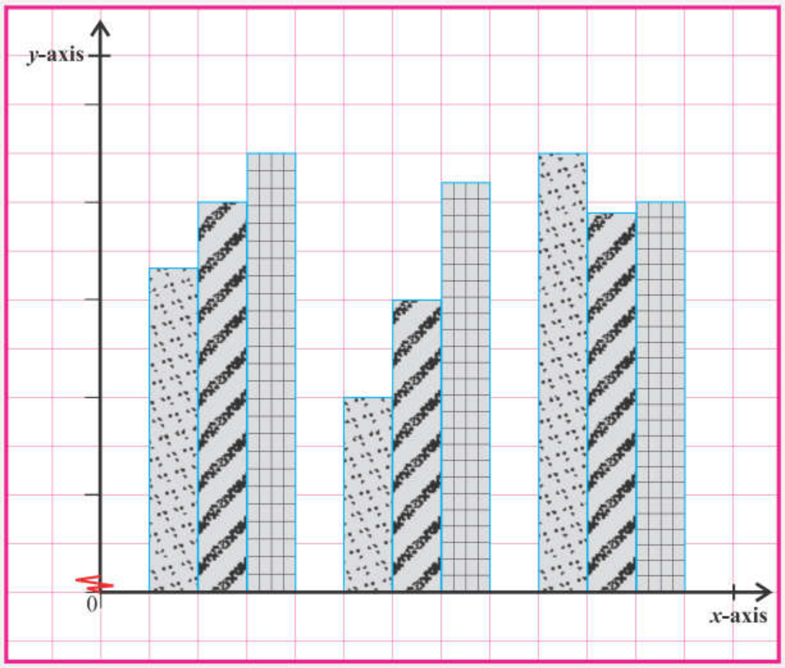
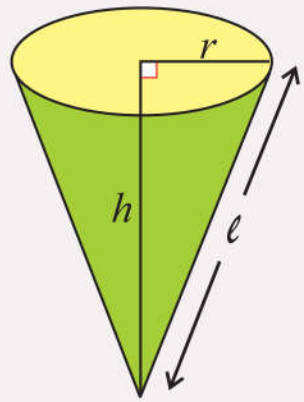
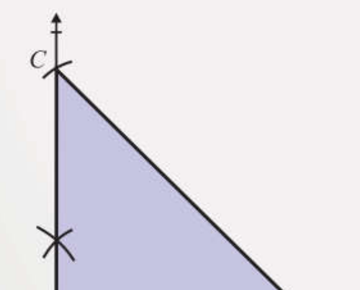
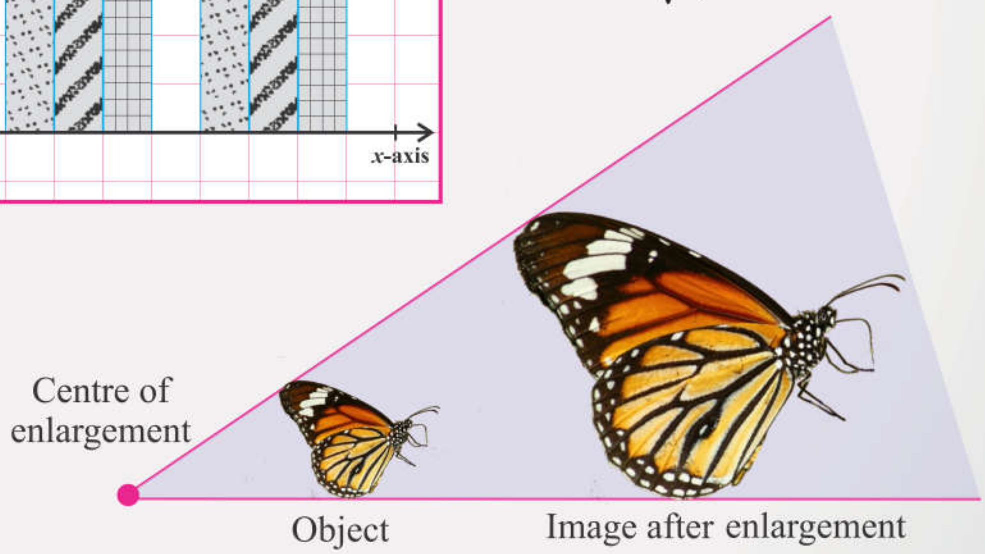
MATHEMATICS 8

Based on Single National Curriculum 2022









Punjab Curriculum and Textbook Board, Lahore

بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

(In the Name of Allah, the Most Compassionate, the Most Merciful)

MATHEMATICS

8

Based on Single National Curriculum 2022

ONE NATION, ONE CURRICULUM

PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

This textbook is based on Single National Curriculum 2022 and has been approved by the Board.

CONTENTS

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Experimental Edition

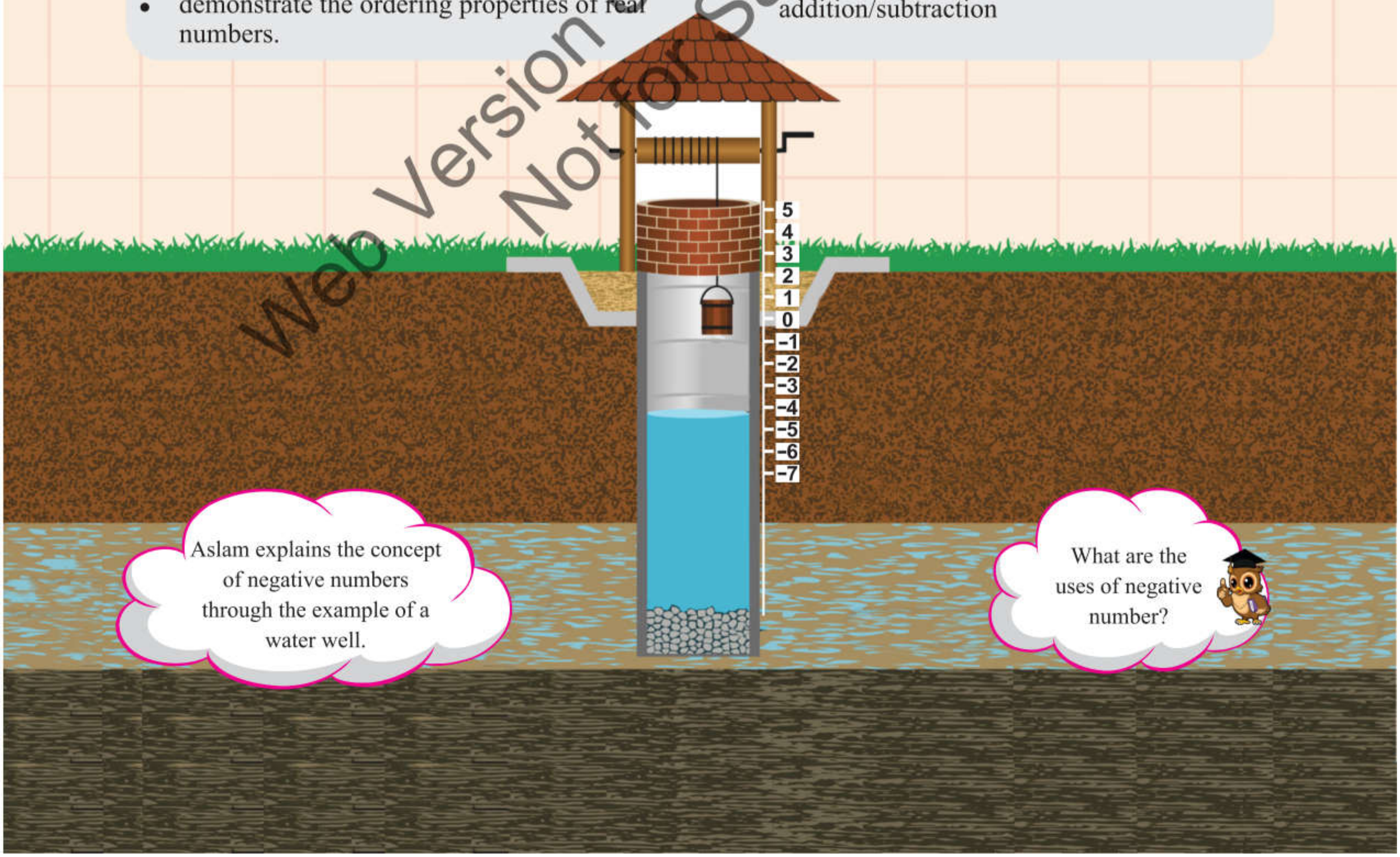
Domain 1 Numbers and Operations

Sub-Domain (i): Real Numbers

Students' Learning Outcomes

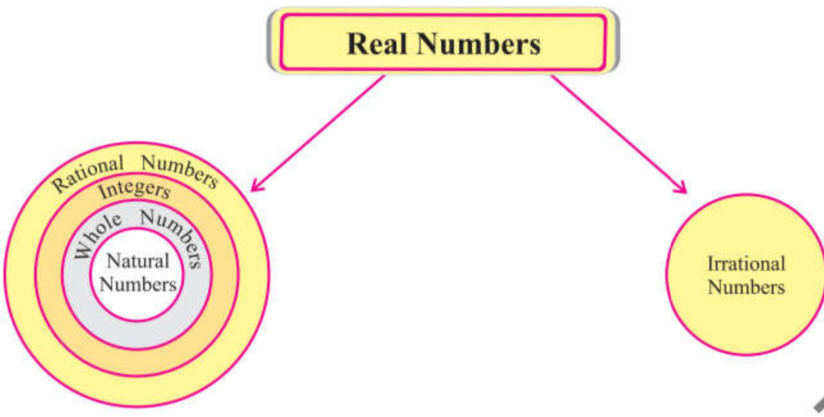
After completing this sub-domain, the students will be able to:

- differentiate rational and irrational numbers
- represent real numbers on a number line
- demonstrate decimal numbers as terminating, non-terminating, recurring and non-recurring
- solve real life situations/word problems involving calculation with decimals and fractions
- identify the absolute value of a real number
- demonstrate the ordering properties of real numbers.
- demonstrate the properties of real numbers and their subsets with respect to addition and multiplication:
  - ❖ closure property
  - ❖ associative property
  - ❖ existence of identity element
  - ❖ existence of inverses
  - ❖ commutative property
- distributive property of multiplication over addition/subtraction



1.1.1 Real Numbers ($\mathbb{R}$)

Numbers are the roots of Mathematics. We use these numbers in our daily life. Before discussing real numbers, we discuss some other numbers.



Need to know!

All the numbers (N, W, Z, all rational and irrational numbers) are real numbers. The set of real numbers is represented by $\mathbb{R}$.

(i) Natural Numbers (N)

Natural numbers start from digit 1 and the set of natural numbers is represented by N. i.e.,

$N = \{1, 2, 3, 4, \ldots\}$

(ii) Whole Numbers (W)

If we include zero (0) in natural numbers, then these numbers become whole numbers and the set of whole numbers is represented by W. i.e., $W = \{0, 1, 2, 3, \ldots\}$

(iii) Integers (Z)

If we include negative numbers in whole numbers, then these numbers become integers and the set of integers is represented by Z. i.e., $Z = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\}$

(iv) Rational Numbers (Q)

The numbers which can be written in the form of $\frac{p}{q}$, where $p, q \in Z$, and $q \neq 0$, are called rational numbers. For example, $\frac{7}{9}, \frac{-5}{3}, \frac{0}{11}, 28, \frac{12}{17}, \frac{6}{23}, \frac{18}{26}, 15$ etc. are all rational numbers. The set of rational numbers is represented by Q. Rational numbers can also be represented in decimal form which has two types.

Key fact!

Every rational number can also be written into decimal form.

(a) Terminating Decimal Numbers

Terminating decimal numbers are the decimals which have finite number of digits in its decimal part. For example: 0.4, 3.14, 0.375 etc. All terminating decimal numbers can also be written in fraction. For example: $0.4 = \frac{2}{5}$, $3.14 = \frac{157}{50}$, and $0.375 = \frac{3}{8}$ etc.

(b) Non-Terminating and Recurring Decimal Numbers

The decimal numbers in which one digit or group of digits repeat again and again infinite times in its decimal part are called recurring decimal numbers. It can also be written in fraction. For example:

$\frac{2}{9} = 0.22222, \ldots$; $\frac{4}{11} = 0.363636, \ldots$; $\frac{43}{99} = 0.434343, \ldots$; $\frac{156}{9} = 17.33333, \ldots$; $\frac{22}{7} = 3.142857142857, \ldots$

(v) Irrational Numbers (Q′)

The numbers which are not rational are called irrational numbers. The set of irrational numbers is represented by Q′.

For example: $\sqrt{2} = 1.41421356, ...$; $\pi = 3.141592654, ...$;

All the non-terminating and non-recurring decimal numbers are irrational numbers.

Important Note!

Value of π and $\frac{22}{7}$ are not equal. $\frac{22}{7}$ is a rational number but value of π and $\frac{22}{7}$ are very very close but we cannot write them equal. It can be written as $\pi \approx \frac{22}{7}$.

Real Numbers
Real numbers are the union of rational and irrational numbers.

- **Rational Numbers**
  - Terminating Decimal Numbers: 0.45, 7.215
  - Non-Terminating and Recurring Decimal Numbers: 0.8181…, 0.373737…
- **Irrational Numbers**
  - Non-Terminating and Non-Recurring Decimal Numbers: 5.23456789… , 0.0289537…

Key facts!

1. Every integer is a rational number.
2. All the rational numbers can be written in decimal fraction but all decimal fractions are not rational numbers.

Example 1: Separate rational and irrational numbers.

(i) $\sqrt{17}$ (ii) 8.04 (iii) π (iv) $\frac{22}{7}$

(v) $\sqrt{2}$ (vi) 63 (vii) 0.33333… (viii) 0

Solution:

(i) Since 17 is not perfect square and $\sqrt{17}$ is non-terminating non-recurring, it is irrational.

(ii) Since 8.04 is terminating decimal number. So, it is a rational number.

(iii) Since the value of π is non-terminating and non-recurring, it is an irrational number.

(iv) $\frac{22}{7}$ is in the form of rational number, it is a rational number.

(v) $\sqrt{2}$ is an irrational number.

(vi) 63 can be written as $\frac{63}{1}$ which is in $\frac{p}{q}$ form so, it is a rational number.

(vii) 0.3333… is recurring decimal number so, it can be written as $\frac{1}{3}$. Therefore, it is a rational number.

(viii) 0 can be written as $\frac{0}{1}$, so it is a rational number.

1.1.2 Representation of Real Numbers on Number Line

To represent the real number $\frac{\ell}{m}$ on the number line, we divide each unit into 'm' equal parts.

For positive number e.g., $\frac{\ell}{m}$, the "ℓth" point of division is represented to the right of the origin.

For negative number e.g., $-\frac{\ell}{m}$, the "ℓth" point of division is represented to the left side of the origin.

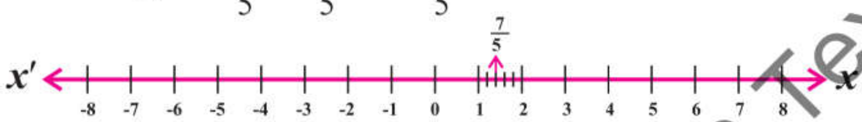
Example 2: Represent the following numbers on the number line.

(i) $\frac{7}{5}$ (ii) $-\frac{1}{3}$

Keep in mind!

A real number line always goes on both directions from zero(0) i.e., left and right directions.

Solution: (i) $\frac{7}{5} = 1\frac{2}{5} = 1 + \frac{2}{5}$



Keep in mind!

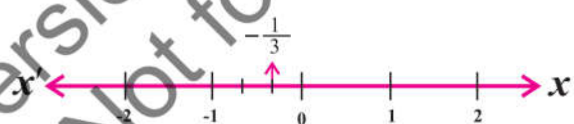
$\frac{7}{5}$ lies between 1 and 2.

To represent the rational number $\frac{7}{5}$ on the number line, divide the distance between 1 and 2 into five equal parts. Take two parts to the right from 1.

(ii) $-\frac{1}{3}$

To represent the rational number $-\frac{1}{3}$ on the number line, divide the distance between 0 and -1 into three equal parts.

Take 1 part to the left from 0



1.1.3 Absolute Value of Real Number

The absolute value (or modulus) of a real number x (represented by $|x|$) is the non-negative value of x. For example, the absolute value of 4 is 4 and absolute value of -4 is also 4. We can write it as:

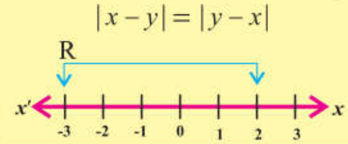
$$|4| = 4$$

$$\text{or } |-4| = 4$$

The absolute value of a real number may be thought of its distance from zero along real number line. Furthermore, the absolute value of difference of two real numbers is the distance between them. For example, the absolute difference between -3 and 2 is:

Showing the absolute difference between two real numbers x and y.

$$|x - y| = |y - x|$$



$$|-3 - 2| = |-5| = 5$$

$$\text{or } |2-(-3)| = |2 + 3| = |5| = 5$$

So, -3 and 2 are at a distance of 5 units apart.

Try yourself!

The temperature of city "A" is 33°C and city "B" is 40°C. Can you find out the absolute difference between them?

1.1.4 Real Life Situations Involving Fractions and Decimals

Example 3: How many pieces of $\frac{1}{4}$ metre of ribbon can be cut from a ribbon which is $\frac{15}{2}$ metres long?

Solution: The length of the ribbon $= \frac{15}{2}$ metres

The required length of each ribbon $= \frac{1}{4}$ metre

To find out the number of pieces of ribbon of required length, we will have to divide $\frac{15}{2}$ by $\frac{1}{4}$.

$$= \frac{15}{2} \div \frac{1}{4}$$

Number of pieces of ribbon $= \frac{15}{2} \times 4 = \frac{60}{2} = 30$

Thus, the total length of ribbon can be cut into 30 equal parts of length $\frac{1}{4}$ metre.

Example 4: Azhar ran 325.650 *km* on a jogging track in the first month and 410.350 *km* in the second month. Find the total distance he covered in both the months. Also tell how many kilometres did he cover per day in the first month?

Solution: To find the total distance, we will have to add both the distances.

$325.650 + 410.350 = 736$ *km*

Per day distance (*km*) covered in the first month $= \frac{325.650}{30} = 10.855\ km$

So, Azhar covered total 736 km in both months

and 10.855 kilometres covered per day in the first month.

Exercise 1.1

1. Identify which of the following fractions are terminating and which are recurring decimal fractions?

(i) $\frac{4}{6}$ (ii) $\frac{11}{12}$ (iii) $\frac{1}{3}$ (iv) $\frac{2}{5}$ (v) $\frac{8}{11}$ (vi) $\frac{5}{12}$

(vii) $\frac{5}{7}$ (viii) $\frac{6}{10}$ (ix) $\frac{16}{20}$ (x) $\frac{9}{10}$ (xi) $\frac{1}{8}$ (xii) $\frac{14}{20}$

(xiii) $\frac{2}{3}$ (xiv) $\frac{4}{5}$ (xv) $\frac{1}{4}$ (xvi) $\frac{7}{9}$ (xvii) $\frac{8}{10}$ (xviii) $\frac{3}{5}$

2. Separate rational and irrational numbers from the following:

(i) $\sqrt{5}$ (ii) $\frac{1}{7}$ (iii) 2.152 (iv) 0.818281828182….. (v) 2.751

(vi) $\frac{22}{7}$ (vii) $\sqrt{14}$ (viii) $\sqrt{25}$

3. Represent the following numbers on the number line.

(i) $\frac{21}{4}$ (ii) $-\frac{53}{3}$ (iii) $\sqrt{11}$ (iv) $\frac{25}{2}$ (v) $-\frac{5}{7}$ (vi) $\sqrt{31}$

4. Find the absolute value of each of the following:

(i) 15 (ii) -21 (iii) -36 (iv) -9 (v) -100 (vi) 88

5. Fine the absolute difference between each of the following:

(i) -7 and -2 (ii) 8 and -3 (iii) -5 and 1 (iv) 2 and 3

6. $50\frac{3}{4}$ kilograms of rice is to be packed equally in $5\frac{1}{2}$ kilograms packets. Find:

(a) how many packets of rice will be packed?

(b) how much rice will be there in 8 packets of mass $5\frac{1}{2}$ kilograms?

7. To decorate a birthday party, Zara used ribbons of two colours. The length of blue ribbon is $15\frac{2}{3}$ metres. The length of green ribbon is $\frac{4}{5}$ times of the length of blue ribbon.

(a) What is the length of green ribbon?

(b) Find the total length of both the ribbons.

8. If 230.75 kilograms of jaggery powder is to be packed equally in the packets, each packet contains 2.5 kg of jaggery powder. Find the number of required packets. Also find how much jaggery powder will be there in 15 packets of mass 2.5 kg.

9. Zeeshan wants to fill 137.5 litres of petrol in bottles. Find the number of bottles if:

(a) The capacity of each bottle is 5.5 litres.

(b) The capacity of each bottle is 6.25 litres.

(c) Also tell how much petrol will be there in 18 bottles of capacity 6.25 litres each.

1.1.5 Properties of Real numbers

The following properties are held in real numbers:

Properties	Examples
(i) Closure Property w.r.t Addition ◎ If we take any two real numbers, then their sum is also a real number. It is called closure property w.r.t addition. $\forall\ a, b \in \mathbb{R} \quad \Rightarrow \quad a + b \in \mathbb{R}$	◎ -5 and $11 \in \mathbb{R}$, $-5 + 11 = 6 \in \mathbb{R}$
(ii) Closure Property w.r.t Multiplication ◎ If we take any two real numbers, then their product is also a real number. It is called closure property w.r.t multiplication. $\forall\ a, b \in \mathbb{R} \quad \Rightarrow \quad a \,.\, b \in \mathbb{R}$	◎ -5 and $11 \in \mathbb{R}$, $(-5) \times (11) = -55 \in \mathbb{R}$

Properties	Examples
(iii) Associative Property w.r.t Addition ◎ If a , b and c are any three real numbers, then $(a+b)+c=a+(b+c)$	◎ $(7+4)+5=7+(4+5)$
(iv) Associative Property w.r.t Multiplication ◎ If a , b and c are any three real numbers, then $(a\cdot b)\cdot c = a\cdot(b\cdot c)$	◎ $(7\cdot 4)\cdot 5 = 7\cdot(4\cdot 5)$
(v) Additive Identity Property ◎ If a is real number, there is a unique number zero (0) called additive identity such that: $a+0=0+a=a$	◎ $2+0=0+2$
(vi) Multiplicative Identity Property ◎ For any real number a, there is a unique number one (1) called multiplicative identity such that: $a\cdot 1=1\cdot a=a$	◎ $2\cdot 1=1\cdot 2$
(vii) Additive Inverse Property ◎ For any real number a, there exists a unique real number $-a$ called additive inverse of a such that: $a+(-a)=0=(-a)+a$	◎ Additive inverse of 8 is -8 Since $8+(-8)=0=-8+8$
(viii) Multiplicative Inverse Property ◎ If $a\neq 0$ is a real number, there exists a unique real number $\frac{1}{a}$ called multiplicative inverse of a such that: $a\cdot\frac{1}{a}=1=\frac{1}{a}\cdot a$ **Remember!** $0\in\mathbb{R}$ has no multiplicative inverse	◎ 7 and $\frac{1}{7}$ are multiplicative inverse of each other i.e., $7\cdot\frac{1}{7}=1=\frac{1}{7}\cdot 7$ or $\frac{3}{5}\cdot\frac{5}{3}=1=\frac{5}{3}\cdot\frac{3}{5}$ $\frac{3}{5}$ and $\frac{5}{3}$ are multiplicative inverse of each other .
(ix) Commutative Property w.r.t Addition ◎ If a and b are any two real numbers, then $a+b=b+a$	◎ $13+7=7+13$; $\sqrt{3}+\sqrt{5}=\sqrt{5}+\sqrt{3}$
(x) Commutative Property w.r.t Multiplication ◎ If a and b are any two real numbers, then $a\cdot b=b\cdot a$	◎ $13\times 7=7\times 13$; $\sqrt{3}\times\sqrt{5}=\sqrt{5}\times\sqrt{3}$

Properties	Examples
(xi) Distributive Property of Multiplication over addition ◎ If a, b and c are real numbers, then $a(b+c) = a\cdot b + a\cdot c$ or $(a+b)c = a\cdot c + b\cdot c$	◎ If $a = 2, b = 3, c = 4$ then $2(3+4) = 2\cdot 3 + 2\cdot 4$ $14 = 14$ or $(2+3)\cdot 4 = 2\cdot 4 + 3\cdot 4$ $20 = 20$

1.1.6 Order (Inequalities) of Real Numbers

Properties	Examples
(i) Trichotomy Property $\forall\ a, b \in \mathbb{R}$ Exactly one of the following holds: ◎ $a < b$ or $a > b$ or $a = b$	**Keep in mind!** $\forall$ stands for "for all" ◎ $7 < 10$, but $7 \not> 10$ and $7 \neq 10$
(ii) Transitive Property $\forall\ a, b, c \in \mathbb{R}$ ◎ If $a > b$ and $b > c$, then $a > c$ or ◎ If $a < b$ and $b < c$, then $a < c$	◎ $8 > 5$ and $5 > 4 \Rightarrow 8 > 4$ or ◎ $2 < 5$ and $5 < 6 \Rightarrow 2 < 6$
(iii) Addition Property $\forall\ a, b, c \in \mathbb{R}$ ◎ If $a < b$, then $a + c < b + c$ or ◎ If $a > b$, then $a + c > b + c$	◎ $5 < 7$ and $c = 2 \Rightarrow 5 + 2 < 7 + 2$ i.e., $7 < 9$ or ◎ $10 > 8$ and $c = 2 \Rightarrow 10 + 2 > 8 + 2$ i.e., $12 > 10$
(iv) Subtraction Property $\forall\ a, b, c \in \mathbb{R}$ ◎ If $a < b$, then $a - c < b - c$ or ◎ If $a > b$, then $a - c > b - c$	◎ $10 < 11$ and $c = 2 \Rightarrow 10 - 2 < 11 - 2$ i.e., $8 < 9$ or ◎ $13 > 9$ and $c = 2 \Rightarrow 13 - 2 > 9 - 2$ i.e., $11 > 7$
(v) Multiplication Property ◎ If $a < b$ and $c > 0$, then $a\cdot c < b\cdot c$ or ◎ If $a < b$ and $c < 0$, then $a\cdot c > b\cdot c$	◎ $5 < 8$ and $c = 2 \Rightarrow 5\cdot 2 < 8\cdot 2$ i.e., $10 < 16$ or ◎ $5 < 8$ and $c = -2 \Rightarrow 5(-2) > 8(-2)$ i.e., $-10 > -16$

Try yourself!

Write the multiplicative property for a > b

Remember!

If we multiply or divide an inequality by a negative number, then the inequality sign is always changed.

Properties	Examples
(vi) Division Property	
If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$	$6 < 9$ and $c = 3 \Rightarrow \frac{6}{3} < \frac{9}{3}$ i.e., $2 < 3$
or	or
If $a < b$, and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$	$6 < 9$ and $c = -3 \Rightarrow \frac{6}{-3} > \frac{9}{-3}$ i.e., $-2 > -3$
Try yourself! Write the division property for a > b.	
(vii) If a and b both are positive (+) or both are negative (–), then $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$ or $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$	$2 < 4 \Rightarrow \frac{1}{2} > \frac{1}{4}$ or $-3 < -2 \Rightarrow -\frac{1}{3} > -\frac{1}{2}$

Exercise 1.2

1. Identify which property of real numbers is used in the following:

(i) $11 + 0 = 11$

(ii) $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

(iii) $12 + (-12) = 0$

(iv) $14 + 20 = 20 + 14$

(v) $\sqrt{5} \cdot \sqrt{5} \in \mathbb{R}$

(vi) $2 \times \frac{1}{2} = 1$

(vii) $22 \times 1 = 22$

(viii) $\left(-\frac{6}{10}\right) \cdot \left(-\frac{10}{6}\right) = 1$

(ix) $(19 + 12) + 8 = 19 + (12 + 8)$

(x) $-3 + (23 + 48) = (-3 + 23) + 48$

(xi) $89 \times 1 = 89$

(xii) $12 \times 46 = 46 \times 12$

(xiii) $\sqrt{27} + 0 = \sqrt{27}$

(xiv) $xy = yx$

(xv) $\pi + (-\pi) = 0$

2. Identify which order property is used in the following:

(i) 15 (ii) -27 (iii) $\frac{7}{9}$ (iv) $-\frac{9}{16}$ (v) $-\frac{20}{23}$

(vi) 1 (vii) $-\frac{2}{3}$ (viii) $\frac{7}{2}$ (ix) -7 (x) 12

3. Identify which order property is used in the following:

(i) $x < y$ or $x > y$ or $x = y$ (ii) $a > b \Rightarrow \frac{a}{c} < \frac{b}{c}$, $c < 0$

(iii) $a > b \Rightarrow a - c > b - c$ (iv) $5 > 2 \Rightarrow 5 \cdot c < 2 \cdot c$, $c < 0$

(v) $7 > z$ and $z > 5 \Rightarrow 7 > 5$ (vi) $10 < 15 \Rightarrow 10 \cdot c > 15 \cdot c$, $c < 0$

(vii) $x < y \Rightarrow x - w < y - w$ (viii) $x < z \Rightarrow \frac{x}{\ell} < \frac{z}{\ell}$, $\ell > 0$

SUMMARY

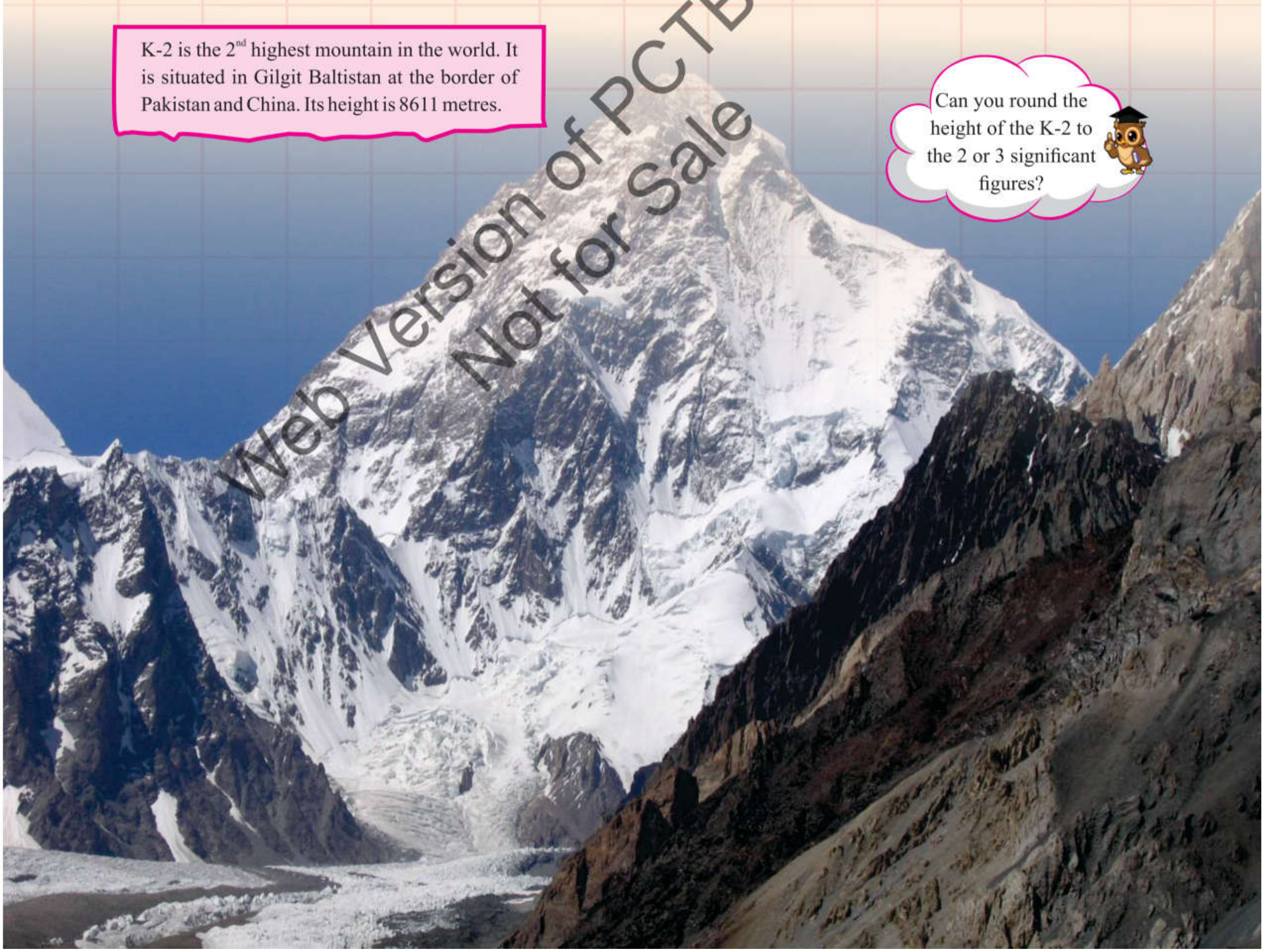
- The numbers which can be written in the form of $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$ are called rational numbers.
- Decimal representations of rational numbers are of two types:
  (i) Terminating decimals (ii) Recurring decimals
- Terminating decimal numbers are the decimals which have finite number of digits in its decimal part.
- The decimal numbers in which one digit or group of digits repeat again and again infinite times in its decimal part are called recurring decimal numbers.
- The numbers which are not rational are called irrational numbers. The set of irrational numbers is represented by Q'.
- All the numbers (N, W, Z, all rational and irrational numbers) are real numbers. The set of real numbers is represented by $\mathbb{R}$.
- The absolute value (or modulus) of a real number x (represented by $|x|$) is the non-negative value of x without keeping in view its sign.
- If we take any two real numbers, then their sum and product is also a real number. It is called closure property.
- If a is a real number, there is a unique number zero (0) called additive identity such that
  $$a + 0 = 0 + a = a$$
- 1 is called multiplicative identity, such that $a \cdot 1 = a = 1 \cdot a$
- The real numbers hold the following properties:
  (i) Closure property (ii) Associative property
  (iii) Additive identity property (iv) Multiliplicative identity property
  (v) Additive inverse property (vi) Multiplicative inverse property
  (vii) Commutative property (viii) Distributive property

Sub-Domain (ii): Estimation and Approximation

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- round whole numbers, integers, rational numbers and decimal numbers to a required degree of accuracy
- analyze approximation error when numbers or quantities are rounded
- solve real life situations involving approximation



K-2 is the 2nd highest mountain in the world. It is situated in Gilgit Baltistan at the border of Pakistan and China. Its height is 8611 metres.

Can you round the height of the K-2 to the 2 or 3 significant figures?

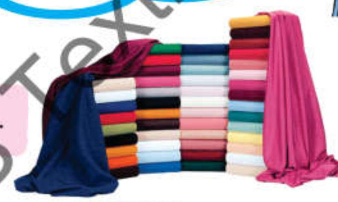
1.2.1 Round Whole Numbers

Sometimes, in our daily life, we are unable to find out the actual value of any item, then we have to estimate/round the values to the required degree of accuracy. For example, if the cost of a biscuit packet is Rs. 19.75, in such situation, we are unable to pay amount in decimal form. That's why, we have to pay Rs. 20 after rounding.

Estimation
A process in which we round the number to the actual value or required degree of accuracy.

Noor purchased 343 metres cloth.

Ayesha also purchased the cloth having the same length.



Noor and Ayesha want to round the given length of cloth.

Noor does round the length of the cloth to the 2-significant figures i.e. 340

Ayesha does round the length of the cloth to the 1-significant figure i.e. 300 metres

Can you tell that who is more accurate?

Noor is more accurate because he rounded the length of the cloth to the 2-significant figures.

Remember!
When a number is rounded to a greater number of significant figures, then that number is called more accurate.

Significant Figure
The first non-zero digit of the number from the left is called the first significant figure. All zeros between non-zero digits are also significant.

Remember!
- s.f. stands for significant figure.
- d.p. stands for decimal place.

Example 1: Round 548735 to the given degree of accuracy.

(i) 1 significant figure.
(ii) 3 significant figures.
(iii) 5 significant figures.

Solution:
(i) 548735 = 500000 (1 s.f.)
(ii) 548735 = 549000 (3 s.f.)
(iii) 548735 = 548740 (5 s.f.)

Key fact!
To round whole number to the required degree of accuracy.
- Look at the digit next to the required degree of accuracy.
- If the digit is 5 or greater than 5, we will add 1 to the digit of the required degree of accuracy.
- If the digit is smaller than 5 we will leave the digit as it is given on the required degree of accuracy.

Ahmad says the cost of one litre petrol is Rs. 150.40

Azra says the cost of one litre petrol is Rs. 150.4

Can you tell that who is more accurate?

Ahmad is more accurate because he rounded the cost of one litre of petrol to the maximum degree of accuracy (5-significant figures) as compare to Azra.

All non-zero digits are significant and all zeros between non-zero digits are significant.

Keep in mind!

In decimal part of a decimal number, all zeros after a non-zero digit are significant and all zeros before a non-zero digit are not significant.

Try yourself!

How many significant figures does 0.00295 have?

1.2.2 Round Decimal Numbers

Example 2:

Round 87.5376 to the given degree of accuracy.

(i) 1 decimal place
(ii) 4 significant figures.
(iii) 3 decimal places.

Solution:

(i) 87.5376 = 87.5 (1 d.p.)
(ii) 87.5376 = 87.54 (4 s.f.)
(iii) 87.5376 = 87.538 (3 d. p.)

The digit next to 2-decimal places i.e., "7" is greater than 5. So, round the digit 3 up to 4 and remove all the next digits towards its right.

Try yourself!

Round 54285 to the 4 significant figures.

Example 3:

Round 0.0785229 to the given degree of accuracy.

(i) 3 significant figures
(ii) 5 significant figures

Solution:

(i) 0.0785229 = 0.0785 (3 s.f.)
(ii) 0.0785229 = 0.078523 (5 s.f.)

The first significant figure is 7, the second is 8 and third is 5. The digit to the right of 5 is 2 and it is less than 5. So, 5 stays the same and remove all the next digits towards its right.

1.2.3 Round Negative Integers

The method to round negative integers is different from the method used to round natural numbers.

Example 4: Round -234576 to the given degree of accuracy.

(i) 3 significant figures

(ii) 5 significance figures

Remember!

In negative integers, the number which is close to the zero (0) is greater number. While the number which is far from the zero (0) is smaller number.

Solution:

(i) $-234576 = -234000$ (3 s.f.) If we want to round the number towards zero (0).

$= -235000$ (3 s.f.) If we want to round the number away from zero (0).

(ii) $-234576 = -234570$(5 s.f.) If we want to round the number towards zero (0).

$= -234580$ If we want to round the number away from zero (0).

Example 5: Round -48.3567 to the given degree of accuracy.

(i) 3 decimal places (ii) 5 significant figures

Solution:

(i) $-48.3567 = -48.357$ (3 d.p.) If we round the number away from zero (0).

$-48.3567 = -48.356$ (3 d.p.) If we round the number towards zero (0).

(ii) $-48.3567 = -48.356$ (5 s.f.) If we round the number towards zero (0).

$-48.3567 = -48.357$ (5 s.f.) If we round the number away from zero (0).

1.2.4 Round Rational Numbers

Example 6: Round $\frac{17}{3}$ to the given degree of accuracy.

(i) 2 significant figures

(ii) 5 significant figures

Solution:

(i) $\frac{17}{3} = 5.66666 = 5.7$ (2 s.f.)

(ii) $\frac{17}{3} = 5.66666 = 5.6667$ (5 s.f.)

Example 7: Round $\frac{235}{28}$ to the given degree of accuracy.

(i) 2 decimal places

(ii) 3 decimal places

Solution:

(i) $\frac{235}{28} = 8.392857 = 8.39$ (2 d.p.)

(ii) $\frac{235}{28} = 8.392857 = 8.393$ (3 d.p.)

1.2.5 Analysis of Approximation Error

Example 8: (i) Work out an approximation of 4.53 + 786,

(ii) Calculate its accurate value,

(iii) Compare your approximate and accurate value and find the approximation error, where $4.53 \approx 5$ and

$786 \approx 800$

Keep in mind!

Approximation error is also known as bias. The symbol ≈ is used for approximation.

Solution: (i) $5 + 800 = 805$

Approximated Value = 805

(ii) $4.53 + 786 = 790.53$

Accurate Value = 790.53

(iii) Approximation error = Approximated value − Accurate value

$= 805 - 790.53$

Approximation error = 14.47

Example 9: Calculate approximated value, accurate value and approximation error of $\frac{1.45 \times 560}{23.5}$.

Solution: Approximated value $= 1.45 \approx 1$; $560 \approx 600$; $23.5 \approx 24$

$$= \frac{1 \times 600}{24} = 25$$

Approximated value = 25

$$\text{Accurate value} = \frac{1.45 \times 560}{23.5} = 34.55$$

Approximation error = Accurate value − Approximated value

$= 34.55 - 25$

Approximation error = 9.45

Keep in mind!

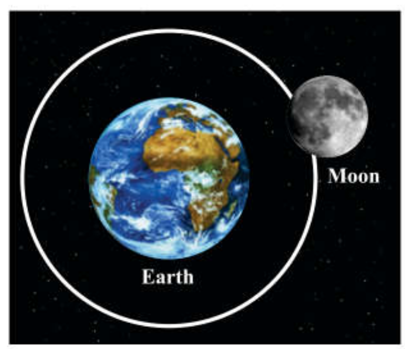
Approximation error is numerical value, which tells us how far is the approximated value from the actual / accurate value.

1.2.6 Real Life Situation Involving Approximation

Example 10: The minimum distance between the earth and the moon is about 363104 km. Round the number according to the given degree of accuracy.

(i) 3 significant figures.

(ii) 5 significant figures.



Solution: (i) $363104 \approx 363000$ (3 s.f.)

(ii) $363104 \approx 363100$ (5 s.f.)

Exercise 1.3

1. Round the following numbers to the given degree of accuracy.

(i) 4763 (2 s.f.) (ii) 785029 (3 s.f.) (iii) 8095321 (4 s.f.)

(iv) 0.0289357 (5 s.f.) (v) 2345987 (5 s.f.) (vi) 0.780247 (4 s.f.)

2. Round the given decimal numbers to the stated number of decimal places or significant figures.
   (i) 2.785 (2 d.p.) (ii) 88.9856 (4 s.f.)
   (iii) 15.3456 (2 d.p.) (iv) 10.4579 (3 s.f.)

3. Round the integers away from zero (0) as given.
   (i) − 23.456 (1 d.p.) (ii) − 45325 (3 s.f.)
   (iii) − 17.7898 (4 s.f.) (iv) 1172859 (5 s.f.)

4. Round the rational numbers to the stated number of decimal places or significant figures.
   (i) $\frac{23}{3}$ (3 d.p.) (ii) $\frac{55}{3}$ (2 d.p.) (iii) $\frac{47}{7}$ (2 d.p.)
   (iv) $\frac{22}{6}$ (3 d.p.) (v) $\frac{25}{29}$ (5 s.f.) (vi) $\frac{5}{521}$ (4 s.f.)

5. Write the number of significant figures in each of the following:
   (i) 0.0028 (ii) 25901 (iii) 0.1080 (iv) 0.000235

6. Compare your approximated value with the accurate value and also find an approximation error.
   (i) $3.78 \div 1.98$ (ii) $\frac{330 \times 7.62}{2.53}$ (iii) $\frac{28.35 + 30.15}{4593 - 3690}$ (iv) $\frac{133 + 8.95}{5.36}$

7. The total length of the Great Wall of China is 21196.18 km. Round the number to 1 decimal place and 5-significant figures.

8. The diameter of earth is 12742 km. Round the number to 3 and 4 significant figures.

SUMMARY

- To round whole number to the required degree of accuracy:
  - Look at the digit next to the degree of accuracy.
  - If the digit is 5 or greater than 5, we will add 1 to the digit of the required degree of accuracy.
  - If the digit is smaller than 5, we will leave the digit as it is given on the required degree of accuracy.
- s.f. stands for significant figure.
- d.p. stands for decimal place.
- The method to round negative integers is different from the method used to round natural numbers.
- In negative integer, the number which is close to the zero (0) is greater number. While the number which is far from the zero (0) is smaller number.
- Approximation error is numerical value, which tells us how far is the approximated value from the actual / accurate value.

Sub-Domain (iii): Square Roots and Cube Roots

$\sqrt{x}$

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- recall squares and cubes of natural numbers up to 3- digits

Square Roots

compute square root of:

- a natural number
- a common fraction
- a decimal
- given in perfect square form by prime factorization method up to 5- digits
- calculate square roots up to 4 digits with maximum 2-decimal places which is not a perfect square
- apply square and square roots in real life situations

Cubes and Cube Roots

- calculate cube roots of a number up to 5-dgits which are perfect cubes by prime factorization method
- apply cubes and cube roots in real life situations/word problems

Advanced/Additional

compute square root of:

- a natural number
- a common fraction
- a decimal
- given in perfect square form by division method up to 5-digits

Nida wants to plant 12 rose plants in rows and 12 rose plants in columns in her home garden as shown in figure.

1.3.1 Squares of Natural Numbers

When a number is multiplied by itself, then the product is known as the square of the number i.e, the square of x is $x \times x = x^2$

For Example: $3 \times 3 = 3^2 = 9$

Read as square of 3 is 9

Similarly,

$5 \times 5 = 5^2 = 25$ (square of 5 is 25)

Finding perfect square of a number

A natural number is called a perfect square, if it is the square of another natural number. e.g., the number 4 is a perfect square because $4 = 2^2$.

Similarly, 25 is a perfect square because $25 = 5^2$ and so on.

Now, we learn to find a perfect square of a number:

Try yourself!

Can you find out the perfect square of -4?

Example 1: Find the perfect square of 13

Solution: The perfect square of 13 is

$13^2 = 13 \times 13$

$= 169$

Establish patterns for the squares of natural numbers

We know that $4^2 = 4 \times 4 = 16$

We can also write the square of 4 in a pattern form as

$4^2 = 1+2+3+4+3+2+1 = 16$

Similarly $5^2 = 1+2+3+4+5+4+3+2+1 = 25$

And $6^2 = 1+2+3+4+5+6+5+4+3+2+1 = 36$

So, we observed that the square of any natural number can be found with the help of summation of above patterns.

$1^2 \quad 1 = 1$

$2^2 \quad 1 + 2 + 1 = 4$

$3^2 \quad 1 + 2 + 3 + 2 + 1 = 9$

$4^2 \quad 1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$

$5^2 \quad 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$

$6^2 \quad 1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36$

$7^2 \quad 1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 49$

$8^2 \quad 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 64$

$9^2 \quad 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 81$

$10^2 \quad 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 100$

In the above pattern, we notice that:

(i) Each row starts and ends with digit 1.
(ii) The digits increase up to the number whose square is required and then decrease.
(iii) The number of digits in each row increases by two digits.
(iv) The difference of any two consecutive squares is an odd number.
(v) The number of digits in a particular row is the addition of the number and the previous consecutive numbers whose squares are to be found.

Consider another pattern of squares of natural numbers.

		Sum of odd numbers
1^2	1	$= 1$
2^2	$1 + 3$	$= 4$
3^2	$1 + 3 + 5$	$= 9$
4^2	$1 + 3 + 5 + 7$	$= 16$
5^2	$1 + 3 + 5 + 7 + 9$	$= 25$
6^2	$1 + 3 + 5 + 7 + 9 + 11$	$= 36$
7^2	$1 + 3 + 5 + 7 + 9 + 11 + 13$	$= 49$
8^2	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$	$= 64$
9^2	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$	$= 81$
10^2	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$	$=100$

In the above pattern, we notice that:

(i) The summation is in ascending order.
(ii) The square of each number is written as the sum of odd numbers only.
(iii) Each row of the pattern starts from an odd number 1.
(iv) The number of odd numbers in each row is equal to the number whose square is to be found.
(v) The sum of each row is equal to the required square.
(vi) The last odd number in each row is one less than the double of the given number.

Exercise 1.4

1. Find the square of the following numbers:
   (i) 7 (ii) 11 (iii) 19 (iv) 25 (v) 37 (vi) 75
2. Write the summation patterns for the following squares.
   (i) 6^2 (ii) 7^2 (iii) 4^2 (iv) 5^2 (v) 3^2 (vi) 8^2

1.3.2 Square Roots

Finding the square root of (a) a natural number (b) a common fraction (c) a decimal given in perfect square form, by prime factorization and division method

(a) Finding square root of a natural number

- **By Prime Factorization Method**

First of all, find prime factors, then make pairs of these factors. Choose one prime number from each pair and then find the product of all those prime factors, which will be the square root of the given number.

Example 2: Find the square root of 12100

Solution: $12100 = 2\times 2\times 5\times 5\times 11\times 11$

$$\sqrt{12100} = \sqrt{\overline{2\times 2}\times\overline{5\times 5}\times\overline{11\times 11}}$$

$$= \sqrt{2^2\times 5^2\times 11^2}$$

$$= 2\times 5\times 11$$

$$\therefore \sqrt{12100} = 110$$

2	12100
2	6050
5	3025
5	605
11	121
11	11
	1

Key fact!

Square root and square are inverse of each other. The symbol used for square root is " $\sqrt{\ }$ ". It is called radical sign.

Example 3: Find the square root of 225

Solution: $225 = 3\times 3\times 5\times 5$

$$\sqrt{225} = \sqrt{\overline{3\times 3}\times\overline{5\times 5}}$$

$$= 3\times 5$$

$$= 15$$

$$\therefore \sqrt{225} = 15$$

3	225
3	75
5	25
5	5
	1

Think!

Is the index of each prime factor should be even to find out square root? Why?

Example 4: Find the square root of 1600

Solution: $1600 = 2\times 2\times 2\times 2\times 2\times 2\times 5\times 5$

$$\sqrt{1600} = \sqrt{\overline{2\times 2}\times\overline{2\times 2}\times\overline{2\times 2}\times\overline{5\times 5}}$$

$$= 2\times 2\times 2\times 5$$

$$= 40$$

$$\therefore \sqrt{1600} = 40$$

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Try yourself!

Can you solve?

$\sqrt{-81} = \boxed{?}$

- **By Division Method**

To find the square root of natural numbers by division method, we will proceed as under:

(i) Make pairs of digits from right to left. If the number of digits is even, we have to complete pairs. If the number of digits is odd, the last digit on extreme left will remain single.

(ii) Look for the numbers whose square is equal to or less than the number of extreme left, which may be a single digit or a pair. This number will be divisor as well as quotient.

(iii) Subtract the product. Bring down the next pair to the right of the remainder.

(iv) Double the quotient and write as divisor as ten's digit.

(v) Look for the number whose square will be equal to or less than the dividend. Write that number with the right side of the quotient as well as with divisor at unit place.

Example 5: Find the square root of 1024

Solution:

$$
\begin{array}{r|l}
 & \quad 32 \\ \hline
3 & \overline{10}\ \overline{24} \\
 & -9 \downarrow \\ \hline
62 & \quad 124 \\
 & -124 \\ \hline
 & \quad\ \ 0
\end{array}
$$

Example 6: Find the square root of 15129

Solution:

$$
\begin{array}{r|l}
 & \quad 123 \\ \hline
1 & 1\overline{51}\ \overline{29} \\
 & -1 \downarrow \\ \hline
22 & \quad 51 \\
 & -44 \downarrow \\ \hline
243 & \quad 729 \\
 & -729 \\ \hline
 & \quad\ \ 0
\end{array}
$$

Exercise 1.5

1. Find the square root of the following by prime factorization method:

(i) 784 (ii) 1225 (iii) 2500 (iv) 4225 (v) 5184 (vi) 7744
(vii) 1296 (viii) 1764 (ix) 29241 (x) 51529 (xi) 418609 (xii) 240100

2. Find the square root of the following by division method:

(i) 1369 (ii) 13689 (iii) 50625 (iv) 103041

(b) Finding square root of a common fraction

We know that in fraction $\frac{4}{9}$, 4 is numerator and 9 is denominator. The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator. This is illustrated with the help of following examples:

Example 7: Find the square root of $1\frac{11}{25}$

Solution:

$$1\frac{11}{25}=\frac{36}{25}=\frac{2\times 2\times 3\times 3}{5\times 5}$$

$$\sqrt{1\frac{11}{25}}=\sqrt{\frac{36}{25}}$$

$$=\frac{\sqrt{\overline{2\times 2}\times\overline{3\times 3}}}{\sqrt{\overline{5\times 5}}}=\frac{2\times 3}{5}=\frac{6}{5}=1\frac{1}{5}$$

Example 8: Find the square root of $9\dfrac{67}{121}$

Solution: $9\dfrac{67}{121} = \dfrac{1156}{121}$

Now $\sqrt{9\dfrac{67}{121}} = \sqrt{\dfrac{1156}{121}}$

$= \dfrac{\sqrt{1156}}{\sqrt{121}}$

$= \dfrac{34}{11}$

$= 3\dfrac{1}{11}$

$$\begin{array}{r|l} & \ \ 3\,4 \\ \hline 3 & \overline{11}\,\overline{56} \\ & -9 \downarrow \\ \hline 64 & \ \ 2\,5\,6 \\ & -2\,5\,6 \\ \hline & \ \ \ \ 0 \end{array} \qquad \begin{array}{r|l} & \ \ 1\,1 \\ \hline 1 & \ \ 1\,\overline{21} \\ & -1 \downarrow \\ \hline 21 & \ \ \ 2\,1 \\ & -2\,1 \\ \hline & \ \ \ 0 \end{array}$$

$\therefore$ $\sqrt{9\dfrac{67}{121}} = 3\dfrac{1}{11}$

Exercise 1.6

1. Find the square root of the following fractions by prime factorization:

(i) $\dfrac{49}{64}$ (ii) $\dfrac{121}{625}$ (iii) $\dfrac{196}{441}$ (iv) $1\dfrac{13}{36}$ (v) $\dfrac{676}{729}$ (vi) $12\dfrac{24}{25}$

2. Find the square root of the following fractions by division method:

(i) $\dfrac{144}{225}$ (ii) $\dfrac{169}{256}$ (iii) $5\dfrac{41}{64}$

(c) Finding square root of a decimal

- **By Prime Factorization**

We convert the decimal to common fraction and then find square root.

Example 9: Find the square root of decimal 2.25

Solution: $2.25 = \dfrac{225}{100}$

$\sqrt{\dfrac{225}{100}} = \dfrac{\sqrt{225}}{\sqrt{100}} = \dfrac{\sqrt{3\times 3\times 5\times 5}}{\sqrt{2\times 2\times 5\times 5}}$

$= \dfrac{\sqrt{\overline{3\times 3}\times \overline{5\times 5}}}{\sqrt{\overline{2\times 2}\times \overline{5\times 5}}}$

$$= \frac{3\times 5}{2\times 5} = \frac{15}{10}$$

$$= 1.5$$

$$\therefore \quad \sqrt{2.25} = 1.5$$

Example 10: Find the square root of 1239.04

Solution:

$$1239.04 = \frac{123904}{100}$$

$$\sqrt{\frac{123904}{100}} = \sqrt{\frac{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 11\times 11}{2\times 2\times 5\times 5}}$$

$$= \sqrt{\frac{\overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{11\times 11}}{\overline{2\times 2}\times \overline{5\times 5}}}$$

$$= \frac{2\times 2\times 2\times 2\times 2\times 11}{2\times 5} = \frac{352}{10}$$

$$\therefore \quad \sqrt{1239.04} = 35.2$$

- **By Division Method**

For using this method, the following steps will be taken:

(i) Make pairs of digits on the left side of the decimal point from right to left.
(ii) Make pairs of digits on the right side of the decimal point from left to right.
(iii) If the number of digits is odd on the right side of the decimal point, then write 0 along with the last digit while making pairs from left to right.
(iv) Place the decimal point in the quotient while bringing down the pair after the decimal point.
(v) While bringing down two pairs at a time, place a zero in the quotient.

This method is illustrated with the help of following examples:

Example 11: Find the square root of 180.9025

Solution:

$$\begin{array}{r|l}
 & 13.45 \\ \hline
1 & 1\overline{80}.\overline{90}\,\overline{25} \\
 & -1 \\ \hline
23 & 80 \\
 & -69 \\ \hline
264 & 1190 \\
 & -1056 \\ \hline
2685 & 13425 \\
 & -13425 \\ \hline
 & 0
\end{array}$$

$1\times 21 = 21$
$2\times 22 = 44$
$3\times 23 = 69$
$4\times 24 = 96$

$1\times 261 = 261$
$2\times 262 = 524$
$3\times 263 = 789$
$4\times 264 = 1056$
$5\times 265 = 1325$

$1\times 2681 = 2681$
$2\times 2682 = 5364$
$3\times 2683 = 8049$
$4\times 2684 = 10736$
$5\times 2685 = 13425$

$$\therefore \quad \sqrt{180.9025} = 13.45$$

Example 12: Find the square root of 0.053361

Solution:

$$
\begin{array}{r|l}
 & 0.231 \\
\hline
2 & 0.\overline{05}\,\overline{33}\,\overline{61} \\
 & -0\,4 \\
\hline
43 & \quad 1\,33 \\
 & \;-1\,29 \\
\hline
461 & \qquad 4\,61 \\
 & \quad\; -4\,61 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore \quad \sqrt{0.053361} = 0.231$

Exercise 1.7

1. Find the square root of the following decimals by prime factorization:

(i)	1.21	(ii)	0.64	(iii)	7.29
(iv)	1.44	(v)	1.69	(vi)	12.25

2. Find the square root of the following decimals by division method:

(i)	0.3249	(ii)	0.5184	(iii)	10.24
(iv)	20.5209	(v)	3856.41	(vi)	1866.24
(vii)	0.680625	(viii)	0.797449	(ix)	1281.64

1.3.3 Find Square Root of a Number Which is Not a Perfect Square

Example 13: Find the square root of 2 up to 2 decimal places.

Solution:

$$
\begin{array}{r|l}
 & 1.414 \\
\hline
1 & 2.\,00\,00\,00 \\
 & -1 \\
\hline
24 & 1\,00 \\
 & \;-96 \\
\hline
281 & \quad 4\,00 \\
 & \; -2\,81 \\
\hline
2824 & \quad 1\,19\,00 \\
 & \;-1\,12\,96 \\
\hline
 & \qquad\; 604 \\
 & \qquad \ldots\ldots
\end{array}
$$

$\therefore \; \sqrt{2} \approx 1.414$, rounded to 1.41 (2 d.p.)

Key fact!

If we cannot find out a rational number whose square is x, then $\sqrt{x}$ is an irrational number.

We observe that:

The process is non-terminating, so we cannot get zero as remainder. In the quotient after the decimal point, there is no group of integers which is repeating itself, as in the case of rational numbers.

$$\frac{2}{3} = 0.666\ldots \quad \text{and} \quad \frac{7}{9} = 0.777\ldots$$

Example 14: Find the square root of 2.5 up to 2 decimal places.

Solution:

$$
\begin{array}{r|l}
 & 1\,.\,5\,8\,1 \\ \hline
1 & 2\,.\,\overline{50}\;\overline{00}\;\overline{00} \\
 & -1 \\ \hline
25 & 1\;50 \\
 & -1\;25 \\ \hline
308 & 2\,5\,00 \\
 & -2\,4\,64 \\ \hline
3161 & 36\,00 \\
 & -31\,61 \\ \hline
 & 4\;39 \\
 & \ldots\ldots
\end{array}
$$

$\therefore \quad \sqrt{2.5} \approx 1.581$, rounded to 1.58 (2 d.p.)

In such cases, we restrict the process after some decimal places. Here we shall restrict it up to 2 decimal places.

Example 15: Find the square root of 2723.56 up to 3 decimal places.

Solution:

$$
\begin{array}{r|r}
 & 52.1877 \\ \hline
5 & \overline{27}\,\overline{23}.\overline{56}\,\overline{00} \\
 & -25 \\ \hline
102 & 223 \\
 & -204 \\ \hline
1041 & 1956 \\
 & -1041 \\ \hline
10428 & 91500 \\
 & -83424 \\ \hline
104367 & 807600 \\
 & -730569 \\ \hline
 & 77031 \\
 & \ldots\ldots
\end{array}
$$

$\therefore \quad \sqrt{2723.56} \approx 52.1877$, rounded to 52.188 (3 d.p.)

Exercise 1.8

1. Find the square root of the following up to 3 decimal places:

   (i) 2 (ii) 3 (iii) 5 (iv) 7 (v) 11 (vi) 21

2. Find the square root of the following up to 2 decimal places:

   (i) 3.6 (ii) 6.4 (iii) 28.9 (iv) 63.34 (v) 921.25 (vi) 5921.25

1.3.4 Use the Rule to Determine the Number of Digits in the Square Root of a Perfect Square

Rule: Let n be the number of digits in the perfect square, then its square root contains:

(i) $\frac{n}{2}$ digits if n is even (ii) $\frac{n+1}{2}$ digits if n is odd

Now, we apply the above rule for finding the number of digits in the square root of a perfect square with the help of following examples:

Example 16: Find the number of digits in the square root of 49729

Solution: Number of digits of the given number = 5

$n = 5$ is odd, so above mentioned rule (ii) will be applied.

Thus, the number of digits in the square root will be $\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$

To check the answer, we proceed as under:

$$\begin{array}{r|r}
 & 223 \\ \hline
2 & \overline{4}\ \overline{9\ 7}\ \overline{2\ 9} \\
 & -4 \\ \hline
42 & 9\ 7 \\
 & -8\ 4 \\ \hline
443 & 1\ 3\ 2\ 9 \\
 & -1\ 3\ 2\ 9 \\ \hline
 & 0
\end{array}$$

$\therefore$ $\sqrt{49729} = 223$

Example 17: Find the number of digits in the square root of 10329796.

Solution: Number of digits (n) = 8

Now $n = 8$ is even, so part (i) of the rule will be applied.

The number of digits in the square root $= \frac{n}{2} = \frac{8}{2} = 4$

Now, we can verify it

$$\begin{array}{r|r}
 & 3214 \\ \hline
3 & \overline{1\ 0}\ \overline{3\ 2}\ \overline{9\ 7}\ \overline{9\ 6} \\
 & 9 \\ \hline
62 & 1\ 3\ 2 \\
 & 1\ 2\ 4 \\ \hline
641 & 8\ 9\ 7 \\
 & 6\ 4\ 1 \\ \hline
6424 & 2\ 5\ 6\ 9\ 6 \\
 & 2\ 5\ 6\ 9\ 6 \\ \hline
 & 0
\end{array}$$

$\therefore$ $\sqrt{10329796} = 3214$

Exercise 1.9

1. Find the number of digits in the square root of the following perfect square and verify it by finding square root:

(i)	63504	(ii)	66564	(iii)	50625	(iv)	837225
(v)	839056	(vi)	1054729	(vii)	1577536	(viii)	2119936
(ix)	3283344	(x)	614656	(xi)	7778521	(xii)	12880921

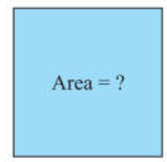
1.3.5 Real Life Problems Involving Square Roots

Example 18: The length of side of a square is $55m$. Find the area of the square.

Solution: Let 'x' be the area of the square

As we know that

$$\begin{aligned} Area &= (\text{length of a side})^2 \\ x &= (55m)^2 = 55m \times 55m \\ x &= 3025m^2 \end{aligned}$$



Length = $55m$

The area of the square is $3025m^2$

Example 19: 1225 students stand in rows in such a way that the number of rows is equal to the number of students in a row. How many students are there in each row?

Solution: Since the number of students in a row is the same as the number of rows. So, square root of 1225 will be found.

$$\begin{array}{r|l} & 3\ 5 \\ \hline 3 & \overline{1\,2}\ \overline{2\,5} \\ & \ \ 9 \\ \hline 65 & 3\ 2\ 5 \\ & 3\ 2\ 5 \\ \hline & \ \ \ \ 0 \end{array}$$

Thus, there are 35 students in each row.

Example 20: Find the least number which, when subtracted from 58780, the answer is a complete square.

Solution: To find which number is subtracted from the given number, we find the square root of 58780 and the remainder will be the required number.

$$\begin{array}{r|l} & 2\ \ 4\ \ 2 \\ \hline 2 & 5\ \ \overline{8\ \ 7}\ \ \overline{8\ \ 0} \\ & 4 \\ \hline 44 & 1\ \ 8\ \ 7 \\ & 1\ \ 7\ \ 6 \\ \hline 482 & \ \ \ \ 1\ \ 1\ \ 8\ \ 0 \\ & \ \ \ \ \ \ \ \ 9\ \ 6\ \ 4 \\ \hline & \ \ \ \ \ \ \ \ 2\ \ 1\ \ 6 \end{array}$$

Remaining Number = Given number – Remainder = 58780 – 216 = 58564

Thus, if 216 is subtracted from 58780, the remaining number 58564 will be a complete square.

Exercise 1.10

1. The length of a squared shaped box is 324cm. Find the area of the box.
2. The length of a square shaped field is 36m. Find the area of the square field.
3. The length of a square shaped lawn is 250 metres. What will be the area of the lawn and also tell what will be its cost of growing grass at the rate of Rs. 75 per sq. metres?
4. The area of a square shaped field is 14400 sq. metres. Find the length of the side of the square.
5. The area of a square shaped field is 422500 sq. metres. How much string is required for fixing along the sides as a fence?
6. A gardener wants to plant 122500 trees in his field in such a way that the number of trees in a row is equal to the number of rows. How many trees will he plant in each row?
7. The area of a rectangular field is 10092 sq. metres. Its length is three times as long as its width. Find its perimeter.
8. A rectangular field has an area 28800 sq. metres. Its length is twice as long as its width. What is the length of its sides?
9. Find that least number which, when subtracted from 109087, the answer is a complete square.
10. The cost of levelling the ground of a circular region at a rate of Rs.2 per square metre is Rs.4928. Find the radius of the ground.
11. The cost of ploughing in a square field is Rs.2450 at the rate of Rs.2 per 100 sq. metres. Find the length of the side of the square.
12. The area of a square shaped lawn is 62500 sq. metres. A wooden fence is to be laid around the lawn. What is the length of a wooden fence required? What will be its cost at the rate of Rs.50 per metre?

1.3.6 Cubes and Cube Roots

Recognition of cubes and perfect cubes

- **Cubes**

Cube of a number means to multiply the number by itself three times.

Let x be any number,

then, $x \times x \times x = x^3$ is the cube of x.

For example, $2 \times 2 \times 2 = 2^3$

$3 \times 3 \times 3 = 3^3$

$4 \times 4 \times 4 = 4^3$ and so on.

Try yourself!

Can you find out the perfect cube of –5?

- **Perfect cubes**

Perfect cube is a number that is the result of multiplying an integer by itself three times. In other words it is an integer to the third power of another integer.

Example 21: Show that 8 and 27 are perfect cubes.

Solution: $8 = 2 \times 2 \times 2 = 2^3$ $\therefore$ 8 is a perfect cube of 2

$27 = 3 \times 3 \times 3 = 3^3$ $\therefore$ 27 is a perfect cube of 3

Try yourself!

.... is a perfect cube of 4.

Example 22: Find cube of 1.2

Solution: $(1.2)^3 = (1.2) \times (1.2) \times (1.2)$
$= (1.44) \times (1.2)$
$= 1.728$

Try yourself!
Can you solve?
$\sqrt[3]{-64} = \boxed{?}$

Finding cube roots of numbers which are perfect cubes

In Mathematics, a cube root of a number, denoted by $x^{\frac{1}{3}}$, is a number 'a' such that $a^3 = x$. i.e. $a = x^{\frac{1}{3}}$
e.g., 216, 64 and 512 are perfect cubes of 6, 4 and 8.

Remember!
Symbol of cube root is $\sqrt[3]{}$
3 is a part of the symbol.

Example 23: Find the cube root of 125

Solution:

$125 = 5 \times 5 \times 5 = 5^3$

$\sqrt[3]{125} = \sqrt[3]{5\times5\times5}$
$= (5^3)^{1/3}$
$= 5$

5	125
5	25
5	5
	1

Example 24: Find the cube root of 9261

Solution: $9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$
$= 3^3 \times 7^3$

$\therefore \quad \sqrt[3]{9261} = \sqrt[3]{3^3 \times 7^3}$
$= (3^3 \times 7^3)^{1/3}$
$= (3^3)^{1/3} \times (7^3)^{1/3}$
$= 3 \times 7$
$= 21$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Think!
Is the index of each prime factor should be a multiple of 3 to find out the cube root? Why?

Recognition of properties of cubes of numbers

(i) Cube of a positive number is positive. e.g., $3^3 = 27$
(ii) Cube of a negative number is negative. e.g.,$(-4)^3 = -64$
(iii) Cube of an even number is even, e.g., $6^3 = 216$
(iv) Cube of an odd number is odd. e.g., $7^3 = 343$
(v) Properties under (a) multiplication and (b) division
(a) $(a \times b)^3 = a^3 \times b^3$ (b) $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
(vi) Cube root of the perfect cubes
$6^3 = 216$, $4^3 = 64$, $8^3 = 512$
$\sqrt[3]{216} = 6$ $\sqrt[3]{64} = 4$ $\sqrt[3]{512} = 8$

$$
\begin{aligned}
216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
&= 2^3 \times 3^3 \\
&= (2 \times 3)^3 \\
&= 6^3
\end{aligned}
$$

$\therefore$ 216 is a perfect cube of 6

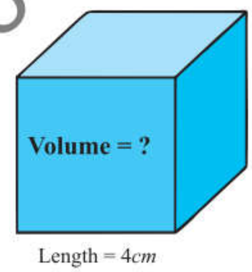
1.3.7 Real Life Problems Involving Cube and Cube Roots

Example 25: Find the volume of cube shaped gift box having the length of a side $4cm$.

Solution:

Let "ℓ" is the length of the cube shaped gift box.

$$\ell = 4cm$$



Volume of the cube shaped gift box $= \ell \times \ell \times \ell$

$$
\begin{aligned}
&= 4cm \times 4cm \times 4cm \\
&= 64\ cm^3
\end{aligned}
$$

Thus, the volume of the cube shaped gift box is $64\ cm^3$

Example 26: Find the length of a cube shaped jewelry box if the volume of the jewelry box is $216cm^3$.

Solution:

Let "ℓ" is the length of the jewelry box.

As we know that

Volume of the cube $= \ell \times \ell \times \ell$

Volume of the cube $= \ell^3$

$216cm^3 = \ell^3$

To find out the length of one side of the jewelry box, we will take cube root on both sides.

$$
\begin{aligned}
\ell^3 &= 216cm^3 \\
\sqrt[3]{\ell^3} &= \sqrt[3]{216} \\
\ell^{\cancel{3} \times \frac{1}{\cancel{3}}} &= (2 \times 2 \times 2 \times 3 \times 3 \times 3)^{\frac{1}{3}} \\
\ell &= (2^3 \times 3^3)^{\frac{1}{3}} \\
\ell &= 2^{\cancel{3} \times \frac{1}{\cancel{3}}} \times 3^{\cancel{3} \times \frac{1}{\cancel{3}}} \\
\ell &= 6cm
\end{aligned}
$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Thus, the length of one side of the cube shaped jewelry box is $6cm$.

Exercise 1.11

1. Which numbers are perfect cubes?
   (i) 512 (ii) 1100 (iii) 6859 (iv) 729 (v) $\frac{1000}{125}$
2. Find the cube roots of the following:
   (i) 729 (ii) 15625 (iii) 13824
3. Find the cubes of the following:
   (i) 1.4 (ii) 0.4 (iii) 0.8
4. Find the cube roots of the following:
   (i) $\frac{27}{216}$ (ii) 35937 (iii) 3375
5. If a cube has length of $6cm$, then what will be the volume of the cube?
6. Find the volume of a cube if the length of one side is $8cm$.
7. If the volume of a cube shape is $3375cm^3$, then what is the measure of the edge of the cube?
8. The volume of a cube is $729m^3$. What will be the length of a side of the cube?
9. How many cubical boxes of dimensions $5cm \times 5cm \times 5cm$ can be packed in a large cubical case having volume $875cm^3$?

SUMMARY

- When a number is multiplied by itself, then the product is known as the square of that number i.e., the square of x is $x \times x = x^2$
- Square root and square are inverse of each other. The symbol used for square root is "$\sqrt{\ }$". It is called radical sign.
- If n be the number of digits in the perfect square, then its square root contains:

  (i) $\frac{n}{2}$ digits if n is even (ii) $\frac{n+1}{2}$ digits if n is odd
- Cube of a number means to multiply the number by itself three times.
- Perfect cube is a number that is the result of multiplying an integer by itself three times. In other words, it is an integer to the third power of another integer.
- A cube root of a number, denoted by $x^{\frac{1}{3}}$, is a number such that $a^3 = x$. i.e., $a = x^{\frac{1}{3}}$
- Symbol of cube root is $\sqrt[3]{\ }$. Remember that 3 is the part of symbol.
- Cube of an even number is even, e.g., $6^3 = 216$
- Cube of an odd number is odd. e.g., $7^3 = 343$

Review Exercise 1 (a)

1. Four options are given against each statement. Encircle the correct one.

i. Real number is:
(a) Difference of rational number and irrational numbers
(b) Intersection of rational numbers and irrational numbers
(c) Union of rational number and irrational numbers
(d) Complete set of natural numbers.

ii. Which of the following is true about $\sqrt{8}$?
(a) Natural number (b) Whole number
(c) Rational number (d) Irrational number

iii. Irrational numbers are non terminating and __________ numbers.
(a) rational number (b) non-recurring
(c) recurring (d) decimal

iv. Round 0.0234589 to the 4 significant figures:
(a) 0.02346 (b) 0.02345 (c) 2346 (d) 0.0235

v. The difference between approximated value and actual accurate value is called:
(a) round (b) significant figure
(c) approximation error (d) estimated value.

vi. Which one of the following is perfect square?
(a) 25.6 (b) 0.256 (c) 2.56 (d) 2560

vii. Square of 0.9 is:
(a) 0.081 (b) 8.10 (c) 0.81 (d) 81.027

viii. $\left(\frac{7}{9}\right)^2 = ?$
(a) $\frac{49}{6}$ (b) $\frac{7}{81}$ (c) $\frac{49}{81}$ (d) $\frac{7}{3}$

ix. 1+2+3+4+5+6+7+8+7+6+5+4+3+2+1=?
(a) 8^2 (b) 9^2 (c) 65 (d) 81

x. If the side length of a square is 0.5m, then its area is:
(a) 0.50 m^2 (b) 2.5 m^2 (c) 0.25 m^2 (d) 25 m^2

xi. $\sqrt{0.04} = ?$
(a) 0.02 (b) 2.0 (c) 0.2 (d) 20

xii. $\sqrt{1^2 \times 4^2} = ?$
(a) 4 (b) 14 (c) 41 (d) 2

xiii. $\sqrt{\frac{4}{9}} = ?$

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{4}{3}$ (d) $\frac{3}{2}$

xiv. $\sqrt{\frac{a}{b}} = ?$

(a) $\frac{a}{b}$ (b) ab (c) $\sqrt{\frac{b}{a}}$ (d) $\frac{\sqrt{a}}{\sqrt{b}}$

2. Identify terminating, non-terminating and recurring decimal fractions from the following:

(i) $\frac{17}{5}$ (ii) $\frac{23}{3}$ (iii) $\frac{49}{8}$ (iv) $\frac{8}{15}$

3. Separate rational and irrational numbers.

(i) $\frac{25}{6}$ (ii) $\frac{55}{7}$ (iii) 8.111111 (iv) $\sqrt{21}$

4. Identify the property of real numbers used in the following:

(i) $25+(7+5)=(25+7)+5$ (ii) $17+(-17)=0$ (iii) $\sqrt{z}+0=\sqrt{z}$ (iv) $25\times 5=5\times 25$

5. Round the following to the given degree of accuracy:

(i) 598235 (4 s.f.) (ii) 0.002859 (3 s.f.) (iii) 45.35946 (2 d.p.)

(iv) $\frac{25}{8}$ (2 d.p.) (v) $\frac{15}{7}$ (2 d.p.) (vi) 0.078952 (4 s.f.)

6. Javeria bought 25.75 metres of cloth and Salma bought 78.28 metres of cloth. Find the total length of cloth. Also round the answer too.

(i) 3 s.f. (ii) 1 d.p.

7. The cost of 10 kg rice is Rs. 1557 and the cost of 15 kg sugar is Rs. 1278. What will be cost of both the items. Also round the answer up to 2 significant figures.

8. Find the number of digits in the square root of the following numbers. Also find the square root.

(i) 9604 (ii) 30625 (iii) 12544 (iv) 422500

9. Find the square root of the following:

(i) $28\frac{4}{9}$ (ii) $17\frac{128}{289}$ (iii) $101\frac{92}{169}$ (iv) 0.053361

(v) 0.204304 (vi) 152.7696 (vii) 0.316406 (viii) 38.01 (2 d.p.)

(ix) 6422.53 (2 d.p.)

10. If the area of a square field is 161604 m^2. Find the length of its one side.

11. Saeeda has 196 marbles that she is using to make a square formation. How many marbles should be in each row?

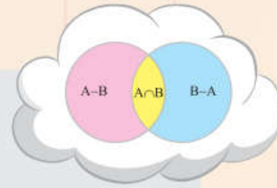
12. Find the cube root of the following numbers:

(i) 1728 (ii) 3375 (iii) $\frac{216}{125}$

13. The volume of a cube is 512 mm^3. What will be the length of a side of the cube?

14. What will be the volume of cube, whose side is 10 cm long?

Sub-Domain (iv): Sets



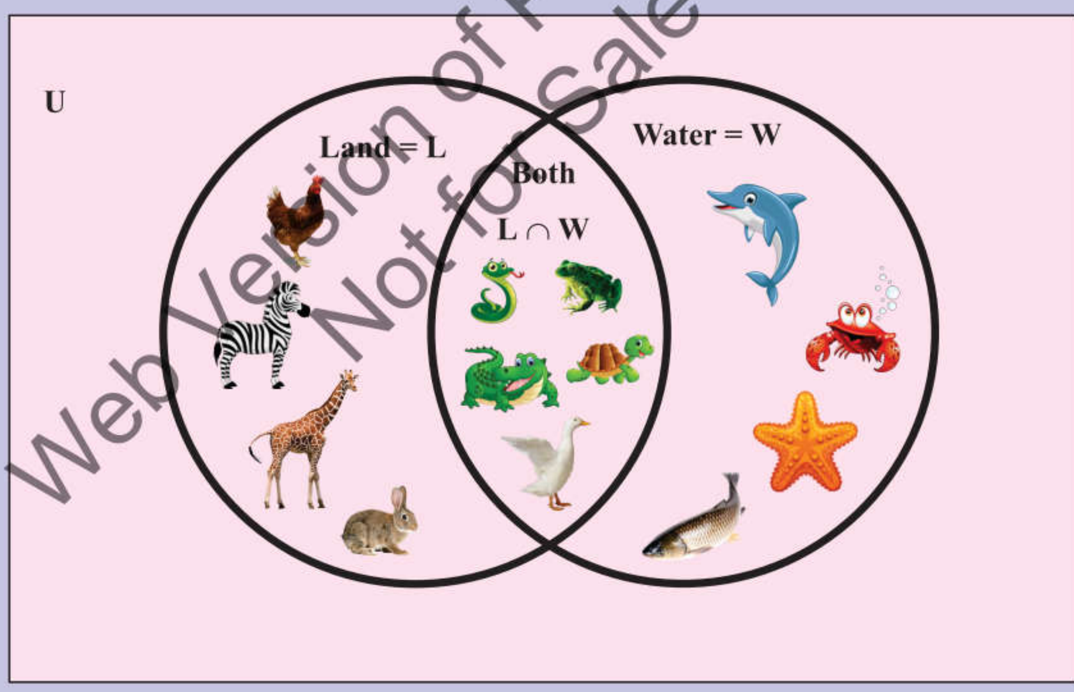
Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- discover sets in natural numbers
- express sets using tabular, descriptive and set-builder notations
- differentiate equivalent, and equal sets
- write subsets
- write power set P(A) of a set A, where A has up to four elements
- describe operations on sets tabular form:
  - ❖ union of two sets
  - ❖ intersection of two sets
  - ❖ difference of two sets
  - ❖ complement of a set
  - ❖ apply sets in real life situations
- describe operations on sets by using Venn diagram
  - ❖ union of two sets
  - ❖ intersection of two sets
  - ❖ difference of two sets
  - ❖ complement of two sets
- `Verify commutative, associative and distributive laws w.r.t. union and intersection.

Advanced/ Additional

- use Venn diagram to demonstrate union and intersection of two sets (subsets, overlapping sets and disjoint sets)
- describe operations on sets by using Venn diagram:
- verify De Morgans's Laws and represent through Venn diagram.



Do you know?

Frog is an amphibian. Amphibians are cold-blooded vertebrates (Vertebrates have backbone). Amphibian comes from a Greek word means "both lives". Frogs start their lives in the water and then live on land.

Can you write the set of the names of animals which live in water and on land?

1.4.1 Sets

The set of eight planets revolve around the sun. The names of these planets are: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune.

A set is a collection of well-defined distinct objects or symbols.

Some important sets and their notations

Set of natural numbers	$N = \{1, 2, 3, 4, \ldots\}$
Set of whole numbers	$W = \{0, 1, 2, 3, \ldots\}$
Set of integers	$Z = \{0, \pm1, \pm2, \pm3, \ldots\}$
Set of prime numbers	$P = \{2, 3, 5, 7, \ldots\ldots\}$
Set of odd integers	$O = \{\pm1, \pm3, \pm5, \ldots\}$
Set of even integers	$E = \{0, \pm2, \pm4, \pm6, \ldots\}$
Set of rational numbers	$Q = \{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0\}$

Key fact!

The objects of a set are called its members or elements.

Expression of a set

A set can be expressed in three ways.

(a) Tabular Form or Roster Form

(b) Descriptive Form

(c) Set Builder Notation

(a) Tabular or Roster Form:

In this form, all the elements of a set are expressed within braces "{}" and each element of the set is separated by comma ",".

For Example:

(i) $A = \{2, 4, 6, 8, 10\}$

(ii) $B = \{1, 3, 5, 7, 9\}$

(iii) $C = \{0, \pm1, \pm2, \pm3, \ldots\}$

(b) Descriptive Form:

In this form, a set is described by a given statement (sentence). The statement indicates all the elements of that set.

For Example:

(i) L = Set of five odd numbers

(ii) N = Set of natural numbers

(iii) S = Set of provinces of Pakistan

(c) Set-Builder Notation

In this form, all the members of a set are expressed by an alphabet showing all the properties of the members of that set.

For Example:

(i) $A = \{1, 2, 3, 4, \ldots, 8\}$ is expressed in set builder notation as:
$A = \{x \mid x \in N \wedge x \leq 8\}$

(ii) $B = \{\pm 1, \pm 3, \pm 5, \ldots\}$ is written in set builder notation as:
$B = \{x \mid x \in O\}$

(iii) $C = \{0, 1, 2, 3, \ldots, 10\}$
$C = \{y \mid y \in W \wedge y \leq 10\}$

Remember!

- The symbol ":" or "|" is read as "such that".
- The symbol "$\wedge$" is used for "and".
- The symbol "$\vee$" is used for "or".
- The symbol "$\in$" is used for "belongs to".

1.4.2 Difference between equivalent and equal sets

Equivalent sets

Equivalent sets are the sets with an equal number of elements. These sets do not have exactly the same elements, just these sets have the same number of elements.

For Example:

$D = \{1, 2, 3, 4, 5\}$
$E = \{L, M, N, O, P\}$

Although set D and set E have completely different elements but they have same number of elements. So, these are equivalent sets and also written as D~E.

Remember!

Equal sets are always equivalent but equivalent sets are not always equal sets.

Do you know?

"~" indicates equivalent sign.

Equal sets

When two sets have the same elements, these sets are called equal sets. It does not matter what order the elements are in.

For Example:

$A = \{2, 4, 6, 8\}$
$B = \{8, 6, 4, 2\}$

These are equal sets i.e., $A = B$

Remember!

- Two sets are equal if they contain the same elements.
- Two sets are equivalent if these sets have the same number of elements.

Do you know?

$\{a, b, c\} = \{c, b, a\}$

Exercise 1.12

1. Write the following sets in descriptive form:

(i) $A = \{1, 3, 5, 7, 9\}$	(ii) $B = \{2, 4, 6, 8, 10\}$
(iii) $C = \{0, \pm 1, \pm 2, \ldots, \pm 10\}$	(iv) $D = \{a, e, i, o, u\}$
(v) $E = \{x \mid x \in N \wedge x \leq 100\}$	(vi) $F = \{y \mid y \in P\}$

2. Write the following sets in tabular form:

(i) A = Set of whole numbers. (ii) B = Letters of the word "football".

(iii) C = Set of integers. (iv) D = Set of solar months start with letter "J".

(v) E = Set of English alphabet. (vi) F = Numbers less than 30 and divisible by 5.

3. Write the following sets in set builder notation:

(i) A = {0, 1, 2, 3, …, 10} (ii) B = Set of odd numbers

(iii) C = {3, 6, 9, 12, 15} (iv) D = {a, b, c, …, z}

(v) E = Set of days in a week. (vi) F = Set of prime numbers between 10 and 30.

4. Choose equivalent sets from the following:

(i) A = {a, b, c, d} , B = {2, 4, 6} (ii) C = {1, 3, 5, 7} , D = {0, 1, 2, 3}

(iii) E = Set of vowel letters , F = {L, M, N, O, P}

(iv) G = {1, 2, 3, 4, 5} , H = {0, 1, 2, 3, 4, 5}

5. Choose equal sets from the following:

(i) A = {3, 6, 9, 12} , B = {9, 12, 3, 6} (ii) C = {a, e, i, o, u} , D = { a, e, i, o}

(iii) E = {2, 3, 5, 7} , F = Set of first 4 prime numbers

(iv) G = {a, b, c, d} , H = {2, 4, 6, 8}

1.4.3 Subset

Consider that A and B are any two sets, if all elements of set A are also elements of set B, then set A is called subset of set B.

For Example:

A = {2, 4, 6}, B = {2, 4, 6, 8}

From these sets it is clear that all the elements of set A are the elements of set B. So, set A is subset of set B. Mathematically, it can be written as $A \subseteq B$.

1.4.4 Power Set

A set consisting of all possible subsets of a given set A is called the power set of A and is denoted by P(A).

For Example,

If A = {a, b}, then all its subsets are: ϕ, {a}, {b}, {a, b}

So, power set of A, P(A) = {ϕ, {a}, {b}, {a, b}}

Example 1: Write the power set of B = {3, 6, 9, 12}

Solution: P(B) = {ϕ, {3}, {6}, {9}, {12}, {3,6}, {3,9}, {3,12}, {6,9}, {6,12}, {9,12}, {3,6,9}, {3,6,12}, {3,9,12}, {6,9,12}, {3, 6, 9, 12}}

Can you tell?

If a set A consists of 4 elements, then how many elements are in $P(A)$?

Remember!

If a set contains 'n' elements, then the number of all its subsets will be 2^n:

For example, if X = {1, 2, 3}, then all its subsets are ϕ, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3} which are 8 in number and $2^3 = 8$

Note that:

- The members of P(A) are all subsets of set A i.e., $\{a\} \in P(A)$ but $a \notin P(A)$
- The power set of ϕ is not empty as number of subsets of ϕ is $2^0 = 1$

  i.e. $P(\phi) = \{\phi\}$ or {{}}

Exercise 1.13

1. Write all subsets of the following sets:

   (i) $\{\}$ (ii) $\{1\}$ (iii) $\{a, b\}$

2. Write the power set of the following sets:

   (i) $\{-1, 1\}$ (ii) $\{a, b, c\}$ (iii) $\{+, -, \times, \div\}$

1.4.5 Describe Operations on Sets in Tabular Form

Union of two sets

The union of two sets is a set containing all elements of both the sets and is denoted by the symbol $\cup$.

For Example:

If $A = \{1, 2, 3\}, B = \{4, 5, 6\}$

then
$$A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\}$$
$$= \{1, 2, 3, 4, 5, 6\}$$

Intersection of two sets

The intersection of two sets is the set of elements that are common in both the sets and is denoted by the symbol $\cap$.

For Example:

If $X = \{1, 2, 3, 4\}, \quad Y = \{2, 4, 6, 8\}$

then
$$X \cap Y = \{1, 2, 3, 4\} \cap \{2, 4, 6, 8\}$$
$$= \{2, 4\}$$

Difference of two sets

If A and B are any two sets, then their difference is given by A–B or B–A. A–B means the set of those elements of A which are not in B.

For Example:

If $A = \{a, b, c, d\}, B = \{a, e, i, o\}$

then
$$A - B = \{a, b, c, d\} - \{a, e, i, o\}$$
$$= \{b, c, d\}$$

and
$$B - A = \{a, e, i, o\} - \{a, b, c, d\}$$
$$= \{e, i, o\}$$

Remember!

We can write $A-B$ as $A \backslash B$ for difference of two sets.

Universal set

A set that consists of all the elements of the sets under consideration, including its own elements, is denoted by the symbol U.

Let us consider an example with two sets A and B.

Here $A = \{a, b, c, d\}, B = \{e, f, g, h\}$

Thus, the universal set U of A and B may be given by

$$U = \{a, b, c, d, e, f, g, h\}$$

Complement of a set

The complement of a set A is defined as a set that contains the elements present in the universal set but not in set A.

For Example:

If $U = \{1, 2, 3, 4, \ldots, 10\}$ and $A = \{2, 4, 6, 8\}$, then the complement of set A,

$A' = U - A$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - \{2, 4, 6, 8\}$

$A' = \{1, 3, 5, 7, 9, 10\}$

The complement of a set A is denoted by A^c or A'.

Remember!

We can write A' as A^c for complement of a set A.

1.4.6 De Morgan's Laws

If A and B are the subsets of a universal set U, then

(a) $(A \cup B)^c = A^c \cap B^c$ (b) $(A \cap B)^c = A^c \cup B^c$

Augustus De Morgan (1806-1871), a British Mathematician who formulated De Morgan's laws.

Example 2: Verify De Morgan's Laws if:

$U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$

Solution: **(a)** L.H.S $= (A \cup B)^c$

$A \cup B = \{2, 4, 6\} \cup \{1, 2, 3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$\therefore \quad (A \cup B)^c = U - (A \cup B) = \{8, 9, 10\}$ .... (i)

R.H.S $= A^c \cup B^c$

$A^c = U - A$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

$B^c = U - B$

$B^c = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 4, 5, 6, 7\}$

$B^c = \{8, 9, 10\}$

$A^c \cap B^c = \{1, 3, 5, 7, 8, 9, 10\} \cap \{8, 9, 10\}$

$= \{8, 9, 10\}$ .... (ii)

Thus, from (i) and (ii) we have $(A \cup B)^c = A^c \cap B^c$

(b) L.H.S $= (A \cap B)^c$

$A \cap B = \{2, 4, 6\} \cap \{1, 2, 3, 4, 5, 6, 7\}$

$= \{2, 4, 6\}$

$(A \cap B)^c = U - (A \cap B)$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$ .... (iii)

R.H.S $= A^c = U - A = \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

and $B^c = U - B = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 4, 5, 6, 7\}$

$$= \{8, 9, 10\}$$

$$A^c \cup B^c = \{1, 3, 5, 7, 8, 9, 10\} \cup \{8, 9, 10\}$$

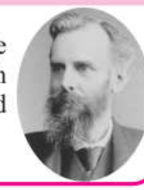
$$= \{1, 3, 5, 7, 8, 9, 10\} \quad \text{.................} \ (iv)$$

Thus, from (iii) and (iv), we have $(A \cap B)^c = A^c \cup B^c$

1.4.7 Venn Diagram

Do you know?

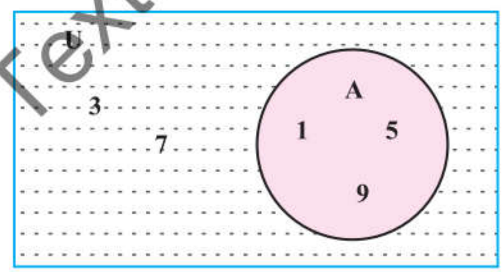
An English logician and the Mathematician John Venn (1834 - 1883A.D) used Venn Diagram first time.



A Venn diagram is an illustration that uses circles or ovals to show the relationships among sets in a perspective way. In Venn diagram, a universal set is usually represented by a rectangle and its subsets are represented by closed figures inside the rectangle, e.g.;

$U = \{1, 3, 5, 7, 9\}$, $A = \{1, 5, 9\}$ and $B = \{5, 9, 11, 13\}$

The rectangular region shown in the figure represents the universal set U and the region enclosed by a closed circle inside the rectangular region represents the set A.

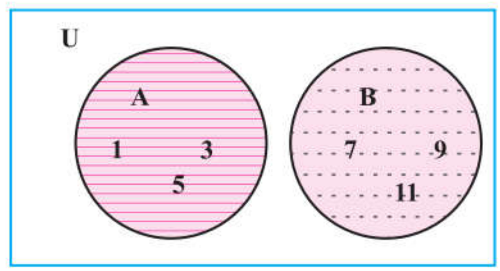


The dotted region of U outside set A represents complement of A i.e. A' or A^c

Thus, $A' = \{3, 7\}$

Disjoint sets

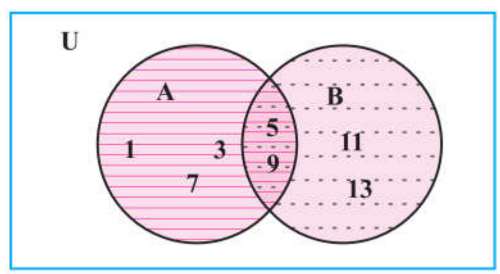
In the given figure, no element of set A and set B is common. Therefore, these sets are disjoint sets. So, the lined portion and the dotted portion represents $A \cup B$. In the same way the dotted portion and the lined portion represents $B \cup A$.



For disjoint sets, $A \cap B = \phi$

Overlapping sets

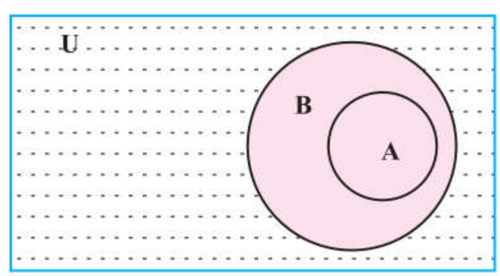
In the given figure, only a small portion is common in both the sets A and B. So, the dotted and the lined common portion between set A and set B is called overlapping sets. This portion is also called $A \cap B$ or $B \cap A$.



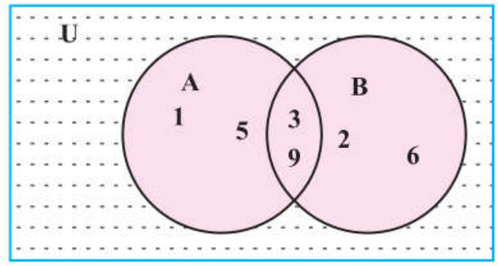
Subsets of a set

In the given figure, the rectangular region represents U (Universal set) and set A and B represent its subsets.

Here $A \subset B$ means all the elements of set A are present in set B.



Union of two sets

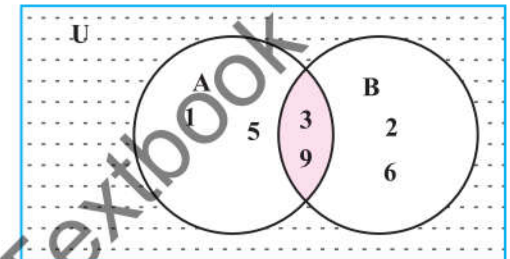


In the given figure, the total region bounded by set A and set B represents $A \cup B$.

Therefore, $A \cup B = \{1, 3, 5, 9\} \cup \{2, 3, 6, 9\}$
$= \{1, 2, 3, 5, 6, 9\}$

The shaded part represents $A \cup B$.

Intersection of two sets

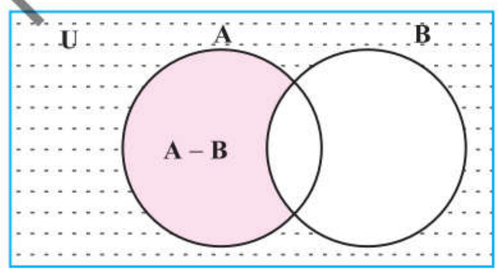


In the given figure, the common region between the two sets A and B represents $A \cap B$.

Therefore, $A \cap B = \{1, 3, 5, 9\} \cap \{2, 3, 6, 9\}$
$= \{3, 9\}$

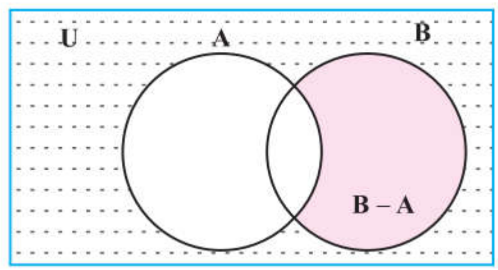
The shaded part represents $A \cap B$.

Difference of two sets



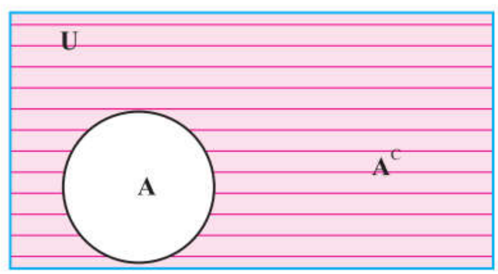
The set of those elements of A which are not in B, is called difference of two sets A and B. It is denoted by $A - B$.

In the given figure, the shaded portion represents $A - B$.



The set of those elements of B which are not in set A is denoted by $B - A$. In the given figure, the shaded portion represents $B - A$.

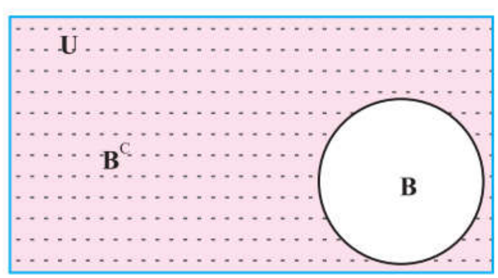
Complement of a set



The given figure represents complement of set A.

$A^c = U - A$

The shaded part represents A^c.



The given figure represents complement of set B.

$B^c = U - B.$

The shaded part represents B^c.

1.4.8 Operations on Sets

Verification of Commutative and Associative Laws with respect to Union and Intersection

- **Commutative Laws of Union and Intersection on Sets**

If A and B are any two sets, then the commutative laws with respect to union and intersection are written as:

(i) $A \cup B = B \cup A$ (Commutative law over union)

(ii) $A \cap B = B \cap A$ (Commutative law over intersection)

Remember!

To find union / intersection of three sets, first we find the union / intersection of any two of them and then the union / intersection of the third set with the resultant set.

Example 3: If $A = \{1, 2, 3, ..., 10\}$ and $B = \{3, 5, 7, 9\}$

(i) Verify the commutative law of union

(ii) Verify the commutative law of intersection

Solution: $A = \{1, 2, 3, ..., 10\}$, $B = \{3, 5, 7, 9\}$

(I) $A \cup B = \{1, 2, 3 ... 10\} \cup \{3, 5, 7, 9\}$

$= \{1, 2, 3, ..., 10\}$

$B \cup A = \{3, 5, 7, 9\} \cup \{1, 2, 3, ..., 10\}$

$= \{1, 2, 3, ..., 10\}$

Therefore, $A \cup B = B \cup A$

(ii) $A \cap B = \{1, 2, 3 ... 10\} \cap \{3, 5, 7, 9\}$

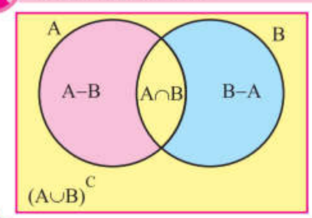
$= \{3, 5, 7, 9\}$

$B \cap A = \{3, 5, 7, 9\} \cap \{1, 2, 3, ..., 10\}$

$= \{3, 5, 7, 9\}$

Therefore, $A \cap B = B \cap A$

Remember!



- **Associative Laws of Union and Intersection**

If A, B and C are any three sets, then the associative laws with respect to union and intersection are written respectively as:

(I) $A \cup (B \cup C) = (A \cup B) \cup C$

(II) $A \cap (B \cap C) = (A \cap B) \cap C$

Example 4: Verify the associative law of union

$A \cup (B \cup C) = (A \cup B) \cup C$

where $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$ and $C = \{6, 7, 8, 9, 10\}$

Solution: L.H.S $= A \cup (B \cup C) = \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6, 7, 8\} \cup \{6, 7, 8, 9, 10\})$

$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ………………… (i)

R.H.S $= (A \cup B) \cup C = (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}) \cup \{6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ………………… (ii)

Thus, from (i) and (ii), we conclude that $A \cup (B \cup C) = (A \cup B) \cup C$

Example 5: Verify the associative law of intersection

$A \cap (B \cap C) = (A \cap B) \cap C$ for sets given in example (4).

Solution: L.H.S $= A \cap (B \cap C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6, 7, 8\} \cap \{6, 7, 8, 9, 10\})$

$= \{1, 2, 3, 4\} \cap \{6, 7, 8\} = \phi$ ………………… (i)

R.H.S $= (A \cap B) \cap C = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}) \cap \{6, 7, 8, 9, 10\}$

$= \{3, 4\} \cap \{6, 7, 8, 9, 10\} = \phi$ ………………… (ii)

Thus, from (i) and (ii), we conclude that $A \cap (B \cap C) = (A \cap B) \cap C$

Verification of Distributive Laws

If A, B and C are any three sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is called the distributive law of union over intersection.

If A, B and C are three sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is called the distributive law of intersection over union.

Example 6: Verify:

(I) Distributive law of union over intersection

(II) Distributive law of intersection over union

where $A = \{1, 2, 3,..., 20\}$, $B = \{5, 10, 15,..., 30\}$ and $C = \{3, 9, 15, 21, 27, 33\}$

Solution: (I) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C) = \{1, 2, 3,..., 20\} \cup (\{5, 10, 15,..., 30\} \cap \{3, 9, 15, 21, 27, 33\})$

$= \{1, 2, 3, ..., 20\} \cup \{15\}$

$A \cup (B \cap C) = \{1, 2, 3,..., 20\}$ ………………… (i)

R.H.S $= (A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 2, 3, ..., 20\} \cup \{5, 10, 15,..., 30\}$

$= \{1, 2, 3,..., 20, 25, 30\}$

and $A \cup C = \{1, 2, 3,...., 20\} \cup \{3, 9, 15, 21, 27, 33\}$

$= \{1, 2, 3,..., 20, 21, 27, 33\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4,..., 20, 25, 30\} \cap \{1, 2, 3 ,..., 20, 21, 27, 33\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3,...., 20\}$ ………………… (ii)

Thus, from (i) and (ii), we conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(II) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C) = \{1,2, 3,..., 20\} \cap (\{5, 10, 15,... , 30\} \cup (3, 9, 15, 21, 27, 33\})$

$= \{1, 2, 3,..., 20\} \cap \{3, 5, 9, 10,15, 20, 21, 25, 27, 30, 33\}$

$A \cap (B \cup C) = \{3, 5, 9, 10, 15, 20\}$ ………………… (i)

R.H.S $= (A \cap B) \cup (A \cap C)$

$A \cap B = \{1, 2, 3,..., 20\} \cap \{5, 10, 15,..., 30\}$

$= \{5, 10, 15, 20\}$

and $\quad A \cap C = \{1, 2, 3, ..., 20\} \cap \{3, 9, 15, 27, 33\}$

$\qquad\qquad\qquad = \{3, 9, 15\}$

$(A \cap B) \cup (A \cap C) = \{3, 5, 9, 10, 15, 20\} \cup (3, 9, 15\}$

$(A \cap B) \cup (A \cap C) = \{3, 5, 9, 10, 15, 20\}$ .... (ii)

Thus, from (i) and (ii), we conclude that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Exercise 1.14

1. Verify:

 (a) $A \cup B = B \cup A$ and (b) $A \cap B = B \cap A$, when

 (i) $A = \{1, 2, 3, ..., 10\}$, $\quad B = \{7, 8, 9, 10, 11, 12\}$

 (ii) $A = \{1, 2, 3, ..., 15\}$, $\quad B = \{6, 8, 10, \ldots, 20\}$

2. Verify:

 (a) $X \cup (Y \cup Z) = (X \cup Y) \cup Z$ and (b) $X \cap (Y \cap Z) = (X \cap Y) \cap Z$, when

 (i) $X = \{a, b, c, d\}$, $Y = \{b, d, c, f\}$ and $Z = \{c, f, g, h\}$

 (ii) $X = \{1, 2, 3, ..., 10\}$, $Y = \{2, 4, 6, 7, 8\}$ and $Z = \{5, 6, 7, 8\}$

 (iii) $X = \{-1, 0, 2, 4, 5\}$, $\quad Y = \{1, 2, 3, 4, 7\}$ and $\quad Z = \{4, 6, 8, 10\}$

 (iv) $X = \{1, 2, 3, ..., 14\}$, $\quad Y = \{6, 8, 10, ..., 20\}$ and $\quad Z = \{1, 3, 5, 7\}$

3. If $A = \{a, b, c\}$, $B = \{b, d, f\}$ and $C = \{a, f, c\}$, then show that:

 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. If $A = \{0\}$, $B = \{0, 1\}$ and $C = \{\}$, then show that:

 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

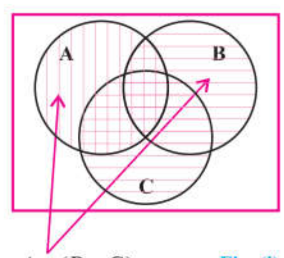
5. Verify De Morgan's Laws if:

 $U = N$, $\quad A = \phi$ and $\quad B = P$

1.4.9 Demonstration of Union and Intersection of three overlapping sets through Venn diagram

(i) $A \cup (B \cup C)$

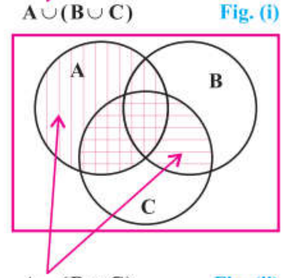
In fig. (i) set $B \cup C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cup (B \cup C)$ is represented by (horizontal, vertical) lines and squares.



$A \cup (B \cup C)$ Fig. (i)

(ii) $A \cup (B \cap C)$

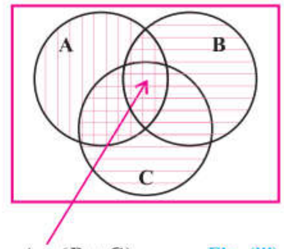
In fig. (ii) set $B \cap C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cup (B \cap C)$ is represented by (horizontal, vertical) lines and squares.



$A \cup (B \cap C)$ Fig. (ii)

(iii) $A\cap(B\cup C)$

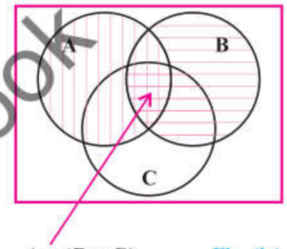
In fig. (iii) set $B \cup C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cap (B \cup C)$ is represented only by squares i.e, small boxes.



$A\cap(B\cup C)$ Fig. (iii)

(iv) $A\cap(B\cap C)$

In fig. (iv) set $B \cap C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cap (B \cap C)$ is represented only by squares i.e, small boxes.



$A\cap(B\cap C)$ Fig. (iv)

Verify associative and distributive laws through Venn diagram

- **Associative Laws**

(a) Associative Law of Union

$A\cup(B\cup C)=(A\cup B)\cup C$

Let $A = \{1, 3, 5, 7, 9, 10\}$, $B = (2, 4, 6, 8, 9, 10\}$ and

$C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A\cup(B\cup C)$

$B\cup C = \{2, 4, 6, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$A\cup(B\cup C) = \{1, 3, 5, 7, 9, 10\} \cup \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$=\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

R.H.S $= (A\cup B)\cup C$

$A\cup B = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(A\cup B)\cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

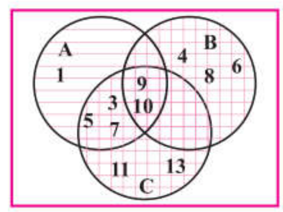


Fig. (v)

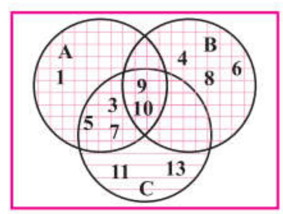


Fig. (vi)

From fig. (v) and (vi), it is clear that:

$A\cup(B\cup C)=(A\cup B)\cup C$

(b) Associative Law of Intersection

$A\cap(B\cap C)=(A\cap B)\cap C$

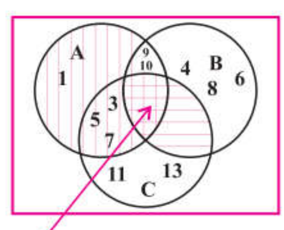
$A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and $C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A\cap(B\cap C)$

$B\cap C = \{2, 4, 6, 8, 9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{2\}$

$A\cap(B\cap C) = \{1, 3, 5, 7, 9, 10\} \cap \{2\} = \{\ \}$

Horizontal lines represent $B\cap C$ and vertical lines represent A, then squares (boxes) represent $A\cap(B\cap C)$. Thus, $A\cap(B\cap C) = \{\ \}$.



$A\cap(B\cap C)$ Fig. (vii)

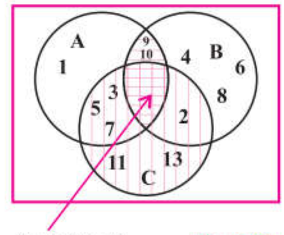
R.H.S $= (A \cap B) \cap C$

$A \cap B = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\} = \{9, 10\}$

$(A \cap B) \cap C = \{9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{\ \}$

Horizontal lines represent $A \cap B$ and vertical lines represent C, then squares represent $(A \cap B) \cap C$. Thus, $(A \cap B) \cap C = \{\ \}$

From fig. (vii) and (viii), it is clear that $A \cap (B \cap C) = (A \cap B) \cap C$



$(A \cap B) \cap C$ Fig. (viii)

- **Distributive Laws**

(a) Distributive Law of Intersection over Union

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

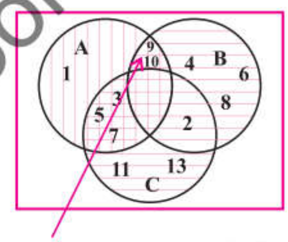
Let $A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and $C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A \cap (B \cup C)$

$B \cup C = \{2, 4, 6, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$B \cup C = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$A \cap (B \cup C) = \{1, 3, 5, 7, 9, 10\} \cap \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\} = \{3, 5, 7, 9, 10\}$



$A \cap (B \cup C)$ Fig. (ix)

Horizontal lines represent $B \cup C$ and vertical lines represent A. Then the squares represent $A \cap (B \cup C)$.

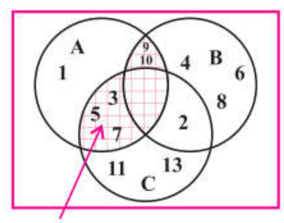
Thus, $A \cap (B \cup C)$ is represented by squares (both horizontal and vertical lines).

R.H.S $= (A \cap B) \cup (A \cap C)$

$A \cap B = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\}$
$= \{9, 10\}$

$A \cap C = \{1, 3, 5, 7, 9, 10\} \cap \{2, 3, 5, 7, 11, 13\}$
$= \{3, 5, 7\}$

$(A \cap B) \cup (A \cap C) = \{9, 10\} \cup \{3, 5, 7\} = \{3, 5, 7, 9, 10\}$



$(A \cap B) \cup (A \cap C)$ Fig. (x)

Horizontal lines represent $A \cap B$, vertical lines represent $A \cap C$ and squares (both horizontal and vertical lines) represent $(A \cap B) \cup (A \cap C)$.

From fig. (ix) and (x), it is clear that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
Hence distributive law of intersection over union holds.

(b) Distributive Law of Union over Intersection

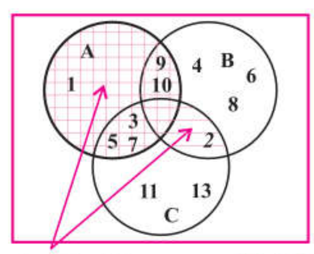
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and
$C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A \cup (B \cap C)$

$B \cap C = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{2\}$

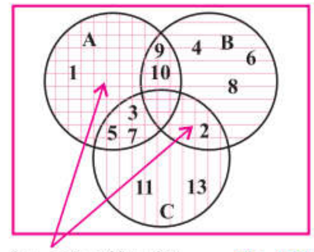
Horizontal lines represent $B \cap C$, vertical lines represent A. Then the squares (horizontal and vertical lines) represent $A \cup (B \cap C)$.



$A \cup (B \cap C)$ Fig. (xi)

$A\cup(B\cap C) = \{1, 3, 5, 7, 9, 10\} \cup \{2\}$

$= \{1, 2, 3, 5, 7, 9, 10\}$



$(A\cup B)\cap(A\cup C)$ Fig. (xii)

R.H.S $= (A\cup B)\cap(A\cup C)$

$A\cup B = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A\cup C = \{1, 3, 5, 7, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{1, 2, 3, 5, 7, 9, 10, 11, 13\}$

$(A\cup B)\cap(A\cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{1, 2, 3, 5, 7, 9, 10, 11, 13\}$

$= \{1, 2, 3, 5, 7, 9, 10\}$

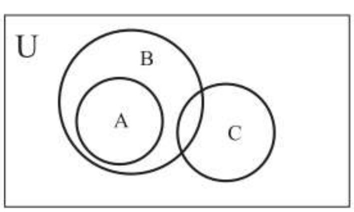
Horizontal lines represent $A\cup B$, vertical lines represent $A\cup C$ and squares represent $(A\cup B)\cap(A\cup C)$. From fig. (xi) and (xii), it is clear that $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$. Hence, distributive law of union over intersection holds.

Exercise 1.15

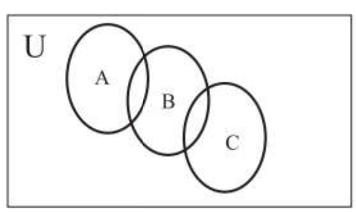
1. Verify the commutative law of union and intersection of the following sets through Venn diagrams.
   - (i) $A = \{3, 5, 7, 9, 11, 13\}$
   - (ii) The sets N and Z.
   - (iii) $C = \{x \mid x \in N \wedge 8 \leq x \leq 18\}$
     $D = \{y \mid y \in N \wedge 9 \leq y \leq 19\}$
   - (iv) The sets E and O.

2. For the given sets, verify the following laws through Venn Diagram.
   - (i) Associative law of Union of sets.
   - (ii) Associative law of Intersection of sets.
   - (iii) Distributive law of Union over Intersection of sets.
   - (iv) Distributive law of Intersection over Union of sets.
     - (a) $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{1, 3, 5, 7, 9, 11\}$ and $C = \{3, 6, 9, 12, 15\}$
     - (b) $A = \{x \mid x \in Z \wedge 8 \leq x \leq 25\}$ and $B = \{y \mid y \in Z \wedge -2 < y < 6\}$ and $C = \{z \mid z \in Z \wedge Z \leq 8\}$

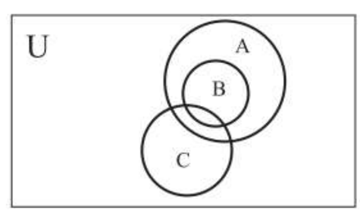
3. Copy the following Venn diagrams and shade according to the operation, given below each diagram.



$(A\cap B)\cup C$



$(A\cup B)\cap C$



$(A\cap B)\cup C$

1.4.10 De Morgans's Laws through Venn diagram

Following are the De Morgans's Laws:

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(i) $(A \cup B)' = A' \cap B'$

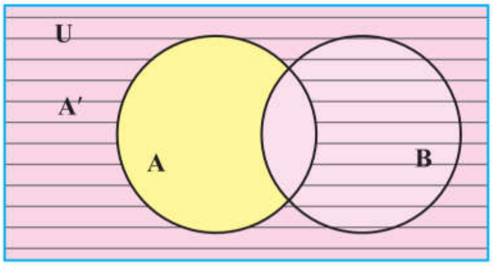


Fig. (i): A′ is shown by horizontal line segments

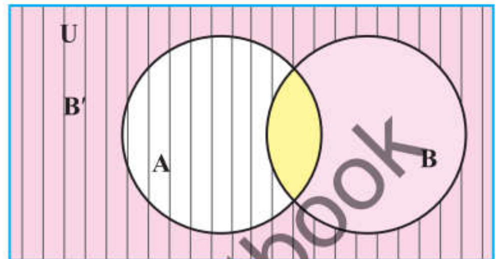


Fig. (ii): B′ is shown by vertical line segments

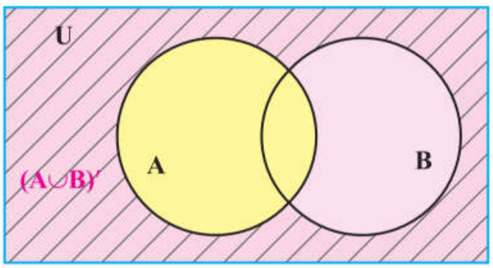


Fig. (iii): $(A \cup B)'$ is shown by slanting line segments

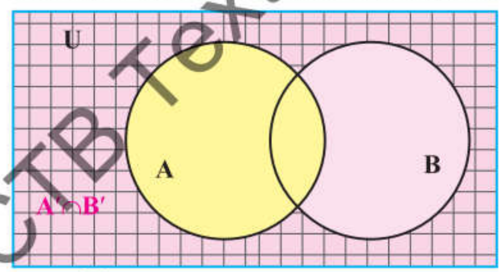


Fig. (iv): $A' \cap B'$ is shown by squares

From Fig. (iii) and (iv), it is clear that $(A \cup B)' = A' \cap B'$.

(ii) $(A \cap B)' = A' \cup B'$

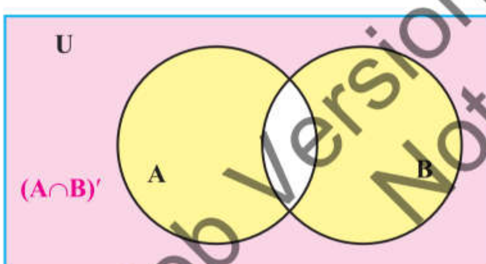


Fig. (v): $U - (A \cap B) = (A \cap B)'$ is shown by shading

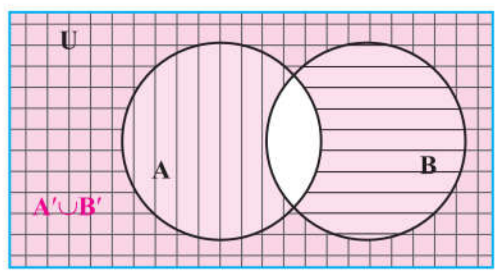


Fig. (vi): $A' \cup B'$ is shown by squares, horizontal and vertical line segments

From Fig. (v) and (vi), it is clear that $(A \cap B)' = A' \cup B'$.

1.4.11 Sets in Real Life Situations

Example: In a class of 50 students, 35 students like mangoes, 25 like bananas and 5 students like neither. Find the number of students who like both.

Solution:

Let U = Total number of students, i.e., $n(U) = 50$

M = Students who like mangoes. $n(M) = 35$

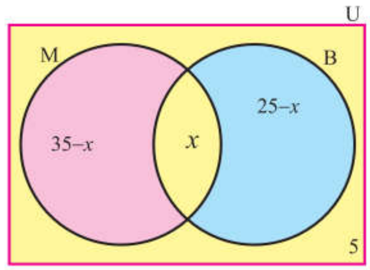
B = Students who like bananas. $n(B) = 25$

x = The number of students who like both mangoes and bananas.

Total number of students $= 50$

$$n(\text{U}) = 50$$
$$35 - x + x + 25 - x + 5 = 50$$
$$65 - x = 50$$
$$x = 65 - 50$$
$$x = 15$$

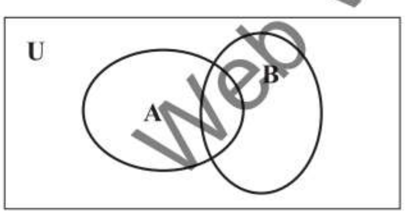
So, 15 students like both mangoes and bananas.



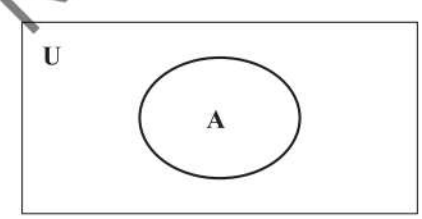
Note:
$n(\text{M}\cup\text{B}) = 15$
$n(\text{M}\backslash\text{B}) = 35 - x$
$= 35 - 15 = 20$
$n(\text{M}\backslash\text{B}) = 25 - x$
$= 25 - 15 = 10$
$n(\text{M}\cup\text{B})' = 5$

Exercise 1.16

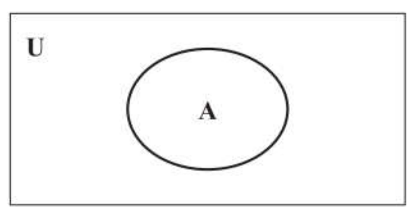
1. Find the Union of the following sets:
   (i) A = {1, 3, 5, 7} , B = {2, 4, 6, 8} (ii) C = {a, b, c}, D = {x, y, z}
   (iii) E = {3, 6, 9, 12} , F = {4, 8, 12}
2. Find the Intersection of the following sets:
   (i) A = {0, ±1, ±2, ±3} , B = {0, 1, 2, 3}
   (ii) C = {3, 6, 9, 12, 15}, D = {1, 3, 5, 7, 9}
   (iii) E = {a, b, c, d}, F = {c, d, e, f}
3. If U = {0, 1, 2, 3, ……………, 10}, A = {2, 4, 6, 8 10}, B = {1, 3, 5, 7, 9},
   C = {0, 3, 6, 9} and D = {5, 6, 7, 8, 9, 10}, then find:
   (i) A^C (ii) B^C (iii) C^C (iv) D^C
4. If A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8} then find:
   (i) A – B (ii) B – A
5. Verify De-Morgan's Laws if U = {1, 2, 3, …………, 15}, A = {1, 3, 5, 7, 9,11,13} and B = {2, 4, 6, 8, 10,12,14}. Verify through Venn Diagrams also.
6. Copy the following figures and shade according to the operation mentioned below each:



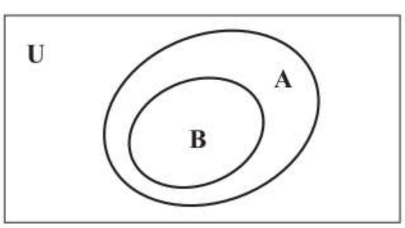
(i) A ∪ B



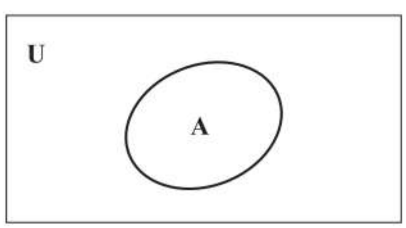
(ii) U ∪ A



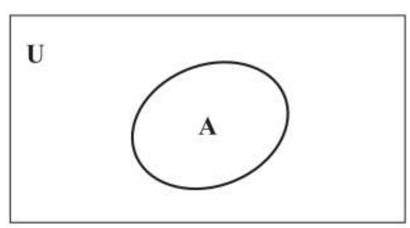
(iii) A ∪ A



(iv) A ∩ B



(v) U ∩ A



(vi) A ∩ A

7. 65 customers choose either red colour shirt or blue colour (or both). If 34 customers choose red colour and 41 customers choose blue colour. How many chose both colours?
8. In a class of 50 students, 22 students have pencils, 32 students have pens and 8 have both. How many don't have both of them?

SUMMARY

- A set can be expressed in three ways.
  (a) Tabular Form or Roster Form (b) Descriptive Form (c) Set Builder Notation
- The objects of a set are called its members or elements.
- Equivalent sets are the sets with an equal number of elements. These sets do not have exactly the same elements, just these sets have the same number of elements.
- Two sets are equal if the sets contain the same elements.
- A set that consists of all the elements of the sets under consideration, including its own elements is called universal set. It is denoted by the symbol U.
- We can write $A-B$ as $A\backslash B$ for difference of two sets.
- The complement of a set A is defined as a set that contains the elements present in the universal set U but not in set A.
- A Venn diagram is an illustration that uses circles to show the relationships among sets in a perspective way. In Venn diagram, a universal set is usually represented by a rectangle and its subsets are represented by closed figures inside the rectangle.
- Intersection of two sets, $A\cap B$, is a set which consists of only the common elements of both A and B.
- Union of two sets, $A\cup B$ is a set which consists of elements of both A and B with common elements represented only once.
- If A and B are any two sets, then
  (i) $A\cup B = B\cup A$ (Commutative law over union)
  (ii) $A\cap B = B\cap A$ (Commutative law over intersection)
- Let A, B and C be any three sets, then
  (i) $A\cup(B\cup C) = (A\cup B)\cup C$ (Associative law of union over sets)
  (ii) $A\cap(B\cap C) = (A\cap B)\cap C$ (Associative law of intersection over sets)
- Let A, B and C be any three sets, then distributive laws are given below.
  (i) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$ (Distributive law of union over intersection)
  (ii) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$ (Distributive law of intersection over union)
- Let A and B be any two sets, then according to the De Morhan's laws.
  (i) $(A\cup B)^c = A^c\cap B^c$
  (ii) $(A\cap B)^c = A^c\cup B^c$

Sub-Domain (v): Ratio, Rate and Proportion

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- recall the difference between direct and inverse proportion
- solve problems involving direct proportion of two quantities using:
  - ❖ table ❖ equation ❖ graph
- solve real life situations/word problems involving compound proportion

Advanced / Additional

- solve problems involving inverse proportion of two quantities using:
  - ❖ table ❖ equation ❖ graph

Ratio is written as:

5 is to 3 or 5 : 3 or $\frac{5}{3}$

Equally to the ratios is proportion

$$\frac{\text{Rs.}10}{\text{Rs.}40} = \frac{2 \text{ chocolates}}{x \text{ chocolates}}$$

1.5.1 Proportion

The relation of equality of two ratios is called proportion, it is denoted by "::".

If two ratios $a:b$ and $c:d$ are proportional to each other, then we can write these as:

Extremes

$$a \quad : \quad b \quad :: \quad c \quad : \quad d$$

Means

> **Remember!**
> Ratio is a comparison of two quantities of same kind. Ratio is denoted by : e.g., ratio a is to b is written as $a:b$, ratio c is to d is written as $c:d$.

> **Remember!**
> In fraction form the ratio "a:b" is written as $\frac{a}{b}$.
> $$a:b \neq b:a$$
> $$\frac{a}{b} \neq \frac{b}{a}$$

If four quantities a, b, c and d are written as:

$$a:b::c:d$$

then a is called the first, b is the second, c is the third and d is the fourth proportional.

To simplify proportion, we write the above as:

$$a \cdot d = b \cdot c$$

Product of Extremes = Product of Means

Example 1: The mass of 72 books is 9 kg. What will be the mass of 80 such books?

Solution: Lets x be the mass of books.

Ratio of mass of books :: Ratio of number of books.

$$9 : x :: 72 : 80$$

$$\text{Product of means} = \text{Product of extremes}$$

$$(72)(x) = (9)(80)$$

$$x = \frac{9 \times 80}{72}$$

$$x = 10 \text{ kg.}$$

Thus, the mass of 80 books will be 10 kg.

> **Remember!**
> If three quantities a, b and c are written as:
> $$a:b::b:c$$
> then these quantities are in continued proportion and b is called the mean proportional.

There are two kinds of proportion.

Proportion

Direct Proportion | Inverse Proportion

Direct Proportion

It is a relation in which increases / decreases one quantity causes proportional increases / decreases in the other quantity, e.g., if we increase the number of eggs, then the cost of eggs will increase.

Direct Proportion by Using Table

Example 2: The cost of 2 pencils is Rs. 10 and 4 pencils is Rs. 20, what will be the cost of 6 and more pencils?

No. of pencils	x	2	4	6	8	10
Cost in (Rs.)	y	10	20			

Solution: The number of pencils is directly proportional to the cost of the pencil (Rs.). In other words, x is directly proportional to y.

To complete the table, we will use the pair of values x as 2 and y as 10 will be used.

We multiply 2 by 2 to obtain 4. As these quantities are in direct proportion, which means that x is double, y will also be double.

×2

No. of pencils	x	2	4	6	8	10
Cost in (Rs.)	y	10	20	30		

×2

Step 1: We multiply 2 by 3 to get 6. Which means, we will multiply 10 by 3 to get 30.

×3

No. of pencils	x	2	4	6	8	10
Cost in (Rs.)	y	10	20	30		

×3

Remember!

In direct proportion, we apply the same operation on both the quantities.

Step-2: We multiply 2 by 4 to get 8, which means, we will multiply 10 by 4 to get 40.

No. of pencils	x	2	4	6	8	10
Cost (Rs.)	y	10	20	30	40	

Step-3: We multiply 2 by 5 to get 10, which means we will multiply 10 by 5 to get 50.

No. of pencils	x	2	4	6	8	10
Cost (Rs.)	y	10	20	30	40	50

Direct Proportion by Using Equation

Example 3: A washerman irons 3 shirts in 15 minutes. How many shirts can be ironed in 45 minutes?

Solution: Let x be the number of shirts which can be ironed in 45 minutes.

We can see that when the time is increased the number of ironed shirts is also increased. So, it is a direct proportion.

So,

Ratio in shirts :: Ratio in time

$$3 : x :: 15 : 45$$

$$\frac{3}{x} = \frac{15}{45}$$

$$15 \times x = 3 \times 45$$

$$x = \frac{3 \times 45}{15}$$

$$x = 9 \text{ shirts}$$

Thus, 9 shirts can be ironed in 45 minutes.

In vertical form, it can be solved as:

Shirts	:	Time (min)
↑ 3	:	↑ 15
x	:	45

$$\frac{x}{3} = \frac{45}{15}$$

$$x \times 15 = 3 \times 45$$

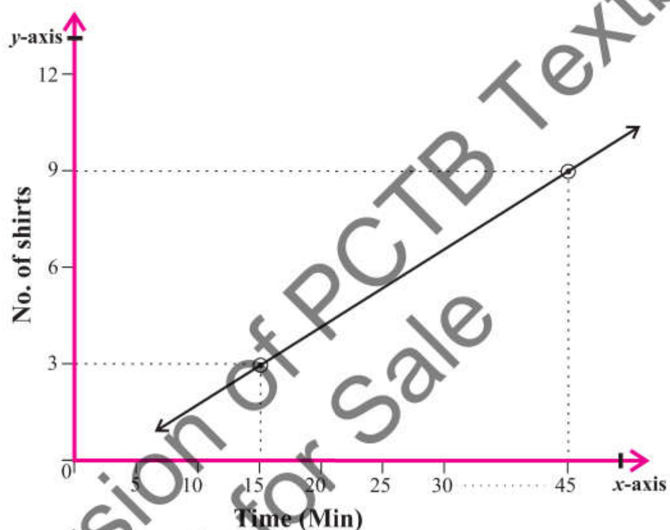
$$x = \frac{3 \times 45}{15}$$

$$x = 9 \text{ shirts}$$

Thus, 9 shirts can be ironed in 45 minutes.

1.5.2 Graphical Representation of Direct Proportion

The given example can also be represented by using graph.



Inverse Proportion

It is a relation in which if one quantity increases / decreases then other quantity decreases / increases proportionally. For example, if we increase the speed of the car, then less time will be required to reach the destination.

Inverse Proportion by Using Table

Example 4: 5 persons build a house in 600 days and 10 persons build the same house in 300 days. How many workers can build the same house in 100 days? Also complete the given table.

No. of workers	x	5	10				
No. of days	y	600	300	200	150	120	100

Solution: As, the number of workers is inversely proportional to the number of days. In other words, x is inversely proportional to y.

To complete the table, the pair of values x as 5 and y as 600.

- As, these variables are in inverse proportion, we will multiply 5 by 2 to get 10 which means we will divide 600 by 2 to get 300.

×2

No. of workers	x	5	10	15			
No. of days	y	600	300	200	150	120	100

÷2

Remember!

In inverse proportion, the variables move in opposite direction. So, If x value is multiplied then y will be divided.

Step 1: We divide 600 by 3 to get 200. As these variables are in inverse proportion, which means we will multiply 5 by 3 to get 15.

× 3

No. of workers	x	5	10	15			
No. of days	y	600	300	200	150	120	100

÷ 3

Step 2: We divide 600 by 4 to get 150, which means we will multiply 5 by 4 to get 20.

× 4

No. of workers	x	5	10	15	20		
No. of days	y	600	300	200	150	120	100

÷ 4

Step 3: We divide 600 by 5 to get 120. Which means we will multiply 5 by 5 to get 25.

× 5

No. of workers	x	5	10	15	20	25	
No. of days	y	600	300	200	150	120	100

÷ 5

Step 4: We divide 600 by 6 to get 100. Which means we will multiply 5 by 6 to get 30

× 6

No. of workers	x	5	10	15	20	25	30
No. of days	y	600	300	200	150	120	100

÷ 6

Thus, 30 workers can build the same house in 100 days.

Inverse Proportion By Using Equation

Example 5: 5 workers take 12 days to weed a field. How many days would 6 workers take to weed it?

Solution: Suppose 6 workers will take x days to weed a field.

We can see that when the number of workers is increased, the less number of days will be required. So, it is an inverse proportion.

Ratio in workers :: Ratio in days

5 : 6 :: 12 : x

$$\frac{5}{6} = \frac{x}{12}$$

$$(6)(x) = (5)(12)$$

$$x = \frac{5\times \cancel{12}^{2}}{\cancel{6}_{1}}$$

$$x = 5\times 2$$

$$x = 10 \text{ days}$$

In vertical, form we can write it as:

Days	:	Workers
12 ↑	:	5 ↓
x	:	6

$$\frac{x}{12} = \frac{5}{6} \quad \text{or} \quad \frac{12}{x} = \frac{6}{5}$$

$$x\times 6 = 12\times 5$$

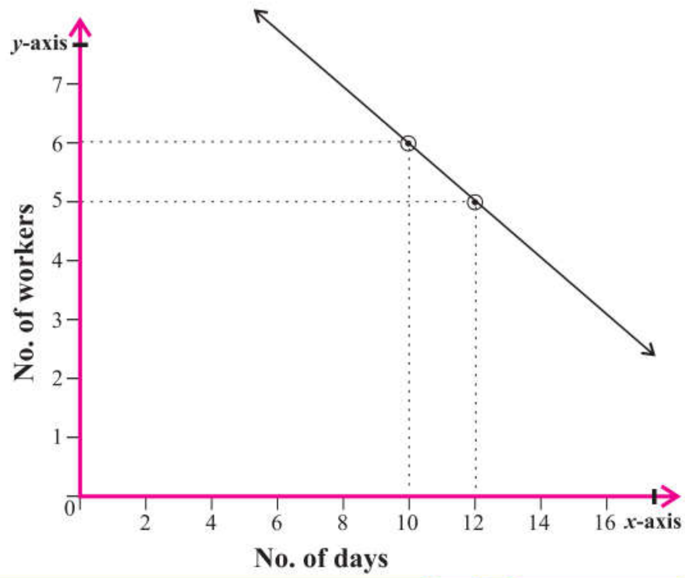
$$x = \frac{\cancel{12}^{2}\times 5}{\cancel{6}_{1}}$$

$$\boxed{x = 10 \text{ days}}$$

Thus, 6 workers will take 10 days.

Graphical Representation of Inverse Proportion

The given example can also be represented by using a graph.



Exercise 1.17

1. Write the ratios in the simplest form.

(i) 75 : 100 (ii) 12 is to 120 (iii) 2 kg is to 800 gm

(iv) $5 : \frac{2}{3}$ (v) $\frac{1}{4} : \frac{1}{6}$ (vi) $\frac{1}{99} : \frac{1}{66} : \frac{2}{33}$

2. Find the unknown in the following proportion:

(i) $12 : p :: 3 : 6$ (ii) $8 : x :: x : 18$ (iii) $5 : 8 :: 15 : m$

(iv) $7 : m :: m : 28$ (v) $x : 2 :: 150 : 100$ (vi) $8 : 12 :: 6 : y$

(vii) $\frac{2}{5} = \frac{p}{20}$ (viii) $\frac{10}{3} = \frac{5}{p}$

3. If 150 shirts can be stitched on 6 sewing machines in a day, how many machines are required to stitch 225 shirts in a day? Also draw its graph.
4. If the price of 12 eggs is Rs. 180, how many eggs can be bought with Rs. 270? Also draw its graph.
5. If 30 labourers dig 800 cm^3 of earth in 3 hours, how many labourers will be required to dig the same earth in 2 hours? Also draw its graph.
6. 10 men have ration for 21 days in a camp. If 3 men leave the camp, for how many days will the ration be sufficient for the remaining men? Also draw its graph.
7. Shayan takes 200 steps for walking a distance of 160m. Find the distance covered by him in 350 steps.
8. In 25 minutes a train travels 20 km, how far will it travel in 5 minutes?
9. A car travelling at 50 km/h makes a journey in 70 minutes. How long will the journey take at 70 km/h? Also draw its graph.
10. A bag of 10 kg potatoes lasts for a week with a family of 7 people. Assuming all eat the same amount. How long will the potatoes last if there are only two in the family? Also draw its graph.
11. If x and y are in direct proportion, then complete the table and draw its graph.

(i)

x	2	6	8	10	12	14
y	8		32		48	

(ii)

x	5	10	15	20		30
y	100			400		

12. If x and y are in inverse proportion, then complete the table and draw its graph.

(i)

x	1	2	4	8	
y	80	40	20	10	

(ii)

x	1	2	4	5	8	
y	200	100			25	20

1.5.3 Compound Proportion

The relationship between two or more proportions is known as compound proportion.

Real Life Problems Involving Compound Proportion

The procedure of solving questions relating to the compound proportion is illustrated below with the help of examples.

Example 6: If 35 labourers dig 805 cm^3 of earth in 5 hours, how much of the earth will 30 labourers dig in 6 hours?

Solution: As the number of labourers decrease, the earth dug will also decrease. It is a direct proportion.

As the working time increase, the earth dug will also increase. It is also a direct proportion.

Let the earth dug be x cm^3

Labourers	:	Hours	:	Earth (cm^3)
35 ↑		5 ↑		805 ↑
30		6		x

$$\Rightarrow \quad \frac{x}{805} = \frac{6}{5} \times \frac{30}{35}$$

$$\Rightarrow \quad x = \frac{6 \times \cancel{30}^{6} \times \cancel{805}^{\cancel{161}^{23}}}{\cancel{5}_{1} \times \cancel{35}_{\cancel{7}_{1}}}$$

$$x = 6 \times 6 \times 23$$

$$x = 828\ cm^3$$

Thus, 828 cm^3 earth will be dug.

Example 7: Rs.8,000 are sufficient for a family of 4 members for 40 days. For how many days Rs. 15,000 will be sufficient for a family of 5 members?

Solution: We see that as amount increases the number of days also increases. So, it is direct proportion.

As the members of a family increase, the number of days decrease. So, it is an inverse proportion.

Let the number of days be x

Rupees	:	Members	:	Days
8,000 ↑		4 ↓		40 ↑
15,000		5		x

$$\Rightarrow \quad \frac{x}{40} = \frac{4}{5} \times \frac{15000}{8000}$$

$$\text{or} \quad = \frac{4 \times 15\,\cancel{000} \times \cancel{40}^{\cancel{5}^{1}}}{\cancel{5}_{1} \times \cancel{8\,000}_{1}}$$

$$x = 4 \times 15 = 60 \text{ days}$$

Thus, the amount shall be sufficient for 60 days.

Example 8: If 4200 men have sufficient food for 32 days at a rate of 12 gram per person, how many men may leave so that the same food may be sufficient for 42 days at a rate of 16 gram per person?

Solution: As the number of days increase, the number of men decreases. So, it is an inverse proportion.

As the quantity of food increases the number of men decreases. So, it is also inverse proportion.

Let the number of men be x.

Days	:	Food	:	Men
32 ↓		12 ↓		4200 ↑
42		16		x

$$\Rightarrow \frac{x}{4200} = \frac{32}{42} \times \frac{12}{16}$$

$$\text{or} \qquad = \frac{\overset{2}{\cancel{32}} \times 12 \times \overset{100}{\cancel{4200}}}{\underset{1}{\cancel{42}} \times \underset{1}{\cancel{16}}}$$

$$x = 2 \times 12 \times 100 = 2400 \text{ men}$$

Thus, the food will be sufficient for 2400 men. So, 4200 − 2400 = 1800 men may leave.

Exercise 1.18

1. 30 men repair a road in 56 days by working 6 hours daily. In how many days 45 men will repair the same road by working 7 hours daily?
2. If 60 women spin 48 kg of cotton by working 8 hours daily, how much cotton will 30 women spin by working 12 hours daily?
3. If the price of a carpet 8 metres long and 3 metres wide is Rs. 6288, what will be the price of 12 metres long and 6 metres wide carpet?
4. If 15 labourers earn Rs. 67,500 in 9 days, how much money will 10 labourers earn in 12 days?
5. 70 men can complete a wall of 150 metres long in 12 days. How many men will complete the wall of length 600 metres in 30 days?
6. If the fare of 12 quintal luggage for a distance of 18 km is 12 rupees, how much fare will be charged for a luggage of 9 quintals for a distance of 20 km?
7. 14 masons can build a wall of 12 metres high in 12 days. How many masons will be needed to build a wall of 120 metres high in 7 days?
8. 15 machines prepare 360 sweaters in 6 days. 3 machines get out of order. How many sweaters can be prepared in 10 days by the remaining machines?

9. 1440 men had sufficient food for 32 days in a camp. How many men may leave the camp so that the same food is sufficient for 40 days when the ration is increased by $1\frac{1}{2}$ times?

[Hint: The 1st element (food) is 1 and the 2nd element (food) is $\frac{3}{2}$]

10. Ten men can assemble 400 cycles in 8 days. How many cycles will 5 men assemble if they work for 16 days?

SUMMARY

- The relation of equality of two ratios is called proportion, it is denoted by "::"
- Ratio is a comparison of two quantities of same kind. Ratio is denoted by ":" e.g., ratio a is to b is written as $a:b$, ratio c is to d is written as $c:d$.
- If four quantities a, b, c and d are written as:

$$a : b :: c : d$$

then a is called the first, b is the second, c is the third and d is the fourth proportional.
- If three quantities a, b and c are written as:

$$a : b :: b : c$$

then these quantities are in continued proportion and b is called the mean proportional.
- There are two types of proportion.

  (i) Direct Proportion (ii) Inverse Proportion.
- Direct proportion is a relation in which increases / decreases one quantity causes proportional increases / decreases in the other quantity, e.g., if we increase the number of eggs, then the cost of eggs will increase.
- Inverse proportion is a relation in which if one quantity increases / decreases then other quantity decreases / increases proportionally. For example, if we increase the speed of the car, then less time will be required to reach the destination.
- Direct proportion in table form, apply the same operation on both the quantities.
- Inverse proportion in table form, the variables move in opposite direction. If x value is multiplied, then y will be divided.

Sub-Domain (vi): Percentage and Financial Arithmetic

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

Currency conversion

- convert Pakistani currency to well-known international currencies and vice versa

Profit, Loss and Discount

- calculate profit percentage and loss percentage
- calculate percentage discount
- solve problems from real life situations involving successive transactions

Profit and Markup

- differentiate profit and markup
- calculate:
  - ❖ the profit/ markup
  - ❖ the principal amount
  - ❖ the profit/ markup rate, time period
- solve problems from real life situation involving profit/markup

Insurance

- solve real life problems involving
  - ❖ Insurance
  - ❖ Partnership
  - ❖ Inheritance (according to Islamic principles)

1.6.1 Conversion of Currencies

A foreign currency exchange rate is a price that represents how much it costs to buy the currency of one country using the currency of another country.

Convert Pakistani Currency to Well-Known International Currencies

Currency conversion rates are not permanent but these change day by day. We use these currency rates to convert Pakistani currency to different international currencies. (rate of one US $ is equal to Rs. 181.70)

Example 1: Saud wants to exchange Pakistani Rupees (PKR) 50,000 to US Dollars. How many US Dollars will he receive? (Rate of one US $ = Rs. 181.70)

Solution:

Amount to be converted = Rs. 50,000

Rate of one US Dollar = Rs. 181.70

Number of US Dollars $= \frac{50,000}{181.70}$ = US $ 275.2

Example 2: Convert Rs. 75,810 into UK £ (1 UK Pound = Rs. 237.5).

Solution:

Amount to be converted = Rs. 75,810

Rate of one UK Pound = Rs. 237.5

Number of UK Pounds $= \frac{75,810}{237.5}$ = UK £ 319.2

Table below shows current exchange rates of some currencies.

Country	Currency	Symbol	Buying (PKR)	Selling (PKR)
US	Dollar ($)	$	181.70	183
UK	Pound (£)	£	237.5	240
Saudi Arabia	Riyal (SR)	SAR	48	48.75
India	Rupee	₹	2.03	2.10

Exercise 1.19

1. Convert Rs. 70,000 into US $ if the conversion rate is 1 US $ = Rs.181.70.
2. Convert Rs. 75,000 into UK £. (Rate 1 UK £ = Rs. 237.5).
3. Converts. 50,000 into Saudi Riyal. (Rate 1 SAR = Rs. 48).
4. Convert Rs. 48,000 into Indian Rupee. (1 INR = Rs. 2.03).
5. Convert Rs. 35,000 into Australian Dollar. (1 Australian Dollar = Rs. 134).
6. Convert Rs. 80,000 into Chinese Yaun. (1 Chinese Yaun = Rs. 23.55).
7. Convert Rs. 50,000 into Canadian Dollar. (1 Canadian Dollar = Rs. 142.50).
8. Convert Rs. 70,000 into Turkish Lira. (1 Turkish Lira = Rs. 12.24).

1.6.2 Percentage

Percentage is widely used in our everyday life. Percentage is another way of expressing fraction or decimal.

The percentage means "per hundred".

The symbol used for percentage is %.

Try yourself!

Change 60% to a fraction.

Change $\frac{4}{5}$ to a percentage.

1.6.3 Profit

If the selling price (S.P.) is higher than the cost price (C.P.) then profit occurs. It can be written as:

Profit = Selling Price – Cost Price

Profit = S.P – C.P

Try yourself!

Change 10% to a decimal.

Profit Percentage

Profit percentage is always expressed in terms of cost price. To find out the profit percentage, we use the following formula.

$$\text{Profit percentage} = \frac{\text{profit}}{\text{cost price}} \times 100$$

Example 3: Abid bought a motor-cycle for Rs. 50,000 and sold it for Rs. 56,000. Find his percentage profit.

Solutions:

Cost Price (C.P.) = Rs. 50,000

Selling Price (S.P.) = Rs. 56,000

Profit = S.P – C.P

= 56,000 – 50,000

= Rs. 6,000

$$\text{Profit\%} = \frac{\text{profit}}{\text{C.P}} \times 100$$

$$= \frac{6000}{50000} \times 100$$

= 12%

1.6.4 Loss

If the cost price (C.P.) is higher than the selling price (S.P.), then loss occurs. It can be written as:

Loss = Cost Price – Selling Price

Loss = C.P. – S.P.

Loss Percentage

Loss percentage is also expressed in terms of cost price. To find out the loss percentage, we use the following formula:

$$\text{Loss percentage} = \frac{\text{loss}}{\text{cost price}} \times 100$$

Example 4: Hameed bought a piece of land worth Rs, 300000 and sold it for Rs. 240000. Find his profit / loss percentage?

Solution:

$$\begin{aligned}
\text{Cost Price (C.P)} &= \text{Rs. } 300000 \\
\text{Sale Price (S.P)} &= \text{Rs. } 240000 \\
\text{Loss} &= \text{C.P.} - \text{S.P.} \\
&= 300000 - 240000 \\
&= \text{Rs. } 60{,}000 \\
\text{Loss percentage} &= \frac{\text{loss}}{\text{C.P.}} \times 100 \\
&= \frac{60{,}000}{300{,}000} \times 100 \\
&= \text{Rs. } 20\%
\end{aligned}$$

1.6.5 Discount

Discount means to reduce the price of an article from its marked price which is also called list price or regular price. After reduction the amount is known as the selling price. The discount is the amount you saved in buying an article.

$$\text{Discount} = \text{Marked price} - \text{Selling price}$$

Percentage Discount

The discount is usually expressed as the percentage discount of the market price. To find the percentage discount. we use the following formula:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{marked price}} \times 100$$

Example 5: Ali bought some articles of worth Rs. 2,500. He was allowed 15% discount on his purchase. Find price he paid of the said articles.

Solution:

$$\begin{aligned}
\text{Marked price} &= \text{Rs. } 2{,}500 \\
\text{Discount} &= 15\% \\
\text{Discount on the articles} &= \frac{25\cancel{00} \times 15}{1\cancel{00}} \\
&= \text{Rs. } 375 \\
\text{So, the selling price} &= 2500 - 375 \\
&= \text{Rs. } 2{,}125
\end{aligned}$$

Example 6: The marked price of an article is Rs.1,700. The selling price of the article is Rs. 1,360. Find the percentage discount.

Solution:

$$\begin{aligned}
\text{Market price} &= \text{Rs. } 1{,}700 \\
\text{Sale price} &= \text{Rs. } 1{,}360 \\
\text{Discount} &= \text{M.P} - \text{S.P} \\
&= 1700 - 1360 \\
&= \text{Rs. } 340 \\
\text{Percentage Discount} &= \frac{\text{Discount}}{\text{Market price}} \times 100 \\
&= \frac{\overset{20}{\cancel{340}}}{\cancel{1700}} \times \cancel{100} \\
&= 20\%
\end{aligned}$$

1.6.6 Solve Problems Involving Successive Transactions

Example 7: The cost price of an article is Rs. 6,000. The shopkeeper writes the market price of the article 15% above the cost price. The selling price of that article is Rs. 4600. Find percentage discount given to the customer.

Solution:

$$\begin{aligned}
\text{Cost Price} &= \text{Rs. } 6{,}000 \\
\text{Percentage increase} &= 15\% \\
\text{Total increase on cost price} &= \frac{6000 \times 15}{100} \\
&= \text{Rs. } 900 \\
\text{Market Price} &= 6000 + 900 \\
&= \text{Rs. } 6900 \\
\text{Selling Price} &= \text{Rs. } 4600 \\
\text{Discount} &= M.P - S.P \\
&= 6900 - 4600 \\
&= \text{Rs. } 2300 \\
\text{Percentage discount} &= \frac{\text{Discount}}{\text{Market price}} \times 100 \\
&= \frac{\overset{1}{\cancel{2300}}}{\underset{3}{\cancel{6900}}} \times 100 \\
&= \frac{100}{3} \\
&= 33\frac{1}{3}\%
\end{aligned}$$

Example 8: A wholesaler sold an article to a retailer at a profit of 10%. The retailer sold it for Rs. 1897.50 at a profit of 15%. What is the cost of wholesaler?

Solution:

Selling price of the retailer $=$ Rs. 1897.50 $=$ Rs. $\frac{3795}{2}$

Profit $=$ 15%

Cost price of retailer $=$?

Let the cost price of the retailer $=$ Rs. 100

Profit $=$ 15%

Sale price of retailer $=$ 100 + 15 = Rs. 115

If the selling price of retailer is Rs. 115, his cost price $=$ Rs. 100

If the selling price of retailer is Rs. 1, his cost price $= \frac{100}{115}$

If the selling price of retailer is Rs. $\frac{3795}{2}$ his cost price $= \frac{\overset{50}{\cancel{100}}}{\underset{1}{\cancel{\underset{23}{\cancel{115}}}}} \times \frac{\overset{33}{\cancel{\overset{759}{\cancel{3795}}}}}{\underset{1}{\cancel{2}}}$

$= 50 \times 33$

$=$ Rs. 1,650

The cost price of retailer $=$ The sale price of wholesaler

Sale price of wholesaler $=$ Rs. 1,650

Let the cost price of the wholesaler $=$ Rs. 100

Profit $=$ 10%

Sale price of wholesaler $=$ 100 + 10 $=$ Rs. 110

If the selling price of wholesaler is Rs. 110, then his cost price $=$ 100

If the selling price of the wholesaler is Rs. 1, then cost price $= \frac{100}{110}$

If the selling price of wholesaler is Rs. 1,650, the cost price is $= \frac{100}{\underset{1}{\cancel{110}}} \times \overset{15}{\cancel{1650}}$

$=$ Rs. 1,500

The cost of wholesaler $=$ Rs. 1,500

Exercise 1.20

1. Haneef bought a car for Rs.550000. He sold it for Rs.605000 after some time. Find his profit percentage.
2. The marked price of an article is Rs.3000. Discount on this article is 20%. Find the selling price of the article.
3. A manufacturer sells an article which cost him Rs. 2500 at 20% profit. The purchaser sells the article at 30% gain. Find the final sale price of the article.
4. The marked price of every article was reduced by12% in sale at a store. A cash customer was given

a further 10% discount. What price would a cash customer pay for an article marked initially as Rs.2000?

5. Tahir purchased two toys for his children. He buys Spider Man and Barbie Doll for Rs.3000, and Rs.5000 respectively. If a discount of 20% is given on all toys, find the amount of discount and the selling price for each toy.
6. Tufail buys some items from a store. A special discount of 15% is offered on food items and 20% on other items. If he purchases food worth Rs. 1250 and other items worth Rs. 750, find the amount of discount and selling price of each separately.
7. A wholesaler sets his selling price by adding 15% to his cost price. The retailer adds 25% to the price he pays to the wholesaler to fix his selling price. At what price would a retailer sell an article which costs the wholesaler Rs. 400.

1.6.7 Profit / Markup

Profit

When we deposit money into a bank, the bank uses our money and in return pays an extra amount alongwith our actual deposit. The extra money which the bank gives for the use of our amount is called profit on the deposit.

Markup

When we borrow money from bank to run a business, the bank in return receives some extra amount alongwith the actual money given. This extra money which the bank receives is known as markup.

Principal Amount

The amount we borrow or deposit in the bank is called principal amount.

Profit / Markup Rate

The rate at which the bank gives share to its account holders is known as profit / markup rate. It is expressed in percentage.

Period

The time for which a particular amount is invested in a business is known as period.

Calculate the profit / markup, the principal amount, the profit / markup rate, the period

Calculate Profit / Markup

For calculation of profit / markup, we use the formula.

$$\text{Profit / markup} = \text{Principal} \times \text{Time} \times \text{Rate}$$

or $$I = P \times R \times T = PRT$$

The use of this formula is illustrated with the help of examples.

Example 9: Younas borrowed Rs. 65,000 from a bank at the rate of 5% for 2 years. Find the amount of markup and the total amount to be paid.

Here principal (P) = Rs. 65,000
Rate (R) = 5%
Time (T) = 2 years

$$\text{Markup} = P \times R \times T$$

$$\text{Markup} = 65,000 \times \frac{5}{100} \times 2$$

$$= 650 \times 5 \times 2$$

$$= \text{Rs. } 6,500$$

So, Younas will have to pay Rs. 6,500 as markup.

Total amount to be paid = 65,000 + 6,500 = Rs. 71,500

Example 10: A student purchased a computer by taking loan from a bank on simple interest. He took loan of Rs. 25,000 at the rate of 10% for 2 years. Calculate the markup to be paid and the total amount to be paid back.

Solution: Here Principal (P) = Rs. 25,000

Rate (R) = 10%

Time (T) = 2 years

$$\text{Markup} = P \times R \times T$$

$$= 25000 \times \frac{10}{100} \times 2$$

$$= 250 \times 20 = \text{Rs. } 5,000$$

He has to pay Rs. 5,000 as markup.

Total amount to be paid = 25,000 + 5,000 = Rs. 30,000

Calculate Principal Amount

We have used formula of markup in the previous examples, we will use the same formula to calculate principal amount.

$$I = P \times R \times T$$

$$P = \frac{I}{R \times T}$$

Example 11: What principal amount is taken to bring in Rs. 640 as profit at the rate of 4% in 2 years?

Solution: Profit = Rs. 640

Rate (R) = 4%

Time (T) = 2 years

$$\text{Principal amount} = \frac{\text{Profit}}{R \times T}$$

$$= \frac{\overset{\overset{80}{\cancel{160}}}{\cancel{640}} \times 100}{\cancel{4} \times \cancel{2}}$$

$$= 80 \times 100$$

$$= Rs. 8,000$$

Thus, the principal amount = Rs. 8,000

Example 12: A person got some loan on which he has to pay Rs. 3,500 as markup at the rate of 10% for 3.5 years. What is the amount of loan?

Solution:

$$\text{Markup} = \text{Rs. } 3{,}500$$

$$\text{Rate (R)} = 10\%$$

$$\text{Time (T)} = 3.5\text{years} = \frac{7}{2}\text{ years}$$

$$\text{Principal (P)} = \frac{\text{Markup}}{\text{Rate} \times \text{Time}}$$

$$= \frac{\overset{50}{\cancel{3500}} \times 100 \times 2}{\underset{1}{\cancel{10}} \times \cancel{7}}$$

$$= 50 \times 200$$

$$= \text{Rs. } 10{,}000$$

$$\text{Thus, the amount of loan} = \text{Rs. } 10{,}000$$

Calculate Profit / Markup Rate

The formula for calculation of profit rate is

$$\text{Rate} = \frac{\text{Markup} \times 100}{\text{Principal} \times \text{Time}}$$

Example 13: At what annual rate percent of markup would the principal amount Rs. 68,000 become Rs. 86,360 in 3 years?

Solution:

$$\text{Total amount to be paid} = \text{Rs. } 86{,}360$$

$$\text{Principal (P)} = \text{Rs. } 68{,}000$$

$$\text{Markup} = 86{,}360 - 66{,}000$$

$$= \text{Rs. } 18{,}360$$

$$\text{Period / Time (T)} = 3 \text{ years}$$

$$\text{Rate (R)} = \frac{\text{Markup} \times 100}{\text{Principal} \times \text{Time}}$$

$$= \frac{\overset{612}{\cancel{18360}} \times 1\cancel{00}}{68\cancel{000} \times \cancel{3}}$$

$$= \frac{612}{68} = 9\%$$

$$\text{Rate of markup} = 9\%$$

Calculate the Period

Example 14: A person got loan from a bank at a rate of 3% per year for some period. In how much period his loan of Rs. 65,000 will become Rs. 68,900.

Solution:

$$\begin{aligned}
\text{Total amount} &= \text{Rs. } 68{,}900 \\
\text{Principal (P)} &= \text{Rs. } 65{,}000 \\
\text{Markup} &= 68{,}900 - 65{,}000 \\
&= \text{Rs. } 3{,}900 \\
\text{Rate} &= 3\% \\
\text{Period / Time (T)} &= ? \\
\text{Period / Time (T)} &= \frac{\text{Markup} \times 100}{\text{Principal} \times \text{Rate}} \\
&= \frac{\cancel{3900} \times \cancel{100}}{\cancel{65000} \times \cancel{3}} \\
&= 2 \text{ years}
\end{aligned}$$

Exercise 1.21

1. Find the profit on Rs. 40,000 at the rate of 3% per year for 4 years.
2. Saud borrowed Rs. 25,000 from bank at the rate of 6% per year for 3 years. Find the markup of the bank.
3. Find the principal amount invested by Riaz in a business if he receives a profit of Rs. 4200 in 3 years at the rate of 10% per year.
4. Ajmal invested some amount in a business. He receives a profit of Rs. 27,000 at the rate of 12% per year for 3 years. Find his original investment.
5. At what annual rate of markup would Rs.6,800 amount become Rs.9,044 in 11 years?
6. At what annual rate of profit would a sum of Rs.5,800 will increase to Rs.7,105 in 3 years' time?
7. How long would Rs. 15,500 have to be invested at a markup rate of 6% per year to gain Rs. 2790.
8. How long would Rs. 25,000 have to be deposited in the bank at 12% per year to receive back Rs. 31,000?

1.6.8 Insurance

Definition of Insurance:

Insurance is a system of protecting or safeguarding against risk or injuries. It provides financial protection for property, life, health, etc. against specified contingencies such as death, loss or damage and involving payment of regular premium in return for a policy guaranteeing. The contract is called the insurance policy. The party bearing the risk is the insurer or assurer and the party whose risk is covered is known as insured or assured.

There are many different types of insurance including health, life, property, etc. We will learn about only two types in this grade namely (i) Life insurance and (ii) Vehicle insurance

1.6.9 Solve Real Life Problems Regarding Life and Vehicle Insurance

Life Insurance

Life insurance is an agreement between the policy owner and the insurance company for an agreed time period. Insurance company agrees to pay back a sum equal to original amount and the profit at the end of agreed period or on the death or critical illness of the policy owner. In return the policy owner agrees to pay regular installments of premium.

Example 15: Saud got a life insurance policy of Rs. 500000. Rate of annual premium is 4.5% of the total amount of the policy whereas the policy fee is at the rate of 0.25%. Find the annual premium of the policy.

Solution:

Policy amount = Rs. 500000

Policy fee @ 0.25% = $\frac{25}{1\cancel{00}} \times 5\,00\cancel{000} \times \frac{1}{1\cancel{00}}$

= Rs. 1250

First premium @ 4. 5% = $\frac{45}{1\cancel{0}} \times \frac{1}{1\cancel{00}} \times 5\,00\cancel{000}$

= Rs. 22,500

Period / Time (T) = $\frac{\text{Markup}}{\text{Principal} \times \text{Rate}}$

Annual premium = First premium + policy fee

= 22,500 + 1,250 = Rs. 23,750

Example 16: A man purchased a life insurance policy for Rs. 300000. The annual premium is 4.5% of the policy amount whereas policy fee is at the rate of 0.25%. Calculate the annual premium and quarterly premium at 27% of the annual premium.

Solution:

Policy amount = Rs. 300000

Policy fee @ 0.25% = $\frac{25}{1\cancel{00}} \times \frac{1}{1\cancel{00}} \times 30\cancel{00}\,\cancel{00}$

= Rs. 750

First premium @ 4. 5% = $\frac{45}{1\cancel{0}} \times \frac{1}{1\cancel{00}} \times 300\,\cancel{00}\cancel{0}$

= Rs. 13,500

Annual premium = First premium + policy fee

= 13,500 + 750

= Rs. 14,250

Quarterly premium = $\overset{285}{\cancel{1425}}\cancel{0} \times \frac{27}{\underset{2}{\cancel{10}}\cancel{0}}$

$= \frac{285 \times 27}{2}$ = Rs. 3847.50

Vehicle Insurance

Vehicle insurance provides a protection against risks to the vehicle. The amount of policy in this case depends upon the actual value of the vehicle.

Example 17: Aslam got his motorcycle insured for one year. The price of his motorcycle is Rs.50,000 and the rate of insurance is 4.5%. Find the amount of premium.

Solution:

The price of motorcycle = Rs. 50,000

Rate of insurance = Rs. 4.5%

Amount of premium = $\frac{4.5}{100} \times 50000$

$$= \frac{4.5}{1\cancel{0}} \times \frac{1}{1\cancel{00}} \times 50\cancel{0}\cancel{00}$$

$= $ Rs. 2,250

Example 18: Khalid purchased an insurance policy for his car. The worth of the car is Rs. 750000. The rate of annual premium is 3% for two years and depreciation rate is 10%. Find the total amount he paid as premium.

Solution:

Worth of car = Rs. 750000

Rate of annual premium = 3%

Depreciation rate = 10%

Time period = 2 years

First premium $= \frac{3}{\cancel{100}} \times 7500\cancel{00} =$ Rs. 22,500

Depreciation after one year = 10% of 750000

$= \frac{10}{\cancel{100}} \times 7500\cancel{00} =$ Rs. 75,000

Depreciation after one year = 750000 − 75000

= Rs. 67,5000

2^{nd} premium = 3% of 675000

$= \frac{3}{\cancel{100}} \times 6750\cancel{00}$

= Rs. 20,250

Depreciation after 2 years = 10% of 675000

$= \frac{10}{\cancel{100}} \times 6750\cancel{00}$

= Rs. 67,500

Depreciation after 2 years = 675000 − 67500

= Rs. 60,7500

Total amount paid as premium = 22,500 + 20,250

= Rs. 42,750

Exercise 1.22

1. Usman purchased a car for Rs. 1250000 and insured it for one year at the rate of 4.5%. Find the annual premium.
2. Hameed got a life insurance policy of Rs.200000. Find the first premium he has to pay when the rate of annual premium is 5.2% and policy fee is 0.25%.
3. Zahid got a life insurance policy of Rs.500000 at the rate of 5.2% and the policy fee is 0.25%. Calculate half yearly premium at 52% of the annual premium.
4. Usama insured his life for Rs. 700000. Find annual premium at 4.5% of the policy amount with policy fee at the rate of 0.25%. Calculate monthly premium at 9% of the annual premium.
5. Saud bought a car for Rs.700000 and got it insured at 4.2% annual premium for 3 years. Calculate how much premium he paid in 3 years if depreciation rate is 12%.

6. A man has a car of worth Rs. 1400000. He got it insured for a period of 2 years at the rate of 4.5%. The depreciation rate is 10% per year. He has to pay the premium yearly. Find the total amount of premium he has to pay for a period of 2 years.

7. Asma insured her life for Rs. 1,000,000 at the rate of 5% per year. Find the amount of annual premium she has to pay.

1.6.10 Partnership

A business in which two or more persons run the business and they are responsible for the profit and loss is called the partnership.

If the partners start the business and close it together with same or different investment capital, this partnership is called a simple partnership.

If the partners contribute different capitals for different time periods or at least one partner contributes two or more capitals for different time periods, then this partnership is called a compound partnership. In this case, the profit or loss is divided in the ratio of monthly investments.

Example 19: Saud and Ammar started a business with capitals of Rs.56,000 and Rs. 64,000 respectively. After one year they earned a profit of Rs.22,500. Find the share of each one.

Solution: The simplified form of capital share ratio:

Saud's share	**:**	**Ammar's share**
56,000	:	64,000
56	:	64
7	:	8

Sum of ratios $= 7 + 8 = 15$

Total profit $=$ Rs. 22,500

Saud's profit $= \dfrac{7}{\cancel{15}_{1}} \times \cancel{22500}^{1500} = 7 \times 1500 =$ Rs. 10,500

Ammar's profit $= \dfrac{8}{15} \times 22500$

$= 8 \times 1500 =$ Rs. 12,000

Example 20: Tahir started a business with a capital of Rs. l5,000. After 5 months Umar also joined him with an investment of Rs. 30,000. At the start of 9 month's Usman joined them by investing Rs.45,000. At the end of the year, they earned a profit of Rs.406000. Find the share of each one.

Solution:

Tahir's investment for 12 months	=	Rs. 15,000
Tahir's effective investment for 1 month	=	15000 × 12
	=	Rs. 18,0000
Umar's investment for 7 months	=	Rs. 30,000
Umar's effective investment for 1 month	=	30,000 × 7
	=	Rs. 21,0000
Usman's investment for 3 months	=	Rs. 45,000
Usman's effective investment for 1 month	=	45,000 × 3
	=	Rs. 13,5000

Tahir	:	Umar	:	Usman
1,80,000	:	2,10,000	:	1,35,000
180	:	210	:	135
12	:	14	:	9

$$\text{Sum of ratios} = 12 + 14 + 9 = 35$$

$$\text{Tahir's share} = \frac{12}{\underset{\underset{1}{\cancel{7}}}{\cancel{35}}} \times \overset{\overset{11600}{\cancel{81200}}}{\cancel{406000}}$$

$$= 12 \times 11600 = \text{Rs. } 139200$$

$$\text{Umar's share} = \frac{14}{\underset{1}{\cancel{35}}} \times \overset{11600}{\cancel{406000}}$$

$$= 14 \times 11600 = \text{Rs. } 162400$$

$$\text{Usman's share} = \frac{9}{\underset{1}{\cancel{35}}} \times \overset{11600}{\cancel{406000}}$$

$$= 9 \times 11600 = \text{Rs. } 104400$$

Example 21: Saud, Ali and Saad started a business with Rs.15,000, Rs.19,000 and Rs. 12,000 respectively. Saud manages the business and receives allowance of Rs. 16,000 for this assignment. After 5 months Ali withdraws Rs.9,000 and business is closed after 9 months. What did each receive in the profit of Rs.58,000.

Solution:

Saud's capital for 9 months	=	Rs. 15,000
Saud's effective capital for 1 months	=	15000×9
	=	Rs. 1,35,000
Ali's capital for 5 months	=	Rs. 19,000
Ali's effective capital for 1 months	=	19000×5
	=	Rs. 95,000
Ali's capital for 4 months	=	Rs. 10,000
Ali's effective capital for 1 months	=	10000×4
	=	Rs. 40,000
Ali's total capital	=	95,000 + 40,000
Saad's capital for 9 months	=	Rs. 12,000
Saad's effective capital for 1 months	=	12000×9
	=	Rs. 10,8,000
Total profit	=	Rs. 58,000
Saud's Allowance	=	Rs. 16,000
Net profit = 58,000 – 16,000	=	Rs. 42,000

Ratios of Capitals	Saud's	:	Ali	:	Saad's
	1,35,000	:	1,35,000	:	1,08,000
	135	:	135	:	108
	15	:	15	:	12
	5	:	5	:	4

Sum of ratios $= 5 + 5 + 4 = 14$

Tahir's share $= \frac{5}{\cancel{14}_{1}} \times \cancel{42000}^{3000}$

$= 5 \times 3000$

$=$ Rs. 15,000

Saud's allowance $=$ Rs. 16,000

Saud received $=$ Total of Saud's Profit + Allowance

$= 15,000 + 16,000$

$=$ Rs. 31,000

Exercise 1.23

1. Aslam and Akram invested Rs. 27,000 and Rs. 30,000 to start a business. If they earned a profit of Rs. 66,500 at the end of the year, find the profit of each one.
2. Amina and Maryam started a business with investment of Rs. 30,000 and Rs. 40,000 respectively in one year. At the end of the year they earned a profit of Rs. 8400. Find the share of each one.
3. Two partners contributed Rs.4000 and Rs. 3000. 1st contributed for 9 months and the 2nd contributed the amount for 7 months. Divide a profit of Rs. 11,590 between the partners.
4. Saad, Saud and Saeed started a business with capital of Rs. 12,000, Rs. 18,000 and Rs. 24,000 respectively. At the end of the year, they suffered with a loss of Rs.13,500. Find the share of each in this loss.
5. Akram and Asghar started a business with Rs. 9,000 and Rs. 11,000 respectively. Akram withdraws Rs. 1000 after 6 months. After 2 months of his withdrawal Asghar invested Rs.1000 more. After a year they earned a profit of Rs. 14,000. Find the share of each in the profit.
6. Three friends A, B and C started a firm with Rs. 20,000, Rs. 16,000 and Rs. 18,000 respectively. A kept his money for 4 months, B for 6 months and C for 8 months. Divide a profit of Rs. 12,000 among these friends.

1.6.11 Inheritance

When a person dies, then the assets left by him are called inheritance and it is distributed among his legal inheritors according to Islamic Shariah Law. In Islam, the principles of distribution of inheritance are given below.

- First of all his/her funeral expenses and all his/her all debt be paid.
- Then execute the will upto 1/3 of his/her property if asked for.
- Then distribute the remaining inheritance accordingly.

The procedure is illustrated with the help of following examples.

Example 22: A man left his property of Rs.640000. A debt of Rs.40,000 was due to him and Rs.5,000 was spent on his burial. Distribute the amount between his widow, 1 daughter and 2 sons according to the Islamic Law.

Solution:

Total amount of property = Rs. 640,000

His debt = Rs. 40,000

Burial Expenses = Rs. 5,000

Total amount paid = 40,000 + 5,000 – 45,000

Remaining amount = 640,000 – 45,000 = Rs. 595,000

Widow's share = $\frac{1}{8}\times 595{,}000$ = Rs. 74,375

Remaining Inheritance = 595,000 – 74,375 = Rs. 520,625

Now, Ratios of shares

Sons	**:**	**Daughter**
2	:	1
$2 \times 2 = 4$	:	$1 \times 1 = 1$

Sum of ratios = $4 + 1 = 5$

Share of 2 sons = $\frac{4}{5}\times 520{,}625$

= $4 \times 104{,}125$

= Rs. 416,500

Share of each son = $\frac{416{,}500}{2} = 208{,}250$

Share of one daughter = $\frac{1}{5}\times 520{,}625$

= Rs. 104,125

Example 23: Mst. Zainab Begum died leaving behind her a property of Rs.802500 which was to be distributed among her husband, her mother and two daughters. The husband got $\frac{1}{4}$, mother got $\frac{1}{6}$ and remaining for 2 daughters. Rs.7,500 was spent on her burial. Find the share of each one.

Solution:

Total amount left = Rs. 802,500

Expenditure on her burial = Rs. 7,500

Remaining amount = 802,500 – 7,500

= Rs. 795,000

Share of her husband = $\frac{1}{4}\times 795{,}000$

= Rs. 198,750

Share of her mother = $\frac{1}{6}\times 795{,}000$

= Rs. 132,500

Total share of her husband and her mother = 198,750 + 132,500

= Rs. 331,250

Remaining Inheritance = 795,000 – 331,250

= Rs. 493,750

$$\text{Share of 2 daughters} = \text{Rs. } 46{,}3750$$

$$\text{Share of each daughter} = \frac{46{,}3750}{2}$$

$$= \text{Rs. } 231{,}875$$

Exercise 1.24

1. A man left Rs. 240000 as inheritance. His heirs are 6 daughters and 2 sons. Find the share of each inheritor so that a son gets twice of his sister's share.
2. Allah Ditta died leaving a property of Rs. 850000. He left a widow, two sons and one daughter. Find the share of each in the inheritance if the burial expenditure was Rs. 50,000.
3. Akram left a wealth of Rs. 780000. His heir are a widow, 3 sons and 4 daughters. Calculate the share of each one if the funeral expenses is Rs. 30,000 and a loan of Rs. 50,000 is due to him.
4. A man died leaving a saving of Rs. 72,000 in the bank. Find the share of each: widow, one son and one daughter.
5. Aslam left a property worth Rs. 650000. He had to pay Rs. 50,000 as debt. The remaining amount was divided among his 2 sons and 2 daughters. Find the share of each.
6. A person died leaving behind inheritance of Rs. 300,000. Distribute the amount among 4 sons and 3 daughters so that each son gets double of what a daughter gets. Find the share of each when a debt of Rs. 80,000 was also to be paid.
7. Wife of Ahmad died leaving behind 2 daughters and a son. If Ahmad gets $\frac{1}{4}$ of the inheritance of Rs. 180,000. The remaining amount is to be distributed among her children so that a son receives twice of a daughter's share. Find the amount of share of each daughter, son and Ahmad.

SUMMARY

- A foreign currency exchange rate is a price that represents how much it costs to buy the currency of one country using the currency of another country.
- If the Selling Price (S.P.) is higher than the Cost Price (C.P.), then profit occurs.
- Profit percentage = $\dfrac{\text{Profit}}{\text{Cost Price}} \times 100\%$
- If the cost price (C.P.) is higher than the selling price (S.P.), then loss occurs.
- Loss percentage = $\dfrac{\text{Loss}}{\text{Cost price}} \times 100\%$
- Percentage Discount = $\dfrac{\text{Discount}}{\text{Market Price}} \times 100\%$

- When we borrow money from bank to run a business, the bank in return receives some extra amount along with the actual money given. This extra money which the bank receives is known as markup.
- The amount we borrow or deposit in the bank is called principal amount.
- The rate at which the bank gives share to its account holders is known as profit / markup rate. It is expressed in percentage.
- The time for which a particular amount is invested in a business is known as period.
- Profit / Markup = Principal × Time × Rate

  Or $I = P \times R \times T = PRT$
- $P = \dfrac{I}{R \times T}$
- $\text{Rate} = \dfrac{\text{Markup}}{\text{Principal} \times \text{Time}}$
- Life insurance is an agreement between the policy owner and the insurance company for an agreed time period.
- Vehicle insurance provides a protection against risks to the vehicle. The amount of policy in this case depends upon the actual value of the vehicle.
- A business in which two or more persons run the business and they are responsible for the profit and loss is called the partnership.
- When a person dies, then the assets left by him are called inheritance and it is distributed among his legal inheritors according to Islamic Shariah Law.
- The relation of equality of two ratios is called proportion, It is denoted by ::
- Ratio is a comparison of two quantities of same kind. Ratio is denoted by : e.g., ratio a is to b is written as $a:b$, ratio c is to d is written as $c:d$.
- If three quantities a, b and c are written as: $a : b :: b : c$, then these quantities are in continued proportion and b is called the mean proportional.

Review Exercise (1b)

1. Four options are given against each statement. Encircle the correct one.

(i) If a is not a member of the set A, then symbolically it is denoted by:

(a) $a \in A$ (b) $a \backslash A$ (c) $a \notin A$ (d) $a \cap A$

(ii) Which of the following is not a set?

(a) $\{1,2,3\}$ (b) $\{a, b, c\}$ (c) $\{2,3,4\}$ (d) $\{1,2,2,3\}$

(iii) A set consisting of all subsets of the set A is called:

(a) subset (b) universal set (c) power set (d) superset

(iv) If $U = \{1,2,3,...10\}$ and $A = \{2,4,6,....,10\}$ and $B=\{1,3,5,....,9\}$, then $(A - B)^c$ is equal to:

(a) U (b) B (c) A (d) ϕ

(v) If $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, then set A is equal to:
(a) ϕ (b) $\{a\}$ (c) $\{b\}$ (d) $\{a, b\}$

(vi) Proportion means:
(a) equality of two ratios (b) equality of quantities
(c) inequality of two ratios (d) inequality of quantities

(vii) Product of extremes is equals to:
(a) direct proportion (b) inverse proportion
(c) product of means (d) division of means

(viii) A relation in which increases one quantity causes proportional increases in the other quantity is called:
(a) compound proportion (b) ratio
(c) inverse proportion (d) direct proportion

(ix) The comparison of the quantities of same kind is called:
(a) proportion (b) ratio
(c) compound proportion (d) inverse proportion

(x) The relation between two or more proportions is called:
(a) compound proportion (b) direct proportion
(c) proportion (d) inverse proportion

(xi) A person who buys life insurance from an insurance company is called;
(a) insured (b) insurer (c) lesser (d) beneficiary

(xii) To reduce the price of an article from its market price is called:
(a) discount (b) profit (c) period (d) insurance

(xiii) The time for which a particular amount is invested in a business is known as:
(a) profit (b) markup rate (c) period (d) principal amount

(xiv) A business in which two or more persons run the business and they are responsible for the profit and loss is called the:
(a) profit (b) partnership (c) markup (d) principal amount

(xv) The cost price of a toy car is Rs. 100. If it is sold at 10% discount then the selling price of the toy car:
(a) Rs. 90 (b) Rs. 110 (c) Rs. 80 (d) Rs. 120

(xvi) When a person dies, then the assets left by him are called:
(a) period (b) principal amount
(c) inheritance (d) discount

2. Define the following:
(i) Direct proportion (ii) Inverse proportion (iii) Compound proportion
(iv) Loss percentage (v) Discount (vi) Markup
(vii) Inheritance (viii) Vehicle insurance (ix) Life insurance

3. Write all subsets of the following sets:
(i) $A = \{1, 2, 3, 4\}$ (ii) $B = \{a, b, c\}$

4. Write the power set of the following sets:
(i) $C = \{2, 4, 6\}$ (ii) $D = \{+, -, \times, \div\}$

5. If U = {1, 2, 3, …………,10}, A = {1, 3, 5, 7} and B = {2, 4, 6, 8, 10}, then verify $(A\cup B)^c = A^c \cap B^c$

6. In a survey of a primary school students, it was found that 20 students have cars, 15 students have motorbikes, 5 students have both and 3 don't have any of them. How many students are there in the survey?

7. Mrs. Razia paid her driver Rs. 2500 for 5 days. What amount will she pay him for 10 days? Also draw its graph.

8. If a motorbike needs 20 litres of petrol for a travel of 160 km. Find how much petrol will be required to travel 360 km. Also draw its graph.

9. If 15 men take 85 days to build a wall, how long would it take for 10 men? Also draw its graph.

10. 10 lions can eat 500 kg meat in 10 days. How many lions will eat the same meat in 5 days? Also draw its graph.

11. A factory can manufacture 200 items in a day using 10 machines. How many items can be manufactured by using 14 machines in 4 days?

12. 25 labourers can construct 5 rooms in 18 days. In how many days can 10 labourers complete 10 rooms of the same size?

13. A factory marked price of the articles 25% above the cost price. The cost price of an article is Rs.5000 and its selling price is Rs. 4500. Find the discount % given to the customer.

14. Saeed invests Rs.12,000 at $8\frac{1}{2}\%$ per year profit. How much would the amount be after 2 years and 6 months?

15. Faheem got his car insured at a rate of 3% for 3 years. The worth of his car is Rs. 850,000. Find the total amount paid as premium if rate of depreciation is 10% per year.

16. Aslam started a business with Rs. 35,000. After 3 months Akram joined the business with Rs.4000 and after 6 months Asghar invested Rs.5000. At the end of the year, they earned a profit of Rs.1620. Find the share of each in the profit.

17. Asghar Ali died leaving assets worth Rs. 655275. Funeral expenses were Rs. 5275. He had to pay Rs. 50,000 as debt. After making these payments, his widow shall get $\frac{1}{8}$ of the remaining property. Find the share of his son and one daughter when share of son is double the share of his daughter.

Domain 2 Algebra

Sub-Domain (i): Number Sequence and Patterns

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- differentiate arithmetic sequence and geometric sequence
- discover terms of an arithmetic sequence using:
  - ❖ term to term rule
  - ❖ position to term rule
- construct the formula for general term (nth term) of an arithmetic sequence
- solve real life problems involving sequence and pattern

Advanced/ Additional

- discover terms of a geometric sequence
- construct the formula for general term (nth term) of geometric sequence

2.1.1 Number Sequence

A number sequence is a list of numbers arranged in an order. For example,

$$2 \xrightarrow{+2} 4 \xrightarrow{+2} 6 \xrightarrow{+2} 8 \xrightarrow{+2} 10, \ldots$$

Each number in a sequence is called a term. Here 2 is the first term, 4 is the second, 6 is the third, 8 is the fourth and 10 is the fifth term of the sequence. Three dots (...) means so on.

In the above sequence, the numbers are arranged in an order i.e., every next number is obtained by adding 2 in the previous term.

Key fact!

The sequence that goes on forever is called an infinite sequence. Otherwise, it is called finite sequence.

Example 1: Identify and complete the following pattern:

(i) 1, 3, 7, 13, ____, 31, ____, 57

(ii) 1, 4, 9, ____, ____, 36, 49, ____, 81

Solution:

(i) $$1 \xrightarrow{+2} 3 \xrightarrow{+4} 7 \xrightarrow{+6} 13 \xrightarrow{+8} \_\_\_\_ \xrightarrow{+10} 31 \xrightarrow{+12} \_\_\_\_ \xrightarrow{+14} 57$$

Here the difference between 1^{st} and 2^{nd} term is 2, i.e. $3 - 1 = 2$ and 2 added continuously in next difference. Similarly,

$$7 - 3 = 4$$
$$13 - 7 = 6$$

So,

5^{th} term $= 13 + 8 = 21$

6^{th} term $= 21 + 10 = 31$

7^{th} term $= 31 + 12 = 43$

8^{th} term $= 43 + 14 = 57$

Hence

1, 3, 7, 13, <u>21</u>, 31, <u>43</u>, 57

(ii) 1, 4, 9, ____, ____, 36, 49, ____, 81

Here

1^{st} term $= 1^2 = 1$

2^{nd} term $= 2^2 = 4$

3^{rd} term $= 3^2 = 9$

So,

4^{th} term $= 4^2 = 16$

5^{th} term $= 5^2 = 25$

and so on.

Hence

1, 4, 9, <u>16</u>, <u>25</u>, 36, 49, <u>64</u>, 81

Try yourself!

$\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \ldots$

- What will be the next term in this pattern?
- What would be the 100^{th} term?

Exercise 2.1

Identify each number pattern and complete the sequence.

1. 28, 24, 20, 16, — , —
2. 1, 2, 4, 7, —, —, 22
3. 77, 66, 55, —, 33, 22, __
4. 50, 100, 150, —, 250, —, 350, —
5. —, —, 243, 81, 27, —, 3, —
6. —, —, 11, 17, 24, —, 41, 51

2.1.2 Arithmetic Sequence

A sequence of numbers such that the difference *'d'* between two consecutive terms is a constant.

$$a_1,\ a_1 + d,\ a_1 + 2d,\ a_1 + 3d,\ \ldots$$

Where a_1 is the first term of the sequence and *'d'* is the common difference.

$$\begin{aligned} a_1 &= a_1 + 0d &&= a_1 + (1-1)d \\ a_2 &= a_1 + d &&= a_1 + (2-1)d \\ a_3 &= a_2 + d &&= a_1 + (3-1)d \\ a_4 &= a_3 + d &&= a_1 + (4-1)d \end{aligned}$$

So, we conclude that

$$a_n = a_1 + (n-1)d$$

Where a_n is the n^{th} term or general term of an arithmetic sequence.

Example 2: Find the n^{th} or general term of an arithmetic sequence 2, 5, 8, 11, ...

Solution:

Here $a_1 = 2, \quad d = 5 - 2 = 3$

We know that

$$a_n = a_1 + (n-1)d$$

Putting $a_1 = 2, d = 3$, we get

$$\begin{aligned} a_n &= 2 + (n-1)(3) \\ a_n &= 2 + 3n - 3 \\ a_n &= 3n - 1 \end{aligned}$$

Thus the general term of the given arithmetic sequence is $3n - 1$

Example 3: Which term of an arithmetic sequence 3, 8, 13, … is 83?

Solution:

Here $a_1 = 3, \quad d = 8 - 3 = 5, \quad a_n = 83$

Using $a_n = a_1 + (n - 1)d$

$83 = 3 + (n - 1)5$ $\quad (a_n = 83, \quad a_1 = 3, \quad d = 5)$

$83 = 3 + 5n - 5$

$83 = -2 + 5n$

$83 + 2 = 5n$

$85 = 5n$

$n = 17$

Hence 83 is the 17^{th} term of the arithmetic sequence 3, 8, 13, ...

Example 4: Find an arithmetic sequence where $a_n = 2n - 7$

Solution:

Given that

$$a_n = 2n - 7$$

Put $n = 1$

$$a_1 = 2(1) - 7 = 2 - 7 = -5$$

Put $n = 2$

$$a_2 = 2(2) - 7 = 4 - 7 = -3$$

Put $n = 3$

$$a_3 = 2(3) - 7 = 6 - 7 = -1$$

Hence the required arithmetic sequence is $-5, -3, -1, \ldots$

2.1.3 Term to Term Rule

A rule which describes how to get from one term to the next.

For example,

2, 6, 10, 14, ...

This is an arithmetic sequence and common difference is 4. So, we can find next term by adding 4 in previous term.

i.e. $a_5 = a_4 + d = 14 + 4 = 18$

$a_6 = a_5 + d = 18 + 4 = 22$

Try yourself!

2, 6, 10, 14, ...

Can we find 200^{th} term of the given sequence?

Hence the general formula for finding next terms by term to term rule in arithmetic sequence is

$$a_n = a_{n-1} + d$$

2.1.4 Position to Term Rule

A position to term rule defines the value of each term with respect to its position.

For example,

3, 5, 7, 9, ...

Let us explain it through table

Position	2 Times Table (2 × Position)	Terms
1	2	3
2	4	5
3	6	7
4	8	9

Rule: Multiply the position by 2 then add 1.

It can be written as $2n + 1$, where n is the position of the term.
We can find any term from this rule.

$$a_5 = 2 \times 5 + 1 = 11$$
$$a_{50} = 2 \times 50 + 1 = 101$$
$$a_{100} = 2 \times 100 + 1 = 201$$

Key fact!
Term to term rule is not helpful if you want to find a term that is far away from the ones that are known. Whereas position to term rule is more powerful.

Example 5: Find 23rd term from position to term rule in the following arithmetic sequences:

(i) 4, 8, 12, 16, ...

(ii) 5, 8, 11, 14, …

Solution:

(i) 4, 8, 12, 16, ...

Position	4 Times Table	Terms
1	4	4
2	8	8
3	12	12
4	16	16

Rule: Multiply the position by 4

So, $a_{23} = 23 \times 4 = 92$

(ii) 5, 8, 11, 14, …

Position	3 Times Table	Terms
1	3	$3 + 2 = 5$
2	6	$6 + 2 = 8$
3	9	$9 + 2 = 11$
4	12	$12 + 2 = 14$

Rule: Multiply the position by 3 then add 2

So, $a_{23} = 23 \times 3 + 2 = 69 + 2 = 71$

Exercise 2.2

1. State whether the following sequences are arithmetic or not. If a sequence is arithmetic, then find the common difference.
   (i) 7, 10, 13, 16, ... (ii) 3, –2, –7, –12, ...
   (iii) 1, 4, 9, 15, ... (iv) 5, 5, 5, 5,...
2. The n^{th} terms of the arithmetic sequences are given. Find the arithmetic sequence in each.
   (i) $a_n = 4n-3$ (ii) $a_n = 2n-5$
   (iii) $a_n = n+2$ (iv) $a_n = 3n-1$
3. Find the general term of the following arithmetic sequences:
   (i) 6, 13, 20, 27,... (ii) 5, 7, 9, 11,...
   (iii) 4, 9, 14, 19,... (iv) 2, 12, 22,...
4. Which term of the arithmetic sequence 1, 4, 7, ... is 91?
5. Which term of the arithmetic sequence 2, 5, 8, ... is 62?
6. Find the next two terms by term to term rule of arithmetic sequence.
   (i) 2, 8, 14, ... (ii) 1, 5, 9, ...
7. Find the indicated term of the arithmetic sequence by position to terms rule.
   (i) $15, 21, 27, 33, ..., a_{13}$ (ii) $2, 9, 16, 23, ..., a_{41}$
8. Find the indicated term of the arithmetic sequence.
   (i) $a=5, d=3, a_6 = ?$ (ii) $a=12, d=2, a_5 = ?$
   (iii) $a=5, d=5, a_{10} = ?$ (iv) $a=9, d=3, a_{15} = ?$

2.1.5 Geometric Sequence

Geometric sequence is a sequence in which each term is obtained by multiplying a non-zero constant 'r' to the preceding term.

$$a_1, a_1r, a_1r^2, a_1r^3, ...$$

Where a_1 is the first term and r is the common ratio

Here

$$a_1 = a_1r^0 = a_1r^{1-1}$$
$$a_2 = a_1r^1 = a_1r^{2-1}$$
$$a_3 = a_1r^2 = a_1r^{3-1}$$
$$a_4 = a_1r^3 = a_1r^{4-1}$$

Hence in general

$$\boxed{a_n = a_1r^{n-1}}$$

Where a_n is the n^{th} term or general term of the geometric sequence.

Example 6: Find the 5^{th} and n^{th} term of the geometric sequence 1, 2, 4, 8, ...

Solution: 1, 2, 4, 8, ...

Here $a_1 = 1,\ r = \frac{2}{1} = \frac{4}{2} = \frac{8}{4} = 2$

Using
$$a_n = a_1 r^{n-1}$$
$$= (1)(2)^{n-1}$$
$$a_n = 2^{n-1}$$

Put $n = 5$
$$a_5 = 2^{5-1} = 2^4 = 16$$

Example 7: Find the geometric sequence where general term is $a_n = 4.(-4)^{n-1}$

Solution:

Given that $a_n = 4.(-4)^{n-1}$

Put n = 1
$$a_1 = 4.(-4)^{1-1} = 4.(-4)^0 = 4$$
Put n = 2
$$a_2 = 4.(-4)^{2-1} = 4.(-4) = -16$$
Put n = 3
$$a_3 = 4.(-4)^{3-1} = 4.(-4)^2 = 64$$
Put n = 4
$$a_4 = 4.(-4)^{4-1} = 4.(-4)^3 = -256$$

Hence the required geometric sequence is 4, −16, 64, −256, ...

2.1.6 Real life Situations Involving Sequence and Pattern

Example 8: There are 50 passengers in the first cabin of train, 100 passengers in the second cabin and 150 passengers in the third cabin. If the number of passengers increases in the same pattern, find the number of passengers in the 8^{th} cabin.

Solution:

The sequence is

50, 100, 150, ...

It is an arithmetic sequence

Here $a = 50,\ d = 100 - 50 = 50,\ n = 8$

Using
$$a_n = a + (n-1)d$$
$$a_8 = 50 + (8-1)(50)$$
$$= 50 + 7(50)$$
$$= 50 + 350 = 400$$

Hence there are 400 passengers in the 8^{th} cabin of train.

Example 9: A car travels $250m$ in the 1^{st} minute, $350m$ in the 2^{nd} minute, $500m$ in the 3^{rd} minute and $700m$ in 4^{th} minute. If the car increased its distance travelled in each minute with the same pattern, find the distance that the car travelled in the 7^{th} minute.

Solution: The sequence is

$$250 \xrightarrow{+100} 350 \xrightarrow{+150} 500 \xrightarrow{+200} 700, \ldots$$

So,

$$a_5 = 700 + 250 = 950$$
$$a_6 = 950 + 300 = 1250$$
$$a_7 = 1250 + 350 = 1600$$

Hence, the car will travel $1600m$ in the 7^{th} minute.

Exercise 2.3

1. Which of the following sequences are geometric? If a sequence is geometric, then find the common ratio.
   (i) $1, 2, 3, 4, \ldots$ (ii) $10, 100, 1000, \ldots$ (iii) $2, 4, 8, \ldots$
   (iv) $16, 8, 4, \ldots$ (v) $6, 18, 24, \ldots$ (vi) $1, 1, 0, 1, \ldots$
2. Find the general term of the following geometric sequences.
   (i) $1, 4, 16, 64, \ldots$ (ii) $6, 12, 24, 48, \ldots$
   (iii) $15, 45, 135, \ldots$ (iv) $200, 100, 50, 25, \ldots$
3. Find the first four terms of geometric sequence. Where the general term is given.
   (i) $a_n = 3.2^{n-1}$ (ii) $a_n = 4.(2)^{n-1}$
   (iii) $a_n = 25.\left(-\frac{1}{2}\right)^{n-1}$ (iv) $a_n = 5.\left(\frac{1}{5}\right)^{n-1}$
4. Find the indicated term of geometric sequence.
   (i) $a = 20, r = 10, a_4 = ?$ (ii) $a = 3, r = 3, a_7 = ?$
   (iii) $a = 512, r = \frac{1}{2}, a_5 = ?$ (iv) $a = 7, r = 2, a_{10} = ?$
5. A writer wrote 120 words on the first day, 240 words on the second day and 480 words on the third day. If the number of written words are increased in the same pattern, find the number of words wrote on the 10^{th} day.
6. The sum of interior angles of a triangle is 180°, of a quadrilateral is 360° and of a pentagon is 540°. Find the sum of the angles of an octagon (8 sides).
7. Abdullah's exercise trainer suggests jogging for 10 minutes each day for the first week. After each week, he suggests to increase that time by 5 minutes per day. At 6^{th} week how many minutes will he jog?

SUMMARY

- A number sequence is a list of numbers arranged in an order.
- If the sequence goes on forever, then it is called an infinite sequence. Otherwise, it is called finite sequence.
- A sequence of numbers such that the difference 'd' between two consecutive terms is a constant is called arithmetic sequence.
  $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$
  Where as, a_1 is the first term of the sequence and d is the common difference.
- A position to term rule defines the value of each term with respect to its position.
- Term to term rule is not helpful if you want to find a term that is far away from the ones that are known. Whereas, position to term rule in more powerful.
- Geometric sequence is a sequence in which each term is obtained by multiplying a non-zero constant r to the previous term $a_1, a_1r, a_1r^2, a_1r^3, \ldots$
  Where a_1 is the first term and 'r' the common ratio.

Sub-Domain(ii): Expansion and Factorization

$x \quad y \quad z$

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- recall the difference between
  - ❖ open and close sentences
  - ❖ expression and equation
  - ❖ equation and inequality
- recall addition, subtraction and multiplication of polynomials

Division of Algebraic Expressions

- divide a polynomial of degree up to 3 by
  - ❖ a monomial
  - ❖ a binomial
- simplify algebraic expression involving addition, subtraction, multiplication and division

Basic Algebraic identities

- recognize algebraic identities and use them to expand expressions
  - ❖ $(a+b)^2 = a^2 + b^2 + 2ab$
  - ❖ $(a-b)^2 = a^2 + b^2 - 2ab$
  - ❖ $(a+b)(a-b) = a^2 - b^2$
- apply algebraic identities to solve problems like $(103)^2$, $(99)^2$, 101×99

Advanced / Additional

- identify base, index/ exponent and its value
- use scientific notation/standard form to express very large and very small numbers
- use positive, negative, fractional and zero indices
- apply the law of exponents/indices
  - ❖ $a^2 - b^2$
  - ❖ $a^2 \pm 2ab + b^2$
  - ❖ $a^2 \pm 2ab + b^2 - c^2$
  - ❖ $ax^2 \pm bxy + cy^2$ (By midterm break)
  - ❖ $(a+b)^3 = a^3 + 3ab(a+b) + b^3$
  - ❖ $(a-b)^3 = a^3 - 3ab(a-b) - b^3$

It is called exponent of x

Expansion

$$x^2(2x - 1) = 2x^3 - x^2$$

It is called variable or base

It is called constant and co-efficient

Factorization

Do you know?

Expansion and Factorization are inverse of each other. Expansion means "removing brackets" and Factorization means "adding brackets".

2.2.1 Closed Sentence

A closed sentence is always true or always false.

For Example:

(i) $12 \div 4 = 3$ (True)

(ii) 6 is an even number. (True)

(iii) $4 \times 9 = 9 + 9 + 9 + 9 + 9$ (False)

2.2.2 Open Sentence

An open sentence is neither true nor false until the variables or unknowns have been replaced by specific values.

For Example:

(i) $x + 4 = 10$

(ii) $3y - 4 = 0.$

The above sentences in (i) and (ii) are neither true nor false because we do not know the values of x and y.

Algebraic Expression

An expression which connects variables and constants by algebraic operations of addition, subtraction, multiplication and division is called an algebraic expression. A few algebraic expressions are given below:

(i) 14 (ii) $x + 2y$ (iii) $4x - y + 5$ (iv) $\frac{-2}{x} + y$ (v) $3y + 7z - \frac{5}{7}$

Equation

A sentence that shows equality (=) between two expressions is called an equation.

For Example:

(i) $3x + 5 = 0$

(ii) $x - 6 = 5 - 3x$

(iii) $\frac{x+3}{2} = \frac{2x-4}{3}$

Key fact!

Expression	Equation
• It is one sided. • It has no relation symbol.	• It is two sided. • It's relation symbol is equality (=).

Inequality

An inequality is a statement that contains one of the symbols: $<, >, \leq$ or $\geq$.

(i) $3 < 4$

(ii) $7 - 3y \geq 9$

(iii) $\frac{2x}{3} - 5 > \frac{x}{7}$

2.2.3 Polynomial

Definitions

Polynomial

A polynomial expression or simply a polynomial is an algebraic expression consisting of one or more terms in each of which the exponent of the variable is zero or a positive integer.

For Example: $13, -x, 5x+3y, x^2-3x+1$ are all polynomials.

The following algebraic expressions are not polynomials.

$$x^{-2},\ \frac{1}{y},\ x^3 - x^{-3} + 3,\ x^2 + y^{-4} - 7 \text{ and } \frac{x}{y} + 5x$$

> **Note!**
>
> Co-efficient ← $2x^3$ → Exponent, Variable / Base

Degree of a Polynomial

Degree of a polynomial is the highest power of a part (term) in a polynomial. Degree of a term in a polynomial is the sum of the exponents on the variables in a single term. The degree of $2x^3y^4$ is 7 as $3 + 4 = 7$

Coefficient of a Variable

In a term, the number multiplied by the variable is called the coefficient of the variable as well as constant. In $4x + 6y$, 4 is coefficient of x, 6 is coefficient of y and both (4 & 6) are constants.

2.2.4 Recognition of Polynomial in One, Two and More Variables

(a) Polynomials in one Variable

Consider the following Polynomials:

(i) $x^4 + 4$ (ii) $x^2 - x + 1$ (iii) $y^3 + y^2 - y + 1$ (iv) $y^2 - y + 8$

In polynomials (i) and (ii) x is the variable and in polynomial (iii) and (iv) y is the variable. All these polynomials are polynomials in one variable.

(b) Polynomials in two Variables

Consider the following Polynomials:

(i) $x^2 + y + 2$ (ii) $x^2y + xy + 6$ (iii) $x^2z + xz + z$ (iv) $x^2z + 8$

In polynomials (i) and (ii) x, y are the variables. In polynomials (iii) and (iv) x, z are the variables. All these polynomials are in two variables.

(c) Polynomials in more Variables

$x^2yz + xy^2z + xy + 1$ is a polynomial in three variables x, y and z.

2.2.5 Recognition of Polynomials of Various Degrees (e.g., Linear, Quadratic, Cubic and Biquadratic Polynomials)

(a) Linear Polynomials ($ax + b$; $a \neq 0$)

Consider the following polynomials:

(i) $x + 2$ (ii) x (iii) $x + 2y$ (iv) $x + z$

In all these polynomials the degree of the variables x, y or z is one. Such types of polynomials are linear polynomials.

(b) Quadratic Polynomials ($ax^2 + bx + c$; $a \neq 0$)

Let us write a few polynomials in which the highest exponent or sum of exponents is always 2.

(i) x^2 (ii) $x^2 - 3$ (iii) $xy + 1$

In the first two polynomials, x is the variable and its degree is 2. In the third polynomial, x and y are the variables and sum of their exponents is $1 + 1 = 2$. Its degree is also 2. Therefore polynomials of the type (i), (ii) and (iii) are quadratic polynomials.

(c) Cubic Polynomials ($ax^3 + bx^2 + cx + d$; $a \neq 0$)

Consider the following polynomials:

(i) $5x^3 + x^2 - 4x + 1$ (ii) $x^2y + xy^2 + y - 2$

The degree of each one of the polynomial is 3. These polynomials are called cubic polynomials.

(d) Biquadratic Polynomials ($ax^4 + bx^3 + cx^2 + dx + e$; $a \neq 0$)

Let us take a few polynomials of degree 4.

(i) $x^4 + x^3y + x^2y^2 + y^3 - 1$ (ii) $y^4 + y^3 - y^2 - y + 8$

These are biquadratic polynomials.

Exercise 2.4

1. Separate the open and closed sentences.
   (i) $3x + 4 = 1$ (ii) $2x^3 - 1 = 6$ (iii) $5 \times 4 = 20$
   (iv) $5y + 7 = y$ (v) $10 + 40 = 50$ (vi) $72 \div 8 = 9$
2. Separate the expressions and equations.
   (i) $2x - 1 = 0$ (ii) $3x - y + 7$ (iii) $x + y = 3$
   (iv) $7y^2 - 2y + 3 = 0$ (v) $x^2 - x - 1 = 0$ (vi) $x = -7$
3. Separate the equations and inequalities.
   (i) $3x + 7 > 10$ (ii) $2x - 5 \leq 1$ (iii) $3x - 4 = 7x$
   (iv) $7x - 5 > 6x$ (v) $5x = 7$ (vi) $\frac{10x}{3} < 5$
4. Separate the polynomial expressions and expressions that are not polynomials
   (i) $x^2 + x - 1$ (ii) $x^2y + xy^2 + 7$ (iii) $x^{-2} + y + 7$
   (iv) $\frac{x}{y^2} + 1 - \frac{y^2}{x}$ (v) $x^3 - x^2 + y - 1$ (vi) $x^4 + x^2 + 5x + \frac{1}{2}$
5. What constants are used in the following expressions?
   (i) $7x - 6y + 3z$ (ii) $5x^2 - 3$ (iii) $8x^2 + 2y + 5$
   (iv) $9y + 3x - 2z$
6. Write the degree of the polynomials given below.
   (i) $x + 1$ (ii) $x^2 + x$ (iii) $x^3 - xy + 1$
   (iv) $x^2y^2 + x^3 + y^2 - 1$

7. Separate the polynomials as linear, quadratic, cubic and biquadratic.

(i)	$3x + 1$	(ii)	$x^2 - 2$	(iii)	$y^2 - y$
(iv)	$x + y$	(v)	$x^3 - x^2 - 2$	(vi)	$x^4 + x^3 + x^2$
(vii)	$x^2y^2 + xy$	(viii)	$x^2 + xy + 8$		

2.2.6 Operations on Polynomials

Addition, Subtraction and Multiplication of Polynomials

(i) Addition of algebraic expressions (Polynomials)

If P and Q are two polynomials, then their sum is represented as $P+Q$. In order to add two or more than two polynomials we first write the polynomials in descending or ascending order and place the like terms in the form of columns. Finally, we add the coefficients of like terms.

Example 1: Add $3x^3 + 5x^2 - 4x$, $x^3 - 6 + 3x^2$ and $6 - x^2 - x$

Solution:

$$\begin{array}{rrrrrrrr} & 3x^3 & + & 5x^2 & - & 4x & + & 0 \\ & x^3 & + & 3x^2 & + & 0x & - & 6 \\ & 0x^3 & - & x^2 & - & x & + & 6 \\ \hline \textit{Sum:} & 4x^3 & + & 7x^2 & - & 5x & & \\ \hline \end{array}$$

(ii) Subtraction of Polynomials

The subtraction of two polynomials P and Q is represented by $P - Q$ or $[P + (-Q)]$. If the sum of two polynomials is zero, then P and Q are called additive inverse of each other.

If $\quad P = x + y \quad$ and $\quad Q = -x - y,$

Then $P + Q = (x + y) + (-x - y) = 0$

Like addition, we write the polynomials in descending or ascending order and then change the sign of every term of the polynomial which is to be subtracted.

Example 2: Subtract $2x^3 - 4x^2 + 8 - x$ from $5x^4 + x - 3x^2 - 9$

Solution: Arrange the terms of the polynomials in descending order.

$$\begin{array}{rl} & 5x^4 + 0x^3 - 3x^2 + x - 9 \\ & \pm 0x^4 \pm 2x^3 \mp 4x^2 \mp x \pm 8 \\ \hline \textit{Difference:} & 5x^4 - 2x^3 + x^2 + 2x - 17 \\ \hline \end{array}$$

(iii) Multiplication of Polynomials

Multiplication of polynomials is explained through examples:

Example 3: Find the product of $4x^2$ and $5x^3$

Solution:

$$\begin{aligned} (4x^2)(5x^3) &= 4 \times 5(x^2 \times x^3) && \text{(Associative Law)} \\ &= (20)(x^2 \times x^3) \\ &= 20x^{2+3} && \text{(Law of Exponents)} \\ &= 20x^5 \end{aligned}$$

Example 4: Find the product of $3x^2 + 2x - 4$ and $5x^2 - 3x + 3$

Solution: Horizontal Method

$$(3x^2 + 2x - 4)(5x^2 - 3x + 3)$$

$$= 3x^2(5x^2 - 3x + 3) + 2x(5x^2 - 3x + 3) - 4(5x^2 - 3x + 3)$$
$$= 15x^4 - 9x^3 + 9x^2 + 10x^3 - 6x^2 + 6x - 20x^2 + 12x - 12$$
$$= 15x^4 + (10 - 9)x^3 + (9 - 6 - 20)x^2 + (6 + 12)x - 12$$
$$= 15x^4 + x^3 - 17x^2 + 18x - 12$$

Example 5: Multiply $2x - 3$ by $5x + 6$

Solution: Vertical Method

$$\begin{array}{rrr} & 5x & +\ 6 \\ \times & 2x & -\ 3 \\ \hline & 10x^2 + 12x & \\ & -\ 15x & -\ 18 \\ \hline & 10x^2 - \ 3x & -\ 18 \\ \hline \end{array}$$

> **Note!**
> The product of two polynomials is also a polynomial whose degree is equal to the sum of the degrees of the two polynomials.

(iv) Division of Polynomials

Division is the reverse process of multiplication.

The method of division of polynomials is explained through examples.

Example 6: Divide $(-8x^2)$ by $(-4x)$

Solution: $(-8x^2) \div (-4x) = (-8x^2) \times \dfrac{1}{-4x}$

$$= 2x^{2-1}$$
$$= 2x$$

Example 7: Divide $x^3 - 2x + 4$ by $x + 2$

Solution:

$$\begin{array}{r|l} & x^2 - 2x + 2 \\ \hline x + 2 & x^3 + 0x^2 - 2x + 4 \\ & \pm x^3 \pm 2x^2 \\ \hline & \quad -2x^2 - 2x \\ & \quad \mp 2x^2 \mp 4x \\ \hline & \qquad\quad 2x + 4 \\ & \qquad\quad \pm 2x \pm 4 \\ \hline & \qquad\qquad 0 \end{array}$$

> **Note!**
> If a polynomial is exactly divisible by another polynomial, then the remainder is zero.

Exercise 2.5

1. Add:

(i) $1 + 2x + 3x^2,\ 3x - 4 - 2x^2,\ x^2 - 5x + 4$

(ii) $a^3 + 2a^2 - 6a + 7,\ a^3 + 2a + 5,\ 2a^3 + 2a - a^2 - 8$

(iii) $a^3 - 2a^2b + b^3,\ 4a^3 + 2ab^2 + 6a^2b,\ 2b^3 - 5a^3 - 4a^2b$

2. Subtract P from Q when,

(i) $P = 3x^4 + 5x^3 + 2x^2 - x$; $Q = 4x^4 + 2x^2 + x^3 - x + 1$

(ii) $P = 2x + 3y - 4z - 1$; $Q = 2y + 3x - 4z + 1$

(iii) $P = a^3 + 2a^2b + 3ab^2 + b^3$; $Q = a^3 - 3a^2b + 3ab^2 - b^3$

3. Find the value of $x - 2y + 3z$ where $x = 2a^2 - a^3 + 3a + 4$, $y = 2a^3 - 3a^2 + 2 - 2a$ and $z = a^4 + 3a^3 - 6 - 5a^2$

4. The sum of two polynomials is $x^2 + 2x - y^2$. If one polynomial is $x^2 - 2xy + 3$, then find the other polynomial.

5. Subtract $4x + 6 - 2x^2$ from the sum of $x^3 + x^2 - 2x$ and $2x^3 + 3x - 7$

6. Solve the following:

(i) $(x + 3)(x^2 - 3x + 9)$ (ii) $(3x^2 - 7x + 5)(4x^2 - 2x + 1)$

(iii) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

7. If $P = x^2 - yz$, $Q = y^2 - xz$ and $R = z^2 - xy$, then find PQ, QR, PR and PQR.

8. Simplify:

(i) $(x^2 + x - 6) \div (x - 2)$ (ii) $(x^3 - 19x - 30) \div (x + 3)$

(iii) $(x^3 - y^3) \div (x - y)$ (iv) $(x^3 + x^2 - 14x - 24) \div (x + 2)$

9. What should be added to $4x^3 - 10x^2 + 12x + 6$ so that it becomes exactly divisible by $2x+1$?

10. For what value of p the polynomial $3x^3 - 7x^2 - 9x + p$ becomes exactly divisible by $x - 3$?

2.2.7 Basic Algebraic Formulas

- $(a + b)^2 = a^2 + 2ab + b^2$

Example 8: Evaluate $(107)^2$ by using formula

Solution:
$$\begin{aligned}(107)^2 &= (100 + 7)^2 \\ &= (100)^2 + 2(100 \times 7) + (7)^2 \\ &= 10000 + 1400 + 49 \\ &= 11449\end{aligned}$$

- $(a - b)^2 = a^2 - 2ab + b^2$

Example 9: Using the formula, evaluate $(87)^2$

Solution:
$$\begin{aligned}(87)^2 &= (90 - 3)^2 \\ &= (90)^2 - 2(90 \times 3) + (3)^2 \\ &= 8100 - 540 + 9 \\ &= 7569\end{aligned}$$

- $a^2 - b^2 = (a + b)(a - b)$

Example 10: Using the formula, evaluate 107×93

Solution:
$$\begin{aligned}107 \times 93 &= (100 + 7)(100 - 7) \\ &= (100)^2 - (7)^2 \\ &= 10000 - 49 \\ &= 9951\end{aligned}$$

Example 11: Find the value of $x^2+\frac{1}{x^2}$, $x^4+\frac{1}{x^4}$ when $x-\frac{1}{x}=2$

Solution: Here, $x-\frac{1}{x}=2$

$(x-\frac{1}{x})^2=(2)^2$ (Taking square of both the sides)

or $x^2-2(x)\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2=4$

or $x^2-2+\frac{1}{x^2}=4$

or $x^2+\frac{1}{x^2}=4+2$

or $x^2+\frac{1}{x^2}=6$

or $\left(x^2+\frac{1}{x^2}\right)^2=(6)^2$ (Again taking square of both the sides)

or $(x^2)^2+2(x^2)(\frac{1}{x^2})+(\frac{1}{x^2})^2=36$

or $x^4+2+\frac{1}{x^4}=36$

or $x^4+\frac{1}{x^4}=36-2$

$x^4+\frac{1}{x^4}=34$

Exercise 2.6

1. Evaluate square of each of the following:
   (i) 53 (ii) 77 (iii) 509 (iv) 1006
2. Evaluate each of the following:
   (i) $(57)^2$ (ii) $(95)^2$ (iii) $(598)^2$ (iv) $(1997)^2$
3. Evaluate:
   (i) 46×54 (ii) 197×203 (iii) 999×1001 (iv) 0.96×1.04
4. (i) Find the value of $x^2+\frac{1}{x^2}$, when $x+\frac{1}{x}=7$
   (ii) Find the value of $x^2+\frac{1}{x^2}$, when $x-\frac{1}{x}=3$
   (iii) Find the value of $x^4+\frac{1}{x^4}$, when $x-\frac{1}{x}=1$

2.2.8 Factorization

Factors of an expression are the expressions whose product is the given expression. The process of expressing the given expressions as a product of its factors is called 'Factorization' or 'Factorizing'.

(i) Type $Ka + Kb + Kc$:

Example 12: Factorize $2x-4y+6z$

Solution: $2x-4y+6z$

$= 2(x-2y+3z)$

Example 13: Factorize $x^2-xy+xz$

Solution: $x^2-xy+xz$

$= x(x-y+z)$

Example 14: Factorize $3x^2-6xy$

Solution: $3x^2-6xy$

$= 3x(x-2y)$

Exercise 2.7

Factorize the following:

1. $3x-9y$
2. $xy+xz$
3. $6ab-14ac$
4. $3m^3np-6m^2n$
5. $30x^3-45xy$
6. $17x^2y^2-51$
7. $4x^3+3x^2+2x$
8. $2p^2-4p^3+8p$
9. $x^3y-x^2y+xy^2$
10. $7x^4-14x^2y+21xy^3$
11. $x^2y^2z^2-xyz^2+xyz$
12. $4x^3y^2-8xy+4xy^3$
13. $xy^4-3xy^3-6xy^2$
14. $x^2y^2z+x^2yz^2+xy^2z^2$
15. $77x^2y-33xy^2-55x^2y^2$
16. $5x^5+10x^4+15x^3$

(ii) Type $ac+ad+bc+bd$:

Consider the following examples for such cases.

Example 15: Factorize: $3x+cx+3c+c^2$

Solution: $3x+cx+3c+c^2$

$= (3x+cx)+(3c+c^2)$

$= x(3+c)+c(3+c)$

$= (3+c)(x+c)$

Example 16: Factorize: $2x^2y-2xy+4y^2x-4y^2$

Solution: $2x^2y-2xy+4y^2x-4y^2$

$= 2y(x^2-x+2yx-2y)$

$= 2y[x(x-1)+2y(x-1)]$

$= 2y(x-1)(x+2y)$

Exercise 2.8

Factorize the following:

1. $ax-by+bx-ay$
2. $2ab-6bc-a+3c$
3. $x^2+2x-3x-6$
4. $x^2+5x-2x-10$
5. $x^2-7x+2x-14$
6. $x^2+3x-4x-12$
7. $y^2-9y+3y-21$
8. $x^2-8x-4x+32$
9. $x^2-7x-5x+35$
10. $x^2-13x-2x+26$
11. $a(x-y)-b(x-y)$
12. $y(y-a)-b(y-a)$
13. $a^2(pq-rs)+b^2(pq-rs)$
14. $ab(x+7)+cd(x+7)$

(iii) Type $a^2 \pm 2ab + b^2$:

Consider the following examples for such cases:

Example 17: Factorize: $9a^2 + 30ab + 25b^2$

Solution:
$9a^2 + 30ab + 25b^2$
$= (3a)^2 + 2(3a \times 5b) + (5b)^2$
$= (3a+5b)^2$

Example 18: Factorize: $16x^2 - 64x + 64$

Solution:
$16x^2 - 64x + 64$
$= 16\,(x^2 - 4x + 4)$
$= 16\,[(x)^2 - 2(2)(x) + (2)^2]$
$= 16\,(x - 2)^2$

Example 19: Factorize: $8x^3y + 8x^2y^2 + 2xy^3$

Solution:
$8x^3y + 8x^2y^2 + 2xy^3$
$= 2xy\,(4x^2 + 4xy + y^2)$
$= 2xy\,[(2x)^2 + 2(2x)(y) + (y^2)]$
$= 2xy\,(2x + y)^2$

Exercise 2.9

Factorize:

1. $x^2 + 14x + 49$
2. $9a^2 + 12ab + 4b^2$
3. $16 + 24a + 9a^2$
4. $25x^2 + 80xy + 64y^2$
5. $a^4 + 84a^2 + 252$
6. $4a^2 + 120a + 900$
7. $x^2 - 34x + 289$
8. $49x^2 - 84x + 36$
9. $x^2 - 18xy + 81y^2$
10. $a^4 - 26a^2 + 169$
11. $2a^2 - 64a + 512$
12. $1 - 6a^2b^2c + 9a^4b^4c^2$
13. $4x^4 + 20x^3yz + 25x^2y^2z^2$
14. $\frac{9}{16}x^2 + xy + \frac{4}{9}y^2$
15. $\frac{49}{64}x^2 - 2xy + \frac{64}{49}y^2$
16. $\frac{a^2}{b^2}x^2 - \frac{2ac}{bd}xy + \frac{c^2}{d^2}y^2$
17. $16x^6 - 16x^5 + 4x^4$
18. $a^4b^4x^2 - 2a^2b^2c^2d^2xy + c^4d^4y^2$

(iv) Type $a^2 - b^2$:

Consider the following examples for such cases:

Example 20: Factorize: $25x^2 - 64$

Solution:
$25x^2 - 64$
$= (5x)^2 - (8)^2$
$= (5x + 8)\,(5x - 8)$

Example 21: Factorize: $16y^2b - 81bx^2$

Solution:
$16y^2b - 81bx^2$
$= b\,(16y^2 - 81x^2)$
$= b\,[(4y)^2 - (9x)^2]$
$= b\,(4y + 9x)\,(4y - 9x)$

Example 22: Factorize: $(3x - 5y)^2 - 49z^2$

Solution:
$(3x - 5y)^2 - 49z^2$
$= (3x - 5y)^2 - (7z)^2$
$= (3x - 5y + 7z)\,(3x - 5y - 7z)$

Example 23: Factorize: $36\,(x + y)^2 - 25(x - y)^2$

Solution:
$36\,(x + y)^2 - 25(x - y)^2$
$= [6(x + y)]^2 - [5(x - y)]^2$
$= [6(x + y) + 5(x - y)]\,[6(x + y) - 5(x - y)]$
$= (11x + y)\,(x + 11y)$

Example 24: Use formula to evaluate: $(677)^2 - (323)^2$

Solution: $(677)^2 - (323)^2$

$= (677 + 323)(677 - 323)$

$= 1000 \times 354$

$= 354000$

Example 25: $\dfrac{0.987\times 0.987 - 0.643\times 0.643}{0.987+0.643}$

Solution: $= \dfrac{0.987\times 0.987 - 0.643\times 0.643}{0.987+0.643}$

$= \dfrac{(0.987)^2 - (0.643)^2}{0.987+0.643}$

$= \dfrac{(0.987+0.643)(0.987-0.643)}{0.987+0.643}$

$= 0.987 - 0.643$

$= 0.344$

Exercise 2.10

Factorize the following expressions:

1. $9 - x^2$
2. $-6 + 6y^2$
3. $16x^2y^2 - 25a^2b^2$
4. $x^3y - xy^3$
5. $16a^2 - 400b^2$
6. $a^2b^3 - 64a^2b$
7. $7xy^2 - 343x$
8. $5x^3 - 45x$
9. $11(a+b)^2 - 99c^2$
10. $75 - 3(a-b)^2$
11. $\left(x-\dfrac{9}{5}\right)^2 - \dfrac{36}{25}y^2$
12. $25\left(x+\dfrac{5}{4}\right)^2 - 16\left(x+\dfrac{7}{4}\right)^2$
13. $16(a+b)^2 - 49(a-b)^2$
14. $36\left(x-\dfrac{1}{4}\right)^2 - 64\left(x-\dfrac{5}{4}\right)^2$

Evaluate the following with the help of formula:

15. $(371)^2 - (129)^2$
16. $(674.17)^2 - (325.83)^2$
17. $\dfrac{(0.567)^2 - (0.433)^2}{0.567 - 0.433}$
18. $\dfrac{(0.409)^2 - (0.391)^2}{0.409 - 0.391}$

(v) Type $a^2 \pm 2ab + b^2 - c^2$:

This type can be explained through the following examples:

Example 26: $a^2 - 2ab + b^2 - 4c^2$

Solution: $a^2 - 2ab + b^2 - 4c^2$

$= (a-b)^2 - (2c)^2$

$= (a - b - 2c)(a - b + 2c)$

Example 27: $4a^2 + 4ab + b^2 - 9c^2$

Solution: $4a^2 + 4ab + b^2 - 9c^2$

$= (2a)^2 + 2(2a)(b) + (b)^2 - 9c^2$

$= (2a+b)^2 - (3c)^2$

$= (2a + b - 3c)(2a + b + 3c)$

Exercise 2.11

Factorize the following expressions:

1. $a^2 + 2ab + b^2 - c^2$
2. $a^2 + 6ab + 9b^2 - 16c^2$
3. $a^2 + b^2 + 2ab - 9a^2b^2$
4. $x^2 - 4xy + 4y^2 - 9x^2y^2$
5. $9a^2 - 6ab + b^2 - 16c^2$
6. $a^2 - b^2 - 4b - 4$

(vi) Type $ax^2 + bxy + cy^2$: (By Middle term Breaking ($a \neq 0, b \neq 0, c \neq 0$))

In order to factorize $ax^2 + bxy + cy^2$, we have to find numbers m and n such that $m + n = b$ and $mn = ac$. This type can be explained with the following examples.

Example 28: Factorize $x^2 + 7x + 12$

Solutions:

$x^2 + 7x + 12$
$= x^2 + 4x + 3x + 12$
$= x(x + 4) + 3(x + 4)$
$= (x + 4)(x + 3)$

Choose one pair.
All possible pairs:
$1 \times 12 = 12$
$2 \times 6 = 12$
$3 \times 4 = 12$
Selected Pair.
$3 \times 4 = 12$

Example 29: Factorize $12x^2 - 17x - 7$

Solutions:

$12x^2 - 17x - 7$
$= 12x^2 + 4x - 21x - 7$
$= 4x(3x + 1) - 7(3x + 1)$
$= (3x + 1)(4x - 7)$

All possible pairs:
$1 \times 84 = 84$
$2 \times 42 = 84$
$3 \times 28 = 84$
$4 \times 21 = 84$
$6 \times 14 = 84$
$7 \times 12 = 84$
Selected Pair.
$4 \times 21 = 84$

Example 30: Factorize $4x^2 + 13xy + 9y^2$

Solutions:

$4x^2 + 13xy + 9y^2$
$= 4x^2 + 4xy + 9xy + 9y^2$
$= 4x(x + y) + 9y(x + y)$
$= (x + y)(4x + 9y)$

All possible pairs:
$1 \times 36 = 36$
$2 \times 18 = 36$
$3 \times 12 = 36$
$4 \times 9 = 36$
$6 \times 6 = 36$
Selected Pair.
$4 \times 9 = 36$

Exercise 2.12

Factorize the following expressions:

1.	$x^2 + 5x + 6$	2.	$x^2 + 10x + 24$	3.	$x^2 - 15x + 56$	4.	$x^2 + 3x - 40$
5.	$4x^2 + 8x + 3$	6.	$4x^2 - 5xy + y^2$	7.	$10x^2 - 13xy - 9y^2$	8.	$x^2 + 12x + 27$
9.	$2x^2 - 17x - 30$	10.	$3x^2 - 10xy + 8y^2$				

Manipulation of Algebraic Expression

- **Formula $(a + b)^3 = a^3 + 3ab(a + b) + b^3$**

Example 31: Expand $(3a + 4b)^3$

Solution:

$(3a + 4b)^3$
$= (3a)^3 + 3(3a)(4b)(3a + 4b) + (4b)^3$
$= 27a^3 + 36ab(3a + 4b) + 64b^3$
$= 27a^3 + 108a^2b + 144ab^2 + 64b^3$

- **Formula $(a - b)^3 = a^3 - 3ab(a - b) - b^3$**

This type can be explained with the following examples.

Example 32: Expand $(2a-3b)^3$

Solution: $(2a-3b)^3$

$= (2a)^3 - 3(2a)(3b)(2a-3b) - (3b)^3$

$= 8a^3 - 18ab(2a-3b) - 27b^3$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

Example 33: If $x+\frac{1}{x}=5$, then find the value of $x^3+\frac{1}{x^3}$

Solution: We have $x+\frac{1}{x} = 5$

$$\left(x+\frac{1}{x}\right)^3 = (x)^3 + 3(x)\left(\frac{1}{x}\right)\times\left(x+\frac{1}{x}\right)+\left(\frac{1}{x}\right)^3$$

$$\left(x+\frac{1}{x}\right)^3 = x^3 + 3\left(x+\frac{1}{x}\right)+\frac{1}{x^3}$$

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right) \qquad \because \left(x+\frac{1}{x}\right)=5$$

$$(5)^3 = x^3 + \frac{1}{x^3} + 3(5)$$

$$125 = x^3 + \frac{1}{x^3} + 15$$

$$x^3 + \frac{1}{x^3} = 125 - 15$$

Thus, $$x^3 + \frac{1}{x^3} = 110$$

Exercise 2.13

1. Find the cube of the following:

(i) $x+4$ (ii) $2m+1$ (iii) $a-2b$ (iv) $5x-1$

(v) $2a+b$ (vi) $3x+10$ (vii) $2m+3n$ (viii) $4-3a$

(ix) $3x+3y$ (x) $7+2b$ (xi) $4x-2y$ (xii) $5m+4n$

2. If $x+\frac{1}{x}=8$, then find the value of $x^3+\frac{1}{x^3}$

3. If $x-\frac{1}{x}=3$, then find the value of $x^3-\frac{1}{x^3}$

4. If $x+\frac{1}{x}=7$, then find the value of $x^3+\frac{1}{x^3}$

5. If $x-\frac{1}{x}=2$, then find the value of $x^3-\frac{1}{x^3}$

6. Find the cube of the following by using formula.

(i) 13 (ii) 103 (iii) 0.99

2.2.9 Exponents and Powers

Introduction

Do you know what is the mass of the Earth?

It is 5, 970, 000, 000, 000, 000, 000, 000, 000 kg.

Can you read this number?

Mass of the moon is 73, 000, 000, 000, 000, 000, 000, 000 kg.

Which has greater mass, Earth or Moon?

These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents.

Exponents: Look at $100,000 = 10\times 10\times 10\times 10\times 10 = 10^5$

The short notation 10^5 stands for the product $10\times 10\times 10\times 10\times 10$.

Here 10^5 is the exponential form of 100, 000. "10" is called the base and "5" is called the exponent or index.

Similarity,

$$125 = 5\times 5\times 5 = 5^3$$

Here 5^3 is the exponential form of 125.

Key fact!

The number 10^5 is read as 10 raised to the power of 5

Try yourself!

Convert 243, 32 and 625 in exponential form. Also identify the base and exponent of each.

Exercise 2.14

1. Identify the base and exponent of each of the following:
   (i) 2^6 (ii) b^2 (iii) t^9 (iv) m^n
2. Find the value of the following:
   (i) 3^4 (ii) 7^3 (iii) 13^2 (iv) 2^8
3. Express the following in exponential form:
   (i) $5\times 5\times 5\times 5$ (ii) $3\times 3\times 3\times 3\times 3\times 3$ (iii) $b\times b\times b\times b$ (iv) $u\times u\times u\times v\times v$
   (v) $7\times 7\times 11\times 11\times 11\times 11$ (vi) $a\times a\times a\times c\times c\times c\times c\times d$
4. Express each of the following numbers using exponential notation:
   (i) 343 (ii) 512 (iii) 729 (iv) 3125

2.2.10 Laws of Exponents

Laws of Sum of Powers

Let us simplify $4^2\times 4^3$

$$4^2\times 4^3 = (4\times 4)\times(4\times 4\times 4)$$
$$= 4\times 4\times 4\times 4\times 4 = 4^5 = 4^{2+3}$$

Thus,

$$4^2\times 4^3 = 4^{2+3}$$

Try yourself!

Simplify and write in exponential form:
(i) $3^4\times 3^5$
(ii) $t^2\times t^7$
(iii) $(-6^{10})\times(-6^{20})$

When multiplying like bases, keep the base same and add the exponents. In general, for any non-zero integer x, where m and n are whole numbers.

$$x^m \times x^n = x^{m+n}$$

Law of quotient of powers with same base

Let us simplify $2^5 \div 2^3$

$$2^5 \div 2^3 = \frac{2^5}{2^3} = \frac{2\times 2\times 2\times 2\times 2}{2\times 2\times 2}$$

$$= 2^5 \times 2^{-3} = 2^{5-3}$$

Thus, $2^5 \div 2^3 = 2^{5-3}$

Try yourself!

Simplify and write in exponential form:

(i) $2^8 \div 2^3$
(ii) $9^{13} \div 9^8$
(iii) $b^{20} \div b^{11}$

When dividing like bases, keep the base same and subtract the denominator exponent from the numerator exponent.

In general, for any non-zero integer x where m and n are whole numbers.

$$x^m \div x^n = x^{m-n}$$

Law of Power of Power

Let us simplify $(6^4)^2$

$(6^4)^2$ means 6^4 is multiplied two times with itself.

$$(6^4)^2 = 6^4 \times 6^4$$

$$= 6^{4+4}$$

$$(x^m \times x^n = x^{m+n})$$

$$= 6^8 = 6^{4\times 2}$$

Thus, $(6^4)^2 = 6^{4\times 2}$

Try yourself!

Simplify and write the answer in exponential form.

i. $(5^4)^3$ ii. $(2^3)^{50}$ iii. $(8^{100})^3$

When raising a base with a power to another power, keep the base same and multiply the exponents.

In general, for any non-zero integer x where m and n are whole numbers.

$$(x^m)^n = x^{mn}$$

Note: $m \times n = mn$

Law of Power of the Product

Let us simplify $3^2 \times 5^2$

$$3^2 \times 5^2 = (3\times 3)\times(5\times 5)$$

$$= (3\times 5)\times(3\times 5)$$

$$= 15\times 15 = 15^2 = (3\times 5)^2$$

Thus, $3^2 \times 5^2 = (3\times 5)^2$

Try yourself!

Simplify and write the answer in exponential form.

i. $4^3 \times 2^3$
ii. $5^6 \times 8^6$
iii. $2^5 \times 7^5$

When raising a product to a power, distribute the power to each factor.

In general, for any non-zero integers x and y, where m is a whole number.

$$(x^m \times y^m) = (x \times y)^m$$

Law of Power of the Quotient

Let us simplify $\frac{2^5}{3^5}$

$$\frac{2^5}{3^5} = \frac{2\times 2\times 2\times 2\times 2}{3\times 3\times 3\times 3\times 3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^5$$

Thus,

$$\frac{2^5}{3^5} = \left(\frac{2}{3}\right)^5$$

$$x^m \div y^m = \left(\frac{x}{y}\right)^m$$

Try yourself!

Convert into another form using

$$x^m \div y^m = \frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$$

(i) $6^5 \div 7^5$

(ii) $(-2)^3 \div t^3$

(iii) $5^6 \div 3^6$

When raising a fraction to a power, distribute the power to each factor.

In general, for any non-zero integers x and y, where m is a whole number.

$$x^m \div y^m = \frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$$

Law of Zero Power

Let us simplify $\frac{2^3}{2^3}$

$$\frac{2^3}{2^3} = \frac{2\times 2\times 2}{2\times 2\times 2} = 1 \quad \cdots\cdots\cdots\cdots \quad (1)$$

By using $\quad x^m \div y^n = x^{m-n}$

$$2^3 \div 2^3 = 2^{3-3} = 2^0 \quad \cdots\cdots\cdots\cdots \quad (2)$$

From result (1) and (2), we have $2^0 = 1$

Try yourself!

Simplify:

(i) b^0

(ii) $(s+t)^0$

(iii) $(5x\text{-}3)^0$

Anything raised to the zero power is one.

In general, for any non-zero integer x

$$x^0 = 1$$

Law of Negative Power

Let us simplify

$$x^3 \div x^7$$

$$x^3 \div x^7 = \frac{x^3}{x^7}$$
$$= \frac{x \times x \times x}{x \times x \times x \times x \times x \times x \times x}$$
$$= \frac{1}{x \times x \times x \times x} = \frac{1}{x^4}$$

Note!
Reciprocal of a is $\frac{1}{a}$

Now $\quad x^3 \div x^7 = x^{3-7} = x^{-4}$ $\quad$ By using $\quad x^m \div x^n = x^{m-n}$

So,

$$x^{-4} = \frac{1}{x^4}$$

When a base is raised to a negative power reciprocal, keep the exponent with the original base and remove the negative sign.

In general, for any non-zero integer x, where m is a whole number.

$$x^{-m} = \frac{1}{x^m}$$

Laws of Fractional Exponents

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$
$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$
$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

Note!
$x^{\frac{1}{2}} = \sqrt{x}$
$x^{\frac{1}{3}} = \sqrt[3]{x}$

2.2.11 Application of Laws of Exponents

Let us solve some examples using rules of exponents.

Example 34:

Simplify and write answers in the exponential form.

(i) $\left(\frac{2^7}{2^3}\right) \times 2^5$ $\quad$ (ii) $3^3 \times 3^2 \times 5^5$ $\quad$ (iii) $\left(7^2 \times 7^4\right) \div 7^3$ $\quad$ (iv) $\left[\left(2^3\right)^2 \times 3^6\right] \times 5^6$

Solution:

(i) $\left(\frac{2^7}{2^3}\right) \times 2^5 = 2^{7-3} \times 2^5$
$= 2^4 \times 2^5$
$= 2^{4+5} = 2^9$

(ii) $3^3 \times 3^2 \times 5^5 = 3^{3+2} \times 5^5$
$= 3^5 \times 5^5$
$= \left(3 \times 5\right)^5 = 15^5$

(iii) $\left(7^2 \times 7^4\right) \div 7^3 = 7^{2+4} \div 7^3$
$= 7^6 \div 7^3$
$= 7^{6-3}$
$= 7^3$

(iv) $\left[\left(2^3\right)^2 \times 3^6\right] \times 5^6 = \left[2^{3\times 2} \times 3^6\right] \times 5^6$
$= \left[2^6 \times 3^6\right] \times 5^6$
$= \left(2\times 3\right)^6 \times 5^6$
$= \left(2\times 3\times 5\right)^6$
$= 30^6$

Example 35: Simplify

(i) $\left(\frac{3}{4}\right)^{-2}$ (ii) $\left(3^7 \times 3^3\right) \div 3^{10}$ (iii) $(64)^{-\frac{2}{3}}$

(iv) $\left(\frac{27x^9}{8x^6}\right)^{\frac{1}{3}}, x \neq 0$ (iv) $\frac{2^5 \times 3^7 \times 4^3}{8^2 \times 9^2 \times 6^2}$

Solution:

(i) $\left(\frac{3}{4}\right)^{-2} = \frac{3^{-2}}{4^{-2}} = \frac{4^2}{3^2}$
$= \frac{4\times 4}{3\times 3} = \frac{16}{9}$

(ii) $\left(3^7 \times 3^3\right) \div 3^{10} = 3^{7+3} \div 3^{10}$
$= 3^{10} \div 3^{10}$
$= 3^{10-10} = 3^0 = 1$

(iii) $(64)^{-\frac{2}{3}} = \left(4\times 4\times 4\right)^{-\frac{2}{3}}$
$= \left(4^3\right)^{-\frac{2}{3}} = 4^{-2} = \frac{1}{4^2}$
$= \frac{1}{4\times 4} = \frac{1}{16}$

(iv) $\left(\frac{27x^9}{8x^6}\right)^{\frac{1}{3}} = \left(\frac{3\times 3\times 3\times x^9}{2\times 2\times 2\times x^6}\right)^{\frac{1}{3}}$
$= \left(\frac{3^3 \times x^9}{2^3 \times x^6}\right)^{\frac{1}{3}}$
$= \frac{3^{3\times\frac{1}{3}} \times x^{9\times\frac{1}{3}}}{2^{3\times\frac{1}{3}} \times x^{6\times\frac{1}{3}}} = \frac{3x^3}{2x^2}$
$= \frac{3}{2}x^{3-2} = \frac{3}{2}x$

(v) $\frac{2^5 \times 3^7 \times 4^3}{8^2 \times 9^2 \times 6^2} = \frac{2^5 \times 3^7 \times (2^2)^3}{(2^3)^2 \times (3^2)^2 \times (2\times 3)^2}$
$= \frac{2^5 \times 3^7 \times 2^6}{2^6 \times 3^4 \times 2^2 \times 3^2} = \frac{2^{5+6} \times 3^7}{2^{6+2} \times 3^{4+2}}$
$= \frac{2^{11} \times 3^7}{2^8 \times 3^6} = 2^{11-8} \times 3^{7-6}$
$= 2^3 \times 3^1$
$= 8 \times 3$
$= 24$

Exercise 2.15

1. Using laws of exponents, simplify and write the answer in exponential form.

(i) $2^4 \times 5^4$ (ii) $\left(2x+3y\right)\left(2x+3y\right)$ (iii) $\left(2^4\right)^3$

(iv) $a^6 \times b^6$ (v) $9^8 \div 9^3$ (vi) $t^4 \times t^3$

(vii) $\dfrac{3^7}{3^4 \times 3^3}$ (viii) $\dfrac{2^8 \times a^5}{2^3 \times a^2}$ (ix) $a^4 \times a^9$

(x) $5^0 \times 3^0$ (xi) $\left(\left(5^2\right)^3 \times 5^4\right) \div 5^7$ (xii) $\dfrac{x^2 y^5}{xy^4}$

2. Simplify:

(i) $\dfrac{2x^4 y^{-4}}{8x^7 y^3}$ (ii) $\dfrac{x^{-4}}{y^{-8}}$ (iii) $\dfrac{6x^7}{2x^4}$

(iv) $(27)^{-\frac{2}{3}}$ (v) $\left(32x^{10}\right)^{\frac{2}{5}}$ (vi) $\left(81a^4\right)^{\frac{3}{2}}$

(vii) $2x^{-2}y^2 \times 2x^3 y$ (viii) $\dfrac{\left(2^5\right)^2 \times 7^3}{8^3 \times 7^{-1}}$ (ix) $\dfrac{3^5 \times a^7 b^4}{3^3 \times a^5 b^2}$

2.2.12 Standard Form

Let us convert the size of Red Blood Cell into standard form, where size of Red Blood cell is $7\times10^{-6}m$.

$$\begin{aligned}
&7\times10^{-6}\\
&=7\times\frac{1}{10^6}\\
&=7\times\frac{1}{1000000}\\
&=0.000007
\end{aligned}$$

Thus, $0.000007m$ is the size of Red Blood Cell in standard form.

Example 36: Express the following numbers in standard form.

(i) 3.52×10^6 (ii) 6.31×10^{-4}

Solution: (i) 3.52×10^6 (ii) 6.31×10^{-4}

$=3.52\times1000000$ $=\dfrac{6.31}{10^4}=\dfrac{6.31}{10000}=0.000631$

$=3520000$

2.2.13 Scientific Notation

We know that the mass of the Earth is 5, 976, 000, 000, 000, 000, 000, 000, 000 kg.

This number is not convenient to read and write. To make it convenient, we use powers.

5, 976, 000, 000, 000, 000, 000, 000, 000 kg.

= 5 976 × 1000, 000, 000, 000, 000, 000, 000 kg.

$= 5976 \times 10^{21} kg$

$= 5.976 \times 10^{3} \times 10^{21} kg$

$= 5.976 \times 10^{3+21} kg$

$= 5.976 \times 10^{24} kg$

Key fact!

Scientific Notation is a way of expressing very large and very small numbers conveniently.

The above form is called scientific notation.

A number in the form of $a \times 10^{b}$ is called scientific notation, where $1 \le a < 10$ and b is an integer.

Example 37: Express in scientific notation.

(i) 78,330 (ii) 0.000000027

Solution: (i) $78,330 = 7.833 \times 10000$

$= 7.833 \times 10^{4}$

(ii) $0.000000027 = \dfrac{27}{1000000000}$

$= \dfrac{27}{10^{9}} = 27 \times 10^{-9} = 2.7 \times 10^{1-9}$

$= 2.7 \times 10^{-8}$

Exercise 2.16

1. Express each numbers in standard form.

(i) 5.18×10^{7} (ii) 9.203×10^{-10} (iii) 2.169×10^{8}

(iv) 7.2×10^{5} (v) 5.4×10^{-6} (vi) 5.4105×10^{-12}

2. Express each number in scientific notation.

(i) 5620000 (ii) 0.0000014 (iii) 874020000

(iv) 7,586,000,000,000,000 (v) 4,460,500,000,000 (vi) 0.0000000000995

SUMMARY

- An open sentence is neither true nor false until the variables or unknowns have been replaced by specific values.
- A sentence that shows equality (=) between two expressions is called an equation.
-

Expression	Equation
• It is one sided. • It has no relation symbol.	• It is two sided. • It's relation symbol is equality (=).

- An inequality is a statement that contains one of the symbols: $<, >, \le$ or $\ge$.

- A polynomial expression or simply a polynomial is an algebraic expression consisting of one or more terms in each of which the exponent of the variable is zero or a positive integer.
- In a term, the number multiplied by the variable is called the coefficient of the variable as well as constant.
- Polynomials in which the highest exponent or sum of exponents is always 2, are called quadratic polynomials.
- If P and Q are two polynomials, then their addition is represented as $P + Q$.
- The subtraction of two polynomials P and Q is represented by $P - Q$ or $[P + (-Q)]$.
- The product of two polynomials is also a polynomial whose degree is equal to the sum of the degrees of the two polynomials.
- If a polynomial is exactly divisible by another polynomial then the remainder is zero.
- The number 10^5 is read as 10 raised to the power of 5.
- **Laws of exponents:**
  - $x^m \times x^n = x^{m+n}$
  - $x^m \div x^n = x^{m-n}$
  - $(x^m)^n = x^{mn}$
  - $(x^m \times y^m) = (x \times y)^m$
  - $x^m \div y^m = \dfrac{x^m}{y^m} = \left(\dfrac{x}{y}\right)^m$
  - $x^0 = 1$
  - $x^{-m} = \dfrac{1}{x^m}$
- **Laws of Fractional Exponents**

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

- Scientific notation is a way of expressing very large and very small numbers conveniently.

Sub-Domain(iii): Linear Equations and Inequalities

$ax + by = c$
$a, b \neq 0$

Students' Learning Outcomes

After completing this domain, the students will be able to:

Graph of Linear Equations

- recall gradient of a straight line
- recall the equations of horizontal and vertical lines i.e., $y = c$ and $x = a$
- find the value of "y" when "x" is given, from the equation and vice versa
- plot graph of linear equations in two variables i.e.,
  $y = mx$ and $y = mx + c$
- interpret the gradient/slope of the straight line
- determine the y-intercept of a straight line

Linear Equations

- change the subject of the formula
- calculate the value of unknown in a given formula by substituting the values of suitable unknown

Simultaneous Linear Equations

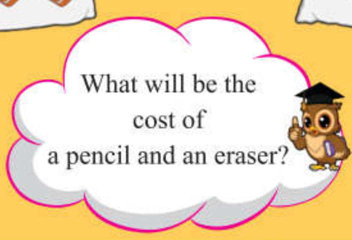
- construct simultaneous linear equations in two variables
- solve simultaneous linear equations in two variables using
  - ❖ elimination method
  - ❖ substitution method
  - ❖ graphical method
- solve problems from real life situations involving two simultaneous linear equations in two variables

Linear Inequalities

- solve simple linear inequalities i.e.,
  - ❖ $ax > b$ or $cx < d$
  - ❖ $ax + b < d$
  - ❖ $ax + b > d$

Advanced/Additional

represent the solution of linear inequality on the number line



Let x = cost of a pencil
y = cost of an eraser

$3x + 4y = 50$ .... (i)

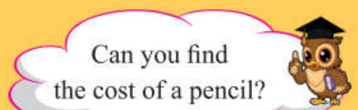
$2x + 2y = 30$ .... (ii)

$x = 15 - y$ (put in (i))

$3(15 - y) + 4y = 50$

$y = 5$

So, the cost of an eraser is Rs. 5



Linear Equation

If a, b and c are real numbers and a and b are not both zero, then $ax + by = c$ is called linear equation in two variables.

The two variables are x and y. The numbers a and b are called coefficients of x and y respectively.

For example: $3x + 7y + 4 = 0$ and $x - 5y = 10$ are linear equations in two variables.

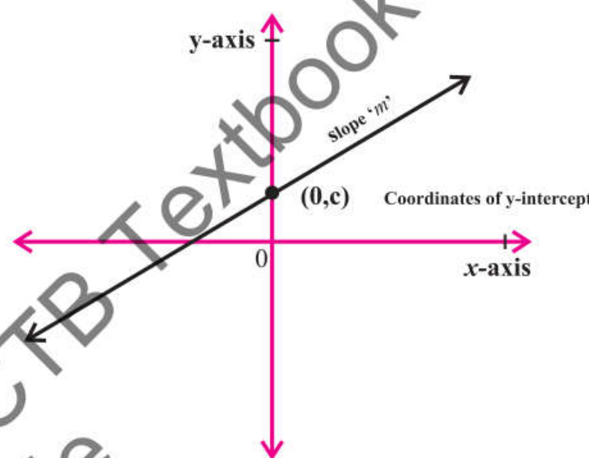
Key fact!

$ax + by = c$ is the standard form of the linear equation in two variables x and y, where $a \neq 0$, $b \neq 0$.

2.3.1 Slope-Intercept Form

If $y = mx + c$, where m and c are constants, then it is called slope-intercept form of the linear equation.

The slope or gradient m represents the steepness of a line and the y-intercept c represents the y-coordinate of the point where the line intersects the *y-axis* i.e., *(0, c)*



For example: $y = 2x + 4$, $y = \frac{1}{3}x - 8$

and $y = -5x - 7$.

Example 1: Find the slope and y-intercept of the equation $6x + 2y = 1$

Solution: Given that $6x + 2y = 1$

To find the slope and y-intercept, convert the given equation into slope-intercept form

$$2y = -6x + 1$$

$$y = -\frac{6}{2}x + \frac{1}{2}$$

$$y = -3x + \frac{1}{2}$$

So, slope $= -3$, y-intercept $= \left(0, \frac{1}{2}\right)$

Example 2: Find the value of y, in the equation $x + 3y = 7$ when $x = 5$

Solution $x + 3y = 7$

Substitute $x = 5$ in the above equation

$$5 + 3y = 7$$

$$3y = 7 - 5$$

$$3y = 2$$

$$y = \frac{2}{3}$$

Example 3: Find the value of x in the equation $6x + 2y = 10$ when $y = 2$

Solution: $6x + 2y = 10$

Substitute $y = 2$ in the above equation

$$\begin{aligned} 6x + 2(2) &= 10 \\ 6x + 4 &= 10 \\ 6x &= 10 - 4 \\ 6x &= 6 \\ x &= 1 \end{aligned}$$

Try yourself!

If $k = 7$ and $\ell = 5$

What will be the value of P

when $P = \frac{3k\ell}{5}$?

Changing the Subject of the Formula

To change the subject of the formula, we rearrange the formula and make the different variable a subject.

For example, in the formula $u = v + at$, we make t the subject.

$$\begin{aligned} u &= v + at \\ u - v &= at \\ t &= \frac{u-v}{a} \end{aligned}$$

Hence, the letter t is separated, so t becomes the subject of the formula.

Example 4: Make d the subject of the formula $\ell = a + (n-1)d$ and find d when $\ell = 10, a = 1$ and $n = 4$

Solution:

$$\begin{aligned} \ell &= a + (n-1)d \\ \ell - a &= (n-1)d \\ d &= \frac{\ell - a}{n-1} \end{aligned}$$

Where 'd' is the required subject

Now, put the values of ℓ, a, n, we get

$$d = \frac{10-1}{4-1}$$

$$d = \frac{9}{3}$$

$$d = 3$$

Exercise 2.17

1. Write each equation in slope - intercept form. Also find the slope and y-intercept.

(i) $10x + 35 = -5y$ (ii) $x + 4y = 8$ (iii) $-8x + y = 5$

(iv) $2y - 8 = 6x$ (v) $3y = 15x + 9$ (vi) $3 = \frac{x}{4} + y$

2. If $2x + y = 4$, then find the value of y when $x = 13$

3. If $2y - 7x = -3$, then find the value of y when $x = 3$

4. If $7y - 10x - 70 = 0$, then find the value of x when $y = -1$
5. Make y the subject of each of the following.

(i) $m - y = s$ (ii) $\dfrac{y+3}{2} = x$ (iii) $\dfrac{x+y}{6} = 2$ (iv) $y^2 + x = 7$

(v) $\dfrac{y}{2} = 4x - 7$ (vi) $10 - 2y = 3x$ (vii) $2 + 3y = ax + b$ (viii) $x = 2(y+3)$

6. If $P = a + b + c$, make b the subject of the formula and find b when $P = 40$, $a = 10$ and $c = 7$.
7. If $A = \dfrac{h}{2}(a+b)$, make h the subject of the formula. Also find h when $a = 4$, $b = 8$ and $A = 24$.
8. If $y = \dfrac{3x-9}{2}$, make x the subject. Also find value of x when $y = 3$.
9. Use the formula $y = 100 - \dfrac{100}{1+x}$ to find the value of y when $x = 19$.

2.3.2 Ordered Pair

An ordered pair is a composition of two elements that are separated by a comma and written inside the parentheses. For example, in an ordered pair (x, y) the first element x is called x-coordinate or abscissa and the second element 'y' is called y-coordinate or ordinate.

Key fact!

(i) (x, y) and (y, x) are two different ordered pairs.
(ii) $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$

Cartesian Plane

Cartesian plane is formed by two perpendicular number lines and is used to plot ordered pairs.

Cartesian plane is also known as xy-plane or coordinate plane. The horizontal line is called x-axis and the vertical line is called y-axis. Both axes meet at a point is called the origin, whose coordinates are (0, 0).

Key fact!

Rene Descartes, the great French mathematician of the 17th century who first used the idea of coordinate plane.

(1596-1650)

Quadrants and Graphing Ordered Pairs on a Cartesian Plane:

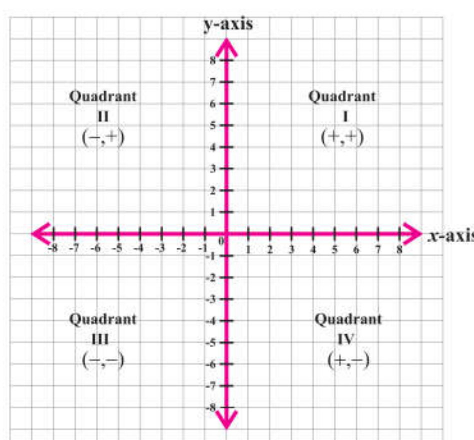
Both axes (x-axis and y- axis) divide a cartesian plane into four regions, called quadrants. The quadrants are identified by Roman numerals, starting from the top right and going around counterclockwise.

(i) In quadrant I, both the x and y-coordinates are positive i.e., $(+,+)$

(ii) In quadrant II, the x-coordinate is negative but the y-coordinate is positive. i.e., $(-,+)$

(iii) In quadrant III, both x and y-coordinates are negative, i.e., $(-,-)$

(iv) In quadrant IV, x-coordinate is positive and y-coordinate is negative. i.e., $(+,-)$



Let us learn the steps to graph ordered pairs:

Step 1: Always start from the origin and move horizontally by $|x|$ units to the right if x is positive and to the left if x is negative. Stay there.

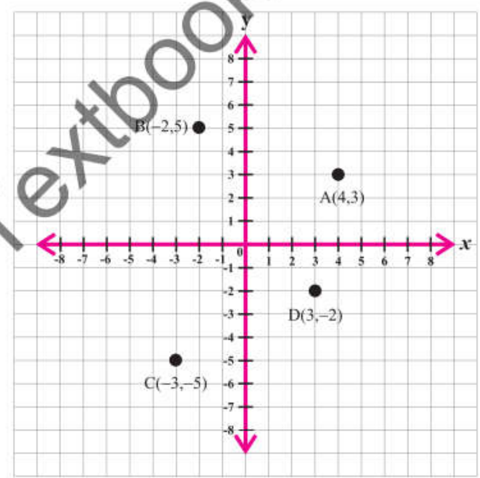
Step 2: Start from where you have stopped in step 1 and move vertically by $|y|$ units to up if y is positive and to down if y is negative. Stay there.

Step 3: Put a dot exactly at the point where you have stopped in step 2 and that dot shows the ordered pair (x, y).

Key fact!
$|5| = 5$
$|-5| = 5$

Example 5: Locate the points A (4, 3), B (−2, 5), C (−3, −5) and D (3, −2) in the cartesian plane.

Solution:



To locate the point A (4, 3), start from the origin and move to the right by 4 units (as 4 is positive) then move up by 3 units (as 3 is positive).
To locate the point B (−2, 5), start from the origin and move to the left by 2 units (as 2 is negative), then move up by 5 units (as 5 is positive).
To locate the point C (−3, −5), start from the origin and move to the left by 3 units (as 3 is negative), then move down by 5 units (as 5 is negative).
To locate the point D (3, −2), start from the origin and move to the right by 3 units (as 3 is positive) then move down by 2 units (as 2 is negative)

Drawing Graphs of the following Equations

(a) $y = c$, where c is constant. (b) $x = a$, where a is constant.
(c) $y = mx$, where m is constant. (d) $y = mx + c$, where m and c both are constants.

To draw the graph of linear equation, we need at least two points.

(a) $y = c$, where c is constant

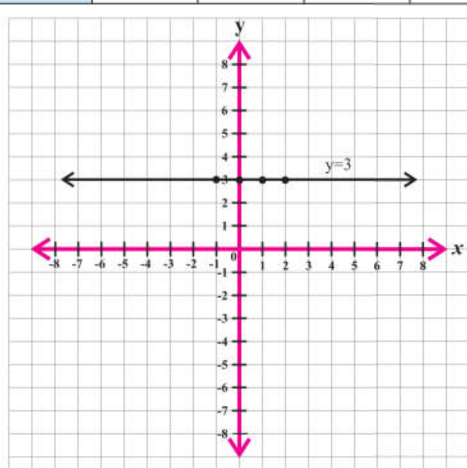
Consider the equation $y = 3$, where $c = 3$

Table for the points of equation $y = 3$ is given below:

x	−1	0	1	2
y	3	3	3	3

Key fact!
The graph of the linear equation is a straight line.

Graph of $y = 3$ is:



The graph of the equation $y = 3$ is the straight line and parallel to x-axis.

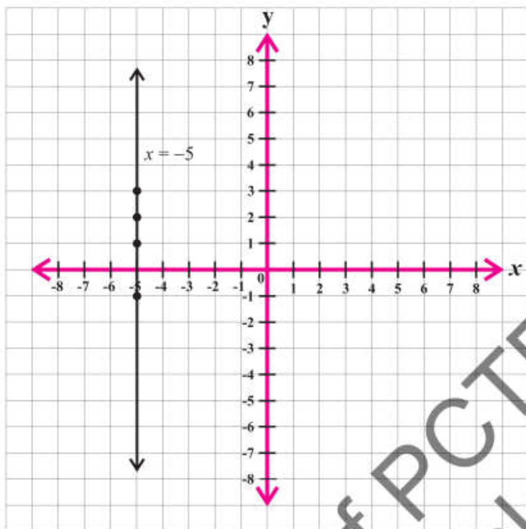
(b) $x = a$, where a is constant,

Consider the equation $x = -5$, where $a = -5$

Table for the points of equation $x = -5$ is given below:

x	-5	-5	-5	-5
y	-1	0	1	2

The graph of the equation $x = -5$ is:



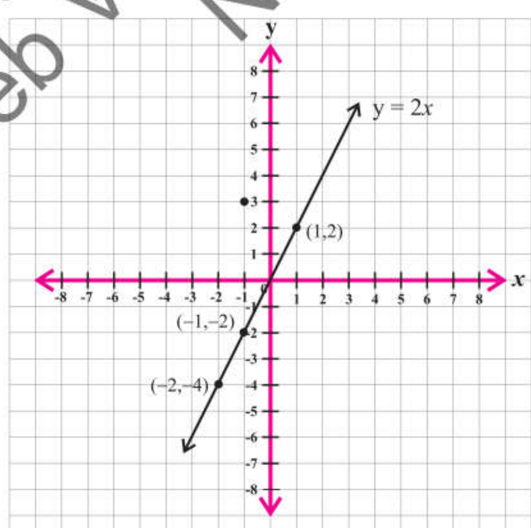
The graph of the equation $x = -5$ is the straight line and parallel to y-axis.

(c) $y = mx$, where m is constant. Consider the equation $y = 2x$, where $m = 2$

Table for the points of equation $y = 2x$ is given below:

x	-2	-1	0	1
y	-4	-2	0	2

The graph of the equation $y = 2x$ is:



The graph of the equation $y = 2x$ is the straight line, it passes through the origin (0, 0) and having slop 2.

(d) $y = mx + c$, where m and c are constants.

Consider the equation $y = 3x - 2$, where $m = 3$, $c = -2$

Put $x = 0, 1, 2, 3$ into $y = 3x - 2$.

$x = 0$	$x = 1$	$x = 2$	$x = 3$
$y = 3(0) - 2$	$y = 3(1) - 2$	$y = 3(2) - 2$	$y = 3(3) - 2$
$y = 0 - 2$	$y = 3 - 2$	$y = 6 - 2$	$y = 9 - 2$
$y = -2$	$y = 1$	$y = 4$	$y = 7$
$(0, -2)$	$(1, 1)$	$(2, 4)$	$(3, 7)$

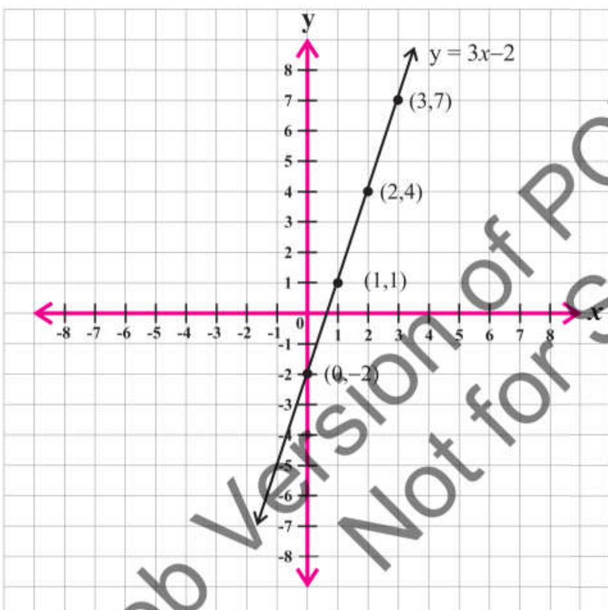
Table for the above points is given below:

x	0	1	2	3
y	-2	1	4	7

Key fact!

y-intercept is the point where the line cuts the y-axis.

Graph of $y = 3x - 2$ is:



We see that $y = 3x - 2$ represents the straight line which does not passes through the origin (0 , 0), having slope $m = 3$ and y-intercept $(0, -2)$.

Slope can also be determined as:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{2-1} = \frac{3}{1} = 3$$

Key fact!

The slope of the line $= \dfrac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two points on the line.

Exercise 2.18

1. Determine the quadrant of the coordinate plane in which the following points lie: J(3, −7), K(5, 9), L(−4, 4), M(−8, −2)
2. Draw the graph of each of the following.

   (i) $y = -2$ (ii) $y = 5$ (iii) $x = -2$ (iv) $x = 0$

(v) $y=0$ (vi) $x=7$ (vii) $y=-3x$ (viii) $y=\frac{1}{2}x$

(ix) $4x-y=0$ (x) $2x-y=4$ (xi) $y=-3x+3$

2.3.3 Simultaneous Linear Equations

If two or more linear equations consisting of same set of variables are satisfied simultaneously by the same values of the variables, then these equations are called simultaneous linear equations.

Simultaneous Linear Equations in One and Two Variables

We know that a linear equation is an algebraic equation in which each term is either a constant or a variable or the product of a constant or a variable. The standard form of linear equation consisting of one variable is:

$$ax+b=c \quad \forall a,b \in \mathbb{R}, a \neq 0 \quad (\forall \text{ means for all})$$

Similarly, a linear equation in two variables is of the form $ax + by = c$, where a, b and c are constants. Two linear equations considered together, form a system of linear equations. For example,

$x + y = 2$ and $x - y = 1$ is a system of two linear equations with two variables x and y. This system of two linear equations is known as the simplest form of linear system which can be written in general form as:

$$a_1x+b_1y=c_1$$
$$a_2x+b_2y=c_2$$

Concept of Formation of Linear Equation in Two Variables

Statements involving two unknowns can be written in algebraic form as explained in the following examples.

Example 6: Write an equation for each statement.

(i) The price of a book and 3 pencils is 90 rupees.
(ii) Sum of two numbers is 5.
(iii) The weight of Iram is half of the weight of Ali.

Solution:

(i) Price of a book and 3 pencils = Rs.90
Let the price of one book $= x$
The price of one pencil $= y$
$\therefore$ The equation can be written as $x + 3y = 90$

(ii) Sum of two numbers = 5
Let the first number $= x$
The second number $= y$
$\therefore$ The equation can be written as $x + y = 5$

(iii) Let the weight of Iram $= x$
The weight of Ali $= y$
$\therefore$ The equation can be written as $x=\frac{y}{2}$

Try yourself!

A piece of wood was 36*cm* long. It was cut into 3 pieces. The lengths in *cm* are $2x-1$, $x+4$ and $x+5$. What is length of the longest piece?

2.3.4 Construction of Simultaneous Linear Equations in Two Variables

Simultaneous linear equations can be constructed in algebraic form from statements involving two unknowns. This can be explained in the following example.

Example 7: The sum of present ages of Huria and Zainab is 12 years, Zainab is elder than Huria by 8 years. Write the statement in equation form.

Solution: Let the present age of Zainab = x years

and

the present age of Huria = y years

sum of their present ages = 12

$\therefore \quad x + y = 12$

and Zainab is elder than Huria by 8 years

$\therefore \quad x - y = 8$

Hence simultaneous linear equations are

$$x + y = 12$$
$$x - y = 8$$

Exercise 2.19

1. Write equations for the following statements:
   - (i) The difference between father's age and daughter's age is 26 years.
   - (ii) The price of 6 biscuits is equal to the price of one chocolate
   - (iii) If a number is added to three times of another number, the sum is 25.
   - (iv) The division of sum of two numbers by their difference is equal to 1 (2^{nd} number is less than 1^{st})
   - (v) Twice of any age increased by 7 years becomes y years.
2. Write simultaneous linear equations for the following statements:
   - (i) The sum of two numbers is 10 and their difference is 2.
   - (ii) If double the age of son is added to the age of father, the sum is 54. But if double the age of father is added to the age of son, the sum is 92.
   - (iii) One pen and two erasers cost is Rs. 45 and 2 pens and 3 erasers cost is Rs. 80.

2.3.5 Solution of Simultaneous Linear Equations

The solution of simultaneous linear equations means finding values for the variables that make them true sentences. Let us learn how to find the solution of simultaneous linear equations.

Solve Simultaneous Linear Equations

There are many methods of solving simultaneous linear equations but here we shall confine ourselves to the following three methods.

- (i) Method of elimination.
- (ii) Method of substitution.
- (iii) Graphical method.

(i) Method of Elimination

Example 8: Find the solution with the method of elimination.

$$9x + 8y = 1$$
$$5x - y = 6$$

Solution: $9x + 8y = 1$ .... (i)

$5x - y = 6$ .... (ii)

Step 1: Convert the given equation into an equivalent equation in such a way that the coefficient of one variable must be same. Multiply both sides of equation (ii) by 8, we have

$8(5x - y) = 8(6)$

$40x - 8y = 48$ .... (iii)

Step 2: Add equations (i) and (iii) to find the value of one variable.

$$\begin{array}{rcl} 9x + 8y & = & 1 \\ 40x - 8y & = & 48 \\ \hline 49x & = & 49 \\ \hline \end{array}$$

$$x = \frac{49}{49} = 1$$

Step 3: Put the value of x in equation (i) or (ii) to find the value of y.

$5x - y = 6$ .... (ii)

$5(1) - y = 6$

$5 - y = 6$

$y = 5 - 6 = -1$

Thus, $x = 1$ and $y = -1$ is the required solution.

Step 4: Check the answer by placing the values of x and y in any equation.

$9x + 8y = 1$

L.H.S $= 9x + 8y$

$= 9(1) + 8(-1)$

$= 9 - 8 = 1 =$ R.H.S

(ii) Method of Substitution

Example 9: Find the solution set with the method of substitution.

$3x + 5y = 5$

$x + 2y = 1$

Solution: $3x + 5y = 5$ .... (i)

$x + 2y = 1$ .... (ii)

Step 1: Find the value of x or y from any of the given equations.

From equation (ii)

$x + 2y = 1 \Rightarrow x = 1 - 2y$ .... (iii)

Step 2: Substitute the value of x in equation (i).

$3x + 5y = 5$

$\Rightarrow \quad 3(1 - 2y) + 5y = 5$

$\Rightarrow \quad 3 - 6y + 5y = 5$

Try yourself!

$x + y = 6$ and $2x + 5y = 18$
What are the values of x and y?

$\Rightarrow \quad 3 - y = 5$

$\Rightarrow \quad y = 3 - 5 = -2$

Step 3: Put the value of y in equation (iii) to find the value of x.

$x = 1 - 2y \quad$ (from (iii))

$x = 1 - 2(-2) = 1 + 4$

$x = 5$

Hence, $x = 5$ and $y = -2$ is the required solution.

Step 4: Check the answer by putting the values in any equation i.e., in (i) or (ii).

$3x + 5y = 5 \quad$ from (i)

L.H.S $= 3(5) + 5(-2)$

$= 15 - 10$

$= 5 =$ R.H.S

Also check by putting the values in equation (ii) $x + 2y = 1$

L.H.S $= (5) + 2(-2)$

$= 5 - 4$

$= 1 =$ R.H.S

Try yourself!

$(-1, 0)$ and $(3, 1)$

Which of the following equation is satisfied by both of these pair of numbers (x, y)?

(i) $x + y = -1$ (ii) $x + 2y = 5$

(iii) $x - 3y = 0$ (iv) $x - 4y = -1$

(iii) Graphical Method

Example 10: Find the solution set with graphical method.

$x + y = 3$

$-x + y = 1$

Solution: Given equations can be written as

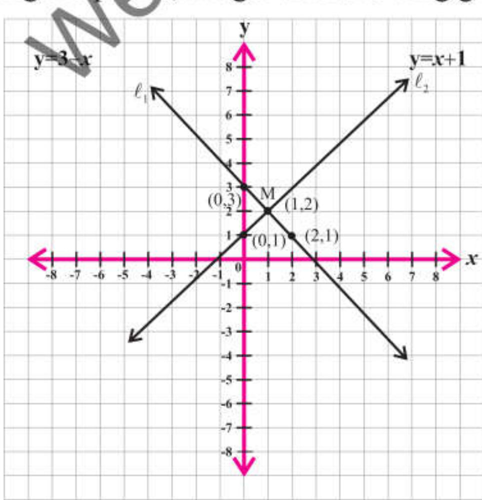
$y = 3 - x \qquad y = x + 1$

Table of values

x	0	2
y	3	1

x	0	1
y	1	2

By plotting the points, we get the following graph.



The solution of the system of equations is the point M where both ℓ_1 and ℓ_2 meet i.e., M (1, 2)

Exercise 2.20

1. Find the solution set by using the method of elimination.

(i) $2x + 5y = -1$, $x - 2y = 4$ (ii) $x + y = 2$, $x - y = 0$ (iii) $2x + 3y = 3$, $x + 5y = 5$

iv) $x - 4y = 4$, $4x - y = 16$ (v) $2x - 3y = 6$, $3x + 5y = 0$ (vi) $3x - 4y = 7$, $5x + y = 27$

2. Find the solution set by using the method of substitution.

(i) $2x + 2y = 5$, $x - 2y = 3$ (ii) $5x + 2y = 15$, $-2x + y = 4$ (iii) $6x + y = 2$, $x - 4y = 15$

(iv) $2x + 7y = 10$, $3x + y = 3$ (v) $2x - 4y = -10$, $y - 5x = -5$ (vi) $x + 8y = 15$, $3x - y = 0$

3. Find the solution set by using the graphical method.

(i) $2x - y = 2$, $x + y = 4$ (ii) $3x - y = 1$, $2x + y = 4$ (iii) $-x + y = 1$, $x + y = 3$ (iv) $x - 4y = 3$, $2x + 4y = 0$

2.3.6 Solving Real Life Problems Involving Two Simultaneous Linear Equations in two Variables

Example 11: A number is half of another number. The sum of 3 times of 1st number and 4 times of 2nd number is 22. Find the numbers.

Solution: Suppose that the numbers are x and y. Then according to given condition.

$$x = \frac{y}{2} \quad \text{................ (i)}$$

$$3x + 4y = 22 \quad \text{................ (ii)}$$

From equation (i), we get

$$x = \frac{y}{2} \Rightarrow y = 2x \quad \text{................ (iii)}$$

Put the value of y in equation (ii)

$$3x + 4(2x) = 22 \Rightarrow 3x + 8x = 22 \Rightarrow 11x = 22 \Rightarrow x = \frac{22}{11} = 2$$

Put the value of x in equation (iii)

$$y = 2x \;\Rightarrow\; y = 2(2) = 4$$

Thus, the numbers are 2 and 4.

Example 12: 11 years ago, Ali's age was 5 times of Waleed's age. But after 7 years, Ali's age will be 2 times of Waleed's age. Find their ages.

Solution: Suppose that Ali's age is x years and Waleed's age is y years. Before 11 years their ages were:

Ali's age = $(x - 11)$ years, Waleed's age = $(y - 11)$ years

Then according to the given condition,

Ali's age = 5 (Waleed's age)

$\Rightarrow \quad x - 11 = 5(y - 11)$

$\Rightarrow \quad x - 11 = 5y - 55$

$\Rightarrow \quad x - 5y = -55 + 11$

$\Rightarrow \quad x - 5y = -44 \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$

After 7 years, their ages will be:

Ali's age = $(x + 7)$ years, Waleed's age = $(y + 7)$ years

Then according to the given condition,

Ali's age = 2 (Waleed's age)

$\Rightarrow \quad x + 7 = 2(y + 7)$

$\Rightarrow \quad x + 7 = 2y + 14$

$\Rightarrow \quad x - 2y = 14 - 7$

$\Rightarrow \quad x - 2y = 7 \quad \ldots\ldots\ldots\ldots\ldots (ii)$

By solving equation (i) and (ii). Subtracting (ii) from (i)

$$\begin{array}{rcrcrl} x & - & 5y & = & -44 & \ldots\ldots\ldots(i) \\ \pm x & \mp & 2y & = & \pm 7 & \ldots\ldots\ldots(ii) \\ \hline & - & 3y & = & -51 & \text{(By subtracting)} \\ & & y & = & 17 & \end{array}$$

Put the value of y in equation (ii)

$x - 2y = 7$

$\Rightarrow \quad x - 2(17) = 7$

$\Rightarrow \quad x - 34 = 7$

$\Rightarrow \quad x = 34 + 7 = 41$

Thus, Ali's age = 41 years and Waleed's age = 17 years.

Example 13: If numerator and denominator of a fraction are increased by 5, the fraction becomes $\frac{1}{2}$ and if numerator and denominator are decreased by 3, the fraction becomes $\frac{2}{5}$. Find the fraction.

Solution: Suppose the numerator is x and denominator is y, therefore the fraction is $\frac{x}{y}$. Then, according to the given condition.

$$\frac{x+5}{y+5} = \frac{1}{2} \Rightarrow 2(x+5) = y+5 \Rightarrow 2x - y = -5$$

$$\Rightarrow \quad y = 2x + 5 \quad \ldots\ldots\ldots\ldots (i)$$

Then, by the second condition.

$$\frac{x-3}{y-3}=\frac{2}{5}$$

$\Rightarrow \quad 5(x-3) = 2(y-3)$

$\Rightarrow \quad 5x-15 = 2y - 6$

$\Rightarrow \quad 5x - 2y = 15 - 6$

$\Rightarrow \quad 5x - 2y = 9 \quad$ …………(ii)

Put the value of y from equation (i), in equation (ii) we have,

$5x - 2(2x + 5) = 9$

$\Rightarrow \quad 5x - 4x - 10 = 9$

$\Rightarrow \quad x - 10 = 9$

$\Rightarrow \quad x = 10 + 9 = 19$

Put the value of x in equation (i),

$y = 2x + 5$ …………(iii)

$\Rightarrow \quad y = 2(19) + 5$

$\Rightarrow \quad y = 38 + 5$

$\Rightarrow \quad y = 43$

Thus, the required fraction is $\frac{19}{43}$

Exercise 2.21

1. Ahmad added 5 in the twice of a number. Then he subtracted half of the number from the result. Finally, he got the answer 8. Find the number.
2. If we add 3 in the half of a number, we get the same result as we subtract 1 from the quarter of the number. Find the number.
3. The sum of two numbers is 5 and their difference is 1. Find the numbers.
4. The difference of two numbers is 4. The sum of twice of one number and 3 times of the other number is 43. Find the numbers.
5. Samia is 7 years older than Amina. Find their ages when $\frac{1}{4}$ of Samia's age is equal to the $\frac{1}{2}$ of Amina's age.
6. 5 years ago Ahsan's age was 7 times of Shakeel's age but after 3 years Ahsan's age will be 4 times of Shakeel's age. Calculate their ages.
7. The denominator of a fraction is 5 more than the numerator. But if we subtract 2 from the numerator and the denominator of the fraction, we get $\frac{1}{6}$. Find the fraction.
8. Fida bought 3kg melons and 4kg mangoes for Rs. 470. Anam bought 5kg melons and 6kg mangoes for Rs.730. Calculate the price of melons and mangoes per kg.
9. The cost of 2 footballs and 10 basketballs is Rs. 2300 and the cost of 7 footballs and 5 basketballs is Rs. 2650. Calculate the price of each football and basketball.

10. If numerator and denominator of a fraction are increased by 1, the fraction becomes $\frac{2}{3}$ and if numerator and denominator of same fraction are decreased by 2, it becomes $\frac{1}{3}$. Find the fraction.

11. If numerator and denominator of a fraction are decreased by 1, the fraction becomes $\frac{1}{2}$. If numerator and denominator of the same fraction are decreased by 3, it becomes $\frac{1}{4}$. Find the fraction.

2.3.7 Linear Inequalities

An inequality in which the variable occurs only to the first power, is called a linear inequality.
The standard form of linear inequality in one variable is $ax + b < 0$, where a and b are real numbers and $a \neq 0$. The symbol $<$ may be replaced by $>$, $\geq$ or $\leq$ also.

Properties of Inequalities

Four properties of inequalities are given with names.

Recall!

1. **Law of Trichotomy**

   The law of Trichotomy says that only one of the following is true:

   $$a < b \text{ or } a = b \text{ or } a > b$$

   where a and b are real numbers.

2. **Transitive Property**

   (i) If $a < b$ and $b < c$, then $a < c$

   (ii) If $a > b$ and $b > c$, then $a > c$

   where $a, b, c \in \mathbb{R}$

3. **Addition Property**

   (i) If $a < b$, then $a + c < b + c$

   (ii) If $a > b$, then $a + c > b + c$

   where $a, b, c \in \mathbb{R}$

4. **Multiplication Property**

   (i) If $a > b$ and $c > 0$, then $ac > bc$

   or

   If $a < b$ and $c > 0$, then $ac < bc$

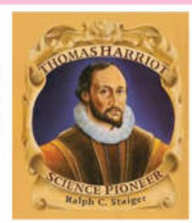
   (ii) If $a > b$ and $c < 0$, then $ac < bc$

   or

   If $a < b$ and $c < 0$, then $ac > bc$

Key fact!

The inequality symbols were introduced by an English Mathematician Thomas Harriot (1560-1621).



Solving Linear Inequalities

Let's explain the method of solving linear inequalities in one variable with the help of following examples.

Example 14: Solve $2x - 5 > 1$

Solution: $2x - 5 > 1$

$2x > 6$ (Adding 5 to each side)

$x > 3$ (Multiplying each side by $\frac{1}{2}$)

Hence the solution set = $\{x \mid x > 3\}$

Try yourself!
Solve:
$9x - 6 < 4x + 9$

Example 15: Solve $-3x \le 6$

Solution:

$$-3x \le 6$$

$$\frac{-3x}{-3} \ge \frac{6}{-3}$$

$$x \ge -2$$

Hence the solution set = $\{x \mid x \ge -2\}$

Representing Inequalities on a Number Line

Representing inequalities on a number line makes it easier to understand the solution of an inequality.

For example,

Key fact!
A number line is a straight line on which numbers are placed at equal intervals.

Example 16: Solve and show the solution on number line
$4x + 1 \le 13$

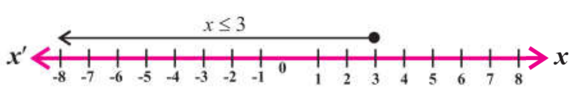
Solutions:

$4x + 1 \le 13$

or $4x \le 12$

or $x \le 3$

Hence the solution set = $\{x \mid x \le 3\}$



Exercise 2.22

1. Solve:
   (i) $x - 3 < 2$ (ii) $-7x < 49$ (iii) $4x \ge 16$ (iv) $5x + 5 < -15$
   (v) $3x - 1 > 17$ (vi) $-x + 3 \le 6$
2. Solve and show the solution on number line.
   (i) $5x > 2x + 6$ (ii) $7x + 6 < 5x$ (iii) $-3x < 21$ (iv) $-9x > -27$

SUMMARY

- If a, b and c are real numbers and a and b are not both zero, then $ax + by = c$ is called linear equation in two variables.
- An ordered pair is a composition of two elements that are separated by a comma and written inside the parentheses.
- Cartesian plane is formed by two perpendicular number lines and is used to plot ordered pairs on it.
- If two or more linear equations consisting of same set of variables are satisfied simultaneously by the same values of the variables, then these equations are called simultaneous linear equations.
- An inequality in which the variable occurs only to the first power, is called a linear inequality.

Review Exercise 2

1. Four options are given against each statement. Encircle the correct one.

(i) The general term of the arithmetic sequence is:

(a) $a + nd$ (b) $a + (n-1)d$ (c) ar^{n-1} (d) ar^n

(ii) Which term of the geometric sequence 3, 6, 12, 24, ………… is 384?

(a) 7 (b) 8 (c) 9 (d) 10

(iii) The 5th term of the sequence $a_n = 2n + 3$ is:

(a) -13 (b) -7 (c) 7 (d) 13

(iv) Find the missing term in 7, 11, 13, 17, 19, ________, 25:

(a) 20 (b) 21 (c) 22 (d) 23

(v) In the arithmetic sequence 7, 10, 13, …, the 20th term is:

(a) 59 (b) 56 (c) 66 (d) 64

(vi) Polynomial $3y^2$ is:

(a) Linear (b) Quadratic (c) Cubic (d) Biquadratic

(vii) The square of 99 by formula is:

(a) $(100)^2 - 2(100)(1) + (1)^2$ (b) $(100)^2 + 2(100)(1) + (1)^2$

(c) $(100)^2 + 2(100)(1) - (1)^2$ (d) $(100)^2 - 2(100)(1) - (1)^2$

(viii) If $x + \frac{1}{x} = 9$, then $x^2 + \frac{1}{x^2} = ?$

(a) 81 (b) 18 (c) 27 (d) 79

(ix) The factorization of $4x^2 - 12xy + 9y^2$ is:

(a) $(2x+3y)(2x-3y)$ (b) $(2x-3y)(2x-3y)$

(c) $(2x+3y)(2x+3y)$ (d) None of these

(x) Simplified form of (100-99°) 100 is:

(a) 10000 (b) 100 (c) 9900 (d) 99000

(xi) If $3y - x = 4$, find the value of y, when $x = 2$.

(a) 4 (b) 3 (c) 2 (d) $\frac{1}{2}$

(xii) Make x the subject in $y = -2x + 1$

(a) $\frac{1+y}{2}$ (b) $\frac{y-1}{2}$ (c) $\frac{1-y}{2}$ (d) $2(1+y)$

(xiii) If $x + y = 6$, $x - y = 2$, then $x = ?$

(a) 4 (b) 2 (c) 6 (d) 8

(xiv) The line $y = -7x$ passes through:

(a) (0,7) (b) (7,0) (c) (0,0) (d) (7,7)

(xv) If $7x < -14$, then:

(a) $x > -2$ (b) $x > -\frac{1}{2}$ (c) $x > \frac{1}{2}$ (d) $x < -2$

2. Find the next two terms.

(i) 12, 14, 17, 21, ____, ____ (ii) 1, 3, 9, ____, ____

3. Find the first four terms.

(i) $a_n = 8 + 2n$ (ii) $a_n = 6.3^{n-1}$

4. Find a_{15} by position to term rule of the arithmetic sequence 1, 4, 7, 10, ...

5. Shahid earned Rs.240 in the 1st day, Rs. 340 in the 2nd day and Rs.440 on 3rd day. How much did he earn in the 9th day?

6. Factorize the following:

(a) $3xy + 6x^2y^2 + 9xz$ (b) $y^4 - 12y^2 + 36$

(c) $x^8 - y^8$ (d) $x^2 + 2x - 63$

7. Solve:

(a) $(6x^3 + x^2 - 26) - (9 + 3x^2 - 5x^3)$ (b) $(y^2 - 5)(-y^2 + 5)$

(c) $(x^3 + x - 2) \div (x - 1)$

8. Simplify:

(a) $\left(\frac{2m}{n}\right)^4$ (b) $\left(x^3y^2\right)^4$ (c) $\frac{12x^2y^6z^9}{2xy^4z^4}$

9. Write each equation in slope-intercept form. Also find slope and y-intercept.

(a) $3y - 7 = 3x$ (b) $4y + 7 = 2x$

10. Make s the subject in $3t + 5s = 10$.

11. If $A = xy$ then make x the subject, also find value of x if $A = 240$, $y = 40$.

12. Draw the graph of each of the following.

(a) $y = 7x$ (b) $y = -2x + 8$

Domain 3 Measurement

Sub-Domain: Mensuration

Students' Learning Outcomes

After completing this domain, the students will be able to:

Pythagorean Theorem

- state the Pythagoras theorem and give its informal proof
- solve right angled triangles using Pythagoras theorem
- solve problems from real life situations using Pythagoras theorem

Circle

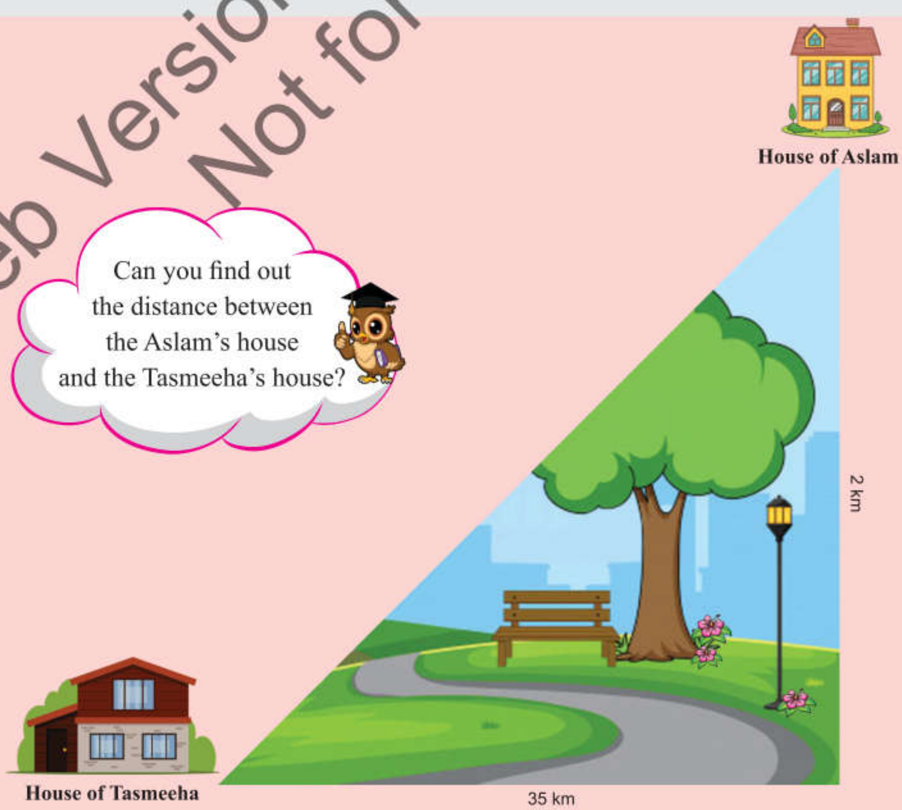
- explain the terms related to the circle:
  - ❖ arc (major and minor arcs)
  - ❖ chord
  - ❖ central angle
  - ❖ Tangent
  - ❖ sector
  - ❖ semi-circle
  - ❖ secant
  - ❖ concentric circles

Surface Area and Volume of pyramid, sphere and cone

- calculate the surface area and volume of pyramid
- calculate the surface area and volume of a sphere and hemi-sphere
- calculate the surface area and volume of a cone
- solve real life problems involving surface area and volume of pyramid, sphere, hemi-sphere and cone

Advanced/ Additional

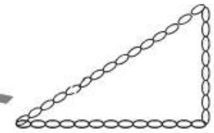
- calculate the arc length of the circle by expressing the arc length as a fraction of circumference of the circle
- calculate the area of the sector of a circle by expressing sector area as a fraction of the area of the circle
- calculate the surface area and volume of composite shapes including pyramid, sphere, hemi-sphere and cone



3.1 Pythagoras Theorem

Pythagoras theorem is an important theorem in geometry. It is named after a Greek **Mathematician Pythagoras** 2500 years ago. He thought of inventing it when he observed a strange method adopted by Egyptians to measure the width of River Nile.

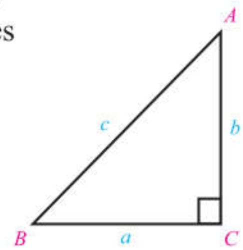
They measure it with the help of a triangle formed by chains with the ratio among its sides as 3 : 4: 5



Statement of Pythagoras Theorem

In a right angled triangle ABC with $m\angle C = 90^\circ$ and a, b, c are lengths of opposite sides of the angles $\angle A$, $\angle B$ and $\angle C$ respectively,

$$a^2 \quad + \quad b^2 \quad = \quad c^2$$
$$(\text{Base})^2 \quad + \quad (\text{Altitude})^2 \quad = \quad (\text{Hypotenuse})^2$$



Informal Proof of Pythagoras Theorem

We shall prove it with the help of an activity.

Activity

Apparatus: Hard paper, pencil, ruler and pair of scissors.

Step I: Draw a right angled triangle ABC with sides of lengths a, b and c, where $m\angle C = 90^\circ$ and $a : b : c = 3 : 4 : 5$

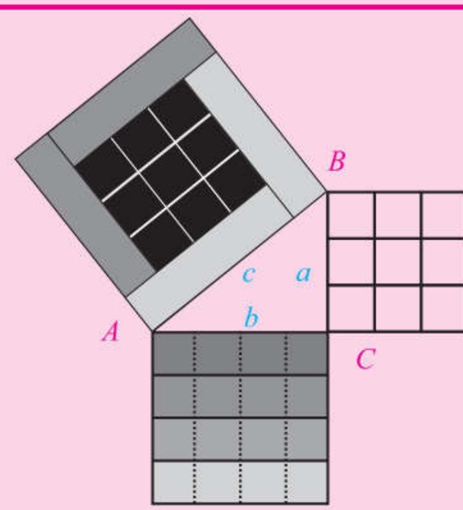
Step II: Draw squares on sides a, b and c adjacent to the respective sides as shown in the figure.

Step III: Since $a : b : c = 3 : 4 : 5$, so divide the lengths of sides of the square a, b and c into 3, 4 and 5 strips of equal width as shown in the figure.

Step IV: Shade the strips as shown in the figure.

Step V: Now cut the square into strips of side b with the help of a pair of scissors.

Step VI: Place the square of side a in the middle and the strips of the square side b on the square side c as shown in the figure.



We can observe that:

The area of the square of side "c" is equal to the total area of the square of side "b" and the square of side "a".

Hence it is proved that:

$$a^2 \quad + \quad b^2 \quad = \quad c^2$$
$$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$$

Remember!

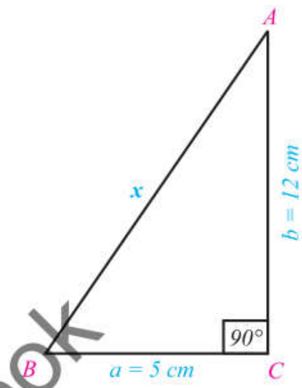
The hypotenuse of a right angled triangle is opposite side to the right angle. The adjacent horizontal side of the right angle is the base, and vertical side is the altitude.

Solution of Right Angled Triangle Through Pythagoras Theorem

Pythagoras theorem is usually applied for finding out the length of the third side of a right angled triangle while the lengths of two sides are known.

If c is the length of side opposite to the right angle, then

$$c^2 = a^2 + b^2 \quad \text{or} \quad a^2 = c^2 - b^2 \quad \text{or} \quad b^2 = c^2 - a^2$$



Example 1: In the given figure of triangle ABC, find the length of side AB.

Solution: Let $m\overline{AB} = x$

By Pythagoras theorem

$$c^2 = a^2 + b^2, \; m\angle C = 90°$$

Here $c = x, \; a = 5cm, \; b = 12cm$

$$x^2 = 5^2 + (12)^2 = 25 + 144$$

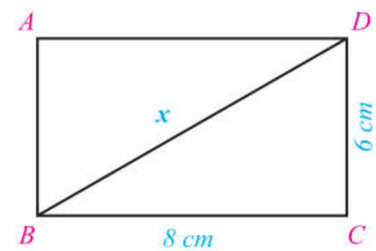
$$x^2 = \sqrt{169} = 13cm$$

So, $m\overline{AB} = 13cm$

Example 2: The length and width of a rectangle are $8cm$ and $6cm$ respectively. Find the length of its diagonals.

Solution: Let $ABCD$ be the rectangle and let $m\overline{BD} = x$cm

In right angled triangle BCD



$$m\angle C = 90°, \text{ Base } = m\overline{BC} = 8cm$$

$$\text{Altitude} = m\overline{CD} = 6cm$$

$$\text{Hypotenuse} = m\overline{BD} = xcm$$

By Pythagoras theorem

$$x^2 = 8^2 + 6^2 = 64 + 36 = 100$$

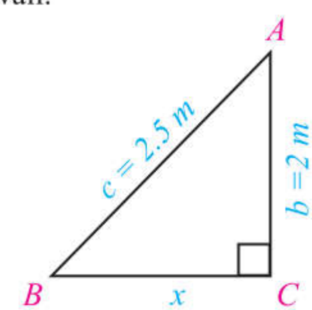
$$x = 10cm \text{ or } m\overline{BD} = 10cm$$

Since, the two diagonals of a rectangle are equal in length, so $m\overline{AC} = 10cm$.

Example 3: A ladder $2.5m$ long is placed against a wall. If its upper end reaches the height of $2m$ along the wall, then find the distance of the foot of the ladder from the wall.

Solution: Let x be the distance of the wall from the foot of the ladder.

Then by Pythagoras theorem



$$c^2 = a^2 + b^2, \; m\angle C = 90°$$

Here $c = 2.5m, \quad a = x, \quad b = 2m$

As $a^2 = c^2 - b^2$

$$x^2 = (2.5)^2 - (2)^2 = 6.25 - 4$$

or $x^2 = 2.25$

$$x = 1.5m$$

Thus, the distance of the wall from foot of the ladder is $1.5m$.

Example 4: Find the area of a rectangular field whose length is $20m$ and the length of its diagonal is $25m$.

Solution: Let us take right angled triangle ABC, then by Pythagoras theorem:

$$b^2 = a^2 + c^2, \; m\angle B = 90°$$

Here $b = 25m, \quad a = 20m$

Let $c = xm$

$(25)^2 = x^2 + (20)^2$

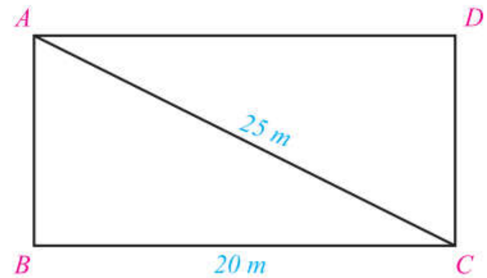
$x^2 = (25)^2 - (20)^2 = 625 - 400 = 225$

$x^2 = 225 \Rightarrow x = \sqrt{225}m \Rightarrow x = 15m$

Width of the rectangle $= 15m$

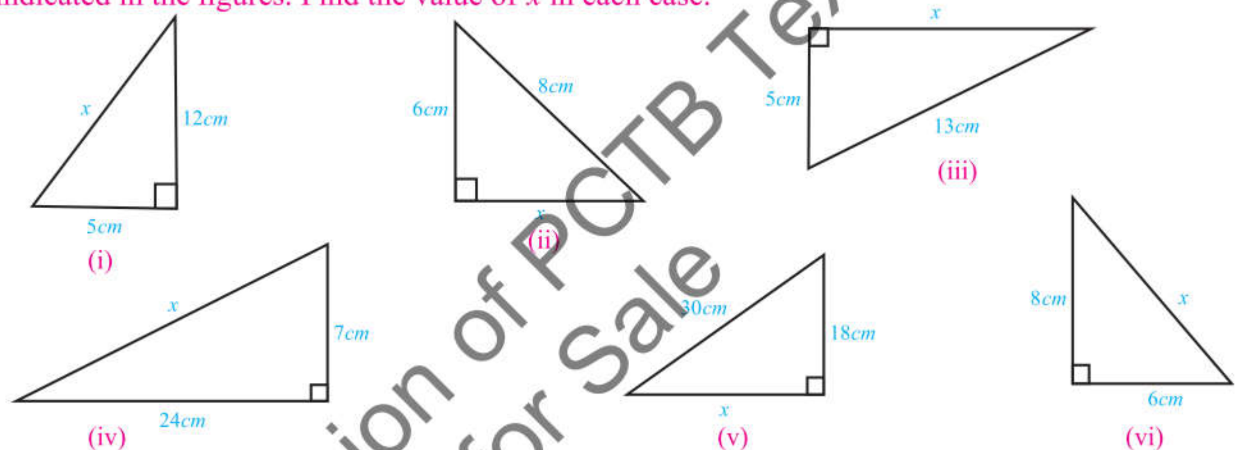
Length of the rectangle $= 20m$

Thus, area of the rectangular field $= 20 \times 15 = 300m^2$



Exercise 3.1

1. In the right angled triangles (not drawn to scale), measurements (in cm) of two of the sides are indicated in the figures. Find the value of x in each case.



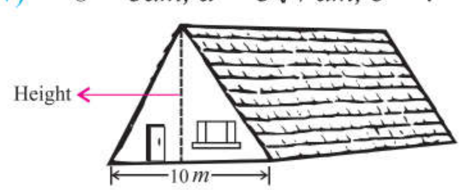
2. In an isosceles right angled triangle, the square of the hypotenuse is $98cm$. Find the length of the equal sides.
3. A ladder $10m$ long is made to rest against a wall. Its lower end touches the ground at a distance of $6m$ from the wall. At what height above the ground the upper end of the ladder rests against the wall?
4. The shadow of a pole measured from the foot of the pole is $2.8m$ long. If the distance from the tip of the shadow to the tip of the pole is $10.5m$, then find the length of the pole.
5. If a, b, c are the lengths of the sides of a triangle ABC, then tell which of the following triangles are not right angled triangles. Any of $\angle A$, $\angle B$ and $\angle C$ may be a right angle.

   (i) $a = 6, b = 5, c = 7$ (ii) $a = 8, b = 9, c = \sqrt{145}$ (iii) $a = 12, b = 5, c = 13$
6. In a right angled triangle ABC with hypotenuse c and sides a and b, find the unknown length.

   (i) $a = 60cm, c = 61cm, b = ?$ (ii) $a = \frac{5}{12}\ cm, c = \frac{13}{12}\ cm, b = ?$

   (iii) $b = 10m, a = 4\sqrt{5}\ m, c = ?$ (iv) $b = 5dm, a = 5\sqrt{7}\ dm, c = ?$

   (v) $c = 10\sqrt{2}\ dm, b = 5\sqrt{3}\ dm, a = ?$
7. The front of a house is in the shape of an equilateral triangle with the measure of side $10m$. Find the height of the house.



3.2 Circle

A circle is a plane figure bounded by one fixed curved line and such that all straight lines drawn from a fixed point within it to the bounding line are equal. That the fixed point is called the Centre of the Circle.

> **Key fact!**
> The term “circle” is derived from a Greek word that means “hoop” or “ring”.

Demonstrate a point lying in the interior and exterior of a circle

A circle divides the plane into two regions: an interior and an exterior. For example, A is outside the circle, B is inside the circle and C lies on the circle.

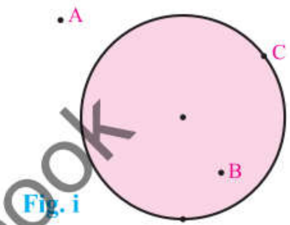


Fig. i

Description of Terms in Circle

Radius:

The distance between the centre and any point on the circle is called radius. $\overline{OA}$ is a radius (Fig. ii).

Diameter

A line segment that passes through the centre of a circle and touches at two points on its edge is called the diameter of the circle. $\overline{CB}$ is a diameter.

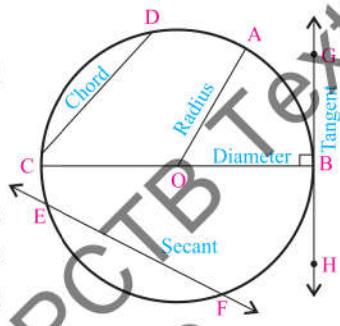


Fig. ii

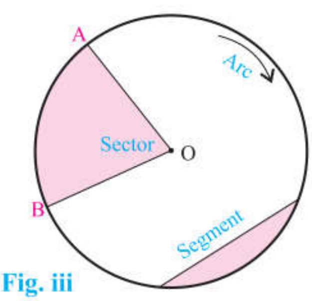


Fig. iii

Arc: A continuous part of the boundary of a circle is called an arc (Fig. iii).

Major Arc: A major arc is the longer arc connecting two endpoints on a circle. The measure of angle of a major arc is greater than 180°. $\overset{\frown}{BDC}$ is a major arc (Fig. iv).

Minor Arc: An arc whose measure of angle is less than 180° is called a minor arc. $\overset{\frown}{BC}$ is a minor arc (Fig. iv).

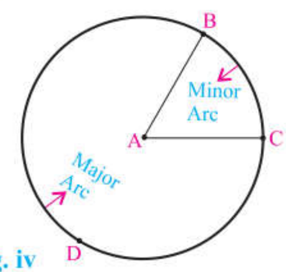


Fig. iv

Sector: A sector is a region of a circle which lies between an arc and the two radii joining the end points of the arc and the centre. OAB is a sector as shown in the Fig. iii.

Chord: A line segment joining two points on a circle is called a chord of the circle. In the Fig. ii, $\overline{CD}$ is a chord of the circle.

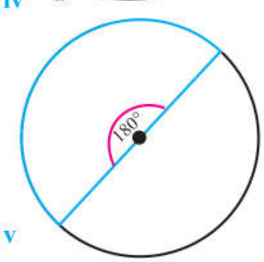


Fig. v

Semi-Circle: An arc whose measure of angle equals 180° degrees is called a semicircle. It is half of a circle (Fig. v).

Segment: The region enclosed between a chord and the boundary of a circle is called a segment (Fig. vi).

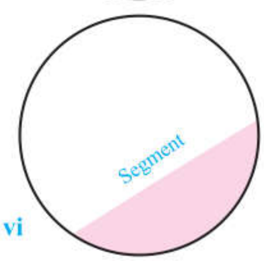


Fig. vi

Secant: A straight line that intersects a circle at two points is called a secant line. $\overleftrightarrow{EF}$ is a secant (Fig. ii).

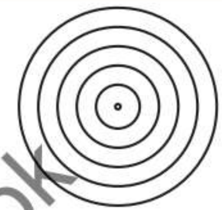
Tangent: A straight line that touches the circle at a single point externally is called a tangent. $\overleftrightarrow{GH}$ is a tangent (Fig. ii).

> **Key fact!**
> This point is called the point of tangency. A tangent never enters the circle.

Concentric Circle: The circles with a common centre are known as concentric circles and have different radii.

Key fact!

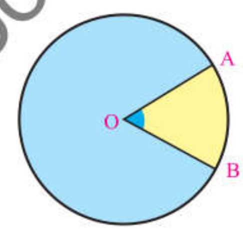
Tangent and radius are always at right angle to each other.



Calculate the arc length of the circle by expressing the arc length as a fraction of circumference of the circle.

Central Angle

Central angle is defined as the angle formed by an arc of the circle at the centre of the circle. In central angle, the two arms are the radii of the circle with the centre of a circle as the vertex. Central angle divides a circle into sectors. In the figure, O is the centre of the circle, AB is an arc and $\overline{OA}$ and $\overline{OB}$ are radii. $\angle AOB$ is the central angle.



Arc Length

A part of a boundary of a circle is called an arc. Arc length is the distance along the curved line that makes up the arc. The circumference of a circle of radius r is $C = 2\pi r$

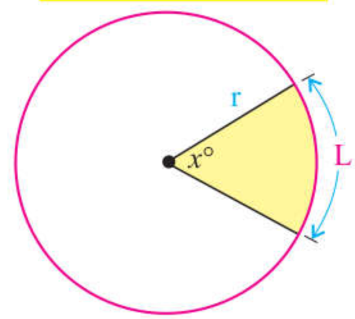
The sector of a circle is a part of a circle. The arc length of a sector of a circle is just the fraction of the circle's circumference.

The formula for arc length is:

The length of an arc is longer than any straight line distance between its endpoints (a chord).

Or $\quad$ Arc length $(L) = \dfrac{x}{360^\circ} \times circumference$

Arc length $(L) = \dfrac{x}{360^\circ} \times 2\pi r$



where $\boldsymbol{x}$ is the central angle measure and r is the radius of the circle.

This formula shows that the arc length is the fraction of the circumference of a circle.

Example 5: Find the arc length of a sector extended by the angle of 45° of a circle with a radius of 14 cm. ($\pi \approx \dfrac{22}{7}$)

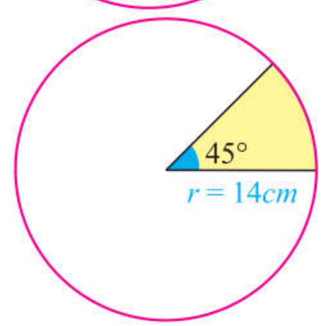
Solution: Radius of circle $= r = 14cm$

Angle $= x = 45^\circ$

Arc length $(L) = \dfrac{x}{360^\circ} \times 2\pi r$

$$L = \frac{45^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 14 = \frac{1}{8} \times 2 \times 22 \times 2$$

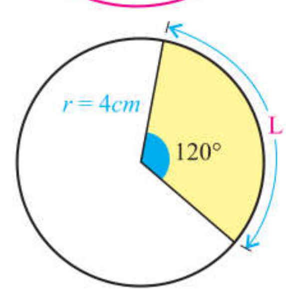
$$= 11\ cm$$



Example 6: Find the value of L given that the angle of the sector is 120° and radius is 4cm.

Solution: Arc length $L = \dfrac{x}{360^\circ} \times 2\pi r$

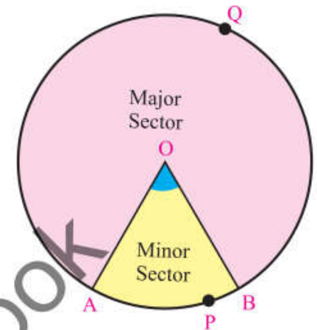
$$L = \frac{120^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 4 = \frac{1}{3} \times 2 \times \frac{22}{7} \times 4 = 8.38cm$$



Calculate the area of the sector of a circle by expressing sector area as a fraction of the area of the circle.

Area of Sector

The area of a sector of a circle is the space enclosed within the boundary of the sector. For example, a pizza slice is an example of a sector that represents a fraction of a pizza. There are two types of sectors: minor and major sectors. Sector is a part of circle and its area can be calculated by taking the fraction of the area of the circle. The area of a sector can be calculated using the following formula:



$$\text{Area of a sector of circle} = \frac{x}{360°} \times area\ of\ circle$$

$$= \frac{x}{360°} \times \pi r^2$$

where x is the sector angle subtended by the arc at the centre, in degrees and r is the radius of the circle.

Key fact!

To calculate the perimeter of the sector, we need to add the arc length to the lengths of the two radii:

Perimeter of a sector = *Arc length* + *radius* + *radius*

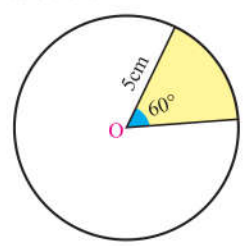
= *Arc length* +2 × *radius*

Example 7: Find the area of sectors subtended by the central angles 160° and 100° respectively, where the radius is 6*cm*.

Solution: Area of sector 1 $= \frac{x}{360°} \times \pi r^2 = \frac{160}{360°} \times \frac{22}{7} \times 6^2 = \frac{4}{9} \times \frac{22}{7} \times 36 = \frac{352}{7} = 50.29cm^2$

Area of sector 2 $= \frac{x}{360°} \times \pi r^2 = \frac{100}{360°} \times \frac{22}{7} \times 6^2 = \frac{5}{18} \times \frac{22}{7} \times 36 = \frac{220}{7} = 31.43cm^2$

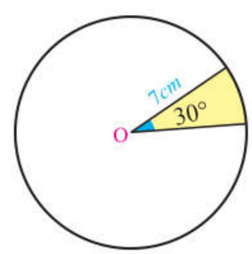
Example 8: Given that the radius of the circle is 5 cm, calculate the area of the shaded sector. (Take $\pi \approx 3.14$).



Solution: Area of sector formula $= \frac{x}{360°} \times \pi r^2$

$$= \frac{60°}{360°} \times 3.14 \times 5^2 = 13.09cm^2$$

Example 9: Find the arc length and area of the sector shown:



Solution: Area of sector $= \frac{x}{360°} \times area\ of\ circle$

Area of sector $= \frac{30°}{360°} \times \frac{22}{7} \times 7^2 = 12.83cm^2$

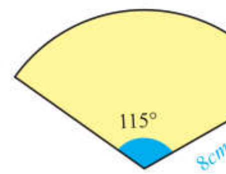
Arc length $= \frac{x}{360°} \times 2\pi r$

$$= \frac{\overset{1}{\cancel{30}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{360}}} \times \overset{1}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{1}{\cancel{7}}$$

$$\text{Arc length} = \frac{11}{3} = 3.66cm$$

Exercise 3.2

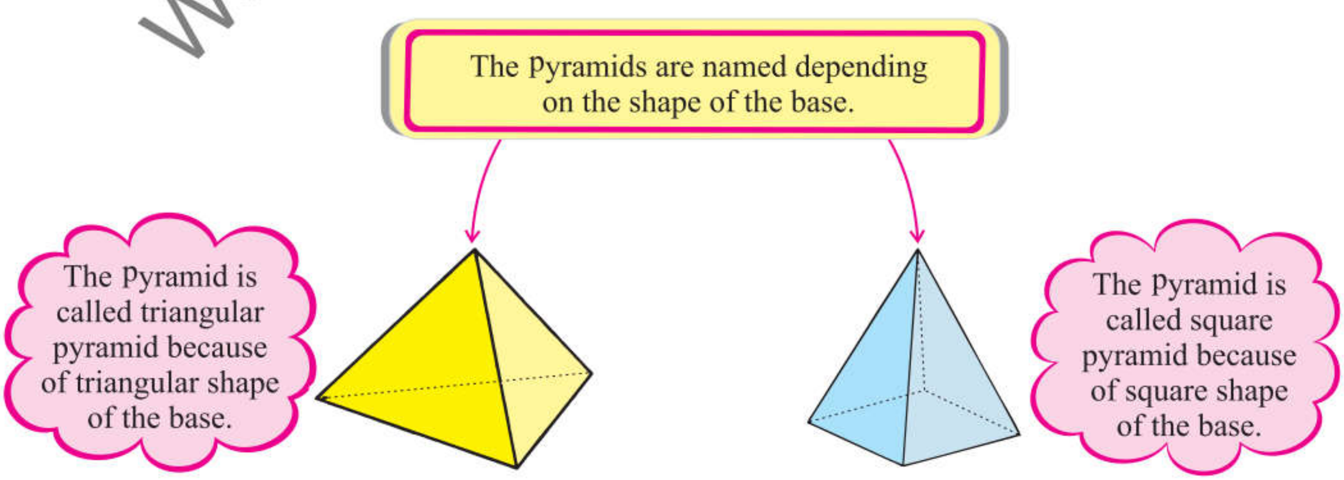
1. Calculate the length of an arc of a circle if the radius of an arc is $8cm$ and the central angle is $40°$.
2. If central angle of the sector of a circle is $36°$ and r is $15cm$, then what will be the arc length?
3. Saleem marks an arc of length $8cm$ and measures its central angle as $120°$ degrees. What is the radius of the arc?
4. Jamal uses a compass to draw an arc of length $11cm$ and a radius of $7cm$. Find the angle of this arc?
5. The radius of the circle is 14 units and the arc subtends an angle of $65°$ at the centre. What is the length of the arc?
6. Calculate the perimeter of the sector of circle to 1 decimal place as shown in the figure.



7. Calculate the perimeter of a sector of circle of radius $10cm$ and central angle is $90°$ in terms of pi?
8. If angle is $130°$ and radius is $28cm$ then what will be the arc length and sector area of the circle?
9. Gulshan wants to create a garden in the shape of a sector of the circle of radius 42 *feet* and having a central angle of $120°$ degrees. Calculate the area of the grass which is required to cover the garden.
10. A sector of a circle with an area of $26\pi cm^2$ and has a radius $6cm$. What is the measure of the central angle?
11. If a clock's face measures 14 inches across, how much area is between the minute and hour hands at 5 O' clock? (Approximate pi to 3.14).

3.3 Surface Area and Volume of Pyramid

Pyramid: A 3D shape with a flat polygon base and three or more triangular sides converging at the top, is called Pyramid.



Parts of Pyramid

Consider a right pyramid.

Slant Height of Pyramid

The slant height is the length from the bottom of one of the faces to the top.

Vertex

The corner point of a pyramid is called vertex.

Apex

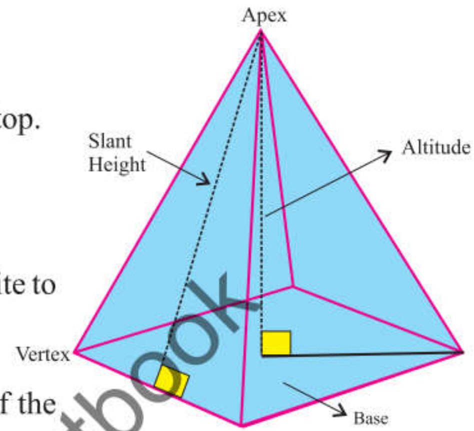
The corner where all faces converge is called the **Apex** and it is opposite to the base.

Lateral Surface Area

Lateral surface area is sum of the surface areas of the lateral sides of the object.

Altitude / Vertical height

The perpendicular distance from base to the apex is called altitude / vertical height.



Parts of a pyramid

Key fact!

Triangular pyramid is also called tetrahedron

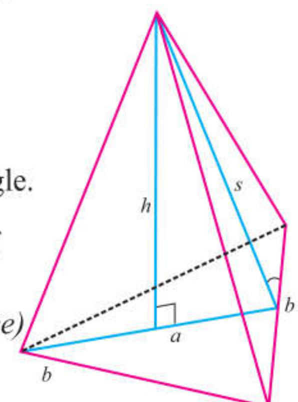
Surface Area and Volumeof Pyramids

Surface area

The region occupied by all the surfaces of any 3D shape is called its surface area.

Surface area of any pyramid $= \textit{area of base} + \textit{lateral surface area}$

Lateral Surface Area of each lateral side $= \frac{1}{2} \times base \times slant\ height$



Surface Area of Triangular Pyramid

Base area of triangular Pyramid $= \frac{1}{2}ab$, when base is an equilateral triangle.

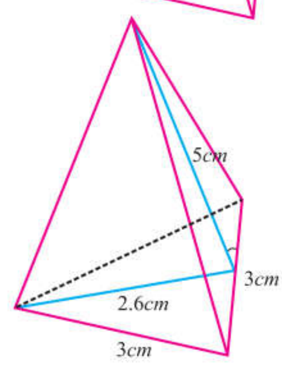
Where a is the altitude of base triangle of side length b. If s is the lateral height of lateral triangle, then

Total surface area of triangular pyramid $= \textit{base area} + 3(\textit{area of each lateral face})$

$$= \frac{1}{2}ab + 3\left(\frac{1}{2}bs\right)$$

$$= \frac{1}{2}ab + \frac{3}{2}bs$$

Example 10: Each side of a triangular based pyramid is of length $3cm$ and the slant height is $5cm$. Find its total surface area.



Solution: The total surface area of a triangular pyramid of side b is $3cm$.

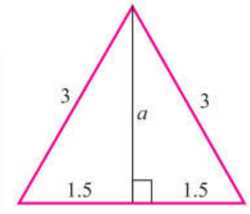
Total surface area of triangular pyramid $= \frac{1}{2}ab + \frac{3}{2}bs$

Total surface area of triangular pyramid $= \frac{1}{2} \times 2.6 \times 3 + \frac{3}{2} \times 3 \times 5$

$= 3.9 + 22.5$

$= 26.4cm^2$

$3^2 = a^2 + 1.5^2$
$a^2 = 9 - 2.25$
$a^2 = 6.75cm$
$a = 2.6cm$



Surface Area of square Pyramids

Base area of square pyramid $= \ell \times \ell$

Lateral surface area of square pyramid (Area of a triangular face)

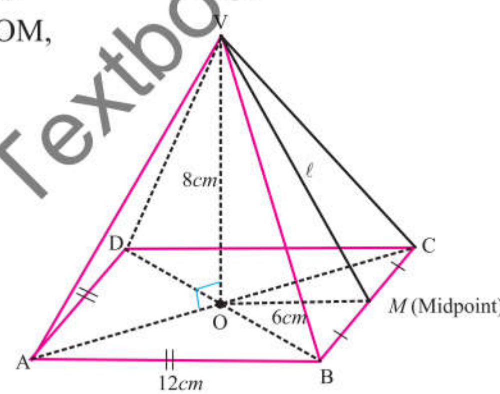
$$= \frac{1}{2} \times base \times slant\ height$$

So,

Surface area of square pyramid = *Area of the base* + 4*(area of a triangular face)*

Example 11: Find the total surface area of a square pyramid with a perpendicular height of 8*cm* and base edge of 12*cm*.

Solution: By Pythagoras Theorem, from right-triangle VOM, we have



$$\ell^2 = 8^2 + 6^2$$
$$= 64 + 36$$
$$= 100$$
$$\ell = \sqrt{100} = 10cm$$

Thus, slant height is 10*cm*

Now, the area of a face of pyramid is

$$\text{Area of } \Delta VBC = \frac{1}{2} \times b \times slant\ height$$
$$= \frac{1}{2} \times 12 \times 10 = 60cm^2$$

$$\text{Area of square base} = 12 \times 12 = 144cm^2$$

$$\text{Total surface area of pyramid} = \text{Area of base} + 4 \times \text{ area of a face}$$
$$= 144 + 4 \times 60 = 144 + 240 = 384cm^2$$

Volume of Pyramid

The volume of a pyramid refers to the space enclosed between its faces.

$$\text{Volume of Pyramid} = \frac{1}{3} \times volume\ of\ prism$$
$$= \frac{1}{3} \times base\ area \times height\ of\ pyramid$$

Key fact!
The volume of any pyramid is always one-third of the volume of a prism.

Example 12: Calculate the volume of tetrahedron whose base area is $21cm^2$ and height 10*cm*. Find the volume of the tetrahedron.

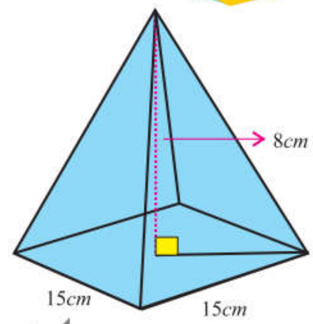
Solution: Volume of pyramid $= \frac{1}{3} \times base\ area \times height\ of\ pyramid$

$$= \frac{1}{3} \times 21 \times 10 = 70cm^3$$

Example 13: A model of a square based pyramid with side length of base is 15*cm*. Its height is around 8*cm*. Calculate its volume.

Solution: Volume of pyramid $= \dfrac{1}{3} \times \textit{base area} \times \textit{ height of pyramid}$

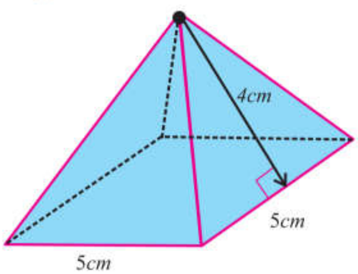
$$= \frac{1}{3} \times 15 \times 15 \times 8 = 600 cm^3$$



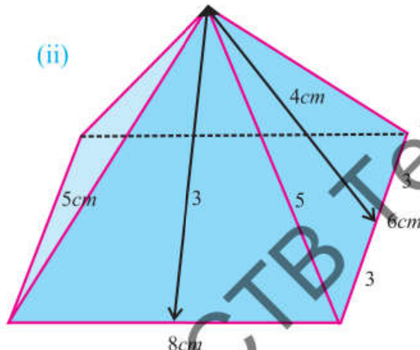
Exercise 3.3

1. Find the total surface area of the following pyramids:

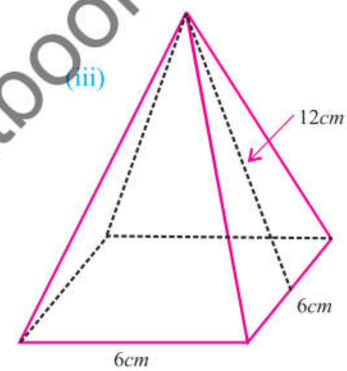
(i)



(ii)



(iii)



2. Find the surface area of a square pyramid of slant height $15cm$ and base length $12cm$.
3. The height of a square pyramid is $25cm$ and the base area of a square pyramid is $256cm^2$. Find its volume and surface area.
4. Find the surface area of a square pyramid with a base length of $5cm$ and a slant height of $10cm$.
5. Find the total surface area of the square based pyramid shown in Fig. (i)
6. Find the total surface area of a rectangular pyramid whose area of the base is 30 square units and lateral surface area of each triangle is 25 square units.

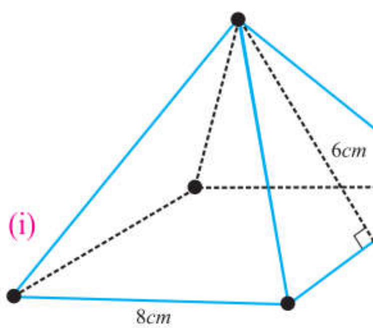


Fig. (i)

7. Find the volume of a rectangular pyramid whose base length and width are $10cm$ and $8cm$ and the lateral height of a triangle of base $8cm$ is $13cm$.
8. Given a pyramid with a square base of side $7cm$. If the total surface area of the pyramid is $161 cm^2$, find its slant height.
9. Each side of a triangular based pyramid is $4cm$ and slant height is $8cm$. Find total surface area.
10. The base of pyramid is a right angled triangle of sides $3cm$, $4cm$ and $5cm$. If height of pyramid is $7cm$, find volume of the pyramid.
11. What is the volume of a triangular pyramid whose base area is 92 inch^2 and height is 4 inches?
12. Find the volume of a tetrahedron whose base area is $15cm^2$ and height is $6cm$.
13. Find the volume of a hexagon based pyramid with base area $23cm^2$ and height $8cm$.
14. The height of the Great Pyramid in Egypt is $146m$ and the base is a square of side $229m$. Find the volume of the pyramid.
15. The pyramid shown in Fig. (ii), has a rectangle base. If its volume is $100 cm^3$, find its vertical height.

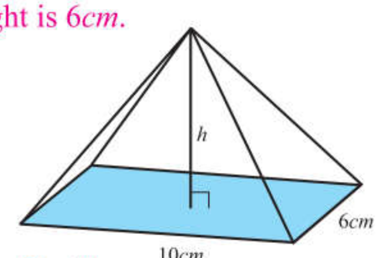
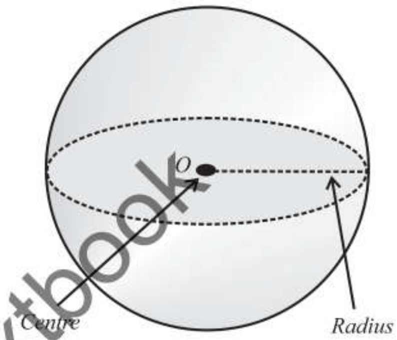


Fig. (ii)

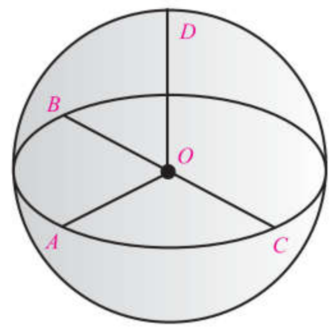
16. A pyramid with a triangular base has a volume of $50cm^3$. If the base and height of the triangle are $5cm$ and $8cm$ respectively, find the height of the pyramid.
17. The height and volume of a square based pyramid are $12cm$ and $100cm^3$ respectively. Find the length of the square base.

3.4 Surface Area and Volume of Sphere



A sphere is a solid bounded by a single curved surface such that all the points on its outer surface are at an equal distance from a fixed point inside the sphere.

The fixed point is called its **Centre**. The distance from centre to its outer surface is called its Radius. In the given figure, the point O is its Centre.



The measurement of line segments OA, OB, OC and OD are all its radii and are equal in length.

Football is an example of a sphere.

Attributes of a Sphere

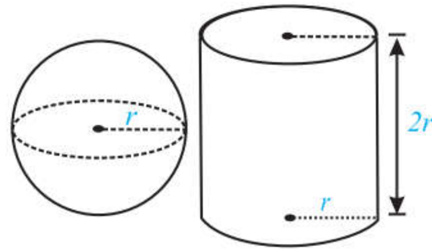
(i) A sphere is perfectly symmetrical
(ii) A sphere is not a polyhedron
(iii) All the points on the surface are equidistant from the centre
(iv) A sphere has only a curved surface, no flat surface, no edges and no vertices

A polyhedron is a three-dimensional shape with flat polygonal faces, straight edges and sharp corners or vertices.

Finding the Surface Area and Volume of a Sphere

Surface Area of a Sphere

A famous scientist Archimedes discovered that the surface area of a sphere is equal to the curved surface area of the cylinder whose radius is equal to the radius of the sphere and its height is equal to the diameter of the sphere (i.e., twice the radius).



Let the radius of the sphere $= r$

Radius of the cylinder $= r$

Height of the cylinder $h = 2r$

Curved surface area of cylinder $= 2\pi rh$

Surface area of sphere $= 2\pi r(2r)$ $\quad (h = 2r)$

$= 4\pi r^2$

Example 14: Find the surface area of a sphere whose radius is $21cm$ $\left(\pi \approx \frac{22}{7}\right)$

Solution: Surface area of a sphere of radius $r = 4\pi r^2$

where $\quad r = 21cm, \quad \left(\pi \approx \frac{22}{7}\right)$

$$\text{Required surface area} = S = 4 \times \frac{22}{7} \times (21)^2$$

$$= 4 \times \frac{22}{7} \times 21 \times 21$$

$$S = 5544cm^2$$

Example 15: Find the radius of a sphere if the area of its surface is $6.16m^2$.

Solution: Let the area of the curved surface = S

$$\text{Radius} = r$$

$$S = 4\pi r^2$$

It is given that $S = 6.16m^2, \left(\pi \approx \frac{22}{7}\right)$

$$4\pi r^2 = 6.16m^2$$

or $r^2 = \dfrac{6.16}{4\pi}$

$$r^2 = \frac{6.16 \times 7}{4 \times 22}$$

$$r^2 = 0.49m^2$$

$$r = \sqrt{0.49}$$

or $r = 0.7m$

Volume of a Sphere

Volume of a sphere V = Two third of the volume of the cylinder (with radius r and height $2r$)

$$V = \frac{2}{3} \times \pi r^2 \times 2r = \frac{4}{3}\pi r^3 \qquad (\boldsymbol{h = 2r})$$

Volume of a sphere $r = V = \dfrac{4}{3}\pi r^3$

Remember!

Volume of cylinder = $\pi r^2 h$

Example 16: How many litres of water a spherical tank can contain whose radius is $1.4m$?

Solution: Volume of a sphere with radius r is given by

$$V = \frac{4}{3} \times \pi r^3, \qquad r = 1.4m$$

$$V = \frac{4}{3} \times \frac{22}{7} \times (1.4)^3$$

$$V = \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$V = 11.499m^3 = 11499\ell \qquad (\therefore \ \ \boldsymbol{1m^3 = 1000\ell})$$

Example 17: Find the volume of a sphere, the surface area of which is $2464cm^2$.

Solution: Surface area of a sphere of radius r is $A = 4\pi r^2$

$$4\pi r^2 = 2464cm^2$$

$$r^2 = \frac{2464}{4\pi}$$
$$= \frac{2464 \times 7}{4 \times 22}$$
$$r^2 = 196$$
$$r = 14cm$$

Let V be the volume of the sphere, then

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \times (14)^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3$$
$$= \frac{34496}{3} = 11498.66cm^3 (approx)$$

Exercise 3.4

1. Find the curved surface area of the spheres whose radii are given below $\left(\text{taking } \pi \approx \frac{22}{7}\right)$

   (i) $r = 3.5cm$ (ii) $r = 2.8m$ (iii) $0.21m$

2. Find the radius of a sphere if its area is given by:

   (i) $154m^2$ (ii) $231m^2$ (iii) $308m^2$

3. Find the volume of a sphere if r is given by:

   (i) $5.8cm$ (ii) $8.7cm$ (iii) $7cm$ (iv) $3.4m$

4. Find the radius and volume of each of the following spheres whose surface areas are given below:

   (i) $201\frac{1}{7}cm^2$ (ii) $2.464cm^2$ (iii) $616m^2$

5. A spherical tank is of radius $7.7m$. How many litres of water can it contain?
6. The radius of sphere A is twice that of a sphere B. Find:

   (i) The ratio among their surface areas. (ii) The ratio among their volumes.

7. The surface area of a sphere is $576\pi\ cm^2$. What will be its volume? If it is melted, how many small spheres of diameter $1cm$ can be made out of it?
8. A solid copper sphere of radius $3cm$ is melted and electric wire of diameter $0.4cm$ is made out of the copper obtained. Find the length of the wire.

3.5 Surface Area and Volume of Hemi-Sphere

Hemi-Sphere: Half of sphere is called Hemisphere. It is formed when a sphere is cut exactly at the centre along its diameter.

Attributes of a Hemi-sphere

i. A hemisphere has one circular flat base and one curved surface.
ii. There are no edges and no vertices in a hemisphere.

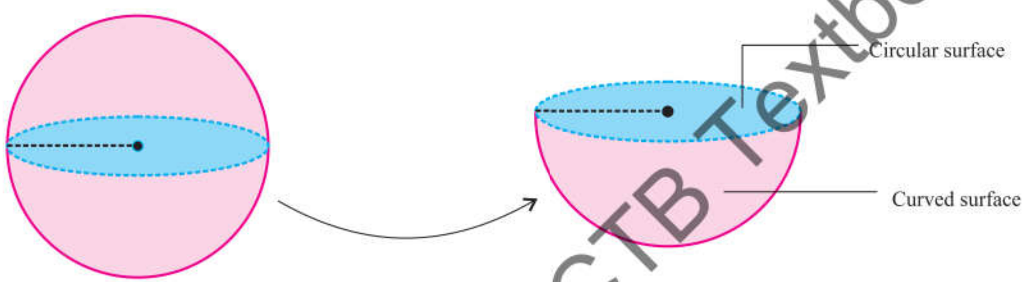
iii. The diameter of a hemisphere is a line segment that passes through the centre and touches the two opposite points on the base of the hemisphere.

Finding the Surface Area and Volume of a Hemi-Sphere

Surface area of a hemisphere $= \frac{1}{2} \times$ surface area of sphere + area of base

$$= \frac{1}{2} \times 4\pi r^2 + \pi r^2$$

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$



Examples 18: Calculate the surface area of a hemisphere with radius $4cm$.

Solution: Surface area of hemi-sphere $= 3\pi r^2$

$$= 3\pi (4)^2$$

$$= 3\left(\frac{22}{7}\right)(4)^2$$

$$= 150.86cm^2$$

Volume of a hemi-sphere

Volume of a hemisphere $= \frac{1}{2}$ of sphere

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$

Examples 19: Calculate the surface area of a hemi-sphere whose radius is $5cm$.

Solution: Volume of hemi-sphere (given $r = 5cm$) $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\left(\frac{22}{7}\right)(5)^3$$

Volume of hemi-sphere $= 261.667cm^3$

Key fact!

Volume of hemi-sphere $= \dfrac{\textit{Volume of sphere}}{2}$

Exercise 3.5

1. Calculate the surface area of a solid hemisphere of radius $10cm$.
2. The surface area of a solid hemispherical object is $150.86ft^2$. What is the diameter of the hemisphere?
3. The surface area of a solid hemispherical object is $1356.48cm^2$. What is the diameter of the hemisphere? (take $\pi \approx 3.14$)
4. What is the total surface area of a solid hemispherical object of radius $2cm$, $\left(\text{considering } \pi \approx \dfrac{22}{7}\right)$
5. A hemisphere has a curved surface of $175cm^2$. Find its radius.
6. Find the volume of a hemisphere whose diameter is:
   (a) $14cm$ (b) $21cm$ (c) $10cm$
7. A hemispherical bowl has a volume of 288π cubic cm. Find the diameter of the bowl.
8. The solid shown in Fig. (i) consists of a cylinder and a hemisphere at the top. Find its volume.

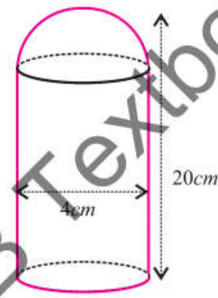


Fig. (i)

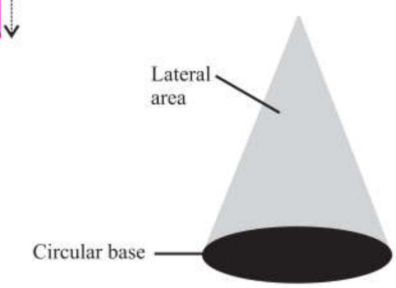
Finding the surface area and volume of a cone

The given figure is of a right circular cone.

Conical solids consist of two parts:

(i) Circular base

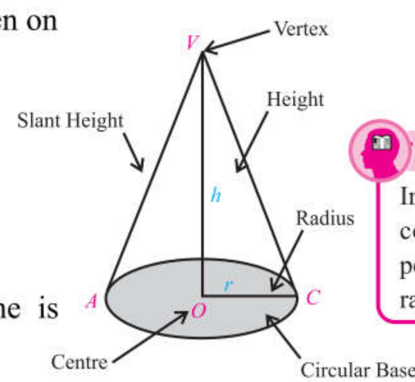
(ii) Curved surface



There are 5 elements of cone as shown in the figure given on the right side.

(i) vertex (the point V)
(ii) radius ($m\overline{OC}$)
(iii) height ($m\overline{OV}$)
(iv) slant height ($m\overline{CV}$) or ($m\overline{AV}$)
(v) centre (the point O)

The line joining the vertex to the centre of the cone is perpendicular to the radial segment of the cone.



Remember!
In right circular cone, height is perpendicular to radius.

Finding the surface area of a cone

We know that the area of the circular base of a cone with radius r, is $Base\ Area = \pi r^2$

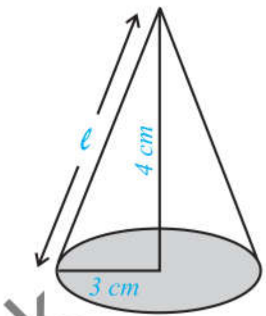
Curved surface area of a cone $= \pi r \ell$ (where r is radius and ℓ is the slant height)

$$\text{Total surface area of a cone} = Base\ area + curved\ surface\ area$$
$$= \pi r^2 + \pi r \ell$$
$$= \pi r (r + \ell)$$

Key fact!

Curved surface area of cone = *Area of sector of circle of radius r and angle of sector x* $= \dfrac{x}{360^0} \times 2\pi r$

Example 20: The radius of the base of a cone is $3cm$ and the height is $4cm$. Find its slant height.



Solution: We know that $\ell = \sqrt{h^2 + r^2}$

where $r = 3cm$ and $h = 4cm$

$$\ell = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5cm$$

Example 21: The radius of the base of a cone is $6cm$, slant height is $10cm$. Find the total surface area of the cone.

Solution: Radius $(r) = 6cm$, $\ell = 10cm$

$$\text{Total surface area} = \pi r(\ell + r)$$
$$= \frac{22}{7}(6)(10+6) = \frac{22}{7} \times 96$$
$$= \frac{2112}{7}cm^2 = 301\frac{5}{7}cm^2$$

Surface area of a cone $= 301\frac{5}{7}cm^2$

Example 22: The base area of a cone is $254\frac{4}{7}cm^2$ and slant height is $15cm$. Find its height.

Solution: Base area $= \pi r^2 = 254\frac{4}{7}cm^2$

$$r^2 = \frac{1782}{7} \times \frac{7}{22} = 81cm^2$$
$$r = 9cm$$
$$Slant\ height = \ell = 15cm$$
$$Height = h = \sqrt{\ell^2 - r^2}$$
$$= \sqrt{(15)^2 - (9)^2} = \sqrt{225 - 81} = \sqrt{144} = 12cm$$

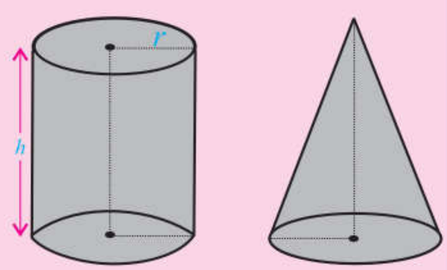
Finding Volume of a Cone

Let us find the volume of a cone through an activity.

Activity

Apparatus
(i) One sided open hollow cylinder with radius r units and height h units (Take r and h as convenient).
(ii) A hollow cone with radius r and height h. (i.e.,) bases and heights of both should be congruent.
(iii) Sand

Step I: Fill up the cone with sand and pour it into the cylinder.
Step II: Fill it up again and pour it into the cylinder.
Step III: Fill it up again and pour it into the cylinder.



We will notice that:

3 times volume of a cone (with radius r and height h) = Volume of the cylinder (with radius r and height h)

Since we know that the volume of a cylinder with radius r is $\pi r^2 h$

Volume of a cone (radius r and height h) $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\text{ (area of the base} \times \text{height)}$$

Example 23: How much sand can a conical container hold whose height is $3.5m$ and radius is $3m$, while $1m^3$ space contains 100kg of sand?

Solution: Radius $(r) = 3m$, $h = 3.5m$

$$= \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 3.5$$

Volume of the container $= 22 \times 3 \times 0.5$

$$= 33m^2$$

Sand in $1m^3$ = 100kg

Sand in $33m^3$ = 3300kg

Example 24: A tent in the form of a cone is $5m$ high and its base is of radius $12m$. Find:

(i) The area of the canvas used to make the tent.

(ii) The volume of the air space in it.

Solution: **(i)** Area of the curved surface of the cone

$$= \pi r \ell$$

$$= 12\pi \times \sqrt{(5)^2 + (12)^2}$$

$$= 12\pi \times \sqrt{25 + 144}$$

$$= 12\pi \times \sqrt{169} = 12\pi \times 13$$

$$= 3.14 \times 156 \qquad (Taking\ \pi = 3.14\ approx)$$

$$= 489.84m^2\ \ (approx)$$

The area of the canvas required for the tent is $489.84m^2$.

(ii) Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi 12^2 \times 5$$

$$= 3.14 \times 4 \times 5 \times 12$$

$$= 3.14 \times 240$$

$$= 753.60m^3$$

Example 25: The radius and height of a metal cone are respectively $2.4cm$ and $9.6cm$. It is melted and re-casted into a sphere. Find the radius of the sphere.

Solution: Let the volume of the cone be $= V_1$

Let the volume of the sphere be $= V_2$

$$V_1 = \frac{1}{3}\pi r^2 h$$

Here $r = 2.4cm$

and $h = 9.6cm$

Let the radius of the sphere to be formed $= R$

Then $V_2 = \frac{4}{3}\pi R^3$

Now $V_2 = V_1$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$4R^3 = r^2 h$$

$$R^3 = \frac{(2.4)^2 \times 9.6}{4} = (2.4)^3$$

$$R = 2.4cm$$

Exercise 3.6

1. Write down the missing elements of cones (all lengths are in cm).

	r	h	ℓ	Curved Surface Area	Base Area	Total Surface Area
(i)	--	8	10	--	--	--
(ii)	3	4	--	--	--	--
(iii)	9	--	25	--	--	--
(iv)	--	--	--	--	$154cm^2$	$375cm^2$

2. Find the volume of the cone if:

   (i) $r = 3cm$, $h = 4cm$ (ii) $r = 7cm$, $h = 10cm$

   (iii) $r = 5cm$, $\ell = 7cm$ (iv) $h = 5cm$, $\ell = 8cm$

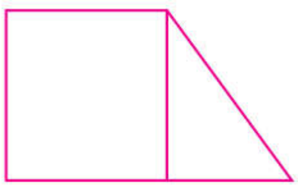
3. A conical cup is full of ice-cream. What will be the quantity of the ice-cream, if the radius and height of the cone are $4cm$ and $5cm$ respectively?
4. What will be the total surface area of a solid cone of height $4cm$ and radius $3cm$?
5. The area of the base of cone is $38.50cm^2$. If its height is three times the radius of the base, find its volume.
6. A conical tent is $8m$ high and its base is of $54dm$ radius. It is to be used to accommodate scouts. How many scouts can be accommodated in the tent if each scout requires $5.832m^3$ of air?

3.7 Calculate the Surface Area and Volume of composite shapes

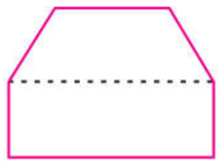
Composite Shapes

A composite 3D figure is a three-dimensional figure made up of basic three-dimensional figures such as cubes, prisms, pyramids, cylinders, cones, etc. To find out the surface area and volume of a composite 3D figure, add the areas and volumes of each geometric figure making up the composite 3D figure.

Example of composite objects



Shape shown is made up of a square and a triangle

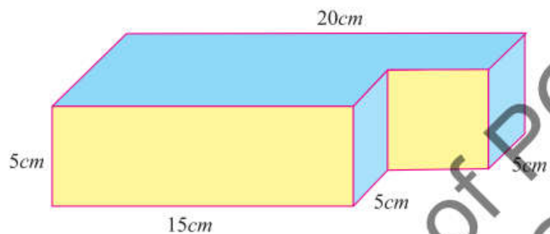


Shape shown is made up of a trapezium and a rectangle

Shape shown is made up of a semicircle and a rectangle

Cylinder and cone

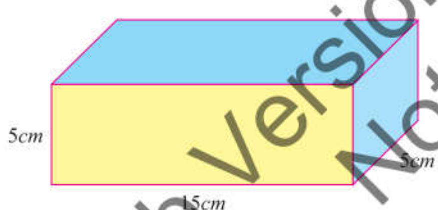
Example 26: Find the surface area and volume of the composite 3D figure below.

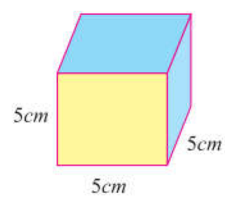


Key fact!

To find out the surface area and volume of a composite 3D figure, add the surface areas and volumes of each geometric figure making up the composite 3D figure.

Solution: The total surface area of the shown object can be divided into walls, top and bottom





Do you know?

What is right circular cylinder?

Area of all walls $= 15\times5 + 5\times5 + 5\times5 + 5\times5 + 20\times5 + 10\times5cm^2$

$= 75 + 25 + 25 + 25 + 100 + 50cm^2$

$= 300cm^2$

Area of top and bottom $= 10\times15 + 5\times5cm^2 + 10\times15 + 5\times5cm^2$

$= 150 + 25cm^2 + 150 + 25cm^2$

$= 175cm^2 + 175cm^2 = 350cm^2$

Total surface area $= 300cm^2 + 350cm^2 = 650cm^2$

Area of base of the object $= 10\times15 + 5\times5cm^2$

$= 150 + 25 = 175cm^2$

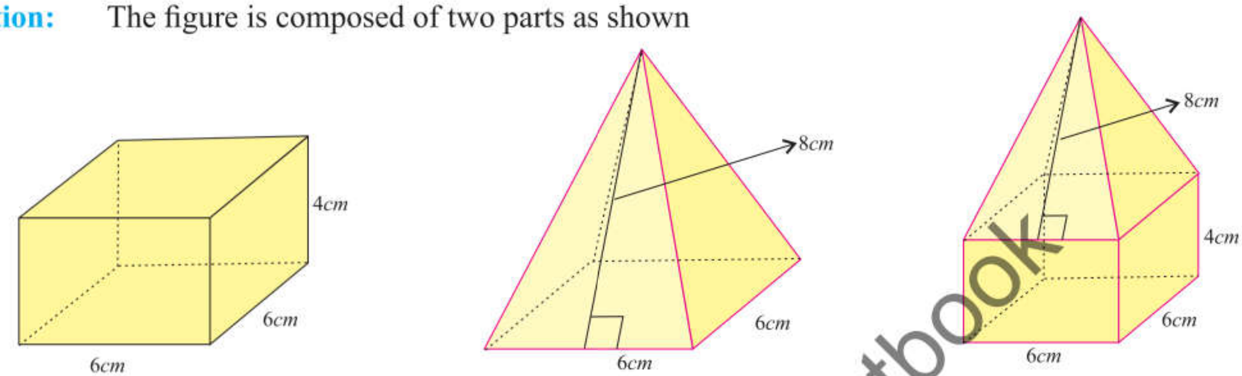
Volume of the object $= \textit{Area of the base} \times \textit{height}$

$= 175\times5cm^3$

$= 875cm^3$

Example 27: Find total surface area and volume of the following where the vertical height of the pyramid is 8.4m.

Solution: The figure is composed of two parts as shown



For the total surface area, the top of the first and the base of the second are not included.

Surface Area:

Surface Area of square prism

$$S_1 = \ell w + 2\ell h + 2wh\ cm^2$$
$$= 6(6) + 2(6)(4) + 2(6)(4)cm^2$$
$$= 36 + 48 + 48cm^2$$
$$= 132cm^2$$

Surface area of square pyramid

$S_2 =$ Areas of four lateral faces

$$S_2 = 4\times\left(\frac{1}{2}\times base\times slant\ height\right)$$
$$= 4\times\left(\frac{1}{2}\times 6\times 8\right) = 96cm^2$$

Total surface area of the object $= 132cm^2 + 96cm^2$
$= 228cm^2$

Volume:

Volume of the square prism

$$V_1 = \text{area of base} \times \text{height}$$
$$= (\ell \times w) \times h$$
$$= (6 \times 6) \times 4$$
$$= 144cm^3$$

Volume of the square pyramid

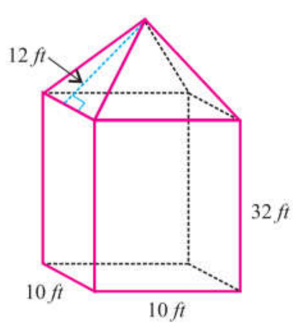
$$V_2 = \frac{1}{3} \times area\ of\ base \times height$$
$$= \frac{1}{3} \times 6 \times 6 \times 8.4$$
$$= 100.8cm^3$$

Total volume $= V_1 + V_2$
$= 144cm^3 + 100.8cm^3$
$= 244.8cm^3$

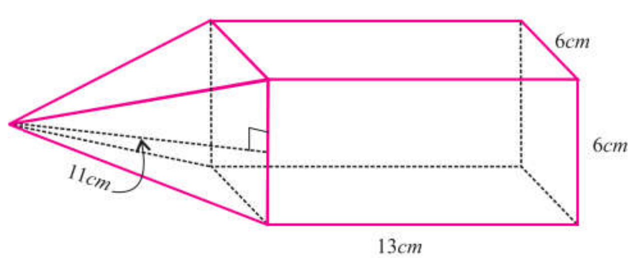
Exercise 3.7

1. Find the volume and surface area of the following:

(a)

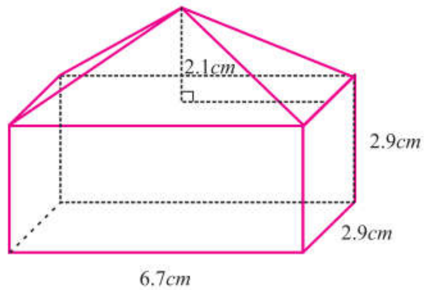


(b)

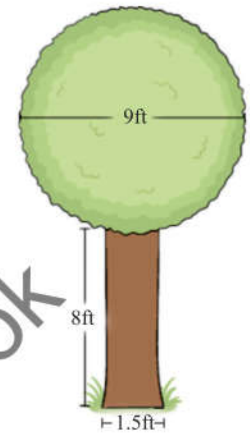


2. Determine the volume of the composite objects shown below:

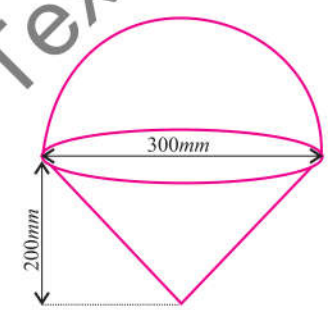
(a)



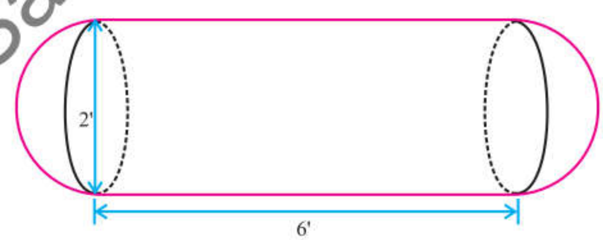
(b)



3. The composite object in the illustration, at right, consists of a hemisphere connected to the top of a cone. Find the object's:
   a. Surface area (in cm^2)
   b. Volume (in cm^3)



4. The composite object in the illustration, at right, consists of two hemispheres which are connected by a cylinder in between. Find the object's:
   a. Surface Area
   b. Volume



SUMMARY

- A circle is a plane figure bounded by one curved line such that all straight lines drawn from a certain point within it to the bounding line are equal. The fixed point is called the centre of the circle.
- A continuous part of the boundary of a circle is called an arc.
- A line segment joining two points on a circle is called a chord of the circle.
- A straight line that intersects a circle at two points is called a secant line.
- A 3D shape with a flat polygon base and three or more triangular sides converging at the top, is called Pyramid.
- Arc length is the distance along the curved line that makes up the arc.

- The sector of a circle is defined as the portion of a circle that is enclosed between its two radii and the arc adjoining them.
- Arc length of a sector of circle $= \dfrac{x}{360°} \times$ circumference
- Area of a sector of circle $= \dfrac{x}{360°} \times$ area of circle
- Volume of pyramid $= \dfrac{1}{3} \times$ volume of prism or $\dfrac{1}{3}$ base area $\times$ height of pyramid
- Surface area of sphere $= 4\pi r^2$
- Volume of sphere $= \dfrac{4}{3}\pi r^3$
- Surface area of a hemisphere $= \dfrac{1}{2} \times$ surface area of sphere + area of base
- Volume of hemi-sphere $= \dfrac{\text{Volume of sphere}}{2}$
- Total surface area of a cone = base area + curved surface area

$$= \pi r^2 + \pi r\ell$$
$$= \pi r(r+\ell)$$

- Volume of a cone $= \dfrac{1}{3}\pi r^2 h$

Review Exercise 3

1. Four options are given against each statement. Encircle the correct one.

(i) If in a right angled triangle ABC, $m\angle C = 90°$, then c is called ___________.
(a) base (b) hypotenuse (c) perpendicular (d) vertex

(ii) If in a right angled triangle ABC, $m\angle C = 90°$ and $\angle A$ is a base angle, then b is called:
(a) base (b) hypotenuse (c) perpendicular (d) vertex

(iii) In a right angled triangle the side opposite to the right angle is called:
(a) perpendicular (b) base (c) hypotenuse (d) right angle

(iv) An arc whose measure is less than 180° is called a ____________.
(a) minor arc (b) major arc (c) arc length (d) sector area

(v) A straight line that touches a circle at a single point externally is called:
(a) chord (b) tangent (c) sector (d) line segment

(vi) The perpendicular distance from base to the apex in a pyramid is called________.
(a) area (b) lateral surface area
(c) slant height of pyramid (d) altitude

(vii) The volume of pyramid is always ____________ of the prism.
(a) two-third (b) one-third (c) one-fourth (d) two-fourth

(viii) Surface area of square pyramid is:
(a) area of the base + 4(area of a face) (b) base area + 3(area of each face)
(c) area of the base + 2(area of a face) (d) base area + area of each face

(ix) Volume of pyramid =

(a) base area × height of pyramid (b) $\frac{1}{3}$ base area + height of pyramid

(c) $\frac{1}{3}$ base area × height of pyramid (d) base area + height of pyramid

(x) Surface area of sphere =

(a) $4\pi r^2$ (b) $4\pi r$ (c) $\frac{4}{3}\pi r^3$ (d) $3\pi r^2$

(xi) Volume of cone:

(a) $\frac{1}{3}$(area of the base + height) (b) $\frac{1}{3}$(area of the base × height)

(c) $\frac{1}{2}$ × surface area of cone + area of base (d) two-third of the volume of cylinder

(xii) The circles with a common center and have different radii are known as:

(a) centre (b) semi-circle (c) chord (d) concentric circle

(xiii) 3 times volume of a cone is equal to:

(a) volume of the cylinder (b) area of the cylinder

(c) volume of sphere (d) volume of hemi-sphere

(xiv) A composite 3D figure is a three-dimensional figure made up of basic ________.

(a) 2D figures (b) 3D figures (c) pyramid (d) 2D and 3D figures

(xv) Arc length (ℓ) =

(a) $\frac{x}{360^0}$ × area of circle (b) $\frac{x}{360^0} + 2\pi r$

(c) $\frac{x}{360^0} \times 2\pi r$ (d) $\frac{x}{360^0}$ + area of circle

2. Write short answers of the following questions.

(i) State Pythagoras theorem.

(ii) Write formula of surface area of a sphere.

(iii) Write the formula of volume of a cone.

(iv) Define:

(a) Sector (b) Chord (c) Concentric circle (d) Central angle

3. (i) Find the volume of a sphere when radius is $3.2cm$.

(ii) Find the volume of the cone if $r = 3cm$ and $h = 4cm$.

(iii) If $a = 2.4m$, $c = 2.6m$, ΔABC is right angled triangle with $m\angle C = 90°$, find b.

(iv) Find the arc length and area of a sector when:

(a) $r = 5cm$, 60^0 (b) $r = 7.5cm$, 45^0

(v) A sector of a circle of radius $8cm$ has an area of $49cm^2$. Find the angle at the centre of the sector.

4. Tariq built a square based pyramid tent for his night camp. The base of the tent is a square of side $2m$ and the height is $3m$. What is the space occupied by the tent?

5. A right pyramid whose base is a right angled triangle has volume $60cm^3$. If base and altitude of the right angle triangle are $6cm$ and $10cm$, find the height of the pyramid.
6. In triangle ABC, right angle is at point C, $m\overline{BC} = 2.1cm$ and $m\overline{AC} = 7.2cm$. What is the length of $\overline{AB}$?
7. What is the volume of a square pyramid with a base area of 255 square cm and a height of 7 cm?
8 The height of the great pyramid is about 455 feet. If you were to walk completely around the square base of the pyramid, you would have gone about 3,024 feet. What is the lateral surface area of the great pyramid today?
9. The cost of leather is Rs.1500 per square metre. Find the cost of manufacturing 100 footballs of radius $0.12m$.
10. The radius of the Earth is $6{,}371km$. What is the surface area of the Earth?
11. What will be the radius of a sphere with the same volume as a rectangular cuboids of length $5cm$, width $3cm$ and height $4cm$?
12. The water level in a cylindrical container of radius $0.5m$ is $1.4m$. If a spherical solid object is completely submerged in the water, the water level rises by $0.3m$. Find the volume of the sphere.
13. The surface area of a hemi-spherical object is $4587.25cm^2$. What will be the diameter of the hemi-sphere?
14. Find the volume of a hemi-sphere whose diameter is $20cm$. Give your answer in m.
15. How much sand can a conical container hold whose height is $2.5m$ and radius is $2m$, while $1m^3$ space contains 120kg of sand?

Domain 4 Geometry

Sub-Domain: Congruency and Similarity, Construction of Triangles and Transformation

Students' Learning Outcomes

After completing this domain, the students will be able to:

Congruent and Similar Figures

- explore congruent and similar figures from surroundings
- apply the properties of congruency and similarity for two figures

Congruent Triangles

- apply following postulates for congruency between triangles:
  - ❖ SAS≅SAS ❖ SSS≅SSS
  - ❖ ASA≅ASA ❖ HS≅HS

Construction of Triangles

- construct a triangle when three sides (SSS) are given
- construct a triangle when two sides and included angle (SAS) are given
- construct a triangle when two angles and included side (ASA) are given
- construct a right-angled triangle when hypotenuse and one side (HS) are given
- construct different types of quadrilaterals(sqaure, rectangle, parallelogram, trapezium, rhombus and kite)
- draw angle and line bisector to divide angles and sides of triangles and quadrilateral

Transformations

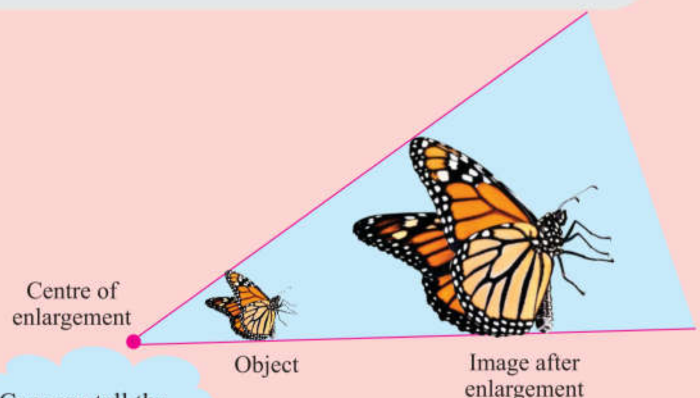
Rotation

- rotate an object and find the centre of rotation by construction

Advanced/ Additional

Enlargement

- enlarge a figure with the given scale factor (positive or negative).
- locate the centre and calculate the scale factor of enlargement given in the original figure and its enlargement

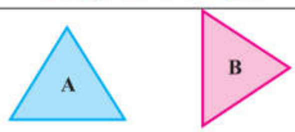
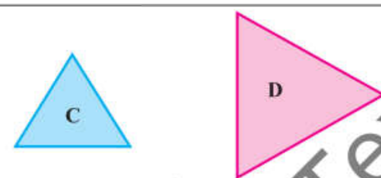
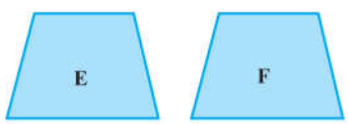
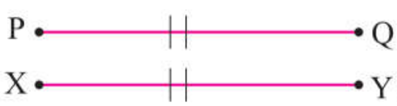
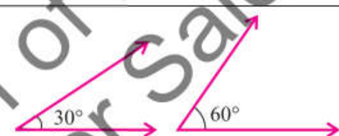
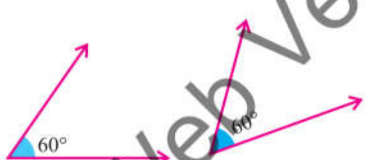
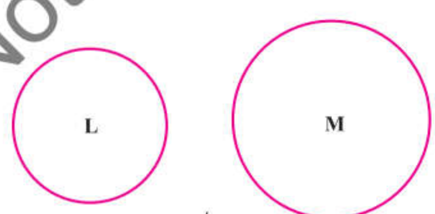


4.1 Congruent and Similar Figures

Congruent

Two figures and objects are congruent if these objects and figures have exactly the same shapes and sizes. These objects and figures need not to be identical. These objects and figures can have different colours and textures. The symbol for congruence is denoted as ≅.

For two lines segments which are congruent to each other are indicated by using the congruence symbol ≅.

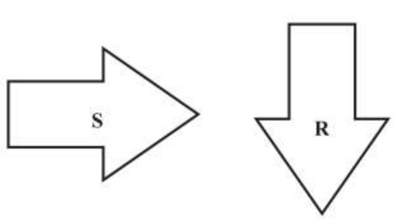
Congruent Shapes	Non Congruent Shapes
 A ≅ B	 C ≇ D
 E ≅ F	 G ≇ H
 PQ ≅ XY	 B ≇ C
 Y ≅ X	 L ≇ M

Key fact!

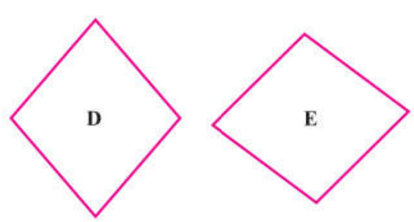
The corresponding angles and the corresponding sides of the congruent shapes are equal.

Example 1: Which shapes in the following figures are congruent?

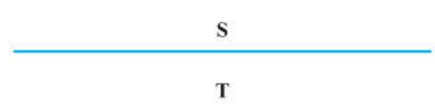
(a)



(b)



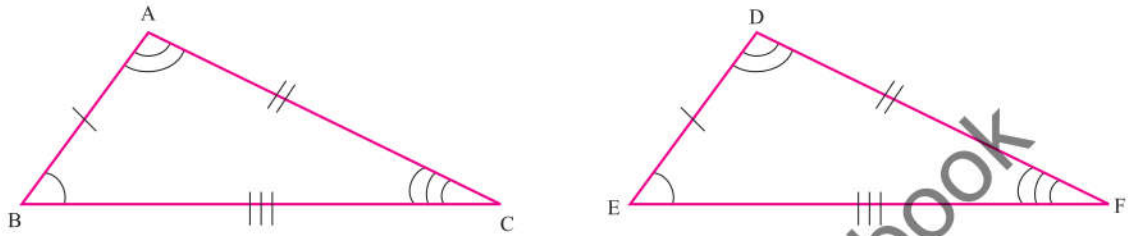
(c)



Solution:

(a) $S \cong R$ (b) $D \cong E$ (c) $S \ncong T$

Example 2: What are the congruent parts of the following two triangles?



Solution:

Since $\Delta ABC \cong \Delta DEF$, then,

Corresponding vertices are: $A \cong D,\ B \cong E,\ C \cong F$

Corresponding angles are: $\angle A \cong \angle D,\ \angle B \cong \angle E,\ \angle C \cong \angle F$

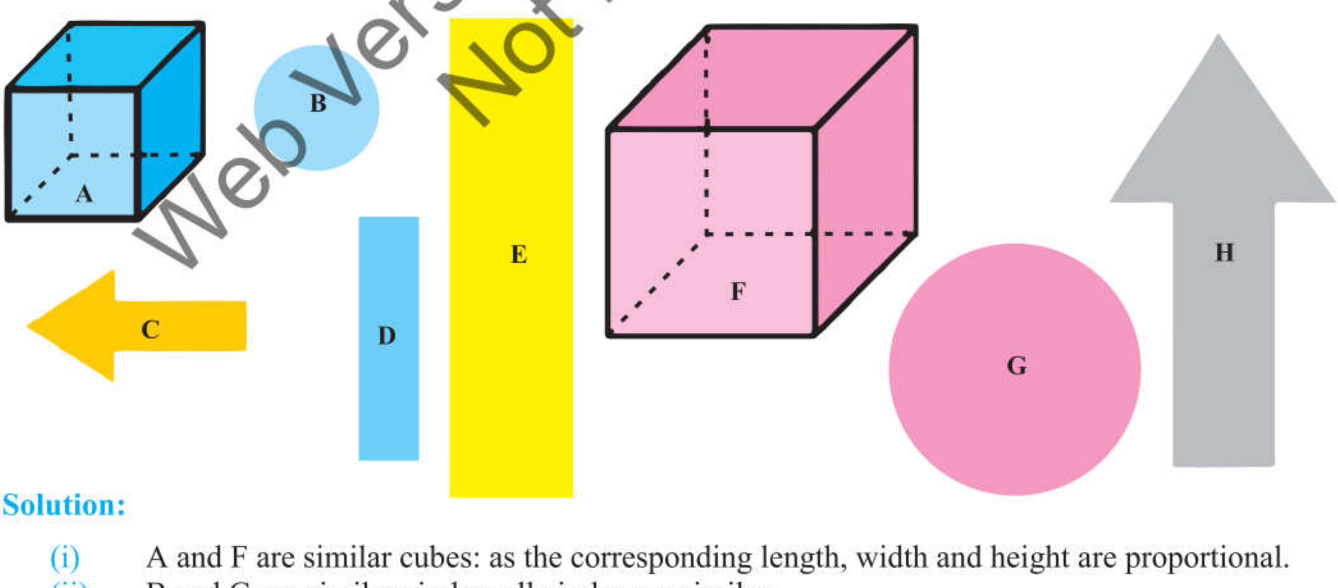
Corresponding sides are: $\overline{AB} \cong \overline{DE},\ \overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$

Similar Shapes

Two shapes are said to be mathematically similar if all of the angles in the shapes are equal, but the shapes are not necessarily of the same size. Enlarging or shortening the original shapes creates a similar shape. Enlarging or shortening is called the Scaling the shape. This means shapes have been enlarged or shortened in the same proportions.

Example 3:

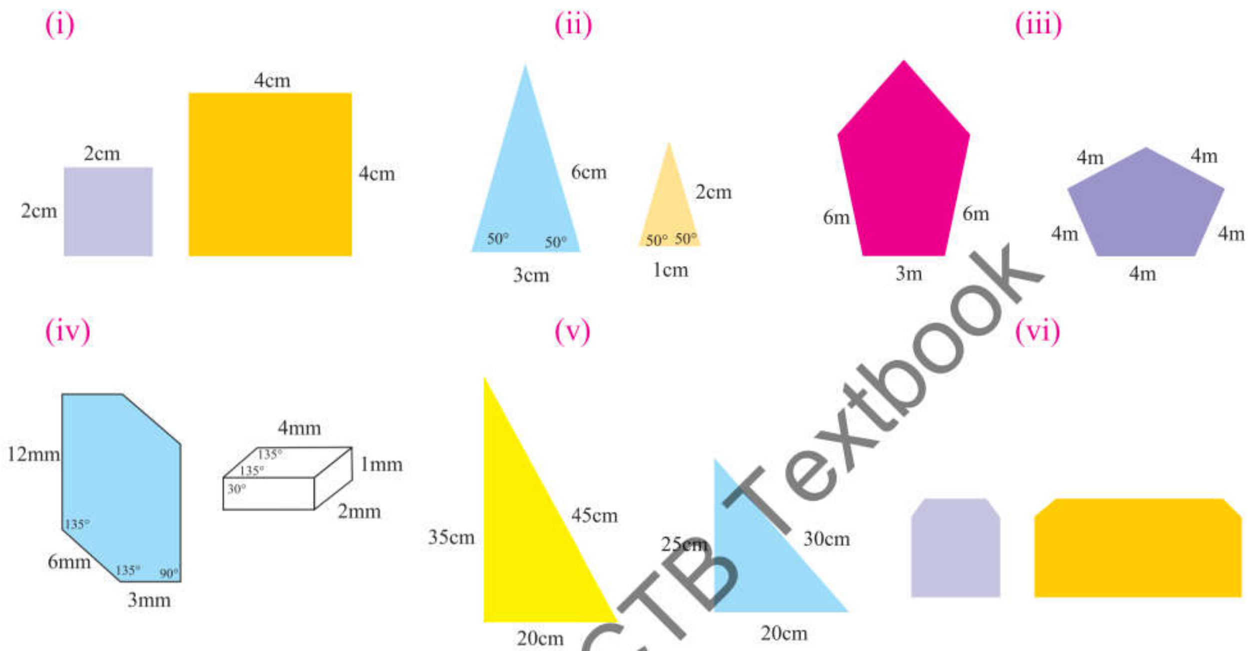
Which of the following figures are similar?



Solution:

(i) A and F are similar cubes: as the corresponding length, width and height are proportional.
(ii) B and G are similar circles: all circles are similar.
(iii) C and H are similar arrows: H is twice as big as C and has been rotated by 90°.
(iv) D and E are not similar: D has been stretched in one direction, but not the other.

Example 4: Are the following shapes similar?



Solution:

(i) Yes (ii) Yes (iii) No (iv) Yes (v) No (vi) No

Congruent Triangles

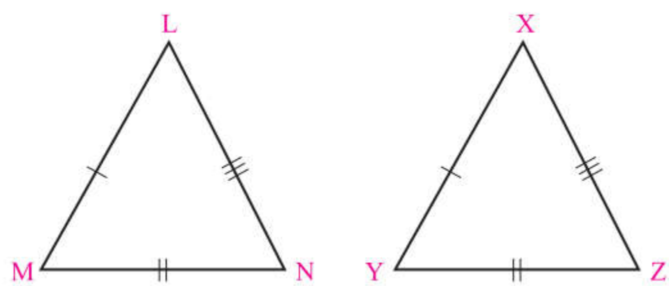
While considering triangles, two triangles will be congruent if:

(a) All the three sides of one triangle are equal to all the three corresponding sides of the other triangles, i.e., $SSS \cong SSS$. For example,

In

$$\Delta LMN \leftrightarrow \Delta XYZ$$
$$m\overline{LM} = m\overline{XY}$$
$$m\overline{MN} = m\overline{YZ}$$
$$m\overline{LN} = m\overline{XZ}$$

Then, $\Delta LMN \cong \Delta XYZ$

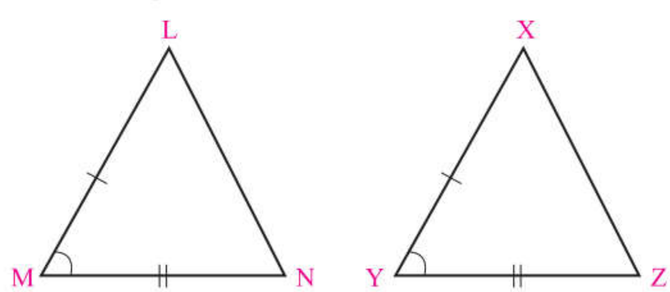


(b) Two sides of one triangle and their included angle are equal to the two corresponding sides and angle of the other triangle, i.e., $SAS \cong SAS$. For example,

In

$$\Delta LMN \leftrightarrow \Delta XYZ$$
$$m\overline{MN} = m\overline{YZ}$$
$$m\angle M = m\angle Y$$
$$m\overline{ML} = m\overline{YX}$$

Then, $\Delta LMN \cong \Delta XYZ$



(c) Two angles of a triangle and their included side are equal to the two corresponding angles and sides of the other triangle, i.e., ASA $\cong$ ASA. For example,

In

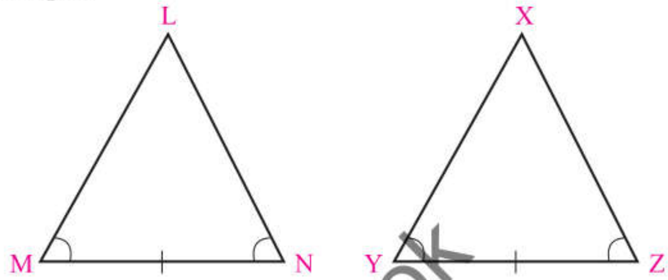
$\Delta LMN \leftrightarrow \Delta XYZ$

$m\angle M = m\angle Y$

$m\overline{MN} = m\overline{YZ}$

$m\angle N = m\angle Z$

Then, $\Delta LMN \cong \Delta XYZ$



(d) The hypotenuse and one side (base or altitude) of a triangle are equal to the corresponding hypotenuse and one side of the other triangle, i.e., $RHS \cong RHS$. For example,

In

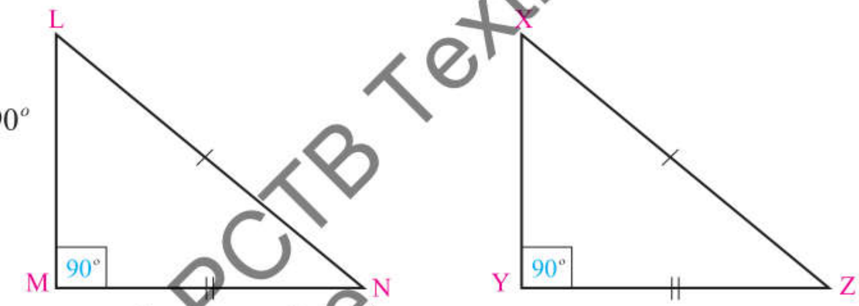
$\Delta LMN \leftrightarrow \Delta XYZ$

$m\angle M = m\angle Y = 90^o$

$m\overline{LN} = m\overline{XZ}$

$m\overline{MN} = m\overline{YZ}$

Then, $\Delta LMN \cong \Delta XYZ$



Remember!

Every triangle is congruent to itself (in the correspondence in which its sides and angles correspond to themselves). Such a congruence is called "Identity Congruence".

Similar Triangles

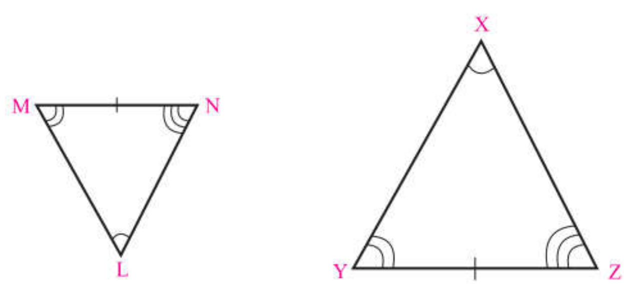
(a) When all corresponding angles are equal.

$m\angle L = m\angle X$

$m\angle M = m\angle Y$

$m\angle N = m\angle Z$

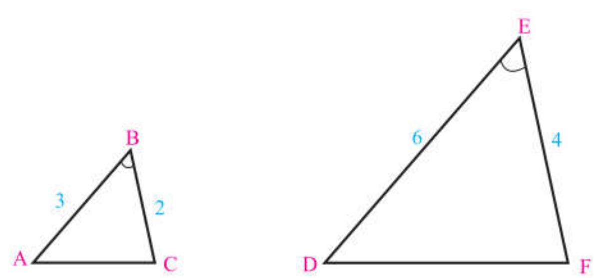
Then, ΔLMN is similar to ΔXYZ



(b) When the ratio of two corresponding sides and their included angle is equal.

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} \text{ and } m\angle B = m\angle E$$

Then, ΔABC is similar to ΔDEF



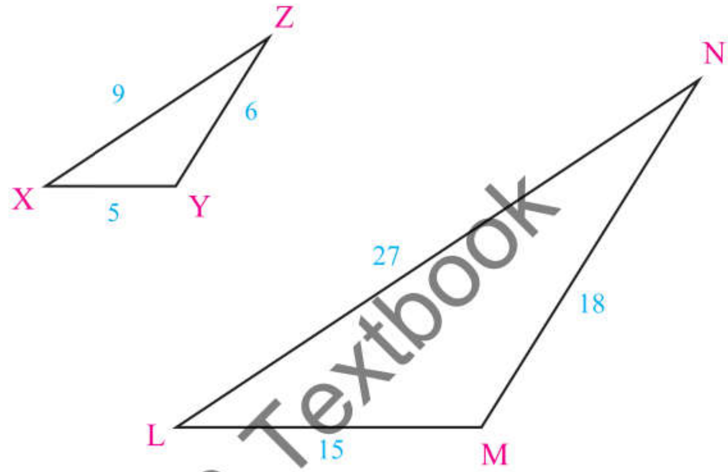
(c) When the ratio of all corresponding sides is equal.

$$\frac{m\overline{XY}}{m\overline{LM}} = \frac{m\overline{YZ}}{m\overline{MN}} = \frac{m\overline{XZ}}{m\overline{LN}}$$

$$\frac{5}{15} = \frac{6}{18} = \frac{9}{27}$$

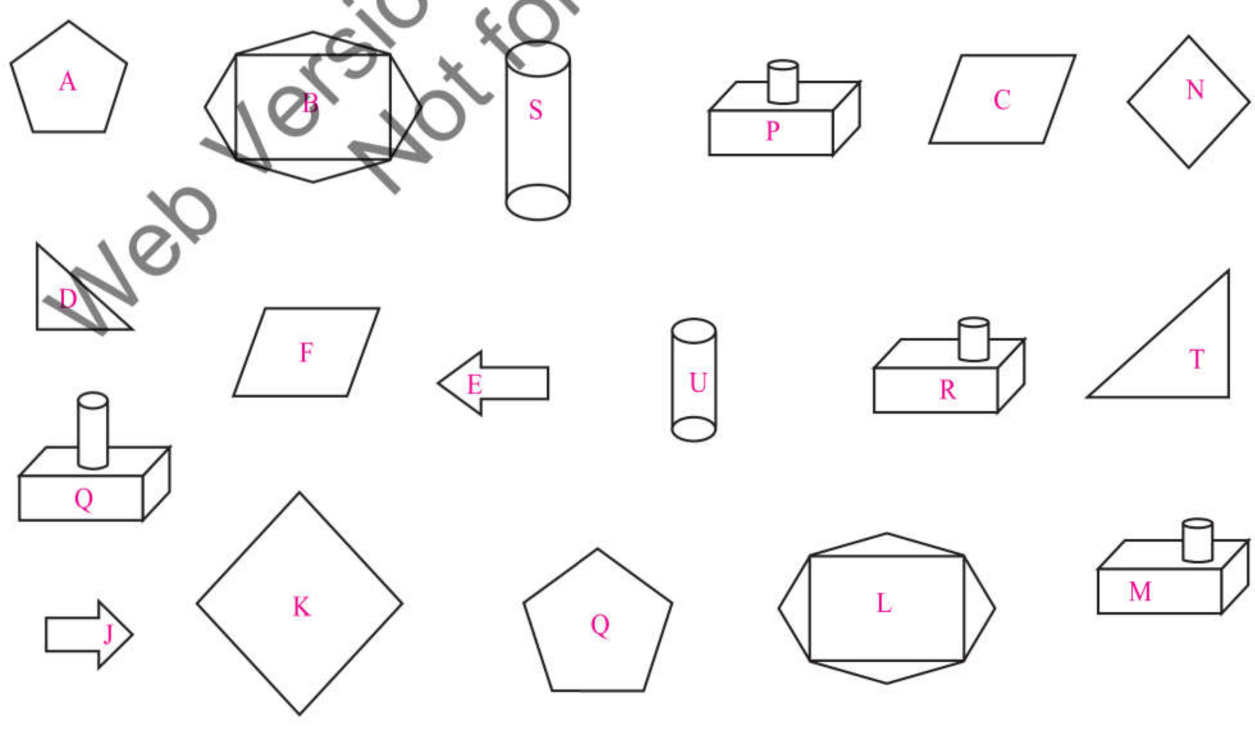
$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

Then, ΔXYZ is similar to ΔLMN

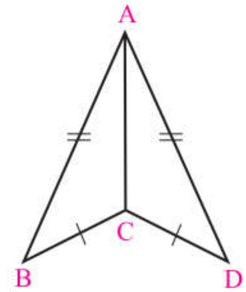


Exercise 4.1

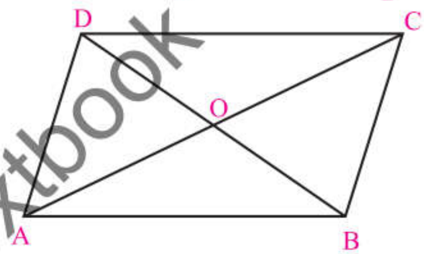
1. Which shapes are congruent and which are similar?



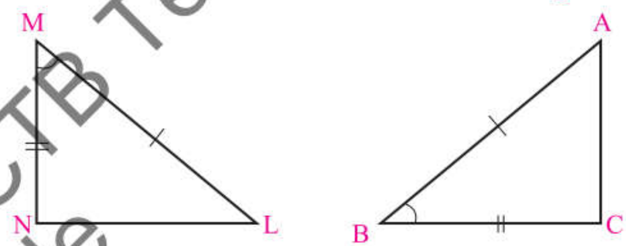
2. If $\Delta ADC \cong \Delta ABC$, then write the name of pairs of congruent angles and congruent sides also write the case of congruency.



3. In the adjoining figure, a parallelogram $ABCD$ and four triangles AOD, DOC, COB, AOB are shown. Which pairs of these triangles appear to be congruent? State the case of congruency.



4. Given that $m\angle B = m\angle M$, then prove that the triangles shown are congruent.



4.2 Construction of Different Types of Quadrilaterals

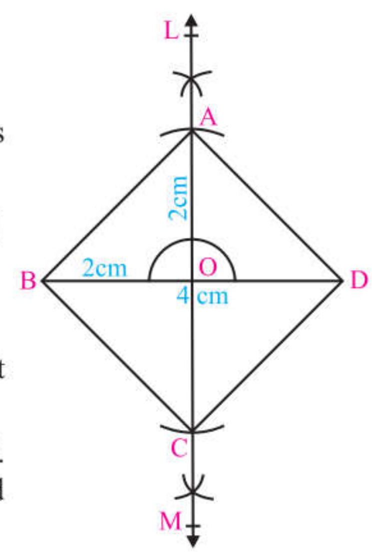
Construct a Square

(a) When its diagonal is given.

Example 5: Draw a square $ABCD$ such that its diagonal is $4cm$.

Solution: One of the diagonals of the square $ABCD$ is $\overline{BD}$ and $m\overline{BD} = 4cm$.

[Note: In a square both the diagonals are of same length]



Steps of Construction:

(i) Draw the diagonal $m\overline{BD} = 4cm$.

(ii) Draw a perpendicular bisector $\overleftrightarrow{LM}$ of the diagonal $\overline{BD}$ cutting it at point O.

(iii) With O as centre and radius $m\overline{OB}$, draw arcs cutting $\overleftrightarrow{LM}$ at A and C.

(iv) Join A with B and D, and C with B and D, which gives the required square $ABCD$.

(b) When the difference between its diagonal and side is given.

Example 6: Draw a square $ABCD$ when the difference between its diagonal and side is equal to $2cm$.

Solution:

Steps of Construction:

(i) Draw $\overleftrightarrow{PQ}$ and mark a point as A on it.

(ii) Construct $m\angle QAN = 90^o$ at A.

(iii) Draw two arcs of radius $2cm$ and centre at A which intersects $\overrightarrow{AQ}$ at M and $\overrightarrow{AN}$ at L.

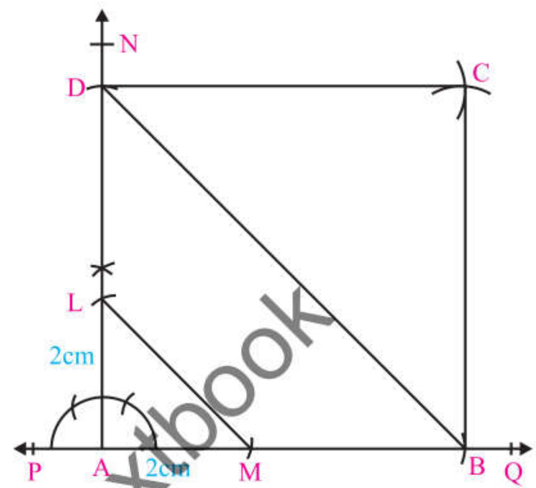
(iv) Draw an arc of radius = $m\overline{LM}$ and centre at M which intersects $\overrightarrow{AQ}$ at B.

(v) Draw an arc of radius = $m\overline{AB}$ and centre at A which intersects $\overrightarrow{AN}$ at D.

(vi) Draw two arcs each of radius = $m\overline{AB}$, one centre at B and second centre at D. These arcs will intersect at point C.

(vii) Join C with D and B.

Hence, $ABCD$ is the required square.



(c) When the sum of its diagonal and side is given

Example 7: Draw a square $ABCD$ when the sum of its diagonal and side is equal to $3cm$.

Solution:

Steps of Construction:

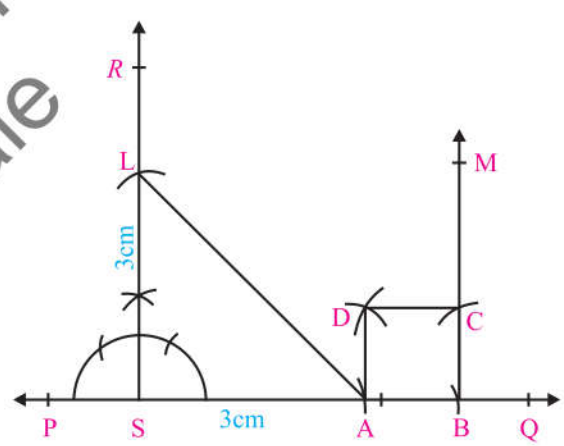
(i) Draw $\overleftrightarrow{PQ}$ and mark a point as S on it.

(ii) Construct $m\angle QSR = 90^o$ at point S.

(iii) Draw an arc of radius $3cm$ and centre at S intersecting $\overrightarrow{SR}$ at L.

(iv) Draw an arc of radius $3cm$ and centre at S intersecting $\overrightarrow{SQ}$ at A.

(v) Draw an arc of radius = $m\overline{AL}$ and centre at S which intersects $\overrightarrow{SQ}$ at B. $\overline{AB}$ is the side of the required square.



(vi) Draw perpendicular $\overrightarrow{BM}$ at B.

(vii) Draw an arc of radius $m\overline{AB}$ and centre at B which intersects $\overrightarrow{BM}$ at C.

(viii) Draw two arcs, each of radius $m\overline{AB}$, one with centre at A and second with centre at C which intersects at D.

(ix) Join C with D and D with A.

Hence, $ABCD$ is the required square.

Construct a Triangle

(a) When two sides are given

Example 8: Construct a rectangle $ABCD$ in which $m\overline{AB} = 4cm$ and $m\overline{BC} = 5cm$.

Solution:

Steps of Construction:

(i) Draw $m\overline{AB} = 4cm$.

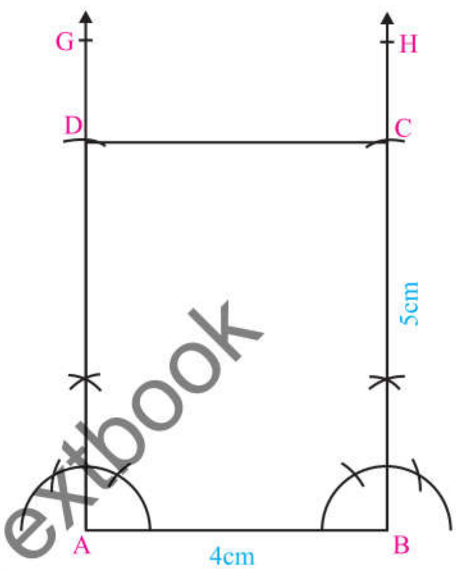
(ii) Construct $m\angle A = m\angle B = 90^o$ and draw $\overrightarrow{AG}$ and $\overrightarrow{BH}$.

(iii) Draw an arc with centre at A and of radius $5cm$ which intersects $\overrightarrow{AG}$ at point D.

(iv) Draw an arc with centre at B and of radius $5cm$ which intersects $\overrightarrow{BH}$ at point C.

(v) Join C with D.

Hence, $ABCD$ is the required rectangle.



Note: Sum of interior angles of a quadrilateral is equal to 360°

(b) When the diagonal and a side are given

Example 9: Construct a rectangle $ABCD$ when

$m\overline{AB} = 3cm$ and $m\overline{AC} = 5cm$

Solution:

Steps of Construction:

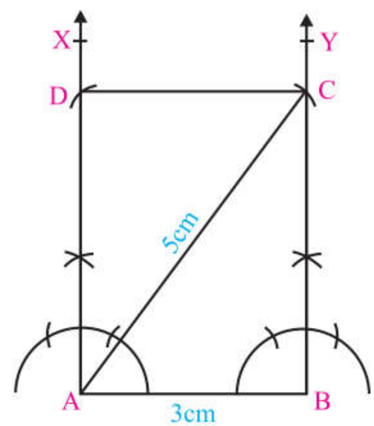
(i) Draw $m\overline{AB} = 3cm$.

(ii) Construct $m\angle A = m\angle B = 90^o$ and draw $\overrightarrow{AX}$ and $\overrightarrow{AB}$.

(iii) With centre at A and radius $5cm$ draw an arc which intersects $\overrightarrow{BY}$ at the point C.

(iv) With centre at B and radius $5cm$ draw an arc which intersects $\overrightarrow{AX}$ at the point D and join C and D.

Hence, $ABCD$ is the required rectangle.



Construct a Rhombus

(a) When one side and the base angle are given.

Example 10: Construct a rhombus $PQRS$ when $m\overline{PQ} = 4cm$ and $m\angle P = 45^0$.

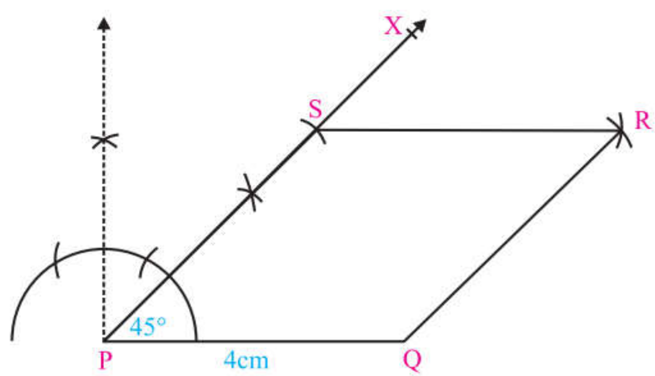
Solution:

Steps of Construction:

(i) Draw $m\overline{PQ} = 4cm$.

(ii) Construct $m\angle P = 45^0$ and draw $\overrightarrow{PX}$.

(iii) Draw an arc with centre at P and radius $4cm$ which intersects $\overrightarrow{PX}$ at S.



(iv) Draw an arc with centre at S and radius 4cm.
(v) Draw an arc with centre at Q and radius 4cm which intersects the previous arc drawn from S at R.
(vi) Join R with S and Q.
Hence, $PQRS$ is the required rhombus.

(b) When one side and a diagonal are given.

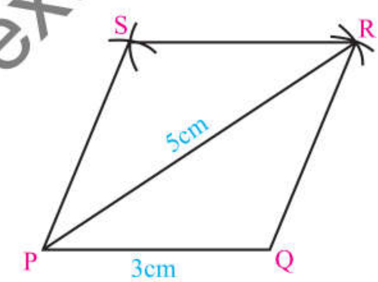
Example 11:

Construct a rhombus $PQRS$, when $m\overline{PQ} = 3cm$ and $m\overline{PR} = 5cm$.

Solution:

Steps of Construction:

(i) Draw $m\overline{PQ} = 3cm$.
(ii) Draw an arc with centre at P and radius 5cm.
(iii) Draw an arc with centre at Q and radius 3cm which intersects the previous arc at R.
(iv) Draw an arc with centre at R and radius 3cm.
(v) Draw an arc with centre at P and radius 3cm which intersects the previous arc at S.
(vi) Join Q with R, R with S and P with S.
Hence, $PQRS$ is the required rhombus.



Construct a Parallelogram

(a) When two diagonals and the angle between them is given

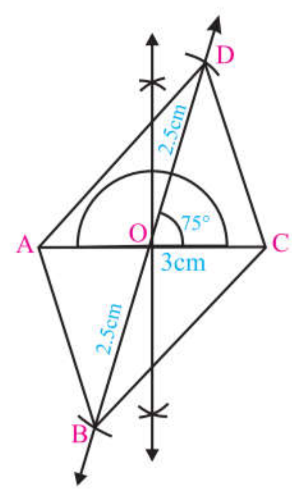
Example 12:

Construct a parallelogram $ABCD$ whose diagonals are 3cm and 5cm and the angle between them is 75°.

Solution:

Steps of Construction:

(i) Draw the diagonal $m\overline{AC} = 3cm$.
(ii) Bisect $\overline{AC}$ with O as the midpoint.
(iii) Construct an angle 75° at the point O and extend the line on both sides.
(iv) From O, draw an arc of radius 2.5cm on both sides of $\overline{AC}$ to cut the above line at B and D.
(v) Join A with B and D.
(vi) Join C with B and D.
Hence, $ABCD$ is the required parallelogram.



(b) When two adjacent sides and the angle included between them are given

Example 13: Construct a parallelogram $PQRS$ when $m\overline{PQ} = 4cm, m\overline{PS} = 7cm$ and included angle between these sides is $m\angle QPS = 60^0$.

Solution:

Steps of Construction:

(i) Draw a line segment $PQ = 4cm$.

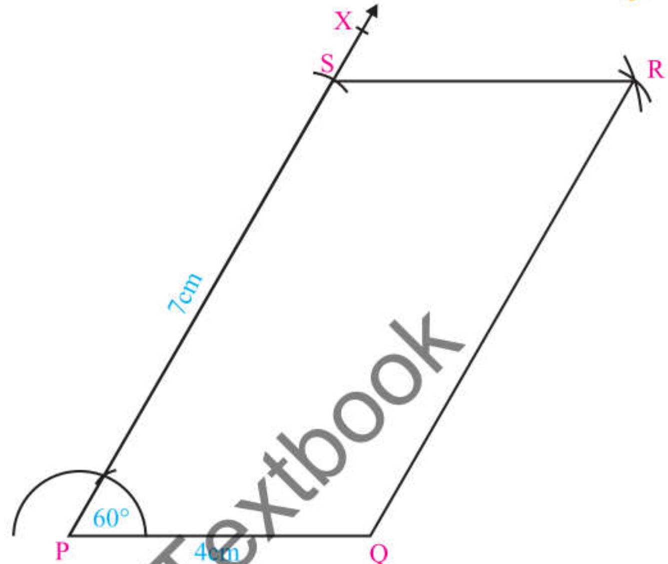
(ii) Construct $m\angle QPS = 60^0$ at point P.

(iii) Draw an arc with centre at P and radius $7cm$ which intersects $\overrightarrow{PX}$ at point S.

(iv) Draw an arc with centre at Q and radius $7cm$ above point Q.

(v) Draw an arc with centre at S and radius $4cm$ which intersects the arc drawn from point Q at R.

(vi) Join R with S and Q to R to form the required parallelogram $PQRS$.



Construct a kite when two unequal sides and a diagonal are given

Example 14: Construct a kite $PQRS$ when $m\overline{PQ} = 4cm$, $m\overline{QR} = 6cm$ and the length of the longer diagonal is $m\overline{PR} = 8cm$.

Solution:

Steps of Construction:

(i) Draw $m\overline{PQ} = 4cm$.

(ii) Draw an arc with centre at Q and radius $6cm$.

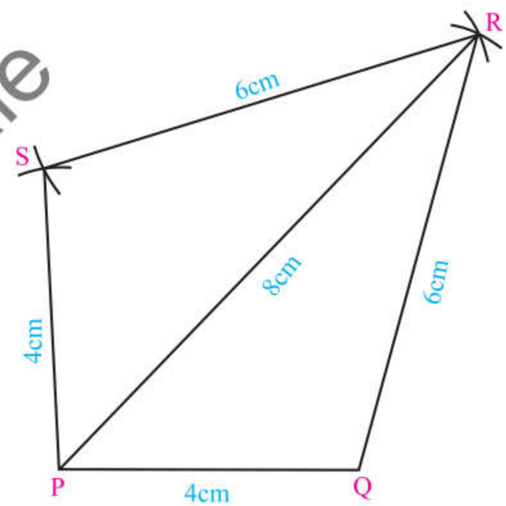
(iii) Draw an arc with centre at P and radius $8cm$. It intersects the previous arc at point R.

(iv) Draw an arc with centre P and radius $4cm$ above P.

(v) Draw an arc with centre at R and radius $6cm$ which intersects the arc drawn from P at S.

(vi) Join R with Q and S and P with S.

Hence, $PQRS$ is the required kite.



Construct a Trapezium

(a) When four sides are given

Example 15: Construct a trapezium $ABCD$ in which

$m\overline{AB} = 5cm, m\overline{BC} = m\overline{CD} = 3cm$ and $m\overline{AD} = 4cm$

Solution:

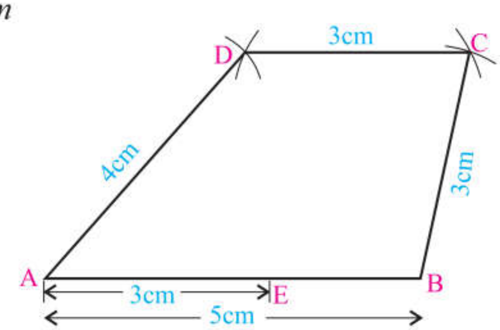
Steps of Construction:

(i) Draw a line segment $AB = 5cm$

(ii) Take point E on AB i.e., $m\overline{AE} = m\overline{CD} = 3cm$.

(iii) Draw an arc of radius $4cm$ at point E.

(iv) Draw another arc of radius $3cm$ with centre at point B which the previous arc at point C.



(v) Draw an arc of radius 3*cm* with centre at point *C*.
(vi) Draw another arc of radius 4*cm* with centre at point *A* which cuts the previous arc at point *D*.
(vii) Join the points *B* and *C, C* and *D, A* and *D*.
Thus, *ABCD* is the required trapezium.

(b) When three sides and one angle are given

Example 16:

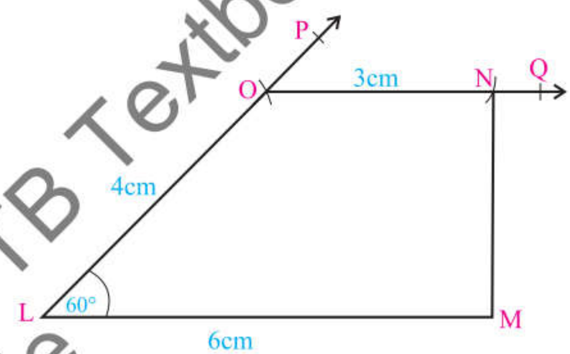
Construct a trapezium *LMNO* when $m\overline{LM} = 6cm, m\overline{ON} = 3cm, m\overline{LO} = 4cm$ and $m\angle L = 60^0$, and $\overline{LM} \parallel \overline{ON}$

Solution:

Steps of Construction:

(i) Draw a line segment $m\overline{LM} = 6cm$.
(ii) By using protractor draw an angle of measure 60^0 at point *L* by a ray *LP*.
(iii) Draw an arc of radius 4*cm* with centre at point *L*.
(iv) By using ruler and set square draw a ray *OQ* at point '*O*' parallel to *LM*.
(v) Draw an arc of radius 3*cm* which intersects the ray *OQ* at point *N*.
(vi) Join *O* with *N* and *M* with *N*.

Thus, *LMNO* is the required trapezium.



Exercise 4.2

1. Construct a square *ABCD*, when a diagonal $m\overline{AC} = 4.5cm$.
2. Construct a square *PQRS*, when its diagonal is 4*cm* more than its side.
3. Construct a square *PQRS*, when the sum of the diagonal and a side of the square is 8*cm*.
4. Construct a rectangle *ABCD*, when $m\overline{AB} = 4cm$ and $m\overline{BC} = 6cm$.
5. Construct a rectangle *ABCD*, when $m\overline{AB} = 5.5cm$ and $m\overline{AC} = 8cm$
6. Construct a rhombus *KLMN*, when the $m\overline{KL} = 5cm, m\angle K = 75^0$
7. Construct a rhombus *STUV*, when $m\overline{ST} = 6cm$ *and* $m\overline{SU} = 9cm$
8. Construct a parallelogram *ABCD* with diagonals 6*cm* and 8*cm* and the angle between them is 70°.
9. Construct a parallelogram *DEFG* where $m\overline{DE} = 5.5cm, m\overline{EF} = 6.5cm$ and $m\angle E = 60^0$.
10. Construct a kite *DEFG* where $m\overline{DE} = 4cm, m\overline{EF} = 8cm$ and the length of the longer diagonal is $m\overline{DE} = 10cm$.
11. Draw a trapezium *EFGH* so, that $\overline{FE} \parallel \overline{GH}, m\angle E = 70^0, m\overline{EF} = 8cm, m\overline{EH} = 5cm, m\overline{HG} = 4cm$
12. Draw a trapezium *PQRS* in which $m\overline{PQ} = 7cm, m\overline{QR} = m\overline{RS} = 5cm, m\overline{PS} = 6cm$

4.3 Construction of Triangles

To construct a triangle, we do not need all its six elements. It can be constructed with three elements only, out of which one should be a side.

We can construct a triangle when.

(i) The measures of three sides are given.

(ii) The measures of two sides and their included angle are given.

(iii) The measures of the hypotenuse and one side are given (in case of right angled triangle).

The result can be verified by measuring the side of different triangles.

Remember!

The sum of the measures of any two sides of a triangle is always greater than the measure of its third side.

The difference of the measure of any two sides of a triangle is always less than the measures of its third side).

• Construct a triangle when three sides (SSS) are given.

Example 17: Construct a ΔABC in which lengths of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ are respectively $4cm$, $3cm$ and $5cm$.

Solution:

Steps of Construction:

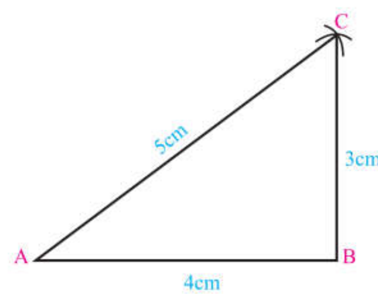
(i) Draw a line segment AB of measure $4cm$.

(ii) From the point A draw an arc of radius $5cm$.

(iii) From the point B draw an arc of radius $3cm$ which interests the first arc.

(iv) Mark C where two arcs intersect.

(v) Join the points A to C and B to C.
Thus, ΔABC is the required triangle.



• Construct a triangle when two sides and one included angle (SAS) is given

Example 18: Construct a triangle when the measure of its two sides are 6.8cm and 4.6cm while their included angle is of measure 120°.

Solution:

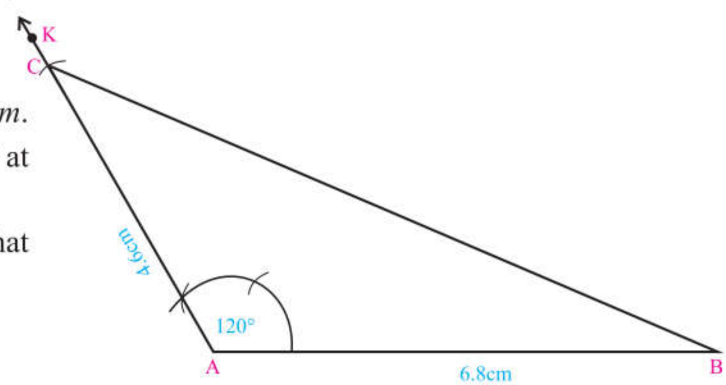
Steps of Constructions:

(i) Draw a line segment AB of measure $6.8cm$.

(ii) Construct an angle BAK of measure 120° at the point A.

(iii) Take a point C on $\overrightarrow{AK}$ such that $m\overline{AC} = 4.6cm$.

(iv) Join the point C to the point B.
Thus, ΔABC is the required triangle.



Exercise 4.3

1. Without construction check, whether or not a triangle can be constructed with given sides. If the answer is negative, give reason.

(i)	$8cm$, $5cm$, $2cm$	(ii)	$6cm$, $3.5cm$, $2cm$	(iii)	$6.7cm$, $4.5cm$, $3.4cm$
(iv)	$9cm$, $4.5cm$, $2.8cm$	(v)	$7cm$, $5.4cm$, $3.5cm$	(vi)	$8.8cm$, $3.6cm$, $5.2cm$

2. Construct the following triangles with the help of pair of compasses and a ruler when the measure of the two sides and their included angle is given:

   (i) 60°, 8.4*cm*, 5.5*cm* (ii) 45°, 7.5*cm*, 4.2*cm*

   (iii) 30°, 6.8*cm*, 4.6*cm* (iv) 120°, 5.4cm, 3.6cm

3. Construct an isosceles triangle, in which the measure of the two congruent sides is 6.4*cm* each and measure of their included angle is 30°.

4. Construct the following triangles with the help of a ruler and a protractor.

   (i) ΔABC, when $m\overline{AB} = 5.8cm$, $m\overline{BC} = 3.6cm$ and $m\angle B = 30°$

   (ii) ΔPQR, when $m\overline{PQ} = 4.8cm$, $m\overline{QR} = 4.8cm$ and $m\angle Q = 120°$

• Construct triangle when two angles and included side are given

Key fact!
The sum of measure of interior angles of a triangle is 180°.

Example 19: Construct a triangle *ABC*, in which The base = $m\overline{BC} = 6.2cm$ long, $m\angle B = 45°$ and $m\angle C = 60°$

Solution:

Steps of Constructions:

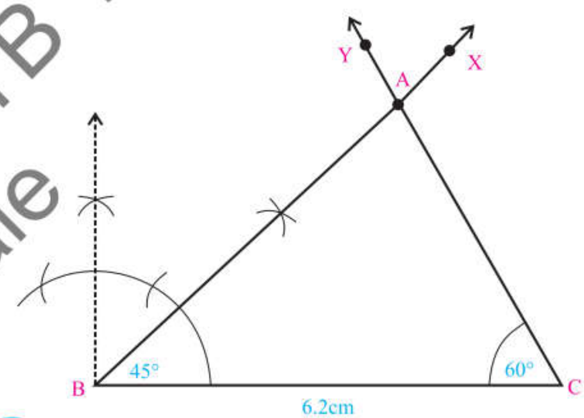
Draw a line-segment BC of measure 6.2*cm*.

(i) Construct $\angle CBX$ of measure 45° at point *B*.

(ii) Construct $\angle BCY$ of measure 60° at point *C*.

(iii) $\overrightarrow{BX}$ and $\overrightarrow{CY}$ intersect each other at point *A*.

Thus, ΔABC is the required triangle.



• Construct a right angled triangle when hypotenuse and one side (HS) are given

Key fact!
If measurements of any two angles of a triangle are known. The measurement of the third angle can be found by subtracting.
The sum of the two angles is subtracted from 180°.
i.e., in ΔABC if $m\angle B$ and $m\angle C$ are known then $m\angle A = 180° - (m\angle B + m\angle C)$

Example 20: Construct a right angled triangle whose hypotenuse is of measure 7.5*cm* and measure of one side is 4.5*cm*.

Solution:

Steps of Construction:

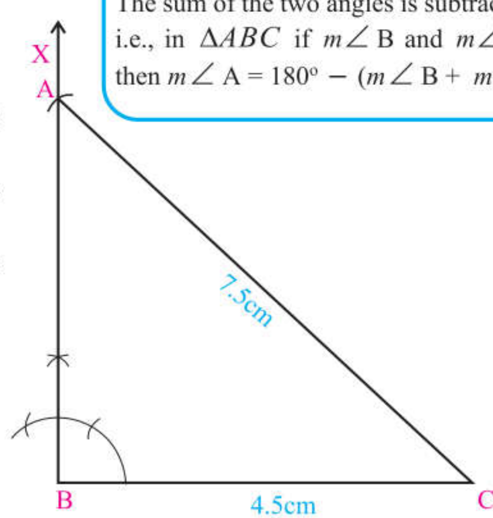
(i) Draw a line-segment BC of measure 4.5*cm*

(ii) Construct $\angle CBX$ of measure 90° at the point *B*.

(iii) From the point *C* draw an arc of radius 7.5*cm* to intersect $\overrightarrow{BX}$ at point A.

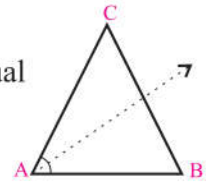
(iv) Join point *A* to *C*.

Thus, ΔABC is the required right angled triangle.



Angle Bisector of a Triangle

When a ray bisects a vertex of a triangle into two equal parts, then it is called angle bisector of a triangle.



Keep in mind!

- Bisect means to divide into two equal parts.
- Angle bisectors of internal angles of a triangle are concurrent.

To Draw Angle Bisector of a Triangle

Steps of Construction:

(i) Draw any triangle ABC.

(ii) Draw an arc of any suitable radius with centre at A which interests $\overline{AC}$ and $\overline{AB}$ at X and Y.

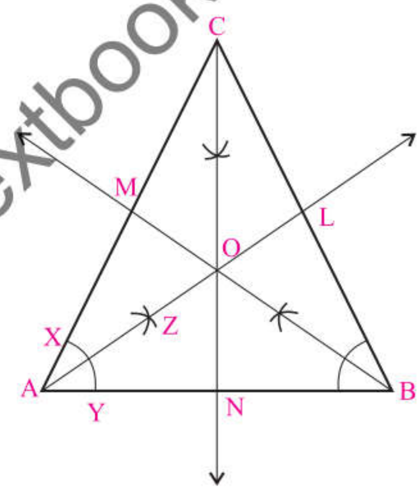
(iii) Draw an arc of any suitable radius with centre at X, another arc of a radius with centre at Y. Which intersects the previous arc at Z.

(iv) Join A with Z and produce it to meet $\overline{BC}$ at L.

(v) Then $\overline{AL}$ is the required angle bisector of $\angle A$.

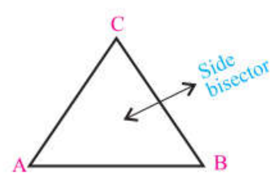
(vi) Draw the angle bisectors $\overline{BM}$ and $\overline{CN}$ for angle B and C on the same steps as given above respectively.

Thus, the point O in figure is called the incentre of the triangle.



Perpendicular Bisector of Sides of a Triangle

When a line segment bisects a side of a triangle, then it is called side bisector of a triangle.



To draw side bisectors of a triangle, we follow the following steps:

Steps of Construction:

(i) Draw any triangle ABC.

(ii) Draw two arcs of any radius (more than half of $\overline{AB}$) over and under $\overline{AB}$ with centre at A.

(iii) Now, draw two arcs of the same radius over and under the $\overline{AB}$ which intersect the previous arcs at points L and M with centre at B.

(iv) Join L with M.

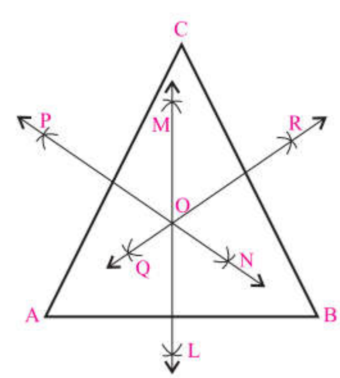
(v) $\overline{LM}$ is the right sector of the side $\overline{AB}$.

(vi) Draw the perpendicular bisectors $\overline{QR}$ and $\overline{NP}$ of $\overline{BC}$ and $\overline{AC}$ respectively, on the same steps as given above.

Thus, the point O in figure is called the circumcentre of the triangle.

Keep in mind!

Perpendicular bisectors of sides of a triangle are concurrent.



Remember!

A single point where three perpendicular bisectors of a triangle intersect is called the circumcentre of the triangle.

Remember!

Can you draw the angle and side bisectors for the quadrilaterals? (square, rectangle, kite, rhombus etc.)

Exercise 4.4

1. Construct the following triangles with the help of a compass and a ruler.

Sr.No.	Measure of angles on the base	Measure of base
(i)	30^o, 45^o	$6.5cm$
(ii)	45^o, 90^o	$5.4cm$
(iii)	30^o, 120^o	$4.8cm$
(iv)	45^o, 60^o	$3.6cm$

2. Construct an isosceles triangle. The length of its base is $8cm$ and the two angles on the base are of measure $45°$ each.
3. Construct the following triangles by constructing angles with the help of a protractor.
   - (i) ΔPQR, when $m\angle Q = 120°$, $m\angle R = 30°$ and $m\overline{QR} = 5.3cm$
   - (ii) ΔGHK, when $m\angle G = 30°$, $m\angle H = 60°$ and $m\overline{GH} = 4.5cm$
4. Construct right angled triangles with the help of a compass.
   - (i) Hypotenuse $= 6.8cm$, Base $= 3.5cm$
   - (ii) Hypotenuse $= 8.8cm$, Base $= 4.7cm$
   - (iii) Hypotenuse $= 7.6cm$, Base $= 5.4cm$
   - (iv) Hypotenuse $= 9cm$, Altitude $= 5cm$
5. The measure of two sides of a triangle are $6.8cm$ and $4.4cm$. If the angle opposite to the side of measure $6.8m$ is a right angle, then construct the triangle.
6. Construct ΔABC, when $m\overline{AB} = 3.8cm$, $m\overline{AC} = 5cm$ and $m\angle B = 90°$
7. Construct ΔPQR, when $m\overline{PQ} = 4.6$cm, $m\overline{QR} = 9.8cm$ and m$\angle$ Q = 90°
8. Draw angle bisectors of any suitable size of a triangle.
9. Draw side bisectors of any suitable size of a triangle.
10. Draw angle and perpendicular bisectors of any quadrilateral.

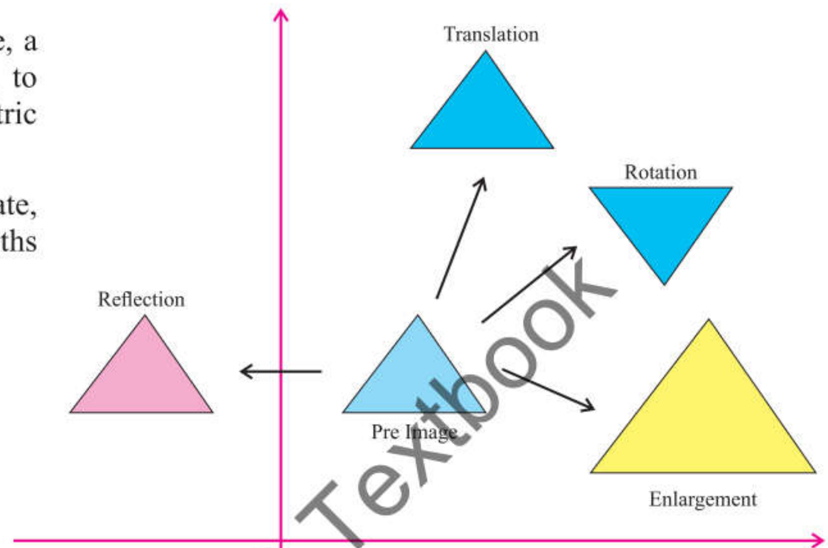
4.4 Transformation

Transformation means to change. Hence, a geometric transformation would mean to make some changes in any given geometric shape.

Even after transforming a shape (translate, reflect or rotate), the angles and the lengths of the sides remain unchanged.

Types of Transformations:

(i) Translation
(ii) Reflection
(iii) Rotation
(iv) Enlargement



Translation

- Translation is a transformation of a shape which gives precise description of transformation

Reflection

- Reflection of an object through a line

Rotation

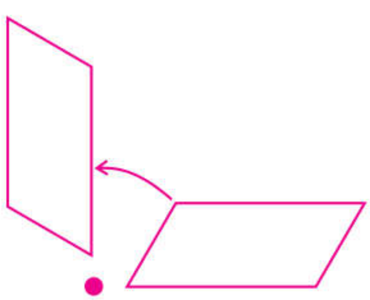
- Rotate an object and find the centre of rotation by construction

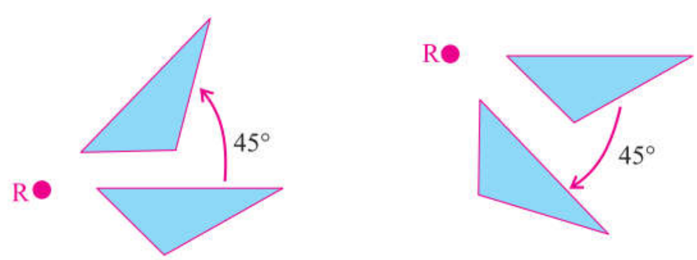
Enlargement

- Enlarge a figure with the given scale factor (positive or negative)
- Locating the centre and scale factor of enlargement given the original figure and its enlargement

Rotation

In geometry, a rotation is a type of transformation where a shape or geometric figure is turned around a fixed point. A rotation is a type in which the size and shape of the figure does not change; the figures are congruent before and after the transformation. A rotation turns each point on a pre-image around a fixed point, called the centre of rotation, a given angle measure. Below are two examples.





A parallelogram rotates around the red dot.

Two triangles are rotated around point R in the above.

Need to know!

(i) In the centre of rotation, the shape is turned around that point.
(ii) The amount of turn, which might be in degrees or given as a fraction.
(iii) The direction of rotation, which will be either clockwise or counterclockwise.

Rotation in Coordinate Geometry

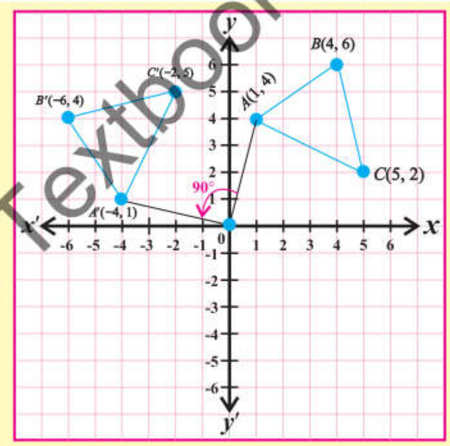
In a coordinate plane, when geometric figures rotate around a point, the coordinates of the points change. While a geometric figure can be rotated around any point at any angle, we will only discuss rotating a geometric figure around the origin at common angles.

90° Rotation

Counterclockwise rotation 90° about origin

A rotation of 90° counterclockwise around the origin changes the position of a point (x, y) such that it becomes $(-y, x)$. Triangle ABC is rotated 90° counterclockwise to land on triangle $A'B'C'$.

Vertices $A(1, 4)$, $B(4, 6)$ and $C(5, 2)$ move to $A'(-4, 1)$, $B'(-6, 4)$ and $C'(-2, 5)$.

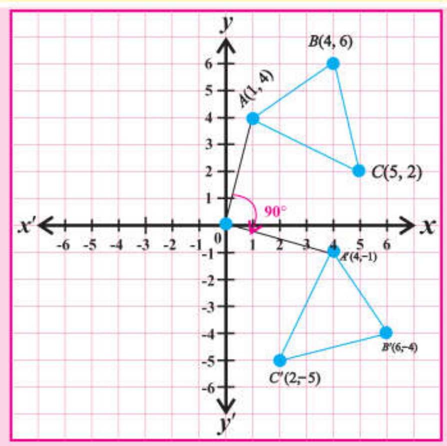


Clockwise Rotation 90° about origin

A rotation of 90° clockwise changes the point such that (x, y) becomes $(y, -x)$.

Triangle ABC is rotated 90° clockwise to land on triangle $A'B'C'$.

Vertices $A(1, 4)$, $B(4, 6)$ and $C(5, 2)$ move to $A'\ (4, -1)$, $B'\ (6, -4)$ and $C'\ (2, -5)$.

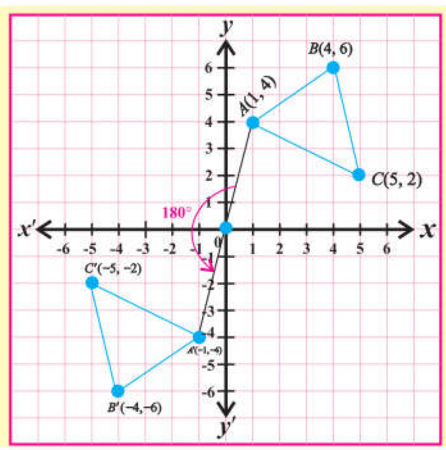


180° Rotation about origin

A rotation of 180° (either clockwise or counterclockwise) around the origin changes the position of a point (x, y) such that it becomes $(-x, -y)$.

Triangle ABC has vertices $A(1, 4)$, $B(4, 6)$ and $C(5, 2)$. It is rotated 180° counterclockwise to land on triangle $A'B'C'$, which has vertices $A'\ (-1, -4)$, $B'(-4, -6)$ and $C'(-5, -2)$.

A clockwise rotation of 180° for triangle ABC also results in triangle $A'B'C'$.

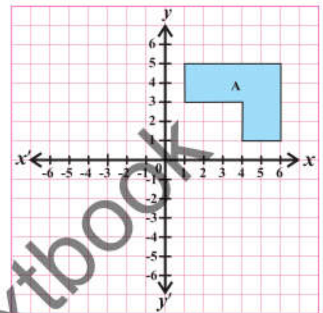


Exercise 4.5

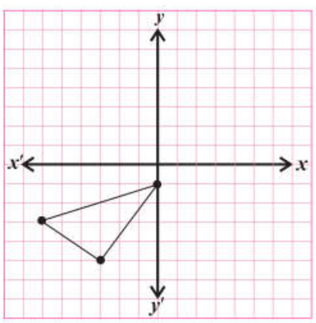
1. Rotate the shape shown by 90° clockwise.

Centre of Rotation

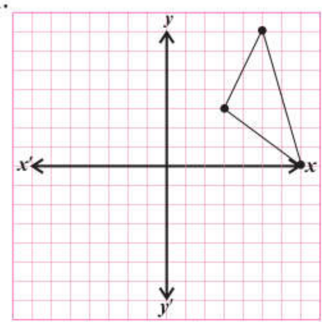
2. Rotate the shape A to 90º counterclockwise, with a centre of rotation at (1, 1).



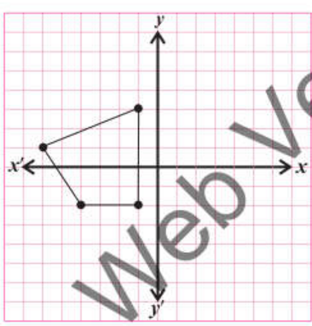
3. Rotate 180° about the origin.



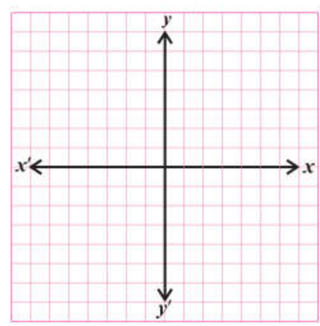
4. Rotate 90° counterclockwise about the origin.



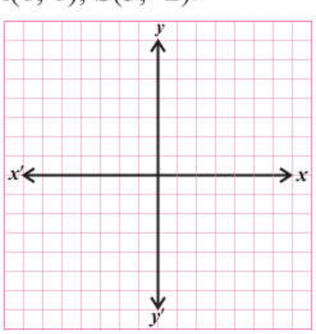
5. Rotate 90° clockwise about the origin.



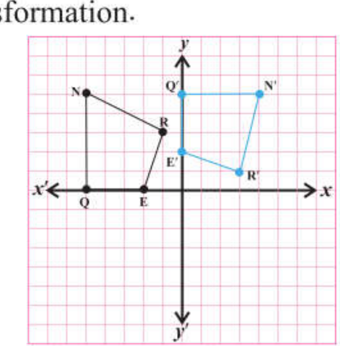
6. Rotate 90° clockwise about the origin A(1, –2), B(0, 2), C(3, 2), D(3, –3).



7. Rotate 180° about the origin P(–1, –5), Q(–1, 0), R(1, 1), S(3, –2).



8. Describe a rule that describe the transformation.



4.5 Enlargement

An enlargement is a type of transformation in which the image is resized to make it larger or smaller. The following are required to enlarge an image.

(i) Scale factor
(ii) Centre of enlargment

Keep in mind!
The image obtained by the enlargement is not congruent but similar.

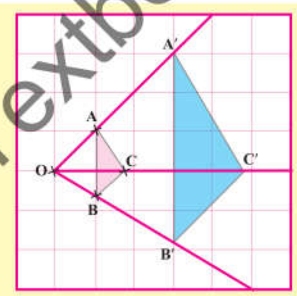
(i) Scale Factor

Scale factor is a number by which a quantity is multiplied, changing the magnitude of the quantity. Scale factor is usually denoted by k.

$$\text{Scale factor} = \frac{\text{Distance of image from centre}}{\text{Distance of object from centre}}$$

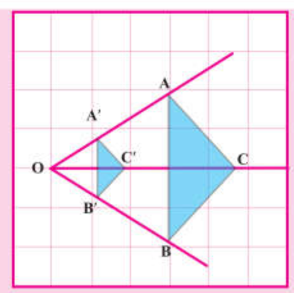
Case-I If $k > 1$

When scale factor is positive, the object and image lies on the same side of the centre. When scale factor is greater than 1, then the image is resized to make it larger. Triangle ABC is the object and triangle A′B′C′ is the image.



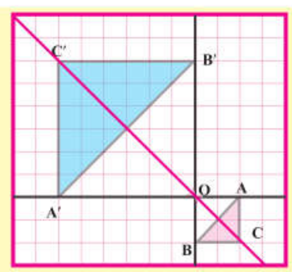
Case-II If $0 < k < 1$

When scale factor is between 0 and 1, then the image is resized to make it smaller. Trianlge ABC is the object and triangle $A'B'C'$ is the image.



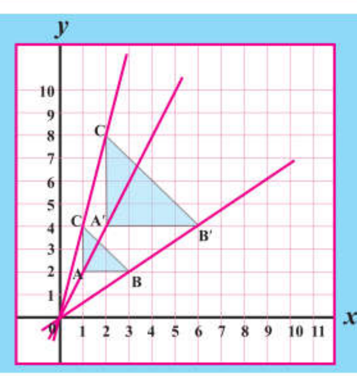
Case-III If $k < 0$

When scale factor is negative, the object and image lies on the opposite side of the centre of enlargement. Traingle ABC is the object and triangle $A'B'C'$ is the image.



(ii) Centre of enlargement

Centre of enlargement is a point which tells us where to draw an enlargement. In the given figure, the triangle $A'B'C'$ is enlarged by a scale factor 2 using the point (0,0) centre of enlargement.



Note!

When centre of enlargment is origin

$m\overline{OA'} = 2 \times m\overline{OA}$ A(1, 2) = 2 × A(1,2) = A′(2, 4)

$m\overline{OB'} = 2 \times m\overline{OB}$ B(3, 2) = 2 × B(3,2) = B′(6, 4)

$m\overline{OC'} = 2 \times m\overline{OC}$ C(1, 4) = 2 × C(1,4) = C′(2, 8)

Enlarge a Shape Using Positive Scale Factor

Example 21: Enlarge the triangle ABC by scale factor 2 with origin as the centre of enlargement.

Solution:

Step 1: Measure the distance from centre of enlargement to point A i.e.,(3,–1). Multiply the distance by scale factor 2. i.e., (6, –2), put point mark for the new point and label as A′.

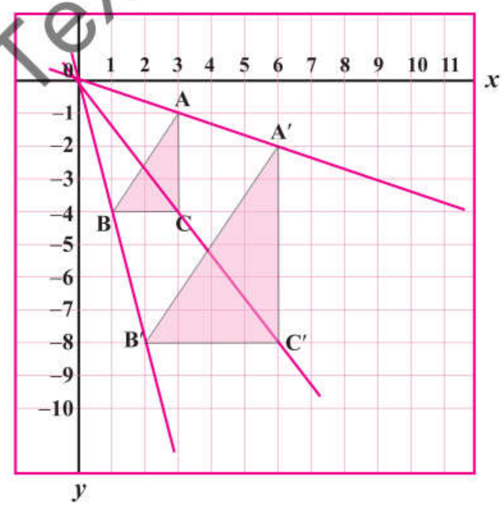
Step 2: Repeat the step-1 for B and C coordinates to put marks B′ and C′ respectively. e.g.,

B(1, –4) = B′(2, –8)

C(3, –4) = C′(6, –8)

Step 3: Draw rays from centre of enlargement (O) through point A to A′, B to B′, C to C′ and extend the line.

Step 4: Join up the points to make the new triangle A′B′C′.



Enlarge a Shape Using Negative Scale Factor

Example 22: Enlarge the square ABCD by scale factor (-2) using O(2,2) as the centre of enlargement.

Solution:

Step 1: Measure the distance from the centre of enlargement to point A i.e., (1,1). Multiply the distance by scale factor (-2), which is (–2, –2). Put point mark for the new point and label as A′ which is (–2, –2) away from the centre of enlargement.

Step 2: Repeat the step-1 for B, C and D coordinates to put points marks B′, C′ and D′ respectively.

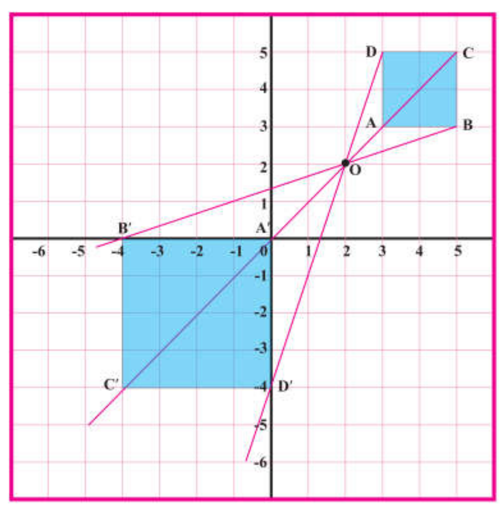
e.g., B(3, 1) = B′(–6, –2)

C(3, 3) = C′(–6, –6)

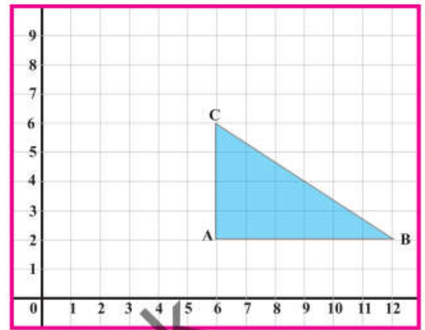
D(1, 3) = D′(–2, –6)

Step 3: Draw rays to join the pair of corresponding vertices through the centre of enlargement.

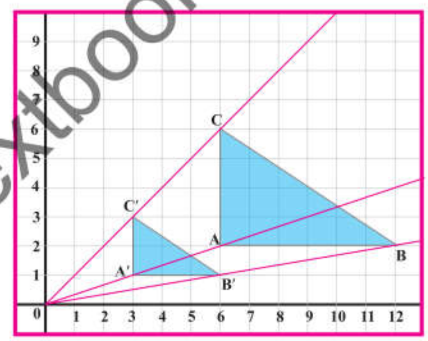
Step 4: Join the points to make the new square A', B', C'.



Enlarge a Shape Using Scale Factor (o<k<1)



Example 23: Enlarge the triangle ABC by scale factor $\frac{1}{2}$ about the point (O).



Solution: Follow all the steps as given in previous examples.

For mark A′ $\frac{1}{2}(6,2) = A'(3,1)$ $\quad$ $1/2(6,2) = A'(3,1)$

For mark B′ $\frac{1}{2}(12,2) = B'(6,1)$ $\quad$ $1/2(12,2) = B'(6,1)$

For mark C′ $\frac{1}{2}(6,6) = C'(3,3)$ $\quad$ $1/2(6,6) = C'(3,3)$

Join $A'B'C'$ as shown.

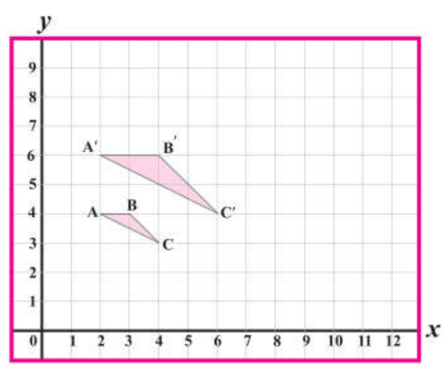
How to find a centre of enlargement

In order to find a centre of enlargement, follow these steps:

(i) Draw a ray through a pair of points (i.e, point and its image).

(ii) Draw other two rays.

(iii) Write down the coordinates of the centre of enlargement.



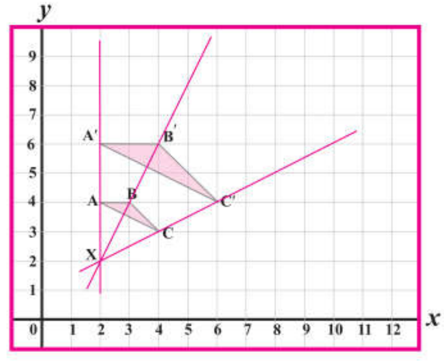
Example 24: Find a centre of enlargement and scale factor when triangle ABC has been enlarged to make triangle A′B′C′.

Solution:

(i) Find a pair of corresponding vertices and draw a ray going through the points.

(ii) Find more pair of corresponding vertices. Draw rays through the pair of points.

(iii) The points at which your rays meet will be the centre of enlargement. The centre of enlargement is (2, 2), mark it as X.

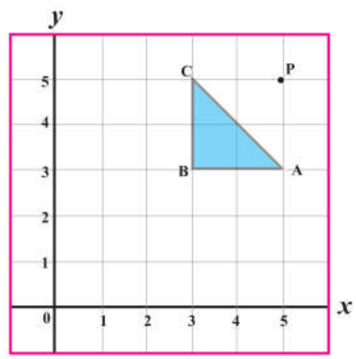


Scale factor of enlargement $= \frac{\overline{XA'}}{\overline{XA}} = \frac{4}{2} = 2$

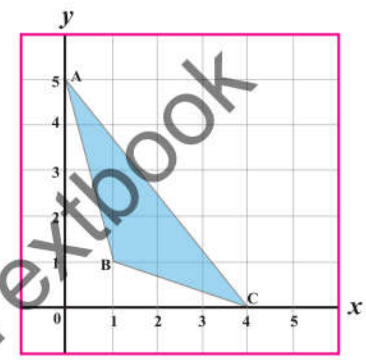
Exercise 4.6

Enlargement

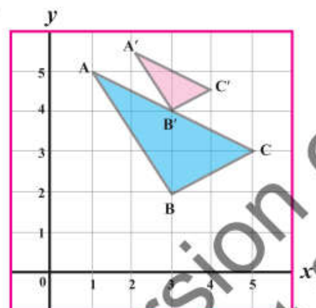
1. Enlarge the triangle ABC by scale factor 2 about the point P(5,5).



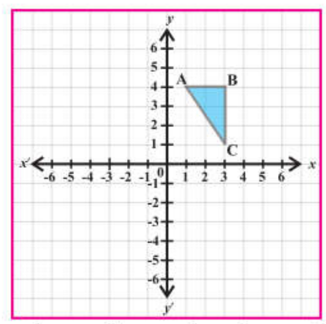
2. Enlarge the triangle ABC by scale factor $\frac{1}{2}$ about O.



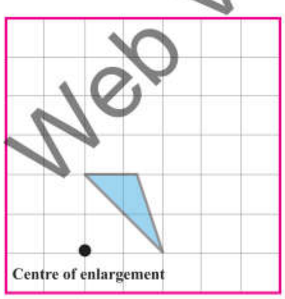
3. Find scale factor and centre of enlargement, Triangle $A'B'C'$ has been enlarged to make triangle ABC.



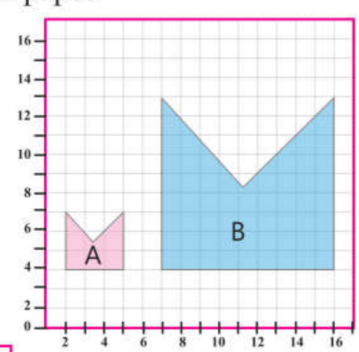
4. Enlarge the triangle ABC by scale factor (-2) about the origin.



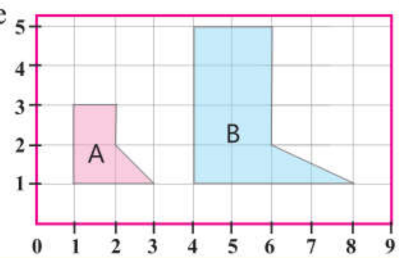
5. Enlarge the shaded shape with scale factor 3 about the point.



6. Shape A has been enlarged to make shape B. Calculate the scale factor. You can draw the shapes on graph paper.

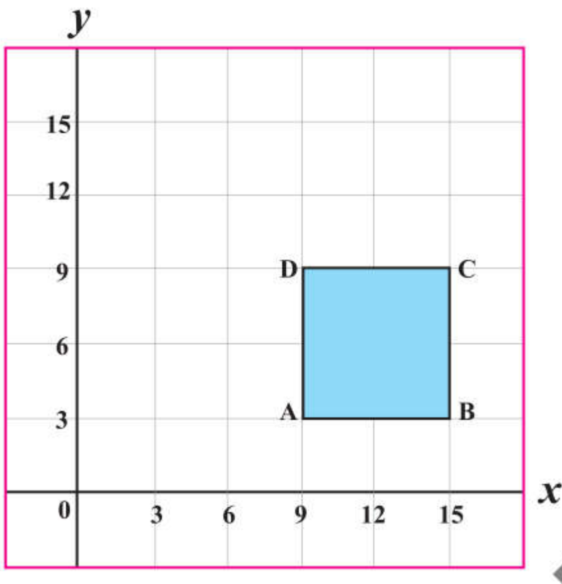


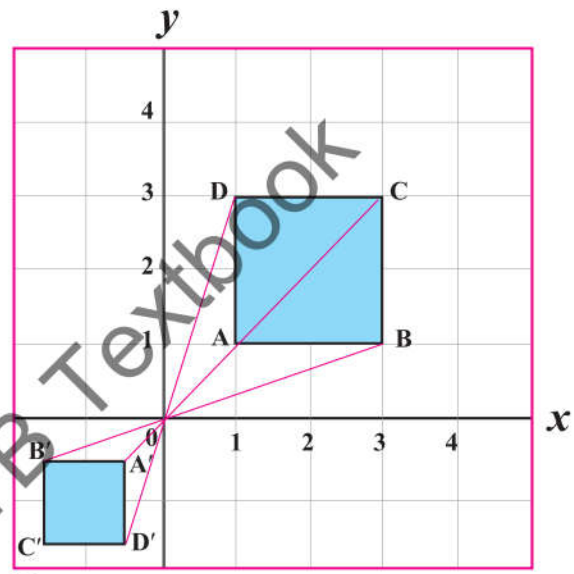
7. Shape A has been enlarged to make shape B. Calculate the scale factor.



8. Draw the shape on graph paper. Then enlarge the square ABCD by scale factor $-\frac{1}{3}$ about origin.

9. Find the centre of enlargement when square ABCD has been enlarged to make square A′B′C′D′.





SUMMARY

- Two shapes are said to be mathematically similar if all of the angles in the shapes are equal, but the shapes are not necessarily of the same size.
- Every triangle is congruent to itself (in the correspondence in which its sides and angles correspond to themselves). Such a congruence is called "Identity Congruence".
- The sum of the measures of any two sides of a triangle is always greater than the measure of its third side.
- The sum of measurement of angles of a triangle is 180^{o}.
- If measurements of any two angles of a triangle are known. Then the measurement of the third angle can be found by subtracting it. The sum of the two angles is subtracted from 180^{o}.
- **Rotation**
  Rotate an object and find the centre of rotation by construction
- **Enlargement**
  - Enlarge a figure with the given scale factor (positive or negative)
  - Locating the centre and scale factor of enlargement given the original figure and its enlargement
- A rotation turns each point on a preimage around a fixed point, called the centre of rotation, on a given angle measure.

Review Exercise 4

1. Four options are given against each statement. Encircle the correct one.

(i) If the objects and figures have exactly the same shapes and sizes then they are called:

(a) similar objects (b) congruent object
(c) transformation (d) rotation.

(ii) Every triangle is congruent to:

(a) congruent (b) similar
(c) rotation (d) itself

(iii) The sum of the measures of any two sides of a triangle is always ______ the measure of its third side.

(a) less than (b) greater than
(c) equal to (d) less than or equal to

(iv) The sum of interior angles of a triangle is:

(a) 60° (b) 90° (c) 180° (d) 360°

(v) In ________the size and shape of the figure does not change.

(a) transformation (b) rotation
(c) enlargement (d) triangle

(vi) A point which tells, where to draw an enlargement is called:

(a) rotation (b) centre of rotation
(c) centre of enlargement (d) transformation

(vii) A factor which is multiplied to change the magnitude of the quantity is called:

(a) transformation (b) scale factor
(c) enlargement (d) rotation

(viii) __________makes a shape larger or smaller.

(a) triangle (b) congruent
(c) rotation (d) enlargement

2. Define the following:

(i) Congruent figures (ii) Similar figures (iii) Transformation
(iv) Rotation (v) Enlargement

3. Construct the following triangles by using the given measurements:

(i) $4.5cm, 2cm, 8cm$ (ii) $5cm, 3cm, 4.5cm$
(iii) $7cm, 3.5cm, 3cm$ (iv) $8.5cm, 3.5cm, 5.5cm$

4. Construct the following triangles with the help of a ruler and a compass:

(i) ΔLMN, when $m\overline{LM} = 5cm, m\angle L = 60^\circ$ and $m\overline{NM} = 8cm$

(ii) ΔPQR, when $m\overline{MQ} = 5.7cm, m\overline{QR} = 3.5cm$ and $m\angle Q = 75^\circ$

5. Construct the following triangles with the help of compass and a ruler.

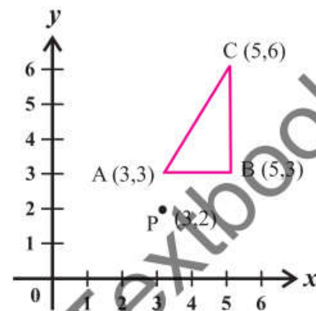
(i) $90^{\circ}, 60^{\circ}, 5.5cm$ (ii) $60^{\circ}, 45^{\circ}, 6.7cm$

6. Construct right angled triangles with the help of a compass:

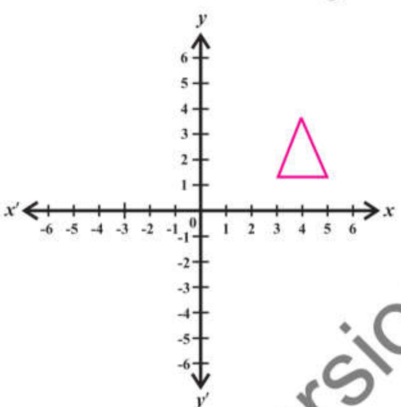
(i) Hypotenuse = $8cm$; Base = $4cm$

(ii) Hypotenuse = $7.8cm$; Altitude = $3.5cm$

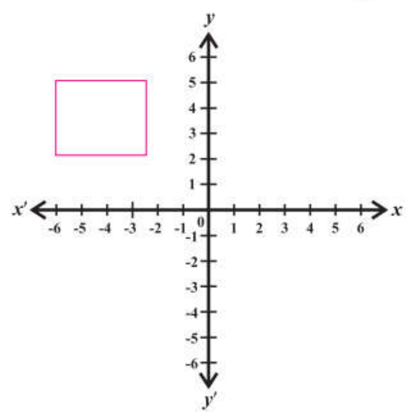
7. Rotate the shape 90° counterclockwise, with a centre of rotation at P(3,2):



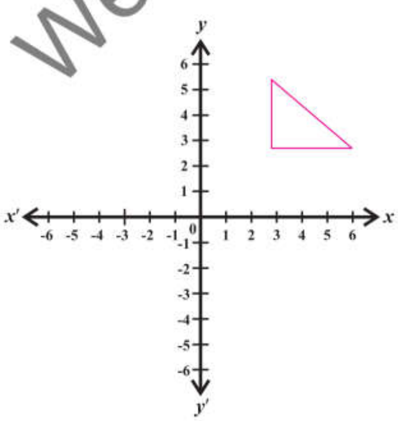
8. Rotate 180° about the origin:



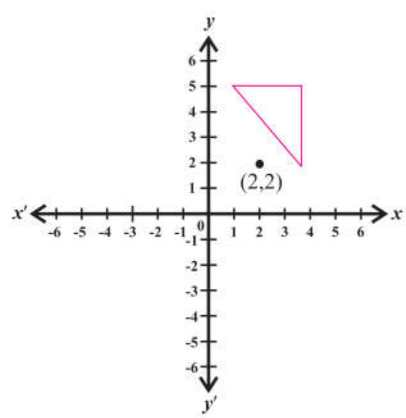
9. Rotate 90° clockwise about the origin:



10. Enlarge the triangle LMN by scale factor 2 about the point (6, 6)



11. Enlarge the shaded shape with scale factor (-3) about the point (2,2)



Domain 5 Statistics and Probability

Sub-Domain (i): Information Handling

Students' Learning Outcomes

After completing this domain, the students will be able to:

- recall difference between discrete and continuous data and grouped and ungrouped data
- reinforce representing the discrete data using suitable graph such as
  - ❖ line graph
  - ❖ bar graph
  - ❖ multiple bar graph
  - ❖ pie chart

Frequency Distribution

- construct cumulative frequency distribution
- represent frequency distribution by constructing:
  - ❖ histogram
  - ❖ frequency polygon

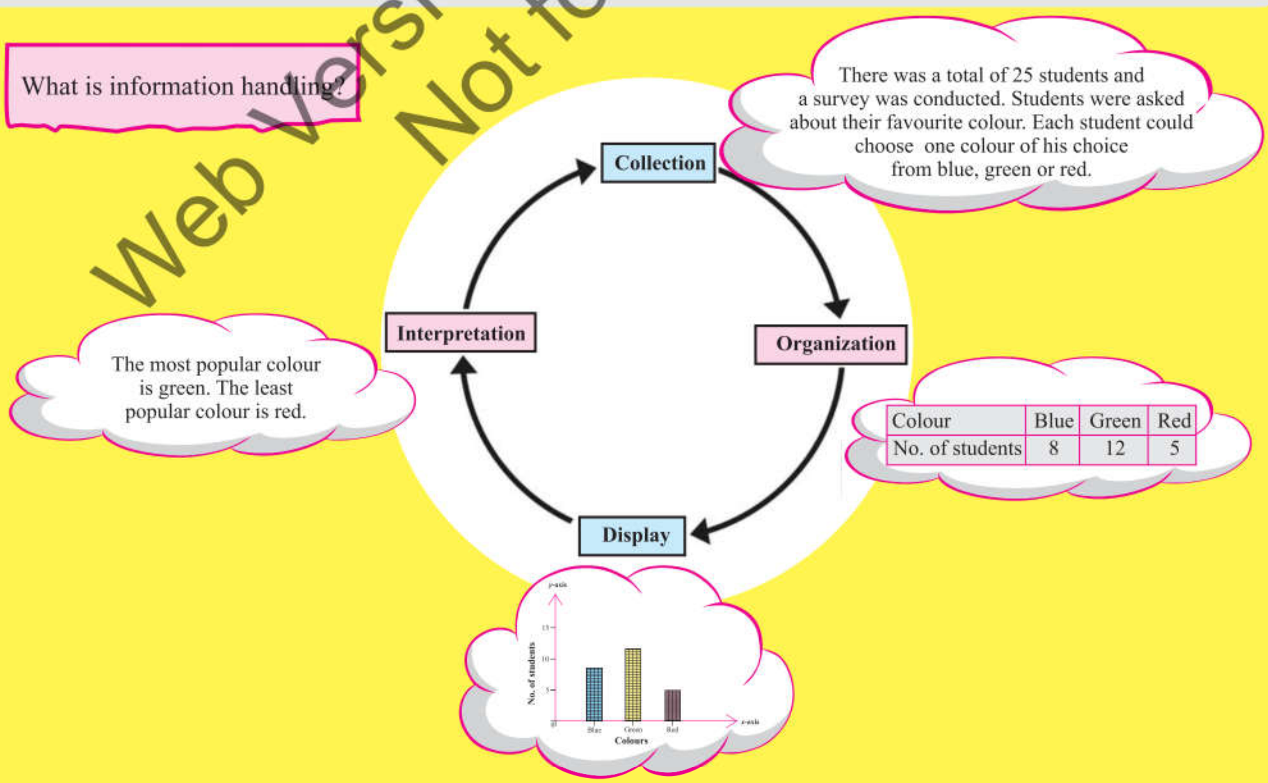
Measure of Central Tendency

- solve real life situations involving mean of grouped and ungrouped data

Advanced/ Additional

- calculate the median and mode for ungrouped data
- solve real life problems involving median and mode of grouped data

What is information handling?



Colour	Blue	Green	Red
No. of students	8	12	5

Data

The collection of information in the form of facts and figures is called data e.g., the height of 20 students of your class, and the marks obtained by the students of your class.

Keep in mind!
Discrete and continuous data is quantitative data.

Type of Data

Discrete data:
It can take only some specific values. Whole numbers are used to write the discrete data e.g.,

- 5 flowers
- 20 students
- 30 caps

Discrete data is further classified into two groups.

- Grouped data
- Ungrouped data

Continuous data:
It can take every possible value in a given interval, decimal numbers are used to write the continuous data. e.g.,

- 5.5 *kg*
- 15.5 litre
- 7.5 *cm*

Continuous data is also classified into two groups.

- Grouped data
- Ungrouped data

Ungrouped Data

Data which is not arranged in any systematic order (groups or classes) is called ungrouped data.

For example, The marks obtained out of 10 in a test by 20 students of a class

3, 10, 9, 8, 7, 6, 5, 4, 1, 8, 9, 5, 6, 7, 8, 5, 6, 7, 8, 5

Grouped Data

Data which is arranged in systematic order (groups or classes) is called grouped data.

For example, 30 students got admission in Grade-8. This data may be divided into 5 sections according to their scores. The detail is given below.

Scores	50-60	60-70	70-80	80-90	90-100
No of students	5	7	12	2	4

For the Teacher
Explain to the students, the difference between the discrete and continuous data (ungrouped and grouped) by using real life examples.

Difference Between Discrete and Continuous Grouped and Ungrouped Data

Discrete Ungrouped and Grouped Data

Let us consider the number of pair of shoes sold by the shopkeeper in a month

10, 11, 12, 10, 11, 14, 15, 13, 14, 10, 11, 11, 12, 13, 15, 15, 14, 10, 11, 13, 10, 10, 10, 14, 13, 15, 14, 14, 15, 10. The data is called discrete ungrouped data.

If we arrange the above given data in groups or classes, then it is called discrete grouped data.

Pair of shoes	Tally marks
10	𝍸 |||
11	𝍸
12	||
13	||||
14	𝍸 |
15	𝍸

Continuous Ungrouped and Grouped Data

Let us consider the distances (in km) of 20 employees from their residence to the factory.

3, 3.5, 4, 4.7, 9.5, 10, 15.5, 16, 11, 10.2, 8.5, 12, 13, 13.5, 7, 8.5, 8, 9, 10.2, 15. The data is called Continuous ungrouped data.

If we arrange the above given data in groups or classes, then it is called continuous grouped data.

Class limits	Tally marks
$3 \leq x < 6$	||||
$6 \leq x < 9$	||||
$9 \leq x < 12$	𝍸 |
$12 \leq x < 15$	|||
$15 \leq x < 17$	|||

Graphical Representation

Graph is another way to represent the data. In graphical representation, the data is shown briefly.

The following graphs are used to represent a data in a meaningful way:

- Line Graph
- Bar Graph
- Multiple Bar Graph
- Pie Chart.

i. Line Graph

Usama sold the following length of cloth (in metre) in five weeks.

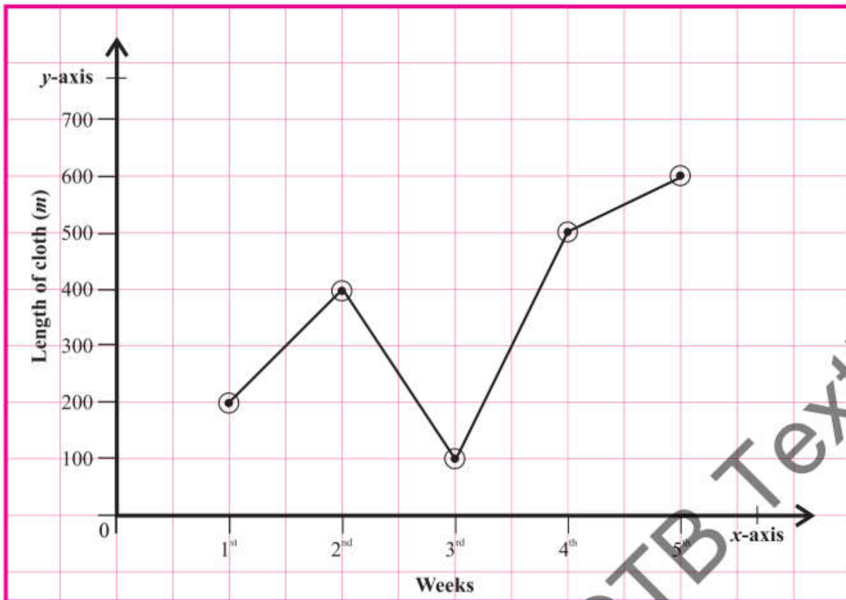
Weeks	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Length of cloth (*m*)	200	400	100	500	600

Key fact!

In line graph, we use line segments to represent the data and line graph is used for continuous data.

Now, we represent the above information by using a line graph.

Title Graph showing the length of cloth (*m*) sold by Usama in 5 weeks.



Scale

Along x-axis:
2 squares represent 1 week
Along y-axis:
1 squares represents 100 metre

Description:

- Usama sold maximum length of cloth on the 5th week.
- Usama sold the least length of cloth in the 3rd week.

ii. Bar Graph

The following data represents the number of students of a high school who passed matriculation examination in the first division during the years 2015-2021.

Years	*2015*	*2016*	*2017*	*2018*	*2019*	*2020*	*2021*
No. of Students	*100*	*150*	*130*	*120*	*145*	*160*	*110*

Now, draw a bar graph for the given data.

Title Graph showing number of students who passed examination in the first division during the years 2015-2021

Key facts!

- In bar graph, we use bars having same width.
- Discrete data is mostly represented by using bar graph.

Think!

In which year, the maximum number of students passed the examination?

Scale

On x-axis:
2 squares represent 1 year
On y-axis:
1 square represents 10 students

Keep in mind!

The height of each bar depends upon frequency.

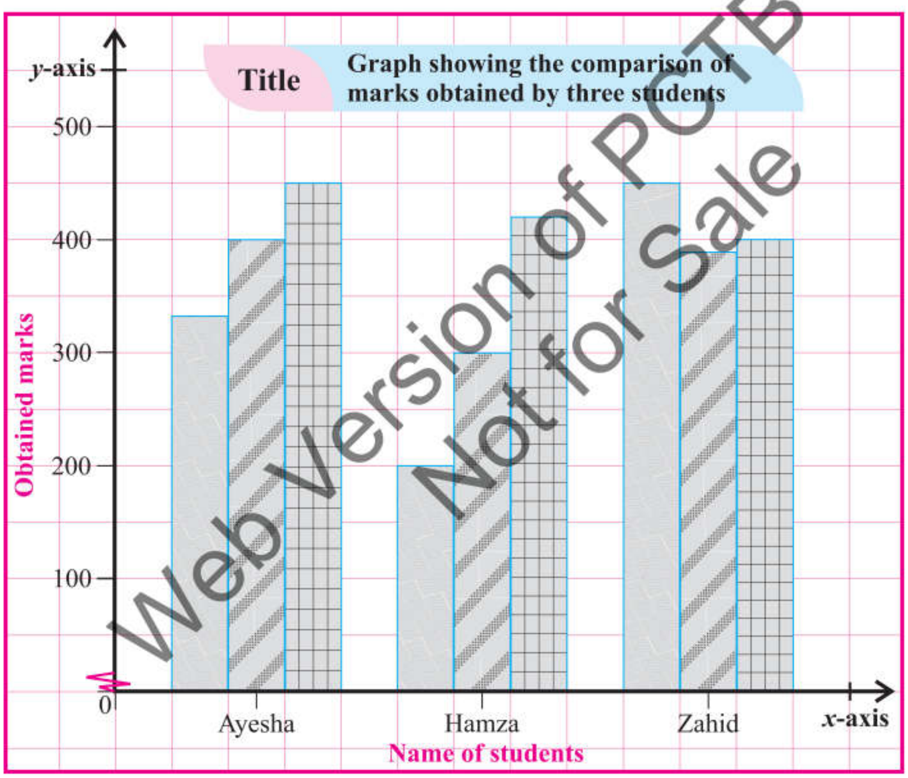
ii. Multiple Bar Graph

The following are the marks obtained out of 500 by three students in quarterly, half yearly and yearly examinations:

Multiple bar graph is an extension of bar graph. It is used for the comparison of two or more variable characteristics.

	Marks out of 500		
Name of Students	**Quarterly**	**Half Yearly**	**Yearly**
Ayesha	*330*	*400*	*450*
Hamza	*200*	*300*	*420*
Zahid	*450*	*390*	*400*

Now, draw a multiple bar graph for the given data.



Key: Quarterly Half yearly Yearly

Remember!
The given graph is called vertical multiple bar graph.

Scale
On *x*-axis:
4 squares represent 1 student
On *y*-axis:
1 square represents 50 students

Try yourself!
Multiple bar graph can also be drawn horizontally. Can you draw it horizontally?

iv. Pie Graph

The table given below shows the total number of students in six grades of a high school.

Pie chart is used for the comparison of values of different items by making the sectors of a circle. Angles are used to represent the sectors.

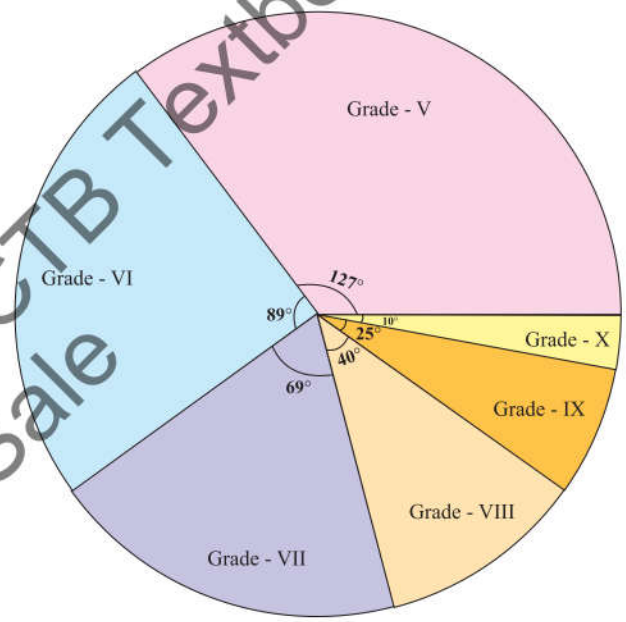
Classes	V	VI	VII	VIII	IX	X	Total
No. of students	300	210	162	95	60	23	850

Need to know!
Pie chart is also known as circular graph.

Now, we represent the data by using a pie chart.

Keep in mind!
Pie graph can be made clockwise or counter clockwise.

Grades	No of Students	Angle of the sectors
V	300	$\frac{300}{850}\times 360^0 = 127^0$
VII	210	$\frac{210}{850}\times 360^0 = 89^0$
VII	162	$\frac{162}{850}\times 360^0 = 69^0$
VIII	95	$\frac{95}{850}\times 360^0 = 40^0$
IX	60	$\frac{60}{850}\times 360^0 = 25^0$
X	23	$\frac{23}{850}\times 360^0 = 10^0$
Total	**850**	**360^0**



Exercise 5.1

1. The marks of a student "Zeeshan" in the grade 6 examinations were 50, 80, 60, 50, 85, 90. Show these marks by drawing an appropriate graph.

2. A shopkeeper sold the following number of toys in 6 weeks.

Weeks	I	II	III	IV	V	VI
No. of Toys	120	80	90	125	110	75

Represent the above information by a simple bar graph. Also tell:

(i) How many toys were sold by the shopkeeper altogether?

(ii) In which week the maximum and least number of toys were sold?

3. Draw an appropriate graph to show the points out of 100 scored by four students in three games.

Students	Game-I	Game-II	Game-III
Ali	72	60	85
Anam	48	50	75
Zara	80	30	60
Umer	50	80	50

4. The following are the runs made in a two days cricket match by the students in a high school. Draw an appropriate graph for the following data.

Name	Javed	Hamza	Husnain	Jamshaid	Arsalan
1^{st} day	55	60	35	80	65
2^{nd} day	90	25	45	50	30

5. Mrs. Imran purchased the following goods and services from a grocery store for her kitchen in the last month. Graph the data using a pie graph.

Goods and Services	Rice	Floor	Sugar	Soap	Oil	Miscellaneous
Expenditure (in Rs.)	600	500	300	150	800	250

6. A shirt making company prepared the following number of shirts with different colours. Draw a pie chart for the following data:

Colour	Red	Blue	Green	Black	White	Yellow
No of Shirts	50	60	35	40	70	80

7. A factory makes fertilizer and sold in 5 cities i.e., 15% sold in city A, 20% sold in city B, 25% sold in city C, 35% sold in city D and 5% sold in city E. Draw a suitable graph to represent this information.

8. The distribution of masses (Ibs) of 100 students is given. Draw a suitable graph to represent the data.

Midpoints of masses	114.5	124.5	134.5	144.5	154.5	164.5
Frequency	35	20	10	5	15	15

Frequency Distribution

Frequency

The number of times a value occurs in a data is called the **frequency** of that value.

Frequency Distribution

To write a data in the form of a table in such a way that the frequency of each class is observed at once is called frequency distribution.

Cumulative Frequency

Cumulative frequency is calculated by adding each frequency from a frequency distribution to the sum of its predecessors.

Construction of Frequency and Cumulative Frequency Distribution

The following are the masses in kg of 50 students selected from a school:

35, 30, 32, 36, 31, 40, 35, 42, 35, 45, 37, 41, 33, 37, 30, 28, 29, 30, 32, 33, 31, 35, 36, 30, 28, 37, 39, 28, 31, 34, 39, 45, 38, 36, 35, 28, 31, 34, 30, 41, 35, 36, 41, 28, 31, 34, 30, 29, 28, 37.

Now, we arrange the data in groups or classes or the form of table as below:

i. Look for the largest and smallest value i.e., 45 and 28 respectively.

ii. Number of classes to be made is 6.

iii. To find out the size of class interval use the given formula.

Keep in mind!

The table below, is called frequency and cumulative frequency distribution.

$$\text{Size of class interval} = \frac{\text{largest value} - \text{smallest value}}{\text{number of classes}}$$

$$= \frac{45 - 28}{6} = \frac{17}{6} \cong 2.8 \approx 3$$

Class Limits	Tally Marks	Frequency (f)	Cumulative Frequency ($c.f.$)
28 – 30	~~||||~~ ~~||||~~ ||||	14	14
31 – 33	~~||||~~ ||||	9	9 + 14 = 23
34 – 36	~~||||~~ ~~||||~~ |||	13	13 + 23 = 36
37 – 39	~~||||~~ ||	7	7 + 36 = 43
40 – 42	~~||||~~	5	5+ 43 = 48
43 – 45	||	2	2 + 48 = 50
Total	———	**50**	

Example 1: Listed below are the scores of 50 students in a 60 marks test.

25, 33, 26.5, 34, 28, 35, 29, 36, 30, 54, 30, 39, 36, 37, 39, 40.5, 37, 34, 27, 41, 37, 41, 38, 42.5, 48, 51, 40, 51, 43, 40, 41, 39, 48, 51, 53, 41, 37.5, 52, 28, 46, 44,37, 39, 52, 51.5, 40, 45, 46, 43, 53

Make a frequency distribution table taking 6 classes of equal size by tally marks. Also make a cumulative frequency distribution.

Solution:

The lowest value = 25

The highest value = 54

Total classes to be made = 6

We take the size of class $= \dfrac{54-25}{6}$

$= \dfrac{29}{6} = 5$ *(approx.)*

Class Limits	Tally Marks	Frequency	Cumulative Frequency
25 − 29	~~IIII~~ I	*6*	*6*
30 − 34	~~IIII~~	*5*	*5 + 6 = 11*
35 − 39	~~IIII~~ ~~IIII~~ III	*13*	*13 + 11 = 24*
40 − 44	~~IIII~~ ~~IIII~~ II	*12*	*12 + 24 = 36*
45 − 49	~~IIII~~	*5*	*5 + 36 = 41*
50 − 54	~~IIII~~ IIII	*9*	*9 + 41 = 50*
	Total:	***50***	

Graphs of Frequency Distribution

The following are the type of graphs which can be used to represent a frequency distribution on a graph.

i. Histogram ii. Frequency polygon

Construction of Histogram

We are familiar with pie and bar graphs. Another common graphic way of representing data is by means of a histogram. A histogram is similar to bar graph but it is constructed for a frequency table.

In a histogram, the values of the data (classes) are represented along the horizontal axis and the frequencies are shown by bars perpendicular to the horizontal axis. Bars of equal width are used to represent individual classes of frequency table.

To draw a histogram from a grouped data, the following steps are followed:

(i) Draw *X-axis* and *Y-axis*.

(ii) Mark class boundaries of the classes along *X-axis*.

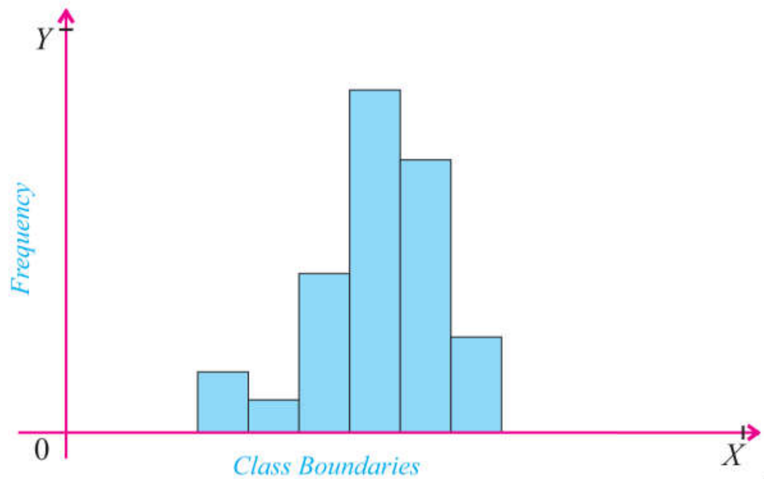
(iii) Mark frequencies along *Y-axis*.

(iv) Draw a bar for each class so that the height of the bar drawn for each class is equal to the frequency of the class.

How to calculate class boundaries?

- Lower class boundaries are obtained by "minus on-half" from the lower class limits.
- Upper class boundaries are obtained by "plus one half" to the upper limits.

The graph is shown below:



> **Key fact!**
> Continuous data is mostly represented by using histogram and frequency polygon.

Construction of Frequency Polygon

To draw a frequency polygon from a grouped data, the following steps are followed:

i. Draw X-axis and Y-axis.

ii. Take midpoints on X-axis and class frequency on Y-axis.

iii. Put a dot mark against each midpoint of the class limits corresponding to its class frequency.

iv. Join all the dots by a line segment to get the required frequency polygon.

The graph is shown below.

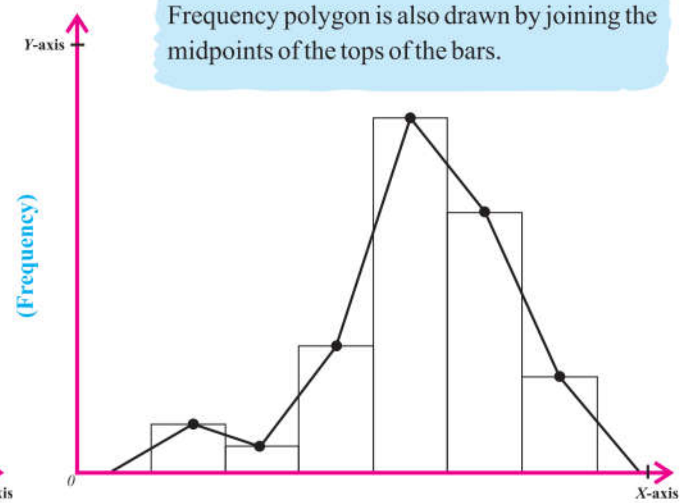
> **Keep in mind!**
> Midpoint is obtained by finding out the average of class limits, e.g., $\frac{\text{Lower limit} + \text{Upper limit}}{2}$

> **Do you know?**
> Midpoints are also known as class marks.

Frequency polygon is also drawn by joining the midpoints of the tops of the bars.

Midpoints (x)

Frequency Polygon



Class Boundaries (C.B)

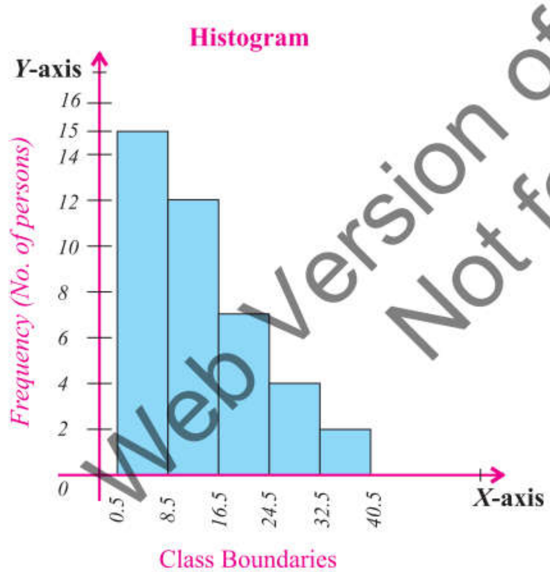
Frequency Polygon on Histogram

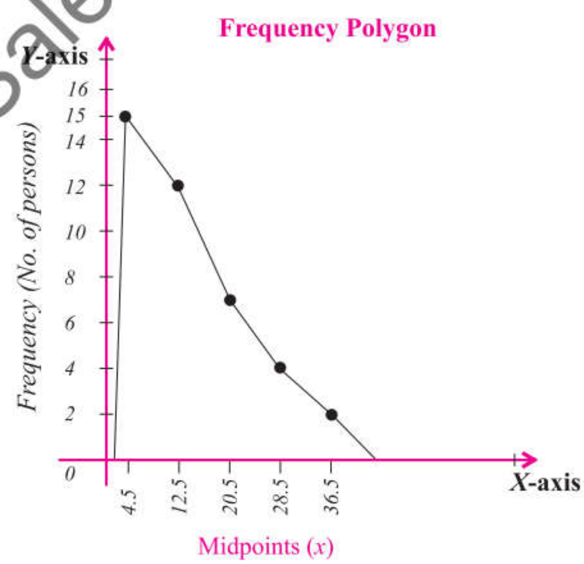
Example 2: The detail of distances travelled daily by the residents of a locality are given below. Construct a histogram and frequency polygon for the following frequency table.

Distance travelled (in *km*)	1-8	9-16	17-24	25-32	33-40
Number of persons	15	12	7	4	2

Solution: Frequency distribution table is:

Distance travelled (km)	Frequency (No. of Persons)	Class boundaries	Midpoints (x)
1-8	15	0.5 - 8.5	$\frac{1+8}{2} = 4.5$
9-16	12	8.5 - 16.5	$\frac{9+16}{2} = 12.5$
17-24	7	16.5 - 24.5	$\frac{17+24}{2} = 20.5$
25-32	4	24.5 - 32.5	$\frac{25+32}{2} = 28.5$
33-40	2	32.5 - 40.5	$\frac{33+40}{2} = 36.5$
Total:	**40**		





Exercise 5.2

1. The following data displays the number of draws of different categories of bonds.
35, 55, 64, 70, 99, 89, 87, 65, 67, 38, 62, 60, 70, 78, 69, 86, 39, 71, 56, 75, 51, 99, 68, 95, 86, 53, 59, 50, 47, 55, 81, 80, 98, 51, 63, 66, 79, 85, 83, 70
Construct a frequency distribution and cumulative frequency distribution for the above data, with seven classes of equal size of class interval 10.

2. Listed below are the number of electricity units consumed by *50* households in a low income group in a locality of Lahore.

55, 45, 64,130, 66,155, 80,102, 62, 60,101, 58, 75, 81, 111, 90, 55,151, 66, 139, 77, 99, 67, 51, 50, 125, 83, 55,136, 91, 86, 54, 78,100, 113, 93, 104, 111, 113, 96, 96, 87,109, 94,129, 99, 69, 83, 97, 97

With *12* classes of equal width of *10,* construct a frequency and cumulative frequency distribution for the electricity units consumed.

3. The following list, is of scores in a mathematics examination. Using the starting class *40 - 44* , set up a frequency and cumulative frequency distribution. List the class boundaries.

63, 88, 79, 92, 86, 87, 83, 78, 40, 67, 68, 76, 46, 81, 92, 77, 84, 76, 70, 66, 77, 75, 98, 81, 82, 81, 87, 78, 70, 60, 94, 79,52,82, 77,81,

4. Construct a frequency and cumulative frequency distribution for the following numbers using *1-10* as the starting class. List the class boundaries.

54, 67, 63, 64, 57, 56, 55, 53, 53, 54, 44, 45, 45, 46, 47, 37, 23, 34, 44,27, 36, 45, 34, 36,15, 23, 43,16, 44, 34, 36, 35, 37, 24, 24,14, 43, 37, 27, 36, 33, 25, 36, 26, 5, 44, 13, 33, 33, 17

5. Following are the number of days that 36 tourists stayed in Hunza valley.

1, 6, 16, 21, 41, 21, 5, 31, 20, 27, 17, 10, 3, 32, 2, 48, 8, 12, 21, 44, 1, 36, 5, 12, 3, 13, 15, 10, 18, 3, 1, 11, 14, 12, 64, 10.

Construct a frequency and cumulative frequency distribution starting with the class *1-7.* Also list the class boundaries.

6. Construct a histogram and frequency polygon for each of the frequency tables in questions *1-5.*

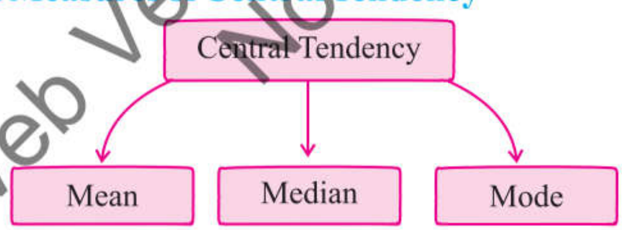
Measures of Central Tendency

Measures of Central Tendency help us to find out the centre of the data or distribution.

Key fact!

The measure that gives the centre of the data is called Measure of Central Tendency.

Description of Measures of Central Tendency



Calculation of Measures of Central Tendency

Mean (Average) for Ungrouped Data

Let $x_1, x_2, \dots, x_n$ be n given quantities. Then their average is the value representing the tendency of these quantities and is called their Mean value or Arithmetic Mean. It can be calculated by the formula:

Key fact!

The average is calculated by adding all the values and then dividing it by the total number of values. It is denoted by $\bar{X}$.

$$\bar{X} = \frac{x_1 + x_2 + \dots, + x_n}{N} = \frac{\Sigma X}{N} \quad \text{(Direct method)}$$

$$\bar{X} = \frac{\text{sum of all values}}{\text{number of values}}$$

Example 3 The scores of a student in eight papers are ***58, 72,65,85,94, 78,87,85.*** Find the mean score.

$$\bar{X} = \frac{58+72+65+85+94+78+87+85}{8}$$

$$\bar{X} = \frac{624}{8} = 78$$

Hence, mean score is ***78***

Mean for Grouped Data

Let $x_1, x_2, x_3, \ldots x_n$ be the midpoints of the class limits with corresponding frequencies say $f_1, f_2, f_3 \ldots f_n$. Then the arithmetic mean is obtained by dividing the sum of the products of f and x by the total of all the frequencies.

Example 4: Given below is a frequency distribution of the masses in kg of 130 students. Find the mean.

Masses (kg)	*40-44*	*44-48*	*48-52*	*52-56*	*56-60*	*60-64*
Frequency	*5*	*13*	*45*	*32*	*26*	*9*

Solution: To calculate the mean of masses, we have to find out the midpoint of each class.

Masses (kg)	Frequency (f)	Midpoints(x)	fx
40-44	*5*	*42*	*210*
44-48	*13*	*46*	*598*
48-52	*45*	*50*	*2250*
52-56	*32*	*54*	*1728*
56-60	*26*	*58*	*1508*
60-64	*9*	*62*	*558*
	$\sum f = 130$	—	$\sum fx = 6852$

Mean (for grouped data) $\bar{X} = \dfrac{\sum fx}{\sum f}$

$$\bar{X} = \frac{6852}{130}$$

$$\bar{X} = 52.707 \approx 52.71\text{kg}$$

Short Formula for Computing Arithmetic Mean

The computation of arithmetic mean, using direct method for ungrouped data as well as for grouped data is no doubt easy for small values of x and f. But when the values of x and f become very large, it becomes difficult to deal with the questions. So, to minimize our time and calculations, we take deviations from an assumed or provisional mean. Let A be considered as the provisional mean, (may be any value from the values of x or any number) and D denotes the deviation of X from A i.e., $D = X - A$. For $X = D + A$, the formula of arithmetic mean becomes:

$$\bar{X} = A + \frac{\Sigma D}{N} \qquad \text{(For ungrouped data)} \qquad (1)$$

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f} \qquad \text{(For grouped data)} \qquad (2)$$

Example 5: Find the arithmetic mean using short formula for the runs made by a batsman Saad in 8 one-day matches. **Runs:** 40, 45, 50, 52, 50, 60, 56, 70.

Solution: Taking deviations from A = 52

X	40	45	50	52	50	60	56	70	
$D = X - A$	−12	−7	−2	0	−2	8	4	18	$\Sigma D = 7$

Now $\Sigma D = 7$

$\because \quad \bar{X} = A + \dfrac{\Sigma D}{N}$

$\therefore \quad \bar{X} = 52 + \dfrac{7}{8} = 52 + 0.875 = 52.88$ or 53 runs

Example 6: Deviations from 12.5 of ten different values are 6, –2, 3.5, 9, 8.7, –5.5, 14, 11.3, –6.8, –4.2. Find the arithmetic mean.

Solution: Deviations from 12.5 are:

$D = 6, -2, 3.5, 9, 8.7, -5.5, 14, 11.3, -6.8, -4.2$

Now, $\Sigma D = 34$. Also, $A = 12.5$, using the formula (1) we have,

$$\bar{X} = A + \frac{\Sigma D}{n} = 12.5 + \frac{34}{10}$$

or $\quad \bar{X} = 12.5 + 3.4 = 15.9$

Example 7: The heights (in inches) of 200 school boys are recorded in the following frequency distribution. Find the mean height by short formula.

Height (inches) X	51	52	53	54	55	56	57	58	59	60
Frequency (f)	2	5	8	24	55	45	38	16	6	1

Solution:

Height (inches) X	Frequency (f)	$A = 55$ $D = X - A$	fD
51	2	−4	−8
52	5	−3	−15
53	8	−2	−16
54	24	−1	−24
A←55	55	0	0
56	45	1	45
57	38	2	76
58	16	3	48
59	6	4	12
60	1	5	5
Total	$\Sigma f = 200$		$\Sigma fD = 135$

Now using the formula (2), we get

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f}$$

or $\quad \bar{X} = 55 + \frac{135}{200}$

or $\quad \bar{X} = 55 + 0.675$

$\therefore \quad \bar{X} = 55.68$ inches approx.

Coding Method of Finding Arithmetic Mean

In fact, this method is applicable when all the class intervals h are of equal size, and hence, the calculation of arithmetic mean becomes more easy.

Now applying the coding variable U, stated as

$$U = \frac{X - A}{h}$$

or $\quad hU = X - A$

or $\quad hU + A = X$

Where A is an assumed mean and h is the class interval size.

The formula of Arithmetic mean becomes:

$$\bar{X} = A + \frac{\Sigma U}{N} \times h \quad \text{(for ungrouped data)} \qquad (3)$$

$$\bar{X} = A + \frac{\Sigma fU}{\Sigma f} \times h \quad \text{(for grouped data)} \qquad (4)$$

Example 8: Five workers do their job at a big general store. Their daily wages are Rs. 300, 350, 400, 450, 500. Calculate the mean wage by coding method.

Solution: By using the formula (3), we have

$$\bar{X} = A + \frac{\sum U}{N} \times h$$

Here, let $A = 450$, and $h = 50$. Then

X	300	350	400	450	500	---
$U = \frac{X - A}{h}$	-3	-2	-1	0	1	$\sum U = -5$

$$\bar{X} = A + \frac{\sum U}{n} \times h$$

or $$\bar{X} = 450 + \frac{-5}{5} \times 50$$

$\therefore$ $$\bar{X} = 450 - 50 = \text{Rs. } 400$$

Example 9: Given below is a frequency distribution of the masses in kg of 130 students. Find the mean of masses using coding method.

Mass (kg)	40-44	44-48	48-52	52-56	56-60	60-64
Frequency	5	13	45	32	26	9

Solution:

Mass (kg)	**Frequency (f)**	X	$U = \frac{X - A}{h}$	fU
40-44	5	42	-2	-10
44-48	13	46	-1	-13
48-52	45	50 (A)	0	0
52-56	32	54	$+1$	32
56-60	26	58	$+2$	52
60-64	9	62	$+3$	27
---	$\sum f = 130$	--	--	$\sum fU = 88$

Now using the formula (4), we get

$$\bar{X} = A + \frac{\sum fU}{\sum f} \times h$$

Taking $A = 50$ and $h = 4$, we have

$$\bar{X} = 50 + \frac{88}{130} \times 4$$

or $\bar{X} = 50 + 2.71$ or $\bar{X} = 52.71\, kg$

Example 10: For the following information, find the arithmetic mean.

(i) If $D = X - 10, \Sigma D = 50$ and $n = 5$

(ii) If $U = \dfrac{X-15}{3}$, $\Sigma U = 25$ and $n = 5$

(iii) If $D = X - 100, \Sigma fD = 200.\quad \Sigma f = 50$

(iv) If $U = \dfrac{X-100}{5}$, $\Sigma fU = 240$ and $\Sigma f = 100$

Solution: (i) Since $D = X - A$, therefore, $A = 10$

$$\text{Hence , } \bar{X} = A + \frac{\Sigma D}{N} = 10 + \frac{50}{5} = 20$$

(ii) Since $U = \dfrac{X-A}{h}$ therefore, $A = 15$ and $h = 3$

$$\text{Hence } \bar{X} = A + \frac{\Sigma U}{N} \times h = 15 + \frac{25}{5} \times 3 = 15 + 15 = 30$$

(iii) Since $D = X - A$, therefore, $A = 100$

$$\text{Hence } \bar{X} = A + \frac{\Sigma fD}{\Sigma f} = 100 + \frac{200}{50} = 104$$

(iv) Since $U = \dfrac{X-A}{h}$ therefore, $A = 100$ and $h = 5$

$$\text{Hence } \bar{X} = A + \frac{\Sigma fU}{f} \times h = 100 + \frac{240}{100} \times 5$$

$$\therefore \quad \bar{X} = 100 + 12 = 112$$

Remember!

Merits of mean
- It is the most simple and reliable form of an average.
- It is easy to find and understand.
- We need only the sum of all observations and number of observations.

Demerits of mean
- It is highly affected by extreme values.
- It provides a high value on having one very large value in the data.

Median for Ungrouped Data

If a data is arranged in ascending or descending order, then median of the data is:

(a) the middle value of the data, if it consists of odd number of values.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{item} \qquad \text{(when } n \text{ is odd number)}$$

(b) the mean of the two middle values of the data, if the number of values in a data is even.

$$\text{Median} = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{item} + \left(\frac{n+2}{2}\right)^{th} \text{item}\right) \qquad \text{(when } n \text{ is even number)}$$

Example 11: The masses in kg of 9 students are as under. Find the median:

29,32,45,27,30,47,35,37,33

Solution: Arranging these values in descending order:

47, 45, 37, 35, 33, 32, 30, 29, 27

The middle value is *33*

So, Median = *33* Kg

$$\left(\frac{9+1}{2} = 5^{th} \text{ item}\right)$$

Try yourself!

Calculate the median when n is even number.

Median for Grouped Data

The median for grouped data is obtained by using the following formula:

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Where, ℓ = the lower class boundary of median class.
h = the size of class of median class.
f = the frequency of the median class.
$n = \sum f$ = Total frequency
c = cumulative frequence of preceding class of the median class.

Keep in mind!

To calculate the median, we need:

- class boundaries
- cumulative frequency
- locate median class $\frac{n}{2}^{\text{th}}$ value in *c.f.*
- underline the median class for taking the values of *f* and *h*.
- It is denoted by $\tilde{X}$

Example 12: Following are the masses (in pounds) of 50 students. Find the median of masses.

Mass (1bs)	110-114	115-119	120-124	125-129	130-134
No of students	5	12	23	6	4

Solution:

Mass (1bs)	Frequency	Class Boundaries	Cumulative Frequency (c. f)
110-114	5	109.5-114.5	5
115-119	12	114.5-119.5	5+12=17
120-124	23	119.5-124.5	17+23=40 (Median class)
125-129	6	124.5-129.5	40+6=46
130-134	4	129.5-134.5	46+4=50
	$\sum f = 50$		$\sum f = 6852$

Here, $n = 50$ so, $\frac{n}{2} = \frac{50}{2} = 25$; 25th item lies in $119.5 - 124.5$

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 119.5 + \frac{5}{23}(25 - 17) \text{ (after putting the values)}$$

$$= 119.5 + \frac{40}{23} = 119.5 + 1.74$$

$$\text{Median } (\tilde{X}) = 121.24 \text{ 1bs}$$

Remember!

Merits of Median

- It is easy to find.
- It is not affected by extreme values.

Demerits of Median

- It cannot be calculated unless the data is arranged in some order.
- It is not based on all items.

Mode for Ungrouped Data

Mode is the value that occurs most frequently in a data . In case no value is repeated in a data, then the data has no mode. If two or more values occur with the same greatest frequency, then each is a mode. It is denoted by $(\hat{X})$.

Example 13: Find the mode of the given data:
12,5,7,8,2,2,4,3,5,7, 2

Solution: The value *2* is repeated the most, so *2* is the mode of this data.

Example 14: Find the mode of the given data:
2,4, 6,8,10,12,14,16,20

Solution: This data has no mode because no value is repeated in the given data.

Remember!
(i) A data can have more than one Mode.
(ii) A data may or may not have a Mode.

Example 15: Find the mode of the data given below:
1,2,2,2,3,4,5,5,5,6,7

Solution: Since **2** is repeated 3 times and 5 is also repeated 3 times so this data has two modes i.e., **2** and **5.**

Remember!
Merits of Mode
- In certain cases mode can be located in the data.
- It is the most common value.

Demerits of Median
- Sometimes, data do not contain any mode.
- Data set may contain more than one mode. In this situation, mode is not most sitable measure.

Mode for Grouped Data

The mode is calculated by using the following formula:

$$\text{Mode }(\hat{X}) = \ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

ℓ = the lower class boundary of modal class
f_m = the frequency of the modal class
f_1 = the frequency preceding the modal class
f_2 = the frequency following the modal class
h = the size of modal class.

Example 16: Following are the heights in (inches) of 40 students in class-8.

Heights (inches)	*48-50*	*50-52*	*52-54*	*54-56*	*56-58*	*58-60*
No of students	*5*	*7*	*10*	*9*	*6*	*3*

Find out the mode of the above given data.

Solution:

Heights (inches)	Frequency
48-50	*5*
50 52	*7*
52-54	*10*
54-56	*9*
56-58	*6*
58-60	*3*
Total	

Model Class

We only assume the class with the highest frequency as the model class.

$$\text{Mode }(\hat{X}) = \ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$\text{Mode }(\hat{X}) = 52 + \frac{10 - 7}{(10 - 7) + (10 - 9)} \times 2$$

$$\text{Mode }(\hat{X}) = 52 + \frac{3}{3 + 1} \times 2 = 52 + 1.5 = 53.5 \text{ inches}$$

Exercise 5.3

1. What is the mean? Explain the methods of calculation of mean for ungrouped data.
2. Compute mean by direct and short method of the following data:
   (i) 10, 8, 6, 0, 8, 3, 2, 5, 8, 4
   (ii) 1, 3, 5, 3, 5, 3, 7, 5, 7, 5, 7
   (iii) 5, 4, 1, 4, 0, 3, 4, 11, 9
   (iv) 5, 9, 8, 6, 8, 7, 6, 7, 9, 8, 7, 9, 8, 7, 9, 7, 8, 5
   (v) –5, 4, 4, 0, –2, –3, –7, –10, 8, 9, –8
3. Calculate the mean from the following information:
   (i) If $D = x - 140$. $\Sigma D = 500$, and $n = 10$
   (ii) If $U = \dfrac{x-130}{6}$, $\Sigma U = -500$, and $n = 15$
   (iii) If $U = \dfrac{x-130}{6}$, $\Sigma U = 60$, and $n = 100$.
4. (i) The mean of 50 numbers is 75. Find their sum.
   (ii) The mean of 4, 5, 10, 9, 8, 11, 9, 11, 5, 7 and z is 8.8 find the value of z.
5. The deviations from 18.5 of 10 different values are 10.2, –17.5, –11.25, 15.50, 14.28, –13.75, 14.5, –12.50, 13.50 and 12.50. Calculate the mean.
6. The mean mass of 10 students is 30.5 *kg*. The masses of 9 students are 25 kg, 29 *kg*, 28 *kg*, 32 *kg*, 33.5 *kg*, 30.5 *kg*, 31.7 *kg*, 30.7 kg and 31.5 *kg*. What is the mass of 10th student?
7. The mean height of 10 women is 1.5 *m*. Find the total height of 10 women.
8. The mean length of 5 ribbons is 15.5 *m*. The mean length of 4 ribbons is 14.75 *m*. Find the length of 5th ribbon
9. Define and explain the method of calculation of mean, median and mode for ungrouped data.
10. Find mean, median and mode for the following data:
    (i) 62, 90, 71, 83, 75
    (ii) 45, 65, 80, 92, 80, 73, 56, 96, 62, 78
    (iii) Number of letters in the first 20 words in a book. 3, 2, 5, 3, 3, 2, 3, 3, 2, 4, 2, 2, 3, 2, 3, 5, 3, 4, 4, 5
    (iv) The number of calories in nine different beverages of 250 mm bottles:
    99, 106, 101, 103, 108, 107, 107, 106, 108
11. The following distances (in km) were travelled by 40 students to reach their school.
    2, 8, 1, 5, 9, 5, 14, 10, 31, 20, 15, 4, 10, 6, 5, 10, 5, 18, 12, 25, 30, 27, 20, 3, 9, 15, 15, 18, 10, 1, 1, 6, 25, 16, 7, 12, 1, 8, 21, 12.
    Compute the mean, median and mode of the distances travelled.
12. Explain the method of calculation of mean, median and mode for grouped data.
13. Compute the mean and mode of the following table:

Size of Family	2	3	4	5	6	7	8
Frequency	51	31	27	12	4	1	1

14. The following is distribution of marks obtained by the class-VIII in a test. Find the mean, median and mode. Also verify $\bar{x} > \tilde{x} > \hat{x}$.

Marks/midpoints (x)	5	6	7	8	9	10
Frequency	15	16	18	10	9	5

15. Find the class mark, mean (by using direct and coding method), median and mode of the following frequency table:

Class limits	1-40	41-80	81-120	121-160	161-200
Frequency	17	41	80	99	4

16. Find the mean, median and mode of the following frequency table:

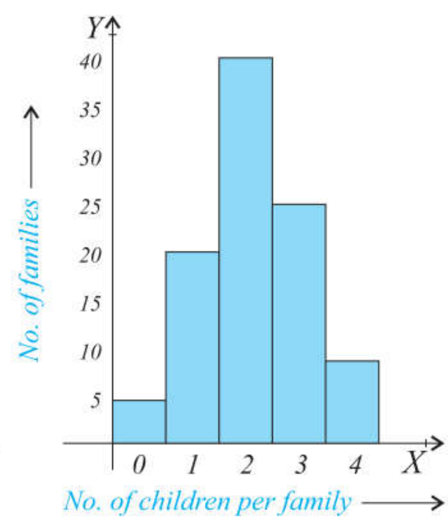
Class limits	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40
Frequency	19	24	18	21	23	20	16	15

17. The diagram illustrates the number of children per family of a sample of *100* families in a certain housing estate:

(i) State the modal number of children per family.

(ii) Calculate the mean number of children per family.

(iii) Find the median number of children per family.



18. Given below is a frequency distribution:

Class limits	5-9	10-14	15-19	20-24	25-29
Frequency	1	8	18	11	2

Find mean (By direct, short and coding), median and mode of the frequency distribution.

19. The following table gives the distribution of marks obtained by two classes. Which class is better on the average?

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Class A	20	32	35	40	15	8
Class B	15	30	40	42	20	10

20. If the mean of the following distribution is 3.78 find the value of a.

x	2	3	4	5	6
f	10	a	11	9	8

Measure of Dispersion

The measure that is used to describe the spreadness of data or the data variation around a central value is called Measure of Dispersion.

The following formulas or techniques are used to measure the behaviour of each unit of the given data around its central value.

i. Range (R) ii. Variance (S^2)

iii. Standard Deviation **(S.D.)**

Remember!

The purpose of measure of dispersion is to check the behaviour of each unit of the given data around the central value i.e., (Mean, Median and Mode).

i. Range (R):

It is the simplest measure of dispersion. Range is the difference between the largest and the smallest values of the data.

$$\text{Range (R)} = X_m - X_0$$

Where,

X_m = The largest value

X_0 = The smallest value

Keep in mind!

Dispersion is the state of getting spreading things..

Example 17: Find the range of the following data:

25, 35, 60, 48, 38, 15, 55

Solution: Range (R) = $X_m - X_0$

$X_m = 60$

$X_0 = 15$

Range $= 60 - 15$

Range $= 45$

Keep in mind!

Range is calculated by using only two extreme data values.

Example 18:

Participant	1	2	3	4	5	6	7	8
Age	25	36	40	33	36	20	25	35

Solution: Range (R) = $X_m - X_0$

$X_m = 40$

$X_0 = 20$

Range (R) $= 40 - 20$

$= 20$

The range of the data is 20 years.

ii. Variance (S^2)

The variance is a measure of variability. It is a single value, obtained by dividing the sum of the squares of the deviations taken from the mean by the number of observations in the data set.

Variance is the average of squared deviation from the mean.

Key fact!

Mostly, variance is used for comparing two sets of data. Smaller variance means that the data is more consistent.

It is denoted by S^2 and is defined for ungrouped data as:

The formula of variance is:

$$S^2 = \frac{\sum(x-\bar{x})^2}{n} \quad \text{(By definition method)}$$

or

$$S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 \quad \text{(By direct method)}$$

Do you know?

- The data is widely spread when the variance of the data is larger.
- The data is clustered when the variance of the data is smaller.

Example 19:

Seven students took a test in Mathematics. They got marks (out of 15) as: 5, 7, 8, 11, 6, 9, 10. Find the variance in their marks by using both the methods.

Solution:

Here, $\bar{x} = \frac{5+7+8+11+6+9+10}{7}$

$\bar{x} = \frac{56}{7} = 8$ marks

$S^2 = \frac{\sum(x-\bar{x})^2}{n}$ (By definition)

(i) $S^2 = \frac{28}{7} = 4$ square marks

(ii) $S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$ (Direct method)

$S^2 = \frac{28}{7} - \left(\frac{28}{7}\right)^2$

$= 68 - (8)^2 = 68 - 64 = 4$

$S^2 = 4$ square marks

x	$x-\bar{x}$	$(x-\bar{x})^2$	x^2
5	−3	9	25
7	−1	1	49
8	0	0	64
11	3	9	121
6	2	4	36
9	1	1	81
10	2	4	100
$\sum x = 56$		$\sum(x-\bar{x})^2 = 28$	$\sum x^2 = 476$

(iii) Standard Deviation (S.D.)

Standard deviation is the square root of the average of squared deviation from the mean. It is abbreviated as S.D. and is denoted by S. It is calculated by the following formula for ungrouped data:

Keep in mind!

S.D. = $\sqrt{\text{variance}}$

$$S = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad \text{(By definition)}$$

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} \quad \text{(Direct method)}$$

Example 20: The height of the students in inches is given. Calculate the standard deviation for the given data.

	x	$x-\bar{x}$	$(x-\bar{x})^2$	x^2
	59	4	16	3481
	56	1	1	3136
	55	0	0	3025
	53	2	4	2809
	52	3	9	2704
Total	275	——	$\sum(x-\bar{x})^2 = 30$	$\sum x^2 = 15155$

(i) $\bar{x} = \frac{\sum x}{n} = \frac{275}{5} = 55$ inches

$$S = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{30}{5}}$$

$S = 2.45$ inches

(ii) $$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{15155}{5} - \left(\frac{275}{5}\right)^2}$$

$$= \sqrt{3031 - 3025} = \sqrt{6} = 2.45$$

$S = 2.45$ inches

Key fact!

- Both measures are used to check the variability in a data set.
- Variance and standard deviation of a constant is zero (0) e.g.,
  x: 5, 5, 5, 5, 5
  $S^2 = 0$; $S = 0$
- Variance and standard deviation is always greater than or equal to 0, i.e., $S^2 \geq 0$; $S \geq 0$
- If data is measured in inches, then the variance will be measured in square inches but the standard deviation will be measured in inches.

Exercise 5.4

1. Define range, variance and standard deviation and explain the purpose of measure of disperation.
2. Find the range, variance and standard deviation for the following sets of value.
   (i) 18, 19, 25, 22, 21, 19, 20, 27
   (ii) 75, 88, 90, 88, 81, 87, 82, 89
   (iii) 220, 225, 195, 320, 322, 235, 250, 253, 200, 310, 300
   (iv) 12.5, 15.25, 7.5, 9.25, 5.75, 10.5, 9.20, 8.53, 8.57, 10.35
3. In a data $n = 5$; $\Sigma x^2 = 10400$ and $\Sigma x = 50$. Find variance and standard deviation.
4. If variance = 5 , $n = 7$, $\Sigma x^2 = 280$. Find the value of Σx.
5. The following are the marks of two student A and B in five test of Mathematics. Calculate mean, variance and standard deviation.

Student A	23	24	30	45	48
Student B	45	35	42	40	43

Hint!
Smaller variance means that data is more consistent.

(i) Who is better based on average?
(ii) Which student is more consistent in his performance?

SUMMARY

- The collection of information in the form of facts and figures is called data.
- Data which is arranged in systematic order/groups or classes in called grouped data.
- In bar graph, we use bars having same width.
- Pie chart is used for comparison of values of different items by making the sectors of a circle. Angles are used to represent the sectors.
- The number of times a value occurs in a data is called the frequency of that value.
- To write a data in the form of a table in such a way that the frequency of each class can be observed at once is called frequency distribution.
- Cumulative frequency is calculated by adding each frequency from a frequency distribution to the sum of its predecessors.
- The mean is calculated by adding all the values and then dividing by the number of values.
- If a data is arranged in ascending or descending order then median of the data is:

  (a) The middle value of the data, if it consists of odd number of values.

  $$\text{Median} = \left(\frac{n+1}{2}\right)^{th} item \quad \text{(when } n \text{ is odd number)}$$

  (b) The mean of the two middle values of the data, if the number of values in data is even.

  $$\text{Median} = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} item + \left(\frac{n+2}{2}\right)^{th} item\right) \quad \text{(when } n \text{ is even number)}$$

- Mode is the value that occurs most frequently in a data.
- Range is the difference between the largest and the smallest values of the data.
- Variance is a single value, obtained by dividing the sum of squares of the deviations taken from the mean by the numbers of observations in the data set.
- Standard deviation is the square root of the average of squared deviation from the mean.

Sub-Domain (ii): Probability

Students' Learning Outcomes

After completing this domain, the students will be able to:

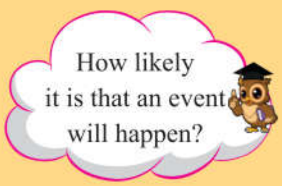
- determine that the probability of an event occurring is 'P' and an event 'not occurring is '1–P'

Experimental and Theoretical Probability

- perform probability experiments (for example tossing a coin, rolling a die, spinning a spinner etc. for each certain number of times) to estimate probability of a simple event
- compare experimental and theoretical probability in simple events
- explain and compute the probability of mutually exclusive, independent, and equally likely events
- calculate combining probabilities with "AND" and "OR"

Probability of a Simple Combined Events

- Predict the outcomes of simple combined events with the help of
  - sample space
  - tree diagram
- calculate probability of simple combined events
- apply the probability concept to real life situations



- **Certain / sure** to a ball
- **Likely** to a yellow ball
- **Unlikely** to a red ball
- **Impossible** to a green ball.

Girolamo Cardano is known as the father of probability.

Probability

Probability tells us how likely it is that an event will occur e.g., can you tell the probability of getting 4 in tossing a die?

Probability?

Probability is the chance of occurrence of an event.

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes}$$

If Usman flips a coin, what will be the probability of occurrence of head?

The result of an experiment is called event.

When a coin is tossed, the sample space is given by:

S= {Head, Tail}

So, the chance of occurrence of head $= \frac{1}{2}$

P(head) $= \frac{1}{2} = 50\%$

The chance of occurrence of tail $= \frac{1}{2}$

P(tail) $= \frac{1}{2} = 50\%$

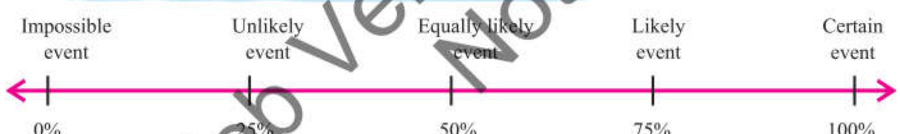
Key fact!

The set of all possible outcomes is called sample space. It is denoted by S.

Remember!

The probability/chance of an event A is denoted by P (A).

Notice that:
The chance of occurrence of head and tail are equal so, these events are called equally likely events.



Key fact!

The probability of an event always lies between 0 and 1 (both 0 and 1 inclusive).

If Zainab rolls a die, what will be the probability of getting 4?

When a die is rolled, the sample space is given by:

Sample Space $= S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event of getting 4,

Where $n(A)$ = Number of elements in $A = 1$

$n(S)$ = Number of elements in sample space $= 6$

$$P(A) = \frac{n(A)}{n(S)}$$

The probability of getting 4 will be:

$$P(A) = \frac{1}{6}$$

Try yourself!

What will be the probability of getting 6?

Now think!

What will be the probability of not getting 4?

The probability of not getting 4 is the complement of getting 4.

To find out the probability of not getting 4, we will subtract the probability of getting 4 from 1.

$$P(A') = 1 - P(A)$$
$$= 1 - \frac{1}{6}$$
$$= \frac{6-1}{6}$$
$$= \frac{5}{6}$$
$$P(A') = P(\text{not getting } 4) = \frac{5}{6}$$

Remember!

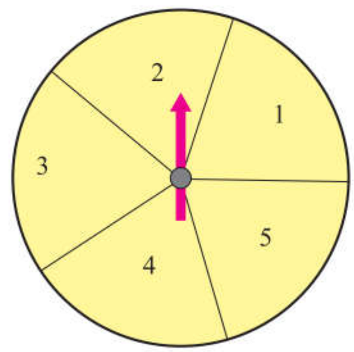
The complement of an event A consists of all outcomes that are not in the event A. It is denoted by $P(A')$. The sum of $P(A)$ and $P(A')$ is always equal to 1.

Need to know!

The sum of the probability of all the outcomes is always equal to one (1).

Example 1:

Ayesha! Look at the spinner given at the right. Find out the probability of getting 2.



Sample Space $= \{1,2,3,4,5\}$

No. of all possible outcomes $= n(S) = 5$

Let A be the event of getting 2.

No. of favourable outcomes $= n(A) = 1$

$$P(A) = P(\text{getting } 2) = \frac{\textit{No. of favourable outcomes}}{\textit{No. of all possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{5}$$

Can you find out the probability of not getting 2?

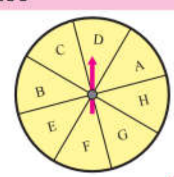
Yes, by subtracting the probability of getting 2 from 1, we can find out the probability of not getting 2.

$$P(\text{getting } 2) = \frac{1}{5}$$

$$P(A') = P(\text{not getting } 2) = 1 - \frac{1}{5}$$
$$= \frac{5-1}{5}$$
$$= \frac{4}{5}$$

Try yourself!

What is the probability of spinner not landing on D.



Key fact!

The event A and its complement are disjoint.

Experimental and Theoretical Probability

The following methods are used to obtain the probabilities.

Types of Probabilities

Experimental Probability	Theoretical Probability
It is based on actual results of an experiment	It is based on assumption
$P(A)=\dfrac{\text{No. of favourable outcomes}}{\text{Total no. of trials}}$	$P(A)=\dfrac{\text{No. of favourable outcomes}}{\text{No. of all possible outcomes}}$
Example: A coin is tossed 25 times. 16 times occurs head and 9 times occurs tail. $P(head) = \dfrac{16}{25}$ $P(tail) = \dfrac{9}{25}$	**Example:** A coin is tossed. $P(head) = \dfrac{1}{2}$ $P(tail) = \dfrac{1}{2}$

Key fact!

Experimental probability is obtained by performing experiment. While the theoretical probability is obtained without performing any experiment.

Hina rolled a die 15 times. The given table is showing the outcomes of the experiment.

Die Outcomes						
Outcomes	1	2	3	4	5	6
No. of Trials	2	1	3	4	3	2

Can you tell the theoretical and experimental probabilities of the experiment?

The theoretical probability on each of the 6 digits is $\frac{1}{6}$

To calculate the experimental probabilities of each digit, we will use the outcomes which are given in the table.

$$P(event) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of trials}}$$

$P(1) = \frac{2}{15}$, $P(2) = \frac{1}{15}$, $P(3) = \frac{3}{15}$,

$P(4) = \frac{4}{15}$, $P(5) = \frac{3}{15}$, $P(6) = \frac{2}{15}$

Activity

Toss a coin 50 times and find the experimental probabilities.

Addition Law of Probability

Addition Law for Mutually Exclusive Events

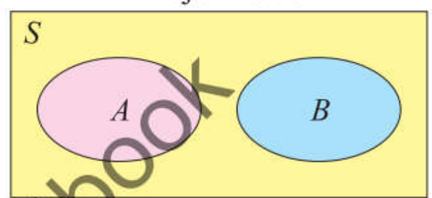
If two events A and B are mutually exclusive, then the addition law of probability is,

$$P(A \cup B) = P(A) + P(B)$$

Or

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

Disjoint sets



$A \cup B$

Need to know!

This law is also known as addition law of probability for mutually exclusive events.

Key fact!

Mutually Exclusive Events:

- When two events do not occur together, these events are mutually exclusive events. i.e.
  $P(A \cap B) = 0$
- Mutually exclusive events are also known as disjoint sets.

Example 2:

A fair die is rolled. What will be the probability of getting 2 or 5?

Solution:

$$\text{Sample Space} = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6$$

Let A be the event of getting 2

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

Let B be the event of getting 5

$$n(B) = 1 \quad ; \quad n(S) = 6$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$$

Keep in mind!

The chance of occurrence of event A and B are equal. So, these events are called equally likely events.

As, events are mutually exclusive.

So, $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Remember!

The events 2 and 5 cannot occur together that's why these events are mutually exclusive.

$P(A \cap B) = 0$

Example 3: A bag contains 5 green balls, 6 yellow balls and 8 blue balls. One ball is selected at random, what will be probability of selecting a yellow ball or blue ball?

Solution: Let A be the event of getting a yellow ball and B be the event of getting a blue ball.

$$n\text{ (yellow balls)} = n(A) = 6$$
$$n\text{ (blue balls)} = n(B) = 8$$
$$n(s) = n\ (Green\ balls) + n(yellow\ balls) + n(blue\ balls) = 5 + 6 + 8 = 19$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{19}$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{19}$$

As, events are mutually exclusive.

So, $P(A\ or\ B) = P(A \cup B) = P(A) + P(B)$

$$= \frac{6}{19} + \frac{8}{19} = \frac{14}{19}$$

Multiplication Law of Probability

If two events A and B are independent, then the multiplication law of probability is,

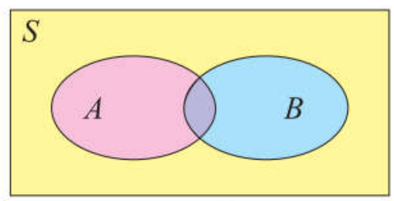
$$P(A \cap B) = P(A) \cdot P(B)$$

Or

$$P(A\ and\ B) = P(A) \cdot P(B)$$

Independent events:

Two or more events are said to be independent if the occurence of one event does not effect the occurence of another event.



$A \cap B$

Example 4: Let "A" and "B" are two independent events,

$P(A) = 0.5$ and $P(B) = 0.38$. Find $P(A \cap B)$.

Solution: As, the events A and B are independent.

$$P(A \cap B) = P(A\ and\ B) = P(A) \cdot P(B)$$
$$= (0.5) \cdot (0.38)$$
$$P(A \cap B) = 0.19$$

Keep in mind!

"OR" is used for union "AND" is used for intersection. $P(A \cap B)$ gives the combined probability of both events A and B.

Example 5: A fair dice is rolled and a fair coin is tossed. Find the probability of getting a tail and 4.

Solution: Let "A" is the probability of getting tail on a coin then,

$$P(A) = \frac{1}{2}$$

Let "B" is the probability of getting "4" on rolling a dice then,

$$P(B) = \frac{1}{6}$$
$$P(\text{Tail and 4}) = P(A \cap B) = P(A) \cdot P(B)$$
$$= \left(\frac{1}{2}\right) \cdot \left(\frac{1}{6}\right)$$
$$P(\text{Tail and 4}) = \frac{1}{12}$$

Example 6: A jar has 2 red beans, 5 green beans and 6 blue beans. A bean is chosen at random. After replacing it, the second bean is chosen. Calculate:

(i) P(red and green beans) in same order

(ii) P(green and blue beans) in same order

(iii) P(red and blue beans) in any order

(iv) P(different colours) in any order

Solution:

Given,

Red beans $(Rb) = 2; \quad n(Rb) = 2$

Green beans $(Gb) = 5; \quad n(Gb) = 5$

Blue beans $(Bb) = 6; \quad n(Bb) = 6$

$$n(S) = n(Rb) + n(Gb) + n(Bb)$$

$$= 2 + 5 + 6 = 13$$

$$P(Rb) = \frac{n(Rb)}{n(S)} = \frac{2}{13} \quad ; \quad P(Gb) = \frac{n(Gb)}{n(S)} = \frac{5}{13} \quad ; \qquad P(Bb) = \frac{n(Bb)}{n(S)} = \frac{6}{13}$$

(i) P(Rb and Gb) = ?

As, both the events are independent

(i) P(Rb and Gb) $= P(Rb) \cdot P(Gb)$

$$= \left(\frac{2}{13}\right) \cdot \left(\frac{5}{13}\right) = \frac{10}{169}$$

(ii) P(Gb and Bb) $= P(Gb) \cdot P(Bb)$

$$= \left(\frac{5}{13}\right) \cdot \left(\frac{6}{13}\right) = \frac{30}{169}$$

(iii) P(Rb and Bb) + P (Bb and Rb) = $P(Rb) \cdot P(Bb) + P(Bb) \cdot P(Rb)$

$$= \left(\frac{2}{13}\right) \cdot \left(\frac{6}{13}\right) + \left(\frac{6}{13}\right) \cdot \left(\frac{2}{13}\right) = \frac{12}{169} + \frac{12}{169} = \frac{24}{169}$$

(iv) P(different colours) = P (Rb and Gb) + P(Gb and Rb) + P(GB and Bb) + P(Bb and Gb) + P(Rb and Bb) + P(Bb and Rb)

$$= \left(\frac{2}{13}\right) \cdot \left(\frac{5}{13}\right) + \left(\frac{5}{13}\right) \cdot \left(\frac{2}{13}\right) + \left(\frac{5}{13}\right) \cdot \left(\frac{6}{13}\right) + \left(\frac{6}{13}\right) \cdot \left(\frac{5}{13}\right) + \left(\frac{2}{13}\right) \cdot \left(\frac{6}{13}\right) + \left(\frac{6}{13}\right) \cdot \left(\frac{2}{13}\right)$$

$$= \left(\frac{10}{169}\right) + \left(\frac{10}{169}\right) + \left(\frac{30}{169}\right) + \left(\frac{30}{169}\right) + \left(\frac{12}{169}\right) + \left(\frac{12}{169}\right) = \frac{104}{169}$$

Simple Combined Events (Tree Diagram)

If Zara toss a fair coin 2 times, what will be the possible outcomes?

The possible outcomes of that experiment can be written by using table.

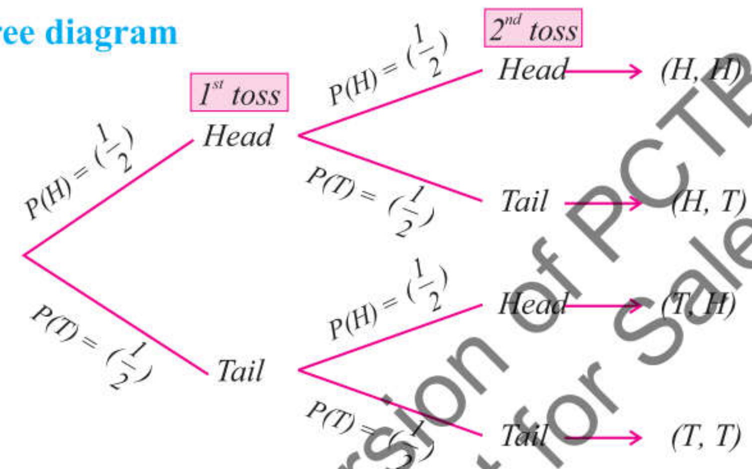
Flip Coin 1 \ Flip Coin 2	H	T
H	(H, H)	(H, T)
T	(T, H)	(T, T)

The total number of possible outcomes of this experiment is 4. So, the sample space will be:

$$\text{Simple space (S)} = \{(H,H), (H,T), (T,H), (T,T)\}$$

$$n(S) = 4$$

Tree diagram



Key fact!

Tree diagram is used to show the combination of two or more events.

The number of possible outcomes of this experiment is *"4"*.

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$

$$n(S) = 4$$

Now, let's find out the probability of getting (i) exactly 2 heads (ii) 1 head and 1 tail.

(i) Exactly *2* heads

Let "A" be the event of getting *2* heads.

$A = \{(H,H)\}$: $n(A) = 1$; $n(S) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

(ii) Let "B" be the event of getting one head and one tail.

$B=\{(H,T),(T,H)\}$: $n(B)=2$; $n(S) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

By using combined events method

(i) $P(\text{Exactly } 2 \text{ heads}) = P(H \cap H) = P(H).P(H)$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

(ii) $P(\text{head and tail}) = P(H \cap T) + P(T \cap H)$

$$= P(H)\ \ P(T) + P(T) \cdot P(H)$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4} = \frac{1}{2}$$

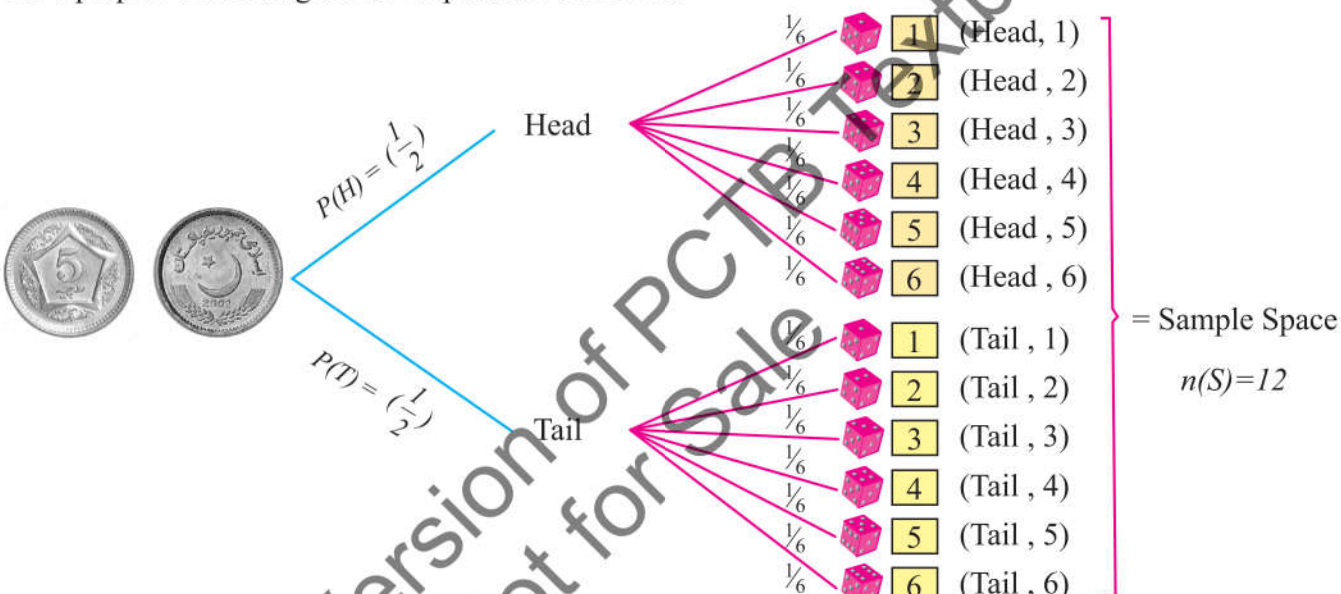
Hamza tossed a fair coin once and rolled a fair dice once.
Let's prepare a table of all possible outcomes.

Coin \ Dice	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

The total number of possible outcomes of this experiment is 12. So, the sample space will be:

Simple space (S) = $\{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

Let's prepare a tree diagram of all possible outcomes.



Now, calculate the probability of the following events:

(i) Head on coin and 5 on dice

Let "A" be the event of occurrence of head on coin and *5* on dice.

$A=\{(Head, 5)\}$; $\quad n(A)=1$; $\quad n(S) = 12$

$$P(A)= \frac{n(A)}{n(S)} = \frac{1}{12}$$

Try yourself!

Solve the given example by using combined events method (independent events).

(ii) Tail on coin and 2, *3, 4, 5, 6* on dice

Let "B" be the event of occurrence of tail on coin and *2, 3, 4, 5, 6* on dice

$B = \{(tail,2), (tail,3), (tail,4), (tail,5), (tail,6)\}$

$n(B) = 5; \; n(S) = 12$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

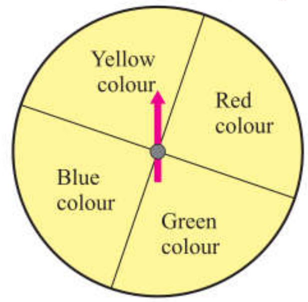
Exercise 5.5

1. Complete the following:

Sr. No.	$n(A)$	$n(S)$	$P(A)$
(i)	5	15	
(ii)		25	$\frac{17}{25}$
(iii)		6	$\frac{1}{6}$

2. Zara rolled a dice. What will be the probability of getting the digit "4". Also find the probability of not getting the digit "4".

3. A spinner is spun. Find the probability that the dice lands on red colour. Also find the probability that the dice lands on yellow or blue colour. (Hint: All events are equally likely).



4. A box contains 8 cards numbered 20 to 27. What is the probability of getting:
   (i) a number 22 from the box.
   (ii) a number 26 from the box
   (iii) all numbers 20 to 27 except 25 from the box

5. Find the probability of each of the following:
   (i) the digit "5" appears on rolling a dice.
   (ii) the digit "5" does not appear on rolling a dice.
   (iii) the digit "3" appears on rolling a dice.

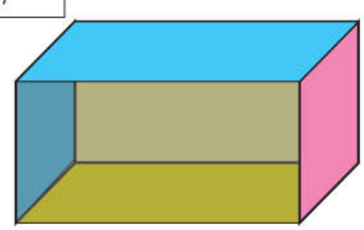
6. The probability that the team will win the match is 0.86. What is the probability that the team will not win the match?

7. The probability that tomorrow's temperature will be 0.45. Find out the probability that the temperature will not be 0.45.

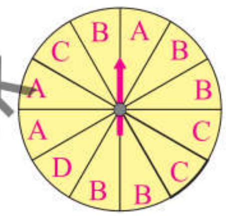
8. Hamza throws the cube shaped cardboard 100 times. Each of the face of the cardboard having different colour.

Colours	Blue	Red	Green	Yellow	Orange	Purple
No. of trials	22	20	15	13	23	7

Find the probability of getting:
   (i) red colour (ii) blue colour
   (iii) red or orange colour (iv) not blue colour
   (v) not yellow colour



9. A group of 15 children consists of 7 boys and 18 girls. If a child is chosen at random from the group, what will be the probability that:
   (i) a boy is chosen (ii) a girl is chosen
   (iii) no boy is chosen (iv) no girl is chosen
10. A basket contains 5 red blocks, 3 green blocks, 2 blue blocks. If one block is picked at random, what will be the probability of getting.
   (i) Red block (ii) Blue block (iii) No blue block
   (iv) No green block (v) No red block



11. A spinner is spun. Find the probability that the alphabet on the landing is:
   (i) B (ii) not B (iii) D
   (iv) A (v) not D (vi) not C
   (**Hint:** all events are equally likely)
12. Three candidates A, B and C are in an examination. The probability of passing the candidate "A" is 0.25, the candidate "B" is 0.28 and the candidate "C" is 0.55. What will be the probability that A or C pass the exam? Also find out the probability of passing C or B.
13. A fair dice is rolled. Ahmad wins the game if the result is either odd or divisible by 6. What will be the probability of winning the game for Ahmad?
14. A dice is rolled. What will be the probability of getting 4 or 5?
15. A bag contains 3 red hairpins, 5 black hairpins, 5 green hairpins and 2 blue hairpins. Find the probability of selecting:
   (i) a red pin or a blue pin (ii) a green pin and a black pin (in same order)
16. A coin is tossed and a dice is rolled. Find the probability of getting head on the coin and the digit "4" on dice.
17. A jar contains 5 yellow marbles, 2 green marbles and 6 blue marbles. A marble is chosen at random from the jar. The second marble is chosen after replacing the first one. What is probability of choosing?
   (i) a green and a blue marble (in same order) (ii) a yellow and a green marble (in any order)
   (iii) P(different colours) in any order
18. A shopkeeper has 100 shirts. 40 are of red colour, 30 are of green colour, 20 are of black colour and 10 are of blue colour. A shirt is chosen at random by the customer. After replacing it, a second shirt is chosen. What is the probability of choosing?
   (i) a red colour shirt and a black colour shirt (in same order)
   (ii) a green colour shirt and a blue colour shirt (in any order)
   (iii) a red colour shirt and a green colour shirt (in any order)
19. Zainab rolled a dice 2 times.
   (i) List all the possible outcomes by using table and tree diagram.
   (ii) Also calculate the probability of getting:
      a) Exactly two 6's. b) Exactly two 4's. c) At least once time 4.

20. Zeeshan spun a spinner having four different colours (red, green, blue and yellow) once and rolled a dice once.
    (i) List all the possible outcomes by using table and tree diagram.
    (ii) Also calculate the probability of getting:
        a) Green on the spinner and the digit "2" on the dice.
        b) Green or yellow on the spinner and the digit "6" on the dice.
        c) Red or blue on the spinner and 2,3 or 4 on the dice.
21. Saima wants to eat ice-cream. There are 3 flavours of ice-cream (Mango, Strawberry and Pistachio). Each flavour has 3 sizes (Small, Medium and large):
    (i) List all the possible outcomes by using table and tree diagram.
    (ii) Also calculate the probability of Selecting/Choosing:
        a) Mango flavour with medium size.
        b) Strawberry flavour with small or large sizes.
        c) Pistachio flavour with medium or large sizes.

SUMMARY

- Probability tells us how likely it is that an event will occur.
- The result of an experiment is called event.
- Probability is the chance of occurrence of an event.
- The set of all possible outcomes is called sample space. It is denoted by "S".
- Experimental probability is obtained by performing experiment. While the theoretical probability is obtained without performing any experiment.
- The probability/chance of an event "A" is denoted by $P(\text{A})$.
- The probability of an event always lies between 0 and 1(both 0 and 1 inclusive).
- The sum of the probabilities of all the outcomes is always equal to one (1).
- Tree diagram is used to show the combination of two or more events.
- When two events do not occur together, these events are mutually exclusive events. i.e., $P(\text{A} \cap \text{B}) = 0$
- If two events A and B are mutually exclusive, then the addition law of probability is:

$$P(A \cup B) = P(A) + P(B)$$

- If two events A and B are independent, then the multiplication law of probability is:

$$P(A \cap B) = P(A) \cdot P(B)$$

Review Exercise 5

1. Four options are given against each statement. Encircle the correct one.

i. What is the value of x i.e., 14, 24, x , 18, 30, when mean is 23.

(a) 28 (b) 29 (c) 30 (d) 31

ii. What is the value of x i.e., 40, 28,16,18, 37, 20, x ,35, when median is 26.

(a) 20 (b) 22 (c) 24 (d) 28

iii. A number which indicates how often a value occurs is called:

(a) frequency (b) mode (c) median (d) average

iv. An arrangement of the values that one or more variables take in data is called:

(a) frequency (b) frequency distribution
(c) median (d) mode

v. A representation of a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies is called:

(a) histogram (b) bar chart (c) pie chart (d) line graph

vi. A measure of central tendency that attempts to describe a data by identifying the central position within that data:

(a) is a single value (b) are multiple values
(a) are duplicate values (d) are repeating values

vii. The statistical measure that identifies a single value as representative of an entire distribution is called:

(a) frequence distribution (b) histogram
(c) mean (d) central tendency

viii. The value which occupies the middle position when all the observations are arranged in an ascending/descending order is called:

(a) frequency distribution (b) median
(c) mode (d) mean

ix. The value that occurs most frequently in the data is called:

(a) frequency distribution (b) median
(c) Mode (d) mean

x. ______________ is based on actual result of an experiment.

(a) probability (b) experimental
(c) theoretical probability (d) independent probability

xi. $P(A) = \dfrac{No.\ of\ favourable\ outcomes}{No.\ of\ all\ possible\ outcomes}$ is called:

(a) theoretical probability (b) experimental Probability
(c) combined probability (d) multiplicative probability

xii. ________ can take only same specific values.

(a) data (b) continuous data

(c) discrete data (d) mode

xiii. ________ is an extension of bar graph

(a) line graph (b) pie chart

(c) multiple bar graph (d) circular graph

xiv. The number of times a value occurs in a data is called:

(a) frequency (b) cumulative frequency

(c) class boundaries (d) distribution

2. A survey was conducted from the students of Grade-VIII and asked about their favourite colour. Draw an appropriate graph for the following data and also interpret the data:

Colours	Blue	Red	Green	Yellow
No. of students	25	50	30	15

3. Following are the heights in inches of 12 students:

55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63

Find the average height (direct and short method) of the given data.

4. Calculate the mean from the following information:

(i) if $U = \frac{X - 50}{}$; $\Sigma U = 25$; $n = 10$

(ii) if $U = \frac{X - 100}{2}$; $\Sigma U = 25$; $n = 10$

5. The price of *10* litres of drinking water was recorded at several stores, and the results are displayed in the table below:

Price *(Rs.)*	*74*	*75*	*76*	*77*	*78*	*79*	*80*

6. Calculate the mean, median and mode for each set of data given below:

(i) 3,6,3, 7,4,3,9 (ii) 11,10,12,12,9,10,14,12,9

(iii) 2,9,7,3,5,5,6,5,4,9 (iv) 6,8,11,5,2,9,7,8

(v) 153.8,154.7,156.9,154.3,152.3,156.1,152.3

Find the average price of drinking water by direct and short method.

7. On a prize distribution day, 50 students brought pocket money as under:

Rupees	5-10	10-15	15-20	20-25	25-30
Frequency	12	9	18	7	4

Calculated the mean, median and mode of the above data. Also draw histogram and frequency polygon.

8. Find the range, variance and standard deviation for the following data.
110, 120, 115, 135, 127, 108, 113, 117, 118, 121, 122, 123, 127

9.

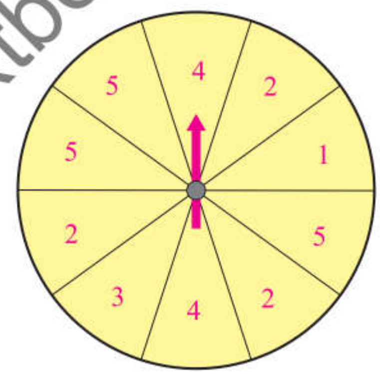
Company A	117	118	120	125	115	113
Company B	200	180	150	120	125	130

(i) Who is better based on average?

(ii) Which company is more consistent in his performance for producing the number of bags in a week.

10. Zara rolled a dice, what will be the probability of getting 1. Also tell what will be the probability of not getting 1.

11. Spin a spinner at right.



i. Write the sample space

ii. Find the probability of getting even numbers.

iii. P(2), P(1) and P(5). (**Hint:** All events are equally likely)

12. In a class of 30 students, 12 are used glasses. Find the probability of students who don't use glasses. Also calculate the probability of students who used glasses.

13. Twenty cards of whole numbers (10-29) are put in a basket. A card is selected at random from the basket. Find the probability that the number on the card.

i. Contains the number 12.

ii. Contains the number 20.

iii. Does not contain the number 20.

14. Imran rolled a dice once and tossed a coin once.

i. List all the possible outcomes by using tree diagram.

ii. Also calculate the probability of getting:

a) 5 on dice and tail on coin

b) 2, 4 or 6 on dice and head on coin

c) Odd numbers on dice and tail on coin.

15. The candidate "A" solved 75% of the problems of mathematics exercise. The candidate "B" solved 10% of the problems of the same exercise. What will be the probability that the candidate A or B solve the problems at random.

16. The probability that monkey will be alive about 25 years is 0.85 and 0.82 will be alive about 28 years. Find the probability that

i. Both of them will alive

ii. Both of them will die.

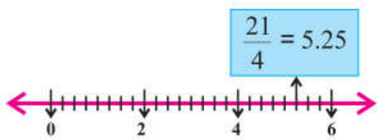
Answers

Exercise 1.1

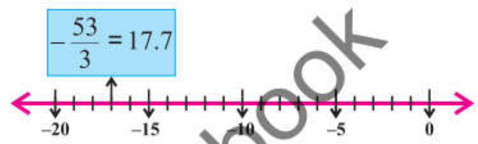
1. (iv), (viii), (ix), (x), (xi), (xii), (xiv), (xv), (xvii), (xviii) are terminating
(i), (iii), (vii), (xiii), (xvi) are recurring (ii) and (vi) are non-terminating

2. (i) and (vii) are irrational
(ii), (iii), (iv), (v) and (viii) are rational.

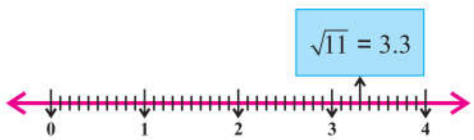
3. (i)



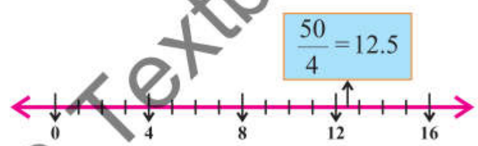
(ii)



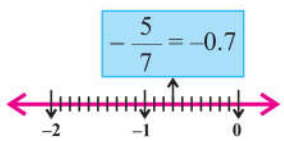
(iii)



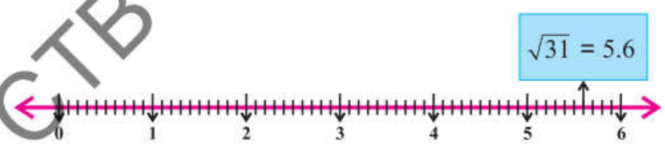
(iv)



(v)



(vi)



4. (i) 15 (ii) 21 (iii) 36 (iv) 9 (v) 100 (vi) 88

5. (i) 5 (ii) 11 (iii) 6 (iv) 5

6. (a) 9.2 ≈ 10 packets (b) 44 kg rice

7. (a) $12\frac{8}{15}$ *metres* (b) $28\frac{1}{5}$ *metres*

8. 92.3 ≈ 93 packets, 37.5 kg

9. (a) 25 bottles (b) 22 bottles (c) 112.5 litres

Exercise 1.2

1. (i) Identity property
(ii) Associative property w.r.t "×"
(iii) Additive inverse property
(iv) Commutative property w.r.t "+"
(v) Closure property w.r.t "×"
(vi) Multiplicative inverse property
(vii) Multiplicative identity property
(viii) Multiplicative inverse property
(ix) Associative property w.r.t "+"
(x) Associative property w.r.t "+"
(xi) Multiplicative identity property
(xii) Commutative property w.r.t. "×"
(xiii) Additive identity property
(xiv) Commutative property w.r.t. "×"
(xv) Additive inverse property

2.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)
Additive inverse	–15	27	$-\frac{7}{9}$	$\frac{9}{16}$	$\frac{20}{23}$	–1	$\frac{2}{3}$	$-\frac{7}{2}$	$-\frac{5}{7}$	7	–12
Multiplicative inverse	$\frac{1}{15}$	$-\frac{1}{27}$	$\frac{9}{7}$	$-\frac{16}{9}$	$-\frac{23}{20}$	1	$-\frac{3}{2}$	$\frac{2}{7}$	$\frac{7}{5}$	$-\frac{1}{7}$	$-\frac{1}{12}$

3. (i) Trichotomy property
(ii) Division property
(iii) Subtraction property
(iv) Multiplicative property if $c < 0$
(v) Transitive property
(vi) Multiplicative property if $c > 0$
(vii) Subtraction property
(viii) Division property if $c > 0$

Exercise 1.3

1. (i) 4800 (ii) 785000 (iii) 8095000 (iv) 0.0289360 (v) 2346000 (vi) 0.780200

2. (i) 2.79 (ii) 88.986 (iii) 15.35 (iv) 10.458
3. (i) –23.5 (ii) –45300 (iii) –17.79 (iv) 1172900
4. (i) 7.667 (ii) 18.33 (iii) 6.71 (iv) 3.667 (v) 0.86207 (vi) 0.009597
5. (i) 2 s.f (ii) 5 s.f (iii) 4 s.f (iv) 3 s.f
7. 21196.2 and 21196
8. 12700 and 12740

Exercise 1.4

1. (i) 49 (ii) 121 (iii) 361 (iv) 625 (v) 1369 (vi) 5625
2. (i) 1+2+3+4+5+6+5+4+3+2+1 (ii) 1+2+3+4+5+6+7+6+5+4+3+2+1 (iii) 1+2+3+4+3+2+1
(iv) 1+2+3+4+5+4+3+2+1 (v) 1+2+3+2+1 (vi) 1+2+3+4+5+6+7+8+7+6+5+4+3+2+1

Exercise 1.5

1. (i) 28 (ii) 35 (iii) 50 (iv) 65 (v) 72 (vi) 88 (vii) 36
(viii) 42 (ix) 171 (x) 227 (xi) 647 (xii) 490
2. (i) 37 (ii) 117 (iii) 225 (iv) 321

Exercise 1.6

1. (i) $\frac{7}{8}$ (ii) $\frac{11}{25}$ (iii) $\frac{14}{21}$ (iv) $1\frac{1}{6}$ (v) $\frac{26}{27}$ (vi) $3\frac{3}{5}$
2. (i) $\frac{4}{5}$ (ii) $\frac{13}{16}$ (iii) $2\frac{3}{8}$

Exercise 1.7

1. (i) 1.1 (ii) 0.8 (iii) 2.7 (iv) 1.2 (v) 1.3 (vi) 3.5
2. (i) 0.57 (ii) 0.72 (iii) 3.2 (iv) 4.53 (v) 25.47 (vi) 54.6
(vii) 87.256 (viii) 0.0932 (ix) 48.73

Exercise 1.8

1. (i) 1.414 (ii) 1.732 (iii) 2.236 (iv) 2.646 (v) 3.317 (vi) 4.583
2. (i) 1.89 (ii) 2.52 (iii) 5.37 (iv) 7.95 (v) 30.35 (vi) 76.94

Exercise 1.9

1. (i) 3 (ii) 3 (iii) 3 (iv) 3 (v) 3 (vi) 4 (vii) 4 (viii) 4 (ix) 4 (x) 3 (xi) 4 (xii) 4

Exercise 1.10

1. 104976 cm^2
2. 1296 m^2
3. 62500 m^2, Rs. 75000
4. 120 m
5. 2600 m
6. 350 trees
7. 464 m
8. 120m, 240 m
9. 187
10. 28 m
11. 350 m
12. 1000 m, Rs. 50, 000

Exercise 1.11

1. (i), (iii), (iv), (v)
2. (i) 9 (ii) 25 (iii) 24
3. (i) 2.744 (ii) 0.064 (iii) 0.512
4. (i) $\frac{1}{2}$ (ii) 33 (iii) 15
5. 64 cm^3
6. 512 cm^3
7. 15 cm
8. 9 m
9. 7 Cubical boxes

Review Exercise 1 (a)

1. (i) c (ii) (iii) b (iv) a (v) c (vi) c (vii) c (viii) c (ix) a
(x) c (xi) c (xii) a (xiii) a (xiv) d
2. (i) terminating (ii) recurring (iii) terminating (iv) non- terminating
3. (i) rational (ii) rational (iii) rational (iv) irrational
4. (i) associative property w.r.t "+" (ii) additive inverse property

(iii) Additive identity property (iv) Commutative property w.r.t. "×"

5. (i) 598200 (ii) 0.00256 (iii) 45.36 (iv) 3.13 (v) 2.14 (vi) 0.07895
6. (i) 104 m (ii) 104.0 = 104 m
7. Rs. 2800
8. (i) 2, 98 (ii) 3, 175 (iii) 3,112 (iv) 3,650
9. (i) $5\frac{1}{3}$ (ii) $4\frac{3}{17}$ (iii) $10\frac{1}{13}$ (iv) 0.231 (v) 0.452 (vi) 12.36 (vii) 0.5625 $\frac{6}{5}$ (viii) 6.17 (ix) 80.14
10. 402m
11. 14 marbles
12. (i) 12 (ii) 15

Exercise 1.12

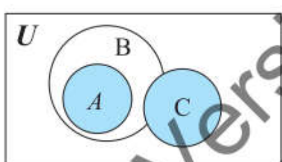
1. (i) A = First five odd numbers. (ii) B = First five even numbers. (iii) C = Set of integers between –10 and +10. (iv) D = Set of vowels of the English alphabet. (v) E = Set of natural numbers equal to 100. (vi) F = Set of Prime numbers.
2. (i) A = {0, 1, 2, 3, …} (ii) B = {f, o, o, t, b, a, l, l} (iii) C = {0, 1, 2, 3, …} (iv) D = {January, June, July} (v) E = {a, b, c, …, z} (vi) F = {5, 10, 15, 20, 25}
3. (i) A = $\{x \mid x \in w \in x \in 10\}$ (ii) B = $\{x \mid x \in D\}$ (iii) C = $\{y \mid y$ is a set of first five number divisible by 3$\}$ (iv) D = $\{z \mid z$ is a set of small English alphabet$\}$ (v) E = $\{x \mid x$ is a set of days of the week$\}$ (vi) F = $\{x \mid x \in P \in 10 < x < 30\}$
4. Equivalent sets (ii) and (iii)
5. Equal sets (i) and (iii)

Exercise 1.13

1. (i) $\{\phi\}$ (ii) $\phi, \{1\}$ (iii) $\phi, \{a\}, \{b\}, \{a, b\}$
2. (i) $\{\phi, \{-1\}, \{1\}, \{-1, 1\}\}$ (ii) $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ (iii) $\{\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+,-\}, \{+,\times\}, \{+,\div\}, \{\times,\div\}, \{-,\times\}, \{-,\div\}, \{+,-,\times\}, \{+,-,\div\}, \{+,\times,\div\}, \{-,\times,\div\}, \{+,-,\times,\div\}\}$

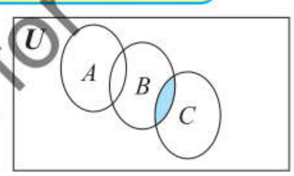
Exercise 1.15

3. (i)



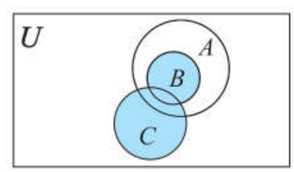
(A∩B)∪C

(ii)



(A∪B)∩C

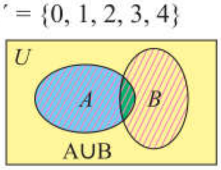
(iii)



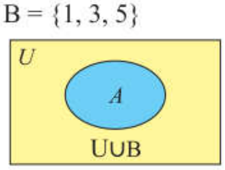
(A∩B)∪C

Exercise 1.16

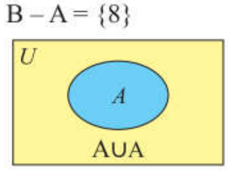
1. (i) A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8} (ii) C ∪ D = $\{a, b, c, x, y, z\}$ (iii) E ∪ F = {3, 4, 6, 8, 9, 12}
2. (i) A ∪ B = {0, 1, 2, 3} (ii) C ∪ D = {3, 9} (iii) E ∪ F = {c, d}
3. (i) A= {0, 1, 3, 5, 7, 9} (ii) B' = {0, 2, 4, 6, 8, 10} (iii) C' = {1, 2, 4, 5, 7, 8, 10} (iv) A' = {0, 1, 2, 3, 4}
4. (i) A – B = {1, 3, 5} (ii) B – A = {8}
6. (i)



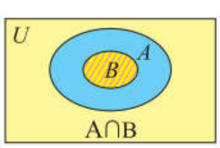
(ii)



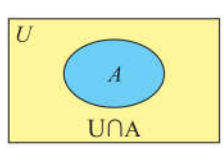
(iii)



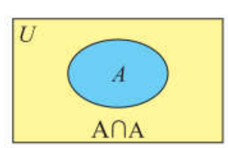
(iv)



(v)



(vi)



7. 10 customers
8. 61 students

Exercise 1.17

1. (i) 15 : 2 (ii) 1 : 10 (iii) 5 : 2 (iv) 3 : 4 (v) 3 : 2 (vi) 2 : 3 : 12
2. (i) P = 24 (ii) $x = \pm 12$ (iii) $m = 24$ (iv) $m = \pm 14$ (v) $x = 3$ (vi) $y = 9$ (vii) $p = 8$
(viii) $\frac{3}{2}$
3. 9 machines 4. 18 eggs 5. 45 labourers 6. 30 days 7. 280 m
8. 4 km 9. 50 minutes 10. 3.5 weeks

11. (i)

x	2	6	8	10	12	14
y	8	24	32	40	48	56

(ii)

x	5	10	15	20	25	30
y	100	200	300	400	500	600

12. (i)

x	1	2	4	8	10
y	80	40	20	10	8

(ii)

x	1	2	4	5	8	10
y	200	100	50	40	25	20

Exercise 1.18

1. 32 days 2. 36 *kg* 3. Rs.18, 864 4. Rs. 60, 000 5. 112 men 6. Rs. 10
7. 240 masons 8. 480 sweaters 9. 672 men 10. 400 bicycles

Exercise 1.19

1. US \$ 385.25 2. UK £ 315.79 3. 1041.67 SAR 4. 23645.32 INR 5. 261.19 Australian Dollar
6. 3397.03 Chinese Yaun 7. 350.88 Canadian Dollar 8. 5718.95 Turkish Lira

Exercise 1.20

1. 10% 2. Rs. 2, 400 3. Rs. 3, 900 4. Rs. 1, 584 5. Discount Rs. 1600, Selling price Rs. 6400
6. Discount of food items = Rs. 187.50, Selling Price = Rs. 1062.50,
Discount of other items = Rs. 150, Selling Price = Rs. 600 7. Rs. 575

Exercise 1.21

1. Rs. 4800 2. Rs. 4500 3. Rs. 14000 4. Rs. 75000 5. 3 % 6 $7\frac{1}{2}\%$.
7. 3 years 8. 2 years

Exercise 1.22

1. Rs. 56250 2. Rs. 10900 included premium fee 3. Rs. 14170 4. Rs. 2992.50
5. Rs. 78039.36 6. Rs. 11970 7. 50,000

Exercise 1.23

1. Aslam's profit : Rs. 31500
Akram's profit : Rs. 35000
2. Amina's profit : Rs. 3600
Maryam's profit: Rs. 4800
3. 1st partner's share : Rs. 7320
2nd partner's share: Rs. 4270
Saeed's loss : Rs. 6000
4. Saad's loss : Rs. 3000
Saud's loss : Rs. 4500
5. Akram's profit :Rs. 6000
Asghar's profit :Rs. 8000
6. A's profit : Rs. 3000
B's profit : Rs. 3600
C's profit : Rs. 5400

Exercise 1.24

1. Son's share Rs. 48000, Daughter's share Rs. 24000
2. Widow Rs. 100,000 Son Rs. 280,000, Daughter Rs. 140,000
3. Widow Rs. 87,500, Son Rs. 122,500, Daughter Rs. 61,250
4. Widow Rs. 9000, Son Rs. 42,000, Daughter Rs. 21,000
5. Son Rs 200,000 , Daughter Rs. 100,000
6. Son Rs. 40,000 , Daughter Rs. 20,000
7. Husband Rs. 45000, Son Rs. 67500 , Daughter Rs. 33750

Review Exercise 1 (b)

1. (i) c (ii) d (iii) c (iv) b (v) d (vi) a (vii) c (viii) d
(ix) b (x) a (xi) b (xii) a (xiii) c (xiv) b (xv) a (xvi) c

3. (i) $\{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}$
(ii) $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$

4. (i) $\phi, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}$
(ii) $\{\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

6. 33 students
7. Rs. 5000
8. 4.5 litres
9. $127.5 \approx 128$ days
10. 20 lions
11. 120 items
12. 90 days
13. 28%
14. Rs.14550
15. Rs. 69105
16. Aslam's profit : Rs. 1400, Akram's profit : Rs. 120, Asghar's profit : 100
17. Widow Rs, 75000, Son Rs. 350000, Daughter Rs. 175000

Exercise 2.1

1. 28, 24, 20, 16, $\underline{12}$, 8
2. 1, 2, 4, 7, $\underline{11}$, $\underline{16}$, 22
3. 77, 66, 55, $\underline{44}$, 33, 22, $\underline{11}$
4. 50, 100, 150, $\underline{200}$, 250, $\underline{300}$, 350, 400
5. 2187, $\underline{729}$, 243, 81, 27, $\underline{9}$, 3
6. $\underline{2, 6,}$ 11, 17, 24, $\underline{32}$, 41, 51

Exercise 2.2

1. (i) yes, $d = 3$ (ii) yes, $d = -5$ (iii) No (iv) yes, $d = 0$
2. (i) 1, 5, 9, … (ii) –3, –1, 1, … (iii) 3, 4, 5, … (iv) 2, 5, 8, …
3. (i) $7n - 1$ (ii) $2n + 3$ (iii) $5n - 1$ (iv) $10n - 8$
4. 31st
5. 21st
6. (i) 20, 26 (ii) 13, 17
7. (i) 87 (ii) 282
8. (i) 20 (ii) 20 (iii) 50 (iv) 51

Exercise 2.3

1. (i) No (ii) yes, $r = 10$ (iii) yes, $r = 2$ (iv) yes, $r = \frac{1}{2}$ (v) No (vi) No
2. (i) 4^{n-1} (ii) $6(2)^{n-1}$ (iii) $15(3)^{n-1}$ (iv) $200\left(\frac{1}{2}\right)^{n-1}$
3. (i) 3, 6, 12, 24 (ii) 4, 8, 16, 32 (iii) $25, \frac{-25}{2}, \frac{25}{4}, \frac{-25}{8}$ (iv) $5, 1, \frac{1}{5}, \frac{1}{25}$
4. (i) 20000 (ii) 2187 (iii) 32 (iv) 3584
5. 61440 words
6. 1080°
7. 35 minutes

Exercise 2.4

1. Open sentences: i, ii, iv
Closed sentences: iii, v, vi
2. Expressions: ii, iv
Expressions: i, iii v, vi
3. Equations: iii, v
Inequalities: i, ii, iv, vi
4. Expressions in (i), (ii), (v) and (vi) are polynomials and expressions in (iii) and (iv) are not polynomials.
5. (i) 7, –6 and 3 (ii) 5 and –3 (iii) 8, 2 and 5 (ii) 9, 3 and –2
6. (i) 1 (ii) 2 (iii) 3 (iv) 4
7. (i) Linear (ii) Quadratic (iii) Quadratic (iv) Linear (v) Cubic (vi) Biquadratic
(vii) Biquadratic (viii) Quadratic

Exercise 2.5

1. (i) $2x+1$ (ii) $4a^3+a^2-2a+4$ (iii) $3b^3+2ab^2$
2. (i) x^4-4x^3+1 (ii) $x-y+2$ (iii) $-5a^2b-2b^3$
3. $3a^4+4a^3-7a^3+7a-18$
4. $2x+2xy-y^2-3$
5. $3x^3+3x^2-3x-13$
6. (i) $3x^3+27$ (ii) $12x^4-34x^3+37x\ -17x+5$ (iii) $a^3+b^3+c^3-3abc$
7. $PQ=x^2y^2-x^3z-y^3z+xyz^2$, $QR=y^2z^2-xy^3-xz^3+x^2yz$, $PR=x^2z^2-x^3y-yz^3+xy^2z$
   $PQR=xyz(x^3+y^3+z^3)-(x^3y^3+y^3z^3+z^3x^3)$
8. (i) $x+3$ (ii) $x^2-3x-10$ (iii) x^2+xy+y^2 (iv) x^2-x-12
9. 3
10. P = 9

Exercise 2.6

1. (i) 2809 (ii) 5929 (iii) 259081 (iv) 1012036
2. (i) 3249 (ii) 9025 (iii) 357604 (iv) 3988009
3. (i) 2484 (ii) 39991 (iii) 999999 (iv) 0.9984
4. (i) 47 (ii) 11 (iii) 7

Exercise 2.7

1. $3(x-3y)$
2. $x(y+z)$
3. $2a(3b-7c)$
4. $3m^2n(mp-2)$
5. $15x(2x^2-3y)$
6. $17(x^2y^2-3)$
7. $x(4x+3x+2)$
8. $2p(p-2p^2+4)$
9. $xy(x^2-x+y)$
10. $7x(x^3-2xy+3y^3)$
11. $xyz(xyz-z+1)$
12. $4xy(x^2y-2+y^2)$
13. $xy^2(y^2-3y-6)$
14. $xyz(xy+xz+yz)$
15. $11xy(7x-3y-5xy)$
16. $5x^3(x^2+2x+3)$

Exercise 2.8

1. $(x-y)(a+b)$
2. $(a-3c)(2b-1)$
3. $(x-3)(x+2)$
4. $(x+5)(x-2)$
5. $(x+2)(x-7)$
6. $(x+3)(x-4)$
7. $(y-9)(y+3)$
8. $(x-8)(x-4)$
9. $(x-5)(x-7)$
10. $(x-13)(x-2)$
11. $(x-y)(a-b)$
12. $(y-a)(y-b)$
13. $(pq-rs)(a^2+b^2)$
14. $(x+y)(ab+cd)$

Exercise 2.9

1. $(x+7)^2$
2. $(3a+2b)^2$
3. $(4+3a)^2$
4. $(5x+8y)^2$
5. $7(a+6)$
6. $4(a+15)^2$
7. $(x-17)^2$
8. $(7x-6)^2$
9. $(x-9y)^2$
10. $(a^2-13)^2$
11. $2(a-16)^2$
12. $(1-3a^2b^2c^2)^2$
13. $x^2(2x-5yz)^2$
14. $\left(\frac{3}{4}x+\frac{2}{3}y\right)^2$
15. $\left(\frac{7}{8}x-\frac{8}{7}y\right)^2$
16. $\left(\frac{ax}{b}-\frac{cy}{d}\right)^2$
17. $4x^4(2x-1)^2$
18. $(a^2b^2x-c^2d^2y)^2$

Exercise 2.10

1. $(3-x)(3+x)$
2. $6(y-1)(y+1)$
3. $(4xy-5ab)(4xy+5ab)$
4. $xy(x-y)(x+y)$
5. $16(a-5b)(a+5b)$
6. $a^2b(b-8)(b+8)$
7. $7x(y-7)(y+7)$
8. $5x(x-3)(x+3)$
9. $11(a+b-3c)(a+b+3c)$
10. $3(5-a+b)(5+a-b)$
11. $\left(x-\frac{9}{5}+\frac{6}{5}y\right)\left(x-\frac{9}{5}-\frac{6}{5}y\right)$
12. $\left(9x+\frac{53}{4}\right)\left(x-\frac{3}{4}\right)$
13. $(11a-3b)(11b-3a)$
14. $\left(14x-\frac{23}{2}\right)\left(-2x+\frac{17}{2}\right)$
15. 121000
16. 348340
17. 1
18. 0.800

Exercise 2.11

1. $(a+b-c)(a+b+c)$ 2. $(a+3b+4c)(a+3b-4c)$ 3. $(a+b+3ab)(a+b-3ab)$
4. $(x-2y-3xy)(x-2y+3xy)$ 5. $(3a-b-4c)(3a-b+4c)$ 6. $(a+b+2)(a-b-2)$

Exercise 2.12

1. $(x+3)(x+2)$ 2. $(x+6)(x+4)$ 3. $(x-7)(x-8)$ 4. $(x+8)(x-5)$
5. $(2x+1)(2x+3)$ 6. $(4x-y)(x+y)$ 7. $(5x-9y)(2x+y)$ 8. $(x+9)(x+3)$
9. $(x-10)(2x+3)$ 10. $(3x-4y)(x-2y)$

Exercise 2.13

1. (i) $x^3+12x^2+48x+64$ (ii) $8m^3+12m^2+6m+1$ (iii) $a^3-6a^2b+12ab^2-8b^3$
(iv) $125x^3-75x^2+15x-1$ (v) $8a^3+12a^2b+6ab^2+b^3$ (vi) $27x^3+270x^2+900x+1000$
(vii) $8m^3+36m^2n+54mn^2+27n^3$ (viii) $64-144a+108a^2-27a^3$ (ix) $27x^3+81x^2y+81xy^2+27y^3$
(x) $343+294b+84b^2+8b^3$ (xi) $64x^3-96x^2y+48xy^2-8y^3$ (xii) $125m^3+300m^2n+240mn^2+64n^3$
2. 488 3. 36 4. 322 5. 14 6. (i) 2197 (ii) 1092727 (iii) 0.970299

Exercise 2.14

1. (i) Base = 2, Exponent = 6 (ii) Base = 1, Exponent = 9 (iii) Base = t, Exponent = 9
(iv) Base = m, Exponent = n
2. (i) 81 (ii) 343 (iii) 169 (iv) 256
3. (i) 5^4 (ii) 3^6 (iii) b^4 (iv) $u^3\times y^3$ (v) $7^2\times 11^4$ (vi) $a^3\times c^4\times d$
4. (i) 7^3 (ii) 2^9 (iii) 3^6 (iv) 5^5

Exercise 2.15

1. (i) $(10)^4$ (ii) $(2x-3y)^2$ (iii) 2^{12} (iv) $(ab)^6$ (v) 9^5 (vi) t^7 (vii) 3^0
(viii) 2^5a^3 (ix) a^{13} (x) $(15)^0$ (xi) 5^3 (xii) xy
2. (i) $\dfrac{1}{4x^3y^7}$ (ii) $\dfrac{y^8}{x^4}$ (iii) $3x^3$ (iv) $\dfrac{1}{9}$ (v) $4x^4$ (vi) $729a^6$
(vii) $4xy^3$ (viii) 4802 (ix) $9a^2b^2$

Exercise 2.16

1. (i) 51800000 (ii) 0.0000000009203 (iii) 216900000 (iv) 720000
(v) 0.0000054 (vi) 0.000000000054105
2. (i) 5.62×10^6 (ii) 1.4×10^{-6} (iii) 8.7402×10^8 (iv) 7.586×10^{15}
(v) 4.4605×10^8 (vi) 9.95×10^{-11}

Exercise 2.17

1. (i) $y=-2x-7$, slope $=-2$, $y-$intrcept $=(0,-7)$
(ii) $y=-\dfrac{1}{4}x+2$, slope $=-\dfrac{1}{4}$, $y-$intercept $=(0,2)$
(iii) $y=8x+5$, slope $=8$, $y-$intercept $=(0,5)$
(iv) $y=3x+4$, slope $=3$, $y-$intercept $=(0,4)$
(v) $y=5x+3$, slope $=5$, $y-$intercept $=(0,3)$

(vi) $y=-\frac{x}{4}+3$, slope $=-\frac{1}{4}$, $y-$intercept $=(0,\ 3)$

2. $y=-22$ 3. $y=9$ 4. $x=-7$

5. (i) $y=m-s$ (ii) $y=2x-3$ (iii) $y=12-x$ (iv) $y=\sqrt{7-x}$

(v) $y=2(4x-7)$ (vi) $y=5-\frac{3}{2}x$ (vii) $y=\frac{1}{3}(ax+b-2)$ (viii) $y=\frac{x}{2}-3$

6. $b=p-a-c$ and $b=23$ 7. $h=\frac{2A}{a+b}$ and $h=4$ 8. $x=\frac{2}{3}y+3$ and $x=5$ 9. $y=95$

Exercise 2.18

1. J in quadrant IV, K in quadrant I, L in quadrant II, M in quadrant III,

Exercise 2.19

1. (i) $x-y=26$ (ii) $6x=y$ (iii) $x+3y=25$ (iv) $\frac{x+y}{x-y}=1$ (v) $2x+7=y$

2 (i) $x+y=10$, $x-y=2$ (ii) $2x+y=54$, $x+2y=92$ (iii) $x+2y=45$, $2x+3y=80$

Exercise 2.20

1. (i) $\{(2,-1)\}$ (ii) $\{(1,\ 1)\}$ (iii) $\{(0,\ 1)\}$ (iv) $\{(4,\ 0)\}$

(v) $\left\{\left(\frac{30}{19},-\frac{18}{19}\right)\right\}$ (vi) $\{(5,\ 2)\}$

2. (i) $\left\{\left(\frac{8}{3},-\frac{1}{6}\right)\right\}$ (ii) $\left\{\left(\frac{7}{9},\ \frac{50}{9}\right)\right\}$

(iii) $\left\{\left(\frac{23}{25},-\frac{88}{25}\right)\right\}$ (iv) $\left\{\left(\frac{11}{19},-\frac{24}{19}\right)\right\}$ (v) $\left\{\left(\frac{5}{3},\frac{10}{3}\right)\right\}$ (vi) $\left\{\left(\frac{3}{5},\frac{9}{5}\right)\right\}$

3. (i) $\{(2,\ 2)\}$ (ii) $\{(1,\ 2)\}$ (iii) $\{(1,\ 2)\}$ (iv) $\left\{\left(1,-\frac{1}{2}\right)\right\}$

Exercise 2.21

1. 2 2. −16 3. 3, 2 4. 11, 7 5. Samia's age = 14 years, Amna's Age = 7 years.

6. Ahsan's age = 61 years, Shakeel's Age = 13 years. 7. $\frac{3}{8}$

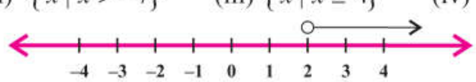
8. Price of melons = Rs. 50 per kg, price of mangoes = Rs. 80 per kg.

9. Football = Rs. 250, Basketball = Rs 180years. 10. $\frac{3}{5}$ 11. $\frac{4}{7}$

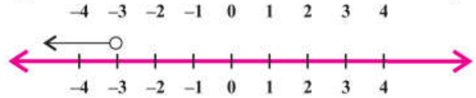
Exercise 2.22

1. (i) $\{x\,|\,x<5\}$ (ii) $\{x\,|\,x>-7\}$ (iii) $\{x\,|\,x\geq 4\}$ (iv) $\{x\,|\,x<-4\}$ (v) $\{x\,|\,x>6\}$ (vi) $\{x\,|\,x\geq -3\}$

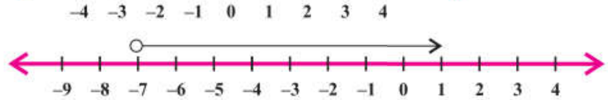
2. (i) $\{x\,|\,x>2\}$



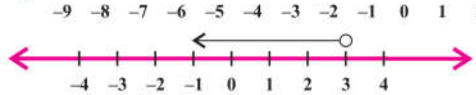
(ii) $\{x\,|\,x>-3\}$



(iii) $\{x\,|\,x>-7\}$



(iv) $\{x\,|\,x>3\}$



Review Exercise 2

1. (i) b (ii) b (iii) d (iv) d (v) d (vi) b (vii) a (viii) d (ix) b (x) c (xi) c (xii) c (xiii) b (xiv) c (xv) d
2. (i) 26,32 (ii) 27,81
3. (i) 10,12,14,16 (ii) 6,18,54,162
4. 43
5. 1040
6. (i) $3x(y+2xy+3z)$ (ii) $(y^2-6)^2$ (iii) $(x^4+y^4)(x^2+y^2)(x+y)(x-y)$ (iv) $(x+9)(x-7)$
7. (i) $11x^3-2x^2-35$ (ii) $10y^2-y-25$ (iii) x^2+x+2
8. (i) $\frac{16m^4}{n^4}$ (ii) $x^{12}y^8$ (iii) $6xy^2z^5$
9. (i) $y=x+\frac{7}{3}$, Slope = 1, y-intercept = $\left(0,\frac{7}{3}\right)$ (ii) $y=\frac{1}{2}x-\frac{7}{4}$, Slope = $\frac{1}{2}$, y-intercept = $\left(0,-\frac{7}{4}\right)$
10. $s=\frac{10-3t}{5}$
11. $x=\frac{4}{y}$ and $x=6$

Exercise 3.1

1. (i) $13cm$ (ii) $2\sqrt{7}cm$ (iii) $12cm$ (iv) $25cm$ (v) $24cm$ (vi) $10cm$
2. $7cm$
3. $8m$
4. $10.12m$
5. (i) is not right angled triangle
6. (i) $11cm$ (ii) $1cm$ (iii) $6\sqrt{5}m$ (iv) $10\sqrt{2}dm$ (v) $5\sqrt{5}dm$
7. $\sqrt{a^2-25}$

Exercise 3.2

1. L = $5.59cm$
2. L = $9.43cm$
3. $r=3.8cm$ or $r=\frac{12}{\pi}$
4. $x=900$
5. L = 15.88units
6. $32.06cm$
7. $15.71cm$
8. Arc length = $63.55cm$, Sector area = $889.77cm^2$
9. Area =1848ft^2
10. Central angle(x) = 2600
11. Area = 64.108 Sq. inches

Exercise 3.3

1. (a) $75cm^2$ (b) $96cm^2$ (c) $180cm^2$
2. $504cm^2$
3. $2133\frac{1}{3}cm^3$, $1095.96cm^2$
4. $125cm^2$
5. $160cm^2$
6. $130cm^2$
7. $320cm^3$
8. $8cm$
9. $54.93cm^2$
10. $21cm^2$
11. $122\frac{2}{3}$ inch^2
12. $30cm^2$
13. $61\frac{1}{3}cm^3$
14. $2552.13m^3$
15. $5cm$
16. $7.5cm$
17. $5cm$

Exercise 3.4

1. (i) $154cm^2$ (ii) $98.56m^2$ (iii) $0.55m^2$
2. (i) $3.5m$ (ii) $4.29m$ (iii) $4.95m$
3. (i) $817.6cm^3$ (ii) $2759.44cm^3$ (iii) $1437.33cm^3$ (iv) $164.70cm^3$
4. (i) $4cm$, $268.19cm^3$ (ii) $0.44cm$, $0.36cm^3$ (iii) $7m$, $1437.33m^3$
5. 1913030.67ℓ
6. (i) 4:1 (ii) 8:1
7. $V=2304\pi cm^3$, 13824(approx.)
8. $900cm$

Exercise 3.5

1. $942.86cm^2$ 2. $8ft$ 3. 24cm 4. $84.86cm^2$ 5. $5.276cm$ 6. (a) $718.66cm^3$
(b) $2425.5cm^3$ (c) $65.49cm^3$ 7. $15.12cm$ 8. $242.83cm^3$

Exercise 3.6

1. (i) 6, 188.57, 113.14, 301.701 (ii) 5, 47.14, 28.28, 75.42 (iii) 23.32, 707.14, 254.57, 961.701
(iv) 7, 7.14, 10, 220 2. (i) $37.7cm^3$ (ii) $513.33cm^3$ (iii) $125.7cm^3$ (iv) $201.35cm^3$
3. $83.79cm^3$(approx.) 4. $75.43cm^3$(approx.) 5. $134.75cm^3$ 6. 44

Exercise 3.7

1. (a) $363.62ft^3$, $1620ft^2$ (b) $127cm^3$, $480cm^2$ 2. (a) $13.6cm^3$ (b) $109.44ft^3$ 3. (a) $2590cm^2$
(b) $11775cm^3$ 4. (a) $50.29cm^2$ (b) $10.48cm^3$

Review Exercise 3

1. (i) b (ii) a (iii) c (iv) a (v) b (vi) d (vii) b (viii) a (ix) c
(x) a (xi) b (xii) d (xiii) a (xiv) d (xv) c
3. (i) $137.31cm^3$ (ii) $37.71cm^3$ (iii) $1cm$ (iv) (a) $5.23cm$, $13.08cm^2$ (b) $11.78cm$, $44.16cm^2$ (v) 87.8^o
4. $4m^3$ 5. $6cm$ 6. $7.5cm$ 7. $595cm^3$ 8. $894348ft^2$ 9. Rs.27130 10. $509805891km^2$ 11. $2.43cm$
12. $0.2355m^2$ 13. $44.2cm$ 14. $20.93m^3$ 15. $1256kg$

Exercise 4.1

1. Congruent shapes: C and F ; P and Q ; M and R ; B and L.
Similar shapes: S and U ; A and Q ; E and J; D and T ; K and N.
2. Congruent sides: $\overline{AB}$ and $\overline{AD}$; $\overline{BC}$ and $\overline{CD}$.
Congruent angles: $\angle BAC$ and $\angle DAC$; $\angle ABC$ and $\angle ADC$; $\angle ACB$ and $\angle ACD$
3. ΔAOD and ΔBOC ; ΔAOB and ΔDOC
$\overline{OD} \cong \overline{OB}; \overline{AD} \cong \overline{BC}; \overline{OA} \cong \overline{OC}$; $\overline{AB} \cong \overline{CD}; \overline{OB} \cong \overline{OD}; \overline{OA} \cong \overline{OC}$
4. $\overline{AC} \cong \overline{LN}; \overline{BC} \cong \overline{MN}$

Review Exercise 4

1. (i) b (ii) d (iii) b (iv) c (v) b (vi) c (vii) b (viii) d

Exercise 5.2

1.

Class limits	Tally marks	f	c.f
31 – 40	|||	3	3
41 – 50	||	2	2 + 3 = 5
51 – 60	卌 |||	8	8 + 5 = 13
61 – 70	卌 卌 |	11	11 + 13 = 24
71 – 80	卌	5	5 + 24 = 29
81 – 90	卌 ||	7	7 + 29 = 36
91 – 100	||||	4	4 + 36 = 40
Total		$\Sigma f = 40$	

2.

Class limits	Tally marks	f	c.f
41 – 50	||	2	2
51 – 60	卌 ||	7	7 + 2 = 9
61 – 70	卌 |	6	6 + 9 = 15
71 – 80	||||	4	4 + 15 = 19
81 – 90	卌 |	6	6 + 19 = 25
91 – 100	卌 卌	10	10 + 25 = 35
101 – 110	||||	4	4 + 35 = 39
111 – 120	||||	4	4 + 39 = 43
121 – 130	|||	3	3 + 43 = 46
131 – 140	||	2	2 + 46 = 48
141 – 150		0	0 + 48 = 48
151 – 160	||	2	2 + 48 = 50
Total		$\Sigma f = 40$	

3.

Class Limits	Tally Marks	f	c.f	Class Boundaries
40 – 44	|	1	1	39.5 – 44.5
45 – 49	|	1	1 + 1 = 2	44.5 – 49.5
50 – 54	|	1	1 + 2 = 3	49.5 – 54.5
55 – 59		0	0 + 3 = 3	54.5 – 59.5
60 – 64	||	2	2 + 3 = 5	59.5 – 64.5
65 – 69	|||	3	3 + 5 = 8	64.5 – 69.5
70 – 74	||	2	2 + 8 = 10	69.5 – 74.5
75 – 79	卌 卌	10	10 + 10 = 20	74.5 – 79.5
80 – 84	卌 |||	8	8 + 20 = 28	79.5 – 84.5
85 – 89	||||	4	4 + 28 = 32	84.5 – 89.5
90 – 94	|||	3	3 + 32 = 35	89.5 – 94.5
95 – 99	|	1	1 + 35 = 36	94.5 – 99.5
Total		$\Sigma f = 36$		

4.

Class Limits	Tally Marks	f	Class Boundaries
01 – 10	|	1	0.5 – 10.5
11 – 20	卌	5	10.5 – 20.5
21 – 30	卌 |||	8	20.5 – 30.5
31 – 40	卌 卌 卌	15	30.5 – 40.5
41 – 50	卌 卌 |	11	40.5 – 50.5
51 – 60	卌 ||	7	50.5 – 60.5
61 – 70	|||	3	60.5 – 70.5
Total		$\Sigma f = 50$	

5.

Class Limits	Tally Marks	f	c.f	Class Boundaries
01 – 07	卌 卌	10	10	0.5 – 7.5
08 – 14	卌 卌	10	10 + 10 = 20	7.5 – 14.5
15 – 21	卌 |||	8	8 + 20 = 28	14.5 – 21.5
22 – 28	|	1	1 + 28 = 29	21.5 – 28.5
29 – 35	||	2	2 + 29 = 31	28.5 – 35.5
36 – 42	||	2	2 + 31 = 33	35.5 – 42.5
43 – 49	||	2	2 + 33 = 35	42.5 – 49.5
50 – 56		0	0 + 35 = 35	49.5 – 56.5
57 – 63		0	0 + 35 = 35	56.5 – 63.5
64 – 70	|	1	1 + 35 = 36	63.5 – 70.5
Total		$\Sigma f = 36$		

Exercise 5.3

2. (i) $\overline{X} = 5.4$ (ii) $\overline{X} = 4.64$ (iii) $\overline{X} = 4.56$ (iv) $\overline{X} = 7.27$ (v) $\overline{X} = -0.91$
3. (i) $\overline{X} = 190$ (ii) $\overline{X} = -70$ (iii) $\overline{X} = 133.6$ 4. (i) $\Sigma X = 3750$ (ii) $z = 17.8$
5. $\overline{X} = 21.048$ 6. The mass of 10[th] students is $33.1kg$ 7. The total of height is $15m$
8. The length of 5[th] ribbon is $18.5m$. 10. (i) $\overline{X} = 76.2, \tilde{X} = 75; \hat{X} =$ No mode (ii) $\overline{X} = 72.9, \tilde{X} = 76.5; \hat{X} = 80$
(iii) $\overline{X} = 3.15, \tilde{X} = 3; \hat{X} = 3$ (iv) $\overline{X} = 105, \tilde{X} = 106; \hat{X} = 106, 107, 108$
11. $\overline{X} = 11.175;\ \tilde{X} = 10; \hat{X} = 1, 5\ and\ 10$ 13. Medium class = 3(Size of family) ; Medium class = 2(Size of family)
14. $\overline{X} = 7.04;\ \tilde{X} = 6.86; \hat{X} = 6.7$ 15. $\overline{X} = 105.81;\ \tilde{X} = 111.75; \hat{X} = 127.11$
16. $\overline{X} = 1954 marks;\ \tilde{X} = 19.55 marks; \hat{X} = 22.5 marks$ 17. (i) Modal number of children per family is 2.
(ii) Mean number of children per family is 2. (iii) Median number of children per family is 2.
18. $\overline{X} = 17.625;\ \tilde{X} = 17.56; \hat{X} = 17.44$ 19. $\overline{X}B > \overline{X}A$ (So, the class–B is better on average)
20. $a = 17$ $58.312 > 56.47$

Exercise 5.4

2. (i) Range = 9 , Var = 8.76, S.D = 2.96 (ii) Range = 15, Var = 23.5, S.D = 4.85
(iii) Range = 127, Var = 2080.27, S.D = 45.61 (iv) Range = 9.5, Var = 6.323, S.D = 2.515
3. Var = 1980, S.D = 44.5 4. $\Sigma X = 41.41$
5. $\overline{X}A = 34; \overline{X}B = 41$, Var(A) = 110.8; Var(B) = 11.6, S.D(A) = 10.53; S.D(B) = 3.41,
(i) Student "B" is better on average (ii) Student "B" is more consistent in his performance

Exercise 5.5

1.

Sr.No.	$n(A)$	$n(S)$	$P(A)$	$P'(A)$
(i)	5	15	$\frac{5}{15} = \frac{1}{3}$	$\frac{2}{3}$
(ii)	17	25	$\frac{17}{25}$	$\frac{8}{25}$
(iii)	1	6	$\frac{1}{6}$	$\frac{5}{6}$

2. $P(4) = \frac{1}{6}; P'(4) = \frac{5}{6}$ 3. $P(red) = \frac{1}{4}; P'(red) = \frac{3}{4}$ 4. (i) $P(22) = \frac{1}{8}$ (ii) $P(26) = \frac{1}{8}$ (iii) $P'(25) = \frac{7}{8}$
5. (i) $P(5) = \frac{1}{6}$ (ii) $P'(5) = \frac{5}{6}$ (iii) $P(3) = \frac{1}{6}$ 6. P(not win the match) = 0.14
7. P(temperature will not be 0.45) = 0.55 8. (i) $P(red) = \frac{20}{100}$ (ii) $P(blue) = \frac{22}{100}$ (iii) $P(red\ or\ orange) = \frac{43}{100}$
(iv) $P'(blue) = \frac{78}{100}$ (v) $P'(yellow) = \frac{87}{100}$ 9. (i) $P(boy) = \frac{7}{15}$ (ii) $P(girl) = \frac{8}{15}$

(iii) $P'(boy)=\frac{8}{15}$ (iv) $P'(girl)=\frac{7}{15}$ 10. (i) $P(red)=\frac{5}{10}$ (ii) $P(blue)=\frac{2}{10}$ (iii) $P(not\,blue)=\frac{8}{10}$

(iv) $P(not\,green)=\frac{7}{10}$ (v) $P(not\,red)=\frac{5}{10}$ 11. (i) $P(B)=\frac{5}{12}$ (ii) $P(not\,B)=\frac{7}{12}$ (iii) $P(D)=\frac{1}{12}$

(iv) $P(A)=\frac{3}{12}$ (v) $P(not\,D)=\frac{11}{12}$ (vi) $P(not\,C)=\frac{9}{12}$ 12. $P(A\cup C)=0.25+0.55=0.80;\ P(C\cup B)=0.83$

13. $P(\text{winning the game})=\frac{4}{6}$ 14. $P(4 \text{ or } 5)=\frac{2}{6}$ 15. (i) $P(\text{red pin or blue pin})=\frac{5}{15}$

(ii) $P(\text{green hairpin and black hairpin})=\frac{25}{225}$ 16. $P(\text{head and digit } 4)=\frac{1}{12}$

17. (i) $P(\text{green and blue})=\frac{12}{169}$ (ii) $P(\text{yellow and green marble})=\frac{20}{169}$ (iii) $P(\text{different colours})=\frac{104}{169}$

18. (i) $P(\text{red and black colour shirt})=\frac{2}{25}$ (ii) $P(\text{green and blue colour shirt})=\frac{3}{50}$

(iii) $P(\text{red and green colour shirt})=\frac{6}{25}$

19. (i) S = Sample space

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

(ii) (a) $P(\text{exactly two 6's})=\frac{1}{36}$ (b) $P(\text{exactly two 4's})=\frac{1}{36}$ (c) $P(\text{At least once times 4})=\frac{11}{36}$

20. (i)

Die/colour	1	2	3	4	5	6
Red (R)	(R,1)	(R,2)	(R,3)	(R,4)	(R,5)	(R,6)
Green (G)	(G,1)	(G,2)	(G,3)	(G,4)	(G,5)	(G,6)
Blue (B)	(B,1)	(B,2)	(B,3)	(B,4)	(B,5)	(B,6)
Yellow (Y)	(Y,1)	(Y,2)	(Y,3)	(Y,4)	(Y,5)	(Y,6)

(ii) (a) $P(\text{green on the spinner and the digit '2' on die})=\frac{1}{24}$

(b) $P(\text{green or yellow on the spinner and the digit '6' on die})=\frac{2}{24}$

(c) $P(\text{red or blue on the spinner and 2, 3 or 4 on the die})=\frac{6}{24}$

21. (i) S = Sample space

Die/colour	Small(S)	Medium(M)	Large(L)
Mango(M)	(M,S)	(M,M)	(M,L)
Strawberry(S)	(S,S)	(S,M)	(S,L)
Pistachio(P)	(P,S)	(P,M)	(P,L)

(ii) (a) *P(mango flavour with medium size)* $= \frac{1}{9}$ (b) *P(strawberry flavour with small or large size)* $= \frac{2}{9}$

(c) *P(pistachio flavour with medium or large)* $= \frac{2}{9}$

Review Exercise 5

1. (i) b (ii) c (iii) b (iv) b (v) a (vi) a (vii) c (viii) b (ix) c (x) b (xi) a (xii) c (xiii) c

(xiv) a 3. $\overline{X}(\textit{Average height}) = 56.8\,\textit{inches}$ 4. (i) $\overline{X} = 55$ (ii) $\overline{X} = 244$

5. Average price of drinking water = $\overline{X} = 77$ 6. (i) $\overline{X} = 5,\ \tilde{X} = 4;\ \hat{X} = 3$ (ii) $\overline{X} = 11,\ \tilde{X} = 11;\ \hat{X} = 12$

(iii) $\overline{X} = 5.5,\ \tilde{X} = 5;\ \hat{X} = 5$ (iv) $\overline{X} = 7;\ \tilde{X} = 7.5;\ \hat{X} = 8$ (v) $\overline{X} = 154.34;\ \tilde{X} = 154.3;\ \hat{X} = 152.3$

7. $\overline{X} = 15.7, \tilde{X} = 16.11;\ \hat{X} = 17.25$ 8. Range = 27; Var = 51.29; S.D = 7.16

9. (i) Company "B" is better based on average. (ii) Company "A" is more consistent in his performance.

10. $P(1) = \frac{1}{6}; P'(1) = \frac{5}{6}$ 11. (i) S = Sample Space = {1, 2, 4, 5, 5, 2, 3, 4, 2, 5} (ii) $P(\text{Even numbers}) = \frac{5}{10}$

(iii) $P(2) = \frac{3}{10}; P(1) = \frac{1}{10}; P(5) = \frac{3}{10}$ 12. $P(\text{do not use glasses}) = \frac{18}{30}$; $P(\text{who uses glasses}) = \frac{12}{30}$

13. (i) $P(12) = \frac{1}{20}$ (ii) $P(20) = \frac{1}{20}$ (iii) $P(not\ 20) = \frac{19}{20}$

14. (i) S = Sample space

Coin \ Die	1	2	3	4	5	6
H	(H,1)	(H_1,2)	(H_1,3)	(H_1,4)	(H_1,5)	(H_1,6)
T	(T,1)	(T_1,2)	(T_1,3)	(T_1,4)	(T_1,5)	(T_1,6)

(ii) (a) $P(\text{5 and tail}) = \frac{1}{12}$ (b) $P(\text{2, 4 or 6 and head}) = \frac{3}{12}$ (c) $P(\text{odd numbers and tail}) = \frac{3}{12}$

15. $P(A \cup B) = 0.85$ 16. (i) $P(A \cap B) = P(\text{both of them will be alive}) = 0.697$ (ii) $P(\text{both of them will be die}) = 0.303$

GLOSSARY

Absolute value (or modulus): The absolute value (or modulus) of a real number x (represented by $|x|$) is the non-negative value of x without keeping in view its sign.

Additive identity: If 'a' is a real number, there is a unique number zero (0) called additive identity such that

$$a + 0 = 0 + a = a$$

Approximation error: It is numerical value, which tells us how far is the approximated value from the actual / accurate value.

s.f.: stands for significant figure.

d.p.: stands for decimal place.

Arc length (L): Arc length of a sector of circle $= \dfrac{x}{360^\circ} \times$ circumference

Arc length: It is the distance along the curved line that makes up the arc.

Arc: A continuous part of the boundary/circumference of a circle is called an arc.

Area of a Sector of Circle: Area of a sector of circle $= \dfrac{x}{360^\circ} \times$ area of circle

Arithmetic sequence: A sequence of numbers such that the difference 'd' between each consecutive term is a constant is called arithmetic sequence.

$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$ Where a_1 is the first term of the sequence.

Cartesian plane: It is formed by two perpendicular number lines and is used to plot ordered pairs on it.

Centre of rotation: A rotation turns each point on a preimage around a fixed point, called the centre of rotation, a given angle measure.

Chord of the circle: A line segment joining two points on a circle is called a chord of the circle.

Circle & Centre of the Circle: A circle is a plane figure bounded by one curved line, and such that all straight lines drawn from a certain point within it to the bounding line, are equal. That point is called the Centre of the Circle.

Closure property: If we take two real numbers, then their sum and product is also a real number. It is called closure property.

Coefficient: In a term the number multiplied by the variable is called the coefficient of the variable as well as constant.

Complement of a set: It is defined as a set that contains the elements present in the universal set but not in set A.

Compound proportion: The relationship between two or more proportions is known as compound proportion.

Continued proportion: If three quantities a, b and c are written as: $a : b :: b : c$ then these quantities are in continued proportion and b is called the mean proportional.

Cube of a number: means to multiply the number by itself three times.

Cube root of a number: It is denoted by $\sqrt[3]{\ }$, is a number such that $a^3 = x$. i.e., $a = \sqrt[3]{x}$

Cumulative frequency: It is calculated by adding each frequency from a frequency distribution to the sum of its predecessors.

Data: The collection of information in the form of facts and figures is called data

Difference of two sets: We can write A-B as A\B for difference of two sets.

Direct proportion: Direct proportion is a relation in which increases / decreases one quantity causes proportional increases / decreases in the other quantity.

Enlargement: Enlarge a figure with the given scale factor (positive or negative).

- The enlargement of a figure with the given scale factor (positive or negative).
- Locating the centre and scale factor of enlargement given the original figure and its enlargement.

Equal sets: Two sets are equal if they contain the same elements.

Equation: A sentence that shows equality (=) between two expressions is called an equation. The equation has fixed and singular solution.

Equivalent sets: Equivalent sets are the sets with an equal number of elements. These sets do not have exactly the same elements, just these sets have the same number of elements.

Event: The result of an experiment is called an event.

Experimental probability: It is obtained by performing an experiment.

Factorization: Expressing polynomials as product of two or more polynomials that cannot be further expressed as product of factors is called factorization.

Foreign currency exchange rate: It is a price that represents how much it costs to buy the currency of one country using the currency of another country.

Fourth proportional: If four quantities a, b, c and d are written as: $a : b :: c : d$ then a is called the first, b is called the second, c is called the third and d is called the fourth proportional.

Frequency distribution: To write a data in the form of a table in such a way that the frequency of each class can be observed at once is called frequency distribution.

Frequency: The number of times a value occurs in a data is called the frequency of that value.

Geometric sequence: It is a sequence in which each term is obtained by multiplying a non-zero constant 'r' to the previous term.

$a_1, a_1r, a_1r^2, a_1r^3, \ldots$ Where 'a_1' is he first term and 'r' the common ratio.

Grouped data: Data which is arranged in systematic order/groups or classes in called grouped data.

Identity Congruence: Every triangle is congruent to itself (in the correspondence in which its sides and angles correspond to themselves). Such a congruence is called "Identity Congruence".

Inequality: It is a statement that contains one of the symbols: $<, >, \leq$ or $\geq$. The inequality has infinite answers.

Inheritance: When a person dies, then the assets left by him are called inheritance and it is distributed among his legal inheritors according to Islamic Shariah Law.

Inverse proportion: Inverse proportion is a relation in which if one quantity increases / decreases then other quantity decreases / increases proportionally.

Irrational numbers: The numbers which are not rational are called irrational numbers. The set of irrational numbers is represented by Q'.

Kinds of proportion: (i) Direct proportion (ii) Inverse proportion

Life insurance: It is an agreement between the policy owner and the insurance company for an agreed time period.

Linear equation: If a, b and c are real numbers and a and b are not both zero, then $ax + by = c$ is called linear equation in two variables.

Loss percentage: Loss percentage $= \dfrac{\text{loss}}{\text{cost price}} \times 100$

Loss: If the cost price (C.P.) is higher than the selling price (S.P.), then loss occurs.

Markup: When we borrow money from bank to run a business, the bank in return receives some extra amount along with the actual money given. This extra money which the bank receives is known as markup.

Mean: It is calculated by adding all the values and then dividing by the number of values.

Median of the data: If a data is arranged in ascending or descending order then median of the data is:

- The middle value of the data, if it consists of odd number of values.
- The mean of the two middle values of the data, if the number of values in data is even.

Members or elements: The objects of a set are called its members or elements.

Mode: It is the value that occurs most frequently in a data.

Multiplicative identity: '1', is called multiplicative identity, such that $a.1 = a = 1.a$

Number sequence: is a list of numbers arranged in an order.

Open sentence: It is neither true nor false until the variables or unknowns have been replaced by specific values.

Ordered pair: It is a composition of two elements that are separated by a comma and written inside the parentheses.

Partnership: A business in which two or more persons run the business and they are responsible for the profit and loss is called the partnership.

Percentage discount: Percentage discount $= \dfrac{\text{discount}}{\text{marked price}} \times 100$

Perfect cube: is a number that is the result of multiplying an integer by itself three times. In other words, it is an integer to the third power of another integer.

Period: The time for which a particular amount is invested in a business is known as period.

Pie chart: It is used for comparison of values of different items by making the sectors of a circle. Angles are used to represent the sectors

Polynomial expression or simply a polynomial: is an algebraic expression consisting of one or more terms in each of which the exponent of the variable is zero or a positive integer.

Polynomials: In which the highest exponent or sum of exponents is always 2.

Position to term rule: defines the value of each term with respect to its position.

Principal amount: The amount we borrow or deposit in the bank is called principal amount.

Probability: It tells us how likely it is that an event will occur.

Profit / markup rate: The rate at which the bank gives share to its account holders is known as profit / markup rate. It is expressed in percentage.

Profit / markup: Profit / markup = Principal × Time × Rate

Profit percentage: $\text{Profit percentage} = \frac{\text{profit}}{\text{cost price}} \times 100$

Profit: If the selling price (S.P.) is higher than the cost price (C.P.) then profit occurs.

Properties of real numbers: It holds the following properties:
(i) Closure property (ii) Commutative property (iii) Associative property (iv) Identity
(v) Inverse (vi) If we cannot find the number whose square is $\sqrt{x}$, then x is an irrational number.

Proportion: The relation of equality of two ratios is called proportion, It is denoted by "::".

Pyramid: A 3D shape with a flat polygon base and three or more triangular sides converging at the top, is called Pyramid.

Radical sign: Square root and square are inverse of each other. The symbol used for square root is "$\sqrt{\ }$". It is called radical sign.

Ratio: It is a comparison of two quantities of same kind. Ratio is denoted by ":" e.g., ratio "a" is to "b" is written as $a:b$, ratio "c" is to "d" is written as $c:d$.

Rational numbers: The members which can be written in the form of $\frac{p}{q}, q \neq 0, p, q \in z$ are called rational numbers.

Real numbers: All the numbers (N, W, Z, all rational and irrational numbers) are real numbers. It is represented by $\mathbb{R}$.

Recurring decimal numbers: The decimal numbers in which one digit or group of digits repeat again and again in its decimal part are called recurring decimal numbers.

Rotation: Rotate an object and find the centre of rotation by construction

Sample space: The set of all possible outcomes is called sample space. It is denoted by "S".

Scientific Notation: It is a way of expressing very large and very small numbers conveniently.

Secant line: A straight line that intersects a circle in two points is called a secant line.

Sector of a circle: It is defined as the portion of a circle that is enclosed between its two radii and the arc adjoining them.

Sequence: It goes on forever is called an infinite sequence. Otherwise, it is called finite sequence.

Set expression: A set can be expressed in three ways.
(a) Tabular Form or Roster Form (b) Descriptive Form (c) Set Builder Notation

Similar Shapes: Two shapes are said to be mathematically similar if all of the angles in the shapes are equal, but the shapes are not necessarily of the same size.

Square of the number: When a number is multiplied by itself then the product is known as the square of the number i.e., the square of x is x^2

Surface area of a hemisphere: $\text{Surface area of a hemisphere} = \frac{1}{2} \times \text{surface area of sphere} + \text{area of base}$

Symbol of cube root: It is $\sqrt[3]{\ }$ remember that 3 is the part of symbol.

Term to term rule: It is not helpful if you want to find a term that is far away from the ones that are known. Position to term rule in more powerful.

Terminating decimal numbers: These are the decimals which have finite digits in its decimal parts.

Theoretical probability: It is obtained without performing any experiment.

Total surface area of a cone: Total surface area of a cone = Base area + curved surface area

$$= \pi r^2 + \pi r\ell$$
$$= \pi r(r + \ell)$$

Types of Rational numbers: These are two:

(i) Terminating decimals (ii) Recurring

Universal set: A set that consists of all the elements of the sets under consideration, including its own elements is called universal set. It is denoted by the symbol U.

Vehicle insurance: It provides a protection against risks to the vehicle. The amount of policy in this case depends upon the actual value of the vehicle.

Venn diagram: It is an illustration that uses circles to show the relationships among sets in a perspective way. In Venn diagram, a universal set is usually represented by a rectangle and its subsets are represented by closed figures inside the rectangle.

Volume of a cone: Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of hemi-sphere: Volume of hemi-sphere $= \frac{\text{Volume of sphere}}{2}$

Volume of Pyramid: Volume of Pyramid $= \frac{1}{3} \times$ volume of prism

Symbols / Notations

Symbol	Stands for	Symbol	Stands for
$<$	is less than	$:$	ratio
$>$	is greater than	$::$	is proportional to
$\leq$	is less than or equal to	$\mid$	tally mark
$\geq$	is greater than or equal to	$\sum$	summation
$=$	is equal to	$\overline{AB}$	line segment AB
$\neq$	is not equal to	$\overrightarrow{AB}$	ray AB
$\nless$	is not less than	$\overleftrightarrow{AB}$	line
$\ngtr$	is not greater than	$\angle$	angle
$\in$	belongs to	Δ	triangle
$\notin$	not belongs to	$\sim$	is similar to
$\forall$	for all	$\cong$	is congruent to
$\sqrt{\ }$	square root	$\approx$	is approximately equal to
$\Rightarrow$	implies that	$\parallel$	is parallel to
$\cup$	union	$\widehat{AB}$	arc AB
$\cap$	intersection	$\leftrightarrow$	correspondence
$\because$	because / as	$\%$	percentage
$\therefore$	therefore / so	ϕ or $\{\ \}$	the empty set / the null set
U	universal set	$\mid$	such that