UNIT 4

Turning Effect of Forces

1 **What is a resultant forces? Describe the head to tail rule for the addition of forces.**

Ans. Resultant force

A resultant force is a single force that has the same effect as the combined effect of all the forces to be added.

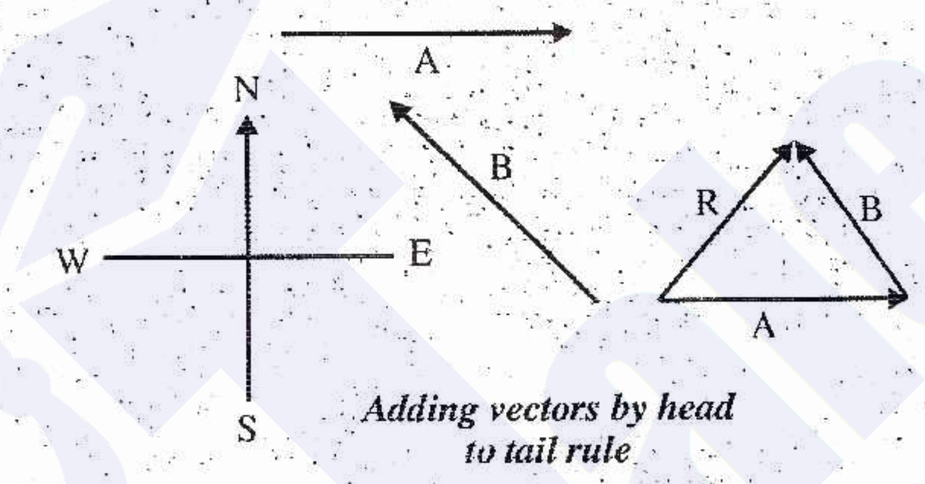
Addition of Forces

One of the methods for the addition of forces is a graphical method. In this method forces can be added by head to tall rule of vector addition.

Head to Tail Rule

By using this rule following steps are involved.

(i) First select a suitable scale. Then draw the vectors of all the forces according to the scale; such as vectors A and B.



Adding vectors by head to tail rule

(ii) Take any one of the vectors as first vector e.g., vector A. then draw next vector B such that its tail coincides with the head of the first vector A. similarly draw the next vector for the third force (if any) with its tail coinciding with the head of the previous vector and so on.

(iii) Now draw a vector R such that its tail is at the tail of vector A, while its head is at the head of vector B, the last vector R represents the resultant force completely in magnitude and direction.

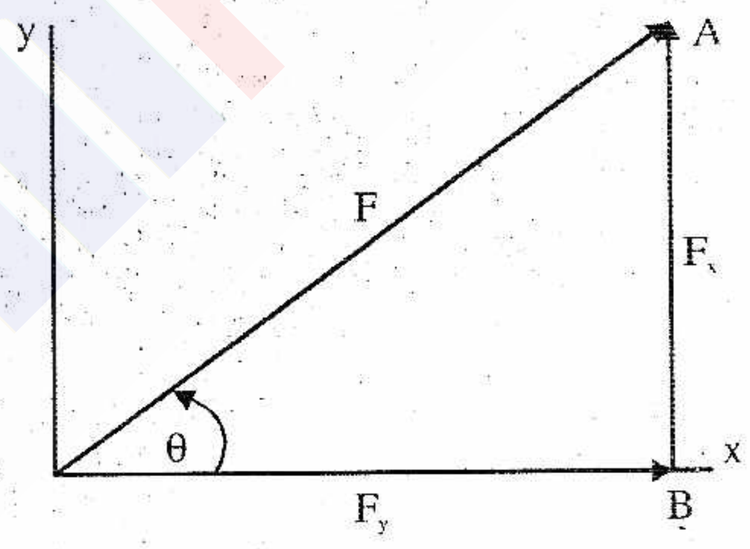
2 **What is meant by resolution of a vector? If a force 'F' makes an angle 'Q' with x-axis, then find the value of its rectangular components.**

Ans. Resolution of a Vector

Splitting up of a force into two mutually perpendicular components is called the resolution of that force.

Explanation

Consider a force F represented by line OA making an angle θ with x-axis. Braw a perpendicular AB on x-axis from A. According to head to tail rule, OA is the resultant of vectors represented by OB and AB.



Thus $OA = OB + BA$ ..... (i)

The components OB and BA are perpendicular to each other. They are called the perpendicular components of OA representing force F. Hence OB represents its x-component F_x and BA represents its y-component F_y. Therefore, equation (i) can be expressed as

$$F = F_x + F_y \qquad \text{..... (ii)}$$

The magnitudes Fx and Fy of forces F_x and F_y can be found using the trigonometric rations. In right angled triangle OBA

Since $$\frac{F_x}{F} = \frac{OB}{OA} = \cos\theta$$

$$F_x = F\cos\theta \qquad \text{..... (iii)}$$

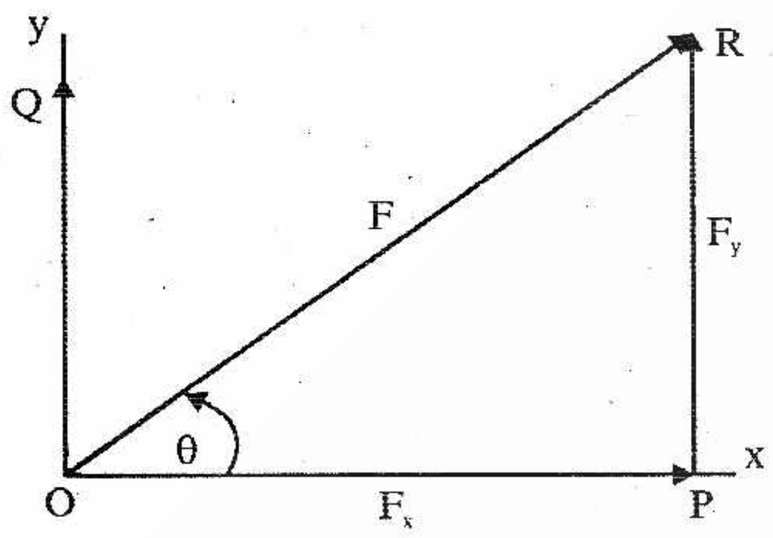
Similarly $$\frac{F}{F_y} = \frac{BA}{OA} = \sin\theta$$

$$\therefore \quad F_y = F\sin\theta \qquad \text{..... (iv)}$$

3 How can you determine the resultant force if rectangular components are given?

Ans. Determination of a Force from its Perpendicular Components:

If rectangular components are given then the resultant force can be determined as. Consider F_x and F_y are the perpendicular components F_x and F_y are represented by line OP and PR respectively as shown in figure.



According to head to tail rule

$$OR = OP + PR$$

Thus OP will completely represent the force F whose x and y-components are F_x and F_y respectively. That is

$$F = F_x + F_y$$

The magnitude of the force F can be determined using the right angled triangle OPR

as $$(OR)^2 = (OP)^2 + (PR)^3$$

$$\tan\theta = \frac{F_y}{F_x}$$

$$\therefore \quad F^2 = F_x^2 + F_y^2$$

Hence $$F = \sqrt{F_x^2 + F_y^2}$$

4 What is meant by torque? Write down its SI unit. Describe the factors on which torque depends.

Ans. Torque

The turning effect of a force is called torque or moment of the force. Thus the moment of the force or torque τ is determined by the product of force F and its moment arm L. Mathematically.

$$\text{Torque } \tau = F \times L$$

SI Unit

SI unit of torque is N.m.

Factors

The torque or moment of a force depends upon.

(i) Force (F)

(ii) Moment arm (L) of the force

Greater the force greater will be the torque. Similarly longer is the moment arm greater is the torque.

5 Describe the method to find the centre of gravity of an irregular shaped this Lamina.

Ans. Centre of Gravity

A point where the whole weight of the body appears to act vertically downward is called centre of gravity.

Method to find the Centre of Gravity

A simple method to find the centre of gravity of a body is by the use of a plumbline. A plumbline consists of a small metal bob (lead or brass) supported by a string. When the bob is suspended freely by the string, it rests along the vertical direction due to its weight acting vertically downward. In this state, centre of gravity of the bob is exactly below its point of suspension.

Experiment

Take and irregular piece of cardboard. Make holes A, B and C as shown in figure near its edge. Fix a nail on a wall. Support the cardboard on the nail through one of the holes (let it be A), so that the cardboard can swing freely about A. The cardboard will come to rest with its centre of gravity just vertically below the nail. Vertical line from A can be located using a plumbline hung from the nail. Mark the line of the cardboard behind the plumbline. Repeat it by supporting the cardboard from hole B. the line from B will intersect at a point G. Similarly, draw another line from the hole C. Note that this line

also passes through G. It will be found that all the vertical line from holes A, B and C have a common point G. This common point G is the centre of gravity of the cardboard.

6 **Prove that the torque produced in a body due to a couple is equal to the product of one of the forces of the couple and the couple arm.**

Ans. Consider two forces F1 and F2 are acting on a rigid body. The perpendicular distance of these forces from the axis 'O' is OA and OB respectively and the perpendicular distance between these forces is r. the forces of the couple produce torque in the same direction about axis 'O' which produces rotational motion in the body. The total torque at the point 'O' can be found as follows:

$$\tau = F_1 \times OA + F_2 \times OB$$

since $F = F_1 = F_2$

$$\tau = F \times OA + F \times OB$$

$$\tau = F(OA + OB)$$

$$\tau = Fr$$

where τ is the perpendicular distance between the two forces of the couple. It is called the couple arm. Hence the torque produced in a body due to a couple is equal to the product of one of the forces of the couple and the couple arm.

7 **What is meant by equilibrium? Describe the conditions for a body to be in equilibrium.**

Ans. **Equilibrium:**

A body is said to be in equilibrium if no net force acts on it. If a body is at rest or moving with uniform velocity is said to be in equilibrium because the net force in such cases is zero.

Conditions for Equilibrium

There are two conditions for a body to be in equilibrium.

First Condition for Equilibrium

A body is said to satisfy first condition for equilibrium if the resultant of all the forces acting on it is zero. Let n number of forces $F_1, F_2, F_3, \ldots F_n$ are acting on a body such that

$$F_1 + F_2 + F_3 + \ldots + F_n = 0$$

or $$\sum F = 0$$

The symbol $\sum$ is a Greek letter called sigma used for summation. Equation (i) is called the first condition for equilibrium. The first condition for equilibrium can also be stated in terms of x and y-components of the force acting on the body as:

$$F_{1x} + F_{2x} + F_{ax} + \ldots + F_{nx} = 0$$

and $$F_{1y} + F_{2y} + F_{3y} + \ldots + F_{ny} = 0$$

or $$\sum F_x = 0$$

$$\sum F_y = 0$$

A book lying on a table or a picture hanging on a wall, are at rest and thus satisfy first condition for equilibrium. A paratroope coming down with terminal velocity (constant velocity) also satisfies first condition for equilibrium and is thus in equilibrium.

Second Condition for Equilibrium

First condition for equilibrium does not ensure that a body is in equilibrium. This is clear from the following example. Consider a body pulling by the forces F_1 and F_2. The two forces are equal but opposite to each other. Both are acting along the same line, hence their resultant will be zero. According to the first condition, the body will be in equilibrium. Now shift the location of the forces as shown in figure (b). In this situation, the body is not is equilibrium although the first condition for equilibrium is still satisfied. It is because the body has the tendency to rotate. This situation demands another condition for equilibrium in addition to the first condition for equilibrium. This is called second condition for equilibrium. According to this, a body satisfies second condition for equilibrium when the resultant torque acting on it is zero. Mathematically,

$$\sum \tau = 0$$

8 **Describe different states of equilibrium.**

Ans. **States of Equilibrium:**

There are three states of equilibrium.

(i) Stable equilibrium

(ii) Unstable equilibrium

(iii) Neutral equilibrium

(i) Stable Equilibrium

A body is said to be in stable equilibrium if after a slight tilt it returns to its previous position. When a body is in stable equilibrium, its centre of

gravity is at the lowest position. When it is tilted, its centre of gravity rises. It returns to its stable state by lowering its centre of gravity. A body remains in stable equilibrium as long as the centre of gravity acts through the base of the body. E.g. A book lying on a table is in a state of stable equilibrium.

(ii) Unstable Equilibrium

If a body does not return to its previous position when sets free after a slightest tilt is said to be in unstable equilibrium. The centre of gravity of the body is at its highest position in the state of unstable equilibrium. As the body topples over about its base (tip), its centre of gravity moves towards its lower position and does not return to its previous position. E.g., A pencil in a vertical position is in a state of unstable equilibrium.

(iii) Neutral Equilibrium

If a body remains in its new position when disturbed from its previous position. It is said to be in a state of neutral equilibrium. In neutral equilibrium, all the new states in which a body is moved, are the stable states and the body, remains in its new state. In neutral equilibrium the centre of gravity of the body remains at the same height, irrespective to its new position. There are various objects which have neutral equilibrium such as a ball, a sphere, a roller, a pencil lying horizontally, an egg lying horizontally on a flat surface etc.

EXERCISE

4.1 Encircle the correct answers from the given choices:

(i) Two equal but unlike parallel forces having different line of action produce.

(a) a torque (b) a couple

(c) equilibrium (d) neutral equilibrium

(ii) The number of forces that can be added by head to tail rule are:

(a) 2 (b) 3

(c) 4 (d) any number

(iii) The number of perpendicular components of a force are:

(a) 1 (b) 2

(c) 3 (d) 4

(iv) A force of 10N is making an angle of 30^{o} with the horizontal. Its horizontal component will be:

(a) 4 N (b) 5 N

(c) 7 N (d) 8.7 N

(v) A couple is formed by

(a) two force perpendicular to each other

(b) two like parallel forces

(c) two equal and opposite forces in the same line

(d) two equal and opposite forces not in the same line

(vi) A body is in equilibrium when its:

(a) acceleration is uniform

(b) speed is uniform

(c) speed and acceleration are uniform

(d) acceleration is zero

(vii) A body is in neutral equilibrium when its centre of gravity:

(a) is at its highest position

(b) is at the lowers position

(c) keeps its height if displaced

(d) is situated at its bottom

(viii) Racing cars are made stable by:

(a) increasing their speed

(b) decreasing their mass

(c) lowering their centre of gravity

(d) decreasing their width

Answers

(i)	(a)	**(ii)**	(d)	**(iii)**	(b)	**(iv)**	(b)
(v)	(d)	**(vi)**	(d)	**(vii)**	(c)	**(viii)**	(c)

4.2. Define the following:

(i) **resultant vector**

(ii) **torque**

(iii) **centre of mass**

(iv) **centre of gravity**

Answer

(i) Resultant Vector:

A single vector that has the same effect as the combine effect of all the vectors is called resultant vector.

(ii) Torque

The turning effect of a force is called torque.

$$\tau = F \times L$$

(iii) Centre of Mass

Centre of mass of a system is such a point where an applied force causes the system to move without rotation.

(iv) Centre of Gravity:

A point where the whole weight of the body appears to act vertically downwards is called centre of gravity of a body.

4.3. Differentiate the following:

(i) **like and unlike forces**

(ii) **torque and couple**

(iii) **stable and neutral equilibrium**

Answer

(i) Like Forces

Like parallel forces are the forces that are parallel to each other and have the same direction.

Unlike Force

Those forces that are parallel but have directions opposite to each other are called un-like forces.

(ii) Torque

The turning effect of a force is called torque.

Couple

A couple is formed by two unlike parallel force of the same magnitude but not along the same line

(iii) Stable Equilibrium

A body is said to be in stable equilibrium if after a slight tilt it returns to its previous position.

Unstable Equilibrium

If a body does not return to its previous position when sets free after a slightest tilt is said to be in unstable equilibrium.

4.4. How head to tail rule helps to find the resultant of forces?

Ans. In head to tail rule the representative lines of all the forces are drawn according to a suitable scale in such a way the head of the first join the tail of the next and soon. The resultant force is obtained by joining the head of the last and the tail of the first force.

4.5. How can a force be resolved into its rectangular components?

Ans. A force is resolved into its rectangular component by drawing its two mutually perpendicular components. Consider a force 'F' is making an angle 'θ' with x-axis. This force can be resolved into two its rectangular components by drawing its two mutually components. The magnitudes of these rectangular components can be calculated as

$$F_x = F.\cos\theta$$

$$F_y = F.\sin\theta$$

4.6. When a body is said to be in equilibrium?

Ans. A body is said to be in equilibrium if no net force acts on it.

4.7. Explain the first condition for equilibrium.

Ans. First Condition of Equilibrium

According to first condition of equilibrium; A body is said to be in equilibrium if the resultant of all the forces acting on it is zero.

Mathematically

$$F_1 + F_2 + F_3 + \ldots\ldots F_n = 0$$

$$\Sigma F = 0$$

In terms of 'X' and y-component of the force acting on the body

$$F_{1x} + F_{2x} + F_{3x} + \ldots\ldots F_{nx} = 0$$

and $$F_{1y} + F_{2y} + F_{3y} + \ldots\ldots F_{ny} = 0$$

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

4.8. Why there is a need of second condition for equilibrium if a body satisfies first condition for equilibrium?

Ans. When two equal but opposite forces act on a body along different lines then they satisfy the first condition but the body is not in equilibrium because the body has the tendency to rotate. This situation demands the another condition for equilibrium.

4.9. What is second condition for equilibrium?

Ans. According to second condition of equilibrium; a body is said to be in equilibrium if the resultant torque acting on it is zero.

$$\Sigma\tau = 0$$

4.10. Give an example of a moving body which is in equilibrium.

Ans. A paratrooper coming down with constant velocity satisfies first condition for equilibrium so it in equilibrium.

4.11. Think of a body which is at rest but not in equilibrium.

Ans. A person sitting in a bus is at rest with respect to its other passengers but he is not in equilibrium.

4.12. Why a body cannot be in equilibrium due to single force acting on it?

Ans. According to second condition of equilibrium a body is in equilibrium if sum of torque acting on it is zero. The torque of a single force can never be zero. There fore a body cannot be in equilibrium under the action of single force.

4.13. Why the height of vehicles is kept as low as possible?

Ans. The height of vehicles is kept as low as possible so that their centre of mass must be kept as low as possible. This increases their stability.

4.14. Explain what is meant by stable, unstable and neutral equilibrium. Give one example in each case.

Ans. Stable Equilibrium

Such a state in which on disturbing a body slightly, the body comes back to its original position. The body is said to be in stable equilibrium. e.g. A book lying on the table.

Unstable Equilibrium

Such a state in which on disturbing the body slightly, the body does not come back to its original position. The body is said to be in unstable equilibrium. e.g. A pencil standing in a vertical position.

Neutral Equilibrium

A state in which on disturbing the body slightly, the centre of gravity does not shift. The body is said to be in the state of neutral equilibrium. e.g. A ball or a sphere lying on the ground.

PROBLEMS

4.1. Find the resultant of the following forces:

(i) 10 N along x-axis

(ii) 6 N along y-axis and

(iii) 4 N along negative x-axis.

Ans. Solution

Scale = 2N = 1 cm

Diagram

OD = F = 4.25 cm

or $F = 4.25 \times 2$

$= 8.50$ N

$Q = 45^\circ$

4.2. Find the perpendicular components of a force of 50N making an angle of 30° with x-axis.

Sol.

$F = 50$ N

$Q = 30^\circ$

$F_x = ?$

$F_y = ?$

$F_x = F.\cos\theta$

$F_x = 50 \times \cos 30$

$F_x = 50 \times 0.866$

$F_x = 43.300$ N

$F_y = F.\sin\theta$

$F_y = 50 \times \sin 30$

$F_y = 50 \times 0.5$

$F_y = 25.0$ N

4.3. **Find the magnitude and direction of a force, if its x-component is 12N and y-component is 5N.**

Sol.

$F_x = 12\ N$

$F_y = 5\ N$

$F = ?, \quad \theta = ?$

$F = \sqrt{(F_x)^2 + (F_y)^2}$

$F = \sqrt{(12)^2 + (5)^2}$

$F = \sqrt{144 + 25}$

$F = \sqrt{169}$

$F = 13\ N$

$\tan\theta = \frac{F_y}{F_x}$

$\tan\theta = \frac{5}{12}$

$\theta = \tan^{-1}\frac{5}{12}$

$= 22.6^\circ$ with x-axis.

4.4. **A force of 100N is applied perpendicularly on a spanner at a distance of 10 cm from a nut. Find the torque produced by the force.**

Sol.

$F = 100\ N$

$L = 10\ cm \quad \Rightarrow \quad 0.1\ m$

$\tau = 3$

$\tau = F \times L$

$\tau = 100 \times 0.1$

$\tau = 10\ N.m.$

4.5. **A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force.**

Sol.

$F_x = 20\ N$

$\theta = 30^\circ$

$F = ?$

$F_x = F.\cos\theta$

$20 = F.\cos 30$

$20 = F \times 0.866$

$\frac{20}{0.866} = F$

$F = 23.1\ N$

4.6. **The steering of a car has a radius 16 cm. Find the torque produced by a couple of 50N.**

Sol.

$R = 16\ cm = 0.16\ m$

$d = 2 \times R$

$d = 0.32\ m$

$F = 50\ N$

$\tau = ?$

$\tau = F \times d$

$= 50 \times 0.32$

$\tau = 16.00\ N.m.$

4.7. **A picture frame is hanging by two vertical strings. The tensions in the strings are 3.8N and 4.4N. Find the weight of the picture frame.**

Sol.

$T_1 = 3.8\ N$

$T_2 = 4.4\ N$

$w = ?$

$w = T_1 + T_2$

$w = 3.8 + 4.4$

$w = 8.2\ N$

4.8. **Two blocks of masses 5 kg and 3 kg are suspended by the two strings as shown. Find the tension in each string.**

Sol.

$m_1 = 5\ kg$

$m_2 = 3\ kg$

weight of body A $= w = m_1 g$

$w_1 = 5 \times 10$

$w_1 = 50\ N$

weight of body B $= w_2 = m_2 g$

$= 3 \times 10$

$= 30\ N$

$T_1 = w_1 + w_2$

$= 50 + 30$

$T_1 = 80N$

$T_2 = w_2$

$T_2 = 30N$

4.9. **A nut has been tightened by a force of 200N using 10cm long spanner. What length of a spanner is required to loosen the same nut with 150N force?**

Sol. $F = 200\ N$

$L = 10\ cm = 0.1\ m$

$\tau = ?$

$\tau = F \times L$

$\tau = 200 \times 0.1$

$F = 150\ N$

$L = ?$

$\tau = F \times L$

$20 = 150 \times L$

$L = 0.133m$ or 13.3cm

4.10. A block of mass 20kg is suspended at a distance of 20cm from the centre of a uniform bar 1m long. What force is required to balance it at its centre of gravity by applying the fore at the other and of the bar?

Sol. $m = 10\ kg$

$w = 10 \times 10$

$= 100\ N$

$OA = 20\ cm \Rightarrow 0.2m$

$OB = 50\ cm \Rightarrow 0.5m$

$F = ?$

$OA \times w = F \times OB$

$0.2 \times 100 = F \times 0.5$

$40 = F$

$F = 40N$