Unit 02

REAL AND COMPLEX NUMBERS

Define the following:

Natural Numbers

The numbers 1, 2, 3, ... Which we use for counting certain objects are called natural numbers or positive integers. The set of natural numbers is denoted by N.

i.e. N = {1, 2, 3, ...}

Whole Numbers

If we include 0 in the set of natural number, the resulting set is the set of whole numbers, denoted by W,

i.e. W = {0, 1, 2, 3, ...}

Integers

The set of integers consist of positive integers, 0 and negative integers and is denoted by Z

i.e. Z = {.....,−3, −2, −1, 0, 1, 2, 3, ...}

Rational Numbers

All numbers of the form *p/q* where *p, q* are integers and *q* is not zero are called rational numbers. The set of rational numbers is denoted by Q,

i.e. $Q=\left\{\frac{p}{q} \mid p,q\in Z\wedge q\neq 0,(p,q)=1\right\}$ or

$$Q=\left\{x \mid x=\frac{p}{q},p,q\in Z\wedge q\neq 0\right\}$$

Irrational Numbers

The numbers which cannot be expressed as quotient of integers are called irrational numbers.

The set of irrational numbers is denoted by Q',

i.e., $Q'=\left\{x \mid x\neq\frac{p}{q},p,q\in Z\wedge q\neq 0\right\}$

For example, the numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π and e are all irrational numbers.

Decimal form of Rational and Irrational number

a) Rational Numbers

The Decimal representation of rational numbers are of two types terminating and recurring

(i) Terminating Decimal Fractions:

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction.

For example $\frac{2}{5}=0.4$ and $\frac{3}{8}=0.375$.

(ii) Recurring and Non-terminating Decimal Fractions

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called a recurring decimal fraction.

For example $\frac{2}{9}=0.2222...$ and $\frac{4}{11}=0.363636...$

b) Irrational Numbers

The decimal representations for irrational numbers are neither terminating nor repeating in blocks. The decimal form of an irrational number would continue forever and never begin to repeat the same block of digits e.g., $\sqrt{2} = 1.414213562$ ..,

Real Number

The Union of the set of rational numbers and irrational numbers is known as the set of real numbers it is deducted by R.

$$R = Q \cup Q'$$

Hence Q and Q' are both subsets of R and

$$Q \cap Q' = \phi$$

Example

Express the following decimals in the form $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$

(a) $0.\bar{3} = 0.333 \ldots$

(b) $0.\overline{23} = 0.232323$

Solution

(a) Let $x = 0.\bar{3}$, which can be rewritten as

$x = 0.3333\ldots$ (i)

We multiply both sides of (i) by 10, and obtain

$10\,x = (0.3333\ \ldots) \times 10$

or $10\,x = 3.3333\ldots$ (ii)

Subtracting (i) from (ii), we have

$10x - x = (3.3333\ldots) - (0.333\ldots)$

or $9x = 3.0000 \Rightarrow x = \frac{1}{3}$

Hence $0.\bar{3} = \frac{1}{3}$

(b) Let $x = 0.\overline{23} = 0.23232323\ldots$

We multiply both sides of (i) by 100.

Then $100\,x = (0.232323\ldots\ldots) \times 100$

$100\,x = 23.232323\ldots\ldots\ldots$(ii)

Subtracting (i) form (ii), we get

$100x - x = (23.232323\ldots) - (0.232323\ldots)$

$99\,x = 23$

$x = \frac{23}{99}$

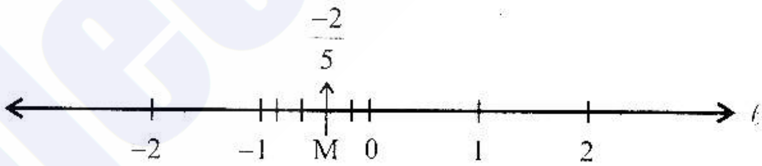
$\Rightarrow$ Thus $0.\overline{23} = \frac{23}{99}$ is a rational number.

Example

Represent the following numbers on the number line.

(i) $\frac{-2}{5}$ (ii) $\frac{15}{7}$

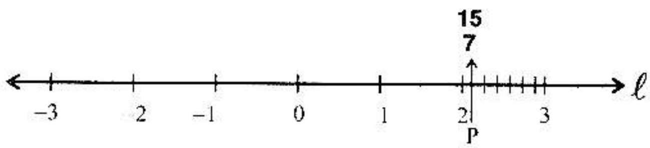
(iii) $-1\frac{7}{9}$



Solution

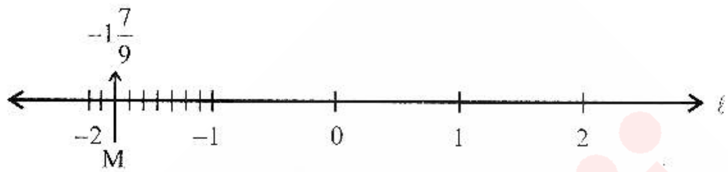
(i) For representing the rational number $-\frac{2}{5}$, on the number line ℓ, divide the unit length between 0 and −1 into five equal parts and take the end of the second part from 0 to its left side. The point M in the following figure represents the rational number $\frac{-2}{5}$.

(ii) $\frac{15}{7} = 2 + \frac{1}{7}$. It lies between 2 and 3.

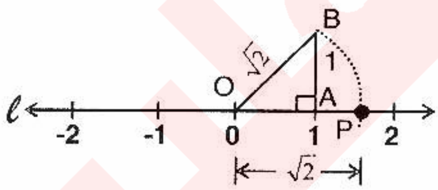


The point P represents the point $\frac{15}{7}=2\frac{1}{7}$.

(iii) For representing the rational number, $-1\frac{7}{9}$, divide the unit length between -1 and -2 into nine equal parts. Take the end of the 7^{th} part from -1. The point M in the following figure represents the rational number, $-1\frac{7}{9}$.



(iv) Irrational number such as $\sqrt{2}$ can be located on the line ℓ by geometric construction the point corresponding to $\sqrt{2}$ may be constructed by forming a right ΔAOB with sides each of length 1 as shown in the figure.



By Pythagoras theorem, OB = $\sqrt{(1)^2+(1)^2}$ = $\sqrt{2}$

By drawing an arc with centre at O and radius OB = $\sqrt{2}$ we get point P representing $\sqrt{2}$ on the number line.

Exercise 2.1

Q1. Identify which of the following are relational and irrational numbers.

(i) $\sqrt{3}$ Irrational Number

(ii) $\frac{1}{6}$ Rational Number

(iii) π Irrational Number

(iv) $\frac{15}{2}$ Rational Number

(v) 7.25 Rational Number

(vi) $\sqrt{29}$ Irrational Number

Q2. Convert the following fractions into decimal fraction.

(i) $\frac{17}{25}$

Sol: $\frac{17}{25}=0.68$

(ii) $\frac{19}{4}$

Sol: $\frac{19}{4}=4.75$

(iii) $\frac{57}{8}$

Sol: $\frac{57}{8}=7.125$

(iv) $\frac{205}{18}$

Sol: $\frac{205}{18}=11.3889$

(v) $\frac{5}{8}$

Sol: $\frac{5}{8} = 0.625$

(vi) $\frac{25}{38}$

Sol: $\frac{25}{38} = 0.65789$

Q2. Which of the following statements are true and which are false?

(i) $\frac{2}{3}$ is an irrational number. False

(ii) π is an irrational number. True

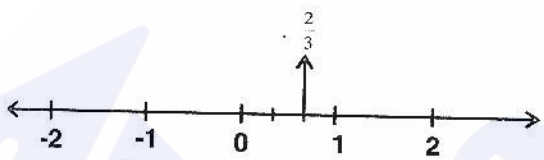
(iii) $\frac{1}{9}$ is a terminating fraction. False

(iv) $\frac{3}{4}$ is a terminating fraction. True

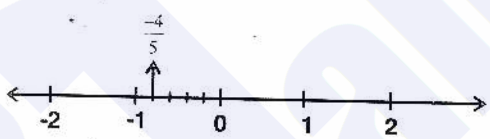
(v) $\frac{4}{5}$ is a recurring fraction. False

Q4. Represent the following numbers on the number line.

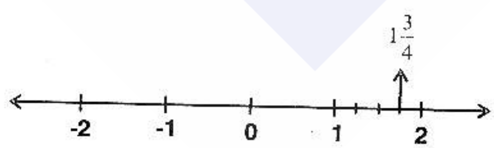
(i) $\frac{2}{3}$



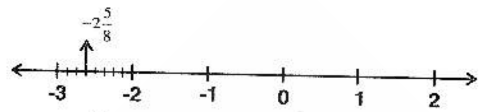
(ii) $-\frac{4}{5}$



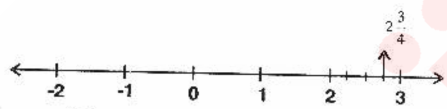
(iii) $1\frac{3}{4}$



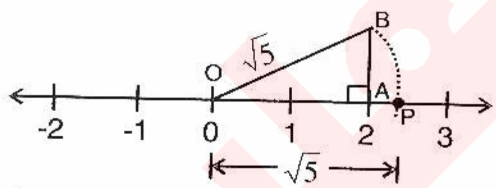
(iv) $-2\frac{5}{8}$



(v) $2\frac{3}{4}$



(vi) $\sqrt{5}$



By Pythagoras theorem

$$OB = \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$$

By drawing an arc with centre at O and radius OB = $\sqrt{5}$ we get point P representing $\sqrt{5}$ on the number line.

Q5. Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Ans. The required rational number is the mean of two given numbers, so the required number

$$= \frac{\frac{3}{4} + \frac{5}{9}}{2}$$

$$= \frac{1}{2}\left(\frac{3}{4} + \frac{5}{9}\right)$$

$$= \frac{1}{2}\left(\frac{27+20}{36}\right)$$

$$= \frac{47}{72}$$

Q6. Express the following recurring decimals as the rational number $\frac{p}{q}$, where p, q are integers and $q \neq 0$

(i) $0.\overline{5}$

Sol: Let $x = 0.\overline{5}$

$x = 0.55555.........$ (i)

Multiplying both sides by 10

$10x = 10(0.5555......)$

$10x = 5.5555........$ (ii)

Subtracting (i) from (ii)

$10x - x = (5.5555...) - (0.5555...)$

$9x = 5$

$x = \frac{5}{9}$

Hence $0.\overline{5} = \frac{5}{9}$

(ii) $0.\overline{13}$

Sol: Let $x = 0.\overline{13}$

$x = 0.13131313.........$ (i)

Multiplying both sides by 100

$100x = 100(0.131313.......)$

$100x = 13.131313.........$ (ii)

Subtracting (i) from (ii)

$100x - x = (13.1313.....) - (0.1313...)$

$99x = 13$

$x = \frac{13}{99}$

Hence $0.\overline{13} = \frac{13}{99}$

(iii) $0.\overline{67}$

Let $x = 0.\overline{67}$

$x = 0.676767.........$ (i)

Multiplying both sides by 100

$100x = 100(0.676767.......)$

$100x = 67.676767.......$ (ii)

Subtracting (i) from (ii)

$100x - x = (67.676767....) - (0.676767...)$

$99x = 67$

$x = \frac{67}{99}$

Hence $0.\overline{67} = \frac{67}{99}$

Properties of Real numbers with respect to Addition and Multiplication

a. Properties of real numbers under addition are as follows:

(i) Closure Property

$a + b \in R, \forall a, b \in R$

e.g., if -3 and $5 \in R$

then $-3 + 5 = 2 \in R$

(ii) Commutative Property

$a + b = b + a, \forall a, b \in R$

e.g., if $2, 3 \in R$

then $2 + 3 = 3 + 2$

or $5 = 5$

(iii) Associative Property

$(a + b) + c = a + (b + c), \forall a, b, c \in R$

e.g., if $5, 7, 3 \in R$

then $(5 + 7) + 3 = 5 + (7 + 3)$

or $12 + 3 = 5 + 10$

or $15 = 15$

(iv) Additive Identity

There exists a unique real number 0 called additive identity such that

$a + 0 = a = 0 + a, \quad \forall a \in R$

(v) Additive Inverse

For every $a \in R$, there exists a unique real number $-a$ called the additive inverse of a such that

$a + (-a) = 0 = (-a) + a$

e.g., additive inverse of 3 is -3 since $3 + (-3) = 0 = (-3) + (3)$

b. Properties of real numbers under multiplication are as follows:

(i) Closure Property

$ab \in R$, $\forall a, b \in R$

e.g., if $-3, 5 \in R$

then $(-3)(5) \in R$

or $-15 \in R$

(ii) Commutative Property:

$ab = ba$, $\forall a, b \in R$

e.g., if $\frac{1}{3}, \frac{3}{2} \in R$

then $\left(\frac{1}{3}\right)\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)\left(\frac{1}{3}\right)$

or $\frac{1}{2} = \frac{1}{2}$

(iii) Associative Property:

$(ab)c = a(bc)$, $\forall a, b, c \in R$

e.g., if $2, 3, 5 \in R$

then $(2 \times 3) \times 5 = 2 \times (3 \times 5)$

or $6 \times 5 = 2 \times 15$

or $30 = 30$

(iv) Multiplicative Identity:

There exists a unique real number 1, called the multiplicative identity such that

$a.1 = a = 1.a \ \forall a \in R$

(v) Multiplicative Inverse

For every non-zero real number, there exists a unique real number a^{-1} or $\frac{1}{a}$, called multiplicative inverse of a, such that

$$aa^{-1} = 1 = a^{-1}a$$

or $a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$

e.g., if $5 \in R$, then $\frac{1}{5} \in R$

such that

$$5 \times \frac{1}{5} = 1 = \frac{1}{5} \times 5$$

So, 5 and $\frac{1}{5}$ are multiplicative inverse of each other.

(vi) Multiplication is Distributive over Addition and Subtraction

For all $a, b, c \in R$

$a(b + c) = ab + ac$ (Left distributive law)

$(a + b)c = ac + bc$ (Right distributive law)

e.g., if $2, 3, 5 \in R$, then

$2(3 + 5) = 2 \times 3 + 2 \times 5$

or $2 \times 8 = 6 + 10$

or $16 = 16$

And for all $a, b, c \in R$

$a(b - c) = ab - ac$ (Left distributive law)

$(a - b)c = ac - bc$ (Right distributive law)

e.g., if $2, 5, 3 \in R$, then

$2(5 - 3) = 2 \times 5 - 2 \times 3$

or $2 \times 2 = 10 - 6$

or $4 = 4$

(b) Properties of Equality of Real Numbers:

Properties of equality of real numbers are as follows:

(i) Reflexive Property

$a = a$, $\forall a \in R$

(ii) Symmetric Property

If $a = b$, then $b = a$, $\forall a, b \in R$

(iii) Transitive Property

If $a = b$ and $b = c$, then $a = c$, $\forall a, b, c \in R$

(iv) Additive Property

If $a = b$, then $a + c = b + c$, $\forall a, b, c \in R$

(v) Multiplicative Property

If $a = b$, then $ac = bc$, $\forall a, b, c \in R$

(vi) Cancellation Property for Addition

If $a+c=b+c$, then $a=b$, $\forall a,b,c\in$ R

(vii) Cancellation property for Multiplication

If $ac = bc$, $c \neq 0$ then $a = b$, $\forall$ $a, b, c \in$ R

(c) Properties of Inequalities of Real numbers

Properties of inequalities of real numbers are as follows:

(i) **Trichotomy Property**

$\forall$ $a, b \in$ R

$a < b$ or $a = b$ or $a > b$

(ii) **Transitive Property**

$\forall$ $a, b, c \in$ R

(a) $a < b$ *and* $b < c \Rightarrow a < c$

(b) $a>b$ *and* $b>c \Rightarrow a > c$

(iii) **Multiplicative Property**

(a) $\forall$ $a, b, c \in$ R and $c > 0$

(i) $a > b \Rightarrow ac > bc$ (ii) $a < b \Rightarrow ac < bc$

(i) $a > b \Rightarrow ca > cb$ (ii) $a < b \Rightarrow ca < cb$

(b) $\forall$ $a, b, c \in$ R and $c < 0$

(i) $a > b \Rightarrow ac < bc$ (ii) $a < b \Rightarrow ac > bc$

(i) $a > b \Rightarrow ca < cb$ (ii) $a < b \Rightarrow ca > cb$

(iv) **Multiplicative Inverse Property:**

$\forall$ $a, b \in$ R and $a \neq 0, b \neq 0$

(a) $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$

(b) $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$

(v) **Additive property:**

$\forall$ **a, b c $\in$ R**

(a) $a < b \Rightarrow a + c < b+c$

$a < b \Rightarrow c + a < c+b$

(b) $a > b \Rightarrow a + c > b + c$

$a > b \Rightarrow c + a > c + b$

Exercise 2.2

Q1. Identify the property used in the following.

(i) $a+b=b+a$

commutative property w.r.t. addition

(ii) $ab(c)=a(bc)$

Associative property w.r.t. multiplication

(iii) $7\times1=7$ Multiplicative Identity

(iv) $x > y$ or $x=y$ or $x<y$

Trichotomy property of inequality

(v) $ab = ba$

Commutative property w.r.t. multiplication

(vi) $a+c=b+c \Rightarrow a=b$

Cancellation property for addition

(vii) $5+(-5)=0$ Additive Inverse

(viii) $7\times\frac{1}{7}=1$ Multiplicative inverse

(ix) $a > b \Rightarrow ac > bc (c > o)$

Multiplicative property of inequality

Q2. Fill in the following blanks by stating the properties of real numbers used.

$3x+3(y-x)$

$=3x+3y-3x$ Distributive property

$=3x-3x+3y$ Commutative property

$=0+3y$ Additive Inverse (3x, -3x)

$= 3y$ Additive Identity (o+a=a)

Q3. Give the name of property used in the following.

(i) $\sqrt{24}+0=\sqrt{24}$ Additive Identity

(ii) $-\frac{2}{3}\left(5+\frac{7}{2}\right)=\left(-\frac{2}{3}\right)(5)+\left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$

Distributive property of multiplication over addition

(iii) $\pi+(-\pi)=0$ Additive Inverse

(iv) $\sqrt{3}.\sqrt{3}$ **is a real number**

Closure property w.r.t. multiplication

(v) $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right)=1$, Multiplicative inverse

Example

Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[5]{-8}$ (ii) $\sqrt[3]{x^5}$

(iii) $y^{3/4}$ (iv) $x^{-3/2}$

Solution:

(i) $\sqrt[5]{-8} = (-8)^{1/5}$

(ii) $\sqrt[3]{x^5}=x^{5/3}$

(iii) $y^{3/4}=\sqrt[4]{y^3}$ or $\left(\sqrt[4]{y}\right)^3$

(iv) $x^{-3/2}=\sqrt{x^{-3}}$ or $\left(\sqrt{x}\right)^{-3}$

Example

Simplify $\sqrt[3]{16x^4y^5}$

Solution:

$\sqrt[3]{16x^4y^5} = \sqrt[3]{(2)(8)(x)(x^3)(y^2)(y^3)},$

$= \sqrt[3]{2xy^2(2^3)(x^3)(y^3)}$

$= \sqrt[3]{2xy^2}\,\sqrt[3]{(2^3)(x^3)(y^3)},$

$= \sqrt[3]{2xy^2}\,\sqrt[3]{(2^3)}\,\sqrt[3]{(x^3)}\,\sqrt[3]{(y^3)} = 2xy\sqrt[3]{2xy^2}$

Exercise 2.3

Q1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[3]{-64}=(-64)^{\frac{1}{3}}$

(ii) $2^{\frac{3}{5}}=\left(2^3\right)^{\frac{1}{5}}=\sqrt[5]{2^3}$

(iii) $-7^{\frac{1}{3}}=-\sqrt[3]{7}$

(iv) $y^{-\frac{2}{3}}=\left(y^{-2}\right)^{\frac{1}{3}}=\sqrt[3]{y^{-2}}$

Q2. Tell whether the following statements are true or false?

(i) $5^{\frac{1}{5}}=\sqrt{5}$ **False**

(ii) $2^{\frac{2}{3}}=\sqrt[3]{4}$ **True**

(iii) $\sqrt{49}=\sqrt{7}$ **False**

(iv) $\sqrt[3]{x^{27}}=x^3$ **False**

Q3. Simplify the following radical expressions.

(i) $\sqrt[3]{-125}=(-125)^{\frac{1}{3}}$

$=\left[(-5)^3\right]^{\frac{1}{3}}=(-5)^{3\times\frac{1}{3}}$

$=-5$

(ii) $\sqrt[4]{32}=\sqrt[4]{16\times2}$

$=\sqrt[4]{16}\times\sqrt[4]{2}$

$$=\left(2^4\right)^{1/4}\sqrt[4]{2}$$

$$=2^{4\times\frac{1}{4}}\sqrt[4]{2}$$

$$=2\left(\sqrt[4]{2}\right)$$

(iii) $\sqrt[5]{\dfrac{3}{32}}$

$$=\frac{\sqrt[5]{3}}{\sqrt[5]{32}}$$

$$=\frac{\sqrt[5]{3}}{\left(2^5\right)^{\frac{1}{5}}}$$

$$=\frac{\sqrt[5]{3}}{2^{5\times\frac{1}{5}}}$$

$$=\frac{\sqrt[5]{3}}{2}$$

(iv) $\sqrt[3]{\dfrac{-8}{27}}$

$$=\left(\frac{-8}{27}\right)^{\frac{1}{3}}$$

$$=\left[\left(\frac{-2}{3}\right)^3\right]^{\frac{1}{3}}$$

$$=\left(\frac{-2}{3}\right)^{3\times\frac{1}{3}} \quad =\frac{-2}{3}$$

Example

Use rules of exponents to simplify each expression and write the answer in terms of positive exponents.

(i) $\dfrac{x^{-2}x^{-3}y^7}{x^{-3}y^4}=\dfrac{x^{-5}y^7}{x^{-3}y^4}=x^{-5+3}y^{7-4}=x^{-2}y^3=\dfrac{y^3}{x^2}$

(ii) $\left(\dfrac{4a^3b^o}{9a^{-5}}\right)^{-2}=\left(\dfrac{4a^{3+5}\times 1}{9}\right)^{-2}=\left(\dfrac{4a^8}{9}\right)^{-2}=\left(\dfrac{9}{4a^8}\right)^2=\dfrac{81}{16a^{16}}$

Example

Simplify the following by using laws of indices:

(i) $\left(\dfrac{8}{125}\right)^{-4/3}$ (ii) $\dfrac{4(3)^n}{3^{n+1}-3^n}$

Solution

Using Laws of Indices.

(i) $\left(\dfrac{8}{125}\right)^{-4/3} = \left(\dfrac{125}{8}\right)^{4/3}=\dfrac{(125)^{4/3}}{(8)^{4/3}}=\dfrac{(5^3)^{4/3}}{(2^3)^{4/3}}=\dfrac{5^4}{2^4}=\dfrac{625}{16}$

(ii) $\dfrac{4(3)^n}{3^{n+1}-3^n} = \dfrac{4(3)^n}{3^n[3-1]}=\dfrac{4(3)^n}{2(3^n)}=\dfrac{4}{2}=2$

Exercise 2.4

Q1. Use laws of exponents to simplify

(i) $\dfrac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$

$=\dfrac{\sqrt{196}}{(243)^{2/3}(32)^{1/5}}$

$=\dfrac{\sqrt{14\times14}}{(3\times3\times3\times3\times3)^{2/3}(2\times2\times2\times2\times2)^{1/5}}$

$=\dfrac{\sqrt{(14)^2}}{\left(3^3\times3^2\right)^{2/3}\left(2^5\right)^{1/5}}$

$=\dfrac{14}{3^{\cancel{3}\times\frac{2}{\cancel{3}}}\times3^{2\times\frac{2}{3}}\times2^{\cancel{5}\times\frac{1}{\cancel{5}}}}$

$=\dfrac{\cancel{14}\,7}{3^2\times3^{2\times\frac{2}{3}}\times\cancel{2}}$

$=\dfrac{7}{3^2\times3^{\frac{4}{3}}}$

$=\dfrac{7}{3^2\times3^{\frac{3+1}{3}}}$

$=\dfrac{7}{3^2\times3^{\frac{3}{3}+\frac{1}{3}}}$

$=\dfrac{7}{3^2\times3\times3^{\frac{1}{3}}}$

$=\dfrac{7}{3^3\times\sqrt[3]{3}}$

$=\dfrac{7}{27\left(\sqrt[3]{3}\right)}$

(ii) $\left(2x^5y^{-4}\right)\left(-8x^{-3}.y^2\right)$

$=2(-8)x^{5-3}.y^{-4+2}$

$=-16x^2.y^{-2}$

$=-16\dfrac{x^2}{y^2}$

(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

$=\left(x^{-2-4}.y^{-1+3}.z^{-4+0}\right)^{-3}$

$=\left(x^{-6}.y^2.z^{-4}\right)^{-3}$

$=x^{-6(-3)}.y^{2(-3)}.z^{-4(-3)}$

$=x^{18}.y^{-6}.z^{12}$

$=\dfrac{x^{18}.z^{12}}{y^6}$

(iv) $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{\left(9^{2n}\right)\left(3^3\right)}$

$=\dfrac{\left(3^4\right)^n.3^5-(3)^{4n-1}\left(3^5\right)}{\left(3^2\right)^{2n}\left(3^3\right)}$

$=\dfrac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}}$

$=\dfrac{3^{4n+3+2}-3^{4n+4}}{3^{4n+3}}$

$=\dfrac{3^{4n+3+2}-3^{4n+3+1}}{3^{4n+3}}$

$=\dfrac{3^{4n+3}.3^2-3^{4n+3}.3^1}{3^{4n+3}}$

$$= \frac{\cancel{3^{4n+3}}\left(3^2-3^1\right)}{\cancel{3^{4n+3}}}$$

$$=9-3$$

$$=6$$

Q2. Show that

$$\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}=1$$

Sol: L.H.S

$$=\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}$$

$$=\left(x^{a-b}\right)^{a+b}\times\left(x^{b-c}\right)^{b+c}\times\left(x^{c-a}\right)^{c+a}$$

$$=x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$

$$=x^{a^2-b^2}\times x^{b^2-c^2}\times x^{c^2-a^2}$$

$$=x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$$

$$=x^0$$

$$=1$$

$$=\text{R.H.S}$$

Q3. Simplify

(i) $$\frac{2^{1/3}\times(27)^{1/3}\times(60)^{1/2}}{(180)^{1/2}\times(4)^{-1/3}\times(9)^{1/4}}$$

$$=\frac{2^{1/3}\times\left(3^3\right)^{1/3}\times\left(2^2\times3\times5\right)^{1/2}}{\left(2^2\times3^2\times5\right)^{1/2}\times\left(2^2\right)^{-1/3}\times\left(3^2\right)^{1/4}}$$

$$=\frac{2^{\frac13}\times3^{3\times\frac13}\times2^{2\times\frac12}\times3^{\frac12}\times5^{\frac12}}{2^{2\times\frac12}\times3^{2\times\frac12}\times5^{\frac12}\times2^{2\left(-\frac13\right)}\times3^{2\times\frac14}}$$

$$=\frac{2^{\frac13}\times3\times2\times3^{\frac12}\times5^{\frac12}}{2\times3\times5^{\frac12}\times2^{\frac{-2}{3}}\times3^{\frac12}}$$

$$=2^{\frac13+1-1+\frac23}\times3^{1+\frac12-1-\frac12}\times5^{\cancel{\frac12}-\cancel{\frac12}}$$

$$=2^{\frac33}\times3^0\times5^0$$

$$=2\times1\times1$$

$$=2$$

(ii) $$\sqrt{\frac{(216)^{\frac23}\times(25)^{\frac12}}{(.04)^{-\frac12}}}$$

$$=\sqrt{\frac{\left(6^3\right)^{\frac23}\times\left(5^2\right)^{\frac12}}{\left(\frac{4}{100}\right)^{-\frac12}}}$$

$$=\sqrt{\frac{6^{\cancel{3}\times\frac{2}{\cancel{3}}}\times5^{\cancel{2}\times\frac{1}{\cancel{2}}}}{\left(\frac{100}{4}\right)^{\frac12}}}\qquad=\sqrt{\frac{6^2\times5}{(25)^{\frac12}}}$$

$$=\sqrt{\frac{6^2\times5}{5^{\cancel{2}\times\frac{1}{\cancel{2}}}}}$$

$$=\sqrt{\frac{6^2\times\cancel{5}}{\cancel{5}}}$$

$$=\sqrt{6^2}$$

$$=6$$

(iii) $$5^{2^3}\div\left(5^2\right)^3$$

$$=5^8\div5^6$$

$$=\frac{5^8}{5^6}$$

$$=5^{8-6}$$

$$=5^2$$

$$=25$$

iv) $\left(x^3\right)^2 \div x^{3^2}$

$= x^6 \div x^9$

$= \frac{x^6}{x^9}$

$= \frac{1}{x^{9-6}} = \frac{1}{x^3}$

Definition of a Complex Number

A number of the form $z = a + bi$ where a and b are real numbers and $i = \sqrt{-1}$, is called a complex number and is represented by z i.e., $z = a + ib$

Set of Complex Numbers

The set of all complex numbers is denoted by C and

$C = \{ z \mid z = a + bi$, where $a, b \in R$ and $i = \sqrt{-1} \}$

The numbers a and b, called the real and imaginary parts of z, are denoted as $a = Re(z)$ and $b = Im(z)$ respectively.

Conjugate of a Complex Number

If we change i to $-i$ in $z = a + bi$, we obtain another complex number $a - bi$ called the complex conjugate of z and is denoted by $\bar{z}$ (read z bar).

Thus, if $z = -1 - i$, then $\bar{z} = -1 + i$.

The number $a + bi$ and $a - bi$ are called conjugates of each other.

Equality of Complex Numbers and its Properties

For all $a, b, c, d \in R$,

$a+bi=c+di$ if and only if $a = c$ and $b = d$.

e.g., $2x + y^2 i = 4 + 9i$

if and only if

$2x = 4$ and $y^2 = 9$, i.e., $x = 2$ and $y = \pm 3$

Properties of real numbers R are also valid for the set of complex numbers.

(i) $Z_1 = Z_2$, (Reflexive Law)

(ii) If $Z_1 = Z_2$, then $Z_2 = Z_1$ (Symmetric law)

(iii) If $Z_1 = Z_2$, and $Z_2 = Z_3$ then $Z_1 = Z_3$ (transitive law)

Exercise 2.5

Q1. **Evaluate**

(i) i^7

$= i^6 . i$

$= \left(i^2\right)^3 . i$

$= (-1)^3 . i$

$= -1 . i$

$= -i$

(ii) i^{50}

$= \left(i^2\right)^{25}$

$= (-1)^{25}$

$= -1$

(iii) i^{12}

$= \left(i^2\right)^6$

$= (-1)^6$

$= 1$

(iv) $(-i)^8$

$= i^8$

$= \left(i^2\right)^4$

$= (-1)^4$

$= 1$

(v) $(-i)^5$

$= -i^5$

$= -(i^4.i)$

$= -((i^2)^2.i)$

$= -((-1)^2.i)$

$-(i)$

$= -i$

(vi) i^{27}

$= i^{26}.\,i$

$= (i^2)^{13}.\,i$

$= (-1)^{13}.\,i$

$= (-1)i$

$= -i$

Q2. Write the conjugate of the following numbers.

(i) $2+3i$

Let $z = 2+3i$

then $\bar{z} = 2-3i$

(ii) $3-5i$

Let $z = 3-5i$

$\bar{z} = 3+5i$

(iii) $-i$

Sol: Let $z = 0-i$

then $\bar{z} = 0+i = i$

(iv) $-3+4i$

Let $z = -3+4i$

then $\bar{z} = -3-4i$

(v) $-4-i$

Let $z = -4-i$

then $\bar{z} = -4+i$

vi) $i-3$

Let $z = -3+i$

then $\bar{z} = -3-i$

Q3. Write the real and imaginar part of the following numbe

(i) $1+i$

Let $z = 1+i$

$\text{Re}(z) = 1,\ \text{Im}(z) = 1$

(ii) $-1+2i$

Let $z = -1+2i$

$\text{Re}(z) = -1,\ \text{Im}(z) = 2$

(iii) $-3i+2$

Let $z = 2-3i$

$\text{Re}(z) = 2,\ \text{Im}(z) = -3$

(iv) $-2-2i$

Let $z = -2-2i$

$\text{Re}(z) = -2,\ \text{Im}(z) = -2$

(v) $-3i$

Let $z = 0-3i$

$\text{Re}(z) = 0,\ \text{Im}(z) = -3$

(vi) $2+0i$

Let $z = 2+0i$

$\text{Re}(z) = 2,\ \text{Im}(z) = 0$

Q4. Find the value of x and y if

$x+iy+1 = 4-3i$

Sol: $x+iy+1 = 4-3i$

$x+iy = 4-1-3i$

$x+iy = 3-3i$

Two complex numbers are equal if the real and imaginary parts are equal

So $x = 3$ and $y = -3$

Basic Operations on Complex Numbers

(i) **Addition:**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers and $a, b, c, d \in \mathbb{R}$.

The sum of two complex numbers is given by

$z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$

i.e., the sum of two complex numbers is the sum of the corresponding real and the imaginary parts.

e.g., $(3 - 8i) + (5 + 2i) = (3 + 5) + (-8 + 2)i = 8 - 6i$

(ii) **Multiplication:**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers and $a, b, c, d \in \mathbb{R}$.

The products are found as

(i) If $k \in \mathbb{R}$, $kz_1 = k(a + bi) = ka + kbi$.
(Multiplication of a complex number with a scalar)

(ii) $Z_1 Z_2 = (a + bi)(c + di) = (ac - bd) + (ad + bc)i$
(Multiplication of two complex numbers)

The multiplication of any two complex numbers $(a + bi)$ and $(c + di)$ is explained as

$$
\begin{aligned}
z_1 z_2 &= (a + bi)(c + di) = a(c + di) + bi(c + di) \\
&= ac + adi + bci + bdi^2 \\
&= ac + adi + bci + bd(-1) && \text{(since } i^2 = -1\text{)} \\
&= (ac - bd) + (ad + bc)i && \text{(combining like terms)}
\end{aligned}
$$

e.g., $(2 - 3i)(4 + 5i) = 8 + 10i - 12i - 15i^2 = 23 - 2i$. (since $i^2 = -1$)

(iii) **Subtraction:**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers and $a, b, c, d \in \mathbb{R}$.

The difference between two complex numbers is given by

$z_1 - z_2 = (a + bi) - (c + di) = (a - c) + (b - d)i$

e.g.., $(-2 + 3i) - (2 + i) = (-2 - 2) + (3 - 1)i = -4 + 2i$

i.e., the difference of two complex numbers is the difference of the corresponding real and imaginary parts.

iv) **Division:**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers and $a, b, c, d, \in \mathbb{R}$.

The division of $a + bi$ by $c + di$ is given by

$$\frac{z_1}{z_2} = \frac{a+bi}{c+di} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$$

(Multiplying the numerator and denominator by $c-di$, the complex conjugate of $c+di$).

$$= \frac{ac + bci - adi - bdi^2}{c^2 - (di)^2}$$

$$= \frac{ac + bci - adi + bd}{c^2 + d^2}, \text{since } i^2 = -1$$

$$= \frac{(ac+bd)+(bc-ad)i}{c^2+d^2} = \frac{ac+bd}{c^2+d^2} + \left(\frac{bc-ad}{c^2+d^2}\right)i$$

Example

Separate the real and imaginary parts of $(-1+\sqrt{-2})^2$

Solution

Let $z = -1+\sqrt{-2}$, then

$z^2 = (-1 + \sqrt{-2})^2 = (-1 + i\sqrt{2})^2$, changing to i-form

$= (-1 + i\sqrt{2})(-1 + i\sqrt{2}) = (-1)(-1 + i\sqrt{2}) + i\sqrt{2}(-1 + i\sqrt{2})$

$= 1 - i\sqrt{2} - i\sqrt{2} + 2i^2 = -1 - 2\sqrt{2}\,i$

Hence $\text{Re}(z^2) = -1$ and $Im(z^2) = -2\sqrt{2}$

Example

Express $\frac{1}{1+2i}$ in the standard form $a+bi$.

Solution

We have $\frac{1}{1+2i} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$

(multiplying the numerator and dominator by $\overline{1+2i}$

$= \frac{1-2i}{1-(2i)^2} = \frac{1-2i}{1-4i^2}$, (simplifying)

$= \frac{1-2i}{5}$, (since $i^2 = -1$)

$= \frac{1}{5} - \frac{2}{5}i$, which is of the form $a + bi$

Example

Express $\frac{4+5i}{4-5i}$ in the standard form $a + bi$.

Solution

$\frac{4+5i}{4-5i} = (4+5i) \cdot \frac{1}{4-5i} \times \frac{4+5i}{4+5i}$

(multiplying and dividing by the conjugate of $(4-5i)$)

$= \frac{(4+5i)^2}{(4)^2-(5i)^2} = \frac{16+40i+25i^2}{16-25i^2}$ (simplifying)

$= \frac{16+40i-25}{16+25}$ (since $i^2 = -1$)

$= \frac{-9+40i}{41} = \frac{-9}{41} + \frac{40}{41}i$

Example

Solve $(3-4i)(x+yi) = 1 + 0.i$ for real numbers x and y, where $i = \sqrt{-1}$.

Solution

We have $(3-4i)(x+yi) = 1+0.i$

or $3x+3iy-4ix-4i^2y = 1+0.i$

or $3x+3iy-4ix-4(-1)y = 1+0.i$

or $3x+4y+(3y-4x)i = 1+0.i$

Equating the real and imaginary parts, we obtain

$3x + 4y = 1$ and $3y - 4x = 0$

Solving these two equations simultaneously, we have $x = \frac{3}{25}$ and $y = \frac{4}{25}$

Exercise 2.6

Q1. Identify the following statements as true or false.

(i) $\sqrt{-3}\times\sqrt{-3}=3$ False

(ii) $i^{73}=-i$ False

(iii) $i^{10}=-1$ True

(iv) Complex conjugate of $\left(-6i+i^2\right)$ is $(-1+6i)$ True

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number. False

(vi) If $(a-1)-(b+3)i=5+8i$ then $a=6$ and $b=-11$. True

(vii) Product of a complex number and its conjugate is always a non-negative real number. True

Q2. Express each complex number in the standard form $a+bi$, where 'a' and 'b' are real numbers.

(i) $(2+3i)+(7-2i)$

$=2+3i+7-2i$

$=(2+7)+(3-2)i$

$=9+i$

(ii) $2(5+4i)-3(7+4i)$

$=10+8i-21-12i$

$=(10-21)+(8-12)i$

$=-11-4i$

(iii) $-1(-3+5i)-(4+9i)$

$=3-5i-4-9i$

$=(3-4)+(-5-9)i$

$=-1-14i$

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$=2(-1)+6i^2.i+3\left(i^2\right)^8-6i^{18}.i+4i^{24}.i$

$=-2+6(-1).i+3(-1)^8-6\left(i^2\right)^9.i+4\left(i^2\right)^{12}.i$

$=-2-6i+3(1)-6(-1)^9.i+4(-1)^{12}.i$

$=-2-6i+3-6(-1)i+4(1).i$

$=-2-\cancel{6i}+3+\cancel{6i}+4i$

$=1+4i$

Q3. Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$

$=21-14i-9i+6i^2$

$=21-23i+6(-1)$

$=21-6-23i$

$=15-23i$

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

$=\left(2-\sqrt{4}.\sqrt{-1}\right)\left(3-\sqrt{4}\sqrt{-1}\right)$

$=(2-2i)(3-2i)$

$=6-4i-6i+4i^2$

$=6-10i+4(-1)$

$=6-10i-4$

$=2-10i$

(iii) $\left(\sqrt{5}-3i\right)^2$

$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$

$=5+9i^2-6\sqrt{5}i$

$=5+9(-1)-6\sqrt{5}i$

$=5-9-6\sqrt{5}i$

$=-4-6\sqrt{5}i$

(iv) $(2-3i)(\overline{3-2i})$

$=(2-3i)(3+2i)$

$=6+4i-9i-6i^2$

$=6-5i-6(-1)$

$=6-5i+6$

$=12-5i$

Q4. Simplify and write your answer in the form of $a+bi$

(i) $\dfrac{-2}{1+i}$

$=\dfrac{-2}{1+i}\times\dfrac{1-i}{1-i}$

$=\dfrac{-2(1-i)}{(1)^2-(i)^2}$

$=\dfrac{-2(1-i)}{1-i^2}$

$=\dfrac{-2(1-i)}{1-(-1)}$

$=\dfrac{-2(1-i)}{1+1}$

$=\dfrac{-\cancel{2}(1-i)}{\cancel{2}}$

$=-(1-i)$

$=-1+i$

(ii) $\dfrac{2+3i}{4-i}$

$=\dfrac{2+3i}{4-i}\times\dfrac{4+i}{4+i}$

$=\dfrac{(2+3i)(4+i)}{(4)^2-(i)^2}$

$=\dfrac{8+2i+12i+3i^2}{16-i^2}$

$=\dfrac{8+14i+3(-1)}{16-(-1)}$

$=\dfrac{8+14i-3}{16+1}$

$=\dfrac{5+14i}{17}$

$=\dfrac{5}{17}+\dfrac{14}{17}i$

(iii) $\dfrac{9-7i}{3+i}$

$=\dfrac{9-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{(9-7i)(3-i)}{(3)^2-(i)^2}$

$=\dfrac{27-9i-21i+7i^2}{9-i^2}=\dfrac{27-30i+7(-1)}{9-(-1)}$

$=\dfrac{27-7-30i}{9+1}$

$=\dfrac{20-30i}{10}$

$=\dfrac{20}{10}-\dfrac{30}{10}i$

$=2-3i$

(iv) $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

$=\dfrac{(2-6i)-(4+i)}{3+i}$

$=\dfrac{2-6i-4-i}{3+i}$

$=\dfrac{-2-7i}{3+i}$

$=\dfrac{-2-7i}{3+i}\times\dfrac{3-i}{3-i}$

$$=\frac{(-2-7i)(3-i)}{(3)^2-(i)^2}$$

$$=\frac{-6+2i-21i+7i^2}{9-i^2}$$

$$=\frac{-6-19i+7(-1)}{9-(-1)}$$

$$=\frac{-6-7-19i}{9+1}$$

$$=\frac{-13-19i}{10}$$

$$=\frac{-13}{10}-\frac{19}{10}i$$

) $\left(\frac{1+i}{1-i}\right)^2$

$$=\frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$$

$$=\frac{1+i^2+2i}{1+i^2-2i}$$

$$=\frac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$$

$$=\frac{2i}{-2i}$$

$$=-1$$

$$=-1+0\,i$$

$$\frac{1}{(2+3i)(1-i)}$$

$$=\frac{1}{2-2i+3i-3i^2}$$

$$=\frac{1}{2+i-3(-1)}$$

$$=\frac{1}{2+i+3}$$

$$=\frac{1}{5+i}$$

$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$

$$=\frac{5-i}{(5)^2-(i)^2}$$

$$=\frac{5-i}{25-i^2}$$

$$=\frac{5-i}{25-(-1)}$$

$$=\frac{5-i}{25+1}$$

$$=\frac{5-i}{26}$$

$$=\frac{5}{26}-\frac{1}{26}i$$

Q5. Calculate (a) $\bar{z}$ **(b)** $z+\bar{z}$ **c)** $z-\bar{z}$ **(d)** $z.\bar{z}$ **for each of the following.**

(i) $z=0-i$

(a) $\bar{z}=0+i$

(b) $z+\bar{z}=0-i+0+i=0$

(c) $z-\bar{z}=0-i-(0+i)$

$=0-i-0-i$

$=-2\,i$

(d) $z.\,\bar{z}=(0-i)(0+i)$

$=(0)^2-(i)^2=0-(-1)$

$=1$

(ii) $z=2+i$

(a) $\bar{z}=2-i$

(b) $z+\bar{z}=2+\cancel{i}+2-\cancel{i}$

$=4$

(c) $z-\bar{z}=(2+i)-(2-i)$

$=\cancel{2}+i-\cancel{2}+i$

$=2i$

(d) $z.\bar{z} = (2+i)(2-i)$

$= (2)^2 - (i)^2$

$= 4 - i^2$

$= 4 - (-1)$

$= 4 + 1$

$= 5$

(iii) $z = \dfrac{1+i}{1-i}$

$= \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{(1+i)^2}{(1)^2 - (i)^2}$

$= \dfrac{(1)^2 + (i)^2 + 2(1)(i)}{1-i^2}$

$= \dfrac{1+i^2+2i}{1-(-1)} = \dfrac{1-1+2i}{1+1}$

$= \dfrac{\cancel{2}i}{\cancel{2}} = i$

$z = 0 + i$

(a) $\bar{z} = 0 - i$

(b) $z + \bar{z} = 0 + i + 0 - i = 0$

(c) $z - \bar{z} = 0 + i - (0 - i)$

$= 0 + i - 0 + i$

$= 2i$

(d) $z.\bar{z} = (0+i)(0-i)$

$= (0)^2 - (i)^2 = 0 - (-1)$

$= 0 + 1 = 1$

(iv) $z = \dfrac{4-3i}{2+4i}$

$= \dfrac{4-3i}{2+4i} \times \dfrac{2-4i}{2-4i}$

$= \dfrac{(4-3i)(2-4i)}{(2)^2 - (4i)^2}$

$= \dfrac{8-16i-6i+12i^2}{4-16i^2}$

$= \dfrac{8-22i+12(-1)}{4-16(-1)}$

$= \dfrac{8-12-22i}{4+16}$

$= \dfrac{-4-22i}{20}$

$= -\dfrac{4}{20} - \dfrac{22}{20}i$

$z = -\dfrac{1}{5} - \dfrac{11}{10}i$

(a) $\bar{z} = -\dfrac{1}{5} + \dfrac{11}{10}i$

(b) $z + \bar{z} = -\dfrac{1}{5} + \cancel{\dfrac{11}{10}i} - \dfrac{1}{5} + \cancel{\dfrac{11}{10}i}$

$= -\dfrac{2}{5}$

(c) $z - \bar{z} = -\dfrac{1}{5} - \dfrac{11}{10}i - \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\cancel{\dfrac{1}{5}} - \dfrac{11}{10}i + \cancel{\dfrac{1}{5}} - \dfrac{11}{10}i$

$= -\dfrac{22}{10}i$

$= -\dfrac{11}{5}i$

(d) $z.\bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right)\left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= \left(-\dfrac{1}{5}\right)^2 - \left(\dfrac{11}{10}i\right)^2$

$= \dfrac{1}{25} - \dfrac{121}{100}i^2$

$$=\frac{1}{25}-\frac{121}{100}(-1)$$
$$=\frac{1}{25}+\frac{121}{100}$$
$$=\frac{4+121}{100}$$
$$=\frac{125}{100}$$
$$=\frac{5}{4}$$

Q6. If $z=2+3i$ and $w=5-4i$, show that:

(i) $\overline{z+w}=\bar{z}+\bar{w}$

Sol: L.H.S $=\overline{z+w}$

$$z+w=2+3i+5-4i$$
$$z+w=7-i$$
$$\overline{z+w}=7+i$$

Now R.H.S $=\bar{z}+\bar{w}$

$$\bar{z}=2-3i$$
$$\bar{w}=5+4i$$
$$\bar{z}+\bar{w}=2-3i+5+4i$$
$$=7+i$$

Hence $\overline{z+w}=\bar{z}+\bar{w}$

(ii) $\overline{z-w}=\bar{z}-\bar{w}$

Sol: L.H.S $=\overline{z-w}$

$$z-w=2+3i-(5-4i)$$
$$=2+3i-5+4i$$
$$=-3+7i$$
$$\overline{z-w}=-3-7i$$

R.H.S $=\bar{z}-\bar{w}$

$$\bar{z}=2-3i$$
$$\bar{w}=5+4i$$
$$\bar{z}-\bar{w}=(2-3i)-(5+4i)$$
$$=2-3i-5-4i$$
$$=-3-7i$$

Hence $\overline{z-w}=\bar{z}-\bar{w}$

(iii) $\overline{z.w}=\bar{z}.\bar{w}$

L.H.S $=\overline{z.w}$

$$z.w=(2+3i)(5-4i)$$
$$=10-8i+15i-12i^2$$
$$=10+7i-12(-1)$$
$$=10+7i+12$$
$$=22+7i$$
$$\overline{z.w}=22-7i$$

R.H.S $=\bar{z}.\bar{w}$

$$\bar{z}=2-3i$$
$$\bar{w}=5+4i$$
$$\bar{z}.\bar{w}=(2-3i)(5+4i)$$
$$=10+8i-15i-12i^2$$
$$=10-7i-12(-1)$$
$$=10-7i+12$$
$$\bar{z}.\bar{w}=22-7i$$

Hence $\overline{z.w}=\bar{z}.\bar{w}$

(iv) $\overline{\left(\frac{z}{w}\right)}=\frac{\bar{z}}{\bar{w}}$, where $w\neq 0$

LHS $=\overline{\left(\frac{z}{w}\right)}$

$$\frac{z}{w}=\frac{2+3i}{5-4i}$$
$$=\frac{2+3i}{5-4i}\times\frac{5+4i}{5+4i}$$
$$=\frac{(2+3i)(5+4i)}{(5)^2-(4i)^2}=\frac{10+8i+15i+12i^2}{25-16i^2}$$

$$= \frac{10+23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-12+23i}{25+16}$$

$$= \frac{-2+23i}{41}$$

$$= -\frac{2}{41}+\frac{23}{41}i$$

$$\overline{\left(\frac{z}{w}\right)} = -\frac{2}{41}-\frac{23}{41}i$$

R.H.S $= \dfrac{\bar{z}}{\bar{w}}$

$$\bar{z} = 2-3i$$

$$\bar{w} = 5+4i$$

$$\frac{\bar{z}}{\bar{w}} = \frac{2-3i}{5+4i}$$

$$= \frac{(2-3i)(5-4i)}{(5)^2-(4i)^2}$$

$$= \frac{10-8i-15i+12i^2}{25-16i^2}$$

$$= \frac{10-23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-12-23i}{25+16}$$

$$= \frac{-2-23i}{41}$$

$$= -\frac{2}{41}-\frac{23}{41}i$$

Hence $\overline{\left(\dfrac{z}{w}\right)} = \dfrac{\bar{z}}{\bar{w}}$

(v) $\dfrac{1}{2}(z+\bar{z})$ is the real part of z

Sol: $z = 2+3i$

Now $\bar{z} = 2-3i$

$$\frac{1}{2}(z+\bar{z}) = \frac{1}{2}(2+\cancel{3i}+2-\cancel{3i})$$

$$= \frac{1}{\cancel{2}}(\cancel{4})$$

$$\frac{1}{2}(z+\bar{z}) = 2$$

$$\frac{1}{2}(z+\bar{z}) = \operatorname{Re}(z)$$

Hence $\dfrac{1}{2}(z+\bar{z})$ is equal to the real part of z.

(vi) $\dfrac{1}{2i}(z-\bar{z})$ is the real part of z.

Sol. $z = 2+3i$

Now $\bar{z} = 2-3i$

$$\frac{1}{2i}(z-\bar{z}) = \frac{1}{2i}[(2+3i)-(2-3i)]$$

$$= \frac{1}{2i}(\cancel{2}+3i-\cancel{2}+3i)$$

$$= \frac{6i}{3i}$$

$$= 3$$

$$\frac{1}{2i}(z-\bar{z}) = R(z)$$

Hence proved that $\dfrac{1}{2i}(z-\bar{z})$ is equal to the real part of z.

Q7. Solve the following equation for real x and y

(i) $(2-3i)(x+yi) = 4+i$

$$(x+yi) = \frac{4+i}{2-3i}$$

$$= \frac{4+i}{2-3i}\times\frac{2+3i}{2+3i}$$

$$=\frac{(4+i)(2+3i)}{(2)^2-(3i)^2}$$

$$=\frac{8+12i+2i+3i^2}{4-9i^2}$$

$$=\frac{8+14i+3(-1)}{4-9(-1)}$$

$$=\frac{8-3+14i}{4+9}$$

$$=\frac{5+14i}{13}$$

$$(x+yi)=\frac{5}{13}+\frac{14}{13}i$$

$\Rightarrow x=\frac{5}{13}$ and $y=\frac{14}{13}$

(ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$

$$3x+3yi-2xi-2yi^2=2x-4yi+2i-1$$

$$3x+(3y-2x)i-2y(-1)=2x-1+(2-4y)i$$

$$(3x+2y)+(3y-2x)i=(2x-1)+(2-4y)i$$

$\Rightarrow 3x+2y=2x-1$ ....(i) and

$3y-2x=2-4y$ ......(ii)

From (i) $3x-2x+2y=-1$

$x+2y=-1$ ....(iii)

From (ii) $-2x+3y+4y=2$

$-2x+7y=2$ ....(iv)

Multiplying (iii) by 2 and adding in (iv)

$$\cancel{2x}+4y=\cancel{-2}$$

$$-\cancel{2x}+7y=\cancel{2}$$

$$11y=0$$

$$y=\frac{0}{11}$$

$$\boxed{y=0}$$

Putting value of y in (iii)

$$x+2y=-1$$

$$x+2(0)=-1$$

$$x+0=-1$$

$$\boxed{x=-1}$$

(iii) $(3+4i)^2-2(x-yi)=x+yi$

$$(3)^2+(4i)^2+2(3)(4i)-2x+2yi=x+yi$$

$$9+16i^2+24i-2x+2yi=x+yi$$

$$9+16(-1)+24i-2x+2yi=x+yi$$

$$9-16+24i-2x+2yi=x+yi$$

$$-7-2x+(24+2y)i=x+yi$$

$\Rightarrow$ $x=-7-2x$

$$x+2x=-7$$

$$3x=-7$$

$$\boxed{x=\frac{-7}{3}}$$

and $24+2y=y$

$$2y-y=-24$$

$$\boxed{y=-24}$$

OBJECTIVE

Q. Select the correct answer.

1. $\left(27x^{-1}\right)^{\frac{-2}{3}} =$ ______

(a) $\frac{\sqrt[3]{x^2}}{9}$ (b) $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ (d) $\frac{\sqrt{x^3}}{8}$

2. Write $\sqrt[7]{x}$ in exponential form …….

(a) x (b) x^7

(c) $x^{\frac{1}{7}}$ (d) $x^{\frac{7}{2}}$

3. Write $4^{\frac{2}{3}}$ with radical sign……

(a) $\sqrt[3]{4^2}$ (b) $\sqrt{4^3}$

(c) $\sqrt[2]{4^3}$ (d) $\sqrt{4^6}$

4. In $\sqrt[3]{35}$ the radicand is

(a) 3 (b) $\frac{1}{3}$

(c) 35 (d) None of these

5. $\left(\frac{25}{16}\right)^{\frac{-1}{2}} =$ ______

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$

(c) $\frac{-5}{4}$ (d) $\frac{-4}{5}$

6. The conjugate of 5 + 4i is ______

(a) $-5+4i$ (b) $-5-4i$

(c) $5-4i$ (d) $5+4i$

7. The value of i^9 is ____

(a) 1 (b) -1

(c) i (d) $-i$

8. Every real number is ____

(a) A positive integer

(b) A rational number

(c) A negative integer

(d) A complex number

9. Real part of $2ab\,(i+i^2)$ is ____

(a) 2ab (b) $-2ab$

(c) 2abi (d) $-2abi$

10. Imaginary part of $-i\,(3i+2)$ is____

(a) -2 (b) 2

(c) 3 (d) -3

11. Which of the following sets have the closure property w.r.t. addition ___

(a) $\{0\}$ (b) $\{0, -1\}$

(c) $\{0, 1\}$ (d) $\left\{1, \sqrt{2}, \frac{1}{2}\right\}$

12. Name the property of real numbers used in $\left(\frac{-\sqrt{5}}{2}\right)\times 1 = \frac{-\sqrt{5}}{2}\times 1$

(a) Additive identity

(b) Additive Inverse

(c) Multiplicative identity

(d) Multiplicative Inverse

13. If $z < 0$ then $x < y \Rightarrow$

(a) $xz < yz$ (b) $xz > yz$

(c) $xz = yz$ (d) none of these

14. If a, b ∈ R then only one of a = b or a < b or a > b holds is called…

(a) Trichotomy property

(b) Transitive property

(c) Additive property

(d) Multiplicative property

15. A non-terminating, non-recurring decimal represents:
(a) A natural number
(b) A rational number
(c) An irrational number
(d) A prime number

16. The union of the set of rational numbers and irrational numbers is known as set of ____
(a) Rational number
(b) Irrational
(c) Real number
(d) Whole number

17. For each prime number A, $\sqrt{A}$ is an ____
(a) Irrational (b) Rational
(c) Real (d) Whole

18. Square roots of all positive non-square integers are ____
(a) Irrational (b) Rational
(c) Real (d) Whole

19. π is an ____ number.
(a) Irrational (b) Rational
(c) Real (d) None

20. $\forall a, b, c \in R$ than $a < b$ and $b < c$ $\rightarrow a < c$ is ____ property.
(a) Transitive
(b) Trichotomy property
(c) Additive property
(d) Multiplicative property

21. Name the property of real numbers used in $x > y$ or $x = y$ or $x < y$.
(a) Trichotomy
(b) Transitive
(c) Additive
(d) Multiplicative

22. Name the property of real numbers used in $\pi + (-\pi) = 0$.
(a) Additive inverse
(b) Multiplicative inverse
(c) Additive identity
(d) Multiplicative identity

23. $\sqrt{3}.\sqrt{3}$ is a ____ number.
(a) Rational (b) Irrational
(c) Real (d) None

24. $\sqrt[n]{ab} =$ ____
(a) $\sqrt[n]{a}\,\sqrt[n]{b}$ (b) $\sqrt{a}\,\sqrt{b}$
(c) $\sqrt[n]{a}\,\sqrt{b}$ (d) $\sqrt{a}\,\sqrt[n]{b}$

25. $\sqrt[5]{-8} =$ ____
(a) $(-8)^{1/5}$ (b) $(-8)^5$
(c) (-8) (d) $(8)^{\frac{1}{5}}$

26. The value of i^{10} is:
(a) -1 (b) 1
(c) $-i$ (d) i

27. The solution set of $x^2 + 1 = 0$ is:
(a) $\{i, i\}$ (b) $\{i, -i\}$
(c) $\{-i, -i\}$ (d) None

28. The conjugate of $2 + 3i$ is ____
(a) $2 - 3i$ (b) $-2 - 3i$
(c) $-2 + 3i$ (d) $2 + 3i$

29. Real part of $\left(-1+\sqrt{-2}\right)^2$ is:
(a) -1 (b) $-2\sqrt{2}$
(c) i (d) $2\sqrt{2}$

30. Imaginary part of $\left(-1+\sqrt{-2}\right)^2$ is
(a) -1 (b) $-2\sqrt{2}$
(c) 1 (d) $2\sqrt{2}$

31.	Product of a complex number and its conjugate is always a non-negative__	(a) Real (b) Irrational (c) Rational (d) None

ANSWER KEY

1.	a	**2.**	c	**3.**	a	**4.**	c	**5.**	b
6.	c	**7.**	c	**8.**	d	**9.**	b	**10.**	a
11.	a	**12.**	c	**13.**	b	**14.**	a	**15.**	c
16.	c	**17.**	a	**18.**	a	**19.**	a	**20.**	a
21.	a	**22.**	a	**23.**	c	**24.**	a	**25.**	a
26.	a	**27.**	b	**28.**	a	**29.**	a	**30.**	b
31.	a								

REVIEW EXERCISE

3. Simplify: (i) $\sqrt[4]{81y^{-12}x^{-8}}$

$$=\left(3^4y^{-12}x^{-8}\right)^{\frac{1}{4}}$$

$$=\left(3^4\right)^{\frac{1}{4}}\left(y^{-12}\right)^{\frac{1}{4}}\left(x^{-8}\right)^{\frac{1}{4}}$$

$$=3y^{-3}x^{-2}$$

$$=\frac{3}{x^2y^3}$$

(ii) $\sqrt{25x^{10n}y^{8m}}$

$$=\left(5^2x^{10n}y^{8m}\right)^{\frac{1}{2}}$$

$$=\left(5^2\right)^{\frac{1}{2}}\left(x^{10n}\right)^{\frac{1}{2}}\left(y^{8m}\right)^{\frac{1}{2}}$$

$$=5x^{5n}y^{4m}$$

(iii) $\left(\dfrac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right)^{\frac{1}{5}}$

$$=\left(x^{3+2}y^{4+1}z^{5+5}\right)^{\frac{1}{5}}$$

$$=\left(x^5y^5z^{10}\right)^{\frac{1}{5}}$$

$$=\left(x^5\right)^{\frac{1}{5}}\left(y^5\right)^{\frac{1}{5}}\left(z^{10}\right)^{\frac{1}{5}}$$

$$=xyz^2$$

(iv) $\left(\dfrac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

$$=\left(\frac{2^5x^{-6}y^{-4}z}{5^4x^4yz^{-4}}\right)^{\frac{2}{5}}$$

$$=\left(\frac{2^5x^{-6-4}y^{-4-1}z^{1+4}}{5^4}\right)^{\frac{2}{5}}$$

$$=\left(\frac{2^5x^{-10}y^{-5}z^5}{5^4}\right)^{\frac{2}{5}}$$

$$=\frac{\left(2^5\right)^{\frac{2}{5}}\left(x^{-10}\right)^{\frac{2}{5}}\left(y^{-5}\right)^{\frac{2}{5}}\left(z^5\right)^{\frac{2}{5}}}{\left(5^4\right)^{\frac{2}{5}}}$$

$$=\frac{2^2x^{-4}y^{-2}z^2}{5^{\frac{8}{5}}}$$

$$=\frac{4z^2}{x^4y^25.5^{\frac{3}{5}}}$$

Q.4. Simplify: $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

$$=\left[\frac{\left(2^3\times3^3\right)^{\frac{2}{3}}\times\left(5^2\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}\right]^{\frac{1}{2}}$$

$$=\left[\frac{\left(2^3\right)^{\frac{2}{3}}\times\left(3^3\right)^{\frac{2}{3}}\times5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$

$$=\left[\frac{2^2\times3^2\times5}{(25)^{\frac{3}{2}}}\right]^{\frac{1}{2}}=\left[\frac{2^2\times3^2\times5}{\left(5^2\right)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$

REVIEW EXERCISE

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$$=\left(3^4y^{-12}x^{-8}\right)^{\frac{1}{4}}$$

$$=\left(3^4\right)^{\frac{1}{4}}\left(y^{-12}\right)^{\frac{1}{4}}\left(x^{-8}\right)^{\frac{1}{4}}$$

$$=3y^{-3}x^{-2}$$

$$=\frac{3}{x^2y^3}$$

(ii) $\sqrt{25x^{10n}y^{8m}}$

$$=\left(5^2x^{10n}y^{8m}\right)^{\frac{1}{2}}$$

$$=\left(5^2\right)^{\frac{1}{2}}\left(x^{10n}\right)^{\frac{1}{2}}\left(y^{8m}\right)^{\frac{1}{2}}$$

$$=5x^{5n}y^{4m}$$

(iii) $\left(\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right)^{\frac{1}{5}}$

$$=\left(x^{3+2}y^{4+1}z^{5+5}\right)^{\frac{1}{5}}$$

$$=\left(x^5y^5z^{10}\right)^{\frac{1}{5}}$$

$$=\left(x^5\right)^{\frac{1}{5}}\left(y^5\right)^{\frac{1}{5}}\left(z^{10}\right)^{\frac{1}{5}}$$

$$=xyz^2$$

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

$$=\left(\frac{2^5x^{-6}y^{-4}z}{5^4x^4yz^{-4}}\right)^{\frac{2}{5}}$$

$$=\left(\frac{2^5x^{-6-4}y^{-4-1}z^{1+4}}{5^4}\right)^{\frac{2}{5}}$$

$$=\left(\frac{2^5x^{-10}y^{-5}z^5}{5^4}\right)^{\frac{2}{5}}$$

$$=\frac{\left(2^5\right)^{\frac{2}{5}}\left(x^{-10}\right)^{\frac{2}{5}}\left(y^{-5}\right)^{\frac{2}{5}}\left(z^5\right)^{\frac{2}{5}}}{\left(5^4\right)^{\frac{2}{5}}}$$

$$=\frac{2^2x^{-4}y^{-2}z^2}{5^{\frac{8}{5}}}$$

$$=\frac{4z^2}{x^4y^2 5.5^{\frac{3}{5}}}$$

Q.4. Simplify: $\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

$$=\left[\frac{\left(2^3\times3^3\right)^{\frac{2}{3}}\times\left(5^2\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}\right]^{\frac{1}{2}}$$

$$=\left[\frac{\left(2^3\right)^{\frac{2}{3}}\times\left(3^3\right)^{\frac{2}{3}}\times5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$

$$=\left[\frac{2^2\times3^2\times5}{(25)^{\frac{3}{2}}}\right]^{\frac{1}{2}}=\left[\frac{2^2\times3^2\times5}{\left(5^2\right)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$

$$=\left[\frac{2^2\times3^2\times5}{5^3}\right]^{\frac{1}{2}}=\left[\frac{2^2\times3^2}{5^2}\right]^{\frac{1}{2}}$$

$$=\frac{(2^2)^{\frac{1}{2}}\times(3^2)^{\frac{1}{2}}}{(5^2)^{\frac{1}{2}}}=\frac{2\times3}{5}=\frac{6}{5}$$

Q.5 Simplify:

$$\left(\frac{a^p}{a^q}\right)^{p+q}.\left(\frac{a^q}{a^r}\right)^{q+r}\div5\left(a^p.a^r\right)^{p-r}$$

$$=\left(a^{p-q}\right)^{p+q}.\left(a^{q-r}\right)^{q+r}\div5\left(a^{p+r}\right)^{p-r}$$

$$=a^{p^2-q^2}.a^{q^2-r^2}\div5a^{p^2-r^2}$$

$$=\frac{a^{p^2-q^2}.a^{q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\frac{a^{p^2-q^2+q^2-r^2-p^2+r^2}}{5}$$

$$=\frac{a^0}{5}=\frac{1}{5}$$

Q.6. Simplify: $\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+l}}\right)$

$$=a^{2l-l-m}\times a^{2m-m-n}\times a^{2n-n-l}$$

$$=a^{l-m}.a^{m-n}.a^{n-l}$$

$$=a^{l-m+m-n+n-l}$$

$$=a^0=1$$

Q.7 Simplify: $\sqrt[3]{\frac{a^l}{a^m}}\times\sqrt[3]{\frac{a^m}{a^n}}\times\sqrt[3]{\frac{a^n}{a^l}}$

$$=\left(\frac{a^l}{a^m}\right)^{\frac{1}{3}}\times\left(\frac{a^m}{a^n}\right)^{\frac{1}{3}}\times\left(\frac{a^n}{a^l}\right)^{\frac{1}{3}}$$

$$=\frac{a^{\frac{l}{3}}}{a^{\frac{m}{3}}}\times\frac{a^{\frac{m}{3}}}{a^{\frac{n}{3}}}\times\frac{a^{\frac{n}{3}}}{a^{\frac{l}{3}}}$$

$$=a^{\frac{l}{3}-\frac{m}{3}}\times a^{\frac{m}{3}-\frac{n}{3}}\times a^{\frac{n}{3}-\frac{l}{3}}$$

$$=a^{\frac{l}{3}-\frac{m}{3}+\frac{m}{3}-\frac{n}{3}+\frac{n}{3}-\frac{l}{3}}$$

$$=a^0=1$$