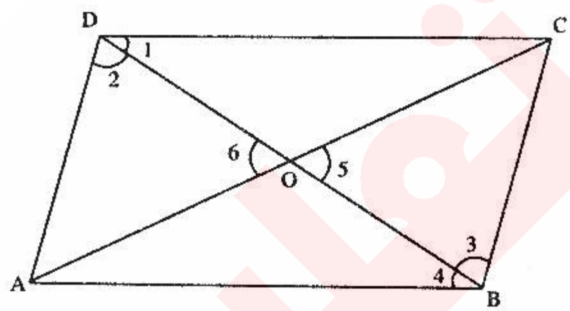
Unit 11

# PARALLELOGRAMS AND TRIANGLES




**Theorem**

In a parallelogram

(i) Opposite sides are congruent.

(ii) Opposite angles are congruent.

(iii) The diagonals bisect each other.

**Given**

In a quadrilateral ABCD, $\overline{AB} \parallel \overline{DC}, \overline{BC} \parallel \overline{AD}$ and the diagonals $\overline{AC}$, $\overline{BD}$ meet each other at point O.

**To Prove**

(i) $\overline{AB} \cong \overline{DC}$. $\overline{AD} \cong \overline{BC}$

(ii) $\angle ADC \cong \angle ABC, \angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC}$. $\overline{OB} \cong \overline{OD}$

**Construction**

In the figure as shown, we label the angles as $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

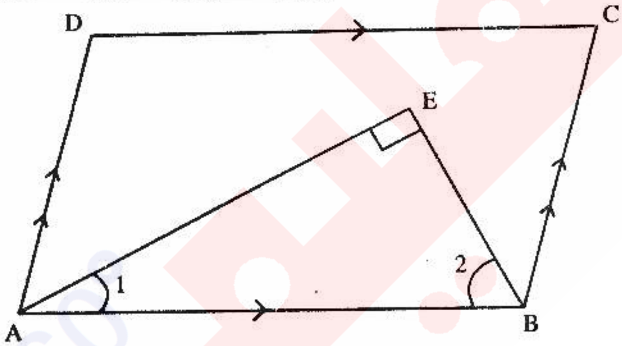
**Proof**

| | Statements | Reasons |
|---|---|---|
| (i) | In $\Delta ABD \leftrightarrow \Delta CDB$ | |
| | $\angle 4 \cong \angle 1$ | Alternate angles |
| | $\overline{BD} \cong \overline{BD}$ | Common |
| | $\angle 2 \cong \angle 3$ | Alternate angles |
| $\therefore$ | $\Delta ABD \cong \Delta CDB$ | A.S.A. ≅ A.S.A. |
| So, | $\overline{AB} \cong \overline{DC}$. $\overline{AD} \cong \overline{BC}$ | (corresponding sides of congruent triangles) |
| and | $\angle A \cong \angle C$ | (corresponding angles of congruent triangles) |
| (ii) | Since | |
| | $\angle 1 \cong \angle 4$ ......(a) | Proved |
| and | $\angle 2 \cong \angle 3$ ......(b) | Proved |
| $\therefore$ | $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$ | From (a) and (b) |
| or | $m\angle ADC = m\angle ABC$ | |
| or | $\angle ADC \cong \angle ABC$ | |

| | |
|---|---|
| and $\angle BAD = \angle BCD$ | Proved in (i) |
| (iii) In $\Delta BOC \leftrightarrow \Delta DOA$ | |
| $\overline{BC} \cong \overline{AD}$ | Proved in (i) |
| $\angle 5 \cong \angle 6$ | Vertical angles |
| $\angle 3 \cong \angle 2$ | Proved |
| $\therefore$ $\Delta BOC \cong \Delta DOA$ | A.A.S $\cong$ A.A.S |
| Hence $\overline{OC} \cong \overline{OA}$, $\overline{OB} \cong \overline{OD}$ | Corresponding sides of congruent triangles) |

**Corollary**

Each diagonal of a parallelogram bisects it into two congruent triangles.




**Example**

The bisectors of two angles on the same side of a parallelogram cut each other at right angles.

**Given**

A parallelogram ABCD, in which

$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

The bisectors of $\angle A$ and $\angle B$ cut each other at E.

**To prove**

$m\angle E = 90^\circ$

**Construction**

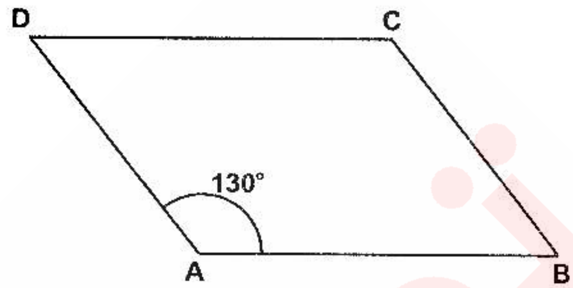
Name the angles $\angle 1$ and $\angle 2$ as shown in the figure.

**Proof**

| Statements | Reasons |
|---|---|
| $m\angle 1 + m\angle 2$<br>$= \frac{1}{2}(m\angle BAD + m\angle ABC)$ | $\left\{ \begin{array}{l} m\angle 1 = \frac{1}{2} m\angle BAD, \\ m\angle 2 = \frac{1}{2} mABC \end{array} \right.$ |
| $= \frac{1}{2}(180^\circ)$<br>$= 90^\circ$ | $\left\{ \begin{array}{l} \text{Int.angles on the same side of } \overline{AB} \\ \text{Which cuts} \parallel \text{ segments } \overline{AD} \text{ and } \overline{BC} \\ \text{are supplementary.} \end{array} \right.$ |
| Hence in $\Delta ABE$, $m\angle E = 90^\circ$ | $m\angle 1 + m\angle 2 = 90^\circ$ (proved) |

# EXERCISE 11.1

**(1) One angle of a parallelogram is $130^\circ$. Find the measures of its remaining angles.**




**Given**

ABCD is a parallelogram that
$m\angle A = 130^\circ$

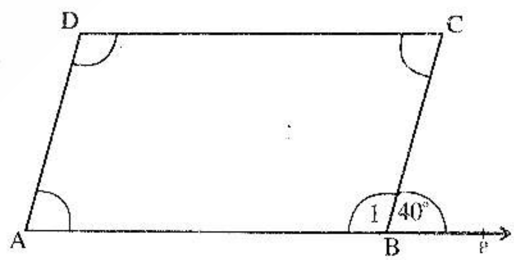
**To Prove**

(Required) To find the measures of $\angle B$, $\angle C$, $\angle D$

**Proof**

| Statements | Reasons |
|---|---|
| $m\angle C = m\angle A$ | Opposite angles of parallelogram. |
| $m\angle C = 130^\circ$ | Given, $m\angle A = 130^\circ$ |
| $m\angle B + m\angle A = 180^\circ$ | $\overline{AD} \parallel \overline{BC}$ and $\overline{AB}$ is transversal. $\therefore$ sum of interior angles. |
| $m\angle B + 130^\circ = 180^\circ$ | Given $m\angle A = 130^\circ$ |
| $m\angle B = 180^\circ - 130^\circ$ | |
| $m\angle B = 50^\circ$ | |
| $m\angle D = m\angle B$ | Opp. angles |
| $m\angle D = 50^\circ$ | As $m\angle B = 50^\circ$ |
| $\therefore$ $m\angle B = 50^\circ$, $m\angle C = 130^\circ$, $m\angle D = 50^\circ$ | |

**(2) One exterior angle formed on producing one side of a parallelogram is $40^\circ$. Find the measures of its interior angles.**




**Given**

ABCD is a parallelogram, side AB has been produced to p to form exterior angle $m\angle CBP = 40^\circ$ and name the interior angles as $\angle 1$, $\angle C$, $\angle D$, $\angle A$.

**Required**

To find the degree measures of $\angle 1$, $\angle C$, $\angle D$, $\angle A$

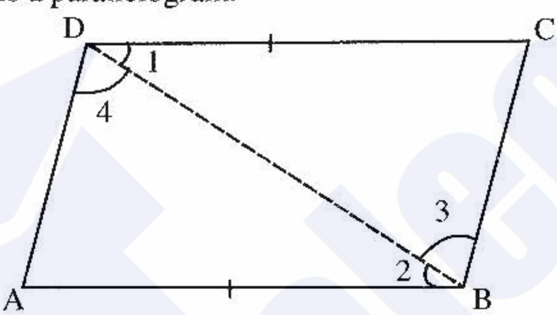
**Proof**

| Statements | Reasons |
|---|---|
| $m\angle 1 + m\angle CBP = 180^\circ$ | Supp.angles. |
| $m\angle 1 + 40^\circ = 180^\circ$ | $m\angle CBP = 40^\circ$ given |

| | |
|---|---|
| $\therefore \quad m\angle 1 = 180^\circ - 40^\circ$ | |
| $m\angle 1 = 140^\circ$ (i) | |
| $m\angle D = m\angle 1$ | Opp.angles of ||m |
| $m\angle D = 140^\circ$.....(ii) | From (i) |
| $m\angle A + m\angle 1 = 180^\circ$ | $\overline{AD} \parallel \overline{BC}$ and $\overline{AB}$ is transversal. (Interior angles) |
| $m\angle A + 140^\circ = 180^\circ$ | From (i) |
| $m\angle A = 180^\circ - 140^\circ$ | |
| $m\angle A = 40^\circ$..............(iii) | |
| $m\angle C = m\angle A$ | Opp. angles |
| $m\angle C = 40^\circ$ | From (iii) |
| Thus $m\angle 1 = 140^\circ$, $m\angle C = 40^\circ$ | |

**Theorem**

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.




**Given**

In a quadrilateral ABCD, $\overline{AB} \cong \overline{DC}$ and $\overline{AB} \parallel \overline{DC}$

**To prove**

ABCD is a parallelogram.

**Construction**

Join the point B to D and in the figure, name the angles as indicated: $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta ABD \leftrightarrow \Delta CDB$ | |
| $\overline{AB} \cong \overline{DC}$ | Given |
| $\angle 2 \cong \angle 1$ | Alternate angles |
| $\overline{BD} \cong \overline{BD}$ | Common |
| $\therefore \quad \Delta ABD \cong \Delta CDB$ | S.A.S. postulate |
| Now $\angle 4 \cong \angle 3$ .....(i) | (corresponding angles of congruent triangles) |
| $\therefore \quad \overline{AD} \parallel \overline{BC}$ ... .(ii) | From (i) |

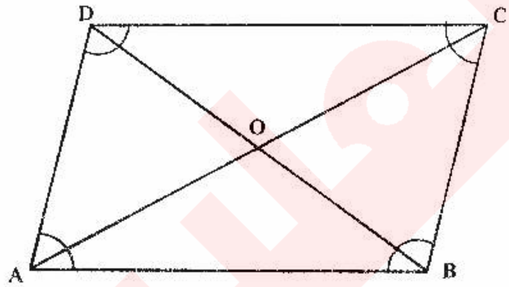
| | |
|---|---|
| and $\overline{AD} \cong \overline{BC}$ ....(iii) | Corresponding sides of congruent $\Delta$s |
| Also $\overline{AB} \parallel \overline{DC}$ ....(iv) | Given |
| Hence ABCD is a parallelogram | From (ii) – (iv) |

# EXERCISE 11.2

**(1) Prove that a quadrilateral is a parallelogram if its**

**(a) Opposite angles are congruent.**

**(b) Diagonals bisect each other.**




**Given** Given ABCD is a quadrilateral.

$m\angle A = m\angle C$,

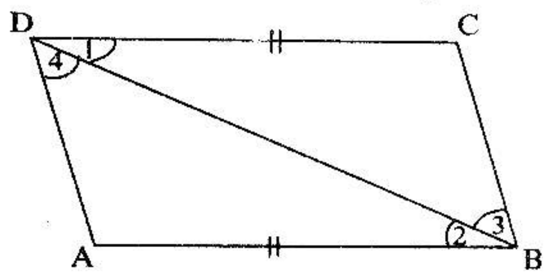
$m\angle B = m\angle D$

**To prove** ABCD is a parallelogram.

**Proof**

| Statements | Reasons |
|---|---|
| $m\angle A = m\angle C$ (i) | Given |
| $m\angle B = m\angle D$ (ii) | Given |
| Now | |
| $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$ | Angles of a quad. |
| $m\angle A + m\angle B + m\angle A + m\angle B = 360^\circ$ | From (i), (ii) |
| $m\angle A + m\angle A + m\angle B + m\angle B = 360^\circ$ | Rearranging |
| $2m\angle A + 2m\angle B = 360^\circ$ | |
| $(m\angle A + m\angle B) = 360^\circ /2 = 180^\circ$ | Dividing by 2 |
| $\therefore$ $\overline{AD} \parallel \overline{BC}$ | As $m\angle A + m\angle B = 180^\circ$ |
| Similarly it can be | (sum of interior angles) |
| Proved that $\overline{AB} \parallel \overline{CD}$ | |
| Hence ABCD is a parallelogram. | |

**(2) prove that a quadrilateral is a parallelogram if its opposite sides are congruent.**




**Given**

In quadrilateral

ABCD, $\overline{AB} \cong \overline{DC}$,

$\overline{AD} \cong \overline{BC}$

**Required**

ABCD is a $\parallel$ gm

$\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{BC}$

**Construction**

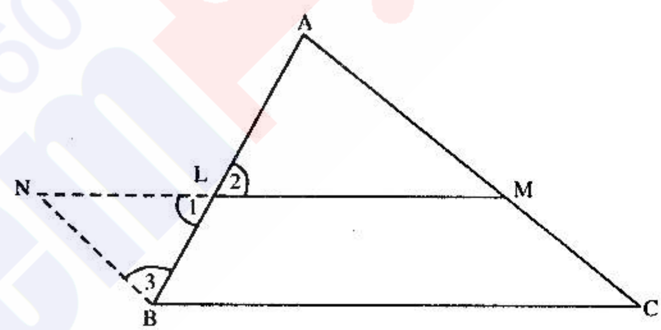
Join point B to D and name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

**Proof**

| Statements | | Reasons |
|---|---|---|
| | $\Delta ABD \leftrightarrow \Delta CDB$ | |
| | $\overline{AD} \cong \overline{CB}$ | Given |
| | $\overline{AB} \cong \overline{CD}$ | Given |
| | $\overline{BD} \cong \overline{BD}$ | Common |
| $\therefore$ | $\Delta ABD \cong \Delta CDB$ | S.S.S $\cong$ S.S.S |
| So | $\angle 2 \cong \angle 1$ (i) | Corresponding angles of Congruent triangles |
| | $\angle 4 \cong \angle 3$ (ii) | Alternate angles |
| Hence | $\overline{AB} \parallel \overline{CD}$ (iii) | $\angle 2$ and $\angle 1$ are congruent |
| Similarly | $\overline{BC} \parallel \overline{AD}$ (iv) | Alternate angles $\angle 3$, $\angle 4$ congruent |
| $\therefore$ | ABCD is a parallelogram. | From iii, iv |

**Theorem**

The line segment, joining the mid-points of two sides of a triangle, is parallel to the third side and is equal to one half if its length.




**Given** In $\Delta ABC$, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

**To Prove**

$$\overline{LM} \parallel \overline{BC} \text{ and } m\overline{LM} = \frac{1}{2} m\overline{BC}$$

**Construction**

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$ .Join N to B. and in the figures name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as shown.

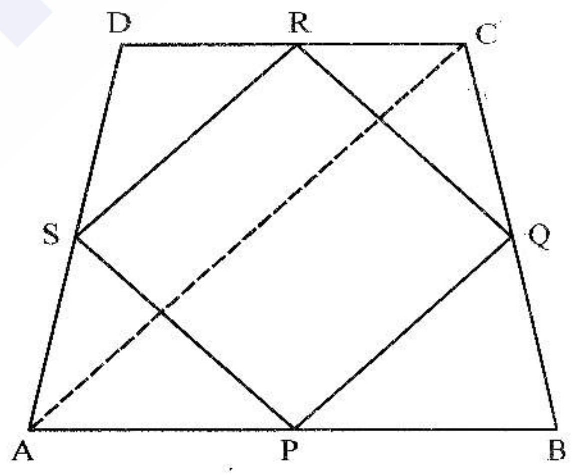
**Proof**

| Statements | | Reasons |
|---|---|---|
| In | $\Delta BLN \leftrightarrow \Delta ALM$ | |
| | $\overline{BL} \cong \overline{AL}$, | Given |
| | $\angle 1 \cong \angle 2$ | Vertical angles |
| | $\overline{NL} \cong \overline{ML}$ | Construction |

| | | |
|---|---|---|
| $\therefore$ | $\Delta BLN \cong \Delta ALM$ | S.A.S. postulate |
| | $\angle A \cong \angle 3$ ......(i) | (corresponding angles of congruent triangles) |
| and | $\overline{NB} \cong \overline{AM}$ .....(ii) | (corresponding sides of congruent triangles) |
| But | $\overline{NB} \parallel \overline{AM}$ | From (i), alternate $\angle$s |
| Thus | $\overline{NB} \parallel \overline{MC}$ ....(iii) | (M is a point of $\overline{AC}$) |
| | $\overline{MC} \cong \overline{AM}$ ....(iv) | Given |
| | $\overline{NB} \cong \overline{MC}$ ... (v) | {from (ii) and (iv)} |
| $\therefore$ | BCMN is a parallelogram | From (iii) and (v) |
| $\therefore$ | $\overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$ | (Opposite sides of a parallelogram BCMN) |
| | $\overline{BC} \cong \overline{NM}$ .....(vi) | (Opposite sides of parallelogram) |
| | $m\overline{LM} = \frac{1}{2} m\overline{NM}$ .....(vii) | Construction |
| Hence | $m\overline{LM} = \frac{1}{2} m\overline{BC}$ | {from (vi) and (vii)} |

**Example**

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.




**Given**

A quadrilateral ABCD, in which P is the mid-point of $\overline{AB}$, Q is the mid-point of $\overline{BC}$, R is the mid-point of $\overline{CD}$, S is the mid-point of $\overline{DA}$.

P is joined to Q, Q is joined to R. R is joined to S and S is joined to P.

**To prove**

PQRS is a parallelogram.

**Construction**

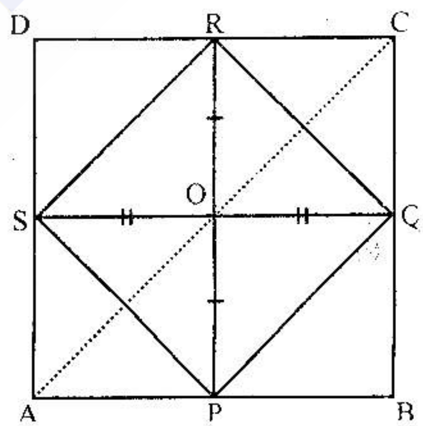
Join A to C.

**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta DAC$, | |
| $\left.\begin{array}{l}\overline{SR} \parallel \overline{AC} \\ m\overline{SR} = \frac{1}{2} m\overline{AC}\end{array}\right\}$ | S is the mid-point of $\overline{DA}$<br>R is the mid-point of $\overline{CD}$ |
| In $\Delta BAC$, | |
| $\left.\begin{array}{l}\overline{PQ} \parallel \overline{AC} \\ m\overline{PQ} = \frac{1}{2} m\overline{AC}\end{array}\right\}$ | P is the mid-point of $\overline{AB}$<br>Q is the mid-point of $\overline{BC}$ |
| $\overline{SR} \parallel \overline{PQ}$ | Each $\parallel$ $\overline{AC}$ |
| $m\overline{SR} = m\overline{PQ}$ | Each $= \frac{1}{2} m\overline{AC}$ |
| Thus PQRS is a parallelogram | $\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved) |

## EXERCISE 11.3

(1) **Prove that the line-segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.**




**Given**

ABCD is a quadrilateral.

P, Q, R, S are the mid-points of $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ respectively.

P is joined to R, Q is joined to S. $\overline{SQ}, \overline{PR}$ intersect at point "O"

**To Prove**

$\overline{OP} \cong \overline{OR}, \overline{OS} \cong \overline{OQ}$

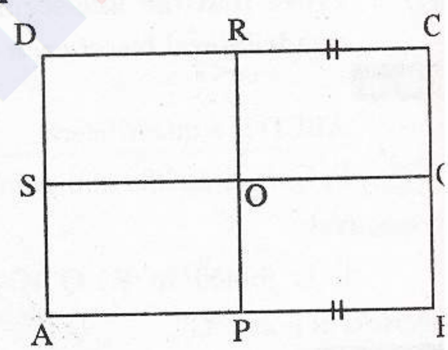
**Construction** Join P, Q, R, S in order, join A to C.

**Proof**

| Statements | Reasons |
|---|---|
| $\overline{SR} \parallel \overline{AC}$ (i)<br>$m\overline{SR} = \frac{1}{2} m\overline{AC}$ (ii) | In $\Delta ADC$. S, R are mid-points Of $\overline{AD}, \overline{DC}$ |

| | |
|---|---|
| And $\overline{PQ} \parallel \overline{AC}$ (iii) | In $\Delta ABC$; P, Q are mid-points of $\overline{AB}, \overline{BC}$ |
| $m\overline{PQ} = \frac{1}{2} m\overline{AC}$ (iv) | |
| $\therefore$ $\overline{PQ} \parallel \overline{SR}$ (v) | from (i), and (iii) |
| $m\overline{PQ} = m\overline{SR}$ (vi) | From (ii) and (iv) |
| Similarly $\overline{PS} \parallel \overline{QR}$ | |
| $m\overline{PS} = m\overline{QR}$ | |
| Hence PQRS is a parallelogram | |
| Now $\overline{PR}$, $\overline{SQ}$ are the diagonals | |
| Of PQRS that intersect at point O. | |
| $\therefore$ $\overline{OP} \cong \overline{OR}$ | |
| $\therefore$ $\overline{OS} \cong \overline{OQ}$ | Diagonals of a parallelogram Bisect each other. |

**(2) Prove that the line-segments joining the mid-points of the opposite sides of a rectangle are the right-bisectors of each other.**




**Given**

ABCD is a rectangle.

and P, Q, R, S are the mid-points of sides $\overline{AB}, \overline{BC}$, $\overline{CD}$ and $\overline{DA}$, respectively.

P is joined to R, S to Q These intersect at "O"

**To Prove**

$\overline{OQ} \cong \overline{OS}, \overline{OR} \cong \overline{OP}$ and $\overline{RP} \perp \overline{SQ}$

**Proof**

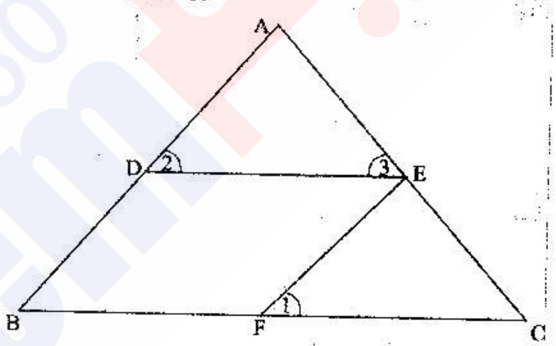
| Statements | Reasons |
|---|---|
| $AB \parallel CD$ | opposite sides of rectangle |
| $\overline{AP} = \overline{DR}$ (i) | |
| $m\overline{AB} = m\overline{CD}$ | |
| $\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$ | |
| $m\overline{AP} = m\overline{DR}$ (ii) | |
| $\therefore$ APRD is rectangle | |

| | |
|---|---|
| $\therefore\ \overline{OR} \cong \overline{OP}$ | As $m\angle A = m\angle D = 90^\circ$ |
| Similarly $\overline{OQ} \cong \overline{OS}$ | |
| Now In rectangle APRD | |
| $m\overline{DA} = m\overline{RP}$ | |
| $\frac{1}{2} m\overline{DA} = m\overline{RP}$ | |
| $m\overline{DS} = m\overline{RO}$ | |
| $\therefore\ \overline{DS} \parallel \overline{RO}$, | |
| Hence SORD is rectangle. | |
| $\therefore\ m\angle SOR = 90^\circ,\ \overline{RP} \perp \overline{SQ}$. | |

[Note: Diagonals of a rectangle are congruent.]

**(9) Prove that the line-segment passing through the mid-point of one side and parallel to another side of a triangle also bisects the third side.**

**Given** In $\Delta ABC$, D is mid-point of $\overline{AB}$, $\overline{DE} \parallel \overline{BC}$ which meets $\overline{AC}$ at E.




**Required** E is mid-point of $\overline{AE}$ and $\overline{EA} \cong \overline{EC}$

**Construction**

Take $\overline{EF} \parallel \overline{AB}$ which meets $\overline{BC}$ at F.

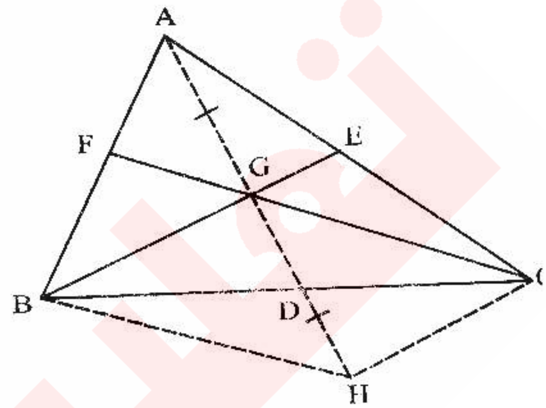
**Proof**

| Statements | | Reasons |
|---|---|---|
| Now BDEF is parallelogram | | $\overline{DE} \parallel \overline{BF}$ given, $\overline{EF} \parallel \overline{DB}$ const. |
| $\therefore\ \overline{EF} \cong \overline{DB}$ | (i) | Opposite sides of parallelogram |
| $\overline{EF} \cong \overline{AD}$ | (ii) | Given |
| $\angle 1 \cong \angle B$ | | Corresponding angles. |
| $\angle 2 \cong \angle B$ | (iii) | Corresponding angles. |
| $\therefore\ \angle 1 \cong \angle 2$ | (iv) | Form (iii) |
| Now In $\Delta ADE \leftrightarrow \Delta EFC$ | | |
| $\angle 1 \cong \angle 2$ | | |
| $\angle 3 \cong \angle C$ | | Form (iv) |
| $\overline{AD} \cong \overline{EF}$ | | Corresponding angles. |
| Hence $\Delta ADE \cong \Delta EFC$ | | Form (ii) |
| | | A.A.S $\cong$ A.A.S |

| | |
|---|---|
| $\therefore \quad \overline{AE} \cong \overline{CE}$ | Corresponding sides of congruent triangles. |

**Theorem**

**The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.**




**Given**

$\Delta ABC$

**To Prove**

The medians of the $\Delta ABC$ are concurrent and the point of concurrency is the point of trisection of each median.

**Construction**

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the $\Delta ABC$ which intersect each other at point G. Join A to G and produce it to point H such that $\overline{AG} \cong \overline{GH}$. Join H to the points B and C. $\overline{AH}$ Intersects $\overline{BC}$ at the point D.

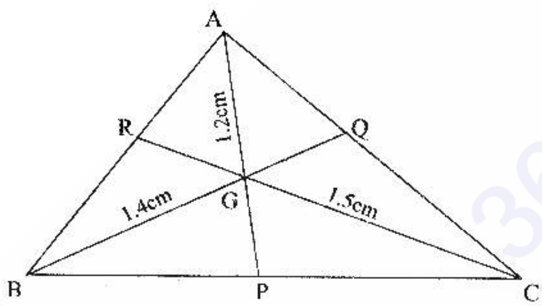
**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta ACH$, $\overline{GE} \parallel \overline{HC}$, | G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively |
| or $\overline{BE} \parallel \overline{HC}$ .....(i) | G is a point of $\overline{BE}$ |
| Similarly $\overline{CF} \parallel \overline{HB}$ .....(ii) | |
| $\therefore$ BHCG is a parallelogram | from (i) and (ii) |
| and $m\overline{GD} = \frac{1}{2} m\overline{GH}$ .....(iii) <br> $\overline{BD} \cong \overline{CD}$ <br> $\overline{AD}$ is a median of $\Delta ABC$ | (Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram BHCG intersect each other at point D). |
| Medians $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ pass through the point G | (G is the intersecting point of $\overline{BE}$ and $\overline{CF}$ and $\overline{AD}$ pass through it.) |
| Now $\overline{GH} \cong \overline{AG}$ .....(iv) | Construction |

| | |
|---|---|
| $\therefore \quad m\overline{GD} = \frac{1}{2} m\overline{AG}$<br>and G is the point of trisection of $\overline{AD}$ –(v)<br>similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$. | from (iii) and (iv) |

## EXERCISE 11.4

**(1) The distances of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2cm; 1.4 cm and 1.5 cm. Find the lengths of its medians.**




**Solution** Let ABC be a triangle with center of gravity at G where $m\overline{AG} = 1.2\text{cm}$, $\overline{BG} = 1.4\text{cm}$, $m\overline{CG} = 1.5\text{cm}$

**Required** To find the length of AP, BQ, CR

**Proof:**

$$m\overline{AP} = \frac{3}{2} \times (mAG)$$

$$= \frac{3}{2} \times 1.2 = 1.8\text{cm}$$

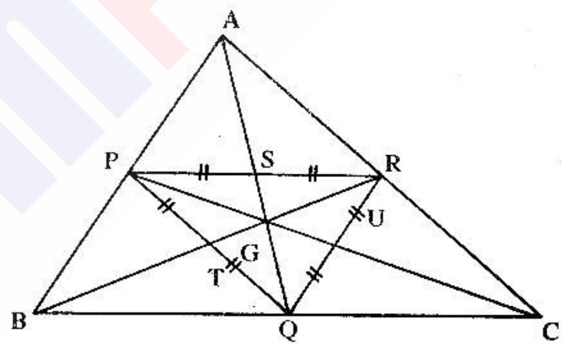
$$m\overline{BQ} = \frac{3}{2} \times (m\overline{BG})$$

$$= \frac{3}{2} \times 1.4 = 2.1\text{cm}$$

$$m\overline{CR} = \frac{3}{2} \times (mCG)$$

$$= \frac{3}{2} \times 1.5 = 2.25\text{cm}$$

**(2) Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the mid-points of its sides is the same.**




**Given**

In $\Delta ABC$, $\overline{AQ}, \overline{BR}, \overline{CP}$ are its medians that are concurrent at point G. $\Delta PQR$ is formed by joining mid-points of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

**To Prove**

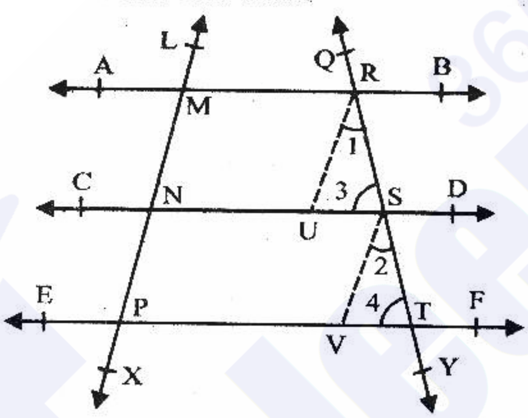
Point G is point of concurrency of triangle PQR.

**Proof**

| Statements | | Reasons |
|---|---|---|
| | $\overline{PR} \parallel \overline{BC}$ | P, R are mid-points of $\overline{AB}$ and $\overline{AC}$ |
| $\Rightarrow$ | $\overline{PR} \parallel \overline{BQ}$ (i) | |
| | $\overline{RQ} \parallel \overline{AB}$ | P, Q are mid-points of $\overline{AB}$ and $\overline{BC}$ |
| $\Rightarrow$ | $\overline{RQ} \parallel \overline{PB}$ (ii) | |
| $\therefore$ | PBQR is a parallelogram. | |
| | $\overline{BR}$, $\overline{PQ}$ are its diagonals, that bisect each other at T. | |
| | T is mid-point $\overline{PQ}$, similarly | |
| | S is mid-point of $\overline{PR}$ and U is mid-point of $\overline{PQ}$. | |

**Theorem**

If three or more parallel lines make congruent segments on a transversal, they also intercept congruent segments on any other line that cuts them.




**Given**

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at points R, S and T respectively.

**To Prove**

$\overline{RS} \cong \overline{ST}$

**Construction**

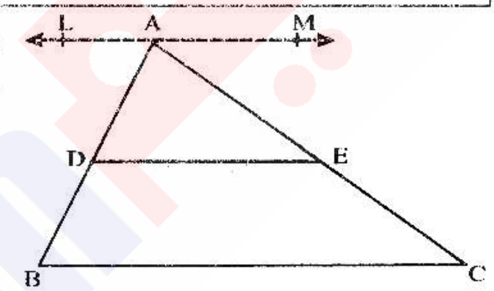
From R, draw $\overline{RU} \parallel \overleftrightarrow{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overleftrightarrow{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as

$\angle 1, \angle 2, \angle 3$ and $\angle 4$

**Proof**

| Statements | Reasons |
|---|---|
| MNUR is a parallelogram | $\overline{RU} \parallel \overleftrightarrow{LX}$ (construction)<br>$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (given) |
| $\therefore$ $\overline{MN} \cong \overline{RU}$ .....(i) | (opposite sides of a parallelogram) |

| | |
|---|---|
| Similarly, | |
| $\overline{NP} \cong \overline{SV}$ .....(ii) | |
| But $\overline{MN} \cong \overline{NP}$ .....(iii) | Given |
| $\therefore$ $\overline{RU} \cong \overline{SV}$ | {from (i), (ii) and (iii)} |
| Also $\overline{RU} \parallel \overline{SV}$ | Each is $\parallel \overleftrightarrow{LX}$ (construction) |
| $\therefore$ $\angle 1 \cong \angle 2$ | Corresponding angles |
| and $\angle 3 \cong \angle 4$ | Corresponding angles |
| In $\Delta RUS \leftrightarrow \Delta SVT$, | |
| $\overline{RU} \cong \overline{SV}$ | Proved |
| $\angle 1 \cong \angle 2$ | Proved |
| $\angle 3 \cong \angle 4$ | Proved |
| $\therefore$ $\Delta RUS \cong \Delta SVT$ | S.A.A. $\cong$ S.A.A. |
| Hence $\overline{RS} \cong \overline{ST}$ | (corresponding sides of a congruent triangles) |




**Corollaries** (i) A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.

**Given** In $\Delta ABC$, D is the mid-point of $\overline{AB}$. $\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

**To prove**

$\overline{AE} \cong \overline{EC}$

**Construction**

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

**Proof**

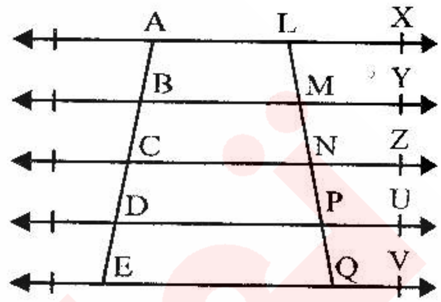
| Statements | Reasons |
|---|---|
| Intercepts cut by $\overleftrightarrow{LM}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BC}$ on $\overleftrightarrow{AC}$ are congruent. i.e., $\overline{AC} \cong \overline{EC}$ | {Intercepts cut by parallels $\overleftrightarrow{LM}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BC}$ on $\overleftrightarrow{AB}$ are congruent (given) |

(ii) The parallel line from the mid-point of one non-parallel side of a trapezium to the parallel sides bisects the other non-parallel side.

(iii) If one side of a triangle is divided into congruent segments, the line drawn from the point of division parallel to the other side will make congruent segments on third side.

## Exercise 11.5

**1.** In the given figure. $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$ if $m\overline{MN} = 1cm$ then find the length of $\overline{LN}$ and $\overline{LQ}$




**Given**

In given figure $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$,

$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$, $m\overline{MN} = 1cm$

**Required:**

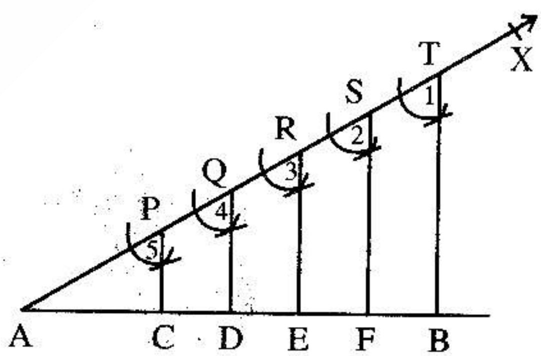
To find $m\overline{LN}$ and $m\overline{LQ}$

| Statement | Reasons |
|---|---|
| $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ | Given |
| $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$ | Given |
| $\overline{BC} \cong \overline{MN}$<br>$\overline{NP} \cong \overline{PQ}$ | $\because \parallel$ lines through A, B, C, D, E cut $\overline{LQ}$ in points L, M, N, P, Q. |
| $m\overline{MN} = 1cm$ | Given |
| $\overline{LN} = 2\overline{MN}$<br>$= 2(1)$<br>$= 2cm$ | $\because \overline{MN} = 1cm$ |
| $\overline{LQ} = 4\overline{MN}$<br>$= 4 \times 1$<br>$= 4cm$ | |

**2.** Take a line segment of length 5cm and divide it into five congruent parts.

[Hint: Draw an acute angle $\angle BAX$. On $\overrightarrow{AX}$ take $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$.

Joint T to B. Draw line parallel to $\overline{TB}$ from the points P, Q, R and S.]




**Construction:**

(i) Take a line segment AB of 5cm long.

(ii) Draw an acute angle $\angle BAX$.

(iii) Mark 5 points on $\overrightarrow{AX}$ at equal distance starting from point A.

(iv) Join the last point (mark)T to B.

(v) Draw $\overline{SF}, \overline{RE}, \overline{QD}, \overline{PC}$ parallel to TB these line segments meet AB at F,E,D,C points.

**Result:** AB has been divided into five equal points

$\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{FB}$

**3.** **Fill in the blanks.**

(i) In a parallelogram opposite sides are….. (Parallel / Congruent)

(ii) In a parallelogram opposite angles are ……. (Equal / Congruent)

(iii) Diagonals of a parallelogram …….. each other at a point. (Intersect)

(iv) Medians of a triangle are ………… (Concurrent)

(v) Diagonal of a parallelogram divides the parallelogram into two ……….. triangles. (Congruent)

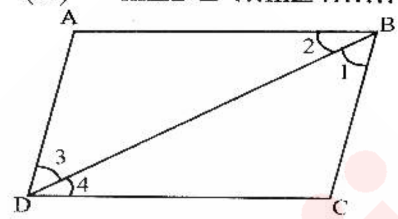
**4.** In parallelogram ABCD

(i) $m\overline{AB}$ …≅….. $m\overline{DC}$

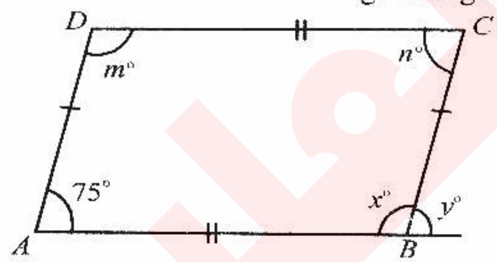
(ii) $m\overline{BC}$ …≅… $m\overline{AD}$

(iii) $m\angle 1 \cong$ …$m\angle 3$……

(iv) $m\angle 2 \cong$ …$m\angle 4$……




**5.** Find the unknowns in the given figure.




**Given:** Let ABCD be the given figure with

$\overline{AB} \cong \overline{CD}$

$\overline{BC} \cong \overline{AD}$

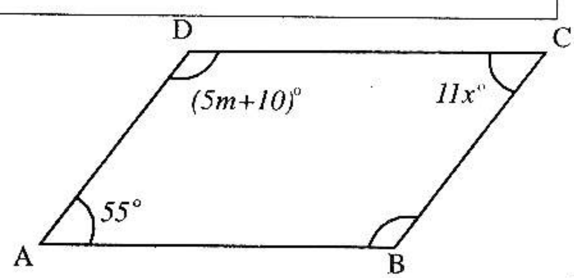
To Find: $m^\circ$, $n^\circ$, $x^\circ$, $y^\circ$

**Proof:**

| Statement | Reasons |
|---|---|
| ABCD is a Parallelogram | $\overline{AB} \cong \overline{CD}$ <br> $\overline{AD} \cong \overline{BC}$ |
| $\angle n = 75^\circ$ | Opposite interior angles |
| $m^\circ + 75^\circ = 180^\circ$ | supplementary angles |
| $m^\circ = 180^\circ - 75^\circ = 105^\circ$ | |
| $x^\circ = m^\circ$ | |
| $x^\circ = 105^\circ$ | |
| $x^\circ + y^\circ = 180^\circ$ | supplementary angles |
| $y^\circ = 180^\circ - x^\circ$ | |
| $y^\circ = 180^\circ - 105^\circ$ | |
| $y^\circ = 75^\circ$ | |

**6.** If the given figure ABCD is a parallelogram, then find *x*, *m*.

**Given:** ABCD is a parallelogram with angles as shown To Find $x^\circ$ and $m^\circ$

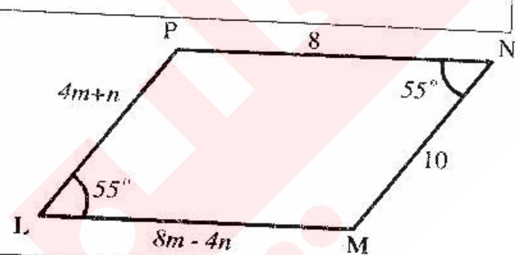


**Proof:**

| Statement | Reasons |
|---|---|
| $11x^\circ = 55^\circ$ | Opposite angles of parallelogram |
| $x^\circ = \frac{55^\circ}{11} = 5^\circ$ | |
| $x^\circ = 5^\circ$ | |
| $(5m + 10)^\circ + 55^\circ = 180^\circ$ | Int. supplementary angles |
| $(5m + 10)^\circ = 180^\circ - 55^\circ$ | |
| $5m^\circ + 10^\circ = 125^\circ$ | |
| $5m^\circ = 125^\circ - 10^\circ$ | |
| $5m^\circ = 115^\circ$ | |
| $m^\circ = 23^\circ$ | |

**7.** The given figure LMNP is a parallelogram. Find the value of *m, n*.

**Given:** The parallelogram LMNP with lengths and angles as shown to find: $m^\circ$ and $n^\circ$




**Proof:**

| Statement | Reasons |
|---|---|
| $4m + n = 10 \ldots\ldots(i)$ | Opposite sides of ‖gm |
| $8m - 4n = 8 \ldots(ii)$ | Opposite side of ‖gm |
| Multiplying (i) by 4 | |
| $16m + 4n = 40$ (iii) | |
| Adding (i) and (iii) | |

$$8m - \cancel{4n} = 8$$
$$16m + \cancel{4n} = 40$$
$$24m = 48$$

$$m = \frac{48}{24} = 2$$

Put in (i)

$4(2) + n = 10$

$8 + n = 10$

$n = 10 - 8 \Rightarrow n = 2$

**8.** In the question 7, sum of the opposite angles of the parallelogram is $110^\circ$, find the remaining angles.

**Given:** LMNP is a parallelogram with angles $55^\circ$, $55^\circ$ as shown

To Find: All angles

**Proof:**

| Statement | Reasons |
|---|---|
| $\angle LPN + 55^\circ = 180^\circ$ | Interior angles |
| $\angle LPN = 125^\circ$ | |
| Also | |
| $\angle m = \angle P$ | Opposite angles |
| $\angle m = 125^\circ$ | $\because \angle P = 125^\circ$ |