UNIT 5

Gravitation

1 **Explain Newton's law of universal gravitation and also explain law of gravitation with respect to Newton's third law of motion.**

Ans. **Law of Gravitation**

According to Newton's law of universal gravitation

Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Consider two bodies of masses m_1 and m_2. the distance between the centres of masses is d as shown in figure

Two messes attract each other with a gravitational force of equal magnitude.

According to the law of gravitation, the gravitational force of attraction F with which the two masses m_1 and m_2 separated by a distance d attract each other is given by:

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{d^2}$$

or $$F \propto \frac{m_1 m_2}{d^2}$$

$$F = G\frac{m_1 m_2}{d^2}$$

Here G is the proportionality constant. It is called the universal constant of gravitation. Its value is same everywhere. In SI units its value is 6.673×10^{-11} Nm^2kg^{-2}. Due to small value of G, the gravitational force of attraction between objects around us is very small and we do not feel it. Since the mass of Earth is very large, it attracts near by objects with a significant force. The weight of an object on the Earth is the result of gravitational force of attraction between the Earth and the object.

Law of Gravitation and Newton's Third law of Motion

It is to be noted that mass m_1 attracts m_2 towards it with a force F while mass m_2 attracts m1 towards it with a force of the same magnitude F but in opposite direction. If the force acting on m_1 is considered as action then the force acting on m_2 will be the reaction. The action and reaction due to force of gravitation are equal in magnitude but opposite in directions. This is in consistence with Newton's third law of motion which states, to every action there is always an equal but opposite reaction.

2 **Explain Gravitational Field.**

Ans. **Gravitational Field**

According to the Newton's law of gravitation, the gravitational force between a body of mass m and the Earth is given by

$$F = G\frac{m M_e}{r^2}$$

Where M_e is the mass of the Earth and r is the distance of the body from the centre of the Earth. The weight of a body is due to the gravitational force with which the earth attracts a body. Gravitational force is a non-contact force. For example, the velocity of a body, thrown up. Goes on decreasing while on return its velocity goes on increasing. This is due to the gravitational pull of the Earth acting on the body whether the body is in contact with the Earth or not. Such a force is called the field force. It is assumed that a gravitational field exists all around the Earth. This field is directed towards the centre of the Earth as shown by arrows in figure. The gravitational field becomes weaker and weaker as we go farther and farther away from the Earth. In the gravitational field of the Earth, the gravitational force per unit mass is called the gravitational field strength of the Earth. At any place its value is equal to the value of g at that point. Near the surface of the Earth, the gravitational field strength is 10 N kg^{-1}.

3 **How the mass of earth can be determine?**

Ans. **Mass of the Earth**

Consider a body of mass m on the surface of the Earth as shown in figure 5.4. Let the mass of the Earth be M_e and radius of the Earth be R. The distance of the body from the centre of the Earth will also be equal to the radius R of the Earth. According to the law of gravitation, the gravitational force F of the Earth acting on a body is given by

$$F = G\frac{m M_e}{R^2}$$

But the force with which Earth attracts a body towards its centre is equal to its weight w. Therefore,

$$F = w = mg$$

or $$mg = G\frac{m M_e}{R^2}$$

$\therefore$ $$g = G\frac{M_e}{R^2}$$

and $$M_e = \frac{R^2 g}{G}$$

Mass M_e of the earth can be determined on putting the values in equation (1)

$$Me = \frac{(6.4 \times 10^6 \text{ m})^2 \times 10 \text{ ms}^{-2}}{6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}}$$

$$= 6.0 \times 10^{24} \text{ kg}$$

Thus, mass of the Earth is 6×10^{24} kg.

4 **Explain how 'g' varies with altitude?**

Ans. **Variation of g with Altitude**

Value of acceleration due to gravity depends on the radius of the Earth at its surface. The value of g is inversely proportional to the square of the radius of the Earth. But it does not remain constant. It decreases with altitude. Altitude is the height of an object or place above sea level. The value of g is greater at sea level than at the hills.

Consider a body of mass m at an altitude h as shown in figure. The distance of the body from the centre of the Earth becomes R + h. Therefore,

$$G_h = G = \frac{M_e}{(R+h)^2}$$

According to the above equation, we come to know that at a height equal to, one Earth radius above the surface of the Earth, g becomes one fourth of its value on the Earth. Similarly at a distance of two Earths radius above the Earth's surface, the value of g becomes one ninth of its value on the Earth.

5 **Calculate the value of g, the acceleration due to gravity at an altitude 1000 km. The mass of the Earth is 6.0×10^{24} kg. The radius of the Earth is 6400 km.**

Sol. Here R = 6400 km

h = 1000 km

$M_e = 6.0 \times 10^{24}$ kg

$g_h = ?$

$$R + h = 6400 \text{ km} + 1000 \text{ km} = 7400 \text{ km}$$

$$= 7.4 \times 10^6 \text{ m}$$

As $$g_h = G\frac{M_a}{(R+h)^2}$$

$\therefore$ $$g_h = \frac{6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} \times 6.0 \times 10^{24} \text{ kg}}{(7.4 \times 10^6 \text{ m})^2}$$

$$= 7.3 \text{ N kg}^{-1} = 7.3 \text{ ms}^{-2}$$

Thus the value of g, the acceleration due to gravity at an altitude of 1000 km will be 7.3 ms^{-2}.

6 **Explain artificial satellites.**

Ans. **Artificial Satellites**

An object that revolves around a planet is called a satellite. The moon revolves around the Earth so moon is a natural satellite of the Earth. Scientists have sent many objects into space. Some of these objects revolve around the Earth. These are called artificial satellites. Most of the artificial satellites, orbiting around the Earth are used for communication purposes. Artificial satellites carry instruments or passengers to perform experiments in space.

A satellite is orbiting around the Earth at a height h above the surface or the Earth.

Large number of artificial satellites have been launched in different orbits around the Earth. They take different time to complete their one revolution around the Earth depending upon their distance h from the Earth. Communication satellites take 24 hours to complete their one revolution around the Earth. As Earth also completes its one rotation about its axis in 24 hours, hence, these communication satellites appear to be stationary with respect to Earth. It is due to this reason that the orbit of such a satellite is called geostationary orbit. Dish antennas sending and receiving the signal from them have fixed direction depending upon their location on the Earth.

7 **How Newton's law of gravitation helps in understanding the motion of satellites?**

Ans. **Motion of Artificial Satellites:**

A satellite requires centripetal force that keeps it to move around the Earth. The gravitational force of attraction between the satellite and the Earth provides the necessary centripetal force.

Consider a satellite of mass m revolving round the Earth at an altitude h in an orbit of radius r_o with orbital velocity v_o. The necessary centripetal force required is given as:

$$F_c = \frac{mv_0^2}{r_o}$$

This force is provided by the gravitational force of attraction between the Earth and the satellite and is equal to the weight of the satellite w' (mg_h). Thus

$$Fc = w' = mg_h$$

$$\text{or} \quad mg_h = \frac{mv_0^2}{r_o}$$

$$\text{or} \quad v_0^2 = g_h r_o$$

$$\text{or} \quad v_o = \sqrt{g_h r_o}$$

$$\text{as} \quad r_o = R + h$$

$$\therefore \quad v_o = \sqrt{g_h (R + h)}$$

Equation gives the velocity, which a satellite must possess when launched in an orbit of radius $r_o = (R + h)$ around the Earth. An approximation can be made for a satellite revolving close to the Earth such that $R >> h$.

$$R + h = R$$

$$\text{and} \quad g_h = g$$

$$\therefore \quad v_o = \sqrt{g R}$$

A satellite revolving around very close to the Earth, has speed V_o nearly 8 kms^{-1} or 29000 kmh^{-1}.

EXERCISE

5.1. Encircle the correct answer from the given choices:

(i) Earth's gravitational force of attraction vanishes at:

(a) 6400 km (b) infinity
(c) 42300 km (d) 1000 km

(ii) Value of g increases with the:

(a) increase in mass of the body
(b) increase in altitude
(c) decrease in altitude
(d) none of the above

(iii) The value of g at a height one Earth's radius above the surface of the Earth is:

(a) 2 g (b) $\frac{1}{2}$ g
(c) $\frac{1}{3}$ g (d) $\frac{1}{4}$ g

(iv) The value of g on moon's surface is 1.6 ms^{-2}. what will be weight of a 100 kg body on the surface of the moon?

(a) 100 N (b) 160 N
(c) 1000 N (d) 1600 N

(v) The altitude of geostationary orbits in which communication satellites are launched above the surface of the Earth's is:

(a) 850 km (b) 1000 km
(c) 6400 km (d) 42,300 km

(vi) The orbital speed of a low orbit satellite is:

(a) zero (b) 8 ms^{-1}
(c) 800 ms^{-1} (d) 8000 ms^{-1}

Answers

(i)	(b)	(ii)	(c)	(iii)	(d)	(iv)	(b)
(v)	(d)	(vi)	(d)				

5.2. What is meant by the force of gravitation?

Ans. The force due to which every body in the universe attracts every other body is called gravitational force.

5.3. Do you attract the Earth or the Earth attracts you? Which one is attracting with a larger force? You or the Earth.

Ans. Both attract to each other. Earth is attracting with larger force because its mass is greater.

5.4. What is a field force?

Ans. The gravitational pull of the earth acts on the body whether the body is in contact with the earth or not this force is called field force.

5.5. Why earlier scientists could not guess about the gravitational force?

Ans. The earlier scientists never thought about the bodies falling towards the earth. The earlier scientists never tried to solve the mystery why planets revolve around the sun. so they could not guess about the gravitational force.

5.6. How can you say that gravitational force is a field force?

Ans. Gravitational force not only acts on the bodies that are contact with the earth but it acts on every body whether the body is in contact or not. It decreases as we go away from the centre of the earth so we can say it is a field force.

5.7. Explain, what is meant by gravitational field strength?

Ans. In the gravitational field of the earth, the gravitational force per unit mass is called the gravitational field strength.

5.8. Why law gravitation is important to us?

Ans. The gravitational law is very important to us because by using this law we can find the mass of the earth. This law also helps us to find the value of 'g' at different height from the surface of earth.

5.9. Explain the law of gravitation.

Ans. Law of Gravitation:

According to this law every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Explanations:

Consider two bodies of masses 'm_1' and 'm_2'. The distance between the centres of masses is 'd'.

According to the law of gravitation

$$F \propto m_1 m_2 \qquad (i)$$

$$F \propto \frac{1}{d^2} \qquad (ii)$$

From eq (i) and (ii)

$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = G\frac{m_1 m_2}{d^2}$$

Here 'G' is the proportionality constant. Its value is $6.673 \times 10^{-11}\ Nm^2\ kg^{-2}$.

5.10. How the mass of earth can be determined?

Ans. See Long Question No. 3.

5.11. Can you determine the mass of our moons? If yes, then what you need to know?

Ans. Yes, In order to find the mass of moon; we should know the radius of the moon as well as the value of 'g' on the surface of moon.

5.12. Why does the value of 'g' vary from place to place?

Ans. The value of 'g' depends on the distance from the centre of the earth. The surface of the earth is not smooth. At different places the distance from the centre of earth is different that is why the value of 'g' vary from place to place.

5.13. Explain how the value of 'g' varies with altitude.

Ans. The value of 'g' is inversely proportional to the square of the radius of the earth. So it decreases with altitudes. The value of 'g' is greater at sea level then at the hills.

5.14. What are artificial satellites?

Ans. The objects that are sent into the space by scientists and they revolve round the earth. These objects are called artificial satellites.

5.15. How Newton's law of gravitation helps in understanding the motion of satellites?

Ans. As we go away from the earth the gravitational force decreases. So, with the help of Newton's gravitational law we can find the required centripetal force for an artificial satellite. So, by using this law we can find the velocity of the satellite which is to be launched in an orbit consider a satellite of mass 'm' revolving round the earth at an altitude 'h' in an orbit or radius 'r_o' the necessary centripetal force required is given as

$$F_c = \frac{mv_0^2}{r}$$

This force is provided by the gravitational force that is equal to the weight of the satellite.

so $\quad F_c = mg_h$

$$mg_h = \frac{mv_0^2}{r_o}$$

$$v_0^2 = g_h r_o$$

$$v_o = \sqrt{g_h r_o}$$

$$r_o = R + h$$

$$v_o = \sqrt{g_h (R + h)}$$

5.16. On what factors the orbital speed of a satellite depends?

Ans. The orbital speed of a satellite depends.

(i) The value of 'g' at the given hight.

(ii) Its value also depends on the altitude.

5.17. Why communication satellites are stationed at geostationary orbits?

Ans. The communication satellites are stationed at geostationary orbit so that they complete their one revolution around the earth in a time as the earth takes to complete its one revolution around its axis. In this way we get the correct information about the atmosphere and about the world. In addition to this we do not need to change the direction of dish antennas.

PROBLEMS

5.1. Find the gravitational force of attraction between two spheres each of mass 1000 kg. The distance between the centres of the spheres is 0.5 m.

Sol. $m_1 = 1000$ kg, $m_2 = 1000$ kg, $d = 0.5$ m

$F = ?$

$$F = \frac{G\, m_1\, m_2}{d^2}$$

$$F = \frac{6.673 \times 10^{-11} \times 1000 \times 1000}{(0.5)^2}$$

$$F = \frac{6.673 \times 10^{-11} \times 10^6}{0.25}$$

$$F = 6.673 \times 10^{-5}$$

$$F = 2.67 \times 10^{-4}\ \text{N}$$

5.2. The gravitational force between two identical lead spheres kept at 1 m apart is 0.006673 N. Find their masses.

Sol. $m_1 = m_2 = ?$

$F = 0.006673$ N

$D = 1$m

$$F = \frac{G\, m_1\, m_2}{d^2}$$

$$F = \frac{6.673 \times 10^{-11} \times m_1^2}{1}$$

$$0.006673 = 6.673 \times 10^{-11} \times m_1^2$$

$$\frac{6673}{1000000 \times 6.673 \times 10^{-11}} = m_1^2$$

$$m_1^2 = 10^{11-3}$$

$$= \sqrt{10^8}$$

$$m = 10^4$$

$\Rightarrow$ $m = 10000$ kg each

5.3. Find the acceleration due to gravity on the surface of the mars. The mass of mars is 6.42×10^{23} kg and its radius is 3370 km.

Sol. $g = ?$

$M = 6.42 \times 10^{23}$ kg

$R = 3370$ km $= 3370 \times 10^3$ m

$$g = \frac{G\, m}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{(337 \times 10^4)^2}$$

$$g = \frac{6.67 \times 6.42 \times 10^{12}}{337 \times 337 \times 10^8}$$

$$g = \frac{667}{100} \times \frac{642 \times 10^{12-8}}{100 \times 337 \times 337}$$

$$g = \frac{428214}{113569} = 3.77\ \text{ms}^{-2}$$

5.4. The acceleration die to gravity on the surface of moon is $1.62\ \text{ms}^{-2}$. The radius of Moon is 1740 km. Find the mass of moon.

Sol. $g_m = 1.62\ \text{ms}^{-2}$

$R = 1740$ km $= 1740 \times 10^3$ m

$m = ?$

$$g = G\frac{m}{R^2}$$

$$1.62 = \frac{6.673 \times 10^{-11} \times m}{(1740 \times 10^3)^2}$$

$$m = \frac{1.62 \times (174 \times 10^4)^2}{6.673 \times 10^{-11}}$$

$$m = \frac{1.62 \times 174 \times 174 \times 10^8}{6.673 \times 10^{-11}}$$

$$m = \frac{162 \times 174 \times 174}{6673} \times 10^{20}$$

$m = 735.00 \times 10^{20}$

$m = 7.350 \times 10^{22}$

5.5. Calculate the value of 'g' at a height of 3600 km above the surface of the earth.

Sol. $g = ?$

$h = 3600 \text{ km} = 3600 \times 10^3 \text{ m}$

$= 36 \times 10^5 \text{ m}$

$R = 6400 \text{ km} = 6400 \times 10^3 \text{ m}$

$= 64 \times 10^5 \text{ m}$

$M_e = 6 \times 10^{24} \text{ kg}$

$$g = G\frac{Me}{(R+h)^2}$$

$$= \frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{(64 \times 10^5 + 36 \times 10^5)^2}$$

$$g = \frac{40.038 \times 10^{13}}{(100 \times 10^5)^2}$$

$$= \frac{40.038 \times 10^{13}}{10000 \times 10^{10}}$$

$$g = \frac{40.038 \times 10^{13}}{10^{14}}$$

$$g = \frac{40.038}{10^{14-13}} = \frac{40.038}{10} = 4.00 \text{ ms}^{-2}$$

5.6. Find the value of g due to the Earth at geostationary satellite. The radius of the geostationary orbit is 48700 km.

Sol. $g = ?$

$r = 48700 \text{ km} = 48700 \times 1000$

$= 487 \times 10^5 \text{ m}$

$M_e = 6 \times 10^{24} \text{ kg}$

$R = 6400 \text{ km} = 6400 \times 1000$

$= 64 \times 10^5 \text{ m}$

$G = 6.673 \times 10^{-11} \text{ N.m.kg}^{-2}$

$$g = G\frac{Me}{(R+r)^2}$$

$$g = \frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{(487 \times 10^5 + 64 \times 10^5)}$$

$$g = \frac{40.038 \times 10^{13}}{(551 \times 10^5)^2}$$

$$g = \frac{40.038 \times 10^{13}}{303601 \times 10^{10}}$$

$$g = 40.038 \times 10^3$$

$$g = \frac{40038}{303601} = 0.17 \text{ ms}^{-2}$$

5.7. The value of g is 4.0 ms^{-2} at a distance of 10000 km from the centre of the Earth. Find the mass of the Earth.

Sol. $g = 4 \text{ ms}^{-2}$

$R = 10000 \text{ km} = 10000 \times 10^3 \text{ m}$

$= 10^7 \text{ m}$

$M_e = ?$

$$M_e = \frac{R^2 \times g}{G}$$

$$M_e = \frac{(10^7)^2 \times 4}{6.673 \times 10^{-11}}$$

$$M_e = \frac{10^{14} \times 4}{6.673 \times 10^{-11}}$$

$$M_e = \frac{10^{14+11} \times 4 \times 1000}{6673}$$

$M_e = 0.000599 \times 10^{28} \text{ kg}$

$= 5.99 \times 10^{24}$

$M_e = 6 \times 10^{24} \text{ kg}$

5.8. At what altitude the value of 'g' would become one fourth than on the surface of the earth.

Sol. $g = \frac{10}{4} = 2.5 \text{ ms}^{-2}$

$h = ?$

$Me = 6 \times 10^{24} \text{ kg}$

$R = 6400 \text{ km} = 64 \times 10^5 \text{ m}$

$G = 6.673 \times 10^{-11} \text{ N.m kg}^{-2}$

$$g = G\frac{Me}{(R+h)^2}$$

$$\frac{10}{4} = \frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{(R+h)^2}$$

$$(R+h)^2 = \frac{40.038 \times 10^{13}}{10} \times 4$$

$$\sqrt{(R+h)^2} = \sqrt{160.152 \times 10^{12}}$$

$6400000 + h = 12.7 \times 10^6$

$h = 12700000 - 6400000$

$= 6300000 \text{ m}$

$h = 6300$ km

That is about one earth's Radius.

5.9. A polar satellite is launched at 850 km above earth. Find its orbital speed.

Sol. $h = 850 \text{ km} = 850 \times 10^3 \text{ m}$

$V_o = ?$

$g_h = ?$

$$g_h = G\frac{Me}{(R+h)^2}$$

$$= \frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{(6400000 + 650000)^2}$$

$$= \frac{40.038 \times 10^{13}}{(725 \times 10^4)^2}$$

$$g_h = \frac{40.038 \times 10^{13}}{525625 \times 10^8}$$

$$= 0.000076 \times 10^5$$

$$g_h = 7.6 \text{ ms}^{-2}$$

$$v_o = \sqrt{g_h (R+h)}$$

$$v_o = \sqrt{7.6 (6400000 + 850000)}$$

$$v_o = \sqrt{55100000}$$

$$v_o = 7431 \text{ ms}^{-1}$$

5.10. A communication satellite is launched at 42000 km above earth. Find its orbital speed.

Sol. $h = 42000$ km

$= 42000 \times 10^3$ m

$= 420 \times 10^5$ m

$R = 6400 \text{ km} = 64 \times 10^5 \text{ m}$

$g_h = ?$

$me = 6 \times 10^{24}$ kg

$G = 6.673 \times 10^{-11} \text{ Nm kg}^{-2}$

$$g_h = G\frac{Me}{(R+g)^2}$$

$$g_h = \frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{(420 \times 10^5 \times 64 \times 10^5)^2}$$

$$g_h = \frac{40.038 \times 10^{13}}{(484 \times 10^5)^2}$$

$$= \frac{40.038 \times 10^3}{234256}$$

$$gh = 0.17$$

$$v_o = \sqrt{g_h (R+h)}$$

$$= \sqrt{0.17 (64 \times 10^5 \times 420 \times 10^5)}$$

$$= \sqrt{0.17 (484 \times 10^5)}$$

$$= \sqrt{822.8 \times 10^4}$$

$$v_o = 2868 \text{ ms}^{-1}$$