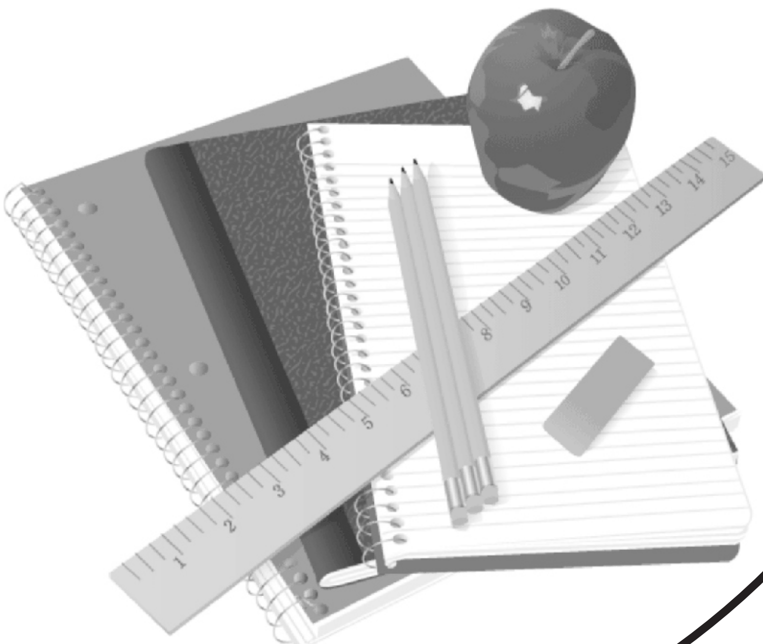
Test Edition

Mathematics

Class 7



Sindh Textbook Board, Jamshoro

Prepared and produced by
The Sindh Textbook Board, Jamshoro

Reviewed by
The Provincial Committee for Review of Textbooks
Bureau of Curriculum and Extension Wing Sindh, Jamshoro

Approved by
Government of Sindh, Education and Literacy, Department, Karachi
Under Notification No. SO(G-I) E&L/CURRICULUM-2014 dated 04-01-2016
as a sole Textbook for English Medium Schools in the Province of Sindh

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CONTENTS

Preface

The Sindh Textbook Board is an orgnization charged with the preparation and publication of textbooks in the province of Sindh. Its prime objective is to develop and produce textbooks which are conductive to equip the new generation with the knowledge and acumen to prepare them to face the challenges of the rapidly changing environment. In this age of knowledge explosion and development of technology not witnessed in the human history, efforts have to be made to ensure that our children do not lag behind. The Board also strives to ensure that Universal Islamic Ideology, culture and traditions are not compromised in developing the textbooks.

To accomplish this noble task, a team of educationists, experts, working teachers and friends endeavor tirelessly to develop text and improve contents, layout and design of the textbooks.

An attempt has made in this textbook to provide horizontal and vertical integration. The efforts of our experts and production personnel can bring about the desired results only if these textbooks are used effectively by teachers and students. Their suggestions will help us in further improving the qualitative contents of textbooks.

Chairman
Sindh Textbook Board

بسم الله الرحمن الرحيم

Unit 1

SETS

1.1 SET

Revolutionary changes are taking place in the field of knowledge. Like other subjects, mathematics too, is enriching with new concepts and topics. New terminologies and notations are evolving. The concept about sets was first introduced by George Cantor in the nineteenth century.

In twentieth century the concept of set used to integrate many branches of mathematics.

Set is collection of 'well-defined' and 'distinct' objects.

1.1.1 Express a set in:

- **descriptive form**
- **set-builder form**
- **tabular form**

Sets are expressed in the following three forms:

(i) Descriptive form
(ii) Tabular form
(iii) Set-builder form

(i) Descriptive form

Consider the following sets:

A = Set of students of class VII of your school
B = Set of first ten natural numbers
C = Set of first ten whole numbers
D = Set of geometrical instruments in a box

We can express the members of each set from the statements mentioned above. As these sets describe the characteristics or properties of their members, so this form of representing sets is called descriptive form.

Following sets are some more examples in descriptive form.

Examples:

A = The set of names of the days of a week
B = The set of first five natural numbers
C = The set of first six whole numbers
D = The set of names of the months of year which start with 'J'
E = The set of integers from 1 to 8

Activity: Write the following sets in descriptive form.

(i) D = {3, 6, 9, 12}

D = Set of first four multiples of 3.

(ii) O = {*a*, *e*, *i*, *o*, *u*}

O = Set of ________________________.

(iii) W = {0, 1, 2, 3, 4, 5, 6, 7, 8}

W = Set of ________________________.

(iv) M = {March, May}

M = Set of ________________________.

(v) B = {*b*, *e*, *a*, *u*, *t*, *i*, *f*, *l*}

B = Set of the ________________________.

Remember:

> In descriptive form of sets, we describe the characteristics of members of a set in any common language.

(ii) Tabular form

Consider the following set which has been represented in tabular form:

A = {*a*, *b*, *c*, *d*, *e*}

Here members of set A are: first five small letters of English alphabets.

i.e. *a*, *b*, *c*, *d* and *e*. Its descriptive form is: 'Set of first five small letters of English alphabets.'

Following are some more examples of sets in tabular form:

(i) C = {1, 2, 3, 4, 5}

Its descriptive form is; 'Set of first five natural numbers.'

(ii) G = {Δ, □, ○}

Its descriptive form is; 'Set of three geometrical figures, triangle, square and circle.'

(iii) F = {1, 2, 3, ..., 100}

Its descriptive form is; 'Set of natural numbers from 1 to 100.'

(iv) $\mathbb{N} = \{1, 2, 3, ...\}$. Its descriptive form is 'set of all natural numbers'.

From the above examples, it is concluded that:

> In tabular form, we tabulate the members of the set within the braces (curly brackets) and separate them by commas.

(iii) Set-builder form

In set-builder form, the set is represented by describing the common characteristics of all members using symbols.

Consider the following examples.

Example 1. A is set of first ten natural numbers.

Write it in three forms.

Descriptive form: A = Set of first ten natural numbers.

Tabular form: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Set-builder form: $A = \{x \,/\, x \in \mathbb{N} \wedge x \leq 10\}$.

The above set represented in set-builder form is read as 'x such that x belongs to set of natural numbers less than or equal to 10'. The symbol ' / ' is read as 'such that'

Example 2: B = Set of all positive even numbers:

Represent it in tabular form and set-builder form.

In tabular form, we write as:

$B = \{2, 4, 6, ...\}$.

In set-builder form, it is written as:

$B = \{x \,/\, x \in \mathbb{E}\}$.

Symbols to remember:	
/	'such that'
$\in$	'belongs to'
$\geqslant$	'greater than or equal to'
$\leqslant$	'less than or equal to'
$\wedge$	'and'
$\vee$	'or'
=	'is equal to'
$\neq$	'is not equal to'

We can read it as, 'Set B is a group of members x such that x belongs to set of all even numbers'.

Example 3: Write set $X = \{a, b, c, d, ..., z\}$ in set-builder form.

Set-builder form: $X = \{x \,/\, x$ is an English alphabet$\}$

Example 4: Write set C = {0, 1, 2, 3, 4, 5} in set-builder form and descriptive form.

Set-builder form: C = $\{x \,/\, x \in \mathbb{W}\ 0 \wedge \leqslant x \leqslant\ 5\}$

Descriptive Form: C = Set of first six whole numbers

Activity:

Match the following sets in column A correctly with column B in set-builder form.

S.No.	Column A	Column B
1.	Set of first five positive even numbers	$\{x \,/\, x \in \mathbb{O},\ 1 \leqslant x \leqslant 9\}$
2.	{1, 3, 5, 7, 9}	$\{x \,/\, x \in \mathbb{N}\}$
3.	Set of first ten whole numbers	$\{x \,/\, x \in \mathbb{E}\ 2 \wedge \leqslant x \leqslant 10\}$
4.	{ – 2, –1, 0, 1, 2 }	$\{x \,/\, x \in \mathbb{W}\ 0 \wedge \leqslant x \leqslant 9\}$
5.	Set of all natural numbers	$\{x \,/\, x \in \mathbb{Z}\ 2- \wedge \leqslant x \leqslant 2\}$

EXERCISE 1.1

I. Write the following sets in descriptive form.

1. $\mathbb{N}$ = { 1, 2, 3, 4, 5, 6, ... }
2. E = { 2, 4, 6, 8 }
3. F = { 1, 2, 5, 10 }
4. J = { 2, 3, 5, 7, 11 }
5. W = { 0, 1, 2, 3, 4, 5 }
6. S = { 5, 10, 15, 20, 25 }
7. T = { 3, 6, 9, 12, ... }

II. Write the following sets in tabular form.

1. A = Set of first seven natural numbers
2. B = Set of first eight positive even numbers
3. C = Set of integers greater than – 4 and less than 2

4. D = Set of English alphabets used in word 'Pakistan'
5. E = Set of first ten multiples of 2
6. F = Set of first ten prime numbers
7. G = Set of all factors of 15

III. Write the following sets in set-builder form.

1. A = { 1, 2, 3, ..., 10 }
2. B = { –2, –1, 0, +1, +2, +3, +4, +5 }
3. C = { Ravi, Chenab, Indus, Jhelum, Sutlej }
4. E = { 2, 4, 6, 8, 10, 12, ..., 20 }
5. D = { 1, 3, 5, 7, 9, ... }
6. S = Set of names of days of a week which start with letter S
7. Z = Set of all integers
8. L = { 4, 8, 12, 16, 20, ... }

IV. Write the following sets in tabular form.

1. $A = \{ x \,/\, x \in \mathbb{N} \wedge x \leqslant 5 \}$
2. $B = \{ y \,/\, y \in \mathbb{E}\ 4 \wedge \leqslant y \leqslant 20 \}$
3. $C = \{ a \,/\, a \in \mathbb{Z}\ 3 - \wedge < a < 3 \}$
4. $D = \{ z \,/\, z \in \mathbb{W} \wedge z < 5 \}$

V. Put the sets in their appropriate column, then write two other forms of each set.

A = { a, e, i, o, u)
B = {2, 4, 6, 8, ...}
C = {1, 2, 3, 4, 5}
D = $\{x \,/\, x \in \mathbb{N} \wedge x > 50\}$
E = {p, a, k, i, s, t, n}
F = Set of names of days of a week

Descriptive form	Tabular form	Set-builder form

1.2 OPERATIONS ON SETS

1.2.1 Define union, intersection and difference of two sets

1. Union of two sets

Consider two sets A and B, the union of set A and set B is a set which consists of all the elements belonging to set A or set B or both. Union is denoted by the symbol '$\cup$'.

Example 1:

If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$ then find $A \cup B$

Solution: $A \cup B = \{1, 2, 3, 4\}\ 7\ ,6\ ,5\{\ \cup\}$

or $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$.

We read $A \cup B$ as 'A union B'.

2. Intersection of two sets

Consider two sets A and B, the intersection of set A and set B is a set which consists of the common members present in both sets A and B.

Intersection is denoted by symbol '$\cap$'.

Example 2: If $A = \{3, 4, 6, 8\}$ and $B = \{3, 5, 6, 7\}$ then find $A \cap B$

Solution: Here 3 and 6 are common in both

$A \cap B = \{3, 4, 6, 8\}\ 7\ ,6\ ,5\ ,3\ \{\cap\}$

So, $A \cap B = \{3, 6\}$

We read $A \cap B$ as 'A intersection B'.

3. Difference of two sets.

Difference of two sets A and B, denoted as $A - B$, is a set which contains all those members of A which are not in B. It is also denoted as $A \backslash B$.

Example 3: If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$ then find $A–B$ and $B–A$

Solution: $A - B = \{1, 2, 3, 4, 5\} - \{2, 4, 6\}$

$= \{1, 3, 5\}$

Also $B - A = \{2, 4, 6\} - \{1, 2, 3, 4, 5\}$

$= \{6\}$

We read A – B as: A difference B.

1.2.2 (A) Find Union of two or more sets

(i) Union of two sets.

Example 1: If $X = \{3, 4, 5, 6\}$ and $Y = \{1, 2, 3, 8\}$ then find $X \cup Y$.

Solution: $X \cup Y = \{3, 4, 5, 6\}$ 8 ,3 ,2 ,1{ $\cup$}
$= \{1, 2, 3, 4, 5, 6, 8\}$

Example 2: If $E = \{ 2, 4, 6 \}$ and $Q = \{ \ \}$ then find $E \cup Q$

Solution: $E \cup Q = \{ 2, 4, 6 \} \cup \{ \ \}$
$= \{ 2, 4, 6 \}$

Example 3: If $A = \{1, 2, 3, 4, 5, \ldots \}$ and $D = \{2, 4, 6, 8\}$ then find $A \cup D$

Solution: $A \cup D = \{1, 2, 3, 4, 5, \ldots \}$ 8 ,6 ,4 ,2{ $\cup$}
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots \}$

(ii) Union of three sets.

Union of three sets is explained with the help of following example.

Example: If $A = \{1, 2\}$, $B = \{2, 3, 4\}$ and $C = \{2, 3, 4, 5\}$ then find $A \cup B \cup C$.

Solution:

There are two ways, to find $A \cup B \cup C$

In case we find $A \cup B$ first, then we write it as $(A \cup B) \cup C$ and find as under.

$A \cup B$ = {1, 2} 4 ,3 ,2 ,1{ = }4 ,3 ,2{ $\cup$}

$(A \cup B) \cup C$ = {1, 2, 3, 4} 5 ,4 ,3 ,2 ,1{ = }5 ,4 ,3 ,2{ $\cup$}

So, $A \cup B \cup C = \{1, 2, 3, 4, 5\}$

In case we find $B \cup C$ first, then we write as $A \cup (B \cup C)$ and find as under:

$B \cup C$ = {2, 3, 4} 5 ,4 ,3 ,2{ = }5 ,4 ,3 ,2{ $\cup$}

Now $A \cup (B \cup C)$ = {1, 2} 5 ,4 ,3 ,2($\cup$}

or $A \cup (B \cup C) = \{1, 2, 3, 4, 5\}$

so, $A \cup B \cup C = \{1, 2, 3, 4, 5\}$

1.2.2 (B) Find intersection of two or more sets

(i) Intersection of two sets

Example 1: If A = {1, 2, 3, 10} and B = {2, 3, 8, 10} then find A ∩ B.

Solution: A ∩ B = {1, 2, 3, 10} 10 ,8 ,3 ,2{ ∩}
Here members 2, 3 and 10 are common.
Therefore, A ∩ B = {2, 3, 10}

Example 2: If C = {a, b, e, f } and D = {2, 4, 8, 10} then find C ∩ D.

Solution: C ∩ D = {a, b, e, f } 10 ,8 ,4 ,2{ ∩}
Therefore C ∩ D = { } or ∅
Because there is no member common in both sets C and D

(ii) Intersection of three sets

We have already learnt the intersection of two sets. In the same way we can perform intersection of three sets.

Intersection of three sets is explained with the help of the following example.

Example: If A = {1, 2}, B = {2, 3} and C = {2, 3, 4} then find A ∩ B ∩ C.
Solution: We can find the solution in following two ways.

In case we find A ∩ B first then we write it as (A ∩ B) ∩ C and find as under:

A ∩ B = { 1, 2 } 2 { = } 3 ,2 { ∩ }

Now (A ∩ B) ∩ C = { 2 } 4 ,3 ,2 { ∩ }
= {2}

So, A ∩ B ∩ C = { 2 }

In case we find B ∩ C first, then we write it as A ∩ (B ∩ C) and find as under:

B ∩ C = { 2, 3 } 4 ,3 ,2 { ∩ }
= { 2, 3 }

Now A ∩ (B ∩ C) = { 1, 2 } 3 ,2 { ∩ }
= { 2 }

So, A ∩ B ∩ C = (2 }

1.2.2 (C) Find the difference of two sets

Example 1: If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 5, 7\}$, then find $A - B$.

Solution:

$A - B = \{1, 2, 3, 4, 5\} - \{2, 4, 5, 7\}$

Here members 1 and 3 are in set A only.

$A - B = \{1, 3\}$

It is clear that the difference of set A and set B is the set of members 1 and 3 which belong to only set A but do not belong to set B.

Again the difference between set B and set A is $B \backslash A$

or $B - A = \{2, 4, 5, 7\} - \{1, 2, 3, 4, 5\}$

Thus $B - A$ or $B \backslash A = \{7\}$

Here also the difference of set B and set A is the set of member 7 only, which belongs to only set B but does not belong to set A.

Therefore $A - B \neq B - A$ or $A \backslash B \neq B \backslash A$

Example 2: If $P = \{2, 3, 4\}$ and $Q = \{1, 2, 3, 4, 5, 6\}$ then find $P - Q$ and $Q - P$.

Solution: $P - Q = \{2, 3, 4\} - \{1, 2, 3, 4, 5, 6\} = \{\ \}$

Therefore $P - Q = \{\ \}$ or $\varnothing$

Again $Q - P = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4\} = \{1, 5, 6\}$

Therefore $Q - P = \{1, 5, 6\}$

It is found that: $\boxed{P - Q \neq Q - P \text{ or } P \backslash Q \neq Q \backslash P}$

EXERCISE 1.2

A. Find the union of the following sets.

1. $A = \{4, 5\}$ and $B = \{4, 6, 8, 9\}$
2. $A = \{0, 1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$
3. $A = \{2, 4, 6, 8, 10\}$ and $B = \{0, 1, 2, 3, 4, 5, 6, 7, 9, 10\}$
4. X = Set of first six prime numbers and
   Y = Set of first six multiples of 3.

5. If D = {2, 4, 6}, E = {1, 3, 5, 6} and F = {1, 3, 6, 9, 10}.

   then find $D \cup E \cup F$.

6. If A = { 1, 2, 3 } , B = { } and C = { 2, 3 }.

   then find $(A \cup B) \cup C$ and $A \cup (B \cup C)$.

B. Find intersection of the following sets.

1. A = {1, 2, 3} and B = {3, 6, 9, 12}.
2. D = {2, 4, 6, 8} and E = {2, 4, 6, 8, 10}.
3. P = {*a*, *b*, *c*, *d*} and Q = {*a*, *e*, *i*, *o*, *u*}.
4. A = Set of integers and B = Set of positive integers.
5. If A = {*a*, *b*, *c*, *d*}, B = { *d*, *e*, *f* } and C = { *f*, *g*, *h* }.

   then find $(A \cap B) \cap C$ and $A \cap (B \cap C)$.
6. If D = {2, 3}, E = {4, 5} and F = { }.

   then find $(D \cap E) \cap F$ and $D \cap (E \cap F)$.

C. Find A – B and B – A if

1. A = { 4, 5, 6 } and B = { 5, 6, 7, 8 }.
2. A = { 1, 3, 5, 9, 11 } and B = { 1, 3, 5, 7, 11 }.
3. A = { 0, 1, 2, 3, 4, 5, 6, ... } and B = {1, 2, 3, 4, 5, ... }
4. A = Set of first six multiples of 3 and B = Set of first six multiples of 4.

1.2.3 Define and identify disjoint and overlapping sets

(i) Disjoint sets.

Two sets A and B are called disjoint sets if they have no common member.

Let A = {1, 2, 3} and B = {4, 5}.

They have no common member, hence set A and B are disjoint sets.

(ii) Overlapping sets

Two sets X and Y are called overlapping sets if they have at least one common member and none of them is a subset of the other set.

If X = { 0, 1, 2, 4 } and Y = { 2, 3, 5, 7 }.

We observe that 2 ∈ X and also 2 ∈ Y;

i.e. 2 is common member in both sets and none of the set is subset of the other. Hence X and Y are overlapping sets.

EXERCISE 1.3

A. Identify the disjoint sets and overlapping sets.

(1) F = {Δ, ▭, ⬡ } and M = {Plate, Jug, Cup}.

(2) W = {0, 1, 2, 3} and N = { 1, 2, 3, 4 }.

(3) C = {*a*, *e*, *i*, *o*, *u*} and D = { *p*, *q*, *r* }.

(4) C = { *a*, *c*, *d*, *f* } and D = { 1, 2, 3 }.

(5) N = {1, 2, 4, 6, 8, 12, 24, 48} and M = { 6, 8, 12, 14, 16, 18 }.

(6) N = Set of natural numbers from 10 to 40 and
W = Set of whole numbers upto 40.

(7) E = {2, 4, 6, ...} and O = {5, 10, 15, ...}

B. Identify the disjoint and overlapping sets in the following:
And write in the given column.

(i)	F = { 2, 3 } and A = { 1, 4, 6, 8 }	
(ii)	H = { 2, 3, 5, 7, 9 } and C = { 1, 3, 5, ... }	....
(iii)	N = { *a*, *e*, *i*, *o*, *u* } and G = { *h*, *o*, *c*, *k*, *y* }	....
(iv)	W = { 0, 1, 2, 3 ... } and B = { *x*, *y*, *z* }	....
(v)	P = { plate, jug, cup } and D = { *f*, *e*, *s*, *l* }	....
(vi)	Q = {✯ , Δ, □, ○ } and E = { 1, 3, 5, 7, 9 }	....
(vii)	A = { 10, 20, 30, ... } and B = { 4, 8, 12, ... }	....

1.2.4 Define a universal set and complement of a set

(i) Universal set

A universal set is the set which consists of all the members of the sets under consideration. It is denoted by 𝕌.

Let us consider natural numbers then all the natural numbers will be the members of the universal set.

Similarly if we consider first six natural numbers, our universal set will consist of all first six natural numbers.

$\mathbb{U} = \{1, 2, 3, 4, 5, 6\}$.

(ii) Complement of a set

If U is a universal set and set A is its subset; then $\mathbb{U} - A$ is called the complement of the set A and is denoted by A', i.e $A' = \mathbb{U} - A$,

Complement of set A means 'set of all members of $\mathbb{U}$ which do not belong to set A'.

Example 1: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4\}$ then find A'.

Solution: $A' = \mathbb{U} - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4\}$

$A' = \{5, 6, 7, 8, 9, 10\}$

Example 2: If $\mathbb{U} = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2\}$ then find B'.

Solution: $B' = \mathbb{U} - B$

$= \{0, 1, 2, 3, 4, 5, 6, 7\} - \{2\}$

$B' = \{0, 1, 3, 4, 5, 6, 7\}$

EXERCISE 1.4

I. Let $\mathbb{U} = \{a, b, c, d, e, f\}$ and $A = \{a, e, f\}$.
Find (i) A' (ii) $A \cup A'$ (iii) $A \cap A'$ (iv) $\mathbb{U}'$ (v) $\varnothing'$

II. Let $\mathbb{U} = \{1, 2, 3, \ldots, 10\}$ and $E = \{2, 4, 6, 8, 10\}$.
Find (i) E' (ii) $E \cup E'$ (iii) $E \cap E'$ (iv) $\mathbb{U}'$ (v) $\varnothing'$

III. If $\mathbb{U} = \{0, 1, 2, 3, \ldots, 12\}$ and $P = \{1, 3, 5, 7, 9, 11\}$, then find:
(i) P' (ii) $P \cup P'$ (iii) $P' \cap P$ (iv) $\mathbb{U} \cap \mathbb{U}'$

IV. If $\mathbb{U} = \{u, v, w, x, y, z\}$ and $B = \{x, y, z\}$, then find:
(i) B' (ii) $B' \cup B$ (iii) $B' \cap B$ (iv) $\mathbb{U} \cup \mathbb{U}'$

V. If $\mathbb{U} = \{1, 2, 3, 4, \ldots, 10\}$, $A = \{1, 2, 3, 5, 6, 7\}$ and $B = \{2, 4, 6, 8, 10\}$.
then find: (i) A' (ii) B' (iii) $A' \cap A$
(iv) $A \cup A'$ (v) $B \cup B'$ (vi) $B \cap B'$

1.2.5 Verify different properties involving union of sets, intersection of sets, difference of sets and complement of a set e.g. $A \cap A' = \varnothing$.

(i) Properties involving union of sets

(a) Commutative property of union of sets

Let A and B are two sets then $A \cup B = B \cup A$. This property is called commutative property of union of sets.

Example 1:

If A = {1, 2} and B = {2, 3} then verify commutative property of union of sets.

Solution: $A \cup B$ = {1, 2} 3 ,2 ,1{ = }3 ,2{∪} ... (i)

$B \cup A$ = {2, 3} 3 ,2 ,1{ = }2 ,1{∪} ... (ii)

Results (i) and (ii) show that $A \cup B = B \cup A$

Hence operation of union of sets is commutative.

(b) Associative property of union of sets

Let A, B and C are any three sets then $A \cup (B \cup C) = (A \cup B) \cup C$.

This property is called associative property of union of sets.

Example. If A = {1, 2}, B = {2, 3} and C = {1, 2, 3, 4}.

Verify associative property of union.

Solution: $A \cup B$ = {1, 2} 3 ,2 ,1{ = }3 ,2{ ∪}
$(A \cup B) \cup C$ = {1, 2, 3} 4 ,3 ,2 ,1{ = }4 ,3 ,2 ,1{∪} ... (i)

Again $B \cup C$ = {2, 3} 4 ,3 ,2 ,1{ = }4 ,3 ,2 ,1{∪}
$A \cup (B \cup C)$ = {1, 2} 4 ,3 ,2 ,1{ = }4 ,3 ,2 ,1{ ∪} ... (ii)

Results (i) and (ii) show that $(A \cup B) \cup C = A \cup (B \cup C)$

Hence operation of union is associative in sets.

(ii) Properties involving intersection of sets

(a) Commutative property of intersection of sets

Let A and B are any two sets then $A \cap B = B \cap A$.

This property is called commutative property of intersection of sets.

Example: If $A = \{ a \}$, $B = \{ a, b \}$ and $C = \{ a, b, c \}$, then verify $A \cap B = B \cap A$.

Solution: $A \cap B = \{ a \} \cap \{ a, b \} = \{ a \}$ … (i)
$B \cap A = \{ a, b \} \cap \{ a \} = \{ a \}$ … (ii)

Results (i) and (ii) show that $A \cap B = B \cap A$

Hence operation of intersection of sets is commutative.

(b) Associative property of intersection of sets

Let A, B and C are any three sets then $A \cap (B \cap C) = (A \cap B) \cap C$.

This property is called associative property of intersection of sets.

Example: Given $A = \{ a, b \}$, $B = \{ a, b, c \}$ and $C = \{ a, c, d \}$ then verify that:

$(A \cap B) \cap C = A \cap (B \cap C)$

Solution: $A \cap B = \{ a, b \} \cap \{ a, b, c \} = \{ a, b \}$
$(A \cap B) \cap C = \{ a, b \} \cap \{ a, c, d \} = \{ a \}$ … (i)

Again $(B \cap C) = \{ a, b, c \} \cap \{ a, c, d \} = \{ a, c \}$
$A \cap (B \cap C) = \{ a, b \} \cap \{ a, c \} = \{ a \}$ … (ii)

Results (i) and (ii) shows that $A \cap (B \cap C) = (A \cap B) \cap C$

Hence operation of intersection is associative in sets.

(iii) Properties involving difference of sets

Example: If $A = \{ 1, 2, 3, 5 \}$, $B = \{ 4, 6, 7, 8 \}$ then verify $A–B$ and $\neq B–A$

Solution: $A – B = \{ 1, 2, 3, 5 \} – \{ 4, 6, 7, 8 \} = \{ 1, 2, 3, 5 \}$ … (i)
$B – A = \{ 4, 6, 7, 8 \} – \{ 1, 2, 3, 5 \} = \{ 4, 6, 7, 8 \}$ … (ii)

Results (i) and (ii) show that $A–B \neq B–A$ or $A \backslash B \neq B \backslash A$

Hence difference of sets is not commutative.

(iv) Properties involving complement of sets

Example: If $\mathbb{U} = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$ and $B = \{ 2, 4, 6, 8 \}$ then find B' . Also verify: (i) $B \cup B' = \mathbb{U}$ (ii) $B \cap B' = \varnothing$

(iii) $\varnothing' = U$ (iv) $\mathbb{U}' = \varnothing$

Solution: $B' = \mathbb{U}–B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 4, 6, 8\}$

Therefore $B' = \{1, 3, 5, 7\}$

(i) $B \cup B' = \{ 2, 4, 6, 8\}$ 8 ,7 ,6 ,5 ,4 ,3 ,2 ,1{ = } 7 ,5 ,3 ,1 { ∪} = $\mathbb{U}$

(ii) $B \cap B' = \{2, 4, 6, 8\}$ 7 ,5 ,3 ,1 { ∩} = { } or $\varnothing$

(iii) $\varnothing' = U–$8 ,7 ,6 ,5 ,4 ,3 ,2 ,1 { = } {–} 8 ,7 ,6 ,5 ,4 ,3 ,2 ,1 { = ∅} = $\mathbb{U}$

(iv) $\mathbb{U}' = \mathbb{U} - \mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 3, 4, 5, 6, 7, 8\} = \{ \ \} = \varnothing$

EXERCISE 1.5

A. **If $A = \{-1, -2, -3\}$, $B = \{-1, -4, -5, -6\}$ and $C = \{0, -1, -2, -3, -4\}$ then verify the following:**

1. $A \cup B = B \cup A$
2. $B - C \neq C - B$
3. $A \cup C = C \cup A$
4. $(A \cup B) \cup C = (B \cup C) \cup A$
5. $(A \cup C) \cup B = (C \cup B) \cup A$
6. $A \cap B = B \cap A$
7. $B \cap C = C \cap B$
8. $A - C \neq C - A$
9. $(A \cap B) \cap C = (B \cap C) \cap A$
10. $(A \cap C) \cap B = (A \cap B) \cap C$

B. **Given $U = \{10, 11, 12, 13, 14, \ldots , 20\}$, $A = \{10, 12, 14, 16, 18\}$ and $B = \{10, 11, 12, 13, 14, 15\}$, Verify the following.**

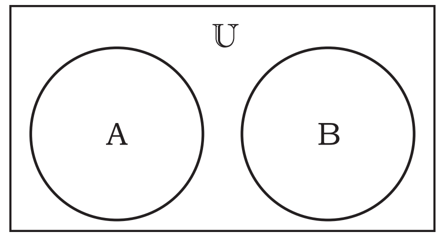
(i) $U' = \varnothing$ (ii) $\varnothing' = U$ (iii) $A \cup A' = U$

(iv) $B \cap B' = \varnothing$ (v) $(A \cup B)' = A' \cap B'$ (vi) $(A \cap B)' = A' \cup B'$

1.3 VENN DIAGRAM

Introduction:

Sets can also be represented graphically using Venn diagrams named after the english mathematician John Venn, who introduced their use in 1881 AD. In Venn diagrams, the universal set $\mathbb{U}$, is usually represented by a rectangle. Inside this rectangle circle or oval represent sets. The Venn diagrams are also used to represent operations on sets.



1.3.1 Represent sets through Venn Diagrams

Let us show the following sets through Venn diagram.

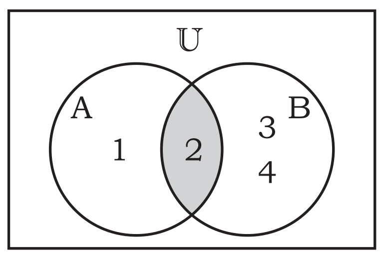
(i) Overlapping sets (ii) Subset (iii) Disjoint sets (iv) Universal set.

(i) Overlapping sets

Example 1:

Given that A = { 1, 2 } and B = { 2, 3, 4 }

Fig (i) represents overlapping sets



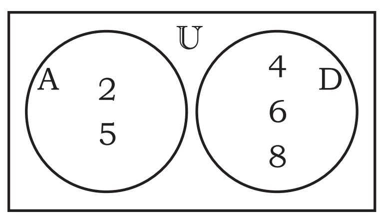
Overlapping Sets

Fig (i)

(ii) *Disjoint sets*

Example 2: Let A = { 2, 5 } and D = { 4, 6, 8 }

Fig. (ii) represents disjoint sets

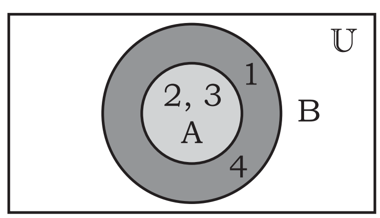


Disjoint Sets

Fig. (ii)

(iii) Subset

Example 3: Let A = { 2, 3 } and B = { 1, 2, 3, 4 }



Subset

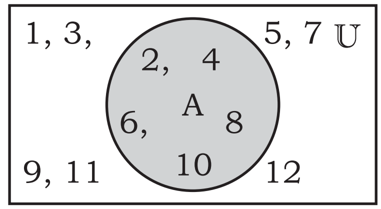
Here, set A is subset of set B. i.e. $A \subseteq B$ **Fig. (iii)**

Fig (iii) represents that $A \subseteq B$

(iv) Universal set

Example 4:

Let $\mathbb{U}$ = { 1, 2, 3, 4, 5, ..., 12} and
A = { 2, 4, 6, 8, 10 }
In Fig (iv). Rectangle represents universal set.
Circle or oval inside the rectangle. represents set A.



Universal Set

Fig (iv)

1.3.2 Perform operations of union, intersection, difference and complement on two sets A and B, when

- A is a subset of B
- B is subset of A
- A and B are disjoint sets
- A and B are overlapping sets.

Representation of operation on sets through Venn diagram

Now we represent these operations on sets through Venn diagram.

(I) UNION OF SETS

(a) When set A is a subset of set B.

Example 1: If A = { 1, 2 } and B = { 1, 2, 3, 4 },

Then A ∪ B = { 1, 2 } 4 ,3 ,2 ,1 { = } 4 ,3 ,2 ,1 {∪}.

In fig. (i) shaded portion represents A ∪ B, through Venn diagr

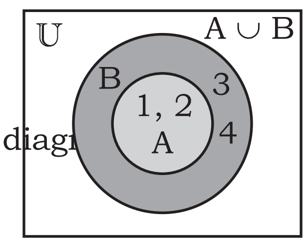


Fig. (i)

(b) When set B is subset of set A

Example 2: If A = { 2, 3, 4 } and B = { 3, 4 },

Then A ∪B = { 2, 3, 4 } 4 ,3 ,2 { = } 4 ,3 { ∪ }

The shaded portion is A ∪ B.

Representation of A ∪ B is shown through Venn diagram in Fig

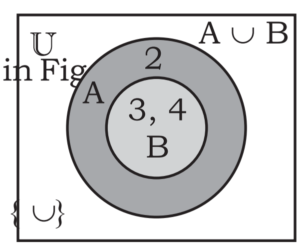


Fig. (ii)

(c) When set A and set B are disjoint sets

Example 3: If A = {1, 2} and B = { 3, 4 },

then A ∪ B = { 1, 2 } 4 ,3 ,2 ,1{ = } 4 ,3 { ∪}

The shaded portion is $A \cup B$.
It is represented in Venn diagram as shown in Fig. (iii)

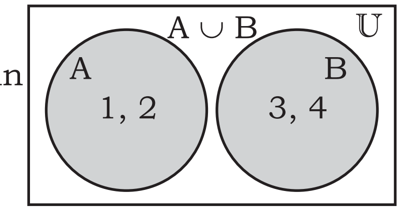


Fig. (iii)

(d) When set A and set B are overlapping sets

Example 4: If A = { 2, 3, 4 } and B = { 3, 6, 9 },
then A ∪ B = { 2, 3, 4 } 9 ,6 ,4 ,3 ,2 { = } 9 ,6 ,3 { ∪ }

The shaded portion is $A \cup B$. It is represented through Venn diagram as shown in Fig. (iv)

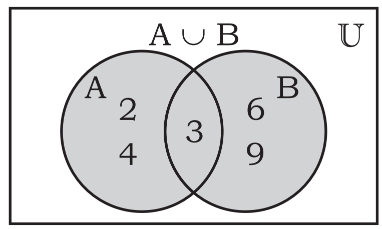


Fig. (iv)

(II) INTERSECTION OF SETS

(a) When set A is a subset of set B

Example 1: If A = {1, 2} and B = {1, 2, 3, 4},
then A ∩ B = {1, 2} 4 ,3 ,2 ,1{ ∩}
= {1, 2} = A

$A \cap B$ is shown in fig. (i).
The shaded portion is $A \cap B$

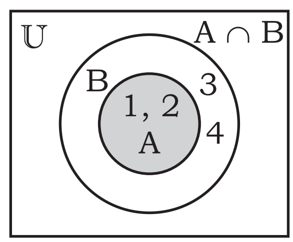


Fig. (i)

(b) When set B is subset of set A

Example 2: If A = { 2, 3, 4 } and B = { 3, 4 },
then A ∩ B = { 2, 3, 4 } 4 ,3{ ∩}
A ∩ B = { 3, 4 } = B

The shaded portion is $A \cap B$.
$A \cap B$ is represented through Venn diagram.
It is shown in fig. (ii).

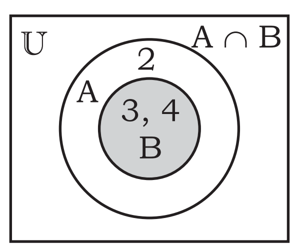


Fig. (ii)

(c) When set A and set B are disjoint sets

Example 3: If A = { 1, 2 } and B = { 3, 4 },

then A ∩ B = { 1, 2 } 4 ,3 { ∩ } = { }

A ∩ B is shown in Fig. (iii) and is represented through Venn diagram

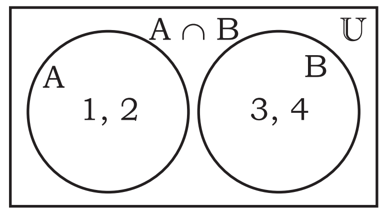


Fig. (iii)

(d) When set A and set B are overlapping sets

Example 4: If A = { 2, 3, 4 } and B = { 3, 6, 9 },

then A ∩ B = { 2, 3, 4 } 9 ,6 ,3 { ∩ }

= { 3 }

The shaded portion is A ∩ B = { 3 }

A ∩ B is represented as shown in fig. (iv) by shaded portion.

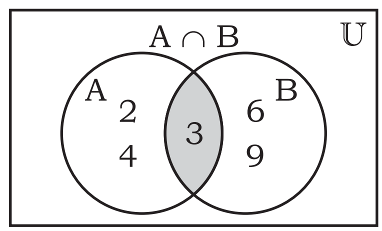


Fig. (iv)

(III) DIFFERENCE OF TWO SETS A AND B

(a) When set A is a subset of set B

Example 1: If A = { 1, 2 } and B = { 1, 2, 3, 4 },

then A–B = { 1, 2 } – { 1, 2, 3, 4 } = { }

A–B is represented through Venn diagram as shown in fig. (i)

Activity. Find B – A and show through Venn Diagram

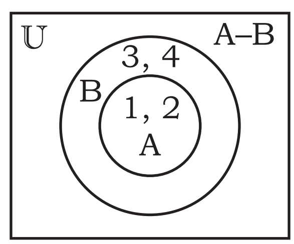


Fig. (i)

(b) When set B is subset of set A

Example 2: Let A = { 2, 3, 4 } and B = { 3, 4 }

then B – A = { 3, 4 } – { 2, 3, 4 } = { }

B–A is represented through Venn diagram as shown in fig. (ii)

Activity. Find A – B and show through Venn diagram

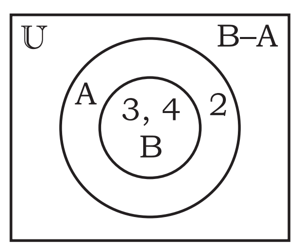


Fig. (ii)

(c) When set A and set B are disjoint sets

Example 3: If A = { 1, 2 } and B = { 3, 4 },

then A–B = { 1, 2 } – { 3, 4 } = { 1, 2 }

The shaded portion is A–B.

It is represented through Venn diagram as shown in fig. (iii)

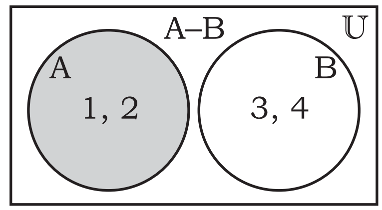


Fig. (iii)

(d) When set A and set B are overlapping sets

Example 4: If A = { 2, 3, 4 } and B = { 3, 6, 9 }

then A–B = { 2, 3, 4 } – { 3, 6, 9 } = { 2, 4 }

The shaded portion is A– B. It is represented through Venn diagram as shown in fig. (iv).

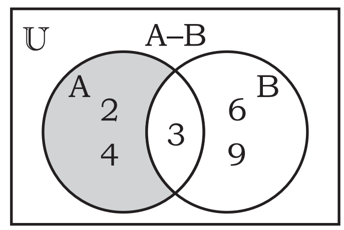


Fig. (iv)

(IV) COMPLEMENT OF A SET

(a) When set B is given and is subset of set A. i.e. B ⊆ A

Example 1: If 𝕌 = { 1, 2, 3, 4, 5, 6, 7, 8 },

A = { 1, 2, 3, 4 } and B = { 2, 3 },

then find B′.

Solution:

B′ = 𝕌–B = { 1, 2, 3, 4, 5, 6, 7, 8 } – { 2, 3 }

B′ = { 1, 4, 5, 6, 7, 8 }

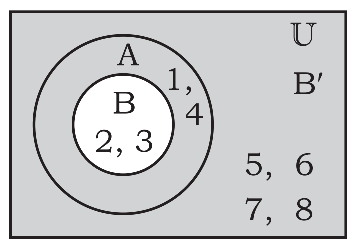


Fig. (i)

Here shaded portion represents B′ as shown in the figure (i).

(b) When Universal set and set A is given

Example 2: Given 𝕌 = { 1, 2, 3, 4, 5, 6, 7, 8 }

and A = { 1, 2, 3, 4}. Find A′

A′ = 𝕌 – A

A′ = { 1, 2, 3, 4, 5, 6, 7, 8 } – { 1, 2, 3, 4 }

= { 5, 6, 7, 8 }

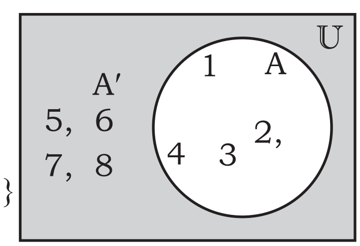


Fig. (ii)

The shaded portion is A′

A′ is represented through Venn diagram

EXERCISE 1.6

A. Find the following and represent operations of union, difference and intersection through Venn diagram.

1. Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8}
   (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$
2. Let W = { a, b, c } and X = { u, v, w }
   (i) $W \cup X$ (ii) $W \cap X$ (iii) $W - X$ (iv) $X - W$
3. Let Y = { Sumair, Raza, Ali } and Z = { Zahid, Sumair, Sabir}
   (i) $Y \cup Z$ (ii) $Y \cap Z$ (iii) $Y - Z$ (iv) $Z - Y$
4. Let Y = {1, 2, 3, ..., 12} and Z = {2, 4, 6, ..., 16}
   (i) $Y \cup Z$ (ii) $Y \cap Z$ (iii) $Y - Z$ (iv) $Z - Y$

B. If A = {8, 10, 12, 14, 16}, $\mathbb{U}$ = {8, 10, 11, 12, 13, 14, 15, 16} and B = {8, 15} then find the following and represent through Venn diagram.

(1) A'	(2) B'	(3) $A-B$	(4) $(A-B)'$
(5) $A \cup B'$	(6) $A' \cup B'$	(7) $A' - B'$	(8) $(A \cap B)'$
(9) $A' \cap B$	(10) $(A \cup B)'$	(11) $A' \cup A$	(12) $B' \cap B$

REVIEW EXERCISE 1

Answer the following:

1. Describe the names of three forms of expressing a set.
2. (a) Express the set of first five whole number into tabular form.
   (b) Express A = { a, b, c, d, ... } in descriptive form.
3. Give three examples of sets in set-builder form.
4. Describe disjoint sets and overlapping sets through examples.
5. If $\mathbb{U}$ = { 1, 2, 3, ... 12 }, A = {1, 2, 3, ..., 10}, B = {2, 4, 6, ..., 12} and C = {1, 3, 5, 7, ..., 11} then find the following:

(i) $A \cup B$	(ii) $A \cap B$	(iii) $A - B$	(iv) $(A \cap C)'$
(v) $(B \cup C)'$	(vi) $B' \cap C'$	(vii) $A - B'$	(viii) $B - A'$

6. **A = { *a, b, c, d* } , B = { *a, c, d* } and C = { *b , d, e* } prove the following.**

(i) $A \cup B = B \cup A$ (ii) $B \cap C = C \cap B$

(iii) $C \cup A = A \cup C$ (iv) $(A \cup B) \cup C = A \cup (B \cup C)$

(v) $(A \cap C) \cap B = A \cap (C \cap B)$ (vi) $(B \cup A) \cup C = B \cup (A \cup C)$

7. **Tick(√) the correct answer.**

(i) If A = {*a, b, c*} and $\mathbb{U}$ = { *a, b, c, d, e* }, then A′ is:

(a) { *a, b* } (b) { *b, c* } (c) { *c, d* } (d) { *d, e* }

(ii) If A = { *l, m, n* } and B = { *l, m, n, o* }, then B – A is:

(a) { *l* } (b) { *o* } (c) { *n* } (d) { *m* }

(iii) If P = { 1, 2, 3 } and Q = { 2, 3, 4 } then $P \cap Q$ is:

(a) { 2, 3 } (b) { 1, 3 } (c) { 3, 4 } (d) { 1, 4 }

(iv) If $\mathbb{U}$ = { 1, 2, 3, 4, 5, 6 } , X = { 2,3,4 } and Y = { 4,5,6 } then $(X \cup Y)'$ is:

(a) $\varnothing$ (b) { 2, 3 } (c) { 1 } (d) { 6 }

(v) If A = { *x, y, z* } and B = { *y, z, x* } then A – B is:

(a) A (b) B (c) $\varnothing$ (d) $A \cap B$

8. **Find the following and show through Venn diagram.**

(i) $C \cap D$ and $C \cup D$ if C = { *c, d, e* } and D = { *d, e, f* }.

(ii) P′ and Q′ if P = { *l, m, n* }, Q = { *n, o, r* } and $\mathbb{U}$ = { *l, m, n, o, r* }.

(iii) $X \cap Y$ and $X \cup Y$ if X = { *r, s* } and Y = { *x, y, z* }.

SUMMARY

Set is a collection of well-defined and distinct objects.

- There are three different ways to express a set. These are:
  (i) Descriptive form (ii) Tabular form (iii) Set builder form.

In tabular form we enclose the members of a set within braces.

- Two sets are said to be disjoint, if there is no member common between

SETS

- If A and B are two sets then union of A and B is denoted by $A \cup B$ and intersection of A and B is denoted by $A \cap B$.
- The union of two sets A and B is a set which consists of all members of A or B or both. The intersection of sets A and B is a set which consists of common members of A and B.
- If A and B are two sets, then B is said to be a subset of A, if every member of B is also the member of A.
- Two sets are called overlapping, if there is at least one member common between them but none of them is a subset of the other.
- A set which contains all possible members of a given situation or discussion is called the universal set.
- Commutative property of union: If A and B are any two sets then $A \cup B = B \cup A$.
- Commutative proper of intersection: If A and B are any two sets, then $A \cap B = B \cap A$.
- If A, B and C are any three sets, then $A \cup (B \cup C) = (A \cup B) \cup C$ is called the associative property of union of three sets.
- If A, B and C are any three sets then $A \cap (B \cap C) = (A \cap B) \cap C$ is called the associative property of intersection of three sets.
- If A and B are any two sets, the difference of sets A and B is the set of all members which belongs to set A but do not belong to set B. It is written as $A - B$ or $A \backslash B$.
- If $\mathbb{U}$ is a universal set and set A is its subset, then $\mathbb{U} - A$ is called the complement of set A and is denoted by A'.
- The union of sets A and A' will be the universal set.
- The intersection of A and A' is null set.

Unit 2

RATIONAL NUMBERS

2.1 RATIONAL NUMBERS

2.1.1 Define a rational number

We use natural numbers and integers in counting, measuring and representing quantities. There are certain quantities which can not be represented by these numbers, for example:

If the price of 1 kilogram of potatoes = 15 rupees

Then the price of half kilogram of potatoes = $\frac{15}{2}$ rupees

We observe that $\frac{15}{2}$ is neither natural number nor integer. It is a rational number. The numbers $\frac{-2}{3}, \frac{1}{7}, \frac{3}{4}, \frac{-15}{17}$ are some more examples of rational numbers.

Rational number is a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

Remember:

Division of any number by zero is undefined. So zero can never be a denominator of fraction.

In rational numbers, numerator and denominator are always integers and denominator can never be zero. For example, $\frac{0}{6}$ is a rational number but $\frac{6}{0}$ is not a rational number, because denominator of rational number is never zero.

And 6, –7 are also rational numbers because the denominator in each number is 1, i.e. $\frac{6}{1} = 6$ and $\frac{-7}{1} = -7$

Remember that

- Every natural number and integer is also a rational number.
- There are infinite rational numbers between any two numbers.

Unit 2 RATIONAL NUMBERS

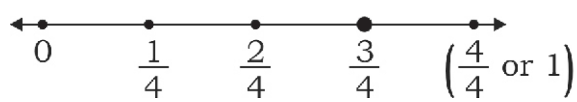
The following diagram describes the relation of different types of numbers.

Rational Numbers
- Integers
  - Non-negative integers
    - Zero
    - Positive integers
  - Negative integers
- Fractions

2.1.2 Represent rational numbers on a number line

We have learnt how to represent integers on a number line in previous class. Now we will learn how to represent a rational number on a number line.

Let us represent $\frac{3}{4}$ on a number line. For this we have to draw a number line and divide the portion between 0 and 1 into four equal parts. Each of the part represents $\frac{1}{4}$ as shown below:



The point at the end of third part is the required number as shown on the number line.

Let us learn how to represent an improper fraction $\frac{11}{4}$ on a number line.

Follow the following:

Step 1. Convert $\frac{11}{4}$ into mixed fraction ,i.e $\frac{11}{4} = 2\,\frac{3}{4}$

$$\begin{array}{r} 2 \\ 4\,\overline{)\,11} \\ \underline{-8} \\ 3 \end{array}$$

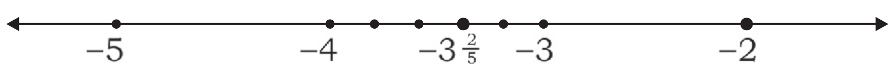
Step 2. Draw a number line showing the whole part 2 of the mixed fraction $2\frac{3}{4}$.

Step 3. Divide portion of line between 2 and 3 into four equal parts as denominator of the fraction is 4.

Step 4. Now start counting these parts from 2, the third point is the required rational number $\frac{11}{4}$ as shown in the figure.

Example: Represent $-\frac{17}{5}$ on a number line.

As $-\frac{17}{5} = -3\frac{2}{5}$, divide the portion of line between –3 and –4 into five equal parts.



Start from –3 to –4, the left end point of second part is the required rational number i.e. $-\frac{17}{5}$

EXERCISE 2.1

1. Write True or False for each of the following statements.

(i) $\frac{0}{5}$ is a rational number. (ii) $\frac{5}{0}$ is a rational number.

(iii) 0 is not a rational number.

(iv) $2\frac{1}{5}$ is a rational number between 5 and 6.

(v) Negative integers are rational numbers.

(vi) Positive integers are not rational numbers.

(vii) Every integer is also a rational number.

(viii) Every natural number is also a rational number.

(ix) In rational number $\frac{p}{q}$, q is always zero.

(x) There is only one rational number between 2 and 3.

(xi) Rational numbers are in the form $\frac{p}{q}$ where p and q are always natural numbers.

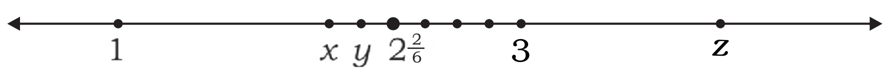
2. Write three rational numbers between 2 and 3.

3. Represent the following rational numbers on a number line.

(i) $\frac{1}{4}$ (ii) $\frac{7}{2}$ (iii) $\frac{11}{3}$ (iv) $-\frac{2}{5}$ (v) $-\frac{12}{5}$

(vi) $2\frac{5}{6}$ (vii) $-3\frac{4}{7}$ (viii) $-\frac{13}{4}$ (ix) $\frac{15}{8}$ (x) $4\frac{2}{3}$

4. For the given number line , write true or false.



(i) The value of x is 0 (ii) The value of x is 2

(iii) The value of y is $2\frac{1}{6}$ (iv) The value of y is $3\frac{2}{5}$

(v) The value of z is 4 (vi) The value of z is -4.

2.2 OPERATIONS ON RATIONAL NUMBERS

There are four fundamental operations on rational numbers. These are addition, subtraction, multiplication and division. We have already learnt the laws for adding, subtracting, multiplying and dividing two fractions in previous classes. Let us generalize these rules on rational numbers.

2.2.1 Add two or more rational numbers.

There are two cases for addition of two or more rational numbers.

Case 1. If denominators of rational numbers are same, then add or subtract the numerators as explained in the following examples:

Example 1. Add the following rational numbers.

(i) $\frac{2}{3}+\frac{5}{3}$ (ii) $\frac{4}{5}+\frac{11}{5}$ (iii) $\frac{16}{7}+\left(-\frac{2}{7}\right)$

Solution:

(i) $\frac{2}{3}+\frac{5}{3}$

$= \frac{2+5}{3}$

$= \frac{7}{3}$

Solution:

(ii) $\frac{4}{5}+\frac{11}{5}$

$= \frac{4+11}{5}$

$= \frac{\cancel{15}^{3}}{\cancel{5}_{1}} = 3$

Solution:

(iii) $\frac{16}{7}+\left(-\frac{2}{7}\right)$

$= \frac{16-2}{7}$

$= \frac{\cancel{14}^{2}}{\cancel{7}_{1}} = 2$

Note: For addition of two rational numbers with same denominator, we solve as: $\boxed{\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}}$

Example 2. Simplify:

(i) $\frac{2}{5}+\frac{3}{5}+\left(-\frac{4}{5}\right)$ (ii) $\frac{17}{10}+\left(-\frac{3}{10}\right)+\frac{1}{10}$

Solution: $\frac{2}{5}+\frac{3}{5}+\left(-\frac{4}{5}\right)$

$= \frac{2+3-4}{5}$

$= \frac{5-4}{5}$

$= \frac{1}{5}$

Solution: $\frac{17}{10}+\left(-\frac{3}{10}\right)+\frac{1}{10}$

$= \frac{17-3+1}{10}$

$= \frac{\cancel{15}^{3}}{\cancel{10}_{2}}$ (Dividing by 5)

$= \frac{3}{2}$

Case 2. Generally in case of rational numbers with different denominators, we calculate LCM of denominators for addition.

Example 3: Simplify: (i) $\frac{5}{6}+\frac{3}{8}$ (ii) $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$

Solution:

(i) $\frac{5}{6}+\frac{3}{8}$

$=\frac{5\times4+3\times3}{24}$ $\left(\text{As } \frac{24}{6}=4 \text{ and } \frac{24}{8}=3\right)$

$=\frac{20+9}{24} = \frac{29}{24}$

$$\begin{array}{c|c} 2 & 6\ ,\ 8 \\ \hline 2 & 3\ ,\ 4 \\ \hline 2 & 3\ ,\ 2 \\ \hline 3 & 3\ ,\ 1 \\ \hline & 1\ ,\ 1 \end{array}$$

LCM = $2\times2\times2\times3 = 24$

Thus LCM of 6 and 8 is 24

Solution:

(ii) $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$

$=\frac{1\times6+3\times3+5\times2}{12}$ $\left(\text{As } \frac{12}{2}=6,\ \frac{12}{4}=3,\ \frac{12}{6}=2\right)$

$=\frac{6+9+10}{12} = \frac{25}{12}$

$$\begin{array}{c|c} 2 & 2\ ,\ 4\ ,\ 6 \\ \hline 2 & 1\ ,\ 2\ ,\ 3 \\ \hline 3 & 1\ ,\ 1\ ,\ 3 \\ \hline & 1\ ,\ 1\ ,\ 1 \end{array}$$

LCM = $2\times2\times3 = 12$

Thus LCM of 2, 4 and 6 is 12

Note: For addition of two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ of different denominators, we solve as:

$$\boxed{\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}}$$

For example $\frac{2}{3}+\frac{4}{5}=\frac{2\times5+4\times3}{3\times5}=\frac{10+12}{15}=\frac{22}{15}$

2.2.2 Subtract a rational number from another rational number.

Case1. If denominators of two rational numbers are same then we just subtract the numerators as explained in the following examples.

Example1:

(i) Subtract $\frac{2}{3}$ from $\frac{5}{3}$

(ii) Simplify: (a) $\frac{7}{2}-\left(-\frac{1}{2}\right)$ (b) $\frac{7}{8}-\frac{1}{8}-\frac{3}{8}$

Solution: (i) $\frac{5}{3}-\frac{2}{3}$ (Subtracting $\frac{2}{3}$ from $\frac{5}{3}$

$=\frac{5-2}{3} = \frac{\cancel{3}^{1}}{\cancel{3}_{1}} = 1$

(ii) (a) $\frac{7}{2}-\left(-\frac{1}{2}\right)$ (b) $\frac{7}{8}-\frac{1}{8}-\frac{3}{8}$

Solution: $\frac{7}{2}-\left(-\frac{1}{2}\right)=\frac{7}{2}+\frac{1}{2}$

$= \frac{7+1}{2}$

$= \frac{\cancel{8}^{4}}{\cancel{2}_{1}} = \frac{4}{1}$

$= 4$

Solution: $\frac{7}{8}-\frac{1}{8}-\frac{3}{8}$

$= \frac{7-1-3}{8}$

$= \frac{6-3}{8}$

$= \frac{3}{8}$

Note: For subtraction of two rational numbers with same denominators, we solve as: $\boxed{\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}}$

Case 2. In case of rational numbers with different denominators, we use LCM of denominators as explained in the following example.

Example 2: Simplify: (i) $\frac{4}{3}-\frac{2}{5}$ (ii) $\frac{7}{4}-\frac{3}{8}-\frac{2}{3}$

Solution:

(i) $\frac{4}{3}-\frac{2}{5}$ LCM = 3 × 5 = 15

3	3 , 5
5	1 , 5
	1 , 1

$= \frac{4\times 5-2\times 3}{15}$ $\left(\text{As } \frac{15}{3}=5 \text{ and } \frac{15}{5}=3\right)$

$= \frac{20-6}{15} = \frac{14}{15}$

Solution:

(ii) $\frac{7}{4}-\frac{3}{8}-\frac{2}{3}$

$= \frac{7\times 6-3\times 3-2\times 8}{24}$ $\left(\text{As } \frac{24}{4}=6, \frac{24}{8}=3 \text{ and } \frac{24}{3}=8\right)$

$= \frac{42-9-16}{24}$

$= \frac{33-16}{24} = \frac{17}{24}$

2	4 , 8 , 3
2	2 , 4 , 3
2	1 , 2 , 3
3	1 , 1 , 3
	1 , 1 , 1

LCM = 2 × 2 × 2 × 3 = 24

Thus, LCM of 4, 8 and 3 is 24

Note: For difference of two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

we solve as: $\boxed{\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}}$

For example, $\frac{5}{3}-\frac{2}{5}=\frac{25-6}{15}=\frac{19}{15}$

2.2.3 Find additive inverse of a rational number:

If the sum of two rational numbers is zero then they are called additive inverse of each other.

Two rational numbers $\frac{a}{b}$ and $\left(-\frac{a}{b}\right)$ are additive inverse of each other.

Because $\left(\frac{a}{b}\right)+\left(-\frac{a}{b}\right)=0$

Example 1. (i) $\frac{5}{2}$ and $-\frac{5}{2}$ are additive inverses of each other.

Because $\left(\frac{5}{2}\right)+\left(-\frac{5}{2}\right)=0$

(ii) $\frac{-6}{7}$ and $\frac{6}{7}$ are additive inverses of each other.

Because $\left(\frac{-6}{7}\right)+\left(\frac{6}{7}\right)=0$

Note: Additive inverse of any rational number (except zero) can be found by changing its sign.

Remember that: **Additive inverse of zero is always zero**

Example 2: Find the additive inverse of each of the following.

(i) $-\frac{3}{7}$ (ii) $\frac{5}{4}$ (iii) $\frac{11}{13}$ (iv) $-\frac{1}{7}$ (v) 8

Solution:

Rational Number	$-\frac{3}{7}$	$\frac{5}{4}$	$\frac{11}{13}$	$-\frac{1}{7}$	8
Additive Inverse	$\frac{3}{7}$	$-\frac{5}{4}$	$-\frac{11}{13}$	$\frac{1}{7}$	-8

2.2.4 Multiply two or more rational numbers.

Product of two rational numbers is found by multiplying numerators and denominators separately. i.e $\left(\frac{a}{b}\right) \times \left(\frac{c}{d}\right) = \frac{ac}{bd}$

Example: Find the product of:

(i) $\frac{2}{7}$ and $\frac{3}{4}$

Solution:

Product of $\frac{2}{7}$ and $\frac{3}{4} = \frac{2}{7} \times \frac{3}{4}$

$= \frac{2 \times 3}{7 \times 4}$

$= \frac{\cancel{6}^{3}}{\cancel{28}_{14}} = \frac{3}{14}$

(ii) $-\frac{5}{6}$ and $\frac{3}{12}$

Solution:

(Product of $-\frac{5}{6}$ and $-\frac{3}{12} = \left(-\frac{5}{6}\right) \times \left(-\frac{3}{12}\right)$

$= \frac{(-5) \times (-3)}{6 \times 12} = \frac{15}{72}$

$= \frac{\cancel{15}^{5}}{\cancel{72}_{24}} = \frac{5}{24}$

Similarly we can multiply more than two rational numbers.

For example, $\frac{2}{3} \times \frac{3}{5} \times \frac{1}{4} = \frac{2 \times 3 \times 1}{3 \times 5 \times 4} = \frac{\cancel{6}^{1}}{\cancel{60}_{10}} = \frac{1}{10}$

2.2.5 Find multiplicative inverse of a rational number.

If the product of two non-zero rational numbers is 1 then they are called multiplicative inverses of each other.

Two non-zero rational numbers $\frac{a}{b}$ and $\frac{b}{a}$ are multiplicative inverses of each other, because $\frac{a}{b} \times \frac{b}{a} = 1$

Example 1. (i) $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses of each other. Because $\frac{2}{3} \times \frac{3}{2} = 1$ or $\frac{3}{2} \times \frac{2}{3} = 1$

(ii) $-\dfrac{5}{6}$ and $-\dfrac{6}{5}$ are multiplicative inverses of each other because

$$\left(-\frac{5}{6}\right)\times\left(-\frac{6}{5}\right)=1 \text{ and } \left(-\frac{6}{5}\right)\times\left(-\frac{5}{6}\right)=1$$

Note: Multiplicative inverse of a non-zero rational number can be found by just interchanging its numerator and denominator.

Remember:

Multiplicative inverse of zero does not exist because denominator of a rational number can never be zero.

Example 2: Find multiplicative inverse of each of the following.

$$\frac{2}{9},\ -\frac{5}{8},\ \frac{111}{3},\ -6,\ \frac{1}{8}$$

Solution:

Rational Number	$\frac{2}{9}$	$-\frac{5}{8}$	$\frac{111}{3}$	-6	$\frac{1}{8}$
Multiplicative inverse	$\frac{9}{2}$	$-\frac{8}{5}$	$\frac{3}{111}$	$-\frac{1}{6}$	$\frac{8}{1}$

2.2.6 Divide a rational number by a non-zero rational number.

Division of a rational number by a non-zero rational number is in fact the multiplication of the given rational number with the multiplicative inverse of the divisor. i.e if a rational number $\dfrac{a}{b}$ is to be divided by a non-zero rational number $\dfrac{c}{d}$ then $\dfrac{a}{b}\div\dfrac{c}{d}=\dfrac{a}{b}\times\dfrac{d}{c}=\dfrac{ad}{bc}$.

Example: Simplify: (i) $\dfrac{2}{5}\div\dfrac{7}{15}$ (ii) $\dfrac{2}{4}\div\dfrac{9}{8}$

(i) $\dfrac{2}{5}\div\dfrac{7}{15}$

Solution:

$$\frac{2}{5}\div\frac{7}{15}=\frac{2}{5}\times\frac{15}{7}$$

$$=\frac{{}^{6}\cancel{30}}{\cancel{35}^{7}} \quad \text{(Dividing by 5)}$$

$$=\frac{6}{7}$$

(ii) $\dfrac{3}{4}\div\dfrac{9}{8}$

Solution:

$$\frac{3}{4}\div\frac{9}{\cancel{8}}=\frac{3}{4}\times\frac{8}{9}$$

$$=\frac{{}^{2}\cancel{24}}{\cancel{36}^{3}} \quad \text{(Dividing by 12)}$$

$$=\frac{2}{3}$$

2.2.7 Find reciprocal of a rational number.

If $\frac{p}{q}$ is a non-zero rational number then $\frac{1}{\frac{p}{q}}$ is its reciprocal and $\boxed{\frac{1}{\frac{p}{q}} = \frac{q}{p}}$

Because $\frac{1}{\frac{p}{q}} = 1 \div \frac{p}{q} = 1 \times \frac{q}{p} = \frac{q}{p}$

Hence reciprocal of a non-zero rational number is the rational number which we get by interchanging the numerator and denominator of given rational number.

Example: Reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$ and reciprocal of 6 is $\frac{1}{6}$.

Note: (i) Reciprocal and multiplicative inverse of a rational number are always equal.

(ii) Reciprocal of zero does not exist.

EXERCISE 2.2

1. Find the reciprocals of the following rational numbers.

(i) $\frac{19}{17}$ (ii) -13 (iii) $\frac{1}{10}$ (iv) $\frac{-6}{25}$

2. Simplify:

(i) $\frac{9}{7} + \frac{15}{7}$ (ii) $\frac{5}{9} - \frac{2}{9}$ (iii) $\frac{3}{5} - \left(-\frac{7}{5}\right)$

(iv) $\frac{5}{6} + \frac{2}{6} - \frac{1}{6}$ (v) $\frac{5}{8} - \frac{4}{5}$ (vi) $-\frac{2}{7} + \frac{7}{6}$

(vii) $\frac{2}{3} + \frac{5}{7} - \frac{1}{6}$ (viii) $\frac{4}{5} - \frac{1}{7} + \frac{3}{70}$ (ix) $\frac{2}{9} - \left(-\frac{3}{6}\right) + \frac{5}{8}$

(x) $\frac{17}{10} + \frac{11}{20} - \frac{4}{5}$

3. Find the additive and multiplicative inverses of each of the following.

(i) $\frac{17}{15}$ (ii) -26 (iii) $-\frac{1}{20}$ (iv) $\frac{-5}{19}$

(v) 0 (vi) 8 (vii) $\frac{1}{9}$ (viii) $\frac{200}{7}$

4. Simplify:

(i) $\frac{5}{8}\times\frac{3}{2}$ (ii) $\frac{6}{7}\div\frac{2}{5}$ (iii) $\frac{2}{3}\times\frac{1}{7}\times\frac{4}{5}$

(iv) $-\frac{6}{7}\div\left(-\frac{7}{6}\right)$ (v) $\frac{14}{5}\div\frac{7}{25}$ (vi) $\frac{9}{4}\times\left(-\frac{4}{9}\right)\times\left(\frac{2}{3}\right)$

(vii) $\frac{1}{8}\times\frac{2}{5}\times-\frac{15}{4}$ (viii) $\left(-\frac{2}{3}\right)\times\left(-\frac{4}{6}\right)\times\left(-\frac{3}{4}\right)$

2.2.8 Verify commutative property of rational numbers with respect to addition and multiplication.

(a) Commutative property w.r.t addition

We know that the sum of any two rational numbers is same irrespective of order, i.e for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\boxed{\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}}$$

This property of rational numbers is called commutative property w.r.t addition.

Example 1: Verify commutative property w.r.t addition for $\frac{2}{3}$ and $\frac{7}{3}$.

Solution: According to commutative property w.r.t addition.

$$\frac{2}{3}+\frac{7}{3}=\frac{7}{3}+\frac{2}{3}$$

Now

LHS $= \frac{2}{3} + \frac{7}{3}$

$= \frac{2+7}{3} = \frac{\cancel{9}^{3}}{\cancel{3}_{1}} = 3$

RHS $= \frac{7}{3} + \frac{2}{3}$

$= \frac{7+2}{3} = \frac{\cancel{9}^{3}}{\cancel{3}_{1}} = 3$

Since LHS = RHS

Therefore $\frac{2}{3} + \frac{7}{3} = \frac{7}{3} + \frac{2}{3}$.

Hence verified.

(b) Commutative property w.r.t multiplication

We know that the product of any two rational numbers is same irrespective of order, i.e for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}}$$

This property of rational numbers is called commutative property w.r.t multiplication.

Example 2: Verify commutative property w.r.t multiplication for $\frac{3}{4}$ and $\frac{1}{4}$

Solution: According to commutative property w.r.t multiplication

$$\frac{3}{4} \times \frac{1}{4} = \frac{1}{4} \times \frac{3}{4}$$

Now

LHS $= \frac{3}{4} \times \frac{1}{4}$

$= \frac{3 \times 1}{16} = \frac{3}{16}$

RHS $= \frac{1}{4} \times \frac{3}{4}$

$= \frac{1 \times 3}{16} = \frac{3}{16}$

Since LHS = RHS

Therefore $\frac{3}{4} \times \frac{1}{4} = \frac{1}{4} \times \frac{3}{4}$.

Hence verified.

2.29 Verify associative property of rational numbers with respect to addition and multiplication.

(i) Associative property w.r.t addition

We know that the sum of any three rational numbers in any order remains same, i.e for three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$

$$\boxed{\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}}$$

This property of rational numbers is called associative property w.r.t addition.

Example 1. Verify associative property w.r.t addition for $\frac{1}{2}$, $\frac{5}{3}$ and $\frac{3}{4}$.

Solution: According to associative property w.r.t addition:

$$\frac{1}{2}+\left(\frac{5}{3}+\frac{3}{4}\right)=\left(\frac{1}{2}+\frac{5}{3}\right)+\frac{3}{4}$$

Now

LHS $= \frac{1}{2}+\left(\frac{5}{3}+\frac{3}{4}\right)$

$= \frac{1}{2}+\left(\frac{5\times 4+3\times 3}{3\times 4}\right)$

$= \frac{1}{2}+\left(\frac{20+9}{12}\right)$

$= \frac{1}{2}+\frac{29}{12}$

$= \frac{1\times 6+29\times 1}{12}$

$= \frac{6+29}{12}=\frac{35}{12}$

RHS $= \left(\frac{1}{2}+\frac{5}{3}\right)+\frac{3}{4}$

$= \left(\frac{1\times 3+2\times 5}{2\times 3}\right)+\frac{3}{4}$

$= \left(\frac{3+10}{6}\right)+\frac{3}{4}$

$= \frac{13}{6}+\frac{3}{4}$

$= \frac{13\times 2+3\times 3}{12}$

$= \frac{26+9}{12}=\frac{35}{12}$

Since LHS = RHS

Therefore $\frac{1}{2}+\left(\frac{5}{3}+\frac{3}{4}\right)=\left(\frac{1}{2}+\frac{5}{3}\right)+\frac{3}{4}$

Hence verified.

(ii) Associative property w.r.t multiplication

We know that the product of any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ in any order remains same.

i.e. $$\boxed{\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}}$$

This property of rational numbers is called associative property w.r.t multiplication.

Example 2. Verify associative property w.r.t multiplication for $\frac{1}{3}, \frac{2}{5}$ and $\frac{3}{4}$

Solution: According to associative property w.r.t multiplication.

$$\frac{1}{3} \times \left(\frac{2}{5} \times \frac{3}{4}\right) = \left(\frac{1}{3} \times \frac{2}{5}\right) \times \frac{3}{4}$$

Now

$$\text{LHS} = \frac{1}{3} \times \left(\frac{2}{5} \times \frac{3}{4}\right) = \frac{1}{3} \times \left(\frac{2 \times 3}{5 \times 4}\right) = \frac{1}{3} \times \frac{6}{20} = \frac{\overset{1}{\cancel{6}}}{\underset{10}{\cancel{60}}} = \frac{1}{10}$$

$$\text{RHS} = \left(\frac{1}{3} \times \frac{2}{5}\right) \times \frac{3}{4} = \left(\frac{1 \times 2}{3 \times 5}\right) \times \frac{3}{4} = \frac{2}{15} \times \frac{3}{4} = \frac{\overset{1}{\cancel{6}}}{\underset{10}{\cancel{60}}} = \frac{1}{10}$$

Since, LHS = RHS

Therefore $$\frac{1}{3} \times \left(\frac{2}{5} \times \frac{3}{4}\right) = \left(\frac{1}{3} \times \frac{2}{5}\right) \times \frac{3}{4}.$$

Hence verified

2.2.10 Distributive Properties

(i) Distributive property of rational numbers w.r.t multiplication over addition:

We know that the product of a given rational number with the sum of two rational numbers is as same as the sum of products of given rational number with each of the two rational numbers.

i.e For $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ any three rational numbers.

$$\boxed{\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}}$$

This property of rational numbers is called distributive property of rational numbers w.r.t multiplication over addition.

Example 1. Verify distributive property of rational numbers w.r.t. multiplication over addition, for $\frac{1}{7}$, $\frac{2}{7}$ and $\frac{3}{4}$.

Solution: According to distributive property of rational numbers w.r.t. multiplication over addition.

$$\frac{1}{7}\times\left(\frac{2}{7}+\frac{3}{7}\right)=\frac{1}{7}\times\frac{2}{7}+\frac{1}{7}\times\frac{3}{7}$$

Now

LHS $=\frac{1}{7}\times\left(\frac{2}{7}+\frac{3}{7}\right)$

$=\frac{1}{7}\times\left(\frac{2+3}{7}\right)$

$=\frac{1}{7}\times\frac{5}{7}=\frac{5}{49}$

RHS $=\frac{1}{7}\times\frac{2}{7}+\frac{1}{7}\times\frac{3}{7}$

$=\frac{1\times2}{7\times7}+\frac{1\times3}{7\times7}=\frac{2}{49}+\frac{3}{49}$

$=\frac{2+3}{49}=\frac{5}{49}$

Since LHS = RHS

Therefore $\frac{1}{7}\times\left(\frac{2}{7}+\frac{3}{7}\right)=\frac{1}{7}\times\frac{2}{7}+\frac{1}{7}\times\frac{3}{7}$.

Hence verified

(ii) Distributive property of rational numbers w.r.t multiplication over subtraction:

We know that the product of a given rational number with the difference of two rational numbers is same as the difference of products of given rational number with each of the two rational numbers.

i.e For any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$

$$\boxed{\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} - \frac{a}{b} \times \frac{e}{f}}$$

This property of rational numbers is called distributive property of rational numbers w.r.t multiplication over subtraction.

Example 2. Verify distributive property of rational numbers w.r.t multiplication over subtraction for $\frac{4}{5}, \frac{3}{5}$ and $\frac{2}{5}$.

Solution: According to distributive property of rational numbers w.r.t multiplication over subtraction.

$$\frac{4}{5} \times \left(\frac{3}{5} - \frac{2}{5}\right) = \frac{4}{5} \times \frac{3}{5} - \frac{4}{5} \times \frac{2}{5}$$

Now LHS $= \frac{4}{5} \times \left(\frac{3}{5} - \frac{2}{5}\right)$

$= \frac{4}{5} \times \left(\frac{3-2}{5}\right)$

$= \frac{4}{5} \times \frac{1}{5} = \frac{4}{25}$

RHS $= \frac{4}{5} \times \frac{3}{5} - \frac{4}{5} \times \frac{2}{5}$

$= \frac{4 \times 3}{5 \times 5} - \frac{4 \times 2}{5 \times 5} = \frac{12}{25} - \frac{8}{25}$

$= \frac{12-8}{25} = \frac{4}{25}$

Since LHS = RHS

Therefore $\frac{4}{5} \times \left(\frac{3}{5} - \frac{2}{5}\right) = \frac{4}{5} \times \frac{3}{5} - \frac{4}{5} \times \frac{2}{5}$.

Hence verified

2.2.11 Compare two rational numbers

In order to compare two rational numbers, we just compare their numerators if denominators are same. If denominators of the two rational numbers are not same then re-write the rational numbers with common denominator by using LCM.

Comparison of two rational numbers is explained in the following examples.

Example: Compare (i) $\frac{3}{4}$ and $\frac{5}{4}$ (ii) $\frac{5}{7}$ and $\frac{3}{8}$

Solution: (i) $\frac{3}{4}$ and $\frac{5}{4}$

Since denominators are same, therefore we just compare numerators.

Here $3 < 5$ So, $\frac{3}{4} < \frac{5}{4}$

(ii) $\frac{5}{7}$ and $\frac{3}{8}$

Solution: Since denominators are not same, therefore we find their LCM.

Now $\frac{5}{7} = \frac{5 \times 8}{7 \times 8} = \frac{40}{56}$

and $\frac{3}{8} = \frac{3 \times 7}{8 \times 7} = \frac{21}{56}$

LCM

2	7 , 8
2	7 , 4
2	7 , 2
7	7 , 1
	1 , 1

LCM = $2 \times 2 \times 2 \times 7 = 56$

Here $40 > 21$

So, $\frac{40}{56} > \frac{21}{56}$

Therefore $\frac{5}{7} > \frac{3}{8}$

2.2.12 Arrange rational numbers in ascending or descending orders.

(I) Two or more rational numbers can be arranged in ascending and descending order with the help of numerators if the denominators are same.

(II) If denominators are not same then we have to re-write all the rational numbers with common denominators using LCM as explained in the following examples.

Example 1. Write the following numbers in ascending and descending order.

$$\frac{2}{5}, \frac{7}{5} \text{ and } \frac{1}{5}$$

Solution: Here denominators are same, So, we just compare their numerators.

Because $1 < 2 < 7$. Therefore $\frac{1}{5} < \frac{2}{5} < \frac{7}{5}$

So, ascending order is: $\frac{1}{5}, \frac{2}{5}, \frac{7}{5}$ and descending order is: $\frac{7}{5}, \frac{2}{5}, \frac{1}{5}$

Example 2 Write the following numbers in ascending and descending order:

$$\frac{3}{7}, \frac{4}{5} \text{ and } \frac{1}{2}$$

Solution: Here denominators are different, so first we find their LCM.

Now $\frac{3}{7} = \frac{3 \times 10}{7 \times 10} = \frac{30}{70}$, $\frac{4}{5} = \frac{4 \times 14}{5 \times 14} = \frac{56}{70}$

and $\frac{1}{2} = \frac{1 \times 35}{2 \times 35} = \frac{35}{70}$.

2	2 , 5 , 7
5	1 , 5 , 7
7	1 , 1 , 7
	1 , 1 , 1

LCM = $2 \times 5 \times 7 = 70$

Here $30 < 35 < 56$

So, $\frac{30}{70} < \frac{35}{70} < \frac{56}{70}$

Therefore $\frac{3}{7} < \frac{1}{2} < \frac{4}{5}$

Hence ascending order is: $\frac{3}{7}, \frac{1}{2}, \frac{4}{5}$ and descending order is: $\frac{4}{5}, \frac{1}{2}, \frac{3}{7}$

EXERCISE 2.3

1. Verify the following

(i) $\frac{2}{3} + \left(-\frac{4}{3}\right) = \left(-\frac{4}{3}\right) + \frac{2}{3}$

(ii) $\frac{4}{7} \times \frac{2}{5} = \frac{2}{5} \times \frac{4}{7}$

(iii) $\frac{3}{4} + \left(\frac{5}{6} + \frac{4}{8}\right) = \left(\frac{3}{4} + \frac{5}{6}\right) + \frac{4}{8}$

(iv) $\frac{7}{15} \times \left(\frac{3}{4} \times \frac{5}{7}\right) = \left(\frac{7}{15} \times \frac{3}{4}\right) \times \frac{5}{7}$

(v) $\frac{2}{7} \times \left(\frac{3}{4} - \frac{1}{2}\right) = \frac{2}{7} \times \frac{3}{4} - \frac{2}{7} \times \frac{1}{2}$

(vi) $\frac{4}{5} \times \left(\frac{7}{8} + \frac{3}{5}\right) = \frac{4}{5} \times \frac{7}{8} + \frac{4}{5} \times \frac{3}{5}$

2. **Verify associative property w.r.t addition for $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$.**

3. **Verify associative property w.r.t multiplication for $\frac{2}{7}, \frac{4}{5}$ and $\frac{3}{4}$.**

4. **Verify distributive property of multiplication over addition for $\frac{3}{4}, \frac{4}{5}$ and $\frac{7}{8}$.**

5. **Verify distributive property of multiplication over subtraction for $\frac{5}{7}, \frac{3}{4}$ and $\frac{1}{5}$.**

6. **Arrange the following rational numbers in descending order.**

   (i) $\frac{1}{2}, \frac{2}{3}, \frac{8}{9}$ (ii) $\frac{1}{6}, \frac{3}{4}, \frac{1}{2}$ (iii) $\frac{4}{7}, \frac{1}{3}, \frac{5}{6}$

7. **Arrange the following rational numbers in ascending order.**

   (i) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ (ii) $\frac{4}{5}, \frac{1}{10}, \frac{2}{15}$ (iii) $\frac{3}{8}, \frac{1}{4}, \frac{5}{6}$

8. **Arrange the following numbers in ascending and descending order.**

   (i) $\frac{4}{3}, \frac{5}{7}, \frac{1}{4}$ (ii) $\frac{1}{2}, \frac{2}{5}, \frac{1}{11}$ (iii) $\frac{2}{9}, \frac{5}{3}, \frac{1}{2}, \frac{5}{6}$ (iv) $\frac{4}{20}, \frac{20}{15}, \frac{1}{2}, \frac{9}{7}$

REVIEW EXERCISE 2

1. **Answer the following questions.**

   (i) Define a rational number and give its examples.

   (ii) Write the additive inverse of the rational number "a".

   (iii) What is the reciprocal of the rational number $\frac{p}{q}$, $q \neq 0$?

   (iv) Write the sum of two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, (q and s are non-zero).

(v) What is the rule of finding the product of two rational numbers?

(vi) What are the inverse operations of addition and multiplication?

2. Fill in the blanks.

(i) Every fraction as well as integer is ________.

(ii) The rational number $\frac{p}{q}$ and $\frac{-p}{q}$ are called _______ inverse of each other.

(iii) A number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a _______ number.

(iv) The rational number 0 has no _______.

(v) The _______ inverse of a rational number is its reciprocal.

3. Tick (√) the correct option.

(i) A number that can be expressed in the form of $\frac{p}{q}$ where $p, q \in Z, q \neq 0$ is called.

(a) integer (b) rational number

(c) whole number (d) positive number.

(ii) The additive inverse of $\frac{2}{3}$ is:

(a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{3}$ (d) $-\frac{3}{2}$

(iii) The multiplicative inverse of $-\frac{4}{7}$ is:

(a) $\frac{4}{7}$ (b) $\frac{7}{4}$ (c) $-\frac{7}{4}$ (d) $\frac{1}{7}$

(iv) $\frac{1}{3}+\frac{1}{2}=$ ______.

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{2}{5}$ (d) $\frac{5}{6}$

(v) $\frac{2}{5}+\left(-\frac{4}{5}\right)=$ ______.

(a) 2 (b) -2 (c) $-\frac{1}{2}$ (d) $-\frac{2}{5}$

4. Draw the number lines and represent the following rational numbers on number lines.

(i) $1\frac{1}{2}$ (ii) $3\frac{1}{3}$ (iii) $-\frac{1}{4}$ (iv) $-1\frac{4}{5}$

5. Find the additive and multiplicative inverse of the following rational numbers.

(i) -14 (ii) $\frac{1}{5}$ (iii) $-\frac{2}{3}$ (iv) $-\frac{11}{27}$

6. Put the correct sign > or < between the following pair of rational number.

(i) $\frac{2}{3},\frac{1}{5}$ (ii) $\frac{10}{13},\frac{11}{14}$ (iii) $\frac{1}{4},-\frac{1}{2}$

(iv) $-\frac{11}{17},\frac{3}{8}$ (v) $-\frac{4}{9},-\frac{2}{5}$ (vi) $-\frac{5}{22},-\frac{11}{25}$

7. Solve the following.

(i) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$ (ii) $\frac{2}{7}+\frac{1}{2}-\frac{3}{14}$ (iii) $\frac{5}{8}-\frac{1}{5}-\frac{3}{4}$

(iv) $\left(-\frac{1}{3}\right)+\left(-\frac{1}{4}\right)+\frac{1}{2}$ (v) $\left(-\frac{11}{15}\right)+\left(-\frac{3}{5}\right)+\frac{5}{4}$

(vi) $\left(-\frac{19}{55}\right)+\frac{51}{55}+\left(-\frac{21}{55}\right)$

8. Simplify the following.

(i) $\left(-\frac{1}{100}\right) \div \left(-\frac{1}{10}\right)$ (ii) $\frac{1}{5} \times \frac{2}{3} \times \left(-\frac{30}{44}\right)$

(iii) $\frac{2}{3} \div \frac{16}{21} \times \frac{27}{49}$ (iv) $\frac{8}{21} \div \frac{7}{12}$

(v) $\frac{1}{6} \times \left(-\frac{2}{3}\right) \div \left(-\frac{11}{63}\right)$ (vi) $-\frac{2}{7} \div \frac{3}{4} \times \frac{63}{100}$

9. Prove that:

(i) $(-1) + \frac{35}{54} = \frac{35}{54} + (-1)$

(ii) $\left(-\frac{121}{169}\right) \times \left(-\frac{13}{11}\right) = \left(-\frac{13}{11}\right) \times \left(-\frac{121}{169}\right)$

(iii) $\frac{4}{9} \times \left(\frac{2}{3} \times \frac{5}{7}\right) = \left(\frac{4}{9} \times \frac{2}{3}\right) \times \frac{5}{7}$

(iv) $-\frac{4}{5} \times \left(\frac{1}{8} + \frac{11}{12}\right) = \left(-\frac{4}{5} \times \frac{1}{8}\right) + \left(-\frac{4}{5} \times \frac{11}{12}\right)$

(v) $-\frac{1}{4} + \left(\frac{1}{6} + \frac{3}{5}\right) = \left(-\frac{1}{4} + \frac{1}{6}\right) + \frac{3}{5}$

(vi) $\frac{5}{12} \times \left(-\frac{2}{7} - 2\right) = \left(\frac{5}{12} \times -\frac{2}{7}\right) - \left(\frac{5}{12} \times 2\right)$

10. Name the properties used in the following.

(i) $\left(-\frac{1}{2}\right) + \frac{1}{3} = \frac{1}{3} + \left(-\frac{1}{2}\right)$

(ii) $-\frac{10}{11} \times \left(-\frac{5}{44}\right) = \left(-\frac{5}{44}\right) \times \left(-\frac{10}{11}\right)$

(iii) $-\frac{2}{3} \times \left(\frac{7}{8} \times \frac{9}{14}\right) = \left(-\frac{2}{3} \times \frac{7}{8}\right) \times \frac{9}{14}$

(iv) $-\frac{3}{5} + \left(\frac{1}{2} + \frac{7}{10}\right) = \left(-\frac{3}{5} + \frac{1}{2}\right) + \frac{7}{10}$

(v) $-\frac{1}{2} + \left(\frac{3}{5} + \frac{1}{4}\right) = \left(-\frac{1}{2} + \frac{3}{5}\right) + \frac{1}{4}$

(vi) $\frac{2}{3} \times \left(\frac{1}{2} + \frac{5}{6}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{5}{6}\right)$

(vii) $\left(-\frac{12}{105}\right)\times\left(-\frac{15}{84}\right)=\left(-\frac{15}{84}\right)\times\left(-\frac{12}{105}\right)$

(viii) $\frac{1}{4}\times\left(\frac{8}{9}-\frac{12}{15}\right)=\left(\frac{1}{4}\times\frac{8}{9}\right)-\left(\frac{1}{4}\times\frac{12}{15}\right)$

(ix) $-\frac{5}{8}\times\left(\frac{4}{7}-\frac{2}{3}\right)=\left(-\frac{5}{8}\times\frac{4}{7}\right)-\left(-\frac{5}{8}\times\frac{2}{3}\right)$

(x) $\frac{24}{49}\times\left(\frac{7}{8}+\frac{14}{6}\right)=\left(\frac{24}{49}\times\frac{7}{8}\right)+\left(\frac{24}{49}\times\frac{14}{6}\right)$

SUMMARY

- Every integer can be divided by another non-zero integer, the number obtained is called a rational number and is written symbolically as $\frac{a}{b}$, where $b \neq o$.
- Addition of rational numbers with:

  Same denominator, solve as: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

  Different denominators, solve as: $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$
- Subtraction of rational numbers with:

  Same denominator, solve as: $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

  Different denominators, solve as: $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$
- To find the product of two rational numbers, multiply the numerator of one rational number by the numerator of the other. Similarly, multiply the denominators.

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

- Division is an inverse operation of multiplication. So, for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(d \neq 0)$.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

- 0 is called additive identity and 1 is called the multiplicative identity.
- $\frac{a}{b}$ is called the reciprocal of $\frac{a}{b}$.
- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then according to the commutative property:

$$\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b} \qquad \frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$$

- If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then according to the associative property:

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right) \text{ and } \left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$$

- If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then according to the distributive property:

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right) \text{ and}$$

$$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)-\left(\frac{a}{b}\times\frac{e}{f}\right)$$

•••

Think Tank

Study the most wonderful knowledge about the number 37.

$37 \times 3 = 111$	$37 \times 6 = 222$	$37 \times 9 = 333$
$37 \times (3 \times 4) = 444$	$37 \times (3 \times 5) = 555$	$37 \times \boxed{} = 666$
$37 \times \boxed{} = 777$	$37 \times \boxed{} = 888$	$37 \times \boxed{} = \boxed{}$

Unit 3

DECIMALS

3.1 Conversion of Decimals to Rational Numbers.

We have already learnt that decimals are those numbers which contain digits whose value is in tenths, hundredths, thousandths and so on.

Let us consider a decimal number 2.5, it comprises of two parts: one '2' which is a whole part and another is '.5' which is a decimal part.

It has one place of decimal because there is only one digit after decimal point.

A decimal may have whole part and decimal part.

as: 2.5, 14.57, 32.163 and 103.0978 etc.

or a decimal may have a decimal part only

as: 0.6, 0.56, 0.10652 and so on.

3.1.1 Convert decimals to rational numbers

We can convert decimals to rational numbers as follows:

Example 1: Convert a decimal 0.5 to rational number.

Solution: $0.5 = \frac{05}{10} = \frac{5}{2 \times 5} = \frac{\cancel{5}^{1}}{2 \times \cancel{5}_{1}} = \frac{1}{2}$

or $0.5 = \frac{\cancel{5}^{1}}{\cancel{10}_{2}} = \frac{1}{2}$

Steps:

(i) Write 1 below the decimal point and remove decimal point.

(ii) Put as many zeros after 1 as the number of digits after decimal point. (As it has one place of decimal, so we put one zero after 1).

(iii) Reduce it to the lowest form (Standard form)

Thus decimal $0.5 = \frac{1}{2}$ (Rational number).

Example 2: Convert a decimal 0.06 to rational number.

Solution: As 0.06 has two place of decimal, so we put two zeros after '1'.

$$0.06 = \frac{06}{100} = \frac{{}^{3}\cancel{6}}{\cancel{100}_{50}} = \frac{3}{50}$$

Thus, we get decimal 0.6 $= \frac{3}{50}$ (Rational number).

Hence $\frac{1}{2}$ and $\frac{3}{50}$ are rational numbers.

Example 3: Convert decimal 2.40 into rational number.

Solution: Given decimal 2.40 comprises of two parts '2' a whole part and '.40' a decimal part.

So, we first convert decimal part '.40' into rational number.

$40 = \frac{40}{100}$ (As .40 has two places of decimal, so we put two zeros after '1').

So, $2.40 = 2 + .40 = 2\frac{40}{100} = 2\frac{4}{10} = 2\frac{\cancel{4}^{2}}{\cancel{10}_{5}} = 2\frac{2}{5} = \frac{12}{5}$

Thus decimal 2.40 = $\frac{12}{5}$ which is a common fraction or a rational number.

Example 4. Convert – 1.60 into rational number.

Here whole part '–1' is kept as a whole and the decimal part '.60' is to be converted into common fraction.

So, $-1.60 = -(1 + .60)$

$$= -\left(1 + \frac{6\cancel{0}}{10\cancel{0}}\right) = -\left(1 + \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}}\right)$$

$$= -\left(1 + \frac{3}{5}\right) = -1\frac{3}{5} = -\frac{8}{5}$$

Thus decimal –1.60 $= -\frac{8}{5}$ which is a rational number.

EXERCISE 3.1

A. Convert the following one place-decimals into rational numbers.

(1) 0.2 (2) 0.4 (3) 0.6 (4) 0.8 (5) 0.9

B. Convert the following two place-decimals into rational numbers.

(1) 0.15 (2) 0.35 (3) 0.48 (4) 0.75 (5) 0.95
(6) – 1.30 (7) – 2.50 (8) 5.05 (9) 5.60 (10) 6.85

C. Convert the following three place-decimals into rational numbers.

(1) 0.024 (2) 3.125 (3) 0.375 (4) 4.065 (5) 5.625

(6) –0.001 (7) –0.0005 (8) –0.005 (9) –5.005

(10) –11.375 (11) –0.00125 (12) –15.00625

3.2 Terminating and Non-terminating Decimals

Basically decimals are of the following two types:

(a) Terminating decimals (b) Non-terminating decimals

3.2.1 Define terminating decimals as decimals having a finite number of digits after the decimal point.

Terminating Decimals:

Let us consider the conversion of some rational numbers

$\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$ and $\frac{3}{8}$

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(i) $\frac{1}{4} = 1 \div 4$

$$\begin{array}{r} 0.25 \\ 4\overline{)\,10} \\ -8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

Thus $\frac{1}{4} = 0.25$

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(ii) $\frac{1}{5} = 1 \div 5$

$$\begin{array}{r} 0.2 \\ 5\overline{)\,10} \\ -10 \\ \hline 0 \end{array}$$

Thus $\frac{1}{5} = 0.2$

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(iii) $\frac{1}{2} = 1 \div 2$

$$\begin{array}{r} 0.5 \\ 2\overline{)\,10} \\ -10 \\ \hline 0 \end{array}$$

Therefore $\frac{1}{2} = 0.5$

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(iv) $\frac{3}{8} = 3 \div 8$

$$\begin{array}{r} 0.375 \\ 8\overline{)\,30} \\ -24 \\ \hline 60 \\ 56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Therefore $\frac{3}{8} = 0.375$

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In above conversion, the process of division is completed and remainder is zero in each case. These rational numbers have been converted into decimals which has limited number of digits after decimal point. These decimals are terminating decimals.

> A decimal which contains finite number of digits after decimal point, is called a terminating decimal.
>
> Hence 0.5, 0.25, 0.75, 0.345, 1.7227, 5.023748 etc are all terminating decimals.

3.2.2 Define recurring decimals as non–terminating decimals in which a single digit or a block of digits repeats itself infinite number of times after decimal point. e.g. $\frac{2}{7}$ = 0. 2 85 71 42 85 71 4 ...

(b) Non-terminating decimals

Consider the conversion of rational number $\frac{7}{9}$, $\frac{1}{6}$ and $\frac{10}{11}$.

(i) $\frac{7}{9} = 7 \div 9$	(ii) $\frac{1}{6} = 1 \div 6$	(iii) $\frac{10}{11} = 10 \div 11$
$\begin{array}{r} 0.777... \\ 9\overline{)\ 70} \\ -63 \\ \hline 70 \\ -63 \\ \hline 70 \\ -63 \\ \hline 7 \end{array}$	$\begin{array}{r} 0.1666... \\ 6\overline{)\ 10} \\ -6 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$	$\begin{array}{r} 0.909090... \\ 11\overline{)\ 100} \\ -99 \\ \hline 100 \\ -99 \\ \hline 100 \\ -99 \\ \hline 1 \end{array}$
So, $\frac{7}{9} = 0.777...$	So, $\frac{1}{6} = 0.1666...$	So, $\frac{10}{11} = 0.909090...$

By converting the given rational numbers to decimals, it is found that

(i) $\frac{7}{9} = 0.777...$ (ii) $\frac{1}{6} = 0.1666...$ and (iii) $\frac{10}{11} = 0.909090....$

Here the process of division is not complete. These decimals 0.777..., 0.1666... and 0.909090... contain infinite number of digits after decimal point. Therefore these decimals are non-terminating.

So,

> A decimal which contain infinite number of digits after decimal point is called non-terminating decimal.

In 0.777... the digit '7' is repeated infinite number of times after decimal.

In 0.1666... the digit '6' is repeated infinite number of times after decimal.

In 0.90909... the block '90' is repeated infinite number of times after decimal.

These non-terminating decimals are recurring.

> A non-terminating decimal in which a 'single digit' or a 'block of digits' is repeated infinite number of times after decimal point, is called recurring decimal.

Example 1: Convert the given rational numbers as decimals and find whether the decimals so obtained are recurring or not..

(i) $\frac{2}{3}$ (ii) $\frac{3}{4}$ (iii) $\frac{7}{6}$ (iv) $\frac{9}{10}$

Solution: We can observe by long division that:

(i) $\frac{2}{3} = 0.666\ ...$ and (iii) $\frac{7}{6} = 1.1666\ ...$

Here in both the numbers, digit '6' repeats infinite number of times after decimal, so both are non-terminating and recurring.

(ii) $\frac{3}{4} = 0.75$ and (iv) $\frac{9}{10} = 0.9$

Here we obtain zero as remainder, so both are terminating.

Example 2: Convert the given rational numbers $\frac{1}{3}$, $\frac{4}{6}$ and $\frac{2}{7}$ into decimals and find whether the decimals so obtained are recurring or not.

Solution:

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(i) $\frac{1}{3} = 1 \div 3$

$$\begin{array}{r} 0.333... \\ 3\overline{)\ 10} \\ -9 \\ \hline 10 \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \end{array}$$

So, $\frac{1}{3} = 0.333...$

Here 3 is repeated infinite number of times.

So, it is a recurring decimal.
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(ii) $\frac{4}{6} = 4 \div 6$

$$\begin{array}{r} 0.666... \\ 6\overline{)\ 40} \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 4 \end{array}$$

So, $\frac{4}{6} = 0.666...$

Here 6 is repeated infinite number times.

So, it is a recurring decimal.
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(iii) $\frac{2}{7} = 2 \div 7$

$$\begin{array}{r} 0.285714... \\ 7\overline{)\ 20} \\ -14 \\ \hline 60 \\ -56 \\ \hline 40 \\ -35 \\ \hline 50 \\ -49 \\ \hline 10 \\ -7 \\ \hline 30 \\ -28 \\ \hline 2 \end{array}$$

So, $\frac{2}{7}$ is equal to 0.285714285714 ...
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Hence 0.333..., 0.666... and 0.285714... are non-terminating and recurring decimals because the single digit or block of digits are repeated infinite number of times after decimal and the process of division is not complete.

Activity: Identify as terminating and non-terminating.

Rational Number	In decimal form	Terminating	Non-Terminating (recurring)
$\frac{4}{3}$	1.333 ...		Non-Terminating
$\frac{4}{9}$	0.444 ...		
$\frac{5}{4}$	1.25	Terminating	
$\frac{25}{7}$	3.5714285 ...		
$\frac{3}{11}$	0.2727 ...		

EXERCISE 3.2

I. Convert into decimals and identify terminating or non-terminating decimals.

(1) $\frac{7}{3}$ (2) $\frac{11}{6}$ (3) $-\frac{13}{9}$ (4) $\frac{15}{7}$ (5) $\frac{14}{9}$

(6) $-\frac{17}{11}$ (7) $\frac{19}{12}$ (8) $\frac{21}{13}$ (9) $-\frac{23}{14}$ (10) $-\frac{29}{15}$

(11) $\frac{31}{17}$ (12) $-\frac{35}{18}$ (13) $\frac{47}{19}$ (14) $\frac{35}{23}$ (15) $-\frac{20}{21}$

(16) $\frac{39}{22}$ (17) $-\frac{41}{29}$ (18) $\frac{40}{31}$ (19) $-\frac{50}{37}$ (20) $\frac{101}{41}$

II. Convert into decimals and identify which is recurring and which is not.

(1) $\frac{19}{20}$ (2) $\frac{17}{6}$ (3) $\frac{15}{8}$ (4) $\frac{25}{10}$ (5) $\frac{2}{11}$

(6) $\frac{10}{3}$ (7) $\frac{11}{12}$ (8) $\frac{16}{7}$ (9) $\frac{2}{13}$ (10) $\frac{4}{25}$

3.2.3 Use the following rule to find whether a given rational number is terminating or not.

RULE: If the denominator of a rational number in standard form has prime factors 2 and 5 or 2, 5 only then that rational number is terminating decimal.

We can identify a rational number whether it is terminating or non-terminating using the above rule without actually dividing.

DECIMALS

Consider the following examples.

Rational Number	Division Method	Using Rule	Type of Decimal
$\frac{9}{20}$	$\frac{9}{20} = 9 \div 20 = 0.45$	$\frac{9}{20} = \frac{9}{2 \times 2 \times 5}$ Denominator has 2 and 5 as prime factors.	Terminating
$\frac{3}{5}$	$\frac{3}{5} = 3 \div 5 = 0.6$	$\frac{3}{5} = \frac{3}{5 \times 1}$ Denominator has 5 as prime factor.	Terminating
$\frac{27}{200}$	$\frac{27}{200} = 27 \div 200 = 0.135$	$\frac{27}{200} = \frac{27}{2 \times 2 \times 2 \times 5 \times 5}$ Denominator has 2 and 5 prime factors.	Terminating
$\frac{40}{90}$	$\frac{40}{90} = 40 \div 90 = 0.4444 \ldots$	$\frac{40}{90} = \frac{2 \times 2 \times 2 \times 5}{2 \times 3 \times 3 \times 5} = \frac{2 \times 2}{3 \times 3}$ Denominator has not 2, 5 as prime factor.	Non-Terminating
$\frac{4}{7}$	$\frac{4}{7} = 0.57142857\ldots$	$\frac{4}{7} = \frac{4}{7 \times 1}$ Denominator has not 2, 5 as prime factor.	Non-Terminating
$\frac{27}{18}$	$\frac{27}{18} = 1.5$	$\frac{27}{18} = \frac{3 \times 9}{6 \times 3} = \frac{3 \times 3 \times 3}{2 \times 3 \times 3} = \frac{3}{2}$ Denominator has 2 as prime factor.	Terminating

EXERCISE 3.3

Without actual division, separate the terminating and non-terminating decimals.

(1) $\frac{12}{25}$	(2) $\frac{43}{21}$	(3) $\frac{5}{12}$	(4) $\frac{17}{20}$	(5) $\frac{117}{125}$
(6) $\frac{5}{7}$	(7) $-\frac{9}{40}$	(8) $\frac{10}{33}$	(9) $\frac{23}{60}$	(10) $\frac{40}{35}$
(11) $\frac{55}{75}$	(12) $-\frac{96}{100}$	(13) $\frac{101}{125}$	(14) $\frac{111}{96}$	(15) $\frac{125}{200}$
(16) $\frac{141}{144}$	(17) $\frac{200}{201}$	(18) $-\frac{210}{147}$	(19) $\frac{372}{400}$	(20) $-\frac{401}{333}$

3.2.4 Express a given rational number as a decimal and indicate whether it is terminating or recurring.

Consider the conversion of following numbers:

(i) $\frac{7}{20}$ (ii) $2\frac{1}{100}$ (iii) $\frac{1}{7}$ (iv) $\frac{5}{9}$

Solution: (i) $\frac{7}{20} = 0.35$ (ii) $2\frac{1}{100} = 2.01$

As the remainder in both $\frac{7}{20}$ and $2\frac{1}{100}$ is zero.; Therefore these are terminating decimals. Now let us convert $\frac{1}{7}$ and $\frac{5}{9}$ in decimals.

(iii) $\frac{1}{7}$	(iv) $\frac{5}{9}$
$$\begin{array}{r} 0.1428... \\ 7\overline{)\,10} \\ -7 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 60 \\ -56 \\ \hline 4 \end{array}$$	$$\begin{array}{r} 0.555... \\ 9\overline{)\,50} \\ -45 \\ \hline 50 \\ -45 \\ \hline 50 \\ -45 \\ \hline 5 \end{array}$$
So, $\frac{1}{7} = 0.1428571...$ and the process of division is not complete.	So, $\frac{5}{9} = 0.555...$ and the process of division is not complete.

We observe that rational numbers $\frac{1}{7}$ and $\frac{5}{9}$ are non-terminating and recurring decimals.

EXERCISE 3.4

Express the following rational numbers as decimals. Also indicate whether each is terminating or recurring.

(1) $\frac{5}{14}$ (2) $-\frac{13}{18}$ (3) $\frac{19}{20}$ (4) $-3\frac{21}{22}$ (5) $6\frac{17}{30}$

(6) $\frac{23}{40}$ (7) $-\frac{25}{34}$ (8) $2\frac{15}{26}$ (9) $3\frac{27}{32}$ (10) $-1\frac{29}{45}$

(11) $2\frac{31}{38}$ (12) $\frac{39}{46}$ (13) $-3\frac{49}{50}$ (14) $2\frac{45}{48}$ (15) $\frac{91}{65}$

(16) $-\frac{153}{85}$ (17) $\frac{266}{171}$ (18) $-\frac{102}{210}$ (19) $\frac{111}{228}$ (20) $\frac{272}{288}$

3.3 APPROXIMATE VALUE

Get an appropriate value of a number called rounding off, to a desired number of decimal places.

In calculations we may come across rational numbers which are non-terminating decimals. So, we round off the decimals to some decimal places and then solve our calculations.

RULE: (i) Check the remaining digits.
If the next digit is less than 5 drop it.

(ii) If the next digit is 5 or more than 5, then add 1 to the last digit of the number.

Example: Consider the decimal 0.555... Round off 0.555... to two decimal places.

Solution: We have to round off up to two decimal places.

Step 1: So, we keep two digits of the given decimal same i.e. 0.55

Step 2: As the next digit to be removed after two decimal places is 5.
So, we add '1' to last digit of 0.55
i.e. 5 which will become 6.

Hence the given decimal 0.555... will be rounded off to two decimal places as 0.56

Example 2. Round off 2.333333 to two decimal places, we get: 2.33

Example 3. Round off 4.8888 to two decimal places, we get: 4.89

Activity: Express the following rational numbers by rounding off up to given number of digits.

S. No.	Number	Up to two decimal places	Up to three decimal places	Up to four decimal places
1.	0.05753	0.06	0.058	0.575
2.	2.421875			
3.	0.83333			
4.	1.76794			
5.	0.9916667			
6.	0.96257			
7.	1.61538			
8.	8.08547			
9.	6.98677			
10.	15.47648			

EXERCISE 3.5

I. Convert the following rational numbers as terminating and non-terminating decimals and express up to three places of decimal.

(1) $\frac{23}{400}$ (2) $\frac{27}{64}$ (3) $\frac{53}{60}$ (4) $-\frac{46}{75}$ (5) $\frac{69}{70}$

(6) $\frac{77}{80}$ (7) $-\frac{43}{90}$ (8) $\frac{119}{120}$ (9) $\frac{246}{300}$ (10) $-\frac{81}{96}$

II. Identify terminating and non-terminating decimals by division and express them up to two places of decimal.

(1) $\frac{4}{15}$ (2) $-\frac{11}{12}$ (3) $\frac{8}{9}$ (4) $\frac{22}{7}$ (5) $-\frac{21}{16}$

(6) $\frac{3}{40}$ (7) $-\frac{18}{45}$ (8) $\frac{21}{66}$ (9) $\frac{33}{90}$ (10) $-\frac{81}{101}$

III. Express the following rational numbers as non-terminating or recurring up to three places of decimal.

(1) $\frac{10}{11}$ (2) $-\frac{41}{90}$ (3) $\frac{21}{13}$ (4) $\frac{17}{19}$ (5) $\frac{9}{7}$

(6) $\frac{15}{23}$ (7) $-\frac{111}{96}$ (8) $\frac{22}{29}$

IV. Round off the following decimals up to two places of decimal.

(1) 0.4555 (2) 0.3648 (3) 0.6666 (4) 0.1054

(5) 1.9091 (6) 3.2479 (7) 2.0518 (8) 8.3381

(9) 10.1036 (10) 3.9995 (11) 13.0967 (12) 15.8159

V. Round off the following decimals up to three places of decimal.

(1) 4.51689 (2) 6.7472 (3) 0.0098 (4) 0.9861

(5) 206.4176 (6) 3.4073 (7) 102.99909 (8) 11.1234

(9) 55.12345 (10) 4.10539 (11) 66.3957 (12) 59.9196

REVIEW EXERCISE 3

1. Convert the following decimals into rational numbers.

(i) 0.325 (ii) 0.75 (iii) 2.5 (iv) 7.75

(v) 0.78 (vi) 1.297 (vii) 2.348

2. Convert the following into decimal fractions and identify terminating and non-terminating fractions.

(i) $\frac{41}{5}$ (ii) $\frac{111}{12}$ (iii) $\frac{28}{9}$ (iv) $\frac{10}{7}$

(v) $\frac{22}{7}$ (vi) $\frac{21}{16}$ (vii) $\frac{73}{10}$

3. Round off the following up to two-decimal places.

(i) 14.5828 (ii) 1078287 (iii) 5.7895

(iv) 6.7989 (v) 25.4897

4. Without actual division, separate the terminating and non-terminating decimals.

(i) $\frac{13}{4}$ (ii) $\frac{17}{25}$ (iii) $\frac{80}{3}$ (iv) $\frac{15}{11}$

(v) $\frac{19}{6}$ (vi) $\frac{25}{15}$ (vii) $\frac{22}{7}$ (viii) $\frac{14}{9}$

5. Express the following rational numbers in termanting decimals.

(i) $\frac{21}{100}$ (ii) $\frac{127}{20}$ (iii) $\frac{13}{25}$ (iv) $\frac{31}{50}$ (v) $\frac{145}{1000}$

(vi) $\frac{25}{8}$ (vii) $\frac{23}{6}$ (viii) $\frac{88}{64}$ (ix) $\frac{48}{32}$

6. Convert the following rational numbers in non-terminating decimals and express up to three places of decimal .

(i) $\frac{40}{3}$ (ii) $\frac{20}{7}$ (iii) $\frac{50}{11}$ (iv) $\frac{80}{13}$ (v) $\frac{100}{6}$

(vi) $\frac{240}{22}$ (vii) $\frac{70}{12}$ (viii) $\frac{52}{91}$

7. Round off the following decimals up to three places of decimal .

(i) 5.71679 (ii) 11.60365 (iii) 0.92598

(iv) 3.40855 (v) 0.74396 (vi) 23.15847

8. Answer the following questions.

(i) What are the terminating and non-terminating decimals?

(ii) Write the method for converting a decimal into a rational number. Give examples.

(iii) Which of the non-terminating decimals are called recurring decimal? Give two examples.

(iv) What is the rule to find whether a given rational number is terminating or not? Give two examples.

(v) What is meant by the term 'rounding off' in decimals? Give two examples.

9. Fill in the blanks.

(i) $\frac{1}{2}$ = 0.5 which is a ______. (ii) $\frac{1}{3}$ = 0.333 ... which is a ______.

(iii) Two parts of a decimal number separated by a dot, are called the _________ and _________ .

(iv) In terminating decimals, division ________ after a finite number of steps.
(v) A fraction will be terminating decimal if the ________ has 2 or 5 or both as factors of denominator.
(vi) In recurring decimals, single digit or ______ of digits repeat themselves.

10. Tick (✓) the correct answer.

(i) To separate a whole number from fractional part in a decimal, we use the symbol.
(a) — (b) . (c) : (d) /

(ii) If we round off the decimal 3.8461 up to two decimal places, we get:
(a) 3.84 (b) 3.81 (c) 3.86 (d) 3.85

(iii) A rational number is terminating decimal, if its denominator has no prime factor other than:
(a) 2 and 5 (b) 3 and 6 (c) 4 and 5 (d) 5 and 7

(iv) When we convert 0.75 into the rational number, we get:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{7}$

SUMMARY

- Decimals are basically of two types:
  (i) Terminating decimals, (ii) Non-terminating decimals.
- Every decimal with finite number of digits after the decimal point is called a terminating decimal.
- A terminating decimal represents a rational number.
- A decimal with infinite number of digits after a decimal point, is called a non-terminating decimal.
- A non-terminating decimal may be recurring or non-recurring.
- A fraction will be terminating decimal if the denominator in standard form has 2 or 5 or both as factors.
- Decimals can be reduced by rounding off the digits upto the desired number of decimal places as under:
  (i) Check the remaining digits.
  If the next digit, it is less then 5 drop it.
  (ii) If the next digit, is 5 or more than 5, than add 1 to the last digit of the number.

Unit 4

EXPONENTS

4.1 EXPONENTS / INDICES

4.1.1 Identify base, exponent and value

We know that if a number is repeatedly multiplied by itself then it can be represented in index or exponential notation.

For example, $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

or $\boxed{2^5 = 32}$

Here 2 is base,

5 is exponent or index.

32 is the value which is the fifth power of 2.

and we read it as "2 to the power 5 is 32".

Now, we define the terms related to index notation as:

Base:

Base is a number which is to be multiplied by itself according to exponent.

Exponent or Index:

Exponent is a number which represents the number of times, a base is to be multiplied by itself.

Value of exponential form:

The product of base with itself according to the exponent is the value of exponential form.

For example.

$$3^4 = 81 \qquad \text{or } 3 \times 3 \times 3 \times 3 = 81$$

Here 81 is the value of "3 to the power 4"

Example 1. Identify base and exponent in the following. Also find the value.

(i) 2^4 (ii) 5^2 (iii) $\left(\frac{4}{7}\right)^3$ (iv) $\left(\frac{2}{3}\right)^4$

Solution:

S. No.	Expression	Base	Exponent	Value
(i)	2^4	2	4	16 because $2 \times 2 \times 2 \times 2 = 16$
(ii)	5^2	5	2	25 because, $5 \times 5 = 25$
(iii)	$\left(\frac{4}{7}\right)^3$	$\frac{4}{7}$	3	$\frac{64}{343}$ because, $\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{64}{343}$
(iv)	$\left(\frac{2}{3}\right)^4$	$\frac{2}{3}$	4	$\frac{16}{81}$ because, $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$

EXERCISE 4.1

1. **Expand and find the value of each exponential form.**

   (i) 3^2 (ii) 2^7 (iii) $2^3 \times 3^4$

   (iv) $5^2 \times 7^2$ (v) $2^3 \times 3^2 \times 5^2$ (vi) $2^4 \times 3^3 \times 7^2$

2. **Identify the base and exponent. Also find the value of each of the following expression˙**

   (i) 2^5 (ii) 5^4 (iii) 6^3 (iv) $\left(\frac{2}{3}\right)^6$ (v) $\left(-\frac{1}{5}\right)^5$

3. **Show that:**

   (i) $(-3)^3 = -27$ (ii) $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$ (iii) $\left(-\frac{1}{4}\right)^4 = \frac{1}{256}$

   (iv) $\left(-\frac{1}{10}\right)^6 = \frac{1}{1000000}$ (v) $\left(-\frac{5}{3}\right)^4 = \frac{625}{81}$ (vi) $\left(\frac{4}{5}\right)^5 = \frac{1024}{3125}$

4. **Express each rational number in exponential form.**

   (i) 1000 (ii) 512 (iii) –343 (iv) 625

   (v) $-\frac{1}{100000}$ (vi) $\frac{32}{243}$ (vii) $-\frac{125}{343}$ (viii) $\frac{729}{64}$

4.2 LAWS OF EXPONENTS / INDICES

4.2.1 Use rational numbers to deduce laws of exponents.

There are some laws of exponents which help us to simplify expressions of exponential form. These laws are given below:

Product law.

(I) When bases are same but exponents are different, then product law is: $a^m \times a^n = a^{m+n}$ where a is a rational number and m, n are natural numbers.

In order to deduce this law, let us consider the following examples.

$$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$$
$$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

i.e $\boxed{2^3 \times 2^2 = 2^{3+2} = 2^5}$

Also $$\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right) \times \left(\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right)$$
$$= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \left(\frac{2}{5}\right)^7$$

ie. $\boxed{\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^7 = \left(\frac{2}{5}\right)^{3+4}}$

Hence we deduce that if two rational numbers with same bases and different exponents are multiplied then we just add their exponents keeping the base same.

i.e. $\boxed{a^m \times a^n = a^{m+n}}$

where a is a rational number and m, n are natural numbers.

Example. Simplify using product law.

(i) $\left(\frac{2}{7}\right)^4 \times \left(\frac{2}{7}\right)^5$ (ii) $\left(\frac{3}{4}\right)^5 \times \left(\frac{3}{4}\right)^6 \times \left(\frac{2}{5}\right)^4 \times \left(\frac{2}{5}\right)^5$

Solution: (i) $\left(\frac{2}{7}\right)^4 \times \left(\frac{2}{7}\right)^5$

$= \left(\frac{2}{7}\right)^{4+5}$ (using product law)

$= \left(\frac{2}{7}\right)^9$

(ii) $\left(\frac{3}{4}\right)^5 \times \left(\frac{3}{4}\right)^6 \times \left(\frac{2}{5}\right)^4 \times \left(\frac{2}{5}\right)^5$

$= \left(\frac{3}{4}\right)^{5+6} \times \left(\frac{2}{5}\right)^{4+5}$ (using product law)

$= \left(\frac{3}{4}\right)^{11} \times \left(\frac{2}{5}\right)^9$

(II) When bases are different but exponents, are same then product law is: $a^n \times b^n = (ab)^n$ where a, b are rational numbers and n is a natural number.

In order to deduce this law, let us consider the following examples.

$3^4 \times 2^4 = 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2$

$= (3 \times 2) \times (3 \times 2) \times (3 \times 2) \times (3 \times 2)$

$= 6 \times 6 \times 6 \times 6 = 6^4$

i.e. $3^4 \times 2^4 = 6^4 = (3 \times 2)^4$

Similarly $\left(\frac{2}{3}\right)^3 \times \left(\frac{1}{5}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$

$= \left(\frac{2}{3} \times \frac{1}{5}\right) \times \left(\frac{2}{3} \times \frac{1}{5}\right) \times \left(\frac{2}{3} \times \frac{1}{5}\right)$

$= \left(\frac{2}{15}\right) \times \left(\frac{2}{15}\right) \times \left(\frac{2}{15}\right) = \left(\frac{2}{15}\right)^3$

i.e. $\left(\frac{2}{3}\right)^3 \times \left(\frac{1}{5}\right)^3 = \left(\frac{2}{15}\right)^3 = \left(\frac{2}{3} \times \frac{1}{5}\right)^3$

Hence we deduce that if two rational numbers with the different bases and same exponents are multiplied then multiply their bases keeping the exponent same.

i.e. $a^n \times b^n = (ab)^n$

where a, b are rational numbers and n is a natural number.

Example. Simplify using product law: (i) $\left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^2$ (ii) $x^3 \times y^3$

(i) $\left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^2$

Solution:

$\left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^2 = \left(\frac{3}{7} \times \frac{4}{5}\right)^2$ (Using product law)

$= \left(\frac{12}{35}\right)^2$

(ii) $x^3 \times y^3$

Solution: $= (x \times x \times x) \times (y \times y \times y)$

$= (x \times y) \times (x \times y) \times (x \times y)$

$= (xy)(xy)\ (xy) = (xy)^3$

(Using product law)

$= x^3\ y^3$

EXERCISE 4.2

1. Simplify using product law.

(i) $\left(\frac{5}{7}\right)^2 \times \left(\frac{5}{7}\right)^4$ (ii) $\left(\frac{3}{4}\right)^6 \times \left(\frac{1}{5}\right)^6$

(iii) $\left(\frac{1}{7}\right)^5 \times \left(\frac{1}{7}\right)^5 \times \left(-\frac{5}{7}\right)^8 \times \left(-\frac{5}{7}\right)^8$ (iv) $p^6 \times p^7$

(v) $y^5 \times x^{11} \times y^6$ (vi) $\left(\frac{4}{3}\right)^5 \times \left(\frac{4}{3}\right)^6 \times \left(\frac{4}{3}\right)^7$

(vii) $x^5 \times y^6 \times x^2 \times y^7$ (viii) $\left(-\frac{1}{3}\right)^6 \times \left(-\frac{1}{3}\right)^5 \times \left(-\frac{1}{3}\right)^3$

(ix) $x^5 \times y^5 \times z^5$ (x) $p^2 \times q^7 \times p^5$

2. Verify that:

(i) $2^5 \times 3^5 = 6^5$ (ii) $p^7 \times q^7 = (pq)^7$

(iii) $\left(\frac{3}{4}\right)^4 \times \left(\frac{3}{4}\right)^8 = \left(\frac{3}{4}\right)^{12}$

(iv) $\left(\frac{1}{7}\right)^4 \times \left(\frac{1}{7}\right)^5 = \left(\frac{1}{7}\right)^9$

(v) $\left(-\frac{1}{8}\right)^6 \times \left(-\frac{3}{5}\right)^6 = \left(\frac{3}{40}\right)^{16}$

Quotient law:

I. When bases are same but the exponents are different, then quotient law is: $a^m \div a^n = a^{m-n}$ where a is rational number and m, n are natural numbers.

In order to deduce this law, let us consider the following examples.

$$3^6 \div 3^2 = \frac{3^6}{3^2}$$

$$= \frac{3\times3\times3\times3\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}}$$

$$= 3\times3\times3\times3 = 3^4$$

i.e. $3^6 \div 3^2 = 3^4 = 3^{6-2}$

Also $\left(\frac{1}{2}\right)^5 \div \left(\frac{1}{2}\right)^2 = \dfrac{\left(\frac{1}{2}\right)^5}{\left(\frac{1}{2}\right)^2} = \dfrac{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}} = \left(\frac{1}{2}\right)^3$

i.e. $\left(\frac{1}{2}\right)^5 \div \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{5-2}$

Hence we deduce that when two rational numbers with same base and different exponents are divided then we just subtract their exponents keeping the base same.

i.e. $a^m \div a^n = a^{m-n}$

where a is a rational number and m, n are natural numbers

Example 2. Simplify using quotient law: $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^2$

Solution: $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^{6-2}$ (using quotient law)

$$= \left(\frac{2}{3}\right)^4$$

II. **When bases are different but exponents are same, then the quotient law is: $a^n \div b^n = \left(\frac{a}{b}\right)^n$ where a, b are rational numbers with $b \neq o$ and n is a natural number.**

In order to deduce this law, let us consider the following examples.

$$5^2 \div 3^2 = \frac{5^2}{3^2} = \frac{5 \times 5}{3 \times 3}$$

$$= \frac{5}{3} \times \frac{5}{3} = \left(\frac{5}{3}\right)^2$$

Also $$7^3 \div 2^3 = \frac{7^3}{2^3} = \frac{7 \times 7 \times 7}{2 \times 2 \times 2}$$

$$= \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \left(\frac{7}{2}\right)^3$$

Hence we deduce that if two rational numbers with different bases and same exponents are divided then we just divide their bases keeping the exponent same.

i.e. $$a^n \div b^n = \left(\frac{a}{b}\right)^n$$

where a, b are rational numbers with $b \neq o$ and n is a natural number.

Example 1. Simplify using quotient law: $5^7 \div 3^7$

Solution: $$5^7 \div 3^7 = \left(\frac{5}{3}\right)^7$$

Example 2. Simplify using quotient law $6^5 \div 2^5$

Solution: $$6^5 \div 2^5 = \left(\frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{2}}}\right)^5 = (3)^5$$

EXERCISE 4.3

1. Simplify by using quotient law:

(i) $5^7 \div 5^4$ (ii) $\left(\frac{5}{7}\right)^5 \div \left(\frac{5}{7}\right)^2$ (iii) $\left(\frac{1}{2}\right)^6 \div \left(\frac{1}{2}\right)^2$

(iv) $x^{10} \div x^4$ (v) $(5)^6 \div (3)^6$ (vi) $(10)^5 \div (2)^5$

(vii) $(-7)^4 \div (-5)^4$ (viii) $\left(\frac{2}{5}\right)^6 \div \left(\frac{3}{5}\right)^6$ (ix) $\left(\frac{7}{11}\right)^6 \div \left(\frac{7}{11}\right)^5$

(x) $x^4 \div y^4$

2. Verify:

(i) $\left(\frac{2}{9}\right)^5 \div \left(\frac{2}{9}\right)^2 = 2^3 \div 9^3$

(ii) $(-3)^8 \div (-5)^8 = \left(\frac{3}{5}\right)^{10} \div \left(\frac{3}{5}\right)^2$

(iii) $\left(\frac{x}{y}\right)^7 \div \left(\frac{x}{y}\right)^5 = x^2 \div y^2$

(iv) $5^{11} \div 5^8 = (20)^3 \div (4)^3$

Power law:

I. If a is a rational number and m, n are natural numbers, then power law is: $(a^m)^n = a^{mn}$.

In order to deduce this law, let us consider the following examples.

$$(7^3)^2 = (7^3) \times (7^3)$$
$$= (7 \times 7 \times 7) \times (7 \times 7 \times 7) = 7^6$$

i.e. $(7^3)^2 = 7^6 = (7)^{3\times 2}$

Similarly

$$\left\{\left(\frac{11}{12}\right)^2\right\}^4 = \left(\frac{11}{2}\right)^2 \times \left(\frac{11}{2}\right)^2 \times \left(\frac{11}{2}\right)^2 \times \left(\frac{11}{2}\right)^2$$
$$= \left(\frac{11}{2} \times \frac{11}{2}\right) \times \left(\frac{11}{2} \times \frac{11}{2}\right) \times \left(\frac{11}{2} \times \frac{11}{2}\right) \times \left(\frac{11}{2} \times \frac{11}{2}\right) = \left(\frac{11}{2}\right)^8$$

i.e. $\left\{\left(\frac{11}{12}\right)^2\right\}^4 = \left(\frac{11}{2}\right)^8 = \left(\frac{11}{2}\right)^{2\times 4}$

Hence we deduce that if a power of a rational number is raised to another power then we have to only multiply the two exponents keeping the base same.

i.e. $(a^m)^n = a^{mn}$

where a, is a rational number and m, n are natural numbers.

Example. Simplify using power law: (i) $(2^5)^3$ (ii) $\left\{\left(\frac{3}{10}\right)^2\right\}^4$

Solution:

(i) $(2^5)^3 = (2)^{5\times3} = 2^{15}$

(ii) $\left\{\left(\frac{3}{10}\right)^2\right\}^4 = \left(\frac{3}{10}\right)^{2\times4} = \left(\frac{3}{10}\right)^8$

Value of exponential expression when exponent is zero.

For zero: $\boxed{a^0 = 1, \text{ where } a \text{ is a rational number.}}$

We know that

$$\frac{a^2}{a^2} = \frac{a\times a}{a\times a} = 1$$

Also $\frac{a^2}{a^2} = a^{2-2} = a^0$

Hence $a^0 = 1$

Similarly $5^0 = 1, \left(\frac{5}{2}\right)^0 = 1$ and $\left(\frac{2}{3}\right)^0 = 1$

Value of exponential expression when exponent is negative.

For exponent as negative integer: $a^{-m} = \frac{1}{a^m}$

We know that

$$\frac{1}{a^2} = \frac{a^o}{a^2}$$

$$= a^{0-2}$$

$$= a^{-2} \text{ (By quotient law)}$$

i.e $\frac{1}{a^2} = a^{-2}$ or $a^{-2} = \frac{1}{a^2}$

Similarly $5^{-3} = \frac{1}{5^3}$ and $6^{-7} = \frac{1}{6^7}$

In general $a^{-n} = \frac{1}{a^n}$

So, $a^{-1} = \frac{1}{a}$

4.2.2 Demonstrate the concept of power of negative integer that is $(-a)^n$ when n is even or odd integer.

Let us consider the following examples

$(-3)^2 = (-3 \times -3) = +9$ (Exponent is even)

$(-3)^3 = (-3)(-3)(-3) = -27$ (Exponent is odd)

$(-3)^4 = (-3)(-3)(-3)(-3) = +81$ (Exponent is even)

$(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$ (Exponent is odd)

From above examples we deduce that

$(-a)^n = +a^n$, if n is even

and $(-a)^n = -a^n$, if n is odd

Example. Which of the following are positive or negative?

(i) $(-5)^{11}$ (ii) $(-3)^{20}$ (iii) $(-8)^{113}$ (iv) $(-5)^{40}$

(v) $(-8)^{o}$ (vi) $(-7)^{-5}$ (vii) $(-110)^{-40}$

Solution:

S.No.	**Expression**	**Positive or Negative**	**Reason**
(i)	$(-5)^{11}$	Negative	The exponent is odd
(ii)	$(-3)^{20}$	Positive	The exponent is even
(iii)	$(-8)^{113}$	Negative	The exponent is odd
(iv)	$(-5)^{40}$	Positive	The exponent is even
(v)	$(-8)^{0}$	Positive	The exponent is even
(vi)	$(-7)^{-5}$	Negative	The exponent is odd
(vii)	$(-110)^{-40}$	Positive	The exponent is even

EXERCISE 4.4

1. Simplify using power law.

(i) $(5^2)^7$ (ii) $\left\{\left(\frac{1}{3}\right)^6\right\}^8$ (iii) $\{(-3)^2\}^{10}$

(iv) $(x^5)^{20}$ (v) $\left\{\left(\frac{4}{7}\right)^9\right\}^2$ (vi $\left\{\left(-\frac{3}{4}\right)^{20}\right\}^2$

2. Convert into negative exponent.

(i) $\frac{1}{2^5}$ (ii) $\frac{1}{3^7}$ (iii) $\frac{1}{x^6}$ (iv) $\frac{1}{\left(\frac{4}{5}\right)^6}$

3. Convert into positive exponent.

(i) 5^{-8} (ii) $\left(\frac{1}{3}\right)^{-7}$ (iii) x^{-5} (iv) y^{-7}

4. Identify which of the following is positive or negative.

(i) $(-7)^6$ (ii) $\left(-\frac{11}{2}\right)^{17}$ (iii) $\left(-\frac{1}{5}\right)^{-26}$

(iv) $(-3)^0$ (v) $\left(-\frac{1}{7}\right)^1$ (vi) $\left(-\frac{1}{5}\right)^{-100}$

4.2.3 Apply laws of exponents to evaluate expressions.

Expressions can be simplified easily with the application of laws of exponents as explained in the following examples.

Example 1. Simplify and write in the simple form.

(i) $(2^5 \div 2^3) \times 2^{-2}$ (ii) $\left(\frac{1}{3}\right)^4 \times \left(\frac{1}{3}\right)^2 \times (3)^6$

(iii) $\left(\frac{2}{3}\right)^2 \times \left(\frac{3}{2}\right)^2 + \left(\frac{5}{4}\right)^3 \times \left(\frac{4}{5}\right)^3$

Solution: (i) $(2^5 \div 2^3) \times 2^{-2}$

$= 2^{5-3} \times 2^{-2} = 2^{5-3-2} = 2^0 = 1$

(ii) $\left(\frac{1}{3}\right)^4 \times \left(\frac{1}{3}\right)^2 \times 3^6$

$= \left(\frac{1}{3}\right)^{4+2} \times 3^6$

$= \left(\frac{1}{3}\right)^6 \times 3^6$

$= \left(\frac{1}{\cancel{3}} \times \cancel{3}\right)^6$

$= (1)^6 = 1$

(iii) $\left(\frac{2}{3}\right)^2 \times \left(\frac{3}{2}\right)^2 + \left(\frac{5}{4}\right)^3 \times \left(\frac{4}{5}\right)^3$

$= \left(\frac{2}{3} \times \frac{3}{2}\right)^2 + \left(\frac{5}{4} \times \frac{4}{5}\right)^3$

$= \left(\frac{6}{6}\right)^2 + \left(\frac{20}{20}\right)^3 = (1)^2 + (1)^3$

$= 1 + 1 = 2$

Example 2. Using laws of exponents, find the value of x.

(i) $2^5 \times 2^3 = 2^x$ (ii) $\left(\frac{4}{5}\right)^6 \div \left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^{2x}$

(iii) $\left\{\left(\frac{3}{4}\right)^2\right\}^x = \left(\frac{3}{4}\right)^8$

Solution:

(i) $2^5 \times 2^3 = 2^x$

or $2^{5+3} = 2^x$

$2^8 = 2^x$

so $8 = x$

or $\boxed{x = 8}$

(ii) $\left(\frac{4}{5}\right)^6 \div \left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^{2x}$

or $\left(\frac{4}{5}\right)^{6-2} = \left(\frac{4}{5}\right)^{2x}$

or $\left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^{2x}$

so $4 = 2x$

or $\frac{4}{2} = x$ i.e. $\boxed{x = 2}$

(iii) $\left\{\left(\frac{3}{4}\right)^2\right\}^x = \left(\frac{3}{4}\right)^8$

or $\left(\frac{3}{4}\right)^{2x} = \left(\frac{3}{4}\right)^8$

so $2x = 8$

or $x = \frac{\cancel{8}^{4}}{\cancel{2}_{1}}$

or $\boxed{x = 4}$

EXERCISE 4.5

1. Simplify and write in simplest form.

(i) $\left(\frac{5}{7}\right)^6 \times \left(\frac{5}{7}\right)^4 \times \left(\frac{7}{5}\right)^{10}$

(ii) $\left\{\left(\frac{2}{3}\right)^2\right\}^3 \div \left(\frac{2}{3}\right)^4$

(iii) $\left(\frac{7}{9}\right)^3 \times \left(\frac{9}{7}\right)^3 + \left(\frac{11}{12}\right)^5 \times \left(\frac{12}{11}\right)^5$

(iv) $x^o + x^5 \times x^{-5}$

(v) $\left\{\left(\frac{4}{5}\right)^7 \times \left(\frac{4}{5}\right)^2\right\} \div \left(\frac{4}{5}\right)^8$

(vi) $\left\{\left(\frac{7}{8}\right)^6 \div \left(\frac{7}{8}\right)^4\right\} \times \left(\frac{8}{7}\right)^2$

2. Find the value of *x* in the following.

(i) $3^6 \times 3^4 = 3^x$

(ii) $\left(\frac{11}{9}\right)^8 \div \left(\frac{11}{9}\right)^6 = \left(\frac{11}{9}\right)^{2x}$

(iii) $\left\{\left(\frac{4}{7}\right)^3\right\}^x = \left(\frac{4}{7}\right)^9$

(iv) $\left(\frac{1}{5}\right)^x \times \left(\frac{1}{5}\right)^5 = \left(\frac{1}{5}\right)^{10}$

REVIEW EXERCISE 4

1. Identify the base, exponent and find the value of following.

(i) 3^5 (ii) 7^4 (iii) $\left(\frac{3}{4}\right)^2$ (iv) $\left(\frac{1}{4}\right)^3$ (v) 8^0 (vi) y^0

2. Simplify using laws of exponent.

(i) $\left(-\frac{1}{2}\right)^4 \times \left(-\frac{1}{2}\right)^2$ (ii) $\left(-\frac{4}{5}\right)^2 \times \left(-\frac{4}{5}\right)^2$ (iii) $6^7 \div 6^4$

(iv) $\left(\frac{2}{7}\right)^5 \div \left(\frac{2}{7}\right)^3$ (v) $\left(3^2\right)^5$ (vi) $\left\{(-4)^2\right\}^5$

(vii) $\left(6^o\right)^2$ (viii) $\left(5^o\right)^2 \times (-5) \times \left(a^o\right)^2$

3. Verify:

(i) $\left(\frac{1}{5}\right)^4 \times \left(\frac{1}{5}\right)^5 = \left(\frac{3}{5}\right)^9 \times \left(\frac{1}{3}\right)^9$ (ii) $(6)^4 \div (2)^4 = 3^8 \div 3^4$

(iii) $6z^o = 6 \times \left(z^o\right)^2$

4. Which one is positive and which one is negative.

(i) $(-2)^7$ (ii) $(-5)^0$ (iii) $(-3)^8$ (iv) $(-6)^{-7}$ (v) $(-8)^{-10}$

5. Simplify: (i) $6^2 \div 3^3$ (ii) $5^0 \div 5^{-2} \times 5^2$

(iii) $\left[\left\{\left(\frac{2}{5}\right)^2\right\}^3 \div \left(\frac{2}{5}\right)^4\right] \times \left(\frac{2}{5}\right)^3$

6. Find the value of *x*.

(i) $25^x = 125$ (ii) $\left(\frac{2}{3}\right)^x \times \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{10} \div \left(\frac{2}{3}\right)^2$

(iii) $\left\{\left(\frac{2}{7}\right)^x\right\}^5 = \left(\frac{2}{7}\right)^{10}$

SUMMARY

Base: Base is a number which is to be multiplied by itself according to exponent.

Exponent: Exponent is a number which represents the number of times a base is to be multiplied.

Unit 4 EXPONENTS

Product law (i) $a^m \times a^n = a^{m+n}$ (ii) $a^n \times b^n = (ab)^n$

where a, b are rational numbers and m, n are natural numbers.

Quotient law (i) $a^m \div a^n = a^{m-n}$ (ii) $a^n \div b^n = \left(\frac{a}{b}\right)^n$

where a, b are rational numbers and m, n are natural numbers.

Power law $(a^m)^n = a^{mn}$

Where a is rational number and m, n are natural number.

Properties (i) $a^0 = 1$ (ii) $a^{-m} = \frac{1}{a^m}$

(iii) $(-a)^n$ is positive if n is even.

(iv) $(-a)^n$ is negative if n is odd.

•••

Think Tank: The Most Wonderful Knowledge.

Let some body asks you: Add the consecutive natural numbers from 1 to say 10, ... You can do it within no time as under:

Multiply 10 by its next consecutive natural number

i.e. 11 and divide it by 2.

$$1 + 2 + 3 + \ldots + 10 = \frac{10 \times 11}{2} = \frac{\overset{5}{\cancel{10}} \times 11}{\cancel{2}_1} = 55$$

Similarly we can do:

$$1 + 2 + 3 + \ldots + 20 = \frac{20 \times 21}{2} = \frac{\overset{10}{\cancel{20}} \times 21}{\cancel{2}_1} = 210$$

$$1 + 2 + 3 + \ldots + 25 = \frac{25 \times 26}{2} = \frac{25 \times \overset{13}{\cancel{26}}}{\cancel{2}_1} = 325$$

Unit 5

SQUARE ROOT OF POSITIVE NUMBER

5.1 PERFECT SQUARES:

There are certain numbers which can be written in exponential form with exponent 2 for example: $16 = 4 \times 4 = 4^2$.

We read it as '16 is square of 4'. Similarly 9 is square of 3 because $9 = 3 \times 3 = 3^2$

5.1.1 Define a perfect square: In the above examples, 9 and 16 are called perfect squares because these are positive numbers and squares of numbers 3 and 4 respectively.

A positive number which is the square of any number is called a perfect square. Some examples of perfect squares are: 1, 4, 9, 16, 25, 36 etc.

Activity: Complete the following table to find some perfect squares.

Number	Square	Perfect square	Read as
1	$1^2 = 1 \times 1$	1	1 is the square of 1
2	$2^2 =$ ________	4	4 is the square of 2
– 3	$(-3)^2 = (-3) \times (-3)$	9	___ is the square of ___
5	$5^2 =$ ________	25	25 is the square of ___
– 6	___ $= (-6) \times (-6)$	___	___ is the square of (–6)
___	$7^2 = 7 \times 7$	___	___ is the square of ___
8	___ $= 8 \times 8$	___	___ is the square of ___
___	___ $= (-9) \times (-9)$	___	___ is the square of ___
___	___ = ___	100	100 is the square of ___

5.1.2 Test whether a number is a perfect square or not.

We can test a number whether it is a perfect square or not by the following procedure.

1. Find and list its prime factors.

2. Group the same factors in pairs.
3. If each prime factor is grouped then the number is a perfect square.
4. If any factor is left from pairing then the number is not a perfect square.

Example: Test whether the following numbers are perfect squares or not.

(i) 5625 (ii) 4563

(i) Solution:

Prime factorization = $\underline{5 \times 5} \times \underline{5 \times 5} \times \underline{3 \times 3}$

Since each prime factor is in pairs,

So, 5625 is a perfect square.

5	5625
5	1125
5	225
5	45
3	9
	3

(ii) Solution:

Prime factorization is : $\underline{3 \times 3} \times 3 \times \underline{13 \times 13}$

As, 3 is left from pairing,

So, 4563 is not a perfect square.

3	4563
3	1521
3	507
13	169
13	13
	1

5.1.3 Properties of perfect square of a number.

Identify and apply some interesting properties of numbers and perfect squares.

(i) The square of even number is even.

We know that the numbers exactly divisible by 2 are called even numbers. Below are the perfect squares of some even numbers.

Even Number	Square	Perfect square
2	$2^2 = 2 \times 2$	4
4	$4^2 = 4 \times 4$	16
6	$6^2 = 6 \times 6$	36
8	$8^2 = 8 \times 8$	64
10	$10^2 = 10 \times 10$	100

Comparing the even numbers and their perfect squares, we conclude that squares of all even numbers are also even.

(ii) The square of an odd number is odd.

We know that odd numbers are not exactly divisible by 2. Let us identify perfect squares of some odd numbers.

Odd Number	Square	Perfect square
1	$1^2 = 1 \times 1$	1
3	$3^2 = 3 \times 3$	9
5	$5^2 = 5 \times 5$	25
7	$7^2 = 7 \times 7$	49
9	$9^2 = 9 \times 9$	81

From the above table it is clear that the squares of all odd numbers are also odd.

(iii) The square of a proper fraction is less than itself:

We know that a fraction is called proper fraction if its denominator is greater that its numerator. Let us learn the process of squaring and comparing a proper fraction by the following examples.

Example. Find the square of $\frac{3}{4}$ and compare with itself.

Solution: Let us find square of $\frac{3}{4}$.

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}$$

Comparing equivalent fractions

Given number is: $\frac{3}{4} = \frac{3\times4}{4\times4} = \frac{12}{16}$, and $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$.

Here $\frac{9}{16} < \frac{12}{16}$, so square of the given number is less than the given number.

i.e. $\frac{9}{16} < \frac{3}{4}$ or $\left(\frac{3}{4}\right)^2 < \frac{3}{4}$

Therefore the square of a fraction is less than itself.

Similarly: $\left(\frac{2}{9}\right)^2 = \left(\frac{2}{9}\right)\times\left(\frac{2}{9}\right) = \frac{2\times2}{9\times9} = \frac{4}{81}$ and $\frac{4}{81} < \frac{2}{9}$

$\left(\frac{1}{7}\right)^2 = \left(\frac{1}{7}\right)\times\left(\frac{1}{7}\right) = \frac{1\times1}{7\times7} = \frac{1}{49}$ and $\frac{1}{49} < \frac{1}{7}$

(iv) The square of a decimal less than 1 is smaller than the given decimal.

Consider the following examples.

Example: Find the square of 0.2 and compare with itself

Solution: Square: $(0.2)^2 = (0.2)\times(0.2) = \frac{2}{10}\times\frac{2}{10} = \frac{4}{100} = 0.04$

It is observed that: $0.04 < 0.2$

or $(0.2)^2 < 0.2$

Therefore the square of a decimal less than one is always smaller than the given decimal.

Similarly: $(0.03)^2 = (0.03)\times(0.03) = \frac{3}{100}\times\frac{3}{100} = \frac{9}{10000} = 0.0009$

So, $0.0009 < 0.03$

or $(0.03)^2 < 0.03$

Also $(0.12)^2 = (0.12) \times (0.12) = \dfrac{12}{100} \times \dfrac{12}{100} = \dfrac{144}{10000} = 0.0144$

So, $0.0144 < 0.12$

or $(0.12)^2 < 0.12$

EXERCISE 5.1

A. Find squares of the following.

(1) 11 (2) 19 (3) 25 (4) 50

(5) 66 (6) 78 (7) 100 (8) 500

B. Test whether each of the following is a perfect square or not.

(1) 81 (2) 95 (3) 121 (4) 169

(5) 224 (6) 3969 (7) 3872 (8) 9845

C. Separate the perfect square of even and odd numbers.

(i) 9 (ii) 25 (iii) 196 (iv) 441 (v) 2704

(vi) 3600 (vii) 9216 (viii) 9801

D. Find the squares of the following proper fractions and compare with themselves.

(1) $\dfrac{1}{5}$ (2) $\dfrac{3}{7}$ (3) $\dfrac{8}{9}$ (4) $\dfrac{5}{6}$ (5) $\dfrac{2}{3}$

E. Find the squares of the following decimal numbers and compare with themselves.

(1) 0.1 (2) 0.5 (3) 0.07 (4) 0.11 (5) 0.15

5.2 SQUARE ROOT

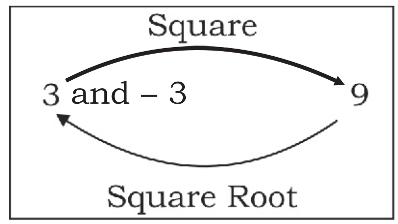
5.2.1 Define square root of a natural number and recognize its notation.

Square root of a number is a numerical value which when multiplied by itself, gives the original number. It is a useful tool to solve the mathematical equations related to our daily life. Square root was introduced by an ancient Egyptian who used it in construction and other activities.

Unit 5 SQUARE ROOT OF POSITIVE NUMBER

We know how to find a perfect square. The process of finding square root is just opposite to it.

As $3^2 = 3 \times 3 = 9$ also $(-3)^2 = (-3) \times (-3) = 9$, we read it as 'the square of 3 and (– 3) is 9'. For square root, we read it as '3 and (– 3) are the square roots of 9'.



Therefore every natural number has two square roots. One is negative and the other is positive, we take only positive root at this stage.

The symbol of radical sign '$\sqrt{\ }$' is used to show square root of a number. The square root of 9 can be written as $\sqrt{9}$ where 9 is called the radicand.

In general square root is used to solve the equation of type $x^2 - y = 0$ or $x^2 = y$ when y is called the square of x and x is called the square root of y.

5.2.2 Find square root by division method and factorization method of natural number, fraction and decimals.

There are two ways to find square root of a number i.e. by factorization method and by division method.

(i) Square root by factorization method:

Example 1: Find square root of the following by factorization method.

(i) 256 (ii) 3969

(i) **Solution:** First of all we find the prime factorization of 256.

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Now $\sqrt{256} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

$= 2 \times 2 \times 2 \times 2$ (Pairing factors)

$= 16$

Therefore $\sqrt{256} = 16$

Verification: $16 \times 16 = 256$

(ii) Solution: First we find the prime factorization of 3969.

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

$3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$

Now $\sqrt{3969} = \sqrt{3 \times 3 \times 3 \times 3 \times 7 \times 7}$

or $\sqrt{3969} = 3 \times 3 \times 7$ (Pairing factors)

$= 63$

Therefore, $\sqrt{3969} = 63$

Verification: $63 \times 63 = 3969$

Example 2:

Find square root of the following fractions by factorization method.

(i) $\dfrac{64}{81}$ (ii) $2\dfrac{113}{256}$

(i) Solution: We find the square root of the numerator and denominator separately as:

Finding the prime factorization of 64 and 81, we get:

Numerator: $64 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$

Denominator: $81 = \underline{3 \times 3} \times \underline{3 \times 3}$

$$\sqrt{\frac{64}{81}} = \frac{\sqrt{64}}{\sqrt{81}} = \frac{\sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}}{\sqrt{\underline{3 \times 3} \times \underline{3 \times 3}}}$$

$$= \frac{2 \times 2 \times 2}{3 \times 3} = \frac{8}{9}$$

Numerator

2	64
2	32
2	16
2	8
2	4
2	2
	1

Denominator

3	81
3	27
3	9
3	3
	1

(ii) Solution: $2\dfrac{113}{256} = \dfrac{625}{256}$ (by changing the mixed fraction into improper fraction.)

Finding prime factorization of 625 and 256, we get:

Numerator: $625 = \underline{5 \times 5} \times \underline{5 \times 5} = 5^2 \times 5^2$

Denominator: $256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$

Numerator

5	625
5	125
5	25
5	5
	1

Denominator

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

So, $\sqrt{2\frac{113}{256}} = \sqrt{\frac{625}{256}}$

or $\sqrt{2\frac{113}{256}} = \frac{\sqrt{625}}{\sqrt{256}} = \frac{\sqrt{\underline{5 \times 5} \times \underline{5 \times 5}}}{\sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}}$

$= \frac{5 \times 5}{2 \times 2 \times 2 \times 2}$

or $\sqrt{2\frac{113}{256}} = \frac{25}{16} = 1\frac{9}{16}$

Example 3: Find square root of the following decimals by factorization method.

(i) 1.44 (ii) 0.81

(i) Solution: $1.44 = \frac{144}{100}$ (By converting decimal into common fraction).

Numerator

2	144
2	72
2	36
2	18
3	9
3	3
	1

Denominator

2	100
2	50
5	25
5	5
	1

Now $\sqrt{1.44} = \sqrt{\frac{144}{100}} = \frac{\sqrt{144}}{\sqrt{100}}$

or $\sqrt{1.44} = \frac{\sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}}}{\sqrt{\underline{2 \times 2} \times \underline{5 \times 5}}} = \frac{2 \times 2 \times 3}{2 \times 5} = \frac{12}{10}$

or $\sqrt{1.44} = 1.2$ (By converting into decimal).

(ii) Solution: $0.81 = \frac{81}{100}$

Numerator

3	81
3	27
3	9
3	3
	1

Denominator

2	100
2	50
5	25
5	5
	1

$\sqrt{0.81} = \sqrt{\frac{81}{100}} = \frac{\sqrt{81}}{\sqrt{100}}$

or $\sqrt{0.81} = \frac{\sqrt{\underline{3 \times 3} \times \underline{3 \times 3}}}{\sqrt{\underline{2 \times 2} \times \underline{5 \times 5}}} = \frac{3 \times 3}{2 \times 5}$

or $\sqrt{0.81} = \frac{9}{10} = 0.9$

(ii) Square root by division method:

This method is very useful to find square root of numbers. First of all the digits of the radicand must be paired. Let us learn to find the square root of perfect square by division method of natural numbers, fractions and decimals with the help of following examples.

Example 1: Find square root of the following natural numbers by division method.

(i) 256 (ii) 1024

(i) Solution:

Step 1: Start pairing the digits by putting a bar over each pair from ones place and so on (In case of odd number of digits the digit on extreme left will be left alone and considered as a pair itself as $\hat{2}\overline{56}$).

Step 2: Consider the left most pair, i.e. '2' find the largest number whose square be either 2 or very near but less than 2 which is '1'.

Step 3: Subtract square of $1(1^2 = 1 \times 1 = 1)$ from the pair i.e, $2 - 1 = 1$ and bring down the next pair which is '56'.

Step 4: Add divisor '1' to itself and make it the digit at left most place of the new divisor.

Step 5: Again find the largest number whose multiplication at ones with the divisor must be equal to or less than that of new dividend (156).

i.e. $\boxed{26 \times 6 = 156}$

When remainder becomes zero and all digits of given number are used then process stops.

	16
1	$\hat{2}\overline{56}$
+ 1	−1
26	156
+ 6	−156
32	0

Step 6: Quotient is the required square root in division method which can be verified by squaring.

Hence, $\sqrt{256} = 16$

(ii) Solution:

	32
3	$\overline{10}\ \overline{24}$
+ 3	−9
62	124
+ 2	−124
64	0

So, $\sqrt{1024} = 32$

Example 2: Find square root of $1\dfrac{1089}{1936}$ by division method.

Solution:

As $\quad 1\dfrac{1089}{1936} = \dfrac{3025}{1936}$

Numerator: $\overline{30}\ \overline{25}$

$$\begin{array}{r|l} & 55 \\ \hline 5 & \overline{30}\ \overline{25} \\ +5 & -25 \\ \hline 105 & 525 \\ +5 & -525 \\ \hline 110 & 0 \end{array}$$

Denominator: $\overline{19}\ \overline{36}$

$$\begin{array}{r|l} & 44 \\ \hline 4 & \overline{19}\ \overline{36} \\ +4 & -16 \\ \hline 84 & 336 \\ +4 & -336 \\ \hline 88 & 0 \end{array}$$

Hence, $\quad \sqrt{1\dfrac{1084}{1936}} = \sqrt{\dfrac{3025}{1936}} = \dfrac{\sqrt{3025}}{\sqrt{1936}} = \dfrac{\cancel{55}^{\,5}}{\cancel{44}_{\,4}} = \dfrac{5}{4} = 1\dfrac{1}{4}$

So, $\quad \sqrt{1\dfrac{1089}{1936}} = 1\dfrac{1}{4}$

Example 3: Find square root of the following decimals by division method.

Solution: (i) 10.24 (ii) 249.64 (iii) 0.0441

(i)

$$\begin{array}{r|l} & 3.2 \\ \hline 3 & \overline{10}.\overline{24} \\ +3 & -9 \\ \hline 62 & 124 \\ +2 & -124 \\ \hline 64 & 0 \end{array}$$

(ii)

$$\begin{array}{r|l} & 15.8 \\ \hline 1 & \hat{2}49.64 \\ +1 & -1 \\ \hline 25 & 149 \\ +5 & -125 \\ \hline 308 & 2464 \\ +8 & -2464 \\ \hline 316 & 0 \end{array}$$

(iii)

$$\begin{array}{r|l} & 0.21 \\ \hline 2 & 0.\overline{04}\ \overline{41} \\ 2 & -4 \\ \hline 41 & 041 \\ +1 & -41 \\ \hline 42 & 0 \end{array}$$

Hence $\sqrt{10.24} = 3.2$ So, $\sqrt{249.64} = 15.8$ So, $\sqrt{0.0441} = 0.21$

Note: Pairing in decimals, start from either sides of the decimal point, '0' can be placed to complete the pair of the decimal part.

EXERCISE 5.2

A. Find square root of each of the following.

(1)	25	(2)	$(4)^2$	(3)	81	(4)	$(36)^2$
(5)	a^2	(6)	y^2	(7)	$(49)^2$	(8)	64

B. Find square root by factorization and division methods.

(1)	676	(2)	169	(3)	484	(4)	961
(5)	4900	(6)	1089	(7)	1600	(8)	2304
(9)	3136	(10)	1681	(11)	1369	(12)	2025

C. Find the square root of the following fractions by factorization and division method.

(1)	$\frac{144}{225}$	(2)	$\frac{324}{441}$	(3)	$\frac{1225}{9801}$	(4)	$1\frac{25}{144}$
(5)	$1\frac{48}{121}$	(6)	$3\frac{13}{36}$	(7)	$\frac{961}{1681}$	(8)	$\frac{1024}{1225}$
(9)	$2\frac{217}{576}$	(10)	$5\frac{55}{169}$	(11)	$1\frac{63}{81}$	(12)	$3\frac{325}{900}$

D. Find the square root of the following decimals by factorization and division methods.

(1)	3.24	(2)	4.41	(3)	5.29	(4)	7.29
(5)	6.25	(6)	37.21	(7)	7.84	(8)	10.24
(9)	30.25	(10)	100.00	(11)	33.64	(12)	34.81

5.2.3 Solve real life problems involving square roots.

In our daily life, there are so many problems, in which we use square root (directly or indirectly) to solve them. Let us` consider the following examples.

Unit 5 SQUARE ROOT OF POSITIVE NUMBER

Example: Students of class VII collected money for picnic. Each student contributed as much rupees as the number of students present. If Rs. 4761 is the total collection, find the amount paid by each.

Solution: Total amount = 4761 rupees

In this case amount paid by each student = number of students

= square root of 4761

(Division Method)

	69
6	$\overline{47}\,\overline{61}$
+ 6	− 36
129	1161
+ 9	− 1161
138	0

3	4761
3	1587
23	529
23	23
	1

(Prime factorization Method)

$$\sqrt{4761} = \sqrt{3\times 3\times 23\times 23}$$
$$= 3\times 23$$
$$= 69$$

Hence, each student contributed Rs 69.

Example 2: There are 529 students in a school. All the students are standing in rows. There are as many rows, as the number of students standing in each row. Find the number of rows.

Solution:

Total number of students = 529

There are as many rows, as the number of students in each row.

Therefore we can find the number of rows, by finding the square root of the total number of students.

Here $\sqrt{529}$ = 23

So, there are 23 rows and in each row there are 23 students.

	23
2	$\hat{5}\,\overline{29}$
+ 2	− 4
43	129
+ 3	129
46	0

Example 3: There are 1225 plants arranged in as many rows, as there are plants in each row. Find the number of plants in each row.

Solution: Total number of plants = 1225 plants.

Number of rows = Number of plants in each row.

Therefore, the number of plants in each row = $\sqrt{1225}$ = 35.

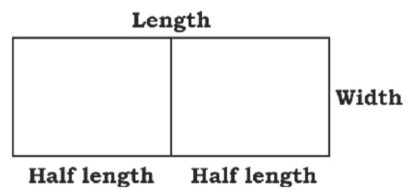
Thus, there are 35 rows of plants and in each row there are 35 plants.

	35
3	$\overline{12}\,\overline{25}$
+3	− 9
65	325
+5	− 325
70	0

Example 4: The length of a rectangular ground is twice its width. If its area is 96800 square metres, find length of the rectangular ground.

Solution:

If the ground is divided in two equal parts, each will be a square.



The area of

each square $= \dfrac{96800}{2}$

$= 48400$ square metres.

Length of each square $= \sqrt{\text{area of square}}$

$= \sqrt{48400}$

$= 220$ m

Hence length of the ground $= 2 \times$ length of square

$= 2 \times 220 = 440$ m.

	220
2	$\hat{4}\ \overline{84}\ \overline{00}$
+ 2	– 4
42	84
+ 2	– 84
440	000
+ 0	– 000
440	0

EXERCISE 5.3

1. 784 chairs are arranged in an examination hall in such a way that the number of chairs in each row is the same as the number of rows. Find the number of rows. Also find the number of chairs in each row.
2. The area of a square field is 240.25 square metres. Calculate the length of its side.
3. The length of a room is three times of its width and the area is 720.75 square are metres. Find its length and width.
4. There are 2116 packets of biscuits packed in a carton. There are as many rows of biscuits as there are biscuits in each row. Find the number of rows of biscuits.
5. School function is to be attended by 1681 students. Find the number of chairs required for seating arrangement in such a way that equal number of students sit in a row as well as in column.

6. There are 625 trucks. All these trucks are standing in rows. There are as many rows, as there are trucks in each row. Find the number of rows and the number of trucks in each row.

7. A drill teacher arranged 1024 students to stand in rows. There are as many rows as there are students in each row. How many number of rows are there and what is the number of students in each row?

8. The length of a rectangular field is thrice as its width. If the area of the field is 995.9052 square metres. Find its width and length.

9. Find the length of boundary of a square shaped school whose area is 3387.24 square metres.

10. In a sugar factory 15,198 bags of sugar are to be arranged into the shape of a solid square. But it is found that 69 bags are in excess. How many bags are arranged in a row?

REVIEW EXERCISE 5

1. Give short answers of the following.

(i) What are the different ways to find the square root of a number?

(ii) What is a perfect square number? Give examples.

(iii) Describe any two properties of perfect square of a number. Give it examples.

2. Fill in the blanks.

(i) The square of 31 is __________.

(ii) The square root of 49 is __________.

(iii) 1, 4, 9, 16,.. are called __________ numbers.

(iv) 144 is the __________ of 12.

(v) $\sqrt{\frac{25}{36}}$ = __________ (vi) Square of 1 is __________

3. Write "T" for true and "F" for false.

(i) Prime factors of 235 are 2, 3 and 5.

(ii) $19^2 = 361$ (iii) The square of an odd number is even.

(iv) $\frac{25}{49} > \frac{5}{7}$ (v) $(4.5)^2 > 4.5$ (vi) $0.7 < (0.7)^2$

4. Choose the correct answer.

(i) If $p = q^2$ then q is called __________ of p.

(a) square root (b) square

(c) prime factor (d) radicand

(ii) The numbers whose squares are equal to their square roots are __________.

(a) 0 and 2 (b) 1 and 3 (c) 0 and 1 (d) 2 and 3

(iii) The perfect square number is __________.

(a) 14 (b) 15 (c) 16 (d) 17

5. Find square root of the following:

(i) 729 (ii) 7056 (iii) 26569 (iv) 42025

(v) $5\frac{71}{121}$ (vi) $\frac{2116}{9604}$ (vii) $1\frac{984}{14641}$ (viii) 0.0256

(ix) 344569 (x) $\frac{6400}{9801}$ (xi) $5\frac{41}{64}$ (xii) 131.1025

6. The area of a square shaped park is 12100 square metres. Find the length of its side.

7. A floor of a square shaped room has an area of 144 square metres. Find the length of the floor.

8. The area of a rectangular field is 7688 square metres. Find the width when length of the field is twice of its width.

9. The length of a window is thrice its width. Find the length and width of the window when its area is 18.75 square metres.

10. Najeeb spent Rs 900 in the month of June in such a way that he spent as many rupees in a day as the number of days in the month. How many rupees did he spend in a day?

Unit 5 SQUARE ROOT OF POSITIVE NUMBER

SUMMARY

- The numerical value which we get by multiplication of a number with itself is called its square.
- Perfect square is a positive number which is the square of any number.
- Prime factors of a perfect square are always in the pairs.
- The square of an even number is even and square of an odd number is odd.
- The square of a proper fraction is less than itself.
- The square of a decimal less than 1 is smaller than the given decimal.
- Square root is a number which when multiplied by itself, gives the original number.
- The process of finding the square root is the reverse operation of squaring a number.
- "$\sqrt{\ }$" is used to show square root. It is called radical sign.
- The number inside radical sign is called radicand.
- We can find the square root of a mixed fraction, by converting it into an improper fraction.
- Let x, y be any two numbers then:

  (i) $\sqrt{x \times y} = \sqrt{x} \times \sqrt{y}$ (ii) $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
- In the equation $x^2 = y$, y is called the square of x and x is called the square root of y.
- There are two ways to find square root of a number; factorization method and division method.
- Square root of a decimal can be found by changing it into a common fraction.

Unit 6

DIRECT AND INVERSE VARIATION

6.1 CONTINUED RATIO

6.1.1 Define continued ratio and recall direct and inverse proportion.

We know that ratio is the comparison of two or more quantities for example ratio of ages of Danish and Rafay is 4:1. Similarly the ratio of ages of Bismah ,Maira and Umaima is 2:3:5, it is called continued ratio.

Continued ratio of three quantities is in fact the combination of two ratios with one common quantity with same size.

Let us see how two ratios combine to make continued ratio in the following example.

Ratio of ages of Tanveer and Taha is 2:5

The ratio of ages of Taha and Rafay is 5:7

Here the common quantity is age of Taha with size 5

So the continued ratio of their ages will be

Tanveer : Taha : Rafay = 2:5:7

So continued ratio of three quantities can be defined as:

> If two ratios containing A, B and C are A : B = $x : y$ and B : C = $y : z$ then their continued ratio is A : B : C = $x : y : z$

Example 1: Find the continued ratio A: B: C if

A : B = 5 : 6 and B : C = 6 : 7

Solution: Here A: B = 5 : 6

and B : C = 6 : 7

Since the size of the common quantity B is same in both ratios i.e 6

Therefore A : B : C = 5 : 6 : 7

In case the size of common quantity is different, we can find the continued ratio as mentioned in the following example.

Example 2: Find the continued ratio X : Y : Z, if X : Y = 6 : 7 and Y : Z = 8 : 9

Solution: **First Method**

First of all we find the product of the sizes of common quantity Y which is 56, Now we make the size of Y same (i.e 56) in each ratio.

Here X : Y = 6 : 7 and Y : Z = 8 : 9

or $\frac{X}{Y} = \frac{6}{7}$ or $\frac{Y}{Z} = \frac{8}{9}$

$= \frac{6 \times 8}{7 \times 8} = \frac{48}{56}$ $= \frac{8 \times 7}{9 \times 7} = \frac{56}{63}$

Hence X : Y : Z = 48 : 56 : 63

Second Method

$$\begin{array}{ccccc}
X & : & Y & : & Z \\
6 & & 7 & & \\
 & & 8 & & 9 \\
\hline
(6\times8) & : & (7\times8) & : & (7\times9) \\
48 & : & 56 & : & 63
\end{array}$$

Hence X : Y : Z = 48 : 56 : 63

We can also find continued ratio of four quantities as explained below:

Example 3. Find A : B : C : D if A : B = 2 : 3 , B : C = 4 : 5 and C : D = 2 : 7

Solution:

$$\begin{array}{ccccccc}
A & : & B & : & C & : & D \\
2 & : & 3 & & & & \\
 & & 4 & : & 5 & & \\
\hline
(2\times4) & & (3\times4) & & (3\times5) & & \\
8 & : & 12 & : & 15 & & \\
 & & & & 2 & : & 7 \\
\hline
(8\times2) & & (12\times2) & & (15\times2) & & (15\times7) \\
16 & & 24 & & 30 & & 105
\end{array}$$

Hence A : B : C : D = 16 : 24 : 30 : 105

Example 4. If the ratio of pocket moneys of Maryam and Bismah is 7:9 and that of Maryam and Hifza is 10:13. Find the continued ratio of their pocket moneys.

Solution :

Maryam	:	Bismah	:	Hifza
7	:	9		
			:	
10				13
(7×10)		(9×10)		(7×13)
70	:	90	:	91

So the required continued ratio is 70 : 90 : 91

Proportion: In previous class we have learnt about proportion and its types ; Let us recall it.

Proportion is the equality of two ratios. For example 2:3=4:6 or 2:3::4:6 is a proportion.

In proportion $a : b = c : d$, the quantities a, d are called ***extremes*** and b, c are called ***means***.

Remember that:

$$\boxed{\text{Product of means = Product of extremes}}$$

Example 1. Find x if $2 : 9 = x : 36$

Solution: We have,

$$\overbrace{2:\underbrace{9 = x}:36}$$

As, product of means = product of extremes

So $9 \times x = 2 \times 36$

or $\dfrac{9\times x}{9} = \dfrac{2\times 36}{9} = \dfrac{2\times \cancel{36}^{\,4}}{\cancel{9}_{\,1}} = 2 \times 4 = 8$

Thus $x = 8$

There are two types of proportion.

(i) Direct proportion (ii) Inverse proportion

Unit 6 DIRECT AND INVERSE VARIATION

Direct Proportion: If two quantities are so related to each other that increase or decrease in one causes corresponding increase or decrease in other.Then they are in direct proportion.

Inverse Proportion: If two quantities are so related to each other that increase or decrease in one causes corresponding decrease or increase in other.Then they are in inverse proportion.

Division of a quantity into a given ratio.

If a quantity is to be divided into a given ratio, then we use the following formula to find the part quantity or share:

$$\text{Share} = \frac{\text{Component of ratio} \times \text{Total quantity}}{\text{Sum of ratios}}$$

Example 1: Divide 180 rupees among A , B , C in the ratio 2 : 3 :4

Solution: Here

Total quantity is 180 rupees

Sum of ratio is = 2+3+4=9

$$\text{Share of A} = \frac{2 \times \cancel{180}^{20}}{\cancel{9}_1} = 2 \times 20 = 40 \text{ rupees}$$

$$\text{Share of B} = \frac{3 \times \cancel{180}^{20}}{\cancel{9}_1} = 3 \times 20 = 60 \text{ rupees}$$

$$\text{Share of C} = \frac{4 \times \cancel{180}^{20}}{\cancel{9}_1} = 4 \times 20 = 80 \text{ rupees}$$

So the amount of shares of A , B and C are 40 rupees , 60 rupees and 80 rupees respectively.

EXERCISE 6.1

1. Find A : B : C if

(i) A : B = 2 : 5 and B : C = 5 : 4

(ii) A : B = 3 : 7 and B : C = 8 : 9

(iii) A : B = 10 : 11 and B : C = 20 : 21

2. Find X : Y : Z if

(i) X : Y = 3 : 5 and X : Z = 4 : 9

(ii) X : Y = 5 : 7 and X : Z = 11 : 13

(iii) X : Z = 3 : 5 and Y : Z = 7 : 8

3. Find A : B : C : D if

(i) A : B = 2 : 3 , B : C = 4 : 5 and C : D = 7 : 10

(ii) A : B = 4 : 3 , B : C = 5 : 7 and A : D = 7 : 9

4. Find x if

(i) $x : 2 = 5 : 6$ (ii) $3 : x = 9 : 7$ (iii) $4 : 5 = x : 7$

5. The ratio of weights of Shazia and Nazia is 5 : 9 and that of Nazia and Madhia is 8 : 11. Find the continued ratio of their weights.

6. Divide 2000 rupees among Saleem, Irfan and Imran in 2 : 3 : 5.

7. The ratio of marks obtained by three friends in an annual examination is as under:

Marks of Akhtar and Aziz are in 3 : 4 and Aziz and Anis are in 5 : 7. Find the continued ratio of their marks.

8. The ratios in the property of a family members are as under: The ratio of property of Father and Mother is 5 : 2, Mother and Daughter is 3 : 4 and Daughter and Son is 6 : 7. Find their continued ratio in property.

9. An amount of Rs 27,400 is used for utility bills of a person; electricity, phone, gas and Water. The ratio among electricity and phone is 5 : 2, phone and gas is 4 : 3 but gas and water is 7:6. Find the actual amount of bills.

10. A student purchased a copy of the books of Mathematics, Science, English and Urdu for class VII for an amount of Rs 675. The ratio among the prices of books is as under:

English: Urdu is 2:3, Urdu : Science is 2:5 and

Science: Mathematics is 3:4. Find the price of each book.

6.1.2 Solve real life problems (involving direct and inverse proportion) using unitary method and proportion method.

Now we learn how to solve real life problems of proportion using unitary method and proportion method as explained in the following examples.

Example 1: If the cost of 10 similar books is 500 rupees, find the cost of 15 books.

Solution: Proportion Method

Let the cost of 15 books be x rupees

Books	Cost (rupees)	
10 ↓	500 ↓	
15	x	(Direct Proportion)

$$\overbrace{10:\underbrace{15=500}:x}$$

As product of means = product of extremes

So, $10\times x = 15\times 500$

or $\dfrac{10\times x}{10}=\dfrac{15\times 500}{10}$

or $x = 750$

So the required cost is Rs 750

Unitary Method

The cost of 10 books $= 500$ rupees

The cost of 1 book $= \dfrac{500}{10}$ rupees (Less, so divide)

$= 50$ rupees

The cost of such 15 books $= 15\times 50$

$= 750$ rupees.

So the required cost of 15 books is Rs 750

Example 2: Three identical pipes can fill a water tank in 10 hours. In how many hours, 5 such pipes, will take to fill the same tank.

Solution: **Proportion Method**

Let the required number of hours be x

Pipes	Hours	
3 $\downarrow$	10 $\uparrow$	
5	x	(Inverse Proportion)

We have

$$\overbrace{3:\underbrace{5=x}:10}$$

As product of means = product of extremes

So $5 \times x = 3 \times 10$

or $\dfrac{5 \times x}{5} = \dfrac{3 \times 10}{5}$

or $x = 6$

So, the required number of hours is 6

Unitary Method

3 pipes can fill the tank in 10 hours

So 1 pipe can fill the tank in 3×10 hours. (As fewer pipes require more time)

Hence, 5 pipes can fill the tank in $\dfrac{3 \times 10}{5}$ hours (As more pipes require fewer hours)

$= \dfrac{3 \times 10}{5} = \dfrac{3 \times \cancel{10}^{2}}{\cancel{5}_{1}}$ hours

= 6 hours

EXERCISE 6.2

Solve the following problems by unitary method and proportion method.

1. The price of 15 chairs is 4500 rupees. Find the price of such 20 chairs.

2. Six identical pipes can fill a water tank in 3 hours .How many such pipes can fill the same tank in 2 hours.

3. If 5 persons can consume the food provisions in 8 days. How many persons can consume the same food provisions in 4 days.

4. The cost of 4 kilograms of mangoes is 300 rupees. Find the cost of 10 kilograms of mangoes.

5. A student reads 30 pages of a book in 4 days. How many pages of same book will he read in 10 days?

6. 20 soldiers can consume a food stock in 8 days. How many soldiers will consume the same food stock in 5 days?

7. If 4 workers complete a job in 8 hours. Find the time in hours to complete the same job by 6 workers.

8. Find the cost of 18 kg of potatoes if 6 kg of potatoes cost Rs 105.

9. If $\frac{4}{7}$ of a piece of metal weighs 28 kg, find the weight of $\frac{8}{9}$ of the same piece of metal.

10. The expenses of 35 guests in a party are Rs 5250. Find the expenses of 85 guests.

6.2 TIME, WORK AND DISTANCE

When we solve the problems related to time and work, it is observed that:

(i) Time is directly proportional to work.

Because more work takes more time and less work takes less time.

(ii) Number of workers is inversely proportional to the time.

Because more workers take less time to complete the job. And less number of workers needs more time to complete the work.

6.2.1 Solve real life problems related to time and work using proportion.

Let us learn to solve real life problems related to time and work using proportion.

Unit 6 DIRECT AND INVERSE VARIATION

Example 1. Ten persons can complete $\frac{1}{2}$ of a work in a certain time. How many persons can complete $\frac{3}{4}$ of the same work in the same time.

Solution: Let the required number of persons be x

$$\begin{array}{cc} \text{Persons} & \text{Work} \\ 10 \downarrow & \frac{1}{2} \downarrow \\ x & \frac{3}{4} \end{array}$$

(Direct Proportion)

We have

$$10 : x = \frac{1}{2} : \frac{3}{4}$$

As product of means = product of extremes

So, $\frac{1}{2} \times x = 10 \times \frac{3}{4}$

or $\frac{x}{2} = \frac{30}{4}$

or $4x = 60$

or $\frac{4x}{4} = \frac{60}{4}$

or $x = 15$

So, the required number of persons is 15

Example 2. Six persons can complete a work in 7 days .How many persons can complete the same work in 6 days.

Solution : Let the required number of persons be x

$$\begin{array}{cc} \text{Persons} & \text{Days} \\ 6 \downarrow & 7 \uparrow \\ x & 6 \end{array}$$

(Inverse Proportion)

We have

$$\overbrace{6 : \underbrace{x = 6}_{} : 7}^{}$$

As product of means = product of extremes

So $6 \times x = 6 \times 7$

or $\dfrac{6 \times x}{6} = \dfrac{6 \times 7}{6}$

or $x = 7$

So, the required number of persons is 7

EXERCISE 6.3

1. 20 workers can complete half of work in a certain time. How many workers are required to complete three-fourth of the work in the same time?

2. If 6 labourers complete a work in 10 days .How many labourers are required to complete the same work in 15 days?

3. If 5 workers can complete a work in 8 days .How many workers are required to complete the same work in 4 days?

4. If three masons construct a wall in four days. In how many days, five such masons can construct the wall.

5. If 5 men complete one third of a work. How much work will be completed by 7 men in the same time?

6. Twelve workers are needed to build a bungalow in 64 days. How many days will 18 workers, working at the same speed, take to build the same bungalow.

7. A consignment of fodder lasts for 75 days for 1500 sheep. Given that sheep consume the fodder at a constant rate, find:

(i) The number of sheep on equal consignment of fodder lasts for 90 days.

(ii) The number of days in equal consignment of fodder for 1200 sheep.

6.2.2 Find relation (i.e. Speed) between time and distance:

In daily life , we notice that distance increases or decreases proportionally with time.

Suppose a car covers 20 km in 10 minutes and 30 km in 15 minutes. If we find relation or ratio of distance with time.

We get

$$\frac{20}{10} = \frac{\cancel{20}^{2}}{\cancel{10}_{1}} = 2 \quad \text{and} \quad \frac{\cancel{30}^{2}}{\cancel{15}_{1}} = 2$$

We see the ratio is same and this ratio of distance to time is called speed

In general we write as:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Example 1. A car travels 15 *km* in 3 hours. Find its speed.

Solution: Here Distance = 15 *km* and time = 3 hours

We know that: $\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{\cancel{15}\,km}{\underset{1}{\cancel{3}}\,\text{h}} = 5\ km\ /\ h$

So, speed of the car is 5 $km\ /\ h$.

Example 2. The speed of a bus is 60 km/h .How much distance will it cover in 4 hours.

Solution: Here Speed = 60 km/h, Time taken = 4 hours and Distance =?

We know that:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{i.e. } 60\ km\ /\ h = \frac{\text{Distance}}{4\,h}$$

$$4\ h \times 6\ km\ /\ h = \text{Distance}$$

$$240\ km = \text{Distance}$$

So, the bus covers 240 *km* in 4 hours.

6.2.3 Covert units of speed (kilometre per hour into metre per second and vice versa).

We know that:

1 kilometre = 1000 metres and 1 hour = 3600 seconds

By using the above relation, we can find the relation between the units of speed i.e between km/h and m/s

The relation is: $1\ km\ /\ h = \dfrac{1000}{3600}\ m/s$

Rules for conversion of units of speed i.e from km/h to m/s and vice versa:

- To convert km/h into m/s, multiply with $\dfrac{1000}{3600}$
- To convert m/s into km/h, divide by $\dfrac{1000}{3600}$

Example 1. Convert $72\ km/h$ into m/s

Solution: $72\ km/h = 72 \times \dfrac{1000}{3600}\ m/s\ \dfrac{720\cancel{00}}{36\cancel{00}} = \dfrac{720}{36} = \dfrac{\overset{20}{\cancel{720}}}{\underset{1}{\cancel{36}}} = 20\ m/s$

Example 2. Convert $5000\ m/s$ into km/h

Solution: $5000\ m\ /\ s = 5000 \div \dfrac{1000}{3600}\ km/h = 5000 \times \dfrac{3600}{1000}\ km/h$

$$= \frac{\overset{5}{\cancel{5000}} \times 3600}{\underset{1}{\cancel{1000}}}$$

$$= 5 \times 3600 \times km/h = 18000\ km/h$$

6.2.4 Solve variation related problems involving time and distance:

Let us learn how to solve variation related problems involving time and distance as explained in the following examples.

Unit 6 DIRECT AND INVERSE VARIATION

Example 1. A vehicle travels 120 km in 2 hours. How much distance will it cover in 6 hours ?

Solution: Let the required distance be x kilometres

Kilometres		Hours		
120	$\downarrow$	2	$\downarrow$	
x		6		(Direct Proportion)

We have $\quad 120 : x = : 2 : 6$

As product of means = product of extremes

So, $\quad 2 \times x = 6 \times 120$

or $\quad \dfrac{\cancel{2}^{1} \times x}{\cancel{2}_{1}} = \dfrac{\cancel{6}^{3} \times 120}{\cancel{2}_{1}}$ or $1 \times x = 3 \times 120$

or $\quad x = 360$

So, the required distance is 360 km

Example 2. A car covers a certain distance in 5 hours with speed of 50 km/h .In what time will it cover the same distance with speed of 60 km/h ?

Solution: Let the required time be x hours

Speed		Hours		
50	$\downarrow$	5	$\uparrow$	
60		x		(Inverse Proportion)

We have $\quad 50 : 60 = : x : 5$

So, $\quad 60 \times x = 50 \times 5$

or $\quad \dfrac{\cancel{60}^{1} \times x}{\cancel{60}_{1}} = \dfrac{5\cancel{0} \times 5}{6\cancel{0}} = \dfrac{25}{6}$

or $\quad x = \dfrac{25}{6} = 4\dfrac{1}{6}$

So, the required time is $4\dfrac{1}{6}$ hours.

EXERCISE 6.4

1. Convert the following into m/s

(i) 50 km/h (ii) 75 km/h (iii) 80 km/h (iv) 50.5 km/h

2. Covert the following into km/h

(i) 30m/s (ii) 12m/s (iii) 42 m/s (iv) 25.5m/s

3. Find the speed of a motor bike which covers a distance of 80 km in 2 hours.

4. How much time will a van take to cover a distance of 180 km with speed of 60 km/h ?

5. If a cyclist covers a distance of 150 m in 30 seconds . How much distance will he cover in 45 seconds?

6. A bus takes 3 hours to reach its destination if speed is 60 km/h. In how much time will it cover the same distance with speed of 70 km/h?

7. A vehicle takes 3 hours to complete the distance of 210 *km*.

(i) What is its speed per hours

(ii) What is its speed in *km*/minute?

(iii) What is its speed in *m*/sec

(iv) What distance will it cover in $4\frac{1}{2}$ hours; with the speed of 70 km/h?

(v) What time will it take to complete the distance of 350 *km*?

REVIEW EXERCISE 6

1. Find $x : y : z$ if $x : y = 2 : 5$ and $x : z = 4 : 7$.

2. Find P : Q : R : S, if P : Q = 3 : 4, Q : R = 5 : 6 and R : S = 7 : 8.

3. The price of ten pens is Rs. 250. Find the price of such 17 pens.

4. Ten workers can complete a work in 7 days. How many workers can complete the same work in 5 days.

5. Find x if $x : 5 = 6 : 15$. 6. Find y if $7 : 8 = y : 24$

7. Covert 60 km/h into m/s. 8. Convert 105 m/s into km/h.

9. A car covers a distance of 30 km in 25 minutes. How much distance can it cover in 55 minutes? Also find its speed in

(i) km / h (ii) km / minute (iii) m / minute (iv) m / sec

10. Quantity of petrol in the fuel tanks of cars A, B and C is in the ratio 7 : 9 : 11. If total quantity of petrol consumed by these three cars is 162 litres; find the consumption of each car.

11. The cost of transporting goods is Rs 100 for a distance of 40 km. At the same rate;

(i) Find the cost of transporting goods for 150 km.

(ii) Find the distance if the cost of transporting is Rs 560.

SUMMARY

Ratio: The comparison of two similar quantities is called ratio.

Continued ratio: It is the combination of two ratios with a common quantity of same size.

Division of a quantity into a given ratio

We divide a quantity into given ratio by the formula.

$$\text{Share} = \frac{\text{Component of ratio} \times \text{Total quantity}}{\text{Sum of ratio}}$$

- **Proportion:** The equality of two ratios is called proportion.

  In Proportion,

  Product of means = product of extremes.

Unit 6 DIRECT AND INVERSE VARIATION

- A relation in which one quantity increases in the same proportion by decreasing the other quantity and vice versa is called the inverse proportion.
- Time is directly proportional to the work and the number of workers is inversely proportional to time.
- **Speed:** The ratio of distance to time is called speed. It can be calculated by the formula.

  $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

  The distance convered per unit time is called speed.
- To understand the relation between distance, speed and time, the formula is:

  Distance = Speed × Time

  Conversion of units of speed
- To convert km/h into m/s, multiply with $\frac{1000}{3600}$
- To convert m/s into km / h, divide by $\frac{1000}{3600}$

Think Tank: The Most Wonderful Knowledge.

Ask your friend to take any 3 digit number in the descending order of digits. Reverse it and then subtract it from the original number. Again reverse the result and add. You tell him, "your answer is 1089". He will be surprised!

$$\begin{array}{r} 732 \\ -\ 237 \\ \hline 495 \\ +\ 594 \\ \hline 1089 \end{array} \qquad \begin{array}{r} 321 \\ -\ 123 \\ \hline 198 \\ +\ 891 \\ \hline 1089 \end{array}$$

Similarly, try with other such numbers.

Unit 7

FINANCIAL ARITHMETIC

Introduction

In our daily life, we use different facilities provided by the state like police for security, government schools for education, government Hospitals for health etc. Government of the state provides such facilities by an amount which is collected from the citizen of the country and this amount is called **tax**.

7.1 Taxes

A fee charged (levied) by the government on a product, income or activity is called tax. If tax is levied directly on personal or corporate income, then it is called **direct tax**. If tax is levied on the price of goods or services, then it is called **indirect tax**.

The purpose of taxation is to finance government expenditure. One of the most important use of taxes is to finance public goods and services.

7.1.1 Property Tax and General Sales Tax (GST).

There are many types of taxes charged by the government, such as wealth tax, income tax, property tax, general sales tax etc.

Here we should study only the following two types of taxes:

(i) Property Tax (ii) General Sales Tax (GST)

(i) Property Tax

Property tax is a levy issued by a government on a person's real or personal property. The property is assessed to give it a value and then that value is taxed.

The amount of tax on a given property may change over time leased on a re-assessment of the property value.

Property tax payments are due annually in most cases.

It is generally paid at a flat rate of 2% but the tax rates vary, depending on the province.

In our province property tax is levied and collected under Sindh Urban Property Tax Act 1958.

Following properties are exempted from tax.

1. Government owned properties.
2. Residential building on land not over 120 square yards.

3. One residential flat of 600 square feet or less on any floor of a building.
4. Libraries, parks, playgrounds, place of worship, orphanages, cemeteries, crematorium etc.
5. Properties owned by widows, orphaned children and disabled individuals within some specified limit.
6. Properties listed as protected heritage under the Sindh Cultural Heritage Preservation Act 1994.

Example 1. Find the property tax on a property of Rs 1,400, 000 at the rate of 3%.

Solution: Worth of the property = Rs 1,400,000

Tax rate = 3%

Property tax = ?

Formula for finding property tax = $\dfrac{\text{Rate} \times \text{value of property}}{100}$

Property tax = 3% of Rs 1,400,000 = $\dfrac{3}{100} \times 1{,}400{,}000$

$= \dfrac{3 \times 1{,}400{,}000}{100}$

$= \dfrac{3 \times 1{,}400{,}0\cancel{00}}{\cancel{100}} = 3 \times 14000 = \text{Rs } 42000$

Thus the required property tax is Rs 42,000.

Example 2. A lady doctor paid Rs 9500 as a property tax at the rate of 2%. Find the worth of her property.

Solution: Property tax paid = Rs 9500

Tax rate = 2%

Worth of the property =?

2% of the worth of property = Rs 9500

1% of the worth of property = $\left(\dfrac{\text{Rs } 9500}{2}\right)$

100% of the worth of property= Rs $\left(\dfrac{9500 \times 100}{2}\right)$ = Rs $\dfrac{9500 \times \overset{50}{\cancel{100}}}{\underset{1}{\cancel{2}}}$

= Rs 9500×50 = Rs 475,000

Thus the required worth of lady doctor's property is Rs 475,000.

EXERCISE 7.1

A. Find property tax on the property at the rate mentioned below:

1. Property worth Rs 286,400, rate = 5%.
2. Property worth Rs 1,657,900, rate = 2%.
3. Property worth Rs 814,500, rate = 3%.
4. Property worth Rs 234,000, rate = 1.4%.
5. Property worth Rs 2,050,000, rate = 1.6%.

B. Find the worth of the property in the following when:

1. Tax paid on the property is Rs 25500 at the rate of 2%.
2. Tax paid on the property is Rs 35900 at the rate of 4%.
3. Tax paid on the property is Rs 4800 at the rate of 5%.
4. Tax paid on the property is Rs 8500 at the rate of 3.5%.
5. Tax paid on the property is Rs 11,700 at the rate 1.5%.

C. Find rate percentage of property tax when:

1. The property tax paid is Rs 30000 and the worth of the property tax paid is Rs 1,800,000.
2. The property tax paid is Rs 33000 and the worth of the property is Rs 3,300,000.
3. The property tax paid is Rs 12000 and the worth of the property is Rs 600,000.
4. The property tax paid is Rs 8676 and the worth of the property is Rs 433,800.
5. The property tax paid is Rs 12000 and the worth of the property is Rs 600,000.

D. Answer the following:

1. What is tax?
2. What is the purpose of taxation?
3. Explain property tax.
4. Mention any four properties which are exempted from property tax.

(ii) General Sales Tax (GST) or Sales Tax

A sales tax is a tax paid to the Government for the sales of certain goods and services. Usually seller is allowed to collect funds for the tax from consumer at the point of purchase.

Sellers some times itemize the tax separately from the price of the goods or services, or include it in the price (tax-intensive).
Often exemption is provided from general sales tax on certain goods or services.

7.1.2 Calculation of General Sales Tax (GST)

Since the year 2014 the rate of GST in Pakistan is 17%.
The GST is calculated as under:

$$\boxed{\begin{aligned}\text{Total GST} &= \text{Item Cost} \times \text{Rate of GST}\\ \text{So, Rate of GST} &= \text{Total GST Tax / Item Cost}\end{aligned}}$$

Example: The cost of a pair of shoes is Rs 2500. The seller charges 17% GST on the cost. Find GST levied.

Solution:

Cost of item = Rs 2,500
Rate of GST = 17%
Value of GST = Item Cost × 0.17
= Rs 2,500 × 0.17 = Rs 425

So, the GST charged is Rs 425.

(i) To Determine the Total Cost

In order to determine the total cost and tax inclusive of an item, the GST is added to the item cost.

So, $\boxed{\text{Total Item Cost} = \text{GST} + \text{Item Cost}}$

In the above example;

Item Cost = Rs 2,500
Value of GST = Rs 425
Therefore, the total cost of pair of shoes
= Rs 2,500 + Rs 425 = Rs 2,925

So, the total cost of shoes is Rs 2,925

$\boxed{\text{GST} = \text{Total Cost} - \text{Item Cost}}$

and $\boxed{\text{Item Cost} = \text{Total Cost} - \text{GST}}$

7.1.3 To Determine Item Cost when Total Cost and Rate of GST is known.

We know that GST is at the rate of 17% of the item cost and is included in total cost.

Let item cost = y

Since Rate of GST = 17% = 0.17

So, GST = $0.17 \times y = 0.1\,7y$

As we know that

$$\text{Item Cost} = \text{Total Cost} - \text{GST}$$

$$\text{Therefore } y = \text{Total Cost} - 0.17 \times y$$

$$\text{or Total Cost} = y + 0.17x = (1 + 0.17)\,y$$

$$\text{or } 1.17\,y = \text{Total Cost}$$

$$\text{or } y = \frac{\text{Total Cost}}{1.17}$$

Hence, $\boxed{\text{Item Cost} = \frac{\text{Total Cost}}{1.17}}$, where Rate of GST is 17%

Example 1: A washing machine is sold for Rs 15,912, inclusive of GST. Find the cost of washing machine excluding GST when rate of GST is 17%.

Solution:

We know that

$$\text{Item Cost} = \frac{\text{Total Cost}}{1.17}$$

In this problem:

Cost of washing machine = Item cost = ?

Total cost = Rs 15,912

Rate of GST = 17% = 0.17

So, Cost washing machine = Rs $\left(\frac{15{,}912}{1.17}\right)$ = Rs 13,600

Thus, the cost of washing machine (exclusive of tax) is Rs 13,600.

Example 2: A company pays Rs 380 as GST on the services of cost Rs 2,180. Find the rate of the GST.

Solution:

We know that: Rate of GST = $\frac{\text{GST}}{\text{Item Cost}}$

Here we have to find Rate of GST = ?

(GST) Paid = Rs 380

Item cost = Service cost = Rs 2,180

So, Rate of GST $= \frac{\text{Rs } 380}{\text{Rs } 2,180} = 0.17 = \frac{17}{100} = 17\%$

Thus the Company paid 17% GST.

Example 3: A Gas company charges Rs 3,212 as gas charges from consumers. Find the amount of GST at the rate of 17%, paid by the consumers.

Solution:

We know that

GST = Item Cost × 17%

Here GST = ?

Item cost = Gas charges = Rs 3,212

Rate of GST = 17%

So, GST = Rs 3,212 × 17% = Rs 3,212 × 0.17

= Rs 546.04 = Rs 546

Hence GST paid by the consumer is Rs 546.

Example 4: Mr. Adam paid Rs 595 as GST at the rate of 17% on the purchase of a shirt. Find the cost of the shirt exclusive of GST.

Solution:

We know that: GST = Item Cost × 17%

Here Cost of shirt = Item Cost = ?

GST = Rs 595

Rate of GST = 17% = 0.17

So, 595 = Cost of shirt × 0.17

or Cost of shirt $= \frac{\text{Rs } 595}{0.17} = \text{Rs } 3,500$

Thus the cost of shirt is Rs 3500.

Example 5: A person purchased some furniture items worth Rs 77,950 including GST. Find the actual cost of the furniture items and amount of GST at the rate of 17% paid by him.

Solution:

Here; Item Cost = Cost of Furniture = ?

Total Cost of Furniture items = Rs 77,950

Rate of GST = 17% = 0.17

Now, Cost of Furniture items $= \frac{\text{Total Cost}}{1.17} = \frac{77{,}950}{1.17}$

= Rs 66,623.93 = Rs 6,6624

Amount GST = 66624×0.17

= Rs 11,326.08 = Rs 11,326

Thus the toal cost of furniture items is Rs 66624 and the amount of GST is Rs 11326.

EXERCISE 7.2

1. The cost of a parker pen is Rs 875. Find the GST charged on it at the rate of 17%.
2. The cost of a mobile is Rs 4,850. Find the GST at the rate of 17%.
3. The total cost of cosmetic items is Rs 14976. If the actual cosmetic item cost is Rs 12,800, find the amount of GST.
4. A microwave costs Rs 15200. Rate of GST is 17% charged on it. Find the amount of GST and total cost of the microwave.
5. Abdullah purchased a three piece suit of cost Rs 7,800. Find the total cost and amount of GST at the rate of 17%.
6. A refrigerator is sold inclusive of 17% GST for Rs 35,375. Find the actual cost of the refrigerator.
7. A person purchased a dishwasher for Rs 61,000 inclusive 17% GST. Find the cost of dishwasher excluding GST. Also find the amount of GST.
8. Ms. Alia paid Rs 2,720 as GST at the rate of 17% on the purchase of a decoration piece. Find the cost of decoration piece exclusive of GST.

9. An electric company charges Rs 1547 GST on the consumption of electricity of the cost Rs 9,100. Find the rate of GST.
10. The cost of a shirt excluding tax is Rs 2,560, and the amount of GST on it is Rs 436. Find rate of GST.

7.2 PROFIT AND MARKUP

7.2.1 Explain profit and markup

Profit

Profit is the amount of gain received from any business, after deducting all cost and expenses from the total sale.

Profit is also referred as net profit or net earning.

So, profit is the difference between Selling Price (SP) and Cost Price (CP).

i.e $\boxed{\text{Profit} = \text{Selling Price} - \text{Cost Price}}$

Markup

In our daily life, we often borrow money from our friends and relatives to buy a thing that we repay them after a certain period. Some banks and retail organization also provide the same services and charge an additional amount called **markup.**

7.2.2 Find the rate of profit and markup per annum

Profit Rate

Profit rate is a profitability ratio that shows the relationship between gross profit and total net sales revenue. It is a common measure to evaluate the operational performance of the business.

In other words, it is the percentage of income that is **profit.**

$$\boxed{\text{Profit Rate} = \frac{\text{Gross Profit}}{\text{Net Sale}} \times 100}$$

Gross profit = Net Sale – Cost of goods

So, Selling Price = Cost Price + Cost Price × Profit Rate

= Cost Price (1 + Profit Rate)

$$\boxed{\text{SP} = \text{CP}\ (1 + \text{Profit Rate})}$$

and

$$\boxed{\text{Cost Price} = \frac{\text{Selling Price}}{1 + \text{Profit rate}}}$$

Example. Find the profit rate when gross profit is Rs 400 and net sale is Rs 1600.

Solution:

$$\text{Profit Rate} = \frac{\text{Gross Profit}}{\text{Net Sale}} \times 100$$

$$= \frac{400}{1600} \times 100$$

$$= \frac{\cancel{400} \times \cancel{100}^{25}}{\underset{1}{\cancel{\underset{\cancel{4}}{1600}}}} = 25$$

Thus Profit Rate = 25%

Markup of per annum

Usually, we calculate the markup as a percentage of the actual amount paid for the things. This is called the **markup rate** and paid amount itself is called the **principal.**

Example: Find the markup on a principal amount of Rs 50,000 for 2 years at the rate of 12%.

Solution:

$$\text{Markup} = \frac{P \times R \times T}{100} = \frac{50000 \times 12 \times 2}{100} = \frac{500\cancel{00} \times 12 \times 2}{\cancel{100}_1} = \text{Rs } 112{,}000$$

Activity 1. Find the missing quantities by using formula.

S.No.	Net Profit	S.P	C.P	Formula
(i)		Rs 1750	Rs 1600	Profit = S.P – C.P
(ii)	Rs 600		Rs 2000	S.P = C.P + Profit
(iii)	Rs 950	Rs 10,450		S.P – Profit = C.P

Activity 2. Find the missing quantities by formula.

S.P	C.P	Net Profit	Profit Rate	Formula
Rs 1000	Rs 800	Rs 200		Profit Rate = $\frac{\text{Profit}}{\text{S.P}} \times 100$
	Rs 1000	Rs 100	10%	S.P = C.P (1 + Profit rate)
Rs 2000	Rs 1600	Rs 400		Profit Rate = $\frac{\text{Profit}}{\text{C.P}} \times 100$
Rs 1200	Rs 1000		20%	
Rs 2750	Rs 2500	Rs 250		

Activity 3:

Find the missing quantities by using the formula.

S.No.	Markup	Principal	Time Period	Markup rate	Formula used
(i)	$\boxed{}$	Rs 500	2 years	12%	$MP = \frac{P \times R \times T}{100}$
(ii)	Rs 205	$\boxed{}$	1 years	8%	$P = \frac{MP \times 100}{R \times T}$
(iii)	Rs 528	Rs 1,650	10 years	$\boxed{}$	$MPR = \frac{MP \times 100}{P \times T}$
(iv)	Rs 350	Rs 3,500	$\boxed{}$	2.5%	$T = \frac{MP \times 100}{P \times R}$
(v)	$\boxed{}$	Rs 100,000	3 years	1.25%	$MP = \frac{P \times R \times T}{100}$
(vi)	Rs 1,050	$\boxed{}$	5 years	4.5%	$P = \frac{MP \times 100}{R \times T}$

7.2.3 (i) Solve real life problems involving profit

Consider the following examples:

Example 1: A person purchased a certain item from wholesaler for Rs 400. Then he sells it to a consumer for Rs 700. What is his profit? Also find his profit rate.

Solution:

We know that Profit = Selling Price – Cost Price

Here, Profit = Rs 700 – Rs 400 = Rs 300

Now Profit Rate $= \frac{\text{Profit}}{\text{Cost Price}} \times 100$

or Profit Rate $= \frac{300}{400} \times 100 = 75\%$

Thus the profit rate is 75% and his profit is Rs 300.

Example 2: A computer software retailers uses a profit rate of 45%. Find the selling price of an electronic computer game that costs the retailer Rs 1250.

Solution:

We know that

Selling price = Cost price (1 + Profit Rate)

Here, Selling price ≡ ?

Cost price Rs 1250

Profit Rate = 45% = 0.45

S.P = C.P (1 + Profit Rate)

So, Selling Price = Rs 1250 (1 + 0.45) = Rs (1250 × 1.45) = Rs 1812.50

Thus the Selling Price of the computer game is Rs 1812.50

Example 3: A shoes store uses a 40% profit rate on cost price. Find the cost of a pair of shoes that is sold for Rs 2,630.

Solution:

We know that

Selling price = Cost price (1 + Profit rate)

Here, Cost price = CP = ?

Selling price = Rs 2630

Profit Rate = 40% = 0.4

S.P = C.P (1 + Profit Rate)

So, 2630 = CP (1 + 0.4)

or $\text{C.P} = \text{Rs } \dfrac{263}{1.4} = \text{Rs } 1{,}878.57$

Hence the cost of a pair of shoes is Rs 1,879.

Remember: The profit is also calculated as a percentage of the original cost price.

$$\text{Profit Rate} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

Profit is used to determine the price in which the goods should be sold.

Thus there are in fact two different ways to analyze the cost and profit of a product or service in small business.

Example 4: A Tile company has Rs 1,500,000 of sales in the previous month. As well as sale returns (means the sale returned back by the consumer due to some reasons) of Rs 45,000, a cost of goods sold for Rs 650,000 and administrative expenses of Rs 470,000. Calculate the net profit percentage of the company.

Solution:

We know that: $\text{Profit Rate} = \dfrac{\text{Gross Profit}}{\text{Net Sale}} \times 100$

Here, Net Sale = Rs 1,500,000 (Sales) – Rs 45,000 (Sale returned)

= Rs 1,455,000

Gross Profit = Rs 1,455,000 (Net Sale) – [Rs 650,000 (Cost of goods) – Rs 470,000 (Administrative expenses)]

= Rs 335,000

Formula: Profit Rate = $\dfrac{\text{Gross Profit}}{\text{Net Sale}} \times 100$

So, Profit Rate = $\dfrac{335{,}000}{1{,}455{,}000} \times 100 \; \dfrac{\overset{67}{\cancel{335{,}000}} \times 100}{\underset{291}{\cancel{1{,}455{,}000}}} = \dfrac{6700}{291} = 23\%$

The profit percentage of Tile Company is 23%.

EXERCISE 7.3

1. A retailer pays to its whole-saler Rs,1860 for a Helmet and then he sells it to consumer for Rs 2,650. What is the markup rate and his profit.
2. A Shopkeeper sells a microwave oven for Rs 6700. Find th markup rate and his profit if the cost price of microwave oven is Rs 6250.
3. A retailer uses markup rate of 65% on the computer accessories. Find the selling price if the cost of a certain accessory is Rs 1520.
4. The cost price of a certain novel is Rs 1,100. Find selling price of the novel at 66% markup rate.
5. A Mobile Phone company uses a markup rate of 55% on the cost of each mobile phones that cost Rs 2,260. Find the selling price of the mobile phone.
6. A mobile phone company uses a 45% markup rate. Find the cost of a unit that sells for Rs 4785.
7. The markup rate on the selling price of a calculator that is Rs 1,230 is 50%. Find the cost price of the calculator.
8. A stationer uses a markup of 60% on each item that he sells. Rs 144. Find the cost price of the item.

9. A home-based small business brings in Rs 25000 a month in sales of baby clothing and accessories. The cost of goods sold including overhead add up to Rs 10550. Calculate the net profit percentage.
10. A certain business has Rs 2,500,000 of sales in one month. Sales returns of Rs 15,000. The goods sold were of cost 505,000 and overhead expenses of Rs 580,000. Calculate the profit rate percent.
11. A medicine company has 1,850,000 of sales in the previous month. The sale returns of Rs 85,000. The medicines sold were of cost 850,000. The administrative expenses are of Rs 550,000. Calculate the net profit percentage of the company.

7.2.3 (ii) Solve real life problems involving markup.

Suppose **'P'** is the principal amount, **'T'** is the time period and **'R'** is the markup rate. Then the amount of **markup** will be:

$$\text{Markup} = \frac{R\times P\times T}{100}$$

In the same way we can find:

$$\text{Principal Amount} = \frac{\text{Mark up} \times 100}{T \times R}, \quad \text{Rate of markup} = \frac{\text{Mark up} \times 100}{P \times T}$$

$$\text{and Time period} = \frac{\text{Mark up} \times 100}{P \times R}$$

Consider the following examples:

Example 1. A person borrowed a loan of Rs 280,000 from a bank at the markup rate of 12% per annum. Find the value of markup, if the time period is 2 years.

Solution:

Principal amount = Rs 280,000Markup rate = 12% annum

Time period (T) = 2 years Value of Markup = ?

Formula:

$$\text{Value of the markup} = \frac{R\times P\times T}{100} = \frac{12 \times 280{,}000 \times 3}{100} = \text{Rs } 67{,}200$$

Thus the value of the markup is Rs 67,200.

Example 2. Aslam borrowed a loan of Rs 450,000 from a retail organiser for 292 days at the rate of 10% per annum. Find the value of markup.

Solution:

Principal (P) = Rs 450,000, Markup rate (R) = 10% per annum, Markup = ?

Time period = 292 days = $\frac{292}{365}$ year = $\frac{4}{5}$ year

By using the formula: Markup= $\frac{R \times P \times T}{100}$

$$= \frac{10 \times 450{,}000 \times \frac{4}{5}}{100} = \text{Rs } \frac{\overset{2}{\cancel{10}} \times 450{,}0\cancel{00} \times 4}{\underset{1}{\cancel{100}} \times \underset{1}{\cancel{5}}} = \text{Rs } 36{,}000$$

Thus the value of markup is Rs 36,000.

Example 3. On a bank loan, calculate the markup on a principal amount of Rs 87,600 for 20 months at the rate of 12.5% per annum.

Solution:

Principal amount (P) = Rs 87600 Markup rate (R) = 12.5%

Time period (T) = 20 months = $\frac{20}{12}$ year = $\frac{5}{3}$ years

Markup = ?

By using the formula: Markup = $\frac{R \times P \times T}{100}$

$$\text{Markup} = \frac{12.5 \times \frac{5}{3} \times P}{100} = \frac{12.5 \times 5 \times \overset{292}{\cancel{87600}}}{\underset{1}{\cancel{100}} \times \underset{1}{\cancel{3}}} = 18250$$

Hence the markup is Rs 18,250

EXERCISE 7.4

1. Qasim bought a car is Rs 450,800 from a bank loan on instalments at the markup rate of 12% per annum. Find the sale price of the car, if the time period is 3 years.

2. Najma bought a golden ornament set for Rs 1,75500, on installments at the markup rate of 10% per annum. Find her sale price of the set if the time period is 9 months.

3. Shazia sold his mobile phone for Rs 16,900 and got 15% markup rate per annum. Find her markup where the period is 219 days.

4. If 15% markup on a colour T.v set is 8,500 for 1½ years. Find the cost price of the colour T.v set.

5. Find the missing quantities by using formula:

S. No.	Markup	Principal	Time period	Markup Rate
(i)	Rs ________	Rs 800,000	1 year	12%
(ii)	Rs 10000	Rs 500,000	________	10%
(iii)	Rs 6,600	________	2 years	11%
(iv)	Rs ________	Rs 200,000	1½ years	8%
(v)	Rs 800	Rs 4,000	2½ years	☐
(vi)	Rs 45,000	________	73 days	10%

7.3 ZAKAT AND USHR

Zakat and Ushr Ordinance 1980

In Pakistan, Zakat Councils are responsible for collecting and distributing Zakat and Ushr. The Ministry of Religious Affairs overseas the Zakat Councils. The collection and distribution of Zakat and Ushr system was enforced in 1980.

7.3.1 Define zakat and ushr

(i) Zakat

Zakat is one of the five pillars of Islam. It has been mentioned along with Salat in Quran.

Zakat is obligatory on every Sahib-e-Nisab muslim who possesses wealth in excess of Nisab that is 87.48 grams or 7.50 tola Gold; or 612.36 grams or 52.50 tola silver, excluding his needs for a complete lunar year.

The zakat is paid after a complete lunar year at the rate of 2.5%.

Zakat = Annual saving × 2.5%.

(ii) Ushr

Ushr is paid on compulsory basis at the rate of 5% (or 10%) of the produce by every land owner, grantee, allottee, lessee, lease-holder or land holder.

An individual farmer is exempted from ushr if he is a 'mustahiq' or his produce is less than 'Five Wasaqs or 948 kg (approximately) of wheat or its equivalent in value'.

If the farm or garden is watered naturally by rain or spring water or river or stream water, tenth part or 10% is obligatory, and if farm or garden is irrigated by artificial means like well, tube well, canals etc. twentieth part or 5% is obligatory.

(iii) Distribution of Zakat and Ushr

Zakat is paid:

- to the poor and needy.
- to those who are in bondage and debt and for wayfarer.
- to those who are employed to administer the funds.

7.3.2 Solved problems related to zakat and ushr.

Example 1: After a whole year a business man has saved Rs 80,000 in cash. He has a cash in Bank of Rs 600,000. Find the Zakat due on him.

Solution:

(i) Saving in cash = Rs 80,000

(ii) Cash in bank = Rs 600,000

Total saving of the businessman = 80,000 + 600,000

= Rs 680,000

Amount of Zakat due on him = Rs 680,000 $\times$ 2.5%

$$= 680{,}000 \times \frac{2.5}{100} = \text{Rs } 17{,}000$$

the Zakat due on the businessman is Rs 17,000.

Example 2: A household woman has gold ornaments of value of Rs 500,000. She also saved an amount of Rs 25,000 from daily life expenses. Find the Zakat due upon her.

Solution:

Total amount = Rs 500,000 + Rs 25,000 = Rs 525,000

So, Zakat = Rs 525,000 $\times$ 2.5 %

$$= \text{Rs } 525{,}000 \times \frac{2.5}{100} = \text{Rs } 13{,}125$$

Thus, she has to pay Rs 13,125 as Zakat.

Example 3: Farah paid an amount of Rs 9,000 as Zakat. Find the saving on which she paid the Zakat.

Solution:

We know that: Amount of Zakat = Annual saving × 2.5%

$$9{,}000 = \text{Annual saving} \times 0.025$$

$$\text{Annual saving} = \frac{9{,}000}{0.025} = \text{Rs } 360{,}000$$

Thus she had an annual saving of Rs 360,000.

Example 4: A landlord produces 3,000 kg of wheat and vegetables of worth Rs 25,000. Find the amount of Ushr due upon him at the rate of 5% and the rate of wheat is Rs 30 per kg.

Solution:

So, Ushr = Total value of Produce × 5%

Value of 3,000 kg wheat at Rs 30 per kg

$$= \text{Rs } (3{,}000 \times 30) = \text{Rs } 90{,}000$$

$$\text{Total value of wheat and vegetable} = \text{Rs } 90{,}000 + \text{Rs } 25{,}000$$

$$= \text{Rs } 115{,}000$$

$$\text{Amount of Ushr} = \text{Rs } 115{,}000 \times 5\%$$

$$= \text{Rs } 115{,}000 \times \frac{5}{100} = 115{,}000 \times 0.05$$

$$= \text{Rs } 5{,}750$$

The landlord has to pay Rs 5,750 as Ushr.

EXERCISE 7.4

1. A person holds an amount of Rs 500,000 for the whole year. Is he sahib-e-nisab? If yes, find the amount of Zakat he has to pay.
2. A salaried person possess Rs 200,000 and 3 tolas of pure gold. Find the Zakat to be paid by him. Rate of gold is Rs 47,400 per tola.
3. A lady doctor possesses ornaments of worth Rs 500,000 and saved Rs 150,000 in the whole year. Find the amount of Zakat due upon her.

4. A business man paid Rs 20,500 as Zakat. Find his saving for the whole year.

5. A farmer pays Rs 10,000 as Ushr. Find the value of the crop he produced. Rate of Ushr is 5%.

6. A farmer produces 15,000 kg of sugarcane. Find his amount of Ushr at the rate of 5%. The rate of sugarcane is Rs 5.25 per kg.

7. Jumman exports oranges of his garden for Rs 5,000,000. Find the amount of Ushr at the rate of 5%.

8. Kamil produces 1,500 kg of wheat on one part of his land and 1,200 kg of rice on the other part of his land. Find the amount of Ushr at the rate of 5%. The rate of wheat is Rs 30 per kg and rate of rice is Rs 65 per kg.

REVIEW EXERCISE 7

1. The cost of a mobile phone is Rs 8,500. The seller charges 17% GST on the cost. Find sales tax levied.
2. An imported LCD TV is sold for Rs 55,800 inclusive of GST. Find the cost of LCD excluding tax. GST rate is 17%.
3. An electric company charges Rs 16,750 as electric charges from consumer. Find the amount of GST at the rate of 17%, paid by the consumer.
4. A person paid Rs 16,989 as GST at the rate of 17% on the purchase of room-set. Find the cost of room set exclusive of GST.
5. A business man gets loan of Rs 1,700,000 from a bank at the markup rate of 12.5% per annum. Calculate the markup for a period of 1½ years.
6. The markup paid to a bank by a lady doctor is Rs 2850. The period is 6 months and the markup rate is 12%. Find principal amount
7. A senior citizen deposits an amount of Rs 500,000 in a bank. Find the his markup when the markup rate is 10% and the time period is 2 ½ years.

8. A car dealer uses 35% markup rate on costs. Find the cost of a car that sells for Rs 1,051,000.

9. After a whole year a business man saved Rs 95,000 and has a stock of Rs 500,000. Find the zakat due on him.

10. A lady teacher has gold ornaments of value 400,000 rupees. Her annual is an amount of Rs 45,000. Find the Zakat due upon her.

11. A gentleman paid an amount of Rs 12,500 az Zakat. Find his amount on which he paid the Zakat.

12. A farmer produces 5,000 kg of wheat and vegetables of worth Rs 48,000. Find the amount of Ushr due upon him at the rate of 5% . The rate of wheat is Rs 35 per kg.

13. A farmer pays Rs 2,000 as Ushr. Find the value of the crop he produced. Rate of Ushr is 5%.

SUMMARY

- TAX is a fee charged by a government on a product, income, or
- THE PURPOSE of taxation is to finance government expenditures.
- PROPERTY TAX is a fee charged by a government on a person's
- GENERAL PROPERTY TAX = 2% of taxable income
- GST = Item Cost × Rate of GST.
- Rate of GST in Pakistan is 17%.
- TOTAL ITEM COST = General Sales Tax (GST) + Item Cost.
- ITEM COST = $\frac{\text{Total Cost}}{1.17}$
- PROFIT = Selling Price – Cost Price.
- Profit Rate = $\frac{\text{Profit}}{\text{Cost Price}} \times 100$

- Markup Rate = $\frac{P \times R \times T}{100}$ where P means principal amount, T means time period. R means markup rate per annum.
- SELLING PRICE = Cost Price (1 + Markup Rate)
- PROFIT RATE = $\frac{\text{Gross Profit}}{\text{Net Sale}} \times 100$
- ZAKAT is obligatory on every sahib-e-nisab muslim. It is an Islamic tax which is paid on the weather which remains with a person for a complete years.
- NISAB: Gold 87.48 grams or 7.5 tola
  or Silver 612.36 grams or 52.5 tola
- ZAKAT RATE is 2.5%
- USHR is paid by the farmer or land owner.
- USHR NISAB is value of 948 kg wheat or 5 wasaqs.
- USHR RATE is 5% or 10% of the land produce.

• • •

Think Tank: The Most Wonderful Knowledge.

Can you imagine that you can get the same result even if you change the style of operation?

It is unimaginable but still you will observe that

$$1\frac{1}{2} \times 3 = 1\frac{1}{2} + 3; \qquad 3\frac{1}{5} + \frac{4}{5} = 3\frac{1}{5} \div \frac{4}{5}$$

$$7\frac{1}{5} \div 6 = 7\frac{1}{5} - 6; \qquad 5 - \frac{5}{6} = \frac{4}{5}\,5 \times \frac{5}{6}$$

Be careful! Don't make it a rule every time.

Unit

8 ALGEBRAIC EXPRESSIONS

Introduction

Algebra is a used to create mathematical models of real-world situations and to handle problems that we can't solve using just arithmetic. Instead of using numbers or words, algebra uses symbols to make statements about things. In algebra, we often use letters to represent numbers.

8.1 ALGEBRAIC EXPRESSIONS

We have already learnt in previous class that an algebraic expression consists of terms which are connected by the operation of addition (+) and subtraction (–).

For examples: $4x+5y+3$, $6a+5b-4$, $5m\div 6\times 2\,p\times q$

8.1.1 Define a constant as a symbol having a fixed numerical value.

A symbol having a fixed numerical value is called a constant.

For example, 3, 8, 12, ... etc are all constants.

8.1.2 Recall variable as a quantity which can take various numerical values.

(a) Variables

These are represented by symbols or letters. For example, x is the number of rupees in the pocket of a student. It can take values in rupees 5, 10, 35, 50 etc.

(b) Coefficients

Coefficient is the number multiplied by a variable or variables.

In $3x$, the coefficient is 3.

In $5yz$, the coefficient is 5.

8.1.3 Recall literal as an unknown number represented by an alphabets

The letters or alphabets that we use to represent unknowns are called literal numbers. For example, area of square can be calculated by multiplying its side by side i.e Area of square A = side × side or A = s × s.

Similarly area of a rectangle can be calculated by multiplying length and breadth i.e

Area = $Length\times Breadth$ or $A = l\times b$. In the above example, s is used for side, l is used for length and b is used for breadth.

Thus s, l and b etc represent the unknown quantities.

8.1.4 Recall algebraic expression as a combination of constants and variables connected by the signs of fundamental operations.

We know that an algebraic expression is the combination of variables and constants connected by the signs of fundamental operations i.e. +, –, ÷, ×. For example,

(a) $2x+3y$ (b) $2x^2+5y^2+3y$ (c) $5a \div b+3c$ and (d) $4m \times n-2q$ are some algebraic expressions.

8.1.5 Define polynomial as an algebraic expression in which the powers of variables are all whole numbers.

In algebra polynomial represents an algebraic expression containing a single term as well as two or more than two terms.

For a polynomial, the exponents of the variables must be the whole numbers. For example 6, 7, $2x$, $4x$, $x^2 + 5$, $y^2 + 3$, $3x + 3x^2 + x + 2$ are polynomials but $y + \frac{1}{y}$, $x^2 + \frac{1}{x}$, $x^3 + y^{\frac{3}{2}} + 3$ etc, are not polynomials because their exponents of variables are not whole numbers.

Thus an algebraic expression in which the exponent is a whole number is called a polynomial.

Therefore we define a Polynomial as:

An algebraic expression consisting of one or more terms, each of which exponent (index) of variable is whole number is called polynomial.

8.1.6 Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms and three terms respectively.

1. Monomial:

An expression having one term is called monomial or simple expression i.e. 2, $3x$, $4ab$ etc.

2. Binomial:

An expression having two terms is called binomial i.e. $x + y$ and $3a - bx^2$, $x^2 + x$ etc.

3. Trinomial:

An expression having three terms is called trinomial i.e. $a^3 + 2a - b$, $2x+3g+4z$ and $x^3 + x^2 + x$.

Examples of polynomials:

Polynomial	Number of terms	Some examples
Monomial	1	$2, x, 5x^3$
Binomial	2	$2x + 5, x^2 - x, x - 5$
Trinomial	3	$x^2 + 5x + 6, x^5 - 3x + 8$

Activity: Identify and tick (√) the given polynomial is a monomial, a binomial, or trinomial.

S. No.	Polynomial	Monomial	Binomial	Trinomial
1)	$3.4 + 3.4x$			
2)	$z^2 + 5z + 6$			
3)	-8			
4)	$2c + 5b + 6$			
5)	$14 + x$			
6)	$5x - 2$			
7)	$4b - 2a$			
8)	$f^2 + 5f + 6$			

8.2 OPERATIONS WITH POLYNOMIALS

8.2.1 Add two or more polynomials:

Addition of polynomials:

Let us solve some examples to understand the addition.

Example 1: Add $7a$ and $3a$

Solution: $7a + 3a = (7 + 3)\,a$, [Distributive Property of Multiplication]
$= 10a$

Example 2: Add $6ab$, $5ab$ and $-7ab$

Solution: Horizontal Method

$$6ab + 5ab - 7ab = (6 + 5 - 7)\ ab = (11 - 7)\ ab = 4ab$$

Vertical Method

$$\begin{array}{r} 6\ ab \\ 5\ ab \\ -7\ ab \\ \hline 4\ ab \\ \hline \end{array}$$

In vertical method, similar terms are written one below the other and their coefficients are added and simplified.

Example 3:

Add $3a + 2b$ and $2a + 5b$

Solution: Horizontal Method

$(3a + 2b) + (2a + 5b)$

$= 3a + 2b + 2a + 5b$

$= 3a + 2a + 2b + 5b$

$= (3 + 2)\ a + (2 + 5)\ b$

$= 5a + 7b$

Vertical Method,

$$\begin{array}{r} 3a + 2b \\ 2a + 5b \\ \hline 5a + 7b \\ \hline \end{array}$$

Example 4:

Add $6a^2b - 3ab^2$ and $4a^2b + 3ab^2$

Solution: Horizontal Method

$(6a^2b - 3ab^2) + (4a^2b + 3ab^2)$

$= (6a^2b + 4a^2b) + (-3ab^2 + 3ab^2)$

$= 10a^2b + 0$

$= 10a^2b$

Vertical Method,

$$\begin{array}{l} 6a^2b - 3ab^2 \\ 4a^2b + 3ab^2 \\ \hline 10a^2b + 0 \\ \hline \end{array} \quad \text{or } 10a^2b$$

In vertical method the given expressions are written one below the other in such a way that the similar terms are in one and the same column. Then these similar terms are added. This gives the addition of the given expressions.

> If the sum of the coefficients is zero, then the whole term becomes zero.

EXERCISE 8.1

A. Add the following:

(1) $5a,\ 8a$ (2) $a,\ a$ (3) $-12x,\ -2x$

(4) $9ab,\ -7ab$ (5) $8y,\ 3y,\ -4y$ (6) $6x^2 + 2x + 4,\ 3x^2 + 5x$

B. Find the sum of:

(1) $2a + 3b,\ 6a + 5b$ (2) $3x - 4y,\ 4x + 6y$

(3) $2a - 3b,\ 4a + 5b,\ 5a - 8b$ (4) $ab + 4cd + 6ad,\ 8ab + 5cd + 6ad$

(5) $4.5a + 5.7b + 3.2c,\ 3.5a + 1.3b + 2.8c$

(6) $1.7a + 3.2b + 4.9c,\ 4.3a + 3.2b + 3.1c,\ 6.0a + 9.6b + 3.0c$

(7) $6a + 11b + 10c,\ 13a + 9b + 5c,\ 8a + 7b + 12c$

(8) $4a + 6b + 5c,\ 7a + 5b + 5c,\ 3a + 7b + 7c$

(9) $2a + 3b + 4c,\ 6a + 8b + 4c,\ 3a + 2b + 3c$

(10) $14e - 15f + 10g,\ 13e + 14f - g,\ -5e + 3g$

8.2.2 Subtract a polynomial from another polynomial

We know that the operation of subtraction is the inverse operation of addition. According to the rules of directed numbers:

$$5 - 2 = 5 + (-2)$$

and $a - b = a + (-b)$

This means that to subtract b from a, we add the additive inverse of b, i.e. $-b$ to a.

Thus, to subtract one expression from the other, we change the signs of the terms of the expression to be subtracted and then add them.

Example 1. Subtract

$2a + 5b$ from $3a + 8b$

Vertical method

Solution:

$$\begin{array}{rcr} 3a & + & 8b \\ -\ 2a & \underline{+} & 5b \\ \hline 1a & + & 3b \\ \hline \end{array} \quad \text{or } a + 3b$$

Horizontal method

$(3a + 8b) - (2a + 5b)$

$= 3a + 8b - 2a - 5b$

$= 3a - 2a + 8b - 5b$

$= (3 - 2)\ a + (8 - 5)\ b$

$= 1a + 3b$

$= a + 3b$

Example 2: Subtract $2x + 3y - 4z$ from $3x - 2y + 5z$

Solution:

$$\begin{array}{rrrrr} & 3x & -\ 2y & + & 5z \\ \pm & 2x & \pm\ 3y & \mp & 4z \\ \hline & x & -\ 5y & + & 9z \\ \hline \end{array}$$

Example 3: Subtract $4x - y + 2$ from the sum of $3x + 8y + 1$ and $2x + 3y - 6$.

Solution:

$$\begin{array}{lrcrcr} & 3x & + & 8y & + & 1 \\ & 2x & + & 3y & - & 6 \\ \cline{2-6} \text{Sum:} & 5x & + & 11y & - & 5 \\ & \pm 4x & & \mp y & \pm & 2 \\ \cline{2-6} \text{Difference:} & x & + & 12y & - & 7 \\ \cline{2-6} \end{array}$$

EXERCISE 8.2

Subtract the second expression from the first expression.

(1) $8a$, $5a$ (2) $-7ab$, $12ab$ (3) $4a$, $-8a$

(4) $6a + 7b$, $2a + 3b$ (5) $9a + 11b + 15c$, $5a + 3b - 6c$

(6) $9c + 8d + 6e$, $3e - 5c + 4d$

(7) $18x + 16y + 12z$, $10x + 9y + 8z$

(8) $14a + 15b + 16c$, $8a + 10b - 2c$

(9) $3x^2 + 25ab + 21bc$, $4x^2 + 18ab - 6bc$

(10) $5a + 4ab + 2b$, $4a + 3ab + b$

(11) Subtract $10a + 10b + 17c$ from the sum of $13a + 8b + 12c$ and $9a + 3b + 5c$

(12) What should be added to $2x + 5y$ to get $6x + 8y$?

8.2.3 Find the product of:

- monomial with monomial
- monomial with binomial and trinomial
- binomial with binomial trinomial.

I. Multiplication of monomial with monomial

While multiplying monomials, the coefficients and variables in each of the corresponding terms are multiplied separately.

Example 1: Find the product of $2a$ and $3a^2$

Solution: $2a \times 3a^2$

$= 2 \times 3 \times a \times a^2 = 6a^{1+2} = 6a^3$

Example 2: Find the product of $3ay^2$ and $7a^2y^3$

Solution: $3ay^2 \times 7a^2y^3 = 3 \times 7 \times a \times a^2 \times y^2 \times y^3$

$= 21 \times a^{1+2} \times y^{2+3} = 21a^3y^5$

II. Multiplication of monomial with binomial.

Example 3: Multiply $(a + b)$ by 3

Solution: Horizontal Method

$3 \times (a + b) = 3 \times a + 3 \times b = 3a + 3b$

Vertical Method

$$\begin{array}{r} a + b \\ \times \quad 3 \\ \hline 3a + 3b \\ \hline \end{array}$$

Example 4: Multiply $2a + 3b$ by a

Vertical Method.

Solution:

$$\begin{array}{r} 2a + 3b \\ \times \quad a \\ \hline 2a^2 + 3ab \\ \hline \end{array}$$

Horizontal Method.

$a \times (2a + 3b) = 2a^2 + 3ab.$

III. Multiplication of monimial with trinomial

Example 5. Multiply $(x + y + z)$ by $4x$

Solution:

Vertical Method.

$$\begin{array}{r} x + y + z \\ \times 4x \\ \hline 4x^2 + 4xy + 4xz \\ \hline \end{array}$$

Horizontal Method.

$4x(x + y + z)$

$= 4x^2 + 4xy + 4xz$

IV. Multiplication of binomial with binomial.

Example 6: Multiply $3e + 5f$ by $4e + 2f$

Solution:

Vertical Method.

$(3e + 5f) \times (4e + 2f)$

$$\begin{array}{rrr} 3e + 5f & & \\ \times\ 4e + 2f & & \\ \hline 12e^2 + 20ef & & \\ + \quad 6ef & + 10f^2 & \\ \hline 12e^2 + 26ef & + 10f^2 & \\ \hline \end{array}$$

$12e^2 + 20ef \rightarrow$ (Multiply both terms of first expression by $4e$)

$6ef + 10f^2 \rightarrow$ (Multiply both term of first expression by $2f$)

Horizontal Method.

$= 3e(4e + 2f) + 5f(4e + 2f)$

$= 12e^2 + 6ef + 20ef + 10f^2$

V. Multiplication of binomial with trinomial.

Example 7: Multiply $2x^2 - 3xy + 4$ by $x + y$

Solution: Vertical Method

$$\begin{array}{rrrrr}
 & 2x^2 - & 3xy & + 4 & \\
\times & & x & + y & \\
\hline
 & 2x^3 - & 3x^2y + & 4x & \\
 & + & 2x^2y & & - 3xy^2 + 4y \\
\hline
 & 2x^3 - & x^2y + & 4x & - 3xy^2 + 4y \\
\hline
\end{array}$$

Horizontal Method

$$\begin{aligned}
(x + y)(2x^2 - 3xy + 4) &= x(2x^2 - 3xy + 4) + y(2x^2 - 3xy + 4) \\
&= 2x^3 - 3x^2y + 4x + 2x^2y - 3xy^2 + 4y \\
&= 2x^3 - 3x^2y + 2x^2y + 4x - 3x^2y + 4y \\
&= 2x^3 - x^2y + 4x - 3xy^2 + 4y
\end{aligned}$$

EXERCISE 8.3

I. Find the product of the following expressions: (Monomial with monomial)

(1) $15a, 8a$ (2) $9a, 6b$ (3) $10a^2, 2a^5$ (4) $4c^5, 8cd^2$

(5) $6a^2b, -5b^2$ (6) $-3a, 2b, -3$ (7) $4m^2, -5m^3, 3m^4$ (8) $-6xy, 3x^3y^2, 2xy$

II. Find the product of following expressions: (Monomial with binomial.

(1) $a + b, 9$ (2) $x - y, -4y$ (3) $2m^2 - 3n^3, -5mn$

(4) $3xy + 2xy^2, -6xy^2$ (5) $6lm - 4mn, 3l^2m^2n$ (6) $-5bc + 3cd, -2bc^2d$

III. Find the product of the following expressions: (Binomial with binomial).

(1) $8c + 12d, -3c + 2d$ (2) $a + b, a - b$ (3) $a^2 + b^2, a^2 - b^2$

(4) $3x - 2y, 2x + 5y$ (5) $2lm + 3mn, -2m + 3n$ (6) $8pq^2 - 3p^2q, 2pq$

IV. Find the product of the following expressions (Monomial with trinomial).

(1) $2a - 3b + 4c, 2abc$ (2) $a^2 + b^2 - c^2$, ab (3) $m^2n + mn^2 - m^2n^2, - mn$

(4) $x^2 - 6xy + 9y^2, - 3xy$ (5) $2p^2 - 3pq + q^2, 4pqr$ (6) $xyz + x^2y^2 - y^2z^2, xyz$

V. Find the product of the following expression (Binomial with trinomial).

(1) $1 + 2x - 3x^2, 1 - x$ (2) $8c + 12d - 6e, c^2 - d^2$ (3) $a^2 - ab - b^2, a - b$

(4) $x^2 - 5x + 4, 3x - 4$ (5) $a^2 - 2a^2b + b^2, a^2 - b^2$ (6) $3p^2q + 2pq^2 - q^2r, p^2 - q^2$

8.2.4 Simplify algebraic expressions involving addition, subtraction and multiplication.

We have already learnt addition, subtraction and multiplication of algebraic expressions. We will learn to simplify the expressions involving these operations through following examples.

Example. Simplify: (i) $3x\ (x + 2y) + 4x^2 - 7xy$

(ii) $2x + \{ (3x + y)\ (2x) - y) - (6x^2 - 2x) \}$

Solution:

(i) $3x\ (x + 2y) + 4x^2 - 7xy$

$= 3x^2 + 6xy + 4x^2 - 7xy$

$= 3x^2 + 4x^2 + 6xy - 7xy$

$= 7x^2 - xy$

Solution:

(ii) $2x + \{ (3x + y)\ (2x - y) - (6x^2 + 2x) \}$

$2x + \{ (3x + y)\ (2x - y) - (6x^2 + 2x) \}$

$= 2x + \{ (3x\ (2x - y) + y\ (2x - y) - 6x^2 - 2x \}$

$= 2x + \{\cancel{6x^2} - 3xy + 2xy - y^2 - \cancel{6x^2} - 2x\}$

$= 2x + \{- xy - y^2 - 2x\}$

$= \cancel{2x} - xy - y^2 - \cancel{2x}$

$= - xy - y^2$

8.3 Algebraic Identities

Recognize and verify the algebraic identities.

An algebraic identity is a simplified form consisting of the algebraic term which provide us with a rule to solve any algebraic expression.

Identities are some such results which are repeatedly used in algebra. We can easily find the product of expressions by using these identities.

8.3.1 Verify: $(x+a)(x+b) = x^2+(a+b)x+ab$

Verification: $(x+a)(x+b) = x^2+(a+b)x+ab$

$$\begin{aligned} \text{LHS} = (x+a)(x+b) &= x\times(x+a)+b\times(x+a) \\ &= x^2+ax+bx+ab \\ &= x^2+(a+b)x+ab = \text{RHS} \end{aligned}$$

Thus LHS = RHS.

Hence verified.

By this identity we can multiply two linear expressions quickly.

Hence, $\boxed{(x+a)(x+b)=x^2+(a+b)x+ab}$

We can also verify it by using geometrical shape.

	10	2
10	100	20
4	40	8

The expression can also be worked out as follows, without drawing a graph

$$\begin{aligned} (10+4)(10+2) &= 10^2+(4+2)\times 10+4\times 2 \\ &= 100+6\times 10+8 \\ &= 100+60+8 \\ &= 168 \end{aligned}$$

The expression $(x+4)(x+2)$ is expanded as shown:

	x	2
x	x^2	$2x$
4	$4x$	8

$$\begin{aligned} (x+4)(x+2) &= x^2+4x+2x+8 \\ &= x^2+6x+8 \end{aligned}$$

Similarly, we can verify:

(i) $(x + a)(x - b) = x^2 + (a - b)x - ab$

(ii) $(x - a)(x + b) = x^2 - (a - b)x - ab$

(iii) $(x - a)(x - b) = x^2 - (a + b)x + ab$

Example 1: Verify: $(x + 2)(x + 3) = x^2 + 5x + 6$

Verification:

LHS: $(x + 2)(x + 3) = x^2 + (2 + 3)x + (2) \times (3)$

$= x^2 + 5x + 6 = \text{RHS}$

Thus LHS = RHS Hence verified

Example 2: Verify: $(x - 2)(x + 3) = x^2 + x - 6$

Verification:

LHS: $(x - 2)(x + 3) = x^2 + (-2 + 3)x + (-2) \times (3)$

$= x^2 + x - 6 = \text{RHS}$

Thus LHS = RHS Hence verified

Example 3: Verify: $(x - 2)(x - 3) = x^2 - 5x + 6$

Verification: Using identity:

LHS: $(x - 2)(x - 3) = x^2 + (-2 - 3)x + (-2) \times (-3)$

$= x^2 - 5x + 6 = \text{RHS}$

Thus LHS = RHS Hence verified

EXERCISE 8.4

I. Simplify:

(1) $5a - 2a(3 + 4b)$

(2) $7m - 4m(5 - 6n)$

(3) $6x\{2x(3x^2 - 4) - (6x^2 + 8y)\}$

(4) $3x - \{(2x + y)(2x - y) - (4x^2 + 2y^2)\}$

II. Verify the following.

(1) $(x + 3)(x + 4) = x^2 + 7x + 12$

(2) $(x + 5)(x + 6) = x^2 + 11x + 30$

(3) $(x + 2)(x - 4) = x^2 - 2x - 8$

(4) $(x - 3)(x + 4) = x^2 + x - 12$

(5) $(x - 5)(x - 4) = x^2 - 9x + 20$

(6) $(x - 7)(x + 8) = x^2 + x - 56$

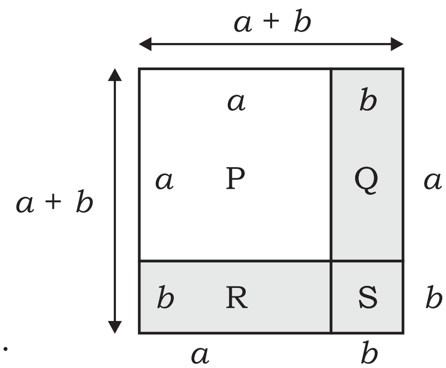
(7) $(x - 9)(x + 2) = x^2 - 7x - 18$

(8) $(x - 1)(x + 9) = x^2 + 8x - 9$

(9) $(x - 2)(x + 5) = x^2 + 3x - 10$

8.3.2 Verify $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

Geometrical verification: Consider the area of a square whose side is $(a + b)$



The area of the square whose sides are $(a + b)$ is equal to the sum of the areas of P, Q, R and S.

$$\begin{aligned} \text{LHS} &= (a + b)^2 \\ &= (a + b)(a + b) = a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \\ &= a^2 + 2ab + b^2 = \text{RHS} \end{aligned}$$

Thus LHS = RHS Hence verified.

Algebraic verification:

$$(a + b)^2 = a^2 + 2ab + b^2$$

This identity also can also be verified as:

Vertical Method

LHS = $(a + b)^2 = (a + b)(a + b)$

$$\begin{array}{rl} a + b & \\ \times\ a + b & \\ \hline a^2 + ab & \text{(Multiply } a + b \text{ by } a\text{)} \\ + ab + b^2 & \text{(Multiply } a + b \text{ by } b\text{)} \\ \hline a^2 + 2ab + b^2 & = \text{RHS} \\ \hline \end{array}$$

Horizontal Method

$$\begin{aligned} \text{LHS} &= (a + b)^2 = (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \text{(because } ab = ba\text{)} \\ &= a^2 + 2ab + b^2 = \text{RHS} \end{aligned}$$

Thus LHS = RHS

Hence verified $\boxed{(a + b)^2 = a^2 + 2ab + b^2}$

This identity can be described in words.

'Square of the sum of two terms is always equal to the square of the first term plus twice the product of first and second terms plus the square of the second term.'

Example 1: Verify: $(a^2 + 5)^2 = a^2 + 2ab + b^2$

Verification:

$$\begin{aligned} (a + 5)^2 &= (a + 5)(a + 5) \\ &= a + 2 \times a \times 5 + 5^2 \\ &= a^2 + 10a + 25 = \text{RHS} \end{aligned}$$

Thus LHS = RHS Hence verified

Example 2: Verify $(3a + 2b)^2 = 9a^2 + 12ab + 4b^2$

Verification: LHS $= (3a + 2b)^2 = (3a + 2b)(3a + 2b)$

$= (3a)^2 + 2(3a)(2b) + (2b)^2$

$= 9a^2 + 12ab + 4b^2 =$ RHS

Thus LHS = RHS Hence verified

EXERCISE 8.5

I. Verify the following:

(1) $(a + 7)^2 = a^2 + 14a + 49$

(2) $(a + 11)^2 = a^2 + 22a + 121$

(3) $(2x + 6)^2 = 4x^2 + 24x + 36$

(4) $(3a + 5)^2 = 9a^2 + 30a + 25$

(5) $(7x + 3)^2 = 49x^2 + 42x + 9$

(6) $(6x + 9)^2 = 36x^2 + 108x + 81$

(7) $(5a + 4)^2 = 25a^2 + 40a + 16$

(8) $(9x + 10)^2 = 81x^2 + 180x + 100$

(9) $(5x + 11)^2 = 25x^2 + 110x + 121$

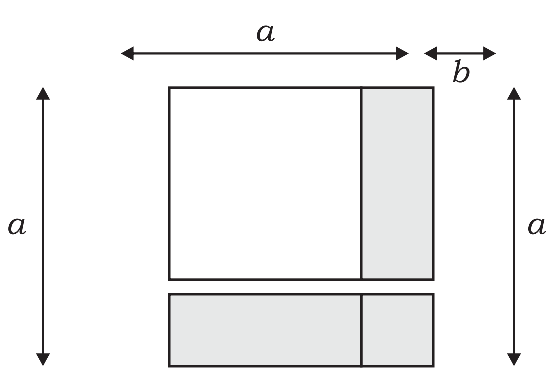
(10) $(6x + 7)^2 = 36x^2 + 84x + 49$

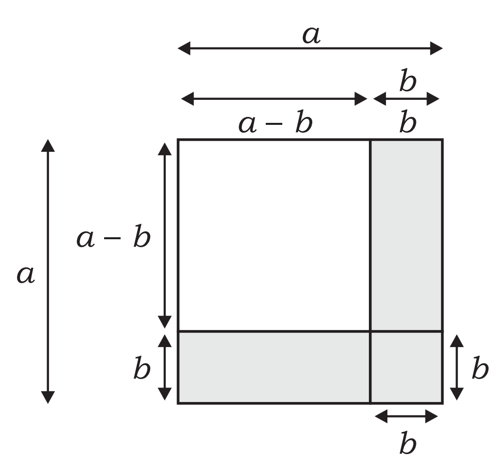
8.3.3 Verify $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Geometrical verification:

Consider the area of a square whose sides are a and the area of two rectangles whose length and width are a and b respectively.

The two rectangles are then placed on the square like this:





The area of the unshaded square is equal to $(a-b)(a-b)$

LHS = $(a-b)^2 = (a-b)(a-b)$ = Area of original square of sides a
– Area of two rectangles of sides a and b (because the area of the square is subtracted twice).
+ Area of small square of sides b

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$ = RHS

Thus LHS = RHS

Hence verified: $\boldsymbol{(a-b)^2 = a^2 - 2ab + b^2}$

Algebraic verification: We can also verify it as under:

Vertical Method

LHS: $(a-b)^2 = (a-b)(a-b)$

or

$$\begin{array}{rrrl}
 & a - b & & \\
\times & a - b & & \\
\hline
 & a^2 - ab & & \text{(Multiply } a-b \text{ by } a\text{)} \\
 & - ab & + b^2 & \text{(Multiply } a-b \text{ by } b\text{)} \\
\hline
 & a^2 - 2ab & + b^2 & = \text{RHS} \\
\hline
\end{array}$$

Horizontal Method

LHS: $(a-b)^2 = (a-b)(a-b)$

$= a(a-b) - b(a-b)$

$= a^2 - ab - ba + b^2$ (because $ab = ba$)

$= a^2 - 2ab + b^2$ = RHS

Thus LHS = RHS Hence verified

The identity can be described in words as:

'Square of the difference of two terms is always equal to the square of the first term minus twice the product of first and second terms plus the square of the second term.'

Example 1: Verify $(a-4)^2 = a^2 - 8a + 16$

Verification: LHS = $(a-4)^2 = (a-4)(a-4)$

$= a^2 - 2 \times a \times 4 + (4)^2$

$= a^2 - 8a + 16$ = RHS

Thus LHS = RHS Hence verified.

Example 2: Verified $\left(\frac{4}{5}x - \frac{1}{3}y\right)^2 = \frac{16}{25} - \frac{8}{15}xy + \frac{1}{9}y^2$

Verification: LHS = $\left(\frac{4}{5}x - \frac{1}{3}y\right)^2 = \left(\frac{4}{5}x - \frac{1}{3}y\right)\left(\frac{4}{5}x - \frac{1}{3}y\right)$

$= \left(\frac{4}{5}x\right)^2 - 2 \times \left(\frac{4}{5}x\right) \times \left(\frac{1}{3}y\right) + \left(\frac{1}{3}y\right)^2$

$= \frac{16}{25}x^2 - \frac{8}{15}xy + \frac{1}{9}y^2$ = RHS

Thus LHS = RHS Hence verified

EXERCISE 8.6

Verify the following:

(1) $(a-3)^2 = a^2 - 6a + 9$

(2) $(a-7)^2 = a^2 - 14a + 49$

(3) $(2a-5)^2 = 4a^2 - 20a + 25$

(4) $(6x-8)^2 = 36x^2 - 96x + 64$

(5) $(4a-7)^2 = 16a^2 - 56a + 49$

(6) $(3a-8)^2 = 9a^2 - 48a + 64$

(7) $(7a-5)^2 = 49a^2 - 70a + 25$

(8) $(9y-10)^2 = 81y^2 - 180y + 100$

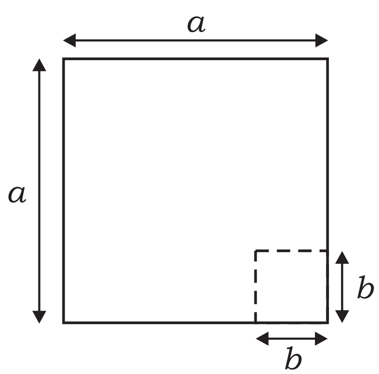
(9) $(8a-9)^2 = 64a^2 - 144a + 81$

(10) $(9a-11)^2 = 81a^2 - 198a + 121$

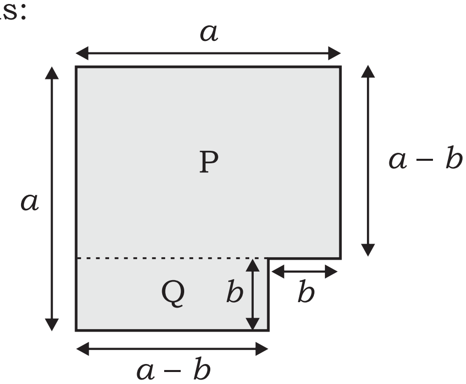
8.3.4 Verify: $a^2 - b^2 = (a-b)(a+b)$

Geometrical verification:

Consider a square of sides a.
A small square of sides b is to be cut out on the square of sides a.



After the square of sides b is cut out, we get a figure which is comprised of rectangle P and a rectangle Q like this:



The rectangle Q is then cut out and placed next to rectangle P like this:
Here the area of the shaded rectangle is: $(a+b) + (a-b)$

RHS = $(a+b)(a-b)$ Area of rectangle P + Area of rectangle Q

$= a(a-b) + b(a-b)$

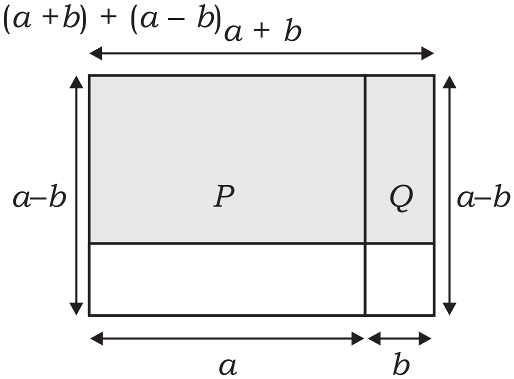
$= a^2 - ab + ab - b^2$

$= a^2 - b^2$ = LHS

Thus RHS = LHS

Hence verified

Therefore $(a+b)(a-b) = a^2 - b^2$



Unit 8 ALGEBRAIC EXPRESSIONS

Given identity: $(a + b)(a - b) = a^2 - b^2$

Algebraic verification:

Multiplying $(a + b)$ and $(a - b)$, we get

Vertical *method:*

LHS = $(a+b)(a-b)$

$$\begin{array}{r} a + b \\ \times \quad a - b \\ \hline a^2 + \cancel{ab} \quad\quad \\ - \cancel{ab} \times b^2 \\ \hline a^2 - b^2 \quad\quad \\ \hline \end{array}$$

Horizontal method:

LHS = $(a + b)(a - b) = a(a - b) + b(a - b)$

$= a^2 - \cancel{ab} + \cancel{ba} - b^2$ (because $ab = ba$ and $ab - ab = 0$)

$= a^2 - b^2$ = RHS

Thus LHS = RHS Hence verified.

The above identity can be described in words as **'product of the sum and difference of two terms is equal to the difference of their squares.'**

Example 1: Verify $(x + 2)(x - 2) = x^2 - 4$

Verification: LHS = $(x + 2)(x - 2) = x^2 - (2)^2$

$= x^2 - 4$ = RHS

Thus LHS = RHS Hence verified.

Example 2: Verify $(2x + y)(2x - y) = 4x^2 - y^2$

Verification: LHS = $(2x + y)(2x - y) = (2x)^2 - (y)^2$

$= 4x^2 - y^2$ = RHS

Thus LHS = RHS Hence verified.

EXERCISE 8.7

I. Verify:

(1) $(3x+2)(3x-2) = 9x^2 - 4$

(2) $(4x+3)(4x-3) = 16x^2 - 9$

(3) $(5x+4)(5x-4) = 25x^2 - 16$

(4) $(3x+7)(3x-7) = 9x^2 - 49$

(5) $(6x+5)(6x-5) = 36x^2 - 25$

(6) $(7x+6)(7x-6) = 49x^2 - 36$

II. Verify the following:

(1) $4a^2 - 9b^2 = (2a + 3b)(2a - 3b)$

(2) $64m^2 - 81 = (8m + 9)(8m - 9)$

(3) $9p^2 - 100q^2 = (3p + 10q)(3p - 10q)$

(4) $100y^2 - 169 = (10y+13)(10y - 13)$

8.4 FACTORIZATION OF ALGEBRAIC EXPRESSIONS

8.4.1 Factorize an algebraic expression (using algebraic identities).

The factors of a number, or an expression, can be expressed as the product of given number or given expressions. Such numbers or expressions are called factors of the given number or given expression.

For example: $12 = 2 \times 2 \times 3$ **,** $4a^2 = 2 \times 2 \times a \times a$ **,** $a^2 + 5a + 6 = (a + 3)(a + 2)$

Unit 8 ALGEBRAIC EXPRESSIONS

We have learnt to find the factors of numbers but here we shall learn to find the factors of an algebraic expression.

(i) Factors of expression of the form: *ka* + *kb* + *kc*

There are three terms in the given expression $ka + kb + kc$ and each term can be divided by k (i.e, the common factor of every term).

As $\frac{ka}{k} = a$, $\frac{kb}{k} = b$, $\frac{kc}{k} = c$,

So, multiplying and dividing each term by k, we get:
$ka + kb + kc = k(a + b + c)$, i.e. there are two factors of the given expression, k and $(a + b + c)$.

Example 1: Find the factors of $2a + 4b + 6c$

Solution: 2 is common factor in all the three terms.

i.e. $\frac{2a}{2} = a$, $\frac{4b}{2} = 2b$, $\frac{6c}{2} = 3c$

Hence, $2a + 4b + 6c = 2(a + 2b + 3c)$

Example 2: Find the factors of $12a^2 - 8ab + 4a$

Solution: a, $2a$ and $4a$ are the common factors in the expression but $4a$ is the highest common factor.

As $\frac{12a^2}{4a} = 3a$, $\frac{8ab}{4a} = -2b$, $\frac{4a}{4a} = 1$

Hence, $12a^2 - 8ab + 4a = 4a(3a - 2b + 1)$

Example 3: Find the factors of $a(x + y) + b(x + y) + c(x + y)$

Solution: $(x + y)$ is the common factor of all the three terms in the given expression.

i.e. $a(x + y) + b(x + y) + c(x + y) = (x + y)(a + b + c)$

EXERCISE 8.8

I. Factorize the following.

(1) $5a^2x - 15a$ (2) $m^4 - m^3 + m^2$ (3) $7a^3 + 14a^2 - 7$

(4) $mx^2 - m^2x - mx$ (5) $13n - 26n^3 + 39n^5$ (6) $a^2bc + ab^2c + abc^2$

(7) $3a(b+c) + 6d(b+c)$ (8) $xy(a + b) + y(a + b)^2 + x(a + b)$

(9) $4lm(x-y) + 8mn(x-y)^2 + 12nl(x-y)$ (10) $pq(p^2-q)^2 + pr(p^2-q)^2 + qr(p^2-q)^2$

II. Factors of expression of the form: $(a^2 + 2ab + b^2)$ or $(a^2 - 2ab + b^2)$

We know that $(a + b)^2 = a^2 + 2ab + b^2$

and $(a - b)^2 = a^2 - 2ab + b^2$

The expression $(a^2 + 2ab + b^2)$ or $(a^2 - 2ab + b^2)$ has three terms.

- The first and last terms are perfect squares
- The middle term is twice the product of the square roots of the first

Example 1. Factorize: $a^2 + 6ab + 9b^2$

Solution: In the given expression:

(i) there are three terms
(ii) the first is the square of a and the last term is the square of $3b$.
(iii) the middle term is according to the condition i.e. $2(a)\ (3b) = 6ab$.

Therefore the factors of the given expression can be written in the form:
$a^2 + 2ab + b^2 = (a + b)^2$

So, $a^2 + 6ab^2 + 9b = a^2 + 2\ (a)\ (3b) + (3b)^2$
$= (a + 3b)^2$
$= (a + 3b)\ (a + 3b)$

Example 2. Factorize: $2x^2 + 12x + 18$

Solution: Take the common factor 2 from the given expression and then factorize accordingly.

i.e. $2x^2 + 12x + 18 = 2\ (x^2 + 6x + 9)$ (2 as common factor)
$= 2\ \{ (x)^2 + 2(x)\ (3) + (3)^2 \}$ (perfect square formula)
$= 2\ (x + 3)^2$
$= 2\ (x + 3)\ (x + 3)$

Example 3. Factorize: $20a^2 - 60a^2b + 45ab^2$

Solution: $20a^2 - 60a^2b + 45ab^2$
$= 5a\ (4a^2 - 12ab + 9b^2)$ (5a is common factor)
$= 5a\ \{(2a)^2 - 2\ (2a)\ (3b) + (3b)^2\}$ (perfect square formula)
$= 5a\ (2a - 3b)^2$ (Because $a^2 - 2ab + b^2 = (a - b)^2$
$= 5a\ (2a - 3b)\ (2a - 3b)$

Example 5. Factorize: $98c^2 - 140cd + 50d^2$

Solution: First and last term are not perfect square. If we take 2 as common

i.e. $98c^2 - 140cd + 50d^2 = 2\,(49c^2 - 70cd + 25d^2)$ (Taking 2 as common)

$= 2\,\{(7c)^2 - 2\,(7c)\,(5d) + (5d)^2\}$

$= 2\,(7c - 5d)^2$

$= 2\,(7c - 5d)\,(7c - 5d)$

EXERCISE 8.9

Factorize the following.

(1) $x^2 + 12x + 36$	(2) $a^2 + 4a + 4$	(3) $4a^2 + 12a + 9$
(4) $9x^2 + 42xy + 49y^2$	(5) $25a^2 + 80ab + 64b^2$	(6) $16b^2 + 40b + 25$
(7) $4x^2 + 2x + \frac{1}{4}$	(8) $144x^2 - 4x + \frac{1}{36}$	(9) $18c^2 + 60cd + 50cd^2$
(10) $4a^2 + 48a + 144$	(11) $5y^2 + 40y + 80$	(12) $3s^2 - 48st + 192t^2$
(13) $9a^2t^2 - 42ats + 49s^2$	(14) $4a^4 - 12a^2 + 9$	(15) $50p^2q^2 - 70pqr + 98r^2$

III. Factors of an algebraic expression of the form $a^2 - b^2$

We already know that $(a + b)\,(a - b) = a^2 - b^2$

or vice versa $a^2 - b^2 = (a + b)\,(a - b)$

The characteristics are

(1) There are two square terms.

(2) Negative sign is between the two terms.

Method for finding the factors

(i) Find the square root of both the terms.

(ii) Write the two terms in two small brackets in the product form,

Example 1. Factorize: $4x^2 - 9y^2$

Solution: There is difference of two squares

Hence, $4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$= (2x + 3y)\,(2x - 3y)$

Example 2. Factorize: $9y^2 - 81$

Solution: Taking first the common factor 9,

$$9y^2 - 81 = 9\ (y^2 - 9)$$
$$= 9\ (y + 3)\ (y - 3)$$

Example 3. Factorize: $15\ (a + b)^2 - 60\ (c + d)^2$

Solution:

$$15\ (a + b)^2 - 60\ (c + d)^2 = 15\ \{(a + b)^2 - 4\ (c + d)^2\}$$
$$= 15\ \{(a + b) + 2\ (c + d)\}\ \{(a + b) - 2\ (c + d)\}$$
$$= 15\ (a + b + 2c + 2d)\ (a + b - 2c - 2d)$$

EXERCISE 8.10

Factorize the following.

(1) $b^2 - c^2$ (2) $a^2 - 36$ (3) $a^2 - 49$ (4) $25 - y^2$

(5) $4x^2 - 9$ (6) $9x^2 - 16y^2$ (7) $6x^2 - 24$ (8) $200 - 18z^2$

(9) $81a^2 - 121b^2$ (10) $a^2c^2 - 16c^2d^2$ (11) $36 - (3a - 2b)^2$

(12) $(2x + 3y)^2 - 100z^2$ (13) $64b^2 - (6b + d)^2$ (14) $(3a - 4b)^2 - (3a - b)^2$

(15) $49\ (2a - 3b)^2 - 16\ (a + 2b)^2$ (16) $225\ (\mathrm{p} - \mathrm{q})^2 - 49\ (\mathrm{p} + \mathrm{q})^2$

(17) $72\ (c + d)^2 - 162\ (c - d)^2$ (18) $48l\ (m - n)^2 - 75l\ (p - q)^2$

8.4.2 Factorize an algebraic expression making groups.

The method of factorizing an algebraic expression by making groups is explained with the help of the following example.

Example 1. Factorize

(i) $x^2 + 5x + 4x + 20$

Solution:

(i) $x^2 + 5x + 4x + 20$

$= (x^2 + 5x) + (4x + 20)$ (Making groups)

$= x\ (x + 5) + 4\ (x + 5)$

$= (x + 5)\ (x + 4)$ [Taking $(x + 5)$ as common factor]

(ii) $x^2 + 4y^2 + xy^2 + 4x$

Solution:

(ii) $x^2 + 4y^2 + 4y^2 + 4x$

$(x^2 + 4x) + (xy^2 + 4y^2)$ (By making groups)

$x\ (x + 4) + y^2\ (x + 4)$

$= (x + 4)\ (x + \mathrm{y}^2)$ [Taking $x + 4$ as common factor]

In case of expression of three terms which is not a complete square, so factorize the last term in such a way that its factors, either added or subtracted, give the coefficient of middle term.

We proceed with such expression as under:

Example 2. Factorize: $x^2 + 5x + 6$

Solution: Factorizing the term 6 in such a way that its two factors when added gives the ceofficient of middle term.

So, $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

It is observed that two factors are 2 and 3 has common variable.

$= (x^2 + 3x) + (2x + 6)$

$= x(x + 3) + 2(x + 3)$ (Making their groups)

$= (x + 3)(x + 2)$ [Take $(x + 3)$ as common factor]

Example 3. Factorize: $x^2 + 5x - 6$

Solution: $x^2 + 5x - 6$

$= x + 6x - x - 6$ (Factorizing the last term)

$= (x + 6x) + (-x - 6)$ (Breaking the middle term)

$= x(x + 6) - 1(x + 6)$ (Making their groups)

$= (x + 6)(x - 1)$ (Taking $(x + 6)$ as common factor)

EXERCISE 8.11

I. Factorize:

(1) $5x^2 + 5x + 4x + 4$ (2) $ax + ay + bx + by$

(3) $b^2 + 4c^2 + bc^2 + 4b$ (4) $p^2 - 6pq - pq + 6q^2$

(5) $a^2 + 5b^2 + ab^2 + 5a$ (6) $7x^2 + py + 7x + pyx$

(7) $2c^2d + 4cd^2 - 2cd - 4d^2$ (8) $2y^2 - 10y + 4y - 20$

(9) $x^2y^2 + 5xy - xy - 5$ (10) $p^2 - pq + q - p$

II. Factorize:

(1) $x^2 + 6x + 8$ (2) $y^2 + 8y + 12$

(3) $a^2 + 7a + 10$ (4) $c^2 + 5c + 6$

(5) $d^2 + 6d + 5$ (6) $p^2 - 4p + 3$

(7) $r^2 + 7r + 12$ (8) $a^2 + 3a - 10$

(9) $m^2 - 5m + 14$ (10) $x^2 - 2x - 15$

(11) $y^2 + 7y - 18$ (12) $x^2 + 10x + 21$

REVIEW EXERCISE 8

1. Answer the following questions.

(i) What is meant by literals? (ii) Define a constant.

(iii) What is a binomial? (iv) What is an algebraic identity?

(v) Define the term factorization of an algebraic expression.

2. Fill in the blanks.

(i) $(a + b)^2 =$ _________ (ii) $(a - b)^2 =$ _________

(iii) $(x + a)(x + b) =$ _________ (iv) $a - b^2 =$ _________

(v) A symbol represented by a literal and can take various numerical values is called a _________.

(vi) A polynomial having only one term is called _________.

3. Tick (✓) the correct answer.

(i) $x^2 - x = ?$

(a) x (b) $x(x - 1)$ (c) x^2 (d) $x - x^2$

(ii) A polynomial having two terms is called a:

(a) factorization (b) monomial (c) binomial (d) trinomial

(iii) A symbol having a fixed value is called a:

(a) term (b) variable (c) constant (d) literal

(iv) The factors of $a^2 - 9$ are:

(a) $(a + 3)(a - 3)$ (b) $(a + 9)(a - 9)$

(c) $(a - 3)(a - 3)$ (d) $(a - 9)(a - 9)$

(v) $(x - y)(x - y) = ?$

(a) $x^2 - y^2$ (b) $x^2 + 2xy + y^2$

(c) $x^2 - 2xy + y^2$ (d) $x^2 + y^2$

4. If $A = 2(x^2 + y^2 + z^2)$, $B = -x^2 + 3y^2 + 2z^2$ and $C = x^2 - y^2 - 3z^2$, then find:

(i) A + B + C (ii) B + C – A (iii) A – B + C

(iv) A + B – C (v) A – B – C (vi) B – C – A

5. Simplify the following polynomials.

(i) $(x - 2y)\ (x + 2y)$ (ii) $(4x^2)\ (3x + 1)$

(iii) $2x\ (x + y) - 2y\ (x - y)$ (iv) $(a^2b^3)\ (2a - 3b)$

(v) $(a^2 - b^2)\ (a^2 + b^2)$ (vi) $(a^2 + 1)\ (a^2 - a - 1)$

(vii) $\mathrm{x}\ (\mathrm{y} + 1) - \mathrm{y}\ (\mathrm{x} + 1) - (\mathrm{x} - \mathrm{y})$

(viii) $a^2\ (b^2 - c^2) + b^2\ (c^2 - a^2) + c^2\ (a^2 - b^2)$

6. Verify the following.

(i) $(3x - 5)\ (3x + 5) = 9x^2 - 25$ (ii) $(2a - 5b)^2 = 4a^2\ 20ab + 25\ b^2$

(iii) $(3x^2 + 4\mathrm{y}^2)^2 = 9x^4 + 24x^2y^2 + y^4$ (iv) $9x^2 - 25y^2 = (3x - 5y)\ (3x + 5y)$

7. Resolve into factors.

(i) $10a^2 - 200a^4b$ (ii) $36x^3y^3z^3 - 27x^2y^4z + 63xyz^4$

(iii) $15x^4y + 21x^3y^2 - 27x^2y^2 - 33xy^4$ (iv) $x\ (a^2 + 11) - 16\ (a^2 + 11)$

(v) $x^2\ (ab + c) + xy\ (ab + c) + z^2\ (ab + c)$

8. Factorize the following.

(i) $a^2 - 26a + 169$ (ii) $1 - 6x^2y^2z + 9x^4y^4z^2$ (iii) $7ab^2 - 343a$

(iv) $75 - 3\ (x - y)^2$ (v) $49\ (x + y)^2 - 16\ (x - y)^2$ (vi) $\frac{9}{16}a + ab + \frac{4}{9}b^2$

(vii) $\frac{a^2}{b^2}\,l^2 - \frac{2ac}{bd}\,lm + \frac{c^2}{d^2}\,m^2$ (viii) $(a - \frac{9}{5})^2 - \frac{36}{25}m^2$

SUMMARY

- Algebra is used to create mathematical models of real world situation.
- The letters or alphabets that we use to represent unknowns /
- A symbol represented by a literal and can take various numerical
- A symbol having a fixed value is called a constant.
- A combination of constants and variables connected by the signs

Unit 8 ALGEBRAIC EXPRESSIONS

- The parts of an algebraic expression separated by the operational

- Co-efficient is the number multiplied by a variable or variables.
- If the sum of the co-efficients its zero then the whole term becomes zero.
- An algebraic expression in which the exponents of variables are

- An expression having one term is called monomial.
- An expression having two terms is called binomial.
- An expression having three terms is called trinomial.
- An algebraic expression can be factorized by making groups.
- A polynomial can be arranged in any order but usually we arrange

- An algebraic equation which is true for all values of the variable

- The process of writing an algebraic expression as the product of two or more expressions which divide it exactly is called the

(i) $(a+b)^2 = (a+b)\,(a-b) = a^2 + 2ab + b^2$
(ii) $(a-b)^2 = (a-b)\,(a-b) = a^2 + 2ab + b^2$
(iii) $a^2 - b^2 = (a\ -b)(\,(a+b)$

• • •

Think Tank: The Most Wonderful Knowledge.

6 is a perfect number because 6 = 1 × 2 × 3 = 1 + 2 + 3 = 6

28 is a perfect number because its divisors are:

1, 2, 4, 7, 14 and 1 + 2 + 4 + 7 + 14 = 28

Find three more perfect numbers.

Unit 9

LINEAR EQUATIONS

9.1 LINEAR EQUATIONS

9.1.1 Define a Linear equation in one variable

Equations play an important role in solving daily life problems in algebra.

In previous class we have learnt, how to construct an equation.

For example, $2x + 3 = 11$ is an equation. Here x is used as variable, therefore it is an equation in one variable. Since in this equation the exponent of x is 1. So, it is an equation of one degree or first degree equation. The equation which contains a single variable with degree 1 is called linear equation in one variable.

Activity: Tick (√) linear equations in one variable.

$y^2 + 3$	$4x + 5 = 2$	$x^2 + 3x + 4 = 0$
$3x^2 + 7 = 0$	$x + 5 = 20$	$x + x = 20$
$x^2 + y^2 = 9$	$m = n + 5$	$x + 7 = 11y$
$7v - 3 = 4$	$a^2 + 2ab + b^2 = 0$	$x = 13 - 4$

9.2 SOLUTION OF LINEAR EQUATIONS

9.2.1 Demonstrate different techniques to solve linear equation.

I. Solving simple linear equation by isolating the variable.

The important thing to remember about any equation is that the sign of equality which represents a balance. So, if we do operation to one side of the equation, we have to do it to the other side of the equation. So long as it's done **exactly** the same way on each side, the balance will be preserved.

Our first step in solving any equation is to gather all the terms involving variables and the constant terms separately.

We do all this by using properties of equation.

Example 1. Solve: $3x + 15 = x + 25$

Solution: $3x + 15 = x + 25$

or $3x - x + 15 = x - x + 25$ (Subtracting x from each side)

or $2x + 15 = 25$

$2x + \cancel{15} - \cancel{15} = 25 - 15$ (Subtracting 15 from both sides)

Thus $2x = 10$

$$\frac{2x}{2} = \frac{10}{2}$$

or $\frac{\cancel{2}x}{\cancel{2}} = \frac{\cancel{10}^{5}}{\cancel{2}_{1}}$ (Dividing each side by 2)

or $x = 5$

So the solution of the equation is $x = 5$.

This solution should be checked by substitution into the original equation, in order to check that both sides are the same. If we do this, the left is 3(5) + 15 = 30. The right is 5 + 25 = 30. So the left equals the right and we have checked that the solution is correct.

Example 2. Solve: $2x + 3 = 6 - (2x - 3)$.

Solution:

$2x + 3 = 6 - (2x - 3)$

(Removing brackets)

or $2x + 3 = 6 - 2x + 3$

or $2x + 3 = 9 - 2x$

$2x + 2x + 3 = 9 - 2x + 2x$

or (Adding $2x$ to both sides)

Thus, $4x + 3 = 9$

(Subtracting 3 from both sides)

or $4x + \cancel{3} - \cancel{3} = 9 - 3$

Therefore, $4x = 6$

(Dividing by 4 on both sides)

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{6}{4}$$

or $x = \frac{6}{4}$

or $x = \frac{3}{2} = 1\frac{1}{2}$

Verification:

Put $x = 1\frac{1}{2}$ in original equation

We have $2x + 3 = 6 - (2x - 3)$

or $2 \times \left(1\frac{1}{2}\right) + 3 = 6 - \left\{2\left(1\frac{1}{2}\right) - 3\right\}$

or $\cancel{2}\left(\frac{3}{\cancel{2}}\right) + 3 = 6 - \left\{2\left(\frac{3}{2}\right) - 3\right\}$

$3 + 3 = 6 - \left\{\cancel{2}\left(\frac{3}{\cancel{2}}\right) - 3\right\}$

$6 = 6 - (\cancel{3} - \cancel{3})$

or $6 = 6$

Hence verified, $x = 1\frac{1}{2}$

So again, the left-hand and right- hand sides are equal. Thus we have checked that the solution of the given equation is correct.

II. Solving linear equations involving brackets.

Example 1. Solve: $8(x-3)-(6-2x)=2(x+2)-5(5-x)$ and verify.

Solution:

We begin by multiplying out the brackets, taking care, in particular, with any signs of terms.

$$8x-24-6+2x=2x+4-25+5x$$

Each side can be tided up by collecting the x terms and the numbers together.

so, $8x+2x-24-6=2x+5x+4-25$

or $10x-30=7x-21$

Now subtract $7x$ from both sides:

$10x-7x-30=\cancel{7x}-\cancel{7x}-21$

or $3x-30=-21$

Add 30 on both sides

$3x-\cancel{30}+\cancel{30}=30-21$

or $3x=9$

or $\frac{3x}{3}=\frac{9}{3}=3$ (Dividing by 3 to both sides)

Therefore $x=3$.

Verification: Put $x=3$ in original equation.

We have, $8(x-3)-(6-2x)=2(x+2)-5(5-x)$

$8(3-3)-(6-2\times3)=2(3+2)-5(5-3)$

$8(0)-(6-6)=2(5)-5(2)$

$0-0=10-10$

$0=0$

Hence verified, $x=3$

Example 2. Solve: $(x+1)(2x+1)=(x+3)(2x+3)-14$ and also verify.

Solution:

First we solve the brackets:

$x(2x+1)+1(2x+1)=x(2x+3)+3(2x+3)-14$

or $2x^2+x+2x+1=2x^2+3x+6x+9-14$

So, $2x^2+3x+1=2x^2+9x-5$

Subtract $2x^2$ from both sides.

$$\cancel{2x^2} - \cancel{2x^2} + 3x + 1 = \cancel{2x^2} - \cancel{2x^2} + 9x - 5$$

so, $3x + 1 = 9x - 5$

or $9x - 5 = 3x + 1$

Subtract $3x$ from both sides.

$$9x - 3x - 5 = \cancel{3x} + 1 - \cancel{3x}$$

or $6x - 5 = 1$

Add 5 on both sides

$$6x - \cancel{5} + \cancel{5} = 1 + 5$$

or $6x = 6$

or $\dfrac{6x}{6} = \dfrac{6}{6}$

Thus, $x = 1$

Verification: Putting $x = 1$ in original equation.

We have, $(x + 1)(2x + 1) = (x + 3)(2x + 3) - 14$

We get: $(1 + 1)(2 \times 1 + 1) = (1 + 3)(2 \times 1 + 3) - 14$

or $2 \times (2 + 1) = 4 \times (2 + 3) - 14$

or $2(3) = 4(5) - 14$

or $6 = 20 - 14$

or $6 = 6$

Hence verified.

III. Solving linear equations with fractional coefficients

Example 1. Solve: $\dfrac{4(x+2)}{5} = 7 + \dfrac{5x}{13}$

Solution:

$$\frac{4(x+2)}{5} = 7 + \frac{5x}{13}$$

Multiplying both sides by the lowest common multiple of denominators, which is $5 \times 13 = 65$.

$$65 \times \frac{4(x+2)}{5} = 65\left(7 + \frac{5x}{13}\right)$$

or $${}^{13}\cancel{65} \times \frac{4(x+2)}{\cancel{5}_1} = 65 \times 7 + {}^{5}\cancel{65} \times \frac{5x}{\cancel{13}_1}$$

or $52(x + 2) = 455 + 25x$

or $52x + 104 = 455 + 25x$

Subtracting $25x$ from both sides.

or $52x - 25x + 104 = 455 + \cancel{25x} - \cancel{25x}$

or $27x + 104 = 455$

Subtracting 104 from both sides.

$$27x + 104 - 104 = 455 - 104$$

or $27x = 351$

Dividing on both sides by 27

Then $\frac{\cancel{27}x}{\cancel{27}} = \frac{\overset{13}{\cancel{351}}}{\cancel{27}_1}$

So, $x = 13$

Example 2. Solve: $\frac{x+5}{6} - \frac{x+1}{9} = \frac{x+3}{4}$ and verify

Solution: $\frac{x+5}{6} - \frac{x+1}{9} = \frac{x+3}{4}$ (LCM of 4, 6 and 9 is 36)

$\frac{\overset{6}{\cancel{36}}(x+5)}{\cancel{6}_1} - \frac{\overset{4}{\cancel{36}}(x+1)}{\cancel{9}_1} = \frac{\overset{9}{\cancel{36}}(x+3)}{4_1}$ (Multiplying both sides by LCM 36)

Therefore, $\frac{6(x+5)}{1} - \frac{4(x+1)}{1} = \frac{9(x+3)}{1}$

or $6(x+5) - 4(x+1) = 9(x+3)$

or $6x + 30 - 4x - 4 = 9x + 27$

or $6x - 4x + 30 - 4 = 9x + 27$

or $2x + 26 = 9x + 27$

or $26 = 9x + 27 - 2x$

or $26 = 7x + 27$

or $26 - 27 = 7x$

or $-1 = 7x$

or $7x = -1$

or $\frac{7x}{7} = -\frac{1}{7}$

Thus, $x = -\frac{1}{7}$

Verification:

Putting $x = -\frac{1}{7}$

$\frac{x+5}{6} - \frac{x+1}{9} = \frac{x+3}{4}$

or $\frac{-\frac{1}{7}+5}{6} - \frac{-\frac{1}{7}+1}{9} = \frac{\frac{-1}{7}+3}{4}$

$\left(\frac{-1+35}{7}\right)\times\frac{1}{6} - \left(\frac{-1+7}{7}\right)\times\frac{1}{9} = \left(\frac{-1+21}{7}\right)\times\frac{1}{4}$

$\frac{34}{42} - \frac{6}{63} = \frac{20}{28}$

$\frac{17}{21} - \frac{2}{21} = \frac{\overset{5}{\cancel{20}}}{\cancel{28}_7}$

or $\frac{17-2}{21} = \frac{5}{7}$

or $\frac{\overset{5}{\cancel{15}}}{\cancel{21}_7} = \frac{5}{7}$

or $\frac{5}{7} = \frac{5}{7}$

Hence verified, $x = -\frac{1}{7}$

IV. Another form of a linear equation in one variable.

In this final section we have a look at some equations which at first sight appear not to be linear equations. However, with some algebraic manipulation they can be recast in more familiar form.

Example 1. Solve $\frac{3}{5} = \frac{6}{x}$ and verify

Solution:

We need a common denominator. We need a quantity that will be divisible by 5 and by x. The obvious choice is $5x$. So let's multiply both sides by $5x$ and simplify:

$$5x \times \frac{3}{5} = 5x \times \frac{6}{x}$$

or $\;{}^{1}\cancel{5}x \times \frac{3}{\cancel{5}_1} = 5\cancel{x}^{1} \times \frac{6}{\cancel{x}_1}$

or $\;3x = 5 \times 6$

or $\;3x = 30$

or $\;\frac{3x}{3} = \frac{30}{3}$ or $x = 10$

Verification:

Put $x = 10$ in original equation:

We have $\;\frac{3}{5} = \frac{6}{x}$

So, $\;\frac{3}{5} = \frac{\cancel{6}^{3}}{\cancel{10}_5}$

or $\;\frac{3}{5} = \frac{3}{5}$

Hence verified, $x = 10$

Example 2. Solve: $\frac{5}{3x} = \frac{25}{27}$

Solution:

We will tackle this by inverting each fraction: $\frac{3x}{5} = \frac{27}{25}$

There is now a common denominator of 25. Multiply both sides by 25 and simplify to get:

$$\frac{3x}{5} \times 25 = \frac{27}{25} \times 25$$

or $\;\frac{3x \times \cancel{25}^{5}}{\cancel{5}_1} = \frac{27 \times \cancel{25}^{1}}{\cancel{25}_1}$

or $\;15x = 27$ or $\frac{\cancel{15}^{1}x}{\cancel{15}_1} = \frac{27}{15}$

or $\;x = \frac{27}{15} = \frac{\cancel{3}^{1} \times 9}{\cancel{3}_1 \times 5}$

so, $\;x = \frac{9}{5}$ or $x = 1\frac{4}{5}$

Verification:

Put $x = \frac{9}{5}$ in original equation:

We have, $\;\frac{5}{3x} = \frac{25}{27}$

$$\frac{5}{3 \times \frac{9}{5}} = \frac{25}{27}$$

$$\frac{5 \times 5}{3 \times 9} = \frac{25}{27}$$

$$\frac{25}{27} = \frac{25}{27}$$

Hence verified, $x = 1\frac{4}{5}$

Example 3. Solve: $\frac{19x}{7} = \frac{57}{49}$

Solution:

The common denominator of 7 and 49 is 49. Multiplying both sides by 49 and simplifying:

$$49 \times \frac{19x}{7} = 49 \times \frac{57}{49}$$

or $$\frac{^{7}\cancel{49} \times 19x}{\cancel{7}_1} = \frac{\overset{1}{\cancel{49}} \times 57}{\underset{1}{\cancel{49}}}$$

So, $7 \times 19x = 57$

Dividing both sides by 19: $\frac{7 \times \overset{1}{\cancel{19}}x}{\cancel{19}_1} = \frac{\overset{3}{\cancel{57}}}{\underset{1}{\cancel{19}}}$

or $7x = 3$

or $\frac{\cancel{7}x}{\underset{1}{\cancel{7}}} = \frac{3}{7}$

or $x = \frac{3}{7}$

Verification:

Put $x = \frac{3}{7}$ in original equation

We have, $\frac{19x}{7} = \frac{57}{49}$

or $\frac{19 \times \frac{3}{7}}{7} = \frac{57}{49}$

or $\frac{19 \times 3}{7 \times 7} = \frac{57}{49}$

or $\frac{57}{49} = \frac{57}{49}$

Thus, $x = \frac{3}{7}$

Hence verified, $x = \frac{3}{7}$

EXERCISE 9.1

A. Solve the following equations.

1. $x + 5 = 9$
2. $12 - x = 7$
3. $5x = 3$
4. $4x + 10 = 2$
5. $5 - 3x = -4$
6. $2 + 14x = 30$
7. $9 + 5x = 3x + 13$
8. $4 - 3x = 8 + x$
9. $4x + \frac{5}{2} = x + 4\frac{1}{2}$

B. Solve the following equations.

1. $5 + 3(x - 1) = 5x - 6$
2. $6 - 4(x + 3) = 2(x - 1)$
3. $5(3 - x) - 2(4 - 3x) = 11 - 2(x - 1)$
4. $5(1 - 2x) + 2(3 - x) = 3(x + 4) + 14$
5. $(x + 2)(x + 3) = (x - 3)(x - 2) + 20$

C. Solve the following equations.

1. $6x + 2 = 29 - 3x$
2. $\frac{1}{3}x + 4 = \frac{4x - 1}{5}$
3. $\frac{3x}{4} = \frac{2}{5}$
4. $\frac{8}{x} = 2$
5. $\frac{7}{3x} = 2$
6. $\frac{3}{x+1} = \frac{6}{5x-1}$

D. Solve the equations.

1. $5 + \frac{x}{3} = 7$
2. $\frac{x}{2} - 1 = 5$
3. $\frac{3x}{4} - 2 = \frac{x}{3} + 3$
4. $4 - \frac{2x}{3} = \frac{x+2}{5}$
5. $\frac{x+2}{3} = \frac{1-2x}{5}$
6. $\frac{4x+3}{3} = \frac{x+7}{2}$
7. $\frac{5x+1}{2} - \frac{x+2}{6} = \frac{2x+4}{3}$
8. $\frac{x-3}{3} - \frac{7}{2} = \frac{4x-3}{3} - \frac{2}{3}$

(ii) Solve linear equations of the type.

- $ax + b = c$
- $\frac{ax+b}{cx+d} = \frac{m}{n}$

We solve above mentioned types of linear equations through examples.

Example 1. Solve: $5x + 10 = 25$

Solution:

$$5x + 10 = 25$$

Subtract 10 from both sides.

$$5x + 10 - 10 = 25 - 10$$

or $5x = 15$

Dividing by 5 on both sides.

$$\frac{\cancel{5x}^{1}}{\cancel{5}_{1}} = \frac{\cancel{15}^{3}}{\cancel{5}_{1}}$$

or $x = 3$

Verification:

Put $x = 3$ in original equation

$$5x + 10 = 25$$

$$5 \times (3) + 10 = 25$$

$$15 + 10 = 25$$

$$25 = 25$$

Thus, $x = 3$

Hence verified, $x = 3$

Example 2. Solve: $\dfrac{2x+4}{5x+6} = \dfrac{1}{2}$

Solution: We have, $\dfrac{2x+4}{5x+6} = \dfrac{1}{2}$

By cross multiplication

$2(2x+4) = 1(5x+6)$

or $4x + 8 = 5x + 6$

Subtract $4x$ from both sides.

$\cancel{4x} + 8 - \cancel{4x} = 5x + 6 - 4x$

or $8 = x + 6$

or $8 - 6 = x$

or $2 = x$ or $x = 2$

Verification:

Put $x = 2$ in original equation

We have $\dfrac{2x+4}{5x+6} = \dfrac{1}{2}$

$\dfrac{2(2)+4}{5(2)+6} = \dfrac{1}{2}$

or $\dfrac{4+4}{10+6} = \dfrac{1}{2}$

or $\dfrac{\overset{1}{\cancel{8}}}{\cancel{16}_2} = \dfrac{1}{2}$

or $\dfrac{1}{2} = \dfrac{1}{2}$

Thus, $x = 2$

Hence verified, $x = 2$

EXERCISE 9.2

Solve and verify the following equations.

(1)	$3x + 9 = 18$	(2)	$2x - 5 = 9$	(3)	$4x - 5 = 11$
(4)	$5x - 20 = 30$	(5)	$3x + -17 = 2$	(6)	$13 - 3x = -2$
(7)	$4x + 8 = 2$	(8)	$\dfrac{x+3}{x-1} = \dfrac{6}{7}$	(9)	$5x + 1 = \dfrac{6}{5}$
(10)	$\dfrac{x-1}{x+2} = \dfrac{4}{3}$	(11)	$\dfrac{2x+3}{7x+1} = \dfrac{7}{15}$	(12)	$\dfrac{3x-8}{5x+2} = 1$

9.2.3 Solve real life problems involving linear equation.

Problem Solving Strategy for solving Linear Equation.

Step 1: Read the problem carefully. Identify what you are trying to find unknown and determine what information is available to help you find it.

Step 2: Choose a variable to assign to an unknown quantity in the problem.

Step 3: Use the relationships among the known and unknown quantities to form an equation.

Step 4: Solve the Equation: Determine the value of the variable and use the result to find any other unknown quantities in the problem.

Step 5: Verify your answer.

Step 6: Write a clear statement that is the answer of question.

Example 1. Teacher asks Sania to divide a 25 metres long wire in two pieces, such that one piece is 1 metre longer than the other. Find the length of each piece.

Solution:

Let the length of two pieces be x and $x + 1$ metres respectively.

So Sania can set up the following linear equation:

Given that $x + (x + 1) = 25$

$x + x + 1 = 25$

or $2x + 1 = 25$

or $2x + 1 - 1 = 25 - 1$

or $2x = 25 - 1 = 24$

or $x = \frac{\cancel{24}^{12}}{\cancel{2}_1} = 12$

Verification:

Length of one piece is $13m$

Length of another piece is $12m$

Total length of wire is $13m + 12m = 25m$

Hence verified

Hence one piece of wire is 12 metres long.

So the other piece of wire is $x + 1 = 12 + 1 = 13$ metres long.

Therefore, the two pieces of wire are 12 metres and 13 metres respectively.

Example 2. 20 years from now, Nazia will become three times as old as she is now. Find her present age.

Solution:

Let the present age of Nazia be: x years old now.

20 years from now, she will be: $x + 20$ years.

But after 20 years from now, she become thrice of her present age, i.e. $3x$ years.

So, $x + 20 = 3x$, or $3x = x + 20$

Hence $3x - x = 20$

or $2x = 20,$

or $x = \frac{\cancel{20}^{10}}{\cancel{2}_1} = 10.$

Verification:

Present age of Nazia is 10 years

After 20 years she becomes:

10 + 20 = 30 years

which is thrice of her present age

i.e. $3 \times 10 = 30$ years

Hence verified

Therefore the present age of Nazia is 10 years.

Example 3. The sum of two consecutive odd number is 36. Find the numbers.

Solution: Let one odd number is x.

Then the consecutive next odd number will be $x + 2$.

Now their sum is 36.

So, the required linear equation is

$x + (x + 2) = 36$

or $x + x + 2 = 36$

or $2x + 2 = 36$

or $2x + 2 - 2 = 36 - 2$

or $2x = 34$

or $\frac{2x}{2} = \frac{\cancel{34}^{17}}{\cancel{2}_1} = 17$

or $x = 17$

Verification:

The first odd number, we get is 17.

The next consecutive odd number is 19.

Their sum is 17 + 19 = 36

Hence verified

Therefore first odd number is 17.

And the next consecutive odd number will be $x + 2 = 17 + 2 = 19$

Thus, the required odd numbers are 17 and 19.

EXERCISE 9.3

1. Aslam had Rs 35 in his pocket. He purchased three pencils. Still he has Rs 5 in his pocket. Find the price of pencils.

2. Ali has some amount of rupees. If Rs 7 is added to thrice of that amount it makes Rs 22. What amount is with him?

3. The length of a room is $1.5m$ more than its breadth. If the perimeter of the room is $63m$, find the length and breadth of the room.

4. The perimeter of a triangle ABC is 11 cm. Sides $\overline{AB}$ and $\overline{BC}$ differ in length by 1cm and the sides $\overline{AC}$ and $\overline{BC}$ differ in length by 3 cm. Find the length of each side.

5. A 32 cm long wire is bent to form a square. What is the length of each side of the square?

6. Hassan and Asghar had 800 rupees. Hassan had four times as many as Asghar had. How many rupees did each of Hassan and Asghar have?

7. Ask your friend to think of a number. Add 3 to it and then multiply by 2. Now subtract 6 from it and then divide the result by 2. What is the number? Did he get the same number that he initially thought of?

8. Which of the two consecutive numbers add up to give 43?

9. What are three consecutive whole numbers, whose sum is 48?

10. The sum of one-third and one fourth of an amount is Rs 14. What is the amount?

11. The sum of two numbers is 9. The difference between five times of the first and four times of the second is 9. What are the numbers?

12. The sum of four consecutive odd numbers is 120. What are the numbers?

REVIEW EXERCISE 9

1. **Write linear equation of the following:**
   (i) Sum of two consecutive even numbers is 42.
   (ii) Sum of three consecutive odd numbers is 21.
   (iii) Sum of four consecutive natural numbers is 46.

2. **Write three different linear equations with different variables in different forms.**

3. **What is meant by the solution of an equation? Give two examples.**

4. **Fill in the blanks.**
   (i) The equation which contains a single variable with the degree 1 is called the linear equation in _______________.
   (ii) A solution is also called a _______________ of the equation.
   (iii) Process of finding the value of a variable is to make a sentence true is called its _______________.
   (iv) Addition of the _______________ to both sides of linear equation does not affect its equality.

5. **Tick (√) the correct answer.**
   (i) A linear equation consist of polynomial of degree.
   (a) one (b) two (c) three (d) four
   (ii) $1\frac{1}{2}x = 1$, then $x = ?$
   (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{2}{3}$
   (iii) The equation $2x + 1 = 6$ is a linear equation with degree.
   (a) 1 (b) 2 (c) 3 (d) 4
   (iv) The solution of $1 + 6x = 7$ is:
   (a) $x = 0$ (b) $x = 1$ (c) $x = 2$ (d) $x = 3$

6. Solve the following equations.

(i) $\frac{x}{2} - 4 = 2$ (ii) $\frac{x-7}{4} = 3$ (iii) $\frac{x-5}{2}$

(iv) $\frac{x+1}{x-3} = \frac{5}{3}$ (v) $\frac{x-2}{x-1} = \frac{3}{4}$ (vi) $\frac{x+3}{3} = \frac{x+6}{2}$

7. A 120 *cm* long wire is given the shape of a rectangle such that its length is 10 *cm* more than the breadth. Find the length and breadth of the rectangle.

SUMMARY

- An equation which contains a single variable with the exponent '1' is called the Linear Equation in one variable. For example $ax + b$=c or $2x + 3 = 4$ etc.

- The value of the variable that makes the equation a true sentence is called the solution of the equation.

  For example: $2x + 3 = 5$

  Then $2x = 5 - 3 = 2$

  or $2x = 2$, then $x = 1$, is its solution.

- A number can be added, subtracted, multiplied and divided on both the sides of an equation and it does not affect the equality of the equation. But in case of division the number should be non zero.

- There are four steps for solving a real life problem, involving linear equations.

  (i) What is the required thing?

  (ii) Represent the required thing by a variable?

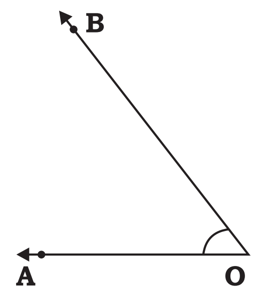
  (iii) Write an equation according to the statement.

  (iv) Solve the equation and verify the solution.

Unit 10

FUNDAMENTALS OF GEOMETRY

10.1 PROPERTIES OF ANGLES



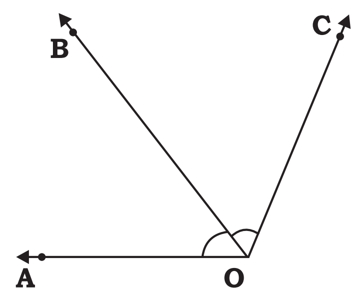
Look at the figure of an angle AOB.

It is formed by two non-collinear rays, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ with a common end point O.

Point O is called vertex. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are sides or arms of $\angle AOB$.

10.1.1 Define Adjacent, Complementary and Supplementary angles.

(i) Pair of adjacent angles.



Look at the adjoining figure.

$\angle AOB$ and $\angle BOC$ are formed by

$\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OB}$, $\overrightarrow{OC}$ respectively originating from the point O.

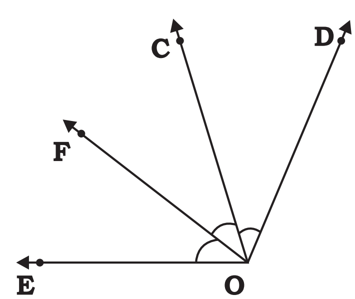
We can observe:

Point O is the common vertex of both the angles: $\angle AOB$ and $\angle BOC$.

$\overrightarrow{OB}$ is the common arm of these angles.

$\angle AOB$ and $\angle BOC$ have no any other common point in their interiors.

Thus $\angle AOB$ and $\angle BOC$ is a pair of **adjacent angles**.



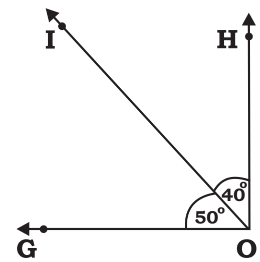
Now look at the figure.

It has four pairs of adjacent angles.

We can write the names of four pairs of adjacent angles; as follows:

($\angle EOF$, $\angle COF$), ($\angle COF$, $\angle COD$), ($\angle EOF$, $\angle FOD$) and ($\angle EOC$, $\angle COD$).

(ii) Complementary angles.



If the sum of the measures of two angles is 90°, they are said to be **complementary angles**. Each one is said to be the complement of the other.

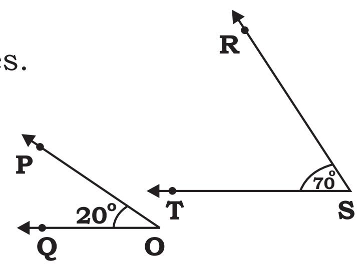
From figure, we see that:

$m\angle GOI + m\angle HOI = 50^\circ + 40^\circ = 90^\circ$

Therefore, ∠GOI and ∠HOI are complementary angles.

And ∠GOI is complement of ∠HOI

or ∠HOI is complment of ∠GOI.



Now look at the figure.

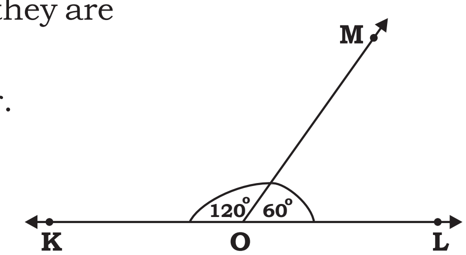
There are two angles such that $m\angle POQ = 20^\circ$ and $m\angle RST = 70^\circ$.

As the sum of $m\angle POQ$ and $m\angle RST = 20^\circ + 70^\circ = 90^\circ$.

Therefore ∠POQ and ∠RST are complementary angles.

(iii) Supplementary angles.

If the sum of measures of two angles is 180°, they are said to be **supplementary angles**.



Each one is said to be supplement of the other.

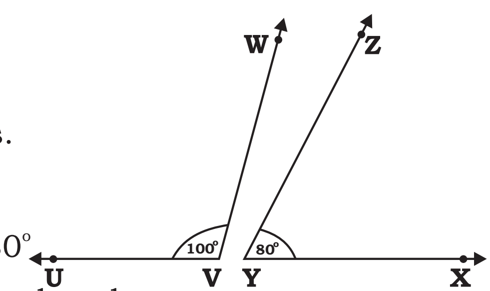
From figure, we see that

$m\angle MOK + m\angle MOL = 120^\circ + 60^\circ = 180^\circ$.

Hence ∠MOK and ∠MOL are supplementary angles.

And ∠MOK is supplement of ∠MOL.

or ∠MOL is supplement of ∠MOK.



Again look at the figure. There are two angles.

$m\angle UVW = 100^\circ$ and $m\angle XYZ = 80^\circ$

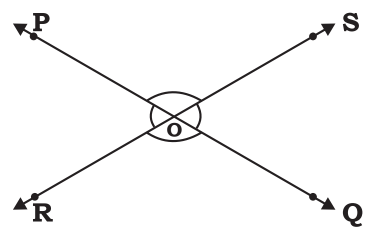
The sum of these two angles = $100^\circ + 80^\circ = 180^\circ$

Therefore both are supplementary angles and each one is supplement of the other.

10.1.2 Define Vertically opposite angles.

Look at the figure.

$\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ intersect each other at point O.



Four angles are formed.

$\angle$POS, $\angle$QOR, $\angle$POR and $\angle$QOS.

We can observe that:

(i) Point O is the common vertex of four angles.

(ii) The arms of $\angle$POS and $\angle$QOR are opposite rays.

(iii) Arms of $\angle$POR and $\angle$QOS are opposite rays.

(iv) $\angle$POS and $\angle$QOR is a pair of vertically opposite angles and these are equal in measure.

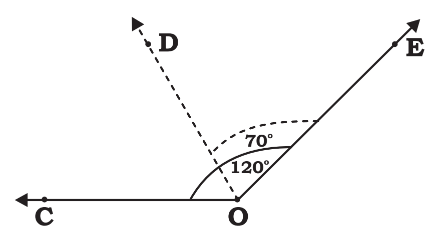
(v) Similarly $\angle$POR and $\angle$QOS is another pair of vertically opposite angles and these are equal in measure.

10.1.3 Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.

We have learnt the definition of adjacent, complementary, supplementary and vertically opposite angles. Now we can calculate their unknown angles.

Example 1. Look at the figure. Write the names of a pair of adjacent angles.

Also calculate the unknown angle in it.



Solution: Looking into the figure, it is observed that $\angle$COD and $\angle$EOD is a pair of two adjacent angles. It is also given that $m\angle EOD = 70^\circ$, $m\angle COE = 120^\circ$ and $m\angle COE = m\angle COD + m\angle EOD$

or $120^\circ = m\angle COD + 70^\circ$

or $m\angle COD = 120^\circ - 70^\circ$

or $m\angle COD = 50^\circ$

Thus the unknown angle ($m\angle COD$) is 50°.

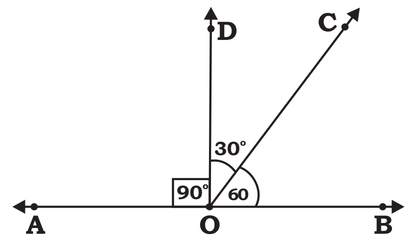
Example 2. Look at the figure.

Write the names of the:

(i) Pair of adjacent angles.

(ii) Pair of complementary angles.

(iii) Pair of supplementary angles.



Solution: Looking into the figure, it is observed that:

(i) There are four pairs of adjacent angles ... ($\angle$AOC, $\angle$BOC), ($\angle$COD, $\angle$BOC), ($\angle$AOD, $\angle$BOD), ($\angle$AOD, $\angle$COD).

(ii) There is one pair of complementary angles ($\angle$COD, $\angle$BOC).

(iii) There are two pairs of supplementary angles ($\angle$AOD, $\angle$BOD) and (AOC, $\angle$BOC).

Activity: Find remaining angles when:

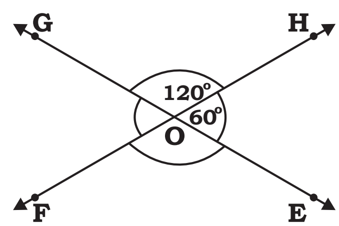
$m\angle HOG = 120^\circ$ and $m\angle EOH = 60^\circ$

Therefore $m\angle FOG$ = ________, because

$m\angle EOH$ = ________ (Vertically opposite angles)

Similarly $m\angle EOF$ = ______, because

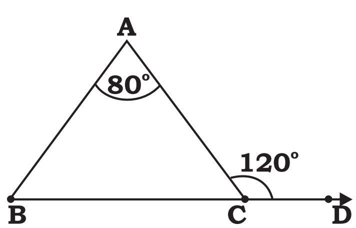
$m\angle EOF$ = ________ (Vertically opposite angles)



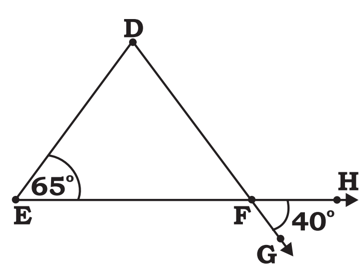
10.1.4 Calculate unknown angles of a triangle.

We have learnt that the sum of the measures of three angles of a triangle is always equal to 180°. Using the above sum property of a triangle, we can find the unknown angles of a triangle.

Example. Find the missing angles in the given figure.



Activity: Find missing angles.



Solution:

In the given figure $m\angle ACD = 120^\circ$, $m\angle BAC = 80^\circ$.

We have to find.

(a) $m\angle ABC = ?$ **(b)** $m\angle ACB = ?$

From the figure, it is observed. $m\angle ACB + m\angle ACD = 180^\circ$

(Supplementary angles).

But $m\angle ACD = 120^\circ$ (Given)

So, $m\angle ACB = 180^\circ - m\angle ACD$

$= 180^\circ - 120^\circ = 60^\circ$...(i)

Also,

$m\angle ABC + m\angle BAC + m\angle ACB = 180^\circ$ (Sum of angles of a Δ is 180°)

Hence,

$m\angle ABC + m\angle BAC + m\angle ACB = 180^\circ$

So, $m\angle ABC + 80^\circ + 60^\circ = 180^\circ$

or $m\angle ABC + 140^\circ = 180^\circ$

or $m\angle ABC = 180^\circ - 140^\circ = 40^\circ$...(ii)

Thus, $m\angle ABC = 40^\circ$

and $m\angle ACB = 60^\circ$

Let us solve the activity:

From the figure

$m\angle DEF =$ _____. It is also clear from the figure that $\overrightarrow{DG}$ and $\overrightarrow{EF}$ intersect each other at point ____ and form two vertically opposite angles:

_______ and ________

So $m\angle DFE =$ _______

As $m\angle HFG =$ ________

Therefore, $m\angle DFE =$ _______ (i)

Sum of the angles of a triangle = _____

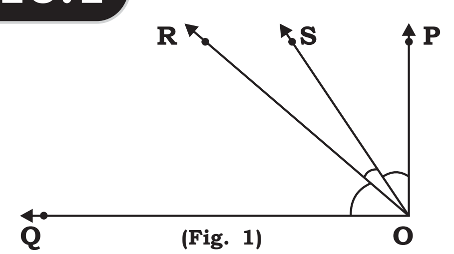
i.e., $m\angle DEF + m\angle DFE + m\angle EDF =$ ____

or $65^\circ + 40^\circ + m\angle EDF =$ ____

or $m\angle EDF = 180^\circ - (65^\circ + 40^\circ) =$ ___ (ii)

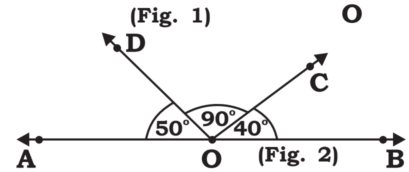
EXERCISE 10.1

1. Write the names of all the adjacent angles, shown in Fig. 1.



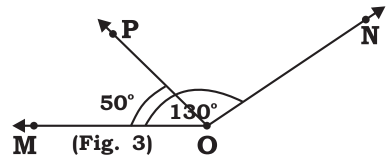
(Fig. 1)

2. Write pairs of all the supplementary angles from Fig. 2.



(Fig. 2)

3. In Fig. 3, $m\angle MON = 130^\circ$ and $m\angle MOP = 50^\circ$. Find the measure of adjacent angle $\angle PON$.



(Fig. 3)

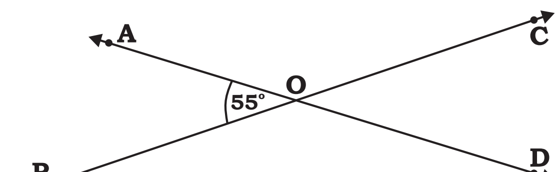
4. Write the measures of the complement of the following:

Measure of the given angle.	49°	62°	54°	36°	67°	81°	15°
Measure of its complement.							

5. Write the measure of the supplement of the following supplementary angles.

Measure of the given angle.	80°	132°	49°	76°	125°	103°	68°
Measure of its supplement.							

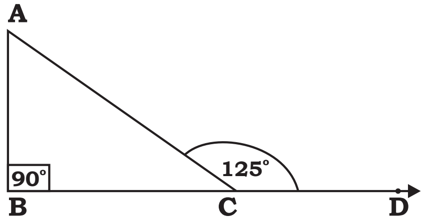
6. Look at the Fig. 4, write the measures of all the pairs of vertically opposite angles.



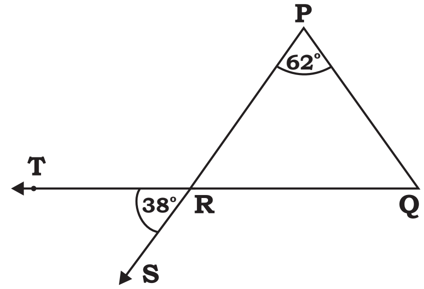
(Fig. 4)

7. Find measurement of the unknown angles in the following triangles.

(i)



(ii)



8. Choose the correct answer and fill in the blanks.

(i) If an angle is equal to its complement, then its measure is ___________. (90°, 45°, 180°)

(ii) If an angle is equal to its supplement, then its measure is ________. (45°, 90°, 180°)

(iii) Supplement of an acute angle is ________ angle. (an acute, a right, an obtuse)

(iv) If an angle of a pair of supplementary angles is a right angle, then the other is ________ angle. (An acute, a right, an obtuse).

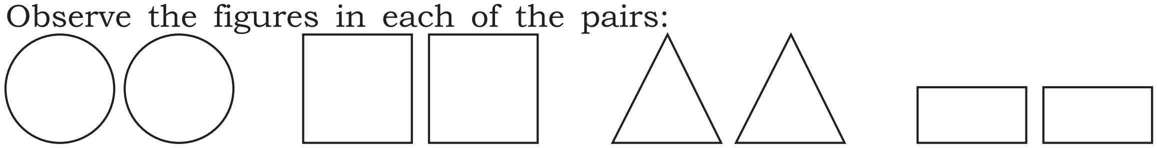
(v) Each angle of the pair of complementary angles is ________ angle. (acute, obtuse, right)

(vi) One of four angles formed by two lines intersecting each other is a right angle. Therefore the other angles are ________ angles. (acute, right, obtuse).

10.2 CONGRUENT AND SIMILAR FIGURES

10.2.1 Identify congruent and similar figures

Observe the figures in each of the pairs:

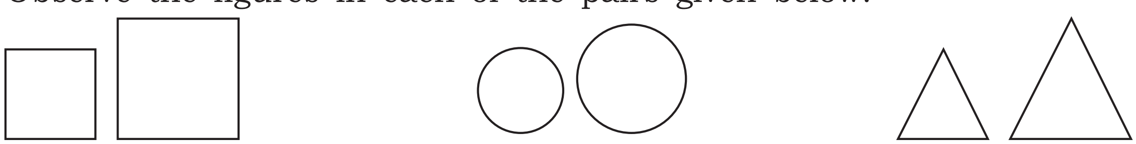


Circles **Squares** **Triangles** **Rectangles**

Congruent Figures: That have exactly the same size and same shape are called congruent figures, they need not be identical. The may have different colours or textures.

Example: All the squares, rectangles, circles and equilateral triangles having exactly the same size and same shape are congruent figures.

Observe the figures in each of the pairs given below:



Squares **Circles** **Equilateral Triangles**

Similar Figures: That have exactly the same shape but not necessarily the same size are called similar figures.

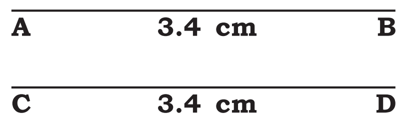
Example: All squares, circles and equilateral triangles are examples of similar figures.

10.2.2 Recognize the symbol of Congruency:

(i) **Congruent Figures:** Two or more geometrical figures are said to be congruent if these have exactly the same shape and same size.

The symbol '≅' stands for 'is congruent to'.

For example: (i) Two line segments are congruent if their lengths are same

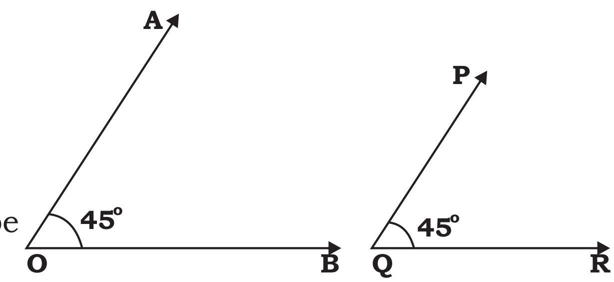


Here $m\overline{AB} = 3.4$ cm $= m\overline{CD}$.

Both have exactly the same shape and same size.

Then $m\overline{AB} = m\overline{CD}$ and $\overline{AB} \cong \overline{CD}$.

(ii) Now consider angles:
$\angle AOB$ and $\angle PQR$
Here $m\angle AOB = 45^\circ$ and
$m\angle PQR = 45^\circ$
Both have exactly the same shape and same size.



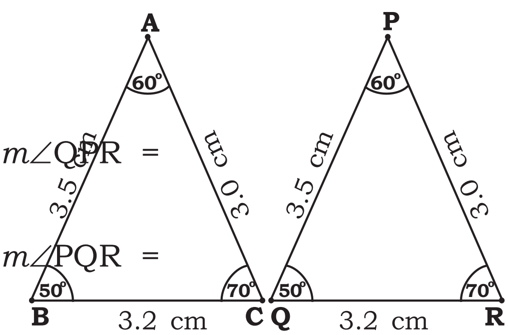
(iii) So, the above two angles are congruent i.e $\angle AOB \cong \angle PQR$

Again consider ΔABC and ΔPQR

Here:

$m\overline{AB} = m\overline{PQ} = 3.5$ cm $\quad m\angle BAC = m\angle QPR =$ — 60° —

$m\overline{BC} = m\overline{QR} = 3.2$ cm $\quad m\angle ABC = m\angle PQR =$

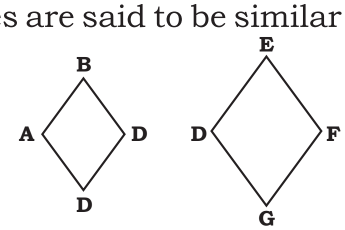


Both the triangle have exactly same shape and same zide.

Hence $\Delta ABC \cong \Delta PQR$

(iii) Similar Figures: Two or more geometrical figures are said to be similar if their shape is same.

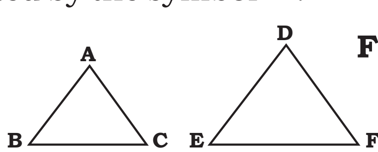
The similarity of the geometrical figures is represented by the symbol '~'.



For example.

Quad. ABCD ~ Quad. DEFG

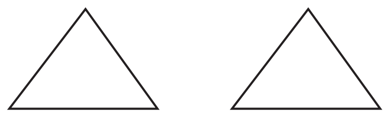
A —— B
C ——— D

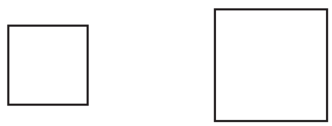


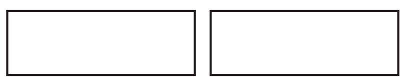
or $\overline{AB} \sim \overline{CD}$

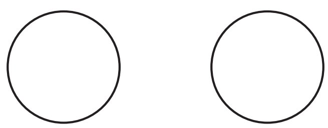
EXERCISE 10.2

1. Write congruent or similar figures in the following.

i.  ______________________

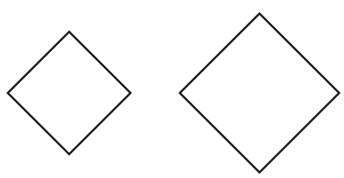
ii.  ______________________

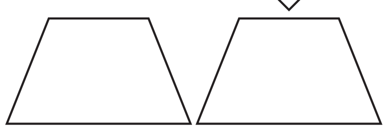
iii.  ______________________

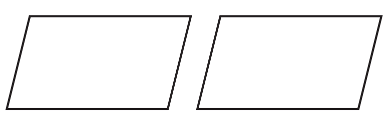
iv.  ______________________

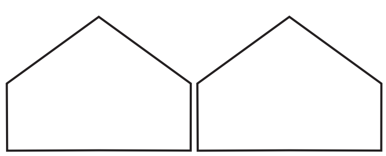
v. ______________________

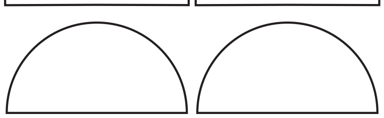
3 cm 5 cm

vi.  ______________________

vii.  ______________________

viii.  ______________________

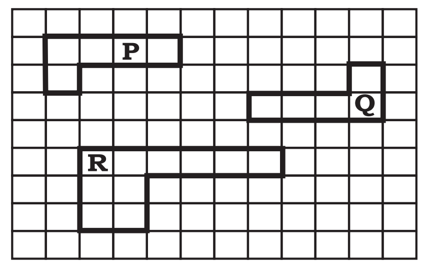
ix.  ______________________

x.  ______________________

2. Draw the following geometrical shapes.

(i) Draw two rectangles of different sizes. Are they similar?
(ii) Draw two circles of different radii. Are they congruent?
(iii) Draw two equilateral triangles of same measures. Are they similar?
(iv) Draw two squares of different size. Are they congruent?
(v) Draw two rectangles of same size. Are they congruent?

3. Look at the graph and consider figure P, Q and R and answer the following:



(i) Has figure P the same shape as figure R?
(ii) Has figure P the same size as figure R?
(iii) Are they congruent?.
(iv) Are they similar?
(v) What is the relation for figures P and Q.

4. Write T for true and F for false.

(i) The picture of Rs 10 currency note has the same shape as the picture of Rs 100 currency note; so these are similar figures.
(ii) The picture of Rs 20 currency note has not same size as the picture of Rs 50 currency note; so these are not similar figures.
(iii) The picture of Rs 500 currency note is congruent to the picture of Rs 1000 currency note.
(iv) The picture of Rs 100 currency note is not congruent to the picture of Rs 50 currency note.
(v) The picture of Rs 20 currency note and picture of Rs 500 currency note are similar figures.

10.2.3 Apply the properties for two figures to be congruent or similar.

We have already discussed about the congruent and similar figures. We know that:

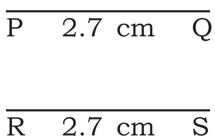
(i) Congruent figures have the same shape necessary also they must have the same size.
(ii) Similar figures have the same shape i.e same angles and same ratio of sides very necessary but not necessary the same size.

Now we use the above two properties for the figures to be congruent or similar.

Unit 10 FUNDAMENTALS OF GEOMETRY

Example 1. Look at the pair of figures. Identify them as congruent or similar.

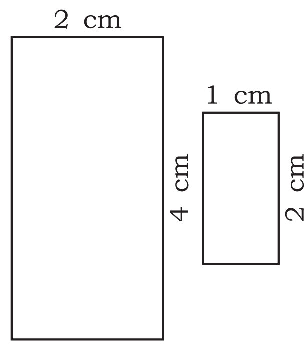
(i)



Both line segments have same shape and same size.

Therefore $\overline{PQ} \cong \overline{RS}$

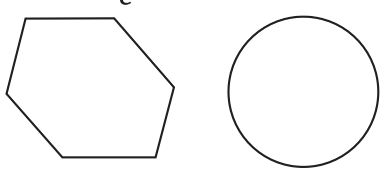
(ii)



Here both are rectangles. They have same shape but different sizes. They also have same ratio in the measure of lengths of the sides.

Therefore these are similar figures.

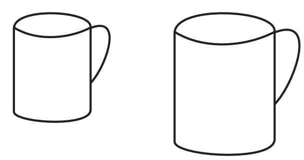
(iii)



Both these figures are of different shapes.

Therefore these figures are neither similar nor congruent.

(iv)



Both these figures represent to same type of object.

Both of them have almost same shape but have different sizes.

Therefore these are similar figures.

Example 2. Identfiy and write similar and congruent triangles.

Let us observe the following triangles.

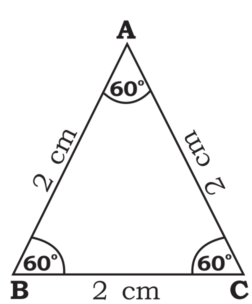


Fig. (i)

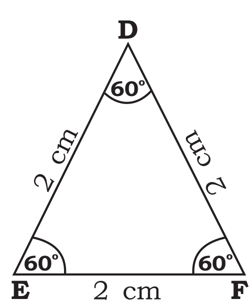


Fig. (ii)

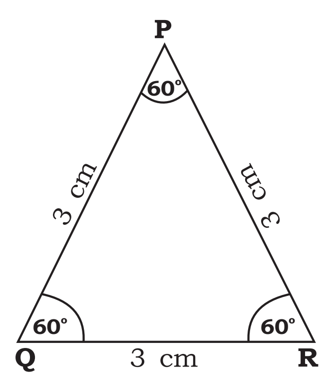


Fig. (iii)

Enlarged Triangle

In ΔABC and ΔDEF,

all the corresponding sides and angles are equal.

$m\overline{AB} = m\overline{DE}$, $m\overline{BC} = m\overline{EF}$ and

$m\overline{CA} = m\overline{FD}$

$m\angle ACB = m\angle DFE$,

$m\angle ABC = m\angle DEF$,

$m\angle BAC = m\angle EDF$

Therefore, $\Delta ABC \cong \Delta DEF$ read as ΔABC is congruent to ΔDEF.

Example 4.

Consider Fig (ii) and (iii) ΔDEF is original triangle and ΔPQR is triangle enlarged.

Therefore, ΔDEF is similar to ΔPQR.

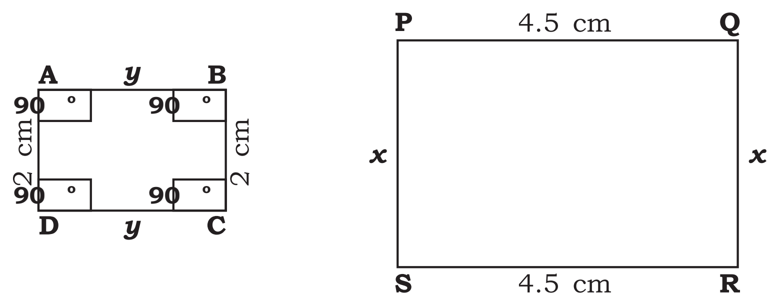
Here the corresponding angles are the same and the length of each side is increased by the same ratio. i.e the corresponding sides are in same proportion:

$$\frac{\overline{DE}}{\overline{PQ}} = \frac{\overline{EF}}{\overline{QR}} = \frac{\overline{FD}}{\overline{RP}} = \frac{2}{3}$$

Hence $\Delta DEF \sim \Delta PQR$

Read as Δ DEFis similar to ΔPQR.

If both rectangles are similar to one another, find the measure of unknowns, when the ratio in the measure of sides is $\frac{3}{2}$.



Solution: Since Rectangle ABCD ~ Rectangle PQRS.

Then: (i) The corresponding sides are proportional.

(ii) Corresponding angles are equal.

$m\angle P = m\angle A = 90^\circ$, $m\angle Q = m\angle B = 90^\circ$, $m\angle R = m\angle C = 90^\circ$, $m\angle S = m\angle D = 90^\circ$

Also $\quad \frac{m\overline{PQ}}{m\overline{AB}} = \frac{4.5}{y} = \frac{3}{2}$ (Corresponding ratio in sides)

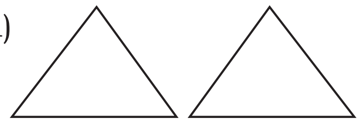
Therefore $y = \frac{(4.5) \times 2}{3} = \frac{9}{3} = 3$, Hence $m\overline{AB} = 3$ cm

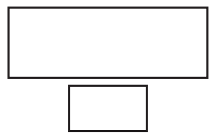
Again $\quad \frac{m\overline{QR}}{m\overline{BC}} = \frac{x}{2} = \frac{3}{2}$

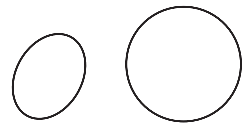
Therefore $x = \frac{2 \times 3}{2}$, then $x = 3$ or $m\overline{QR} = 3$ cm.

EXERCISE 10.3

1. Decide whether the following figures, given in each pair are congruent or similar or neither similar nor congruent.
(First three parts have been solved as under):

(i)


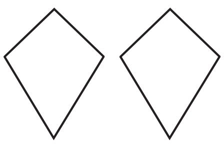
(ii)


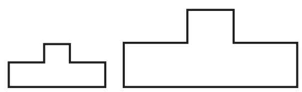
(iii)


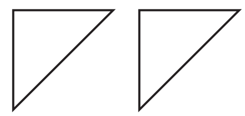
Solution
(i) Same shape, same size (Congruent).

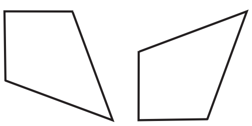
(ii) same shape, but not same size (Similar).

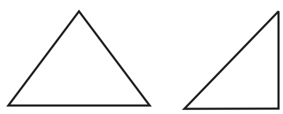
(iii) no same shape nor same size. Neither similar nor congruent

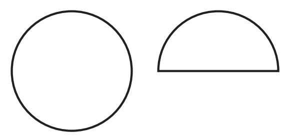
(iv)


(v)


(vi)


(vii)


(viii)


(ix)


2. Write in symbols the congruent segments from the following:

$m\overline{AB} = 3.2$ cm	$m\overline{CD} = 4.5$ cm
$m\overline{DE} = 5.1$ cm	$m\overline{GH} = 4.8$ cm
$m\overline{MN} = 4.5$ cm	$m\overline{XZ} = 5.1$ cm
$m\overline{YZ} = 4.8$ cm	$m\overline{PQ} = 5.4$ cm
$m\overline{RS} = 5.4$ cm	$m\overline{EF} = 3.9$ cm
$m\overline{YZ} = 3.9$ cm	$m\overline{XY} = 3.2$ cm

3. Write in symbol the congruent angles from the following.

$m\angle ABC = 60^\circ$	$m\angle DEF = 70^\circ$
$m\angle XYZ = 110^\circ$	$m\angle PQR = 45^\circ$
$m\angle COD = 45^\circ$	$m\angle PRQ = 60^\circ$
$m\angle AOB = 70^\circ$	$m\angle MNS = 110^\circ$

4. Fill in the blanks.

(i) $m\angle ABC = 100^\circ$, $\angle ABC \cong \angle DEF$, Therefore $m\angle DEF$ = ______

(ii) $m\angle PQR = m\angle XYZ$ Therefore $\angle PQR$ ______ $\angle XYZ$

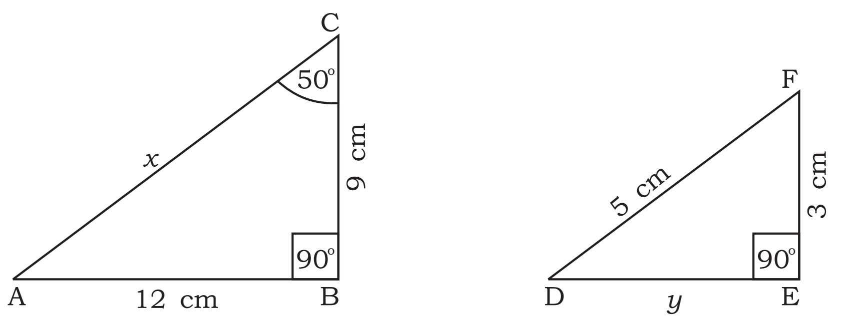
(iii) Line segments having equal lengths are called ______ segments.

(iv) Angles having ______ measures are called congruent angles.

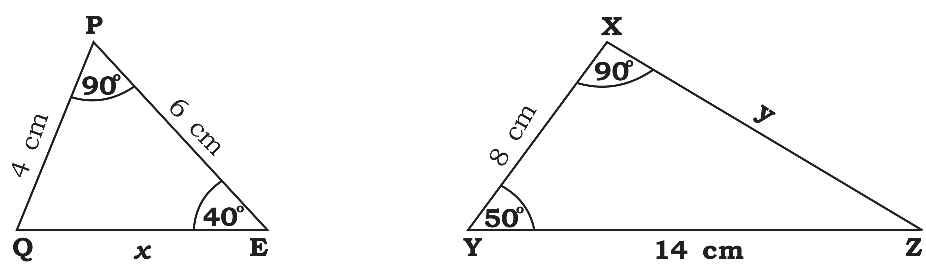
(v) Two circles are congruent if their ______ are equal.

5. Consider the following pair of similar figures. Find un-known elements *x* and *y* in the following:

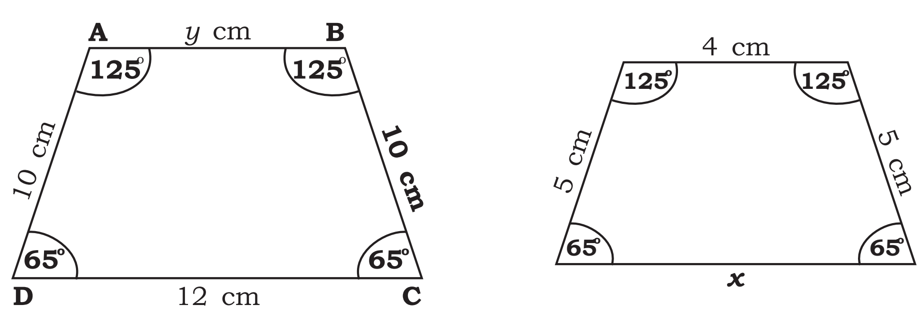
(i)



(ii)



(iii)



10.3 CONGRUENT TRIANGLES

Apply following properties for congruency between two triangles.

(i) SSS $\cong$ SSS

(ii) SAS $\cong$ SAS

(iii) ASA $\cong$ ASA

(iv) HS $\cong$ HS

There are four properties for congruency between two triangles.

10.3.1 First property for congruency of triangles (SSS $\cong$ SSS).

This property describes that all the three sides of one triangle are congruent to all the three corresponding sides of the other triangle, then both the triangles are congruent.

Consider two triangles PQR and XYZ.

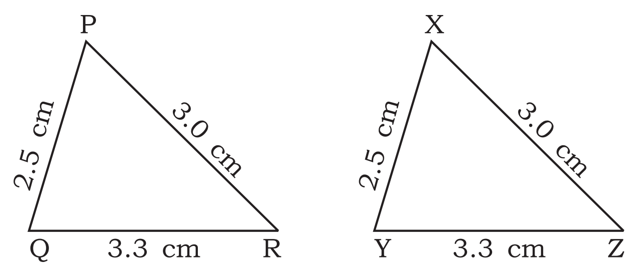
$\Delta PQR \longleftrightarrow \Delta XYZ$

i.e. ΔPQR has one-one correspondence with ΔXYZ, as under:

(i) $m\overline{PQ} = 2.5$ cm, $m\overline{XY} = 2.5$ cm
Therefore, $\overline{PQ} \cong \overline{XY}$

(ii) $m\overline{PR} = 3.0$ cm, $m\overline{XZ} = 3.0$ cm
Therefore, $\overline{PR} \cong \overline{XZ}$

(iii) $m\overline{QR} = 3.3$ cm, $m\overline{YZ} = 3.3$ cm
Therefore, $\overline{QR} \cong \overline{YZ}$
Hence, $\Delta PQR \cong \Delta XYZ$
(SSS $\cong$ SSS)



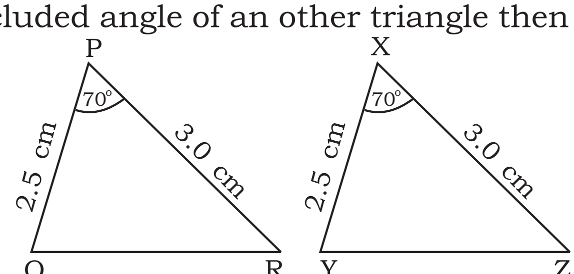
10.3.2 Second Property for congruency of triangles (SAS $\cong$ SAS)

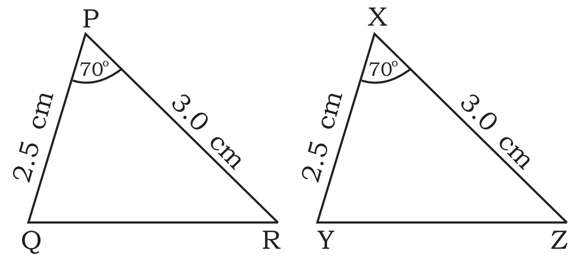
If that two sides and one included angle of a triangle are congruent to corresponding two sides and their included angle of an other triangle then both the triangles are congruent.

Consider two triangles PQR and XYZ.

$\Delta PQR \longleftrightarrow \Delta XYZ$

ΔPQR has one-one correspondence with ΔXYZ as under:





(i) $m\overline{PQ} = 2.5$ cm , $m\overline{XY} = 2.5$ cm,

Therefore, $\overline{PQ} \cong \overline{XY}$

(ii) $m\overline{PR} = 3.0$ cm, : $m\overline{XZ} = 3.0$ cm

Therefore, $\overline{PR} \cong \overline{XZ}$

(iii) $m\angle YXZ = 70^\circ$ and $m\angle QPR = 70^\circ$,

Therefore, $\angle QPR \cong \angle YXZ$

Then $\Delta PQR \cong \Delta XYZ$ (SAS $\cong$ SAS)

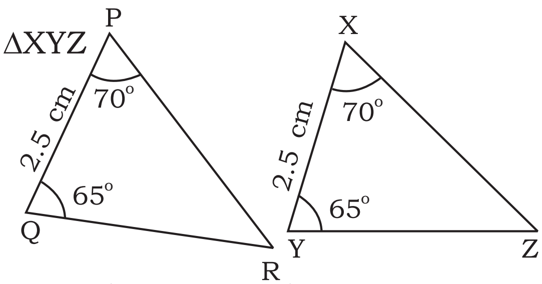
10.3.3 Third property for congruency of triangles (ASA $\cong$ ASA)

This property describes that two angles and one included side of a triangle are congruent to coressponding two angles and included side of another triangle, then both the triangles are congruent.

Consider two triangles PQR and XYZ

$\Delta PQR \longleftrightarrow \Delta XYZ$

ΔPQR has one-one correspondence with ΔXYZ



(ASA $\cong$ ASA)

(i) $m\angle PQR = 65^\circ$ and $m\angle XYZ = 65^\circ$

Therefore, $\angle PQR \cong \angle XYZ$

(ii) $m\overline{PQ} = 2.5$ cm, $m\overline{XY} = 2.5$ cm

Therefore, $\overline{PQ} \cong \overline{XY}$

(iii) $m\angle QPR = 70^\circ$ and $m\angle YXZ = 70^\circ$

Therefore, $\angle QPR \cong \angle XYZ$

Hence, $\Delta PQR \cong \Delta XYZ$ (ASA $\cong$ ASA)

10.3.4 Fourth Property for congruency of triangles. (HS $\cong$ HS)

If the Hypotenuse and one side (Base or Altitude) of right angled triangle are congruent to corresponding Hypotenuse and one side of the other right angled triangle, then the triangles are congruent.

Consider two right angled triangles PQR and XYZ.

$\Delta PQR \longleftrightarrow \Delta XYZ$

ΔPQR has one-one correspondence with ΔXYZ as under:

(i) $m\angle PQR = 90^\circ$, $m\angle XYZ = 90^\circ$ i.e. $m\angle PQR = m\angle XYZ$ = right angle.

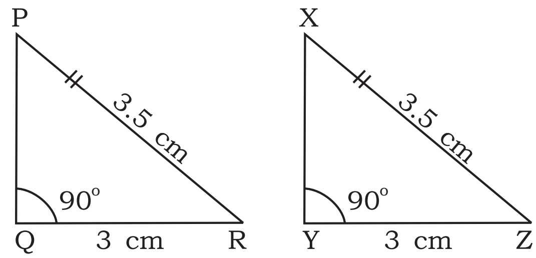
(ii) $m\overline{QR} = 3$ cm, $m\overline{YZ} = 3$ cm

Therefore: $\overline{QR} \cong \overline{YZ}$

(iii) $m\overline{PR} = 3.5$ cm, $m\overline{XZ} = 3.5$ cm

Therefore: $\overline{PR} \cong \overline{XZ}$

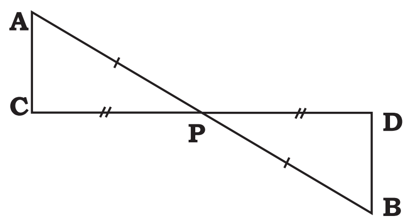
Then $\Delta PQR \cong \Delta XYZ$ (HS $\cong$ HS)



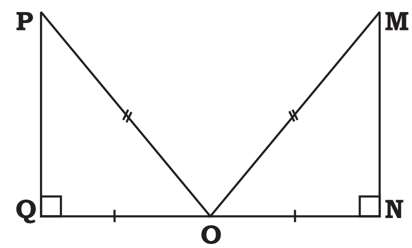
EXERCISE 10.4

In the figures below, the congruent parts of triangles have been indicated by like marking. Write the congruent parts in order and state by which property, the triangles are congruent.

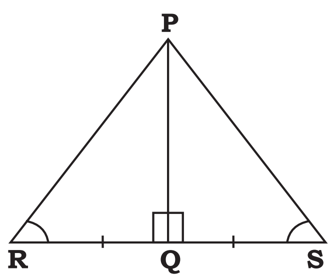
1.



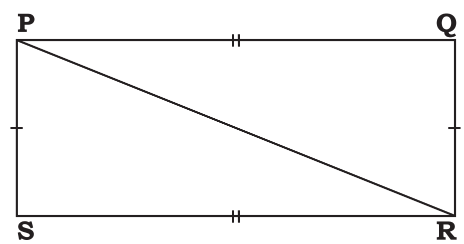
2.



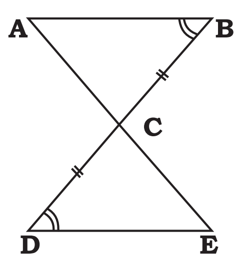
3.



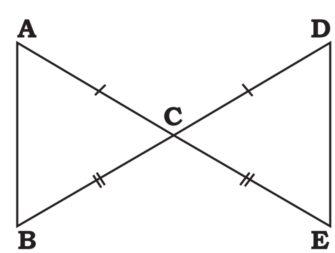
4.



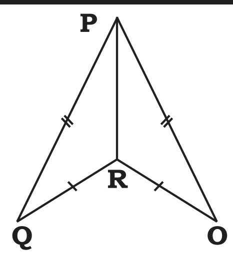
5.



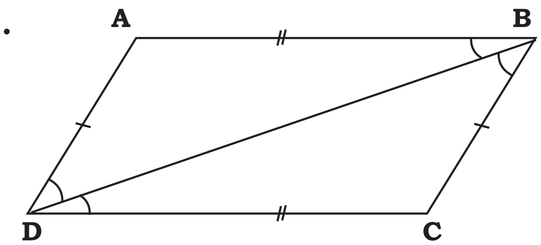
6.



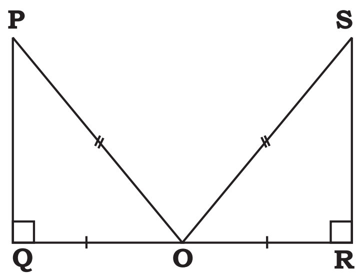
7.



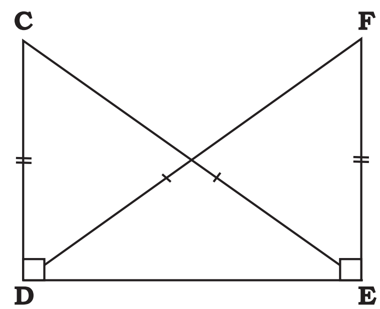
8.



9.



10.



10.4 CIRCLE

10.4.1 Describe a circle and its centre, radius, diameter, chord, arc, major and minor arcs, semi circle and segment of circle.

We have already learnt about a circle in previous classes.

It is most common and familiar shape of the Geometry. A bangle represents a circle, One rupee coin, Two rupee coin and Five rupee coin also represent circle.

(i) A circle and its centre

Take a point O on a paper. Open the pair of compasses and place the metallic end of the pair of compasses on the piont O and rotate the pencil arm. The figure, we get is that of a **circle**.

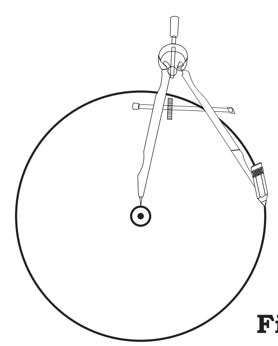


Fig (i)

The point O is called the **centre** of the circle.

Thus a circle consists of only those points which are at equal distance from a point called centre.

(ii) Radius of circle

In Fig. (i) the distance between the ends of the pair of compasses is called the radius of the circle.

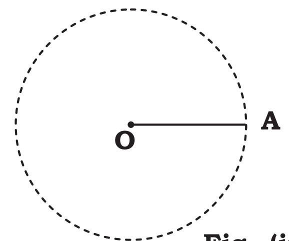


Fig. (ii)

In Fig. (ii), a circle is drawn with centre O. The length of the line segment joining the centre of a circle to any point on the circle is **radius** of the circle.

The term 'Radius of a circle' is used to denote a distance. For example $\overline{OA}$ is the radial segment of the circle or the radius is 2 cm.

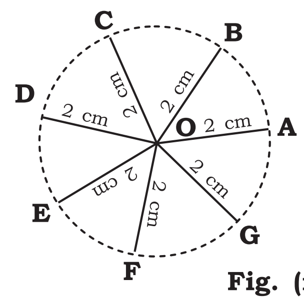


Fig. (iii)

Remember: A circle has an infinite number of radical segments. In Fig. (iii) all these radii are equal in length.

i.e. $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF} = m\overline{OG} = 2$ cm.

(iii) Diameter

A line segment joining any two points on the circle, passing through the centre of the circle is called **Diameter** of the circle. Diameter is also used to denote a line segment as well as distance.

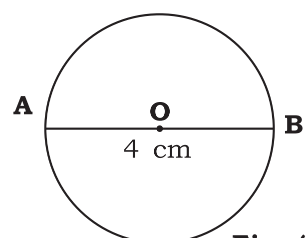


Fig. (iv)

For example. In Fig. (iv) $\overline{AB}$ is the diameter of the circle. Here diameter of the circle is a distance of 4 cm.

Remember: The diameter of a circle passes through its centre and its length is twice the length of the radius.

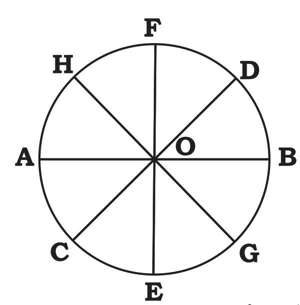


Fig. (v)

A circle has an infinite number of diameters and all these diameters are equal in length. In Fig. (v) $m\overline{AB} = m\overline{CD} = m\overline{EE} = m\overline{GH}$

(iv) Chord

A line segment joining any two points on the circle is called **chord** of the circle. In Fig. (vi) $\overline{RS}$ and $\overline{MN}$ are chords of the circle. $\overline{PQ}$, the diameter of the circle is also a chord.

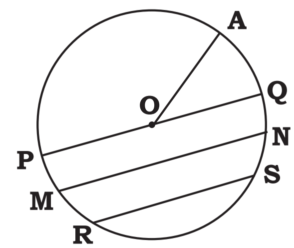


Fig. (vi)

Remember: A circle has an infinite number of chords. Every diameter is the longest chord of the circle.

(v) Arc

Consider a circle. In Fig. (vii) take any two points A and B on the circle. These two points divide the circle into two parts. Each of these parts is called an arc of the circle. Points A and B are the end points of each arc. Hence while naming an arc, we have to use three letters.

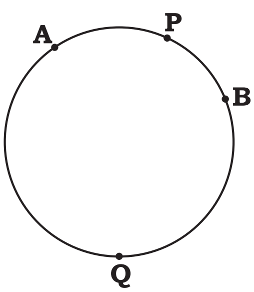


Fig. (vii)

In Fig. (vii) there are two arcs. (i) arc APB and
(ii) arc AQB.

Thus we get two types of arcs.

(a) Minor Arc (b) Major Arc.

Minor Arc: An arc which is smaller than half of the circle is called minor arc. In above figure $\overset{\frown}{APB}$ is minor arc or $\overset{\frown}{AB}$ is minor arc. Minor arc is named with two end points.

Major Arc: An arc which is more than half of the circle is major arc. In above figure $\overset{\frown}{AQB}$ is major arc.

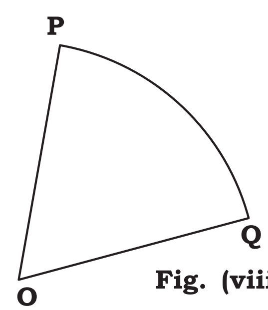


Fig. (viii)

Sector: Let us cut a piece of circular region. It will give us the curved shape as shown in Fig. (viii). This piece of circle is called **Sector**. It consists of an arc and two radii. i.e. $\overset{\frown}{PQ}$ is an arc, $\overline{OP}$ and $\overline{OQ}$ are two radii.

(vi) Semi-Circle.

Semi-circle means half of a circle.

In the adjoining figure, there is semi circle with $\overset{\frown}{PRQ}$.

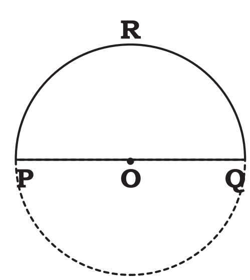


Fig. (ix)

(vii) Segment of a circle

The segment of a circle is a shape whose boundary is an arc of the circle and a chord of the circle.

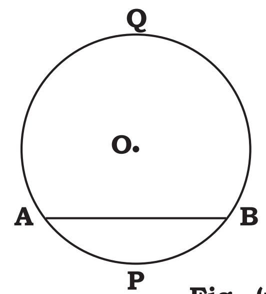


Fig. (x)

Example. A chord normally divides a circle into two segments of different sizes. The smaller segment is called the minor segment and the larger segment is called the major segment.

Look at the Fig (x). $\overline{AB}$ is a chord of the circle.

The chord $\overline{AB}$ divides the circle into two parts. Each part is a segment of the circle.

The segment bounded by the minor arc $\overset{\frown}{APB}$ and the chord $\overline{AB}$ is the minor segment of the circle.

The segment bounded by the major arc $\overset{\frown}{AQB}$ and the chord $\overline{AB}$ is the major segment of the circle.

EXERCISE 10.5

Fill in the blanks with proper words.

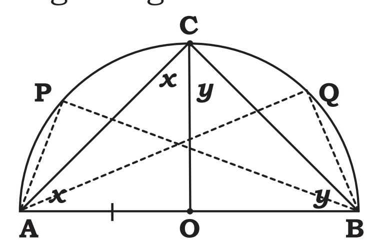
1. The segment joining any two points on the circle is called the ________ of the circle.
2. The distance between the centre of a circle and any point on the circle is called the ________ of the circle.
3. The segment joining the centre of a circle to any point on the circle is called the ________ of the circle.
4. All the points on a circle are equidistant from the ________.
5. A chord passing through the centre of a circle is called the ________ of the circle.
6. The diameter of a circle passes through its ________ and its length is ________ of the length of the radius.
7. The longest chord of a circle is the ________ of the circle.
8. There are ________ number of radial segments of any given circle and they are ________ in length.

9. The radius of a circle is ________ of diameter of the circle.

10. There are ________ number of diameters of any given circle and they are ________ in length.

10.4.2 Draw a semi-circle and demonstrate the property; the angle in a semi-circle is a right angle.

Lets us demonstrate: **'**The angle in a semi circle is a right angle'.



I. Draw a semi-circle with suitable radius.

Mark its centre as O.

Draw a diamter $\overline{AB}$ through the centre O.

II. Make diameter as one side of a triangle.

Draw other two sides from points A and B to meet at a vertex C on the circumference. Thus we get a ΔABC.

Now measure ∠ACB with the protractor; we will see $m\angle ACB = 90^\circ$ i.e. ∠ACB is a right angle.

III. Again take two points P and Q on the semi-circle. Draw triangles APB and AQB.

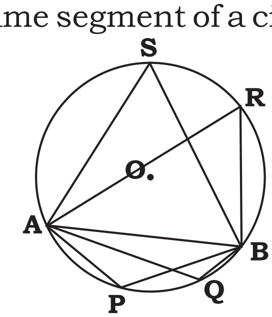
IV. Measure ∠APB and ∠AQB with the protractor. We will see that: $m\angle APB = m\angle AQB = 90^0$.

Thus the angle in a semi-circle is a right angle.

Hence proved that the angle in a semi-circle is a right angle.

10.4.3 Draw a segment of a circle and demonstrate the property, the angles in the same segment of a circle are equal.

Let us demonstrate the property: 'The angles in the same segment of a circle are equal'.



Steps of Demonstration:

I. Draw a circle with suitable radius.

II. Draw a chord $\overline{AB}$ of the circle, other than diameter.

III. We have two segments (i) Minor (ii) Major.

IV. In minor segment of the circle, draw two inscribed angles of the smaller segment $\angle$APB and $\angle$AQB with scale.

V. Now measure them, with protractor, we will see that:

$m\angle APB = m\angle AQB$

VI. Again draw two angles in the major segment of the circle, $\angle$ASB and $\angle$ARB, with scale.

VII. Measure them with the proctor, we will see that:

$m\angle ASB = m\angle ARB$

It is concluded that the angles in the same segment of a circle are equal.

EXERCISE 10.6

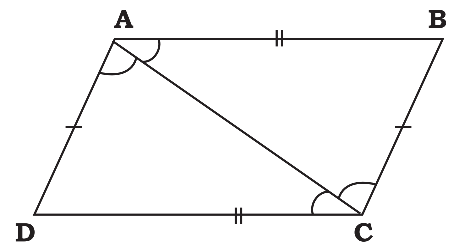
1. Draw two semi-circles of different radial segments say, 2.5 cm and 3 cm. Draw an angle in every semi-circle and verify it is a right angle.
2. Draw a semi-circle with radius 3.3 cm. Demonstrate the property of a semi-circle. 'The angle in a semi-circle is of measure 90°.'
3. Draw a circle of radius 2.7 cm, demonstrate the property: 'The angles in the same segment of a circle are equal.'
4. Draw a circle of radius 2.8 cm. Draw the angles in the same segment of a circle. Verify that these angles are equal in measure.
5. Draw a circle of radius 3.4 cm. Draw the angles in the same segment of a circle. Then verify that these angles are equal in measure.

REVIEW EXERCISE 10

1. Define adjacent angles and show the pair of adjacent angles in a figure.
2. Define complementary and supplementary angles. Show the pairs of complementary angles and supplementary angles in a figure.
3. What is vertically opposite angles. Show two pairs of vertically opposite angles in a figure.

4. What are similar figures? Show them by drawing the diagram and also show in symbolic form.
5. What are congruent figures? Show them by drawing the diagram and also show in symbolic form.
6. Draw figure of a circle in which show the centre, radius, diameter, chord, minor arc and major arc. Also show semi circle and segment of a circle.
7. ABCD is a parallelogram in which $\overline{AC}$ is a diagonal.

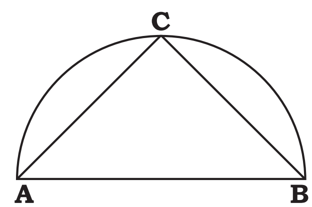
   State the various properties for congruency of the two triangles formed.



8. Draw a semi-circle of 3.3 cm radius.

   Demonstrate the property of a semi-circle that the angle in a semi - circle is a right angle.
9. Draw a semi-circle with radius 4.4 cm. Draw a ΔABC as shown. Measure three angles of the triangle.

   Is $m\angle ACB$ a right angle?



10. Draw a circle with radius 3.6 cm. Draw a segment of a circle and demonstrate the property: ' The angles made in the same minor segment of a circle are equal.'
11. Draw a circle with radius 4 cm. Cut the circle along a chord other than diameter. Demonstrate the property that 'The two angles in the major segment of the circle are equal'.

12. Fill in the blanks.

(i) The angles with common vertex and a common arm are called ______.

(ii) If sum of two angles is 90°, then these angles are called __________.

(iii) Two figures are similar, if they are __________ in __________.

(iv) If two lines intersect each other then __________ angles are always __________ in measure.

(v) In a right angled triangle the __________ and one side (base or altitude) of a triangle are congruent to the corresponding __________ and one side of the other right angled triangle; then the triangles are __________.

SUMMARY

- Two angles are said to be adjacent if they have a common vertex and a common arm.
- If the sum of the measure of two angles is 90°, they are said to be complementary angles. Each is said to be the complement of the other.
- If the sum of the measures of two angles is 180°, they are said to be supplementary angles. Each is said to be the supplement of the other.
- The angles opposite to the common vertex formed by the intersection of two lines are called vertical opposite angles.
- Sum of the three angles of a triangle is 180°. With the help of this property we can calculate unknown angles of a triangle.
- Geometrical figures which have exactly the same shape and same size are called congruent figures. The symbol used for congruent figures is $\cong$.
- Geometrical figures which have exactly the same shape are called similar figures. The symbol used for similar figures is $\sim$.
- Two triangles are said to be congruent if any of the following property hold. (a) SSS $\cong$ SSS (b) SAS $\cong$ SAS (c) ASA $\cong$ ASA and (d) HS $\cong$ HS.
- Circle is most familiar geometrical figure. It is traced on paper by a point moving in such a way that its distance from another fixed point on the plane is always constant.

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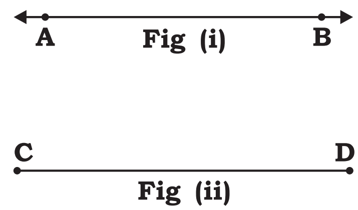
Unit 11

PRACTICAL GEOMETRY

11.1 LINE SEGMENT



We have learnt that a line can be extended to any length on both sides. Here in Fig. (i) AB is a line.

A line has no end points.

A portion of a line is a line segment. Here in Fig (ii) CD is a line segment.

A line segment has two end points.

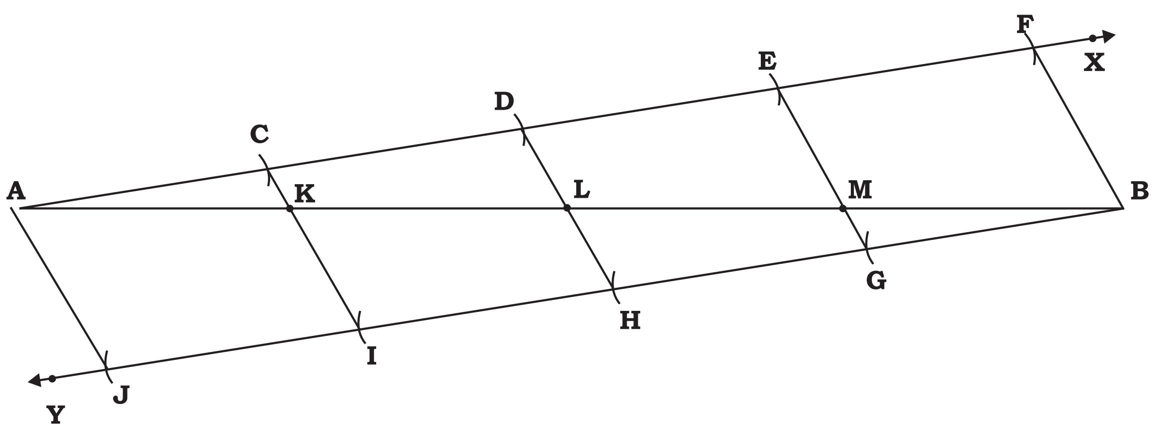
Therefore a line segment has length.

In geometry, the word **"construction"** means to draw a correct and an accurate figure from the given data. We need this ability in all walks of life.

To acquire this skill, we shall learn to draw some figures using a ruler, a set-square, a protractor and a pair of compasses.

11.1.1 Divide a line segment into a given number of equal segments.

Example 1. Divide a line segment AB of length 7 cm into four equal parts.



Steps of construction:

1 Draw $\overline{AB}$ of length 7 cm.

2. Draw $\overrightarrow{AX}$ making any suitable acute angle say 40° with $\overline{AB}$, with protractor.

3. At point B, draw $\overrightarrow{BY}$, making same acute angle such that $m\angle BAX = m\angle ABY = 40^\circ$.
4. Using pair of compasses, draw four arcs of suitable measure to cut $\overrightarrow{AX}$ at points C, D, E and F such that all the small pieces of lengths are equal in measure.
5. Similarly using pair of compasses draw four arcs of same measure to cut $\overrightarrow{BY}$ at points G, H, I and J such that the small pieces of lengths are equal in measure.
6. Join points A to J, C to I, D to H, E to G and F to B in order to draw, $\overline{AJ}$, $\overline{CI}$, $\overline{DH}$, $\overline{EG}$ and $\overline{FB}$. These line segments cut $\overline{AB}$ at points A, K, L, M and B.
7. In this way, the given $\overline{AB}$ is divided in the required four equal parts such that $m\overline{AK} = m\overline{KL} = m\overline{LM} = m\overline{MB}$.

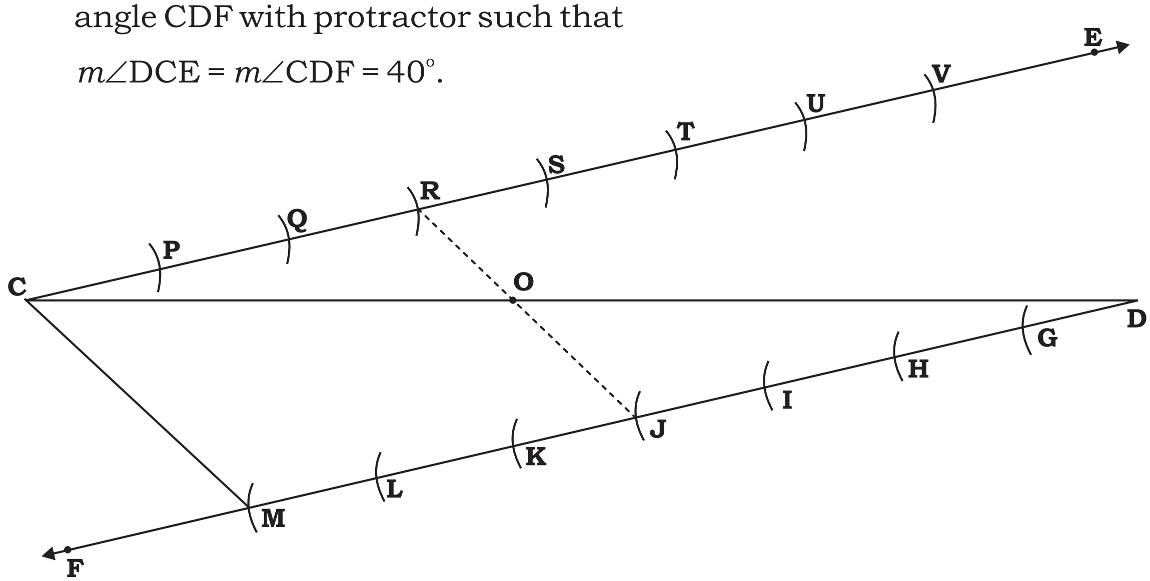
11.1.2 Divide a line segment internally in a given ratio.

Example 1. Divide a line segment CD of length 7 cm in the ratio 3:4.

Steps of construction:

1. Draw $\overline{CD}$ of length 7 cm.
2. At point C, draw $\overrightarrow{CE}$ making an acute angle DCE say 40° with protractor.
3. At point D, on the opposite side of $\overline{CD}$, draw $\overrightarrow{DF}$ making same acute angle CDF with protractor such that

   $m\angle DCE = m\angle CDF = 40^\circ$.

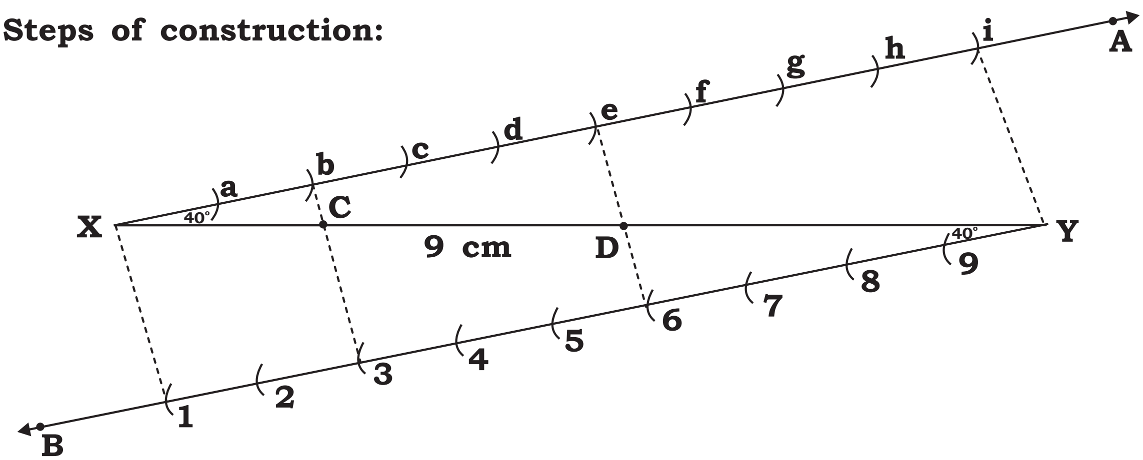


4. Using compasses draw seven (3 + 4 = the sum of ratios) arcs of suitable measure to cut $\overrightarrow{CE}$ at points P, Q, R, S, T, U and V.
5. Similarly using compasses of same measure, draw seven (3 + 4 = sum of ratios) arcs to cut $\overrightarrow{DF}$ at points G, H, I, J, K, L and M.
6. From point C mark 3rd. point R.
7. From point D mark 4th. point J.
8. Now join points R and J and draw $\overline{RJ}$ to cut $\overline{CD}$ at point O.
9. In this way $\overline{CO}$: $\overline{OD}$ = 3:4.

It means the given $\overline{CD}$ of measure 7 cm is divided in the required ratio of 3:4 at point O or $m\overline{CO}$: $m\overline{OD}$ = 3:4.

Example 2. Divide a given line segment XY of measure 9 cm in the ratio of 2:3:4.

Steps of construction:



1. Draw $\overline{XY}$ of measure 9 cm.
2. At point X, draw $\overrightarrow{XA}$ making suitable acute angle YXA (say 40°) with protractor.
3. At point Y, draw $\overrightarrow{YB}$ making same suitable acute angle XYB, such the $m\angle YXA = m\angle XYB = 40°$.
4. Using compasses draw nine (2 + 3 + 4 = the sum of ratios) arcs of suitable measure to cut $\overrightarrow{XA}$ at points a, b, c, d, e, f, g, h and i.
5. Similarly using compasses of same measure draw nine another arcs to cut $\overrightarrow{YB}$ at points: 9, 8, 7, 6, 5, 4, 3, 2 and 1.
6. Join b with 3, e with 6 to cut $\overline{XY}$ at points C and D.
7. In this way, the given $\overline{XY}$ is divided in three parts in the ratio of 2:3:4.

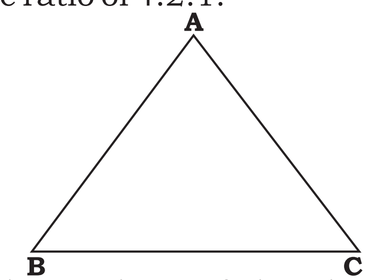
   Thus $m\overline{XC}$: $m\overline{CD}$: $\overline{DY}$ = 2:3:4.

EXERCISE 11.1

1. Divide a line segment of length 6 cm into four equal parts.
2. Divide $m\overline{AB}$ = 7.5 cm in five equal parts.
3. Divide $m\overline{CD}$ = 9 cm in six equal parts.
4. Divide a line segment 5 cm in the ratio of 2:3.
5. Divide a line segment measuring 8.4 cm in the ratio of 1:5.
6. Divide $m\overline{MN}$ = 8 cm in the ratio of 3:5.
7. Divide a line segment measuring 6.6 cm in the ratio of 1:2:3.
8. Divide $m\overline{XY}$ = 7 cm in the ratio of 2:2:3.
9. Divide a line segment measuring 8.4 cm in the ratio of 4:2:1.

11.2 TRIANGLES

We know that a triangle has three sides and three angles.



It is a ΔABC in which $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are its sides. $\angle A$, $\angle B$ and $\angle C$ are its three angles.

In order to construct a triangle, we require to know three of the above elements. Now let us learn some more ways for constructing a triangle.

11.2.1 Construct a triangle when perimeter and ratio among the lengths of sides are given.

Example 1. Construct a triangle ABC, whose perimeter is 9 cm and the ratio among the three sides is 1:2:3.

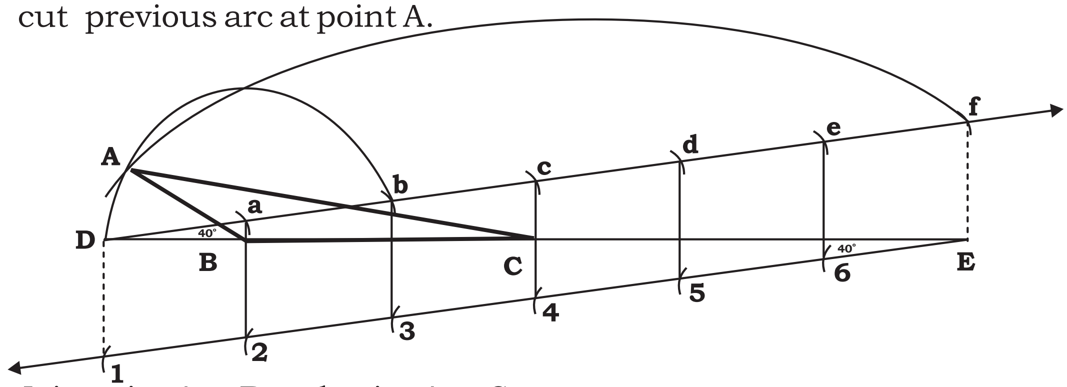
Given: ΔABC, its perimeter is 9 cm. Ratio of the sides is $\overline{AB} : \overline{BC} : \overline{CA}$ = 1:2:3.

Required to construct: To construct a ΔABC from the given data.

Steps of construction:

1. With the help of ruler, draw $\overline{DE}$ measuring 9 cm = perimeter of the triangle.
2. Divide $\overline{DE}$ in the given ratio 1:2:3, we get the lengths of three sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ as under: $m\overline{AB} = m\overline{DB}$ (the shortest side), $m\overline{BC} = m\overline{BC}$ and $m\overline{CA} = m\overline{CE}$ (the greatest side).

3. Take point B as centre, draw an arc with radius $m\overline{DB}$.
4. Take another point C as centre, draw an other arc with radius $m\overline{CE}$ to cut previous arc at point A.



5. Join point A to B and point A to C.
   Thus we get the triangle ABC which is the required triangle. Its perimeter is 9 cm and its sides are in the ratio of 1:2:3.

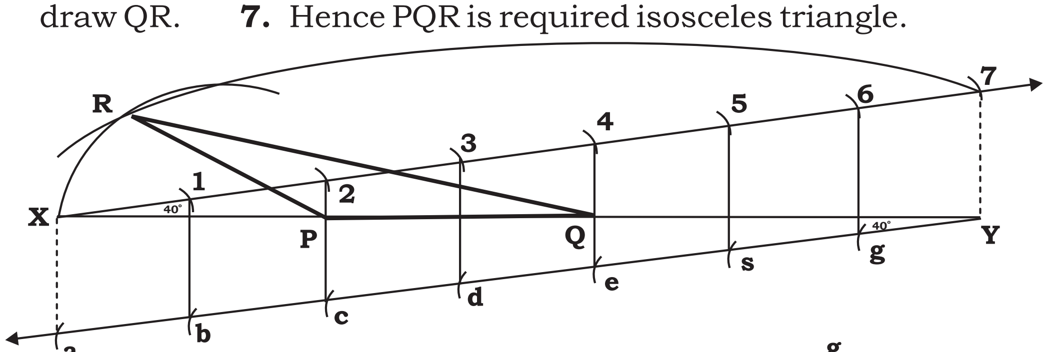
Example 2. Construct an isosceles triangle when its perimeter is 10.5 cm and ratio of the three sides is 2:2:3.

Given: ΔPQR is an isosceles triangle. Its perimeter is 10.5 cm and the ratio among the sides is 2:2:3.

Required to construct: Draw an isosceles triangle PQR from above data.

Steps of construction:

1. Draw a line segment XY measuring 10.5 cm with the help of ruler.
2. Divide the $\overline{XY}$ in the ratio of 2:2:3. We get the measure of three sides of the triangle as $\overline{XP}$, $\overline{PQ}$ and $\overline{QY}$.
3. Take point P as centre and radius = $m\overline{XP}$ (one side of Δ), draw an arc.
4. Again take point Q as centre and radius = $m\overline{QY}$ (3rd side of the Δ), draw an other arc, to cut previous arc at point R.
5. Join points P and R to draw $\overline{PR}$. **6.** Again join points R and Q to draw $\overline{QR}$. **7.** Hence PQR is required isosceles triangle.



EXERCISE 11.2

Construct the following:

1. A triangle ABC in which the ratio among the sides of the triangle is 1:2:3 and the perimeter is 9.6 cm.
2. The perimeter of a ΔDEF is 8.4 cm and the ratio among sides of the triangle is 2:3:2.
3. A triangle LMN whose perimeter is 90 mm and the ratio among sides of the triangle is 2:3:4.
4. A triangle PQR in which ratio among the sides is 5:3:2. The perimeter of the triangle is 10 cm.
5. A triangle XYZ in which ratio among the sides of the triangle is 3:3:2 and the perimeter is 88 mm.

11.2.2 Construction of an equilateral triangle

(a) Construct an equilateral triangle when base is given.

We know that if all the three sides of a triangle are congruent, the triangle is said to be an equilateral triangle.

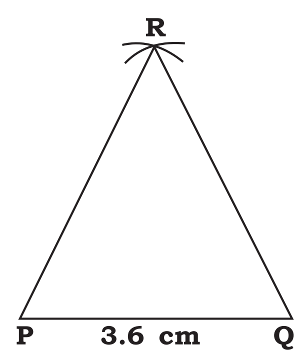
An equilateral triangle is always an acute angled triangle. All its angles are congruent. Each of its angle measures 60°.

Example: Construct an equilateral triangle PQR such that the measure of its base is 3.6 cm.

Given: ΔPQR is an equilateral triangle and the measure of its base is 3.6 cm.

Required to construction: ΔPQR an equilateral triangle in which all the three sides are congruent.

Steps of construction:

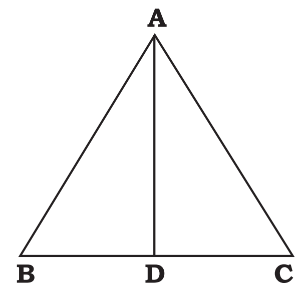


1. Draw a line segment PQ measuring 3.6 cm.
2. Take point P as centre, draw an arc with radius 3.6 cm.
3. Again take point Q as centre, draw another arc with same radius to intersect the previous arc at point R.
4. Join points P and R and draw $\overline{PR}$.
5. Join points Q and R and draw $\overline{QR}$.
6. In this way we get the required equilateral triangle PQR whose every side is of 3.6 cm.

(b) Construct an equilateral triangle when its altitude is given.

Altitude of a triangle is a perpendicular segment drawn from the vertex of a triangle to the base of the triangle. Here ABC is an equilateral triangle in which $\overline{AD}$ is a perpendicular distance to the base $\overline{BC}$.

Symbolically $\overline{AD} \perp \overline{BC}$ or $\overline{AD}$ is an altitude of equilateral triangle ABC.



Remember: Altitude of an equilateral triangle bisects the angle at vertex.

Every angle of an equilateral triangle is of 60°.

We can also construct an equilateral triangle when its altitude is given.

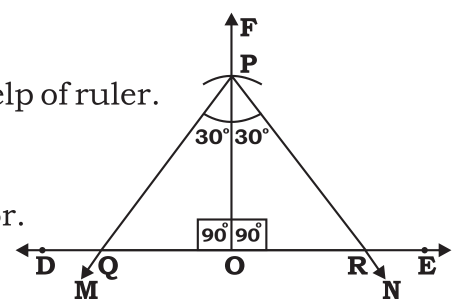
Example: Construct an equilateral triangle PQR when an altitude ($\overline{PO} \perp \overline{QR}$) where $m\overline{PO}$ = 4.3 cm

Given: PQR is an equilateral triangle in which

$\overline{PO}$ is an altitude, $\overline{PO} \perp \overline{QR}$ and $m\overline{PO}$ = 4.3 cm.

Required to construct: To construct an equilateral triangle PQR.

Steps of construction:



1. Draw $\overleftrightarrow{DE}$ of suitable measure with the help of ruler.
2. Take any point O on it.
3. Draw $\overrightarrow{OF} \perp \overleftrightarrow{DE}$ with the help of protractor.
4. Take point O as centre and radius = 4.3 cm, draw an arc to cut $\overrightarrow{OF}$ at point P, to get $\overline{OP}$ an altitude.
5. We know that every angle of equilateral triangle is of 60° and an altitude of the given equilateral triangle bisects the angle at vertex.
   Draw angles of 30° with the help of protractor at point P such that
   $m\angle OPM = m\angle OPN = 30^\circ$
6. Let $\overrightarrow{PM}$ intersect $\overleftrightarrow{DE}$ at point Q and $\overrightarrow{PN}$ intersect $\overleftrightarrow{DE}$ at point R.
   Thus PQR is the required equilateral triangle.

EXERCISE 11.3

1. Construct the following equilateral triangles when the measure of base is as follows:

   (i) 2.9 cm (ii) 3.2 cm (iii) 4.1 cm

   (iv) 3.8 cm (v) 2.8 cm (vi) 4.3 cm

2. Construct the following equilateral triangles, when the measurement of its altitude is as follows:

   (i) 3.6 cm (ii) 3.9 cm (iii) 4.3 cm

   (iv) 4.5 cm (v) 4.7 cm (vi) 4.8 cm

3. Construct an equilateral triangle PQR in which the measure of its base is 50 mm long.

4. Construct an equilateral triangle MNO in which its altitude is 36 mm long.

11.2.3 Construction of an Isosceles Triangle

(a) Construct an isosceles triangle when base and a base angle are given.

We have already learnt that any two sides of a triangle are congruent, the triangle is said to be an **isosceles triangle**.

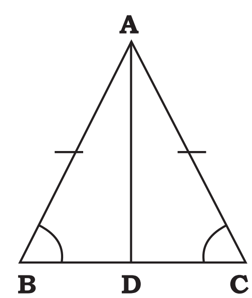


Figure ABC is an isosceles triangle, in which

$\overline{AB} \cong \overline{AC}$. Also $m\angle ABC \cong m\angle ACB$

Thus in an isosceles triangle, the congruent sides are called arms of the triangle and the third side is called the base.

We can construct an isosceles triangle when its base and base angles are given.

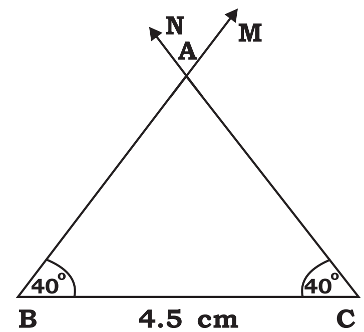
We know that in an isosceles triangle, two sides are congruent. Therefore the angles related with congruent sides are also congruent.

Example: Construct an isosceles triangle ABC in which its base measures 4.5 cm and the measure of base angle is 40°.

Given: ABC is an isosceles triangle in which $\overline{AB} \cong \overline{AC}$ and $m\angle ABC = 40^\circ$.

Required to construct: Construct an isosceles triangle ABC.

Steps of construction:

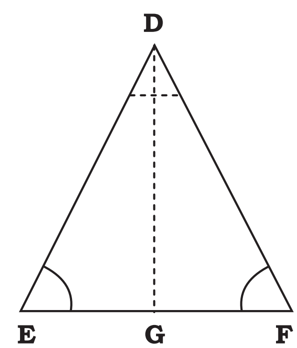


1. Draw $\overline{BC}$ measuring 4.5 cm.
2. At point B, draw an angle CBM measuring 40°.
3. At point C, also draw an angle BCN measuring 40°.
4. Let arms of two angles intersect each other at point A.
5. Thus ΔABC is the required isosceles triangle.

(b) Construct an isosceles triangle when vertical angle and an altitude are given.

Look at the picture. It is an isosceles triangle. DEF, in which $m\overline{DE} = m\overline{DF}$ and

$m\angle DEF = m\angle DFE$ (base $\angle$s). $\angle EDF$ is called vertex angle.



$\overline{DG}$ is an altitude drawn from point D to $\overline{EF}$.

Therefore $\overline{DG} \perp \overline{EF}$.

We have already learnt that an altitude is drawn from vertex to the base of an isosceles triangle, it bisects the vertex angle.

So $m\angle EDG = m\angle FDG$.

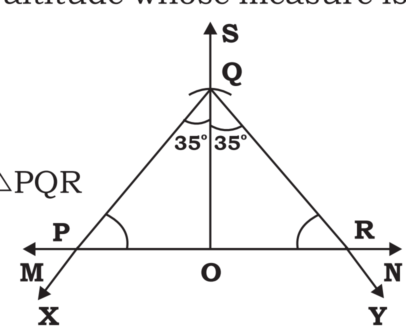
Example: Construct an isosceles triangle PQR in which congruent sides $\overline{QP} \cong \overline{QR}$ (arms of an isosceles triangle) $\overline{QO}$ is an altitude whose measure is 3.9 cm and its vertex angle measures 70°.

Given: PQR is an isosceles triangle.

$m\overline{QP} = m\,\overline{QR}$, $m\angle PQR = 70^\circ$ and $\overline{QO} \perp \overline{PR}$

Required to construct: Construct an isosceles $\triangle PQR$

Steps of construction:



1. Draw $\overleftrightarrow{MN}$ with the help of scale.
2. Take any point O on it.

3. Draw $\overrightarrow{OS} \perp \overleftrightarrow{MN}$

4. Take point O as centre, draw an arc with radius = 3.9 cm to cut $\overrightarrow{OS}$ at point Q.

5. Now as the vertex angle is 70° and altitude of the triangle bisects the vertex angle. Therefore perpendicular $\overline{QO}$ shows the angle of $\frac{70}{2} = 35^\circ$ on both the sides of $\overline{QO}$.

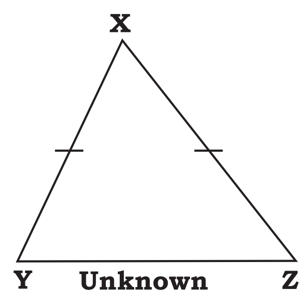
6. Draw $\angle OQX$ and $\angle OQY$ measuring 35° each.

7. Let $\overrightarrow{QX}$ intersect $\overrightarrow{OM}$ at point P and $\overrightarrow{QY}$ intersect $\overrightarrow{ON}$ at point R.

8. Thus ΔPQR is the required isosceles triangle.

(c) Construct an isosceles triangle when an altitude and a base angle are given.

Here is a picture of an isosceles triangle.



Arms of an isosceles triangle are congruent

i.e $\overline{XY} \cong \overline{XZ}$ or $m\overline{XY} = m\overline{XZ}$.

Therefore $m\angle XYZ = m\angle XZY$ i.e. base

angles are congruent ($\angle XYZ \cong \angle XZY$)

We know that the sum of the three angles of triangle is 180°.

With this property we can easily find the measurement of vertex angle as follows:

Vertex angle = 180° – (sum of two base angles)

Example: Construct an isosceles triangle XYZ, when the measurement of its altitude $\overline{XR}$ is 3.8 cm and a base angle XYZ is 55°.

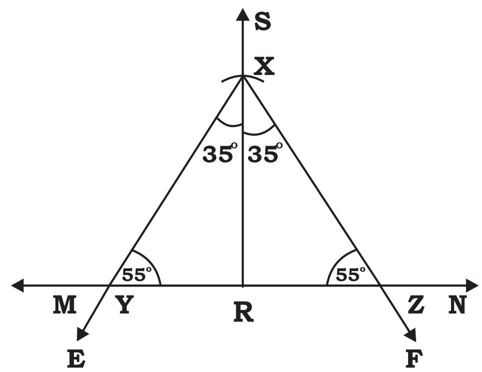
m*XR = 3.8 cm***Given:** $\overline{\Delta XYZ \text{ is}}$ an $\overline{\text{iso}}$sceles triangle. $XR \perp YZ$,

$XZY = 55^\circ . \angle XYZ = m\angle XZY$ or $m\angle \cong XYZ \angle$

Required to construct: Construct an isosceles triangle XYZ

Steps of construction:

First we find the vertex angle. Vertex angle = 180° – (sum of two angles)



Vertex angle = 180° – (55° + 55°)

Vertex angle = 180° – 110°

Vertex angle = 70°

1. Draw $\overleftrightarrow{MN}$.
2. Take any point R on it.
3. Draw $\overrightarrow{RS} \perp \overleftrightarrow{MN}$.
4. Take point R as centre and radius = 3.8 cm, draw an arc to cut $\overrightarrow{RS}$ at point X.
5. At point X, draw $\angle RXE = \dfrac{\text{vertex angle}}{2} = \dfrac{70^\circ}{2} = 35^\circ$
6. At point X, draw $\angle RXF = \dfrac{\text{vertex angle}}{2} = \dfrac{70^\circ}{2} = 35^\circ$
7. Let $\overrightarrow{XE}$ intersect $\overrightarrow{RM}$ at point Y
8. And $\overrightarrow{XF}$ intersect $\overrightarrow{RN}$ at point Z
9. Thus ΔXYZ is the required isosceles triangle.

EXERCISE 11.4

1. Construct isosceles triangle for following data.

(i) Base = 4.2 cm, Base angle = 42° (ii) Base = 4.8 cm, Base angle = 48°

(iii) Base = 38 mm, Base angle = 38° (iv) Base = 45 mm, Base angle = 45°

2. Construct the following isosceles triangles when the measure of vertical angles and altitude is given.

(i) Vertical angle = 76°. Altitude = 4 cm

(ii) Vertical angle = 80°. Altitude = 5 cm

(iii) Vertical angle = 90°. Altitude = 5.3 cm

(iv) Vertical angle = 100°, Altitude = 60 mm

3. Construct the following isosceles triangles when the measure of altitude and base angle are given.

(i) Altitude = 4.5 cm. Base angle = 54°

(ii) Altitude = 39 mm. Base angle = 65°

(iii) Altitude = 4.7 cm. Base angle = 70°

(iv) Altitude = 59 mm. Base anlge = 75°

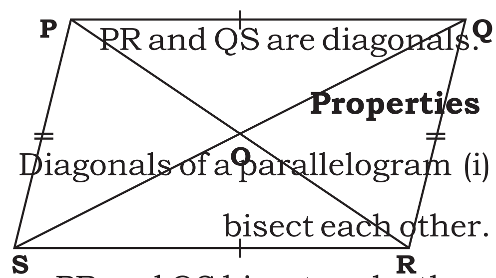
11.3 PARALLELOGRAM

Parallelogram is a quadrilateral whose opposite sides are congruent and parallel.

Look at the picture.

PQRS is a parallelogram in which $\overline{PQ} \parallel \overline{SR}$ and $\overline{PS} \parallel \overline{QR}$.

$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$



PR and QS are diagonals.

Properties

Diagonals of a parallelogram (i)

bisect each other.

PR and QS bisect each other.

i.e. PO ≅ RO , SO ≅ QO

PQ and QR are two adjacent sides.(ii)

PQR is included in the above two adjacent sides.∠

Similarly QR and SR are two adjacent sides.

SRQ is included in the above two adjacent sides.∠

Opposite angles of a parallelogram are also congruent.(iii)

i.e. $\angle PSR \cong \angle PQR$, $\angle SPQ \cong \angle SRQ$

11.3.1 Construction of a parallelogram

I. Construct a parallelogram when two adjacent sides and their included angle are given.

Here we have to construct a parallelogram when pair of its two adjacent sides and an included angle is given.

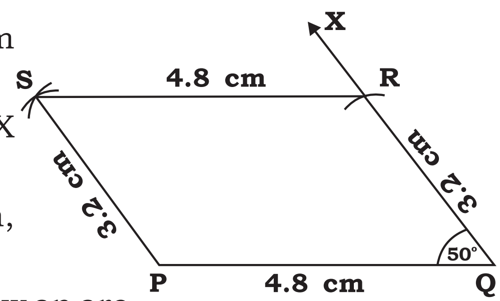
Example: Construct a parallelogram PQRS in which $m\overline{PQ} = 4.8$ cm , $m\overline{QR} = 3.2$ cm and $m\angle PQR = 50^\circ$.

Given: PQRS is a llogram, $m\overline{PQ} = 4.8$ cm, $m\overline{QR} = 3.2$ cm

$m\angle PQR = 50^\circ$.

Required to construct: Construct a parallelogram PQRS from given data.

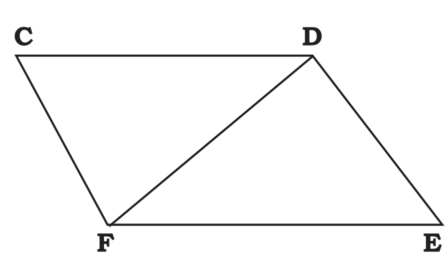
Steps of construction:



1. A line segment PQ measuring 4.8 cm is drawn.
2. At point Q, draw an angle PQX measuring 50° with protractor.
3. With centre Q and radius 3.2 cm, draw an arc to cut $\overrightarrow{QX}$ at point R.
4. With centre R and radius 4.8 cm, draw an arc.
5. Again with centre P and radius 3.2 cm, cut previous arc at point S.
6. Join points P and S to draw $\overline{PS}$.
7. Join points R and S to draw $\overline{RS}$.

In this way, we get the required parallelogram PQRS.

II. Construct a parallelogram when two adjacent sides and a diagonal are given.



Consider a parallelogram CDEF in which $\overline{DF}$ is a diagonal of the parallelogram CDEF.

And $\overline{CD}$, $\overline{CF}$ are adjacent sides.

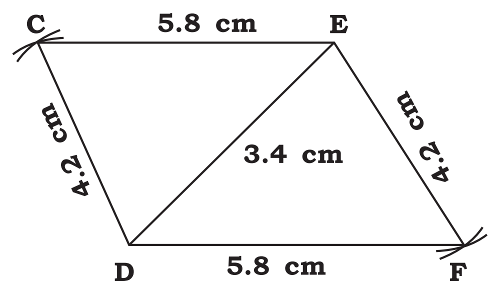
We can construct a parallelogram when measure of its diagonal $\overline{FD}$ and two adjacent sides $\overline{CD}$ and $\overline{CF}$ are given.

Example: Construct a parallelogram CDEF in which length of its diagonal $\overline{DF}$ = 5.8 cm and the lengths of its two adjacent sides $\overline{DE}$ = 3.4 cm and $\overline{EF}$ = 4.2 cm.

Given: CDEF is a parallelogram. Measurement of its adjacent sides $m\overline{DE}$ = 3.4 cm, $m\overline{EF}$ = 4.2 cm and the diagonal $m\overline{DF}$ = 5.8 cm.

Required to construct: Construct a parallelogram CDEF from given data.

Steps of construction.



1. Draw $\overline{DF}$ diagonal measuring 5.8 cm.
2. With centre D and radius 3.4 cm, draw an arc on the upper side of diagonal $\overline{DF}$.
3. With centre F and radius 4.2 cm, draw an other arc on the upper side of $\overline{DF}$ to cut the previous arc at point E.
4. Join points D and E to draw $\overline{DE}$.
5. Join points E and F to draw $\overline{EF}$.
6. Now with centre D and radius 4.2 cm, draw an arc on the lower side of the diagonal $\overline{DF}$.
7. Again with centre F and radius 3.4 cm, draw an other arc on the lower side of the diagonal $\overline{DF}$ to cut previous arc at point C.
8. Join points C and D to make $\overline{CD}$.
9. Join points C and F to make $\overline{CF}$.
10. In this way, we get the required parallelogram CDEF.

EXERCISE 11.5

A. Construct the following parallelograms in which the lengths of two adjacent sides and the measurement of their included angle are given as under:

1. Parallelogram PQRS , $m\overline{PQ}$ = 5.9 cm, $m\overline{QR}$ = 4.1 cm , $m\angle PQR = 65^\circ$.

2. Parallelogram CDEF , $m\overline{CD}$ = 63 mm , $m\overline{DE}$ = 39 mm , $m\angle CDE = 70^\circ$.

3. Parallelogram ABCD , $m\overline{AB}$ = 6.6 cm, $m\overline{AD}$ = 4 cm , $m\angle BAD = 50^\circ$.

4. Parallelogram RSTU, $m\overline{RS}$ = 70 mm , $m\overline{ST}$ = 45 mm , $m\angle RST = 80^\circ$.

5. Parallelogram ACEG , $m\overline{AC}$ = 7.5 cm , $m\overline{CE}$ = 3.4 cm , $m\angle ACE = 75^\circ$.

6. Parallelogram LMNO, $m\overline{ON}$ = 6 cm , $m\overline{MN}$ = 3.5 cm , $m\angle MNO = 55^\circ$

B. Construct the following parallelograms in which the measurement of two adjacent sides and the length of one diagonal are given as under:

1. Parallelogram ABCD , $m\overline{BC}$ = 6 cm , $m\overline{CD}$ = 4 cm , $m\overline{BD}$ = 7.3 cm.

2. Parallelogram BDFH , $m\overline{BD}$ = 50 mm , $m\overline{BH}$ = 30 mm , $m\overline{HD}$ = 64 mm.

3. Parallelogram ACEG , $m\overline{AC}$ = 6.6 cm , $m\overline{AG}$ = 3.3 cm , $m\overline{CG}$ = 7.9 cm.

4. Parallelogram PQRS , $m\overline{RS}$ = 4.8 cm , $m\overline{RQ}$ = 2.9 cm , $m\overline{SQ}$ = 7 cm.

5. Parallelogram RSTU , $m\overline{RS}$ = 59 mm , $m\overline{ST}$ = 36 mm , $m\overline{TR}$ = 7.5 cm.

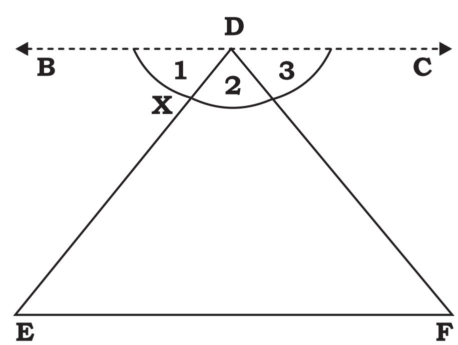
6. Parallelogram UVWX , $m\overline{UX}$ = 7 cm , $m\overline{UV}$ = 5 cm , $m\overline{VX}$ = 9 cm

11.3.2 (a) Verify practically that the sum of the measures of the angles of a triangle is 180°.

Given: DEF is a triangle.

There are three angles of the ΔDEF

(i) $\angle DEF$, (ii) $\angle DFE$ and (iii) $\angle EDF$.



We have to verify:

$m\angle DEF + m\angle DFE + m\angle EDF = 180^\circ$.

Required to verify: The sum of the measures of angles of given ΔDEF is 180°.

SETPS

Step 1. Draw $\overleftrightarrow{BC} \,||\, \overline{EF}$ passing through point D.

Step 2. According to the properties of parallel lines.

(a) $m\angle BDE = m\angle DEF$ or $\angle \cong 1\angle E$

(b) $m\angle CDF = m\angle DFE$ or $\angle \cong 3\angle F$

Step 3. From figure, it is observed that:

$m\angle DEF + m\angle EDF + m\angle DFE = m1\angle + m2\angle + m3\angle$

or $m\angle E + m\angle D + m\angle F = m1\angle + m2\angle + m3\angle$

Step 4. We know that the angles on a straight line are supplementary.

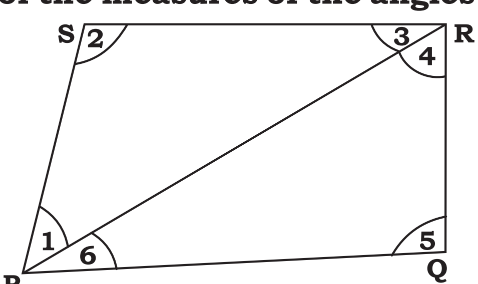
i.e. $m1\angle + m2\angle + m180 = 3\angle^{\circ}$

Hence $m\angle D + m\angle E + m\angle F = m1\angle + m2\angle + m180 = 3\angle^{\circ}$.

In this way, it is verified that the sum of the angles of a triangle is 180°.

Thus we can practically observe that the sum of the measures of the three angles of a triangle is 180° by drawing any three or more triangles. Then measure three angles in each of the triangle with protractor. Also find sum of the three angles in each case. Thus verified the sum of three angles of a triangle is 180°.

11.3.2 (b) Verify parctically that the sum of the measures of the angles of a quadrilateral is 360°.



Given: PQRS is a quadrilateral.

There are four angles of the quadrilateral.

(i) $\angle SPQ$ (ii) $\angle PQR$, (iii) $\angle QRS$ and (iv) $\angle RSP$

or $\angle P$, $\angle Q$, $\angle R$ and $\angle S$

We have to verify that $m\angle SPQ + m\angle PQR + m\angle QRS + \angle RSP = 360^{\circ}$

or $m\angle P + m\angle Q + m\angle R + m\angle S = 360^{\circ}$

Required to verify: The sum of the angles of a given quadrilateral PQRS is 360°

STEPS

Step 1. In above given figure of quadrilateral PQRS, join points, R and P to draw the diagonal of the quadrilateral PQRS.

Step 2. The diagonal PR divides the given quadrilateral in two triangles: ΔPSR and ΔPQR.

Step 3. We know that the sum of the angles in a triangle is 180°.

So sum of the angles of the given quadrilateral = (sum of the angles of ΔPSR) + (Sum of the angles of ΔPQR).

Step 4. In ΔPSR , we observe that:

$m\angle SPR + m\angle PSR + m\angle SRP = 180^\circ$

or $m1\angle + m2\angle + m180 = 3\angle^\circ$ -------------------- (A)

In ΔPQR, we observe that:

Sum of the angles of trhe given quadrilateral =

$m\angle QPR + m\angle PRQ + m\angle PQR = 180^\circ$

or $m6\angle + m4\angle + m180 = 5\angle^\circ$ -------------------- (B)

Step 5. Now we have to find sum of all the angles of the given quadrilateral PQRS.

i.e. Adding the above results of (A) and (B), we get:

$m1\angle + m2\angle + m3\angle + m6\angle + m4\angle + m180 = 5\angle^\circ + 180^\circ = 360^\circ$

or $m1\angle + m6\angle + m2\angle + m3\angle + m4\angle + m360 = 5\angle^\circ$

or $m\angle P + m\angle S + m\angle R + m\angle Q = 360^\circ$

or $m\angle P + m\angle Q + m\angle R + m\angle S = 360^\circ$

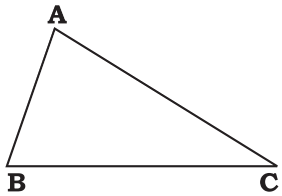
Hence verified that the sum of the four angles of the given quadrilateral PQRS is 360°.

It can practically be observed by drawing the pictures of two or more quadrilaterals. Then measure the four angles of the quadrilateral in each case. Also find the sum of four angles of each quadrilateral. Thus the sum of measures of angles of a quadrilateral is 360°.

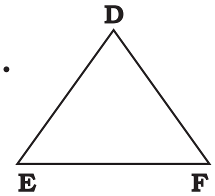
EXERCISE 11.6

I. Verify in the following that the sum of the measures of the angles of a triangle is 180°. Measure three angles of each triangle by protractor. Find their sum.

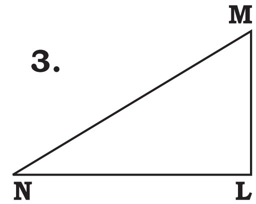
1.



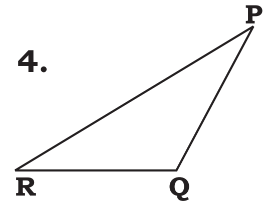
2.



3.



4.

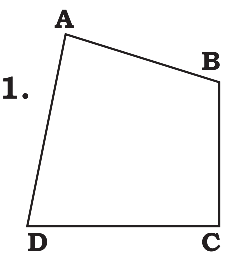


II. Draw any three different triangles and measure the angles of each triangle. Also verify that the sum of the angles of a triangle is 180°.

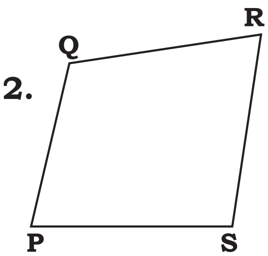
III. Verify in the following that the sum of the measures of four angles of a quadrilateral is 360°.

First we measure the angles with the help of protractor, and then find their sum.

1.



2.



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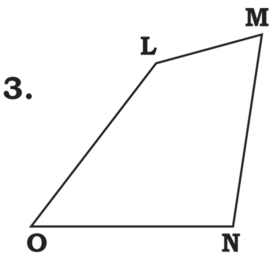

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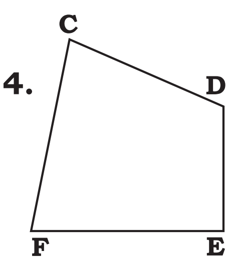
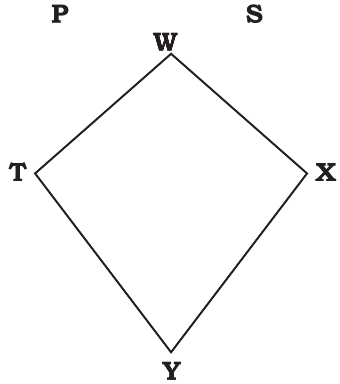
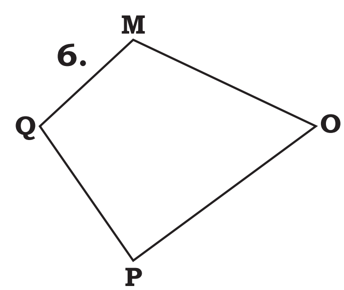


5.



6.



IV. Draw any two quadrilaterals. Measure all the angles of each and every quadrilateral. Verify that the sum of the angles of a quadrilateral is 360°.

REVIEW EXERCISE 11

A. Answer the following short questions.

1. What is a line segment? Describe its properties.
2. What is triangle? Describe its properties.
3. Name the different kinds of a triangle.
4. When two line segments will be congruent?
5. What is the sum of the angles of a triangle? Verify it with the help of a protractor.
6. What is a quadrilateral describe only the names of different kinds.
7. What are the properties of an equilateral triangle?
8. Is square a rectangle, a parallelogram or a rohmbus?

9. What is the sum of the angles of a quadrilateral? Verify it with proof.
10. Define an altitude and vertex angle of a triangle?

B. Construct the following with neat and clean figure.

1. Divide a line segment of 10.5 cm in the ratio of 3:4.
2. Divide a line segment 11.9 cm in the ratio of 2:4:2.
3. Construct an isosceles triangle in which base measures 52 mm and measure of base angle is 70°.
4. Construct a parallelogram PQRS in which $m\overline{SR} = 4.1$ cm, $m\overline{QR} = 3.1$ cm and $m\angle QRS = 70^\circ$.
5. Construct an equilateral triangle whose altitude is of length 6 cm.
6. Construct a triangle whose perimeter is 129 mm. The ratio between the lengths of the sides is 3:4:5.
7. Construct an isosceles triangle in which altitude is 6.2 cm and the vertex angle is 40°.
8. Construct a parallelogram in which the lengths of two adjacent sides are 43 mm and 32 mm. The length of one of its diagonal is 6.4 cm.

C Tick (√) the correct answer.

1. A polygon with three sides, three vertices with a sum of interior angles 180° is called ____________________.
   (a) quadrilateral (b) line segment (c) triangle (d) circle
2. Base angle of an isosceles triangle is _________. When the vertex angle is 56°.
   (a) 67° (b) 70° (c) 60° (d) 90°
3. A parallelogram with each of its angle 90° is__________.
   (a) Circle (b) Rectangle (c) Trapezium (d) Triangle
4. Two line segments $\overline{OP}$ and $\overline{QR}$ are congruent, if
   (a) $m\overline{OP} = m\overline{QR}$ (b) $m\overline{OP} > m\overline{QR}$ (c) $m\overline{OP} < m\overline{QR}$
   (d) $m\overline{OP} \neq m\overline{QR}$.
5. The sum of the interior angles of a quadrilateral is ________.
   (a) 90° (b) 180° (c) 270° (d) 360°

6. A quadrilateral whose opposite sides are congruent and parallel is called _______.
   (a) Trapezium (b) Parallelogram (c) Square (d) Rectangle
7. In a triangle, the sum of two base angles is 90°. Then the measure of its vertex angle is _______.
   (a) 45° (b) 90° (c) 135° (d) 180°
8. In a quadrilateral the sum of the three angles of a quadrilateral is 270°, then the measure of its fourth angle is _______.
   (a) 90° (b) 180° (c) 270° (d) 360°

SUMMARY

1. Two line segments are congruent, if they are of equal length.
   i.e. $\overline{AB} \cong \overline{CD}$ when $m\overline{AB} = m\overline{CD}$.
2. A line segment can be divided into any given number of equal segments; for example $m\overline{CD}$ = 120 mm.
   (i) When divided into six equal segments, the measurement of each segment is $\frac{12}{6}$ = 2 cm.
   (ii) When the above said line segment CD is divided in the ratio 1:2:3.
   We get three small line segments. l_1, l_2 and l_3 as under:

   C 2 cm E 4 cm F 6 cm D

   $$\frac{m\overline{CD} \times \text{ratio of } l_1}{\text{Sum of ratios}}, \quad \frac{m\overline{CD} \times \text{ratio of } l_2}{\text{Sum of ratios}} \text{ and } \frac{m\overline{CD} \times \text{ratio of } l_3}{\text{Sum of ratios}}$$
3. A triangle is a polygon having three sides and three angles.
   (i) We can construct a triangle when at least its three elements are given.
   (ii) We can construct a triangle when its perimeter and ratio among the lengths of its sides is given.
4. An equilateral triangle has three congruent sides.
   (i) We can construct an equilateral triangle when the measure of its one side say base is given.
   (ii) We can construct an equilateral triangle when its altitude is given. Altitude of a triangle is a perpendicular segment from the vertex.

5. Isosceles triangle has two sides congruent.
   (i) We can construct an isosceles triangle, when its base and two equal base angles are given.
   (ii) We can also construct an isosceles triangle when its vertical angle and an altitude is given. Altitude is drawn from vertex to the base and it bisects the vertex angle.

In an isosceles triangle, we can easily find the measurement of unknown vertex angle as follows: Vertex Angle = 180° – (Sum of two equal base angles).

Similarly in an isosceles triangle, we can find the measurement of unknown Base Angle as follows:

$$\text{Base Angle} = \frac{(180^\circ - \text{Vertex Angle})}{2}$$

6. Parallelogram is a quadrilateral whose opposite sides are congruent and parallel. Properties of a parallelogram are as follows:
   (i) Diagonals of a parallelogram bisect each other.
   (ii) Opposite angles of a parallelogram are congruent.
   (iii) A square, a rectangle and a rhombus is a parallelogram. But every parallelogram is not a square, a rectangle and a rhombus.

• • •

Think Tank: The Most Wonderful Knowledge.

Let us observe the magic of the number 123456789. Multiply it by any multiple of 9 up to 81 (i.e. multiply it by 9; 18; 27; 36; ... ; 81).

We will be pleased to arrive at the result (Product).

$$\begin{array}{r} \overset{2}{1}\ \overset{3}{2}\ \overset{4}{3}\ \overset{5}{4}\ \overset{6}{5}\ \overset{7}{6}\ \overset{8}{7}\ 9 \\ \times\ 9 \\ \hline 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ \hline \end{array}$$

Similarly we can multiply 12345679 by 18, ...

Hence 12345679 × 18 = 222222222.

Unit 12 CIRCUMFERENCE, AREA AND VOLUME

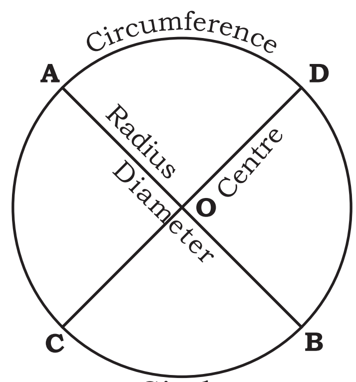
12.1 CIRCUMFERENCE AND AREA OF CIRCLE

12.1 Circumference of a Circle

Circle is a familiar shape. O is centre of the circle. $\overline{OA}$ is radius. $\overline{AB}$ is diameter. **Circle** is a closed figure and every point on it is at the same distance from a given point O, the centre.

The measure of the length of the boundary of a circle is called its **circumference.**

Whereas AOD, BOD, AOC and BOC are sector of the circle. $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ and $\overline{OD}$ all represent the radii of the given circle.



Circle

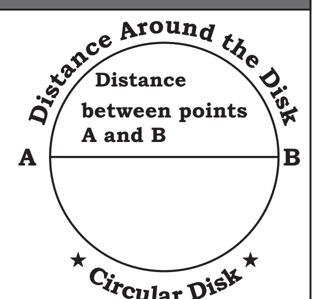
$\overline{AB}$ and $\overline{CD}$ are diameters. Since a diameter is a line segment, so it can easily be measured.

12.1.1 Express π as the ratio between the circumference and the diameter of a circle.

Activity: Let us work with a partner or in a group to find the ratio between the circumference and the diameter of a circle. We will need several circular objects, a calculator, a ruler, and a measuring tape and string.

Step 1

Choose one circular disk and answer the following.
Consider a circular disk.



(a) Which do you think is greater:
the distance around the circular disk or
the distance between A and B

Let us try to find the answer of second question.

It is observed that the distance around the disk is greater.

(b) How many times as great do you think it is?

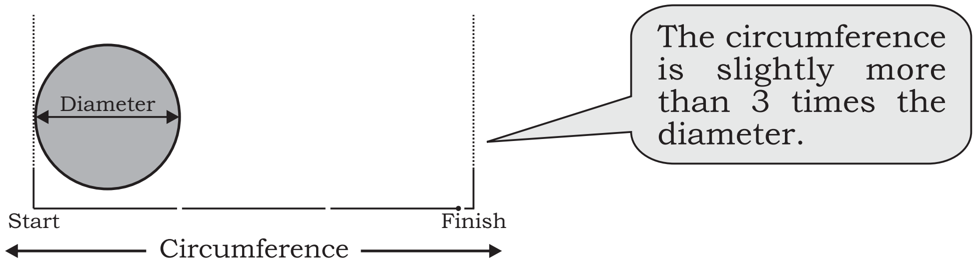
Step 2
Measure and record the distance around each disk (its **circumference**), to the nearest millimetre, using string.

Step 3
Measure and record the distance across each disk (its **diametre**). Using ruler or tape.

1. Repeat Steps 2 and 3 for the other disk also. Record your results in a table given below: Include the results of your partner or other group members.

2. Also compare the numbers in the circumference column with those in the diameter column.

We find the ratio of circumference to diameter in all the above cases:



The table below shows the measure of diameters and approximate size of circumferences of four circles. Find the value of ratio $\left(\frac{\text{Circumference}}{\text{Diameter}}\right)$ for each circle. What did you notice?

Circle	Diameter	Circumference (Approximate value)	$\frac{\text{Circumference}}{\text{Diameter}}$ (Approximate value)
(i)	3.5 cm	11 cm	
(ii)	28 mm	88 mm	
(iii)	4.2 m	13.2 m	
(iv)	21 mm	66 mm	

Unit 12 CIRCUMFERENCE, AREA AND VOLUME

If we consider circles of different radii and find their circumferences, we will see that the ratio of each circumference to its diameter is approximately the same. This ratio is denoted by π.

The value of $\frac{\text{Circumference}}{\text{Diameter}}$ or π is the same for all circles. It is always about 3.14 or $\frac{22}{7}$.

Therefore $\boxed{\text{Circumference} = \pi \times \text{Diameter}}$

i.e. Circumference is π times greater than diameter.

12.1.2 Find the circumference of a circle using formula

There are different methods of measuring the circumference of a circle.

(a) Measurement of the circumference of a circle through activities.

Activity 1:

Steps: (i) Let circle is a cross section of a cylinder.

(ii) We want to measure the circumference of this circle.

(iii) Wrap a strip of paper or a string round the cylinder.

(iv) When the strip or the string reaches the initial end point, put a mark on that place.

(v) Measure the length of the strip or the string from the initial point to the place marked.

(vi) This is the measure of length of the circumference.

The perimeter of a circle is also known as the **circumference** of the circle.

Measurement of the circumference of a circle using formula.

We can use the following formula to find the circumference of a circle:

Circumference of the circle = π × Length of the diameter.

We know that diameter = $2\ r$ = Twice of radius.

Circumference $C = 2\pi r$, also $C = \pi d$, where d is the length of diameter.

Example 1. A circle has a radius of 35 mm. Find (a) its diameter, (b) its circumference. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution:

(a) Diameter = 2 × Radius

$= 2 \times 35$ mm

$= 70$ mm

Its diameter is 70 mm.

(b) Circumference = π × Diameter

$= \frac{22}{7} \times \frac{70\text{mm}}{1}$

$= \frac{22 \times \cancel{70}^{10}}{\cancel{7}} = 220$ mm

Its circumference is 220 mm.

Example 2. The length of the diameter of a circle is 14 cm. Find the circumference of the circle?

Solution: The length of the diameterd = 14 cm and $\pi = \frac{22}{7}$

Thus, the circumference of the circle = π × d

$$= \frac{22}{7} \times 14 = \frac{22 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}}} = 44 \text{ cm}$$

Example 3: Find the diameter and radius of the circle, when its circumference is 99 cm.

Solution: We know that:

Circumference $= \pi \times d$

or $C = \pi d$

or $\frac{C}{\pi} = d$

So,

Diameter $= d = \frac{C}{\pi} = \frac{99}{\frac{22}{7}} = \frac{99 \times 7}{22}$

or $d = \frac{\overset{9}{\cancel{99}} \times 7}{\underset{2}{\cancel{22}}} = \frac{63}{2} = 31.5$ cm

and radius $= r = \frac{d}{2} = \frac{63/2}{2} = \frac{63}{4} = 15.75$ cm

EXERCISE 12.1

A. Find the circumference of the circle when its diameter is:

(1) d = 28 cm (2) d = 35 cm (3) d = 42 mm (4) d = 56 mm

B. Find the circumference of the circle when its radius is:

(1) 10.5 cm (2) 28 cm (3) 38.5 cm (4) 49 cm

(5) 59.5 cm (6) 63 cm (7) 80.5 cm (8) 77 cm

C. Find the radius of the circle when its circumference is:

(1) 22 cm (2) 66 cm (3) 88 cm (4) 110 cm

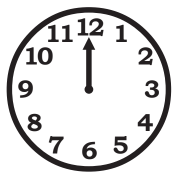
(5) 132 cm (6) 176 cm (7) 220 cm (8) 198 cm

D. Find the diameter of the circle, when its circumference is:

(1) 44 cm (2) 154 cm (3) 242 mm (4) 264 mm

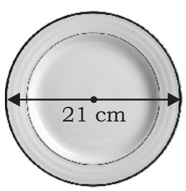
(5) 48.4 mm (6) 30.8 cm (7) 52.8 cm (8) 39.6 mm

E. Complete the table. Minute hand completes one round in 1 hour.

S.No.	Length of minute hand a clock is r	Find the distance covered by the minute hand
1.	r = 1.4 cm 	(i) Distance = $2\pi r$ $= \dfrac{2 \times 22 \times 1.4}{7} = 8.8$ cm
2.	r = 21 mm	
3.	r = 35 mm	
4.	r = 2.8 cm	
5.	r = 42 mm (***Note:*** Minute hand completes one round in 1 hours)	

F. Find the minimum length of the string required to measure the circular boundary in each of the following:

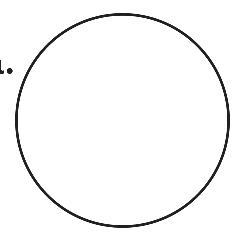
(1)



(2)

(3)

12.1.3 Find the area of a circular region using formula.



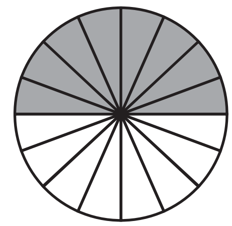
Let us find the Area of Circular Region through activity:

A**ctivity:**

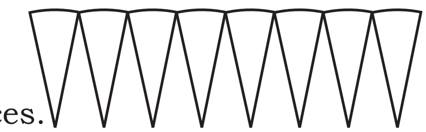
(i) Draw a circle with suitable radius.

(ii) Cut the circle with a pair of scissors.

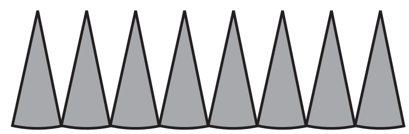
(iii) Fold the circle into two parts.
(iv) Keep on folding the circle, till we get 16 equal parts.
(v) Unfold the circle.
(vi) Shade half of the circle, as shown in the figure
(vii) Now cut the circle with a pair of scissors along the marks formed into 16 parts.



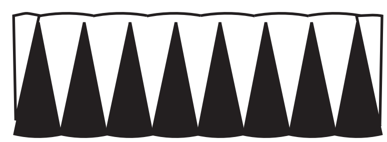
(viii) The length of every segment of the circle is equal to the radius.
(ix) Divide these 16 parts into two equal pieces.



(xi) Arrange the white parts cut in the upper half and the others in lower half.
(xii) The figure so formed is almost a rectangular region.



(xiii) The length of the rectangular region is equal to half of the circumference.
(xiv) The width of the rectangular region is equal to the radius of the circle.
(xv) The area of rectangular region is equal to the area of the circle.



Thus a circular region = Area of the rectangular region.

= Length of the rectangle × Breadth of the rectangle

= Length × Breadth

= (Half of the circumference) × (Radius of the circle)

$$= \left[\frac{1}{2}(2\pi r)\right] \times r$$

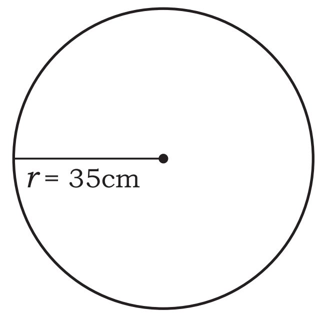
$$= \frac{1 \times \cancel{2} \times \pi r \times r}{\cancel{2}} = \pi r^2$$

Hence formula to find the area of a circular region is:

$$\boxed{\text{Area of a Circular Region} = \pi r^2}$$

where r is the radius of the circle.

Example 1: The radius of a circle is 35 cm. Find its area.



Solution: Area of the circular region = πr^2

Here radius = r = 35 cm; and $\pi = \frac{22}{7}$

$$\text{Area} = \pi r^2 = \frac{22}{7} \times (35)^2 = \frac{22 \times 35 \times 35}{7}$$

$$= \frac{22 \times 35 \times \overset{5}{\cancel{35}}}{\underset{1}{\cancel{7}}} = 3,850 \text{ sq. cm}$$

Example 2: The area of a circular region is 1,386 sq. cm. Find its radius.

Solution: Given area A of the circular region is 1386 sq. cm.
We know that the formula for finding area of circular region is: A = πr^2.

Therefore $1386 = \pi r^2$

or $\frac{22}{7} r^2 = 1386$

or $r^2 = 1386 \div \frac{22}{7}$

So, $r^2 = \frac{1386 \times 7}{22} = \frac{11 \times 63 \times 2 \times 7}{22}$

Therefore $r^2 = 63 \times 7 = 3 \times 3 \times 7 \times 7$

Thus radius = $r = 3 \times 7 = 21$ cm

EXERCISE 12.2

A. Find the area of circular region when its radius is:

(1) 14 cm (2) 10.5 cm (3) 21 cm (4) 17.5 mm

(5) 35 mm (6) 24.5 cm (7) 71.4 cm (8) 38.5 cm

B. Find the radius of the circular region when its area is:

(1) 154 sq. mm (2) 2,464 sq. cm (3) 7546 sq. mm

(4) 5,544 sq. mm (5) 6.16 sq. mm (6) 962.5 sq. cm

(7) 38.5 sq. m (8) 75.46 sq. mm (9) 13.86 sq. m

C. Find the area of circular region when its diameter is:

(1) 21 cm (2) 28 cm (3) 42 cm (4) 56 cm

(5) 8.4 cm (6) 9.8 cm (7) 11.2 cm (8) 12.6 cm

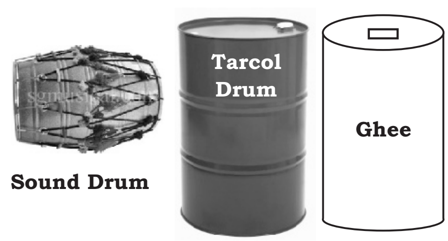
D. Find the diameter of the circular region when its area is as under:

(1) 154 cm^2 (2) 1,386 (mm^2) (3) 3,850 (mm^2) (4) 9,856 mm^2

(5) 124.74 cm^2 (6) 186.34 cm^2 (7) 221.76 cm^2 (8) 260.26 cm^2

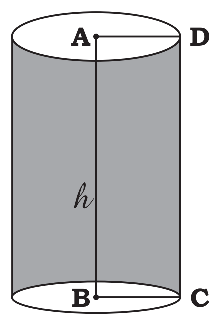
12.2 SURFACE AREA AND VOLUME OF A CYLINDER

Cylinders: We have already learnt about cylinders in previous class. Ghee tins, cooking oil tins, cold drink tins, drums of tarcol etc are the examples of cylinder.



Activity: Staking of the coins. Let us keep on putting one rupee, two rupee and five rupee coins one above the other. What type of shapes, we get?

12.2.1 Find surface area of a cylinder using formula.



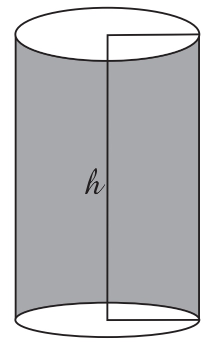
Surface of cylinder consists of three parts. Two equal circular surfaces and a third curved surface.

Let us find the area of these three surfaces. Let the radius of each of the circular surface be r and height of cylinder surface be h.

(i) Area of the two circular regions of the cylinder = Area of the upper base of the cylinder + Area of the lower base of the cylinder

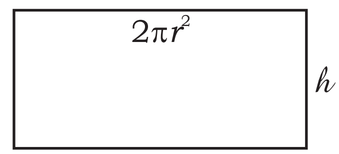
$$= \pi r^2 + \pi r^2 = 2\pi r^2$$

(ii) To find the area of the curved surface of the cylinder: We Take sheet of paper whose width is equal to the height of the cylinder.



Wrap it round the cylinder and cut of the extra piece, this will take the shape of the rectangle. The width of this paper will be h and the length will be equal to the circumference of the base, the circular region of the cylinder.

Area of the curved surface of the cylinder



$= $ Area of the rectangle

$= $ Length × Breadth

$= 2\pi r \times h$

$= 2\pi rh$

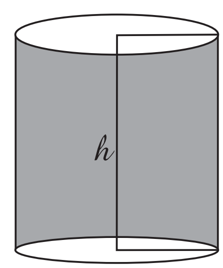
Thus the total area of the surface of the cylinder

= The area of the two circular surfaces
+ Area of the curved surface of the cylinder.

$= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r + h)$

Thus the surface area of the cylinder $= 2\pi r(r + h)$



Example 1: The radius of the base of a cylinder is 14 cm. And the height of the cylinder is 20 cm. Find the total surface area of the cylinder.

Solution: Radius of the base $= r = 14$ cm

Height of cylinder $= h = 20$ cm $\left(\text{Take } \pi = \dfrac{22}{7}\right)$

Formula for finding the total surface area of the cylinder $= 2\pi r(r + h)$

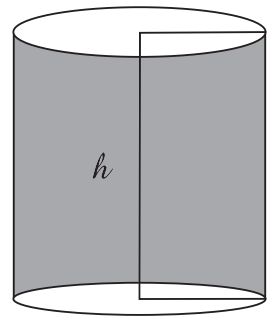
i.e. $A = 2\pi\, r(r + h) = 2 \times \dfrac{22}{7} \times 14\ (14 + 21)$

$= \dfrac{2 \times 22 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}}} \times (35)$

$= 88 \times 35 = 3{,}080$ sq. cm

Hence the total surface area of the sylinder is $3{,}080\ cm^2$.

Example 2: Find the height of a cylinder when its radius is 10.5 cm. Total surface area is 1650 cm^2.



Solution: Height of the cylinder $= h = ?$

radius of circular base $= r = 10.5$ cm

A = Total surface area = 1650 cm^2

Formula: $A = 2\pi r\,(r + h)$

$$\text{or } 1650 = 2 \times \frac{22}{7} \times 10.5\,(10.5 + h)$$

$$\text{or } 1650 = \frac{2 \times 22 \times \cancel{10.5}^{\,1.5}}{\cancel{7}_{\,1}} \times (10.5 + h)$$

$$\text{or } 1650 = 693 + 66h$$

$$\text{or } 1650 - 693 = 66h$$

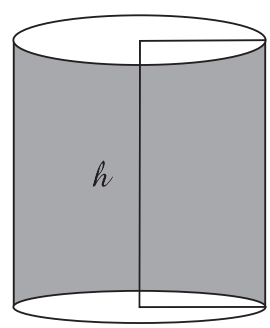
$$\text{or } 957 = 66h$$

$$\text{or } h = \frac{957}{66} = \frac{\cancel{\cancel{957}^{319}}^{29}}{\cancel{\cancel{66}_{22}}_{2}} = \frac{29}{2} = 14.5$$

$$\text{or } h = 14.5 \text{ cm}$$

Thus the required height of the cylinder is 14.5 cm

Example 3: Find the value of radius and height of a cylinder whose total surface area is 2200 sq. cm. And the sum of radius and height is 35 cm.



Solution: Total surface area = A = 2,200 cm^2

A = Total surface area = 2,200 cm^2

$r + h = 35$ cm

We have to find the value of radius and height of cylinder.

Formula: $A = 2\pi r\,(\,r + h\,)$

$$2200 = 2 \times \frac{22}{7} \times r\,(35)$$

or $\quad 2200 = \left(\frac{2 \times 22 \times 35}{7}\right) r$

or $\quad 2200 = \frac{44 \times \cancel{35}^{5}}{\cancel{7}_{1}} r$

or $\quad 2200 = 220r$

or $\quad \frac{2200}{220} = r$

or $\quad 10 = r \quad$ or $\quad r = 10$ cm

Therefore radius $= r = 10$ cm

Again $r + h = 35$

$10 + h = 35$

Therefore $h = 35 - 10 = 25$ cm

Thus the required radius of the cylinder is 10 cm and height is 25 cm.

EXERCISE 12.3

A. Find the total surface area of the cylindrical region when the radius of the base of cylinder and height of the cylinder are given.

(1) Radius = 14 cm, height = 26 cm

(2) Radius = 10 cm, height = 18 cm

(3) Radius = 21 mm, height = 29 mm

(4) Radius = 17.5 cm, height = 22.5 cm

(5) Radius = 30 cm, height = 40 cm

(6) Radius = 25 mm, height = 41.5 mm

B. Find the height of the cylindrical region when its radius and total surface area are given.

(1) Radius = 10.5 cm, total surface area = 1980 cm^2

(2) Radius = 17.5 cm, total surface area = 4400 cm^2

(3) Radius = 12 mm, total surface area = 2112 mm^2

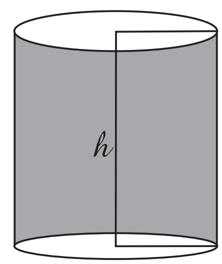
(4) Radius = 15 cm, total surface area = 3960 cm^2

(5) Radius = 28 cm, total surface area = 10560 cm^2

C. Solve the following.

1. Find the value of radius and height of a cylinder. The sum of its radius and height is 28 cm. Also total surface area of the cylinder is 1408 cm^2.
2. The sum of radius and height of a cylinder is 35 cm. The total area of the cylinder is 3,300 cm^2. Find its radius and height separately.
3. Find the value of radius and height of a cylinder when sum of its radius and height is 49 cm and total surface area is 5,852 cm^2.
4. The sum of radius and height of a cylinder is 56 mm. The total surface area is 6,864 $(mm)^2$. Find the radius and height of the cylinder.

II. Find the volume of a cylindrical region using formula



We know that the base of a cylinder is a circular surface. We denote its radius by r. We have already learnt to find the area of the circular region.

Area of circular region = Area of the base of the cylinder

$$\mathbf{A} = \pi r^2$$

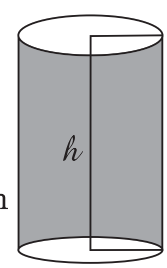
If the height of the cylinder be denoted by h, then:

Volume of cylinder = Area of circular region × height of cylinder

$$\mathbf{V} = \pi r^2 \times h = \pi r^2 h$$

Hence the volume of the cylinder = $\pi r^2 h$ where r represents the radius of the circular base and h the height of the cylinder.

Example 1: If the length of the diameter of a cylinder is 84 cm and height is 55 cm; what is its volume?



Solution:

d = Diameter of the given cylinder = 84 cm

Therefore the radius r of the given cylinder = $\frac{d}{2} = \frac{84}{2}$ cm

Thus r = 42 cm

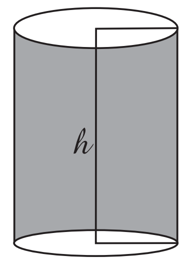
Now formula for finding volume of the cylinder is V = $\pi r^2 h$

Then $V = \frac{22}{7} \times (42)^2 \times 55$

$= \frac{22}{\cancel{7}_{1}} \times \cancel{42}^{6} \times 42 \times 55$

$= 22 \times 6 \times 42 \times 55 = 304{,}920$ cubic centimetres

Example 2: The volume of a cylinder is 184,800 cubic centimetres and the radius of the circular base of the cylinder is 35 cm. Find the height



Solution: The formula of volume of the cylinder is $V = \pi r^2 h$

$V = 184{,}800$ cubic centimetres

$r = 35$ cm

$h = ?$

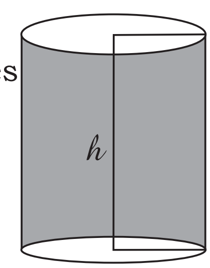
$184800 = \frac{22}{7} \times (35)^2 \times h$

$\frac{7 \times 184800}{22 \times 35 \times 35} = h$

$h = \frac{\cancel{7} \times \cancel{184800}^{\cancel{7392}\ \cancel{1056}\ \cancel{528}\ 48}}{\cancel{22}_{\cancel{11}\ 1} \times \cancel{35}_{\cancel{7}\ 1} \times \cancel{35}_{\cancel{7}\ 1}} = 48$ cm

Thus the required height of the given cylinder is 48 cm.

Example 3: Find the radius of the cylinder, when its volume is 4,400



Solution:

V = Volume of the cylinder = 4,400 cubic centimetres

h = Height of the cylinder = 14 cm

r = radius of circular base of the cylinder = ?

Formula for finding volume of cylinder = $V = \pi r^2 h$

$4{,}400 = \frac{22}{7} \times r^2 \times 14$

$4{,}400 = \frac{22 \times \cancel{14}^{2}}{\cancel{7}_{1}} r^2$

$4400 = 44r^2$

or $r^2 = \frac{4400}{44} = 100$

Therefore $r = \sqrt{100} = 10$

Hence radius of circular base of the cylinder is 10 cm.

EXERCISE 12.4

A. In each of the following, the radius of the base of cylinder and its height is given. What is the volume in each case?

(1) Radius = 14 cm and height = 20 cm
(2) Radius = 25 cm and height = 35 cm
(3) Radius = 28 cm and height = 40 cm
(4) Radius = 20 cm and height = 28 cm
(5) Radius = 1.05 m and height = 2.5 m
(6) Radius = 63.5 cm and height = 75 cm
(7) Diameter = 70 mm and height = 50 mm
(8) Diameter = 60 mm and height = 63 mm

B. In each of the following, the radius of the base of the cylinder and its volume is given. What is the height of cylinder in each case?

(1) Radius = 14 cm and volume = 15,400 cubic centimetres
(2) Radius = 20 cm and volume = 35,200 cubic centimetres
(3) Radius = 25 mm and volume = 82,500 cubic millimetres
(4) Radius = 21.5 mm and volume = 305,085 cubic millimetres
(5) Radius = 30 mm and volume = 99,000 cubic millimetres
(6) Radius = 28 cm and volume = 98,560 cubic centimetres

C. In each of the following, the height of the cylinder and its volume is given. What is the radius of the base of the cylinder in each case?

(1) Height = 25 cm and volume = 34,650 cubic centimetres
(2) Height = 20 m and volume = 12,320 cubic metres
(3) Height = 28 cm and volume = 35,200 cubic centimetres
(4) Height = 42 cm and volume = 118,800 cubic centimetres
(5) Height = 56 mm and volume = 281,600 cubic millimetres
(6) Height = 30 cm and volume = 41,580 cubic centimetres

12.2.3 (a) Solve real life problems involving Circumference and Area of Circle

Example 1: The radius of the wheel of a machine is 84 cm. How much wire will be needed to wound round ten full rounds.

Solution:

r = radius of the wheel = 84 cm = 0.84 m

Formula for finding circumference is $C = 2\pi r$

Thus we have circumference = 5.28 m

The wire required for one round is 5.28 m

Therefore the wire required for ten rounds is (5.28×10)m = 528 m

$$C = \left(2 \times \frac{22}{7}\ 0.84\right) \text{m}$$

$$\text{or } C = \frac{2 \times 22 \times 0.84 \text{ m}}{7}$$

$$\text{or } C = \frac{2 \times 22 \times \overset{0.12}{\cancel{0.84}} \text{ m}}{\underset{1}{\cancel{7}}}$$

$$\text{or } C = 5.28 \text{ m}$$

Hence the length of the required wire is 528m.

Example 2: The diameter of a round table is 4.2 m. How many metres of lace are required for its border? Also find its cost at the rate Rs 40 per metre.

Solution:

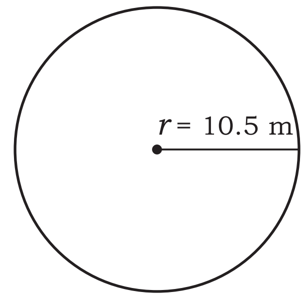
First we have to find the circumference of round table.

Circumference = $\pi d = \frac{22}{7} \times 4.2 \text{ m} = \frac{22}{\cancel{7}} \times \overset{0.6}{\cancel{4.2}} = 22 \times 06 = 13.2$ m

Thus the length of the lace required for round the circular cloth is 13.2 metres.

The cost of the length of lace = Rate × length
= Rs 40 × 13.2
= Rs 528

Example 3: What is area of a circular floor and the cost of flooring it at the rate of Rs 150 per square metre. When the radius of circular floor is 10.5 m.



Solution: The radius of circular floor is 10.5 m.

Formula for finding area of circular floor is πr^2

i.e. $A = \pi r^2 = \frac{22}{7} \times 10.5 \times 10.5$

or $A = \frac{22 \times \overset{1.5}{\cancel{10.5}} \times 10.5}{\underset{1}{\cancel{7}}} = 346.5 \text{ m}^2$

Thus the area of circular floor is 346.5 m^2.

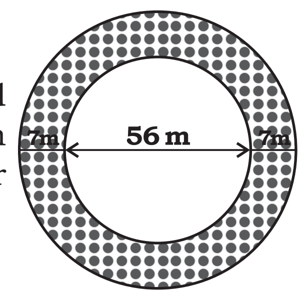
The cost of flooring it is: Rate × Area

= Rs (150 × 346.5)

= Rs 51975 . 00

Hence cost of flooring the circular floor is Rs 51,975.

Example 4: The diameters of two circular grounds and the width of the road encircling it is given in the figure. Find the area of the road and cost for cementing it a Rs 50 m^2.



Solution: There are two circles:

(i) Outer circle having diameter = (56 + 14) m = 70 m

Therefore the radius of outer circle = $\frac{70}{2}$ m = 35 m = R

(ii) Inner circle having diameter = 56 m

Therefore the radius of inner circle = 28 m = r

Now we find the area of both circles.

Area of outer circle = $\pi R^2 = \frac{22}{7} \times 35 \times 35 = \frac{22 \times \overset{5}{\cancel{35}} \times 35}{\underset{1}{\cancel{7}}} = 3{,}850 \text{ m}^2$

Area of inner circle = $\pi r^2 = \frac{22}{7} \times 28 \times 28 = \frac{22 \times \overset{4}{\cancel{28}} \times 28}{\underset{1}{\cancel{7}}} = 2{,}464 \text{ m}^2$

Hence area of road = Area of outer circle – Area of inner circle

$$= 3{,}850\ m^2 - 2{,}464\ m^2 = 1{,}386\ m^2$$

The cost of cementing the road is: Rate × Area of the road

$$= \text{Rs } 50 \times 1386 = 69{,}300 \text{ rupees}$$

Hence the cost of cementing the circular road is Rs 69,300.

EXERCISE 12.5

1. The diameter of a circular garden is 70 m. The garden has 5 rounds of wire fencing. If the wire costs Rs 10 per metre. Find

2. Find the radius of a circular ring prepared from a wire used for a square with side 22 cm.

3. The radius of a bicycle wheel is 42 cm. Each wheel completed 1000 rotations when Ahmed travelled from home to school. Find

4. The total cost of the wire required for four rounds of a circular fencing is Rs 3,960/- If the wire costs Rs 5 per metre; find the

5. A piece of wire is bent in the shape of an equilateral triangle, having each side 13.2 m. It is again re-bent to form a circular ring.

6. What is the area of a circular garden and the cost of gardening it at the rate of Rs 20 per sq. metre? When the radius of the

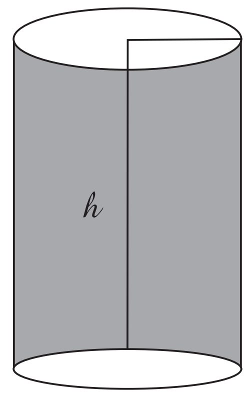
7. There are two circular grounds. The outer circular ground has diameter 49 m. The inner circular ground has diameter 35 m. Find the area of the road and the cost of cementing it at Rs 100

8. There is a round wooden table of 2.8 m diameter. Find the area of the surface of the table and the cost of polishing it at the rate

9. A circular fountain is built in the centre of crossing. Its radius is 3.5 m. What is the area of the fountain? Also find the cost of

10. A table cover is made to cover a round dinning table. The diameter of the table cover is 3.5 m. What is area of the table cover? Also

12.2.3 (b) Solve real life problems involving Surface Area and Volume of a Cylinder



Example 1: An open cylindrical tank has a circular base of diameter 2.1 m. The height of the tank is 4 m. Find how many square metres of steel sheet are required for the tank. Also find the cost of tank at the rate of Rs 400 per m^2.

Solution:

Diameter of circular base = 2.1 m

Radius of circular base = $r = \dfrac{2.1}{2}$ m

or $r = 1.05$ m

Area of circular base = Area of circle = πr^2

$$A = \pi r^2$$

$$= \left(\frac{22}{7} \times 1.05 \times 1.05\right) m^2 = \left(\frac{22 \times \overset{0.15}{\cancel{0.15}} \times 1.05}{\cancel{7}_1}\right) m^2$$

$$= (22 \times 0.15 \times 1.05)\ m^2 = (3.30 \times 1.05)\,\text{m}^2 = 3.465\,\text{m}^2$$

Curved surface area of cylinder $= 2\pi rh = \left(2 \times \frac{22}{7}\ 1.05 \times 4\right)\text{m}^2$

$$= \left(\frac{2 \times 22 \times \overset{0.15}{\cancel{1.05}} \times 4}{\cancel{7}_1}\right)\text{m}^2 = (44 \times 0.60)\ \text{m}^2 = 26.40\ \text{m}^2$$

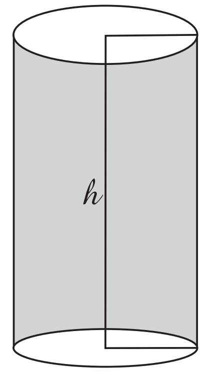
Now total area of steel sheet required to make the tank is

$$= 3.465\ \text{m}^2 + 26.40\ \text{m}^2$$

$$= 29.865\ \text{m}^2$$

The cost of tank = Rate × Area = 400 × 29.865 = Rs 11946.000 = Rs 11,946.

Example 2: The radius of a circular well is 1.4 m and its depth is 20 m. What will be the cost of designing the well at the rate of Rs 300 per cubic metre?



Solution:

Radius of the base of the circular well = r = 1.4 m

Depth of the well = h = 20 m

Volume of the well = V = $\pi r^2 h$

$$\text{Volume} = V = \frac{22}{7} \times 1.4 \times 1.4 \times 20 = \frac{22 \times \overset{0.2}{\cancel{1.4}} \times 1.4 \times 20}{\underset{1}{\cancel{7}}} = 4.4 \times 28.0$$

$$= 123.2 \text{ cubic metres}$$

Cost of designing = Rate × Volume

= 300 × 123.2 = Rs 36960.0 = Rs 36,960

Hence the volume of the well = 123.2 cubic metres and the cost of

EXERCISE 12.6

1. The radius of an open pond in a cylindrical shape is 10.5 m and depth is 8 m. Find the cost of cementing its floor and curved

2. Find the total surface area of a cylidrical steel tank 8.4 m long. The radius of its circular base is 7 m. Also find its cost at the rate of Rs 100 per sq. metre.

3. An open cylindrical tank has a base of diametre 2.8 m. The height of the tank is 5m. Find how many square metres of steel sheet are required for the tank. Also find the cost of the tank at

4. The length of a steel pipe is 2.8 m and radius 7 cm. Calculate its

5. Find the total surface area of oil drum whose length is 1.5 m and

6. Find the capacity of a circular water tank in litres when the height of the tank is 5 m and its diameter is 4.2 m. (Hint: Convert metres into centimetres and 1 litre = 1000 cm^3).

7. A cylindrical tin can is 63 cm high and its radius is 25 cm. Find

8. A well is 16 m deep and its diameter is 7 m. How much soil is

9. A tin pack of a soft drink is 10.5 cm long and the radius of the

10. The diameter of a circular well is 3.5 m and its depth is 24 m. What will be the cost of digging it at the rate of Rs 250 per

11. Find the radius of an oil drum whose volume is 11 m^3.

12. The capacity of an oil drum is 3850 litres. What is its volume in cm^3. Find its height in metres when its radius is 70 cm.

13. The internal diameter of a round cylindrical shape mosque is 28 m and height of curved walls is 8.75 m. Find the cost of tiles using on the walls only at the rate of Rs 200 per m^2.

14. The internal diameter of a swimming pool is 42 m. The depth is 5 m. Calculate the cost of tiles using round the wall at the rate of Rs 100 per m^2.

REVIEW EXERCISE 12

A. Answer the following:

1. What is circumference of a circle. Give formula to find circumference.
2. What is relation between diameter and circumference?
3. What is diameter of a circle? Give relation between radius and diameter.
4. What is radius of a circle? Give formula for finding area of a circle.

5. Write the formula for total surface area of a cylinder.
6. What is volume of a cylinder? Express its formula.
7. What is the formula for finding diameter from circumference of a circle?
8. In a cylinder when radius = height, what is the formula for finding volume of a cylinder?

B. Fill in the blanks:

1. The formula for finding curved surface area of the cylinder is ___________.
2. Tin pack of soft drink is an example of __________.
3. Take ten coins of Rs 5 and stake up a file then __________ type of shape is obtained.
4. When radius of cylinder is in metres; the unit for volume of cylinder will be ____________.
5. The value of π upto three decimal places is __________.

C. Solve the following mentally and (✓) the correct answer.

1. The circumference of a circle is ____ cm when its diameter is 7 cm.
   (i) $\frac{22}{7}$ (ii) $\frac{7}{22}$ (iii) 7 (iv) 22
2. The area of a circle is ________ sq. cm when its radius is 1 cm.
   (i) $\frac{22}{7}$ (ii) $\frac{7}{22}$ (iii) $\frac{1}{7}$ (iv) $\frac{1}{22}$
3. The area of circular base of cylinder is ___ sq.cm when the value of $r^2 = \frac{1}{22}$ sq.cm.
   (i) 7 (ii) $\frac{1}{7}$ (iii) 22 (iv) $\frac{1}{22}$
4. Area of curved surface of a cylinder is _______cm^2 when $r = 7$ cm and $h = \frac{1}{22}$cm.
   (i) 1 (ii) 2 (iii) 3 (iv) 4
5. Total surface area of a cylinder is __________ cm^2 when $r = 1$ cm and $h = 1$ cm.
   (i) 2π (ii) 3π (iii) 4π (iv) π
6. Volume of a cylinder is ______ cm^3 when $r = 1$ cm and $h = 1$ cm.
   (i) $\frac{22}{7}$ (ii) $\frac{7}{22}$ (iii) 22 (iv) 7

7. Area of circular upper base and lower base of cylinder is ___ cm^2 when $r = 1$ cm.

(i) $\frac{22}{7}$ (ii) $\frac{44}{7}$ (iii) $\frac{7}{22}$ (iv) $\frac{7}{44}$

8. Height of an oil drum is 1 m and radius of the drum is also 1 m. Then its volume is _______ m^3.

(i) 7 (ii) 22 (iii) $\frac{22}{7}$ (iv) $\frac{7}{22}$

9. The length of the minute hand of a clock is 3.5 cm. The distance covered in 1 hour is ________cm.

(i) 22 cm (ii) 7 cm (iii) 3.5 cm (iv) 2.2 cm

10. In a circle the radius is increased by twice in length. The area of the circle will be increased by _____________.

(i) Twice (ii) Thrice (iii) Four times (iv) None of these

SUMMARY

- Circumference is the length of the boundary of a circle.
  Circumference $C = 2\pi r$ or $C = \pi d$
- Ratio between circumference and diameter of a circle is denoted by π i.e $\frac{\text{Circumference of the circle}}{\text{Diameter of the circle}} = \frac{22}{7} = 3.14 = \pi$.
- Area of a circle is the area of the circular region that is the number of square units inside the circle. $A = \pi r^2$ square units.
- The surface area of a cylinder consists of three parts; two equal circular surfaces and a third curved surface. Total surface area of cylinder $= \pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$
- Volume of a cylinder = Area of the circular region × Height of the cylinder.

$$V = \pi r^2 \times h$$

$$V = \pi r^2 h \text{ cubic units}$$

Unit 13 INFORMATION HANDLING

Introduction

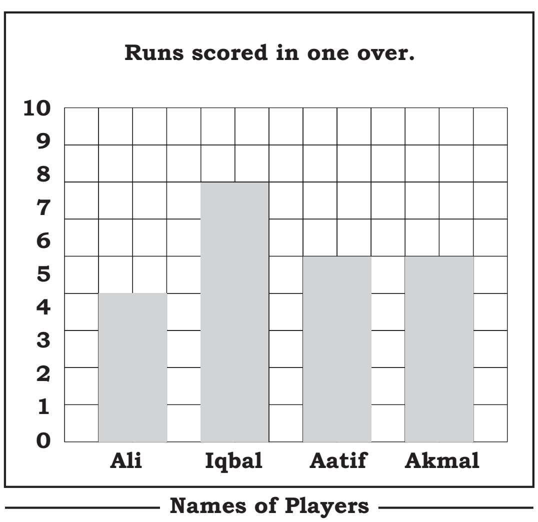
Information handling plays an important role in the subject of statistics. This subject has been constantly expanding during the last few centuries and its definition is also changing. This subject originated due to the desires of the rulers in ancient times to find how much manpower was available in their kingdoms. For this they used to hold census in an organized manner. This way they used to be well informed about their power and the capability to wage wars. That is why statistics used to be called the rulers' science and political arithmetic.

13.1 FREQUENCY DISTRIBUTION

13.1.1 Demonstrate data presentation

In our daily life we collect information and present it in different ways bar graphs, charts etc so that it could be understood easily and the useful conclusions can be drawn.

I



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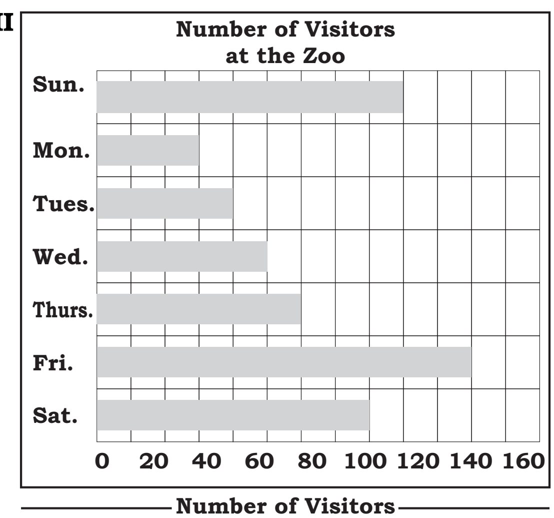


In previous class, we have learnt that 'Data' means, the facts about information that are the results of measurement, observation or experiments.

The results help us in reviewing our past performance and future planning. After collection of data, the most important step is its presentation.

Tally charts are used for describing, counting, quantity of the grouped data.

Table below represents the data of students of class VII; how they go to school.

How do you go to school?		**Number of students**
By foot	卌 IIII	9
BY car	卌 I	6
By bus	卌 卌 III	13

13.1.2 Define frequency distribution (i.e frequency, lower class limit, upper class limit and class-interval).

I. Frequency

The number of values that occurs for a particular group of a data is called its frequency.

Example 1. A group of twenty students of VII class obtained the following marks in a Science test.

16, 11, 40, 27, 38, 55, 45, 70, 50, 65, 60, 41, 100, 71, 80, 75, 82, 85, 92, 89

Steps:

(i) Arrange the data in ascending order:

11, 16, 27, 38, 40, 41, 45, 50, 55, 60, 65, 71, 70, 75, 80, 82, 85, 89, 92 and 100.

(ii) Make interval of suitable number of three groups.

First, students who obtained marks from 11 to 40. Second, who obtained the marks from 41 to 70. Third, who secured the marks from 71 to 100.

(iii) Make frequency chart as under:

Name of Group	**Period or class interval**	**Marks obtained**	**No. of students**	**Tally mark**
First	(11 – 40)	11, 16, 27, 38 and 40	5	卌
Second	(41 – 70)	41, 45, 50, 55, 60, 65 and 70	7	卌 II
Third	(71 – 100)	71, 75, 80, 82, 85, 89, 92 and 100	8	卌 III

The frequency chart is as under:

The frequency of (11 – 40) is 5. Here 5 students obtained marks 11 to 40.

The frequency of (41 – 70) is 7. Here 7 students obtained marks 41 to 70.

The frequency of (71 – 100) is 8. Here 8 students obtained marks 71 to 100.

Class Intervals: Each group of a data is also known as the class interval. For example, (11 – 40), (41 – 70) and (71 – 100) are class intervals represent the values of a group.

Upper Class Limit: The greatest value of a class interval is called the upper class limit. For example, in the class interval (41 – 70), 70 is the upper class limit.

Lower Class Limit: The smallest value of a class interval is called the lower class limit. For example, in the class interval (71 – 100), 71 is the lower class limit.

Size of the Class Interval: The number of values in a class interval is called its size or length. For example, the size or length of class interval (11 – 40) is 30.

Example. There are 40 students in the class VII who got the following marks in an English test. Make a frequency table or frequency distribution table by using 5 classes of an equal size.

26, 41, 17, 15, 18, 60, 46, 33, 24, 15, 52, 39, 28, 89, 74, 68, 56, 38, 92, 49, 28, 82, 19, 21, 34, 23, 43, 77, 65, 64, 21, 59, 15, 33, 66, 29, 33, 65, 35, 39.

Solution: Arrange the marks obtained in ascending order as:

9, 15, 15, 15, 17, 18, 19, 21, 21, 23, 24, 26, 28, 28, 29, 33, 33, 33, 34, 35, 38, 39, 39, 43, 46, 46, 49, 52, 56, 59, 60, 64, 65, 65, 66, 68, 74, 77, 82, 89, 92.

We know that,

$$\text{Size of class} = \frac{\text{Greatest value} - \text{Lowest value}}{\text{Number of class intervals}}$$

We can see from the above un-grouped data (arranged in ascending order) that of:

Greatest value = 92

Lowest value = 9

No. of class intervals = 5

Therefore size of class interval $= \frac{92-9}{5} = \frac{83}{5} = 16.6 \approx 17$

Class Interval	Tally Marks	Frequency
9 – 25	$\cancel{||||}$ $\cancel{||||}$	10
26 – 42	$\cancel{||||}$ $\cancel{||||}$ |||	13
43 – 59	$\cancel{||||}$ |	6
60 – 76	$\cancel{||||}$ ||	7
77 – 93	||||	4

EXERCISE 13.1

1. A book store sold the following number of story books, last week. Construct a frequency distribution table.

Name of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Bbooks sold	24	25	27	28	22	30	21

2. The electricity bills paid by 12 consumers are given below. Make a frequency distribution table of 5 classes of an equal size.

Rs 5100, Rs 7000, Rs 3560, Rs 6030, Rs 4220, Rs 6740, Rs 4810, Rs 5450, Rs 7180, Rs 5920, Rs 6850 and Rs 5690.

3. In an annual examination, 20 students of a school got the following marks out of 850 marks. Construct a frequency distribution table by taking 100 as a class interval.

551, 786, 678, 725, 788, 580, 720, 690, 750, 651, 599, 609, 719, 760, 625, 775, 646, 667, 753, 675.

4. The following data shows the number of children who visited the zoo in last month.

134, 167, 145, 130, 155, 142, 130, 180, 162, 130, 120, 165, 170, 110, 200, 185, 132, 140, 110, 170, 100, 160, 133, 150, 125, 110, 145, 155, 160, 200.

Construct a frequency distribution table by taking 10 as a class interval.

5. There are 30 students in class VII who secured following marks in a Mathematics Test.

45, 19, 49, 17, 37, 10, 25, 28, 50, 46, 43, 34, 25, 50, 48 46, 42, 34, 33, 11, 28, 19, 30, 17, 13, 11, 39, 41, 35, 34.

Make a frequency distribution table using 5 as a class interval of an equal size.

13.2 PIE GRAPH

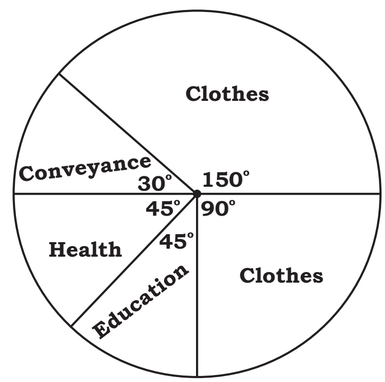
Interpret and draw pie-graph

A pie graph is generally used for the comparison of some numerical facts classified in different classes. In this graph, the central angle measures 360^{o} which is sub-divided into the ratio of the sizes of the groups to be shown through this graph. Following example will help to understand the concept of a pie graph.

Unit 13 INFORMATION HANDLING

Example 1. The expenditure made by a student in a week period is Rs 3600. It is represented through the following pie graph.

Read the pie graph and answer.



(i) Draw a chart showing the expenditure on each item.

(ii) At what items, he spent most amount? And how much?

(iii) At what item, he spent the least amount? And how much?

(iv) At what items, he spend equal amount? And how much?

Solution: Here whole circle is divided in 360°.

There are five sectors. The biggest sector has an angle of 150°.

It show the expenditure made on food.

(i) Therefore: amount spent on food $= \dfrac{\text{Rs } 3600 \times 150^\circ}{360^\circ} = \dfrac{\text{Rs } \overset{10}{\cancel{3600}} \times 150^\circ}{\cancel{360}_1}$

= Rs 1500

(ii) The amount spent on clothes $= \dfrac{\text{Rs } 3600 \times 90^\circ}{368} = \dfrac{\text{Rs } \overset{10}{\cancel{3600}} \times 90^\circ}{\cancel{360}_1}$

= Rs 900

(iii) The amount spent on education $= \dfrac{\text{Rs } 3600 \times 45^\circ}{360^\circ} = \dfrac{\text{Rs } \overset{10}{\cancel{3600}} \times 45}{\cancel{360}_1}$

= Rs 300

Name of item	Food	Clothes	Education	Health	Conveyance
Amount spent	Rs 1500	Rs 900	Rs 450	Rs 450	Rs 300

Pie-graph shows, the most amount is spent on food i.e Rs 1500.

Pie-graph shows, the least amount is spend on conveyance i.e Rs 300.

Pie-graph shows, the equal amount is spent on education and heath i.e Rs 450.

Example 2. In District Sports competition of 1800 students 750 played cricket, 200 played badminton, 400 played hockey and 450 played football. Draw a pie graph of the students playing the games.

Solution:

Total number of students = 1800

(i) Find the angle for each sector by using the following formula.

$$\text{Required angle} = \frac{\text{No. of students played a game}}{\text{Total number of students}} \times 360^{\circ}$$

$$\text{Measure of angle associated with badminton} = \frac{200}{1800} \times 360^{\circ} = 40^{\circ}$$

$$\text{Measure of angle associated with cricket} = \frac{750}{1800} \times 360^{\circ} = 150^{\circ}$$

$$\text{Measure of angle associated with hockey} = \frac{400}{1800} \times 360^{\circ} = 80^{\circ}$$

$$\text{Measure of angle associated with football} = \frac{450}{1800} \times 360^{\circ} = 90^{\circ}$$

***(ii) To draw a pie graph*.**

Step 1: Draw a circle of suitable radius.

Step 2: Draw an angle of 40° representing the badminton: (40°)

Step 3: Draw an angle of 150° representing the cricket: (150°)

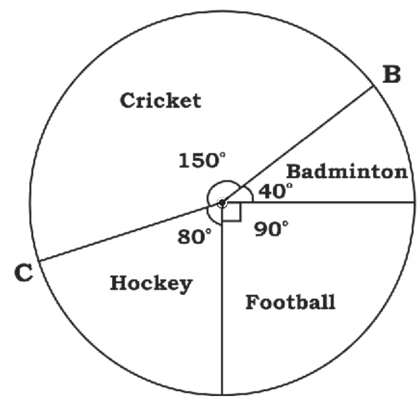
Step 4: Draw an angle of 80° representing the hockey: (80°)

Step 5: Remaining angle representing the football will be of: (90°)

Total (360°)

(iii) Label each sector according to the figure.

(iv) Answer the following:



1. Which is the most favourite game?

   Cricket.

2. Which game is liked by the least number of students?

   Badminton

3. How many more students played football than hockey?

   50 students

4. How many less number of students played badminton than hockey.

   200 students

EXERCISE 13.2

1. In mathematics quiz 80 students appeared. 60 students passed and 20 students failed. Represent it through pie chart.
2. In class VII there are 36 students. The favourite colour of 16 is red, 9 is blue. 7 students like pink colour and four liked white. Make a pie chart of it.
3. Sara and her friends eat fruits in a week as shown in the table.

Name of fruit	Apple	Banana	Mango	Peach
No. of fruits	12	24	16	8

By using the table draw a pie graph.

4. In a party, Mr. Jamal served the guest by following food items.

Food Items	Cold drink	Sandwich	Burger	Samosa
Quantity	180	124	330	86

Use the table to draw a pie graph.

5. The following data shows the distance in km that Hooria travelled in last month.

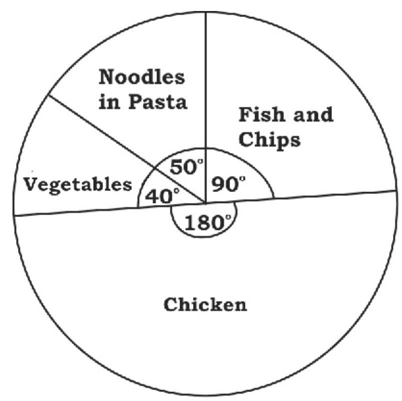
   90, 44, 55, 49, 28, 9, 92, 27, 18, 84, 50, 60, 79, 69, 24, 89, 63, 74, 35, 48, 39, 80.

   Construct the frequency table using 6 as class interval of an equal size. Also draw its pie chart.

6. Hina asked 180 students, about their favourite meal.

   She drew a pie graph.

Read the pie-graph and answer the following:



(i) What is the most favourite meal? And how many students liked it?

(ii) Which food is liked by the least number of students? And how many students liked it?

(iii) How many students liked noodles in pasta?

(iv) Use the pie graph and complete the table.

Favourite meals	Frequency	Angle
Chicken	90	180°
Noodles in pasta		50°
Vegetables		
Fish and Chips	40	
Total	180	

REVIEW EXERCISE 13

1. Fill in the blanks.

(i) Information are called __________ which are normally the results of measurements, observations or experiments.

(ii) The number of values that occurs in a group of data is called __________.

(iii) A data is collected in __________ form and it provides us information about individuals.

(iv) The smallest value of a class interval is called __________.

(v) The table which shows the frequency of class intervals is called __________.

(vi) After collection of data, the most important step is _________.

(vii) The method which is used to record the result is called __________.

(viii) The greatest value of a class interval is called the __________ limit.

(ix) The number of values in a class interval is called its __________.

(x) The representation of a numerical data in the form of sectors of a circle is called a __________.

2. Tick (√) the correct answers.

(i) In pie graph, the measure of central angle is:

(a) 90^{o} (b) 18^{o} (c) 24^{o} (d) 360^{o}

(ii) In the class interval (10 – 50), the upper class limit is:

(a) 11 (b) 10 (c) 50 (d) 51

(iii) In the class interval (9 – 12), the lower class limit is:

(a) 9 (b) 7 (c) 13 (d) 14

(iv) In a grouped data, greatest value = 21, lowest value = 3, and number of interval = 3, then the size of the class is:

(a) 3 (b) 6 (c) 18 (d) 21

3. The ages of visitor in park during a week are given below. Group the data taking 10 as the size of an interval.

25, 50, 49, 47, 26, 10, 2, 1, 15, 17, 18, 19, 27, 28, 30, 35, 17, 32, 31, 3, 4, 9, 10, 15, 12, 13, 17, 24, 20, 22, 24, 26, 30, 17, 35, 40, 36, 32, 31, 37.

SUMMARY

1. Data means groups of information that are normally the results of measurements, observations experiments.
2. Data is collected in raw form and it provides us information about individuals. Such form of the data is called ungrouped data.
3. The number of values that occurs in a class interval is called its frequency.
4. In a grouped data, each group is also known as the class interval.
5. The greatest value of a class interval is called the upper class limit.
6. The smallest value of a class interval is called the lower class limit.
7. The table which shows the frequency of class intervals is called frequency table.
8. The representation of a numerical data in the form of disjoint sectors of a circle is called a pie graph.
9. In pie graph, the central angle measures 360^o which is divided into the ratio of the sizes of the groups.

Think Tank: The Most Wonderful Knowledge.

A teacher asked the students to simplify $\frac{16}{64}$ to its lowest term. Abid cancelled 6 in the numerator with 6 in the denominator and brought the answer $\frac{1}{4}$. The answer is no doubt correct! Should the teacher give him marks for solving the example? Not at all. Again try with $\frac{19}{95}$. The answer is correct and yet it does not merit marks.
Similarly try with other numbers also.

GLOSSARY

Additive inverse:	Two rational numbers whose sum is equal to zero, are additive inverses of each other ($-\frac{2}{3}$ is additive inverse of $\frac{2}{3}$).
Adjacent angles:	Two angles that have a common vertex, a common arm and no. point common in their interiors.
Associative property of union of sets:	$A \cup (B \cup C) = (A \cup B) \cup C$
Associative property of intersection of sets:	$A \cap (B \cap C) = (A \cap B) \cap C$
Algebraic expression:	It is a combination of constants and variables connected by the signs of fundamental operations (+, – , ×, ÷).
Arc:	A part of the circle.
Area of circle:	Area of circle = πr^2
Base:	When a number is written in exponential form, the number which is to be multiplied by itself according to exponent. e.g In $2^3 = 8$, the base is 2.
Binomial:	A polynomial having two terms. e.g. $x + y$, $3a - bc^2$ and $y^2 + z$.
Complement of set A:	It is a set of all members of U which do not belong to A and is denoted by A′. (A′ = U – A).
Commutative property: of union of sets	$A \cup B = B \cup A$.
Commutative property of intersection of sets:	$A \cap B = B \cap A$.
Circumference:	It is the length of the boundary of a circle.
Complementary angles:	Two angles are complementary when their sum is 90°.
Continued ratio of three quantities:	It is the combination of two ratios with at least one common quantity with same size:
Class interval:	In grouped data, each group is class interval.
Descriptive form:	A form of a set which describes characteristics or properties of the members of the set in any common lenguage e.g. A = set of days of a week.
Difference of two sets:	Difference of A and B, denoted as A – B, is the set which contains all the members of A which are not in B. $A - B \neq B - A$. A – B is also represented by A\B.
Disjoint sets:	Those sets which have no common member. e.g. A = { 1, 2, 3 } and B = { p, q, r } are dispoint sets.
Direct proportion:	A relation in which increase or decrease in one quantity causes corresponding increase or decrease in the other.
Decimals:	It is a number with decimal point.
Exponent:	It is a number which represents the number of times; a base is to be multiplied by itself.
Frequency:	It is the number of values that occur for a particular group of data. e.g The frequency of (11 – 40) is 5. It means only 5 values are between 11 to 40.
General sales tax (GST):	It is a tax paid to the Government for sales of certain goods and services. The rate is 17% since 2014.
Hypotenuse:	In right angled triangle, it is the longest side opposite to right angle.

GLOSSARY

Hypotenuse: In right angled triangle, it is the longest side opposite to right angle.

Height of triangle: In a triangle, it is a perpendicular length (altitude) from a vertex to the base.

Intersection of two sets: It is a set consisting of the common members present in both the set the symbol for intersection of two sets is $\cap$

$A \cap B = \{ x \,/\, x \in A \wedge \in B \}$.

Identity property of intersection: $A \cap \mathbb{U} = A$.

Identity property of union: $A \cup \varnothing = A$

Inverse proportion: A relation in which one quantity increases in the same proportion by decreasing the other and vice versa.

Linear equation one variable: It is the equation which contains a single variable with degree 1.

Lower class limit: It is the smallest value of class interval.

Literals: These are the letters or alphabets used to represent unknowns.

Multiplicative inverse of a rational number: If the product of two non-zero rational numbers is 1, then they are called multiplicative inverses of each other. ($\frac{4}{3}$ and $\frac{3}{4}$ are multiplicative inverses of each other).

Markup: It is an amount added to C.P to determine S.P.

Monomial: A polynomial having only one term i.e. simple expression.

Minor arc: It is smaller than half of the circle.

Major arc: It is an arc which is greater than half of a circle.

Non-terminating decimals: The decimals in which a single digit or a block of digits repeats itself infinite number of times after decimal point.

Negative exponent: The exponent which is less than zero, e.g. 3^{-2}, 5^{-7}.

Overlapping sets: Two sets are overlapping if they have at least one common member. e.g $A = \{1, 2, 3,\}$ and $B = \{3, 4, 5,\}$ are overlapping sets.

Perfect square: It is a positive number which is the square of any number, e.g. 4, 9, 16, 25 etc.

Pie: It is the ratio between the circumference and the diameter of a circle. It is denoted by π and $\pi = 3.14$ (approx).

Polynomial: An algebraic expression consisting of one or more terms and the exponents of variables are all whole numbers.

Property tax: It is a levy issued by the government on a person's real or personal property.

GLOSSARY

Rational number: It is a number that can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. e.g. $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{0}{8}$.

Set-builder form: A form of a set which describes the common characteristics of all members using symbols, e.g. $A = \{ x \mid x \in \mathbb{N} \wedge 1 \leq x \leq 10 \}$

Segment of a circle: It is a shape whose boundary is an arc of the circle and a chord of the circle.

Semi-circle: Half of a circle.

Similar Figures: That have exactly the same shape but not necessarily the same size.

Square root: It is a number which can be multiplied by itself to give the original number, the symbol for square root is '$\sqrt{\ }$'.

Tabular form: A form of a set in which the members of a set are tabulated within braces (curly brackets) and separated by commas.

e.g. $A = \{ a, b, c, d \}$; $P = \{ 0, 1, 2, \ldots \}$

Terminating decimal: A decimal having a finite number of digits after the decimal point.

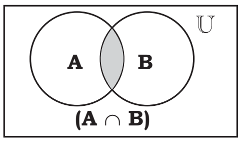
Trinomial: A polynomial having three terms, e.g. $a^3 + 2a - b$, $x^2 + 2x + 1$.

Union of two sets: It is a set which consists of all the members belonging to set A or set B. The symbol for union of sets is '$\cup$'.

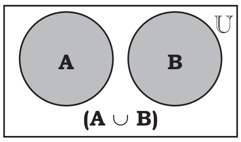
Universal set: It consists of all members under consideration. Usually it is denoted by $\mathbb{U}$.

Upper class limit: It is the greatest value of a class interval.

Venn Diagram: These are used to represent sets and operation on sets, graphically.



$(A \cap B)$



$(A \cup B)$

Zakat: It is one of the five pillars of Islam. It has been mentioned along with Salat in Quran. Zakat = Annual saving × 2.5%.

ANSWERS

EXERCISE 1.1

I. **1.** $\mathbb{N}$ = The set of natural numbers.
2. E = The set first four even numbers.
3. F = The set of first five odd numbers.
4. J = The set of all divisors of 10.
5. $\mathbb{W}$ = The set of first six whole numbers.
6. S = The set of first five multiples of 5.
7. T = The set of multiples of 3.

II. **1.** A = {1, 2, 3, 4, 5, 6, 7}.
2. B = {0, 2, 4, 6, 8, 10, 12, 14}.
3. C = {–3, –2, –1, 0, 1}.
4. D = {*p, a, k, i, s, t, n*}.
5. E = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}.
6. {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
7. G = {1, 3, 5, 15}.

III. **1.** A = $\{x/x \in \mathbb{N} \wedge 1\ x \leq 10\}$.
2. B = $\{x\ /\ x \in Z \wedge -2 \leq x \leq 5\}$.
3. C = {*x*/*x* is the name of river of Pakistan.
4. E = $\{x\ /\ x \in E \wedge 2 \leq x \leq 20\}$.
5. D = $\{x\ /\ x \in O\}$
6. S = {*x*|*x* is a name of a day of a week start with letter S}
7. $\{x\ /\ x \in \mathbb{Z}\}$
8. L = {*x* /*x* is multiple of 4}.

IV. **1.** A = {1, 2, 3, 4, 5}
2. B = {4, 6, 8, 10, 12, 14, 16, 18, 20}
3. C = {–2, –1, 0, 1, 2,}
4. D = {0, 1, 2, 3, 4}

Descriptive form	Tabular form	Set-builder form
A = set of vowels of English alphabet	A = {*a, e, i, o, u*}	A = {*x* / *x* is a vowel of English alphabets}
B = set of all even numbers	B = {2, 4, 6, 8, ...}	B = $\{x\ /\ x \in \mathbb{E}\}$
C = set of first five natural numbers	C = {1, 2, 3, 4, 5}	C = $\{x\ /\ x \in \mathbb{N} \wedge x \leq 5\}$
D = Set of natural numbers which are greater than 50	D = {50, 51, 52, ...}	D = $\{x\ /\ x \in N \wedge x \geq 50\}$
E = Set of letters of English alphabet used in the word 'Pakistan'	E = {*p, a, k, i, s, t, n*}	E = {*x*/*x* is an English alphabets used in the word 'Pakistan'
F = Set of names of days of a week	F = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}	F = {*x* / *x* is a name of a day of a week}

EXERCISE 1.2

A. **1.** A∪B = {4, 5, 6, 8, 9}
2. A∪B = {0, 1, 2, 3, 4, 5, 6, 8}
3. A∪B = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
4. X∪Y = {2,3,5,6,7,9,11,12,15,18}
5. D∪E∪F = {1, 2, 3, 4, 5, 6, 9, 10}
6. (A∪B) ∪ C={1,2,3}, A∪ (B∪C)={1, 2, 3}

B. **1.** A∩B = {3}
2. D∩E = {2, 4, 6, 8}
3. P∩Q = { *a* }
4. A∩B = {1, 2, 3, ...}
5. (A∩B) ∩ C = { } , A ∩ (B∩C) = { }
6. (D∩E) ∩ F = { } , D ∩ (E∩F) = { }

C. **1.** A – B = {4} , B – A = {7, 8}
2. A – B = {9} , B – A = {7}
3. A – B = {0} , B – A = { }
4. A – B = {3, 6, 9, 15, 18} , B – A = {4, 8, 16, 20, 24}

EXERCISE 1.3

A. **1.** Disjoint sets
2. Overlapping sets
3. Disjoint sets
4. Disjoint sets
5. Overlapping sets
6. Overlapping sets
7. Overlapping sets

B. **(i)** Overlapping sets
(ii) Overlapping sets
(iii) Overlapping sets
(iv) Disjoint sets
(v) Disjoint sets
(vi) Disjoint sets
(vii) Overlapping sets

ANSWERS

EXERCISE 1.4

I. **(i)** A = {b, c, d} **(ii)** A∪A′ = {a, b, c, d, e, f} **(iii)** A∩A′ = { }
(iv) U′ = { } **(v)** ∅′ = {a, b, c, d, e, f}

II. **(i)** E′ = {1, 3, 5, 7, 9} **(ii)** E∪E′ = {1, 2, 3, …, 10} **(iii)** F∩E′ = ∅
(iv) U′ = ∅ **(v)** ∅′ = {1, 2, 3, …, 10}

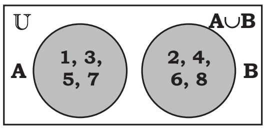
III. **(i)** P′ = {0, 2, 4, 6, 8, 10, 12} **(ii)** P∪P′ = {0, 1, 2, 3, …, 12} **(iii)** P′ ∩ P = { }
(iv) U ∩ U′ = { }

IV. **(i)** B′ = {u, v, w } **(ii)** B′ ∪ B = {u, v, w, x, y, z} **(iii)** B′ ∩ B = { }
(iv) U ∪ U′ = {u, v, w, x, y, z}

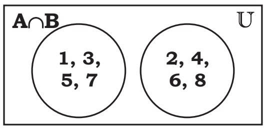
V. **(i)** A′ = {8, 9, 10} **(ii)** B′ = {1, 3, 5, 7, 9} **(iii)** A′ ∩ A = { }
(iv) A ∪ A′ = {1, 2, 3, …, 10} **(v)** B ∪ B′ = {1, 2, 3, …, 10} **(vi)** B ∩ B′ = { }

EXERCISE 1.6

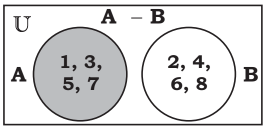
A. 1. (i)



(ii)



(iii)



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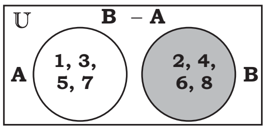
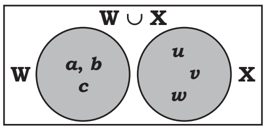
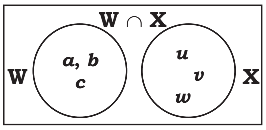


2. (i)



(ii)



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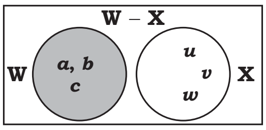

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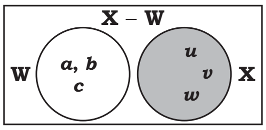
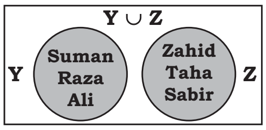
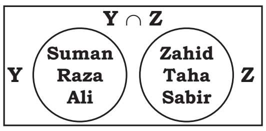


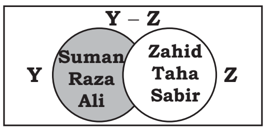
3. (i)



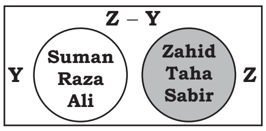
(ii)



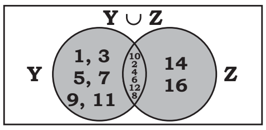
(iii)



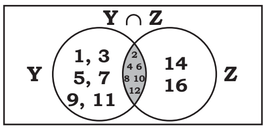
(iv)



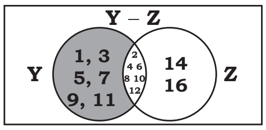
4. (i)



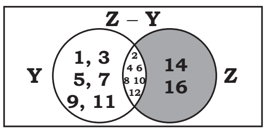
(ii)



(iii)



(iv)



B (1) A′ = {11, 13, 15}
(2) B′ = {10, 11, 12, 13, 14, 16}
(3) A–B = {10, 12, 14, 16}
(4) (A–B)′ = {8, 11, 13, 15}
(5) A∪B′ = {8, 10, 11, 12, 13, 14, 16}
(6) A′∪B′ = {10, 11, 12, 13, 14, 15, 16}
(7) A′–B′ = {15}
(8) (A∩B)′ = {10, 11, 12, 13, 14, 15, 16}
(9) A′∩B = {15}
(10) (A∪B)′ = {11, 13}
(11) A′∪A = {3, 10, 11, 12, 13, 14, 15, 16}
(12) B′∩B = { }

Students should draw Venn diagrams themselves.

ANSWERS

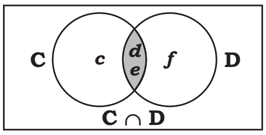
REVIEW EXERCISE 1

2. **(a)** {0, 1, 2, 3, 4} **(b)** Set of English Alphabets

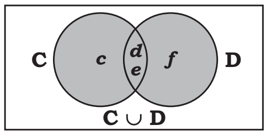
5. **(i)** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$ **(ii)** $A \cap B = \{2, 4, 6, 8, 10\}$
(iii) $A - B = \{1, 3, 5, 7, 9\}$ **(iv)** $(A \cap C)' = \{2, 4, 6, 8, 10, 11, 12\}$
(v) $(B \cup C)' = \{\ \ \}$ **(vi)** $B' \cap C' = \{2, 4, 6, 8\}$
(vii) $A - B' = \{2, 4, 6, 8, 10\}$ **(viii)** $B - A' = \{2, 4, 6, 8, 10\}$
(ix) $A' - C = \{12\}$ **(x)** $(B - C)' = \{1, 3, 5, 7, 9, 11\}$
(xi) $(C - A)' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$ **(xii)** $A' - C' = \{11\}$

7. **(i)** (a) **(ii)** (b) **(iii)** (a) **(iv)** (c) **(v)** (c)

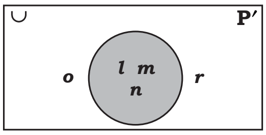
(i) $C \cap B = \{ d, c \}$



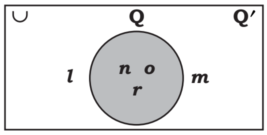
$C \cup D = \{ c, d, e, f \}$



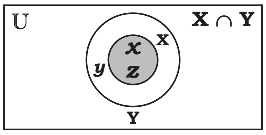
(ii) $P' = \{ o, r \}$



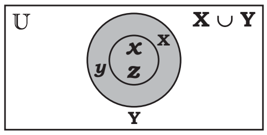
$Q' = \{ l, m \}$



(iii) $X \cap Y = \{ x, z \}$



$X \cup Y = \{ x, y, z \}$

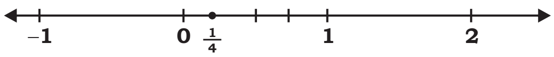


EXERCISE 2.1

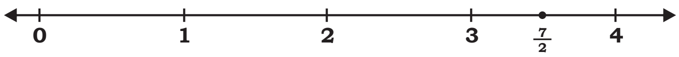
1. **(i)** True **(ii)** False **(iii)** False **(iv)** False **(v)** True **(vi)** False
(vii) True **(viii)** True **(ix)** False **(x)** False **(xi)** False

2. $2\frac{1}{2}$, $2\frac{1}{3}$, $2\frac{1}{4}$

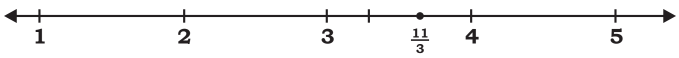
3. **(i)**



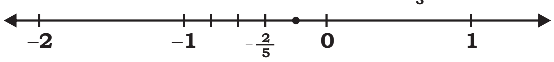
(ii)



(iii)

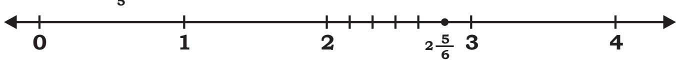


(iv)



(v)

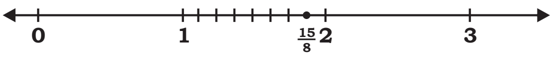
(vi)

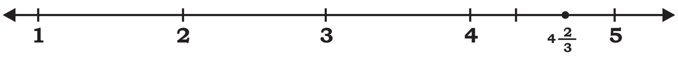


(vii)

(viii)

ANSWERS

(ix) 

(x) 

4. **(i)** False **(ii)** True **(iii)** True **(iv)** False **(v)** True **(vi)** False

EXERCISE 2.2

1. **(i)** $\frac{17}{19}$ **(ii)** $-\frac{1}{13}$ **(iii)** 10 **(iv)** $-\frac{25}{6}$

2. **(i)** $\frac{24}{7}$ **(ii)** $\frac{1}{3}$ **(iii)** 2 **(iv)** 1 **(v)** $-\frac{7}{40}$

(vi) $\frac{37}{42}$ **(vii)** $\frac{17}{14}$ **(viii)** $\frac{7}{10}$ **(ix)** $\frac{97}{72}$ **(x)** $\frac{29}{20}$

3. **(i)** $-\frac{17}{14}$ and $\frac{15}{17}$ **(ii)** $+26$ and $-\frac{1}{26}$ **(iii)** $+\frac{1}{20}$ and -20 **(iv)** $+\frac{5}{19}$ and $-\frac{19}{5}$

(v) 0 and multiplicative inverse does, not exist **(vi)** -8 and $\frac{1}{8}$

(vii) $-\frac{1}{9}$ and 9 **(viii)** $-\frac{200}{7}$ and $\frac{7}{200}$

4. **(i)** $\frac{15}{16}$ **(ii)** $\frac{15}{7}$ **(iii)** $\frac{8}{105}$ **(iv)** $-\frac{12}{7}$ **(v)** 10

(vi) $-\frac{2}{3}$ **(vii)** $-\frac{3}{16}$ **(viii)** $-\frac{1}{3}$

EXERCISE 2.3

6. **(i)** $\frac{8}{9}, \frac{2}{3}, \frac{1}{2}$ **(ii)** $\frac{3}{4}, \frac{1}{2}, \frac{1}{6}$ **(iii)** $\frac{5}{6}, \frac{4}{7}, \frac{1}{3}$

7. **(i)** $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ **(ii)** $\frac{1}{10}, \frac{2}{15}, \frac{4}{5}$ **(iii)** $\frac{1}{4}, \frac{3}{8}, \frac{5}{6}$

8. **(i)** Ascending order: $\frac{1}{4}, \frac{5}{7}, \frac{4}{3}$

Descending order: $\frac{4}{3}, \frac{5}{7}, \frac{1}{4}$

(ii) Ascending order: $\frac{1}{11}, \frac{2}{5}, \frac{1}{2}$

Descending order: $\frac{1}{2}, \frac{2}{5}, \frac{1}{11}$

(iii) Ascending order: $\frac{2}{9}, \frac{1}{2}, \frac{5}{6}, \frac{5}{3}$

Descending order: $\frac{5}{3}, \frac{5}{6}, \frac{1}{2}, \frac{2}{9}$

(iv) Ascending order: $\frac{4}{2}, \frac{1}{2}, \frac{9}{7}, \frac{20}{15}$

Descending order: $\frac{20}{15}, \frac{9}{7}, \frac{1}{2}, \frac{4}{20}$

ANSWERS

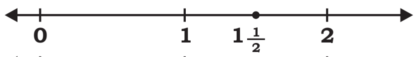
REVIEW EXERCISE 2

1. **(ii)** $-a$ **(iii)** $\frac{q}{p}$ **(iv)** $\frac{ps+qr}{qs}$ **(vi)** Subtraction and division

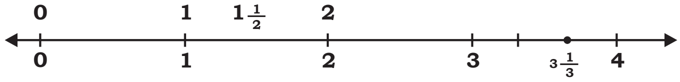
2. **(i)** rational number **(ii)** additive **(iii)** rational number
(iv) multiplicative inverse **(v)** multiplicative

3. **(i)** b **(ii)** a **(iii)** c **(iv)** d **(v)** d

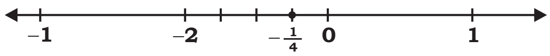
4. **(i)**



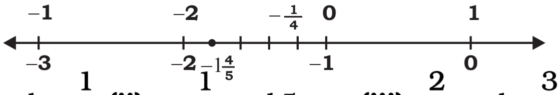
(ii)



(iii)



(iv)



5. **(i)** 14 and $-\frac{1}{14}$ **(ii)** $-\frac{1}{5}$ and 5 **(iii)** $\frac{2}{3}$ and $-\frac{3}{2}$ **(iv)** $\frac{11}{27}$ and $-\frac{27}{11}$

6. **(i)** $>$ **(ii)** $<$ **(iii)** $>$ **(iv)** $<$ **(v)** $<$ **(vi)** $>$

7. **(i)** 1 **(ii)** $\frac{4}{7}$ **(iii)** $-\frac{13}{40}$ **(iv)** $-\frac{1}{12}$ **(v)** $-\frac{11}{60}$ **(vi)** $\frac{1}{5}$

8. **(i)** $\frac{1}{10}$ **(ii)** $\frac{-1}{11}$ **(iii)** $-\frac{27}{56}$ **(iv)** $\frac{32}{49}$ **(v)** $\frac{7}{11}$ **(vi)** $-\frac{6}{25}$

10. **(i)** Commutative property w.r.t addition **(ii)** Commutative property w.r.t multiplication.
(iii) Associative property w.r.t multiplication.
(iv) Associative property w.r.t addition
(v) Associative property w.r.t addition.
(vi) Distributive property of multiplication over addition.
(vii) Commutative property w.r.t multiplication.
(viii) Distributive property of multiplication over subtraction.
(ix) Distribution property of multiplication over subtraction.
(x) Distributive property of multiplication over addition

EXERCISE 3.1

A. **(1)** $\frac{1}{5}$ **(2)** $\frac{2}{5}$ **(3)** $\frac{3}{5}$ **(4)** $\frac{4}{5}$ **(5)** $\frac{8}{10}$

B. **(1)** $\frac{3}{20}$ **(2)** $\frac{7}{20}$ **(3)** $\frac{12}{25}$ **(4)** $\frac{3}{4}$ **(5)** $\frac{18}{20}$
(6) $1\frac{3}{10}$ **(7)** $2\frac{1}{2}$ **(8)** $5\frac{1}{2}$ **(9)** $5\frac{3}{5}$ **(10)** $6\frac{17}{20}$

C. **(1)** $\frac{3}{125}$ **(2)** $3\frac{1}{8}$ **(3)** $\frac{3}{8}$ **(4)** $4\frac{13}{200}$ **(5)** $5\frac{27}{40}$
(6) $-\frac{1}{1000}$ **(7)** $-\frac{1}{2000}$ **(8)** $-\frac{1}{200}$ **(9)** $-5\frac{1}{200}$ **(10)** $-11\frac{3}{8}$
(11) $-\frac{1}{800}$ **(12)** $-1\frac{87}{160}$

ANSWERS

EXERCISE 3.2

N = Non terminating
T = Terminating

I.	**(1)** N	**(2)** T	**(3)** N	**(4)** N	**(5)** N	**(6)** N	**(7)** T
	(8) N	**(9)** T	**(10)** T	**(11)** N	**(12)** T	**(13)** N	**(14)** T
	(15) N	**(16)** T	**(17)** N	**(18)** N	**(19)** N	**(20)** N	

N = Not recurring
R = Recurring

II.	**(1)** N	**(2)** R	**(3)** N	**(4)** N	**(5)** R	**(6)** R	**(7)** R
	(8) N	**(9)** N	**(10)** N				

EXERCISE 3.3

Terminating

$\frac{12}{25}, \frac{5}{12}, \frac{17}{20}, \frac{117}{125}, -\frac{9}{40}, \frac{23}{60}$

$-\frac{96}{100}, \frac{101}{125}, \frac{111}{96}, \frac{125}{200}, \frac{55}{75}$

$\frac{372}{400}$

Non-terminating

$\frac{43}{21}, \frac{5}{7}, \frac{10}{33}$

$\frac{40}{35}, \frac{141}{144}$

$\frac{200}{201}, -\frac{210}{147}, -\frac{401}{333}$

EXERCISE 3.4

I.	(1) 0357142... (T)	(2) – 0.722222... (R)	(3) 0.95 (T)
	(4) – 3.95454545... (R)	(5) 6.566666... (R)	(6) 0575 (T)
	(7) – 0.735294117... (T)	(8) 2.576923... (T)	(9) 2.84375 (T)
	(10) – 1.6444444... (R)	(11) 2.815789473... (T)	(12) 0.847826081... (T)
	(13) – 3.98 (T)	(14) 2.9375 (T)	(15) 1.4
	(16) – 1.8 (T)	(17) 1.5555555... (R)	(18) –0.495714281... (T)
	(19) 0.486842105... (T)	(20) 0.944444444... (R)	

EXERCISE 3.5

	(1) 0.0575	(2) 0421875	(3) 0.883333...
	(4) – 0.613333...	(5) 0.98571428...	(6) 0.9625
	(7) 0.47777778...	(8) 0.99166667...	(9) 0.82
	(10) 0.84375		
II.	(1) 0.26666...	(2) – 0.916666...	(3) 6.888888...
	(4) 3.142887143...	(5) 1.3125...	(6) 0.075
	(7) – 0.4	(8) 0.3181818...	(9) 0.366666...
	(10) – 0.801980198...		
III.	(1) 0.90909090...	(2) – 0.455555...	(3) 1.6183846...
	(4) 0.894736...	(5) 1.285714...	(6) 0.6521735...
	(7) – 1.15625	(8) 0.78620689...	

IV.	(1) 0.46	(2) 0.36	(3) 0.67	(4) 0.11	(5) 1.91
	(6) 3.25	(7) 2.05	(8) 8.34	(9) 10.10	(10) 4.00
	(11) 13.10	(12) 12.82			
V.	(1) 4.517	(2) 6.747	(3) 0.010	(4) 0.986	(5) 206.418
	(6) 3.407	(7) 102.999	(8) 11.123	(9) 55.123	(10) 4.105
	(11) 66.396	(12) 59.920			

ANSWERS

REVIEW EXERCISE 3

1. (i) $\frac{13}{40}$ (ii) $\frac{3}{4}$ (iii) $2\frac{1}{2}$ (iv) $7\frac{3}{4}$ (v) $\frac{39}{50}$
   (vi) $1\frac{297}{1000}$ (vii) $2\frac{87}{250}$
2. (i) 8.2 (T) (ii) 9.25 (T) (iii) 3.11111... (N) (iv) 1.428571 ... (N)
   (v) 3.142857... (N) (vi) 1.3125 (T) (vii) 7.3 (T)
3. (i) 14.58 (ii) 10.78 (iii) 5.79 (iv) 6.80 (v) 25.49
4. Terminating: $\frac{13}{4}, \frac{17}{25}$
   Non-terminating: $\frac{80}{3}, \frac{15}{11}, \frac{19}{6}$, $\frac{25}{15}, \frac{22}{7}, \frac{14}{9}$
5. (i) 0.21 (ii) 6.35 (iii) 0.52 (iv) 0.62 (v) 0.145
   (vi) 3.125 (vii) 3.8333 ... (viii) 1.375 (ix) 1.5
6. (i) 13.333... (ii) 2.8571 ... (iii) 4.54545... (iv) 6.153846 ... (v) 16.6666...
   (vi) 10.9090... (vii) 5.8333 (viii) 0.57142...
7. (i) 5.717 (ii) 11.604 (iii) 0.926 (iv) 3.409 (v) 0.744
   (vi) 23.158
9. (i) Terminating decimal (ii) Non-Terminating decimal
   (iii) Whose number – decimal part (iv) Stops
   (v) Fraction (vi) Block
10. (i) b (ii) d (iii) a (iv) c

EXERCISE 4.1

1. **(i)** 3×3 and 9 **(ii)** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ and 128
(iii) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$ and 648 **(iv)** $5 \times 5 \times 7 \times 7$ and $1 \times 2 \times 2 \times 5$
(v) $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
(vi) $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7$ and 21168

2. **(i)** 2, 5 and 32 **(ii)** 5, 4 and 625 **(iii)** 6, 3 and 216 **(iv)** $\frac{2}{3}$, 6 and $\frac{64}{729}$
(v) $-\frac{1}{5}$, 7 and $-\frac{1}{78125}$

4. **(i)** 10^3 **(ii)** 2^9 **(iii)** $(-7)^3$ **(iv)** 5^4 **(v)** $\left(-\frac{1}{10}\right)$ **(vi)** $\left(\frac{2}{3}\right)^5$
(vii) $\left(-\frac{5}{7}\right)^3$ **(viii)** $\left(\frac{9}{4}\right)^3$ or $\left(\frac{3}{2}\right)^6$

EXERCISE 4.2

1. **(i)** $\left(\frac{5}{7}\right)^6$ **(ii)** $\left(\frac{3}{20}\right)^6$ **(iii)** $\left(\frac{1}{7}\right)^{10} \times \left(\frac{5}{7}\right)^{16}$ **(iv)** p^{13} **(v)** $(xy)^{11}$ **(vi)** $\left(\frac{4}{3}\right)^{18}$
(vii) $x^7 y^{13}$ **(viii)** $\left(\frac{1}{3}\right)^{14}$ **(ix)** $(xyz)^5$ **(x)** $p^8 q^7$

ANSWERS

EXERCISE 4.3

1. **(i)** 5^3 or 125 **(ii)** $\left(\frac{5}{7}\right)^3$ or $\frac{125}{343}$ **(iii)** $\left(\frac{1}{2}\right)^4$ or $\frac{1}{16}$ **(iv)** x^6

(v) $\left(\frac{5}{3}\right)^6$ or $\frac{15625}{729}$ **(vi)** 5^5 or 3125 **(vii)** $\left(\frac{7}{5}\right)^4$ or $\frac{2401}{625}$

(viii) $\left(\frac{2}{3}\right)^4$ or $\frac{64}{729}$ **(ix)** $\frac{7}{11}$ **(x)** $\frac{x^4}{y^4}$

EXERCISE 4.4

1. **(i)** 5^{14} **(ii)** $\left(\frac{1}{3}\right)^{48}$ **(iii)** 3^{20} **(iv)** x^{100} **(v)** $\left(\frac{4}{7}\right)^{18}$ **(vi)** $\left(\frac{3}{4}\right)^{40}$

2. **(i)** 2^{-5} **(ii)** 3^{-7} **(iii)** x^{-6} **(iv)** $\left(\frac{4}{5}\right)^{-6}$

3. **(i)** $\frac{1}{5^8}$ **(ii)** 3^7 **(iii)** $\frac{1}{x^5}$ **(iv)** $\frac{1}{y^7}$

4. **(i)** Positive **(ii)** negative **(iii)** positive **(iv)** positive **(v)** negative **(vi)** positive

EXERCISE 4.5

1. **(i)** 1 **(ii)** $\frac{4}{9}$ **(iii)** **2** **(iv)** 2 **(iv)** $\frac{4}{5}$ **(vi)** 1

2. **(i)** 10 **(ii)** 1 **(iii)** 3 **(iv)** 5

REVIEW EXERCISE 4

1. **(i)** 3, 5 and 243 **(ii)** 7, 4 and 2401 **(iii)** $\frac{3}{4}$, 2 and $\frac{9}{16}$ **(iv)** $\frac{1}{4}$, 3 and $\frac{1}{64}$

(v) 8,0 and 1 **(vi)** y, 0 and 1 **2.** **(i)** $\frac{1}{64}$ **(ii)** $\frac{256}{625}$ **(iii)** 216 **(iv)** $\frac{4}{49}$ **(v)** 3^{10} **(vi)** 4^{10}

(vii) 1 **(viii)** –5

4. **(i)** negative **(ii)** positive **(iii)** positive **(iv)** negative **(v)** positive

5. **(i)** $\frac{4}{3}$ **(ii)** 5^4 **(iii)** $\left(\frac{2}{5}\right)^5$ or $\frac{32}{3125}$ **6.** **(i)** $\frac{3}{2}$ **(ii)** 4 **(iii)** 2

EXERCISE 5.1

A. **(1)** 121 **(2)** 361 **(3)** 625 **(4)** 500 **(5)** 4356 **(6)** 6084
(7) 10000 **(8)** 250000

B. **(i)** Perfect square **(ii)** Not perfect square **(iii)** Perfect square
(iv) Perfect square **(v)** Not perfect square **(vi)** Perfect square
(vii) Not perfect square **(viii)** Not perfect square

ANSWERS

C. **(1)** , **(2)** , **(4)** and **(8)** are perfect square of odd numbers
(3) , **(5)** , and **(6)** , **(7)** are perfect square of even numbers.

D. **(1)** $\frac{1}{25}$ and $\frac{1}{25} < \frac{1}{5}$ **(2)** $\frac{9}{49}$ and $\frac{9}{49} < \frac{3}{7}$ **(3)** $\frac{64}{81}$ and $\frac{64}{81} < \frac{8}{9}$
(4) $\frac{25}{36}$ and $\frac{25}{36} < \frac{5}{6}$ **(5)** $\frac{4}{9}$ and $\frac{4}{9} < \frac{2}{3}$

E. **(1)** 0.01 and 0.01 < 0.1 **(2)** 0.25 and 0.25 < 0.5 **(3)** 0.0049 and 0.0049 < 0.07
(4) 0.0121 and 0.012 < 0.11 **(5)** 0.0225 and 0.0225 < 0.15

EXERCISE 5.2

A. **(1)** 5 **(2)** 4 **(3)** 9 **(4)** 36 **(5)** *a* **(6)** *y* **(7)** 49 **(8)** 8

B. **(1)** 26 **(2)** **13** **(3)** 22 **(4)** 31 **(5)** 70 **(6)** 33 **(7)** 40 **(8)** 48
(9) 56 **(10)** 41 **(11)** 37 **(12)** 45

C. **(1)** $\frac{4}{5}$ **(2)** $\frac{6}{7}$ **(3)** $\frac{35}{99}$ **(4)** $1\frac{1}{12}$ **(5)** $1\frac{2}{11}$ **(6)** $1\frac{5}{6}$ **(7)** $\frac{31}{41}$ **(8)** $\frac{32}{35}$
(9) $1\frac{13}{24}$ **(10)** $2\frac{4}{13}$ **(11)** $1\frac{3}{9}$ **(12)** $1\frac{5}{6}$

D. **(1)** 1.8 **(2)** 2.1 **(3)** 2.3 **(4)** 2.7 **(5)** 2.5 **(6)** 6.1 **(7)** 2.8 **(8)** 3.2
(9) 5.5 **(10)** 10.0 **(11)** 5.8 **(12)** 5.9

EXERCISE 5.3

(1) 28 rows and 28 chairs **(2)** 15.5 metres
(3) Length = 46.5 m and width = 15.5 m **(4)** 46 rows **(5)** 41 chairs
(6) 25 rows and 25 trucks **(7)** 32 rows and 32 students
(8) Width = 18.22 m and length = 54.66 m **(9)** 58.2 m **(10)** 123 bags

REVIEW EXERCISE 5

2. **(i)** 961 **(ii)** 7 **(iii)** perfect square **(iv)** square of **(v)** $\frac{5}{6}$ **(vi)** 1

3. **(i)** False **(ii)** True **(iii)** False **(iv)** False **(v)** True **(vi)** False

4. **(i)** *a* **(ii)** *c* **(iii)** *c*

5. **(i)** 27 **(ii)** 84 **(iii)** 163 **(iv)** 205 **(v)** $2\frac{4}{11}$ **(vi)** $\frac{23}{49}$
(vii) $1\frac{4}{121}$ **(viii)** 0.16 **(ix)** 587 **(x)** $\frac{80}{99}$ **(xi)** $2\frac{3}{8}$ **(xi)** 11.15

6. **(6)** 110 m **(7)** 12 m **(8)** 62 m **(9)** width = 2.5 m and length = 7.5 m
(10) 30 rupees

EXERCISE 6.1

1. **(i)** 2 : 5 : 4 **(ii)** 24 : 56 : 63 **(iii)** 200 : 220 : 231
2. **(i)** 12 : 20 : 27 **(ii)** 55 : 77 : 65 **(iii)** 24 : 35 : 40
3. **(i)** 56 : 84 : 105 : 150 **(ii)** 140 : 105 : 147 : 180
4. **(i)** $\frac{5}{3}$ **(ii)** $\frac{7}{3}$ **(iii)** $\frac{28}{5}$ **5.** 40 : 72 : 99

ANSWERS

6. Share of Saleem = Rs 400
Share of Irfan = Rs 600
Share of Imran = Rs 1000

7. 15 : 20 : 28

8. 90 : 36 : 48 : 56

9. Electricity bill = Rs 14000
Phone bill = Rs 5600
Gas bill = Rs 4200
Water bill = Rs 3600

10. Price of English book = Rs 60
Price of Urdu book = Rs 90
Price of Science book = Rs 225
Price of Maths book = Rs 300

EXERCISE 6.2

1. Rs 6000 **2.** 9 pipes **3.** 10 persons **4.** Rs 750 **5.** 75 pages
6. 32 soliders **7.** $5\frac{1}{3}$ hours **8.** Rs 315 **9.** $\frac{392}{9}$ kg **10.** Rs 12750

EXERCISE 6.3

1. 15 workers **2.** 4 labourers **3.** 10 workers **4.** $2\frac{2}{5}$ days
5. $\frac{7}{15}$ of the work **6.** $42\frac{2}{3}$ days **7. (i)** 1250 sheep **(ii)** $93\frac{3}{4}$ days

EXERCISE 6.4

1. (i) 13.89 m/s **(ii)** 20.83 m/s **(iii)** 22.22 m/s **(iv)** 14.03 m/s
2. (i) 108 km/h **(ii)** 43.2 km/h **(iii)** 151.2 km/h **(iv)** 91.8 km/h
3. 40 km/h **4.** 3 hours **5.** 225 m **6.** $2\frac{4}{7}$ hours
7. (i) 70 km/h **(ii)** $1\frac{1}{6}$ km/m **(iii)** 19.4 m/s **(iv)** 315 km **(v)** 5 hours

REVIEW EXERCISE 6

1. 4 : 10 : 7 **2.** 105 : 140 : 168 : 192 **3.** 425 rupees **4.** 14 workers
5. 2 **6.** 21 **7.** 16.7 m/s **8.** 378 km/h
9. 66 km and speed = 1.2 km/ minute, **(i)** 72 km/h **(ii)** 1.2 km / minute
(iii) 1200 m/minute **(iv)** 20 m/sec.
10. Car A = 42 litres Car B = 54 litres Car C = 66 litres **11. (i)** Rs 375 **(ii)** 224 km

EXERCISE 7.1

A. **1.** Rs 14,320 2. Rs 33,158 3. Rs 24,435 4. Rs 9,360 5. Rs 32,800
B. 1. Rs 1,27,5000 2. Rs 897,500 3. Rs 96,000 4. Rs 283,333 5. Rs 78,0000
C. 1. 1.67% 2. 10% 3. 2% 4. 2% 5. 2%
D. Answers should be given by the students themselves.

EXERCISE 7.2

1. Rs 1,118.75 2. Rs 824.50 (3) Rs 2,176
4. GST Rs 2,584, Total cost Rs 17,784 5. GST Rs 1,326, Total cost Rs 9126
6. Rs 30,235 7. Cost of Dishwasher Rs 52,137, GST Rs 8,863
8. Rs 16,000 9. 17% 10. 17%

ANSWERS

EXERCISE 7.3

1.	Profit Rs 790, Markup rate = 42.7%						
2.	Profit Rs 450, Markup rate 7.2%			3.	Rs 2,508	4.	Rs 1,826
5.	Rs 3503	6.	Rs 3300	7.	Rs 820	8.	Rs 90
9.	57.8%	10.	77.34%	11.	20.67%		

Activity (Page 124)

i.	Rs 96,000	ii.	Rs 0.2 years	iii.	Rs 30,000	iv.	Rs 24,000
v.	8%	vi.	Rs 2,250,000				

EXERCISE 7.4

1.	Rs 613,088	2.	Rs 188,662.5	3.	Rs 1,431	4.	Rs 113,333.3
5.	(i) Rs 192,000	(ii)	0.2 years	(iii)	Rs 30,000	(iv)	Rs 24,000

EXERCISE 7.5

1.	Rs 12,500	2.	He is not Sahib-e-Nisab	3.	Rs 16,250	4.	Rs 820,000
5.	Rs 200,000	6.	Rs 39,37.5	7.	Rs 25,0000	8.	Rs 5,850

REVIEW EXERCISE 7

1.	Rs 1445	2.	Rs 47692.308	3.	Rs 2847.5	4.	Rs 99935.3	
5.	Rs 318750	6.	Rs 47500	7.	Rs 12500	8.	Rs 778518.52	
9.	Rs 14875	10.	Rs 11125	11.	Rs 500000	12.	Rs 11150	13. Rs 40,000

EXERCISE 8.1

A. **(1)** 13 **(2)** $2a$ **(3)** $-12x$ **(4)** $2ab$ **(5)** $7y$ **(6)** $9x^2 + 7x + 4$

B.

(1)	$8a + 8b$	**(2)**	$7x + 2y$	**(3)**	$11a - 6b$
(4)	$9ab + 9cd + 12ad$	**(5)**	$8a + 7b + 6c$	**(6)**	$12a + 16b + 11c$
(7)	$27a + 27b + 27c$	**(8)**	$14a + 18b + 17c$	**(9)**	$11a + 13b + 11c$
(10)	$22c - f + 12g$				

EXERCISE 8.2

(1)	$3a$	**(2)**	$-19ab$	**(3)**	$12a$	**(4)**	$32c^6d^2$
(5)	$4a + 8b + 21c$	**(6)**	$14c + 4d + 3c$	**(7)**	$8x + 7y + 4z$	**(8)**	$6a + 5b + 18c$
(9)	$-x^2 + 7ab + 27bc$	**(10)**	$a + ab + b$	**(11)**	$12a + b$	**(12)**	$4x + 3y$

EXERCISE 8.3

I.

(1)	$120a^2$	**(2)**	$54ab$	**(3)**	$20a$	**(4)**	$4a + 4b$
(5)	$-30a^2b^3$	**(6)**	$18ab$	**(7)**	$60m^9$	**(8)**	$36x^5y^4$

II.

(1)	$9a + 9b$	**(2)**	$-4xy + 4y^2$	**(3)**	$-10m^3n + 15mn^4$
(4)	$-18x^2y^3\ 12\ x^2y^4$	**(5)**	$18l^3m^3n - 12\ l^2m^3n^2$	**(6)**	$10bc^3d^2$

III.

(1)	$24c^2 - 20cd + 24d^2$	**(2)**	a^2b^2	**(3)**	$a^4 - b^4$
(4)	$6x^2 + 11xy - 10y^2$	**(5)**	$-4lm^2 + 6lmn - 6m^2n + 9mn^2$	**(6)**	$16\ p^2q^3 - 6\ p^3q^2$

IV.

(1)	$4a^2bc - 6ab^2c + 8abc^2$	**(2)**	$a^3b + ab^3 - abc^2$	**(3)**	$-m^2n^3 - m^3n^2 + m^3n^3$
(4)	$-3x^3y + 18x^2y - 27xy^3$	**(5)**	$8\ pqr - 12p^2q^2r + 4pq^3r$	**(6)**	$x^2y^2z^2 + x^3y^3z - xy^3z^3$

ANSWERS

V. **(1)** $1 + x - 5x^2 + 3x^3$ **(2)** $8c^3 - 8cd^2 + 12c^2d - 12d^3 - 6c^2e + 6d^2e$
(3) $a^3 - 2a^2b + b^3$ **(4)** $3x^3 - 19x^2 + 32x - 16$ **(5)** $a^4 - 2a^4b + 2a^2b^3 - b^4$
(6) $3p^4q - 3p^2q^3 + 2p^3q^2 - 2pq^4 - p^2q^2r + q^4r$

EXERCISE 8.4

I. **(1)** $-a - 8ab$ **(2)** $-13m + 24mn$ **(3)** $36x^4 - 48x^2 - 36x^3 - 48xy$ **(4)** $3x + 3y^2$
(5) $12d^2l^2m^2 - 12l^2m^4 - 8lm^4n + 8lm^2n^3$ **(6)** $6p^4q^2 + 6p^2q^4 + 12p^2q^2r^2 - 12pq^2r^2$

EXERCISE 8.8

(1) $5a\,(ax - 3)$ **(2)** $m^2\,(m^2 - m + 1)$ **(3)** $7\,(a^3 + 2a^2 - 1)$
(4) $mx\,(x - m - 1)$ **(5)** $n\,(13 - 2n^2 + 39n^4)$ **(6)** $abc\,(a + b^2 + c)$
(7) $(b + c)\,(3a + 6d)$ **(8)** $(a + b)\,(xy + ay + by + x)$
(9) $(x - y)\,(4lm + 8mnx - 8mny + 12nl)$ **(x)** $(p^2 - q)^2\,(pq + pr + qr)$

EXERCISE 8.9

(1) $(x + 6)\,(x + 6)$ **(2)** $(a + 2)\,(a + 2)$ **(3)** $(2a + 9)\,(2a + 9)$
(4) $(3x + 7y)\,(3x + 7y)$ **(5)** $(5a + 8b)\,(5a + 8b)$ **(6)** $(4b + 5)\,(4b + 5)$
(7) $(2x + \frac{1}{2})\,(2x + \frac{1}{2})$ **(8)** $(12x - \frac{1}{6})\,(12x - \frac{1}{6})$ **(9)** $2\,(3c + 5d)\,(3c + 5d)$
(10) $4\,(a + 6)\,(a + 6)$ **(11)** $5\,(y + 4)\,(y + 4)$ **(12)** $3\,(s - 8t)\,(s - st)$
(13) $(3at + 7s)\,(3at + 7s)$ **(14)** $(2a^2 - 3)\,(2a^2 - 3)$ **(15)** $2\,(5pq - 7r)\,(5pq - 7r)$

EXERCISE 8.10

(1) $(b + c)\,(b - c)$ **(2)** $(a + 6)\,(a - 6)$ **(3)** $(a + 7)\,(a - 7)$
(4) $(5 + y)\,(5 - y)$ **(5)** $(2x + 3)\,(2x - 3)$ **(6)** $(3x + 4y)\,(3x - 4y)$
(7) $6\,(x + 2)\,(x - 2)$ **(8)** $2\,(10 + 3z)\,(10 - 3z)$ **(9)** $(9a + 11b)\,(9a - 11b)$
(10) $(ac + 4cd)\,(ac - 4cd)$ **(11)** $(6 + 3a - 2b)\,(6 - 3a + 2b)$
(12) $(2x + 3y + 10z)\,(2x + 3y\ 10z)$ **(13)** $(8b + 6b + d)\,(8b - 6b - d)$
(14) $-3b\,(6a - 5b)$ **(15)** $-13b\,(8a - 29b)$ **(16)** $(22p - 8q)\,(8p - 22q)$
(17) $18\,(5c - d)\,(-c + 5d)$ **(18)** $31\,(4m - 4n + 5p - 5q)\,(4m - 4n - 5p + 5q)$

EXERCISE 8.11

I. **(1)** $(x + 1)\,(5x + 4)$ **(2)** $(x + y)\,(a + b)$ **(3)** $(b + c^2)\,(b + 4)$
(4) $(p - 6q)\,(p - q)$ **(5)** $(a + b^2)\,(a + 5)$ **(6)** $(7x + py)\,(x + 1)$
(7) $2\,(cd - d)\,(c + 2d)$ **(8)** $2\,(y + 2)\,(y - 5)$ **(9)** $(xy - 1)\,(xy + 5)$
(10) $(p - q)\,(p - 1)$

II. **(1)** $(x + 2)\,(x + 4)$ **(2)** $(y + 2)\,(y + 6)$ **(3)** $(a + 2)\,(a + 5)$
(4) $(c + 1)\,(c + 6)$ **(5)** $(d + 1)\,(d + 5)$ **(6)** $(p - 1)\,(p - 3)$
(7) $(r + 4)\,(r + 3)$ **(8)** $(a - 2)\,(a + 5)$ **(9)** $(m + 2)\,(m - 7)$
(10) $(x + 3)\,(x + 7)$ **(11)** $(y - 2)\,(y + 9)$ **(12)** $(x + 3)\,(x + 7)$

REVIEW EXERCISE 8

2. **(i)** $a^2 + 2ab + b^2$ **(ii)** $a^2 - 2ab + b^2$ **(iii)** $x^2 + (a + b)\,x + b^2$
(iv) Letter of alphabet **(vi)** Monomial

ANSWERS

3. **(i)** b **(ii)** c **(iii)** c **(iv)** a **(v)** a

4. **(i)** $2x^2 + 4y^2 + z^2$ **(ii)** $-2x^2 - 3z^2$ **(iii)** $4x^2 - 2y^2 - 3z^2$
(iv) $6y^2 + 7z^2$ **(v)** $2x^2 + 3z$ **(vi)** $-4x^2 + 2y^2 + 3z^2$

5. **(i)** $x^2 - 4y^2$ **(ii)** $12x^3 + 4x^2$ **(iii)** $2x^2 + 2y^2$
(iv) $2a^3b^3 - 3a^2b^4$ **(v)** $a^4 - b^4$ **(vi)** $a^4 - a^3 + a^2 - 2a - 1$
(vii) 0 **(viii)** 0

6. **(i)** $9x^2 + x - 20$ **(ii)** $4a^2 - 20ab + 25b^2$ **(iii)** $9x^4 + 24x^2y^2 + 16y^4$
(iv) $(3x)^2 - (5y)^2$

7. **(i)** $10a^2 (1 - 20a^2b)$ **(ii)** $9xyz (4x^2y^2z^2 - 3x^3y - 7z^3)$
(iv) $(a^2 + 11)(x - 16)$ **(v)** $(ab + c)(x^2 + xy + z^2)$

8. **(i)** $(a - 13)(a - 13)$ **(ii)** $(1 - 3x^2y^2z)(1 - 3x^2y^2z)$ **(iii)** $7a(b + 7)(b - 7)$
(iii) $3xy(5x^3 + 7x^2y - 9xy - 11y^3)$ **(iv)** $3(5 + x - y)(5 - x + y)$ **(v)** $(11x + 3y)(3x + 11y)$

(vi) $\left(\frac{3}{4}a + \frac{2}{3}b\right)\left(\frac{3}{4}a + \frac{2}{3}b\right)$ **(vii)** $\left(\frac{a}{b}l - \frac{c}{d}m\right)\left(\frac{a}{b}l - \frac{c}{d}m\right)$

(viii) $\left(a - \frac{9}{5} + \frac{6}{5}m\right)\left(a - \frac{9}{5} - \frac{6}{5}m\right)$

EXERCISE 9.1

A. **(1)** $x = 4$ **(2)** $x = 5$ **(3)** $x = \frac{3}{5}$ **(4)** $x = -4$ **(5)** $x = 3$ **(6)** $x = 2$
(7) $x = 2$ **(8)** $x = -1$ **(9)** $x = \frac{2}{3}$

B. **(1)** $x = 4$ **(2)** $x = -\frac{2}{3}$ **(3)** $x = 6\frac{2}{3}$ **(4)** $x = -1$ **(5)** $x = 2$

C. **(1)** $x = 3$ **(2)** $x = 9$ **(3)** $x = \frac{8}{15}$ **(4)** $x = 4$ **(5)** $x = 1\frac{1}{6}$ **(6)** $x = 1$

D. **(1)** $x = 6$ **(2)** $x = 12$ **(3)** $x = 12$ **(4)** $x = 4\frac{2}{13}$ **(5)** $x = \frac{7}{11}$ **(6)** $x = 3$
(7) $x = \frac{7}{10}$ **(8)** $x = -2\frac{5}{6}$

EXERCISE 9.2

(1) $x = 3$ **(2)** $x = 7$ **(3)** $x = 4$ **(4)** $x = 10$ **(5)** $x = 6$ **(6)** $x = 5$
(7) $x = -1\frac{1}{2}$ **(8)** $x = -27$ **(9)** $x = \frac{1}{25}$ **(10)** $x = -11$ **(11)** $x = 2$ **(12)** $x = -5$

EXERCISE 9.3

(i) He paid Rs 30
(ii) Rs 5
(iii) 15 m and 16.5 m
(iv) $m\overline{AB} = 2$cm, $m\overline{BC} = 3$ cm $m\overline{AC} = 6$ cm
(v) 8cm
(vi) Asghar has Rs 160 and Hussain has Rs 640
(vii) Yes
(viii) 21 and 22 are the numbers
(ix) 15, 16 and 17 are the required odd numbers.
(x) 24
(xi) 4 and 5 are he required numbers
(xii) 27, 29, 31, and 33

ANSWERS

REVIEW EXERCISE 9

1. **(i)** x + (x + 2) = 42 **(ii)** x + (x + 2) = 21 **(iii)** x + (x + 1) + (x + 2) + (x + 3) = 46
4. **(i)** One variable **(ii)** Root **(iii)** Root **(iv)** Value
5. **(i)** a **(ii)** d **(iii)** a **(iv)** b
6. **(i)** $x = 12$ **(ii)** $x = 19$ **(iii)** $x = 3$ **(iv)** $x = 9$
(v) $x = 5$ **(vi)** $x = -12$
7. Length is 35 cm and breadth is 25 cm

EXERCISE 10.1

1. **(i)** (∠POS, ∠SOR) **(ii)** (∠SOR, ∠QOR) **(iii)** (∠POR, ∠QOR) **(iv)** ∠POS, ∠QOS
2. (∠BOD, ∠AOD), (∠AOC, ∠BOC) **3.** $m\angle PON = 80^o$
4. $(49^o, 41^o)$, $(62^o, 28^o)$, $(54^o, 36^o)$, $(36^o, 54^o)$, $(67^o, 23^o)$, $(81^o, 9^o)$, $(15^o, 75^o)$
5. $(80^o, 100^o)$, $(132^o, 48^o)$, $(49^o, 131^o)$, $(76^o, 104^o)$, $(125^o, 55^o)$, $(103^o, 77^o)$, $(68^o, 112^o)$
6. $(m\angle AOP = m\angle COD = 55^o)$, $(m\angle AOC = m\angle POD = 125^o)$
7. **(i)** $m\angle ACB = 55^o$, $m\angle BAC = 25^o$
(ii) $m\angle PRQ = 38^o$, $m\angle PQR = 80^o$ $m\angle PRT = 142^o = m\angle QRS$
8. **(i)** 45^o **(ii)** 90^o **(iii)** an obtuse **(iv)** right **(v)** acute **(vi)** right

EXERCISE 10.2

1. Similar figures
(i) and (*a*) , **(ii)** and (*a*)
(iii) and (*b*) , **(iv)** and (*e*)
(vi) and (*h*) , **(vii)** and (*f*)
(viii) and (*g*) , **(x)** and (*i*)

Congruent figures
(*v*) and (*d*) **(ix)** and (*j*)

3. **(i)** Fig. P and R have same shape. **(ii)** Fig P and Fig. R have not the same size.
(ii) Fig. P and Fig. R are not congruent **(iv)** Fig. P and Fig. R are similar
(v) Fig. P and Fig. Q are congruent.
4. **(i)** True **(ii)** False
(iii) No, they are not congruent because they have only the same shape but not have the same size.
(iv) Yes they are not congruent; because they have only same shape and not have the same size. **(v)** Yes

EXERCISE 10.3

1. **(iv)** Same shape, same size (congruent)
(v) Same shape, but not same size (similar)
(vi) Same shape, same size (congruent)
(vii) Same shape, same size (congruent)
(viii) No same shape, nor same size (Neither similar nor congruent)
(ix) No same shape, nor same size (Neither similar nor congruent)
2. $\left(\overline{AB} \cong \overline{XY}\right)$, $\left(\overline{DE} \cong \overline{XZ}\right)$, $\left(\overline{MN} \cong \overline{CD}\right)$, $\left(\overline{YZ} \cong \overline{GH}\right)$, $\left(\overline{RS} \cong \overline{PQ}\right)$, $\left(\overline{YZ} \cong \overline{EF}\right)$
3. (∠ABC ≅ ∠PRQ), (∠XYZ ≅ ∠MNS), (∠COD ≅ ∠PQR), (∠AOB ≅ ∠DEF)
4. **(i)** $m\angle DEF = 100^o$ **(ii)** ∠PQR ≅ ∠XYZ **(iii)** Congruent
(iv) equal **(v)** radii
5. **(i)** $x = 15, y = 4$ **(ii)** $b = 12, x = 7$ **(iii)** $x = 6, y = 8$

ANSWERS

EXERCISE 10.4

(1) $\overline{AP} \cong \overline{BP}$
$\overline{CP} \cong \overline{BP}$
$\angle APC \cong \angle BPD$
SAS ≅ SAS

(2) $\overline{PO} \cong \overline{MO}$
$\overline{QO} \cong \overline{NO}$
$m\angle PQO = m\angle MNO = 90^{\circ}$
RHS ≅ RHS

(3) $\angle PRQ \cong \angle PSQ$
$\angle PQR \cong \angle PQS$
$\overline{RQ} \cong \overline{SQ}$
ASA ≅ ASA

(4) $\overline{PQ} \cong \overline{SR}$
$\overline{QR} \cong \overline{PS}$
$\overline{PR} \cong \overline{PR}$
SSS ≅ SSS

(5) $\angle ABC \cong \angle EDC$
$\overline{BC} \cong \overline{DC}$
$\angle ACB = m\angle DCE$
ASA ≅ ASA

(6) $\overline{AC} \cong \overline{DC}$
$\angle ACB \cong \angle DCE$
$\overline{BC} \cong \overline{EC}$
SAS ≅ SAS

(7) $\overline{PQ} \cong \overline{PO}$
$\overline{QR} \cong \overline{OR}$
$\overline{PR} \cong \overline{PR}$
SSS ≅ SSS

(8) $\angle ABD \cong \angle CDB$
$\overline{BD} \cong \overline{BD}$
$\angle ADB = m\angle DBC$
ASA ≅ ASA

(9) $\overline{PO} \cong \overline{SO}$
$\angle PQO \cong \angle SRO$
$\overline{QO} \cong \overline{RO}$
RHS ≅ RHS

(10) $\overline{CE} \cong \overline{FD}$
$\angle CDE = m\angle FED$
$\overline{CD} \cong \overline{FE}$
RHS ≅ RHS

EXERCISE 10.5

(1) chord **(2)** radius **(3)** radial segment **(4)** centre
(5) diameter **(6)** centre, twice **(7)** diameter **(8)** infinite, equal
(9) half **(10)** infinite, equal

REVIEW EXERCISE 10

7. **(1)** $\overline{AB} \cong \overline{DC}$
$\overline{BC} \cong \overline{AD}$
$\overline{AC} \cong \overline{AC}$
SSS ≅ SSS

(2) $\overline{AB} \cong \overline{DC}$
$\angle BAC = m\angle ACD$
$\overline{AC} \cong \overline{AC}$
SAS ≅ SAS

(3) $\angle BAC \cong \angle ACD$
$\overline{AC} \cong \overline{AC}$
$\angle BCA \cong \angle DAC$
ASA ≅ ASA

12. **(i)** adjacent angles **(ii)** complementary angles **(iii)** same, shape
(iv) vertical equal **(v)** Hypotenuse, Hypotenuse, congruent

REVIEW EXERCISE 11

A. **(viii)** Yes, a square is a rectangle; square is a parallelogram a square is a rhombus.

C. **(i)** triangle **(ii)** 67° **(iii)** rectangle **(iv)** (*a*) **(v)** (*d*)
(vi) (*b*) **(vii)** (*b*) **(viii)** (*a*)

ANSWERS

EXERCISE 12.1

A. **(1)** 88 cm **(2)** 110 cm **(3)** 132 mm **(4)** 176 mm
B. **(1)** 66 cm **(2)** 176 cm **(3)** 242 cm **(4)** 308 cm
(5) 374 cm **(6)** 396 cm **(7)** 506 cm **(8)** 484 cm
C. **(1)** 3.5 cm **(2)** 10.5 cm **(3)** 14 cm **(4)** 17.5 cm
(5) 21 cm **(6)** 28 cm **(7)** 35 cm **(8)** 31.5 cm
D. **(1)** 14 cm **(2)** 49 cm **(3)** 77 mm **(4)** 84 mm
(5) 15.4 mm **(6)** 9.8 cm **(7)** 16.8 cm **(8)** 12.6 cm
E. **(1)** 8.8 cm **(2)** 132 mm **(3)** 220 mm **(4)** 417.6 cm **(5)** 264 mm
F. **(1)** 66 cm **(2)** 176 cm **(3)** 4.4 m

EXERCISE 12.2

A. **(1)** 616 sq cm **(2)** 346.55q cm **(3)** 1,386 sq cm **(4)** 962.5 sq mm
(5) 3,850 sq mm **(6)** 1,886.5 sq cm **(7)** 16,022.16 cm **(8)** 4,658.5 sq cm
B. **(1)** 7 mm **(2)** 28 cm **(3)** 49 mm **(4)** 42 mm
(5) 1.4 mm **(6)** 17.5 cm **(7)** 3.5 m **(8)** 4.9 mm
(9) 2.1 m
C. **(1)** 346.5 sq cm **(2)** 616 sq cm **(3)** 1,336 sq cm **(4)** 2462 sq cm
(5) 55.44 sq cm **(6)** 75.46 sq cm **(7)** 98.568 q cm **(8)** 124.74 sq cm
D. **(1)** 14 cm **(2)** 42 mm **(3)** 70 mm **(4)** 112 mm
(5) 12.6 cm **(6)** 15.4 cm **(7)** 16.8 cm **(8)** 18.2 cm

EXERCISE 12.3

A. **(1)** 3,520 sq cm **(2)** 1,760 sq cm **(3)** 6,600 sq cm **(4)** 4,400 sq cm
(5) 13,200 sq cm **(6)** 10,450 sq cm

B. **(i)** 19.5 cm **(ii)** 22.5 cm **(iii)** 44 mm **(iv)** 27 cm **(v)** 32 cm
C. **(i)** r = 8 cm, h = 20 cm **(ii)** r = 15 cm, h = 20 cm
(iii) r = 19 cm, h = 30 cm **(iv)** r = 19.5 cm, h = 36.5 cm

EXERCISE 12.4

A. **(i)** 12,320 cm³ **(ii)** 68,750 cm³ **(iii)** 98,560 cm³ **(iv)** 35,200 cm³
(v) 8.6625 m³ **(vi)** 956,455.5 cm³ **(vii)** 192,500 mm³ **(viii)** 178,200 mm³
B. **(i)** 25 cm **(ii)** 28 cm **(iii)** 42 mm **(iv)** 210 mm **(v)** 35 mm
C. **(i)** 21 cm **(ii)** 14 cm **(iii)** 20 cm **(iv)** 30 cm **(v)** 40 mm **(vi)** 21 mm

EXERCISE 12.5

(i) Rs 11000 **(ii)** 14 cm **(iii)** 264000 m or 264 km **(iv)** 31.5 cm
(v) 1.8 m **(vi)** Area = 616 m², cost = Rs 12,320
(vii) Area = 3696 m², cost = Rs 369,600 **(viii)** Area = 6.16 m², cost = Rs 616
(ix) Area = 38.5 m², cost = Rs 2,887.50 **(x)** Area = 0.625 m², cost = Rs 770

EXERCISE 12.6

(i) Area 874.5m², cost = Rs 43,725 **(ii)** Area 677.6m², cost Rs 67,760
(iii) Area 50.16m², cost Rs 15.048 **(iv)** Area = 1.232 m²
(v) Area = 4.07 m² **(vi)** Volume = 69,300,000 cm³, capacity = 69,300*l*

ANSWERS

(vii) Volume = 123,750 cm^3, capacity = 123.75l **(viii)** Volume = 616 m^3 = soil

(ix) Volume = 404.25 cm^3, capacity = 0.40425 l = 0.4 l (approx)

(x) Volume = 231 m^3, cost = Rs 57,750 **(xi)** $r = 1\ m$

(xii) Volume = 3850,000 cm^3, height = 2.5 m

(xiii) Area (curved walls) = 770m^2, cost = Rs 154,000

(xiv) Area (Base) = 1386m^2, Area (curved surface-walls) = 660m^2
Total Area = 2046 m^2, cost Rs 204,600

REVIEW EXERCISE 12

A. **(viii)** $v = \pi r^3$

B. **(ii)** Cylinder **(iii)** Cylinder **(iv)** m^3 **(v)** 3.142

C. **1.** (i) **2.** (i) **3.** (ii) **4.** (ii) **5.** (iii)

6. (iv) **7.** (ii) **8.** (iii) **9.** (i) **10.** (iii)

EXERCISE 13.1

(1)

Group	Period or Class Interval	Tally Marks	Frequency
1st	21 – 22	II	2
2nd	23 – 24	I	1
3rd	25 – 26	I	1
4th	27 – 28	II	2
5th	29 – 30	I	1

(2)

Group	Period or Class Interval	Tally Marks	Frequency
First	3001 – 4000	I	1
Second	4001 – 5000	II	2
Third	5001 – 6000	IIII	4
Fourth	6001 – 7000	III	3
Fifth	7001 – 8000	II	2

(3)

Group	Period or Class Interval	Tally Marks	Frequency
First	551 – 650	~~IIII~~ I	6
Second	651 – 750	~~IIII~~ IIII	9
Third	751 – 850	~~IIII~~	5

(4)

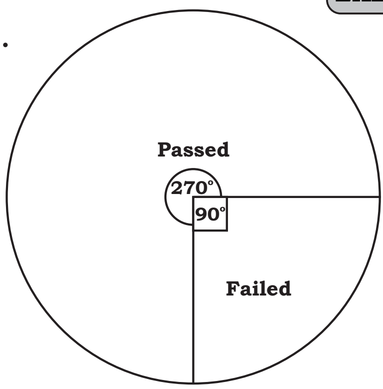
Group	Period or Class Interval	Tally Marks	Frequency
1st	100 – 120	𝍸	5
2nd	121 – 140	𝍸 III	8
3rd	141 – 160	𝍸 III	8
4th	161 – 180	𝍸 I	6
5th	181 – 200	III	3

(5)

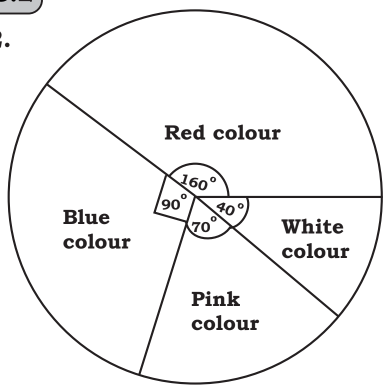
Group	Period or Class Interval	Tally Marks	Frequency
1st	10 – 17	𝍸 I	6
2nd	18 – 25	IIII	4
3rd	26 – 33	IIII	4
4th	34 – 41	𝍸 II	7
5th	42 – 50	𝍸 IIII	9

EXERCISE 13.2

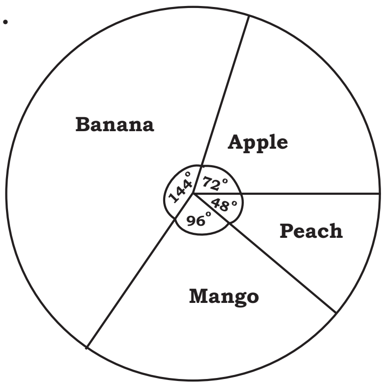
1.



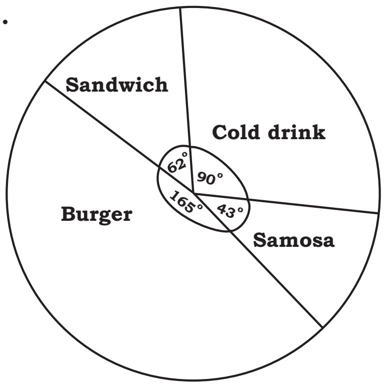
2.



3.



2.

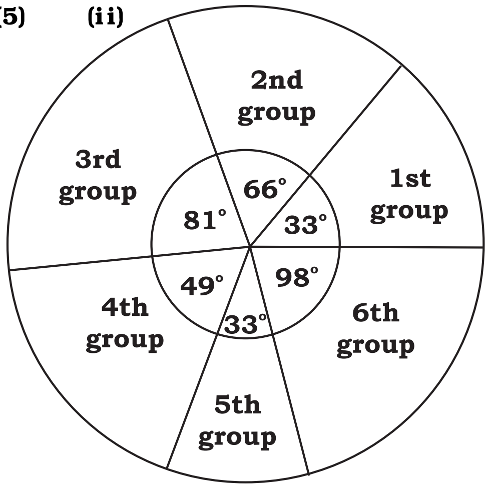


ANSWERS

(5) **(i)**

Group	Period or Class Interval	Tally Marks	Frequency
1^{st}	9 – 22	$\|\|$	2
2^{nd}	23 – 36	$\|\|\|\|$	4
3^{rd}	37 – 50	$\cancel{\|\|\|\|}$	5
4^{th}	51 – 64	$\|\|\|$	3
5^{th}	65 – 78	$\|\|$	2
6^{th}	79 – 92	$\cancel{\|\|\|\|}\ \|$	6

(5) **(ii)**



6. **(i)** Chicken; 90 students

(ii) Vegetables, 20 students

(iii) 25 students

Favourit Meals	Frequency	Angle
Chicken	90	180°
Noodles in pasta	25	50°
Vegetables	20	40°
Fish and chips	45	90°
Total	180	360°

REVIEW EXERCISE 13

1. **(i)** data (ii) frequency (iii) raw (iv) lower class limit
(v) frequency table (vi) presentation (vii) tallying (viii) upper class limit
(ix) size or length (x) pie-graph

2. **(i)** d (ii) c (iii) a (iv) b

3.

Group	Period or Class Interval	Tally Marks	Frequency
1^{st}	1 – 10	$\cancel{\|\|\|\|}\ \|\|$	7
2^{nd}	11 – 20	$\cancel{\|\|\|\|}\ \cancel{\|\|\|\|}\ \|$	11
3^{rd}	21 – 30	$\cancel{\|\|\|\|}\ \cancel{\|\|\|\|}$	10
4^{th}	31 – 40	$\cancel{\|\|\|\|}\ \|\|\|\|$	9
5^{th}	41 – 50	$\|\|\|$	3