(iii) Urdu and Sindhi, 50 marks each
(iv)

Subject	Sindhi	Urdu	English	Science	Social Studies	Islamiat	Maths
Marks	50	50	60	70	40	78	79

2. (i) True (ii) True (iii) False (iv) False
3. (i) (ii) (iii) Vertically (iv) Data (v) Data