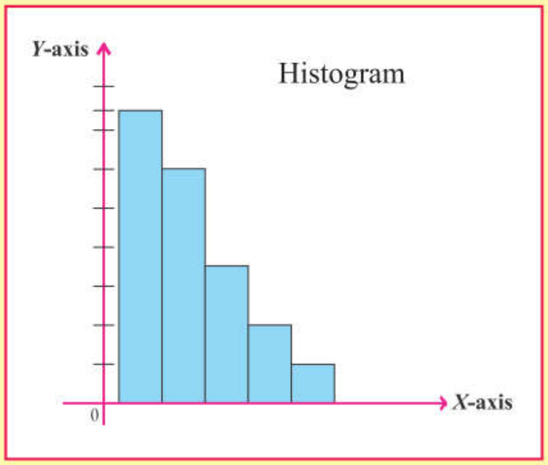
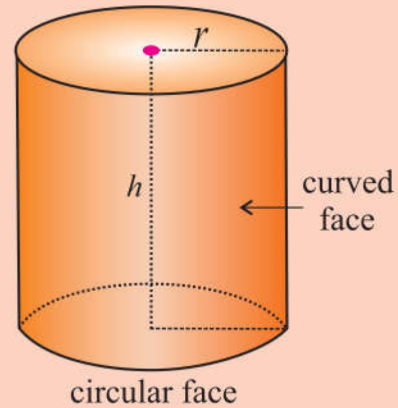
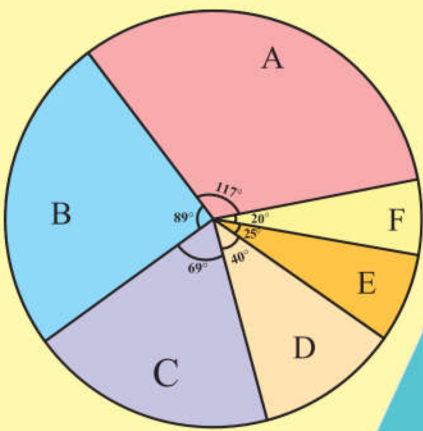
MATHEMATICS

7

Based on Single National Curriculum 2022







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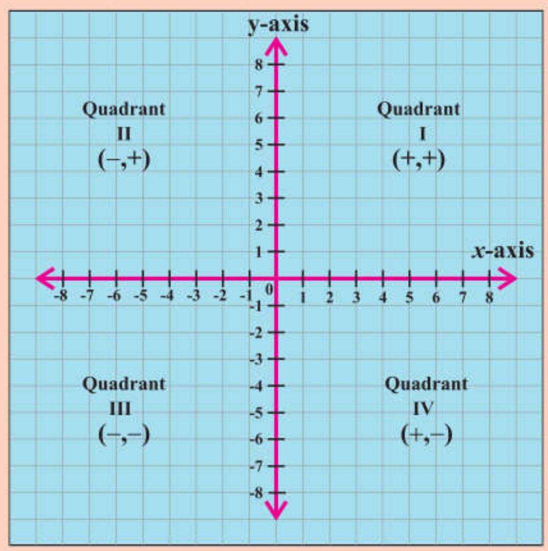

Punjab Curriculum and Textbook Board, Lahore

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِيْمِ

(In the Name of Allah, the Most Compassionate, the Most Merciful)

MATHEMATICS

7

Based on Single National Curriculum 2022

ONE NATION, ONE CURRICULUM

PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

This textbook is based on Single National Curriculum 2022 and has been approved by the Board.

CONTENTS

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Experimental Edition

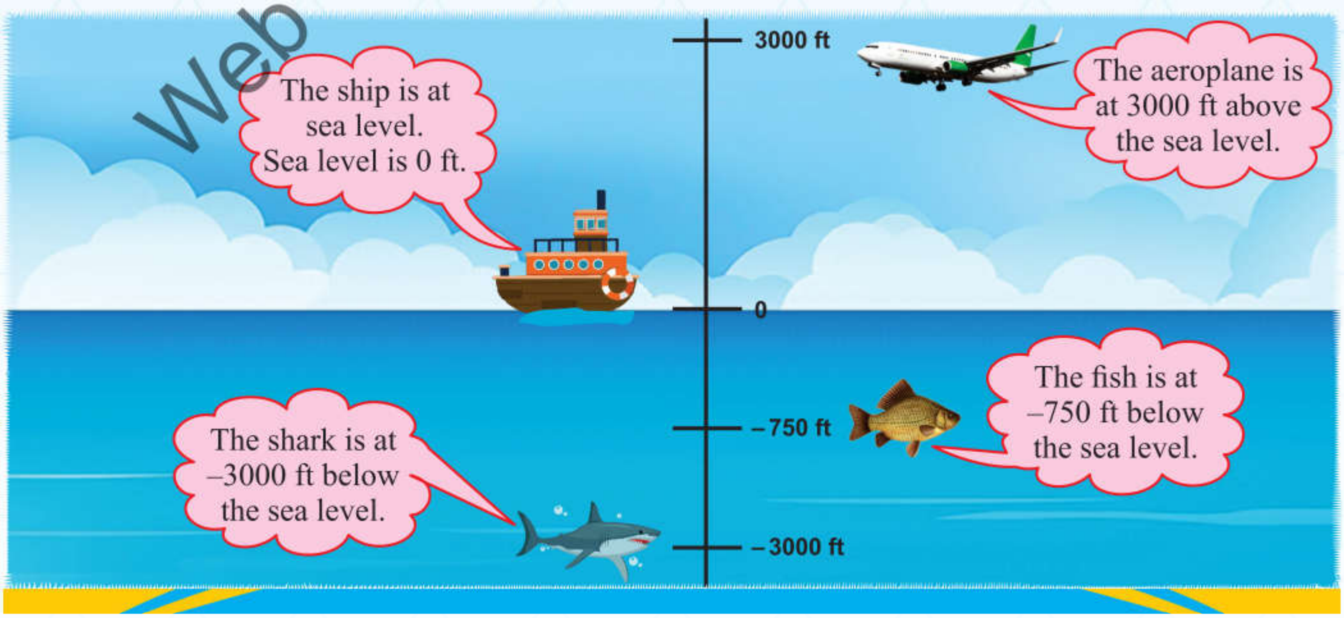
Domain 1 NUMBERS AND OPERATIONS

Sub-domain (i) Rational Numbers and Decimal Numbers

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recall - Recognise, identify and represent integers (positive, negative and neutral integers) and their absolute or numerical value.
- Represent whole numbers, integers and decimal numbers on a number line.
- With increasing degree of challenge, use the concept of place value for whole numbers, integers, rational numbers and decimal numbers.
- Compare (using symbols $<, >, =, \leq$ and $\geq$) and arrange (in ascending or descending order) whole numbers, integers, rational numbers and decimal numbers.
- Identify and convert between various types of fractions.
- Recall H.C.F and L.C.M.
- Identify and represent (on a number line) rational numbers.
- Solve real-world word problems involving operations on rational numbers.
- Verify commutative, associative and distributive properties of rational numbers
- Round whole numbers, integers, rational numbers and decimal numbers to a required degree of accuracy, significance or decimal places (up to 3 decimal places).
- Use knowledge of rounding to give an estimate to a calculation; to check the reasonableness of the solution.



RECALL

You have already learnt about:

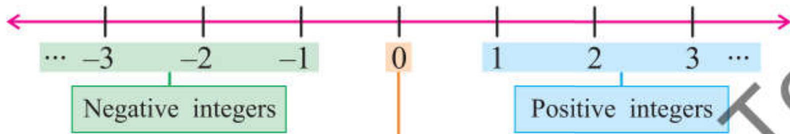
- Natural numbers (1, 2, 3, 4, ...) are also called positive integers and –1, –2, –3, ... are called negative integers. The positive integers and negative integers along with zero (0) are called integers. i.e.,

  The set of integers $Z = \{0, \pm1, \pm2, \pm3, ...\}$.

- Natural numbers along with zero (0) are called whole numbers i.e.,

  The set of whole numbers $W = \{0, 1, 2, 3, 4, ...\}$.

When we arrange these integers on a line, then the line is called number line.



- 0 is neither positive nor negative.
- 0 is also known as neutral number/integer.

History

In 1563, Arbermouth Holst introduced integers. He was a German Mathematician.

Think!

Give any five real life examples in which you use negative integers.

Keep in mind!

The distance from 0 to –3 and 0 to 3 is same. So, –3 and 3 have the same numerical or absolute value. The absolute value of –3 is denoted by $|-3|$.

So, $|-5| = 5$; $-|-6| = -6$

Skill Practice

Solve the following:

$|-10|, -|-17|, |19|$ and $|-6|$

Also arrange them in ascending and descending order.

Search out

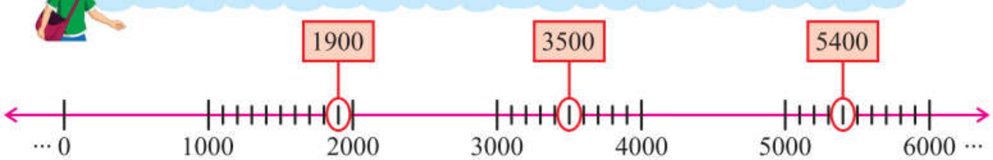
https://www.math-play.com/Millionaire-Game-Absolute-Value/Millionaire-Game-Absolute-Value_html5.html

Activity

Display different temperature cards in front of the students, they will be asked to arrange them in order from the smallest to the greatest after taking their absolute values.

1.1.1 Representation of Whole Numbers on Number Line

Let us represent 1900, 3500 and 5400 on the number line.



Skill Practice

Represent the following whole numbers on the number line.

(i) 880, 730, 540, 990, 670

Teachers' Guide

Recall and clear the difference among the natural numbers, whole numbers, integers and the concept of absolute value by using real life situations, in classroom.

1.1.2 Place Value of Whole Numbers

Look at the following place value chart:

Third period			Second period			First period		
Millions			Thousands			Ones		
Hundred million	Ten million	Million	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
			9	8	7	4	3	6
			9	2	3	5	3	0
			9	8	7	2	1	9

Let us consider 987436.

- The digit 9 is at "hundred thousands place". Its value is 900000.
- The digit 8 is at "ten thousands place". Its value is 80000.
- The digit 7 is at "thousands place". Its value is 7000.
- The digit 4 is at "hundreds place". Its value is 400.
- The digit 3 is at "tens place". Its value is 30.
- The digit 6 is at "ones place". Its value is 6.

Challenge

Find the 7-digit number with the help of the given clues: 2 is at ones place. The place value of 1 is 100. 7 is not at the tens place. 4 is at thousand place. The digit at the hundred thousands place is sum of digits at thousands and ones place. The digit at ten thousands is one more than the digit at thousand place. 7 digit are 1, 2, 3, 4, 5, 6 and 7 to make a 7-digit number.

1.1.3 Comparing and Ordering Whole Numbers

Let us consider the numbers 987219, 987436 and 923530 for comparing.

(i) In 923530, the digit at ten thousands place is smaller than the remaining two numbers. Therefore, 923530 is the smallest number.

(ii) In 987436 and 987219, the digit at the hundreds place "4" is greater than "2". Therefore, 987436 is greater than 987219.

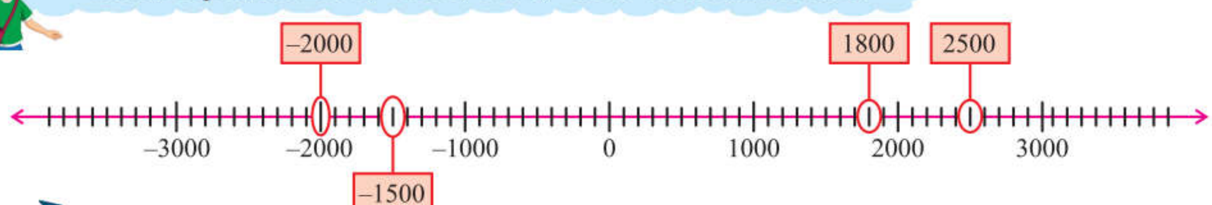
987436 > 987219

So, 987436 > 987219 > 923530

Ascending order: 923530, 987219, 987436
Descending order: 987436, 987219, 923530

1.1.4 Representation of Integers on the Number Line

Let us represent 2500, –1500, 1800, –2000 on a number line.



Teachers' Guide

Draw a number line on the writing board and ask the students to represent some whole numbers on the number line.

1.1.5 Comparing and ordering of Integers

Let us compare the given integers and write in ascending and descending order.

–436, –538, 483, 836

Integer that has negative sign, it will be smaller.

So, 836 > 483 > –436 > –538

Ascending order: –538, –436, 483, 836

Descending order: 836, 483, –436, –538

Skill Practice

Represent the following on the number line:

(i) –700, 100, 900, –800, –500

(ii) 2200, – 1800, –900, 1500, 2500

Remember!

When we write the negative sign (–) with a whole number, then it becomes negative integer expect zero (0)

Search out

https://www.mathplayground.com/mobile/numberballs_fullscreen.htm

Skill Practice

Represent the following integers on number line:

–50, 60, –30, –20, 10, 40

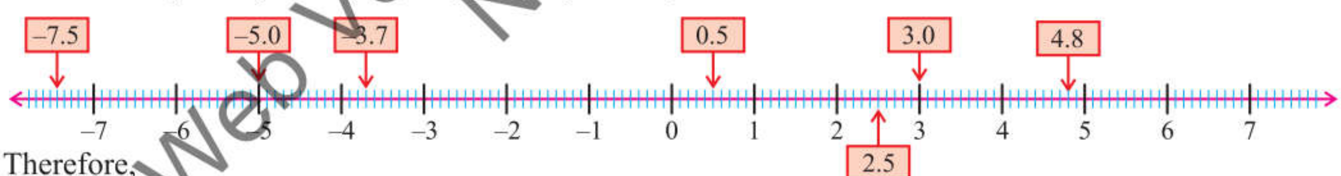
Activity

- Teacher will provide a handout with some blank spaces for 2-digit or 3-digit numbers with positive and negative signs and an empty box in between number blanks for comparison of integers.
- Player 1 will roll dice and fill one blank of question and then player 2 will roll dice and fill the other blank then according to positive and negative signs, players will determine which sign goes between numbers(< , > or =).
- The player with greater number gets 1 point.

1.1.6 Representation of Decimal Numbers on the Number Line

Let us represent the given decimal numbers on the number line.

–7.5 , 2.5 , 0.5 , –3.7 , 3.0 , –5.0 , 4.8



Therefore,

Ascending order: –7.5, –5.0, –3.7, 0.5, 2.5, 3.0, 4.7, Descending order: 4.7, 3.0, 2.5, 0.5, –3.7, –5.0, –7.5

1.1.7 Place Value of Decimal Numbers

Look at the following place value chart for decimal numbers:

Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
2	3	5	.	8	9	
2	3	5	.	7	0	
2	3	5	.	7	4	

1.1.8 Comparing and ordering of Decimal Numbers

Let us compare the given decimal numbers.

(i) Compare the digits at the greatest place.

As, the digits at hundreds, tens and ones are same.
Now we move at tenths place.

(ii) At tenths place digit 8 is greater than 7.

(iii) So, 235.89 is greater than 235.70 and 235.74

(iv) At hundredths place, the digit 4 is greater than 0.

So, 235.74 is greater than 235.70

By using symbols, 235.89 > 235.74 > 235.70

Let us arrange these decimal numbers in ascending and descending order.

Ascending order: 235.70, 235.74, 235.89, Descending order: 235.89, 235.74, 235.70

Skill Practice

Write the following integers on your notebook and arrange them in ascending and descending order.

–890.550, –230.430, –410.890

Think!

On which day the price of one litre petrol was maximum and minimum?

EXERCISE 1.1

1. Represent the following on the number line:

(i) 230, 130, 150, 100, 170, 120 (ii) 70, 80, 95, 35, 40, 45, 60
(iii) 1600, 1000, 1700, 1500, 900, 2000 (iv) –50, 60, –25, –70, –10, –35, 45
(v) –225, 440, –400, 100, –150, 120, 110 (vi) –1100, 2100, –1500, –2500, 1600
(vii) –15.5, 20.7, 11.2, –10.5, –12.5 (viii) 0.5, –0.7, –0.1, 0.3, 0.9, –0.5

2. Compare by using symbol >, <. Also arrange them in ascending and descending order.

(i) 235691, 235090, 245091, 245192 (ii) 578901, 579803, 679807, 679817
(iii) 10028, 100026, 110028, 110027 (iv) 562389, 562399, 572390, 572381

3. Compare by using symbol >, <. Also arrange them in ascending and descending order.

(i) –8395, –8496, –8491, –8394 (ii) 503, 530, 556, –563
(iii) –137, –138, –1886, 1308 (iv) –87650, –78432, –78402, 78401

4. Compare by using symbol >, <. Also arrange them in ascending and descending order.

(i) 28.356, 28.343, 28.357, 28.532 (ii) 120.08, 130.08, 120.80, 120,01
(iii) 131.01, 131.08, 103.78, 113.08 (iv) 236.089, 236.219, 236.217, 236.207

Teachers' Guide

Ask the students to come in front of writing board, write four to five decimal numbers on the writing board then ask them to compare these numbers and arrange them in ascending and descending order.

1.1.9 Identification and Conversion of Various Types of Fractions

Hence, there are total seven types of fractions such as:

Types of Fractions

Proper Fractions	Improper Fractions	Mixed Fractions	Unit Fractions	Like Fractions	Unlike Fractions	Equivalent Fractions
e.g., $\frac{1}{2}, \frac{5}{6}$ etc.	e.g., $\frac{7}{5}, \frac{8}{3}$ etc.	e.g., $3\frac{1}{2}, 2\frac{3}{5}$ etc.	e.g., $\frac{1}{5}, \frac{1}{3}$ etc.	e.g., $\frac{1}{5}, \frac{2}{5}$ etc.	e.g., $\frac{2}{5}, \frac{1}{3}$ etc.	e.g., $\frac{2}{6}, \frac{1}{3}$ etc.

The concept of fraction is illustrated as:

A whole pizza is divided into six equal parts. Here one piece of pizza is represented as $\frac{1}{6}$, where 1 is the numerator and 6 is the denominator.

Remember!

The first four fractions are defined for a single fraction but other three fractions are used to compare two or more fractions.

Important Information

Improper fractions are sometimes called the top heavy numbers.

Remember!

An improper fraction is always ≥ 1.

To solve the unlike fractions and equivalent fractions, we will have to use LCM and HCF of the denominators.

Important Information

Mixed fractions can be converted into improper fractions and vice versa.

Important Information

Mixed number is basically an improper fraction > 1.

RECALL

Least Common Multiples (LCM) and Highest Common Factors (HCF)

We have already learnt in previous grades to find out the LCM of two or more than two numbers.

LCM

2	72 , 48 24
2	36 , 24 , 12
2	18 , 12 , 6
2	9 , 6 , 3
3	9 , 3 , 3
3	3 , 1 , 1
	1 , 1 , 1

LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

HCF

$$
\begin{array}{r|l}
24 & 36 \;(1 \\
 & \underline{-24} \\
 & 12 \,)\, 24 \,(2 \\
 & \quad \underline{-24} \\
 & \quad\ \ 0
\end{array}
\qquad
\begin{array}{r|l}
12 & 12 \;(1 \\
 & \underline{-12} \\
 & \ \ 0
\end{array}
$$

HCF = 12

Keep in mind!

- Highest common factor is also known as greatest common divisor (GCD).
- The smallest common multiple of two or more than two numbers is called least common multiple (LCM).

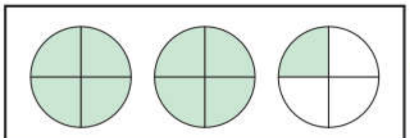
i Conversion of Improper Fractions into Mixed Numbers

Example 1 Convert $\frac{9}{4}$ into mixed number.

Solution To convert $\frac{9}{4}$ into mixed number, we will divide 9 by 4.

$$\begin{array}{r|l} 4 & \begin{array}{l} \ \ 2 \\ \overline{)\ 9} \\ \underline{-8} \\ \ \ 1 \end{array}\end{array}$$

$$\frac{9}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$$



$$\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$
$$= 1 + 1 + \frac{1}{4}$$
$$= 2 + \frac{1}{4} \Rightarrow 2\frac{1}{4}$$

Keep in mind!

In $2\frac{1}{4}$

2 → Whole part, $\frac{1}{4}$ → Fractional part

ii Conversion of Mixed Number into Improper Fraction

We follow the following steps to convert a mixed number into an improper fraction

Rule 1 Write the mixed number e.g., $2\frac{3}{5}$

Rule 2 Multiply the whole number by the denominator i.e., $2 \times 5 = 10$

In this case, we multiply 2 by 5

Rule 3 Add the product to the numerator i.e., add $10 + 3 = 13$

So, the improper fraction is $\frac{13}{5}$.

Activity

Students will work in groups of 4. Teacher provide cards of improper fractions and cards of mixed fraction equivalent fraction cards. Prepare at least 8 extra cards of improper fraction and mixed fraction also that do not match. Distribute the sets of cards to each group . Students will match the mixed fraction and improper fraction that are equal. The group who will match all the cards correctly will win.

1.1.10 Rational Numbers

We have learnt about different types of numbers such as natural numbers, whole numbers, integers and common fractions. All these numbers are called rational numbers.

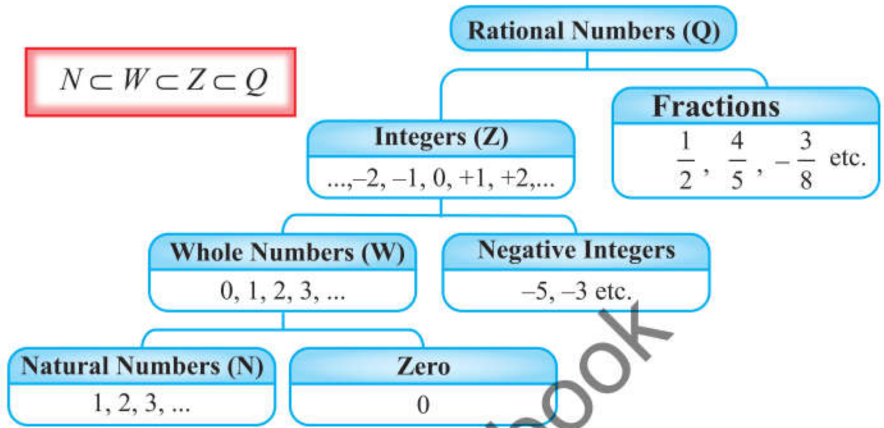
A number that can be expressed in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called a rational number.

Important Information

There are many situations involve in fractional numbers. To include such numbers we need to extend our number system by introducing rational numbers.

The relationship among different types of numbers is illustrated by the figure given below:

$N \subset W \subset Z \subset Q$



Important Information

The set of rational number is denoted by Q.

Important Information

In $\frac{p}{q}$, the integer p is the numerator and the integer $q \neq 0$ is the denominator.

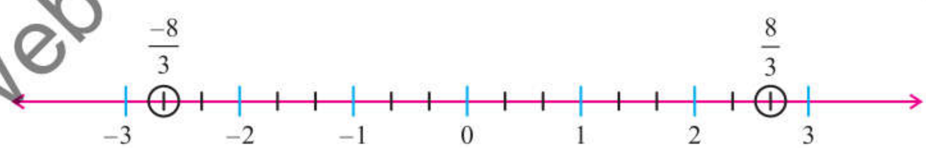
1.1.11 Representation of Rational Numbers on the Number Line

Example 2 Represent $\frac{8}{3}$ and $\frac{-8}{3}$ on number line.

Solution Since $\frac{8}{3}$ and $\frac{-8}{3}$ are improper fractions and $\frac{8}{3}$ is positive, so it will lie on right side of 0 on the number line and $-\frac{8}{3}$ is negative so, it will lie on left side of 0 on the number line. Let us first convert $\frac{8}{3}$ and $\frac{-8}{3}$ into mixed number.

$$\frac{-8}{3} = -2\frac{2}{3} \qquad \Bigg| \qquad \frac{8}{3} = 2\frac{2}{3}$$

So, $\frac{8}{3}$ lies between 2 and 3. We will divide the line between 2 to 3 and –2 to –3 into 3 equal parts.



The rational numbers $\frac{-8}{3}$ and $\frac{8}{3}$ are represented on the number line as shown in the figure.

1.1.12 Comparison of Two Rational Numbers

Sadia used $\frac{5}{8}$ metre ribbon to decorate the gift box.

Tahir used $\frac{13}{16}$ metre ribbon to decorate the gift box.

Who did use more ribbon to decorate the gift box?

Remember!

- Every positive rational number is greater than zero.
- Every positive rational number is greater than every negative rational number.

Keep in mind!

- If the denominators of two rational numbers are same, then compare the numerators, the rational number with greater numerator is greater than other.
- If the denominators of two rational numbers are different, then convert them into like fractions by taking LCM of the denominators.

To solve this problem, we will compare both the rational numbers.

As the denominators of both the rational numbers are different, so we convert them into like fractions. Multiply all the fractions by a number so that their denominators become equal to their LCM

$$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$$

$$\frac{13}{16} = \frac{13 \times 1}{16 \times 1} = \frac{13}{16}$$

Now, compare numerators

As, $13 > 10$ So, $\frac{13}{16} > \frac{10}{16}$ i.e., $\frac{13}{16} > \frac{5}{8}$

Hence, Tahir used more ribbon than Sadia.

Working

Find LCM of 8 and 16

2	8 , 16
2	4 , 8
2	2 , 4
2	1 , 2
	1 , 1

LCM = 2 × 2 × 2 × 2 = 16

1.1.13 Arrange the Rational Numbers in Ascending or Descending Order

To arrange the rational numbers in order, we will have the following steps:

Step 1 Write the given rational numbers with positive denominator.

Step 2 Take LCM of positive denominators.

Step 3 Write each rational number with LCM as the common denominators.

Example 3 Arrange the following rational numbers in ascending and descending order.

$$\frac{4}{5}, \frac{7}{30}, \frac{7}{15}, \frac{3}{10}$$

Solution $\frac{4}{5}, \frac{7}{30}, \frac{7}{15}, \frac{3}{10}$

2	5 – 30 – 15 – 10
3	5 – 15 – 15 – 5
5	5 – 5 – 5 – 5
	1 – 1 – 1 – 1

LCM = 2 × 3 × 5 = 30

Multiply all the rational numbers by a number so that their denominators become equal to their LCM

$$\text{Now, } \frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

$$\frac{7}{30} = \frac{7}{30}$$

$$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$$

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$$

Now, since $24 > 14 > 9 > 7$

So, $\frac{24}{30} > \frac{14}{30} > \frac{9}{30} > \frac{7}{30} \Rightarrow \frac{4}{5} > \frac{7}{15} > \frac{3}{10} > \frac{7}{30}$

Ascending order: $\frac{7}{30}$, $\frac{3}{10}$, $\frac{7}{15}$, $\frac{4}{5}$ Descending order: $\frac{4}{5}$, $\frac{7}{15}$, $\frac{3}{10}$, $\frac{7}{30}$

Activity

Students will work in pairs. Provide each pair with blank paper chits (4 or 5). All pairs of students will write some rational numbers with different denominators. Each pair will now make all the denominators equal to their LCM. Now, each pair will arrange these rational numbers in the ascending or descending order.

EXERCISE 1.2

1. Represent the following rational numbers on number line:

(i) $\frac{1}{3}$ (ii) $\frac{3}{7}$ (iii) $\frac{-3}{5}$ (iv) $\frac{5}{3}$

(v) $\frac{5}{8}$ (vi) $\frac{3}{10}$ (vii) $-2\frac{1}{2}$ (viii) $-2\frac{3}{4}$

2. Convert:

(i) $1\frac{2}{3}$, $3\frac{4}{5}$, $2\frac{5}{11}$, $8\frac{3}{7}$ into improper fractions

(ii) $\frac{18}{7}$, $\frac{63}{14}$, $\frac{43}{13}$, $\frac{115}{9}$ into mixed fractions

3. Put the correct sign (>, < or =) between the following rational numbers:

(i) $\frac{3}{5} \square \frac{6}{25}$ (ii) $\frac{7}{10} \square \frac{-7}{10}$ (iii) $\frac{-3}{8} \square \frac{-5}{8}$

(iv) $1\frac{1}{7} \square 2\frac{3}{14}$ (v) $1\frac{5}{12} \square \frac{7}{12}$ (vi) $\frac{-19}{25} \square \frac{-7}{25}$

4. Compare the following rational numbers:

(i) $\frac{2}{7}$, $\frac{2}{5}$ (ii) $\frac{5}{9}$, $\frac{-3}{5}$ (iii) $\frac{1}{12}$, $\frac{-1}{12}$

(iv) $\frac{-2}{7}$, $\frac{-3}{11}$ (v) $\frac{-5}{11}$, $\frac{-7}{9}$ (vi) $\frac{3}{22}$, $\frac{-5}{22}$

(vii) $3\frac{5}{9}$, $4\frac{5}{18}$ (viii) $-1\frac{7}{25}$, $\frac{-3}{5}$ (ix) $3\frac{1}{2}$, $2\frac{1}{14}$

5. Arrange the following rational numbers in ascending order:

(i) $\frac{1}{4}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{5}{4}$ (ii) $\frac{6}{7}$, $\frac{3}{14}$, $\frac{-9}{7}$, $\frac{13}{-14}$

(iii) $\frac{-7}{12}$, $\frac{5}{-4}$, $\frac{-2}{3}$, $\frac{5}{8}$ (iv) $\frac{7}{8}$, $\frac{-5}{4}$, $\frac{-7}{5}$, $\frac{13}{8}$

6. Arrange the following rational numbers in descending order:

(i) $\frac{2}{3}$, $\frac{4}{7}$, $\frac{-4}{-7}$, $\frac{-3}{4}$ (ii) $\frac{4}{5}$, $\frac{-13}{-30}$, $\frac{7}{-10}$, $\frac{7}{-15}$

(iii) $\frac{5}{8}$, $\frac{13}{24}$, $\frac{17}{-12}$, $\frac{9}{48}$ (iv) $\frac{-5}{11}$, $\frac{-9}{22}$, $\frac{-4}{33}$, $\frac{-41}{44}$

1.1.14 Operations on Rational Numbers

i Addition of Rational Numbers

Addition of rational numbers is similar to the addition of fractions.

Case I Rational numbers having the same denominators

To add rational numbers with the same denominators, we add their numerators and the denominator remains the same.

In other words, if $\frac{p}{q}$ and $\frac{r}{q}$ are two rational numbers, then

$$\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$$

Skill Practice

Solve the following:

$\frac{1}{2} + \frac{7}{9} + \frac{1}{9} = \boxed{}$

For example

$$\frac{5}{7} + \frac{1}{7} = \frac{5+1}{7} = \frac{6}{7}$$

$$\frac{2}{13} + \frac{4}{13} + \frac{3}{13} = \frac{2+4+3}{13} = \frac{9}{13}$$

Case II Rational numbers having different denominators

To add rational numbers with different denominators, first make the denominators same by taking the LCM as their common denominators.

Danish purchased a cake. He eats $\frac{3}{10}$ parts of the cake. Danish's friend eats $\frac{1}{5}$ parts of the cake and his sister eats $\frac{4}{15}$ parts of the cake. How much cake is eaten by them altogether?

Challenge

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers then:

$\frac{p}{q} + \frac{r}{s} = \boxed{}$

To solve this problem, we will have to add these rational numbers i.e., $\frac{3}{10} + \frac{1}{5} + \frac{4}{15}$

As, the rational numbers are unlike fractions. First of all, we will convert them into like fractions.

Step 1 Find LCM of all the denominators. LCM of 10, 5 and 15 is 30.

Step 2 Now, multiply all the rational numbers by a number to make their denominators equal to their LCM.

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30} \;;\; \frac{1}{5} = \frac{1 \times 6}{5 \times 6} = \frac{6}{30} \;;\; \frac{4}{15} = \frac{4 \times 2}{15 \times 2} = \frac{8}{30}$$

These become like fractions.

Step 3 Now, add these like fractions

$$\frac{9}{30} + \frac{6}{30} + \frac{8}{30} = \frac{9+6+8}{30} = \frac{23}{30}$$

Hence, $\frac{23}{30}$ parts of the cake is eaten by them.

Skill Practice

Can you find out how much cake is left with Danish?

ii Subtraction of Rational Numbers

Case I Rational numbers having the same denominators

To subtract rational numbers with the same denominators, we subtract the numerators, the denominators remain the same.

If $\frac{p}{q}$ and $\frac{r}{q}$ are rational numbers then,

$$\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$$

For example

Look at the following examples:

(i) $\frac{5}{9} - \frac{1}{9} = \frac{5-1}{9} = \frac{4}{9}$ (ii) $\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8} = \frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$

Case II Rational numbers having different denominators

In this case, make denominators same by taking the LCM as their common denominator.

Consider the following example:

Skill Practice

Solve the following:

$\frac{11}{15} - \frac{9}{15} - \frac{2}{15} = \boxed{}$

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then

$$\frac{p}{q} - \frac{r}{s} = \frac{p \times s - r \times q}{q \times s}$$

Example 4 The sum of two rational numbers is $\frac{-3}{5}$. If one of them is $\frac{9}{10}$, find the other.

Solution Sum of the numbers = $\frac{-3}{5}$; One of the number = $\frac{9}{10}$

The other number = Sum of numbers – one of the number

$$= \frac{-3}{5} - \frac{9}{10} = \frac{-3 \times 2 - 9 \times 1}{10} = \frac{-6-9}{10} = \frac{-15}{10}$$

Hence, the other number is $\frac{-15}{10}$.

iii Additive Inverse

An additive inverse of a rational number is the same original number with the opposite sign. For example, the additive inverse of $\frac{5}{6}$ is $\frac{-5}{6}$.

For each rational number $\frac{p}{q}$ there exist a number $\frac{-p}{q}$ such that

$$\frac{p}{q} + \left(\frac{-p}{q}\right) = \left(\frac{-p}{q}\right) + \frac{p}{q} = 0$$

Here, $\frac{-p}{q}$ is called additive inverse of $\frac{p}{q}$.

Skill Practice

Find the additive inverse: (i) $\frac{2}{3}$ (ii) $\frac{-4}{7}$

iv Multiplication of Rational Numbers

To find the product of two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$.

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then $\frac{p}{q} \times \frac{r}{s} = \frac{p \times r}{q \times s} = \frac{pr}{qs}$

i.e., Product of two rational numbers = $\frac{\text{Product of their numerators}}{\text{Product of their denominators}}$

Example 5 Solve: $\frac{2}{5} \times \left(\frac{-3}{4}\right) \times \left(\frac{-9}{5}\right)$

Solution $\frac{2}{5} \times \left(\frac{-3}{4}\right) \times \left(\frac{-9}{5}\right) = \frac{2 \times (-3) \times (-9)}{5 \times 4 \times 5}$

$$= \frac{2 \times 27}{100} = \frac{54}{100} = \frac{27}{50}$$

Remember!

To multiply two rational numbers, multiply the numerator by numerator and denominator by the denominator.

Noor and Nazia planted their plants. The height of Noor's plant was $\frac{15}{4}$ cm while height of Nazia's plant was $\frac{7}{5}$ times of Noor's plant. Find the height of Nazia's plant.

Teachers' Guide

Write some real life examples on the writing board to clear how to find out the difference of two or more rational numbers having different denominators.

To find the height of Nazia's plant, we will have to multiply $\frac{7}{5}$ by $\frac{15}{4}$.

Height of Nazia's plant $= \frac{7}{{}_1\cancel{5}} \times \frac{\cancel{15}^{3}}{4} = \frac{21}{4}$

So, the height of Nazia's plant was $\frac{21}{4}$ cm.

Skill Practice

Ahmad's friend left $\frac{3}{8}$ of a whole pizza for him, Ahmad ate $\frac{1}{3}$ of what was left. How much of the whole pizza did Ahmad left?

v Division of Rational Numbers

Division of rational numbers is the same as division of a fraction by another fraction.

Important Information

Division is the inverse process of multiplication.

If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers such that $\frac{r}{s} \neq 0$, then:

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{p \times s}{q \times r} = \frac{ps}{qr}$$

Remember!

Division by 0 is undefined.

Example 6 Solve: $-4\frac{2}{5} \div \left(\frac{-6}{5}\right)$

Solution $-4\frac{2}{5} \div \left(\frac{-6}{5}\right) = \frac{-22}{5} \times \left(\frac{-5}{6}\right) = \frac{(-22) \times (-5)}{5 \times 6}$

$$= \frac{\cancel{22}^{11} \times \cancel{5}^{1}}{{}_1\cancel{5} \times \cancel{6}_3} = \frac{11}{3} = 3\frac{2}{3}$$

Important Information

It is important to note that when divide rational numbers is the order matters i.e.,

$$\frac{p}{q} \div \frac{r}{s} \neq \frac{r}{s} \div \frac{p}{q}$$

How many pieces of $\frac{15}{7}$ metres of ribbon can be cut from the ribbon which is $\frac{150}{7}$ metres long?

To find out the number of pieces of ribbon, we will have to divide $\frac{150}{7}$ by $\frac{15}{7}$.

Number of pieces of ribbon $= \frac{150}{7} \div \frac{15}{7}$

$$= \frac{\cancel{150}^{10}}{{}_1\cancel{7}} \times \frac{\cancel{7}^{1}}{\cancel{15}_1} = 10$$

So, the total length of ribbon can be cut into 10 equal parts of length $\frac{15}{7}$ metres.

Skill Practice

Solve the following:

$$\left(\frac{2}{3} \times \frac{9}{10}\right) \div \frac{3}{10} = \boxed{}$$

EXERCISE 1.3

1. Add the following rational numbers:

(i) $\frac{5}{9}$ and $\frac{4}{9}$ (ii) $\frac{-13}{4}$ and $\frac{9}{14}$ (iii) $\frac{-7}{11}$ and $\frac{-3}{22}$

(iv) $\frac{-6}{13}$ and $\frac{-8}{13}$ (v) $\frac{3}{4}$ and $\frac{-6}{5}$ (vi) 4 and $\frac{-3}{5}$

2. Solve the following:

(i) $\frac{3}{-40} + \left(\frac{-5}{14}\right)$ (ii) $\frac{-5}{9} + \left(\frac{-1}{3}\right)$ (iii) $\frac{3}{4} + \left(\frac{-2}{5}\right) + \left(\frac{-7}{20}\right)$

(iv) $-2 + \frac{7}{11} + \left(\frac{-4}{15}\right)$ (v) $\frac{-1}{8} + \frac{5}{12} + \left(\frac{-7}{24}\right)$

3. Subtract:

(i) $\frac{3}{4}$ from $\frac{1}{2}$ (ii) $\frac{-7}{6}$ from $\frac{5}{18}$ (iii) $\frac{-4}{9}$ from $\frac{-3}{5}$ (iv) $\frac{-7}{8}$ from -2

4. Solve the following:

(i) $\frac{4}{7} - \frac{5}{14}$ (ii) $1\frac{1}{5} - \frac{7}{14}$ (iii) $\frac{-4}{15} - \frac{8}{25}$

(iv) $\frac{-2}{5} - \frac{3}{14}$ (v) $\frac{4}{9} - \left(\frac{-5}{-9}\right)$ (vi) $1\frac{1}{12} - \left(\frac{-4}{9}\right)$

(vii) $\frac{-5}{-8} - \left(\frac{-3}{7}\right)$ (viii) $\frac{4}{11} - \left(\frac{-5}{22}\right)$

5. Solve the following:

(i) $\frac{7}{11} \times \frac{5}{6}$ (ii) $\frac{3}{7} \times \left(\frac{-4}{5}\right)$ (iii) $\frac{-2}{9} \times \frac{5}{12}$

(iv) $\frac{-3}{14} \times \frac{-5}{7}$ (v) $\frac{-7}{6} \times (-18)$ (vi) $\frac{-13}{5} \times \frac{-25}{26}$

(vii) $\frac{-5}{11} \times \left(-1\frac{7}{22}\right)$ (viii) $-4\frac{1}{2} \times \left(-1\frac{3}{4}\right)$ (ix) $\frac{-5}{8} \times 48$

6. Solve the following:

(i) $\frac{5}{6} \div \frac{7}{12}$ (ii) $\frac{-7}{16} \div \frac{1}{8}$ (iii) $\frac{-6}{5} \div 8$

(iv) $\frac{16}{15} \div (-18)$ (v) $\frac{-3}{10} \div \left(\frac{-21}{40}\right)$ (vi) $4\frac{1}{16} \div -1\frac{5}{8}$

(vii) $-12 \div \left(\frac{-5}{6}\right)$ (viii) $-1\frac{10}{19} \div \left(\frac{-5}{58}\right)$

7. The product of two rational numbers is $\frac{-1}{10}$. If one number is $\frac{-3}{5}$, find the other.

8. By what rational number should we multiply $\frac{-15}{14}$ to get $\frac{-3}{7}$.

9. Find the additive inverse of:

(i) $\frac{3}{5}$ (ii) $\frac{-7}{12}$ (iii) $-\left(\frac{-8}{15}\right)$ (iv) $-7\frac{5}{12}$ (v) $-2\frac{1}{13}$

10. Amna purchased $\frac{25}{3}$ m cloth to stitch a shirt. If she used $\frac{13}{9}$ m cloth to stitch the shirt. How much cloth was left with her?

11. If $\frac{75}{4}$ kilogram rice is to be packed in $\frac{25}{8}$ kilogram packets, then find:

(a) How many packets of rice will be packed?

(b) How much rice will be there in 20 packets of mass $\frac{24}{15}$ kilograms each?

12. Zara bought $\frac{235}{7}$ litres cooking oil. She used $\frac{2}{14}$ litres oil for cooking. How much oil was left with her?

1.1.15 Commutative, Associative and Distributive Properties of Rational Numbers

Rational numbers have several distinct properties, including commutative, associative and distributive properties.

i Commutative Property

In commutative property, the order of operations doesn't matter. To understand this concept better, let's look at an example: If Rakib has three apples in his bag and adds six more, then it is the same as if Rakib has six apples and adds three more.

(a) Commutative Property of Rational Numbers w.r.t Addition

The addition of two rational numbers is commutative in general. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then:

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$$

Example 7 Prove that: $\frac{1}{2} + \frac{4}{5} = \frac{4}{5} + \frac{1}{2}$

Solution

Left Hand Side	Right Hand Side
L.H.S $= \frac{1}{2} + \frac{4}{5}$	R.H.S $= \frac{4}{5} + \frac{1}{2}$
L.H.S $= \frac{5+8}{10} = \frac{13}{10}$	R.H.S $= \frac{8+5}{10} = \frac{13}{10}$

So, L.H.S = R.H.S

(b) Commutative Property of Rational Numbers w.r.t Multiplication

The commutative property of multiplication means changing the order of the numbers while multiplying does not change the answer. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then:

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

Keep in mind!

Commutative property does not hold in subtraction and division in nature.

Example 8 Prove that: $\frac{3}{4} \times \frac{5}{7} = \frac{5}{7} \times \frac{3}{4}$

Solution

Left Hand Side	Right Hand Side
L.H.S $= \frac{3}{4} \times \frac{5}{7}$	R.H.S $= \frac{5}{7} \times \frac{3}{4}$
L.H.S $= \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$	R.H.S $= \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$

So, L.H.S = R.H.S

ii Associative Property

The associative property means that you can join or combine things in any order and still get the same answer. For example, Rakib went to the supermarket and bought ice cream for 12 rupees, bread for 8 rupees, and milk for 15 rupees. He gave the cashier 35 rupees for everything. He always got the same answer not matter in which order he added them.

(a) Associative Property of Rational Numbers w.r.t Addition

If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to associative properties of addition, we have:

$$\left(\frac{p}{q} + \frac{r}{s}\right) + \frac{t}{u} = \frac{p}{q} + \left(\frac{r}{s} + \frac{t}{u}\right)$$

Example 9 Prove that: $\left(\frac{3}{4} + \frac{5}{2}\right) + \frac{1}{2} = \frac{3}{4} + \left(\frac{5}{2} + \frac{1}{2}\right)$

Solution

Left Hand Side	Right Hand Side
L.H.S $= \left(\frac{3}{4} + \frac{5}{2}\right) + \frac{1}{2}$	L.H.S $= \frac{3}{4} + \left(\frac{5}{2} + \frac{1}{2}\right)$
$= \left(\frac{3+10}{4}\right) + \frac{1}{2}$	$= \frac{3}{4} + \left(\frac{5+1}{2}\right)$
$= \frac{13}{4} + \frac{1}{2}$	$= \frac{3}{4} + \frac{6}{2}$
L.H.S $= \frac{13+2}{4} = \frac{15}{4}$	L.H.S $= \frac{3+12}{4} = \frac{15}{4}$
So, L.H.S = R.H.S	

(b) Associative Property of Rational Numbers w.r.t Multiplication

When we multiply three numbers, it does not matter how we group them because the answer will be the same. If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to associative property of multiplication, we have

$$\left(\frac{p}{q} \times \frac{r}{s}\right) \times \frac{t}{u} = \frac{p}{q} \times \left(\frac{r}{s} \times \frac{t}{u}\right)$$

Example 10 Prove that: $\left(\frac{5}{7} \times \frac{3}{2}\right) \times \frac{1}{3} = \frac{5}{7} \times \left(\frac{3}{2} \times \frac{1}{3}\right)$

Solution

Left Hand Side	Right Hand Side
L.H.S $= \left(\frac{5}{7} \times \frac{3}{2}\right) \times \frac{1}{3}$	L.H.S $= \frac{5}{7} \times \left(\frac{3}{2} \times \frac{1}{3}\right)$
$= \left(\frac{5 \times 3}{7 \times 2}\right) \times \frac{1}{3}$	$= \frac{5}{7} \times \left(\frac{3 \times 1}{2 \times 3}\right)$
$= \frac{15}{14} \times \frac{1}{3}$	$= \frac{5}{7} \times \frac{3}{6}$
L.H.S $= \frac{15 \times 1}{14 \times 3} = \frac{15}{42} = \frac{5}{14}$	L.H.S $= \frac{5 \times 3}{7 \times 6} = \frac{15}{42} = \frac{5}{14}$
So, L.H.S = R.H.S	

iii Distributive property of Rational Numbers w.r.t Multiplication over Addition and Subtraction

If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to distributive property of multiplication over addition and subtraction, we have:

(i) $\frac{p}{q} \times \left(\frac{r}{s} + \frac{t}{u}\right) = \left(\frac{p}{q} \times \frac{r}{s}\right) + \left(\frac{p}{q} \times \frac{t}{u}\right)$ (ii) $\frac{p}{q} \times \left(\frac{r}{s} - \frac{t}{u}\right) = \left(\frac{p}{q} \times \frac{r}{s}\right) - \left(\frac{p}{q} \times \frac{t}{u}\right)$

Example 11 Prove that: $\frac{2}{3} \times \left(\frac{1}{3} + \frac{4}{5}\right) = \left(\frac{2}{3} \times \frac{1}{3}\right) + \left(\frac{2}{3} \times \frac{4}{5}\right)$

Solution

Left Hand Side	Right Hand Side
L.H.S $= \frac{2}{3} \times \left(\frac{1}{3} + \frac{4}{5}\right)$	L.H.S $= \left(\frac{2}{3} \times \frac{1}{3}\right) + \left(\frac{2}{3} \times \frac{4}{5}\right)$
$= \frac{2}{3} \times \left(\frac{5+12}{15}\right)$	$= \left(\frac{2 \times 1}{3 \times 3}\right) + \left(\frac{2 \times 4}{3 \times 5}\right)$
$= \frac{2}{3} \times \frac{17}{15}$	$= \frac{2}{9} + \frac{8}{15}$
L.H.S $= \frac{2 \times 17}{3 \times 15} = \frac{34}{45}$	L.H.S $= \frac{10+24}{45} = \frac{34}{45}$
So, L.H.S = R.H.S	

Example 12 Prove that: $\frac{1}{4} \times \left(\frac{2}{6} - \frac{1}{8}\right) = \left(\frac{1}{4} \times \frac{2}{6}\right) - \left(\frac{1}{4} \times \frac{1}{8}\right)$

Solution

Left Hand Side	Right Hand Side
L.H.S $= \frac{1}{4} \times \left(\frac{2}{6} - \frac{1}{8}\right)$	L.H.S $= \left(\frac{1}{4} \times \frac{2}{6}\right) - \left(\frac{1}{4} \times \frac{1}{8}\right)$
$= \frac{1}{4} \times \left(\frac{8-3}{24}\right)$	$= \left(\frac{1 \times 2}{4 \times 6}\right) - \left(\frac{1 \times 1}{4 \times 8}\right)$
$= \frac{1}{4} \times \frac{5}{24}$	$= \frac{2}{24} - \frac{1}{32}$
L.H.S $= \frac{1 \times 5}{4 \times 24} = \frac{5}{96}$	L.H.S $= \frac{8-3}{96} = \frac{5}{96}$
So, L.H.S = R.H.S	

EXERCISE 1.4

1. Verify the following commutative properties:

(i) $\frac{-3}{5} + \frac{6}{7} = \frac{6}{7} + \frac{-3}{5}$ (ii) $\frac{7}{10} + \frac{-2}{5} = \frac{-2}{5} + \frac{7}{10}$

(iii) $\frac{3}{-2} \times \frac{-5}{7} = \frac{-5}{7} \times \frac{3}{-2}$ (iv) $\frac{15}{26} \times \frac{15}{45} = \frac{15}{45} \times \frac{15}{26}$

2. Verify the following associative properties:

(i) $\left(\frac{4}{3}+\frac{2}{5}\right)+\frac{3}{7}=\frac{4}{3}+\left(\frac{2}{5}+\frac{3}{7}\right)$ (ii) $\left(\frac{10}{12}+\frac{11}{-2}\right)+\frac{-3}{4}=\frac{10}{12}+\left(\frac{11}{-2}+\frac{-3}{4}\right)$

(iii) $\left(\frac{15}{20}+\frac{-5}{10}\right)+\frac{12}{40}=\frac{15}{20}+\left(\frac{-5}{10}+\frac{12}{40}\right)$ (iv) $\left(\frac{-5}{10}+\frac{10}{-20}\right)+\frac{15}{30}=\frac{-5}{10}+\left(\frac{10}{-20}+\frac{15}{30}\right)$

3. Verify the following distributive properties:

(i) $\frac{-2}{5}\times\left(\frac{3}{9}+\frac{8}{6}\right)=\left(\frac{-2}{5}\times\frac{3}{9}\right)+\left(\frac{-2}{5}\times\frac{8}{6}\right)$ (ii) $\frac{10}{11}\times\left(\frac{6}{7}+\frac{9}{4}\right)=\left(\frac{10}{11}\times\frac{6}{7}\right)+\left(\frac{10}{11}\times\frac{9}{4}\right)$

(iii) $\frac{15}{16}\times\left(\frac{8}{12}-\frac{2}{6}\right)=\left(\frac{15}{16}\times\frac{8}{12}\right)-\left(\frac{15}{16}\times\frac{2}{6}\right)$

(iv) $\frac{-4}{6}\times\left(\frac{8}{10}-\frac{4}{5}\right)=\left(\frac{-4}{6}\times\frac{8}{10}\right)-\left(\frac{-4}{6}\times\frac{4}{5}\right)$

Activity

Provide a handout of associative property with blanks. Students will roll 3 dice at once and will write the numbers in the same sequence on both sides and then they will solve both sides to find out both sides give the same answer.

1.1.16 Round Whole Numbers, Integers, Rational Numbers and Decimal Numbers

Nimra wants to buy the pack of tea. The price of the tea pack is Rs. 239.75. In such situation, how much amount does she pay?

To solve this situation, she will have to pay Rs. 240. This procedure is known as rounding.

Skill Practice

If the price of a packet of chips is Rs. 15.15, tell the price of packet of chips after rounding.

i Round Whole Numbers

Keep in mind!

- First of all look at the digit next to the required degree of accuracy while rounding.
- If the digit is 5 or greater than 5, then we will add 1 to the digit of the required degree of accuracy.
- If the digit is less than 5, then we will leave the digit as it is given on the required degree of accuracy.

Let us round 3925 to the 1 significant figure, 2 significant figures, and 3 significant figures.

3925 = 4000 1 significant figure

3925 = 3900 2 significant figures

3925 = 3930 3 significant figures

Key fact!

The number which is rounded to a greater number of significant figures that number is more accurate.

Skill Practice

Round 48163 to the given degree of accuracy: (i) 2 significant figures (ii) 3-significant figures

ii Round Decimal Numbers

Let us round 24.3528 to the given degree of accuracy.
(i) 2 decimal places (ii) 3 significant figures

(i) 24.3528 = 24.35 (2 decimal places) (ii) 24.3528 = 24.4 (3 significant figures)

Example 13 Round 0.028635 to the given degree of accuracy.

(i) 3 significant figures (ii) 3 decimal places

Solution (i) 0.028635 = 0.0286 (ii) 0.028635 = 0.029

Skill Practice

Hamza purchased two litres of petrol for Rs. 438.75. Can you round the number to the given degree of accuracy?
(i) 3 significant figures
(ii) 1 decimal place

iii Round Negative Integers

While rounding the negative integer, the number which is close to the zero (0) is greater number and which is far from the zero (0) is smaller number.

Keep in mind!

In decimal part of a decimal number, all zeros before a non-zero digit are not significant but all zeros after a non-zero digit are significants.

Let us round –43566 to the given degree of accuracy.
(i) 2 significant figures
(ii) 3 significant figures

Remember!

In negative integers, the procedure to round integers is different from the procedure to round whole numbers.

(i) –43566 = –43000 (2 significant figures) When round towards zero (0)
= –44000 (2 significant figures) When round away from zero (0)

(ii) –43566 = –43500 (3 significant figures) When round towards zero (0)
= –43600 (3 significant figures) When round away from zero (0)

Skill Practice

How many significant does 0.0789 have?

Skill Practice

Round –53278 to the 2 significant figures and 3 significant figures.

Teachers' Guide

Write some negative integers and decimal numbers on writing board. Call out students in front of writing board. Ask them to round the numbers to the required degree of accuracy.

iv Round Rational Numbers

Aslam bought $\frac{25}{18}$ kilogram of rice. How can we round the number to the given degree of accuracy?

(i) 3 decimal places (ii) 2 significant figures (iii) 3 significant figures

(i) $\frac{25}{18} = 1.388889 = 1.389$ (3 decimal places)

(ii) $\frac{25}{18} = 1.3888889 = 1.4$ (2 significant figures)

(iii) $\frac{25}{18} = 1.38889 = 1.39$ (3 significant figures)

Skill Practice

Round $\frac{72}{33}$ to the:

(i) 3 decimal places
(ii) 2 significant figures
(iii) 3 significant figures

1.1.17 Use Knowledge of Rounding to Give an Estimate to a Calculation; to check the Reasonableness of the Solution

Sometimes, in our real life, we are unable to find the actual value of any object or thing, then to solve this situation we have to round or estimate the value to the required degree of accuracy. When round a number, we obtain an estimated or approximated value that may or may be differ from the actual value.

If the difference between the approximated / estimated value and actual value is small, then we can say that the result or solution is reasonable.

Error = Actual value – Approximated value

Keep in mind!

The difference between the actual value and approximated / estimated value is called an error.

Example 14 Find the sum of 24.8356 and 15.7156 after rounding 3 decimal places. Also check the solution is reasonable or not?

Solution First of all round 24.8356 and 15.7156 to the 3 decimal places.

24.8356 = 24.836 (3 decimal places) ; 15.7156 = 15.716 (3 decimal places)

Approximated sum = 24.836 + 15.716
= 40.552

Actual sum = 24.8356 + 15.7156
= 40.5512

Error = |Actual sum – Approximated sum|
= |40.5512 – 40.552|
= |–0.008|

Error = 0.008

The value of error is small. Hence, the approximated sum is reasonable.

Skill Practice

28.3 × 15.2

Find the approximated value and error. Also explain regarding the reasonableness of the solution.

Activity

Students will work in pairs. One student will roll the dice to make a 4-digit number. Then his/her partner will roll the dice to make another 4-digit number. Estimate the sum of the first and the second number. Add the numbers. Make a table of 6 by 2 (6 rows and 2 columns) on the sheet with headings estimated sum and actual sum. Students will record their estimated sum and actual sum. Repeat steps 2 to 7 five times. Compare the answers with the estimated sums. Are the answers reasonable?

EXERCISE 1.5

1. Round the following to the given degree of accuracy:
   - (i) 87932 (2 significant figures)
   - (ii) 3890 (3 significant figures)
   - (iii) 790253 (2 significant figures)
   - (iv) 10025 (3 significant figures)
   - (v) 18954 (3 significant figures)
   - (vi) 25980 (3 significant figures)
2. Round the numbers away from zero (0) as given.
   - (i) –27.3289 (3 significant figures)
   - (ii) –59058 (2 significant figures)
   - (iii) –12569 (3 significant figures)
   - (iv) –17.238 (2 significant figures)
   - (v) 0.003857 (3 significant figures)
   - (vi) 0.0467453 (3 significant figures)
3. Round the following to the stated number of decimal places or significant figures:
   - (i) $\frac{47}{8}$ (2 decimal places)
   - (ii) $\frac{5}{27}$ (3 significant figures)
   - (iii) $\frac{22}{27}$ (3 decimal places)
   - (iv) $\frac{27}{19}$ (3 significant figures)
   - (v) $\frac{37}{53}$ (3 decimal places)
   - (vi) $\frac{77}{83}$ (3 significant figures)
4. Round the following to the required degree of accuracy:
   - (i) 25.359 (2 decimal places)
   - (ii) 0.002897 (3 significant figures)
   - (iii) 0.0878 (3 decimal places)
   - (iv) 17.00597 (3 significant figures)
   - (v) 0.0005246 (3 significant figures)
   - (vi) 13.5876 (3 decimal places)
5. Compare your approximated value and accurate value. Also check is the solution reasonable or not?
   - (i) $4.281 \div 2.157$ (2 decimal places)
   - (ii) $3.348 - 2.104$ (2 decimal places)
   - (iii) 2.281×3.567 (2 decimal places)

SUMMARY

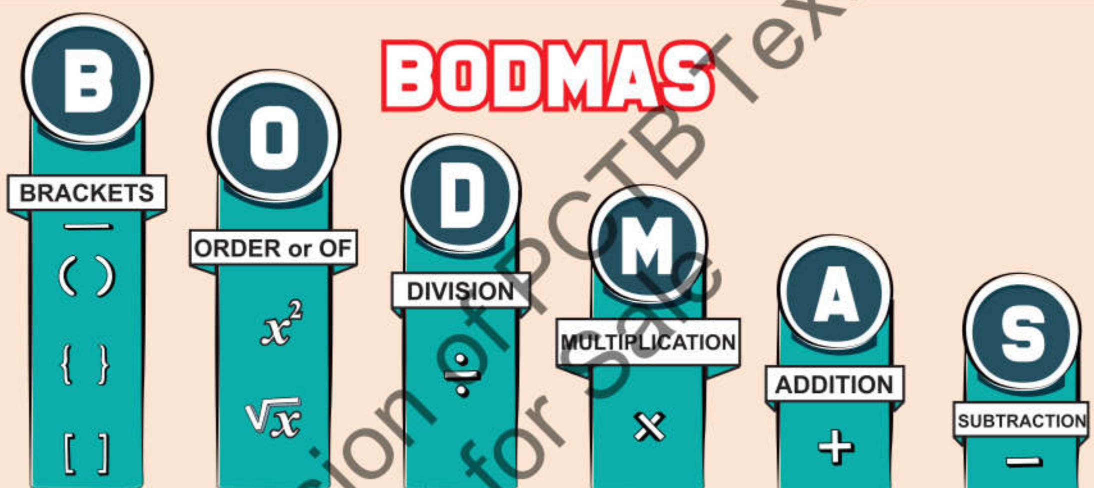
- The positive integers and negative integers along with zero (0) are called integers. i.e.,
  $Z = \{0, \pm1, \pm2, \pm3, ...\}$
- The only whole number zero (0) is not a natural number.
- 0 is neither positive nor negative. 0 is also known as neutral number/integer.
- A number that can be expressed in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called a rational number. The set of rational number is denoted by Q.
- In $\frac{p}{q}$, the integer p is the numerator and the integer $q \neq 0$ is the denominator.
- If the denominators of two rational numbers are same, then compare the numerators, the rational number with greater numerator is greater than other.
- If the denominator of two rational numbers are different, then convert them into like fractions by taking LCM of the denominators.
- The smallest common multiple of two or more than two numbers is called least common multiple (LCM).
- Every positive rational number is greater than zero. Every positive rational number is greater than every negative rational number.
- The difference between the actual value and approximated/ estimated value is called an error.
- The following are the properties of rational numbers:
  - $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ (commutative property of rational numbers w.r.t addition)
  - $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ (commutative property of rational numbers w.r.t multiplication)
  - $\left(\frac{p}{q} + \frac{r}{s}\right) + \frac{t}{u} = \frac{p}{q} + \left(\frac{r}{s} + \frac{t}{u}\right)$ (Associative property of rational numbers w.r.t addition)
  - $\left(\frac{p}{q} \times \frac{r}{s}\right) \times \frac{t}{u} = \frac{p}{q} \times \left(\frac{r}{s} \times \frac{t}{u}\right)$ (Associative property of rational numbers w.r.t multiplication)
- Distributive property of rational numbers w.r.t multiplication over addition and subtraction.
  - $\frac{p}{q} \times \left(\frac{r}{s} + \frac{t}{u}\right) = \left(\frac{p}{q} \times \frac{r}{s}\right) + \left(\frac{p}{q} \times \frac{t}{u}\right)$
  - $\frac{p}{q} \times \left(\frac{r}{s} - \frac{t}{u}\right) = \left(\frac{p}{q} \times \frac{r}{s}\right) - \left(\frac{p}{q} \times \frac{t}{u}\right)$

Sub-domain (ii) Simplification

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise the order of operations and use it to solve mathematical expressions involving whole numbers, decimals, fractions and integers.



Mathematics involves directly to business, health and finance. A mathematical expression consist of fractions, integers, whole numbers and decimals along with mathematical operations. To solve the mathematical expression a special technique and conceptual procedure is required. technique. In a mathematical expression, the order to apply the mathematical operations plays a vital role. In order to solve the mathematical expression, BODMAS rule is used. It guides us which operation is used first.

Step 1	B	Brackets	$[\{(\overline{\quad})\}]$
Step 2	O	order or of or power	x^2 or $\sqrt{x}$
Step 3	D	Division	÷
Step 4	M	Multiplication	×
Step 5	A	Addition	+
Step 6	S	Subtraction	−

Remember!

The procedure to simplify the mathematical expression is known as simplification.

History

Achilles Reselfelt introduced the "BODMAS" rule.

1.2.1 Brackets

In a mathematical expression, brackets are used to group/gather along with mathematical operations.

i.e., +, –, × and ÷. To simplify a mathematical expression, the given order of brackets is used.

Step 1	Vinculum or bar	$\overline{\quad}$
Step 2	Round, curved brackets	( )
Step 3	Curly brackets or braces	{ }
Step 4	Box brackets or square brackets	[ ]

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Let us solve the given mathematical expression using BODMAS rule and order of brackets.

Example 1 Simplify: $18 \div 2 - [3 \times 5 - \{12 - 6 - (-6 + \overline{8 - 5})\}]$

Solution

$18 \div 2 - [3 \times 5 - \{12 - 6 - (-6 + \overline{8 - 5})\}]$

$= 18 \div 2 - [3 \times 5 - \{12 - 6 - (-6 + (8 - 5))\}]$ solved bar or vinculum

$= 18 \div 2 - [3 \times 5 - \{12 - 6 - (-3)\}]$

$= 18 \div 2 - [3 \times 5 - \{12 - 6 + 3\}]$ solved round brackets

$= 18 \div 2 - [3 \times 5 - \{9\}]$

$= 18 \div 2 - [3 \times 5 - 9]$ solved curly brackets

$= 18 \div 2 - [15 - 9]$ applied BODMAS rule

$= 18 \div 2 - 6$ solved box brackets

$= 9 - 6$ applied BODMAS rule

$= 3$

Find Answer

$12 + 3 - 8 + 3 - 5 - 6 = \boxed{?}$

Keep in mind!

When expression has subtraction and addition, then it can be solved from left to right.

Example 2 Simplify: $2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (5.2 - \overline{3.2 + 5.2})\}]$

Solution

$2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (5.2 - \overline{3.2 + 5.2})\}]$

$= 2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (5.2 - 3.2 - 5.2)\}]$ solved bar

$= 2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (-3.2)\}]$

$= 2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 + 3.2\}]$ solved round brackets

$= 2.25 \div 8.5 \times [2 \times 1.37 + 6.4]$ solved curly brackets

Solve

$68 \div 2 \times 2 \div 4 = \boxed{?}$

Teachers' Guide

Clear the concept by applying mathematical operations when the expression has subtraction and addition by writing some examples on writing board.

$= 2.25 \div 8.5 \times [2.74 + 6.4]$

$= 2.25 \div 8.5 \times 9.14$ solved square brackets

$= 0.265 \times 9.14$

$= 2.4221$

Keep in mind

- When expression has multiplication and division, then solve division first.
- When expression has multiplication and division, then it can be solved from left to right.

Example 3 Simplify:

Solution

$$2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{2 \times 5\frac{1}{2} - \left(\frac{1}{2} - \overline{\frac{1}{3} + \frac{1}{5}}\right)\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{2 \times 5\frac{1}{2} - \left(\frac{1}{2} - \frac{1}{3} - \frac{1}{5}\right)\right\}\right]$$ solved bar or vinculum

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{2 \times 5\frac{1}{2} - \left(\frac{15 - 10 - 6}{30}\right)\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{2 \times 5\frac{1}{2} - \left(-\frac{1}{30}\right)\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{2 \times 5\frac{1}{2} + \frac{1}{30}\right\}\right]$$ solved round brackets

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{2 \times \frac{11}{2} + \frac{1}{30}\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{11 + \frac{1}{30}\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{\frac{330 + 1}{30}\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{\frac{331}{30}\right\}\right]$$

$$= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \frac{331}{30}\right]$$ solved curly brackets

$$= 2\frac{1}{3} \div \left[\frac{4}{3} - \frac{11}{5} - \frac{331}{30}\right]$$

$$= 2\frac{1}{3} \div \left[\frac{40 - 66 - 331}{30}\right] = 2\frac{1}{3} \div \left[\frac{-357}{30}\right]$$

Challenge

Use each of the digits 1, 2, 3, 4, 5, 6, ..., 9. Place appropriate operation symbol and brackets to drive the given answers.

$4 \times 2 + (1 \times 3 \times 5)$	23
	14
	16
	2
	39

Teachers' Guide

Quick mental test by writing some question on mixed operation e.g., $22 \div 11 \times 10$, $50 + 60 \times 5$, $50 \div 5 + 4 \times 3$ etc., on the writing board to clear the concept of BODMAS.

$= 2\frac{1}{3} \times \frac{-30}{357}$ solved square brackets

$= \frac{\cancel{7}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{-30}^{10}}{\cancel{357}_{51}}$

$= \frac{-10}{51}$

Find Answer

$12 \times 14 \div 2 = \boxed{?}$

EXERCISE 1.6

1. Simplify the following:

(i) $70 + [10 + 20 - 2\{8 - 2 + 3\}]$

(ii) $25 - [5 + \{28 - (16 \div 4 + 12)\}]$

(iii) $12 \times 8 - [64 - \{18 \div (9 - 6 - 3)\}]$

(iv) $9 \times 3 - [28 - \{12 \div (10 - \overline{6 - 2})\}]$

(v) $8 \times 9 - [32 - \{24 \div (8 - \overline{4 - 2})\}]$

(vi) $2\frac{3}{12} \div \left[1\frac{4}{5} \times \left\{1\frac{1}{3} + \left(2\frac{1}{2} + \overline{1\frac{1}{3} - 2\frac{1}{6}}\right)\right\} \text{of } 1\frac{2}{3}\right]$

(vii) $1\frac{1}{2}\left[\frac{7}{6} + \left\{\frac{245}{2} - \left(\frac{4}{3} + 121 \div \frac{11}{8}\right)\right\}\right]$

(viii) $\left[3\frac{2}{3} \times \left\{2\frac{1}{4} \div \left(1\frac{1}{8} + \overline{2\frac{1}{4} - 1\frac{1}{2}}\right)\right\}\right] - 2\frac{1}{3}$

(ix) $1\frac{3}{5} \div \left[\frac{1}{25} \times \left\{1\frac{1}{4} + \left(3\frac{1}{3} \div 2\frac{1}{2} \text{ of } 1\frac{5}{16}\right)\right\}\right] \times \frac{1}{3}$

(x) $\left[3\frac{2}{3} \div \left\{1\frac{1}{3} + \left(1\frac{2}{3} + \overline{3\frac{2}{5} - 2\frac{1}{5}}\right)\right\} \times 2\frac{3}{5}\right]$

(xi) $[3 + \{1.25 \times 3.85 \div (5.64 - \overline{2.7 + 1.4})\}]$

(xii) $[2.35 + \{12.099 \div (1.45 - 2.1 \times 1.23)\}]$

(xiii) $15.165 \div [2.125 + \{3.04 - (2.75 \times \overline{2.06 - 2.02})\}]$

(xiv) $0.8 \times [4.9 \times \{0.555 \div (0.2 + 0.02 + 0.002)\}]$

(xv) $5.2383 \div [1.026 + \{1.123 \times (9.261 \div 2.345 + 5.432)\}] \times 2.03$

SUMMARY

- The procedure to simplify a mathematical expression is known as simplification.
- When expression has subtraction and addition, it can be solved from left to right.
- When expression has multiplication and division, then it can be solved from left to right.
- BODMAS rule:

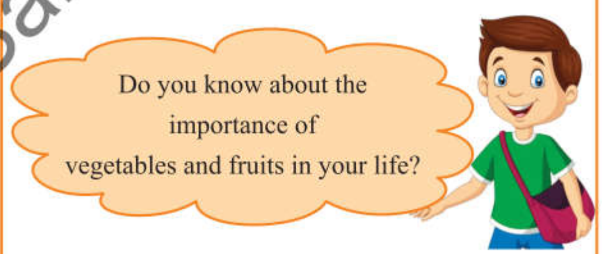
B	O	D	M	A	S
Brackets	Orders	Division	Multiplication	Addition	Subtraction
$[\{(\overline{\ \ })\}]$	$\sqrt{x}$, x^2	$\div$	$\times$	$+$	$-$

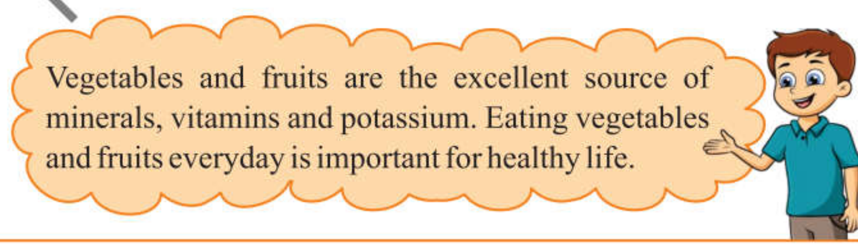
Sub-domain (iii) Sets

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Use language, notation, and Venn Diagrams to represent different sets and their elements. (natural numbers, whole numbers, integers, even numbers, odd numbers, prime numbers)
- Identify and differentiate between:
  - i. subset and superset
  - ii. proper and improper
  - iii. equal and equivalent
  - iv. disjoint and overlapping.
- Describe and perform operations on sets (union, intersection, difference and complement).
- Verify the following:
  - $A \cap A^c = \phi$
  - $A \cup A^c = U$
  - $(A \cup B)^c = A^c \cap B^c$
  - $(A \cap B)^c = A^c \cup B^c$





Introduction

In this world, there are many things or objects which exist in collections or groups form. We use many words in our daily life to explain the collections or groups of things or objects e.g., a hive of bees, bunch of flowers, collection of different kinds of coins, a team of cricket players, group of animals etc. But, in Mathematics, for collection of objects or things, we use the word "Set".

Set is a fundamental mathematical concept that allows us to group objects. It is an important part of the study of mathematics and computer science as well as other fields like probability and statistics. Let's understand the concept of a set and its types.

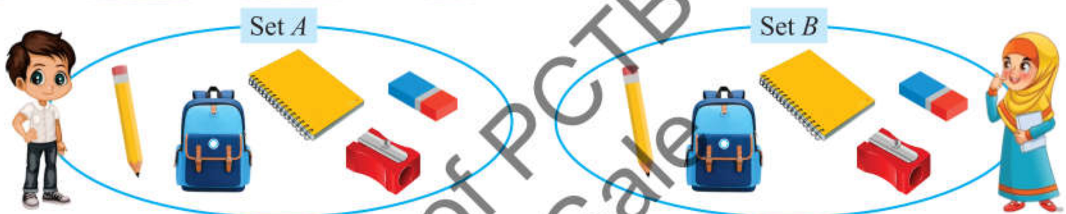
History

George Cantor (1845 – 1918) was a German Mathematician. He played an important role in the creation of the set theory. He is known as the founder of the set theory.

Activity

Visit the nearest market with your parents and find out the different sets of objects/things and note down on your notebooks.

Zain went to the market to buy some items/objects for his 2 children for going to the school.



In set A and set B, all things are well defined and distinct.

Remember!

A set is a collection of well defined and distinct objects.

Well defined: An object is clearly described and there is nothing to describe it more.

Distinct: Each object of the set must be unique.

Keep in mind!

- The objects of a set are called elements or members of the set
- Object may be numbers, names, symbols, geometrical shapes etc.

1.3.1 Notation of a Set

- Sets are represented by capital English alphabet A, B, C etc. The elements of a set are written in curly brackets "{ }" and separated by commas (,) .

For example,

Curly bracket or braces.

It is a set and represented by "A".

$$A = \{a, b, c, d\}$$

The elements a, b, c and d are called the elements of the set.

$a \in A$
$b \in A$
$c \in A$
$d \in A$
but
$\{a\} \notin A$

Symbol

$\in$ belongs to, $\notin$ does not belong to

… so on or continue on

Teachers' Guide

Clear the concept of set by using the set of real life objects. Also tell them the difference between the well defined and distinct by using real life objects.

Example 1 $A = \{1, 2, 3, 1\}$ is a set or not?

Solution The elements of the set A are not distinct, because "1" comes twice.

So, A is not a set.

Brain Teaser!

Tick (✓) which are sets and cross (×) which are not sets.

$B = \{a, b, c, d, d\}$, ☐ $C = \{\triangle, \bigcirc, \square, ▯\}$ ☐

D = {Red, Blue, Green, Blue} ☐

Skill Practice

If $A = \{2, 7, 8\}$, put $\in$ or $\notin$ in the given boxes.

(i) 2 ☐ A (ii) 6 ☐ A (iii) 8 ☐ A

Activity

In the classroom, find out the different sets of objects or things and note down on your notebooks.

1.3.2 Some Families of Sets

- Set of natural numbers = N = {1,2,3,…}
- Set of whole numbers = W = {0,1,2,3,…}
- Set of integers = Z = {0, ±1, ±2, ±3,…}
- Set of odd numbers = O = {±1, ±3, ±5,…}
- Set of even numbers = E = {0, ±2, ±4,…}
- Set of prime numbers = P = {2, 3, 5, 7, 11, …}

Brain Teaser!

The collection of five months is a set or not? If no then explain.

Remember!

- An empty set has no element and it is represented by { } or ϕ. It is also called null set.
- If a set has only one element, then it is called singleton set.

1.3.3 Expressing of a Set

A set can be expressed in three ways.

(a) Tabular form or Roster form (b) Descriptive form (c) Set builder notation

(a) Tabular Form

When all the elements of a set are written within the curly bracket "{ }" and elements are separated by using commas (,), then it is called tabular form. For example,

$A = \{0, 1, 3, 5, 7\}$; $B = \{a, b, c, d, e, f\}$

$C = \{10, 5, 15, 25, 20\}$; D = {Raza, Hamza, Ayesha}

Keep in mind!

Order of the elements in a set does not matter. e.g.,

$A = \{a, c, d, f, k\}$ or $A = \{f, d, a, c, k\}$

Both are same.

Skill Practice

Write the descriptive form of sets in the tabular form.

(b) Descriptive Form

When all the elements of a set are described in words form, then it is called descriptive form. For example, A = Set of multiples of 5 B = Set of vowels in English alphabet

C = Set of days of a week D = Set of the first five solar months

(c) Set Builder Notation

When all the elements of a set are expressed by using mathematical notations, stating all the properties of elements given in a set then it is called set builder notation. For example, $A = \{0, \pm2, \pm4, \pm6, \pm8, ...\}$ is expressed in set builder notation as: $A = \{x \mid x \in E\}$.

$B = \{0, 1, 2, 3, 4, 5, ..., 15\}$ is expressed in set builder notation as: $B = \{x \mid x \in W \wedge x \leq 15\}$

Remember!

- The symbol "|" is read as "such that"
- The symbol "$\wedge$" is read as "and"
- The symbol "$\vee$" is read as "or"
- The symbol "$\in$" is read as "belongs to"

Skill Practice

Write the given sets in other form (Descriptive form).

EXERCISE 1.7

1. Write the following sets in tabular form:
   - (i) A = Set of the first five odd numbers.
   - (ii) B = Set of the days of a week.
   - (iii) C = Set of the last five English alphabet.
   - (iv) D = Set of the even numbers greater than 9 and less than 25.
   - (v) $E = \{x \mid x \in Z \wedge -9 \leq x \leq 12\}$
   - (vi) $F = \{x \mid x \in O\}$
   - (vii) $G = \{x \mid x \in N \wedge x < 15\}$
   - (viii) Set of the first seven prime numbers

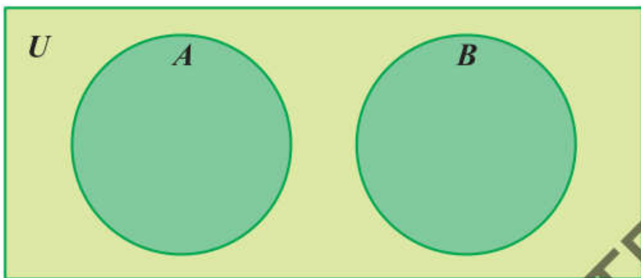
2. Write the following sets in descriptive form:
   - (i) $A = \{3, 6, 9, 12, 15, 18, 21\}$
   - (ii) $B = \{a, b, c, d, e, f, g, h, i, j, k, \ell, m\}$
   - (iii) $C = \{y \mid y \in E\}$
   - (iv) $D = \{\pm4, \pm5, \pm6, ..., \pm20\}$
   - (v) $E = \{x \mid x \in N \wedge x \leq 50\}$
   - (vi) $F = \{0, 1, 2, 3, 4, 5, ...\}$
   - (vii) $G = \{13, 17, 19, 23, 29, 31, 37\}$
   - (viii) $H = \{25, 30, 35, 40, 45, 50, 55\}$

3. If $A = \{1, 2, 3, 4, 5, 6\}$, then fill in the blanks by using symbols $\in$ or $\notin$.
   - (i) 2 ____ A
   - (ii) 7 ____ A
   - (iii) 4 ____ A
   - (iv) 6 ____ A
   - (v) 3 ____ A
   - (vi) -5 ____ A
   - (vii) 10 ____ A
   - (viii) 5 ____ A

4. Write the following sets in set builder notation:
   - (i) A = Set of natural numbers less than 50 and divisible by 4.
   - (ii) B = Set of integers between -5 and 5.
   - (iii) $C = \{1, 3, 5, 7, 9, 11, 13, 15\}$
   - (iv) $D = \{10, 11, 12, 13, 14, ..., 25\}$
   - (v) E = Set of the last five solar months.
   - (vi) $F = \{-20, -19, -18, -17, ..., 0, 1, 2, 3, 4, ..., 20\}$
   - (vii) G = Set of odd numbers between 24 and 38.
   - (viii) $H = \{10, 20, 30, 40, 50, ..., 100\}$

1.3.4 Venn Diagram

The Venn diagram is a pictorial way of representing sets and their relationship. Circles, ovals and rectangles are used to make a Venn diagram. Rectangle is used to show the universal set and circles or ovals are used to show the sets under consideration, inside the rectangle. Venn diagram is often used in mathematics, but it can also be used in every day life to represent relationships between things.



Venn diagram

> **History**
>
> John Venn (1834 - 1923) was the inventor of Venn diagram. In 1880, the Venn diagram was introduced by John Venn.

> **Important Information**
>
> **Universal Set:** A set is called universal set which has all the elements of the sets under consideration. It is denoted by U.
> For example,
> If $A = \{2, 4, 6, 8\}$
> $B = \{1, 2, 3, 5, 7, 8\}$, then universal set (U) is
> $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

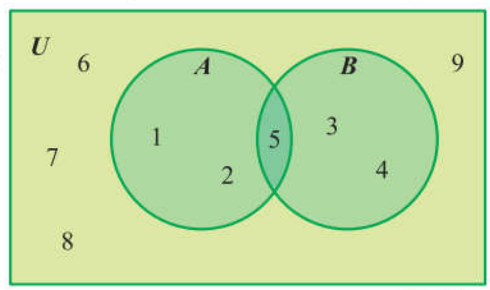
Example 2 Draw a Venn diagram to represent the following sets: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 5\}$ and $B = \{3, 4, 5\}$.

Solution

Step 1 Draw a rectangle and label it U to represent the universal set.

Step 2 Draw circles within the rectangle to represent the other sets. Label the circles and write the relevant elements in each circle.

Step 3 Write the remaining elements outside the circles but within the rectangle.



Search out https://www.transum.org/software/sw/starter ofttheday/students/venn diagram.asp

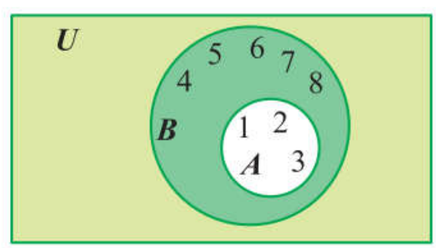
> **Important Information**
>
> A set is called an empty set which has no element. It is written as:
> { } or ϕ

1.3.5 Types of Sets

Sets are classified by how many elements they have and how they relate to other sets.

Subset If all the elements of a set A are also the elements of a set B, then the set A is called the subset of the set B and we write it as $A \subseteq B$. The symbol '$\subseteq$' stands for 'is a subset of'. For example,

$A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$



Here, we can see that all the elements of the set A are elements of the set B. Therefore, $A \subseteq B$

We can find many examples of subsets in everyday life such as:

1. If we consider school items in a bag as one set, then the set of books is a subset.
2. If all the classes from 1 to 8 in a school form a set, then the sets of class 1 and class 2 are its subsets.

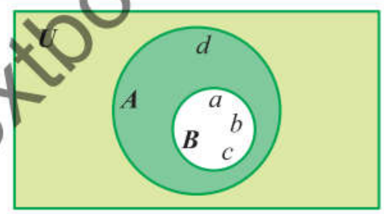

Super Set If a set A contains all the elements of a set B, then the set A is a superset of a set B and we write it as $A \supseteq B$. For example, $A = \{a, b, c, d\}$ and $B = \{a, b, c\}$

Here, we can see that the set A contains all the elements of the set B. Therefore, $A \supseteq B$ or A is a super set of set B.

There are two types of subset, namely, proper subset and improper subset.

Remember!

i. Every set is a subset of itself.
ii. Empty set is a subset of every set.



Search out

Use the following online video link to explain types of sets: https://www.youtube.com/watch?v=8dup8yGwBhM

Proper Subset A set A is a proper subset of a set B if all the elements of the set A contain in the set B but at least one element of the set B is not an element of set A. It is denoted by $A \subset B$ and read as set A is a proper subset of set B. For example, if $A = \{b, c, d\}$ and $B = \{a, b, c, d, e\}$

Here, we can see that all the elements of the set A contain in the set B but set A does not contain all elements of the set B. So, the set A is a proper subset of the set B or $A \subset B$.

Improper Subset If set A is a subset of set B and set B is a subset of set A, then A and B are improper subsets of each other. It is denoted by $A \subseteq B$ and read as set A is an improper subset of the set B. For example, if $A = \{0, 2, 4, 6\}$ and $B = \{4, 6, 2, 0\}$, Here, we can see that A and B are subsets of each other because all the elements of the set A are contained in set B and all the elements of the set B are contained in set A. Therefore, A and B are improper subsets of each other.

Example 3 Write three subsets of the set $\{0,1,2,3\}$.

Solution Three subsets of $\{0, 1, 2, 3\}$ are ϕ, $\{1, 2\}$, $\{0, 1, 3\}$.

Example 4 If $X = \{a, e, i, o, u\}$, then write any four proper subsets and an improper subset.

Solution Four proper subsets of $\{a, e, i, o, u\}$ are ϕ, $\{a\}$, $\{b\}$ and $\{a, e, i, u\}$

Improper subset of $\{a, e, i, o, u\}$ is $\{a, e, i, o, u\}$.

Note

All the subsets of any set except the set itself are proper subsets. Empty set is a subset of every set. Empty set has no proper subset.

Teachers' Guide

Clear the concept between proper subset and improper subset, equivalent and equal sets, disjoint and overlapping sets by using real life objects.

Try yourself

Write the set which has no proper subset? Which set has only one subset? What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

Equivalent Sets Two sets A and B are said to be equivalent if they have the same number of elements. We write it as $A \leftrightarrow B$. For example, $A = \{a, e, i, o, u\}$ and $B = \{1, 3, 5, 7, 9\}$ We can see that number of elements in set A and set B is 5. So, the sets A and B are equivalent sets or $A \leftrightarrow B$.

Brain Teaser!

Tick (✓) which are equal sets and cross (×) which are equivalent sets.

(i) $A = \{\triangle, \bigcirc, \square\}$; $B = \{1, 4, 5\}$ ☐

(ii) A = {Amna, Ali} ; B = {Ali, Amna} ☐

Search out

https://www.transum.org/software/sw/starteroftheday/students/venn diagramMatching.asp

Skill Practice

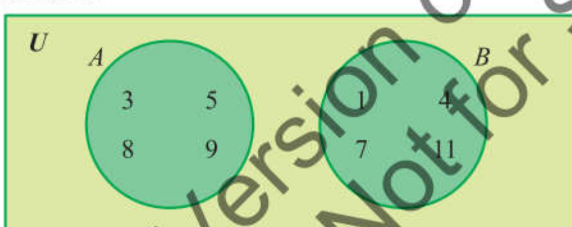
Make at least two pairs of equivalent sets using real life objects.

Equal Sets If sets A and B contain the same elements, then the sets A and B are equal sets. We write it as $A = B$. e.g., $A = \{2, 3, 5, 7, 9\}$ and $B = \{5, 2, 7, 3, 9\}$. The sets A and B have the same elements. So, the sets A and B are equal sets or $A = B$. Equal sets are always equivalent sets. Equivalent sets may or may not be equal.

Disjoint sets Two or more sets are disjoint, if they do not have common elements. For example, $A = \{3, 5, 8, 9\}$ and $B = \{1, 4, 7, 11\}$ are disjoint sets because they have no common elements.

Keep in mind!

In a set, order of the elements does not matter. It can be changed e.g., $\{a, e, i, o, u\} = \{o, e, i, a, u\}$

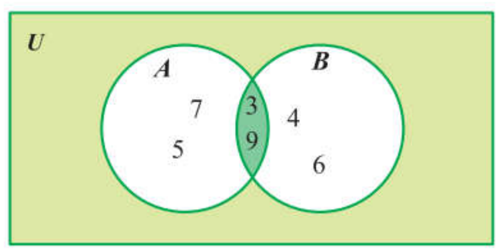


Skill Practice

Think any two disjoint sets and write on your notebook.

Overlapping sets Two or more sets are overlapping sets, if they have at least one common element. For example,

$A = \{3, 5, 7, 9\}$ and $B = \{3, 4, 6, 9\}$ are overlapping sets because they have 3 and 9 common elements.



EXERCISE 1.8

1. Write two proper and improper subsets of each of the following sets:

(i)	$A = \{a, c, d\}$	(ii)	$B = \{2, 4, 6. 8\}$	(iii)	$C = \{-1, 0, +1\}$
(iv)	$D = \{4, 8, 12, 16\}$	(v)	$E = \{\triangle, \bigcirc, \square\}$	(vi)	F = {Zara, Aslam, Zeeshan}

2. Which is the super set in each of the following pair of sets:

(i) $A = \{a, c, d\}$, $B = \{a, b, c, d\}$ (ii) $C = \{0, 2, 4, 6, 8\}$, $D = \{2, 4\}$

(iii) $N = \{1, 2, 3, \ldots\}$, $W = \{0, 1, 2, \ldots\}$ (iv) $E = \{1, 2, 3, \ldots, 10\}$, $F = \{0, \pm1, \pm2, \ldots, \pm10\}$

(v) $L = \{u, v, w, x\}$, $M = \{u, v, w, x, y, z\}$

3. Look at each pair of sets and separate the equal and equivalent sets.

(i) $A = \{1, 3, 5, 7, 9\}$, $B = \{a, b, c, d, e\}$ (ii) $C = \{0, 2, 4, 6, 8\}$, $D = \{0, 2, 4, 6, 8\}$

(iii) $E = \{1, 2, 6, 7, 8\}$, $F = \{6, 7, 8, 1, 2\}$ (iv) $G = \{1, 2, 3, \ldots, 20\}$, $H = \{1, 2, 3, \ldots, 20\}$

(v) $I = \{\text{Sunday, Monday}\}$, $J = \{\text{Waqar, Ali}\}$

4. Look at each pair of sets and separate the disjoint and overlapping sets.

(i) $A = \{7, 9, 8, 9\}$, $B = \{a, b, c, d, e\}$ (ii) $C = \{0, 5, 10, 15\}$, $D = \{0, 5, 8, 9, 15\}$

(iii) $E = \{1, 2, 3, 4, 5, 6\}$, $F = \{1, 3, 2, 5, 6\}$ (iv) $P = \{2, 3, 5, 7, 11\}$, $Q = \{4, 6, 8, 9, 10, 12\}$

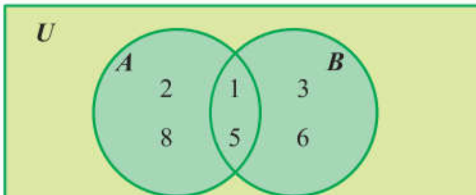
(v) $G = \{x, y, z\}$, $H = \{u, v, w, x, y, z\}$ (vi) $I = \{3, 6, 12\}$, $J = \{\text{January, February, April}\}$

1.3.6 Operations on Sets

Set operations are the ways to combine sets. They operate on the elements of each set, combining them according to a specific set of rules. The operations are union ($\cup$), intersection ($\cap$) and difference ($-$ or $\backslash$).

i Union of Sets

The union of the sets A and B is the set that contains all elements that are in both sets. The union of two sets is written as $A \cup B$ and read as 'A union B'.



$A \cup B$ (Shaded part)

Example 5 If $A = \{1, 2, 5, 8\}$ and $B = \{1, 3, 5, 6\}$, then find the union of sets A and B.

Solution $A \cup B = \{1, 2, 5, 8\} \cup \{1, 3, 5, 6\}$

$= \{1, 2, 3, 5, 6, 8\}$

Note

In the union set, all common elements of two sets are written once.

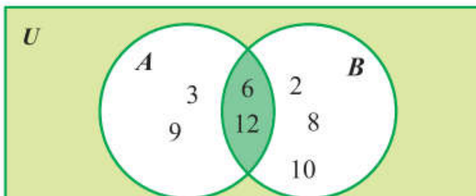
ii Intersections of Sets

The intersection of the sets A and B is the set that contains all common elements of the sets A and B. The intersection of two sets is written as $A \cap B$ and is read as 'A intersection B'.

Example 6 If $A = \{3, 6, 9, 12\}$ and $B = \{2, 6, 8, 10, 12\}$, then find the intersection of sets A and B.

Solution $A \cap B = \{3, 6, 9, 12\} \cap \{2, 6, 8, 10, 12\}$

$= \{6, 12\}$



$A \cap B$ (Shaded part)

iii Difference of Sets

The difference between the sets A and B is the set that contains all elements of the set A that are not in the set B. It is written as $A - B$ or $A \backslash B$.

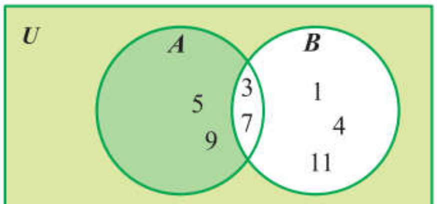
Skill Practice

If A = {Monday, Tuesday, Friday} and V = {Tuesday} then find $A \cap B$. Also draw venn diagram.

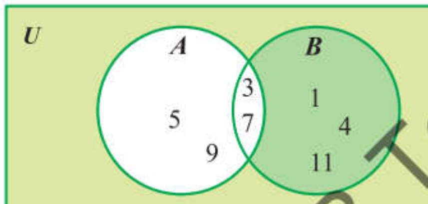
Example 7 If $A = \{3, 5, 7, 9\}$ and $B = \{1, 3, 4, 7, 11\}$, then find $A - B$ and $B - A$.

Solution $A - B = \{3, 5, 7, 9\} - \{1, 3, 4, 7, 11\} = \{5, 9\}$.

$B - A = \{1, 3, 4, 7, 11\} - \{3, 5, 7, 9\} = \{1, 4, 11\}$.



$A - B$ (Shaded part)



$B - A$ (Shaded part)

Try yourself

Write two sets whose difference is empty set.

Skill Practice

Prove that: $A - B \neq B - A$, If

$A = \{2, 4, 6, 8, 9\}$ and

$B = \{2, 8, 11, 12\}$

Activity

Teacher explain the concept of representing sets using the Venn diagram by this activity. Teacher make 3 sets of hobbies; sports, reading and computer games. Set A is for pupils who like sports, set B is the pupils who like reading and set C is for pupils who like computer games. Teacher write the names of students in each set and construct the Venn diagram on writing board.

Keep in mind!

- If the intersection of sets A and B is empty, then the sets A and B are disjoint sets.
- If the intersection of sets A and B is non-empty, then the sets A and B are overlapping sets.

iv Universal Set

A universal set is a set that contains all elements of sets under consideration and it is denoted by U. For example, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set of the sets $A = \{1, 3, 5, 7\}$ and $A = \{2, 4, 6, 8\}$.

v Complement of a Set

The complement of a set A is defined as a set that contains the elements present in the universal set but not in set A. It is denoted by A' or A^C.

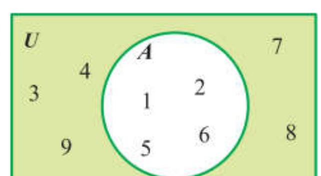
Example 8 If U = $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 5, 6\}$, then find A^C.

Solution $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 5, 6\}$

$A^C = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 5, 6\}$

$= \{3, 4, 7, 8, 9\}$



A' or A^C (Shaded part)

Skill Practice

If $U = \{a, b, c, d, e, f\}$ and $A = \{d, e\}$, then find the complement of the set A.

EXERCISE 1.9

1. Find the union of the following sets:

   (i) $A = \{1, 4, 7, 10, 13\}$, $B = \{2, 3, 5, 8, 9\}$ (ii) $C = \{0, 2, 4, 6, 8, 10\}$, $D = \{0, 1, 2, 3, 4, 6, 8\}$

   (iii) $E = \{a, b, c, d, e\}$, $F = \{a, e, i, o, u\}$ (iv) $G = \{1, 2, 3, \ldots, 10\}$, $H = \{0, 1, 2, 3, 4, 6, 8, 9, 10\}$

2 Find the intersection of the following sets:

   (i) $S = \{s, t, u, d, e, n, t\}$, $T = \{t, e, a, c, h, r\}$ (ii) $U = \{0, -1, -2, -3\}$, $V = \{0, 1, 2, 3\}$

   (iii) $W = \{2, 3, 5, 7, 11, 13\}$, $X = \{0, 2, 4, 6, 8, 9, 10\}$ (iv) $Y = \{0, 5, 10, 15, 20\}$, $Z = \{0, 1, 2, \ldots\}$

3. If $N = \{1, 2, 3, \ldots\}$ and $W = \{0, 1, 2, \ldots\}$, then find $N \cup W$ and $N \cap W$.

4. If $E = \{0, 2, 4, \ldots\}$ and $O = \{1, 3, 5, \ldots\}$, then find $E \cup O$ and $E \cap O$.

5. If P = set of prime numbers and C = set of composite numbers, then find $P \cup C$ and $P \cap C$.

6. If $U = \{1, 2, 3, \ldots, 10\}$, $X = \{0, 3, 6, 9\}$, $Y = \{0, 4, 8\}$ and $Z = \{0, 2, 4, 6, 8, 10\}$, then find:

   (i) X^C (ii) Y^C (iii) Z^C (iv) U^C

7. If $X = \{0, 2, 6, 9, 10\}$, $Y = \{1, 2, 3, 4, 5\}$ and $Z = \{1, 3, 5, 7\}$, then find:

   (i) $X \setminus Y$ (ii) $Y \setminus Z$ (iii) $X \setminus Z$ (iv) $Z \setminus Y$

1.3.7 Properties of the Complement of Sets

Properties involving the complement of sets are given below:

(i) $A \cup A^C = U$ (ii) $A \cap A^C = \phi$ (iii) $(A \cup B)^C = A^C \cap B^C$ (iv) $(A \cap B)^C = A^C \cup B^C$

Example 9 If $U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 2, 3, 6, 7, 8, 9\}$, then verify the following properties:

(i) $A \cup A^C = U$ (ii) $A \cap A^C = \phi$ (iii) $(A \cup B)^C = A^C \cap B^C$ (iv) $(A \cap B)^C = A^C \cup B^C$

Solution $U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 2, 3, 6, 7, 8, 9\}$

First, we find A^C and B^C

$A^C = U - A$

$A^C = \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9\}$

$B^C = U - B$

$B^C = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 6, 7, 8\} = \{4, 5, 9, 10\}$

Skill Practice

If U = Set of natural numbers and A = Set of even numbers then prove that: $A \cup A^C = U$, $A \cap A^C = \phi$

Activity

Explain to students about the complement of sets. Let us say A is a set of all coins which is a subset of a universal set that contains all coins and notes. So, the complement of set A is a set of notes, which does not include coins.

Teachers' Guide

Ask the students to choose a random universal set and two subsets and find intersection and complement.

(i) $A \cup A^c = U$

L.H.S $= A \cup A^c$

$= \{2, 4, 6, 8, 10\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{1, 2, 3, \ldots, 10\} = U =$ R.H.S

Hence, L.H.S = R.H.S

(ii) $A \cap A^c = \phi$

L.H.S $= A \cap A^c$

$= \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$= \{\ \}$ or $\phi =$ R.H.S

Hence, L.H.S = R.H.S

(iii) $(A \cup B)^c = A^c \cap B^c$

L.H.S $= (A \cup B)^c$

$= (\{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 6, 7, 8\})^c$

$= (\{1, 2, 3, 4, 6, 7, 8, 10\})^c$

$= U - \{1, 2, 3, 4, 6, 7, 8, 10\}$

$= \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 4, 6, 7, 8, 10\} = \{5, 9\}$

R.H.S $= A^c \cap B^c$

$= \{1, 3, 5, 7, 9\} \cap \{4, 5, 9, 10\} = \{5, 9\}$

Hence, L.H.S = R.H.S

(iv) $(A \cap B)^c = A^c \cup B^c$

L.H.S $= (A \cap B)^c$

$= (\{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 6, 7, 8, 9\})^c$

$= \{2, 6, 8\}^c$

$= U - \{2, 6, 8\}$

$= \{1, 2, 3, \ldots, 10\} - \{2, 6, 8\}$

$= \{1, 3, 4, 5, 7, 9, 10\}$

R.H.S $= A^c \cup B^c$

$= \{1, 3, 5, 7, 9\} \cup \{4, 5, 9, 10\}$

$= \{1, 3, 4, 5, 7, 9, 10\}$

Hence, L.H.S = R.H.S

History

A British Mathematician Augustus De Morgan (1806 – 1871) introduced De Morgan's law.
$(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ are called De Morgan's laws.

Skill Practice

If U = Set of the name of months of a year, A = {January, June, July}, B = {March, July, October, November, December}, then verify:

(i) $(A \cup B)^c = A^c \cap B^c$ (ii) $(A \cap B)^c = A^c \cup B^c$

Search out

https://www.w3schools.blog/properties-of-complement-sets

EXERCISE 1.10

1. If $U = \{0, 1, 2, 3, 4, 5, ..., 25\}$, A = {1, 3, 5, 7, 9, 21, 24}, B = {5, 10, 15, 20, 25}, C = {2, 4, 6, 8, 14, 16, 18, 24} and D = {15, 16, 18, 21, 24, 25} then verify the following:

   (i) $A \cup A^c = U$ (ii) $A \cap A^c = \phi$ (iii) $B \cup B^c = U$ (iv) $B \cap B^c = \phi$

   (v) $C \cup C^c = U$ (vi) $C \cap C^c = \phi$ (vii) $D \cup D^c = U$ (viii) $D \cap D^c = \phi$

2. Verify $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ where $U = \{1, 2, 3, 4, 5, ..., 20\}$

   (i) $A = \{1, 7, 9\}$; $B = \{2, 4, 7, 9, 12, 15\}$

   (ii) $A = \{2, 4, 6, 8, 10\}$; $B = \{11, 12, 13, 14, 15, 16\}$

   (iii) $A = \{1, 3, 9, 11, 13, 15\}$; $B = \{11, 13, 15, 17\}$

   (iv) $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$; $B = \{2, 4, 8, 10, 12, 16, 18, 20\}$

SUMMARY

- A set is a well defined and distinct collection of objects/elements.
- If all the elements of a set A are also the elements of a set B, then the set A is called the subset of the set B.
- If a set A contains all the elements of a set B, then the set A is a super set of the set B.
- A set A is a proper subset of a set B if all the elements of the set A contain in the set B but at least one element of the set B is not an element of set A.
- If set A is a subset of set B and set B is a subset of set A, then A and B are improper subsets of each other.
- Two sets A and B are said to be equivalent, if they have the same number of elements.
- If sets A and B contain the same elements, then the sets A and B are equal.
- Two or more sets are disjoint, if they do not have common elements.
- Two or more sets are overlapping sets, if they have at least one common element.
- Laws of complement:

  (i) $A \cup A^C = U$ (ii) $A \cap A^C = \phi$ (iii) $(A \cup B)^C = A^C \cap B^C$ (iv) $(A \cap B)^C = A^C \cup B^C$

REVIEW EXERCISE 1(a)

1. Choose the correct option.

(i) The number that can be expressed in the form of $\frac{p}{q}$, $q \neq 0$ is called:

(a) mixed number (b) rational number (c) whole number (d) natural number

(ii) The set of rational number is denoted by:

(a) W (b) Q (c) N (d) Q'

(iii) $\frac{1}{3}(18)(-6) =$ ________

(a) –36 (b) 36 (c) 18 (d) –18

(iv) $\frac{-(17-17)}{20} =$ ________

(a) $\frac{-34}{20}$ (b) 0 (c) $\frac{-1}{20}$ (d) $\frac{1}{20}$

(v) $\frac{-5}{6} + \frac{3}{7} = \frac{3}{7} + \boxed{?}$

(a) $\frac{1}{2}$ (b) $\frac{6}{7}$ (c) $\frac{5}{6}$ (d) $\frac{-5}{6}$

(vi) The difference between the actual value and estimated value is called:

(a) error (b) approximation (c) rounding (d) reasonableness

(vii) Who introduced BODMAS:

(a) Achilles Reselfelt (b) Al-Khawarzmi (c) John (d) William

(viii) In BODMAS rule, "of" stands for:

(a) addition (b) subtraction (c) division (d) multiplication

(ix) The set is written in the bracket.

(a) ( ) (b) { } (c) [ ] (d) [}

(x) To write an empty set, we use the symbol:

(a) $\subset$ (b) ϕ (c) $\cap$ (d) $\{\phi\}$

(xi) If sets A and B are disjoint sets, then:

(a) $A \cup B = \phi$ (b) $A \setminus B = B$ (c) $A \cap B = A$ (d) $A \cap B = \phi$

(xii) If $A = \{1, 2\}$ and $B = \{4, 5\}$, then:

(a) $A \cup B = \phi$ (b) $A \leftrightarrow B$ (c) $A \cap B = A$ (d) $A = B$

(xiii) In the Venn diagram, the universal set is represented by:

(a) rectangle (b) circle (c) square (d) quadrilateral

2. Represent the following on the number line:

(i) 250, 230, 140, 90, 100, 120 (ii) 50.5, 55.5, 57.6, –55.5, –53.5

(iii) 2200, 2000, 1800, 2500, 2100, 1500 (iv) –40, –15, –30, –25, –65, –70

(v) $\frac{1}{5}$ (vi) $\frac{9}{11}$ (vii) $\frac{-2}{5}$ (viii) $\frac{-7}{13}$

3. Compare the given numbers by using symbol > or <. Also arrange them in ascending and descending order.

(i) 326781, 326681, 336281, 336291 (ii) –55451, –55540, –56580, –56508

(iii) 108.01, 180.08, 111.78, 111.70 (iv) $\frac{7}{10}$, $\frac{-3}{5}$, $\frac{-7}{10}$, $\frac{13}{15}$

4. Convert:

(i) $1\frac{7}{9}$, $3\frac{1}{5}$, $2\frac{5}{12}$, $8\frac{5}{7}$ into improper fractions (ii) $\frac{27}{7}$, $\frac{48}{14}$, $\frac{43}{13}$, $\frac{68}{9}$ into mixed fractions

5. Solve the following rational numbers:

(i) $\frac{5}{13} + \frac{4}{13}$ (ii) $\frac{13}{8} - \frac{9}{16}$ (iii) $\frac{-7}{11} \times \frac{-3}{21}$ (iv) $\frac{-8}{11} \div \frac{-2}{121}$

6. The product of two numbers is $\frac{77}{56}$. If the first number is $\frac{13}{8}$, then find the other number.

7. Ahmed had to travel $\frac{8}{9}$ km from his house to the library and then to school. The distance between his house and the library was $\frac{1}{3}$ km. How much was the distance between the library and the school?

8. Verify the following properties:

(i) $\left(\frac{4}{7} + \frac{2}{9}\right) + \frac{3}{5} = \frac{4}{7} + \left(\frac{2}{9} + \frac{3}{5}\right)$ (ii) $\frac{-1}{5} \times \left(\frac{5}{9} + \frac{7}{18}\right) = \left(\frac{-1}{5} \times \frac{5}{9}\right) + \left(\frac{-1}{5} \times \frac{7}{18}\right)$

9. Round the following to the given degree of accuracy:

(i) 97852 (2 significant figures) (ii) –16.578 (2 significant figures)

(iii) 0.003757 (3 significant figures) (iv) 0.0657953 (3 significant figures)

(v) $\frac{113}{8}$ (2 decimal places) (vi) $\frac{27}{83}$ (3 significant figures)

10. Compare your approximated value and accurate value. Also check is the solution reasonable?

(i) $3.481 \div 1.287$ (2 decimal places) (ii) $10.59 - 7.24$ (2 decimal places)

(iii) 10.52×5.57 (2 decimal places)

11. Simplify the following:

(i) $[12 + \{ 23 \times 75 \div (16 - \overline{22 + 11})\}]$ (ii) $\frac{3}{5} \div \left[\frac{1}{15} \times \left\{ 1\frac{1}{2} + \left(3\frac{1}{3} \div 2\frac{1}{2} \times 3\frac{7}{16} \right) \right\} \right] \times \frac{1}{3}$

(iii) $18.3 \div [2.430 + \{3.245 - (2.7 \times \overline{7.06 - 7.02})\}]$

12. State whether each of the following collections is a well defined set. Give a reason for each answer.

(i) The collection of good students in your class. (ii) The objects that can be worn.

(iii) Prime numbers between 0 and 10. (iv) The students in your class have one brother.

13. Which of the following are set? Justify your answer.

(i) The collection of all the months in a year starting with the letter "A".

(ii) The collection of good stories books.

(iii) $A = \{s, c, h, o, o, l\}$. (iv) $B = \{b, a, g\}$ (v) $C = \{1, 2, 3, 4, 5\}$

14. Write the following sets in other two forms:

(i) $A = \{4, 8, 12, 16, 20, 24, 28, 32\}$ (ii) $B = \{x \mid x \in N \wedge 10 \leq x \leq 20\}$

(iii) $C = \{0, \pm 1, \pm 2, \pm 3, ...\}$ (iv) D = Set of all perfect square numbers between 5 and 100.

(v) $F = \{y \mid y \in N \wedge y \leq 15\}$ (vi) $G = \{x \mid x \in W \wedge x < 10\}$

15. If $U = \{0, \pm 1, \pm 2, \ldots\}$, then represent the following sets in Venn diagram:

(i) $X = \{-1, -2, -3, -4\}$, $Y = \{1, 2, 3, 4, 5\}$ (ii) $A = \{-1, 2, -3, 4\}$, $B = \{1, -2, -3, 4, 5\}$

16. Find the union and intersection of the following sets:

(i) $U = \{-1, -3, -5, -7, -9\}$, $V = \{-1, -2, -3, \ldots, -10\}$ (ii) $X = \{-1, -2, -3, -4\}$, $Y = \{1, 2, 3, 4, 5\}$

17. If $Y = \{a, b, c, \ldots, z\}$, $Z = \{a, e, i, o, u\}$, then find: (i) $Y - Z$ (ii) $Z \setminus Y$

18. If $U = \{21, 22, 23, 24, ..., 40\}$, $A = \{21, 23, 24, 26, 27\}$ and $B = \{25, 26, 27, 28, 29, 30, 35, 36\}$ then verify the following:

(i) $A \cup A^C = U$ (ii) $A \cap A^C = \phi$ (iii) $B \cup B^C = U$ (iv) $B \cap B^C = \phi$

19. Verify $(A \cup B)^C - (A^C \cap B^C) = \phi$ and $(A \cap B)^C - (A^C \cup B^C) = \phi$ where $U = \{0, 1, 2, 3, 4, 5, .., 18\}$

(i) $A = \{2, 4, 6, 7, 12, 14, 15, 17\}$; $B = \{3, 9, 13, 16, 18\}$

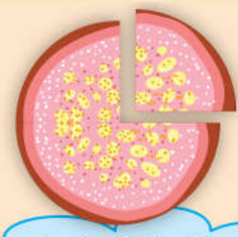
(ii) $A = \{1, 2, 3, 4, 5, 10, 12, 14\}$; $B = \{5, 10, 15\}$

Sub-domain (iv) Rate, Ratio and Proportion

Students' Learning Outcomes

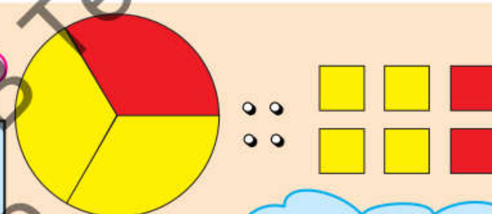
After studying this sub-domain, students will be able to:

- Calculate rate and average rate of quantities.
- Calculate increase and decrease in a ratio based on change in quantities.
- Explain and calculate direct and inverse proportion and solve real-world word problems related to direct and inverse proportion.



1 : 4 (Ratio Form)
$\frac{1}{4}$ (Fraction Form)
1 is to 4 (Word Form)

THINK!
Are the ratio of coloured part of circle and the coloured parts of square are in proportion?

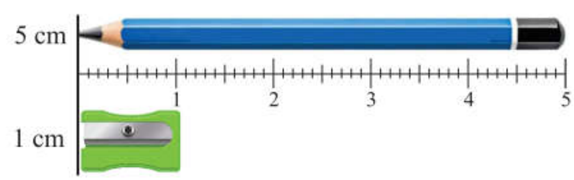


$\frac{1}{3} :: \frac{2}{6}$

Here, we can compare the first ratio to the second ratio.

1.4.1 Ratio

In our daily life, there are many situations in which we compare two quantities. For example, The length of pencil is 5 times of the length of sharpener. In other words, we can also say that the length of sharpener is $\frac{1}{5}^{\text{th}}$ of the length of pencil.



The length of pencil = 5 (Length of sharpener) OR The length of sharpener = $\frac{1}{5}$ (Length of pencil)
We can also compare the above objects by using ratio.
In the given case, we write the ratio of the length of sharpener to the length of pencil as:

Length of sharpener	:	Length of pencil
1	:	5

Or the ratio of the length of pencil to the length of sharpener is as:

Length of pencil	:	Length of sharpener
5	:	1

Remember!
Ratio is changed when we change the order of quantities e.g., 1 : 5 and 5 : 1 are different from each other.

The first term in the ratio is known as antecedent and the second term is known as consequent. e.g., In 5 : 1 (5 = antecedent and 1 = consequent). To compare two quantities in the ratio, the units of the quantities must be same. A ratio has no units. Ratio is a dimension less number or quantity.

Skill Practice

Find ratio of:
- 5 km to 950 m
- 2 kg to 800 g
- 7 weeks to 21 days

Keep in mind!

When the ratio is multiplied or divided by the same (non zero) number, then the ratio is not changed.

Example 1 The price of a mobile is Rs. 21000 and the price of a LED is Rs. 42000. Find the ratio of the price of mobile to LED and LED to price of mobile.

Solution

The ratio of the price of mobile to the price of LED is written as:

$$21000 : 42000$$

After simplifying to the lowest form:

$$3 : 6 \qquad 1 : 2$$

The ratio of the price of LED to the price of mobile is written as:

$$42000 : 21000$$

$$6 : 3 \qquad 2 : 1$$

Remember!

In general, the ratio of a to b is written as:

$a : b$ or $\frac{a}{b}$; $(b \neq 0)$

Skill Practice

Can you tell the price of LED how many times greater than the price of mobile?

Example 2 If $a : b = 3 : 5$ and $b : c = 6 : 7$, then find $a : c$

Solution

$a : b = 3 : 5$

$\frac{a}{b} = \frac{3}{5}$

$b : c = 6 : 7$

$\frac{b}{c} = \frac{6}{7}$

Now, $\frac{a}{c} = \frac{a}{b} \times \frac{b}{c}$

$= \frac{3}{5} \times \frac{6}{7} = \frac{18}{35}$

$a : c = 18 : 35$

Challenge

Find ratio of $\frac{1}{5}$ to $\frac{3}{15}$

Skill Practice

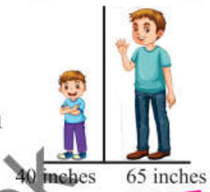
If $x : y = 10 : 8$ and $y : z = 5 : 9$, then find $x : z$.

i Increase and Decrease in Ratio

Sara went to the market. She observed that the price of 1 kg rice was Rs. 120 but now the price of 1 kg rice is Rs. 180 of the same kind while purchasing.

Can you tell how much price is increased in ratio? In such situation, increase and decrease in ratio play significant role to solve this problem.

(a) Increase in Ratio



Example 3 If the height of Umer is increased from 40 inches to 65 inches in 13 years, then find the ratio of increased height.

Solution The required ratio = New height : Old height

= 65 : 40

= 13 : 8

Thus, the height of Umer increased in the ratio of 13 : 8.

Skill Practice

If old price of an object is Rs. 800 and new price is Rs. 1100. Can you find out the ratio of increased price?

Example 4 Increase 1200 in the ratio of 8 : 5

Solution First of all convert the ratio into a fraction, then multiply it by 1200.

$$= \frac{8}{\cancel{5}_{1}} \times \cancel{1200}^{240}$$

$$= 1920$$

Hence, we can say that increase of 1200 in the ratio of 8 : 5 is 1920.

Keep in mind!

The first term of the ratio (antecedent) is always greater than consequent in increase in ratio.

(b) Decrease in Ratio

Example 5 If the price of wheat is decreased from Rs. 180 to 150, then find the ratio of decreased price.

Solution The required ratio can be calculated by this method.

The required ratio = New value : Old value

= 150 : 180

= 5 : 6

Thus, price of wheat decrease in the ratio of 5 : 6.

Skill Practice

- The length of the ribbon was originally 30 cm. It was reduced in the ratio of 3 : 5. What is its length now?
- If old price of an item was Rs. 750 and new price of the item is Rs. 550. Find out the ratio of decreased prices.

Keep in mind!

The first term of the ratio (antecedent) is always less than consequent in decrease in ratio.

Skill Practice

Decrease: • 2 litres in the ratio of 3 : 5 • 500 days in the ratio of 7 : 15

Example 6 Decrease 800 in the ratio of 7 : 10.

Solution First convert 7 : 10 into fraction and multiply it by 800.

$$= \frac{7}{10} \times 800 = 560$$

Hence, we can say that decrease in the ratio of 7 : 10 is 560.

Remember!

In increasing ratio, the numerator is greater than denominator. In other words, the fraction is always greater than 1. In decreasing ratio, the numerator is less than the denominator. In other words, the fraction is less than 1.

EXERCISE 1.11

1. Write the following in the simplest form of ratio:
   (i) 15 : 75 (ii) $\frac{2}{15} : \frac{3}{5}$ (iii) $\frac{1}{16} : \frac{25}{8}$
   (iv) $\frac{2}{15} : \frac{3}{5}$ (v) 5 kg and 500 g (vi) 2 km and 700 m
   (vii) 8 weeks and 14 days (viii) 240 minutes and 2 hours
   (ix) 180 days and 1 month (x) 5 years and 24 months.
2. Find increase:
   (i) 40 in the ratio 5 : 4 (ii) 36 in the ratio 8 : 3 (iii) 175 in the ratio 11 : 5
   (iv) 48 in the ratio 17 : 12 (v) 360 in the ratio 14 : 9 (vi) 425 in the ratio 7 : 5
3. Find decrease:
   (i) 30 in the ratio 2 : 3 (ii) 35 in the ratio 5 : 7 (iii) 50 in the ratio 2 : 5
   (iv) 70 in the ratio 2 : 7 (v) 160 in the ratio 5 : 8 (vi) 420 in the ratio 7 : 15
4. If old price was Rs. 990 and new price is Rs. 1200, then find the ratio of increased price.
5. If old price was Rs. 1500 and new price is Rs. 1300, then find the ratio of decreased prices.
6. If new mass of Raza is 80 kg and old mass was 120 kg, then find the ratio of decreased masses.
7. The price of cotton box has increased in ratio of 7: 6. If the old price of the cotton box was Rs. 1836, then find the new price of cotton box.
8. The height of Ali increased in ratio of 11 : 6 and the height of Ali was 1220 cm, then what will be the new height of Ali?
9. Mohsin's mass was 140 kilograms. If he reduces his mass in the ratio of 6 : 7, find his new mass.
10. Ahmad mass was 140 kilograms and reduced his mass to 112 kilograms. In what ratio did Ahmad reduce his mass?

1.4.2 Rate and Average Rate

i Rate

We have already discussed application of rate in our daily life in previous grade. We compare two quantities with different units. e.g.,

The speed of tiger is 60 km/hour

The speed of a bus is 80 km/hour

Remember!

Rate is a ratio of two quantities which have different units.

Important Information

Unit rate is a rate that is reduced to 1 unit. In other words, the 2nd term is always 1.

We normally read these as:

(i) the speed of tiger is 60 kilometres per hour

(ii) the speed of bus is 80 kilometres per hour

All the above examples are of unit rate.

Unit Rate
- Calories per serving (calories/serving)
- Miles per hour (mph)
- Blink per second (blink/second)
- 8000 steps per day (8000 steps/day)

Search out

Following Online game links can be shared with students for practice of unit rate: https://www.brainpop.com/games/unitrates/?topic_id=

ii Average Rate

Average rate is a rate in which an object changes its rate according to the period of time. Let us consider some examples.

Example 7 Suppose a truck travels 90 km in the first hour and 80 km in the second hour. What is average speed of the truck?

Solution Total distance covered by the truck = 90 + 80 = 170 km

Total time taken by the truck = 1 + 1 = 2 hours

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{170 \text{ km}}{2 \text{ h}}$$

$$= 85 \text{ km/h}$$

Hence, the average speed of the truck is 85 km/h.

Example 8 A bus is to cover 630 km. It covers first 180 km of the journey at speed of 60 km/h and covers next 450 km at speed of 90 km/h. What is its average speed?

Solution

For first 180 km

As, we know that:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$60 = \frac{180}{\text{Time}}$$

$$\text{Time} = \frac{180}{60}$$

$$\text{Time} = 3 \text{ hours}$$

For next 450 km

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$90 = \frac{450}{\text{Time}}$$

$$\text{Time} = \frac{450}{90}$$

$$\text{Time} = 5 \text{ hours}$$

Total distance covered by the bus = 630 km

Total time taken by the bus = 3 + 5 = 8 hours

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{630 \text{ km}}{8 \text{ hours}}$$

$$= 78.75 \text{ km/h}$$

Hence, the average speed of the bus is 78.75 km/h.

EXERCISE 1.12

1. Hassan earns Rs. 3500 in 5 days. What is his pay for one day?
2. Zain bought one dozen oranges for Rs. 240. What is the price of one orange?
3. A car covers 220 km in 11 litres. Find the rate of kilometre per litre. If car travels 400 km, then what will be the fuel consumption in litres?
4. Moeen cooked 10 dishes in 2 hours. What will be the speed of cooking dishes per hour? How much dishes will he cook in 8 hours?
5. The cost of 25 units of electricity is Rs. 375. Find the cost of 75 units of electricity.
6. Hamza covers the distance of 225 km in 3 hours by car. At this rate, how far can he drive in 8 hours?
7. The cost of 20 kg apples is Rs. 4000. What is the cost of 25 kg apples?
8. In the first cricket match, Hamza made 52 runs in 6 overs and in the second cricket match he made 68 runs in 8 overs. What is the average run rate per over?

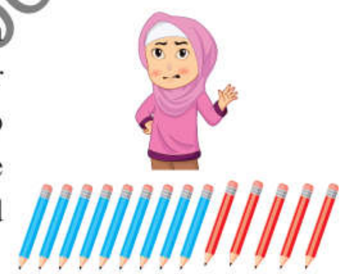
9. Atif sold 40 pens in 15 minutes on Monday and 95 pens in 30 minutes on Tuesday. What is the average sale of pens per minute?

10. A train is to cover 1180 km. If it covers first 550 km at the speed of 100 km/hour and the remaining distance at the speed of 90 km/hour. Find the average speed of the train.

1.4.3 Proportion

Let us consider the given situation.

They want to distribute these pencils between themselves. Zainab gave 5 red pencils to her sister Tabassum. Tabassum gave 9 blue pencils to Zainab. Tabassum was not happy. She felt that she gave more blue pencils to Zainab than the red pencils gave by zainab to her.



What do you think? Is Tabassum right?

To solve this problem, she went to her mother. Tabassum's mother explained that Zainab gave 5 red pencils out of 10 red pencils to Tabassum. The ratio is 5 : 10 or 1 : 2.

Tabassum gave 9 blue pencils out of 18 blue pencils to Zainab. The ratio is 9 : 18 or 1 : 2.

As, both the ratios are same, so the distribution is fair.

If two ratios are equal, then they are in proportion.

A proportion is a mathematical comparison between two ratios. In other words " The equality of two ratios originates proportion. It is denoted by "::"

Skill Practice

Whether the given ratios are in proportion. If "yes" then write them in proportion form.

(i) 6, 8, 9, 12 (ii) 5, 4, 10, 4

In the above example, we say 5, 10, 9 and 18 are in proportion which is written as: 5 : 10 :: 9 : 18

If a : b and c : d are two ratios, then its proportion is written as:

$$\underbrace{a : \overbrace{b :: c}^{\text{Means of proportion}} : d}_{\text{Extremes of proportion}}$$

$$\frac{a}{b} \times \frac{c}{d}$$

$ad = bc$

- "a" is called the 1st element/term of proportion.
- "b" is called the 2nd element/term of proportion.
- "c" is called the 3rd element/term of proportion.
- "d" is called the 4th element/term of proportion.

Skill Practice

Separate means and extremes from the following:

(i) 5 : 10 :: 25 : 50 (ii) 6 : 48 :: 10 : 80

Remember!

In proportion,

Product of extremes = Product of means

Example 9 Whether the following are in proportion?

(i) 16 : 10 :: 32 : 20 (ii) 4 : 8 :: 10 : 2

Solution (i) 16 : 10 :: 32 : 20

$$(16)(20) = (10)(32)$$

$$320 = 320$$

As, Product of means = Product of extremes

Hence, these ratios are in proportion.

(ii) 4 : 8 :: 10 : 2

$$(4)(2) = (8)(10)$$

$$8 \neq 80$$

As, Product of means ≠ Product of extremes

Hence, these ratios are not in proportion.

Example 10 Find the value of the x $12 : x :: 3 : 6$

Solution As, Product of extremes = Product of means

$$(12)(6) = 3x$$

$$3x = 72$$

$$x = \frac{72}{3}$$

$$x = 24$$

Skill Practice

Find the 3rd term in the given proportion:

$12 : 32 :: x : 8$

Kinds of Proportion

There are two kinds of proportion.

Proportion
- Direct proportion
- Indirect / inverse proportion

(a) Direct Proportion

Two quantities are said to be in direct proportion if increase or decrease in one quantity causes the increase or decrease in the other quantity in the same ratio. The concept of direct proportion is widely used in our daily life. e.g., as, the quantity of books is increased, the price of books will also be increased. As, the quantity of envelopes is increased, the cost of envelopes will also be increased.

Example 11 If the cost of 15 pens is Rs. 150, then find the cost of 20 such pens.

Solution Let the cost of 20 pens = x

Cost of pens (Rs.)	:	Number of pens
150 ↑	:	15 ↑
x	:	20

Teachers' Guide

Explain to the students that a change in the proportion of one quantity means a change in the proportion of the other. For example, when you buy more apples, you will have to pay more money.

As, if the number of pens increased, then the price will also be increased.

Therefore, it is a direct proportion

$$\frac{150}{x} = \frac{15}{20}$$

$$(15)(x) = (150)(20)$$

$$(15x) = 3000$$

$$x = \frac{3000}{15} = 200$$

By Using Proportion Method

Rs. : Rs. :: No. of pens : No. of pens

150 : x :: 15 : 20

In proportion, Product of means = Product of extremes

$$(x)(15) = (150)(20)$$

$$x = \frac{3000}{15} = 200$$

So, the cost of 20 such pens is Rs. 200.

(b) Indirect / Inverse Proportion

Two quantities are said to be indirect proportion if the increase or decrease in one quantity causes the decrease or increase in the other quantity in the same ratio.

This concept is also widely used in our daily life. e.g.,

- If the speed of car is increased, then the travelling time will be decreased.
- If the number of workers is increased, then less time will be required to complete the same task.

An indirect proportion is written as:

$$a : b :: \frac{1}{c} : \frac{1}{d}$$

$$a : b :: \frac{1}{c}cd : \frac{1}{d}cd$$

$$a : b :: d : c$$

Example 12 If 10 men do a work in 24 days. In how many days 15 men can do the same task?

Solution Let the required number of days = x

No. of days	:	No. of men
24 ↓	:	10 ↑
x	:	15

As, the number of men is increased then less time will be required to complete the same task.

So, it is an indirect proportion.

$$\frac{24}{x} = \frac{15}{10}$$

$$(x)(15) = (24)(10)$$

$$(15x) = 240$$

$$x = 16$$

Thus, 15 men will complete the same task in 16 days.

Skill Practice

Solve the example by using proportion method.

EXERCISE 1.13

1. Which of the following are in proportion?

(i) 7 : 2 :: 35 : 10 (ii) 2 : 6 :: 5 : 20 (iii) 16 : 12 :: 4 : 3

(iv) 10 : 8 :: 70 : 56 (v) 15 : 50 :: 9 : 30 (vi) 20 : 4 :: 3 : 40

2. Find the value of variable in the following proportions.

(i) $7 : 6 :: x : 50$ (ii) $y : 8 :: 45 : 20$

(iii) $z : 9 :: 15 : 35$ (iv) $w : 2 :: 24 : 18$

3. If 7 kg potatoes cost 140 rupees, how much we pay for 12 kg potatoes?

4. For 10 horses, 12 kg 400 g of food is required daily. In the same proportion, how much will be needed for 16 horses?

5. The cost of 16 quintals of bean is 40,400 rupees. How much will 4 quintals cost?

6. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many each will get if the number of children is reduced by 4?

7. If 2 men take 10 days to cut a tree, how many days will take 4 men to do the same job?

8. If 1 person takes 10 days to pick the apples from a garden, how many days will take 5 people to pick the apples from a garden?

9. If 1 person takes 7 days to clean a house from bugs, how many days will take 7 people take to clean the house from bugs?

SUMMARY

- The rate is a ratio of two quantities having different units.
- An equality of two ratios is called a proportion.
- Two quantities are said to be in direct proportion if increase or decrease in one quantity causes the increase or decrease in the other quantity in the same ratio.
- Two quantities are said to be indirect proportion if the increase or decrease in one quantity causes the decrease or increase in the other quantity in the same ratio.

Sub-domain (v) Financial Arithmetic

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Identify and differentiate between selling price, cost price, loss, discount, profit percentage and loss percentage.
- Explain income tax, property tax, general sales tax, value-added tax, zakat and ushr.
- Solve real world word problems involving profit, loss, discount, commission, tax, zakat and ushr.

INTRODUCTION

In our daily life, we are familiar with the words profit and loss. Mostly, we use these words in business and while selling and purchasing the objects or items.

Zain purchased two laptops of Rs. 65000 each. He sold one laptop for Rs. 66000 and other for Rs. 63000. How can we find out that Zain is in profit or loss?

Remember!

If the S.P. is greater than the cost price, then Zain will be in profit.

If the S.P. is less than the cost price, then Zain will be in loss.

1.5.1 Selling Price

The price at which an item is sold is called its selling price. It is denoted by S.P. e.g., the selling price of the first laptop is Rs. 66000. The selling price of the second laptop is Rs. 63000.

1.5.2 Cost Price

The price at which an item is bought is called its cost price. It is denoted by C.P. e.g., the cost price of a laptop is Rs. 65000

Activity

Write C.P. and S.P. of any object on the writing board. Call a student in front of writing board and ask the student to find profit or loss.

1.5.3 Profit

The cost price of a laptop is Rs. 65000 and the selling price of the first laptop is Rs. 66000.

As, while selling the first laptop, selling price (S.P.) is greater than cost price (C.P.)

Zain is in profit while selling the first laptop. The profit of Zain can be calculated by their difference.

Profit = Selling price – Cost price
= S.P. – C.P.
= 66000 – 65000
Profit = Rs. 1000

Keep in mind!
In profit, S.P. > C.P.

So, the profit of Zain is Rs. 1000

1.5.4 Loss

The cost price of the other laptop is also Rs. 65000, and the selling price of the second laptop is Rs. 63000. As, while selling the second laptop, cost price is greater than selling price.

So, Zain is in loss while selling the second laptop. The loss of Zain can also be calculated by their differences.

Loss = Cost price – Selling price
= C.P. – S.P.
= 65000 – 63000
Loss = Rs. 2000

Keep in mind!
In loss, S.P. < C.P.

So, the loss of Zain is Rs. 2000

Hence, Zain is in profit while selling the first laptop and he is in loss while selling the second laptop.

1.5.5 Profit Percentage and Loss Percentage

Recall

Percentage is a mixture of two words i.e., "per" and "centum". It means "out of hundred". It is denoted by %. We can say that 80% flower is coloured.

$$\text{Percentage of coloured flower} = \frac{4}{5} \times 100 = 80\%$$

$$\text{Percentage of uncoloured flower} = \frac{1}{5} \times 100 = 20\%$$

Skill Practice

Solve the following:

(i) $\frac{15}{20} \times 100 =$ ☐

(ii) $\frac{200}{300} \times 100 =$ ☐

(iii) 5% of 50 kg = ☐

(iv) 10% of 100 ℓ = ☐

Profit percentage and loss percentage are widely used in comparison of objects. For example,

A shopkeeper purchases a bag for Rs. 500 and sells it for Rs. 550. He buys a book for Rs. 250 and sells it for Rs. 300. On which product does the shopkeeper get more profit?

To solve this problem, we will find the profit percentage of both the items. Which item has greater profit percentage, the shopkeeper will get more profit on that item.

Skill Practice

A shopkeeper bought 200 kg onion and 180 kg potatoes. He found 5% of onion and 10% of potatoes were rotten. How many vegetables are in good condition?

Teachers' Guide

Ask the students to write some examples on their notebooks regarding selling price and cost price of any object and calculate profit or loss of these objects.

i Profit Percentage

The profit percentage is calculated by the formula:

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} \times 100\%$$

Keep in mind!

The profit in terms of percentage is called profit percentage.

$$\text{Profit on bag} = \text{S.P.} - \text{C.P.}$$
$$= 550 - 500 = \text{Rs. } 50$$
$$\text{Profit \% on bag} = \frac{50}{500} \times 100 = 10\%$$
$$\text{Profit on book} = \text{S.P.} - \text{C.P.}$$
$$= 300 - 250 = \text{Rs. } 50$$
$$\text{Profit \% on book} = \frac{50}{250} \times 100 = 20\%$$

As, the profit percentage of book is greater than the bag. So, the shopkeeper gets more profit on book.

Example 1 A women purchased a dress for Rs. 4300. She sold it at a profit of 5%. Find the selling price of the dress.

Solution The cost price of a dress = Rs. 4300

$$\text{Profit} = 5\% \text{ of Rs. } 4300$$
$$= \frac{5}{100} \times 4300 = \text{Rs. } 215$$
$$\text{Selling price} = \text{C.P.} + \text{Profit}$$
$$= 4300 + 215 = \text{Rs. } 4515$$

So, the selling price of the dress is Rs. 4515.

Skill Practice

A LED was bought for Rs. 67000 and sold at a profit of 10%. Find the selling price of the LED.

ii Loss Percentage

The loss percentage is calculated by the formula:

$$\text{Loss \%} = \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

Saleem bought a watch for Rs. 1500. After few months he sold his watch for Rs. 1200. Find his loss percentage.

Keep in mind!

The loss in terms of percentage is called loss percentage.

The cost price of the watch (C.P.) = Rs. 1500

The selling price of watch (S.P.) = Rs. 1200

$$\text{Loss} = \text{Cost price} - \text{Selling price}$$
$$= 1500 - 1200 = \text{Rs. } 300$$
$$\text{Loss \%} = \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$
$$= \frac{300}{1500} \times 100\% = 20\%$$

Hence, the loss percentage of the watch is 20%.

Skill Practice

In a furniture shop, 15 chairs were bought at the rate of Rs. 280 per chair. The shopkeeper sold 10 of them at rate of Rs. 400 per chair and the remaining at the rate of Rs. 250 per chair .Find his profit or loss percentage

1.5.6 Discount

The reduction or cut offered on the marked price is called discount.

Zaheer went to an electronic shop and bought an oven. The marked price of the oven was Rs. 14000. The shopkeeper sold the oven for Rs. 12500. How much discount did the shopkeeper give to Zaheer? To solve this problem, we will find out the difference of marked price and selling price.

Discount = Marked price – Selling price

= 14000 – 12500 = Rs. 1500

So, the shopkeeper gave the discount of Rs. 1500 to Zaheer.

Can you find out how much discount did the shopkeeper give to Zaheer in percentage?

Remember!

The difference between the marked price and selling price is known as discount. In discount, selling price is always less than the marked price.

To answer the given question, we will have to find out the discount percentage.

i Discount Percentage

$$\text{Discount percentage} = \frac{\text{Discount}}{\text{Marked price}} \times 100\%$$

$$= \frac{1500}{14000} \times 100\%$$

$$\text{Discount } \% = 10.71\%$$

Hence, the shopkeeper gave 10.71% discount to Zaheer.

Skill Practice

Can you find the selling price of the shirt?

Example 2 The marked price of an electric heater is Rs. 4800. A discount of 15% is announced on sale. What is the amount of discount and its selling price?

Solution Marked price of an electric heater = Rs. 4800

Discount = 15% of Rs. 4800

$= \frac{15}{100} \times 4800 = \text{Rs. } 720$

Selling price of the electric heater = Marked price – Discount amount

= 4800 – 720 = Rs. 4080

Hence, the amount of discount is Rs. 720 and its selling price is Rs. 4080.

Skill Practice

Aslam purchased the following items whose marked price and discount % are given below:

Item	Quantity	Marked price	Discount %	Selling Price
Oven	1	4700	5%	
Iron	1	2400	10%	
Food factory	1	8500	17%	
Washsing machine	2	12000	12%	

Find the total amount of the bill he has to pay.

EXERCISE 1.14

1. Fill in the boxes.

(i)	Cost Price = 450,	Selling Price = 560,	Profit =	
(ii)	Cost Price = 1230,	Selling Price = 1180,	Loss =	
(iii)	Marked Price = 4550,	Selling Price = 3950,	Discount =	

2. A bakery sold 1 kg of sweets for Rs. 1250. 1kg of sweets costs Rs. 1100 to the owner. Did the owner make a profit or loss? Calculate the amount of profit or loss.
3. Majid bought a book for Rs. 2500 from a bookstore. The bookstore bought this book for Rs. 2200. Did the bookstore make a profit or loss? Calculate the amount of profit or loss.
4. The Price of one bicycle is Rs. 7000. Adnan bought two bicycles and sold them for Rs. 13000. Calculate the amount of profit or loss.
5. Hamza bought a sack of rice for Rs. 12000. Find selling price if he got a profit of 4%.
6. Atif purchased a sofa set for Rs. 48000. Find selling price if he bear a loss of 10%.
7. A shopkeeper bought an aquarium for Rs. 7000 and sold it for Rs. 10000. Find his profit percentage.
8. Samar bought an old motorbike for Rs. 35000 and spent Rs. 15000 on its repairing. After few months he sold this bike for Rs. 60000. Find his profit percentage.
9. Salman bought a sofa set for Rs. 25000 and sold it for Rs. 20000. Find his loss percentage.
10. The price of a cycle is Rs. 12000 in market. Ahmad ordered this online and got this cycle for Rs. 13000. Find his loss percentage.
11. If the marked price of an item is Rs. 7690 and selling price is Rs. 7000. Find the discount offered on this item.
12. Ramaisa sells an item having marked price Rs. 370. Find its selling price if the profit is 15%.
13. Minahil bought a dress for Rs. 3200. If the loss is 8%, then find the selling price of the dress.
14. Find marked price of a LED at a discount of 13% having selling price Rs. 39000.
15. Eman purchases a heater for Rs. 4600 and sells it for Rs. 4500. She buys an oven for Rs. 14500 and sells it for Rs. 14000. On which item does Eman bear less loss?
16. The marked price of a computer table is Rs. 6700. It is sold at a discount of 12%. Find the selling price of the computer table.

1.5.7 Tax

In the 14th century, the word "tax" is appeared first time in English language. It is derived from a Latin word taxare and its meaning is "to assess".

A tax is an amount that government imposes on public to give them facilities, like education, health, security, justice, roads, electricity etc.

Tax is the most important source of government income. Some taxes are paid directly and some indirectly by the public.

Need to Know!

In Pakistan, Federal Board of Revenue (FBR) is a main department which is responsible for tax collection.

i Direct Tax

A tax in which the tax payer pays directly to the government. For example income tax, property tax, etc. Direct tax is different for everyone.

ii Indirect Tax

In such taxes, taxes are charged on goods and services or on commodities. Indirect tax is same for everyone.

Think!

On which commodities, indirect tax is paid by us? Give at least five examples.

The following different types of taxes, we will discuss and learn one by one.

(a) Income Tax

Income tax is the tax imposed by the government on the income of individuals exceeding a certain amount. It means Federal Government fixes a certain limit of income above which tax has to be paid by the individual. Federal Government announces the rates of income taxes annually. The rates of income tax for financial year 2022 – 2023 are given below:

Remember!

Exempt Income: The amount which is exempted from income tax is called exempt income.

Taxable Income: The income above a certain limit for which tax has to be paid is called the taxable income.

Sr. #	Taxable Income	Income Tax
1	Taxable income exceeds Rs. 600,000 but does not exceed Rs. 1,200,000	2.5% of the amount exceeding Rs. 600,000
2	Taxable income exceeds Rs. 1,200,000 but does not exceed Rs. 2,400,000	Rs. 15,000 + 12.5% of the amount exceeding Rs. 1200,000
3	Taxable income exceeds Rs. 2,400,000 but does not exceed Rs. 3,600,000	Rs 165,000 + 20% of the amount exceeding Rs. 2,400,000
4	Taxable income exceeds Rs. 3,600,000 but does not exceed Rs. 6,000,000	Rs. 405,000 + 25% of the amount exceeding Rs. 3,600,000
5	Taxable income exceeds Rs. 6,000,000 but does not exceed Rs. 12,000,000.	Tax rate Rs. 1,005,000 + 32.5% of the amount exceeding Rs. 6,000,000
6	Taxable income exceeds Rs. 12,000,000	Rs. 2,955,000 + 35% of the amount exceeding Rs. 12,000,000)

Think!

How much amount did your parents pay as income tax in the year 2022?

Keep in mind!

For paying tax, financial year starts from the 1st July and ends on 30th June of next year.

Example 3 Ahmad earns Rs. 80000 per month. Calculate the income tax on Ahmad's annual income.

Solution

Ahmad's monthly income = Rs. 80000

Ahmad's annual income = 80000 × 12

= Rs. 960000

Exempted amount = Rs. 600000 — According to the given income slab

Taxable income = Gross Income – Exempted amount

Taxable income = 960000 – 600000

Taxable income = Rs. 360000

Rate of income tax = 2.5% — According to the given taxable income slab

$$\text{Income tax} = \frac{25}{1000} \times 360000$$

= Rs. 9000

Therefore, Ahmad has to pay Rs. 9000 as income tax at the end of the financial year.

(b) Property Tax

Property Tax is a tax imposed by Government on the properties such as house, land and shops. Government imposes property tax on the annual value of a property. The amount of this tax depends on the location of the property and varies from location to location. The value of the property is assessed by the Government Departments.

Example 4

Ahmad owns a house of worth Rs. 4500000. Calculate the amount of property tax at the rate of 4.5%.

Solution

Total value of the house = Rs. 4500000

Rate of tax = 4.5%

$$\text{Amount of tax} = \frac{4.5}{100} \times 4500000$$

= Rs 202, 500

Activity

Shows the sample of utility bills and ask the students to check the rate of tax which is charged on the amount of bill.

Skill Practice

Discuss with your parents about the income tax. Ask your parents how much income tax did they pay last year and calculate the rate of income tax.

Example 5

Aslam owns a property. If he has to pay property tax of Rs. 22000 at the rate of 2%. Find the total worth of the property.

Solution

Let the rate of property $= x$

Amount of tax = Rs. 22000

Rate of tax = 2%

2% of x = Rs. 22000

$$\frac{2}{100} \cdot x = \text{Rs. } 22000$$

$$x = \text{Rs. } 22000 \times \frac{100}{2}$$

$$x = \text{Rs. } 1100000$$

Hence, the total worth of the property is Rs. 1100000.

(c) General Sales Tax

When a customer purchases an item, he pays an extra amount in addition to the original price of the item. This extra amount is called General Sales Tax.

Usually, Government imposes this tax on expensive items. In Pakistan, rate of GST varies from 0% to 25% depending on the type of items.

Key fact!

- General sales tax is normally read as (GST).
- Usually we pay 17% GST on our daily purchasing.
- The value of GST varies from (0 – 25)%.
- GST on any item is same for everyone.

Example 6 The price of a motorcycle is Rs. 110000. Find the GST on it at the rate of 17%.

Solution

Price of the motorcycle = Rs. 110000

Rate of GST = 17%

$$\text{Amount of GST} = \frac{17}{100} \times 110000$$

$$= \text{Rs. } 18700$$

Hence, the GST on this motorcycle is Rs. 18700.

Skill Practice

The marked price of a packed tea bag is Rs. 350 including 5% GST. What will be the original price of tea bag?

Example 7

The price of 1 fan is Rs. 6000. Find the price of 10 fans including GST.

Solution

The price of 1 fan = Rs. 6000

The price of 10 fans = 6000 × 10

The price of 10 fans = 60000

Rate of GST = 17%

$$\text{Amount of GST} = \frac{17}{100} \times 60000$$

$$= \text{Rs. } 10{,}200$$

Total Price Payable = 60000 + 10200

= Rs. 70200

Brain Teaser!

Think and tell the name of any five products, in which we pay GST while purchasing commodities. Can you guess whether GST is direct or indirect?

(d) Value Added Tax

A tax in which the price of an object is increased during production or distribution at each stage. Mostly, we write it as VAT. In Pakistan, GST and VAT are normally same. VAT is normally known as goods and services tax.

Do you know?

In Pakistan, the standard rate of VAT is 17% for goods and (13% – 18%) for services.

The price of a packet of cakes is Rs. 175. Find value added tax on it at the rate of 17%. To calculate the value added tax (VAT), we will have to use the following formula:

Amount of VAT = Rate of VAT × The value of an item

Amount of VAT = 17% × 175

$$= \frac{17}{100} \times 175 = \text{Rs. } 29.75$$

Hence, the value added tax on the packet of cakes is Rs. 29.75

Think!

Can you guess value added tax whether a direct tax or indirect tax?

Teachers' Guide

By using the examples of real life objects, clear the concept of GST and explain them, is it direct or indirect?

1.5.8 Commission

Commission is an amount of money which is paid by the seller or purchaser to the agent for his services. In other words, commission is an amount of money paid to an employee or agent by the seller or purchaser for selling or purchasing something.

i Rate of Commission

The rate at which an agent or person gets the commission for doing a service is called rate of commission. Rate of commission is always expressed in percentage form i.e., 5%, 2%, 10% etc. Commission is calculated by the given formula:

Keep in mind!

Rate of commission is decided between the agent / person and the buyer / seller that's why it can vary.

$$\text{Commission} = \text{Rate of commission} \times \text{Selling price}$$

If an agent gets 5% commission on selling a property of worth Rs. 3500000, then how much amount does he receive from the seller as a commission? We can find out his commission by using the following formula:

$$\begin{aligned}\text{Commission} &= \text{Rate of commission} \times \text{Selling price}\\ &= 5\% \times 3500000\\ &= \frac{5}{100} \times 3500000\\ &= \text{Rs. } 175{,}000\end{aligned}$$

Hence, the agent received commission of Rs. 175,000 from the seller.

Skill Practice

A real estate agent receive Rs. 75000 as commission. Which is 6% of the selling price. At what price does the agent sell the property.

Example 8 If a property dealer gets Rs. 100000 as commission on the sale of a shop for Rs. 4000000. Find the rate of commission.

Solution

$$\begin{aligned}\text{Selling price of shop} &= \text{Rs. } 4000000\\ \text{Amount of commission} &= \text{Rs. } 100000\\ \text{Commission} &= \text{Rate of commission} \times \text{selling price}\\ 100000 &= \text{Rate of commission} \times 4000000\\ \text{Rate of commission} &= \frac{100000}{4000000}\\ \text{Rate of commission} &= 0.025\\ &= 2.5\%\end{aligned}$$

EXERCISE 1.15

1. Zawar has annual income of Rs. 1000000. Find the amount of income tax.
2. If the income tax paid by Ali is Rs. 10000, then Find the total annual income of Ali.
3. Asra owns a plot of worth Rs. 400000. Find the amount of property tax at the rate of 3%.

4. Faheem owns one plot and a house of values Rs. 4000000 and Rs. 7000000 respectively. Find the amount of property tax at the rate of 4%.
5. A man paid Rs. 17000 as property tax at the rate of 1%. Find the total value of the property.
6. Ammar wants to buy an air conditioner of price Rs. 200000. If the rate of GST is 17%, then find how much money Ammar has to pay?
7. The price of one chair is Rs. 5000. Find the amount of GST on 50 such chairs at the rate of 17%.
8. Waqar purchased ten packets of biscuits for Rs. 350. He paid VAT of Rs. 42 on ten packets. Find the rate of GST on these packets.
9. Naveed purchased a shop of worth Rs. 6500000. He paid property tax worth Rs. 520000. What was the rate of property tax?
10. Ali sold a plot for Rs. 5000000 by an agent who received 2.5% commission. Find amount of commission.
11. Salman sold 50 articles of a company. If each article is sold for Rs. 2500. Find the amount of commission earned by Salman at the rate of 2%.
12. An online car service charges 20% commission from its drivers on each ride. If a driver got Rs. 5000 from all rides in a day. Calculate how much commission did he pay to car service?
13. A travel agency got 5% commission on the sale of air tickets. If he sold tickets for Rs. 2000000 in a day. Find the amount of commission.
14. An online market place charges 7% on all the sale from its sellers. If the sale of 1000000 rupees is made on Blessed Friday Sale. Calculate the amount of commission earned by online market place owner.
15. A consultant charges 10% of the visa fee for its consultancy. If visa fee is 1000000 rupees. Calculate the commission of consultant.

1.5.9 Zakat

There are five pillars of Islam. Zakat is one of the pillars of Islam. It imposes on those muslims who have certain amount of wealth the whole year. There are eight types of recipients of zakat. The purpose of zakat is to help the poor and needy among the Muslims to create a welfare muslim state. The muslims pay zakat if their annual savings reaches a certain level(Nisab).

Nisab of Zakat Nisab is the minimum amount of annual savings on which zakat has to be paid. Nisab for zakat is 7.5 tola (87.48 grams) gold or 52.5 tola (612.36 grams) silver or equivalent amount.

Rate of Zakat The rate of zakat is 2.5% of the total wealth.

The amount of zakat is calculated by the folloiwng formula:

$$\text{Amount of zakat} = \text{Rate of zakat} \times \text{Total amount}$$

Example 9

Aslam saved Rs. 2000000 for one year in his account. Calculate the amount of zakat Aslam has to pay.

Solution

$$\text{Total Amount} = \text{Rs. } 2000000$$
$$\text{Rate of zakat} = 2.5\%$$
$$\text{Amount of zakat} = \text{Rate of zakat} \times \text{Total amount}$$
$$\text{Amount of zakat} = \frac{2.5}{100} \times 2000000 = \text{Rs. } 50000$$

Hence, Aslam has to pay Rs. 50000 as a zakat

Remember!

Mostly, people pay zakat during the month of Ramadan but it is an annual duty and can be paid any time of the year when your wealth exceeded from the amount of Nisab.

Example 10

Sadia paid zakat of Rs. 15000 on gold. Find the total price of the gold.

Solution Let the total price of the gold = x

$$\text{Amount of zakat paid} = \text{Rs. } 15000$$
$$\text{Amount of zakat} = \text{Rate of zakat} \times \text{Total amount}$$
$$\text{Rs. } 15000 = 2.5\% \times \text{Total amount}$$
$$\text{Rs. } 15000 = \frac{2.5}{100} \times \text{Total amount}$$
$$\text{Total amount} = 15000 \times \frac{100}{2.5} = \text{Rs. } 600000$$

Hence, the total price of the gold is Rs. 600000

Important Information

Zakat helps to reduce the financial gap between rich and poor, it reduces poverty and purifies or cleans the Muslim's wealth.

1.5.10 Ushr

Ushr is paid on agricultural yield (vegetable, fruits, grains etc.).

Rate of Ushr

- The rate of ushr is 10%, when the land is irrigated by natural sources like rain, river and streams etc.
- The rate of ushr is 5%, when the land is irrigated by artificial resources like wells, tube-wells and artifical canals etc.

Example 11 Aslam has crop of worth Rs. 700000 irrigated by natural sources. Find the amount of ushr on it.

Solution Total worth of the crop = Rs. 700000

Rate of ushr = 10%

We can find the amount of ushr by using the given formula:

$$\text{Amount of ushr} = \text{Rate of ushr} \times \text{Total worth of crop}$$
$$= 10\% \times 700000$$
$$\text{Amount of ushr} = \frac{10}{100} \times 700000 = \text{Rs. } 70000$$

Hence, the amount of ushr is Rs. 70000.

Teachers' Guide

Gather students in the middle of the room, and read multiple-choice questions and their possible answers aloud. Students then move to the corner that represents what they believe is the correct answer. The top left room corner can be option *A*, the bottom-left can be *B* and so on.

Example 12 Find ushr on 2300 kg onion irrigated by natural source at the rate of Rs. 140 per kg and 1400 kg ginger irrigated by artificial source at the rate of Rs. 220 per kg.

Solution The total quantity of onion = 2300 kg

Price of onion per kg = Rs. 140

The total amount of onion = 2300×140

= Rs. 322000

Ushr on 2300 kg onion = Rate of ushr × Total worth of onion

$= 10\% \times 322000$

As, onion is irrigated by natural source, that's why rate of ushr is 10%.

$= \frac{10}{100} \times 322000$

Ushr on 2300 kg onion = Rs. 32200

The total quantity of ginger = 1400 kg

The amount of ginger = 1400×220 = Rs. 308000

Ushr on 2300 kg ginger = Rate of ushr × Total worth of ginger

$= 5\% \times 308000$

As, ginger is irrigated by artificial resource, that's why rate of ushr is 5%.

$= \frac{5}{100} \times 308000$

Ushr on 1400 kg ginger = Rs. 15400

Total ushr on both the items = 32200 + 15400

= Rs. 47600

Hence, total ushr on both the items is Rs. 47600.

Skill Practice

A farmer produces 18000 kg corn. Find his amount of ushr at the rate of 5%. The rate of corn is Rs. 110 per kg.

EXERCISE 1.16

1. Find the amount of the zakat on 15 tola gold if value of 1 tola gold is Rs. 130000.
2. Find the amount of zakat on 80 tola silver if value of 1 tola silver is Rs. 1500.
3. Raheel paid zakat worth Rs. 47000, on gold and his savings. Find the price of gold if his savings is Rs. 1000000.
4. Find the amount of zakat on total of 9 tola gold and 50 tola silver if the values of 1 tola gold and 1 tola silver are Rs. 130000 and Rs. 1500 respectively.
5. Hina has jewelry of 11 tola gold. Find the amount of zakat on it if the value of 1 tola of gold is Rs. 130000.

6. Zahid paid zakat of Rs. 23500. Find his savings.

7. Ahsan has a crop wheat of worth Rs. 400000 irrigated by natural sources. Find the amount of ushr on it.

8. Hamid has rice crop of worth Rs. 800000 irrigated by artificial resources. Find the amount of ushr on it.

9. Suleman paid ushr of Rs. 6500 on cotton. If the land is irrigated by rain, then find the worth of the cotton.

10. Find ushr on 850 kg wheat irrigated by natural source, the rate is Rs. 110 per kg and 1250 kg potatoes irrigated by artificial resource, the rate is Rs. 60 per kg.

SUMMARY

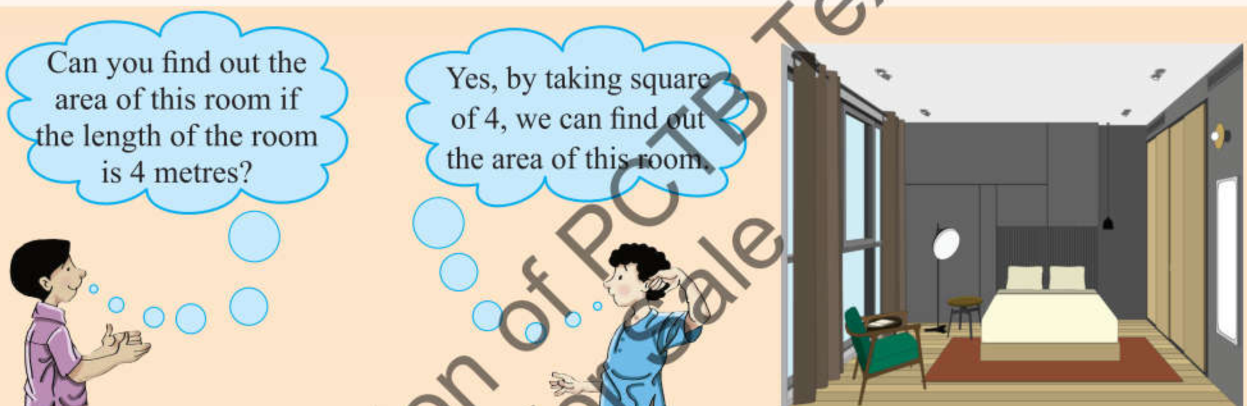
- The price at which an item is bought is called its cost price.
- The price at which an item is sold is called its selling price.
- Profit = Selling Price – Cost Price
- Loss = Cost Price – Selling Price
- Profit % = $\dfrac{\text{Profit}}{\text{Cost Price}} \times 100\,\%$
- Loss % = $\dfrac{\text{Loss}}{\text{Cost Price}} \times 100\,\%$
- The reduction or cut offered on the marked price is called the discount.
- Income tax is the tax imposed by the government on the income of individuals exceeding a certain amount.
- Property tax is a tax imposed by government on the properties such as house, land and shops.
- When a customer purchases an item, he pays an extra amount in addition to the original price of the item. This extra amount is called General Sales Tax.
- The rate of zakat is 2.5% of the total wealth.
- The rate of ushr is 5% and 10% depending on the land irrigated by natural sources or artificial resources respectively.

Sub-domain (vi) Squares and Square Roots

Students' Learning Outcomes

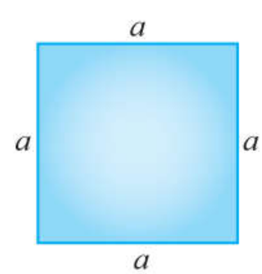
After studying this sub-domain, students will be able to:

- Recognise and calculate squares of numbers up to 3-digits.
- Find the square roots of perfect squares of (up to 3-digits) natural numbers, fractions and decimals.
- Solve real-world word problems involving squares and square roots.



1.6.1 Square of a Number

As we know that the area of a square shape is calculated by multiplying its length of a side by itself as given below:



$$\text{Area of a square} = \text{length} \times \text{length}$$
$$= a \times a = a^2$$

Here a^2 is the square of a. That is, the square of $a = a^2$

When a number is multiplied by itself, the resultant number is known as its square. For example, when we multiply $6 \times 6 = 6^2$, we get 36. Here, 36 is a squared number.

Remember!

$6^2 = 36$ can be read as:
- The square of 6 is 36.
- 6 squared is 36.
- 6 to the power of 2 is 36.

Example 1 Write 25 in terms of square of a number.

Solution Recall the table of 5, we can write

$$5 \times 5 = 25$$

Applying the law of exponent, $a \times a = a^2$

$$5^2 = 25$$

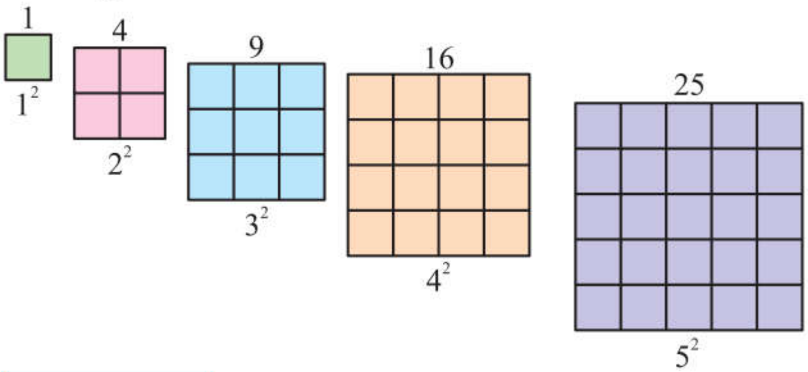
Therefore 25 is the square of 5.

Note

Perfect squares are used frequently in math. Try to remember these familiar numbers so that you can recognize them as they are used in most of the math problems.

1.6.2 Perfect Square of a Number

Perfect squares are the natural numbers that are squares of natural numbers. The first five squares of the counting numbers are shown below:



Square Notation	Perfect Square
1^2 (1-Squared)	1
2^2 (2-Squared)	4
3^2 (3-Squared)	9
4^2 (4-Squared)	16
5^2 (5-Squared)	25

Example 2 Find the perfect squares of the numbers: (i) 23 (ii) 331

Solution

Skill Practice

Find the perfect squares of the following numbers:

(i) 37 (ii) 58
(iii) 429 (iv) 900

i

$$\begin{array}{r} 23 \\ \times\ 23 \\ \hline 69 \\ 460 \\ \hline 529 \\ \hline \end{array}$$

$(23)^2 = 23 \times 23$
$= 529$

ii

$$\begin{array}{r} 331 \\ \times\ 331 \\ \hline 331 \\ 9930 \\ 99300 \\ \hline 109561 \\ \hline \end{array}$$

$(331)^2 = 331 \times 331$
$= 109561$

Example 3 Check whether the numbers are perfect squares or not:

(i) 25 (ii) 105 (iii) 1225

Solution

(i) 25

Factorize 25

The prime factors of 25 = $\overline{5 \times 5}$

Prime factors of 25 form a pair.

So, 25 is a perfect square.

5	25
	5

(ii) 105

Factorize 105

The prime factors of 105 = 3 × 5 × 7

We can see factors of 105 cannot be paired.

So, 105 is not a perfect square.

3	105
5	35
	7

(iii) 1225

Factorize 1225

The prime factors of 1225 = $\overline{5 \times 5} \times \overline{7 \times 7}$

Each prime factor of 1225 forms a pair.

So, 1225 is a perfect square.

5	1225
5	245
7	49
	7

Remember!

If each prime factor of a natural number forms a pair of prime numbers then natural number is a perfect square.

Teachers' Guide

Explain the concept of square with the help of examples. Ask students to draw multiplication table chart and highlight perfect squares.

Properties about Perfect Squares

- The square of an even number is always even. For example, the square of 12 is 144.
- The square of an odd number is always odd. For example, the square of 25 is 625.
- A number ends with 1 or 9, its square will end with 1. For example, the square of 11 is 121 and the square of 9 is 81.
- A number ends with 4 or 6, its square will end with 6. For example, the square of 14 is 196 and the square of 26 is 676.
- The square of a proper fraction is less than itself. For example, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ Now, $\frac{1}{4} < \frac{1}{2}$
  Hence, the square of a proper fraction is less than the given fraction.
- The square of decimal less than 1 is smaller than itself. For example, the square of 0.15 is 0.0225.

Example 4 Find the square of given numbers and write the property of perfect squares of a number which it satisfy: (i) 325 (ii) 1026 (iii) $\frac{12}{15}$ (iv) 0.25

Solution

(i) 325

Number 325 is an odd number

Square of 325 $= 325 \times 325$

$= 105625$ is an odd number

Therefore, square of odd number is odd.

(ii) 1026

Number 1026 is an even number

Square of 1026 $= 1026 \times 1026$

$= 1052676$ is an even number

Therefore, square of even number is even and 1026 ends with 6 so its square 1052676 ends with 6.

(iii) $\frac{12}{15}$

$\frac{12}{15}$ is a proper fraction

Square of $\frac{12}{15} = \frac{12}{15} \times \frac{12}{15} = \frac{12 \times 12}{15 \times 15} = \frac{144}{225}$

To compare $\frac{12}{15}$ and $\frac{144}{225}$,

by cross multiplication

$12 \times 225 = 2700$ and $15 \times 144 = 2160$

As $2700 > 2160$

Therefore, square of proper fraction is less than itself, i.e. $\frac{144}{225} < \frac{12}{15}$

(iv) 0.25

0.25 is a decimal less than 1

Square of $0.25 = 0.25 \times 0.25 = \frac{25}{100} \times \frac{25}{100} = \frac{625}{10000}$

Hence, 0.0625 is smaller than 0.25, i.e. $0.0625 < 0.25$

Challenge

Write five numbers which you can decide by looking at their ones digit that they are not square numbers.

Teachers' Guide

Ask students whether strength of classroom students and classroom furniture represents perfect squares or not.

Skill Practice

- Which of 123^2, 77^2, 83^2, 161^2, 109^2 would end with digit 1?
- Check whether the square of proper fraction $\frac{21}{35}$ is less than itself or not.
- Check whether the square of decimal 0.35 is smaller than itself or not.

EXERCISE 1.17

1. Find the squares of the following natural numbers:

(i) 49	(ii) 66	(iii) 75	(iv) 111
(v) 230	(vi) 313	(vii) 279	(viii) 400

2. Check whether the following numbers are perfect squares or not:

(i) 900	(ii) 912	(iii) 1764	(iv) 1600
(v) 1650	(vi) 2100	(vii) 3025	(viii) 710

3. Find the square of following numbers by using the property of perfect square:

(i) 72	(ii) 80	(iii) 63	(iv) 355
(v) 524	(vi) 4216	(vii) 821	(viii) 629
(ix) 0.33	(x) 0.18	(xi) 0.23	(xii) 0.04
(xiii) $\frac{14}{16}$	(xiv) $\frac{2}{18}$	(xv) $\frac{19}{28}$	(xvi) $\frac{25}{28}$

1.6.3 Square Root

The power " $\frac{1}{2}$ " of a perfect square of any positive number is called square root of that positive number.

Square root symbol is denoted by radical sign $\sqrt{}$, it is equivalent to power $\frac{1}{2}$.

We know that: $4^2 = 4 \times 4 = 16$. We say square root of 16 is 4. This is written as: $\sqrt{16} = 4$

Let us see some more examples $7^2 = 49 \longrightarrow \sqrt{49} = 7$; $5^2 = 25 \longrightarrow \sqrt{25} = 5$

i Finding Square Root by Prime Factorization Method

To find the square root of a perfect square by prime factorization, we follow the below steps:

(i) Find the prime factors of the given number.

(ii) Make pairs of prime factors.

(iii) Write each pair of prime factors in square form .

(iv) Take square root or power $\frac{1}{2}$ of each square term.

(v) Simplify/cancel powers of terms.

(vi) Find the product of terms, which will be the required square root.

Teachers' Guide

Explain the concept of square root by using real life situation.

Let us take an example to find the square root by prime factorization method.

Example 5 Find the square root of 784 by prime factorization method.

Solution Factorize 784

2	784
2	392
2	196
2	98
7	49
	7

$$\text{The prime factors of } 784 = \overline{2\times 2}\times\overline{2\times 2}\times\overline{7\times 7}$$
$$= 2^2\times 2^2\times 7^2$$
$$\text{Square root of } 784 = \sqrt{784}$$
$$= (784)^{\frac{1}{2}}$$
$$= (2^2\times 2^2\times 7^2)^{\frac{1}{2}}$$
$$= (2^2)^{\frac{1}{2}}\times (2^2)^{\frac{1}{2}}\times (7^2)^{\frac{1}{2}} \quad \left(\text{Take power } \frac{1}{2} \text{ of each square term separately}\right)$$
$$= 2^{2\times\frac{1}{2}}\times 2^{2\times\frac{1}{2}}\times 7^{2\times\frac{1}{2}} \quad \text{(Cancel the powers)}$$
$$= 2\times 2\times 7 = 28$$

Here, 28 is the square root of 784.

ii Finding Square Root by Long Division Method

We use long division method, when the numbers are very large. Because the method to find out their square root by prime factorization becomes very lengthy.

Let us find the square root of 484 by long division method.

Step 1 Place a bar over the pair of numbers starting from the ones place or right hand side of the number.

$\overline{4}\ \overline{84}$

Step 2 Take the largest number as the divisor whose square is less than or equal to the number on the extreme left of the number.

$$\begin{array}{r|l} & \ \ 2 \\ \hline 2 & \overline{4}\ \overline{84} \\ & \underline{-4} \\ & \ \ 0 \end{array}$$

Step 3 Bring down the number, which is under the bar, to the right side of the remainder.

$$\begin{array}{r|l} & \ \ 2 \\ \hline 2 & \overline{4}\ \overline{84} \\ & \underline{-4} \\ & \ \ 0\ 84 \end{array}$$

Step 4 Double the quotient and enter it with a blank space on the right side.

$$\begin{array}{r|l} & \ \ 2 \\ \hline 2 & \overline{4}\ \overline{84} \\ & \underline{-4} \\ 4\_ & \ \ 0\ 84 \end{array}$$

Step 5 Next, we have to select the largest digit for the ones place of the divisor (4_) such that new number 2, when multiplied by the new digit at the ones place, is equal to or less than dividend (84).

$$\begin{array}{r|l} & \ \ 2\ 2 \\ \hline 2 & \overline{4}\ \overline{84} \\ & \underline{-4} \\ 42 & \ \ 0\ 84 \\ & \underline{-\ 84} \\ & \ \ \ \ \ 0 \end{array}$$

The remainder is 0 and we have no number left for division.

Therefore, $\sqrt{484} = 22$

Challenge

What will be the square root of 0?

Skill Practice

Find the square root of 225 by long division method.

iii Square Root of Fractions

Let us find the square root of fraction $\frac{4}{9}$

$$\sqrt{\frac{4}{9}} = \left(\frac{4}{9}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2 \times 2}{3 \times 3}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^2}{3^2}\right)^{\frac{1}{2}} = \frac{2^{2 \times \frac{1}{2}}}{3^{2 \times \frac{1}{2}}} = \frac{2}{3}$$

Here, $\frac{2}{3}$ is the square root of $\frac{4}{9}$

Example 6 Find the square root of $\frac{196}{400}$ by prime factorization method.

Solution To find the square root of $\frac{196}{400}$, we will find square roots of numerator and denominator separately.

Factorize 196

2	196
2	98
7	49
	7

Prime factors of 196 = $\overline{2 \times 2} \times \overline{7 \times 7}$

$= 2^2 \times 7^2$

Square root of 196 = $\sqrt{196}$

$= (196)^{\frac{1}{2}}$

$= (2^2 \times 7^2)^{\frac{1}{2}}$

$= (2^2)^{\frac{1}{2}} \times (7^2)^{\frac{1}{2}}$

$= 2^{2 \times \frac{1}{2}} \times 7^{2 \times \frac{1}{2}}$

$= 2 \times 7 = 14$

Here, 14 is the square root of 196.

Also factorize 400

2	400
2	200
2	100
2	50
5	25
	5

Prime factors of 400 = $\overline{2 \times 2} \times \overline{2 \times 2} \times \overline{5 \times 5}$

$= 2^2 \times 2^2 \times 5^2$

Square root of 400 = $\sqrt{400}$

$= (400)^{\frac{1}{2}}$

$= (2^2 \times 2^2 \times 5^2)^{\frac{1}{2}}$

$= (2^2)^{\frac{1}{2}} \times (2^2)^{\frac{1}{2}} \times (5^2)^{\frac{1}{2}}$

$= 2^{2 \times \frac{1}{2}} \times 2^{2 \times \frac{1}{2}} \times 5^{2 \times \frac{1}{2}}$

$= 2 \times 2 \times 5 = 20$

Here, 20 is the square root of 400. So,

So, square root of $\frac{196}{400} = \sqrt{\frac{196}{400}} = \frac{14}{20}$ or $\frac{7}{10}$ (Put values of $\sqrt{196} = 14$ and $\sqrt{400} = 20$)

Teachers' Guide

Provide steps for finding square root of fractions.

Example 7 Find the square root of $\frac{289}{625}$ by division method.

Solution To find the square root of $\frac{289}{625}$, we will find square roots of numerator and denominator separately.

First we find square root of numerator 289.
Consider 1 as a divisor and quotient
$\because$ $2 \times 2 = 4$ is greater than 2
Sum of divisor and quotient is 2, write 2 at tens place of new divisor, whereas 7 is at ones place of new divisor
$\because$ $27 \times 7 = 189$
Here, 17 is the square root of 289.

$$\begin{array}{r|r} & 17 \\ \hline 1 & \overline{2}\ \overline{89} \\ & -1 \\ \hline 27 & 189 \\ & -189 \\ \hline & 0 \end{array}$$

Now, we find square root of denominator 625.
Consider 2 as a divisor and quotient
$\because$ $3 \times 3 = 9$ is greater than 6
Sum of divisor and quotient is 4, write 4 at tens place of new divisor, whereas 5 is at ones place of new divisor.
$\because$ $45 \times 5 = 225$
Here, 25 is the square root of 625.

$$\begin{array}{r|r} & 25 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline 45 & 225 \\ & -225 \\ \hline & 0 \end{array}$$

Square root of $\frac{289}{625} = \sqrt{\frac{289}{625}}$

$= \frac{17}{25}$ (Put values of $\sqrt{289} = 17$ and $\sqrt{625} = 25$)

Skill Practice

Find square root of following decimals by prime factorization and long division method.

(i) 17.64 (ii) 1.21

iv Square Root of Decimal

Example 8 Find the square root of 0.49 by prime factorization method.

Solution To find the square root of 0.49 , we shall find square root of $\frac{49}{100}$.

Factorize 49

The prime factors of $49 = \overline{7 \times 7}$
$= 7^2$

7	49
	7

The square root of $49 = \sqrt{49}$
$= (49)^{\frac{1}{2}}$
$= (7^2)^{\frac{1}{2}}$
$= 7^{2\times\frac{1}{2}}$ (Cancel the powers)
$= 7$

Here, 7 is the square root of 49.

Also factorize 100

The prime factors of $100 = \overline{2 \times 2} \times \overline{5 \times 5}$
$= 2^2 \times 5^2$

2	100
2	50
5	25
	5

Square root of $100 = \sqrt{100}$
$= (100)^{\frac{1}{2}}$
$= (2^2 \times 5^2)^{\frac{1}{2}}$
$= (2^2)^{\frac{1}{2}} \times (5^2)^{\frac{1}{2}}$
$= 2^{2\times\frac{1}{2}} \times 5^{2\times\frac{1}{2}}$
$= 2 \times 5$
$= 10$

Here, 10 is the square root of 100.

So, the square root of $0.49 = \sqrt{0.49} = \frac{\sqrt{49}}{\sqrt{100}} = \frac{7}{10}$ or 0.7 (Put values of $\sqrt{49} = 7$ and $\sqrt{100} = 10$

Example 9 Find the square root of 2.56 by division method.

Solution To find the square root of 2.56

Consider 1 as a divisor and quotient

$\because$ $2 \times 2 = 4$ is greater than 2

Sum of divisor and quotient is 2,

write 2 at tens place of new divisor,

whereas, 6 is at ones place of new divisor

$\because$ $26 \times 6 = 156$

Here, 1.6 is the square root of 2.56.

$$\begin{array}{r|r} & 1.6 \\ \hline 1 & \overline{2}.\overline{56} \\ & -1 \\ \hline 26 & 156 \\ & -156 \\ \hline & 0 \end{array}$$

Place the bars

You can check $25 \times 5 = 125$ is less than 156 and $27 \times 7 = 189$ is greater than 156.

EXERCISE 1.18

1. Find the square roots of the following natural numbers:

(i)	49	(ii)	100	(iii)	144	(iv)	196
(v)	324	(vi)	784	(vii)	529	(viii)	676

2. Find the square roots of following numbers by prime factorization method:

(i)	900	(ii)	2500	(iii)	1849	(iv)	1444
(v)	$\frac{49}{144}$	(vi)	$\frac{64}{225}$	(vii)	7.29	(viii)	6.76

3. Find the square roots of following numbers by division method:

(i)	576	(ii)	2304	(iii)	1521	(iv)	0.0441
(v)	$\frac{256}{1089}$	(vi)	$\frac{81}{289}$	(vii)	$\frac{676}{841}$	(viii)	26.01

1.6.4 Real-World Word Problems Involving Squares and Square Roots

Now, we discuss real life problems involving square and square root of numbers.

Example 10 Find the length of side of a square shaped park. If its area is 1296 m^2.

Solution Area of square shaped park = 1296 m^2

Length of side = $\sqrt{1296}$

$\because$ Area $= x^2$ and Length $= \sqrt{x^2} = x$

Consider 3 as a divisor and quotient

$\because$ $4 \times 4 = 16$ is greater than 12.

Sum of divisor and quotient is 6,

write 6 at tens place of new divisor,

whereas, 6 is at ones place of new divisor

$\because$ $66 \times 6 = 396$

$$\begin{array}{r|r} & 36 \\ \hline 3 & \overline{12}\,\overline{96} \\ & -9 \\ \hline 66 & 396 \\ & -396 \\ \hline & 0 \end{array}$$

Place the bars

You can check $65 \times 5 = 325$ is less than 396 and $67 \times 7 = 496$ is greater than 396.

Here, 36 is the square root of 1296, so length of side of a square shaped park is 36 m.

Example 11 Find the length of a rectangular field if area of a rectangular field is 19.36 m^2, where length of a rectangular field is four times of its width.

Solution Area of a rectangular field $= 19.36\ m^2$

Length of side $= 4$ (Width)

Let width of rectangular field is x,

$$\text{Area of a rectangular field} = 19.36\ m^2$$
$$\text{Length} \times \text{Width} = 19.36\ m^2$$
$$(4x) \times (x) = 19.36\ m^2$$
$$4x^2 = 19.36\ m^2$$
$$x^2 = \frac{19.36}{4}\ m^2$$
$$x = \sqrt{\frac{19.36}{4}}\ m$$
$$\text{Width} = \frac{\sqrt{19.36}}{\sqrt{4}}\ m$$
$$= \frac{\sqrt{19.36}}{2}\ m$$
$$= \frac{4.4}{2}\ m$$
$$\text{Width} = 2.2\ m$$
$$\text{Length of side} = 4\ (\text{Width})$$
$$= 4(2.2\ m)$$
$$= 8.8\ m$$

	4.4
4	$\overline{19}.\overline{36}$
	−16
84	336
	−336
	0

So, the length of a side of the rectangular field is 8.8 m.

Example 12 Find the area of square shaped shop if length of one side of shop is 33 m.

Solution Area of square shaped shop = Square of side length of shop

$$= (33\ m)^2$$
$$= 33^2\ m^2$$
$$= 33 \times 33\ m^2$$

Area of square shaped shop $= 1089\ m^2$

Teachers' Guide

Ask the students to find square root of 1444 by prime factorization method and long division method. Also check wether the answers are same.

EXERCISE 1.19

1. In a lecture hall, 8649 students are sitting in such a manner that there are as many students in a row as there are rows in the lecture hall. How many students are there in each row of the lecture hall?
2. The students of class-VII of a school donated Rs. 2304 for the Prime Minister's Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in class.
3. Kiran wants to wish her teacher on Eid Day by giving her a self-made greeting card. She chooses a purple coloured square sheet of paper. A side of that paper measures 19.5 cm. Find the area of paper she chooses for the card.
4. The area of a square plot is 900 m^2. Find the length of the side of the plot.
5. 400 students sit in rows in such a way that the number of rows is equal to the number of students in a row. How many students are there in each row?

SUMMARY

- When a number is multiplied by itself, the resultant number is known as its square.
- Perfect squares are the numbers that are squares of natural numbers.
- The power "$\frac{1}{2}$" of perfect square of any positive number is called square root of that positive number.
- Square root of any positive number can be find from the following two methods:
  - Prime Factorization Method
  - Long Division Method

REVIEW EXERCISE 1 (b)

1. Choose the correct option.

(i) The ratio of 2 years to 6 months.

(a) 1 : 4 (b) 2 : 6 (c) 4 : 1 (d) 1 : 3

(ii) The comparison of two quantities with same kinds is known as:

(a) rate (b) ratio (c) average rate (d) proportion

(iii) The value of m in the given proportion: $6 : m :: 5 : 10$

(a) 12 (b) 30 (c) 25 (d) 15

(iv) The equality of two ratios is known as:

(a) ratio (b) increase ratio (c) rate (d) proportion

(v) In proportion, product of means and product of extremes are:
(a) less (b) equal (c) greater (d) not equal

(vi) If the cost price is Rs. 450 and selling price is Rs. 550 then Profit will be:
(a) 50 (b) 100 (c) 150 (d) 200

(vii) If the marked price is Rs. 780 and selling price is Rs. 750 then discount will be:
(a) 10 (b) 20 (c) 30 (d) 40

(viii) The tax imposed on the income of an individual is called __________.
(a) GST (b) VAT (c) income tax (d) property tax

(ix) Rate of Zakat is________.
(a) 1% (b) 1.5% (c) 2.5% (d) 3%

(x) Rate of ushr on a land irrigated by natural sources is _________.
(a) 5% (b) 10% (c) 15% (d) 20%

(xi) Which of the following cannot be at the ones place of a perfect square number?
(a) 1 (b) 4 (c) 8 (d) 9

(xii) The number of zeros in square of 40 is:
(a) 1 (b) 2 (c) 3 (d) 4

(xiii) How many natural numbers lies between 2^2 and 3^2?
(a) 4 (b) 5 (c) 7 (d) 9

(xiv) The ones place in the square of 16 is:
(a) 4 (b) 6 (c) 7 (d) 8

(xv) A square shaped garden has an area of $100m^2$. Find the length of one side of the garden.
(a) 9 (b) 10 (c) 11 (d) 12

2. Write the following in ratio form and reduce it into lowest form:
(i) $\frac{75}{125}$ (ii) $\frac{200}{500}$ (iii) 185 : 85.
(iv) 5 week to 60 days (v) 4 km to 800 m (vi) 3 litres to 1800 mℓ

3. (i) Increase 15 in the ratio of 8 : 5 (ii) Increase 120 in the ratio of 13 : 10

4. (i) Decrease 510 in the ratio of 7 : 10 (ii) Decrease 999 in the ratio of 2 : 9

5. If the price of a shirt is increased from Rs. 3200 to Rs. 3600, what will be the increased in ratio?

6. If old price of an object was Rs. 890 and new price is Rs. 560. What will be decreased in ratio?

7. A shopkeeper purchased 20 packets of almonds for Rs. 16000. How much will he have to pay if he buys 25 packets?

8. A train covers 140 km in 2 hours. What will be the speed of the train per hour. Also find in how much distance will it cover in 4 hours?

9. Find the value of variable in the following proportions.
(i) $5 : 4 :: x : 20$ (ii) $a : 6 :: 55 : 10$ (iii) $e : 10 :: 25 : 50$ (iv) $q : 3 :: 24 : 30$

10. Two watches cost Rs. 16,000. How much money will be required to buy 14 such watches?
11. Mohsin took 5 days to finish a book, reading 100 pages daily. How many pages must he read in a day to finish it in 10 days?
12. If 42 men can reap a field in 16 days, in how many days can 20 men reap the same field?
13. 14 men can dig a well in 8 days. How many men can dig it in 4 days?
14. A fort had enough food for 60 soldiers for 40 days. How long would the food last if 30 more soldiers join after 16 days?
15. Ahmad travelled 300 kilometres in 5 hours. Find the unit rate in kilometres / hour.
16. An international phone call costs Rs.10 for 4 minutes. Find the unit rate in rupees / minute.
17. Hassan reads 18 pages in 9 minutes. Find the unit rate in pages / minute.
18. A car consumes 10 litres of fuel for a distance of 260 km. Find the unit rate in km/litre.
19. A car travels 600 km in 10 hours on Monday and 200 km in 3 hours on Tuesday. What will be the average speed of the car?
20. The cost price of an article is Rs. 3000 and the selling price is 5000. Find the profit percentage.
21. Ahsan bought a car for Rs. 1500000 and sold it for Rs. 1200000. Find loss percentage.
22. If the discount on an item is Rs. 500 and the selling price is Rs. 4000. Find the marked price.
23. Adeel earns Rs. 100000 monthly. Find the amount of income tax on his income.
24. Zawar pays Rs. 10000 as property tax at the rate of 2%. Find the total worth of the property.
25. The price of a washing machine is Rs. 80000. If the rate of GST is 17%, then find the total price of washing machine including GST.
26. Find the amount of zakat on 200000 rupees.
27. Shahwar has two crops of worth Rs. 500000 and Rs. 800000 respectively irrigated by artificial resources. Find the amount of ushr.
28. The price of packet of a rusk is Rs. 270. If the amount of value added tax is Rs. 80, then find the rate of value added tax.
29. Find the squares of the following natural numbers:
(i) 53 (ii) 69 (iii) 288 (iv) 500
30. Check whether the following numbers are perfect squares or not:
(i) 441 (ii) 572 (iii) 425 (iv) 3025
31. Find the square root of following numbers by prime factorization method.
(i) 169 (ii) 289 (iii) $\frac{49}{625}$ (iv) 3025
32. Find the square root of following numbers by division method:
(i) 361 (ii) 729 (iii) $\frac{100}{121}$ (iv) 6.76
33. Hooria has a square shaped mat with an area of 289 square cm. She wants to decorate the mat by putting fringe around the edges. How many cm of fringe, she needs to buy?
34. What is the length of a side of a square having area 441 square metres?

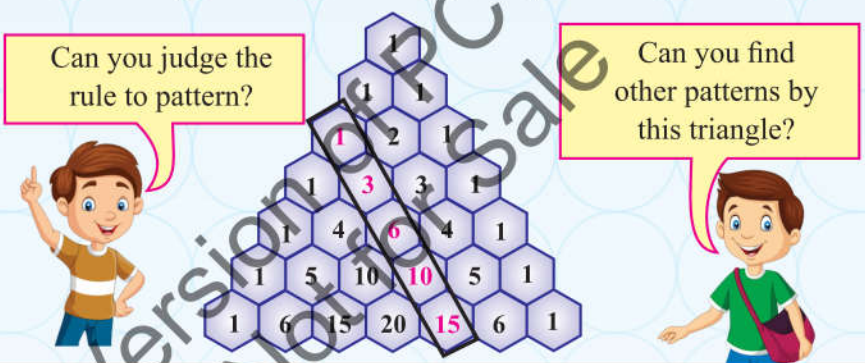
Domain 2 ALGEBRA

Sub-domain (i) Number Sequence and Patterns

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recall recognizing simple patterns from various number sequence.
- Recall how to continue a given number sequence and find ▪ term to term rule ▪ position to term rule
- Find term of sequence when the general term (n^{th} term) is given.
- Solve real life problems involving number sequences and patterns.



2.1.1 Number Sequence

A number sequence is an ordered list of numbers. For example, 1, 3, 5, ... is a number sequence comprising of 1 as first term, 3 as a second term, 5 as third term and three dots (...) means to continue forward in the pattern established.

Consider the following even numbers:

1^{st} term	2^{nd} term	3^{rd} term	4^{th} term	5^{th} term	
2 ,	4 ,	6 ,	8 ,	10 ,	...

$2 \xrightarrow{+2} 4 \xrightarrow{+2} 6 \xrightarrow{+2} 8 \xrightarrow{+2} 10$

In the above sequence, the numbers are arranged in an order i.e., every next number is obtained by adding 2 in the previous term.

Activity

Teacher can provide a hundred square grid or can display it on writing board to ask students to colour squares to explore different number patterns like odd numbers, even numbers, counting in multiples from times tables and ten more and ten less.

Need to Know!

The arrangement of numbers according to a specific rule is known as number sequence.

Remember!

A specific rule which is used to obtain the next term is known as rule of pattern.

i Term to Term Rule

A term to term rule is used for a sequence in which the next term is obtained from the previous term.

Example 1 Identify and complete the following patterns.

(i) 2, 5, 8, 11, _____, _____, _____, 23 (ii) 1, 3, 7, 13, _____, 31

Solution **(i)**

$$2 \xrightarrow{+3} 5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} \_\_ \xrightarrow{+3} \_\_ \xrightarrow{+3} \_\_ \xrightarrow{+3} 23$$

Here, the difference between 1st and 2nd term is 3. i.e., $5 - 2 = 3$ and 3 is added in each term. Therefore,

5^{th} term = 11 + 3 = 14

6^{th} term = 14 + 3 = 17

7^{th} term = 17 + 3 = 20

Hence, 2 , 5 , 8 , 11 , 14 , 17 , 20 , 23

(ii)

$$1 \xrightarrow{+2} 3 \xrightarrow{+4} 7 \xrightarrow{+6} 13 \xrightarrow{+8} \text{—} \xrightarrow{+10} 31$$

Here the difference between 1st and 2nd term is 2. i.e., $3 - 1 = 2$ and 2 is added continuously in next difference. Therefore, 5^{th} term = 4^{th} term + 8 = 13 + 8 = 21

Hence, 1 , 3 , 7 , 13 , 21 , 31

Observe the following number sequence:

$$23 \xrightarrow{\times(-1)} -23 \xrightarrow{\times(-1)} 23 \xrightarrow{\times(-1)} -23 \xrightarrow{\times(-1)} 23$$

We can see that the next term is obtained by multiplying the previous term by (–1). Now, consider another number sequence.

$$500000 \xrightarrow{\div(-10)} -50000 \xrightarrow{\div(-10)} 5000 \xrightarrow{\div(-10)} -500 \xrightarrow{\div(-10)} 50$$

We can see that, the each term is obtained by dividing the previous term by (–10).

Try yourself!

Can you find the next three terms of the given number sequence.

5, 11, 18, 26, ___, ___, ___

Search out

http://www.transum.org/Software/SW/Starter_of_the_day/Similar.asp?ID_Topic=36

Try yourself!

Can you find out the next three terms of the given number sequence?

Try yourself!

Can you find out the next three terms of the given number sequence?

Skill Practice

Can you find out the next two flowers where butterfly will sit? Also tell the rule of pattern.

Teachers' Guide

Write the following questions on the writing board to clear the concept of pattern. Find the term a_5, a_6, a_8 and a_{11}, if $a_1 = 3$, $a_2 = 5$, $a_3 = 8$ and $a_4 = 12$.

ii Position-to-term Rule

A position to term rule defines the value of each term with respect to its position. A number sequence is arranged in table like this.

Position	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	n^{th}
Terms	3	5	7	9	...	...	...	$2n+1$

Rule Multiply the position by 2 and then add 1. We have observed that each position is multiplied by 2 and then add 1.

So, we obtain:

$$\begin{aligned}
a_1 &= 2\times 1+1 = 3\\
a_2 &= 2\times 2+1 = 5\\
a_3 &= 2\times 3+1 = 7\\
a_4 &= 2\times 4+1 = 9\\
a_5 &= 2\times 5+1 = 11\\
a_6 &= 2\times 6+1 \quad 13\\
a_7 &= 2\times 7+1 \quad 15\\
&\vdots\\
a_n &= 2\times n+1 = 2n+1
\end{aligned}$$

Here $a_n = 2n + 1$ is the n^{th} term of the given sequence.

Remember!

- n^{th} term is also known as general term.
- n^{th} is a formula that is used to find out any term in a number sequence.
- n^{th} term of a number sequence is followed a specific rule.

Example 2 Find the n^{th} term of 5, 6, 7, 8, ... by using the position to term rule and calculate:

(i) 15^{th} term (ii) 23^{rd} term (iii) 50^{th} term

Solution

Position	1^{st}	2^{nd}	3^{rd}	4^{th}
Terms	5	6	7	8

$$\begin{aligned}
a_1 &= 1+4 = 5\\
a_2 &= 2+4 = 6\\
a_3 &= 3+4 = 7\\
a_4 &= 4+4 = 8\\
&\vdots\\
a_n &= n+4 = n+4
\end{aligned}$$

Hence, the general or n^{th} term is $n + 4$.

(i) 15^{th} term

Put $n = 15$ in the general term.

$a_{15} = 15 + 4 = 19$

(ii) 23^{rd} term

Put $n = 23$ in the general term.

$a_{23} = 23 + 4 = 27$

(iii) 50^{th} term

Put $n = 50$ in the general term.

$a_{50} = 50 + 4 = 54$

Remember!

a_1 is called the first term of a number sequence.
a_2 is called the second term of a number sequence.
⋮
a_n is called the n^{th} term of a number sequence.

Activity

Look at the given shapes:

N NN NNN

- The number of line segments required to one N is 3.
- The number of line segments required to two Ns is 5.
- To continue this pattern, for three Ns 7 line segments are required.
- Can you develop the n^{th} term or general term for this pattern? Also find how many line segments are required for thirteen Ns.

Teachers' Guide

Give students different number sequences (increasing and decreasing), pictorial patterns and tables and generate discussion on the possible n^{th} term.

Skill Practice

If the n^{th} term of a number sequence is $n^3 - 5$, then find the 2^{nd}, 3^{rd}, 4^{th} and 5^{th} term of the given sequence.

2.1.2 Real Life Problems

Example 3 There are 10 rows of seats in a concert hall. 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats in the 3rd row and so on.

Rows	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Number of Seats	25	27	29	31						43

(i) Complete the table. (ii) Find the total number of seats.

(iii) If the price of per ticket is Rs. 500, then find how much amount the owner received from this concert?

Solution

(i)
$$\begin{aligned}
a_1 &= 25\\
a_2 &= a_1 + 2 = 25 + 2 = 27\\
a_3 &= a_2 + 2 = 27 + 2 = 29\\
a_4 &= a_3 + 2 = 29 + 2 = 31\\
a_5 &= a_4 + 2 = 31 + 2 = 33\\
a_6 &= a_5 + 2 = 33 + 2 = 35\\
a_7 &= a_6 + 2 = 35 + 2 = 37\\
a_8 &= a_7 + 2 = 37 + 2 = 39\\
a_9 &= a_8 + 2 = 39 + 2 = 41\\
a_{10} &= 43
\end{aligned}$$

Search out

https://mathsframe.co.uk/en/resources/resource/42/sequences

Try yourself!

Can you find out the n^{th} term of the given number sequence?

(ii) Total number of seats $= 25 + 27 + 29 + 31 + 33 + 35 + 37 + 39 + 41 + 43 = 340$

Hence, the total number of seats in the hall is 340.

(iii) The owner received total amount from this concert= Total number of seats × price of per ticket

$= 340 \times 500 =$ Rs. 170,000

Hence, the owner received Rs. 170,000 from this concert altogether.

EXERCISE 2.1

1. What number is added to make the sequence?

(i) 4, 8, 12, 16, ..., (ii) 12, 17, 22, 27, ...,

(iii) 28, 34, 40, 46, ..., (iv) 101, 106, 111, 116, ...,

Teachers' Guide

Ask students to make their own number sequences and derive their n^{th} terms and vice-versa.

2. What number is multiplied to make the sequence?
 (i) 3, 6, 12, 24 (ii) 5, 10, 20, 40
 (iii) 6, 18, 54, 162 (iv) 7, 14, 28, 56
3. Use the first three terms in the pattern to find the rule.
 (i) 52, 57, 62, ... (ii) 78, 85, 92, ...
4. Use the pattern rule to make the pattern.
 (i) Start at 8 and add 2 each time. (ii) Start at 3 and multiply 2 each time.
5. Find the missing terms in the given sequences.
 (i) 7, 10, 13, _____, _____, _____, 25 (ii) 6, 12, 18, _____, _____, _____, 42
 (iii) 23, 20, 17, _____, _____, _____, 5 (iv) 98, 96, 94, _____, _____, _____, 86
6. Find the general term of the following number sequences.
 (i) 6, 13, 20, 27, ... (ii) 5, 7, 9, 11, ... (iii) 4, 9, 14, 19, ... (iv) 2, 12, 22, 32, ...
7. Find 30th term from the following general terms:
 (i) $a_n = 2n + 1$ (ii) $a_n = 3n + 4$ (iii) $a_n = 5n - 2$ (iv) $a_n = \dfrac{3}{2n+1}$
8. The general terms of the number sequence are given. List the first four terms for each sequence.
 (i) $a_n = 3n + 5$ (ii) $a_n = 8n + 2$ (iii) $a_n = 5n^2 + 1$ (iv) $a_n = 4n + 12$
9. A plant that is 17 cm tall grows 2 cm each week.
 (a) Complete the sequence:
 17, ______ , ______ , ______
 1 week 2 weeks 3 weeks
 (b) How tall will the plant be after three weeks?
 (c) After how many weeks will the plant be 27 cm tall?
10. Ibrahim solves 5 questions of an exercise in the subject of Mathematics in 20 minutes, 7 questions in 30 minutes, 9 questions in 40 minutes. When will he solve 13 questions and 15 questions if this pattern is continued? Also convert the time into hours and minutes.
11. A company used 10 cartons to pack balls. If the company packed 20 balls in the first carton, 25 balls in the second carton, 30 balls in the third carton, and this pattern is continued, then find:
 (i) The number of balls in fourth, fifth, sixth, seventh, eighth, ninth and tenth cartons.
 (ii) How many balls are there altogether in 10 cartons?
 (iii) The cost of all balls if the price of a ball is Rs. 18.

SUMMARY

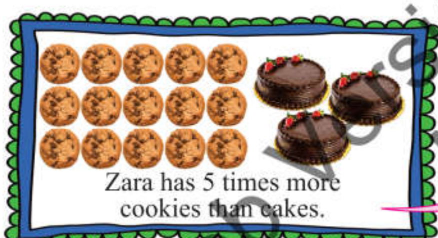
- Number sequence is an ordered list of numbers.
- The term- to-term rule is used for a sequence in which the next term is obtained from the previous terms.
- A position-to-term rule defines the value of each term with respect to its position.
- n^{th} term of the sequence is also known as general term of the sequence and written as a_n.

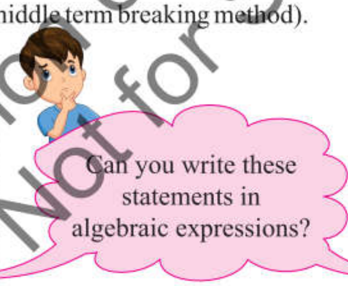
Sub Domain (ii) Algebraic Expressions

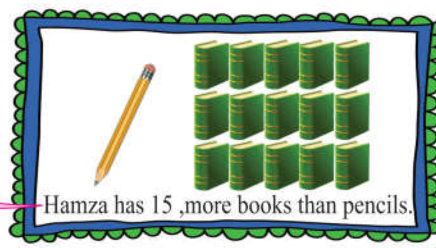
Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Know Muhammad bin Musa Al-Khwarizmi as the founding father of Algebra.
- Recall variables as a quantity which can take various numerical values.
- Recognise open and close sentences, like and unlike terms, variable, constant, expression, equation and inequality.
- Recognise polynomials as algebraic expressions in which the powers of variables are whole numbers.
- Identify a monomial, a binomial and a trinomial as a polynomial.
- Add and subtract two or more polynomials.
- Find the product of:
  - monomial with monomial
  - monomial with binomial/trinomial
  - binomial with binomial/trinomial
- Simplify algebraic expressions (by expanding products of algebraic expressions by a number, a variable or an algebraic expression) involving addition, subtraction, multiplication and division.
- Explore the following algebraic identities and use them to expand expressions:
  - $(a+b)^2 = a^2 + 2ab + b^2$
  - $(a-b)^2 = a^2 - 2ab + b^2$
  - $a^2 - b^2 = (a+b)(a-b)$
- Factorize algebraic expressions (by taking out common terms and by regrouping)
- Factorize quadratic expressions (by middle term breaking method).







2.2.1 Muhammad Bin Musa Al-Khwarizmi (780 - 850 A.D)

Muhammad Bin Musa Al-khwarizmi also known as Al-khwarzmi was an Islamic mathematician who wrote on Hindu-Arabic numerals. He is considered to be the father of modern algebra. His algebra treaties Hisab Al-jabra W'almuqabala gives us the word algebra.

Algebra is a branch of Mathematics, which is related to the mathematical operations, variables, constant as well as concept of equations and algebraic structure.

Important Information

Algebra is an Arabic word. Algebra means "Reduction"

2.2.2 Recognize Open and Close Sentences

i Open Sentence

An open sentence is neither true nor false until the unknown values have been replaced by a specific values. For example:

(i) $\square + 2 = 6$ (ii) $4 \times \square = 8$ (iii) $9 \div \square = 3$ (iv) $10 - \square = 3$

The above statements neither true nor false because we do not know the unknown values. In all four cases the blank boxes represent the open sentences.

Keep in mind!

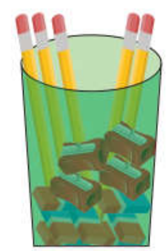
A sentence which has one or more unknowns is called open sentence.

ii Close Sentence

A mathematical close sentence is always true or false. For example:

i	$16 \div 4 = 4$	True mathematical close sentence
ii	8 is an even number	True mathematical close sentence
iii	Square root of 9 is 2	False mathematical close sentence
iv	$8^4 = 8 \times 8 \times 8$	False mathematical close sentence

2.2.3 Algebraic Expression



Zeshan buys some objects and put them in a box altogether.

Now, he wants to put the objects/ things in different bags.

There are 5 pencils + 4 sharpeners + 6 erasers

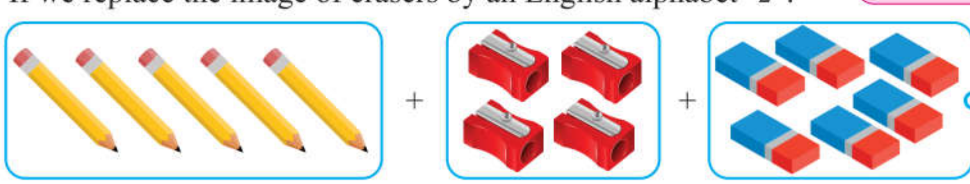
- If we replace the image of pencil by an English alphabet "x".
- If we replace the image of sharpeners by an English alphabet "y".
- If we replace the image of erasers by an English alphabet "z".

Skill Practice

Convert the following word statements into algebraic expressions:

(i) The sum of three bags of rice and five bags of wheat.

(ii) Subtract five times of x from 20 times of y.



$5x + 4y + 6z$

It is called an algebraic expression

Keep in mind!

The expression in which English alphabet along with numbers are used is called algebraic expression.

Important Information

In $5x + 4y + 6z$, x, y, z are called variables.

Algebraic Expression

An expression which connects variables and constants by mathematical operations (+, –, ×, ÷) is called an algebraic expression.

Think!

- Is this an algebraic expression? $ax + by + cz + d$
- Also tell how many terms are there?
- Seperate the variables and constants.

For example:

$4x + 8$

(4) Coefficient — Term ($4x$) — (x) Variable

Term (8) — Constant

Important Information

Terms: Parts of an algebraic expression which are connected by operations (+,×,÷ and –) are called terms of an algebraic expression.

Search out

https://www.khanacademy.org/math/algebra-basics/alg-basics-algebraic-expressions?t=practice

A few examples of algebraic expressions are given below:

(i) $18x + y$ (ii) $2x + 4$ (iii) $8 - 4x + 5y$ (iv) $3x + \frac{1}{2}$

i Variable and Constant

In mathematics, a variable is an English alphabet or term that represents an unknown number or unknown value or unknown quantity. The variables are used in algebraic expressions.

For example , in $2x + y + 1$, $3z + 5$, x, y and z are called variables. 1 and 5 are called constants.

Skill Practice

Separate the variables and constant from each of the given algebraic expressions.

Algebraic Expression	Variables	Constant
$18x + y$		
$2x + 4$		
$8 - 4x + 5y$		
$3x + \frac{1}{2}$		

Keep in mind!

Variable
A quantity which can take various numerical values.

Constant
A quantity which cannot be changed or vary. It has a fixed value.

Challenge

Ask the students to develop the algebraic expression with two variables independently and exchange it with the student sitting beside and seperate the variables and constants as a challenge.

2.2.4 Like Terms and Unlike Terms

i Like Terms

The algebraic terms that contain the same variable, same power (exponent) are called like terms.

For example, in algebraic expression $2x + 5x$, $2x$ and $5x$ are called like terms.

In algebraic expression $7x^2 + 2x^2 + 10x^2$,the terms $7x^2$, $2x^2$ and $10x^2$ are also like terms.

Skill Practice

Seperate the like terms from the following:

(i) $5x^2$, $3x$, $5x$, $10x^2$

(ii) $7x$, $2x$, $7x^2$

(iii) $6x^2$, $8x$, $2x^2$

Teachers' Guide

Teacher can take a simple example like $5x - 6y - 9$ to explain coefficients, variables and constant in algebraic expressions.

Search out

https://www.theproblemsite.com/games/quadratic-rush

ii Unlike Terms

The algebraic terms that contain the different variables with same power (exponent) or same variable with different power (exponent) are called unlike terms.

For example, in algebraic expression $(3x + 4y + 3z)$, $3x$, $4y$ and $3z$ are unlike terms.

In algebraic expression $(7x + 8x^2 + 5)$, $7x$, $8x^2$ and 5 are unlike terms.

2.2.5 Equation and Inequality

i Equation

When two algebraic expressions are connected by equality sign (=).

For example, $\left.\begin{array}{l} 2x + 3 = 2y + 5 \\ 5x - 12 = 5 - 3y \end{array}\right\}$ These are called equations

Keep in mind!

Both the expressions are equal, that's why these are connected by equality sign.

ii Inequality

When two algebraic expressions are not equal and are connected by the symbols ($>$, $<$, $\leq$ or $\geq$) are called inequalities.

For example, $2x + 3y \geq 2x$; $2x - 3 < 5$

Key Fact

Equation is a statement which shows the equal value of both the expressions.

On the other side, an inequality is a statement which shows that an expression is less than or greater than the other.

Activity

Explain to the students, inequalities in real life problem solving skills such as speed limit on the highway, minimum payments on the credit card bills, number of text massages you can send each month from your cell-phone. All of these can be represented as mathematical inequalities.

EXERCISE 2.2

1. What number can replace $\square$ in each of the following open sentences to make a true statement?

   (i) $\square + 10 = 12$ (ii) $15 - \square = 12$ (iii) $10 \div \square = 5$ (iv) $\square \times 4 = 28$

2. Seperate open and close sentences.

   (i) $2x + 7 = 5$ (ii) $z + 4 = 10$ (iii) $10 + 10 = 20$ (iv) $18 \div 2 = 9$
   (v) $10x + 5 = 10$ (vi) $y + z = 16$ (vii) $5a + 3b = 10$ (viii) $9 \div 3 = 3$
   (ix) $7p + 5q = 15$

3. Complete the following table:

Sr. #	Algebraic Expression	Number of Terms	Variables	Constant
(i)	$4xy$			
(ii)	$4x^2 - 2y + 8$			
(iii)	$2xyz + 20$			
(iv)	$x^2 - 2xy + 16$			

4. Seperate like and unlike terms from the given expressions.

   (i) $3x^2 + 4xy + 7 + 9x^2$ (ii) $4x^2y + 5x^2 + 7yx^2 + 4x$
   (iii) $2x^2 + 8xy + 8y + 9x^2$ (iv) $\frac{3}{2}x^2 + 8 + \frac{9}{2}x^2 + 4x$

5. Seperate the equations and inequalities from the following:.

   (i) $3x + 8 > 10$ (ii) $2x - 10 \leq 1$ (iii) $3x - 4 = 7x$
   (iv) $7x - 5 > 6x$ (v) $5x = 7$ (vi) $8x < 5$

Search out

Teacher can share following online game links to practice algebraic expressions:
- https://www.mathgames.com/skill/4.94-write-variable-expressions
- https://www.mathgames.com/skill/6.9-evaluate-multi-variable-expressions

2.2.6 Polynomials

A polynomial is an algebraic expression comprising of whole number as exponent of the variable. Polynomial consists of variable, coefficient and constant. For example,

i $3x^2 + 2x + 4$ is a polynomial
ii $x^2y + y + 17$ is a polynomial
iii $2x^2 + \frac{2}{x} + 11$ is not a polynomial
iv $\sqrt{x} + y + 2$ is not a polynomial

i Degree of Polynomial

The highest power of the variable is called degree of the polynomial. e.g., degree of $8x^3 + 4x^2 + 6x + 9$ is "3" and the degree of $4x^2y^2 + x^2y + 18$ is "4", because the term $4x^2y^2$ has the highest sum of the exponent of variables (x and y) which is "4".

ii Types of Polynomials

There are three types of polynomials based on the number of terms.

(a) Monomials

It is a type of polynomial having a single term. For example, $3x^2$, $4xy^2$ etc.

(b) Binomials

A binomials is a polynomials having two terms.
For example, $2y^2 + 7$, $2xy + 4x$ etc.

Need to Know!

A binomial is the sum of two monomials and thus will have two unlike terms.

(c) Trinomials

A trinomials is a type of polynomials that consists of three terms.
For example, $7x^2 + 2x + 6$, $xy^2 + xy + 7$ etc.

Remember!

A trinomial is the sum of three monomials meaning it will be the sum of three unlike terms.

iii Operation with Polynomials

In this section, we shall learn addition and subtraction of polynomials.

(a) Addition of Polynomials

There are two different methods to add the polynomials.

- **Horizontal Method**

In this method, all expressions are written in a horizontal line (row) and then add the like terms.

- **Vertical Method**

In this method, each expression is written in a separate row such that the like terms are written below the like terms.

Teachers' Guide

Explain the students that a monomial is the product of non-negative powers of variables. A monomial has no variable in its denominators and will have only one term.

Example 1 Add $4x^2 + 2x + 5$ and $3x^2 - 4x + 2$

Solution

Horizontal Method

$$4x^2 + 2x + 5 + 3x^2 - 4x + 2$$
$$= 4x^2 + 3x^2 + 2x - 4x + 5 + 2$$
$$= 7x^2 - 2x + 7$$

Vertical Method

$$\begin{array}{r} 4x^2 + 2x + 5 \\ + 3x^2 - 4x + 2 \\ \hline 7x^2 - 2x + 7 \\ \hline \end{array}$$

So, $(4x^2 + 2x + 5) + (3x^2 - 4x + 2) = 7x^2 - 2x + 7$

Example 2 Add $x^2 + 3x - 2$, $x^3 + 2x^2 + 8x + 9$ and $2x^3 + 4x^2 - 7x + 5$

Solution $x^2 + 3x - 2$, $x^3 + 2x^2 + 8x + 9$, $2x^3 + 4x^2 - 7x + 5$

Horizontal Method

Arrange the given polynomials in descending order:

$$\begin{aligned} & x^2 + 3x - 2 + x^3 + 2x^2 + 8x + 9 + 2x^3 + 4x^2 - 7x + 5 \\ &= x^3 + 2x^3 + x^2 + 2x^2 + 4x^2 + 3x + 8x - 7x - 2 + 9 + 5 \\ &= (1 + 2)x^3 + (1 + 2 + 4)x^2 + (3 + 8 - 7)x + 12 \\ &= 3x^3 + 7x^2 + 4x + 12 \end{aligned}$$

Vertical Method

$$\begin{array}{r} x^2 + 3x - 2 \\ x^3 + 2x^2 + 8x + 9 \\ 2x^3 + 4x^2 - 7x + 5 \\ \hline 3x^3 + 7x^2 + 4x + 12 \\ \hline \end{array}$$

So, $(x^2 + 3x - 2) + (x^3 + 2x^2 + 8x + 9) + (2x^3 + 4x^2 - 7x + 5) = 3x^3 + 7x^2 + 4x + 12$

(b) Subtraction of Polynomials

In order to subtract two polynomials, subtract the co-efficients of terms having same degree. Subtraction of polynomials can also be done through horizontal and vertical methods.

Example 3 Subtract $2x^2 + 5x + 4$ from $5x^2 + 8x + 12$

Solution

Horizontal Method

Arrange the given polynomials in descending order:

$$\begin{aligned} & 5x^2 + 8x + 12 - (2x^2 + 5x + 4) \\ &= 5x^2 + 8x + 12 - 2x^2 - 5x - 4 \\ &= 5x^2 - 2x^2 + 8x - 5x + 12 - 4 \\ &= (5 - 2)x^2 + (8 - 5)x + 8 \\ &= 3x^2 + 3x + 8 \end{aligned}$$

Vertical Method

$$\begin{array}{r} 5x^2 + 8x + 12 \\ \underset{-}{+} 2x^2 \underset{-}{+} 5x \underset{-}{+} 4 \\ \hline 3x^2 + 3x + 8 \\ \hline \end{array}$$

So, $(5x^2 + 8x + 12) - (2x^2 + 5x + 4) = 3x^2 + 3x + 8$

Example 4 Subtract $-8x^2 + 2x^3 + 1$ from $14x + 7x^3 + 10$

Solution

Horizontal Method

Arrange the given polynomials in descending order:

$$\begin{aligned} & 7x^3 + 14x + 10 - (2x^3 - 8x^2 + 1) \\ &= 7x^3 + 14x + 10 - 2x^3 + 8x^2 - 1 \\ &= 7x^3 - 2x^3 + 8x^2 + 14x + 10 - 1 \\ &= (7 - 2)x^3 + 8x^2 + 14x + 9 \\ &= 5x^3 + 8x^2 + 14x + 9 \end{aligned}$$

Vertical Method

$$\begin{array}{rrrr} 7x^3 & + 0x^2 & + 14x & + 10 \\ \underset{-}{+}2x^3 & \underset{+}{-}8x^2 & \underset{-}{+}0x & \underset{-}{+}1 \\ \hline 5x^3 & + 8x^2 & + 14x & + 9 \\ \hline \end{array}$$

So, $(7x^3 + 14x + 10) - (2x^3 - 8x^2 + 1) = 5x^3 + 8x^2 + 14x + 9$

Teachers' Guide

Tell the students that we can add and subtract the algebraic expressions by collecting like terms.

EXERCISE 2.3

1. Which of the following algebraic expressions are polynomials?

(i) $ax^2 + bx + cy$ (ii) $\frac{2}{3}x^2 - \frac{x}{4} + \frac{1}{16}$ (iii) $x^3 - 4x^2 + \frac{1}{x}$ (iv) $2x^2 + 4x + 3$

(iv) $\frac{1}{2}x^2y + 9x^3$ (vi) $-\frac{1}{2}x$ (vii) $2x^3y + 2$ (viii) $z^2 + 2z + 12$

2. Find the degree of polynomials given in question 1.

3. Add the following expressions:

(i) $4x^2 - 8x + 12$, $6x^2 + 5x + 4$ (ii) $5x^2 + 4xy + 7$, $2x^2 + 6xy + 2$

(iii) $x^3 + 4x^2 + 3x + 4$, $3x^3 + 4x + 4$ (iv) $x^2 + 2x + 4$, $2x^2 + 4x + 11$, $3x^2 - 8x + 10$

(v) $x^3 + 2x^2 - x + 2$, $4x^3 + 20x + 4$, $x^2 + 4x + 11$

4. Subtract the first polynomial from the second polynomial.

(i) $5x^2 + 2x - 1$, $10x^2 + 8x + 7$ (ii) $y^2 - 2q^2 + 3r$, $8y^2 + 6q^2 + 7r$

(iii) $2x^3 + 12x^2 + 4x + 12$, $7x^3 + 12x + 24$ (iv) $3x^2 + 4x + 2$, $8x^3 + 12x^2 + 9x + 10$

(v) $x^3 + 2x^2y + 3xy^2 + y^3$, $4x^3 + 3x^2y + 6xy^2 + 4y^3$ (vi) $2x + 3y - 4z - 1$, $4x + 3z + 4y + 12$

5. The sum of two polynomials is $6x^3 + 4x^2 + 8x + 12$. If one polynomial is $x^3 + 2x^2 + 3x + 2$, then find the other polynomial.

6. Subtract $2x^2 - 4x + 4$ from the sum of $4x^2 + 2x + 7$ and $x^2 + 6x + 2$.

(c) Product of Polynomials

To multiply the polynomials, multiply each term in one polynomial with the other polynomial and then add those answers having same terms. There are following general cases for product of polynomials:

- **Product of Monomial by a Monomial**

Challenge

Complete the table.

$\times$	x^2	$-2xy^2$	y^3
$-2x$			
$3x^2y$			

Example 5 Multiply polynomials: $2xy$, $3x^2y$

Solution

Horizontal Method

$(2xy) \times (3x^2y)$

$= (2 \times 3) \times (x \times x^2 \times y \times y)$

$= 6 \times x^3y^2 = 6x^3y^2$

Vertical Method

$$\begin{array}{r} 2xy \\ \times\ 3x^2y \\ \hline 6x^3y^2 \\ \hline \end{array}$$

So, $(2xy) \times (3x^2) = 6x^3y^2$

Example 6 Multiply polynomial: $5x^3y$ by $-2x^5$

Solution

Horizontal Method

$$-2x^5(5x^3y)$$
$$= -2x^5(5x^3y)$$
$$= (-2 \times 5) \times (x^5 \times x^3 \times y)$$
$$= (-10) \times x^8y$$
$$= -10x^8y$$

Vertical Method

$$\begin{array}{r} 5x^3y \\ \times -2x^5 \\ \hline -10x^8y \\ \hline \end{array}$$

So, $(-2x^5) \times (5x^3y) = -10x^8y$

- **Product of Monomial by a Binomial**

Example 7 Find the product of $(2x^3 + 3xy)$ and $2xy$

Solution

Horizontal Method

$$2xy(2x^3 + 3xy)$$
$$= 2xy(2x^3) + 2xy(3xy)$$
$$= (2 \times 2)(x \cdot x^3)(y) + (2 \times 3)(x \cdot x)(y \cdot y)$$
$$= 4x^4y + 6x^2y^2$$

Vertical Method

$$\begin{array}{r} 2x^3 + 3xy \\ \times 2xy \\ \hline 4x^4y + 6x^2y^2 \\ \hline \end{array}$$

So, $(2x^3 + 3xy) \times 2xy = 4x^4y + 6x^2y^2$

- **Product of Monomial by Trinomial**

Example 8 Find the product of $2a^2 - 3b^2 + 5ab$ and $-3ab^2$

Solution

Horizontal Method

$$-3ab^2(2a^2 - 3b^2 + 5ab)$$
$$= -6a^3b^2 + 9ab^4 - 15a^2b^3$$

Vertical Method

$$\begin{array}{r} 2a^2 - 3b^2 + 5ab \\ \times -3ab^2 \\ \hline -6a^3b^2 + 9ab^4 - 15a^2b^3 \\ \hline \end{array}$$

So, $(2a^2 - 3b^2 + 5ab) \times (-3ab^2) = -6a^3b^2 + 9ab^4 - 15a^2b^3$

- **Product of Binomial by a Binomial**

Example 9 Find the product of $(3x^2 - 2xy^2)$ and $(5x^2 - 4xy^2)$

Solution

Horizontal Method

$$(3x^2 - 2xy^2) \times (5x^2 - 4xy^2)$$
$$= 3x^2(5x^2 - 4xy^2) - 2xy^2(5x^2 - 4xy^2)$$
$$= 15x^4 - 12x^3y^2 - 10x^3y^2 + 8x^2y^4$$
$$= 15x^4 - 22x^3y^2 + 8x^2y^4$$

Vertical Method

$$\begin{array}{rrr} 3x^2 & - 2xy^2 & \\ \times\ 5x^2 & - 4xy^2 & \\ \hline 15x^4 & - 10x^3y^2 & \\ & - 12x^3y^2 & + 8x^2y^4 \\ \hline 15x^4 & - 22x^3y^2 & + 8x^2y^4 \\ \hline \end{array}$$

So, $(3x^2 - 2xy^2) \times (5x^2 - 4xy^2) = 15x^4 - 22x^3y^2 + 8x^2y^4$

- **Product of Binomial by Trinomial**

Example 10 Find the product of $(2m^2 + 2n^2 - 3mn)$ and $(3m^2 - 2n^2)$

Solution

Horizontal Method

$$(2m^2 + 2n^2 - 3mn) \times (3m^2 - 2n^2)$$
$$= 2m^2(3m^2 - 2n^2) + 2n^2(3m^2 - 2n^2) - 3mn(3m^2 - 2n^2)$$
$$= 6m^4 - 4m^2n^2 + 6n^2m^2 - 4n^4 - 9m^3n + 6mn^3$$
$$= 6m^4 + 2m^2n^2 - 4n^4 - 9m^3n + 6mn^3$$

Vertical Method

$$\begin{array}{rrrrr} & 2m^2 + 2n^2 - 3mn & & & \\ \times & 3m^2 - 2n^2 & & & \\ \hline & 6m^4 + 6m^2n^2 - 9m^3n & & & \\ & -4m^2n^2 & & -4n^4 + 6mn^3 & \\ \hline & 6m^4 + 2m^2n^2 - 9m^3n - 4n^4 + 6mn^3 & & & \\ \hline \end{array}$$

So, $(2m^2 + 2n^2 - 3mn) \times (3m^2 - 2n^2) = 6m^4 + 2m^2n^2 - 9m^3n - 4n^4 + 6mn^3$

(d) Division of Polynomials

- **Division of Monomial by a Monomial**

Example 11 Divide the polynomial: $8x^4y^5$ by $2x^2y^3$

Solution

$$8x^4y^5 \div 2x^2y^3$$
$$= \frac{\overset{4}{\cancel{8}}x^4y^5}{\cancel{2}x^2y^3}$$
$$= 4x^{4-2} \cdot y^{5-3}$$
$$= 4x^2 \cdot y^2$$

Try yourself!

Complete the table.

$\div$	$4x^3y$	$8x^5y^8$	$16x^3y^2$
$2x$	$2x^2y$		
$4xy$			

- **Division of Binomial by Monomial**

Example 12 Divide the polynomial: $8\ell^2m^2 + 16\ell m^2$ by $2\ell m$

Solution

$$(8\ell^2m^2 + 16\ell m^2) \div 2\ell m$$
$$= \frac{8\ell^2m^2 + 16\ell m^2}{2\ell m}$$
$$= \frac{\overset{4}{\cancel{8}}\ell^2m^2}{\cancel{2}\ell m} + \frac{\overset{8}{\cancel{16}}\ell m^2}{\cancel{2}\ell m}$$
$$= 4\ell^{2-1} \cdot m^{2-1} + 8\ell^{1-1} \cdot m^{2-1}$$
$$= 4\ell m + 8\ell^0 \cdot m \qquad \because \ \ell^0 = 1$$
$$= 4\ell m + 8m$$

Challenge

Find the mistake in the table.

$\div$	$3x^5$	$6x^3y^6$	$12x^5$
$3x^4$	x	$2y^2$	$4x$
$4xy$	$3x^4$	$6x^2y^6$	$12x^4$

- **Division of Trinomial by Monomial**

Example 13 Divide the polynomial: $8a^5b^2 - 10a^5b^4 + 4a^2b^2$ by $2a^2b^2$

Solution $(8a^5b^2 - 10a^5b^4 + 4a^2b^2) \div 2a^2b^2$

$$= \frac{8a^5b^2 - 10a^5b^4 + 4a^2b^2}{2a^2b^2}$$

$$= \frac{8a^5b^2}{2a^2b^2} - \frac{10a^5b^4}{2a^2b^2} + \frac{4a^2b^2}{2a^2b^2}$$

$$= 4a^{5-2} \cdot b^{2-2} - 5a^{5-2} \cdot b^{4-2} + 2a^{2-2}b^{2-2}$$

$$= 4 \cdot a^3 \cdot b^0 - 5 \cdot a^3b^2 + 2a^0b^0$$

$$= 4a^3 - 5a^3b^2 + 2$$

Keep in mind!

$x^0 = a^0 = b^0 = 1$

EXERCISE 2.4

1. Slove the following polynomials:

(i) $(4x)(8x^2y)$ (ii) $(12x^3)(2x^2y^5)$ (iii) $(7x^3)(2x^5y^6)$

(iv) $(2x^4y^2)(4x^3y^5)$ (v) $(2\ell^2m^2)(7\ell^3m^6)$ (vi) $(2x^2 + 5y^2)(3x^3y^2)$

(vii) $(5xy^2 + 8x^2y^3)(2xy)$ (viii) $(3c^2d^2 + 5c^3d)(5c^3d^3)$ (ix) $\left(\frac{3}{4}m^2n\right)\left(\frac{4}{3}mn^2\right)$

(x) $\left(\frac{16}{3}\ell m^2\right)\left(\frac{9}{4}\ell^3m^5\right)$ (xi) $(7a^2 + 8b^2)(2a^2b - 5ab^2)$

(xii) $(2\ell^3m^2 - 5\ell m^3)(5\ell^3m - 2\ell m)$ (xiii) $(2x^2 - 5y^2 - 8x^3y^2)(2x^2y^2)$

(xiv) $(x^2y - 2xy^2 - 3xy)(5x^3y)$ (xv) $(3\ell^2m + 5\ell m^3 - 2\ell m)(2\ell^3m^2)$

(xvi) $(2a^2b^2 - 3a^2b + 4ab^2)(ab^2 - a^2b)$ (xvii) $(2\ell m^2 + 4\ell^2m^2 + 3\ell)(\ell m + 3\ell m^2)$

(xviii) $(5x^2y^2 + 6xy^2 - 2x^2y)(2x^2y^2 - 2xy^3)$ (xix) $(2ab^2 - 5a^3b - a^4b)(ab^2 - 3a^3b)$

2. Divide the polynomials.

(i) $5x^4y^2$ by $5x^3y$ (ii) $10x^9y^5$ by $2x^5y$ (iii) $15\ell^5m^9$ by $3\ell^2m^2$

(iv) $2a^2b^2 + 5a^4b^3$ by $2ab$ (v) $30p^3q^5 + 45p^8q^9$ by $3p^2q^4$ (vi) $8x^5y^5 + 6x^3y^6$ by $2x^3y$

(vii) $25x^5y^9 + 15x^9y^6$ by $5x^2y^5$ (viii) $21a^5b^5 + 14a^4b^3 + 7a^2b^2$ by $7a^2b^2$

2.2.7 Simplification of Algebraic Expressions

Simplification of algebraic expressions will be done with the help of BODMAS rule. Simplification of algebraic expression is explained through following examples:

Example 14 Simplify the algebraic expressions

(i) $4x^2(x - y) + 7y(x + y)$ (ii) $8x - [9y - \{2x - 2(\overline{2x + 3y})\}]$

Solution

(i) $4x^2(x-y)+7y(x+y)$
$= 4x^3 - 4x^2y + 7yx + 7y^2$

(ii) $8x - [9y - \{2x - 2(\overline{2x - 3y})\}]$
$= 8x - [9y - \{2x - 2(2x - 3y)\}]$
$= 8x - [9y - \{2x - 4x + 6y\}]$
$= 8x - [9y - \{-2x + 6y\}]$
$= 8x - [9y + 2x - 6y]$
$= 8x - [2x + 15y]$
$= 8x - 2x - 15y \Rightarrow 6x - 15y$

EXERCISE 2.5

1. Simplify each of the following:

(i) $2x^2(x-y) - 3y(x+y)$

(ii) $4x^2 + 10(x+2) - 5(x-2)$

(iii) $\frac{2}{3}x^2 + \frac{2}{3}(x-2)(x+2)n^2$

(iv) $6x^2 - 3(x+4) + 4(x^2+2)$

(v) $(4m+3)(m^2-4m+4) - (2m-3)(2m^2+3m-4)$

(vi) $4x - [8y - \{4x - 2(2x-3y)\}]$

(vii) $8x + [6y - 2\{2y - 4(z - \overline{3x - 2y})\}]$

(viii) $8m - \{2n + 6 - (4m - \overline{2n - 4}) + 2m^2 - (2m^2 - 2n)\}$

(ix) $ab + [2x - \{2(x - y + 2z) - 2\}]$

(x) $3x - [-3x + 5 - \{(3x - 4y - \overline{3x - 2x})\} - 2x]$

2.2.8 Basic Algebraic Formulae-Identities

In algebra, the basic identities are equations that are always true regardless of the values assigned to the variables. It is used to solve the algebraic equations and to find the values of the unknown algebraic identities. Following are the three algebraic identities:

(i) $(a+b)^2 = a^2 + 2ab + b^2$ (ii) $(a-b)^2 = a^2 - 2ab + b^2$ (iii) $a^2 - b^2 = (a-b)(a+b)$

i Algebraic Identity $(a+b)^2 = a^2 + 2ab + b^2$

Take L.H.S $= (a+b)^2$
$= (a+b)\cdot(a+b)$ $\quad \because \; x^2 = x\cdot x$
$= a\cdot(a+b) + b\cdot(a+b)$
$= a\cdot a + a\cdot b + b\cdot a + b\cdot b$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$
$=$ R.H.S

So, $(a+b)^2 = a^2 + 2ab + b^2$

ii Algebraic Identity $(a-b)^2 = a^2 - 2ab + b^2$

Take L.H.S $= (a-b)^2$
$= (a-b)\cdot(a-b)$ $\quad \because \; x^2 = x\cdot x$
$= a\cdot(a-b) - b\cdot(a-b)$
$= a\cdot a - a\cdot b - b\cdot a + b\cdot b$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$
$=$ R.H.S

So, $(a-b)^2 = a^2 - 2ab + b^2$

iii Algebraic Identity $a^2 - b^2 = (a - b)(a + b)$

Take R.H.S $= (a - b)\cdot(a + b)$ $\quad \because \; x^2 = x \cdot x$

$= a\cdot(a + b) - b\cdot(a + b)$

$= a\cdot a + a\cdot b - b\cdot a - b\cdot b$

$= a^2 + ab - ab - b^2$

$= a^2 - b^2$

$=$ L.H.S

So, $a^2 - b^2 = (a - b)(a + b)$

Teachers' Guide

Guide the students to prepare coloured sheet to note down all the formulae of the algebraic identities. At the end of topics, paste these resources sheet on their notebook.

Example 15 Solve: (i) $(2x + 4y)^2$ (ii) $(103)^2$

Solution (i) $(2x + 4y)^2 = (2x)^2 + 2(2x)(4y) + (4y)^2$

$= 4x^2 + 16xy + 16y^2$

(ii) $(103)^2 = (100 + 3)^2$

$= (100)^2 + 2(100)(3) + (3)^2$

$= 10{,}000 + 600 + 9$

$= 10{,}609$

Example 16 Solve: (i) $(2a - 3b)^2$ (ii) $(87)^2$

Solution (i) $(2a - 3b)^2 = (2a)^2 - 2(2a)(3b) + (3b)^2$

$= 4a^2 - 12ab + 9b^2$

(ii) $(87)^2 = (90 - 3)^2$

$= (90)^2 - 2(90)(3) + (3)^2$

$= 8{,}100 - 540 + 9$

$= 7{,}569$

Example 17 Solve: (i) $(2x - 8y)(2x + 8y)$ (ii) 107×93

Solution (i) $(2x - 8y)(2x + 8y) = (2x)^2 - (8y)^2$

$= 4x^2 - 64y^2$

(ii) $107 \times 93 = (100 + 7)(100 - 7)$

$= (100)^2 - (7)^2$

$= 10{,}000 - 49$

$= 9951$

EXERCISE 2.6

1. Simplify the following by using $(a + b)^2 = a^2 + 2ab + b^2$.

(i) $(2x + y)^2$ (ii) $(2x + 5y)^2$ (iii) $\left(\frac{x}{2} + \frac{y}{2}\right)^2$ (iv) $\left(\frac{1}{m} + \frac{1}{n}\right)^2$

(v) $(5\ell + 2m)^2$ (vi) $\left(\frac{1}{3}a + \frac{1}{2}b\right)^2$ (vii) $(4x + 2y)^2$ (viii) $(7x + y)^2$

2. Simplify the following by using $(a - b)^2 = a^2 - 2ab + b^2$.

(i) $(3x - 4)^2$ (ii) $(5m^2 - n)^2$ (iii) $\left(\frac{2}{x} - \frac{3}{y}\right)^2$ (iv) $(4x^2 - 2y)^2$

(v) $(3x - 2y)^2$ (vi) $\left(\frac{1}{\ell} - \frac{1}{m}\right)^2$ (vii) $\left(\frac{1}{2a} - \frac{1}{3b}\right)^2$ (viii) $(4p - 2q)^2$

3. Simplify the following by using $a^2 - b^2 = (a - b)(a + b)$.

(i) $(x - 2y)(x + 2y)$ (ii) $(m^2 - n^2)(m^2 + n^2)$ (iii) $\left(\frac{1}{2p} + \frac{1}{2q}\right)\left(\frac{1}{2p} - \frac{1}{2q}\right)$

(iv) $(a^4 - b^4)(a^4 + b^4)$ (v) $\left(\frac{1}{5a^2} - \frac{1}{5b^2}\right)\left(\frac{1}{5a^2} + \frac{1}{5b^2}\right)$ (vi) $(5x - 2y)(5x + 2y)$

4. Simplify the following algebraic expressions by using algebraic identities:

(i) $(x - 3)^2 + (2x + 1)^2$ (ii) $(2x + 1)^2 - (3x - 2)^2$

(iii) $(2x - 5)(2x + 5x) - (3x + 1)^2$ (iv) $(7x - 2)^2 + (9x - 1)^2$

(v) $(a - 5b)(a + 5b) - (9a - 3)^2$ (vi) $(2a - 5b)^2 - (a - 3b)^2$

5. Find the missing term in each of the following:

(i) $(4x + 3y)^2 = 16x^2 +$ _____ $+ 9y^2$ (ii) $(a - 2b)^2 = a^2 - 4ab +$ _____

(iii) $\left(a + \frac{1}{2}b\right)^2 = a^2 +$ _____ $+ \frac{b^2}{4}$ (iv) $\left(\frac{x}{2} - \frac{y}{2}\right)^2 = \frac{x^2}{4} +$ _____ $+ \frac{y^2}{4}$

6. Evaluate the following using suitable identity:

(i) $(37)^2$ (ii) $(63)^2$ (iii) 34×26

2.2.9 Factorization of Algebraic Expressions

Factors of an expression are the expressions whose product is the given expression. The process of expressing the given expressions as a product of its factors is called "Factorization" or " Factorizing"

i Factorization of the Expression $Ka + Kb + Kc$

Factorize $Ka + Kb + Kc$

$= K(a + b + c)$ (By taking common factor)

Teachers' Guide

Tell the students factorization as the inverse process of multiplication of an algebraic expression with a number or a variable or both.

Example 18 Factorize $2x^2 - 4xy + 8xz$

Solution $2x^2 - 4xy + 8xz$

$= 2x(x - 2y + 4z)$ Since "$2x$" is a common factor

ii Factorization of the Expression $ac + ad + bc + bd$

Factorize the $ac + ad + bc + bd$

$= a(c + d) + b(c + d)$ (By taking common factor)

$= (c + d)(a + b)$

Example 19 Factorize $4x + ax + 4a + a^2$

Solution

$$4x + ax + 4a + a^2$$
$$= (4x + ax) + (4a + a^2)$$
$$= x(4 + a) + a(4 + a)$$
$$= (4 + a)(x + a)$$

Example 20 Factorize $2x^2y - 2xy + 4y^2x - 4y^2$

Solution

$$2x^2y - 2xy + 4y^2x - 4y^2$$
$$= 2y(x^2 - x + 2xy - 2y)$$
$$= 2y[x(x - 1) + 2y(x - 1)]$$
$$= 2y[(x - 1)(x + 2y)]$$
$$= 2y(x - 1)(x + 2y)$$

Search out

https://www.khanacademy.org/math/algebra-basics/alg-basics-algebraic-expressions?t=practice

EXERCISE 2.7

1. Factorize the following:

(i)	$6xy - 14yz$	(ii)	$30x^4 - 45x^2y$	(iii)	$6x^2y - 24yz$
(iv)	$7x^4 - 14x^2y + 21xy^3$	(v)	$x^2y^2z^2 - xyz^2 + xyz$	(vi)	$5x^5 + 10x^4 + 15x^3$
(vii)	$8a^3b^3c - 2a^2b^3c + abc$	(viii)	$5x^3y^3 - 15x^2y^2 + 5x^3y^4$	(ix)	$4x^3 - 8x^2y^3 + 12x^2y$
(x)	$2\ell^4m^4 - 5\ell^2m^3 - 2\ell^2m^2$	(xi)	$x^2y^5 - x^4y^6 + x^2y^3$	(xii)	$9p^2q^2 - 18pq + 27p^3q^4$

2. Factorize the following:

(i)	$x^2 + 5x - 2x - 10$	(ii)	$2ab - 6bc - a + 3c$	(iii)	$a(x - y) - b(x - y)$
(iv)	$y^2 - ay - by + ab$	(v)	$ab(x + 7) + cd(x + 7)$	(vi)	$a^2pq - a^2rs + b^2pq - b^2rs$
(vii)	$3x^2 + 6y^2 - 3xy^2 - 6x$	(viii)	$5x + 2xy - 2x^2 - 5y$	(ix)	$4x + 6y - 2 - 12xy$
(x)	$3xy + 3 - 3x - 3y$	(xi)	$3a^2 + 9bc - 9ac - 3ab$	(xii)	$2x^4 - 4x^2y + 2x^2 - 4y$

iii Factorization of the Expression $ax^2 + bxy + cy^2$

(By middle term breaking) where $a \neq 0, b \neq 0, c \neq 0$

In order to factorize $ax^2 + bxy + cy^2$, we have to find numbers m and n such that $m + n = b$ and $m \times n = ac$. This type can be explained with the following examples.

Example 21 Factorize $x^2 + 9x + 18$

Solution

$$1 \cdot x^2 + 9x + 18$$
$$= x^2 + 3x + 6x + 18$$
$$= x(x + 3) + 6(x + 3)$$
$$= (x + 3)(x + 6)$$

Working

Co-efficient of x^2 is 1 and constant value is 18. So, $1 \times 18 = 18$
All possible pairs whose product is 18 are:
$1 \times 18 = 18$; $2 \times 9 = 18$; $3 \times 6 = 18$
Now, select the pairs whose sum is 9 which is 3 and 6.

Example 22 Factorize $x^2 + 10 + 21$

Solution

$x^2 + 10 + 21$
$= x^2 + 7x + 3x + 21$
$= x(x + 7) + 3(x + 7)$
$= (x + 3)(x + 7)$

Example 23 Factorize $4x^2 - 4x - 3$

Solution

$4x^2 - 4x - 3$
$= 4x^2 - 6x + 2x - 3$
$= 2x(2x - 3) + 1(2x - 3)$
$= (2x + 1)(2x - 3)$

Teachers' Guide

Recall the concept of factors of a numeral to make students understand the concept of factorization in algebra.

EXERCISE 2.8

1. Factorize the following by using middle term breaking:

(i)	$x^2 - 10x + 24$	(ii)	$x^2 + 3x - 40$	(iii)	$4x^2 + 8x + 3$	(iv)	$x^2 + 7x + 10$
(v)	$x^2 + 2x - 8$	(vi)	$y^2 + 3y - 10$	(vii)	$25x^2 + 5x - 2$	(viii)	$x^2\ 6\ +x13\ - 5$
(ix)	$2x^4 + x^2 - 3$	(x)	$2\ell^2 - 3\ell - 5$	(xi)	$10m^2 - 13m - 3$	(xii)	$14m^2 - 37m + 5$

SUMMARY

- Algebra is an Arabic word. Algebra means "Reduction".
- A sentence which has one or more unknowns is called open sentence.
- An expression which connects variables and constants by mathematical operations (+, –, ×, ÷) is called an algebraic expression.
- The parts of an algebraic expression which are connected by operations (+, × , ÷ and –) are called terms of an algebraic expression.
- A term which can take various numerical values is called variable.
- A term which cannot be changed and has a fixed value is called constant.
- An Equation is a statement which shows the equal value of both the expressions.
- An inequality is a statement which shows that an expression is less than or greater than the other.
- A polynomial is an algebraic expression comprising of whole number as exponent of the variable. Polynomial consists of variables, coefficient and constant.
- There are three types of polynomials i.e. monomials, binomials and trinomials.
- Basic Algebraic Formulae-Identities:
  - $(a + b)^2 = a^2 + 2ab + b^2$
  - $(a - b)^2 = a^2 - 2ab + b^2$
  - $a^2 - b^2 = (a - b)(a + b)$

Activity

Gather the students in the middle of the room, and read multiple-choice questions and their possible answers aloud. Students then move to the corner that represents what they believe is the correct answer. The top left room corner can be option A, the bottom-left can be B and so on.

Sub-domain (iii) Linear Equations

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recall solving linear equations in one variable.
- Construct linear equations in two variables such as; $ax + by = c$, where a and b are not zero.
- Introduction to Cartesian coordinate system.
- Plot the graph of the linear equation $ax + b = 0$ where $a \neq 0$ and of linear equations in two variables.
- Recognise and state the equation of a horizontal line and a vertical line.
- Find values of x and y from the graph.

2.3.1 Recall Solving Linear Equations in One Variable

i Linear Equation

A linear equation is an equation in which the highest power of the variable(s) is always 1.

For example $3x = 24$, $y - 3x = 5$, $-8 - x = x - 4x$, etc.

ii Linear Equation in One Variable

A linear equation in one variable is an equation having only one variable of degree 1. The general or standard form of linear equation in one variable is written as $ax + b = 0$, where a and b are any two real numbers, $a \neq 0$ and x is a variable.

Variable

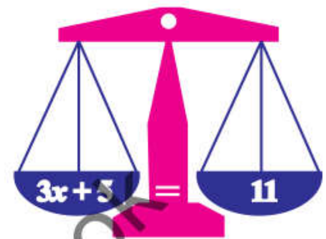
$$ax + b = 0$$

Coefficient　Constant

For example $5x = 7$, $10 - 3x = 7$, $5y + 10 = 5(y + 2)$, etc.

iii Solution of Linear Equation

A solution of linear equation is a number that makes the equation true when we replace it with the variable. Let's check whether 2 is the solution of $3x + 5 = 11$ or not?



Substitute $x = 2$ in $3x + 5 = 11$, we get

$3(2) + 5 = 11$

$6 + 5 = 11$ which is true

So, 2 is a solution.

iv Steps for Solving Linear Equations

The following steps are given for solving linear equation in one variable:

1. Remove parentheses on each side of the equation.
2. Clear fractions by multiplying all terms on both sides by the LCD (Least Common Denominator).
3. Group like terms on each side of =.
4. Isolate the variable on one side of the equation. (By adding or subtracting to combine like terms across the equal sign)
5. Divide both sides of the equation by the coefficient of the unknown.
6. Simplify the answer if necessary.

Example 1 Solve the equation $5x - 1 = 2(x - 5)$

Solution

$5x - 1 = 2(x - 5)$

$5x - 1 = 2x - 10$ (Step 1: Remove brackets)

$-2x + 5x - 1 = 2x - 10 - 2x$ (Step 4: Isolate variable)

$3x - 1 = -10$

$3x - 1 + 1 = -10 + 1$

$3x = -9$

$\frac{3x}{3} = \frac{-9}{3}$ (Step 5: Divide to solve for x)

$x = -3$

Hence, $x = -3$ is the solution of the given equation.

Remember!

Linear equation in one variable has only one solution.

Try yourself!

Verify that $x = -3$ is the solution of $5x - 1 = 2(x - 5)$

Think!

What is that number one third of which added to 5 gives 8?

EXERCISE 2.9

Solve the following linear equations:

1. $3x - 1 = 8$
2. $7x = 60 + 2x$
3. $5x + 4 = 2x + 17$
4. $5x + 11 = 20x - 64$
5. $3(2x - 5) = 21$
6. $3(x + 2) = 5 - 2(x - 3)$

Teachers' Guide

Write the terms "constant" and "variable" on the writing board. Encourage students to tell about these terms.

7. $-3(x+8)=2(x+3)+10x$ 8. $3(2x-8)=2x-2(x+3)$ 9.

10. $6-\frac{2}{3}(x+5)=4x$ 11. $\frac{2}{3}x-\frac{1}{2}=\frac{7}{6}+\frac{1}{2}x$ 12. $\frac{5}{4}x+3=x-\frac{1}{4}$

2.3.2 Real Life Problems Involving Linear Equations in One Variable

A word problem involving unknown numbers can be translated into a linear equation having one unknown quantity. The equation can be formed by using the conditions of the problem. By solving the resulting equation, the unknown quantity can be found.

i Steps to Solve a Real life Word Problem

The steps to solve a real life word problem are given below:

1. Read the statement of the word problem carefully and repeatedly to find the unknown quantity which is to be found.
2. Represent the unknown quantity by a variable.
3. Form an equation in the unknown variable by using the conditions given in the problem.
4. Solve the equation thus obtained.

Think!

If solution of an equation is given, then how many equations can you make?

Try yourself!

When you multiply a number by b and subtract 5 from the product, you get 7. Can you tell what the number is?

Riddle!

I am a number
Take me seven times over
To reach a triple century
Tell my identity!
And add a fifty!
You still need forty

Example 2 Hooria is 5 years younger than Farah. Two years later, Farah will be twice as old as Hooria. Find their present ages.

Solution Suppose Farah's present age be x.

Then Hooria's present age $= x-5$

After 12 years, Farah's age $= x+2$, Hooria's age $= x-5+2$.

According to given condition, Farah will be twice as old as Hooria.

Therefore, $x+2=2(x-5+2)$

$$x+2=2(x-3)$$
$$x+2=2x-6$$
$$x-2x=-6-2$$
$$-x=-8$$
$$x=8$$

Therefore, Hooria's present age $= x-5=8-5=3$

Hence, present age of Farah = 8 years and present age of Hooria = 3 years.

Activity

The teacher tells the class that the highest marks obtained in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?

Skill Practice

Majid is six years older than Kiran. If the sum of their ages is 76 years, find their ages.

Teachers' Guide

Make groups of students, assign linear equations in one variable and ask them to solve.

EXERCISE 2.10

1. When two is added to six more than a certain number, the result is 20. What is the number?
2. If four is subtracted from two times a certain number, the result is 10. What is the number?
3. Seventy more than 8 times a number is the same as two less than ten times the number. What is the number?
4. The sum of three consecutive integers is 123. What are the integers?
5. The sum of the ages of Abdul Hadi and Abdullah is 32. In two years Abdul Hadi will be three times as old as Abdullah. How old are they now?
6. Sakeena's father is 4 times as old as Sakeena. After 5 years, father will be three times as old as Sakeena. Find their present ages.

2.3.3 Construction of Linear Equation in Two Variables

Linear Equation in Two Variables

If a, b and c are real numbers (and if $a \neq 0$, $b \neq 0$) then $ax + by = c$ is called a linear equation in two variables (The "two variables" are the x and the y.) and known as standard form.

The numbers a and b are called the coefficients of the variables in the equation $ax + by = c$. The number c is called the constant of the equation $ax + by = c$. e.g., $11x + 3y = 7$, $2x - 3y = 5$, $\frac{x}{y} - 2 = 1$ etc.

Coefficient of x — Constant

$$\boldsymbol{ax + by = c}$$

Coefficient of y

Example 3 Convert $y - 4 = \frac{1}{3}(x + 3)$ into standard form of linear equation.

Solution $y - 4 = \frac{1}{3}(x + 3)$

Multiplying both sides by 3

$$3(y - 4) = 3 \times \frac{1}{3}(x + 3)$$

$$3(y - 4) = x + 3$$

$$3y - 12 = x + 3 \quad \text{(Remove brackets)}$$

$$-x + 3y - 12 = 3 \quad \text{(Transpose } x \text{ to L.H.S)}$$

$$-x + 3y = 12 + 3 \quad \text{(Transpose 12 to R.H.S)}$$

$$-x + 3y = 15$$

Which is required standard form.

Remember!

We can transpose a number or variable instead of adding or subtracting it form both sides of the equation.

Skill Practice

Write the following equations in standard form.

(a) $x - y = \frac{x}{2} + 3$

(b) $\frac{2x+1}{2} = \frac{y-1}{4}$

Teachers' Guide

Write any linear equation in two variables on the writing board. Ask different students to perform the one step and convert it into standard form.

Steps for Construction of Linear Equation in Two Variables

1. Read the statement of the word problem carefully and repeatedly to find the unknown quantities.
2. Represent the two unknown quantities by variables.
3. Form an equation by using the conditions given in the problem.

Example 4 Hashim buys 3 bats and 5 balls for Rs. 3800. Write the statement in equation form.

Solution Let the price of 1 bat = Rs. x ; The price of 1 ball = Rs. y

So, the price of 3 bats = Rs. $3x$

The price of 5 balls = Rs. $5y$

According to the given condition, $3x + 5y = 3800$

Skill Practice

If numerator and denominator of a fraction are increased by 5, the fraction becomes $\frac{7}{10}$. Construct the linear equation of the above statement.

Example 5 Sum of ages of Amna and Faiza is 25 years. Write the statement in an equation form with two variables.

Solution Let the age of Amna = x years

The age of Faiza = y years

According to the given condition, $x + y = 25$

EXERCISE 2.11

1 Convert the following equations into standard form:

(i) $y = -x - 3$ (ii) $y - 3x - 2 = 0$ (iii) $y - 1 = \frac{5}{3}(x + 2)$

(iv) $x - 1 = \frac{5y}{3} - 3$ (v) $\frac{x-1}{2} = \frac{y+2}{3}$ (vi) $\frac{2x+1}{4} = y - 1$

2 Construct the following statements into linear equations in two variables:

(i) The sum of two numbers is 11. (ii) The price of a book and 2 pencils is Rs.90.

(iii) The weight of Zainab is one-third of the weight of Hamid.

(iv) The sum of 2 times of 1^{st} number and 3 times of 2^{nd} number is 30.

(v) Sum of ages of Hania and Kahaf is 26 years.

(vi) The cost of 3 footballs and 7 basketballs is Rs.3000.

(vii) If numerator and denominator of a fraction are decreased by 3, the fraction becomes $\frac{2}{3}$.

2.3.4 Introduction to Cartesian Coordinate System

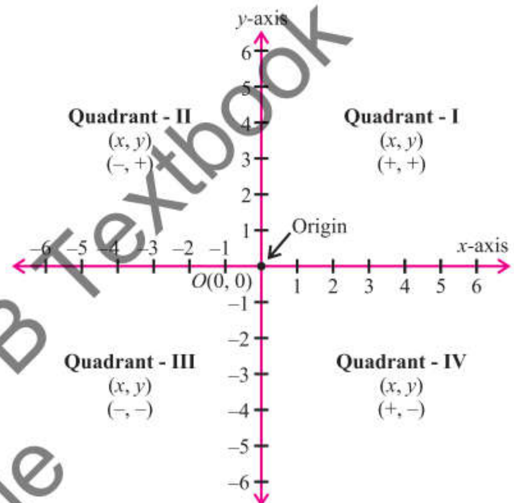
A cartesian coordinate system or rectangular coordinate system is named after the French mathematician and philosopher René Descartes (pronounced day-KART) (1596 - 1650), who is known as the "Father of

Teachers' Guide

Write some statements on the writing board. Ask students to convert these statements in equation form.

Modern Mathematics". He was having problem falling asleep one night. While trying to fall asleep, he seemed up on the tiled ceiling and noticed a fly. His thoughts started to wander and a query popped in his head: should he describe the way of the fly without tracing the real path? From that query came the innovatory invention of the coordinate system- an invention which made it possible to linkage algebra and geometry.

i Parts of Cartesian coordinate system



Axes The two intersecting number lines that are perpendicular to each other in the cartesian plane are known as axes. These are x – axis and y – axis.

- The horizontal number line is called the ***x* – axis.**
- The vertical number line is called the ***y* – axis.**

On the x-axis, the numbers to the right of the origin are positive and the numbers to the left are negative. On the y-axis, the numbers above the origin are positive and the numbers below the origin are negative.

Origin The point of intersection of the horizontal and vertical number lines is called the origin. It is denoted by (0, 0).

Quadrants The regions that divide the cartesian plane into four equal parts are called the quadrants. These quadrants are numbered in sequence as Quadrant I, Quadrant II, Quadrant III and Quadrant IV moving in a counter-clockwise direction starting from the upper right as shown in the above figure.

Remember!
The numbers in the ordered pair are called coordinates.

Ordered Pair Points in the cartesian plane are labelled with ordered pair (x, y).

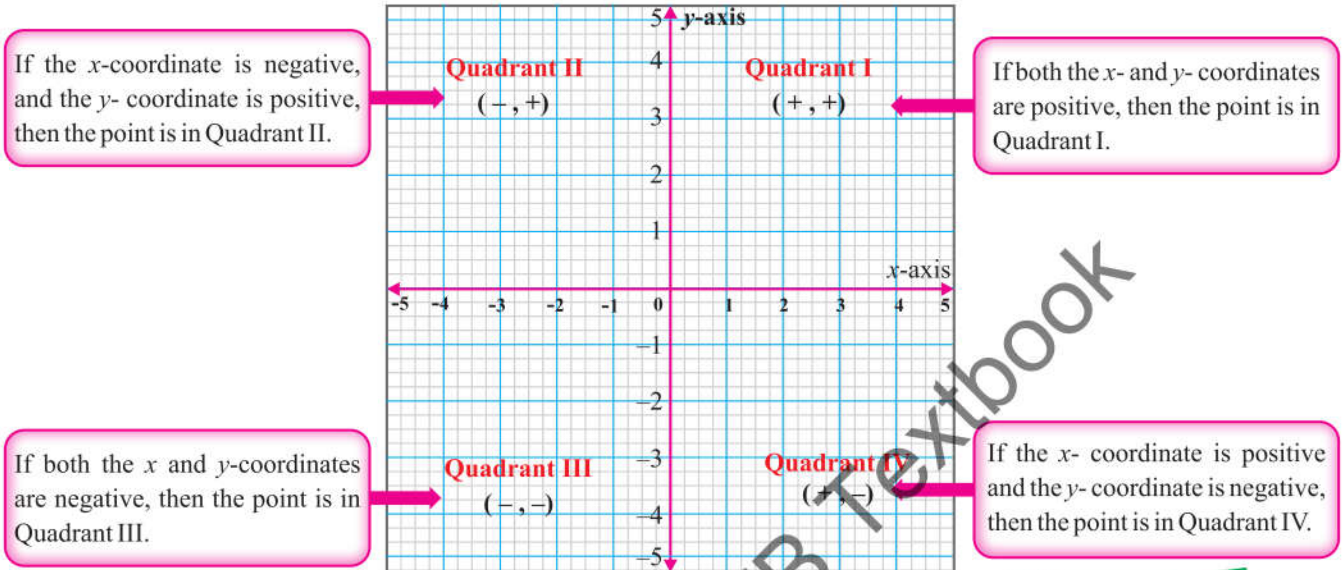
- **Abscissa (x-coordinate):** The first number (called the abscissa or x –coordinate) of an ordered pair is the horizontal distance from the origin.
- **Ordinate (y-coordinate):** The second number (called the ordinate or y–coordinate) of an ordered pair is the vertical distance from the origin.

Keep in mind!
The x-coordinate always comes first and the y-coordinate always comes second whether the numbers are positive or negative.

Remember!
Perpendicular means that two lines intersect each their at a right angle.

Teachers' Guide
Ask students that have you ever heard the name of Rene' Descartes? Also draw cartesian plane on the writing board and explain its all parts.

- The signs of first and second coordinates of a point vary in the four quadrants as indicated below.



This means that you can easily tell that in which quadrant an ordered pair is located by just simply looking at the signs of the coordinates.

Remember!

There are also points which lie on the x and y-axes. The points which lie on the x-axis have coordinates $(x, 0)$ and the points which lie on the y-axis have coordinates $(0, y)$.

Example 6

Identify the quadrants in which each of the following points lie:

(i) (2, 6) (ii) (1,–5) (iii) (–7,–1) (iv) (–1,9)

Solution

(i) Quadrant I, because the x-coordinate and y-coordinate are both positive.

(ii) Quadrant IV, because the x-coordinate is positive and y-coordinate is negative.

(iii) Quadrant III, because the x-coordinate and y-coordinate are both negative.

(iv) Quadrant II, because the x-coordinate is negative and y-coordinate is positive.

ii Plotting Points on the Cartesian Plane

For plotting points in the cartesian plane, the value of abscissa and the ordinates will tell you how to locate the point on the plane.

Here are the steps for plotting a point on the cartesian plane:

1. **Start first with the origin**: The origin will serve as your reference point.
2. **Look into the abscissa**: If the sign is positive, the point moves to right side along the x-axis and if the sign is negative, the point moves to the left side along the x-axis.
3. **Look into the ordinate:** If the sign is positive, the point moves to the upward along the y-axis and if the sign is negative, the point moves to the downwards along the y-axis.

Teachers' Guide

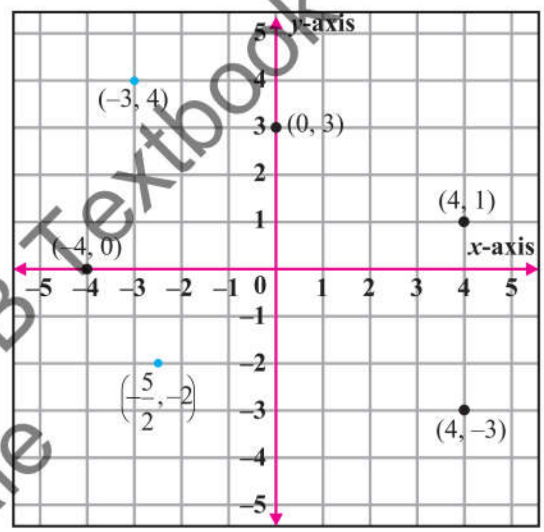
- Write "Cartesian coordinate system" on the writing board. Ask students to brainstorm and tell about it.
- Write different points on the writing board and ask students to identify and tell in which quadrant they lie?

Example 7 Plot each point and state the quadrant or axis where it is located.

(a) (4,1) (b) (–3, 4) (c) (4, –3) (d) $\left(-\frac{5}{2}, -2\right)$ (e) (0, 3) (f) (– 4, 0)

Solution **(a)** (4, 1) means that the point is located 4 units to the right of the y-axis and 1 unit above the *x*-axis. Since the signs of both the coordinates are positive, so the given point lies in Quadrant I.

(b) (–3, 4) means that the point is located 3 units to the left of the y-axis and 4 units above the *x*-axis. Since the sign of the abscissa is negative and the sign of the ordinate is positive, so the given point lies in Quadrant II.



(c) (4, –3) means that the point is located 4 units to the right of the *y*-axis and 3 units below the *x*-axis. Since the sign of the abscissa is positive and the sign of the ordinate is negative, so the given point lies in Quadrant IV.

(d) $\left(-\frac{5}{2}, -2\right)$ means that the point is located units to $\frac{5}{2}$ the left of the y-axis and 2 units below the *x*-axis. Since the signs of both the coordinates are negative, so the given point lies in Quadrant III.

(e) (0, 3) means that the point is located 3 units above *x*-axis. As abscissa is 0, so the given point lies on the *y*-axis.

(f) (– 4, 0) means that the point is located 4 units to the left of the *y*-axis. As ordinate is 0, so the given point lies on the *x*-axis.

2.3.5 Graph of a Linear Equation of the Form $ax + b = 0$

To plot a graph of the linear equation of the form $ax + b = 0$ where $a \neq 0$, first we convert it into the form $x = a$.

Let's draw graph of $2x - 6 = 0$

First of all we will isolate the variable "x"

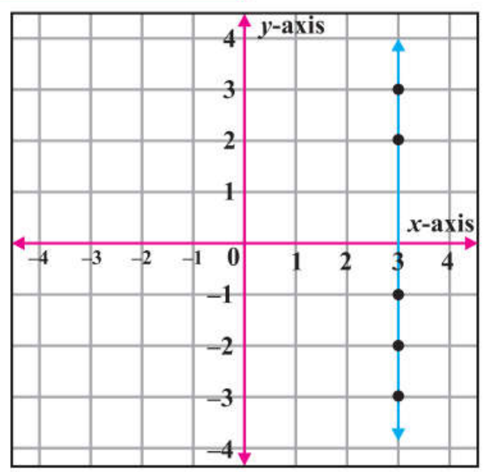
$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

Table

x	y
3	–3
3	–2
3	–1
3	2
3	3



Vertical Line

The graph of $x = a$ is a vertical line passing through the point $(a, 0)$ and parallel to y-axis.

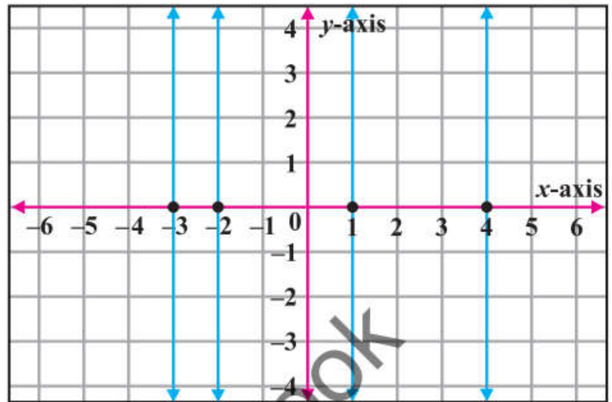
Teachers' Guide

The points (–3, 4) and (4,-3) are in different quadrants. Changing the order of the coordinates changes the location of the point. That is why points are represented by ordered pairs.

Example 8

Plot the graphs of the following linear equations:

$x = -3, x = 1, x = -2, x = 4$



Solution

We observe that all lines

$x = -3, x = 1, x = -2$ and $x = 4$

are parallel to *y-axis*.

2.3.6 Graph of a Linear Equation of the Form $cy + d = 0$

To plot a graph of the linear equation of the form $cy + d = 0$ where $c \neq 0$, first we convert it into the form $y = b$.

Let's draw graph of $3y + 6 = 0$

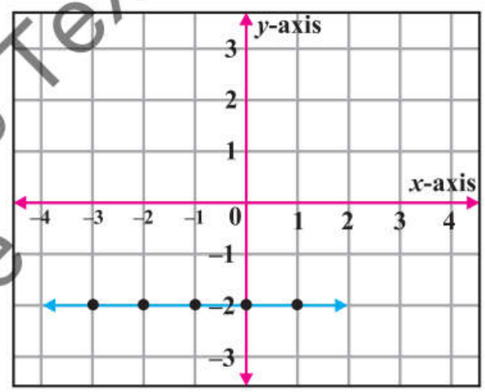
First of all we will isolate the variable "y"

$$3y = -6$$

$$\frac{3x}{3} = \frac{-6}{3}$$

$$y = -2$$

Table	
x	y
1	–2
0	–2
–1	–2
–2	–2
–3	–2



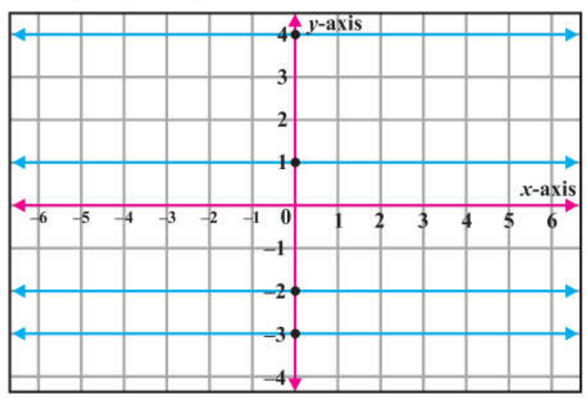
Horizontal Line

The graph of $y = b$ is a horizontal line passing through the point $(0, b)$ and parallel to x-axis.

Example 9

Plot the graphs of the following linear equations:

$y = 4, y = 1, y = -2, y = -3$



Solution

We observe that all lines

$y = 4, y = 1, y = -2$ and $y = -3$

are parallel to *x-axis.*

2.3.7 Graph of a Linear Equation of the Form $ax + by = c$

We can plot the graph of a linear equation in two variables by the following steps given below:

1. Isolate the variable "y" from the given equation.

Teachers' Guide

Plot the above lines on the writing board and explain each step to the students.

2. Find at least three integral values of both variables x and y.
3. Plot the points (ordered pairs) on the graph paper.
4. Join the points.

Example 10 Graph $3x - 2y = -6$

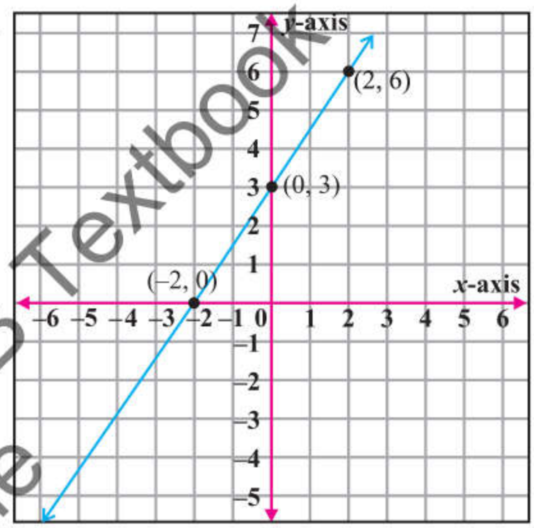
Solution The equation $3x - 2y = -6$ is written in standard form. To graph this equation, first of all isolate the y from the given equation.

$$3x - 2y = -6$$
$$3x - 2y - 3x = -3x - 6$$
$$-2y = -3x - 6$$

Next, choose three values of x and calculate the corresponding y-coordinates.

Plot the points on the graph and draw a line through them.

x	$y = \frac{3}{2}x + 3$	(x, y)
-2	$y = \frac{3}{\cancel{2}}(-\cancel{2}) + 3$ $= -3 + 3 = 0$	$(-2, 0)$
0	$y = \frac{3}{2}(0) + 3$ $= 0 + 3 = 3$	$(0, 3)$
2	$y = \frac{3}{\cancel{2}}(\cancel{2}) + 3$ $= 3 + 3 = 6$	$(2, 6)$



The line intersects the x-axis at $(-2, 0)$ and y-axis at $(0, 3)$.

x-Intercept: The point where the line intersects the x-axis is called the x-intercept. At x-intercept, y-coordinate is 0.

y-Intercept: The point where the line intersects the y-axis is called the y-intercept. At y-intercept, x-coordinate is 0.

Skill Practice

Draw the graph for the following linear equations:

(a) $x = -7$ (b) $y = -9$

(c) $x + 5y = 10$ (d) $5x - 2y = 20$

Example 11 Graph $-2x + 4y = 8$

Solution To graph the given equation, let's find the x and y-intercepts.

Put $y = 0$ in $-2x + 4y = 8$, we get

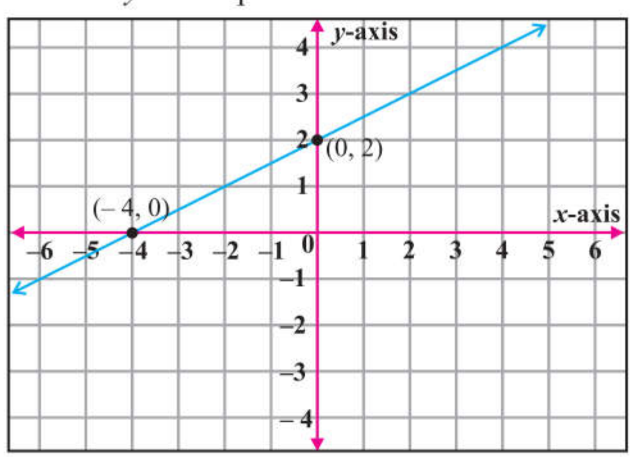
$$-2x + 4(0) = 8$$
$$-2x = 8$$
$$x = -4$$

The x-intercept is $(-4, 0)$.

Put $x = 0$ in $-2x + 4y = 8$, we get

$$-2(0) + 4y = 8$$
$$4y = 8$$
$$y = 2$$

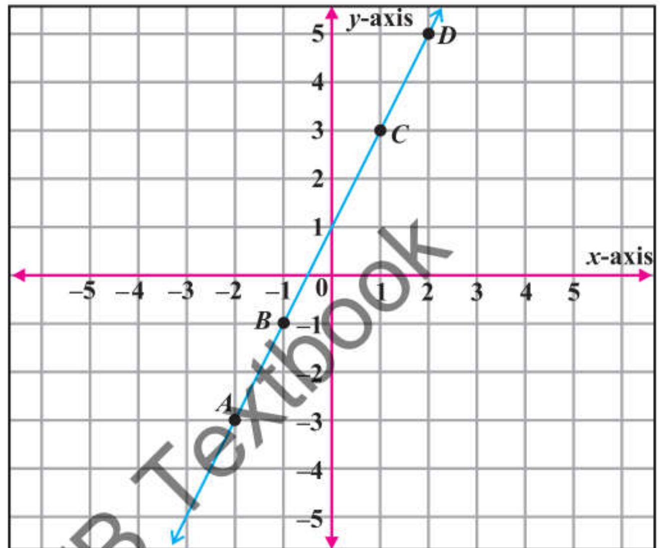
The y-intercept is $(0, 2)$.



Next plot each intercept, label the points and draw a line through them.

2.3.8 Finding the Values of x and y from the Graph

Consider the following graph of the linear equation in two variables:



Let's find the coordinates of the points A, B, C and D as shown on the given line.

To find the x-coordinate of point A, look up on x-axis from A i.e., $x = -2$ and to find the y-coordinate of A, look on y-axis against A i.e., $y = -3$. Thus, $A(-2, -3)$

For the x-coordinate of point B, look up on *x-axis* from B i.e., $x = -1$ and for y-coordinate of B, look on y-axis against B i.e., $y = -1$. Thus, $B(-1, -1)$

For the x-coordinate of point C, look down on x-axis from C i.e., $x = 1$ and for the y-coordinate of C, look on y-axis against C i.e., $y = 3$. Thus $C(1, 3)$

For the x-coordinate of point D, look down on *x-axis* from D i.e., $x = 2$ and for the y-coordinate of D, look on *y-axis* against D i.e., $y = 5$. Thus, $D(2, 5)$

EXERCISE 2.12

1. Plot each point and state the quadrant or axis where it is located:

(i) $A(-6, 3)$ (ii) $B(-3, -3)$ (iii) $C(-2, 7)$
(iv) $D(6, -10)$ (v) $E(3, 4)$ (vi) $F(-4, -2)$

2. Draw the graph of the following linear equations in one variable:

(i) $y = -10$ (ii) $x = 6$ (iii) $3x + 7 = 9$
(iv) $2x - 8 = 3x$ (v) $2y + 9 = 23$ (vi) $y = \frac{5}{2}$

3. Draw the graph of the following linear equations in two variables:

(i) $y = -3x + 2$ (ii) $y = 2x - 3$ (iii) $2x - y = 4$
(iv) $3x - 5y = 15$ (v) $y = \frac{1}{2}x - 5$ (vi) $y = -\frac{2}{3}x + 4$

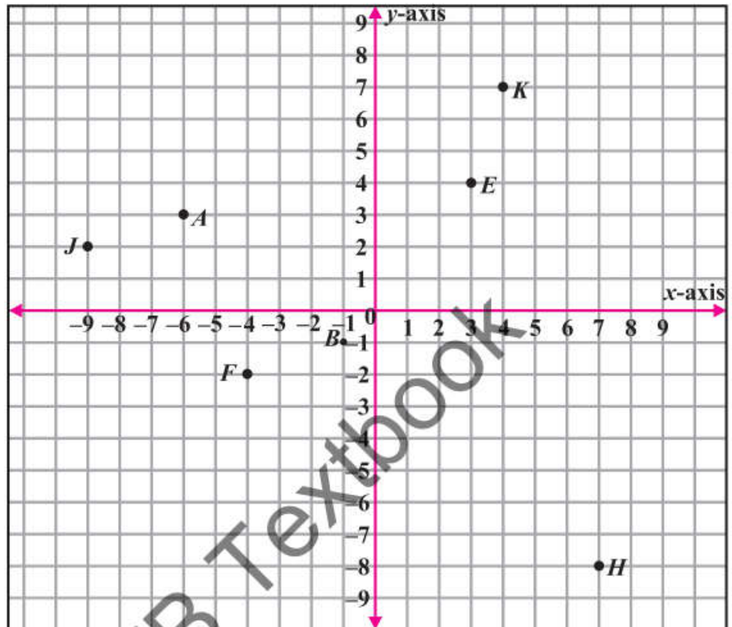
4. Recognize which of the following is an equation of horizontal line or vertical line:

(i) $y = \frac{7}{3}$ (ii) $x = 18$ (iii) $2x = 50 - x$ (iv) $y = 12 - 2y$

Teachers' Guide

Explore x-intercept and y-intercept by drawing the line on the writing board.

5. Find the values of x and y of the points A, E, F, H, J and K for the given graph.



SUMMARY

- A linear equation is an equation in which the highest power of the variables is always 1.
- A linear equation in one variable is an equation having only one variable of degree 1.
- The general form of linear equation in one variable is $ax + b = 0$, where a and b are any two real numbers, $a \neq 0$ and x is a variable.
- If a, b, and c are real numbers (and if $a \neq 0$, $b \neq 0$) then $ax + by = c$ is called a linear equation in two variables.
- The cartesian coordinate system is composed of two intersecting number lines that are perpendicular to each other at the point zero.
- The two intersecting number lines that are perpendicular to each other in the cartesian plane are known as axes.
- The point of intersection of the horizontal and vertical number lines is called the origin.
- The regions that divide the cartesian plane into four equal parts are called the quadrants.
- The first number (called the abscissa or x-coordinate) of an ordered pair is the horizontal distance from the origin.
- The second number (called the ordinate or y-coordinate) of an ordered pair is the vertical distance from the origin.
- The graph of $x = a$ is a vertical line passing through the point $(a, 0)$ and parallel to *y-axis*.
- The graph of $y = b$ is a horizontal line passing through the point $(0, b)$ and parallel to *x-axis*.
- The point where the line intersects the *x-axis* is called the x-intercept.
- The point where the line intersects the *y-axis* is called the y-intercept.

REVIEW EXERCISE 2

1. Choose the correct option.

(i) What number was subtracted to make the sequence 58, 56, 54, 52?

(a) 1 (b) 3 (c) 2 (d) 4

(ii) In sequence: 30, 33, 36, ... the next term is:

(a) 38 (b) 39 (c) 40 (d) 43

(iii) Which of the following sequence starts at 36 and having common difference 2?

(a) 34, 35, 36 (b) 36, 36, 36 (c) 36, 39, 42 (d) 36, 38, 40

(iv) The n^{th} term of the sequence 1, 3, 5, 7, ... is:

(a) $n^2 + 1$ (b) $3n + 4$ (c) $2n - 1$ (d) $2n + 1$

(v) $(z^4)^3 \div z^4$

(a) z^{16} (b) z^{12} (c) z^3 (d) z^8

(vi) Which one of the following is polynomial

(a) $8x^{-2} + 2$ (b) $9x^2 + 5x^{-1}$ (c) $2x^2 + 5$ (d) $\frac{5}{x} + 3$

(vii) The factorization of $x^2 - 4$ is:

(a) $(x-4)^2$ (b) $(x-2)^2$ (c) $(x-4)^3$ (d) $(x+2)(x-2)$

(viii) The example of linear equation in two variables is:

(a) $7x + 3y - 4z = 0$ (b) $6x + 5y = 10$

(c) $8x = 2 + 9$ (d) $y = 1 + 3y$

(ix) (9, –6) lies in __________ quadrant.

(a) I (b) II (c) III (d) IV

(x) The solution of $2x - 3 = 7$ is:

(a) 5 (b) 7 (c) 11 (d) 12

2. Write the first three terms of the following general terms.

(i) $4n - 9$ (ii) $3n + 7$ (iii) $7n + 2$

3. Find 15^{th} term of $a_n = \frac{1}{2n+4}$.

4. Ali has Rs. 50. He spends Rs. 5 each day. How much money does he have left after 5 days?

5. Aslam walks 2 kilometres on Monday, 4 kilometres on Tuesday, 6 kilometres on Wednesday, how much distance will he cover on Friday?

6. A painter used 2 litres paint in first week, 6 litres paint in the second week, 10 litres in the third week. How much paint did he use on the 6th week, if pattern is continued?

7. Add the following:

(i) $7x^2 + 8x + 3,\ 12x^2 + 2x + 6$ (ii) $15x^2 - 5x + 2,\ 7x^2 - 3x - 8$

(iii) $20x^2 + 8x - 10,\ 8x^2 - x - 8$

8. Subtract the first polynomial from the 2^{nd} polynomial:

(i) $10x^2 - 8x - 5,\ 15x^2 + 2x + 8$ (ii) $8x^2 - 5x + 2,\ 7x^2 + 2x - 8$

9. Solve the following:

(i) $(8x^2y^2 + 3xy^2 + 2)(3xy^2)$ (ii) $(5x^3y + 8)(-2x^2 + y)$ (iii) $\left(\frac{1}{2}\ell^2 m + \frac{1}{3}\right)(2\ell m^2 + 2)$

10. Divide the polynomials.

(i) $10x^3y^2$ by $5xy$ (ii) $15\ell m^3$ by $3\ell m^2$ (iii) $(28a^3b^3 + 7a^2b^2 + 14a^4b^3)$ by $7a^2b^2$

11. Simplify each of the following:

(i) $2x^2 - \{4(2x-2) - (4x^2 - \overline{3+2})\}$ (ii) $5x - [-2x + 5 - \{2x^2 - 5x + 2\}]$

12. Simply the following:

(i) $(7x - 3y)(7x + 3y)$ (ii) $(3a - 5b)(3a + 5b) - (4a - 2)^2$

13. Factorize the following:

(i) $x^8y^6 - x^4y^3 + x^2y^2$ (ii) $20x^2 + 10x - 210$ (iii) $4x^3 + 2x^2 + 4x^2y + 2xy$

14. Solve the following linear equations:

(i) $3x = 72 - 3x$ (ii) $6x + 3 = 23 + x$ (iii) $28 - x = 17 + 3x$

(iv) $3(4 + x) = 5(10 + x)$ (v) $\frac{4}{3} = \frac{x+10}{15}$ (vi) $\frac{x-2}{3} + \frac{1}{6} = \frac{5}{6}$

15. When 18 is subtracted from six times a certain number, the result is −42. What is the number?

16. A certain number added twice to itself equals 96. What is the number?

17. Construct the following statements into linear equations:

(a) The difference of the two numbers is 13.

(b) In a two digit number. The units digit is thrice the tens digit.

18. Plot each point and state the quadrant or axis where it is located.

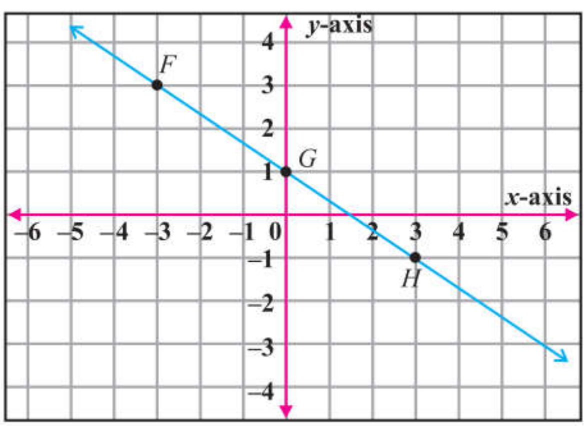
(i) $L(-9, 2)$ (ii) $M(4, 7)$ (iii) $N(8, -1)$

19. Draw the graph of the following linear equations:

(i) $x + 1 = 8$ (ii) $y = 6$ (iii) $y - 3 = 7$

(iv) $x = -5$ (v) $2x + 4y = 8$ (vi) $3x + 2y = 12$

20. Find the values of x and y of the points F, G and H for the following graph:



21. Recognize which of the following equations represent horizontal or a vertical line.

(i) $3y - 7 = 88$ (ii) $4x - 8 = x + 11$ (iii) $2(x + 2) = 42$

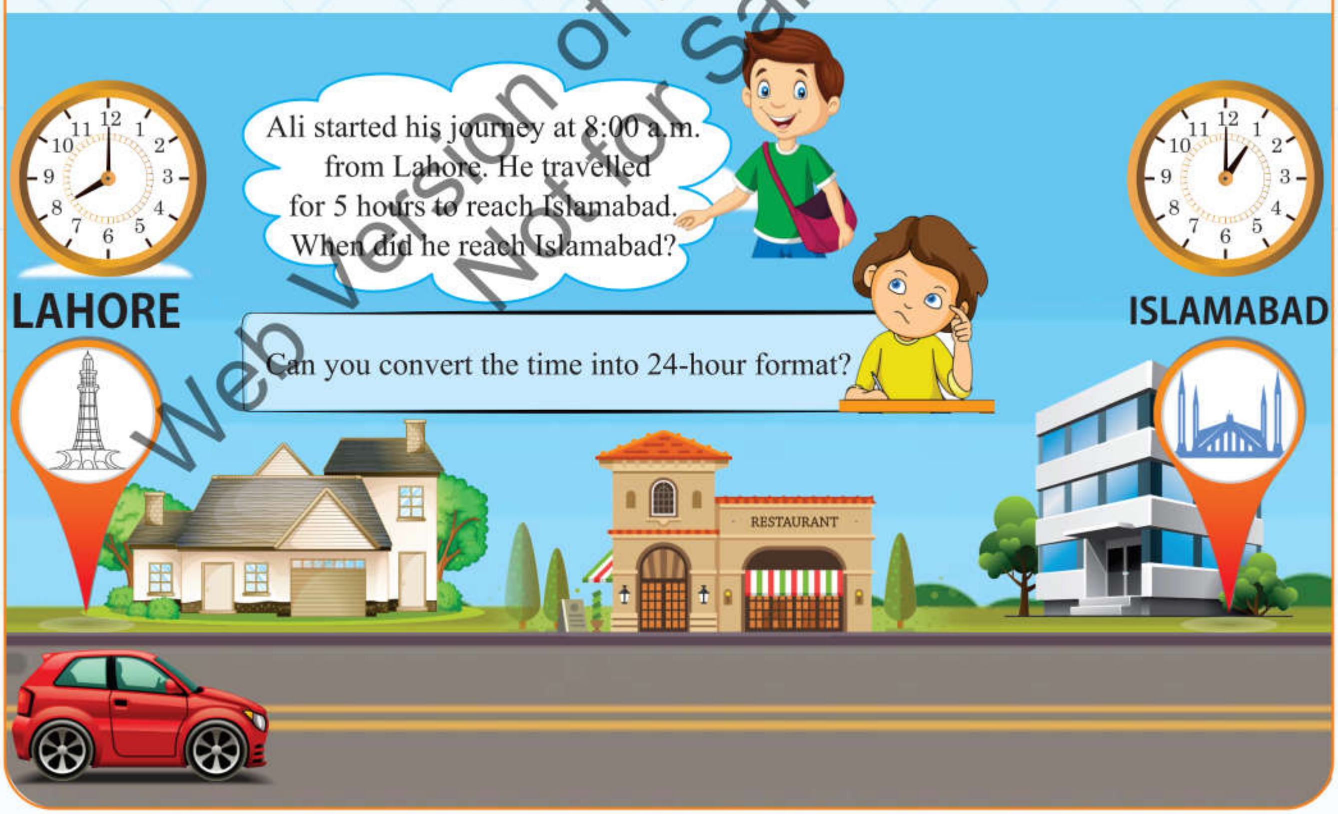
Domain 3 MEASUREMENTS

Sub-domain (i) Distance, Speed and Time

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Convert different units of distance.
- Convert 12 hour clock to 24 hour clock and vice versa.
- Convert between different units of time and speed.
- Calculate arrival time, departure time and journey time in a given situation (on the previous day and the next day).
- Solve real-world word problems involving distance, time and average speed.
- Differentiate between uniform and average speeds.



INTRODUCTION

We frequently use the terms like distance, speed and time in our daily life. For example, seeing remaining time of the bus at the metro station, speed limit shown on the road, distance shown from one city to the other city on the road, etc.

3.1.1 Distance

The length of space between two points is known as distance. Large distances are usually measured in kilometres and small distances are measured in metres. There are other smaller units (dm, cm, mm) which are used to measure the length or height of small objects like pencils, books and erasers etc.

Do you know?

The other units for measuring distances are inches, feet and yards etc.

Now, we learn about conversion between units.

i Conversion of Units of Distance

Important Information

Conversion of Distance Units
(From larger to smaller units)

1 m = 10 dm
1 m = 100 cm
1 m = 1000 mm
1 km = 1000 m

Example 1 A trip of school students is going to the museum from school. They cover the distance of 15 km 200 m. How much distance do they cover in metres ?

Solution To find out the distance in metres, we will multiply 15 km by 1000 and then add 200 into it.

$$15 \text{ km } 200 \text{ m} = 15 \times 1000 + 200$$
$$= 15000 + 200$$
$$= 15200 \text{ m}$$

Hence, they cover 15200 m from school to the museum.

Remember!

To convert from larger to smaller units, we multiply the given number by the number of smaller units it contains.

Teachers' Guide

Give flashcards of different units of distance to the students and ask them to convert the units as per given instructions.

Example 2 A tailor used 1 m 20 cm of cloth to stitch a shirt and 1 m 15 cm to stitch another shirt. What is total length of the cloth used to stitch two shirts in centimetres.

Solution To solve this problem, first of all we will add the lengths of both cloth, then covert it into cm.

	m	cm
Length of 1^{st} cloth =	1	20
Length of 2^{nd} cloth = +	1	15
Total length =	2	35

Now, we will multiply 2 m by 100, and then add 35 into it.

$$2 \text{ m } 35 \text{ cm} = 2 \times 100 + 35$$
$$= 200 + 35$$
$$= 235 \text{ cm}$$

Hence, the length of both the cloth is 235 cm.

Important Information

Conversion of Distance Units

$$1 \text{ dm} = \frac{1}{10} \text{ dm} = 0.1 \text{ m}$$
$$1 \text{ mm} = \frac{1}{1000} \text{ m} = 0.001 \text{ m}$$
$$1 \text{ m} = \frac{1}{1000} \text{ km} = 0.001 \text{ km}$$
$$1 \text{ mm} = \frac{1}{10} \text{ cm} = 0.1 \text{ cm}$$

Remember!

To convert from smaller to larger units, we divide the given number by the number of smaller units (the larger unit contains).

Example 3 Convert 15 km into m.

Solution As we know that

$$1 \text{ km} = 1000 \text{ m}$$
$$15 \text{ km} = 15 \times 1 \text{ km}$$
So, $$15 \text{ km} = 15 \times 1000 \text{ m}$$
$$= 15000 \text{ m}$$

Example 4 Convert 72 m into cm.

Solution As we know that

$$1 \text{ m} = 100 \text{ cm}$$
$$72 \text{ m} = 72 \times 1\text{m}$$
So, $$72 \text{ m} = 72 \times 100 \text{ cm}$$
$$= 7200 \text{ cm}$$

Example 5 Convert 4000 m into km.

Solution As we know that

$$1 \text{ m} = \frac{1}{1000} \text{ km}$$
So, $$4000 \text{ m} = 4000 \times \frac{1}{1000} \text{ km}$$
$$= 4 \text{ km}$$

Example 6 Convert 8 m 25 cm into m

Solution As we know that

$$1 \text{ cm} = \frac{1}{100} \text{ m}$$
$$= 25 \times \frac{1}{100} \text{ m}$$
$$= 0.25 \text{ m}$$

Hence, $$8 \text{ m } 25 \text{ cm} = 8 \text{ m} + 0.25 \text{ m}$$
$$= 8.25 \text{ m}$$

EXERCISE 3.1

1. Convert:

(i)	5 km 313 m into m	(ii)	10 km 200 m into m
(iii)	120 m 78 cm into cm	(iv)	200 m into cm
(v)	54 m into cm	(vi)	95 km into m

2. Zahid goes for a walk daily. The distance from his house to the park is 5100 m. Find the distance between house and park in km and m.

3. Sana bought 20 m 75 cm of cloth and she used 15 m 85 cm from it. How much cloth is left with her in cm?

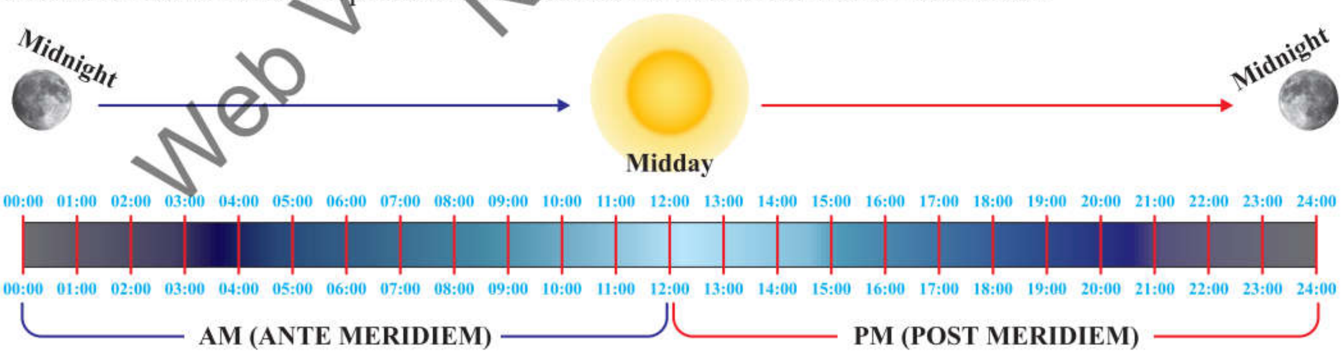
4. The distance between Azra's house and Masjid is 4500 m. The distance between Ayesha's house and Masjid is 7600 m. Whose house is nearer to the Masjid and how much in km?

5. The length of two ropes are 20 cm 2 mm and 15 cm 9 mm. What is the difference between the lengths of two ropes in mm?

3.1.2 Time

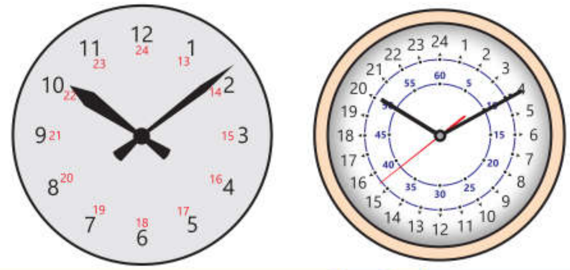
Time can be defined as the duration of a period in which an action, process or condition exists or continues. It is a sequence of events that occurs in succession from the past through present to the future. In daily life, time is usually measured by using analogue or digital clocks.

There are two formats to represent time based on 12 hour clock and 24 hour clock.



Time from midnight to midday is written with a.m. and time from midday to midnight is written with p.m.

We observe that 12 hour clock repeats itself after every 12 hours and 24 hour clock repeats itself after every 24 hours.



i Conversion of 12 hour Time into 24 hour Time

For the time between midnight to midday, we add nothing and write time as it is without a.m. and p.m.

For the time between midday to midnight, we add 12 to the original time and write without a.m. and p.m.

Important Information

24 hour time is also known as Military time. Military time is written using four digits without colon ":" e.g., 16:00 is written as 16 hours.

Example 7 Convert 5:45 a.m. into 24 hour time.

Solution **Step 1** Remove a.m. i.e., 5:45

Step 2 Since time is between midnight and midday, so we write it as it is.
Therefore, the 24 hour time is 05:45.

Need to Know!

Midnight time in 24 hour clock is written as 00:00.

Activity

Guide the students to search the time table of a specific bus / train / aeroplane moving from one destination to another. Ask the students to find out time differences between different destinations and convert the time into 12 hour or 24 hour.

Example 8 Convert 4:15 p.m. into 24 hour time.

Solution **Step 1** Remove p.m. i.e., 4:15

Step 2 Since time is between midday and midnight, so we add 12 hours to the original time.
Therefore, the 24 hour time is

$$4:15 + 12 = 16:15$$

Important Information

Sun dial is the earliest type of time keeping device, which indicates the time of the day by the position of the shadow of some object exposed to the sun rays. As the day progresses, the sun moves across the sky, causing the shadow of the object to move and indicating the passage of time.

ii Conversion of 24 hour Time into 12 hour Time

For the time between 00:00 to 11:59, we do nothing and write a.m. with the original time.
For the time between 12:00 to 23:59, we subtract 12 hours from the original time and write with p.m.

Example 9 Convert 06:15 into 12 hour time.

Solution **Step 1** Since time is between 00:00 and 11:59, so we write it as it is. 06:15

Step 2 Write a.m. with the original time. 6:15 a.m.

Example 10 Convert 16:30 into 12 hour time.

Solution **Step 1** Since time is between 12:00 and 23:59, so we subtract 12 hours from the original time. i.e., $16:30 - 12 = 4:30$

Step 2 Write p.m. with the calculated time. Therefore, the 12 hour time is 4:30 p.m.

Skill Practice

Write the times of prayers and convert it into 24 hour clock.

iii Conversion of Units of Time

Kalsoom completes a project in the subject of science in 3 hours 20 minutes. In how many minutes does she complete the

To solve this problem, we will multiply 3 hours to 60 and then add 20 minutes to it.

$$\begin{aligned} 3 \text{ hours } 20 \text{ min} &= 3 \times 60 + 20 \\ &= 180 + 20 \\ &= 200 \text{ min} \end{aligned}$$

Skill Practice

Convert 200 minutes into seconds

Hence, Kalsom completes the project in 200 minutes.

Important Information

Conversion of Units of Time

1 hour = 60 minutes

1 minute = 60 seconds

OR

1 minute = $\frac{1}{60}$ hour

1 second = $\frac{1}{60}$ minutes

Zeeshan needs 10 months 25 days to complete a house. How many days will be required for the construction of the house?

To solve this problem, we will multiply 10 months to 30 and add 25 days to it.

$$\begin{aligned} 10 \text{ months } 25 \text{ days} &= 10 \times 30 + 25 \text{ days} \\ &= 300 + 25 \\ &= 325 \text{ days} \end{aligned}$$

Hence, 325 days will be required to construct the house.

Important Information

Conversion Units of Time

1 week = 7 days	1 month = 30 days
1 day = $\frac{1}{7}$ week	1 day = $\frac{1}{30}$ month
1 year = 12 months	1 month = $\frac{1}{12}$ year

Skill Practice

Convert:
- 5 months 15 days into days
- 420 days into months and days

Example 11 Convert 1 year 6 months into months.

Solution As, we know that:

$$\begin{aligned} 1 \text{ year} &= 12 \text{ months} \\ 1 \text{ year } 6 \text{ months} &= 1 \times 12 + 6 \\ &= 18 \text{ months} \end{aligned}$$

Example 12 Convert 7 weeks 5days into days.

Solution As, we know that:

$$\begin{aligned} 1 \text{ week} &= 7 \text{ days} \\ 7 \text{ weeks } 7 \text{ days} &= 7 \times 7 + 5 \\ &= 49 + 5 = 54 \text{ days} \end{aligned}$$

Example 13 Convert 30 months into years and months.

Solution As, we know that: 1 month = $\frac{1}{12}$ year

Divide 30 months by 12 to convert it into years

$$30 \text{ months} = \frac{30}{12} \text{ years} = 2.5 \text{ years} = 2 \text{ years } 6 \text{ months}$$

Working

$$\begin{array}{r} 2 \\ 12\overline{)\,30} \\ -24 \\ \hline 6 \end{array}$$

Example 14 Convert 108 days into weeks and days.

Solution As, we know that: $1 \text{ day} = \frac{1}{7} \text{ week}$

Divide 108 days by 7 to convert it into weeks.

$$108 \text{ days} = \frac{108}{7} \text{ weeks} \quad = 15 \text{ weeks } 3 \text{ days}$$

Working

$$\begin{array}{r} 15 \\ 7\overline{)108} \\ -7 \\ \hline 38 \\ -35 \\ \hline 3 \end{array}$$

EXERCISE 3.2

1. Convert:
   - (i) 15 hours 30 minutes into minutes
   - (ii) 15 minutes 55 seconds into seconds
   - (iii) 12 months 15 days into days
   - (iv) 18 weeks 3 days into days
   - (v) 56 months into years and months
   - (vi) 60 days into weeks and days
   - (vii) 870 days into years and days
   - (viii) 3900 seconds into minutes and seconds

2. Complete the following table:

Sr. #	12 hour time	24 hour time
(i)	4:50 a.m.	
(ii)		09:30
(iii)	7:10 p.m.	
(iv)		21:05
(v)	Midnight	
(vi)		16:00
(vii)	Midday	

3. Ahmad works in a factory for 135 months. For how many years does he work in the factory?
4. Zain spends 315 weeks 5 days in his grandmother's house. How many days does he spend in his grandmothers's house?
5. Hameed takes 3 weeks 6 days to complete a science model. How many days does he spend to complete the science model? Also convert the days into hours.

Teachers' Guide

Give flashcards of units of time to the students. Ask them to convert the units as per given instruction.

iv Calculation of Arrival Time, Departure Time and Journey Time

We often see arrival time and departure time on the metro stations, railway stations and airports. Time tables of different airlines and trains also contain the information about the journey time.

In this topic we will learn about calculating arrival time, departure time and journey time from the given information.

The following equations are used to calculate arrival, journey or departure time:

Arrival time = Departure time + Journey time

Departure time = Arrival time – Journey time

Journey time = Arrival time – Departure time

The time when a journey ends or someone (something) arrives is called arrival time.

The time when a journey starts or someone (something) departs, is called the departure time

The time which someone (something) takes for the journey is called the journey time.

Example 15 A bus departs from Lahore at 3:30 p.m. It takes 6 hours 30 minutes to reach Islamabad. At which time the bus reaches Islamabad?

Solution First, convert 12 hour time into 24 hour time.

Departure time = 3:30 p.m.

= 3:30 + 12 = 15:30

Journey time = 6 hours 30 minutes

Remember!

Add hours into hours and minutes into minutes.

	Hours	Minutes
Departure time =	15	30
Journey time =	+ 6	30
Arrival time =	21	60

So, Arrival time = 21 hours 60 minutes = 22:00

60 minutes = 1 hour So, 21 + 1 = 22

= 10:00 p.m.

By converting 24 hour time to 12 hour time (22:00–12hr = 10:00)

Example 16 Salman reaches home at 2:15 p.m. after making journey of 4 hours 45 minutes. When did he start the journey?

Solution First, convert 12 hour time into 24 hour time.

Arrival time = 2:15 p.m.

= 2:15 + 12 = 14:15

Journey time = 4 hours 45 minutes

So, Departure time = 09:30

= 9:30 a.m.

	Hours	Minutes
Arrival time =	14	15 60
Journey time =	– 4	45
Departure time =	9	30

Example 17 A ship started its journey from port A at 6:15 a.m. and reached the port B at 12:45 p.m. How long did it take to reach form port A to port B?

Solution First, convert 12 hour time into 24 hour time.

Departure time = 6:15 a.m. = 06:15

Arrival time = 12:45 p.m. = 12:45

	Hours	Minutes
Arrival time =	12	45
Departure time =	–06	15
Journey time =	6	30

So, Journey time = 6 hours 30 minutes

Similarly, we can calculate:

Departure Time	Arrival Time	Journey Time
4:00 a.m.	8:15 p.m.	16 h 15 min
8:20 p.m.	11:50 a.m. (next day)	15 h 30 min
03:20	5:20 a.m. (next day)	26 hours
9:40 a.m. (previous day)	21:40	36 hours
2:30 p.m. (previous day)	07:55	17 h 25 min

We can use this concept in the following example:

Example 18 Ahmad started his journey to move abroad at 08:30 a.m. and reached at 04:50 p.m. next day. Find his journey time.

Solution First, convert 12 hour time into 24 hour time.

Departure Time = 8:30 a.m.
= 08:30

Arrival Time = 4:50 p.m. (next day)
= 16:50 (next day)
= 16:50 + 24 = 40:50

	Hours	Minutes
Arrival time =	40	50
Departure time =	– 8	30
Journey time =	32	20

So, Journey time = 32 hours 20 minutes

Remember!

Add hours into hours and minutes into minutes.

Activity

Ask the students to write their arrival time to the school and departure time back to the house. Calculate your time to stay in the school and convert your arrival time and departure time into 12 hour and 24 hour formats.

EXERCISE 3.3

1. A bus starts travelling at 6:15 a.m. and stops at 8:50 p.m. Find the journey time.
2. Ali left home for school at 8:00 a.m. and arrived school at 8:30 a.m. He remained at school for 5 hours 15 minutes. When did he get off from the school? How much time he remained outside home?
3. Salman started his journey at 4:50 p.m. and travelled for 8 hours. When did he finish his journey? Write answer in 12 hour and 24 hour format.
4. Ahmad reached expo center at 09:00 a.m. to attend book fair. He left book fair at 03:40 p.m. How much time did he spend at book fair?
5. Rehan attend an online lecture for 1 h 40 min which ended at 03:45 p.m. When did he start to attend the lecture?
6. Usman went to a charity program at 09:50 a.m. on Sunday. He returned at 11:25 a.m. How much time did he spend over there?

3.1.3 Speed

We have heard this word frequently in our daily life e.g. speed of a car, speed of a train, speed of a ball and speed of an animal.

Speed is the rate of change of distance per unit time. It can be thought off as the measure how fast you move.

Usually speed is measured in km/h and m/s.

Mathematically, speed can be represented as:

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

Speed of Animals

Name of animals	Speed (approx.)
Peregrine falcon	389 km/h
Golden eagle	320 km/h
Gyrfalcon	209 km/h
Horse fly	140 km/h
Sailfish	120 km/h
Cheetah	120 km/h
Springbok	100 km/h
Wildebeest	80.5 km/h
Kangaroo	71 km/h
Lion	60 km/h
Red fox	50 km/h
African elephant	40 km/h
Polar bear	30 km/h
Garden snail	0.05 km/h

A train covers distance of 480 km in 8 hours. Find its speed.

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{480\text{ km}}{8\text{ h}} = 60\text{ km/h}$$

i Conversion of Units of Speed

Remember!

1 km = 1000 m

1 hr = 60 min = 60 × 60 s = 3600 s

(a) Km/h to m/s

Example 19 Convert 72 km/h into m/s.

Solution $72\text{ km/h} = \frac{72 \times 1\text{ km}}{1\text{ hr}} = \frac{72 \times 1000\text{ m}}{3600\text{ s}} = 20\text{ m/s}$

Skill Practice

Convert 150 km/h into m/s

(b) m/s to km/h

Example 20 Convert 70 m/s into km/h.

Solution

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

$$70\text{ m/s} = \frac{70 \times \frac{1}{1000}\text{ km}}{\frac{1}{3600}\text{ h}} = \frac{70 \times 3600}{1000} = 252\text{ km/h}$$

Skill Practice

If a car travels 60 km in one hour. Find its speed.

Skill Practice

Convert 120 m/s into km/h

ii Average speed

Average speed is the average of two or more different speeds. It is calculated by dividing the total distance covered by the total time taken.

$$\text{Average Speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Example 21 A bus moves at the speed of 32 km/h for an hour and 50 km/h for 2 hours. Find the average speed of the bus.

Solution Since time intervals are different, so first we calculate total distance by using the formula:

Distance covered = Speed × Time

Distance covered in first hour = 32 × 1 = 32 km

Distance covered in next 2 hours = 50 × 2 = 100 km

Total distance covered = 32 + 100 = 132 km

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$\text{Average speed} = \frac{132}{3} = 44\text{ km/h}$$

Skill Practice

If Hamza solves 5 questions in 1 hour and 15 question in 4 hour. What was his average speed to solve the questions per hour?

Skill Practice

Convert 44 km/h into m/s.

iii Uniform Speed

A body is said to have uniform speed if its speed remains same in different intervals of time.

In other words, if a body covers equal distances in equal intervals of time then it is said to have uniform speed. For example, in 5 hours, if a body covers 50 km in each hour then it is said to have uniform speed of 50 km/h.

Example 22 A loaded truck is travelling at uniform speed. If it covers 60 km in 1 hour. How much distance will it cover in 8 hours.

Solution In an hour distance covered = 60 km

In 8 hours distance covered = 60 × 8 = 480 km

Hence, the truck will cover 480 km in 8 hours at uniform speed.

Important Information

The earth revolves around the sun and completes one revolution in every 24 hours. It is an example of uniform speed.

Example 23 If a car travels 80 km in an hour, then how much distance is covered in 12 hours at the same speed?

Solution Distance covered in an hour = 80 km

At the same speed, distance covered in 12 hours = 80 × 12 = 960 km

Hence, the car will cover 960 km in 12 hours, at the uniform speed.

Skill Practice

If a train covers 10 km in 10 min, 20 km in 20 min, 30 km in 30 min, find its average speed.

Difference Between Average Speed and Uniform Speed

Average speed	Uniform speed
• Speed of the object varies or changes throughout the journey. e.g., bus moves at 50 km/h in the first hour, and 60 km/h in the second hour and so on.	• Speed of an object remains constant throughout the journey. e.g., ▪ Movement of clock hands. ▪ The motion of earth around the sun.

EXERCISE 3.4

1. Complete the following table:

(a)

	m/s	km/h
(i)		35
(ii)	54	
(iii)		20

(b)

	m/s	km/h
(i)	18	
(ii)		15
(iii)		55

2. If a horse covers 40 km in every 30 minutes, then find his speed in km/h.
3. If a car covers 20 m, 25 m and 45 m in 7s, 8s and 10s respectively, then find the average speed of the car and convert it into km/h.
4. The speed of a train is 42 km/h in 3 hours and 63 km/h in next 2 hours. Find the average speed of the train in 5 hours.

Summary

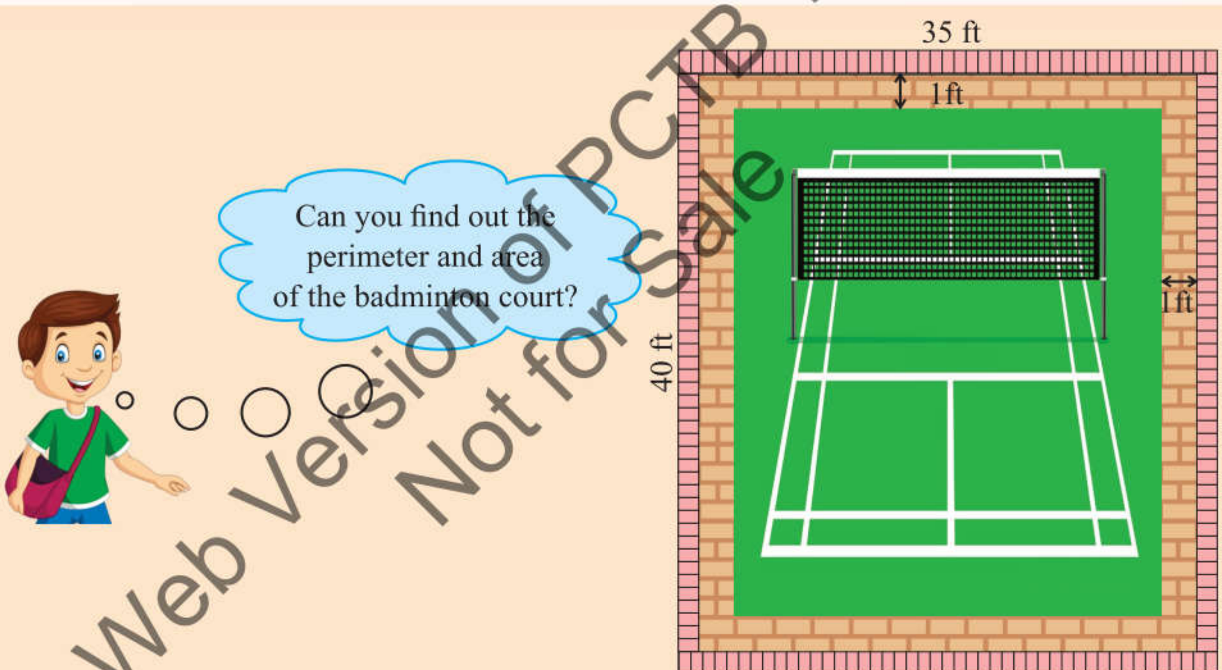
- The length of space between two points is called distance.
- Time is the duration of a period in which an action, process or condition exists or continues.
- Arrival time = Departure time + journey time
- Departure time = Arrival time – journey time
- Journey time = Arrival time – departure time
- Speed is the rate of change of distance per unit time.

Sub-domain (ii) Perimeter and Area

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Calculate the area and perimeter of the shaded/unshaded region in composite shapes.
- Calculate the circumference and area of a circle.
- Calculate the surface area and volume of any simple 3D shape including right prisms and cylinders.
- Convert between standard units of area (m^2, cm^2, mm^2 and vice versa) and volume (m^3, cm^3 and mm^3 and vice versa)
- Solve real life word problems involving the surface area and volume of right prisms and cylinders.

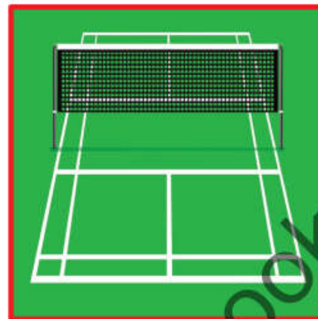


RECALL

We have studied about the concept of perimeter, area and volume in previous grade. Perimeter, area and volume are measures of different aspects of 2D and 3D objects. Perimeter gives the length of outer boundary of a 2D shape whereas, area gives the measurement of interior region of a closed shape or surface.

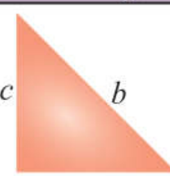
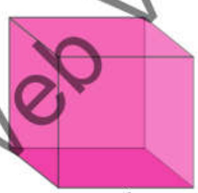
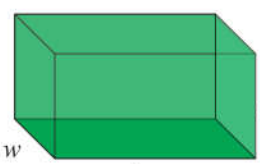
For example, in the following figures the total length of red line is the perimeter of the figures whereas the measure of green interior region shows the area of the shape.

The distance covered by Amina in the circular track in one round is called circumference of the track.



Similarly, the sum of lengths of all sides of the above bedminton court is the perimeter and the green region occupied by the bedminton court is its area.

3.2.1 Perimeter, Area and Volume of 2D and 3D-shapes

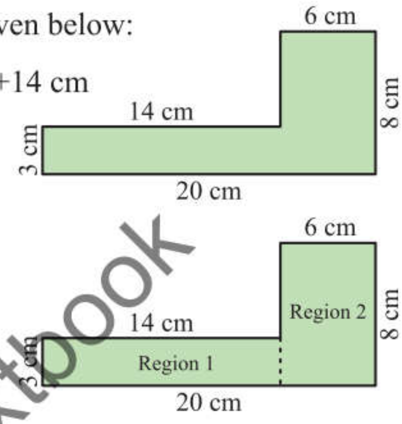
	Rectangle	Square	Triangle
2D-shapes	 Perimeter = 2(length + breadth) $= 2\times(\ell + b)$ unit Area = length × breadth (unit)^2	 Perimeter = 4ℓ (unit) Area = $\ell \times \ell$ $= \ell^2$ (unit)^2	 Perimeter = $(a + b + c)$ unit Area = $\frac{1}{2}$(base) × (height) (unit)^2
3D-shapes	**Cube**  Volume = ℓ^3(unit)^3 Surface area = $6\ell^2$ (unit)^2	**Cuboid**  Volume = $\ell \times w \times h$ (unit)^3 Surface area = $2(\ell w \times wh + \ell h)$ (unit)^2	

3.2.2 Introduction

A composite 2D shape is a two dimensional figure made up of two basic 2D figures. To find out the perimeter and area of composite 2D shape, add the perimeter and area of each figure making up the composite 2D shape. Let's find the perimeter and area of shaded region of composite shapes.

Example 1 Find the area and perimeter of the shaded region in the following composite shape:

Solution First, We divide the shaded region into two regions as given below:



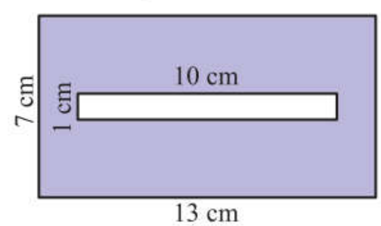
Perimeter of shaded region = 3 cm + 20 cm + 8 cm + 6 cm + 5 cm +14 cm
= 56 cm

Area of Region 1 = 14 cm × 3 cm
= 42 cm^2

Area of Region 2 = 8 cm × 6 cm
= 48 cm^2

Total area of the shaded region = Area of Region 1 + Area of Region 2
= 42 cm^2 + 48 cm^2
= 90 cm^2

Example 2 Find the perimeter and area of the shaded region. Also convert the perimeter and area of shaded region into mm.



Solution Perimeter of big rectangle = 2(7+13) = 2(20) = 40 cm

Perimeter of small rectangle = 2(10+1) = 2(11) = 22 cm

Perimeter of shaded region = 40 + 22 = 62 cm

Area of big rectangle = 7 × 13 = 91 cm^2

Area of small rectangle = 10 × 1 = 10 cm^2

Area of the shaded region = 91 – 10 = 81 cm^2

Now, convert the perimeter and area into mm;

Perimeter of shaded region = 62 cm = 62 × 10 = 620mm

Area of the shaded region = 81 cm^2 = 81 × 100
= 8100mm^2

Important Information

Conversion of Units
(From larger to smaller units)

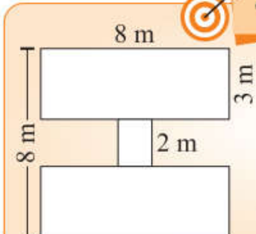
1 m^2 = (100 cm)^2 = 10000 cm^2
1 cm^2 = (10 mm)^2 = 100 mm^2
1 m^2 = (1000 mm)^2 = 1000000 mm^2

Important Information

Conversion of Units
(From smaller to larger units)

1 cm^2 = $\frac{1}{10000}$ m^2	1 mm^2 = $\frac{1}{100}$ cm^2	1 mm^2 = $\frac{1}{1000000}$ m^2

Challenge



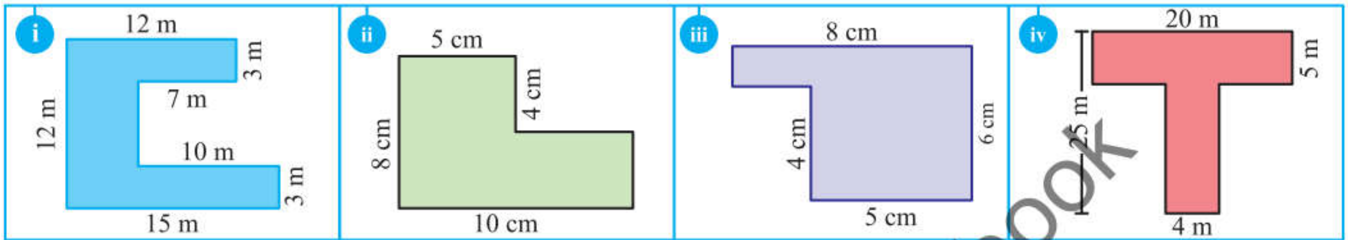
Can you find out the area of the given shape and convert into millimtres?

Teachers' Guide

Generate discussion on volume and surface area with the students. Explain that as square units are used to measure surface area and cubic units are used to measure volume.

EXERCISE 3.5

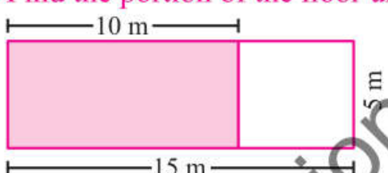
1. Find the area and perimeter of the following composite figures:



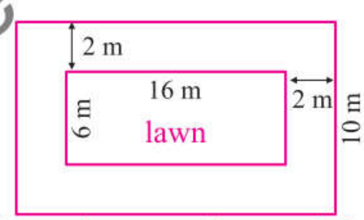
2. Complete the following table:

	mm^2	cm^2	m^2
i	1400		
ii		100	
iii			5

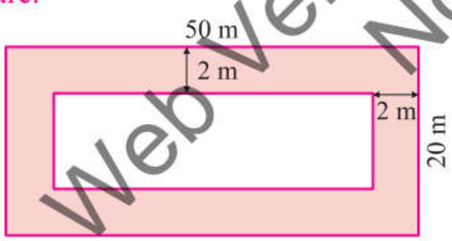
3. Some part of a floor of a hall is covered with carpet. Find the portion of the floor uncovered.



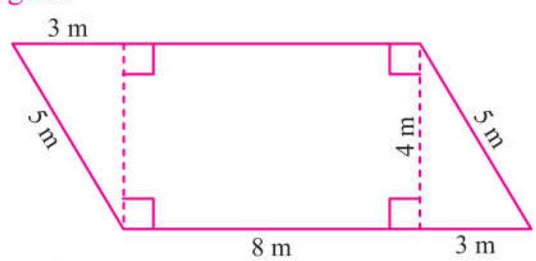
4. Find the area of walking track around the lawn.



5. Find the area of unshaded region in the given figure.



6. Find the perimeter and area of the following figure:



3.2.3 Circumference and Area of a Circle

We have learnt to find the area and perimeter of rectangles, squares and triangles. Now we will learn to find the area and perimeter of the circle. Perimeter of the circle is called its circumference.

Circle A circle is the union of all points fixed at constant distance from a fixed point. The fixed point is called the centre of the circle. A circle has one and only one centre.

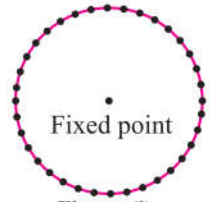


Figure (i)

Radius The distance from the centre of the circle to any point of the circle is constant. This constant distance is called radius of circle. In figure (ii) $m\overline{OA}$ is a radius.

Chord A line segment joining two different points of a circle is called the chord. In figure (ii) $\overline{BC}$ is a chord of a circle.

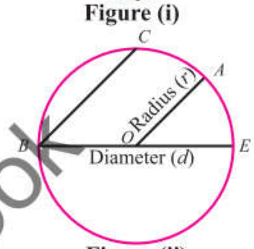


Figure (ii)

Diameter The length of the chord passing through the centre of the circle is called diameter of the circle. In figure (ii) $m\overline{BE}$ is a diameter.

Arc An arc is a part of the circumference of circle. In figure (iii), LM is an arc of the circle. It is written as $\widehat{LM}$. Rainbow is an example of an arc.

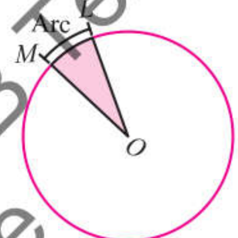


Figure (iii)

i Circumference of a Circle

The perimeter of a circle is called circumference of a circle. In other words, the length of the outer boundary of a circle is called the circumference of a circle and region occupied by the circular shape is its surface area. The circumference of the circle can be found by using the following formula:

$$C = 2\pi r \quad \left[\pi = \frac{22}{7}\right]$$

Or $$C = \pi d \quad [d = 2r]$$

[Where π is a constant and is the ratio of circumference to the diameter of the circle which is taken approximately equal to $\frac{22}{7}$ or 3.14]

Example 3 If the radius of the circle is 7 cm then find the circumference of the circle.

Solution $C = 2\pi r$

$$C = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$

Skill Practice

Calculate the radius if circumference is 240 cm. Take $\pi = \frac{22}{7}$

Example 4 If the diameter of the circle is 10 cm, then find the circumference of the circle.

Solution $C = \pi d$

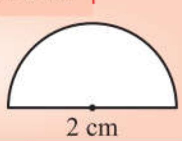
$$C = 3.14 \times 10 = 31.4 \text{ cm}$$

Keep in mind!

Circumference of semi-circle = $\frac{1}{2}$(circumference of the circle.)

Try yourself!

Find the circumference of the given shape.

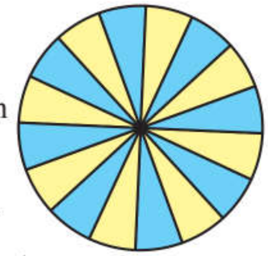


Skill Practice

Measure the radius and diameter of a variety of tins and circular objects. For each circle workout a way to measure the area and circumference. List all the results in a table.

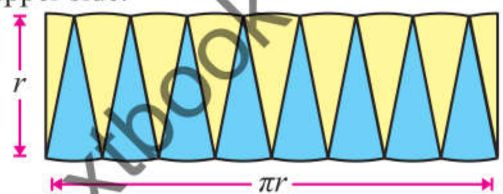
Objects	Radius	Diameter	Surface area	Volume

ii Area of a Circle



We find the area of a circle as follows:

1. Divide the circle into large number of sections (as given) and arrange them in the following way.
2. When the number of sections is arranged, it approximately equals to a rectangle.
3. As half of the length of the circle is at lower side and half on upper side.
   So, length of the circle is half of $2\pi r$ that is πr.
4. The breadth of the circle is r.
5. Hence, area of circle $= \pi r \times r = \pi r^2$



Example 5 If the radius of the circle is 5 cm then find the area of the circle.

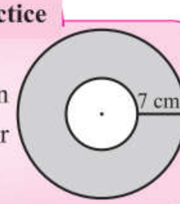
Solution Area of a circle $= \pi r^2$

$$= \frac{22}{7} \times (5)^2 \quad = \frac{22}{7} \times 25 \quad = \frac{550}{7}$$

$A = 78.5 \text{ cm}^2$

Skill Practice

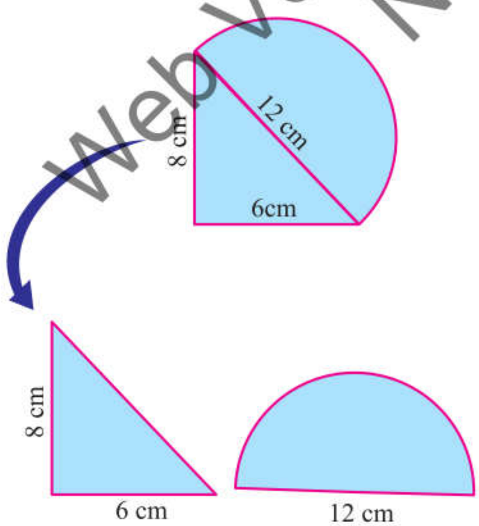
Find the area of shaded region when the area of smaller circular region is $130 cm^2$



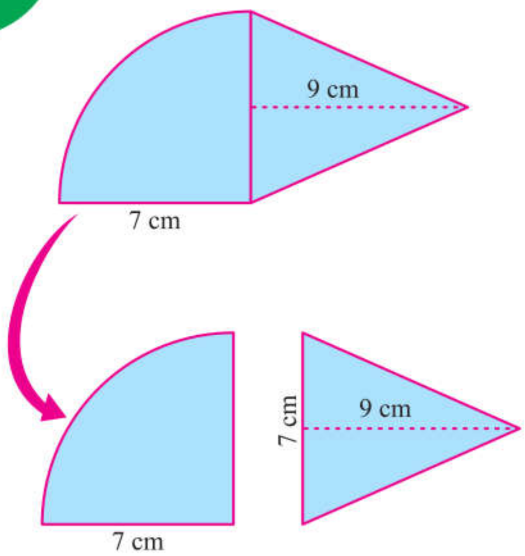
Important Information

A circle can be divided into sectors which rearrange to form an approximate parallelogram.

Example 6 Find the area of the following:

i Figure	Solution
	In triangle, Base = 6 cm ; Height = 8 cm Area of triangle $= \frac{1}{2}$ base × height $= \frac{1}{2}(6)(8) = 24 \text{ cm}^2$ Radius $= r = \frac{d}{2} = \frac{12}{6} = 6$ cm Area of semi-circle $= \frac{1}{2}\pi r^2$ $= \frac{1}{2} \cdot \frac{22}{7} \cdot (6)^2$ $= 56.571 \text{ cm}^2$ Area of shaded region = 24 + 56.571 $= 80.571 \text{ cm}^2$

ii

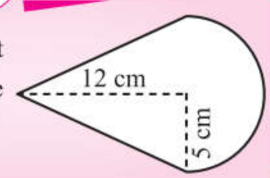
Figure	Solution
	Area of quadrant $= \frac{1}{4}\pi r^2$ As, radius = 7 cm $= \frac{1}{4} \cdot \frac{22}{7} \cdot (7)^2$ $= 38.5 \text{ cm}^2$ As, base = 7 cm ; height = 9 cm Area of triangle $= \frac{1}{2}$ base × height Area of triangle $= \frac{1}{2} \times 7 \times 9$ $= 31.5 \text{ cm}^2$ Area of shaded region $= 38.5 + 31.5$ $= 70 \text{ cm}^2$

Activity

(i) Cut out circular regions of different diameters from a piece of cardboard.
(ii) Measure the circumference of each circle by using thread and measuring tape.
(iii) Also calculate the area of these circles.

Skill Practice

Can you find out the area of the given object?



EXERCISE 3.6

1. Find the circumference and area of the following circles having:

(i) $r = 3$ cm	(ii) $d = 8$ cm	(iii) $r = 5$ cm
(iv) $r = 7$ cm	(v) $r = 2$ cm	(vi) $r = 10$ cm

2. Fill in the blanks.

i. $r = 3.1\ m, C =$ ________ ii. $d = 5\ cm, C =$ ________

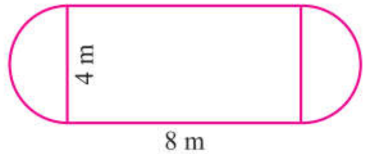
iii. $r = 6\ cm, A =$ ________ iv. $d = 11\ m, A =$ ________

3. There are three circular ponds having radii 3.2 m, 4.3 m and 5.4 m respectively in a park. Find the area covered by these three ponds.

4. The circumference of a circular ground is 88 m. Find the area of the ground. (take $\pi = \frac{22}{7}$)

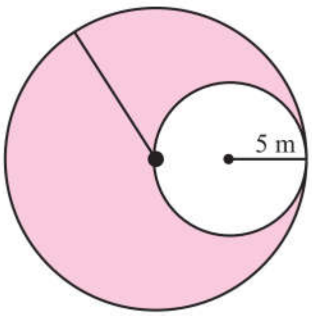
5. The diameter of a circular region is 9 m. Find the area and circumference of the region.

6. Find the perimeter and area of the given figure

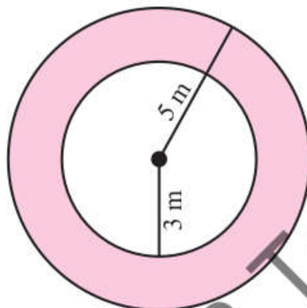


7. Find the area of the shaded regions.

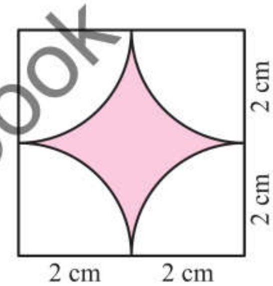
(i)



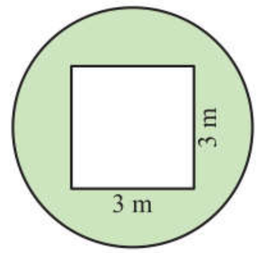
(ii)



(iii)



8. There is a square platform inside a circular park. Find the area of the grassy land ($r = 21m$)

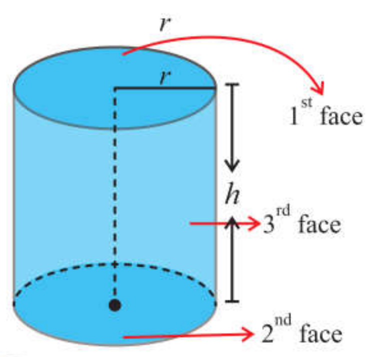


3.2.4 Cylinder

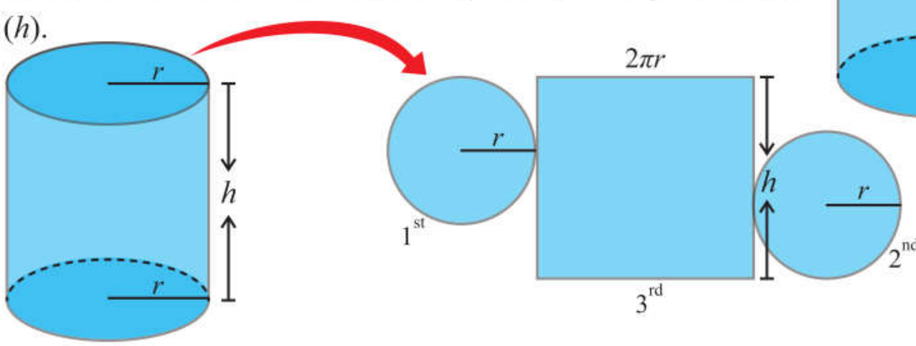
A cylinder is a closed solid that has two parallel (circular) bases which are connected by a curved suface.

All the above real life objects have cylindrical shape.

i Circumference of a Cylinder



Consider a cylinder. We can see that a cylinder has three faces,2 circular based faces and 1 curved face. If we cut this cylinder, having radius (r) and height (h).



Area of cylinder = Area of 1^{st} face + Area of 2^{nd} face + Area of 3^{rd} face

$$= \pi r^2 + \pi r^2 + (2\pi r \times h)$$

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r\,(r + h)$$

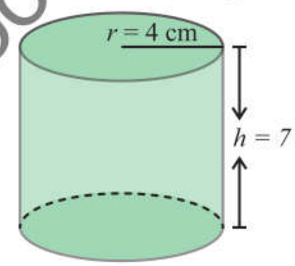
Hence, surface area of cylinder = $2\pi r(r + h)$

Activity

- Take a coin.
- Place another coin on the first coin and then place 3^{rd} coin on the second coin and so on. You will obtain a 3D shape (cylinder). Try to find the volume and surface area of this cylinder.

Example 7

If the radius and height of the cylinder are 4 cm and 7 cm respectively. Find the surface area of the given cylinder.



Solution Surface area of the Cylinder = $2\pi r(r + h)$

$$= 2 \times 3.14 \times 4 \text{ cm } (4 \text{ cm} + 7\text{cm})$$

$$= 25.12 \text{ cm} \times 11 \text{ cm} = 276.32 \text{ cm}^2$$

ii Volume of a Cylinder

Volume of the cylinder is the space occupied by the cylinder in three dimension. If the cylinder is divided into large number of 2 dimensional disks then its volume can be calculated by multiplying the area of one disk by the height h that is:

Volume of the Cylinder = Area of one disk × Height

$$= \pi r^2 \times h$$

$$= \pi r^2 h$$

Example 8 If the radius and height of a cylinder are 3 cm and 9 cm respectively then, find the volume of the cylinder.

Solution Volume of the cylinder = $\pi r^2 h$

Volume of the cylinder = $3.14 \times (3 \text{ cm})^2 \times 9 \text{ cm}$

$$= 3.14 \times 9 \text{ cm}^2 \times 9 \text{ cm}$$

$$= 254.34 \text{ cm}^3$$

Skill Practice

If the volume = 270.75 cm^3 and r = 4 cm, find out the height of the cylinder.

Example 9 If the radius and height of a cylinder are 2.5 cm and 6 cm respectively. Find the area of curved surface, surface area and volume of the cylinder. Also calculate the cost of painting the cylinder at the rate of Rs. 15 per square centimetre.

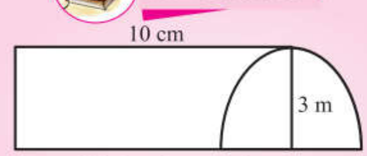
Solution Given, $r = 2.5$ cm, $h = 6$ cm

Area of curved surface = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.5 \times 6 = 94.286 \text{ cm}^2$$

So, Area of curved surface is 94.286 cm^2

Skill Practice



What will be the surface area of the given figure?

Total surface area of cylinder = $2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 2.5(2.5 + 6)$$

$$= 2 \times \frac{22}{7} \times 21.25 = 133.571 \text{ cm}^2$$

So, Total surface area of the cylinder is 133.571 cm^2

The volume of other cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (2.5)^2 \times 6$$

$$= \frac{22}{7} \times 37.5 = 117.857 \text{ cm}^3$$

So, The volume of the cylinder is 117.857 cm^3

Cost of painting for 1 cm^2 = Rs. 15

Cost of painting for 133.571 cm^2 = 15×133.571

= Rs. 2003.565

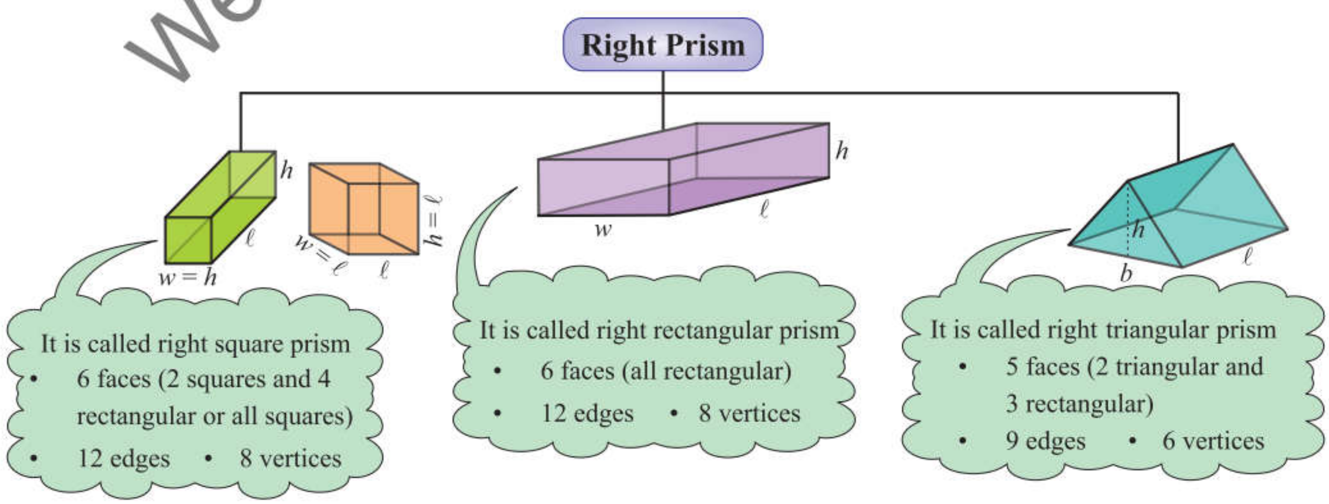
The cost of painting the cylinder at the rate of Rs. $15/\text{cm}^2$ is Rs. 2003.565

3.2.5 Prism

A prism is a solid 3D object that is bounded on all its sides by plane faces. The top and bottom faces are identical and are called bases. A prism is named after the shape of these bases. A triangular prism has triangular base and rectangular prism has rectangular base. A prism in which its bases are identical and parallel and all other faces are perpendicular to the bases, is called right prism.

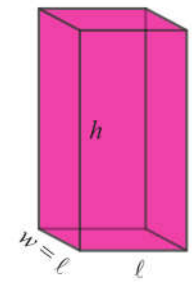
Right Prism

There are three kinds of right prims



(a) Surface Area and Volume of Right Square Prism

Right Square Prism



A right square prism is a sold which has 8 vertices, 6 flat faces and 12 straight edges. In right square prism, two faces (bases) are square and other four can be square or rectangular.

Surface area of right square prism = Area of square faces + Area of rectangular faces

$= 2(\text{area of square faces}) + 4(\text{area of rectangular faces})$

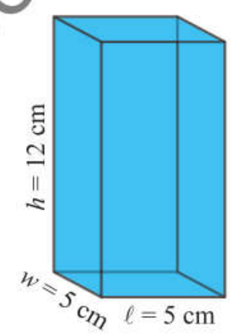
$= 2(\ell \times \ell) + 4(\ell \times h)$

Surface area of right square prism $= 2\ell^2 + 4\ell h$ unit^2

Volume of right square prism = length × width × height

Volume of right square prism $= \ell^2 h$ unit^3

Example 10 Find the surface area and volume of the given right square prism.



Solution Given; $h = 12$ cm, $w = \ell = 5$ cm

Surface area of right square prism $= 2\ell^2 + 4\ell h$ unit^2

$= 2\ell^2 + 4\ell h$ unit^2

$= 2(5)^2 + 4(5)12)$

$= 50 + 240 = 290$ cm^2

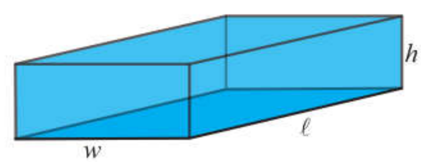
Volume of right square prism $= \ell^2 h$ unit^3

$= (5)^2(12)$ cm^3 $= 300$ cm^3

> **Remember!**
> When all the 6 faces of the right square prism are square, then it is cube.

(b) Surface Area and Volume of Right Rectangular Prism

Right Rectangular Prism



A right rectangular prism is also known as cuboid. All the faces of the right rectangular prism are rectangular. So,

$$\begin{pmatrix}\text{Surface area of the right rectangular prism} \\ \text{or} \\ \text{Surface area of the cuboid}\end{pmatrix} = 2(\ell w + wh + h\ell)$$

$$\begin{pmatrix}\text{Volume of the rectangular prism} \\ \text{or} \\ \text{Volume of the of the cuboid}\end{pmatrix} = \ell \times w \times h$$

Example 11 Find the surface area and volume of the given shape.

Solution Given; $h = 5$ cm, $\ell = 15$ cm and $w = 8$ cm

Surface area of the right rectangular prism $= 2\ell w + wh + h\ell$

$= 2[(15)(8) + (8)(5) + (15)(5)]$

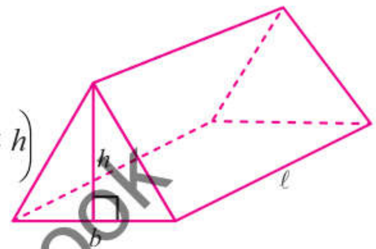
$= 2[120 + 40 + 75] = 470$ cm^2

Volume of the right rectangular prism $= \ell \times w \times h \quad = [(15)(8)(5)] = 600$ cm^3

(c) Surface Area and Volume of Right Triangular Prism

i Surface area of a Right Triangular Prism

Surface area of a triangular prism is the sum of areas of all its faces.



Surface area of triangular prism = Area of lateral surfaces + Area of bases

$$= (\ell \times b + \ell \times b + \ell \times b) + \left(\frac{1}{2}b \times h + \frac{1}{2}b \times h\right)$$

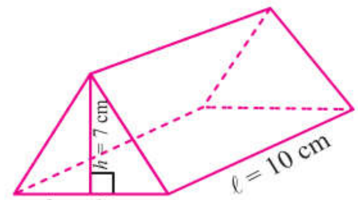
$$= 3(\ell \times b) + 2\left(\frac{1}{2}b \times h\right) = 3\ell b + bh$$

Example 12 If the length, base and height of the triangular prism are 10 cm, 8 cm and 7 cm respectively then, find the surface area of the right triangular prism.

Solution Surface Area of the triangular prism $= 3\ell b + bh = 3(10\text{ cm})(8\text{ cm}) + (8\text{ cm})(7\text{ cm})$

$$= 240 + 56$$

$$= 296\text{ cm}^2$$



ii Volume of a Right Triangular Prism

Volume of right triangular prism is the space occupied by the prism in three dimensions. If the triangular prism is divided into large number of 2 dimensional solid tringles then its volume can be calculated by multiplying the area of one triangular plate by the length ℓ that is

Volume of the prism = Area of one triangular plate × length $= \frac{1}{2}b \times h \times \ell = \frac{1}{2}bh\ell$

Example 13 If the length, base and height of the right triangular prism are 9 cm, 8 cm and 7 cm respectively then, find the volume of right triangular prism.

Solution Volume of right triangular prism $= \frac{1}{2}\ b \times h \times \ell$

$$= \frac{1}{2}(8)(7)(9) = 252\text{ cm}^3$$

Try yourself!

If the volume of prism is 144 cm^3, base = 5cm and height = 4.3 cm, then find the length of the prism.

Example 14 The length of a cubic box is 5 cm. Find Volume of the box and convert it into m^3 and mm^3.

Solution Volume of the cubic box = 5 cm × 5 cm × 5 cm $= 125\text{ cm}^3$

$$= 125 \times \frac{1}{1000000}\text{ m}^3$$

$$= 0.000125\text{ m}^3$$

$$= 125 \times 1000\text{ mm}^3$$

$$= 125000\text{ mm}^3$$

Teachers' Guide

Guide the students to prepare coloured sheets to note down all the formulae of the circle, cylinder and prism. At the end of topics, paste these resources sheet on their notebook.

Important Information

Conversion of Units
(From larger to smaller units)

$1\text{ m}^3 = (100\text{ cm})^3 = 1000000\text{ cm}^3$

$1\text{ cm}^3 = (10\text{ mm})^3 = 1000\text{ mm}^3$

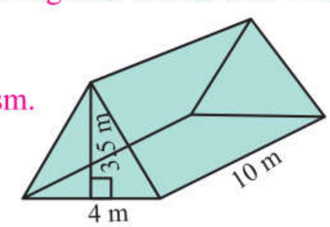
$1\text{ m}^3 = (1000\text{ mm})^3 = 1000000000\text{ mm}^3$

EXERCISE 3.7

1. Complete the following table:

Sr. #	mm^3	cm^3	m^3
(i)	300		
(ii)		350	
(iii)			7

2. The length of a cubic aquarium is 50 cm. Find the surface area and volume of the aquarium.
3. If the radius and height of a cylinder are 1 m and 5 m respectively. Find the volume of the cylinder.
4. If the length, base and height of a triangular prism are 10 cm, 5 cm and 3.5 cm respectively. Find the surface area and volume of prism.
5. The length of a cubic tank is 10 m. Find the volume of the tank.
6. The radius and height of a cylindrical pipe is 25 cm 250 cm. Find the volume of the water in this pipe.
7. The length, breadth and height of a room are 7 m, 5 m and 5 m respectively. Find the volume of the room.
8. A well of 2 m radius is dig 100 m deep. Find the volume of the well.
9. If three cylinders of radius 5 cm and height 20 cm are joined together end to end. Then find the total surface area and volume of resulting cylinder.
10. Find the surface area and volume of the given triangular prism.



SUMMARY

- Perimeter is the measurement of boundary of closed plain figure.
- Area is the measurement of interior region of a closed shape.
- Perimeter of rectangle = $2(\ell + b)$
- Area of rectangle = $\ell \times b$
- Circumference of circle = $2\pi r$
- Area of circle = πr^2
- Surface area is the sum of outer surfaces of a 3D shape.
- Surface area of cube = $6 \times (\ell \times \ell)$
- Volume of a cube = $\ell \times \ell \times \ell$
- Surface area of cylinder = $2\pi r(r + h)$
- Volume of cylinder = $\pi r^2 h$
- Surface area of right triangular prism = $3\ell b + bh$
- Volume of right triangular prism = $\frac{1}{2} b\ell h$

REVIEW EXERCISE 3

1. Choose the correct option.

(i) 1 km = __________.

(a) 10m (b) 100 m (c) 1000 m (d) 10000 m

(ii) 1mm = __________.

(a) 0.01 cm (b) 0.1 cm (c) 0.1 m (d) 0.0001 m

(iii) 16 km = __________.

(a) 0.16 m (b) 16 cm (c) 16000 cm (d) 16000 m

(iv) 14:00 in 12-hour clock is:

(a) 01:00 a.m (b) 2:00 a.m (c) 01:00 p.m (d) 02:00 p.m

(v) If arrival time = 2:40 p.m and journey time = 4 hours then departure time = __________.

(a) 08:40 a.m (b) 09:40 a.m (c) 10:40 a.m (d) 11:40 a.m

(vi) If a car covers 10 m in 5 s then its speed is:

(a) 1 m/s (b) 2m/s (c) 3m/s (d) 11:40 a.m

(vii) 25 m/s = __________ km/h.

(a) 60 (b) 70 (c) 80 (d) 90

(viii) Time 03:48 p.m in 24-hour clock is:

(a) 03:48 (b) 13:48 (c) 14:48 (d) 15:48

(ix) The perimeter of a square of length 4 cm is:

(a) 8 cm (b) 12 cm^2 (c) 16 cm (d) 16 cm^2

(x) The area of circle with radius 3 cm is:

(a) 9π cm^2 (b) 18π cm^2 (c) 3π cm^2 (d) 16π cm^2

(xi) 1 cm^2 = __________.

(a) 10 mm^2 (b) 100 mm^2 (c) 1000 mm^2 (d) 10000 mm^2

(xii) 1 mm^3

(a) $\frac{1}{10}$ cm^3 (b) $\frac{1}{100}$ cm^3 (c) $\frac{1}{1000}$ cm^3 (d) $\frac{1}{10000}$ cm^3

(xiii) The surface area of cylinder with radius = 2 cm and h = 10 cm.

(a) 100 cm^2 (b) 150.8 cm^2 (c) 150.8 cm^3 (d) 150 cm^3

(xiv) The surface area of triangular prism with ℓ = 8 cm, b = 4 cm, h = 3 cm:

(a) 78 cm^2 (b) 96 cm^2 (c) 108 cm^2 (d) 120 cm^2

(xv) 1 m^3 = __________.

(a) 1000 cm^3 (b) 10000 cm^3 (d) 100000 cm^3 (d) 1000000 cm^3

2. Convert:

(i) 75 km 880 m into m (ii) 75 cm into mm

(iii) 585 mm into cm and mm (iv) 5700 m into km and m

3. The park near Hamnah's house is 1 km 200 m. What is the length of park in m?

4. Complete the following table:

Sr. #	12 hour time	24 hour time
(i)	5:00 a.m	
(ii)		22:35
(iii)		18:15
(iv)	2:30 a.m	

5. Convert:

(i) 560 seconds into min and sec (ii) 35 months into year and months

(iii) 98 week 5 days into days (iv) 490 days into months and days

6. Umair takes 2 hours 35 minutes to complete his home work. Convert the time into seconds.

7. Complete the following table:

Sr. #	Departure Time	Journey Time	Arrival Time
(i)	5:50 a.m	4 h 15min	
(ii)	8:30 a.m		7:48 p.m
(iii)	07:50	3 hours	
(iv)		1 h 20 min	18:20
(v)		13 h 20 min	10:00 a.m (next day)

8. Ahmed reached science museum at 10:20 a.m and left at 03:10 p.m. How much time did he spend over there?

9. Ahmad left house at 12:00 p.m and reached the Masjid at 01:00 p.m. He left Masjid at 02:00 p.m and went to bazar. After spending 1 h and 30 min in bazar, he reached back at home:

(a) Find arrival time. (b) How much time did Ahmad spend outside the house?

10. Complete the following tables:

(a)

Sr. #	cm	m	km
(i)	100		
(ii)			2
(iii)		500	
(iv)	1000		
(v)		100	

(b)

Sr. #	km/h	m/s
(i)	108	
(ii)		50
(iii)	144	
(iv)		100

11. Complete the following tables:

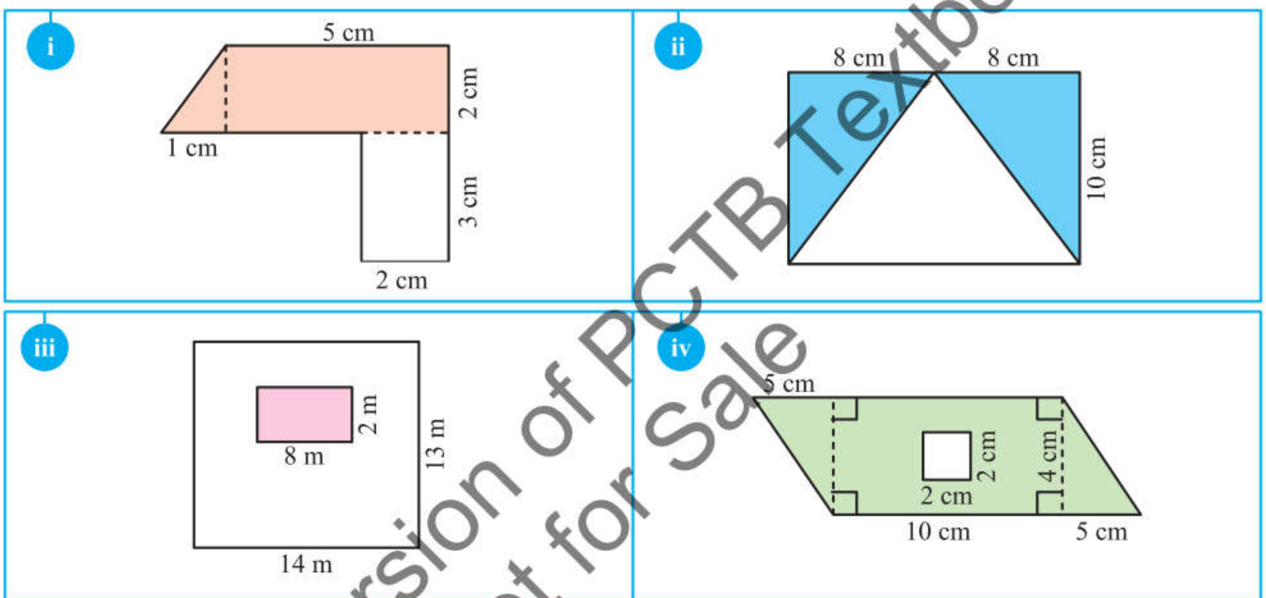
(a)

Sr. #	m^2	cm^2	mm^2
(i)	5		
(ii)		7000	
(iii)			100000
(iv)	3		

(b)

Sr. #	m^3	cm^3	mm^3
(i)	4		
(ii)		800000	
(iii)	20		
(iv)			900000000

12. Find the shaded and unshaded area of the following composite shapes.



13. A circular flower bed has diameter 4 m. Find the circumference and area of the bed.
14. The circumference of a circular region is 31.4 m. Find the radius of the region.
15. The radius of a cylindrical pillar of a bridge is 1 m and h = 10 m. Find:
    (a) Curved surface area of the pillar (b) Base area of the pillar
    (c) Volume of the pillar
16. The dimensions of a shoe box are 15 cm, 30 cm and 10 cm. Find:
    (a) Surface area of the shoe box (b) Volume of the shoe box
17. The dimensions of a wooden right triangular prism are ℓ = 8 cm, b = 5 cm and h = 5 cm. Find the surface area and volume of the prism.
18. The length of a underground water tank is 7 m. Find the volume of the cube shaped tank.
19. The length, base and height of a right triangular prism like a hut are 5 m, 4 m, 3.5 m respectively. Find the surface area and volume of the hut.
20. There are seven 1 cm^3 cubes in an aquarium with ℓ = 30 cm, b = 20 cm, h = 20 cm. Find the free space in terms of volume inside the aquarium.

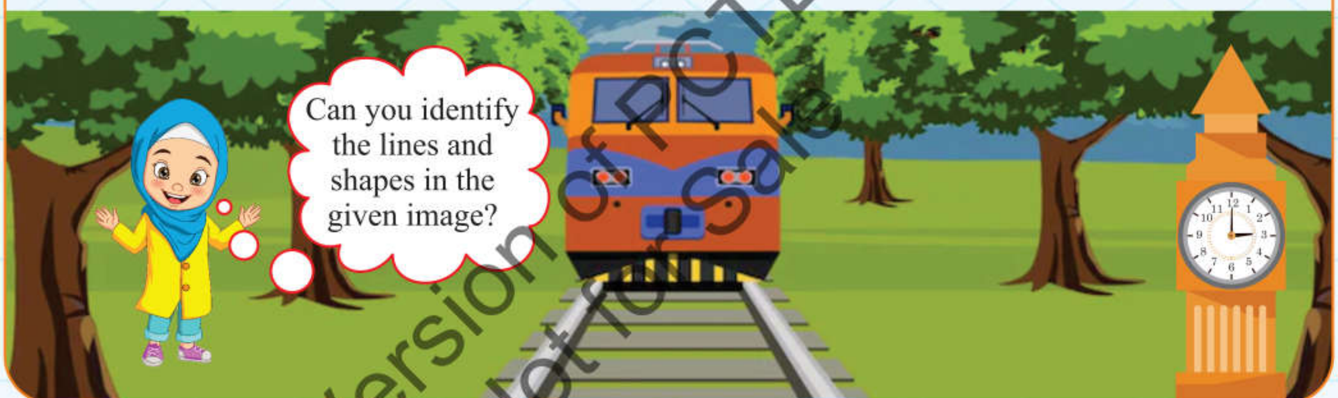
Domain 4 GEOMETRY

Sub-domain (i) Practical Geometry

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Know that the perpendicular distance from a point to a line is the shortest distance to the line.
- Construct different types of triangles. (equilateral, isosceles, scalene, acute-angled, right-angled and obtuse-angled)

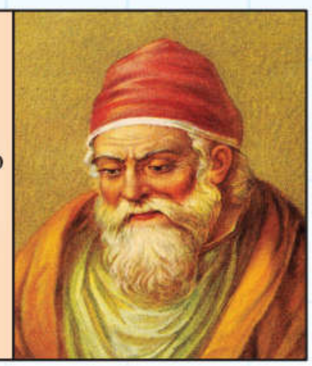


INTRODUCTION OF GEOMETRY

Geometry is the branch of mathematics which deals with the study of points, lines, surfaces and solids.

Geometry helps us what design to make which plays an important role in the construction of houses, buildings, dams etc. Art and architecture are based on geometry.

Euclid (300 BC) was the first mathematician who made remarkable achievements in geometry.



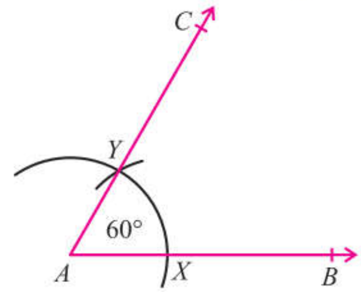
RECALL

Construction of Angles Using Compass and Ruler

We have learnt in previous class about the construction of angles 30°, 45°, 60°, 75°, 90°, 105° and 120°. We will revise here the construction of 60° and 90°.

Example 1 **Construction of an angle of 60°**

Solution **Steps of construction**



i. Draw a ray AB of any suitable length.

ii. Taking point A as centre, draw an arc of any suitable radius with compass intersecting $\overrightarrow{AB}$ at point X.

iii. Taking point X as centre, draw an arc of same radius cutting the previous arc at point Y.

iv. Draw $\overrightarrow{AC}$ passing through point Y. We get an angle of 60° i.e., $m\angle BAC = 60°$

Try yourself

In previous class we have also learnt how to bisect the given angle. Bisect the angle of 60°.

Important Information

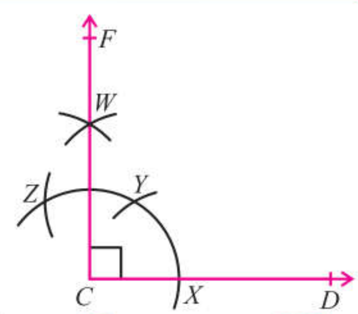
Bisection of angle 60° is 30°. Bisection of an angle means to find an angle, half of the given angle

Example 2 **Construction of an angle of 90°**

Solution **Steps of construction**

Remember!

For the construction of angle 90°, we use the angles 60° and 120°.



i. Draw a ray CD of any length.

ii. With point C as centre, draw an arc of any suitable radius with compass which intersecting $\overrightarrow{CD}$ at point X.

iii. With point X as centre, draw an arc of same radius intersecting the previous arc at point Y.

iv. Taking point Y as centre, draw an arc of same radius intersecting the first arc at point Z.

v. Taking points Y and Z as centres draw two arcs of any suitable radius cutting each other at point W.

vi. Draw $\overrightarrow{CF}$ through point W. We get an angle of 90°. i.e., $m\angle DCF = 90°$.

Do you know?

If we draw a ray through Z we will get an angle of 120°.

Try yourself

What is bisection of angles 30°, 60° and 90°?

Brain Teaser!

If we bisect the angle 90° which angle do we get?

4.1.1 Triangles and their Construction

Triangles are closed figures with three line segments (called sides) and three angles inside it. Triangles are used in patterns for design and in construction.

Remember!

A triangle has 3 sides, 3 angles and 3 vertices.
Egyptians built pyramids that are triangular in shape.

i Types of Triangles

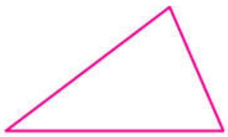
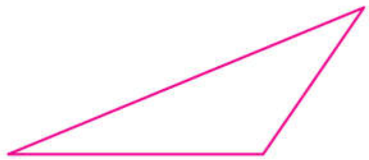
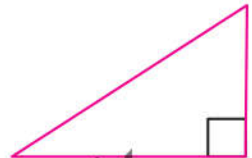
Triangles are mainly divided into two types:

(a) Triangles with respect to sides (b) Triangles with respect to angles

(a) Triangles with respect to sides

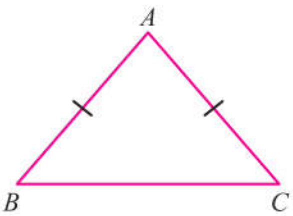
Scalene Triangle

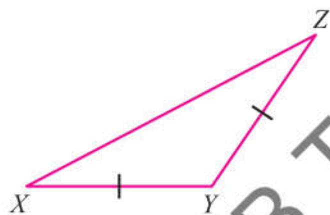
A triangle with all sides of different measure is called scalene triangle.

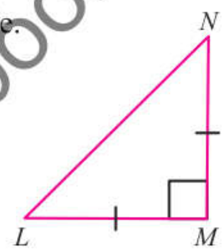




Isosceles Triangle

A triangle with two sides of equal measure is called an isoscales triangle.







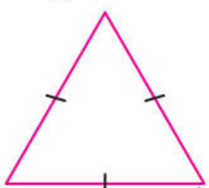
In ΔABC, $m\overline{AB} = m\overline{AC}$ and $\angle B$ and $\angle C$ are called base angles. $\overline{BC}$ is base and $\angle A$ is called vertical angle.

Try yourself

In ΔXYZ and ΔLMN, name the base angles and vertical angle.

Equilateral Triangle

A triangle with all sides of equal measure is called an equilateral triangle.



Remember!

Each interior angle in an equilateral triangle is 60°.

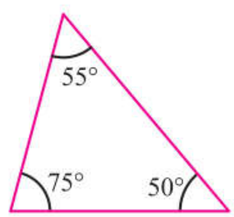
Need to Know!

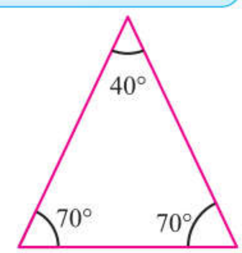
The sum of all interior angles of a triangle is 180°.

(b) Triangles with respect to angles

Acute Angled Triangle

A triangle is called an acute angled triangle if all angles are less than 90°.





Need to Know!

An acute angled triangle can be equilateral or isosceles.

Remember!

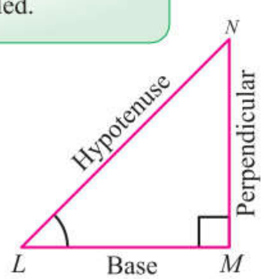
If sum of two angles in a triangle is greater than 90°, the triangle is acute angled.

Right Angled Triangle

A triangle is called a right angled triangle if exactly one angle is equal to 90°.

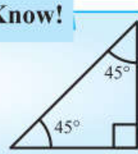
In right angled triangle LMN,

$\overline{LN}$ is the hypotenuse, $\overline{LM}$ is base and $\overline{MN}$ is perpendicular.



Need to Know!

A right angled triangle can be isosceles.

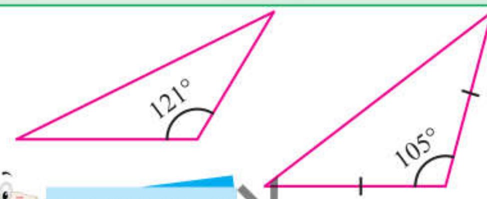


Remember!

In right angled triangle, one angle is 90° and the sum of the other two is 90°. i.e., other two angles are complementary.

Obtuse Angled Triangle

A triangle is called obtuse angled triangle if exactly one angle is of measure greater than 90°.



Remember!

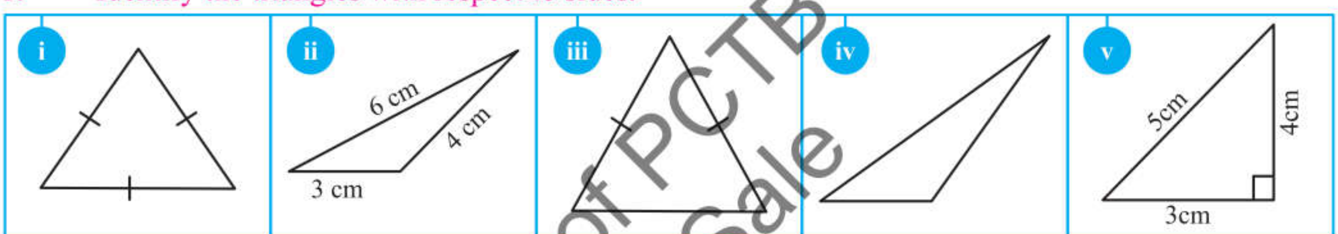
- An obtuse angled triangle can not be equilateral.
- An obtuse angled triangle may be isosceles or scalene.

Need to Know!

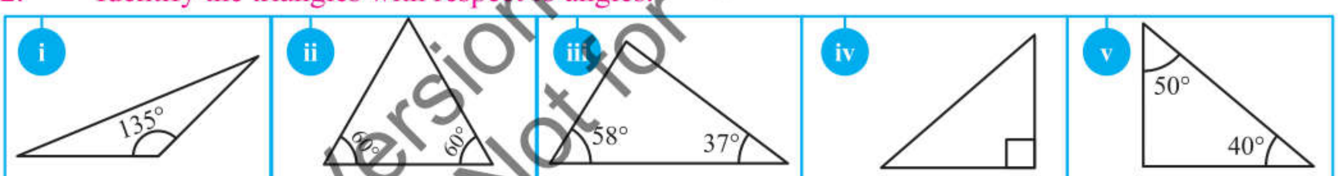
If the sum of two angles is less than 90°, then the triangle will be obtuse angled triangle.

EXERCISE 4.1

1. Identify the triangles with respect to sides.



2. Identify the triangles with respect to angles.



4.1.2 Construction of Triangles

It is very important to know how triangles are constructed. We construct triangles with respect to sides and angles.

Need to Know!

The sum of the length of any two sides in a triangle is always greater than the length of the third side.

Example 3 **Construction of Equilateral Triangle**

Construct an equilateral triangle *ABC* of a side length 5cm.

1st Method

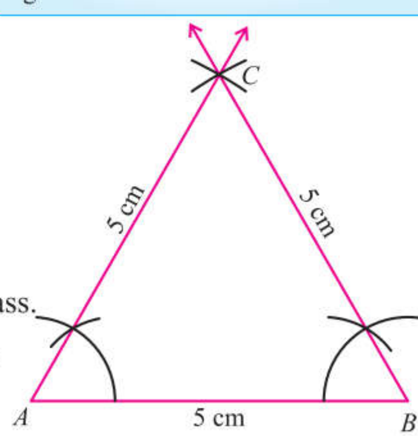
Solution **Steps of construction**

i. Draw a line segment *AB* of length 5*cm*.

ii. Construct an angle of 60° at points *A* and *B* with the help of compass.

iii. Draw two rays from points *A* and *B* meeting each other at point *C*.

Therefore, ΔABC is the required an equilateral triangle.



2nd Method

> **Remember!**
> We can construct the following angles with compass:
> 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°

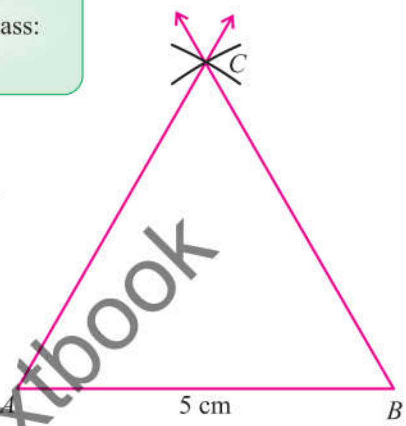
Steps of construction

i. Draw a line segment AB of length 5*cm*.

ii. With centres at points A and B, draw arcs of radius 5*cm* with compass which intersecting each other at point C.

iii. Draw rays AC and BC.

Therefore, ΔABC is the required an equilateral triangle.



Example 4 **Construction of an isosceles triangle.**

Construct a triangle ABC with $m\overline{AB}$ = 6 cm as base and base angle 50°.

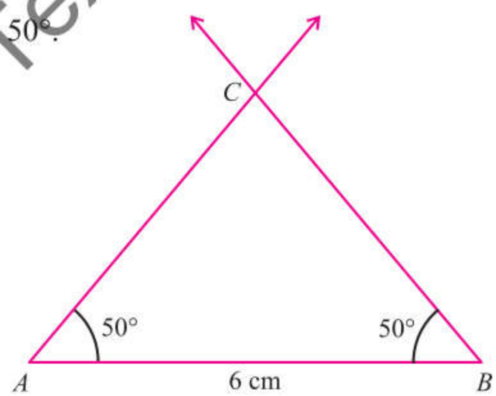
Solution **Steps of construction**

i. Draw $m\overline{AB}$ = 6*cm* with ruler.

ii. Using protractor, draw an angle of 50° at points A and B.

iii. Draw rays from points A and B meeting at point C.

Therefore, ΔABC is the required an isosceles triangle with $m\overline{AC} = m\overline{BC}$



Example 5 **Construction of a right angled triangle.**

Construct a right angled triangle XYZ.

(a) $m\overline{XY}$ = 5.2 cm, $m\overline{YZ}$ = 4.5 cm and $m\angle Y = 90°$

(b) length of hypotenuse $\overline{YZ}$ = 7 cm and $m\overline{XY}$ = 6 cm

> **Brain Teaser!**
> Can you find $m\angle C$?

Solution **(a) Steps of construction**

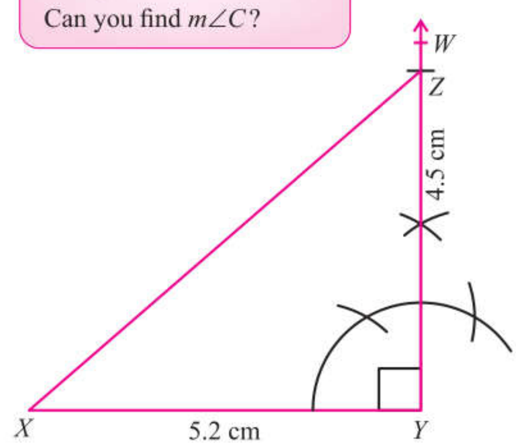
i. Draw $m\overline{XY}$ = 5.2 cm with ruler.

ii. Taking centre at point Y, construct an angle of 90° and draw a ray YW i.e., $m\angle XYW = 90°$.

iii. Draw an arc of radius 4.5 cm with centre at point Y cutting $\overrightarrow{YW}$ at Z.

iv. Join points X and Z.

Therefore, We get a right angled triangle XYZ with hypotenuse $\overline{XZ}$.



> **Brain Teaser!**
> Measure $\overline{XZ}$

Solution **(b) Steps of construction**

i. Draw $m\overline{YZ} = 7$ cm.

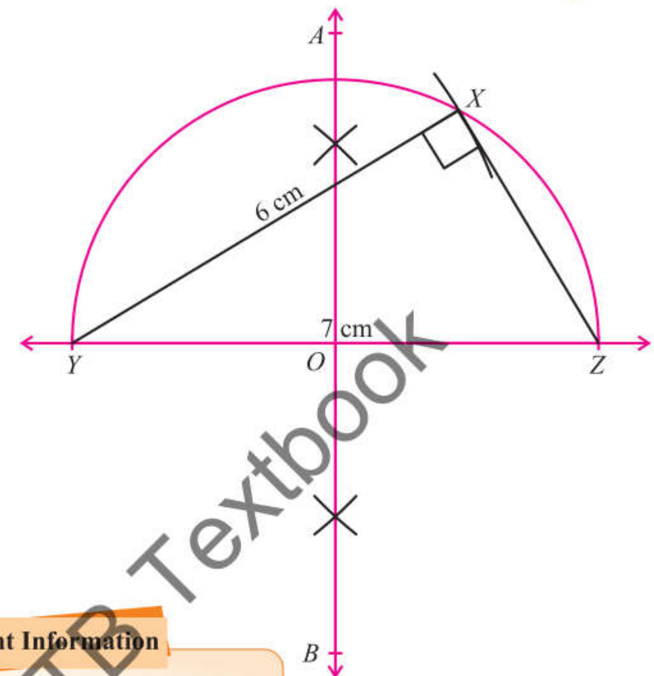
ii. Draw perpendicular bisector $\overleftrightarrow{AB}$ of $\overline{YZ}$ with the help of compass and ruler and mark the point O where $\overleftrightarrow{AB}$ intersects $\overline{YZ}$ (Point O is on $\overline{YZ}$).

iii. Taking the point O as centre, draw a semi-circle of radius $m\overline{OZ}$ using compass.

iv. Taking the point Y as centre draw an arc of radius 6 cm cutting the semi-circle at point X.

v. Join point X with points Y and Z.

We get ΔXYZ with $m\angle X = 90°$



Important Information

Since point O is the midpoint of $\overline{YZ}$, so $m\overline{OY} = m\overline{OZ}$

4.1.3 Construction of Scalene Triangles

We construct a scalene triangle when

(a) Length of two sides and their included angle are given.

(b) Length of two sides and any angle are given.

(c) Two angles and their included side are given.

Remember!

A scalene triangle can be acute angled, right angled or obtuse angled.

(a) Length of two sides and their included angle are given:

Example 6 Construct a triangle ABC when $m\overline{AB} = 5.5$ cm, $m\overline{BC} = 4.5$ cm and $m\angle B = 45°$

Solution **Steps of construction**

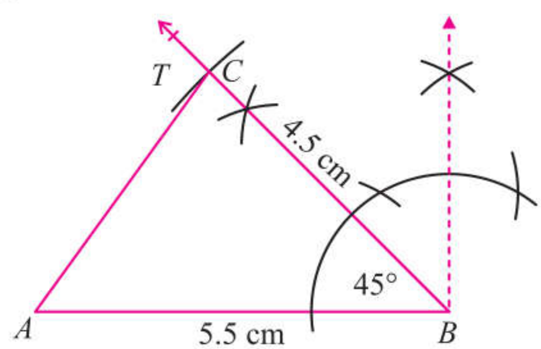
i. Draw a line segment AB of 5.5 cm long.

ii. Construct an angle of 45° at point B with compass and draw a ray BT.

iii. With centre at point B, draw an arc of radius 4.5 cm with compass which intersecting $\overrightarrow{BT}$ at point C.

iv. Join point A with point C.

We get a scalene triangle ABC.



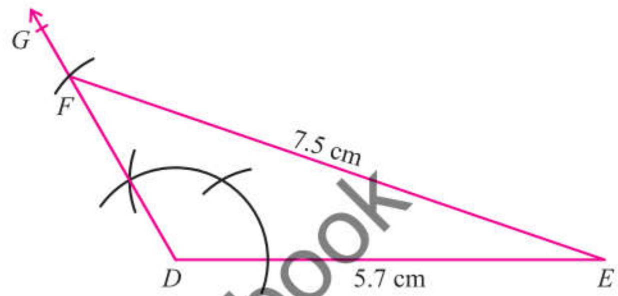
Try yourself

Construct a triangle with measure of sides 3.8 cm, 4.2 cm and 5 cm using compass and ruler.

(b) Length of two sides and an angle are given

Example 7 Draw a ΔDEF such that $m\overline{DE} = 5.7$ cm, $m\overline{EF} = 7.5$ cm and $m\angle D = 120°$

Solution **Steps of construction**



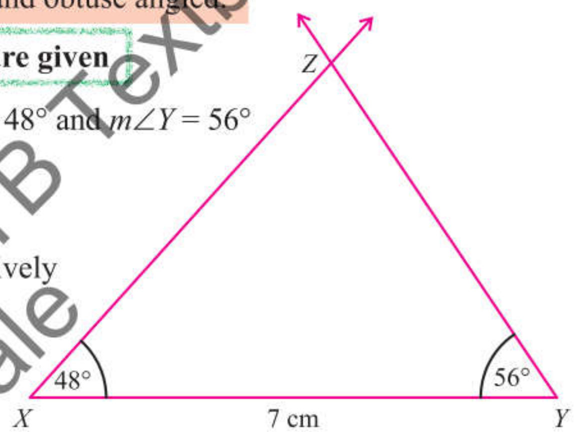
i. Draw $m\overline{DE} = 5.7$ cm.
ii. Construct an angle 120° at point D using a compass and draw $\overrightarrow{DG}$ such that $m\angle EDG = 120°$
iii. With centre at point E, draw an arc of radius 5.7 cm with compass which intersecting $\overrightarrow{DG}$ at point F.
iv. Join points E and F.

We get ΔDEF, which is scalene and obtuse angled.

(c) Two angles and the length of their included side are given

Example 8 Draw a ΔXYZ such that $m\overline{XY} = 7$ cm, $m\angle X = 48°$ and $m\angle Y = 56°$

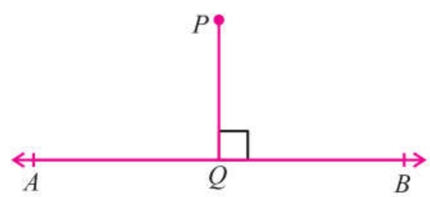
Solution **Steps of construction**



i. Draw a line segment XY of measure 7 cm.
ii. Construct angles 48° and 56° at points X and Y respectively using protractor and draw two rays from points X and Y.
iii. These rays intersect each other at point Z.

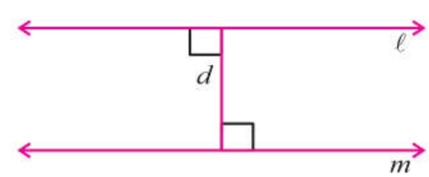
We get ΔXYZ which is scalene and acute angled.

Shortest Distance We have learnt in the previous class how to draw perpendicular from a point to a line i.e., in the figure. $\overline{PQ}$ is perpendicular to the line $\overline{AB}$ at point Q and $m\angle PQB = 90°$.



It is clear from the figure that $m\overline{PQ}$ is the shortest distance from point P to $\overleftrightarrow{AB}$. Hence we conclude that:

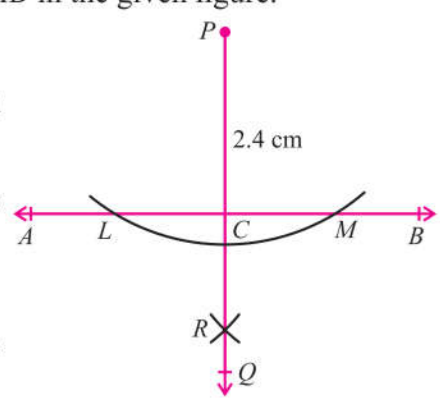
The shortest distance is always perpendicular.



In parallel lines ℓ and m, the shortest distance is d which is perpendicular to ℓ and m.

Example 9 Find the shortest distance of a point P from the line AB in the given figure.

Solution **Steps of construction**



i. With centre at point P draw an arc of any suitable radius which intersecting $\overleftrightarrow{AB}$ at points L and M.
ii. With centres at points L and M draw two arcs of any suitable radius which intersecting each other at point R.
iii. Draw a ray PQ passing through point R intersecting $\overleftrightarrow{AB}$ at point C.
iv. $\overline{PQ}$ is the perpendicular and $m\overline{PC} = 2.4$ cm (measured by ruler) is the shortest distance from point P to the line AB.

Important Information

A perpendicular bisector is a line which divides a line segment into two equal parts.

Activity

Draw the perpendicular bisector on side AB of a triangle ABC taking any suitable measures of its sides.

EXERCISE 4.2

1. Using compass and ruler, construct the following angles:

   (i) 30° (ii) 45° (iii) 75° (iv) 105° (v) 150°

2. Construct a right angled triangle XYZ in which $m\overline{XY} = 6.5$ cm, $m\angle Y = 40°$ and right angle at point X.
3. Construct a right angled triangle ABC with hypotenuse AB of measure 7.1 cm and measure of angle A is 52°.
4. Construct an equilateral triangle of side length 4.7 cm.
5. Construct an isosceles triangle DEF given that $m\overline{DE} = 8.2$ cm is the base and equal sides are $\overline{EF}$ and $\overline{DF}$ each of length 6.5 cm.
6. Construct an isosceles triangle GHI with vertical angle at point G of measure 100° and $m\overline{GH} = 6$ cm.
7. Construct a triangle XYZ with $m\overline{YZ} = 5.3$ cm, $m\angle Y = 45°$ and $m\angle Z = 30°$.
8. Construct a triangle PQR with $m\overline{PQ} = 7$ cm, $m\overline{QR} = 5.5$ cm and $m\angle Q = 75°$.
9. Construct a triangle FGH with $m\overline{GH} = 6.3$ cm, $m\angle H = 55°$ and $m\overline{FG} = 6.9$ cm.
10. Draw a line segment PQ of measure 6 cm. Take a point A above $\overline{PQ}$ and draw a perpendicular from the point A to $\overline{PQ}$. What is the shortest distance of point A from $\overline{PQ}$?

SUMMARY

- Right angle is of measure 90°.
- A triangle is a 3-sided closed figure.
- Types of triangle:
  - with respect to angles are: acute angled , right angled and obtuse angled.
  - with respect to sides are: scalene, isosceles and equilateral.
- Triangle can be constructed if:
  - (a) length of all sides are given.
  - (b) when length of two sides and their included angle are given.
  - (c) when two angles and their included side are given.
  - (d) when length of two sides and any angle are given.
- The shortest distance from a point to a line is always perpendicular.

Sub-domain (ii) Angle Properties of Polygons

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise quadrilaterals and their characteristics (parallel sides, equal sides, equal angles, right angles, lines of symmetry etc.) square, rectangle, parallelogram, rhombus, trapezium and kite.
- Differentiate between convex and concave polygons.
- Calculate unknown angles in quadrilaterals using the properties of quadrilaterals. (square, rectangle, parallelogram, rhombus, trapezium and kite).
- Understand the relationship between interior and exterior angles of polygons and between opposite interior and exterior angles in a triangle.
- Calculate the interior and exterior angles of a polygon and the sum of interior angles of a polygon.
- Calculate unknown angles in a triangle.

Can you see geometrical patterns on snake's skin?

4.2.1 Introduction

There are many 2D figures around us in triangular, square, rectangular, pentagonal or hexagonal in our daily life. These figures are used to make patterns in different 3D objects.

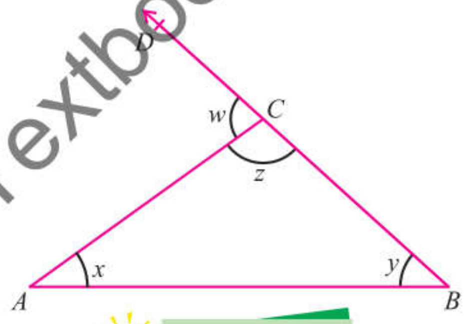
These figures are base of geometry which are useful to teach in elementary education.

4.2.2 Angle Properties of Triangle

The sum of all interior angles in a triangle is equal to 180°.

i.e., $x + y + z = 180°$ in ΔABC

If we extend the side $\overline{BC}$ to $\overrightarrow{BD}$ we get $m\angle ACD = w$ as exterior angle of ΔABC.



We see that interior and exterior angles at point C are supplementary

$$z + w = 180° \rightarrow \text{supplementary angles}$$

$$\Rightarrow \quad w = 180° - z$$

$$\text{But} \quad x + y + z = 180°$$

$$x + y = 180° - z$$

$$\because \quad x + y = w \quad \because w = 180° - z$$

Exterior angle = Sum of two interior opposite angles

Remember!

Angle formed between a side and extended side is called exterior angle.

Skill Practice

In ΔABC, find x, if $y = 36°$ and $w = 100°$

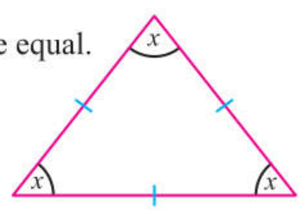
Important Information

$m\angle A = 180° - 2m\angle B$ (vertical angle)

$m\angle B = \dfrac{180° - m\angle A}{2}$ (base angle)

i Equilateral Triangle

In equilateral triangle all angles and sides are equal.



In the figure $\quad x + x + x = 180°$

$$3x = 180°$$

$$x = 60° \quad \text{(each angle is equal to 60°)}$$

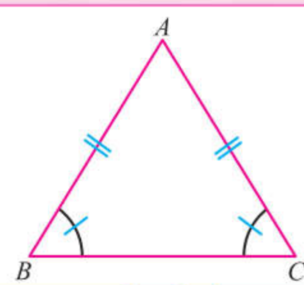
Skill Practice

Find vertical angle of an isosceles triangle if base angle is 75°.

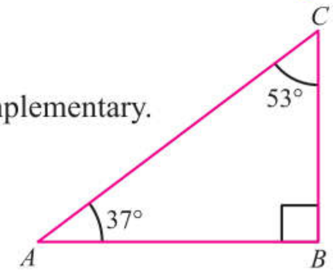
ii Isosceles Triangle

In ΔABC, $\quad m\overline{AB} = m\overline{AC}$ and $m\angle B = m\angle C$

$\angle B$ and $\angle C$ are called base angles and $\angle A$ is called vertical angle of isosceles triangle.



iii Right Angled Triangle



In right angled triangle, exactly one angle is 90° and other two angles are complementary.

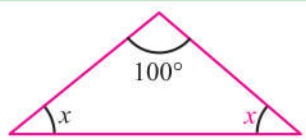
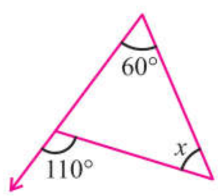
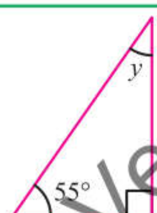
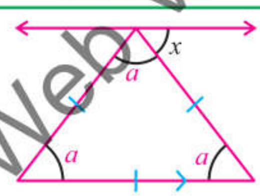
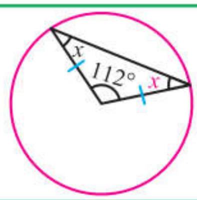
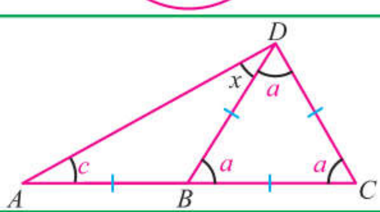
In $\Delta\, ABC$, $m\angle B = 90°$, $m\angle A = 37°$ and $m\angle C = 53°$

Where $\angle A$ and $\angle C$ are complementary angles.

Skill Practice

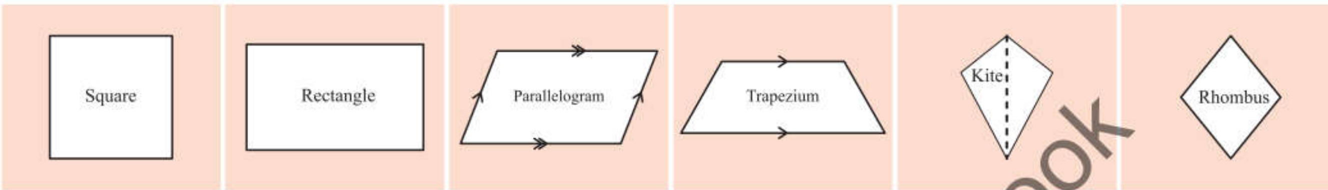
If complementary angles in a right angle triangle are equal to each other, then find these angles.

Example 1 Find unknown angles in the following figures:

	Figure	Solution
i		Sum of angles in a triangle = 180° $x + x + 100° = 180°$ $2x = 180° - 100° \Rightarrow x = \dfrac{80}{2} = 40°$
ii		Exterior angle = Sum of two interior opposite angles $110° = 60° + x$ $x = 110° - 60°$ $x = 50°$
iii		Triangle is right angled, So $y + 55° + 90° = 180°$ $y + 145° = 180°$ $y = 180° - 145° \Rightarrow y = 35°$
iv		The given triangle is equilateral $\because\ a + a + a = 180° \Rightarrow 3a = 180° \Rightarrow a = 60°$ a and x are alternate. i.e., $x = a = 60°$
v		The given triangle is isosceles $x + x + 112 = 180°$ $2x = 180° - 112° \Rightarrow 2x = 68° \Rightarrow x = 34°$
vi		ΔBCD is equilateral $3a = 180° \Rightarrow a = 60°$ also $c = x$ (ΔABD is isosceles) $c + x = a \Rightarrow x + x = 60°$ (The exterior angle is equal to the sum of two opposite interior angles) $2x = 60° \Rightarrow x = 30°$

4.2.3 Identification of Quadrilaterals

Any four sided closed figure is called quadrilateral. It has four angles and four vertices. There are following different types of quadrilaterals:

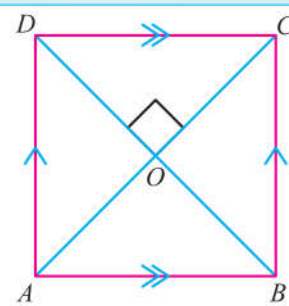


Now we discuss their properties in detail.

Need to Know!

The sum of interior angles of any quadrilaterals is equal to 360°.

Square



(i) All four sides are of equal length.

i.e., $m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{AD}$

(ii) Four angles, each of measure equal to 90°.

i.e., $m\angle A = m\angle B = m\angle C = m\angle D = 90°$

(iii) Two pairs of parallel sides i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$.

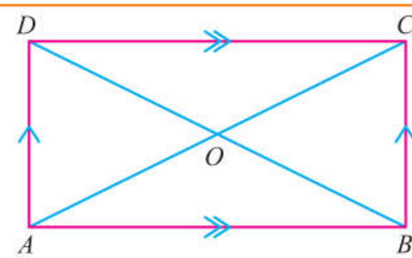
(iv) Diagonals are of equal length i.e., $m\overline{AC} = m\overline{BD}$.

(v) Diagonals bisect each other at point O i.e., midpoint of both diagonals is same.

(vi) Diagonals are perpendicular to each other i.e., $m\angle AOB = m\angle BOC = m\angle COD = m\angle AOD = 90°$

(vii) Since $\overline{AB} \parallel \overline{DC}$, so alternate angles formed are equal i.e., $m\angle CAB = m\angle ACD$ etc.

Rectangle



(i) It has four sides. Opposite sides are of equal length.

i.e., $m\overline{AB} = m\overline{DC}$ and $m\overline{AD} = m\overline{BC}$

(ii) Each interior angle is equal to 90°.

i.e., $m\angle A = m\angle B = m\angle C = m\angle D = 90°$

(iii) Two pairs of parallel sides i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$.

(iv) The following alternate angles are formed with the diagonals

$m\angle BAC = m\angle DCA$ and $m\angle ABD = m\angle CDB$ etc.

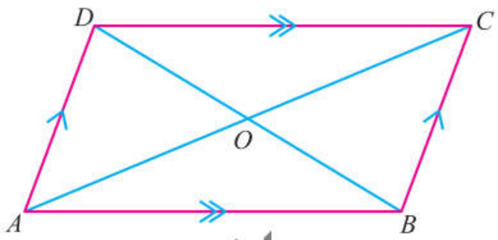
(v) Diagonals are of equal length i.e., $m\overline{AC} = m\overline{BD}$.

(vi) Diagonals bisect each other at point O and the midpont of diagonal AC = midpoint of diagonal BD.

Think

Is every square a rectangle?

Parallelogram



(i) It has four sides. Opposite sides are of equal length.

i.e., $m\overline{AB} = m\overline{DC}$ and $m\overline{AD} = m\overline{BC}$

(ii) Two pairs of parallel sides. (Opposite sides are parallel)

i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

(iii) The following alternate angles are formed with the diagonals

$m\angle BAC = m\angle DCA$ and $m\angle ABD = m\angle CDB$

Remember!

Every square is a parallelogram and every rectangle is a parallelogram.

(iv) Opposite angles are equal i.e., $m\angle A = m\angle C$ and $m\angle B = m\angle D$

(v) Adjacent angles are supplementary due to parallel lines i.e.,

$m\angle A + m\angle D = 180°$ and $m\angle B + m\angle C = 180°$

($m\angle A$, $m\angle D$) and ($m\angle B$, $m\angle C$) are two pairs of interior angles of parallel lines.

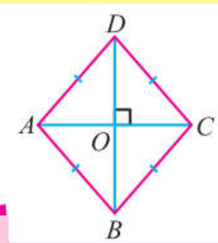
(vi) Diagonals bisect each other at point O and the mid point of diagonal AC = mid point of diagonal BD.

Rhombus

(i) All four sides are of equal length. i.e., $m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{AD}$

(ii) Opposite sides are parallel. i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$. Alternate and interior angles are formed due to parallel lines.

(iii) Diagonals are not of equal length and they bisect the angles $m\angle A$, $m\angle B$, $m\angle C$ and $m\angle D$.



(iv) Diagonals bisect each other at right angle at point O and midpoint of diagonal AC = midpoint of diagonal BD.

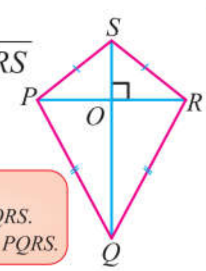
(v) Opposite angles are equal in measure

i.e., $m\angle A = m\angle C$ and $m\angle B = m\angle D$

Think

What is the difference between a rhombus and a square?

Kite



(i) Two pairs of adjacent sides are of equal length i.e., $m\overline{PQ} = m\overline{QR}$ and $m\overline{PS} = m\overline{RS}$

(ii) One pair of opposite angles are equal. i.e., $m\angle P = m\angle R$

(iii) Diagonals are perpendicular to each other.

(iv) One diagonal QS bisect the other diagonal PR.

Try yourself!

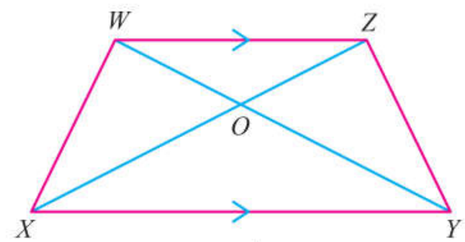
- Name two isosceles triangles in kite $PQRS$.
- Name two right angled triangles in kite $PQRS$.

Trapezium

(i) Only one pair of parallel sides i.e., $\overline{XY} \parallel \overline{WZ}$

(ii) Two pairs of interior angles of parallel lines
i.e., $m\angle WXY + m\angle XWZ = 180°$ and
$m\angle XYZ + m\angle WZY = 180°$

(iii) The pairs of alternate angles of parallel lines.
i.e., $m\angle YXZ = m\angle WZX$ and $m\angle XYW = m\angle ZWY$

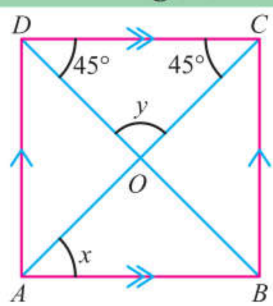


(iv) Vertically opposite angles at point O are: $m\angle XOY = m\angle ZOW$; $m\angle XOW = m\angle ZOY$

Example 2 Find the unknown angles *x, y* or *z* in the following:

i

Figure



ABCD is a square

Solution

Since the diagonals bisect the angles of square, so

$$x = \frac{90°}{2} = 45°$$

In ΔCOD, $m\angle C = m\angle D = 45°$

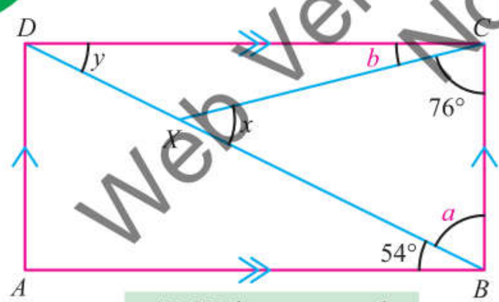
So, $y + 45° + 45° = 180°$

$y + 90° = 180°$

$y = 90°$

ii

Figure



ABCD is a rectangle

Solution

Since each angle is 90°,

So, $m\angle B = 90°$

$a = 90° - 54°$

$= 36°$

Mark angles *a* and *b* as shown

From ΔBCX: $x + 76° + a = 180°$

So, $x + 76° + 36° = 180°$

$x = 180° - 112°$

$= 68°$

Since $m\angle BCD = 90°$

So, $b + 76° = 90°$

$b = 90° - 76°$

$= 14°$

In ΔCDX,

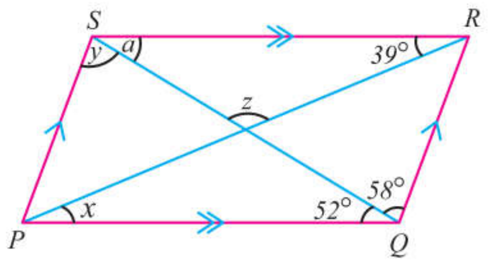
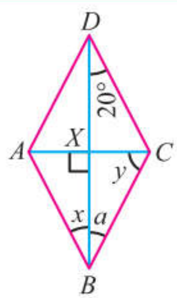
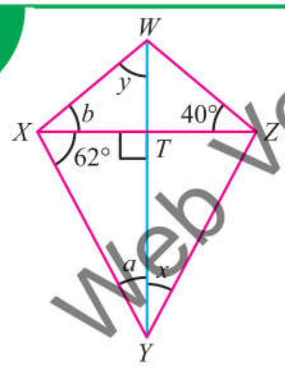
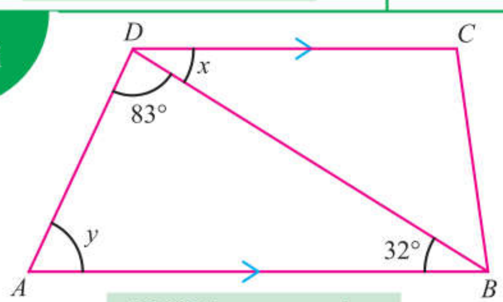
Interior angle = Sum of two opposite interior angles

$\because$ $x = b + y$

So, $68° = 14 + y$

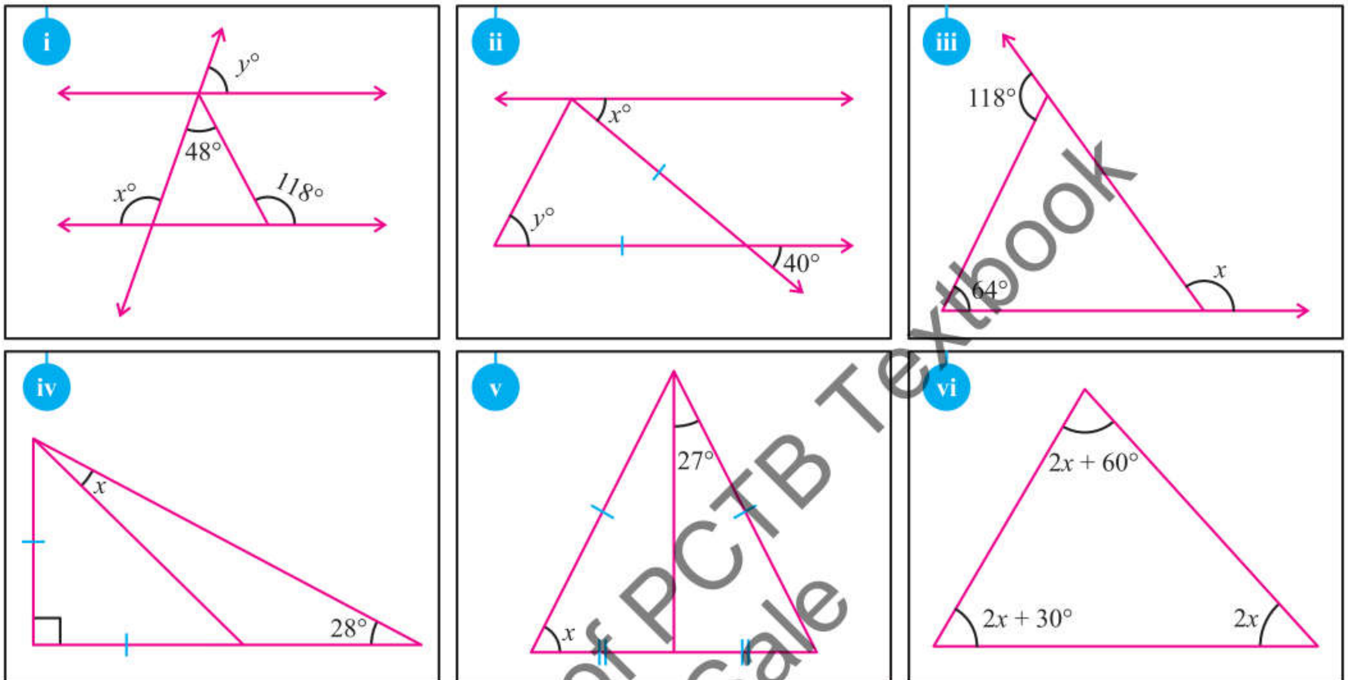
$y = 68° - 14°$

$= 54°$

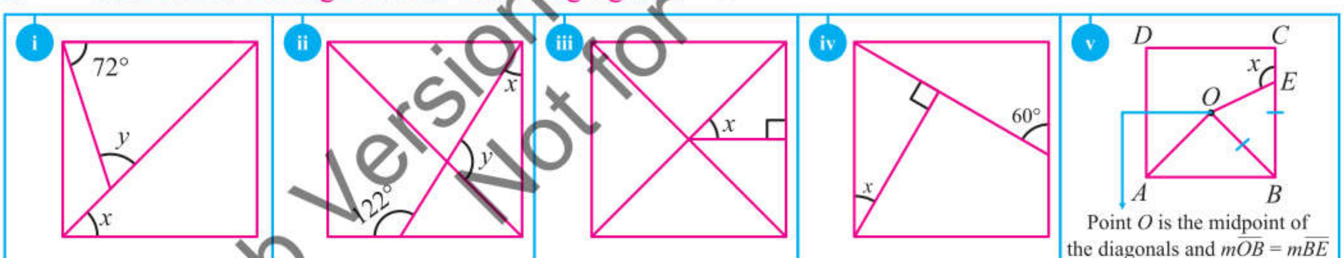
	Figure	Solution
iii	 *PQRS* is a parallelogream	Mark angle a as shown Since, $\overline{PQ} \parallel \overline{SR}$ So, $x = 39°$ Alternate angles of parallel lines Since, opposite angles in parallelogram are equal, so $m\angle PSR = m\angle PQR$ $y = 52° + 58° = 110°$ $a = 52°$ Alternate angles of parallel lines In ΔSRX, $a + z + 39° = 180°$ $52° + z + 39° = 180°$ $z = 180° - 91° = 89°$
iv	 *ABCD* is a rhombus	Mark angle a as shown Since, diagonals bisect the angles so, $x = 20°$ They are also alternate angles of parallel lines Also ΔBXC is right angled $y + a + 90° = 180°$ $y + 20° + 90° = 180°$ $y = 180° - 110° = 70°$
v	 *XYZW* is a kite	Mark angles a and b as shown. Diagonals are at right angle to each other. So, a and $62°$ are complementary angles in right angled ΔXTY $a + 62 = 90°$ $a = 90° - 62° = 28°$ Since ΔXZW is isosceles, so $b = 40°$ In right angled ΔXWT, b and y are complementary $b + y = 90°$ $40° + y = 90°$ $y = 50°$
vi	 *ABCD* is a trapezium	Diagonals intersect parallel lines $\overline{AB}$ and $\overline{DC}$ $x = 32°$ Alternate angles of parallel lines In ΔABD, $y + 32° + 83° = 180°$ $y = 180° - 115°$ $y = 65°$

EXERCISE 4.3

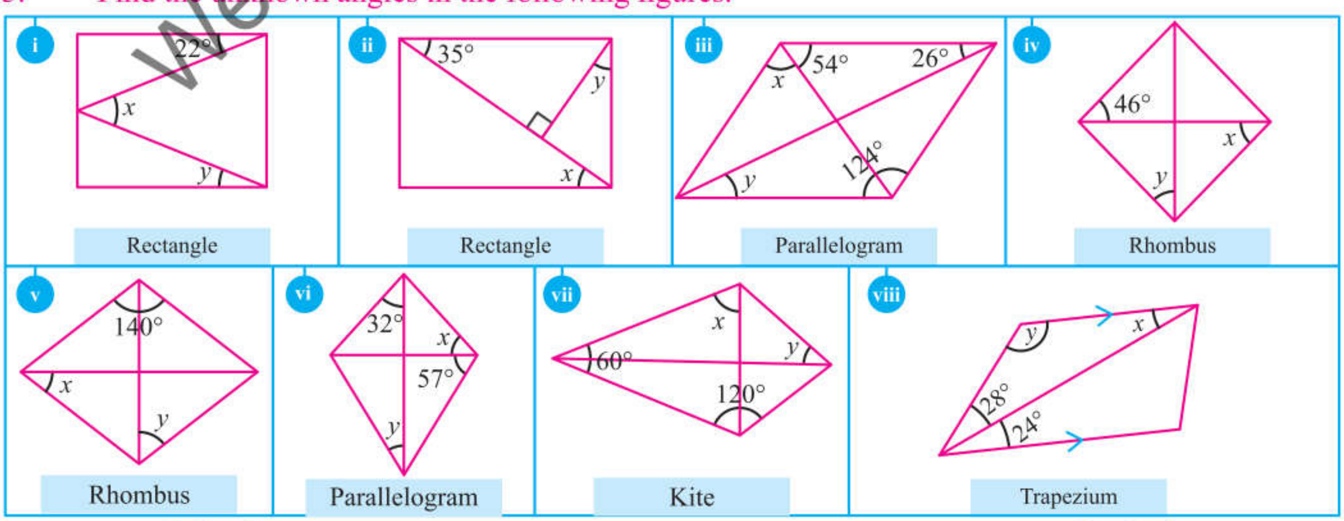
1. Find unknown angles in the following figures.



2. Find unknown angles in the following figures.



3. Find the unknown angles in the following figures.



4.2.4 Polygons

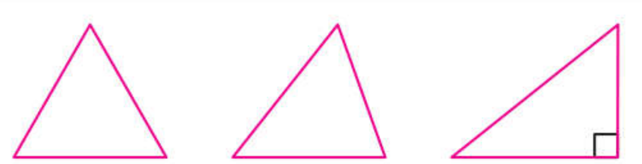
The design of floor or walls are usually square, triangular, pentagonal or hexagonal. Due to symmetry, their designs are fascinating.

Any closed 2D shape with three or more sides is called polygon. If a polygon has n-sides, then this polygon has n-angles and n-vertices.

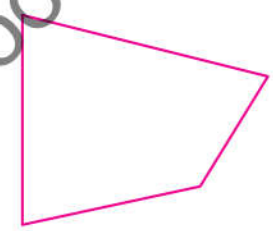
Remember!

The sides of polygon are line segments.

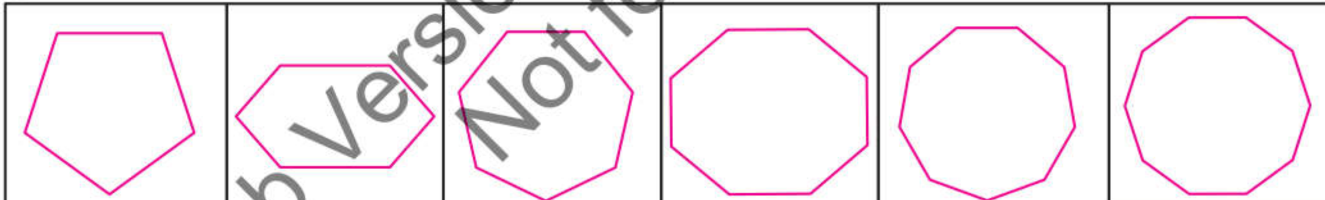
Polygon with three sides is called a triangle.



Polygon with four sides is called quadrilateral

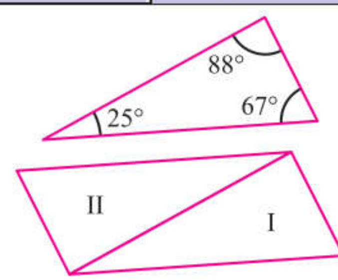


We have already studied different types of quadrilateral.



Pentagon (5-sides) | Hexagon (6-sides) | Heptagon (7-sides) | Octagon (8-sides) | Nonagon (9-sides) | Decagon (10-sides)

i The Sum of Interior Angles of a Polygon



The number of sides of a polygon is equal to the number of angles. We know that the sum of interior angles of a triangle is 180°.

We can find the sum of interior angles of a quadrilateral by dividing the figure into two triangles.

The sum of interior angles of the quadrilateral = Sum of interior angles of triangle-I and II

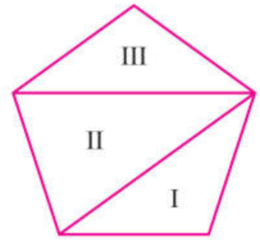
$$= 180° + 180°$$

or $$= 2 \times 180°$$

or $$= (4 - 2) \times 180°$$

$$= 360°$$

If we divide the pentagon into three triangles then the sum of interior angles of pentagon = Sum of interior angles of triangles I, II and III.



$$= 180° + 180° + 180°$$
$$= 3 \times 180° = 540°$$
$$= (5 - 2) \times 180°$$

We have seen that

Shape	Sum of interior angles	
Triangle	$180° = 1 \times 180°$	$(3 - 2) \times 180°$
Quadrilateral	$360° = 2 \times 180°$	$(4 - 2) \times 180°$
Pentagon	$540° = 3 \times 180°$	$(5 - 2) \times 180°$
n-sided polygon		$(n - 2) \times 180°$

Hence, the sum of interior angles of n-sided polygon = $(n - 2) \times 180°$

Example 3 Find the sum of interior angles of:

(i) Hexagon (ii) Octagon

Solution

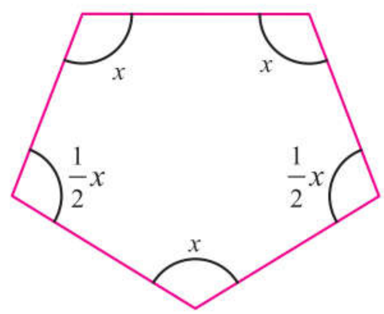
(i) Here $n = 6$ (Hexagon has 6 sides)

Sum of interior angles $= (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 4 \times 180°$
$= 720°$

(ii) Here $n = 8$ (Octagon has 8 sides)

Sum of interior angles $= (n - 2) \times 180°$
$= (8 - 2) \times 180°$
$= 6 \times 180°$
$= 1080°$

Example 4 Find unknown value x.



Solution The given figure is a pentagon

Sum of interior angles of a pentagon $= (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 3 \times 180° = 540°$

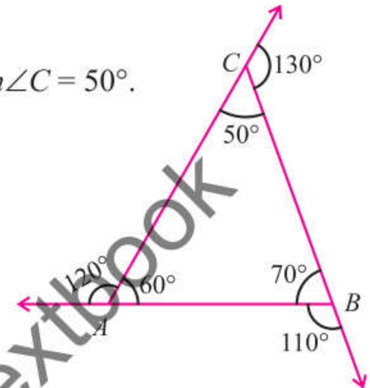
Sum of interior angles $= x + x + x + \frac{1}{2}x + \frac{1}{2}x$
$= 3x + x$
$= 4x$

$\because \quad 4x = 540°$

$x = \frac{540}{4}$
$= 135°$

ii The Sum of Exterior Angles of a Polygon

An exterior angle of polygon is formed when one of its side is extended. Then the angle formed with the adjacent side is called exterior angle.



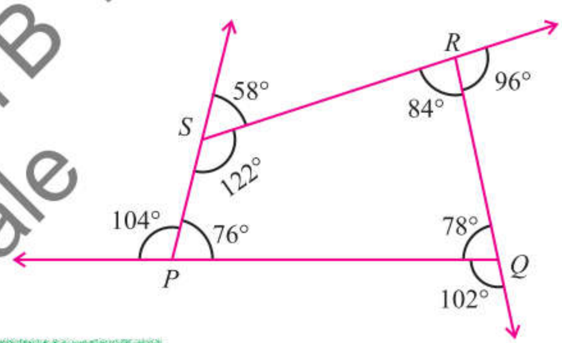
For example in ABC, interior angles are $m\angle A = 60°$, $m\angle B = 70°$ and $m\angle C = 50°$.

Exterior angles at point A, B, C are 120°, 110° and 130° respectively.

We notice that sum of exterior angles in a triangle

$$= 120° + 110° + 130°$$
$$= 360°$$

In case of quadrilateral $PQRS$ the exterior angles at P, Q, R and S are 104°, 102°, 96° and 58° respectively.



Sum of these exterior angles $= 104° + 102° + 96° + 58°$

$$= 360°$$

We have seen that the sum of exterior angle is 360° for both triangle and quadrilateral. So,

Sum of exterior angles of any polygon = 360°

(a) Rotation between Interior and Exterior Angles of a Polygon

From the above examples we noted that interior and exterior angles are adjacent and they are supplementary. So, the sum of interior and exterior angles at a point is equal to 180°.

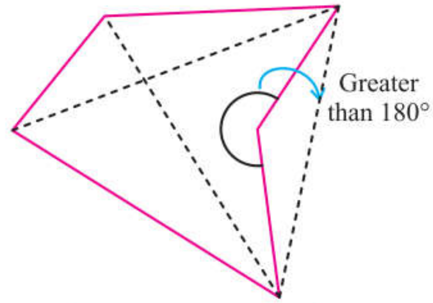
i.e., Interior angle + Exterior angle = 180°

4.2.5 Types of Polygons

There are two main types of polygons (a) Concave polygon (b) Convex polygon

(a) Concave Polygon

A polygon which has at least one reflex angel is said to be a concave polygon.

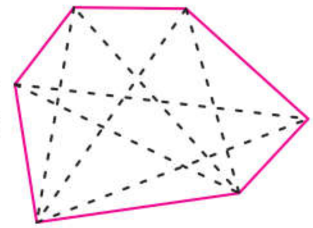


Concave polygon

For example: In the figure, one angle is of measure greater than 180° (i.e., one angle is reflex). So, it is a concave polygon. If we draw diagonals, we see that one of its diagonals lies outside the polygon.

(b) Convex Polygon

A polygon which has all of its angles less than 180° is called convex polygon. All of its diagonals lie inside the polygon as shown in the figure. Each angle is less than 180°.



Convex polygon

Activity

The number of diagonals in a polygon can be calculated by the formula $\frac{n(n-3)}{2}$ where n is the number of sides of the polygon. Find the number of diagonals of pentagon and octagon and also draw these figures with their diagonals.

Brain Teaser!

Draw a polygon which has two reflex angles.

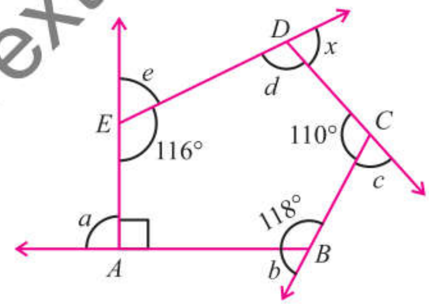
Example 5

(i) Identify the exterior angles of pentagon $ABCDE$.

(ii) Find the exterior angles at vertices A, B, C and E.

(iii) Find the value of x.

(iv) Identify the polygon (concave or convex)



Solution

(i) The angles marked at vertices A, B, C, D and E as a, b, c, x and e are exterior angles of pentagon $ABCDE$. Because they are formed with the extended side of the polygon.

(ii) The exterior angle at point A, $a = 180° - 90° = 90°$

The exterior angle at point B, $b = 180° - 118° = 62°$

The exterior angle at point C, $c = 180° - 110° = 70°$

The exterior angle at point E, $e = 180° - 116° = 64°$

Important Information

The exterior angle at point D can be written as

$x = 180° - d$

(iii) To find the value of x, first we find the value of d.

The sum of interior angles of pentagon $= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 3 \times 180° = 540°$

Explore

Find whether the sum of exterior angles of pentagon $ABCDE$ is equal to 360°.

Adding all the interior angles of pentagon

$$90° + 118° + 110° + d + 116° = 540°$$

$$434° + d = 540° \quad \Rightarrow \quad d = 106°$$

Since x is exterior angle, so $x = 180° - 106°$

$= 74°$

(iv) Since each of the interior angle of the pentagon is less than 180°, so it is a convex polygon.

4.2.6 Regular Polygon

A polygon is said to be regular if all of its sides are equal in measure. A regular polygon also has all of its interior angles (or exterior angles) equal in measure.

An equilateral triangle and square are regular polygons because each side is of equal length and all of the interior angles are equal.

Important Information

A regular pentagon and a regular hexagon are:

(a) Interior Angle of Regular Polygon

Interior angle can be found by dividing sum of the interior angles by number of sides (n) i.e.,

$$\text{Interior angle} = \frac{\text{Sum of all interior angles}}{\text{Number of sides}}$$

$$= \frac{(n-2) \times 180^\circ}{n}$$

$\because$ Each interior angle is equal in measure

Skill Practice

Draw a pattern made of repeating equilateral triangle.

(b) Exterior Angle of Regular Polygon

We know that sum of exterior angles of every polygon with n-sides is equal to 360°. In regular polygon all exterior angles are equal in measure. So,

$$\text{Exterior angle} = \frac{360^\circ}{n}$$

n is the number of sides

Example 6 Find interior and exterior angles of a regular polygon with given number of sides (n).

(i) $n = 6$ (ii) $n = 9$

Solution **(i) $n = 6$ (Hexagon)**

Sum of interior angles of hexagon $= (n-2) \times 180^\circ$

$= 4 \times 180^\circ$

$= 720^\circ$

$$\text{Interior angle} = \frac{720^\circ}{6} = 120^\circ$$

$$\text{Exterior angle} = \frac{360^\circ}{6} = 60^\circ$$

Working

Exterior angle = 180° – interior angle

= 180° – 120° = 60°

Working

The sum of all exterior angles of hexagon = 6 × 60° = 360°

(i) $n = 9$ (Nonagon)

$$\text{Interior angle} = \frac{(n-2) \times 180^\circ}{n}$$

$$= \frac{(9-2) \times 180^\circ}{9}$$

$$= \frac{7 \times \cancel{180^\circ}^{20}}{\cancel{9}_1} = 140^\circ$$

Exterior angle = 180° – interior angle

= 180° – 140° = 40°

Working

Sum of all exterior angles of nonagon = 9 × 40° = 360°

Important Information

Bee honeycomb is a network of regular hexagon

Example 7

(i) The interior angle of a regular polygon with n-sides is 144°. Find the value of n.

(ii) The exterior angle of a regular polygon is 30°. Find the number of sides of the polygon.

Solution

(i) Interior angle = 144°

Number of sides = n

Interitor angle = $(n-2) \times 180°$

$$144° = \frac{(n-2) \times 180°}{n}$$

$$144n = 180n - 360$$

$$180n - 144n = 360$$

$$n = \frac{360}{36}$$

$$n = 10$$

(ii) Exterior angle $= \frac{360°}{n}$

$$30° = \frac{360°}{n}$$

$$n = \frac{360°}{30°}$$

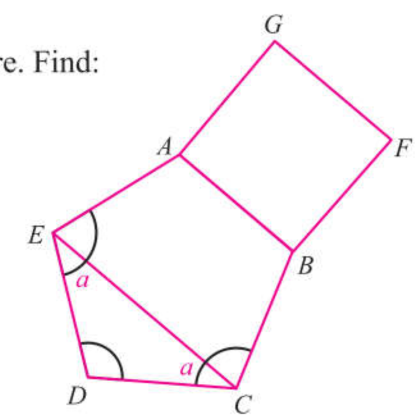
$$n = 12$$

Example 8

ABCDE is a regular pentagon and *ABFG* is a square. Find:

(i) Interior angle of pentagon

(ii) $m\angle CBF$ (iii) $m\angle DCE$



Solution

(i) A pentagon has $n = 5$ sides.

$$\text{Interior angle} = \frac{(n-2) \times 180°}{n}$$

$$\text{Interior angle} = \frac{(5-2) \times 180°}{5}$$

$$= \frac{3 \times \cancel{180°}^{36}}{\cancel{5}_1}$$

$$= 108°$$

(ii) $m\angle ABC = 108°$

$$m\angle ABF = 90°$$

Reflex angle *CBF*

$$m\angle ABC + m\angle ABF = 108° + 90°$$

$$= 198°$$

$$m\angle CBF = 360° - 198°$$

$$= 162° \text{ (which is obtuse)}$$

(iii) ΔDCE is isosceles because of regular pentagon

Let $m\angle CDE = 108°$ and $m\angle DCE = a$

So, $108° + a + a = 180°$

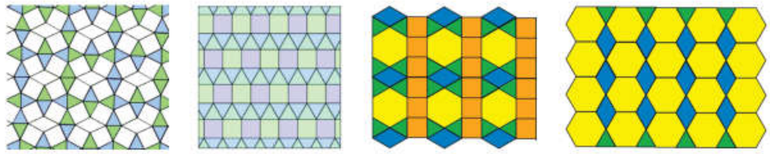
$$2a = 180° - 108°$$

$$a = \frac{72°}{2}$$

$$m\angle DCE = 36°$$

Important Information

A tessellation is a repetitive pattern of polygons that fit together without overlapping and without gaps between them. Find a tessellation anywhere in your house or city.



EXERCISE 4.4

1. Find the sum of interior angles of a polygon with given number of sides (n)

   (i) $n = 7$ (ii) $n = 9$ (iii) 13

2. Find an interior angle of a regular polygon with given number of sides (n):

   (i) $n = 6$ (ii) $n = 8$ (iii) $n = 15$

3. Find exterior angle of a regular polygon with given number of sides (n):

   (i) $n = 7$ (ii) $n = 8$ (iii) $n = 10$

4. Find number of sides (n) of a regular polygon with given an interior angle:

   (i) 108° (ii) 140° (iii) 150°

5. Find number sides (n) of regular polygon with given an exterior angle:

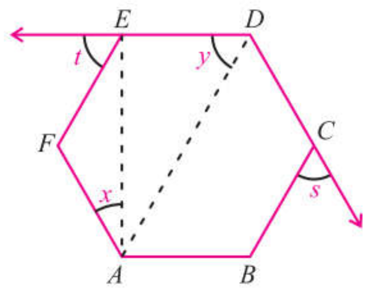
   (i) $32\frac{8}{11}°$ (ii) $25\frac{5}{7}°$ (iii) 24°

6. The figure *ABCDEF* is a regular hexagon:

   (i) Find the value of x and y

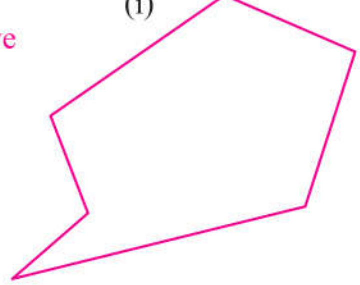
   (ii) Identify two exterior angles in the polygon.

   (iii) Find the value of s and of t.

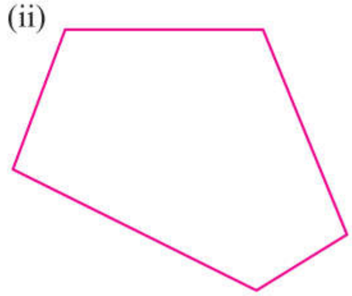


7. Draw diagonals in the given figures to identify which polygon is concave and which is convex?

(i)



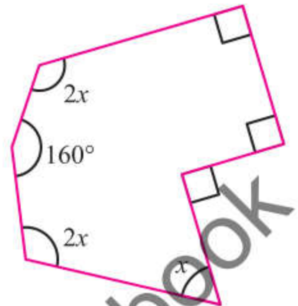
(ii)



8. The interior angles of a pentagon are in the ratio 1:2:3:4:2. Find these angles.

9. The size of each interior angle is 8 times the exterior angle in a regular polygon. Find the number of sides of the polygon.

10. The given polygon is a heptagon. Find the value of x.



SUMMARY

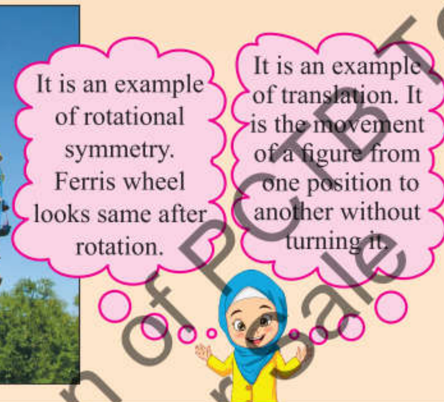
- In square, rectangle, parallellogram and rhombus, diagonals bisect each other and midpoint of one diagonal = midpoint of the other diagonal.
- In trapezium, there are two parallel lines.
- Any closed 2D figure with three or more sides is called a polygon.
- A polygon which has at least one reflex angel is said to be a concave polygon.
- A polygon is said to be a regular polygon if all the sides (or angles) are of equal measure. Equilateral triangle and square are regular polygon.
- The sum of interior angles of any polygon is $(n-2) \times 180$, with n-sides.
- The sum of exterior angles of any polygon is 360°.
- The size of interior angle of a regular polygon is $\frac{(n-2) \times 180°}{n}$.
- The size of exterior angle of a regular polygon is $\frac{360°}{n}$.
- The sum of each interior and corresponding exterior angles of any polygon is 180°.
- The exterior angle of triangle is equal to the sum of two opposite interior angles.

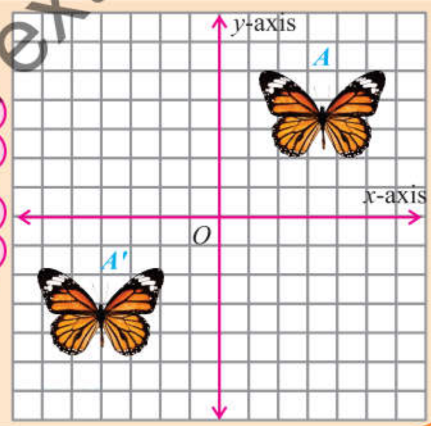
Sub-domain (iii) Transformation

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise identity and draw lines of symmetry in 2D shapes and rotate objects using rotational symmetry and find the order of rotational symmetry.
- Translate an object and give precise description of transformation





4.3.1 Symmetry

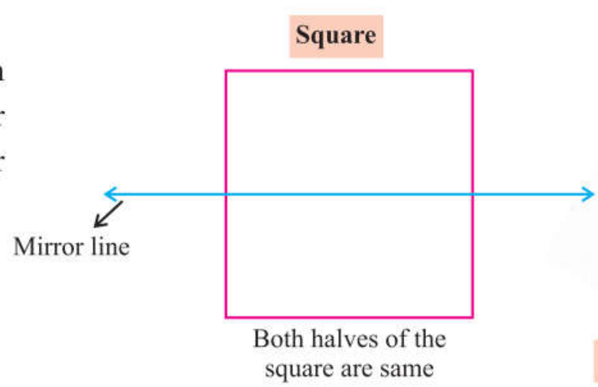
Most of the things in our life are symmetrical. Symmetry looks beautiful. Allah made symmetrical objects which creates harmony and order. We will discuss here line symmetry, rotational symmetry and translation symmetry.

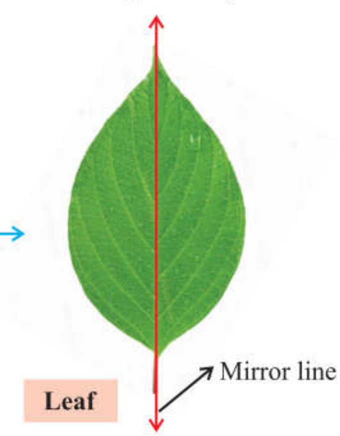
i Types of Symmetry

Two types of symmetry are mostly used in daily life which are reflective and rotational symmetry.

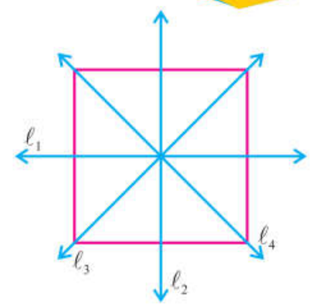
(a) Reflective Symmetry

It is a type of symmetry in which half of the shape reflects the other half. It is also known as mirror symmetry.





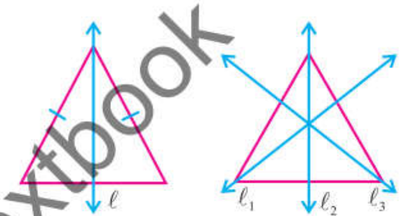
Lines of Symmetry in a Square



Mirror line is also called line of symmetry. There are following lines of symmetry that can be drawn in a square.

ℓ_1 is horizontal, ℓ_2 is vertical, ℓ_3 and ℓ_4 are diagonal lines of symmetry.

Lines of Symmetry in Isosceles and Equilateral Triangles



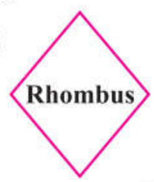
In an isosceles triangle there is only one line of symmetry. The equilateral triangle has three lines of symmetry.

Consider the line of symmetry ℓ_1 in equilateral triangle. If we fold on the line ℓ_1 we see that half of the figure matches the other half.

Example 1 Draw lines of symmetry in the following figures:

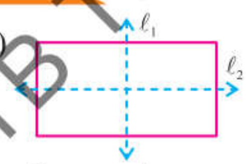
(i)

(ii)

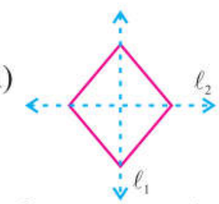


Solution

(i)



(ii)



We can draw only two lines of symmetry in a rectangle and rhombus.

Identify Line of Symmetry

The figure shows a rectangle. If we fold the rectangle over the line ℓ_1, then upper half of figure does not match with lower half and also same in case of ℓ_3. So ℓ_1 and ℓ_3 are not lines of symmetry.

If the figure is folded over the line ℓ_2 both halves match each other. So ℓ_2 is the line of symmetry.

(b) Rotational Symmetry

We have seen that many figures have lines of symmetry but many have no line of symmetry. Some figures look the same after rotation.

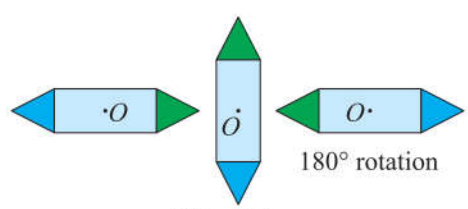
If a figure is rotated around its centre and it looks same as it was before rotation, it is said to be a rotational symmetry.

The point about which the figure is rotated is called centre of rotation.

Ferris wheel looks same after rotation

Identify Rotational Symmetry

State whether the figure has rotational symmetry or not? We see that 90° rotation clockwise or anticlockwise about point O we do not get the original figure. If we rotate 180° about point O, we will get the original figure again. So, the figure has rotational symmetry because it gets its shape two times when rotated in full rotation.

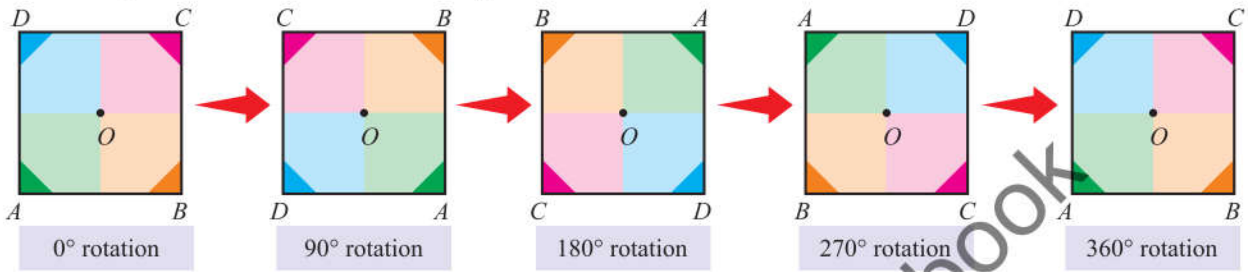


180° rotation

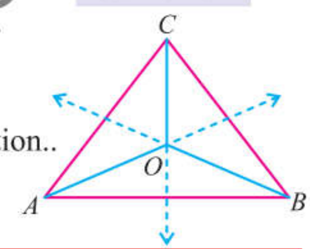
90° rotation

Order of Rotation

The order of rotation is the number of times a shape becomes same in one full rotation. We rotate the square about point O anticlockwise through 90°.



After 360° rotation the square is rotated four times and looks same after each of 90° anticlockwise rotation. So, the square has rotational symmetry of order 4.



Example 2 In the given equilateral triangle ABC, find centre and angle of rotation..

Solution $\overrightarrow{AO}$, $\overrightarrow{BO}$ and $\overrightarrow{CO}$ are lines of symmetry.

In the ΔOAB,

$m\angle OAB = m\angle OBA = 30°$

So, $m\angle AOB = 180° - 30° - 30° = 120°$

Similarly, $m\angle BOC = m\angle AOC = 120°$

If we rotate $\Delta\ ABC$ about point O through 120°, we get the original figure again. So,

Angle of rotation = 120°

Centre of rotation = point O.

Order of rotational symmetry $= \dfrac{360°}{120°} = 3$

> The centre of rotation can be found by drawing lines of symmetry. The point where the lines of symmetry meet, is the point of rotation.

> **Need to Know!**
> In regular polygon with n-sides, the number of lines of symmetry are n and order of rotational symmetry is also n.
> Angle of rotational symmetry $= \dfrac{360°}{n}$

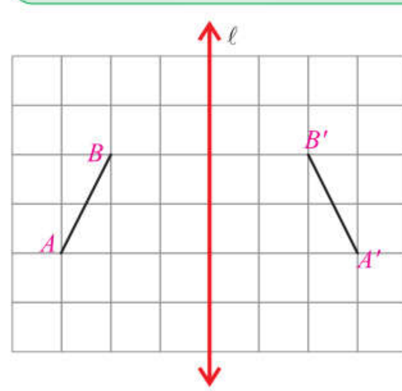
4.3.2 Transformation

It is the process of moving a geometric figure from one place to another in a plane. The image is the figure which is obtained after transformations. Reflection, rotation and translation are all types of transformation. We first discuss reflection and then translation.

> **Do you know?**
> Transformation is the mapping of a geometric figure.

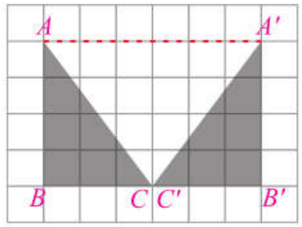
i Reflection

The mirror image produced by flipping a figure over a line is called reflection. The distance of any point on the object is equal to the distance of the corresponding point on the image.

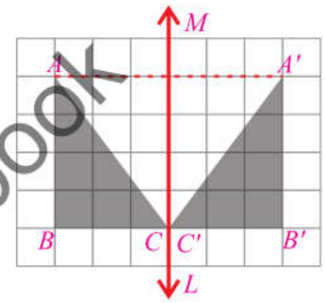


We see that A and A' are 3 units from line of reflection ℓ and B and B' are 2 units from the line of reflection ℓ. $\overline{A'B'}$ is the image of $\overline{AB}$ in the line of reflection ℓ.

Example 3 Draw line of reflection in the following figure.



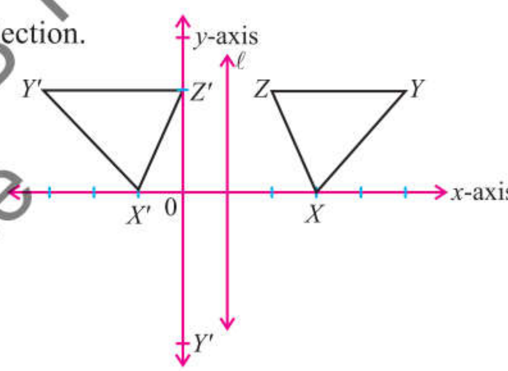
Solution



ΔABC has an image $\Delta A'B'C'$. We see that $m\overline{BC} = m\overline{B'C'}$. So, points C or C' is the midpoint of $\overline{BB'}$. Similarly midpoint of AA' is M. Draw $\overleftrightarrow{ML}$ which is the required line of reflection.

If a point lies on the line of reflection, the image of that point is same as the point. i.e., C and C' are same points.

Example 4 Complete the figure using the given line of reflection.



Solution

Image of X will be X' both 2 units away from line of reflection ℓ. Similarly Z , Z' are 1 unit away ℓ and Y , Y' are 4 units away from ℓ.

ii Translation

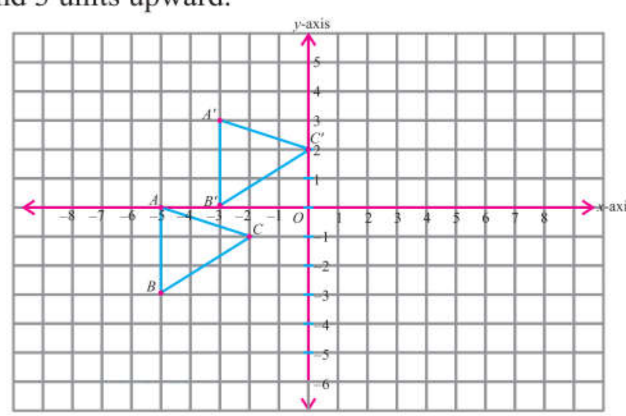
It is the movement of a figure from one position to another without turning it. In translation each point of the object is moved along straight line.

Need to Know!

- Translation is sometimes called slide.
- Translation is a type of transformation.

Example 5 Move the triangle ABC, 2 units right and 3 units upward.

Solution



The triangle ABC has coordinates $A(-5, 0)$, $B(-5, -3)$ and $C(-2, -1)$. Now move the point A, 2 units right and 3 units upward. We get $A'(-3, 3)$. Similarly the images of B and C are $B'(-3, 0)$ and $C'(0, 2)$.

So, $\Delta A'B'C'$ is the image of ΔABC under a translation 2 units right and 3 units upward.

We denote the translation by $T = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, point by P and image by I then:

$P + T = I$ [i.e., Point + Translation = Image] i.e., for point A, $\begin{pmatrix}-5\\0\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\3\end{pmatrix}$

For point B, $\begin{pmatrix}-5\\-3\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\0\end{pmatrix}$

For point C, $\begin{pmatrix}-2\\-1\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}$

Remember!

- 2 units upward is written as +2
- 2 units down is written as –2
- 2 units right is written as +2
- 2 units left is written as –2

Example 6 Find translation when the point $P(-1, 1)$ is mapped on to $P'(3, -2)$

Solution We know that $P + T = P'$

$$T = P' - P$$
$$=\begin{pmatrix}3\\-2\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}$$
$$=\begin{pmatrix}3+1\\-2-1\end{pmatrix}=\begin{pmatrix}4\\-3\end{pmatrix}$$

So, translation is 4 units right and 3 units downward.

Skill Practice

The image of a point P is $P'(-6, 8)$ under a translation $T=\begin{pmatrix}4\\-4\end{pmatrix}$. Find the point P.

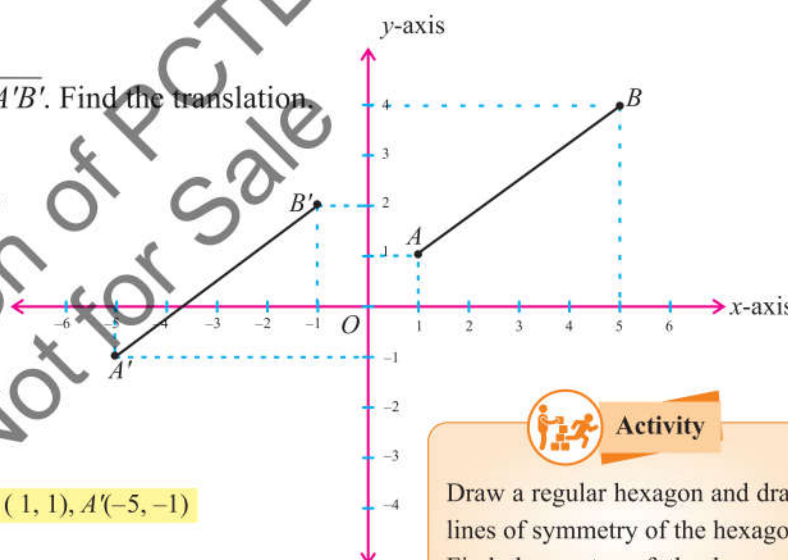
Example 8

Given a line segment AB and its image $\overline{A'B'}$. Find the translation.

Solution

By looking at points A and A' (also B and B'), we see that point A' is 6 units left and 2 units down from point A.

So, translation is $\begin{pmatrix}-6\\-2\end{pmatrix}$. We can also find translation T as:



T = Image – Point $A = (1, 1), A'(-5, -1)$

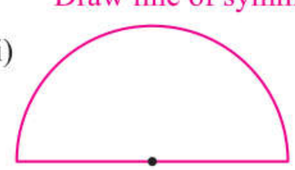
$$=\begin{pmatrix}-5\\-1\end{pmatrix}-\begin{pmatrix}1\\1\end{pmatrix}$$
$$=\begin{pmatrix}-5-1\\-1-1\end{pmatrix}=\begin{pmatrix}-6\\-2\end{pmatrix}$$

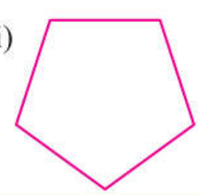
Activity

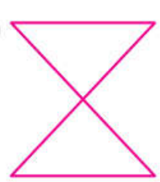
Draw a regular hexagon and draw lines of symmetry of the hexagon. Find the centre of the hexagon. What is the angle of rotation?

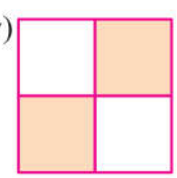
EXERCISE 4.5

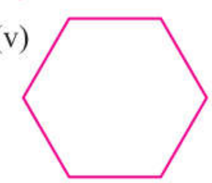
1. Draw line of symmetry and order of rotational symmetry in each of the following:

(i)


(ii)


(iii)


(iv)


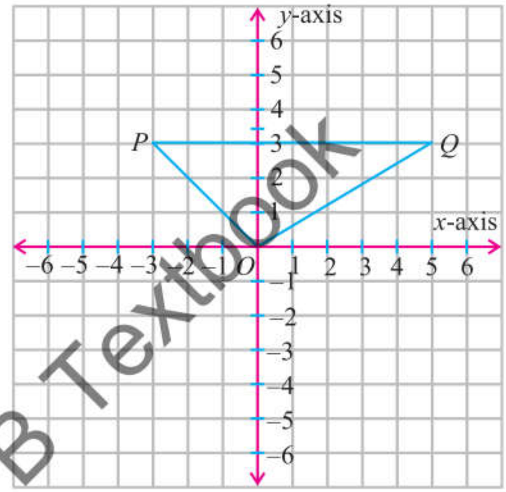
(v)


2. Find the point of rotation of regular hexagon and decagon. Also find the angle of rotation.

3. Find the translation if the point $B(-3, 2)$ is mapped on to the point $B'(2, 6)$.

4. A triangle ABC with vertices $A(-3, 0)$, $B(0, -3)$ and $C(3, 1)$ is mapped on to the triangle DEF with a translation of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

5. Translate the ΔOPQ with a translation of $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$



SUMMARY

- In reflective symmetry, half of the shape is the mirror image of the other half.
- The line which divides the shape into two equal halves is called line of symmetry.
- The point and image are at equal distances from the line of symmetry.
- When a shape is rotated about its centre through an angle and the shape attains its original position after rotating, this type of symmetry is known as rotational symmetry.
- Order of rotational symmetry is the number of times, the shape remains same between 0° and 360°.
- The centre of rotation in regular polygons is the point of intersection of its lines of symmetry.
- Transformation is the process of moving a geometric figure from one position to another. Reflection, rotation and translation are types of transformation.

REVIEW EXERCISE 4

1. Choose the correct option.

(i) A right angled triangle can not be __________ triangle.

(a) isosceles (b) equilateral

(c) scalene (d) both isosceles and scalene

(ii) In right angled triangle, one angle is 90° and the other two angles are __________ .

(a) complementary (b) supplementary (d) obtuse (d) reflex

(iii) In an isosceles triangle, if base angles are equal to 42° each, then vertical angle is________.

(a) 42° (b) 76° (c) 84° (d) 96°

(iv) Which of the following angles so formed with transversal and parallel lines are supplementary?

(a) alternate angles (b) interior angles

(c) vertically opposite angles (d) corresponding angles

(v) If two angles of an isosceles triangle are 40° and 100° the third angle is __________.

(a) 40° (b) 80° (c) 100° (d) 140°

(vi) The diagonals in the quadrilateral __________ do not bisect each other.

(a) square (b) rectangle (c) kite (d) rhombus

(vii) In which quadrilateral, there are no parallel lines?

(a) square (b) rectangle (c) kite (d) rhombus

(viii) In which quadrilateral, the opposite angles are not equal?

(a) square (b) rectangle (c) rhombus (d) trapezirm

(ix) A polygon is said to be __________ if atleast one angle is reflex.

(a) regular (b) concave (d) convex (d) closed

(x) The exterior angle of regular octagon is __________.

(a) 35° (b) 45° (c) 90° (d) 135°

(xi) If a figure is divided into the two equal parts, it is known as __________.

(a) reflection (b) rotation (c) translation (d) image

(xii) The order of rotational symmetry of hexagon is __________.

(a) 2 (b) 4 (c) 16 (d) 8

(xiii) Which of the following quadrilateral has no line of symmetry?

(a) rectangle (b) rhombus (c) kite (d) square

(xiv) Which of the following quadrilateral has no rotational symmetry?

(a) rectangle (b) rhombus (c) kite (d) square

(xv) The movement of an object from one position to another along straight line is called __________.

(a) translation (b) rotation (c) reflection (d) measurement

2. Using compass and ruler construct the following angles:

(i) 15° (ii) $22\frac{1}{2}^\circ$ (iii) $37\frac{1}{2}^\circ$ (iv) 135° (v) 165°

3. Construct a right angled triangle with side lengths 4.5 cm, 6 cm, 75 mm.

4. Construct an isosceles triangle with base length 5.4 cm and base angles each of measure 75°.

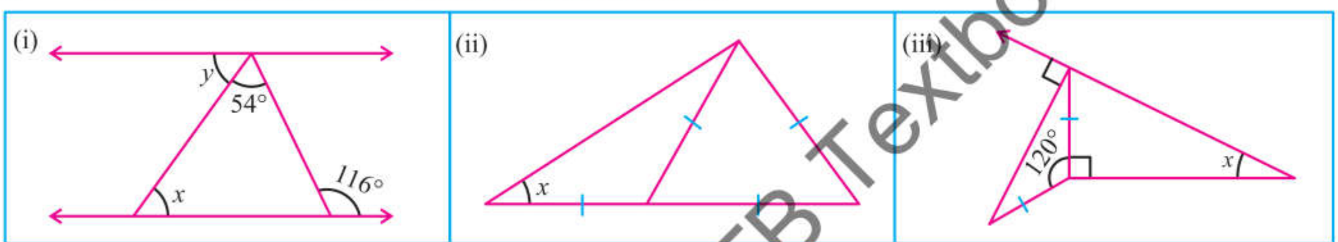
5. Construct triangle PQR if:

(i) $m\overline{PQ} = 7$ cm, $m\overline{PR} = 6.2$ cm and $m\angle QPR = 60°$

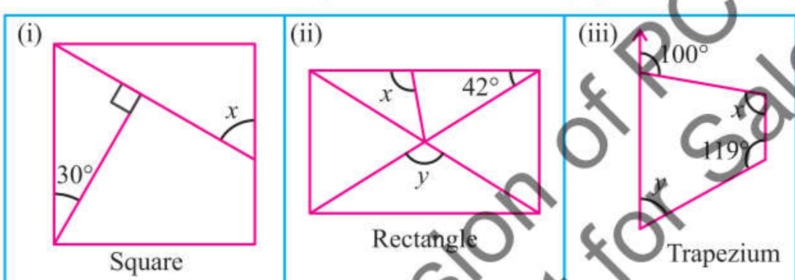
(ii) $m\overline{QR} = 6$ cm, $m\angle Q = 50°$ and $m\angle R = 120°$

(iii) $m\overline{PR} = 5.5$ cm, $m\angle R = 45°$ and $m\overline{PQ} = 6.7$ cm

6. Find the unknown angles in the following figures:



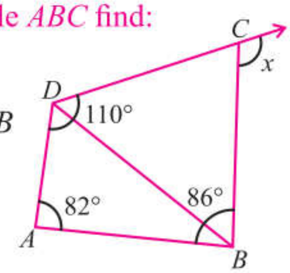
7. Find the unknown angles in the following quadrilaterals:



8. $ABCD$ is a quadrilateral and BD bisects the angle ABC find:

(a) x

(b) $m\angle ADB$

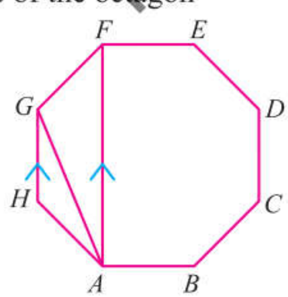


9. $ABCDEFGH$ is a regular octagon. Find

(i) Interior angle of the octagon

(ii) $m\angle HAG$

(iii) $m\angle AFG$

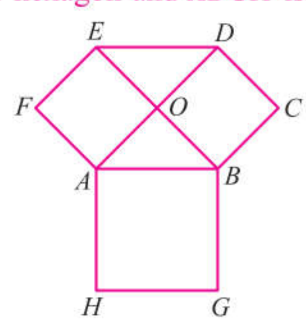


10. $ABCDEF$ is a regular hexagon and $ABGH$ is a square. Find:

(i) $m\angle ABC$

(ii) $m\angle CBG$

(iii) $m\angle AOB$



11. Find interior angle of regular

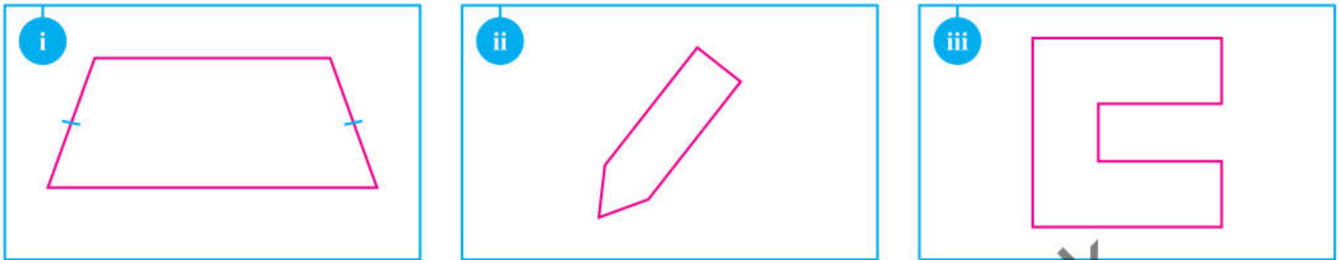
(i) 24-gon (ii) 27-gon

12. Find number of sides of a regular polygon with given interior angle:

(i) 162° (ii) 170°

13. The size of each interior angle is 14 times the exterior angle of a regular polygon with n-sides. Find the value of n.

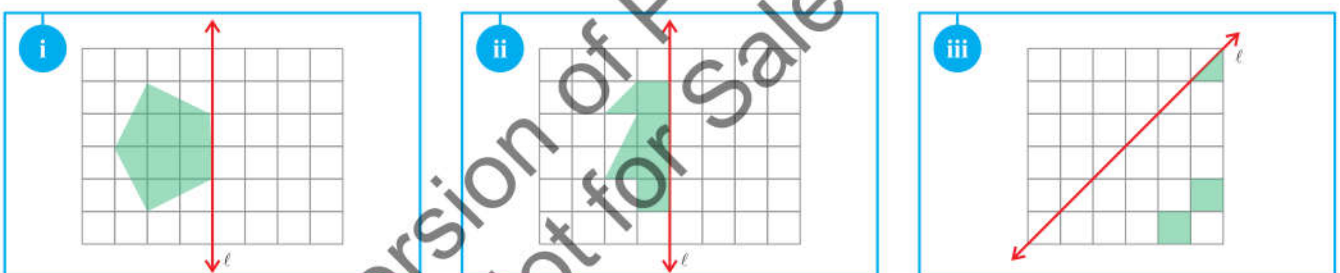
14. Draw one line of symmetry in each of the following:



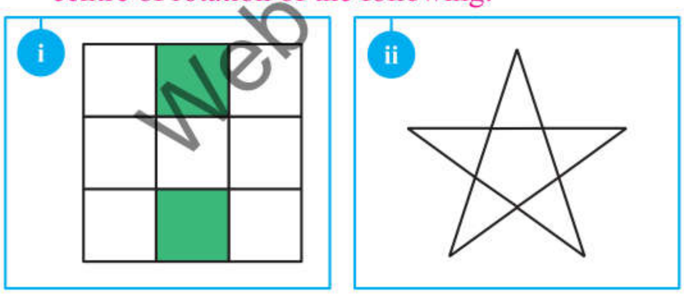
15. Which of the following letters have:

(a) line symmetry (b) rotational symmetry

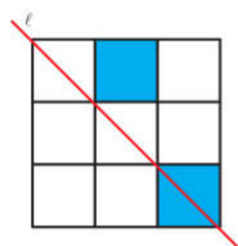
16. Complete the following diagrams using the given line of reflection:



17. Find the order of rotational symmetry and the centre of rotation of the following:

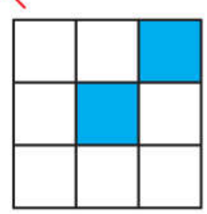


18. Shade one square so that the diagram has the given line of symmetry ℓ.



19. (i) How many squares should be shaded in the given diagram to get a rotational symmetry of order 4? Draw the figure.

(ii) Shade one square so that the diagram has two lines of symmetry



20. Find a translation when a point $A(1, -2)$ is moved to the point $A'(-3, 5)$.

21. Translate ΔABC with vertices $A(-2, 0)$, $B(-5, 2)$ and $C(-3, -6)$, 7 units right and 2 units upward.

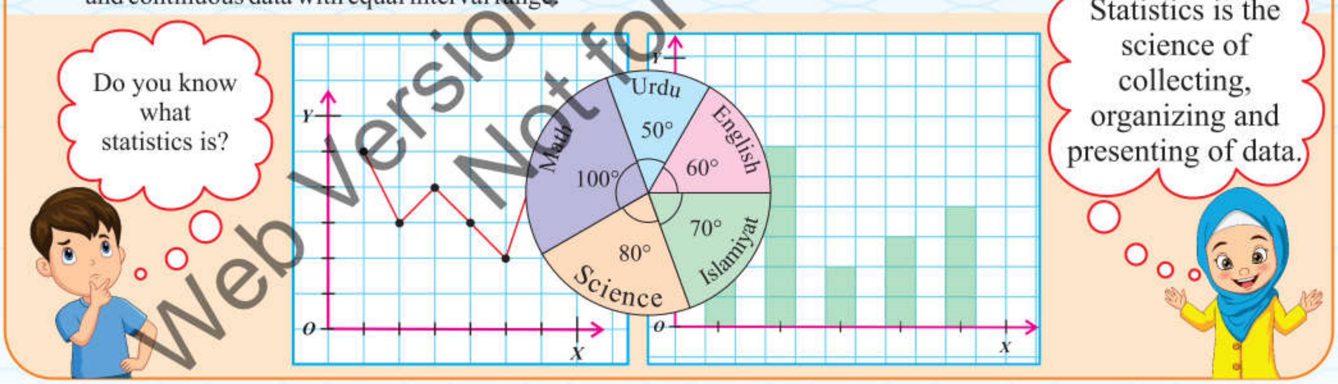
Domain 5 DATA MANAGEMENT

Sub-domain (i) Statistics

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise the difference between discrete, continuous, grouped and ungrouped data.
- Construct frequency distribution tables for given data (i.e., frequency, lower class limit, upper class limit, class interval and mid-point) and solve related real-world problems.
- Recognise drawing and interpreting of bar graphs, line graphs and pie charts.
- Construct and compare histograms for both discrete and continuous data with equal interval range.
- Differentiate between a histogram and a bar graph.
- Select and justify the most appropriate graph(s) for a given data set and draw simple conclusions based on the shape of the graph.
- Calculate the mean, median and mode for ungrouped data and the mean for grouped data and solve related real-world problems; Compare, choose and justify the appropriate measures of central tendency for a given set of data.



INTRODUCTION

Statistics is the collection of data and then organizing, presenting, analyzing describing and drawing conclusion based on data. Statistics plays vital role in our daily life. We come across the information in the form of shapes, figures, graphs, tables etc. In this way, we can get information quickly and easily.

Tasneem wants to know about the mass of each student in the classroom. She records the mass by each student i.e., 28.5 kg, 25 kg, 30 kg, 25.6 kg, 24.5 kg, 26.4 kg, 23.5 kg, 14 kg, 28 kg, 27 kg, etc., Such collection is known as data.

5.1.1 Data

The collection of information in the form of facts and figures is known as data. For example, the collection of height of 30 students of your class and the record of temperatures of 30 days etc.

i Discrete Data

This type of data contains only whole numbers. For example,

- the number of toys sold by the shopkeeper in a particular weak.
- number of students in a class i.e., 30 students.
- the number of persons visited a department during a month. i.e., 215 persons, 320 persons etc.
- the number of accounts being operated in a branch of a bank.

In the above examples, we can see that only certain finite and non-negative numbers are taken as the data which cannot be further divided into parts. This type of data is known as discrete data.

ii Continuous Data

This type of data contains measures. These measures can be broken down into smaller individual parts. For example,

- The height of a boy i.e., 5.3 feet.
- The mass of the bag of a student i.e., 15.5kg.
- The speed of wind at the hill i.e, 10.8 km/hr.

In the above examples, we observe that the data can be measured on an infinite scale. It can take any value between two values

Data is represented by the following two methods:

Representation of Data

Ungrouped Data

The data which is not arranged in any systematic order is called ungrouped data. For example, the marks obtained by 10 students are:

9, 16, 11, 19, 14, 13, 17, 20, 15, 16

Grouped Data

- The data which is arranged in a systematic order is called grouped data. For example,

Marks obtained	Number of students
9 – 11	2
12 – 14	2
15 – 17	4
18 – 20	2
Total	10

Keep in mind!

Data is a plural form of datum.

Remember!

Discrete data is obtained by counting.

Types of data

Discrete data | Continuous data

Skill Practice

Think any two real life examples of discrete data and continuous data.

Remember!

Continuous data is obtained by measuring.

Challenge

Separate discrete data and continuous data

(i) The speed of wind.
(ii) The players in a game.
(iii) The fish in a pond.
(iv) The height of a hill.
(v) The mass of a boy.

Teachers' Guide

Clear the difference between discrete data and continuous data by using real life examples.

(a) Difference Between Discrete and Continuous; Grouped and Ungrouped Data

Discrete Data
(Ungrouped and grouped data)

A shopkeeper sold the following number of toys in 15 days:

10, 15, 12, 13, 16, 12, 15, 10,
13, 16, 12, 15, 12, 12, 15

The data is called discrete ungrouped data.

When we arrange the discrete ungrouped data in grouped or classes form, then it is called discrete grouped data.

No. of toys sold	No. of days
10	2
11	0
12	5
13	2
14	0
15	4
16	2
Total	**15**

Continuous Data
(Ungrouped and grouped data

A shopkeeper sold the following lengths of cloth (in metres) in 15 days:

2.5, 3, 4.5, 10.5, 11, 12, 15.5, 8.5,
7, 13, 5, 3.5, 6.5, 3.5, 12.5

The data is called continuous ungrouped data.

When we arrange the continuous ungrouped data in groups or classes form, then it is called continuous grouped data.

Length of cloth (m)	No. of days
$1 \le x < 4$	4
$4 \le x < 7$	3
$7 \le x < 10$	2
$10 \le x < 13$	4
$13 \le x < 16$	2
Total	**15**

5.1.2 Frequency Distribution

A distribution / table that represents classes along with their respective class frequencies is called frequency distribution / table.

In the other words, a listing of classes / groups and their frequencies is called frequency distribution.

Remember!

Frequency is the number of times of a repeated observation or value occurs in any data. It is denoted by 'f'.

Let us consider an example to learn how frequency distribution is constructed.

Example 1 The scores of 25 students in a test of 40 marks are given below:

26, 33, 28, 35, 29, 30, 38, 37, 36, 39, 37, 34, 27, 37, 40, 38, 40, 38, 32, 38, 39, 28, 37, 39, 33.

Construct a frequency distribution table with number of classes 5 .

Solution We arrange the data in groups or classes in the form of a table. Look at the given steps carefully. (i) The largest value in the data is 40. (ii) The smallest value in the data is 26.
(iii) The number of classes to be made is 5. (iv)The size of the class interval is found by the formula.

$$\text{Size of class interval} = \frac{\text{Largest value} - \text{Smallest value}}{\text{Number of classes}} = \frac{40-26}{5} = \frac{14}{5} = 2.8 \approx 3$$

So, the size of class interval will be 3.

(i) Make a table of five columns.

(ii) Write class intervals in the 1st column, tally marks in the 2nd column, frequency in the 3rd column, midpoints in the 4th column and class boundaries in the 5th column as headings.

(iii) Make class intervals having size of 3. Start from the smallest value (see column 1). i.e.,

26 – 28 and so on. In this group 26 is called the lower limit of this group and 28 is called the upper limit of this group.

Remember!

- A tally is a mark made to count the number of values written against the class interval. Usually tally marks are grouped together as a bundle of five (卌).
- Frequency is the number of times a value is repeated in a certain class interval.

(iv) Tally marks are used to count the given values, fall in the given interval (see column 2).

(v) The number of times a value is repeated in a class interval is known as frequency of that group or class interval (see column 3).

(vi) Midpoint of the class interval is the average of upper and lower limits of a class interval.

$$\text{Midpoint} = \frac{\text{Lower limit} + \text{Upper limit}}{2} = \frac{26+28}{2} = \frac{54}{2} = 27 \quad \text{(see column 4)}$$

So, 27 is a midpoint of the first group. Midpoint is also known as class mark. It is denoted by x.

(vii) Class boundaries:

- Lower class boundaries are obtained by "subtracting 0.5" from the lower class limit.
- Upper class boundaries are obtained by "adding 0.5" to the upper class limit.

(Lower limit – 0.5)	**(Upper limit + 0.5)**	
(26 – 0.5)	(28 + 0.5)	(See column 5)
25.5	28.5	

Keep in mind!

Class boundaries is mostly written as C.B.

Class intervals	Tally Marks	Frequency	Midpoint	C.B
26 – 28	||||	4	27	25.5 – 28.5
29 – 31	||	2	30	28.5 – 31.5
32 – 34	||||	4	33	31.5 – 34.5
35 – 37	卌 |	6	36	34.5 – 37.5
38 – 40	卌 ||||	9	39	37.5 – 40.5
Total		**25**		

Example 2 The maximum temperature (in °C) of Lahore for 30 days in a year is given below:

36, 21, 18, 32, 15, 27, 18, 15, 27, 15, 26, 16, 39, 25, 16,
39, 20, 25, 23, 39, 30, 15, 37, 16, 31, 28, 37, 36, 24, 32,

Construct the frequency distribution table for the given data with number of classes 5. Also find out midpoints and class boundaries of this distribution.

Solution The largest value = 39 ; The smallest value = 15

The number of classes as given = 5

$$\text{The size of class interval} = \frac{\text{Largest value} - \text{Smallest value}}{\text{Number of classes}} = \frac{24}{5} = 4.8 \simeq 5$$

Midpoint of the class interval = $\frac{15+19}{2} = \frac{34}{2} = 17$ and so on for each class interval.

Class boundaries = (15 – 0.5) — (19 + 0.5)
14.5 — 19.5 and so on for each class interval.

Class intervals	Tally Marks	Frequency	Midpoint	C.B.
15 – 19	~~||||~~ ||||	9	17	14.5 – 19.5
20 – 24	||||	4	22	19.5 – 24.5
25 – 29	~~||||~~	5	27	24.5 – 29.5
30 – 34	~~||||~~	5	32	29.5 – 34.5
35 – 39	~~||||~~ ||	7	37	34.5 – 39.5
Total		30		

EXERCISE 5.1

1. Write the difference between discrete data and continuous data.
2. Write the difference between grouped data and ungrouped data.
3. Separate discrete data and continuous data.
   (i) The number of students in 7^{th} class.
   (ii) The weight of a bag
   (iii) The number of players in the ground
   (iv) The speed of storm.
   (v) The temperature of a room.
4. Read the frequency table given below and answer the following questions:

Class intervals	Frequency
20 – 24	5
25 – 29	4
30 – 34	6
35 – 39	5

(i) What is the size of class interval?
(ii) What is the lower limit of the class interval 20 – 24?
(iii) What is the upper limit of the class interval 35 – 39?
(iv) What the midpoint of the class interval 30 – 34?
(v) What the frequency of class interval 25 – 29?

5. The marks obtained by 35 students in a Mathematics test out of 100 marks are given below:
50, 49, 49, 47, 48, 34, 39, 78, 73, 67, 58, 56, 62, 46, 38, 42, 84, 60,
83, 50, 68, 60, 90, 70, 57, 57, 61, 49, 61, 58, 49, 79, 89, 77, 54.
Construct frequency distribution table with 10 number of classes. Also find midpoints and class boundaries for each class interval

6. The number of electricity units consumed by 50 households in a low income group in a locality of Lahore are given below:
125, 55, 83, 45, 55, 64, 136, 130, 91, 66, 86, 155, 54, 80, 78, 102, 100, 62,
113, 60, 93, 101, 104, 58, 111, 75, 113, 81, 96, 111, 96, 90, 87, 55, 109, 155,
94, 66, 129, 139, 99, 77, 83, 67, 69, 99, 97, 51, 97, 50
Construct a frequency distribution table with 10 number of classes. Also calculate midpoints and class boundaries.

7. The scores in Science of 45 students are given below:
75, 61, 89, 65, 68, 75, 84, 67, 75, 74, 82, 62, 68, 95, 90, 78, 62, 63, 88, 72, 76, 66, 93,
78, 73, 82, 79, 75, 88, 94, 73, 77, 60, 69, 93, 74, 71, 68, 59, 60, 85, 96, 75, 78, 61
Construct a frequency distribution table with 5 number of classes. Find midpoints and class boundaries.

8. The masses of 40 students at a university are given below:
153, 144, 164, 138, 152, 135, 150, 168, 140, 135, 126, 132, 144, 148
138, 161, 145, 176, 125, 149, 163, 135, 142, 119, 157, 146, 154,
150, 156, 165, 158, 140, 146, 145, 128, 173, 147, 136, 142, 147
Find out midpoints and class boundaries after constructing frequency distribution with 6 number

5.1.3 Drawing and Interpreting of Bar Graphs, Line Graph and Pie Graph

Another way to represent the data effectively is by means of graphic representation. Graphs bring salient features of the data at a glance. The purpose of graph is to show the numerical data in visual form for easy, clear and quick understanding. The different types of graph are bar graph, line graph, pie graph and histogram. Let us learn these graphs one by one.

i Bar Graph

A bar graph is a graphical representation of numerical data of different categories. In this presentation, bars of different heights proportional to the values they represent are used. The bar graph is of two types i.e., vertical bar graph and horizontal bar graph.

Example 3 A fruit seller sold the fruits on a day as given in the table.

Fruits	Apple	Mangoes	Coconut	Pine apple	Papaya
Kilograms	50	60	20	30	40

Bar Graph
Vertical bar graph | Horizontal bar graph

Draw the vertical and horizontal bar graphs for the above data.

Solution

Steps of drawing a vertical bar graph:

(i) Draw two lines OX and OY perpendicular to each other i.e., X-axis and Y-axis.

(ii) Give the title for the graph.

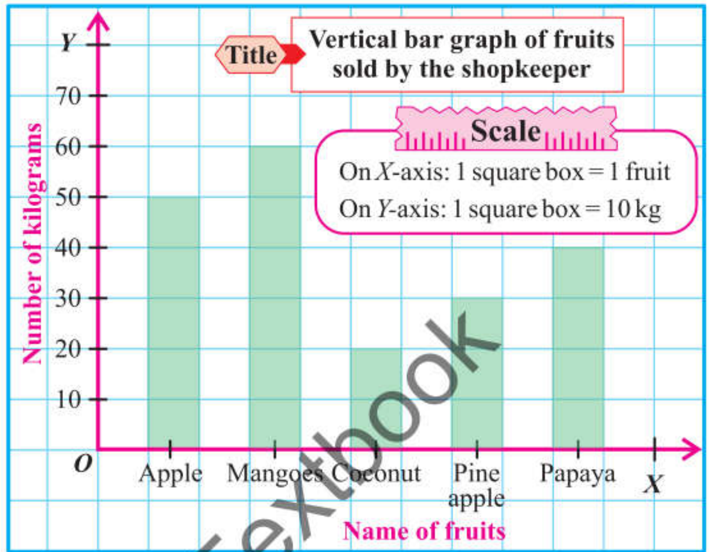
(iii) Write the name of fruits along $\overrightarrow{OX}$ (horizontal line).

(iv) Write the number of kilograms along $\overrightarrow{OY}$ (vertical line).

(v) Draw bars for each fruit. The heights of bars are equal to number of kilograms.

(vi) The width of bars are equal.

Since the bars are vertical, so the graph is called vertical bar graph.



Keep in mind!

In vertical bar graph, bars are drawn vertically on X-axis. In horizontal bar graph, bars are drawn horizontally on Y-axis.

Steps of drawing a horizontal bar graph:

(i) Draw two lines OX and OY which are perpendicular to each other i.e., X-axis and Y-axis.

(ii) Write the title for the graph.

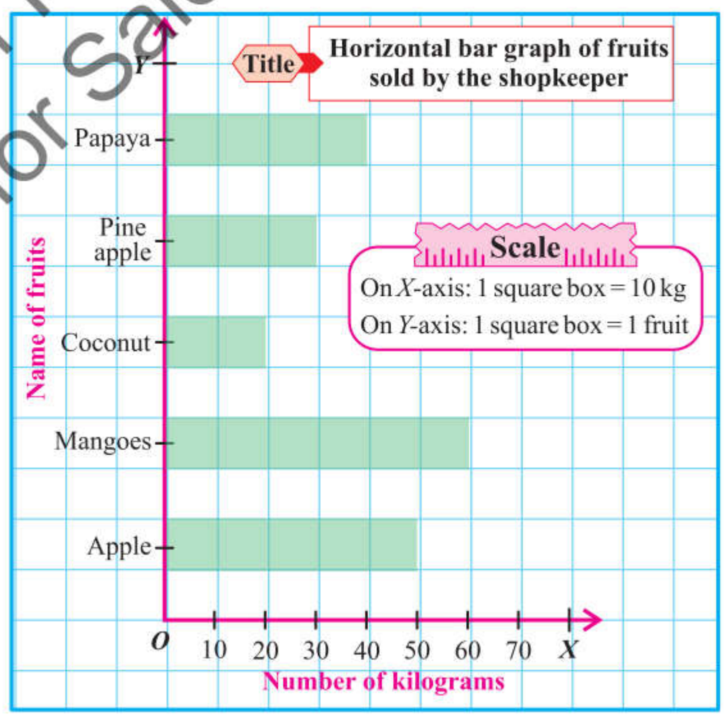
(iii) Write the number of kilograms along $\overrightarrow{OX}$ (horizontal line).

(iv) Write the name of fruits along $\overrightarrow{OY}$ (vertical line).

(v) Draw bars for each fruit. The heights of bars are equal to number of kilograms.

(vi) The width of bars are equal.

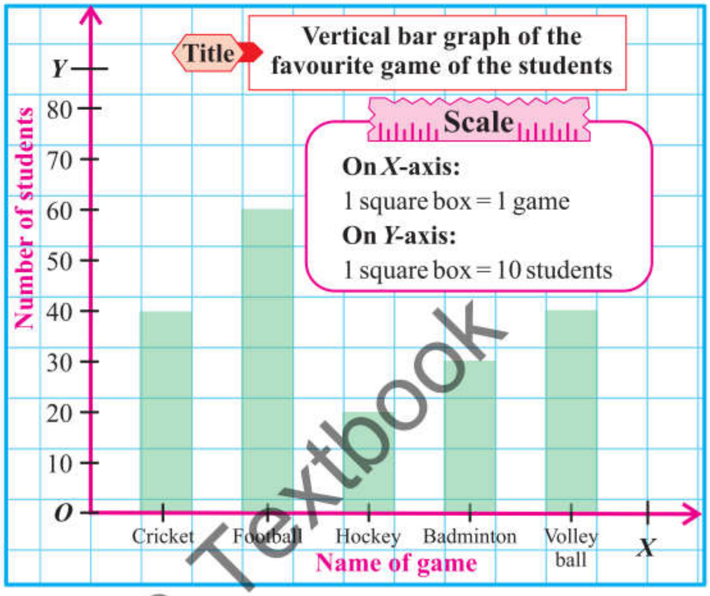
Since, the bars are horizontal, the graph is known as horizontal bar graph.



Activity

Students will work in groups of 3 or 4. Students will sort the buttons by colour and will record the number of buttons of each colour they received. Later they will create a bar graph that will display the data and answer the following simple questions. (i) What is the colour of buttons that are most in number? (ii) Which colour of buttons did you get least? (iii) Did you get any colour of buttons that are same in number?

(a) Reading a Bar Graph



Example 4 Read the vertical bar graph. Answer the following questions:

(i) How many students do like cricket?

(ii) Which game is the most favourite for the students?

(iii) Which games are liked equally by the students?

(iv) How many students do like hockey?

(v) Which game is the least liked?

Solution (i) 40 students like cricket.

(ii) Football is the most favourite game for the students.

(iii) Cricket and volleyball are liked equally by the students.

(iv) 20 students like hockey.

(v) Hockey is the least liked by the students.

Keep in mind!

In short, frequency is along *Y*-axis in vertical bar graph and along *X*-axis in horizontal bar graph.

ii Line Graph

It is a type of graph that shows information which is changed over the period of time. A line graph is drawn by using several points which are connected by line segments.

Example 5 The sale of a shop from January to June is given below in a table and interpret it.

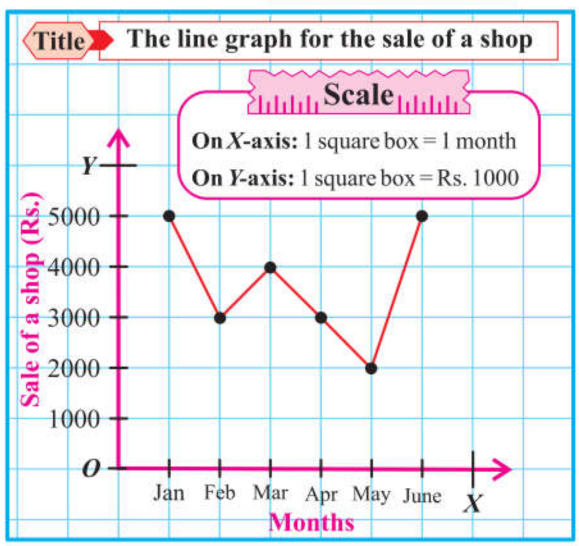
Months	January	February	March	April	May	June
Selling (Rs.)	5000	3000	4000	3000	2000	5000

Keep in mind! Line graph is used for continuous data.

Draw a line graph.

Solution

Draw $\overrightarrow{OX}$ and $\overrightarrow{OY}$ which are perpendicular to each other. *X*-axis represents the months and *Y*-axis represents the sale of a shop. Points (dots) on the graph represent data. The points are the sale of the shop that match with the months on the *X*-axis. From the drawn graph we can read the information about sale of the shop, e.g.,



(i) The maximum sale of the shop was in the months of January and June

(ii) The minimum sale of the shop was in the month of May.

(iii) The sale of shop in the month of January and June are equal, February and April are also equal.

iii Pie Chart / Pie Graph

This type of graph represents the data in a circular shape. The arc length of slices of pie shows the proportional size of the data. This is a pictorial representation of data.

Keep in mind!

Pie chart is also known as circular graph.

Example 6 The marks out of 50 obtained by Humnah in the yearly examination is given below:

Solution

Subject	English	Urdu	Math	Science	Islamiyat
Obtained marks	30	25	50	40	35

Now, represent the data by using a pie graph.

Step I Add all the values to get the total

Total = 30 + 25 + 50 + 40 + 35 = 180

Step II To know the degree of each pie sector, we will take full central angle i.e., 360°.

$$\text{Angle of each sector} = \frac{\text{Value of each component}}{\text{Sum of values of all component}}$$

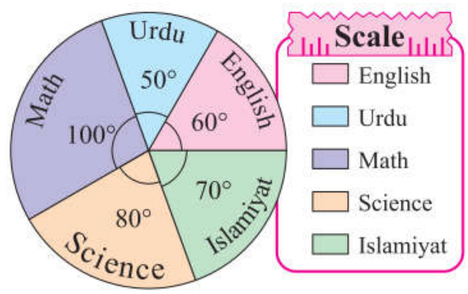
Subject	Number of graph	Angle of the sector
English	30	$\frac{30}{180} \times 360° = 60°$
Urdu	25	$\frac{25}{180} \times 360° = 50°$
Math	50	$\frac{50}{180} \times 360° = 100°$
Science	40	$\frac{40}{180} \times 360° = 80°$
Islamiyat	35	$\frac{35}{180} \times 360° = 70°$

Skill Practice

- In which subject, Humnah got the highest marks?
- In which subject, Humnah got the least marks?

Keep in mind!

Angle can be drawn clockwise or counter clockwise.



Step III Draw a circle of any suitable radius and by using a protractor, draw angles of each sector.

Search out The following is the online game link for interpreting and constructing pie chart:
https://www.transum.org/software/SW/Starter_of_the_day/Students/Pie_Charts.asp

5.1.4 Histogram

Histogram is another graphical way to represent the data. Histogram is similar to bar graph, but it is constructed for a frequency distribution. Histogram is a graph which consists of rectangular bars of equal width without any gaps. Histogram can be drawn for continuous and discrete data.

Comparison between graphs

Bar graph	Histogram
Bar graph is drawn for categorical or qualitative data and gaps exist within the bars.	Histogram is drawn only for numerical data or quantitative data (discrete and continuous data) and there is no gaps with the bars.

Steps of drawing a histogram:

(i) Draw two lines OX and OY perpendicular to each other.

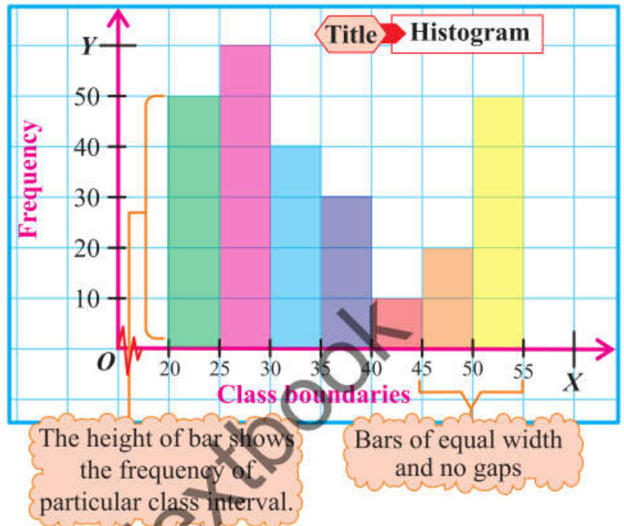
(ii) Give the title for the histogram.

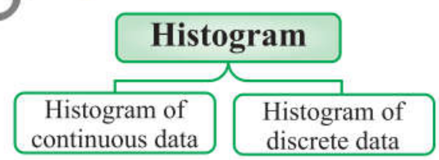
(iii) Make class boundaries of each class interval along X-axis.

(iv) Write frequency along Y-axis.

(v) The width of bars are equal.

(vi) The height of each bar is equal to the frequency of that class interval.



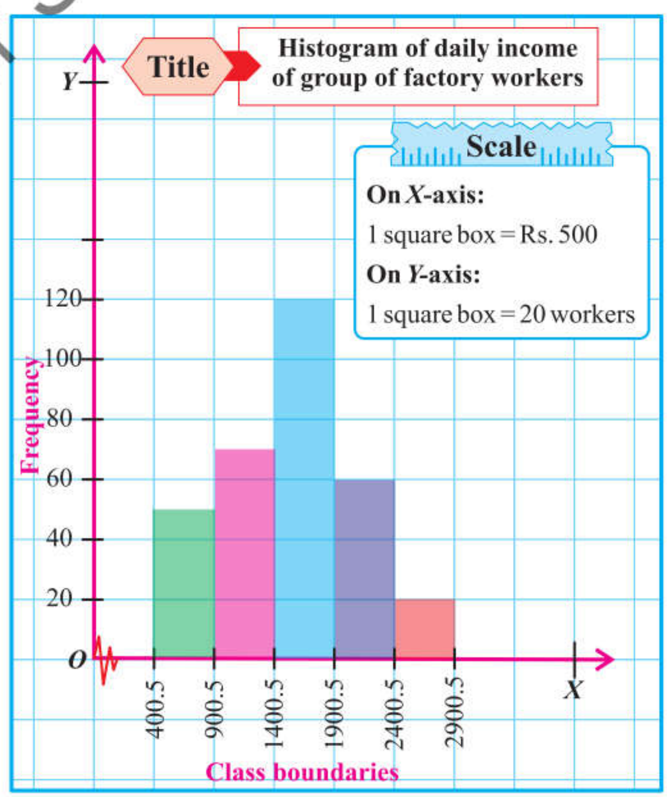


i Histogram of Continuous Data

The table given below shows the daily income (in rupees) of workers.

Daily income (in Rupees)	401 – 900	901 – 1400	1401 – 1900	1901 – 2400	2401 – 2900
Number of workers	50	70	120	60	20

Daily Income (in Rupees)	**Number of workers**	**C.B.**
401 – 900	50	400.5 – 900.5
901 – 1400	70	900.5 – 1400.5
1401 – 1900	120	1400.5 – 1900.5
1901 – 2400	60	1900.5 – 2400.5
2401 – 2900	20	2400.5 – 2900.5
Total	**320**	



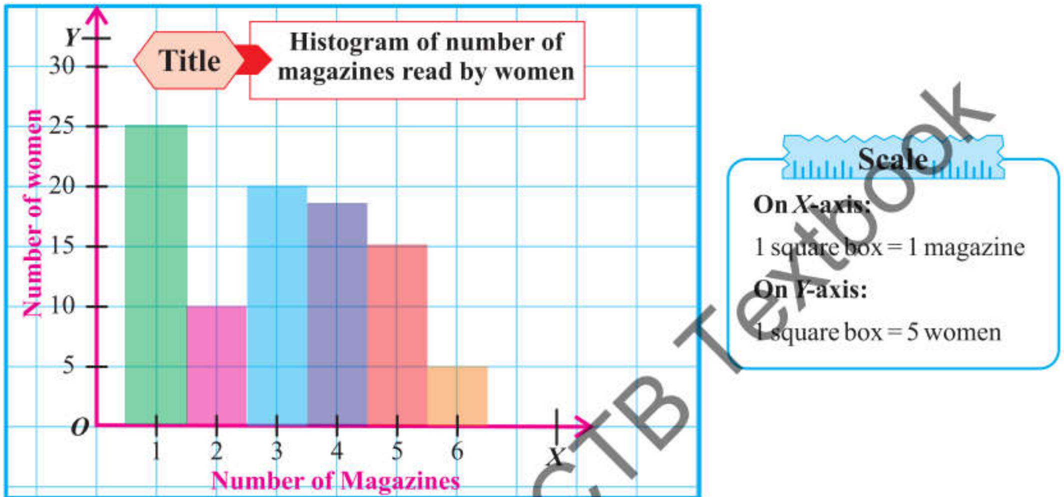
Teachers' Guide

Set up groups that collect, arrange and display different kinds of data through different methods including questionnaires, experiments, databases and electronic media. Ask students to construct frequency tables from that data and choose the most appropriate graph to represent it. The groups may give a presentation at the end of their data collection.

ii Histogram of Discrete Data

The table given below shows the number of magazines read by the women in a month is recorded:

Number of magazines	1	2	3	4	5	6
Number of women	25	10	20	18	15	5



5.1.5 Selection of an Appropriate Graph for a Given Data

Bar Graph Bar graph is used to represent the quantities in different categories. It is used for qualitative data. e.g., favourite colour of the students, colour of eyes, favourite fruits of the students etc.

Line Graph Line graph is used to represent the data which is changed over the period of time. It shows the trend in a particular time period. e.g., the production of potatoes in the last five years, the number of toys sold in the last four weeks etc.

Pie Graph Pie graph is used for the comparison of parts of a whole. It tells us how a whole is divided into different sectors. Pie graph is also used for the qualitative and ordinal data. e.g., the colour of car in a parking, positioned the different number of chairs in an order (1^{st}, 2^{nd}, 3^{rd}, ... so on) etc.

Histogram Histogram is a special type of bar graph. It is a way of summarizing the quantitative data that is measured on an interval scale. Quantitative data can be discrete or continuous. e.g., the height of the students in the class, the number of times a head occurred in tossing 50 times.

EXERCISE 5.2

1. A survey was conducted from the students of grade-7 and asked the students about their favourite season. Draw horizontal bar graph and vertical bar graph for the following table:

Number of seasons	Winter	Summer	Spring	Autumn
Number of students	38	20	45	25

Answer the following questions:

(a) How many students are there altogether in the class? (b) How many students chose summer?

(c) How many more students chose winter than autumn?

(d) Which season is the most liked by the students? (e) Which season is the least liked by the students?

2. Use the information given in the table to make a line graph and answer the following questions.

Time	5 P.M	6 P.M	7 P.M	8 P.M	9 P.M	10 P.M	11 P.M
Number of diners	19	25	15	30	35	22	10

(a) During which 1 hour interval did the greatest decrease in the number of diners occur?

(b) What was the difference in the number of diners between the hours when the restaurant was the most crowded and the least crowded?

3. The table shows the vehicles that Tahir saw on the road during a particular time. Draw a pie graph.

Name of Vehicles	Car	Bicycle	Motorbike	Rickshaw	Van	Bus
Number of Vehicles	100	20	130	80	10	25

Answer the following questions:

(a) How many more cars did Tahir see than the bus? (b) Which vehicle did Tahir see the most?

(c) Which vehicle did Tahir see the least? (d) How many vehicles were there altogether?

4. The following frequency table shows the number of goals made by a player in 70 football matches. Represent the following table on histogram and interpret it.

Number of goals	1	2	3	4	5	6	7
Number of matches	20	10	13	18	5	9	5

5. The following frequency table shows the number of school bags sold by shopkeepers. Also tell how many shopkeepers are there altogether? Draw histogram for the given table.

Number of school bags	241 – 250	251 – 260	261 – 270	271 – 280	281 – 290	291 – 300
Number of shopkeepers	15	10	20	13	7	18

6. The following table shows the lengths of 320 ribbons. Construct histograms for the given data:

Length (m) of ribbons	$5 < x \leq 10$	$10 < x \leq 15$	$15 < x \leq 20$	$20 < x \leq 25$	$25 < x \leq 30$
Number of ribbons	80	60	50	35	95

7. Make a table and draw the appropriate graph for the following data. Also explain why did you choose this graph?

(i) A bag contains 150 balls of different colours. There are 50 red balls, 20 blue balls, 30 green balls, 15 black balls and 35 yellow balls.

(ii) Azhar rolled a dice 200 times. The number "1" was occured 20 times, the number "2" was occured 35 times, the number "3" was occured 40 times, the number "4" was occured 25 times, the number "5" was occured 53 times and the number "6" was occured 27 times.

5.1.6 Measures of Central Tendencey

Measure of central tendency are used to describe the middle or centre value of a data set. In real life, the collection of data is around us, e.g., the height of 10 people, the wages of 15 employees and record the marks obtained by the 80 students. But how can you take further information e,g., if we want to find out the mean height of the 10 people, the average wages of the 15 employees and the average of obtained marks by 80 students. In such situations, mean, median and mode are used to find out the average height, average wages and average marks of the given data set. Now we learn how to calculate mean, median and mode.

i Mean

Mean or arithmetic mean is usually used in most of the situations in our daily life. It tells us the middle value of the given data set. e.g., average weight of the students in a class, mean/average of annual production of a country, average/mean of income of employees in a factory, mean of annual report of a country.

$$\text{Average or mean} = \frac{\text{Sum of all values}}{\text{Total number of values}}$$

If $x_1, x_2, x_3, ..., x_n$ are values of the data and the number of values is n, then

$$\text{Mean}(\bar{x}) = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

Let us apply this formula for finding the mean in the following examples:

Keep in mind!

$\bar{x} = \frac{\Sigma x}{n}$ ($\bar{x}$ is a symbol for mean)

Σ is a symbol used for sum. It is read as sigma or summation.

Skill Practice

Collect data of ages of 10 students and find the mean of that data.

Example 7 The number of eggs collected in 7 days from a small poultry form are: 92, 110, 90, 95, 115, 105 and 100. Find the mean of the eggs per day.

Solution We know that,

$$\text{Mean}(\bar{x}) = \frac{\Sigma x}{n} = \frac{\text{Sum of all values}}{\text{Total number of values}}$$

$$\bar{x} = \frac{92 + 110 + 90 + 95 + 115 + 105 + 100}{7}$$

$$\bar{x} = \frac{707}{7} = 101 \text{ eggs}$$

Thus, the mean of the eggs collected per day is 101.

Skill Practice

- Find the mean of the first ten whole numbers.
- Find the mean of the first 5 prime numbers.

Example 8 A player scored runs in 10 cricket matches as:
105, 98, 75, 100, 105, 125, 56, 112, 80 and 64. Find the mean runs he scored in these matches.

Solution We know that, $\text{Mean}(\bar{x}) = \frac{\Sigma x}{n}$

$$= \frac{105 + 98 + 75 + 100 + 105 + 125 + 56 + 112 + 80 + 64}{10}$$

$$= \frac{920}{10} = 92 \text{ runs}$$

Thus, mean score per match is 92 runs.

Skill Practice

The mean of 8, 11, 6, 14, x and 13 is 66. Find the value of the observation x.

ii Median

If the values of a data are arranged in an ascending or descending order, the middle value is called the median of the data set. In the given data,

(i) If number of values is odd, the middle value is the median of the data set and can be found as:

$\left(\frac{n+1}{2}\right)^{\text{th}}$ term, where n = Total number of terms

(ii) If number of values is even, the mean of two middle values is the median of the data set.

Example 9 Find the median of the given data: 39, 33, 37, 41, 43, 36, 34

Solution We arrange the given data in an ascending order.

$\underbrace{33,\ 34,\ 36,}\ \ \underline{37,}\ \ \underbrace{39,\ 41,\ 43}$

Since, the total number of values is 7. i.e., odd, the middle value is median. Median is found as:

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term} = \left(\frac{7+1}{2}\right)^{\text{th}} \text{term}$$

$$= \left(\frac{8}{2}\right)^{\text{th}} \text{term} = 4^{\text{th}} \text{ term} = 37$$

Thus, median of the given data is 37.

Example 10 Find the median of the following data:

39, 33, 41, 36, 43, 37

Solution We arrange the given data in an ascending order.

$\underbrace{33,\ 36,}\ \ \underline{37,\ 39,}\ \ \underbrace{41,\ 43}$

Here, the number of terms is 6, i.e., even. So, the median is the mean of two middle terms. The 3^{rd} term and 4^{th} term are middle values of the data set.

$$\text{Median} = \left(\frac{3^{\text{rd}} \text{ term} + 4^{\text{th}} \text{ term}}{2}\right) = \left(\frac{37+39}{2}\right) = \left(\frac{76}{2}\right) = 38$$

Thus, median of the given data is 38.

Skill Practice

Find the median of the following data.

- 27, 39, 49, 20, 21, 28, 38
- 10, 19, 54, 80, 15, 16
- 47, 41, 52, 43, 56, 35, 49, 55, 42
- 12, 17, 3, 14, 5, 8, 7, 15

Skill Practice

Find mean and median of given data sets:

- First 10 even numbers.
- Odd numbers between 50 and 70.
- Multiples of 15 below 100.

iii Mode

Mode is the most frequently occurred value in the given data.

- If number of values occur not more than once, then the data has no mode.
- If two or more values occur with same number of times, then there is more than one mode.

Example 11 Find the mode of the given data: (i) 2, 4, 6, 8, 9, 10, 12

(ii) 3, 4, 3, 5, 6, 3, 7 (iii) 3, 6, 8, 9, 8, 10, 12, 18, 6, 18

Solution (i) 2, 4, 6, 8, 9, 10, 12

In the above data, none of the value occur more than once. Therefore, the given data has no mode.

(ii) 3, 4, 3, 5, 6, 3, 7

In this data, the most occurred value is 3 (occurred three times). Therefore, 3 is the mode of the given data set.

(iii) 3, 6, 8, 9, 8, 10, 12, 18, 6, 18

In this data, the values 6, 8 and 18 is occurred two times. Therefore, 6, 8 and 18 are three modes of the given data set.

Choose an Appropriate Measure of Central Tendency

The mass (in kg) of 10 persons, are given: 65, 70, 68, 72 , 79, 79, 75, 67, 79, 66.Find the average mass of a person. To find the average mass of the persons, we will use the following formula:

$$\text{Mean}(\bar{x}) = \frac{\Sigma x}{n} = \frac{65+70+68+72+79+79+75+67+79+66}{10}$$

$$= \frac{720}{10} = 72$$

Thus, the average mass of a person is 72 kg.

Why we use mean?

In such situation, mean is the best average because there is no extreme value, either large or small value in the data set.

The marks of students in a class are: 1, 2, 4, 5, 3, 2, 25. Find the average marks of a student.

$$\text{Mean}(\bar{x}) = \frac{\Sigma x}{n} = \frac{1+2+4+5+3+2+25}{7} = \frac{42}{7} = 6$$

Median: We arrange the given data in an ascending order.

$$\underbrace{1, \quad 2, \quad 2,} \quad \underline{3,} \quad \underbrace{4, \quad 5, \quad 25}$$

Since, the total number of values is 7. i.e., odd, the middle value is median. Median is found as:

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term} = \left(\frac{7+1}{2}\right)^{\text{th}} \text{term}$$

$$= \left(\frac{8}{2}\right)^{\text{th}} \text{term} = 4^{\text{th}} \text{term} = 3, \text{Thus median is } 3$$

Why we use median?

In such situation, median is better measure because 25 is much greater than other values and because of 25 the mean has come out 6, which is away from the other values. So, it is better to take median.

Mode: Sana has 50 coloured pencils. She has 25 red pencils, 5 blue pencils, 8 green pencils, 2 black pencils, 10 yellow pencils. The mode would be the most amount of the type of pencil. This would be the red pencil in this case. So, the mode is 50.

Why we use mode?

In such situation the mode would be an appropriate measure because the mode is mostly appropriate for qualitative data.

iv Mean for Grouped Data

If $x_1, x_2, x_3, ..., x_n$ are the class marks for n values in data and $f_1, f_2, f_3, ..., f_n$ are their respective frequencies, then mean is calculated as:

$$\text{Mean} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n}$$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}, \text{ where } i = 1, 2, 3, ..., n$$

We calculate the mean of grouped data with the help of the example.

Teachers' Guide

Generate a discussion on situations where mean, median or mode is most appropriate choice of average.

Example 12 The following data shows the marks obtained by 50 students in Mathematics test.

Obtained marks	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50
Number of students	3	7	12	16	12

Calculate the average of obtained marks.

Solution

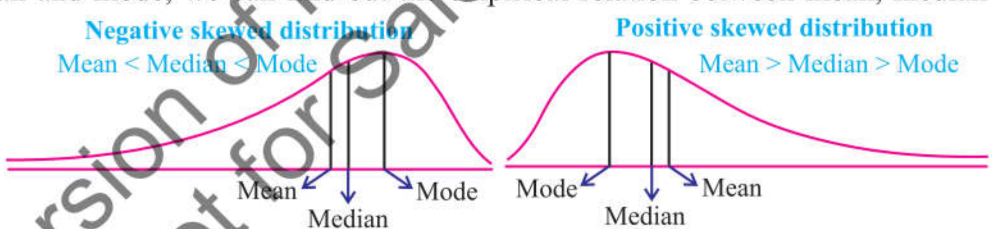
Marks	Number of student (f)	Class Marks (x)	fx
26 – 30	3	28	84
31 – 35	7	33	231
36 – 40	12	38	456
41 – 45	16	43	688
46 – 50	12	48	576
Total	**$\Sigma f = 50$**		**2035**

Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{2035}{50} = 40.7$ Hence, the average of obtained marks is 40.7 marks.

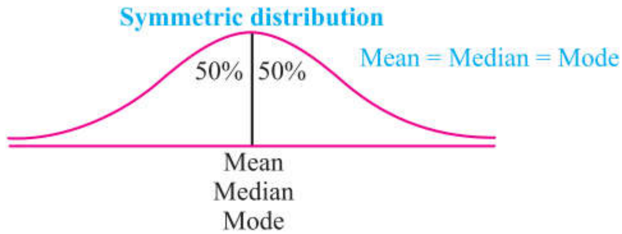
Comparison of Mean, Median and Mode

By comparing mean, median and mode, we can find out the empirical relation between mean, median and mode.

If the values of mean, median and mode differ, then the distribution is skewed or asymmetric.



If the values of mean, median and mode are equal, then the distribution is symmetric.



Example 13 Find mean, median and mode of the given data set, compare and tell the shape of distribution. 5, 6, 7, 8, 9, 10, 11, 10, 9

Solution After calculating its mean, median and mode, the values are:

Mean = 8.33, Median = 9, Mode = 10

As, mean < median < mode. So, the distribution is negatively skewed.

Keep in mind!

Mean is preferred over median and mode because all values in the data are used in it.
Median is useful when data has extreme values.
Mode is used when most common value is required.

EXERCISE 5.3

1. Find the mean of the following data:
   (i) 42, 40, 47, 35, 41, 50, 55, 30, 32, 45 (ii) 25, 29, 12, 15, 31, 36, 38, 40, 30
   (iii) 56, 71, 78, 67, 76, 62, 56, 77, 76, 63
2. Compare mean, median and mode, and also tell the shape of the distribution of the given data:
   (i) 20, 25, 21, 24, 22, 18, 32, 20 (ii) 9, 13, 9, 19, 21, 15, 30, 35, 91
   (iii) 24, 80, 50 55, 66, 68, 79, 80, 80 ,95 (iv) 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6
3. Find average of the given data by using an appropriate measure of central tendency.
   (i) 10, 15, 7, 8, 11, 12, 5, 3, 4, 5, 9, 11, 12, 14, 8, 7, 6, 5, 4, 9 (ii) 5, 4, 9, 6, 3, 9, 45
   (iii) In a class, 40 students have black eyes and 50 students have brown eyes.
   Compute mean, median and mode.
4. The mean mass of 10 sacks of rice is 50.25kg. The masses of 9 sacks of rice (in kg) are:
   49.5, 55.75, 50.5, 51.75, 48.25, 47.5, 54.23, 55.26, 53.25
   What is the mass of 10^{th} sack? Also find median and mode.
5. The average height of 15 students is 5.3ft, The height of 14 students (in ft) are:
   4.8, 5.2, 5.1, 4.7, 4.5, 5.2, 5.4, 5.5, 5.7, 5.8, 4.8, 4.9, 4.5, 4.6
   Find the height of 15^{th} student. Also find median and mode.
6. Find the mean of the given data.

Class Interval	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30
Frequency	17	20	25	15	10

7. Find the mean of the given data.

x	12	14	16	18	20
f	100	120	105	95	115

SUMMARY

- Statistics is the collection of data and then organizing, presenting, analyzing describing and drawing conclusion based on data.
- A distribution / table that represents classes along with their respective class frequencies is called frequency distribution / table.
- Frequency is the number of times of a repeated observation or value occurs in any data.
- Bar graph is used for qualitative data, line graph is used to represent the data which is changed our the period of time and pie graph is used for the comparison of parts of a whole. Histogram is a way of summarizing the quantitative data that is measured on an interval scale.
- Mean is preferred over median and mode because all values in the data are used in it.
- Median is preferred because it is not affected by the extreme values of the data.
- Mode is used where the most common or model value is required.

Sub-domain (ii) Probability

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Explain and compute the probability of; certain events, impossible events and complement of an event. (including real-world word problems).

Introduction

In our daily life, we usually make statements which cannot be predicted with total certainty such as:

- I will win this game.
- I shall probably go Karachi this month.
- It is likely to rain today.

All the above statements lack prediction with total certainty. Mathematics makes it possible for us to represent the chance of an event occurring numerically using the idea of probability.

Probability

Probability is the chance of occurence of an event.

Important Information

The word "Probability" is derived from the Latin word "Probabilitas". It means "probity".

History

Girolamo Cardano is known as the father of probability

5.2.1 Basic Concepts of Probability

i Experiment

In statistics, the term experiment describes any process which generates outcomes. For example,

- The tossing of a coin is considered as experiment.
- The rolling of a dice is another example of experiment.

ii Outcomes

The results of an experiment are known as outcomes. e.g., the possible outcomes of tossing a coin are head or tail and the possible outcomes of rolling a dice are 1or 2 or 3 or 4 or 5 or 6.

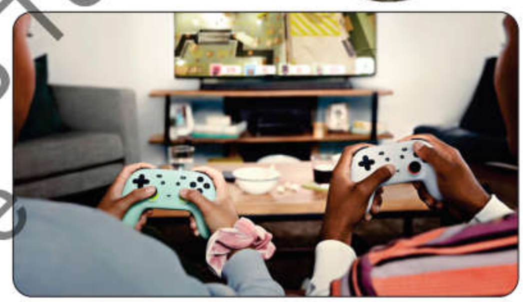
Remember!

- The possible number of outcomes in tossing a coin is 2.
- The possible number of outcomes in rolling a dice is 6.

iii Favourable outcome

An outcome that represents how many times we expect the things to be happen, is called favourable outcome. e.g.,

- When we toss a coin, there is one (1) favourable outcome of getting “head”
- When we roll a dice, there are three (3) favourable outcomes of getting even numbers i.e., {2,4,6}
- When we participate in a game there is one (1) favourable outcome to win.



iv Sample space

The set of all possible outcomes of an experiment is called sample space. It is denoted by "S". For example,

- If we toss a coin once, the possible outcomes of that experiment are head and tail then the sample space will be:

  $S = \{H, T\}$, where H = head and T = tail

  $n(S) = 2$

- If we roll a dice, the possible outcomes of that experiment are 1, 2, 3, 4, 5 and 6, then the sample space will be:

  $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Remember!

The number of favourable outcomes is always less than or equal to total number of outcomes.

Keep in mind

When we write all the possible outcomes within the curly bracket “{ }" is known as sample space.

v Event

The set of outcomes of an experiment is called an event. It is denoted by E. For example,

- While tossing a coin getting head is an event, so $E = \{H\}$
- While rolling a dice, getting an odd number is an event. So, $E = \{1, 3, 5\}$, $n(E) = 3$
- While participating in a game getting win is also an event. So, $E = \{\text{win}\}$, $n(E) = 1$

Key fact!

Each element of the sample space is called sample point.

Do you know?

Certain event is also known as sure event.

Experiment	Outcomes	Sample space	Event
Tossing a coin	H, T	$S = \{H, T\}$	$E = \{H\}, E = \{T\}$
Rolling a dice	1, 2, 3, 4, 5, 6	$S = \{1, 2, 3, 4, 5, 6\}$	$E = \{1\}, E = \{2\}, E = \{2, 3\}$ $E = \{4, 5, 6\}$,
Pick a card at random	$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}$ $\boxed{7}\ \boxed{8}\ \boxed{11}\ \boxed{12}$	$S = \{\boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}$ $\boxed{7}, \boxed{8}, \boxed{11}, \boxed{12}\}$	$E = \{\boxed{1}\}, E = \{\boxed{2}\}, E = \{\boxed{3}, \boxed{4}\}, E = \{\boxed{4}\}$ $E = \{\boxed{7}\}, E = \{\boxed{8}, \boxed{11}, \boxed{12}\}$

5.2.2 Type of Events

(i) Certain event (ii) Impossible event (iii) Likely event
(iv) Unlikely event (v) Equally likely event

i Certain Event

An event which is sure to occur in any given experiment is called a certain event. e.g., if today is Thursday, then tomorrow is "Friday" is a certain event.

Key fact!
The probability of a certain event is always equal to one (1).

ii Impossible Event

When an event cannot occur in any given experiment, it is called an impossible event. For example

- While tossing a coin, the outcomes "both head and tail" is an impossible event.
- While rolling a dice, the outcome of the number "7" is an impossible event.

Brain Teaser!
Think any two more examples regarding impossible events.

Search out
Teachers can share the following online quiz link of probability.
- https://www.ixl.com/math/grade-3/certain-probable-unlikely-and-impossible

Key fact!
The probability of an impossible event is always Zero (0).

iii Likely event

An event is called likely event which will probably occur. In other words, there will be a greater chance that the event will occur. e.g., it is likely to pick out red balls.

Skill Practice
A weather forecaster forecasts that there is 90% chance of rain today. Is it more likely to rain for today?

Key fact!
The probability of a likely event is close to 1.

iv Unlikely event

An event is called unlikely event which will not probably occur. In other words, there will be less chance that the event will occur. e.g., it is unlikely to pick out pink ball.

Teachers' Guide
Explain two or more examples on certain event by using real life situation.

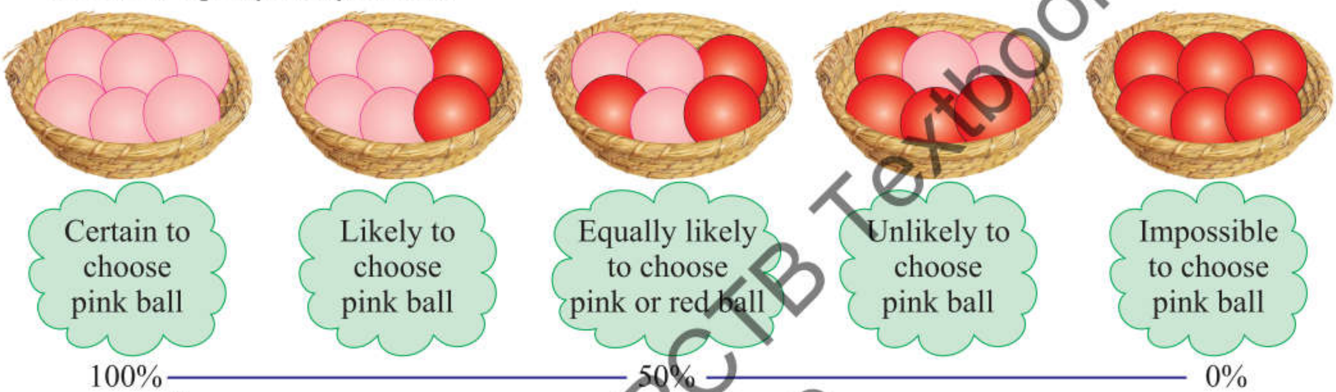
v Equally likely events

The events are called equally likely events which have equal chance of occurrence. For example,

- In tossing a fair coin, the chance of occurrence of head or tail is equal. These events are called equally likely events.
- In rolling a fair dice, the chance of occurence of the number 1, 2, 3, 4, 5 or 6 is equal. These events are called equally likely events.

Brain Teaser!

Is this equally likely to pick out green balls?



Challenge

- Write five things which you are likely to do in a day and 5 things which you are unlikely to do in your complete day.
- Write 7 events which will have zero chances to happen.

5.2.3 Computation of Probability

The probability of an event is calculated by the following formula:

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$

$P(E)$ = Probability of the event E

$n(E)$ = Number of favourable outcomes

$n(S)$ = Total number of possible outcomes i.e., sample space (S).

Brain Teaser!

In tossing a 5 rupees coin, are the events equally likely?

Remember!

The probability of an event is always between 0 and 1. (0 and 1 inclusive) i.e., $0 \le P(A) \le 1$

i Probability of Certain Event

If an event is sure to occur, its probability is 1. It means that the probability of a certain event is 1. e.g.,

Suppose a dice has the number "2" on all faces if we roll this dice, what is the probability to get the number 2?

Teachers' Guide

Clear the concept of likely event, unlikely event, certain event, impossible event and equally likely event by using different colours of balls, pencils etc.

The number "2" appears every time, if we roll the dice several times. Thus, the number of favourable outcomes is 1. The total number of possible outcomes is also 1. i.e., $n(S) = 1$

Hence, the probability of getting the number 2 is: $P(E) = \dfrac{n(E)}{n(S)} = 1$

ii Probability of Impossible Event

If an event cannot occur, its probability is 0. It means that the probability of impossible event is 0. e.g.,

When we roll a dice, what is the probability to get number "8" ?

As we know that the number "8" will never appear on this dice in any situation.

Thus, the number of favourable outcomes is 0 *i.e.* $n(E) = 0$ and the number of possible outcomes is 6.

i.e., $n(S) = 6$. Hence the probability of getting the number 8 is: $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{0}{6} = 0$

iii Probability of Complement Event

A complement of an event "E" is the event that "E" does not occur. It is denoted by E' or E^C. e.g., tossing a coin to get head i.e, $E = \{\text{Head}\}$ and the complement of event E is not head i.e., $E' = \{\text{Tail}\}$

E	E'
{Head}	{Trail}

When we roll a dice, what is the probability of:
(i) getting the number 2 (ii) not getting the number 2

(i) P(getting number 2)

Let E be the event of getting the number 2

$$E = \{2\},\ n(E) = 1$$
$$S = \{1,2,3,4,5,6\}$$
$$n(S) = 6$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

(ii) P(not getting number 2)

Let E' be the event of not getting the number 2.

$$P(E') = 1 - P(E)$$
$$P(E') = 1 - \frac{1}{6}$$
$$P(E') = \frac{6-1}{6} = \frac{5}{6}$$

Remember!

The complement of an event is obtained / calculated by subtracting the probability of event "A" from 1.

$$P(A') = 1 - P(A)$$

Keep in mind

The sum of $P(A)$ and $P(A')$ is always equal to 1.

$$P(A) + P(A') = P(S)$$
As $$P(S) = 1$$
$$P(A) + P(A') = 1$$

Activity

Bring a dice to the classroom and allow the students to play with it. Then ask the following questions:

(i) What is the probability of rolling a 3?
(ii) What is the probability of rolling less than 5?
(iii) What is the probability of rolling not more than 4?
(iv) What is the probability of rolling an even number?
(v) What is the probability of rolling an odd number?
(vi) What is the probability of rolling not a prime number?
(vii) What is the probability of rolling not an even prime number?

Example 1 Two fair coins are tossed. What is the probability of getting 2 heads? Also find the complement of this event.

Solution When two fair coins are tossed, then sample space will be:

$S = \{HH, HT, TH, TT\}$; $n(S) = 4$

1st coin \ 2nd coin	H	T
H	HH	HT
T	TH	TT

Let "E" be the event of getting 2 heads

$E = \{HH\}$; $n(E) = 1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

The complement of the event is: $P(E') = 1 - P(E)$

$$P(E') = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

Hence, $P(E) = \frac{1}{4}$ and $(P(E') = \frac{3}{4}$

Skill Practice

Can you find out the total number of possible outcomes when 3 coins are tossed?

Remember!

To know the possible outcomes of an experiment we can use the following formula:

$(\text{Total possible outcomes})^{\text{Number of experiments}}$

e.g., $2^2 = 4$ (When 2 times a coin is tossed)

$2^3 = 8$ (When 3 times a coin is tossed)

Example 2 If Ahmad rolled two fair dice, find the probability of getting:

(i) odd numbers on both dice

(ii) no odd number on both the dice

(iii) odd number on the 1st dice and even number on 2nd dice.

(iv) at least the number 4 on the 1st dice and number 6 on the 2nd dice.

(v) Find the complement of the event part (iv).

Solution When two fair dice are rolled, then the sample space will be:

1st dice \ 2nd dice	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$n(S) = 36$

(i) Odd numbers on both the dice.

Let A be the event of getting odd numbers on both the dice.

$A = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

$n(A) = 9$

$$P(A) = \frac{n(A)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) no odd number on both the dice.

Let A' be the event of not getting odd number on both dice.

$$P(A') = 1 - P(A)$$

$$P(A') = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

Teachers' Guide

Give examples to explain complements of events. e.g., if the desired outcome is head on a flipped coin, the complement is tail. The complement rule states that the sum of the probabilities of an event and its complement must be equal to 1.

(iii) Odd number on the 1^{st} dice and even number on 2^{nd} dice.

Let B be the event of getting odd number on 1^{st} dice and even numbers on 2^{nd} dice.

$$B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\}$$

$$n(B) = 9$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(iv) at least the number 4 on the 1^{st} dice and the number 6 on the 2^{nd} dice.

Let C be the event of getting at least the number 4 on 1^{st} dice and the number 6 on the 2^{nd} dice.

$$C = \{(4, 6), (5, 6), (6, 6)\}$$

$$n(C) = 3$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(v) Find the complement of the event in part (iv).

Let C' be the complement of event C.

$$P(C) = \frac{1}{12}$$

$$P(C') = 1 - P(C)$$

$$P(C') = 1 - \frac{1}{12} = \frac{12-1}{12} = \frac{11}{12}$$

Skill Practice

Out of 300 students in a school, 95 play cricket, 120 play football, 80 play volleyball and 5 don't play any games. If one student is chosen at random, find the probability that

- he/she plays volleyball
- he/she plays cricket
- he/she plays football

Example 3 An urn contains 10 blue cards, 8 green cards and 12 yellow cards. A card is chosen at random from the urn. What is the probability of choosing:

(i) blue card (ii) green card (iii) not blue card

(iv) yellow card (v) not yellow card

Solution Total number of cards = 10 +8 + 12 = 20

(i) P(Blue card)

Let A be the event of choosing blue card

Blue cards = 10

$n(A) = 10$

Total number of cards = $n(S) = 30$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{30} = \frac{1}{3}$$

(ii) P(Green card)

Let B be the event of choosing green card

Green cards = 8

$n(B) = 8$

Total number of cards = $n(S) = 30$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{30} = \frac{4}{15}$$

(iii) P(Not blue card)

Let A' be the event of choosing not blue card

$$P(A') = 1 - P(A)$$

$$P(A') = 1 - \frac{1}{3}$$

$$P(A') = \frac{3-1}{3} = \frac{2}{3}$$

Challenge

- Can you find out the complement of choosing green card?
- Find the probability of choosing a red card. Also tell the name of the event.
- Find the probability of choosing a card. Also tell the name of the event.

(iv) P(Yellow card)

Let C be the event of choosing yellow card

Yellow card $= 12$

$n(C) = 12$

Total number of cards $= n(S) = 30$

$$P(C) = \frac{n(C)}{n(S)} = \frac{12}{30}$$

$$P(C) = \frac{6}{15} = \frac{2}{5}$$

(v) P(Not yellow card)

Let C' be the event of choosing not yellow card

$$P(C') = 1 - P(C)$$

$$P(C') = 1 - \frac{2}{5}$$

$$P(C') = \frac{5-2}{5}$$

$$P(C') = \frac{3}{5}$$

Skill Practice

A bag contains 50 marbles out of which 28 are red and 22 are blue. If a marble is picked at random from the bag. What is the probability that it will be:

(i) a red marble (ii) a blue marble (iii) not a blue marble (iv) a marble (v) a green marble

EXERCISE 5.4

1. Complete the following:

Sr. No.	$n(A)$	$n(S)$	$P(A)$	$P(A')$
(i)	6	18		
(ii)			$\frac{2}{5}$	
(iii)				$\frac{7}{19}$
(iv)				$\frac{1}{6}$

2. Zain rolled a fair dice. What will be the probability of getting number divisible by 3? Also find the probability of number not divisible by 3.

3. Shahzad throws a pair of fair dice. What will be the probability of getting:
   (i) number 6 on the 1^{st} dice and at least 4 on the 2^{nd} dice.
   (ii) no odd number on both the dice.
   (iii) sum of dots on both the dice is at least 7.
   (iv) difference between the dots is equal to 3.
   (v) sum of dots on both the dice is equal to 15.

4. A letter is chosen at random from the word "PROBABILITY" Find the probability of the following:

(i) $P(B)$ (ii) $P(A)$ (iii) $P(I)$ (iv) $P(B')$

(v) $P(\text{vowel})$ (vi) $P(\text{consonant})$ (vii) $P(G)$ (viii) $P(\text{a letter})$

5. A pair of fair coin is tossed, Find the probability of getting

(i) at least one head (ii) at least one tail

(iii) two tails (iv) three heads.

6. A box has 15 pencils, 8 sharpeners, 12 erasers and 2 rulers. Find the probability of choosing:

(i) a pencil (ii) a ruler

(iii) not a sharpener (iv) not an eraser

(v) a book (vi) an eraser

7. The probability that the team will win the cricket match is 0.79. What will be the probability of the team will not win the cricket match?

8. A box has 8 red balls, 9 white balls, 10 green balls and 5 blue balls. Find the probability of obtaining.

(i) a white ball (ii) a blue ball (iii) a green ball

(iv) not a red ball (v) a ball (vi) a black ball (vii) not a white ball

SUMMARY

- The word "Probability" is derived from the Latin word "Probabilitas". It means "probity"
- Probability is the chance of occurence of an event.
- The results of an experiment are known as outcomes.
- An outcome that represents how many times we expect the things to be happen, is called favourable outcome.
- The set of all possible outcomes of an experiment is called sample space.
- The set of outcomes of an experiment is called an event.
- An event which is sure to occur in any given experiment is called a certain event.
- When an event cannot occur in any given experiment, it is called an impossible event.
- An event is called likely event which will probably occur.
- An event is called unlikely event which will not probably occur.
- The events are called equally likely events which have equal chance of occurrence.
- The probability of an event is calculated by the following formula:

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} \Rightarrow P(E) = \frac{n(E)}{n(S)}$$

REVIEW EXERCISE 5

1. Choose the correct option:

(i) The singular of data is:

(a) graph (b) datum (c) values (d) observations

(ii) Ungrouped data is also known as:

(a) qualitative data (b) grouped data

(c) raw data (d) quantitative data

(iii) Pie graph is also called:

(a) circular graph (b) bar graph (c) line graph (d) histogram

(iv) Which of the following graph is suitable, when the data is given in continuous frequency distribution?

(a) Bar graph (b) Line graph

(c) Histogram (d) Pie graph

(v) Grouped data can be in the form of:

(a) frequency table (b) raw form

(c) ungrouped data (d) discrete table

(vi) The value of probability lies between:

(a) $0 < P(A) \leq 1$ (b) $0 \leq P(A) \leq 1$

(c) $0 < P(A) < 1$ (d) $0 \leq P(A) < 1$

(vii) The probability of getting number 6 in rolling a dice is:

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

(viii) The probability of not getting number 1 in rolling a dice is:

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

(ix) The mean of five numbers is 15.75. If the first four numbers are 16.25, 14.25, 15.50, 15.73 then find the 5^{th} number.

(a) 16.25 (b) 17.02 (c) 15.73 (d) 14.25

(x) What is the mode of the data: 2, 5, 9, 4, 3, 6, 10, 11, 12?

(a) 2 (b) 9 (c) no mode (d) 12

2. The following list, is of scores in a mathematics examination:

45, 75, 60, 63, 62, 85, 40, 95, 96, 80, 82, 81, 67, 57, 48, 91, 89, 90, 95, 90, 61, 52, 45, 46, 98, 99, 57, 60, 63, 63, 67, 68, 70, 58, 61, 49, 53, 98, 89, 81, 79, 62, 59, 49.

Construct a frequency distribution table of the given data by using 5 number of classes. Also find midpoints and class boundaries of the given data.

3. The table below shows Saad's yearly income in Rs. from 2015 to 2023. Draw a line graph for the following data:

Year	2015	2016	2017	2018	2019	2020	2021	2022	2023
Yearly income	25000	23000	30000	28000	38000	32000	35000	45000	

(a) His yearly income in 2023 was Rs. 12000 more than his yearly income in 2019. What was his yearly income for 2023?

(b) In which year did Saad earn the least?

(c) In which year did he earn 2 times as much as he earn in 2015?

(d) How much did he earn from 2015 to 2023?

4. The percentage of expenditure of a company under different heads is as follows. Represent the following information on the pie chart:

Heads of expenditure	Salary	Electricity	Conveyance	Machines
Percentage	40%	20%	15%	25%

5. Draw a vertical and horizontal bar graph for the following data:

Favourite Fruit	Mango	Strawberry	Banana	Watermelon	Peach
Number of Students	8	10	15	20	5

6. The following frequency table shows the number of toys sold by shopkeepers in the last six months of a year. Draw a histogram for the following data:

Number of toys	101 – 500	501 – 900	901 – 1300	1301 – 1700	1701 – 2100
Number of shopkeepers	50	70	120	80	90

7. Draw a histogram for the following frequency distribution table.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Number of Students	25	40	38	27	40	18

8. Find the mean, median, mode of the following data:

(i) 57, 69, 68, 74, 57, 58, 65, 69, 70, 80

(ii) 100.25, 85.35, 89.75, 80.50, 84.95, 99.5, 98.6, 101.5, 88.25, 99.25, 97.35, 111.5, 100.25

9. The mean of 8, 9, 10, 14, and x is 10. Find the value of the observation x.

10. The mean of 15 observations was calculated 200. It was found on rechecking that the value 125 was wrongly copied as 152. Find the correct mean.

11. Find the mean of the following data:

(i)

Length (mm)	30 – 50	50 – 70	70 – 90	90 – 110	110 –130
Frequency	15	18	35	20	13

(ii)

Length (mm)	6-10	11-15	16-20	21-25	26-30
Frequency	48	53	120	135	110

12. Complete the following:

Sr. No	$n(A)$	$n(S)$	$P(A)$	$P(A')$
(i)	11	17		
(ii)			$\frac{1}{21}$	
(iii)				$\frac{2}{3}$
(iv)	21	36		

13. Tahir throws a pair of fair dice. Find the probability of getting:

(i) even number on both the dice. (ii) product of dots between 10 – 30.

(iii) number 5 on the 1^{st} dice and at least 3 on the 2^{nd} dice.

(iv) no even number on both the dice.

14. A letter is chosen at random from the word "STATISTICS". Find the probability of getting:

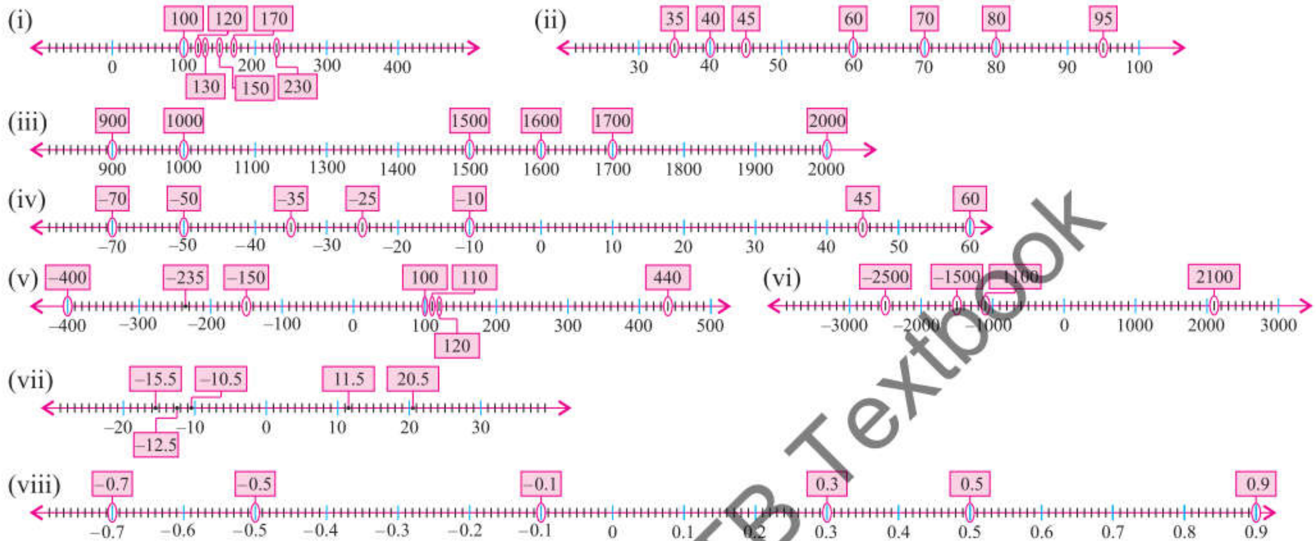
(i) P(Vowel) (ii) P(consonant) (iii) $P(T)$ (iv) $P(T')$

(v) $P(A)$ (vi) P (an alphabet) (vii) $P(G)$

ANSWERS

EXERCISE 1.1

1.



2. (i) 235090 < 235691 < 245091 < 245192
Ascending Order = 235090, 235691, 245091, 245192
Descending Order = 245192, 245091, 235691, 235090

(ii) 579803 < 578901 < 679807 < 679817
Ascending Order = 579803, 578901, 679807,679817
Descending Order = 679817, 679807, 578901,579803

(iii) 10028 < 100026 < 110027 < 110028
Ascending Order = 10028, 100026, 110027, 110028
Descending Order = 110028, 110027, 100026, 10028

(iv) 562389 < 562399 < 572381 < 572390
Ascending Order = 562389,562399,572381,572390
Descending Order = 572390,572381,562399,562389

3. (i) –8496 < –8494 < –8491 < – 8395
Ascending Order = –8496, –8494, – 8491, –8395
Descending Order = –8395,–8491, – 8494, –8496

(ii) –563 < –503 < –530 < –556
Ascending Order = –563, –503, –530, –556
Descending Order = –556, –530, –503, –563

(iii) –1886 < –138 < –137 < 1308
Ascending Order = –1886, –138, –137, 1308
Descending Order = 1308, –137, –138, –1886

(iv) –87650 < –78432 < – 78402 < –78401
Ascending Order = –87650, –78432, –78402, –78401
Descending Order = –78401, –78402, –78432, –87650

4. (i) 28.343 < 28.356 < 28.357 < 28.532
Ascending Order = 28.343, 28.356, 28.357, 28.532
Descending Order = 28.532, 28.357, 28.356, 28.343

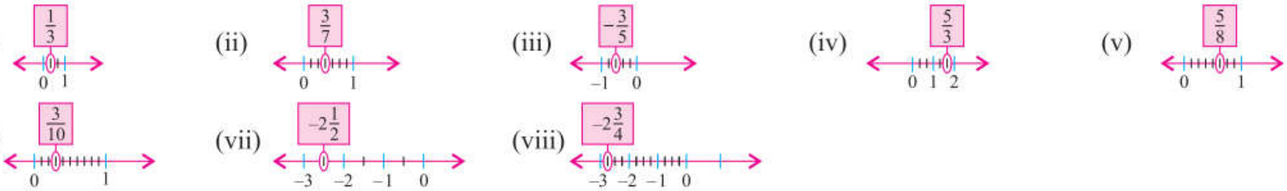
(ii) 120.01 < 120.08 < 120.80 < 130.08
Ascending Order = 120.01, 120.08, 120.80, 130.08
Descending Order = 130.08, 120.80, 120.08, 120.01

(iii) 103.78 < 113.08 < 131.01 < 131.08
Ascending Order = 103.78, 113.08, 131.01, 131.08
Descending Order = 131.08, 131.01, 113.08, 103.78

(iv) 236.089 < 236.207 < 236.217 < 236.219
Ascending Order = 236.089,236.207,236.217,236.219
Descending Order = 236.219,236.217,236.207,236.089

EXERCISE 1.2

1.



2. (i) $\frac{5}{2}, \frac{19}{5}, \frac{27}{11}, \frac{59}{7}$ (ii) $2\frac{4}{7}, 4\frac{7}{14}, 3\frac{4}{13}, 12\frac{7}{9}$

3. (i) > (ii) > (iii) > (iv) < (v) > (vi) <

4. (i) $\frac{2}{7} < \frac{2}{5}$ (ii) $\frac{5}{9} > \frac{-3}{5}$ (iii) $\frac{1}{12} > \frac{-1}{12}$ (iv) $\frac{-2}{7} < \frac{-3}{11}$ (v) $\frac{-5}{11} < \frac{-7}{9}$ (vi) $\frac{3}{22} > \frac{-5}{22}$

(vii) $3\frac{5}{9} < 4\frac{5}{18}$ (viii) $-1\frac{7}{23} < \frac{-3}{5}$ (ix) $3\frac{1}{2} > 2\frac{1}{14}$

5. (i) $\frac{1}{4}, \frac{3}{5}, \frac{7}{8}, \frac{5}{4}$ (ii) $\frac{-9}{7}, \frac{13}{-14}, \frac{3}{14}, \frac{6}{7}$ (iii) $\frac{5}{-4}, \frac{-7}{12}, \frac{-2}{3}, \frac{5}{8}$ (iv) $\frac{-7}{5}, \frac{-5}{4}, \frac{7}{8}, \frac{13}{8}$

6. (i) $\frac{2}{3}, \frac{4}{7}, \frac{-4}{-7}, \frac{-3}{4}$ (ii) $\frac{4}{5}, \frac{-13}{-30}, \frac{7}{-15}, \frac{7}{-10}$ (iii) $\frac{5}{8}, \frac{13}{24}, \frac{9}{48}, \frac{17}{-12}$ (iv) $\frac{-4}{33}, \frac{-9}{22}, \frac{-5}{11}, \frac{-41}{44}$

EXERCISE 1.3

1. (i) $\frac{9}{9}$ or 1 (ii) $\frac{-73}{28}$ (iii) $\frac{-17}{22}$ (iv) $\frac{-14}{13}$ (v) $\frac{-9}{20}$ (vi) $\frac{17}{5}$ **2.** (i) $\frac{-121}{280}$

(ii) $\frac{-8}{9}$ (iii) 0 (iv) $\frac{-269}{165}$ (v) 0 **3.** (i) $\frac{1}{4}$ (ii) $\frac{13}{9}$ (iii) $\frac{-7}{45}$ (iv) $\frac{-9}{18}$

4. (i) $\frac{3}{14}$ (ii) $\frac{7}{10}$ (iii) $\frac{-44}{75}$ (iv) $\frac{-43}{70}$ (v) $\frac{-1}{9}$ (vi) $\frac{55}{36}$ (vii) $\frac{59}{56}$ (viii) $\frac{13}{22}$

5. (i) $\frac{35}{66}$ (ii) $\frac{-12}{35}$ (iii) $\frac{-5}{54}$ (iv) $\frac{15}{98}$ (v) 21 (vi) $\frac{5}{2}$ (vii) $\frac{145}{242}$

(viii) $\frac{-63}{8}$ (ix) –30 **6.** (i) $\frac{10}{7}$ (ii) $\frac{-7}{2}$ (iii) $\frac{-3}{20}$ (iv) $\frac{-8}{135}$ (v) $\frac{4}{7}$ (vi) $-2\frac{1}{2}$

(vii) $14\frac{2}{15}$ (viii) $17\frac{67}{95}$ **7.** $\frac{1}{6}$ **8.** $\frac{2}{5}$ **9.** (i) $\frac{-3}{5}$ (ii) $\frac{7}{12}$ (iii) $\frac{-8}{15}$ (iv) $7\frac{5}{12}$

(v) $2\frac{1}{13}$ **10.** (i) $\frac{62}{9}$ m **11.** (a) 6 packets (b) 32kg **12.** $\frac{234}{7}$ litres

EXERCISE 1.5

1. (i) 88000 (ii) 3890 (iii) 790000 (iv) 10000 (v) 19000 (vi) 26000 **2.** (i) –27.3 (ii) –59000
(iii) –12600 (iv) –17 (v) 0.00386 (vi) 0.0468 **3.** (i) 5.9 (ii) 0.185 (iii) 0.815 (iv) 1.42
(v) 0.698 (vi) 0.928 **4.** (i) 25.36 (ii) 0.00290 (iii) 0.088 (iv) 17.0 (v) 0.000525 (vi) 13.588
5. (i) 1.99 (ii) 1.24 (iii) 8.14

EXERCISE 1.6

1. 82 **2.** 8 **3.** 34 **4.** 1 **5.** 44 **6.** $\frac{1}{4}$ **7.** $9\frac{1}{2}$ **8.** $2\frac{1}{15}$ **9.** $4\frac{4}{9}$

10. $2\frac{17}{63}$ **11.** 6.125 **12.** 5.35 **13.** 3 **14.** 9.8 **15.** 4.4993

EXERCISE 1.7

1. (i) $\{1,3,5,7\}$ (ii) {Monday, Tuesday, Wednesday,...} (iii) $\{v, w, x, y, z\}$ (iv) $\{10,12,...,24\}$
(v) $\{-9,-8,...,12\}$ (vi) $F = \{\pm1, \pm2, \pm3, ...\}$ (vii) $\{1,2,3...,14\}$ (viii) $\{2,3,5,7,11,13,17\}$

2. (i) Set of multiples of 3 less than 24 (ii) Set of English alphabet from *a* to *m*. (iii) Set of even numbers
(iv) Set of integers greater than –21 and less than 21 (v) Set of natural numbers less than 51 (vi) Set of whole numbers
(vii) Set of prime numbers between 10 and 40 (viii) Set of multiples of 5 greater than 20 and less than 60

3. (i) $\in$ (ii) $\notin$ (iii) $\in$ (iv) $\in$ (v) $\in$ (vi) $\notin$ (vii) $\notin$ (viii) $\in$

4. (i) $\{x \mid x \in N, x \text{ is multiple of } 4 \wedge x < 50\}$ (ii) $\{x \mid x \in N \wedge -5 < x < 5\}$ (iii) $\{x \mid x \in O \wedge x \leq 15\}$
(iv) $\{x \mid x \in N \wedge 10 \leq x \leq 25\}$ (v) $\{x \mid x \text{ is a name of last five solar months}\}$ (vi) $\{x \mid x \in Z \wedge -20 \leq x \leq 20\}$
(vii) $\{x \mid x \in O \wedge 24 < x < 38\}$ (viii) $\{x \mid x \in N, x \text{ is multiple of } 10 \wedge x \leq 100\}$

EXERCISE 1.8

1. (i) Proper subsets $= \{a\}, \{a,c\}$, Improper subsets $= \phi, \{a,c,d\}$

(ii) Proper subsets = {2,4},{6,8}, Improper subsets = ϕ,{2,4,6,8}
(iii) Proper subsets = {–1},{0}, Improper subsets = {},{–1, 0, +1}
(iv) Proper subsets = {4, 8,12},{8,12,16}, Improper subsets = {},{4,8,12,16}
(v) Proper subsets = { Δ },{○,□} Improper subsets = ϕ,{ Δ,○,□}
(vi) Proper subsets = {Zara},{Zeeshan}, Improper subsets = ϕ,{Zara, Aslam, Zeeshan}

2. (i) B (ii) C (iii) W (iv) F (v) M **3.** (i) $A \leftrightarrow B$ (ii) C = D (iii) E = F (iv) G = H (v) $I \leftrightarrow J$

4. (i) Sets A and B are disjoint (ii) Sets C and D are overlapping (iii) Sets E and F are overlapping
(iv) Sets P and Q and disjoint (v) Sets G and H are overlapping (vi) Sets I and J are disjoint

EXERCISE 1.9

1. (i) {1,2,3,4,5,7,8,9,10,13} (ii) {0,1,2,3,4,6,8,10} (iii) {a, b, c, d, e, i, o, u} (iv) {0,1,2,3,...,10}

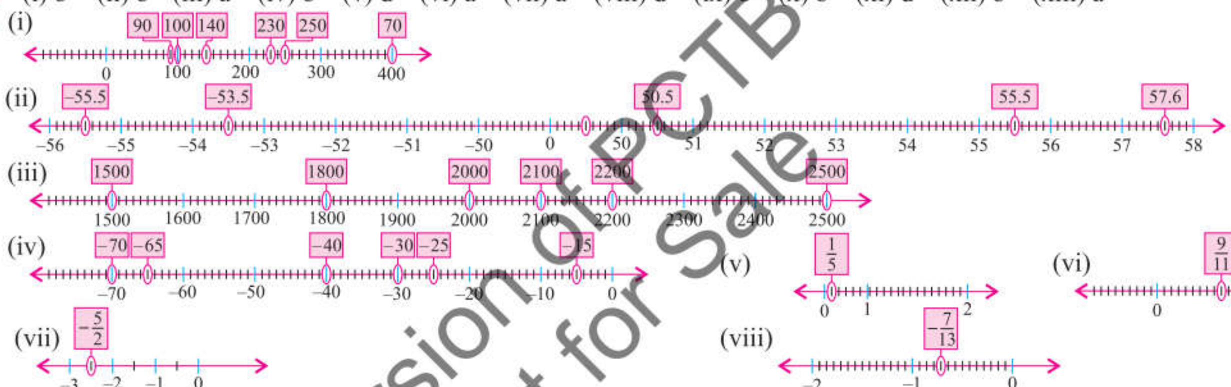
2. (i) $\{t, e\}$ (ii) {2} (iii) {0} (iv) {0,5,10,15,20}

3. $N \cup W = \{0,1,2,...\}$ and $N \cap W = \{1,2,3,...\}$ **4.** $E \cup O = \{0,\pm 1,\pm 2,...\}$ and $E \cap O = \{\}$ or ϕ

5. $P \cup C = \{2,3,4,...\}$ and $P \cap C = \{\}$ or ϕ **6.** (i) {1,2,4,5,7,8,10} (ii) {1,2,3,5,6,7,9,10} (iii) {1,3,5,7,9}
(iv) ϕ **7.** (i) {0,6,9,10} (ii) {2,4} (iii) {0,2,6,9,10} (iv) {7}

REVIEW EXERCISE 1 (a)

1. (i) b (ii) b (iii) a (iv) b (v) d (vi) a (vii) a (viii) d (ix) b (x) b (xi) d (xii) b (xiii) a

2.


3. (i) $326681 < 326781 < 336281 < 336291$
Ascending Order = 326681, 326781, 336281, 336291
Descending Order = 336291, 336281, 326781, 326681

(ii) $-55451 > -55540 > -56508 > -56580$
Ascending Order = $-56580, -56508, -55540, -55451$
Descending Order = $-55451, -55540, -56508, -56580$

(iii) $108.01 < 180.08 < 111.70 < 111.78$
Ascending Order = 108.01, 111.70, 111.78, 180.08
Descending Order = 180.08, 111.78, 111.70, 108.01

(iv) $\frac{-7}{10} < \frac{-3}{5} < \frac{7}{10} < \frac{13}{15}$ Ascending Order = $\frac{-7}{10}, \frac{-3}{5}, \frac{7}{10}, \frac{13}{15}$
Descending Order = $\frac{13}{15}, \frac{7}{10}, \frac{-3}{5}, \frac{-7}{10}$

4. (i) $\frac{16}{9}, \frac{16}{5}, \frac{29}{12}, \frac{61}{7}$ (ii) $3\frac{6}{7}, 3\frac{6}{14}, 3\frac{4}{13}, 7\frac{5}{9}$ **5.** (i) $\frac{9}{13}$ (ii) $\frac{17}{16}$ (iii) $\frac{1}{11}$ (iv) 44 **6.** $\frac{231}{364}$

7. $\frac{5}{9}$ **9.** (i) 98000 (ii) 4580 (iii) –54000 (iv) –16 (v) 0.00376 (vi) 0.0658
(vii) 14.13 (viii) 0.325 **11.** (i) 357 (ii) $\frac{36}{73}$ (iii) 3.28723 **12.** (i) Not well-defined
(ii) Well-defined (iii) Well-defined (iv) Well-defined **13.** (i) Set because all the elements are well-defined
(ii) Not a set because not well-defined (iii) A is not a set because '0' comes two times
(iv) Set because all the elements are well-defined and distinct (v) same as (iv)

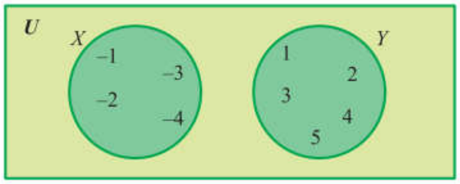
14. (i) Descriptive form = Set of multiple of 4 greater than 3 and less than 33
Set builder : $\{x | x \in N, x \text{ is multiple of } 4 \wedge 3 < x < 33\}$
(ii) Tabular form : {10,11,12,.........,20} ; Descriptive form : Set of natural numbers greater than 9 and less than 21
(iii) Descriptive form = Set of integers ; Set builder = $\{x \mid x \in Z\}$

(iv) Tabular form = $\{9,16,25,36,49,64,81\}$; Set builder = $\{x \mid x \text{ is perfect square number} \wedge 5 < x < 100\}$

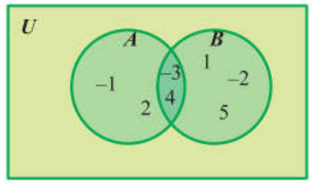
(v) Descriptive form: set of natural numbers less than or equal to 15. ; Tabular form: $\{1,2,3,4,5,\ldots, 15\}$

(vi) Descriptive form: set of whole numbers less than 10. ; Tabular form: $\{0,1,2,3,\ldots, 9\}$

15. (i)



(ii)



16.
(i) $U \cup V = \{-1, -2, -3, ..., -10\}$
$U \cap V = \{-1, -3, -5, -7, -9\}$
(ii) $X \cup Y = \{\pm 1, \pm 2, \pm 3, ..., \pm 4, \pm 5\}$
$X \cap Y = \{\ \}$

EXERCISE 1.11

1. Same as given
2. (i) 50 (ii) 96 (iii) 407 (iv) 68 (v) 560 (vi) 595 **3.** (i) 20 (ii) 25 (iii) 20 (iv) 20
(v) 100 (vi) 196 **4.** 40:33 **5.** 13:15 **6.** 2:3 **7.** Rs. 2142 **8.** 2277cm **9.** 120kg **10.** 4:5

EXERCISE 1.12

1. Rs. 700/day **2.** Rs.20/orange **3.** 20 km/ℓ, 20 litres **4.** 5 dishes/h, 40 dishes **5.** Rs. 15/unit, Rs. 1125
6. 75 km/h, 600 km **7.** Rs. 5000 **8.** 10 runs/over **9.** 3 pens/minute **10.** 94.4 km/h

EXERCISE 1.13

1. (i) in proportion (ii) not in proportion (iii) in proportion (iv) in proportion (v) in proportion
(v) Not in proportion **2.** (i) 58.33 (ii) 18 (iii) 3.9 (iv) 2.7 **3.** 240 **4.** 19kg 840g
5. 10, 100 **6.** 6 **7.** 5 **8.** 6 **9.** 1

EXERCISE 1.14

1. (i) Rs. 110 (ii) Rs. 50 (iii) Rs. 600 **2.** Owner made a profit Rs.15 **3.** Shopkeeper made a profit Rs. 300
4. Loss = Rs. 1000 **5.** Selling price = Rs. 124 **6.** Selling Price = Rs. 43200 **7.** 42.86%
8. 20% **9.** 20% **10.** 8.33% **11.** Rs. 690 **12.** Rs.300, Rs. 345 **13.** Rs. 2944
14. 22%, Rs. 50000 **15.** Eman bears less loss on heater **16.** Rs. 5896

EXERCISE 1.15

1. Rs. 10000 **2.** Rs. 1000000 **3.** Rs. 12000 **4.** Property tax = Rs. 440000 **5.** Rs. 1700000
6. Rs. 234000 **7.** Rs. 42500 **8.** Rs.12% **9.** 8% **10.** Rs. 125000 **11.** Rs. 2500 **12.** Rs. 1000
13. Rs. 50000 **14.** Rs. 70000 **15.** Rs. 100000

EXERCISE 1.16

1. Rs. 48750 **2.** Rs. 3000 **3.** Rs. 880000 **4.** Rs. 31125 **5.** Rs. 35750 **6.** Rs. 940000 **7.** Rs. 40000
8. Rs.40000 **9.** Rs.65000 **10.** Rs.13100

EXERCISE 1.17

1. (i) 2401 (ii) 4356 (iii) 5625 (iv) 12321 (v) 52900 (vi) 97969 (vii) 77841 (viii) 160000
2. (i) Yes (ii) No (iii) Yes (iv) Yes (v) No (vi) No (vii) Yes (viii) No
3. (i) 5184 (ii) 6400 (iii) 3969 (iv) 16384 (v) 126025 (vi) 274576 (vii) 17774656 (viii) 516961
(ix) 848241 (x) 395641 (xi) 10310521 (xii) 103041 **5.** (i) $\frac{196}{256}$ (ii) $\frac{4}{324}$ (iii) $\frac{361}{784}$ (iv) $\frac{625}{784}$

EXERCISE 1.18

1. (i) 7 (ii) 10 (iii) 12 (iv) 14 (v) 18 (vi) 28 (vii) 23 (viii) 26
2. (i) 30 (ii) 50 (iii) 43 (iv) 38 (v) $\frac{7}{12}$ (vi) $\frac{8}{15}$ (vii) 2.7 (viii) 2.6
3. (i) 24 (ii) 48 (iii) 39 (iv) 0.21 (v) $\frac{16}{33}$ (vi) $\frac{9}{17}$ (vii) $\frac{26}{29}$ (viii) 5.1

EXERCISE 1.19

1. 1. 93 students **2.** 48 students **3.** 380.25 cm^2 **4.** 30 m **5.** 20 students

REVIEW EXERCISE 1 (b)

1. (i) d (ii) b (iii) a (iv) d (v) b (vi) b (vii)c (viii) c (ix) c (x) b (xi) c (xii)b (xiii) a (xiv) d (xv) b **2.** (i) 3:5 (ii) 2:5 (iii) 37:17 (iv) 7:12 (v) 5:1 (vi) 5:2 **3.** (i) 24 (ii) 156
4. (i) 357 (ii) 222 **5.** 9:8 **6.** 56:89 **7.** Rs. 20,000 **8.** 70 km/h, 280 km **9.** (i) 25 (ii) 33 (iii) 5 (iv) 24 **10.** Rs. 112000 **11.** 200 pages **12.** 2.85 ≈ 3 men **13.** 7 men
14 26,67 ≈ 27 days **15.** 60 km/h **16.** Rs. 2.5/min **17.** 2 pages/min **18.** 26 km/ℓ
19. 61.54km/h **20.** 67% **21.** 20% **22.** Rs. 4500 **23.** Rs. 150,000 **24.** Rs. 500,000
25. Rs. 9360 **26.** Rs. 5000 **27.** Rs. 65000 **28.** 29.6 ≈ 30% **29.** (i) 2809 (ii) 4761 (iii) 82944 (iv) 250000 **30.** (i) yes (ii) No (iii) No (iv) yes **31.** (i) 13 (ii) 17 (iii) $\frac{7}{25}$ (iv) 2.9 **32.** (ii) 19 (ii) 27 (iii) $\frac{10}{11}$ (iv) 2.6 **33.** 17 inches 34. 21 metres

EXERCISE 2.1

1. (i) 4 (ii) 5 (iii) 6 (iv) 5 **2.** (i) 2 (ii) 5 (iii) 3 (iv) 2
3. (i) Adding 5 (ii) Adding 7 **4.** (i) 8, 10, 12, … (ii) 3, 6, 12, …
5. (i) 16, 19, 22, (ii) 24, 30, 36 (iii) 14, 11, 8 (iv) 92, 90, 88
6. (i) 7n – 1 (ii) 2n + 3 (iii) 5n – 1 (iv) 10n – 8 **7.** (i) 38 (ii) 94 (iii) 148 (iv) 212
8. (i) 8, 11, 14, 17 (ii) 10, 17, 26, 34 (iii) 6, 21, 46, 81 (iv) 16, 20, 24, 28
9. (a) 19, 21, 23 (b) 23 (c) 5 weeks **10.** (a) 60 minutes (1 hour) (b) 70 minutes (1 hour and 10 minutes)
11. (i) 35, 40, 45, 50, 55, 60, 65 (ii) 425 balls altogether (iii) Rs. 7650

EXERCISE 2.2

1. (i) 2 (ii) 3 (iii) 2 (iv) 7
2. (i), (ii), (v), (vi), (vii), (ix) are open sentences and (iii), (iv), (viii) are close sentences.
3.

Sr. #	Algebraic Expression	Number of Terms	Variables	Constant
(i)	$4xy$	1	x, y	0
(ii)	$4x^2 - 2y + 8$	3	x, y	8
(iii)	$2xyz + 20$	2	x, y, z	20
(iv)	$x^2 - 2xy + 16$	3	x, y	16

4. (i) $3x^2$ and $8x^2$ are like terms
(ii) $4x^2y$, $7yx^2$ are like terms
(iii) $2x^2$, $9x^2$ are like terms
(iv) $\frac{3}{2}x^2$ and $\frac{9}{2}x^2$ are like terms
5. (i), (ii), (iv) and (vi) are inequalities
(iii), (v) are equations

EXERCISE 2.3

1. (i), (ii), (iv), (vi) and (vii) are polynomials **2.** (i) 2, (ii) 2 (iii) 3 (iv) 2 (v) 3 (vi) 1 (vii)4 (viii) 2
3. (i) $10x^2 - 3x + 16$ (ii) $7x^2 + 10xy + 9$ (iii) $4x^3 + 4x^2 + 7x + 8$ (iv) $6x^2 - 2x + 25$ (v) $5x^3 + 3x^2 + 23x + 17$
4. (i) $5x^2 + 6x + 8$ (ii) $7y^2 + 8q^2 + 4r$ (iii) $5x^3 - 12x^2 + 8x + 12$ (iv) $8x^3 + 9x^2 + 5x + 8$ (v) $3x^3 + x^2y + 3xy^2 + 3y^3$ (vi) $7z + y + 13$ **5.** $5x^2 + 2x^2 + 5x + 10$ **6.** $3x^2 + 12x + 5$

EXERCISE 2.4

1. (i) $32x^3y$ (ii) $24x^5y^5$ (iii) $14x^8y^6$ (iv) $8x^7y^7$ (v) $14\ell^5m^8$ (vi) $6x^5y^2 + 15x^3y^4$ (vii) $10x^2y^3 + 16x^3y^4$
(viii) $15c^5d^4 + 25c^6d^4$ (ix) m^3n^3 (x) $12\ell^4m^7$ (xi) $14a^4b - 35a^3b^2 + 16a^2b^3 - 40ab^4$
(xii) $14a^4b - 35a^3b + 16a^2b^3 - 40ab^4$ (xiii) $4x^4y^2 - 10x^2y^4 - 16x^5y^4$ (xiv) $5x^5y - 10x^4y^3 - 15x^4y^2$
(xv) $6\ell^5m^3 + 10\ell^4m^5 - 4\ell^4m^3$ (xvi) $2a^3b^4 - 7a^3b^3 + 4a^2b^4 - 2a^4b^3 + 3a^4b^2$
(xvii) $2\ell^2m^3 + 4\ell^3m^3 + 3\ell^2m + 6\ell^2m^4 + 12\ell^3m^4 + 9\ell^2m^2$ (xviii) $10x^4y^4 + 16x^3y^4 - 4x^4y^3 - 10x^3y^5 - 12x^2y^5$
(xix) $2a^2b^4 - 11a^4b^3 - a^5b^3 + 15a^6b + 3a^7b^2$

2. (i) xy (ii) $5x^4y^4$ (iii) $5\ell^3m^7$ (iv) $ab+\frac{5}{2}a^3b^2$ (v) $10pq+15p^6q^5$ (vi) $4x^2y^4+3y^5$
(vii) $5x^3y^4+3x^7y$ (viii) $3a^3b^3+2a^2b+1$

EXERCISE 2.5

1. $2x^3-2x^2y-3xy-3y^2$ (ii) $4x^2+x-8$ (iii) $\frac{4}{3}(x^2-2)$ (iv) $10x^2-3x-4$ (v) $-13m^2+21m+24$
(vi) $4x-2y$ (vii) $-16x+8z+18y$ (viii) $4m-6n-2$ (ix) $ab+2y-4z+2$ (x) $6x-4y-5$

EXERCISE 2.6

1. (i) $4x^2+4xy+y^2$ (ii) $4x^2+20xy+25y^2$ (iii) $\frac{x^2}{4}+\frac{xy}{2}+\frac{y^2}{4}$ (iv) $\frac{1}{m^2}+\frac{2}{mn}+\frac{1}{n^2}$
(v) $25\ell^2+20\ell m+4m^2$ (vi) $\frac{1}{9}a^2+\frac{ab}{3}+\frac{1}{4}b^2$ (vii) $16x^2+16xy+4y^2$ (viii) $49x^2+14xy+y^2$

2. (i) $9x^2-24x+16$ (ii) $25m^4-10m^2n+n^2$ (iii) $\frac{4}{x^2}-\frac{12}{xy}+\frac{9}{y^2}$ (iv) $9x^2-12xy+4y^2$
(v) $\frac{1}{\ell^2}-\frac{2}{\ell m}+\frac{1}{m^2}$ (vi) $\frac{1}{4a^2}-\frac{1}{3ab}+\frac{1}{9b^2}$ **3.** (i) x^2-4y^2 (ii) m^4-n^4 (iii) $\frac{1}{4p^2}-\frac{1}{4q^2}$
(iv) a^8-b^8 (v) $\frac{1}{25a^4}-\frac{1}{25b^4}$ (vi) $5x^2-4y^2$ **4.** (i) $5x^2-2x+10$ (ii) $-5x^2+16x-3$
(iii) $-5x^2-6x-26$ (iv) $130x^2-46x+5$ (v) $80a^2-25b^2+54a-9$ (vi) $3a^2-10ab+16b^2$

5. (i) $24xy$ (ii) $4b^2$ (iii) ab (iv) $\frac{xy}{2}$ **6.** (i) 1369 (ii) 3969 (iii) 884

EXERCISE 2.7

1. (i) $2y(3x-7z)$ (ii) $15x^2(2x^2-3y)$ (iii) $6y(x^2-4z)$ (iv) $7x(x^3-2xy+3y^3)$
(v) $xyz(xyz-z+1)$ (vi) $5x^3(x^2-2x+3)$ (vii) $abc(8a^2b^2-2ab^2+1)$ (viii) $5x^2y^2(xy-3+y)$
(ix) $4x^2(x-2y^3+3y)$ (x) $\ell^2m^2(2\ell^2m^2-5m-2)$ (xi) $x^2y^3(y^2-x^2y^3+1)$ (xii) $9pq(pq-2+3p^2q^3)$

2. (i) $(x-2)(x+5)$ (ii) $(a-3c)(2b-1)$ (iii) $(a-b)(x-y)$ (iv) $(y-b)(y-a)$
(v) $(ab+cd)(x+7)$ (vi) $(pq-rs)(a^2+b^2)$ (vii) $3(x-y^2)(x-2)$ (viii) $(-x+y)(2x-5)$
(ix) $(4x-2)(1-3y)$ (x) $(3-3y)(1-x)$ (xi) $3(a-3c)(a-b)$ (xii) $2(x^2-2y)(1+x^2)$

EXERCISE 2.8

1. (i) $(x-4)(x-6)$ (ii) $(x-5)(x+8)$ (iii) $(2x+1)(2x+3)$ (iv) $(x+2)(x+5)$
(v) $(x-2)(x+4)$ (vi) $(y-2)(y-5)$ (vii) $(5x-1)(5x+2)$ (viii) $(3x-1)(2x+5)$
(ix) $(x-1)(x+1)(2x^2+3)$ (x) $(\ell+1)(2\ell-5)$ (xi) $(5m+1)(2m-3)$ (xii) $(7m-1)(2m-5)$

EXERCISE 2.9

1. 9 **2.** 12 **3.** $\frac{13}{3}$ **4.** 5 **5.** 6 **6.** -1 **7.** -2 **8.** 3 **9.** 3
10. $\frac{4}{7}$ **11.** 10 **12.** -13

EXERCISE 2.10

1. 12 **2.** 7 **3.** 36 **4.** 40, 41, 42 **5.** Abdullah=7, Abdul Hadi=25
6. Sakeena's age = 10 years, Father's age = 40 years

EXERCISE 2.11

1. (i) $x+y=-3$ (ii) $-3x+y=2$ (iii) $5x-3y=-13$ (iv) $3x-5y=0$ (v) $3x-2y=7$ (vi) $2x-4y=-5$

2. (i) $x+y=11$ (ii) $x+2y=90$ (iii) $x=\frac{1}{3}y$ (iv) $2x+3y=30$ (v) $x+y=37$ (vi) $3x+7y=3000$ (vii) $\frac{x-3}{y-3}=\frac{2}{3}$

EXERCISE 2.12

1. (i) II (ii) III (iii) II (iv) IV (v) I (vi) III
4. $A(-6,3)$, $E(3,4)$, $F(-4,-2)$, $H(7,-8)$, $J(-9,2)$, $K(4,7)$

REVIEW EXERCISE 2

1. (i) c (ii) b (iii) d (iv) c (v) d (vi) c (vii) d (viii) b (ix) d (x) a
2. (i) –5, –1, 3 (ii) 7, 10, 13 (iii) 9, 16, 23
3. $a_{15} = \frac{1}{34}$
4. Rs. 25
5. 10 kilometres
6. 12 litres
7. (i) $19x^2 + 10x + 9$ (ii) $22x^2 - 8x - 6$ (iii) $28x^2 + 7x - 10$
8. (i) $5x^2 + 10x + 13$ (ii) $-x^2 + 7x - 10$
9. (i) $24x^3y^4 + 9x^2y^4 + 6xy^2$ (ii) $-10x^5y - 16x^2 + 5x^3y^2 + 8y$ (iii) $\ell^3m^3 + \frac{2}{3}\ell m^2 + \ell^2m + \frac{2}{3}$
10. (i) $2x^2y$ (ii) $5m$ (iii) $4ab + 1 + 2a^2b$
11. (i) $2x^3 + 4x^2 - 8x + 3$ (ii) $2x^2 + 2x - 3$
12. (i) $49x^2 - 9y^2$ (ii) $-7a^2 - 25b^2 + 16a - 4$
13. (i) $x^2y^2(x^6y^4 - x^2y + 1)$ (ii) $(x - 3)(2x + 7)$ (iii) $2x(x + y)(2x + 1)$
14. (i) 12 (ii) 4 (iii) $\frac{11}{4}$ (iv) –19 (v) 10 (vi) 4
15. –4
16. 32
17. (a) $x - y = 13$ (b) $12x$
18. (i) II (ii) I (iii) IV
20. F (–3, –3), G (0, 1), H (–1, 3)
21. (i) horizontal line (ii) vertical line (iii) vertical line

EXERCISE 3.1

1. (i) 5313 m (ii) 10200 m (iii) 12078 cm (iv) 20000 cm (v) 5400 cm (vi) 95000 m
2. 5 km 100m
3. 5m 57 cm
4. Azra's house is nearer and 3100 m.
5. 4 cm 3 mm

EXERCISE 3.2

1. (i) 930 min (ii) 955 seconds (iii) 375 days (iv) 147 days (v) 4 years 6 months (vi) 8 weeks 4 days (vii) 2 years 140 days (viii) 65 minutes
2.

Sr. #	12 hour time	24 hour time
(i)	4:50 a.m	4:50
(ii)	9:30 a.m	9:30
(iii)	7:10 pm	19:10
(iv)	9:05 p.m	21:05
(v)	6:00 a.m	06:00
(vi)	4:00 p.m	16:00
(vii)	12:00 a.m	00:00

3. 11 years 3 months
4. 2210 days
5. 27 days, 648 hours

EXERCISE 3.3

1. 14 h 35 min
2. 1:45 p.m., 5 h 45 min
3. 12:50 a.m; 00:50
4. 6 h 40 min
5. 2:05 p.m.
6. 1 h 35 min

EXERCISE 3.4

1. (a)

Sr.#	m/s	km/h
(i)	126	35
(ii)	54	15
(iii)	72	20

(b)

Sr.#	m/s	km/h
(i)	18	5
(ii)	54	15
(iii)	198	55

2. 80 km/h
3. 29.6 m/s, 106.56 km/h
4. 60 km/h

EXERCISE 3.5

1. (i) $A = 111\ m^2$, $P = 68$ m (ii) $A = 60\ cm^2$, $P = 36$ cm (iii) $A = 36\ cm^2$, $P = 28$ cm (iv) $A = 180\ m^2$, $P = 90$ m
2.

Sr.#	mm^2	cm^2	m^2
(i)	1400	14	0.0014
(ii)	10000	100	0.01
(iii)	5000000	50000	5

3. $25\ m^2$
4. $104\ m^2$
5. $736\ m^2$
6. $P = 32$ m, $A = 44\ m^2$

EXERCISE 3.6

1. (i) $C = 18.86$ cm , $A = 28.27$ cm^2 (ii) $C = 25.14$ cm , $A = 50.27$ cm^2 (iii) $C = 31.43$ cm , $A = 78.54$ cm^2
(iv) $C = 44$ cm , $A = 154$ cm^2 (v) $C = 12.57$ cm , $A = 12.57$ cm^2 (vi) $C = 62.86$ cm , $A = 314.16$ cm^2

2. (i) $C = 19.48$ m (ii) $C = 15.71$ cm (iii) $A = 113.10$ cm^2 (iv) $A = 95.03$ m^2

3. 32.17 m^2 , 58.09 m^2 , 91.61 m^2 **5.** $A = 63.62$ m^2 , $C = 28.27$ m **6.** Area = 44.57m^2, $P = 36.57$ m

7. (i) 235.62 m^2 (ii) 50.27 m^2 (iii) 3.43 cm^2 **8.** 1376.44 m^2

EXERCISE 3.7

1.

Sr.#	mm^3	cm^3	m^3
(i)	300	14	0.0014
(ii)	10000	100	0.01
(iii)	5000000	50000	5

2. Square Area = 15000 cm^2 , Volume = 125000 cm^3

3. 15.71 m^3 **4.** Square Area = 167.5 cm^2 , Volume = 87.5 cm^3

5. 1000 m3 **6.** 490873.85 cm^3 **7.** 175 m^3 **8.** 1256.64 m^3

9. Square Area = 167.5 cm^2 , Volume = 87.5 cm^3

10. Square Area = 2356.19 cm^2 , Volume = 4712.39 cm^3

REVIEW EXERCISE 3

1. (i) c (ii) b (iii) d (iv) d (v) c (vi) b (vii)d (viii) d (ix) d
(x) a (xi) b (xii)c (xiii) b (xiv) b (xv) d **2.** (i) 75880 m (ii) 750 mm
(iii) 58 cm 5mm (iv) 5 km 700m **3.** 1200 m

4.

Sr. #	12 hour time	24 hour time
(i)	5:00 a.m	17:00
(ii)	10:35 p.m	22:35
(iii)	6:15 p.m	18:15
(iv)	2:30 a.m	14:30

5. (i) 9 min 20 sec (ii) 2 years 11 minutes (iii) 691 days
(iv) 16 months 10 days. **6.** 9300 seconds

7.

Sr. #	Departure Time	Journey Time	Arrival Time
(i)	5:50 a.m	4 h 15 min	10:05 a.m
(ii)	8:30 a.m	11h 18 min	7:48 p.m
(iii)	07:50	3 hours	10: 50
(iv)	17:00	1 h 20 min	18:20
(v)	8:40 p.m	13 h 20 min	10:00 a.m (next day)

8. 4 h 50 min

9. (i) 3:30 p.m (ii) 3 h 30 min

10. (a)

Sr. #	cm	m	km
(i)	100	1	0.001 km
(ii)	200,000	2000	2
(iii)	50,000	500	0.5
(iv)	1000	10	0.01 km
(v)	10,000	100	0.1 km

(b)

Sr. #	km/h	m/s
(i)	108	30
(ii)	180	50
(iii)	144	40
(iv)	360	100

11. (a)

Sr. #	mm^2	cm^2	m^2
(i)	5000000	50000	50
(ii)	700000	7000	0.7
(iii)	100000	1000	0.1
(iv)	3000000	30000	3

(b)

Sr. #	m^3	cm^3	mm^3
(i)	4	4000000	4000000000
(ii)	0.8	800000	800000000
(iii)	20	20000000	20000000000
(iv)	0.9	900000	900000000

12. (i) Shaded area = 11 cm^2 , Unshaded area = 6 cm^2 (ii) Shaded area = 80 cm^2 , Unshaded area = 80 cm^2
(iii) Shaded area = 16 cm^2 , Unshaded area = 166 cm^2 (iv) Shaded area = 56 cm^2 , Unshaded area = 4 cm^2

13. $C = 12.57$ m , $A = 12.57$ m^2 14. $r = 5$m **15.** (a) 69.12 m^2(b) 3.14 m^2 (c) 31.42m^3

16. (a) 1800 cm^2 (b) 4500 cm^3 **17.** Surface Area = 145 cm^2 , Volume = 100 cm^3

18. 343 m^3 **19.** Surface area = 74 m^2 , Volume = 35 m^3 **20.** 11993 cm^3

EXERCISE 4.1

1. (i) Equilateral triangle (ii) Scalene triangle (iii) Isosceles triangle (iv) Scalene triangle (v) Scalene triangle

2. (i) Obtus angled triangle (ii) Acute angled / Equilateral triangle (iii) Acute angle triangle (iv) Right angled triangle (v) Right angled triangle **3.** (i) 41° and 49° are complementary angles (ii) 61° and 119° are supplementary angles (iii) 60° and 120° are supplementary angles (vi) 90° and 90° are supplementary angles (v) 70° and 110° are supplementary angles

EXERCISE 4.3

1. (i) $x = 110^\circ, y = 70^\circ$ (ii) $x = 40^\circ, y = 70^\circ$ (iii) $x = 126^\circ$ (iv) $x = 17^\circ$ (v) $x = 63^\circ$ (vi) $x = 15^\circ$ **2.** (i) $x = 45^\circ, y = 63^\circ$ (ii) $x = 32^\circ, y = 103^\circ$ (iii) $x = 45^\circ$ (iv) $x = 30^\circ$ (v) $x = 112.5^\circ$ **3.** (i) $x = 44^\circ, y = 22^\circ$ (ii) $x = 35^\circ, y = 35^\circ$ (iii) $x = 70^\circ, y = 30^\circ$ (iv) $x = 46^\circ, y = 44^\circ$ (v) $x = 20^\circ, y = 70^\circ$ (vi) $x = 58^\circ, y = 33^\circ$ (vii) $x = 60^\circ, y = 30^\circ$ (viii) $x = 24^\circ, y = 128^\circ$

EXERCISE 4.4

1. (i) 900° (ii) 1260° (iii) 1980° **2.** (i) 120° (ii) 135° (iii) 156° **3.** (i) $51\frac{3}{7}^\circ$ (ii) 45° (iii) 36°

4. (i) 5 (ii) 9 (iii) 12 **5.** (i) 11 (ii) 14 (iii) 15 **6.** (i) $x = 30^\circ,\ y = 60^\circ$ (ii) $\angle s$ and $\angle t$ (iii) $s = 60^\circ,\ t = 60^\circ$ **8.** $45^\circ, 90^\circ, 135^\circ, 180^\circ, 90^\circ$ **9.** 18 **10.** $x = 58^\circ$

EXERCISE 4.5

2. (i) $51\frac{3}{7}^\circ$ (ii) 36° **3.** $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ **4.** ΔDEF has vertices: $D(0, 2)$, $E(3, -1)$ and $F(6, 3)$

REVIEW EXERCISE 4

1. (i) b (ii) a (iii) d (iv) b (v) a (vi) c (vii) c (viii) d (ix) b (x) b (xi) a (xii) c (xiii) d (xiv) c (xv) a

7. (i) $x = 62^\circ,\ y = 62^\circ$ (ii) $x = 30^\circ$ (iii) $x = 32^\circ$ **7.** (i) $x = 111^\circ, y = 96^\circ$ (ii) $x = 100^\circ, y = 61^\circ$ (iii) $x = 100^\circ,\ y = 61^\circ$ **8.** (a) $x = 104^\circ$ (b) $m\angle ADB = 55^\circ$

9. (i) 135° (ii) $22\frac{1}{2}^\circ$ (iii) 45° **10.** (i) $m\angle ABC = 120^\circ$ (ii) $m\angle CBG = 210^\circ$ (iii) $m\angle AOB = 60^\circ$

11. (i) 165° (ii) $166\frac{2}{3}^\circ$ **12.** (i) 20 (ii) 36 **13.** 30

15. (i) (a) One line symmetry (b) No rotational symmetry
(ii) (a) One line symmetry (b) No rotational symmetry
(iii) (a) No line symmetry (b) No rotational symmetry
(iv) (a) Two-line symmetries (b) Rotational symmetry of order 2
(v) (a) No line symmetry (b) Rotational symmetry of order 2

17. (i) Rotational symmetry of order 2 centre of big square.
(ii) Rotational symmetry of order 4 centre of the figure.

20. Three squares should be shaded **21.** $T = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$ **22.** (b) $T = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

EXERCISE 5.1

3. (i), (iii), are discrete data ; (ii), (iv), (v) are continuous data

4. **(i)** 5 **(ii)** 20 **(iii)** 39 **(iv)** 32 **(v)** 4

5.

Class intervals	Tally marks	Frequency	Midpoint	C.B
34 – 39	|||	3	36.5	33.5 – 39.5
40 – 45	|	1	42.5	39.5 – 45.5
46 – 51	~~||||~~ ||||	9	48.5	45.5 – 51.5
52 – 57	||||	4	54.5	51.5 – 57.5
58 – 63	~~||||~~ ||	7	60.5	57.5 – 63.5
64 – 69	||	2	66.5	63.5 – 69.5
70 – 75	|||	3	72.5	69.5 – 75.5
76 – 81	||	2	78.5	75.5 – 81.5
82 – 87	||	2	84.5	81.5 – 87.5
88 – 93	||	2	90.5	87.5 – 93.5
Total		$\Sigma f = 35$		

6.

Class intervals	Tally marks	Frequency	Midpoint	C.B
45 – 58	~~||||~~ |||	8	51.5	44.5 – 58.5
59 – 72	~~||||~~ ||	7	65.5	58.5 – 72.5
73 – 86	~~||||~~ |||	8	79.5	72.5 – 86.5
87 – 100	~~||||~~ ~~||||~~ ||	12	93.5	86.5 – 100.5
101 – 114	~~||||~~ |||	8	107.5	100.5 – 114.5
115 – 128	|	1	121.5	114.5 – 128.5
129 – 142	||||	4	135.5	128.5 – 142.5
143 – 156	||	2	149.5	142.5 – 156.5
Total		$\Sigma f = 50$		

7.

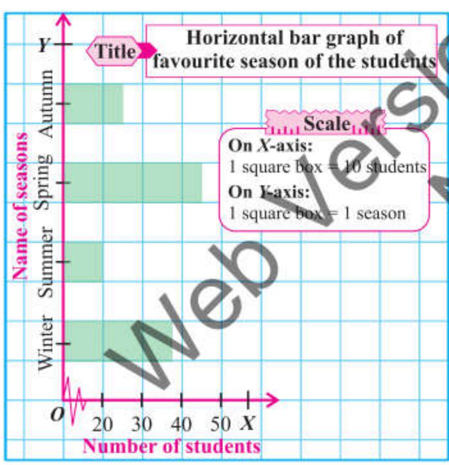
Class intervals	Tally marks	Frequency	Midpoint	C.B
59 – 66	~~||||~~ ||||	9	62.5	58.5 – 66.5
67 – 74	~~||||~~ ~~||||~~ ||	12	70.5	66.5 – 74.5
75 – 82	~~||||~~ ~~||||~~ |||	13	78.5	74.5 – 82.5
83 – 90	~~||||~~ |	6	86.5	82.5 – 90.5
91 – 98	~~||||~~	5	94.5	90.5 – 98.5
Total		$\Sigma f = 35$		

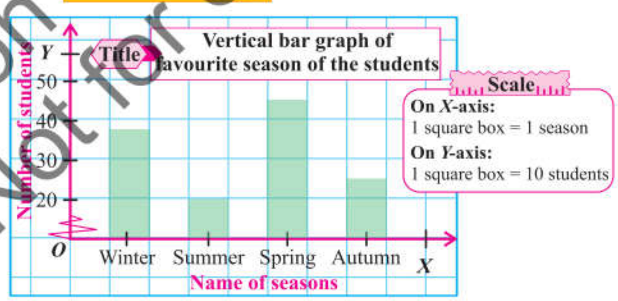
8.

Class intervals	Tally Marks	Frequency	Midpoint	C.B
119 – 128	||||	4	123.5	118.5 – 128.5
129 – 138	~~||||~~ ||	7	133.5	128.5 – 138.5
139 – 148	~~||||~~ ~~||||~~ |||	13	143.5	138.5 – 148.5
149 – 158	~~||||~~ ||||	9	153.5	148.5 – 158.5
159 – 168	~~||||~~	5	163.5	158.5 – 168.5
169 – 178	||	2	173.5	168.8 – 178.5

EXERCISE 5.2

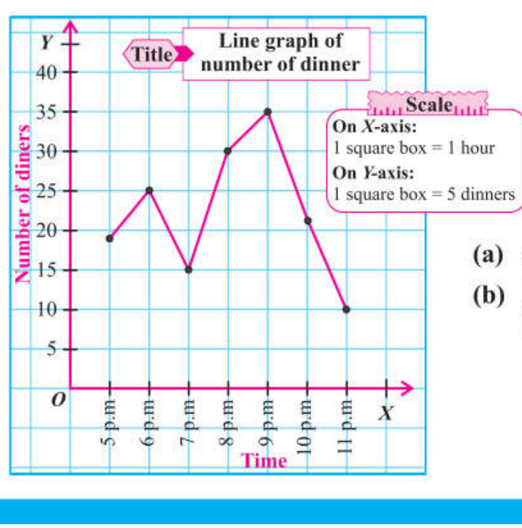
1.





(a) 128 students
(b) 20 students
(c) 13 students
(d) spring
(e) summer

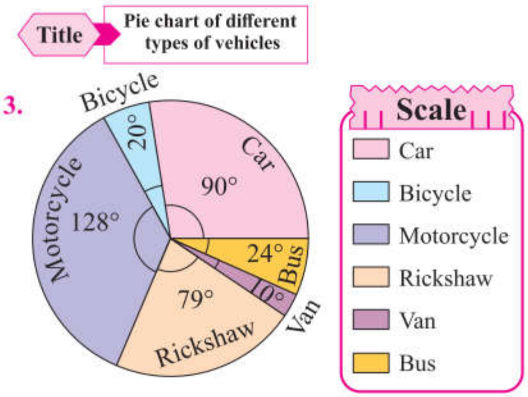
2.



(a) At 11 p.m

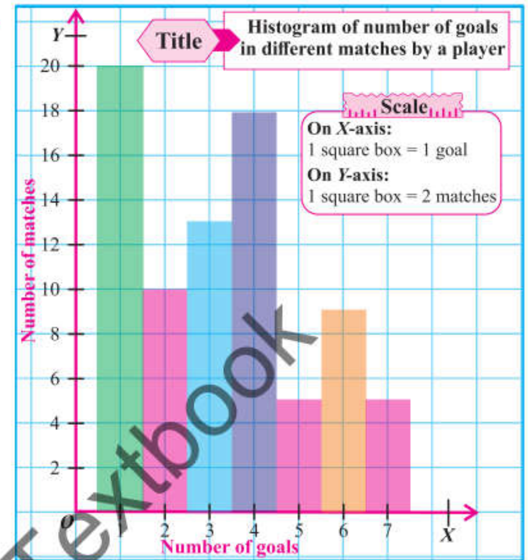
(b) Most crowded at 8 p.m
Least crowded at 11 p.m

3.

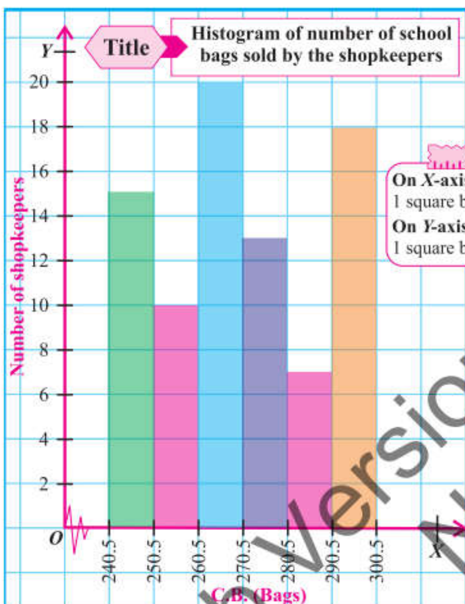


(a) 75 more than bus
(b) Motorbikes
(c) Van
(d) 365 vehicles altogether

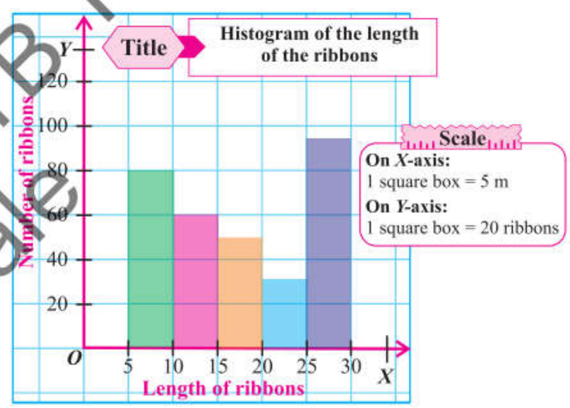
4.



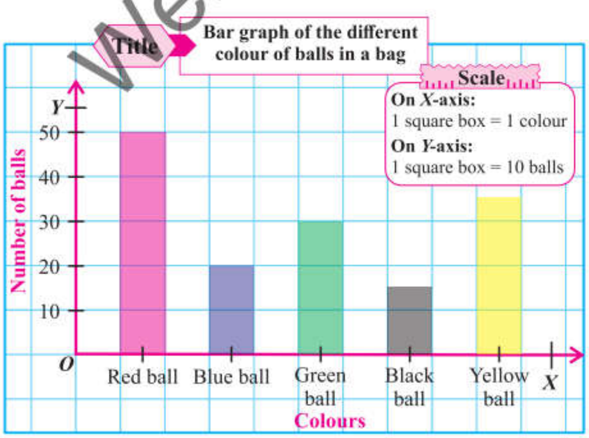
5.



6.

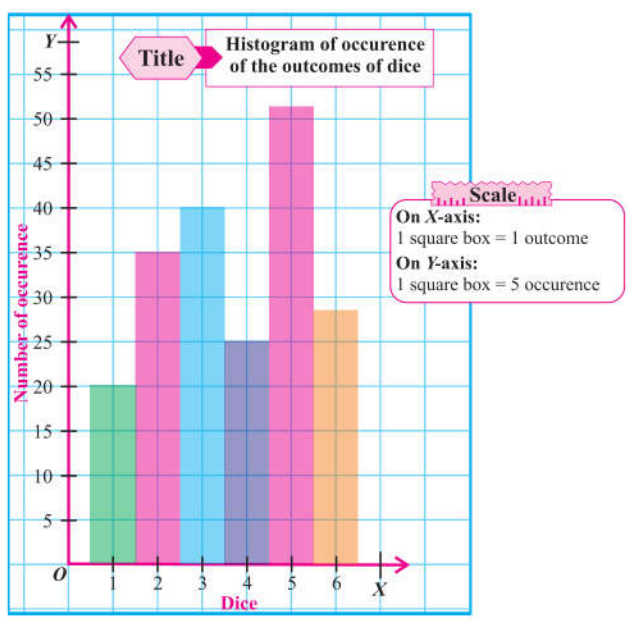


7. (i)



As, the given is qualitative, so the bar graph is a suitable graph.

7. (ii)



The data is discrete data and both the variables are quantitative. So, the discrete histogram is suitable for the given data

EXERCISE 5.3

1. (i) 41.7 (ii) 28.44 (iii) 68.2
2. (i) Mean = 22.75, Median = 21.5, Mode = 20, positive skewed
   (ii) Mean = 26.88, Median = 19, Mode = 9, positive skewed
   (iii) Mean = 67.7, Median = 73.5, Mode = 80, negative skewed
   (iv) Mean = 6, Median = 6, Mode = 6, symmetric distribution
3. (i) Mean = 8.25, (ii) Median = 6 (iii) Mode = 50 students
4. Mass of 10^{th} sack is 36.51 kg, Median = 51.125, Mode = no mode
5. The height of 15^{th} student is 5.8 ft, Median = 5.1 ft, Mode = 4.5 ft, 5.2 ft, 5.8 ft
6. Mean = 17.5
7. 16.01

EXERCISE 5.4

1.

Sr. #	$n(A)$	$n(S)$	$P(A)$	$P(A')$
(i)	6	18	$\frac{1}{3}$	$\frac{2}{3}$
(ii)	2	5	$\frac{2}{5}$	$\frac{3}{5}$
(iii)	7	19	$\frac{12}{19}$	$\frac{7}{19}$
(iv)	5	6	$\frac{5}{6}$	$\frac{1}{6}$

2. $\frac{1}{3}$, $\frac{2}{3}$
3. (i) $\frac{1}{12}$ (ii) $\frac{1}{4}$ (iii) $\frac{5}{9}$ (iv) $\frac{1}{6}$ (v) 0
4. (i) $\frac{2}{11}$ (ii) $\frac{1}{11}$ (iii) $\frac{2}{11}$ (iv) $\frac{9}{11}$ (v) $\frac{4}{11}$ (vi) $\frac{7}{11}$ (vii) 0 (viii) 1
5. (i) $\frac{3}{4}$ (ii) $\frac{3}{4}$ (iii) $\frac{1}{4}$ (iv) 0
6. (i) $\frac{15}{37}$ (ii) $\frac{2}{37}$ (iii) $\frac{29}{37}$ (iv) $\frac{25}{37}$ (v) 0 (vi) $\frac{12}{37}$
7. 0.21
8. (i) $\frac{9}{32}$ (ii) $\frac{5}{32}$ (iii) $\frac{5}{16}$ (iv) $\frac{3}{4}$ (v) 1 (vi) 0 (vii) $\frac{23}{32}$

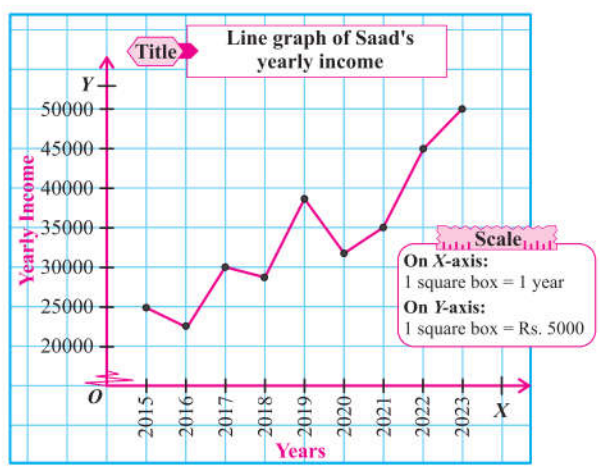
REVIEW EXERCISE 5

1. (i) *b* (ii) *c* (iii) *a* (iv) *c* (v) *a* (vi) *b* (vii) *d* (viii) *c* (ix) *b* (x) *c*

2.

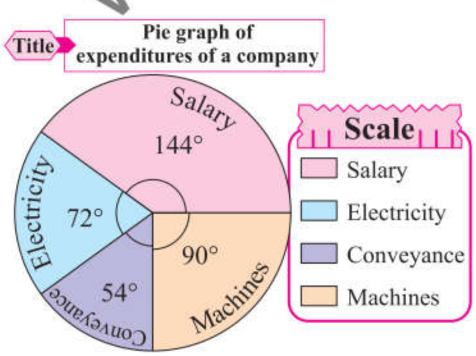
Class intervals	Tally marks	Frequency	Midpoint	C.B
40 – 51	卌 ||	7	45.5	39.5 – 51.5
52 – 63	卌 卌 卌	15	57.5	51.5 – 63.5
64 – 75	卌	5	69.5	63.5 – 75.5
76 – 87	卌 |	6	81.5	75.5 – 87.5
88 – 99	卌 卌 |	11	93.5	87.5 – 99.5
Total		$\Sigma f = 44$		

3.

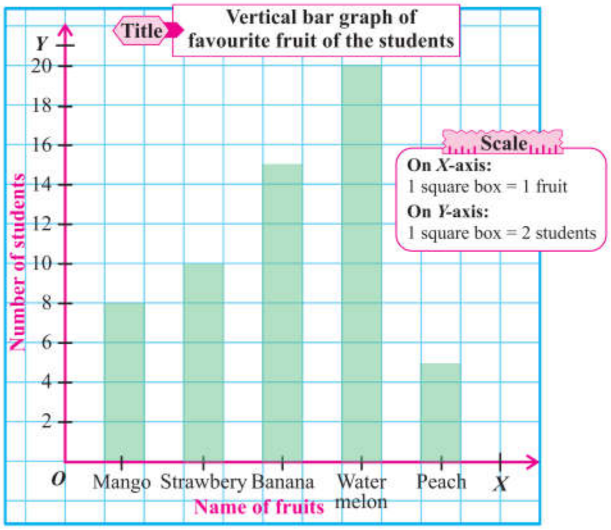


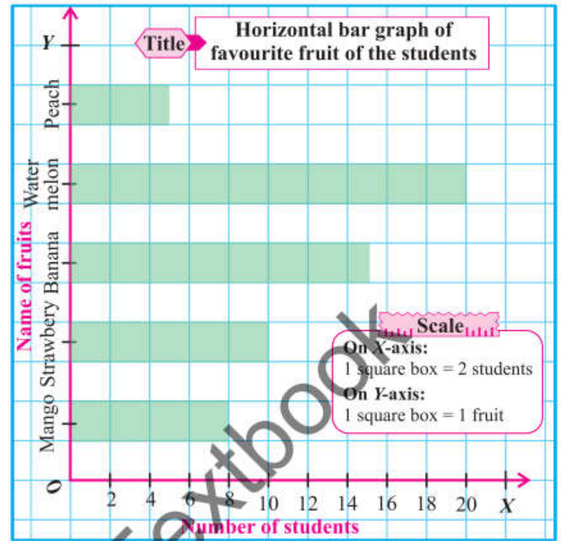
(a) Rs. 50000 **(b)** 2016 **(c)** 2023 **(d)** 306,000

4.



5.





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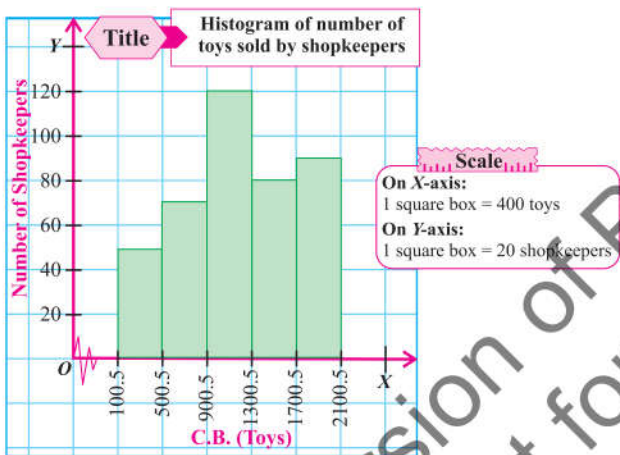

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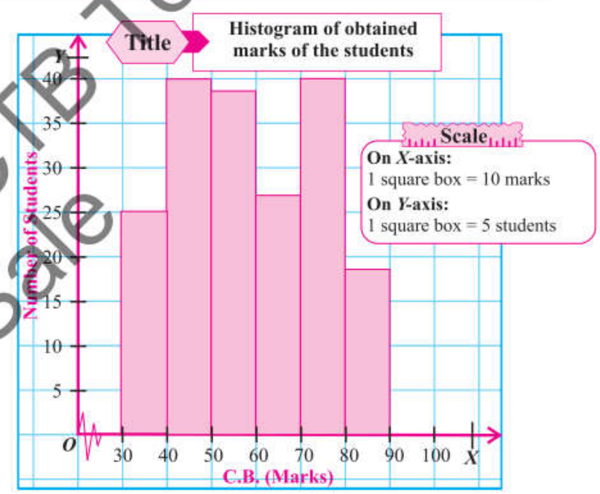

8. (i) Mean = 66.7, Median = 68.5, Mode = 57 (ii) Mean = 95.15, Median = 98.6, Mode = 100.25

9. $x = 9$ **10.** $\bar{x} = 198.2$ **11.** (i) 79.60 (ii) 20.21

12.

Sr. #	$n(A)$	$n(S)$	$P(A)$	$P(A')$
(i)	11	17	$\frac{11}{17}$	$\frac{6}{17}$
(ii)	1	21	$\frac{1}{21}$	$\frac{20}{21}$
(iii)	1	3	$\frac{1}{3}$	$\frac{2}{3}$
(iv)	21	36	$\frac{7}{12}$	$\frac{5}{12}$

13. (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{9}$ (iv) $\frac{3}{4}$

14. (i) $\frac{3}{10}$ (ii) $\frac{7}{10}$ (iii) $\frac{3}{10}$ (iv) $\frac{7}{10}$ (v) $\frac{1}{10}$ (vi) 1 (vii) 0

GLOSSARY

Integers: The positive integers and negative integers along with zero (0) are called integers. i.e.,
$Z = \{0, \pm1, \pm2, \pm3, ...\}$

Rational number: A number that can be expressed in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called a rational number. The set of rational number is denoted by Q.

LCM: The smallest common multiple of two or more than two numbers is called least common multiple (LCM).

Error: The difference between the actual value and approximated/ estimated value is called an error.

Simplification: The procedure to simplify a mathematical expression is known as simplification.

Set: A set is a well-defined and distinct collection of objects/elements.

Subset: If all the elements of a set A are also the elements of a set B, then the set A is called the subset of the set B.

Super set: If a set A contains all the elements of a set B, then the set A is a super set of the set B.

Proper subset: A set A is a proper subset of a set B if all the elements of the set A contain in the set B but at least one element of the set B is not an element of set A.

Improper subsets: If set A is a subset of set B and set B is a subset of set A, then A and B are improper subsets of each other.

Equivalent sets: Two sets A and B are said to be equivalent if they have the same number of elements.

Equal sets: If sets A and B contain the same elements, then the sets A and B are equal.

Disjoint sets: Two or more sets are disjoint if they do not have common elements.

Overlapping sets: Two or more sets are overlapping sets if they have at least one common element.

Laws of complement: (i) $A \cup A^C = U$ (ii) $A \cap A^C = \phi$ (iii) $(A \cup B)^C = A^C \cap B^C$ (iv) $(A \cap B)^C = A^C \cup B^C$

Rate: The rate is a ratio of two quantities having different units.

Proportion: An equality of two ratios is called a proportion.

Direct proportion: Two quantities are said to be in direct proportion if increase or decrease in one quantity causes the increase or decrease in the other quantity in the same ratio.

Inverse proportion: Two quantities are said to be indirect proportion if the increase or decrease in one quantity causes the decrease or increase in the other quantity in the same ratio.

Cost price: The price at which an item is bought is called its cost price.

Selling price: The price at which an item is sold is called its selling price.

Discount: The reduction or cut offered on the marked price is called the discount.

Income tax: Income tax is the tax imposed by the government on the income of individuals exceeding a certain amount.

Property Tax: Property tax is a tax imposed by government on the properties such as house, land and shops.

Sales Tax: When a customer purchases an item, he pays an extra amount in addition to the original price of the item. This extra amount is called general sales tax.

Zakat: The rate of zakat is 2.5% of the total wealth.

Ushr: The rate of ushr is 5% and 10% depending on the land irrigated by natural sources or artificial resources respectively.

Term rule: The term-to-term rule is used for a sequence in which the next term is obtained from the previous terms.

General term: n^{th} term of the sequence is also known as general term of the sequence and written as a_n.

Open sentence: A sentence which has one or more unknown is called open sentence.

Algebraic expression: An expression which connects variables and constants by mathematical operations (+, –, ×, ÷) is called an algebraic expression.

Variable: A term which can take various numerical values is called variable.

Constant: A term which cannot be changed and has a fixed value is called constant.

An Equation: An equation is a statement which shows the equal value of both the expressions.

An inequality: An inequality is a statement which shows that an expression is less than or greater than the other.

Polynomial: A polynomial is an algebraic expression comprising of whole number as exponent of the variable. Polynomial consists of variables, coefficient and constant.

Types of polynomials: There are three types of polynomials i.e. monomials, binomials and trinomials.

Linear equation: A linear equation is an equation in which the highest power of the variables is always 1.

Origin: The point of intersection of the horizontal and vertical number lines is called the origin.

Quadrants: The regions that divide the cartesian plane into four equal parts are called the quadrants.

Speed: Speed is the rate of change of distance per unit time.

Perimeter: Perimeter is the measurement of boundary of closed plain figure.
Area: Area is the measurement of interior region of a closed shape.
Surface area: Surface area is the sum of outer surfaces of a 3D shape.
Perpendicular: The shortest distance from a point to a line is always perpendicular.
Transversal: When a line intersects two parallel lines is called transversal.
Polygon: Any closed 2D figure with three or more sides is called a polygon.
Convex polygon: A polygon in which all the diagonals lie inside the polygon is called convex polygon and if atleast one diagonal lie outside the polygon is called concave polygon.
Reflective symmetry: In reflective symmetry, half of the shape is the mirror image of the other half.
Line of symmetry: The line which divides the shape into two equal halves is called line of symmetry.
Rotational symmetry: When a shape is rotated about its centre through an angle and the shape attains its original position after rotating, this type of symmetry is known as rotational symmetry.
Order of rotational symmetry: Order of rotational symmetry is the number of times, the shape remains same between 0° and 360°.
Transformation: Transformation is the process of moving a geometric figure from one position to another.
Tipes of Transformation: Reflection, rotation and translation are types of transformation.
Statistics: Statistics is the collection of data and then organizing, presenting, analyzing describing and drawing conclusion based on data.
Frequency distribution / table: A distribution / table that represents classes along with their respective class frequencies is called frequency distribution / table.
Frequency: Frequency is the number of times of a repeated observation or value occurs in any data.
Mean: Mean is preferred over median and mode because all values in the data are used in it.
Median: Median is preferred because it is not affected by the extreme values of the data.
Mode: Mode is used where the most common or model value is required.
Probability: Probability is the chance of occurence of an event.
Sample space: The set of all possible outcomes of an experiment is called sample space.
Event: The set of outcomes of an experiment is called an event.
Certain event: An event which is sure to occur in any given experiment is called a certain event.
Impossible event: When an event cannot occur in any given experiment, it is called an impossible event.
Likely event: An event is called likely event which will probably occur.
Unlikely event: An event is called unlikely event which will not probably occur.
Equally likely event: The events are called equally likely events which have equal chance of occurrence.

Symbols / Notations

Symbol	Stands for	Symbol	Stands for	Symbol	Stands for
$<$	is less than	:	ratio	$\parallel$	is parallel to
$>$	is greater than	: :	is proportional to	$\widehat{AB}$	arc AB
$\leq$	is less than or equal to	I	tally mark	$\leftrightarrow$	correspondence
$\geq$	is greater than or equal to	$\sum$	summation	%	percentage
$=$	is equal to	$\overline{AB}$	line segment AB	ϕ or { }	the empty set / the null set
$\neq$	is not equal to	$\overrightarrow{AB}$	ray AB	$\mid$	such that
$\nless$	is not less than	$\overleftrightarrow{AB}$	line	$\Rightarrow$	implies that
$\ngtr$	is not greater than	$\angle$	angle	$\cup$	union
$\in$	belongs to	Δ	triangle	$\cap$	intersection
$\notin$	not belongs to	$\sim$	is similar to	$\because$	because / as
$\forall$	for all	$\cong$	is congruent to	$\therefore$	therefore / so
$\sqrt{\ }$	square root	$\approx$	is approximately equal to	U	universal set

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