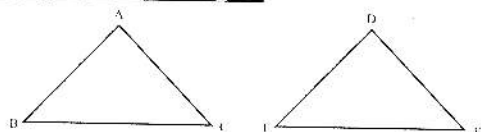


CONGRUENT TRIANGLES

Congruent Triangle



Let there be two triangles ABC and DEF. Out of the total six (1 - 1) correspondences that can be established between $\triangle ABC$ and $\triangle DEF$. One of the choices is explained below.

In the correspondence $\triangle ABC \leftrightarrow \triangle DEF$ it means.

$\angle A \leftrightarrow \angle D$ ($\angle A$ corresponds to $\angle D$)

$\angle B \leftrightarrow \angle E$ ($\angle B$ corresponds to $\angle E$)

$\angle C \leftrightarrow \angle F$ ($\angle C$ corresponds to $\angle F$)

$\overline{AB} \leftrightarrow \overline{DE}$ (\overline{AB} corresponds to \overline{DE})

$\overline{BC} \leftrightarrow \overline{EF}$ (\overline{BC} corresponds to \overline{EF})

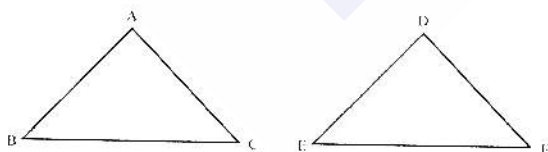
$\overline{CA} \leftrightarrow \overline{FD}$ (\overline{CA} corresponds to \overline{FD})

Congruency of Triangles

Two triangles are said to be congruent written symbolically as, \cong , if there exists a correspondence between them such that all the corresponding sides and angles are congruent i.e.

If $\begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases}$ and $\begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$

Then $\triangle ABC \cong \triangle DEF$



Note

(i) These triangles are congruent w.r.t. the above mentioned choice of the (1 - 1) correspondence.

(ii) $\triangle ABC \cong \triangle ABC$

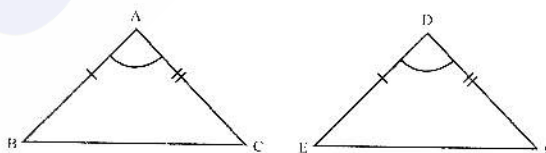
(iii) $\triangle ABC \cong \triangle DEF \Leftrightarrow \triangle DEF \cong \triangle ABC$

(iv) If $\triangle ABC \cong \triangle DEF$ and $\triangle ABC \cong \triangle PQR$, then $\triangle DEF \cong \triangle PQR$

In any correspondence of two triangles, if two sides and their included angle of one triangle are congruent to the corresponding two sides and their included angle of the other, then the triangles are congruent.

In $\triangle ABC \leftrightarrow \triangle DEF$, shown in the following figure.

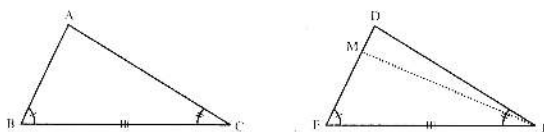
If $\begin{cases} \overline{AB} \cong \overline{DE} \\ \angle A \cong \angle D \\ \overline{AC} \cong \overline{DF} \end{cases}$



Then $\triangle ABC \cong \triangle DEF$ (S.A.S. Postulate)

Theorem

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding, side and angles of the other, then the triangles are congruent. (A.S.A \cong A.S.A)



Given

In $\triangle ABC \leftrightarrow \triangle DEF$

$$\angle B \cong \angle E$$

$$\overline{BC} \cong \overline{EF}$$

Proof

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle MEF$	Construction
$\overline{AB} \cong \overline{ME}$(i)	Given
$\overline{BC} \cong \overline{EF}$(ii)	Given
$\angle B \cong \angle E$(iii)	S.A.S. postulate
$\therefore \triangle ABC \cong \triangle MEF$	(Corresponding angles of congruent triangles)
So, $\angle C \cong \angle MFE$	Given
But $\angle C \cong \angle DFE$	Both congruent to $\angle C$
$\therefore \angle DFE \cong \angle MFE$	
This is possible only if D and M are the same points, and $\overline{ME} \cong \overline{DE}$	
So, $\overline{AB} \cong \overline{DE}$(iv)	$\overline{AB} \cong \overline{ME}$ (construction) and
Thus from (i), (iii) and (iv), we have	$\overline{ME} \cong \overline{DE}$ (proved)
$\triangle ABC \cong \triangle DEF$	S.A.S. postulate

Example

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the correspondence side and angles of the other, then the triangles are congruent. (S.A.A \cong S.A.A.)

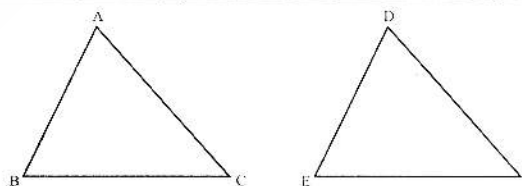
$$\angle C \cong \angle F$$

To prove

$$\triangle ABC \leftrightarrow \triangle DEF$$

Construction

Suppose $\overline{AB} \not\cong \overline{DE}$, take a point M on \overline{DE} such that $\overline{AB} \cong \overline{ME}$. Join M to F



Given

In $\triangle ABC \leftrightarrow \triangle DEF$

$$\overline{BC} \cong \overline{EF}, \angle A \cong \angle D, \angle B \cong \angle E$$

To Prove

$$\triangle ABC \cong \triangle DEF$$

Proof

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle DEF$ $\angle B \cong \angle E$ $\overline{BC} \cong \overline{EF}$ $\angle C \cong \angle F$ $\therefore \triangle ABC \cong \triangle DEF$	Given Given $\angle A \cong \angle D, \angle B \cong \angle E, (\text{Given})$ A.S.A. \cong A.S.A

Example

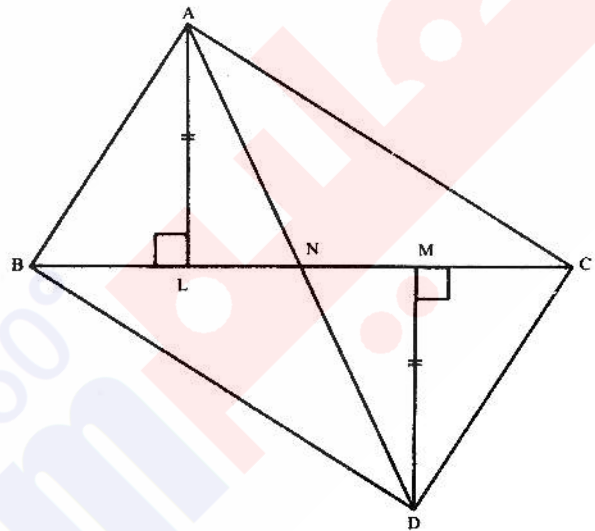
If $\triangle ABC$ and $\triangle DCB$ are on the opposite sides of common base \overline{BC} such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$, then \overline{BC} bisects \overline{AD} .

Given

$\triangle ABC$ and $\triangle DCB$ are on the opposite sides of \overline{BC} such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$ and \overline{AD} is cut by \overline{BC} at N .

To Prove

$\overline{AN} \cong \overline{DN}$



Statements	Reasons
In $\triangle ALN \leftrightarrow \triangle DMN$ $\overline{AL} \cong \overline{DM}$ $\angle ALN \cong \angle DMN$ $\angle ANL \cong \angle DNM$ $\triangle ALN \cong \triangle DMN$ Hence $\overline{AN} \cong \overline{DN}$	Given Each angle is right angle Vertical angles S.A.A. \cong S.A.A Corresponding sides of \cong Δ s.

Exercise 10.1

1. In the given figure.

$$\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$$

Prove that

$$\triangle ABD \cong \triangle CBE$$

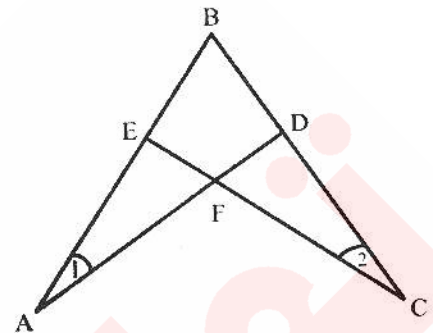
Given

$$\overline{AB} \cong \overline{CB}$$

$$\angle 1 = \angle 2$$

To Prove

$$\triangle ABD \cong \triangle CBE$$



Statements	Reasons
<p>In $\triangle ABD \leftrightarrow \triangle CBE$</p> <p>$\overline{AB} \cong \overline{CB}$</p> <p>$\angle 1 \cong \angle 2$</p> <p>$\angle ABD \cong \angle CBE$</p> <p>$\therefore \triangle ABD \cong \triangle CBE$</p>	<p>Given</p> <p>Given</p> <p>Common angle</p> <p>A.S.A \cong A.S.A</p>

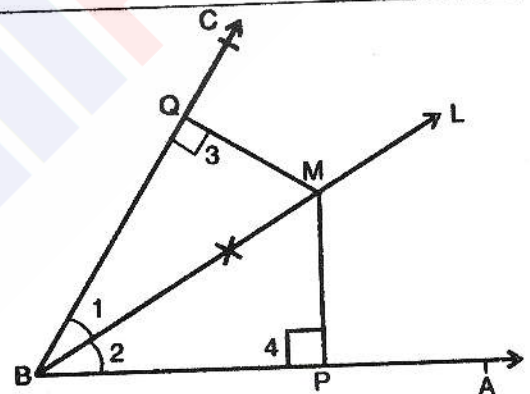
- (2) From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

Given

$\angle ABC$, \overline{BL} the bisector of $\angle ABC$, M any point on \overline{BL} , \overline{MP} perpendicular on \overline{AB} , $\overline{MQ} \perp \overline{BC}$.

To Prove

$$\overline{MP} \cong \overline{MQ}$$



Statements	Reasons
<p>In $\triangle BMP \leftrightarrow \triangle BMQ$</p> <p>$\angle 1 \cong \angle 2$</p> <p>$\angle 3 \cong \angle 4$</p> <p>$\overline{BM} \cong \overline{BM}$</p> <p>$\triangle BMP \cong \triangle BMQ$</p> <p>$\overline{PM} \cong \overline{QM}$</p>	<p>\overline{BL} bisects $\angle PBQ$</p> <p>Each = 90°</p> <p>Common</p> <p>A.S.A \cong A.S.A</p> <p>Corresponding sides of the congruent triangles.</p>

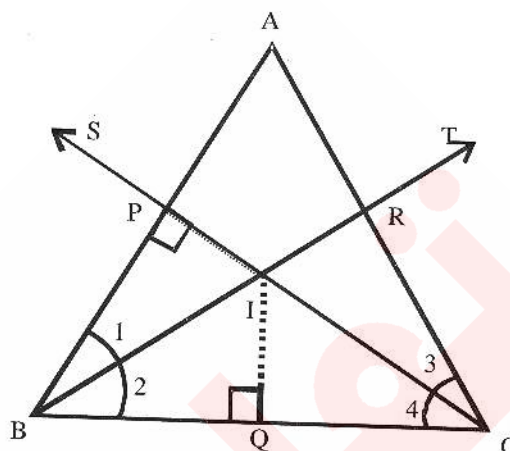
(3) In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of $\triangle ABC$.

Given

In $\triangle ABC$, \overline{BT} , \overline{CS} are the bisectors of the angles B and C respectively.

To Prove

I is equidistant from the three sides of $\triangle ABC$ i.e. $\overline{IP} \cong \overline{IQ} \cong \overline{IR}$



Construction

$\overline{IR} \perp \overline{AC}$, $\overline{IQ} \perp \overline{BC}$, $\overline{IP} \perp \overline{AB}$

Statements	Reasons
In $\triangle IPB \leftrightarrow \triangle IQB$	
$\angle 1 \cong \angle 2$	Given
$\angle P \cong \angle Q$	Each = 90°
$\overline{IB} \cong \overline{IB}$	Common
$\triangle IPB \cong \triangle IQB$	A.S.A \cong A.S.A
$\overline{IP} \cong \overline{IQ}$(i)	Corresponding sides of congruent triangles
Similarly $\triangle IRC \cong \triangle IQC$	
$\overline{IQ} \cong \overline{IR}$(ii)	Corresponding sides of congruent triangles
$\overline{IP} \cong \overline{IQ} \cong \overline{IR}$	By (i) and (ii)

Theorem

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.

Given

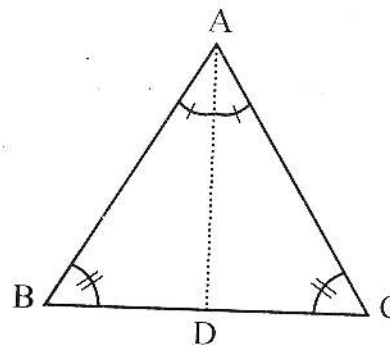
In $\triangle ABC$, $\angle B \cong \angle C$

To Prove

$\overline{AB} \cong \overline{AC}$

Construction

Draw the bisector of $\angle A$, meeting \overline{BC} at the point D.



Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore \triangle ABD \cong \triangle ACD$	S.A.A. \cong S.A.A.
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

Example

If one angle of a right triangle is of 30° , the hypotenuse is twice as long as the side opposite to the angle.

Given

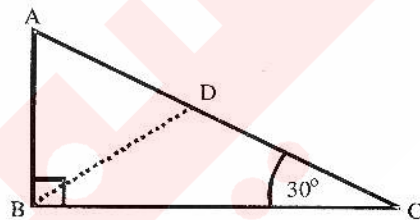
In $\triangle ABC$, $m\angle B = 90^\circ$ and $m\angle C = 30^\circ$

To Prove

$$m\overline{AC} = 2m\overline{AB}$$

Construction

At B, construct $\angle CBD$ of 30° . Let \overline{BD} cut \overline{AC} at the point D.

**Proof**

Statements	Reasons
In $\triangle ABD$, $m\angle A = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC - m\angle CBD = 60^\circ$	
$\therefore m\angle ADB = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle CBD = 30^\circ$
$\therefore \triangle ABD$ is equilateral	Sum of measures of \angle s of a \triangle is 180°
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{AD}$	Each of its angles is equal to 60°
In $\triangle BCD$, $\overline{BD} \cong \overline{CD}$	Sides of equilateral \triangle
Thus	$\angle C = \angle CBD$ (each of 30°).
$\left. \begin{aligned} m\overline{AC} &= m\overline{AD} + m\overline{CD} \\ &= m\overline{AB} + m\overline{AB} \\ &= 2(m\overline{AB}) \end{aligned} \right\}$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$

Example

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.

Given

In $\triangle ABC$, \overline{AD} bisects $\angle A$ and $\overline{BD} \cong \overline{CD}$

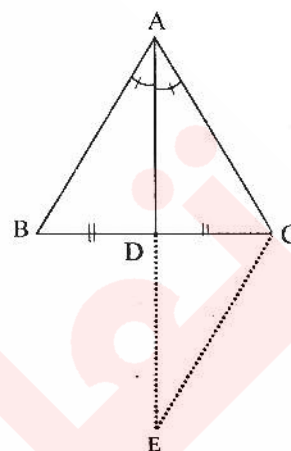
To Prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce \overline{AD} to E, and take $\overline{ED} \cong \overline{AD}$.

Join C to E

Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore \triangle ADB \cong \triangle EDC$	S.A.S. Postulate
$\therefore \overline{AB} \cong \overline{EC} \dots\dots\dots(1)$	Corresponding sides of $\cong \Delta s$
and $\angle BAD \cong \angle E$	Corresponding angles of $\cong \Delta s$
But $\angle BAD \cong \angle CAD$	Given
$\therefore \angle E \cong \angle CAD$	Each $\cong \angle BAD$
In $\triangle ACE$, $\overline{AC} \cong \overline{EC} \dots\dots\dots(2)$	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (1) and (2)

Exercise 10.2

Prove that a point, which is equidistant from the end points of a line segment, is on the right bisector of the line segment.

Given

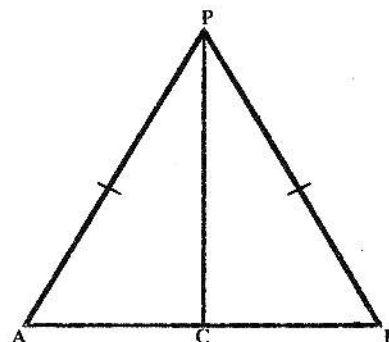
\overline{AB} is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$

To Prove

Point P is on the right bisector of \overline{AB} .

Construction

Join P to C, the midpoint of \overline{AB}



Proof

Statements	Reasons
In $\triangle ACP \leftrightarrow \triangle BCP$	Given
$\overline{PA} \cong \overline{PB}$	Common
$\overline{PC} \cong \overline{PC}$	Construction
$\overline{AC} \cong \overline{BC}$	S.S.S \cong S.S.S
$\triangle ACP \cong \triangle BCP$	Corresponding angles of congruent triangles
$\angle ACP \cong \angle BCP$(i)	supplementary angles,
But $m\angle ACP + m\angle BCP = 180^\circ$... (ii)	From (i) and (ii)
$m\angle ACP = m\angle BCP = 90^\circ$	$m\angle ACP = 90^\circ$ (proved)
or $\overline{PC} \perp \overline{AB}$(iii)	construction
Also $\overline{CA} \cong \overline{CB}$(iv)	
$\therefore \overline{PC}$ is a right bisector	from (iii) and (vi)
Of \overline{AB} i.e, the point P is on the right bisector of \overline{AB} .	

Theorem

In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent.

(S.S.S. \cong S.S.S.)

Given

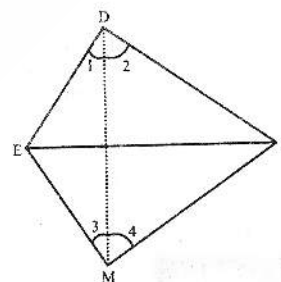
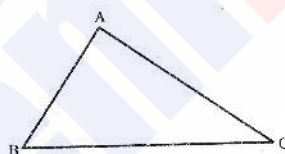
In $\triangle ABC \leftrightarrow \triangle DEF$
 $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove

$\triangle ABC \cong \triangle DEF$

Construction

Suppose that in $\triangle DEF$ the side \overline{EF} is not smaller than any of the remaining two sides. On \overline{EF} construct a $\triangle MEF$ in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. As shown in the above figures we label some of the angles as 1, 2, 3 and 4.



Proof

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore \triangle ABC \cong \triangle MEF$	S.A.S postulate
and $\overline{CA} \cong \overline{FM}$(i)	(Corresponding sides of congruent triangles)
Also $\overline{CA} \cong \overline{FD}$(ii)	Given
$\therefore \overline{FM} \cong \overline{FD}$	From (i) and (ii)
In $\triangle FDM$	
$\angle 2 \cong \angle 4$(iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly $\angle 1 \cong \angle 3$(iv)	
$\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	{ from (iii) and (iv) }
$\therefore m\angle EDF = m\angle EMF$	
Now, In $\triangle DEF \leftrightarrow \triangle MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
And $m\angle EDF \cong m\angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one $\cong \overline{AB}$
$\therefore \triangle DEF \cong \triangle MEF$	S.A.S postulate
Also $\triangle ABC \cong \triangle MEF$	Proved
Hence $\triangle ABC \cong \triangle DEF$	Each $\triangle \cong \triangle MEF$ (Proved)

Example

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

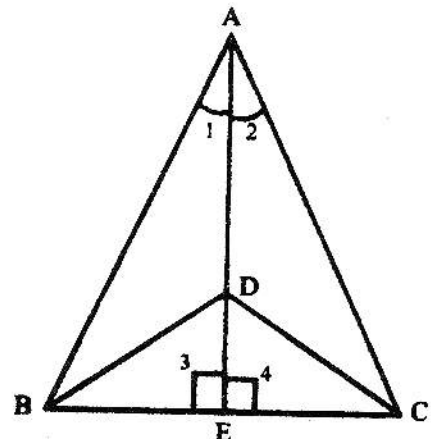
Given

$\triangle ABC$ and $\triangle DBC$ are formed on the same side of \overline{BC} such that

$\overline{AB} \cong \overline{AC}$, $\overline{DB} \cong \overline{DC}$, \overline{AD} meets \overline{BC} at E .

To prove

$\overline{BE} \cong \overline{CE}$, $\overline{AE} \perp \overline{BC}$



Proof

Statements	Reasons
In $\triangle ADB \leftrightarrow \triangle ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \triangle ADB \cong \triangle ADC$	S.S.S. \cong S.S.S.
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of $\cong \Delta$ s
In $\triangle ABE \leftrightarrow \triangle ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\overline{AE} \cong \overline{AE}$	Common
$\therefore \triangle ABE \cong \triangle ACE$	S.A.S. postulate
$\therefore \overline{BE} \cong \overline{CE}$	Corresponding sides of $\cong \Delta$ s
$\angle 3 \cong \angle 4$I	Corresponding angles of $\cong \Delta$ s
$m\angle 3 + m\angle 4 = 180^\circ$II	Supplementary angles Postulate
$\therefore m\angle 3 = m\angle 4 = 90^\circ$	From I and II
Hence $\overline{AE} \perp \overline{BC}$	

Corollary: An equilateral triangle is an equiangular triangle.

Exercise 10.3

- Q1. In the figure, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$.
Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$.

Given

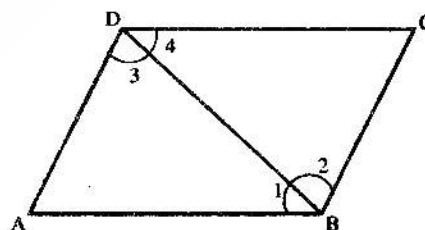
$\overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$

To prove

$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle CBD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given

$\overline{BD} \cong \overline{BD}$ $\therefore \triangle ABD \cong \triangle CBD$ $\angle A \cong \angle C$ $\angle 1 \cong \angle 4 \dots (i)$ $\angle 2 \cong \angle 3 \dots (ii)$ $\angle 1 + \angle 2 = \angle 3 + \angle 4$ $\angle ABC \cong \angle ADC$	Common S.S.S \cong S.S.S Corresponding angles of congruent triangles Corresponding angles of congruent triangles Adding (i) and (ii)
---	--

2. In the figure, $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$.
Prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$.

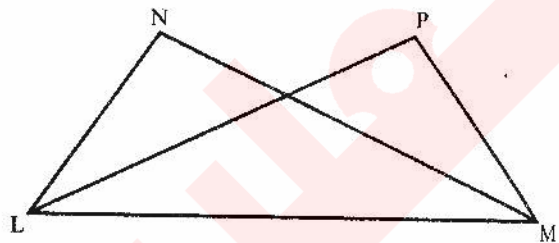
Given

$$\overline{LN} \cong \overline{MP}$$

$$\overline{LP} \cong \overline{MN}$$

To prove

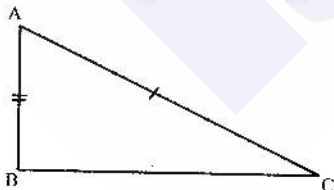
$$\angle N \cong \angle P, \quad \angle NML \cong \angle PLM$$



Statements	Reasons
In $\triangle LMN \leftrightarrow \triangle LPM$	
$\overline{LM} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{LM}$	Common
$\triangle LMN \cong \triangle LPM$	S.S.S \cong S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

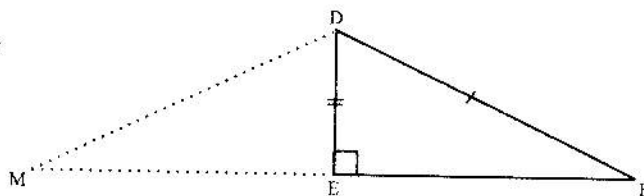
Theorem

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent. (H.S \cong H.S)



Given

$$\text{In } \triangle ABC \leftrightarrow \triangle DEF$$



$$\angle B \cong \angle E \text{ (right angles)}$$

$$\overline{CA} \cong \overline{FD}, \quad \overline{AB} \cong \overline{DE}$$

To Prove $\triangle ABC \cong \triangle DEF$

Construction

Produce \overline{FE} to a point M such that $\overline{EM} \cong \overline{BC}$ and join the points D and M.

Proof

Statements	Reasons
In $m\angle DEF + m\angle DEM = 180^\circ \dots (i)$	(Supplementary angles)
Now $m\angle DEF = 90^\circ \dots (ii)$	(Given)
$\therefore m\angle DEM = 90^\circ$	{from (i) and (ii)}
In $\triangle ABC \leftrightarrow \triangle DEM$	
$\overline{BC} \cong \overline{EM}$	(construction)
$\angle ABC \cong \angle DEM$	(each \angle equal to 90°)
$\overline{AB} \cong \overline{DE}$	(given)
$\therefore \triangle ABC \cong \triangle DEM$	S.A.S. postulate
And $\angle C \cong \angle M$	(Corresponding angles of congruent triangles)
$\overline{CA} \cong \overline{MD}$	(Corresponding sides of congruent triangles)
But $\overline{CA} \cong \overline{FD}$	(given)
$\therefore \overline{MD} \cong \overline{FD}$	
In $\triangle DMF$	Each is congruent to \overline{CA}
$\angle F \cong \angle M$	$\overline{FD} \cong \overline{MD}$ (Proved)
But $\angle C \cong \angle M$	(proved)
$\angle C \cong \angle F$	(each is congruent to $\angle M$)
In $\triangle ABC \leftrightarrow \triangle DEF$	
$\overline{AB} \cong \overline{DE}$	(given)
$\angle ABC \cong \angle DEF$	(given)
$\angle C \cong \angle F$	(proved)
Hence $\triangle ABC \cong \triangle DEF$	(S.A.A \cong S.A.A)

Example

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.

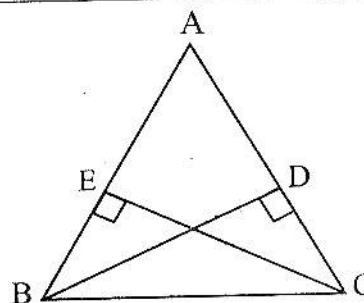
Given

In $\triangle ABC$, $\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To Prove

$\overline{AB} \cong \overline{AC}$

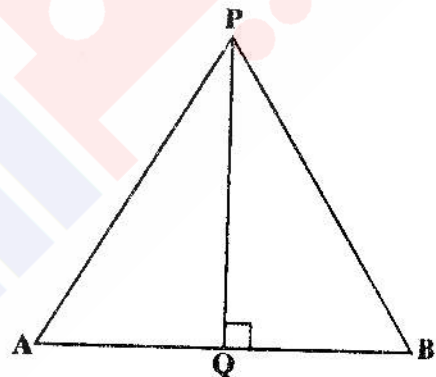


Proof

Statements	Reasons
In $\triangle ABCD \leftrightarrow \triangle CBE$ $\angle BDC \cong \angle BEC$	$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ (given) \Rightarrow each angle = 90° Common hypotenuse
$\overline{BC} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{CE}$	H.S. \cong H.S.
$\triangle ABCD \cong \triangle CBE$	Corresponding angles of $\cong \Delta$ s.
$\angle BCD \cong \angle CBE$	
Thus $\angle BCA \cong \angle CBA$	
Hence $\overline{AB} \cong \overline{AC}$	In $\triangle ABC, \angle BCA \cong \angle CBA$

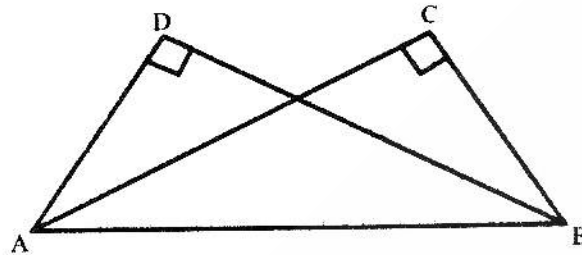
Exercise 10.4

1. In $\triangle PAB$ of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$, prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$.

GivenIn $\triangle PAB, \overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ **To Prove** $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$ **Proof**

Statements	Reasons
In $\triangle APQ \leftrightarrow \triangle BPQ$ $\overline{PA} \cong \overline{PB}$ $\overline{PQ} \cong \overline{PQ}$	Given Common
$\therefore \triangle PAQ \cong \triangle PBQ$	H.S. \cong H.S.
$\therefore \overline{AQ} \cong \overline{BQ}$ $\angle APQ \cong \angle BPQ$	Corresponding sides of congruent triangles Corresponding angles of the congruent triangles.

2. In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$. Prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$.



Given

$$m\angle C = m\angle D = 90^\circ$$

$$\overline{BC} \cong \overline{AD}$$

To Prove

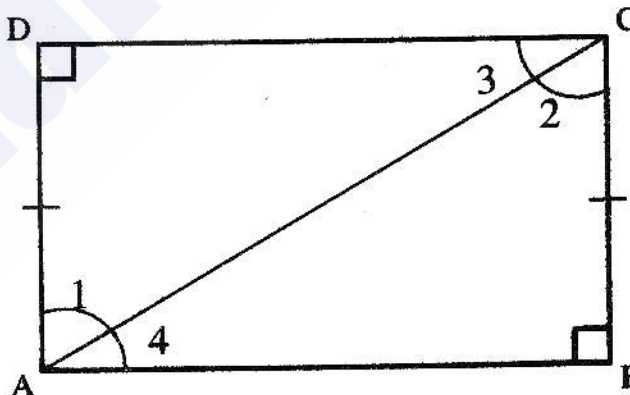
$$\overline{AC} \cong \overline{BD}$$

$$\angle BAC \cong \angle ABD$$

Proof

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle ABD$ $m\angle C \cong m\angle D$ $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{AB}$	Each of 90° Given Common H.S \cong H.S
$\therefore \triangle ABC \cong \triangle ABD$ $\overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\therefore \angle BAC \cong \angle ABD$	Corresponding angles of the congruent triangles

3. In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is a rectangle.



Given

$$m \angle B = m \angle D = 90^\circ, \overline{AD} \cong \overline{BC}$$

Proof

ABCD is a rectangle

Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle ADC$	
$m \angle B \cong m \angle D$	Each of 90°
$\overline{AD} \cong \overline{BC}$	Given
$\overline{AC} \cong \overline{AC}$	Common
$\therefore \triangle ABC \cong \triangle ADC$	H.S \cong H.S
$\overline{AB} \cong \overline{DC}$	
$\angle 1 \cong \angle 2 \quad \dots(i)$	
$\angle 4 \cong \angle 3 \quad \dots(ii)$	
$\angle 1 + \angle 4 = \angle 2 + m \angle 3$	
$\angle A = \angle C = 90^\circ$	
ABCD is a rectangle	By (i) and (ii)

4. Which of the following are true and which are false?

- (i) A ray has two end points.
- (ii) In a triangle, there can be only one right angle.
- (iii) Three points are said to be collinear if they lie on same line.
- (iv) Two parallel lines intersect at a point.
- (v) Two lines can intersect only in one point.
- (vi) A triangle of congruent sides has non-congruent angles.

Answers

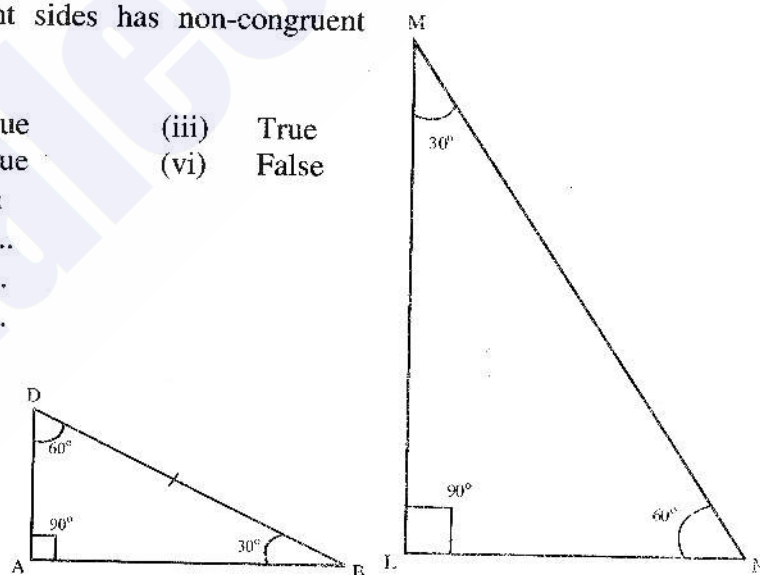
- (i) False (ii) True (iii) True
- (iv) False (v) True (vi) False

5. If $\triangle ABC \cong \triangle LMN$, then

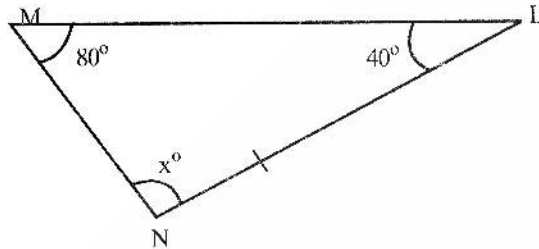
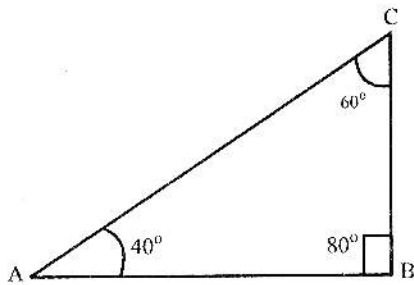
- (i) $m \angle M \cong \dots\dots\dots$
- (ii) $m \angle N \cong \dots\dots\dots$
- (iii) $m \angle A \cong \dots\dots\dots$

Answers

- (i) $m \angle M \cong m \angle B$
- (ii) $m \angle N \cong m \angle C$
- (iii) $m \angle A \cong m \angle L$



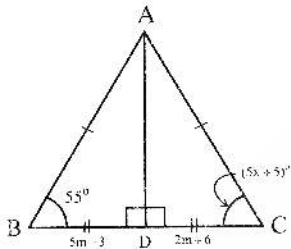
6. If $\triangle ABC \cong \triangle LMN$, then find the unknown x .



Answers

$$x = 60^\circ$$

7. Find the value of unknowns for the given congruent triangles.



$$\triangle ABD \cong \triangle ACD$$

$$\overline{BD} \cong \overline{DC}$$

$$\Rightarrow 5m - 3 = 2m + 6$$

$$5m - 2m = 3 + 6$$

$$3m = 9$$

$$m = \frac{9}{3} = 3$$

Also

$$\angle ACD \cong \angle ABD \Rightarrow$$

Angles opposite to congruent sides are congruent

$$5x + 5 = 55$$

$$5x = 55 - 5$$

$$5x = 50$$

$$x = \frac{50}{5}$$

$$x = 10$$

8. If $\triangle PQR \cong \triangle ABC$

, then find the unknowns.

$$\triangle PQR \cong \triangle ABC$$

$$\overline{PQ} \cong \overline{AB}$$

$$x = 3$$

$$\overline{BC} \cong \overline{QR}$$

$$\Rightarrow z = 4 \text{ cm}$$

$$\overline{AC} \cong \overline{PR}$$

$$y - 1 = 5$$

$$y = 5 + 1$$

$$y = 6 \text{ cm}$$

$$\therefore x = 3 \text{ cm}, y = 6 \text{ cm}, z = 4 \text{ cm}$$

