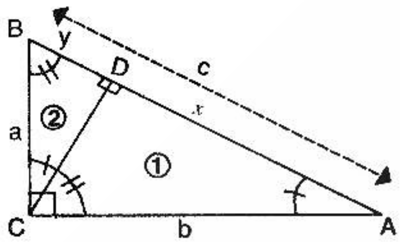
Unit
15

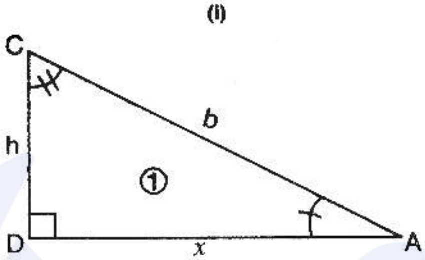
PYTHAGORAS THEOREM

Pythagoras Theorem

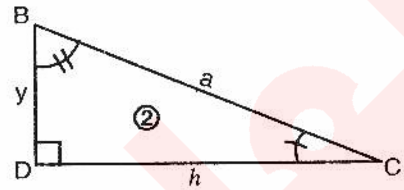
In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



(i)



(ii) a



(ii)-b

Given

ΔACB is a right angled triangle in which $m\angle C = 90^{\circ}$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$.

To Prove

$$c^2 = a^2 + b^2$$

Construction

Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$.

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two Δs ADC and BDC which are separately shown in the figures (ii)-a and (ii)-b respectively.

Proof (Using similar Δs)

Statements	Reasons
In $\Delta ADC \longleftrightarrow \Delta ACB$	Refer to figure(ii)-a and (i)
$\angle A \cong \angle A$	Common – self congruent
$\angle ADC \cong \angle ACB$	Construction – given, each angle = 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$, complements of $\angle A$.
$\therefore$ $\Delta ADC \sim \Delta ACB$	Congruency of three angles
$\therefore$ $\dfrac{x}{b} = \dfrac{b}{c}$	(Measures of corresponding sides of similar triangles are proportional)
or $x = \dfrac{b^2}{c}$ ....(i)	

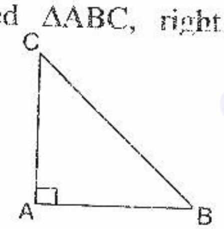
Again in $\Delta BDC \longleftrightarrow \Delta BCA$	Refer to figure (ii)-b and (i)
$\angle B \cong \angle B$	Common–self congruent
$\angle BDC \cong \angle BCA$	Construction –given, each angle = 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$, complements of $\angle B$
$\therefore$ $\Delta BDC \sim \Delta BCA$	Congruency of three angles.
$\therefore$ $\frac{y}{a} = \frac{a}{c}$	(Corresponding sides of similar triangles are proportional).
or $y = \frac{a^2}{c}$ ....(ii)	
But $y + x = c$	Supposition.
$\therefore$ $\frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	Multiplying both sides by c.
i.e., $c^2 = a^2 + b^2$	

Corollary

In a right angled ΔABC, right angled at A.

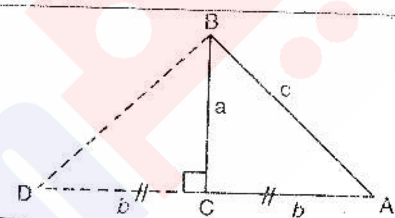
(i) $\overline{AB}^2 = \overline{BC}^2 - \overline{CA}^2$

(ii) $\overline{AC}^2 = \overline{BC}^2 - \overline{AB}^2$



Converse of Pythagoras' Theorem

If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a right angled triangle.



Given In a ΔABC, $m\overline{AB} = c, m\overline{BC} = a$ and $m\overline{AC} = b$ such that $a^2 + b^2 = c^2$.

To Prove ΔACB is a right angled triangle.

Construction Draw $\overline{CD}$ perpendicular to $\overline{BC}$ such that $\overline{CD} \cong \overline{CA}$. Join the points B and D.

Proof

Statements	Reasons
ΔDCB is a right –angled triangle.	Construction
$\therefore$ $(m\overline{BD})^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore$ $(m\overline{BD})^2 = c^2$	Taking square root of both sides.
or $m\overline{BD} = c$	
Now in $\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction

$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each side = c.
$\therefore$ $\Delta DCB \cong \Delta ACB$	S.S.S. $\cong$ S.S.S.
$\therefore$ $\angle DCB \cong \angle ACB$	(Corresponding angles of congruent triangles)
But $m\angle DCB = 90^\circ$	Construction
$\therefore$ $m\angle ACB = 90^\circ$	
Hence the ΔACB is a right-angled triangle.	

Corollary: Let c be the longest of the sides a, b and c of a triangle.

- If $a^2 + b^2 = c^2$, then the triangle is right.
- If $a^2 + b^2 > c^2$, then the triangle is acute.
- If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Exercise 15

1. Verify that the Δs having the following measures of sides are right-angled.

(i) $a = 5$ cm, $b = 12$ cm, $c = 13$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$

$(13)^2 = (12)^2 + (5)^2$

$169 = 144 + 25$

$169 = 169$

$\therefore$ The triangle is right angled.

(ii) $a = 1.5$ cm, $b = 2$ cm, $c = 2.5$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$

$(2.5)^2 = (1.5)^2 + (2)^2$

$625 = 2.25 + 4$

$6.25 = 6.25$

$\therefore$ The triangle is right angled.

(iii) $a = 9$ cm, $b = 12$ cm, $c = 15$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$

$(15)^2 = (12)^2 + (9)^2$

$225 = 144 + 81$

$225 = 225$

$\therefore$ The triangle is right angled.

(iv) $a = 16$ cm, $b = 30$ cm, $c = 34$ cm

Ans. $(\text{Hyp})^2 = (\text{Perp.})^2 + (\text{Base})^2$

$(34)^2 = (30)^2 + (16)^2$

$1156 = 900 + 256$

$1156 = 1156$

$\therefore$ The triangle is right angled.

2. Verify that $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are the measures of the sides of a right angled triangle where a and b are any two real numbers ($a > b$).

Ans. In right angle triangle.

$H^2 = P^2 + B^2$

$(a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2$ ....(i)

$(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2$ ....(ii)

$(2ab)^2 = 4a^2b^2$ .... (iii)

Adding (ii) and (iii) we get

$(a^2 - b^2)^2 + (2ab)^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$

$= a^4 + b^4 + 2a^2b^2$ ....(iv)

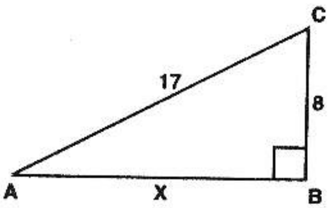
Comparing (i) and (iv), we get

$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$

Hence $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are measures of the sides of a right angled triangle where $a^2 + b^2$ is Hypotenuse.

3. The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right angled triangle?

Ans:



Consider a right angled triangle

With $\overline{AB} = x$

$\overline{BC} = 8$

and $\overline{AC} = 17$

If x is the base of right angled ΔABC then we know by Pythagoras theorem that

$$(hyp)^2 = (Base)^2 + (perp)^2$$
$$(17)^2 = x^2 + (8)^2$$
$$289 = x^2 + 64$$
$$x^2 + 64 = 289$$
$$x^2 = 289 - 64$$
$$x^2 = 225$$
$$x = \sqrt{225}$$

As x is measure of side

So $x = 15$ units

4. In an isosceles Δ, the base $\overline{BC} = 28$ cm, and $\overline{AB} = \overline{AC} = 50$cm.

If $\overline{AD} \perp \overline{BC}$, then find:

(i) Length of $\overline{AD}$

(ii) Area of ΔABC

Given

$m\overline{AC} = m\overline{AB} = 50\,cm$

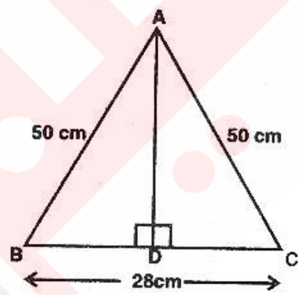
$m\overline{BC} = 28cm$

$\overline{AD} \perp \overline{BC}$

To Prove

$m\overline{AD} = ?$

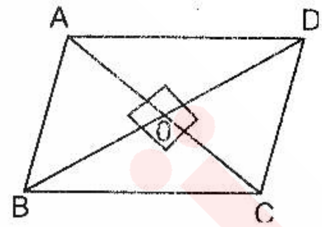
Area of $\Delta ABC = ?$



Proof

Statements	Reasons
In right angled triangle	
$m\overline{CD} = 14cm$	$\overline{CD} = \frac{1}{2} m\overline{BC}$
$m\overline{AC} = 50cm$	Given
$(mAD)^2 = (mAC)^2 - (mCD)^2$	
$(mAD)^2 = (50)^2 - (14)^2$	$(mAC)^2 = (mAD)^2 - (mCD)^2$ (by Pythagoras theorem)
$= 2500 - 196$	
$= 2304$	
$\sqrt{(mAD)^2} = \sqrt{2304}$	Taking square root of both sides
$m\overline{AD} = 18$ cm	
(ii) Area of $\Delta ABC = \dfrac{\text{Base} \times \text{Altitude}}{2}$	
$= \dfrac{28 \times 48}{2}$	
$= 14 \times 28$	
$= 672$ sq.cm	

In a quadrilateral ABCD, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. Prove that:



$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2.$$

Given: Quadrilateral ABCD diagonal $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.

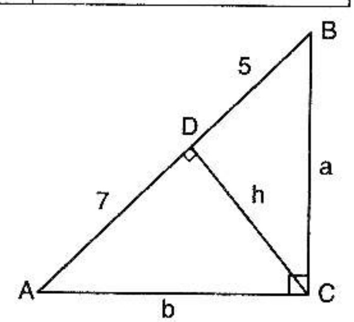
To Prove:

$$m\left(\overline{AB}\right)^2 + m\left(\overline{CD}\right)^2 = m\left(\overline{AD}\right)^2 + m\left(\overline{BC}\right)^2$$

Proof

Statements	Reasons
In right triangle AOB	
$m\left(\overline{AB}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OB}\right)^2$ ....,(i)	By Pythagoras theorem
In right triangle COD	
$m\left(\overline{CD}\right)^2 = m\left(\overline{OC}\right)^2 + m\left(\overline{OD}\right)^2$ ....,(ii)	By Pythagoras theorem
In right triangle AOD	
$m\left(\overline{AD}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OD}\right)^2$ ...,(iii)	By Pythagoras theorem
In right triangle BOC	
$m\left(\overline{BC}\right)^2 = m\left(\overline{OB}\right)^2 + m\left(\overline{OC}\right)^2$ ....,(iv)	By Pythagoras theorem
$m\left(\overline{AB}\right)^2 + m\left(\overline{CD}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OB}\right)^2 + m\left(\overline{OC}\right)^2 + m\left(\overline{OD}\right)^2$ ...,(v)	By adding (i) and (ii)
$m\left(\overline{AD}\right)^2 + m\left(\overline{BC}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OD}\right)^2 + m\left(\overline{OB}\right)^2 + m\left(\overline{OC}\right)^2$ ....(vi)	By adding (ii) and (iv)
$\left(m\overline{AB}\right)^2 + \left(m\overline{CD}\right)^2 = \left(m\overline{BC}\right)^2 + \left(m\overline{AD}\right)^2$	By adding (v) and (vi)

6. (i) In the ΔABC as shown in the figure, $m\angle ACB = 90^0$ and $\overline{CD} \perp \overline{AB}$. Find the lengths a, h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units.



Given: A ΔABC as shown

$m\ \angle ACB = 90^0$

and $\overline{CD} \perp \overline{AB}$

To prove : a, h and b.

In right angled ΔBDC

$a^2 = 25 + h^2$ .... (i)

in right angled ΔADC

$b^2 = 49 + h^2$ .... (ii)

in right angled ΔABC

$a^2+b^2 = 144$ .... (iii)

adding (i) and (ii)

$a^2+b^2 = 74+2h^2$ .... (iv)

from (iii) and (iv)

$$74 + 2h^2 = 144$$
$$2h^2 = 144-74$$
$$2h^2 = 70$$
$$h^2 = 35$$
$$h = \sqrt{35}$$

Now we will find a and b

Put $h^2 = 35$ (in Eq. 1)

$$a^2 = 25+35$$
$$a^2 = 60$$
$$a = \sqrt{60}$$
$$= \sqrt{4\times15}$$
$$a = 2\sqrt{15}$$

now put $h^2 = 35$ (in Eq. 2)

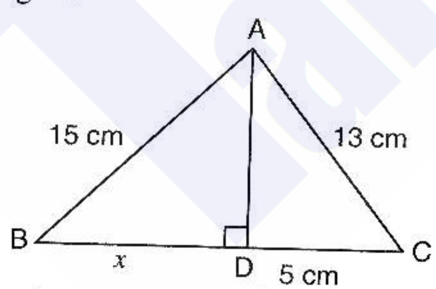
$$b^2 = 49+35$$
$$b^2 = 48$$
$$b = \sqrt{84}$$
$$b = \sqrt{4\times21}$$
$$b = 2\sqrt{21}$$

SO $a = 2\sqrt{15}$

$h = \sqrt{35}$

$b = 2\sqrt{21}$

(ii) Find the value of x in the shown in the figure.



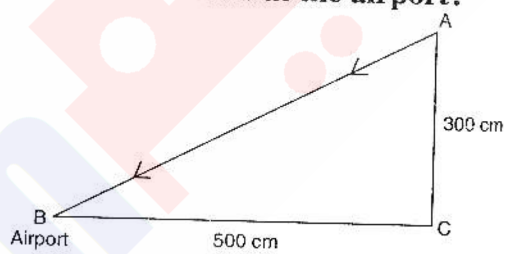
In right angled triangle ADC

$$m(\overline{AC})^2 = m(\overline{AD})^2 + m(\overline{DC})^2$$
$$(13)^2 = (AD)^2 + (5)^2$$
$$169 = (AD)^2 + 25$$
$$(AD)^2 = 169 - 25$$
$$(AD)^2 = 144$$
$$AD = \sqrt{144}$$
$$AD = 12cm$$

In right angled triangle ABD

$$(AB)^2 = (AD)^2 + (BD)^2$$
$$(15)^2 = (12)^2 + x^2$$
$$225 = 144 + x^2$$
$$x^2 = 225 - 144$$
$$x^2 = 81$$
$$x = 9 \text{ cm}$$

7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport?



Here A be the position of plane and B be the position of airport.

$$m\overline{AC} = 500m$$
$$m\overline{BC} = 300m$$
$$m\overline{AB} = ?$$

Applying Pythagoras theorem on right angled triangle ABC

$$|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$
$$= (500)^2 + (300)^2$$
$$= 250000 + 90000$$
$$= 34000$$
$$|\overline{AB}|^2 = 34\times10000$$

so $|\overline{AB}| = \sqrt{34\times10000}$

$$= \sqrt{34\times100\times100}$$

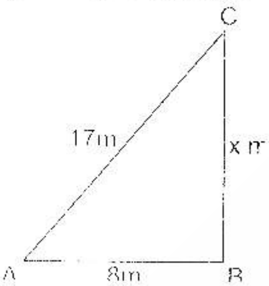
$= 100\sqrt{34}m$

So required distance is $100\sqrt{34}m$

8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach?

Ans. Let the height of ladder = x m

in right angled triangle



$(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(17)^2 = (x)^2 + (8)^2$

$289 = x^2 + 64$

$x^2 = 289 - 64$

$x^2 = 225$

$x = \sqrt{225} = 15m$

9. A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

According to figure, $m\overline{AB} = 2km$

$m\overline{BC} = 6km$

$m\overline{CD} = 3km$

Here $m\overline{AB}$ and $m\overline{CD}$ are perpendicular

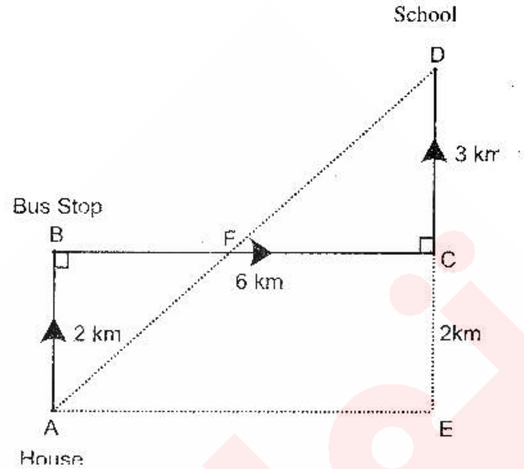
Perpendicular $= \overline{AB} + \overline{CD}$

$= 2 + 3$

$= 5km$

According to Pythagoras theorem

$(H)^2 = P^2 + B^2$



$(m\overline{AD})^2 = (5)^2 + (6)^2 = 25 + 36$

$(m\overline{AD})^2 = 61$

$m\overline{AD} = \sqrt{61}$ Km

10. Which of the following are true and which are false?

(i) In a right angled triangle greater angle is $90°$. (T)

(ii) In a right angled triangle right angle is $60°$. (F)

(iii) In a right triangle hypotenuse is a side opposite to right angle. (T)

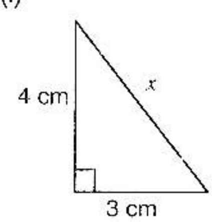
(iv) If a, b, c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$. (T)

(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm. (T)

(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm.(F)

11. Find the unknown value in each of the following figures.

(i)



By Pythagoras theorem

$(Hyp)^2 = (Perp.)^2 + (Base)^2$

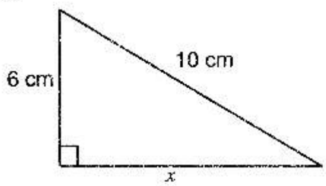
$x^2 = (4)^2 + (3)^2$

$x^2 = 16 + 9$

$x^2 = 25 \Rightarrow x = \sqrt{25}$

$x = 5cm$

(ii)



By Pythagoras theorem

$(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(10)^2 \;=\; (6)^2 + (x)^2$

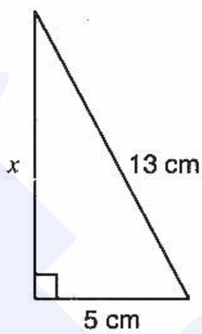
$100 \;= 36 + x^2$

$x^2 \;= 64$

$x = \sqrt{64}$

$X = 8cm$

(iii)



By Pythagoras theorem

$(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(13)^2 = (x)^2 + (2)^2$

$169 = x^2 + 25$

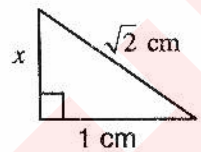
$x^2 = 169 - 25$

$x^2 = 144$

$x = \sqrt{144}$

$x = 12cm$

(iv)



By Pythagoras theorem

$(Hyp.)^2 = (Perp.)^2 + (Base)^2$

$(\sqrt{2})^2 = (x)^2 + (1)^2$

$(\sqrt{2})^2 = (x)^2 + (1)^2$

$2 = x^2 + 1$

$x^2 = 2 - 1$

$x^2 = 1$

$x = \sqrt{1} \;= 1cm$

OBJECTIVE

1. In a right angled triangle, the square of the length of hypotenuse is equal to the _____ of the squares of the lengths of the other two sides

(a) Sum
(b) Difference
(c) Zero
(d) None

2. If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a _____ triangle.

(a) Right angled
(b) Acute angled
(c) Obtuse angled
(d) None

3. Let c be the longest of the sides a, b and c of a triangle. If $a^2 + b^2 = c^2$, then the triangle is ___:
(a) Right
(b) Acute
(c) Obtuse
(d) None

4. Let c be the longest of the sides a, b and c of a triangle. If $a^2 + b^2 > c^2$ then triangle is:
(a) Acute
(b) Right
(c) Obtuse
(d) None

5. Let c be the longest of the sides a, b and c of a triangle of $a^2+b^2 < c^2$, then the triangle is:
(a) Acute
(b) Right
(c) Obtuse
(d) None

6. If 3cm and 4cm are two sides of a right angled triangle, then hypotenuse is;
(a) 5cm
(b) 3cm
(c) 4cm
(d) 2cm

7. In right triangle ____ is a side opposite to right angle.
(a) Base
(b) Perpendicular
(c) Hypotenuse
(d) None

ANSWER KEY

1.	a	**2.**	a	**3.**	a	**4.**	a	**5.**	c
6.	a	**7.**	c						