MATHEMATICS

Textbook for Grade **VI**

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$A=\{0, 1, 2, 3, \ldots\}$

Khyber Pakhtunkhwa Textbook Board, Peshawar

نام تیمور علی 6th

MATHEMATICS

A Textbook for Grade

VI

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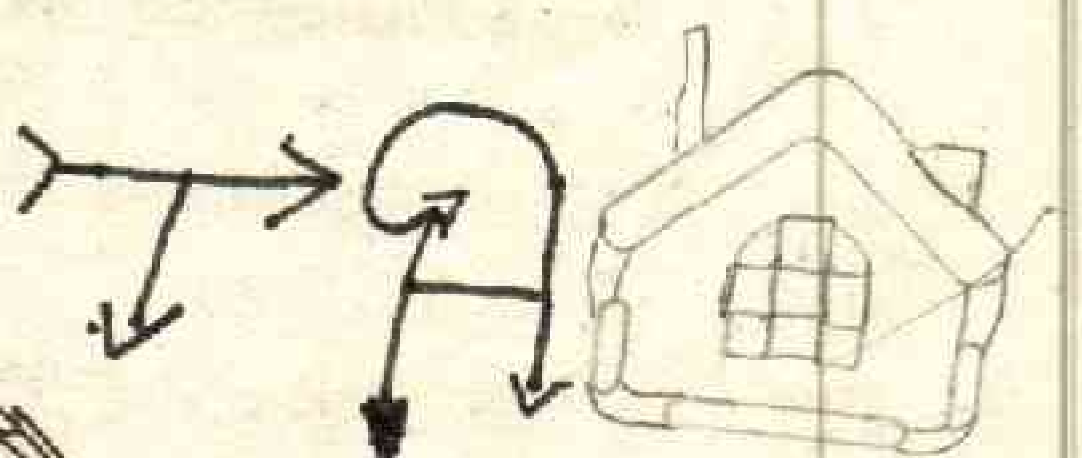
6th

(i) 2, 4
2 < 4

0 1 2 3 4

(ii) 3
3 <

1 2 3 4 5 6



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Contents

Hom work

Unit No.	Topic	Page No
1	Sets	1
2	Whole Numbers	12
3	Factors and Multiples	23
4	Integers	45
5	Simplifications	61
6	Ratio and Proportion	72
7	Financial Arithmetic	84
8	Introduction to Algebra	97
9	Linear Equations	109
10	Geometry	119
11	Perimeter and Area	141
12	Three Dimensional Solids	159
13	Information Handling	168
	Answers	180

UNIT 01 SETS

What You'll Learn

- ❖ Definition of a set. Recognize notation of a set and its objects/elements.
- ❖ Describe tabular form of a set and demonstration through examples.
- ❖ Define
  - Finite and infinite sets.
  - Empty/Void/Null set.
  - Singleton set.
  - Equal and equivalent sets.
  - Subset and superset of a set.
  - Proper and improper subsets of a set, and demonstration through examples.

Why It's important

Every area of mathematics utilizes sets in some way. Set theory was developed in the 19th century by a German Mathematician, George Cantor. The concept of set has helped to integrate various branches of Mathematics. It is due to this generality that this concept has become important.

How is the concept of set use in daily life?

Every language has an abundance of words that mean "a collection" or "a grouping." For example, we speak of a herd of cattle, a flock of birds, a school of fish, a stamp collection, and a set of dishes. All these grouping words serve the same purpose, and in mathematics we use the word set to refer to any collection of objects. A collection of books is an example of a set.

1.1(a) Sets

A set is a collection of well-defined and distinct objects. The objects are called its elements or members.

The word well-defined means that there should be such an explanation of the characteristics of its numbers that one may clearly find out whether a number is contained in the set or not. The word distinct means no two numbers of a set are identical.

1. A collection of 'lovely flowers' is not a set, because the objects (flowers) to be included are not well-defined.

Reason: The word "lovely" is a relative term. What may appear lovely to one person may not be so to the other person.

2. A collection of "Yellow flowers" is a set, because all yellow flowers will be included in this set i.e., the objects of the set are well-defined.

Examples of Sets:

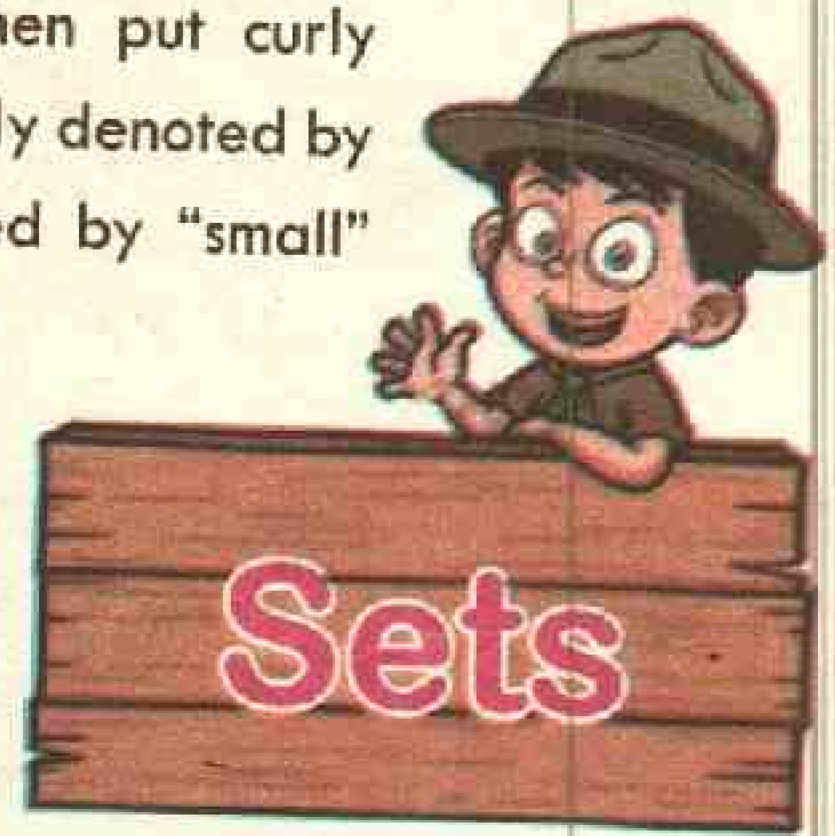
1. The set of Hockey players of your school.
2. The set of days of a week.
3. The set of whole numbers less than 10.
4. The set of boys of Class 6th of your school.

Days of the Week

- Sunday
- Monday
- Tuesday
- Wednesday
- Thursday
- Friday
- Saturday

(b) Tabular form of a Set

In tabular form of a set we simply list each element (or "member") separated by a comma, and then put curly brackets around the whole thing. Sets are usually denoted by "capital" letters and its elements are denoted by "small" letters.



Note

The curly brackets { } are also called "braces".

Example 1 write tabular form

Write the following sets in tabular form.

(i) Set of first three natural numbers (ii) Set of first four alphabets

Solution:

(i) Let A denote the set. Then
$A = \{1, 2, 3\}$.

(ii) Let B denote the set. Then
$B = \{a, b, c, d\}$

Example 2 find cardinality

Count the number of elements in the following sets:

(i) $X = \{8\}$ (ii) $P = \{7, 9, 11\}$

(iii) $S = \{1, 2, 3, \ldots, 10\}$ (iv) $Q = \{w, x, y, z\}$

Solution:

(i) The set X has one element.

(ii) The set P has 3 elements.

(iii) The set S has 10 elements.

(iv) The set Q has 4 elements.

(c) Notation for Membership of a Set

Consider $P = \{x, y, z\}$, we see that x is a member of P, y is a member of P and z is a member of P. In Mathematics, where possible, we use symbols to represent certain statements. In the above cases

'x is a member of P' is written as $x \in P$.

'y is a member of P' is written as $y \in P$.

'z is a member of P' is written as $z \in P$.

The symbol $\in$ stands for 'is a member of ' or 'belongs to'. If m, does not belong to P, then we write it as $m \notin P$.

Example 1 write in symbols

Represent symbolically:

(i) p is a member of Q (ii) r is not a member of S

(iii) 0, 1, 2, 3 are members of F (iv) d, e, f are not members of A

Solution:

(i) $p \in Q$ (ii) $r \notin S$

(iii) $0 \in F, 1 \in F, 2 \in F, 3 \in F$ (iv) $d \notin A, e \notin A, f \notin A$

Example 2 true, false

If a set $A = \{3, 6, 9, 10, 13, 18\}$. State whether the following statements are 'true' or 'false':

(i) $7 \in A$ (ii) $10 \notin A$ (iii) $13 \in A$ (iv) $9, 12 \in A$

Solution:

(i) $7 \in A$

False, since the element 7 does not belongs to the given set A.

(ii) $10 \notin A$

False, since the element 10 belongs to the given set A.

(iii) $13 \in A$

True, since the element 13 belongs to the given set A.

(iv) $9, 12 \in A$

True, since the elements 9 and 12 both belong to the given set A.

Exercise 1.1

1. State, giving reason, whether the following objects form a set or not:
   - (i) All problems of this book, which are difficult to solve.
   - (ii) All problems of this book, which are difficult to solve for Amjad.
   - (iii) All the objects heavier than 26 kg.
   - (iv) Students in a class-room.
   - (v) Books in your school-bag.
   - (vi) Counting numbers from 5 to 15.
   - (vii) Students of your class, who are taller than you.

2. If set $A = \{4, 6, 8, 10, 12, 14\}$. State which of the following statements are 'correct' and which are 'wrong':
   - (i) $5 \in A$
   - (ii) $12 \in A$
   - (iii) $14 \in A$
   - (iv) $9 \in A$
   - (v) 4, 6 and 10 are members of the set A.

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3. Write the following in tabular form of the set:
   - (i) The set of days in a week.
   - (ii) The set of first ten natural numbers.
   - (iii) The set of vowels in English alphabet.
   - (iv) The set of first six months of Islamic calendar.
   - (v) The set of months in English calendar.
   - (vi) The set of prime numbers.
   - (vii) The set of seasons in a year.

1.2 Types of Sets

There are different types of sets according to their number of elements. These sets are defined below.

1) Finite Set

A set whose elements can be listed is called finite set. The number of elements in a set S is often called its **cardinality and is** denoted by n(S) or |S|.

Examples of finite set:

1. Let $P = \{5, 10, 15, 20, 25, 30\}$
   Then, P is a finite set and $n(P) = 6$.
2. Let $Q = \{\text{natural numbers less than } 25\}$
   Then, Q is a finite set and $n(P) = 24$.
3. Let $R = \{\text{whole numbers between 5 and 45}\}$
   Then, R is a finite set and $n(R) = 38$.
4. The set of all persons in Pakistan is a finite set.
5. The set of all birds in KP is a finite set.

2) Infinite Set

A **set** that is not **finite** is called **infinite.**

Examples of infinite set:

1. Set of all positive integers which is multiple of 3 is an infinite set.
2. $W = \{0, 1, 2, 3, \ldots\}$ i.e. set of all whole numbers is an infinite set.
3. $N = \{1, 2, 3, \ldots\}$ i.e. set of all natural numbers is an infinite set.
4. $Z = \{\ldots -2, -1, 0, 1, 2, \ldots\}$ i.e. set of all integers is an infinite set.

3) Empty Set

A set which has no element is called an **empty set, void or null set** and is written as { } or $\varnothing$.

Examples of empty set:

1. The set of months with 32 days.
2. Set of whole numbers less than 0.

4) Singleton Set

A set having only one element is called a singleton set.

Examples of singleton set:

$\{0\}, \{10\}, \{a\}, \{\text{Nasir}\}$

5) **Equal Sets**

Two sets A and B are said to be **equal** if they have the same elements and denoted as $A = B$.

For Example:

If $A=\{1,2,3\}$, $B=\{1,2,3\}$ then $A=B$.

6) **Equivalent Sets**

Two sets A and B are said to be **equivalent** if they have the same number of elements but not necessarily the same members. Symbolically it is written as $A \simeq B$.

For Example:

If $A=\{1,2,3\}$, $B=\{a,b,c\}$ then $A \simeq B$.

Example 1 Equal and equivalent sets

Which of the following sets are equivalent? Are any equal?

{○, △, □, ⊙}, {5, 8, 11, 14}, {♥, ♣, ♦, ♠}, {•, ⊙, ★, ⊞}, {1, 2, 3, 4}

Solution:

All of the given sets are equivalent. Notice that no two of them are equal.

Example 2 Equal sets

Which of the following sets are equal.

$P=\varnothing$

S=Set of whole numbers less than 0.

R={Colors in the rainbow}

Q={red, orange, yellow, green, blue, indigo, violet}

Solution:

$P = S$, $\therefore$ S is also empty

$R = Q$

because Q contains all colours of rainbow.

7) Subsets

A Set A is said to be a **subset** of the set B if all the elements of A are the elements of B. It is symbolically written as $A \subseteq B$. It is important to note that the null set { } is the subset of all sets. If A is not subset of B, then we write $A \nsubseteq B$.

8) Proper Subset

If all the elements of A are in B, but there is at least one element of B which is not the element of A, then A is called proper subset of B. Symbolically it is written as $A \subset B$.

9) Improper Subsets

If $A \subseteq B$ and $A = B$ then A is called improper subset of B.

Example 1 Find subsets

Find all possible subsets of C= {5,7}

Solution:

Subsets of C= {5,7} are ∅,{5},{7}, {5,7}.

Note: There is one improper subset of C= {5,7}.

Example 2 Find proper and improper subsets

Find the proper and improper subsets of A = {2, 4, 6, 8}

Solution:

There is one improper subset: {2, 4, 6, 8}. The proper subsets are as follows:

{ },
{2}, {4}, {6}, {8},
{2,4}, {2,6}, {2,8}, {4,6}, {4,8}, {6,8},
{2,4,6}, {2,4,8}, {2,6,8}, {4,6,8}

Practice

Find the proper and improper subsets of A = {a, b, c, d}

Exercise 1.2

1. Which one of the following sets is infinite?

   i) The set of whole numbers less than 10.

   ii) The set of prime numbers less than 10.

   iii) The set of whole numbers.

   iv) The set of factors of 10

   v) {7, 8, 10, 13} A 7-13 B 8, 10

   vi) {1, 3, 5, ... }.

2. Write the following sets, in tabular form:

   i) The set of 5 first five natural numbers.

   ii) The set of 5 first six letters of English alphabets.

   iii) The set of all odd numbers less than 9. NO

   iv) The set of all natural numbers which divide 12. 6

   v) The set of all letters in the word MATHEMATICS.

   vi) The set of last four months of the year.

3. Name the following sets:

   i) Set of triangles with four vertices.

   ii) {Umar}

   iii) {105}

4. Which of the following sets are equal sets?

   A = {0, 1, 2}, B = {0, 1, 2, 3}

   C = {1,2,0}, D = {1, 0, 3, 2}

5. Which of the following are equivalent sets? Use the symbols ≃.

   A = {a, b, c,} B = {i, j, k}

   C = {2, 10} D = {0, 1}

Review Exercise 1

1. Fill in the following blanks.
   (i) {1, 2} is a tabular form of set.
   (ii) {0} has one member, this set is called singleton.
   (iii) {a, b, c} has Three members.
   (iv) {1, 2, ... 7} is a Finite set.
   (v) {1, 2, 3, ...} is an Infinite set.
   (vi) {p, q, r, s} and {q, r, s, p} are equivalent sets.
   (vii) {chair, desk, table} and {Samina, Shazia, Irum} are equal sets.

2. Fill in the following blanks in the following using symbols $\subseteq$ or $\nsubseteq$.
   (i) $\{0\} \nsubseteq \{1, 0\}$
   (ii) $\{a, b\} \subseteq \{a, b, c, d\}$
   (iii) $\{-1, -2\} \nsubseteq \{-1, -3, -5\}$
   (iv) $\{\} \subseteq \{0\}$
   (v) $\{1, 2, 3, \ldots\} \subseteq \{0, 1, 2, \ldots\}$
   (vi) $\{1, 2, 3, \ldots\} \subseteq \{-1, -2, -3, \ldots\}$

3. Is A = {1,3,5,...} subset of B = {1,3,5,7,9,11,13,15}? $\boxed{\text{No}}$

4. List all possible subsets of the given set.
   i) G = {}
   ii) $H = \{6\}$.
   iii) $I = \{6, 7\}$ H={6} {}={7}
   iv) $J = \{6,7,8\}$
   v) K = {6, 7, 8, 9} H= K= {9}

5. Look for a pattern in Question 4, how many subsets the set $L = \{6, 7, 8, 9, 10\}$ has? 32

6. Find all possible subsets of S = {a,b,c}

7. Find the cardinality (no. of elements) of each of the following sets.
   i) $P = \{\omega, \gamma, \pi, \Delta\}$
   ii) $S = \{\ \}$
   iii) T ={states of the United States}

Summary

- **Set:** A set is a well-defined collection of distinct objects.
- **Tabular form :** A form of a set is that in which each member is listed separately.
- **Notation for membership of set:** If p is member of a set A, then symbolically we write $p \in A$, where $\in$ stands for 'is a member of' or 'belong to'. If q does not belong to A, then we write that $q \notin A$.
- **Finite set:** A set whose elements can be listed is called finite set. The number of elements of a finite set is a natural number. The number of elements in a set S is often called its cardinality and is denoted by n(S) or $|S|$.
- **Infinite set:** A set that is not finite is called infinite.
- **Empty set:** A set which has no element is called an empty set, void or null set and is written as { } or $\varnothing$.

- **Singleton:** A set having only one element is called a singleton set.
- **Equal sets:** Two sets A and B are said to be equal if they have the same elements and denoted as $A = B$.
- **Equivalent sets:** Two sets L and M are said to be **equivalent** if they have the same number of elements but not necessarily the same members. Symbolically it is written as $L \simeq M$.
- **Subset:** Set A is said to be a subset of the set B if all the elements of A are the elements of B. It is symbolically written as $A \subseteq B$. It is important to note that the null set { } is the subset of all sets. If A is not subset of B, then we write $A \nsubseteq B$.
- **Proper subset:** If all the elements of A are in B, but there is at least one element of B which is not the element of A, then A is called proper subset of B. Symbolically it is written as $A \subset B$.
- **Improper subset:** If $A \subseteq B$ and $A = B$ then A is called improper subset of B.

UNIT 02 WHOLE NUMBERS

What You'll Learn

- ❖ Differentiate between natural and whole numbers.
- ❖ Identify natural and whole numbers, and their notations.
- ❖ Represent
  - a given list of whole numbers,
  - whole numbers $<$ (or $>$) a given whole number,
  - whole numbers $\geq$ (or $\leq$) a given whole number,
  - whole numbers $>$ but $<$ a given whole number,
  - whole numbers $\geq$ but $\leq$ a given whole number,
  - sum of two or more given whole numbers, on the number line.
- ❖ Add and subtract two given whole numbers.
- ❖ Verify commutative and associative law (under addition) of whole numbers.
- ❖ Recognize '0' as additive identity.
- ❖ Multiply and divide two given whole numbers.
- ❖ Verify commutative and associative law (under multiplication) of whole numbers.
- ❖ Recognize '1' as multiplicative identity.
- ❖ Verify distributive law of multiplication over addition.
- ❖ Verify distributive law of multiplication over subtraction (with positive difference).

Why It's important

Many of us take whole numbers for granted, not fully appreciating that whole numbers have many significant applications and that they provide the basis for working with fractions, decimals, and integers. Whole numbers help us locate streets and houses. They help us keep track of the federal budget and the amount of unemployment.

2.1 Natural and Whole Numbers

The natural numbers (or counting numbers) are the numbers in the set:

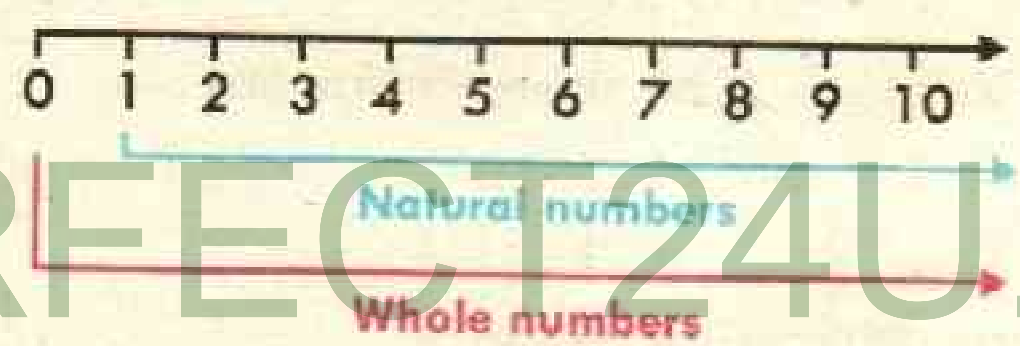
$$N = \{1, 2, 3, 4, 5, 6, 7, 8, ...\}$$

When you include the number 0 with the set of natural numbers, you have the set of whole numbers:

$$W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, ...\}$$

The three dots indicate that the pattern continues without end.

You can represent the natural numbers and whole numbers as equally spaced points on a number line, increasing endlessly in the direction of the arrow, as shown in Figure 2.1.



Symbols used to Compare Whole Numbers

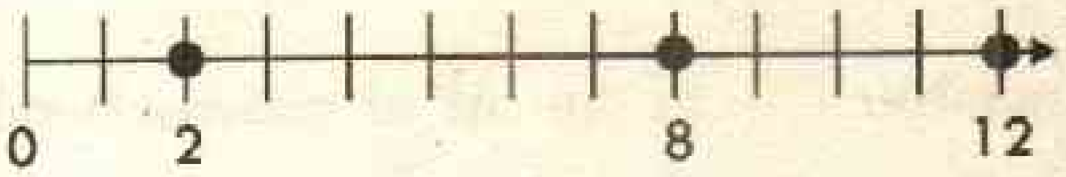
Let x be any whole number and a, b be given whole numbers.

- $x > a$ means all whole numbers greater than a.
- $x < b$ means all whole numbers less than b.
- $x \geq a$ means all whole numbers greater than or equal to a.
- $x \leq b$ means all whole numbers less than or equal to b.
- $a < x < b$ means all whole numbers greater than a but less than b.
- $a \leq x \leq b$ means all whole numbers greater than or equal to a but less than or equal to b.

Example 1 arrange on number line

Arrange the numbers 2, 8, and 12 on the number line.

Solution:

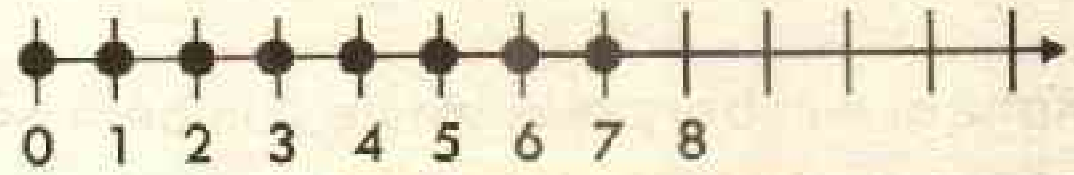


Just count to the right of 0 by 2, 8 and 12 steps.

Example 2 express on number line

Express all whole numbers < 8 on the number line.

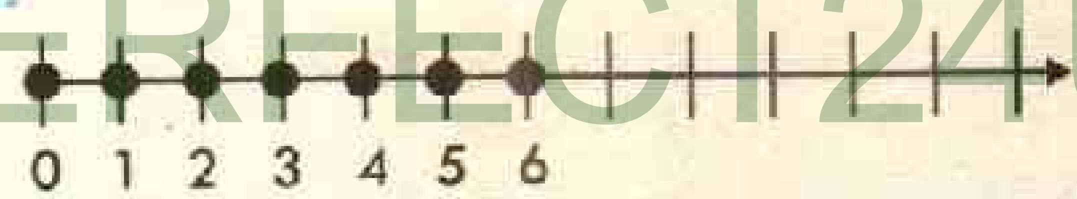
Solution:



Example 3 express on number line

Express all whole ≤ 6 on the number line.

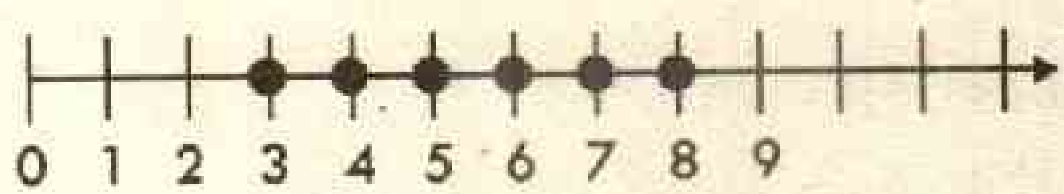
Solution:



Example 4 express on number line

Express all whole numbers > 2 but < 9 on the number line.

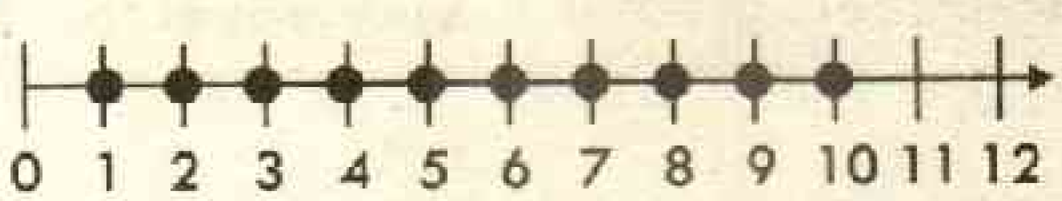
Solution:



Example 5 express on number line

Express all whole numbers ≥ 1 but ≤ 10 on the number line.

Solution:



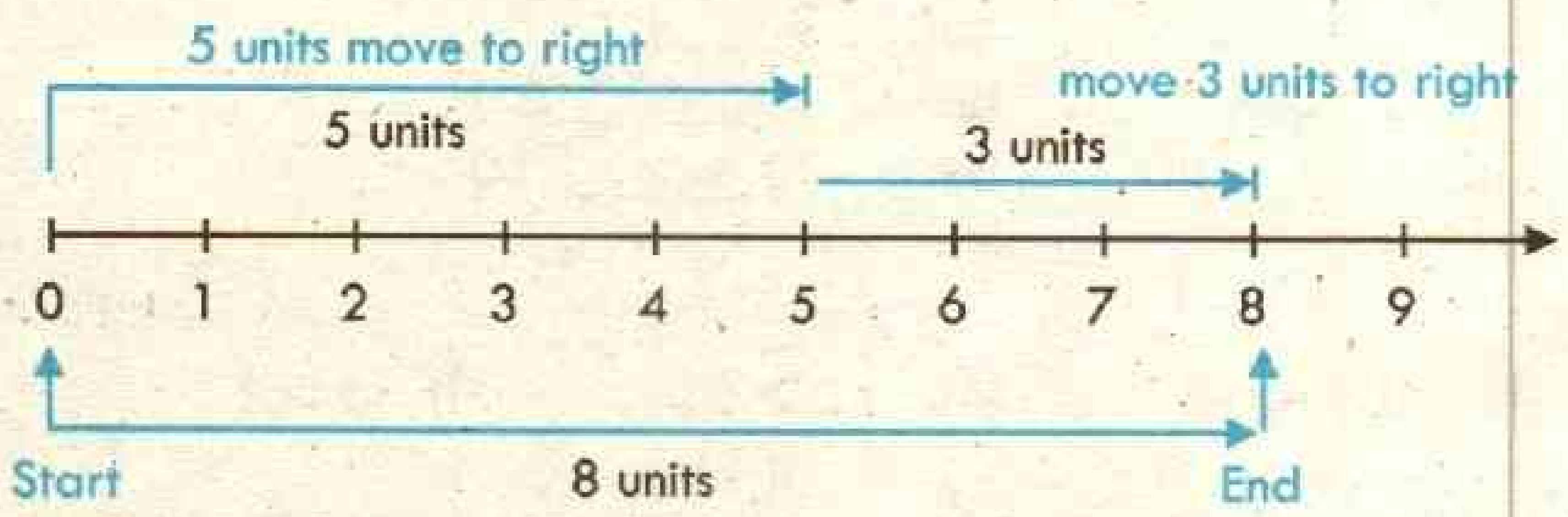
2.1.1 Represent the sum and difference of two or more given whole numbers on the number line

The procedure is explained in the following example.

Example 1 add on number line

To add 5 and 3. First we move from "0" to 5 and then 3 units ahead of 5 we get 8 on the number line so, 5 + 3 = 8.

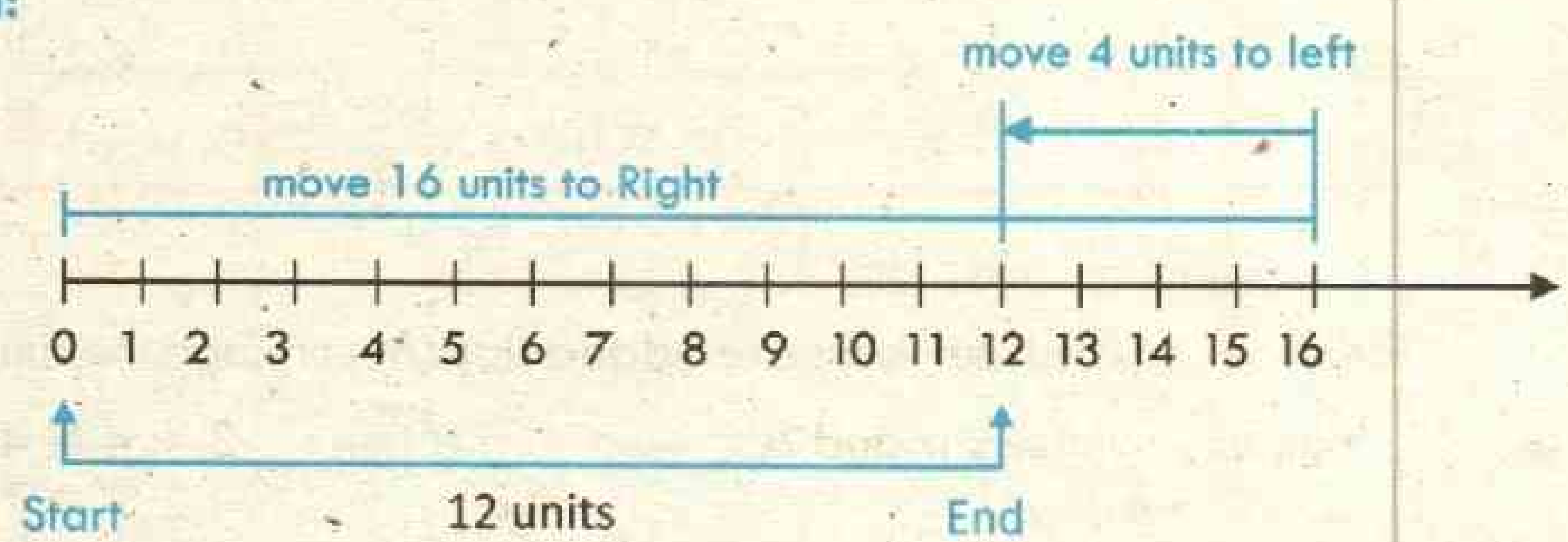
Solution:



Example 2 subtract on number line

Subtract 4 from 16.

Solution:



In order to subtract 4 from 16, first we move 16 units to the right of '0'. From 16 we count back 4 units which is number 12.

So, 16 – 4 = 12.

Exercise 2.1

1. Replace each box with < or >.

   (i) 46 ☐ 35 (ii) 5 ☐ 3 (iii) 49 ☐ 70

2. Fill in the blanks by using the symbol < or >.

   (i) 6 __>__ 7 (ii) 9 __<__ 8 (iii) 0 __>__ 1

3. Fill in the blanks by using the symbol < or >.

   (i) 3 < 4 but 3 __>__ 0
   (ii) 10 > 9 but 10 __<__ 30
   (iii) 11 < 20 but 11 __>__ 10

4. Which of the following statements are true and which are false?

   (i) 18 < 19 (ii) 12 > 20 (iii) 0 > 1

5. Find the sum of the following numbers on the number line.

   (i) 10, 5 (ii) 8, 7 (iii) 5, 6 (iv) 9, 2 (v) 0, 8

6. Subtract the first number from the second on the number line.

   (i) 2, 4 (ii) 3, 6 (iii) 4, 7 (iv) 5, 9 (v) 6, 10

2.2 Commutative Law of Whole Numbers w.r.t Addition

Commutative Property of Addition

The order in which numbers are added does not change the sum.

For any numbers a and b,
$a + b = b + a$.

Example: $3 + 4 = 4 + 3$
$7 = 7$

Associative Property of Addition

The way you group three or more numbers when adding does not change their sum.

For any numbers a, b and c,
$(a + b) + c = a + (b + c)$

Example: $(2+5)+6=2+(5+6)$
$13 = 13$

A counterexample is an example that shows a conjecture is not true.

Example 2 true,false

State whether the following conjecture is true or false, provide a counterexample.

Solution:

Write two subtraction expressions using the Associative Property, and then check to see whether they are equal.

$$9-(5-3)=(9-5)-3$$
$$9-2=4-3$$
$$7\neq 1$$

We found a counterexample. That is. $9-(5-3)\neq(9-5)-3$.

So, subtraction is not associative. The conjecture is false.

For any number a, the sum of a and 0 is a.

$a+0=a$, $5+0=5$

Exercise 2.2

1. Find each difference.

(i) $53-17=$ 36 (ii) $104-82=$ 22 (iii) $152-123=$ 29

2. Fill in the $\square$.

(i) $2+3=\boxed{2}+3$ (ii) $4+\boxed{5}=5+4$

(iii) $2+\boxed{0}=2$ (iv) $2+(3+4)=(\boxed{2}+3)+4$

3. Tick T for true and F for false in each of the following statements.

(i) $3+(5+7)=(3+5)+7$ T ✓ F

(ii) $6+7=6+5$ T ✗ F

(iii) $0+2=2$ T ✓ F

2.3 Multiplication And Division of Whole Numbers

Commutative Property of Multiplication

The order in which numbers are multiplied does not change the product.

For any numbers a and b, $a \times b = b \times a$.

Example: $3 \times 4 = 4 \times 3$
$12 = 12$

Associative Property of Multiplication

The way you group three or more numbers when multiplied does not change their product.

For any numbers a, b and c, $(a \times b) \times c = a \times (b \times c)$

Example: $(2\times5)\times6=2\times(5\times6)$
$60 = 60$

We can use the properties of numbers to find sums and products of mentally. Look for sums and products that end in zero.

Example 1 Mental Math

Find $4 \times (25 \times 11)$ mentally.

Solution:

Group 4 and 25 together because $4 \times 25 = 100$. It is easy to multiply by 100 mentally.

$4 \times (25 \times 11) = (4 \times 25) \times 11$ (Associative Property of Addition)
$= 100 \times 11$ (Multiply 4 and 25 mentally)
$= 1100$ (Multiply 100 and 11 mentally)

Important Properties of Division:

(i) Any number divided by 1 is equal to the number itself.
If a is any whole number then $a \div 1 = a$. For example $3 \div 1 = 3$

(ii) Any non -zero number divided by itself is equal to 1.
For any $a \neq 0$, $a \div a = \frac{a}{a} = 1$
For example $2 \div 2 = \frac{2}{2} = 1$

(iii) Zero divided by any non-zero whole number is 0. For any $a \neq 0$, $\frac{0}{a} = 0$.

2.3.1 Multiplicative Identity Property

There exists a whole number 1, called the multiplicative identity, such that for any whole number a, $a \times 1 = a$ and $1 \times a = a$.

For the whole number 5, we have $5 \times 1 = 5 = 1 \times 5$.

2.3.2 Distributive Laws of Whole Numbers

To multiply a number by a sum, multiply each number inside the parentheses by the number outside the parentheses.

For example:

i) $a(b + c) = ab + ac$
$3(5 + 2) = 3 \times 5 + 3 \times 2$

ii) $(b + c)a = ba + ca$
$(5 + 2)3 = 5 \times 3 + 2 \times 3$

2.3.3 Distributive Law of Multiplication Over Subtraction

If a, b, c are any three whole numbers then $a(b - c) = a \times b - a \times c$, then this property is called distributive law of multiplication over subtraction.

For example:

$$2(4-3) = 2 \times 4 - 2 \times 3$$

or, $$2 = 2$$

Exercise 2.3

1. Find product.
   (i) 48×5 (ii) 106×13 (iii) 59×127
2. Verify the commutative law under multiplication.
   (i) 3, 7 (ii) 4, 5 (iii) 7, 8
3. Name the property shown by each statement.
   (i) $5 \times 3 = 3 \times 5$ (ii) $1 \times 4 = 4$ (iii) $6(2+1)=6\times2+6\times1$
   (iv) $(12 \times 8) \times 3 = 12 \times (8\times3)$
4. Find each sum or product mentally.
   (I) $17 + 5 + 33$ (ii) $15 \times 0 \times 2$ (iii) $5 + 18 + 15 + 2$

Review Exercise 2

1. **Complete the following statements.**

(i) '0' is a __________ number.

(ii) '2' is a __________ than 3.

(iii) $24 \div 3 =$ __________.

(iv) $9 + (6 + 3) =$ __________.

(v) If x is a whole number, the distributive property of multiplication over addition says that $7 \times x + 3 \times x =$ __________.

2. **Encircle 'T' for true and 'F' for False in each of the following statements.**

(i) $6 < 8$ T – F

(ii) $17 \leq 20$ T – F

(iii) For any three whole number a, b, c, $a \times (b - c) = a \times c - a \times b$ T – F

(iv) For any whole number a, b, c, $a \times (b \times c) = (a \times b) \times c$ T – F

(v) $(x + y) + z = x + (y + z)$ for all whole numbers T – F

3. **Choose the correct answer.**

(i) Name of the property $2 + 3 = 3 + 2$ is

a. Commutative property of addition
b. Associative property of addition
c. Distributive property of addition

(ii) '0' in whole numbers is called __________

a. Multiplicative identity b. Additive identity
c. Additive inverse

(iii) If $25 \div 5 = 5$, then the quotient is

a. Zero b. 5 c. 25

(iv) The statement $e + (f + g) = (f + g) + e$ is an example of which property of addition?

a. Commutative b. Associative c. Identity

4. A wagon manufacturing plant in Japan, can produce 8000 wagons a day at top production. Which of the following is a reasonable amount of wagons that can be produced in a month?

   a. 24,000 b. 240,000 c. 2,400,000 d. 240,000,000

5. Name the property used in the following expressions.

   (i) $3 \times 5 = 5 \times 3$ (ii) $5 + (3 + 4) = (5 + 3) + 4$

   (iii) $4\,(2 + 5) = 4\,(2) + 4\,(5)$ (iv) $6\,(7 - 2) = 6\,(7) - 6\,(2)$

6. Find each sum or product mentally.

   i) $13 + 87$ ii) $6 \times 9 \times 5$

   iii) $8 + 11 + 22 + 4$ iv) $2 \times 5 \times 10$

7. A storekeeper buys 24 pens for Rs.99 each. A method for mentally multiplying 24×99 is to compute $(24 \times 100) - (24 \times 1)$. Use a distributive property to show why these two expressions are equal.

8. Amjad and Najma are using the Associative Properties of Addition and Multiplication to rewrite expressions.

Amjid

$(4 + 3) + 6 = 4 + (3 + 6)$

Najma

$(2 + 7) \times 5 = 2 + (7 \times 5)$

Who is correct?

Summary

- **Whole numbers:**

  The number 0, 1, 2, 3, 4… are called whole numbers where '…' means that the whole numbers continue indefinitely.

- **Identity element of addition:**

  The number '0' is called the identity element of addition, since for any whole number a, $a+0=a$ or $0+a=a$

- **Commutative property of addition:**

  If a and b are any two whole numbers, then $a+b=b+a$

- **Associative property of addition:**

  If a, b and c are any three whole numbers, then $a+(b+c)=(a+b)+c$

- **Identity element of multiplication:**

  The number '1' is called identity element of multiplication, since for any whole number a, $a.1=a$ and $1.a=a$

- **Commutative property of multiplication:**

  If a and b be any two whole numbers then $a \times b=b \times a$

- **Associative property of multiplication:**

  If a, b and c are any three whole numbers then $a \times (b \times c)=(a \times b) \times c$

- **Distributive property of multiplication over addition:**

  If a, b and c are any three whole numbers then $a \times (b+c)=a \times b+a \times c$

UNIT 03 FACTORS AND MULTIPLES

What You'll Learn

- ❖ Define a factor as a number which divides the dividend completely leaving no remainder.
- ❖ Define a multiple as a dividend into which a factor can divide.
- ❖ Define even numbers as the numbers which are multiples of 2.
- ❖ Define odd numbers as the numbers which are not multiples of 2.
- ❖ Define prime numbers as numbers which have only two factors (i.e., 1 and itself).
- ❖ Define composite numbers as numbers which have more than two factors.
- ❖ Know that 1 is neither prime nor composite as it has only one factor which is 1 itself.
- ❖ Know that 1 is a factor of every number.
- ❖ Know that 2 is the only even prime number whereas all other prime numbers are odd.
- ❖ Test by inspection whether the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15 and 25 can divide a given number.
- ❖ Define prime factorization as the process of factorizing a number into its prime factors.
- ❖ Recognize index notation.
- ❖ Factorize a given number and express its factors in the index notation.

- ❖ Define HCF as the greatest number which is a common factor of two or more numbers.
- ❖ Find HCF of two or more than two numbers by
  - prime factorization,
  - long division method.
- ❖ Define LCM as the smallest number which is a common multiple of two or more numbers.
- ❖ Find LCM of two or more numbers by
  - prime factorization,
  - division method.
- ❖ Solve real life problems related to HCF and LCM.

Why factors and multiples are important

Factors are parts of numbers that, multiplied together, give a larger number. Every number has at least two factors, one and the number itself. On the other hand, multiples are numbers where the same number is repeated, as if you were counting by that number. For example, the multiples of 2 start with 2 and are: 2, 4, 6, 8, 10 . . . and so on. Each additional number is a multiple of 2.

Factors and multiples are especially important in working with expanding and reducing fractions, as well as finding the greatest common factor, least common multiple and prime factors of a number.

multiple of 4

$5 \times 4 = 20$

factor of 20 | factor of 20 | multiple of 5

3.1 Factors

3.1.1 Definition of Factors

A factor is a number which divides the dividend completely leaving no remainder . For example,

$6 \div 1 = 6$
$6 \div 2 = 3$
$6 \div 3 = 2$
$6 \div 6 = 1$

$$1\overline{)6} = 6,\; -6,\; 0 \qquad 2\overline{)6} = 3,\; -6,\; 0 \qquad 3\overline{)6} = 2,\; -6,\; 0 \qquad 6\overline{)6} = 1,\; -6,\; 0$$

Here 1, 2, 3 and 6 divide 6 completely i.e. remainder in each case is 0. We say that 1, 2, 3 and 6 are the factors of 6.

Example 1 List Factors

List all the factors of 72. Use the divisibility rules to determine whether 72 is divisible by 2, 3, 4 and so on. Then use division to find other factors of 72.

Solution:

Number	72 Divisible by Number?	Factor Pairs
1	yes	1×72
2	yes	2×36
3	yes	3×24
4	yes	4×18
5	no	—
6	yes	6×12
7	no	—
8	yes	8×9
9	yes	9×8

Use division to find the other factor in each factor pair. $72 \div 2 = 36$

You can stop finding factor when the number start repeating.

So, the factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

Practice

List all the factors of each numbers.

a) 203 b) 80 c) 115 d) 145 e) 350

3.1.2 Prime Number

A Prime number is a natural number that has exactly two factors, 1 and itself.

A composite number is a natural number that has more than two factors. 1 is neither prime nor composite.

Example 1 Identify Numbers as Prime number

a) Determine whether 19 is prime or composite.

Find factors of 19 by listing the whole number pairs whose product is 19.

Solution:

$19 = 1 \times 19$

The number 19 has only two factors. Therefore, 19 is a prime number.

b) List all prime numbers and composite numbers up to 9.

Solution:

	Whole Numbers	Factors	Number of Factors
Prime Numbers	2	1, 2	2
	3	1, 3	2
	5	1, 5	2
	7	1, 7	2
Composite Numbers	4	1, 2, 4	3
	6	1, 2, 3, 6	4
	8	1, 2, 4, 8	4
	9	1, 3, 9	3
Neither Prime nor Composite	1	1	1

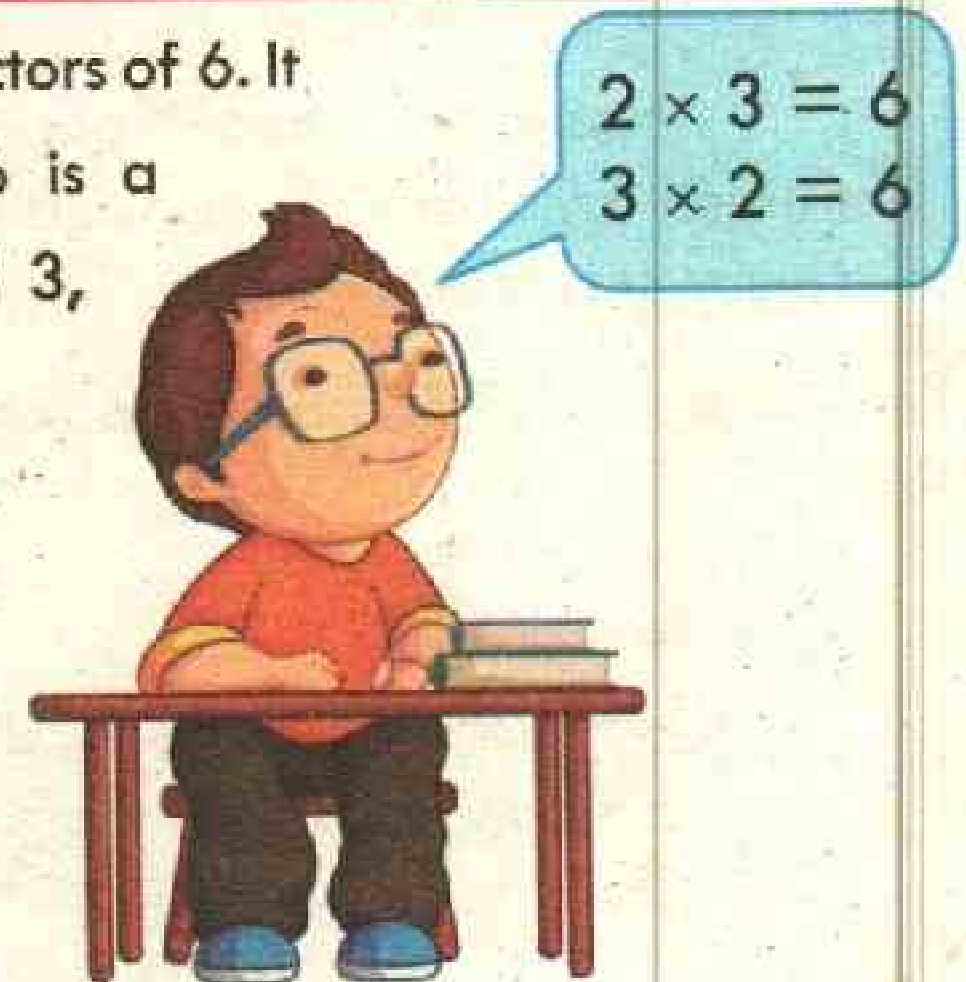
Practice

a) Extend the above table up to 20.

b) List All the primes up to 50.

3.1.3 Multiples

We know that 2 and 3 are the factors of 6. It means that 2 divides 6 completely or 6 is a multiple of 2. Similarly 6 is a multiple of 3, because 3 divides 6 completely.



> Multiple is the dividend into which a factor can divide.

To find the multiples of a number we have to find the dividends for which a divisor is available. Multiples can be obtained by multiplying the divisors by 1, 2, 3, 4,...

So multiples of 2 are 2, 4, 6, 8,...

Multiples of 3 are 3, 6, 9, 12,...

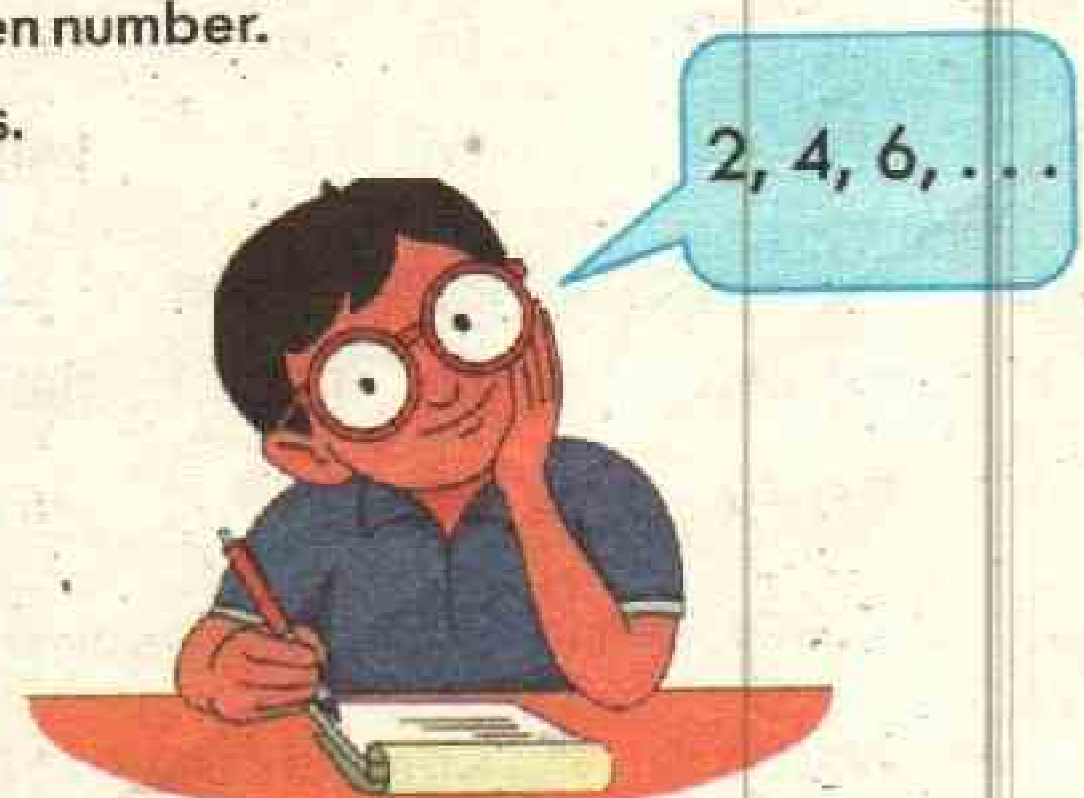
Multiples of 5 are 5, 10, 15, 20,...

and multiple of 12 are 12, 24, 36, 48,...

3.1.4 Even Numbers

Numbers that can be divided completely by 2 are called Even numbers, i.e., 2 is the factor of every even number.

For example 2,4,6,... are even numbers.



> Numbers which are the multiples of 2 are called Even numbers.

Remember

'2' is the only even prime number, whereas all other prime numbers are odd.

3.1.5 Odd Numbers.

Numbers that cannot be divided completely by 2 are called Odd numbers. 2 is not the factor of any odd number.

For example, 1,3,5... are odd numbers.

Remember

Numbers which are not the multiples of 2 are called Odd numbers.

Exercise 3.1

1. Which of the following numbers are prime numbers and which of them are composite numbers?

(i) 17	(ii) 25	(iii) 21	(iv) 29	(v) 35
(vi) 37	(vii) 40	(viii) 47	(ix) 65	(x) 71

2. Write prime numbers between 20 and 40.
3. Write composite numbers between 40 and 50.
4. Find the factors of:

(i) 20	(ii) 26	(iii) 48	(iv) 30
(v) 42	(vi) 16	(vii) 32	(viii) 49

5. Write 3 multiples of each of the following numbers.

(i) 7	(ii) 11	(iii) 17	(iv) 19
(v) 12	(vi) 15	(vii) 18	(viii) 20

6. Write even numbers between 50 and 70.
7. Write odd numbers between 80 and 100.

3.2 Divisibility Rules

A number is divisible by **2**
if its digit at unit place is even.
e.g. 72, 114, 396, 518

A number is divisible by **3**
if the sum of its digits is divisible by 3.
e.g. 621, 8754, 7515

A number is divisible by **4**
if its last two digits are either 0 or divisible by 4.
e.g. 72, 114, 396, 518

A number is divisible by **5**
if the last digit is either 5 or 0.
e.g. 735, 1905, 9370

A number is divisible by **6**
if it is divisable by 2 and it is divisible by 3.
e.g. 72, 540, 414, 7896

A number is divisible by **8**
if the last three digits are Zero or divisible by 8.
e.g. 3040, 15008, 560

A number is divisible by **9**
if the sum of its digits is divisible by 9.
e.g. 55008, 17460

A number is divisible by **10**
if the last digit is zero.
e.g. 440, 7650, 13830

A number is divisible by **11**
if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is either 0 or divisible by 11.
e.g. 5247, 6314, 18271, 936276

A number is divisible by **12**

if sum of the digits is divisible by 3 and last two digits are either 0 or divisible by 4.

e.g. 4524, 271800, 27684

A number is divisible by **15**

if sum of the digits is divisible by 3 and last digit is either 0 or 5.

e.g. 4524, 271800, 27684

A number is divisible by **25**

if the last two digits are either 0 or divisible by 25.

e.g. 4525, 271800, 27675

Example 1 use divisibility rules

Determine whether 138 is divisible by 2, 5, 6, 10.

Solution:

Number	Divisible	Reason
2	yes	The ones digit is 8, and 8 is divisible by 2.
3	yes	The sum of the digits is 1+3+8 or 12 and 12 is divisible by 3.
5	no	The ones digit is 8, not 0 or 5.
6	yes	138 is divisible by 2 and 3.
10	no	The ones digit is not 0.

So, 138 is divisible by 2, 3 and 6.

Practice

Determine whether each number is divisible by 2, 3, 5, 6 or 10.

a) 51 b) 146 c) 876 d) 3050 e) 72183

Exercise 3.2

1. Using the tests of divisibility, find out which of the following numbers are divisible by (a) 2, (b) 3, (c) 4, (d) 5:

(i) 5320 a	(ii) 706 a	(iii) 7230 a
(iv) 5421 a	(v) 425	(vi) 9128
(vii) 4773	(viii) 1048 a	(ix) 8005
(x) 7072 a	(xi) 5217	(xii) 12960 a

2. Using the tests of divisibility, find out which of the following numbers are divisible by (a) 6, (b) 9, (c) 10, (d) 8:

(i) 423	(ii) 960	(iii) 4125
(iv) 2340	(v) 7260	(vi) 7821
(vii) 3920	(viii) 1542	(ix) 9823
(x) 5553	(xi) 7860	(xii) 2970

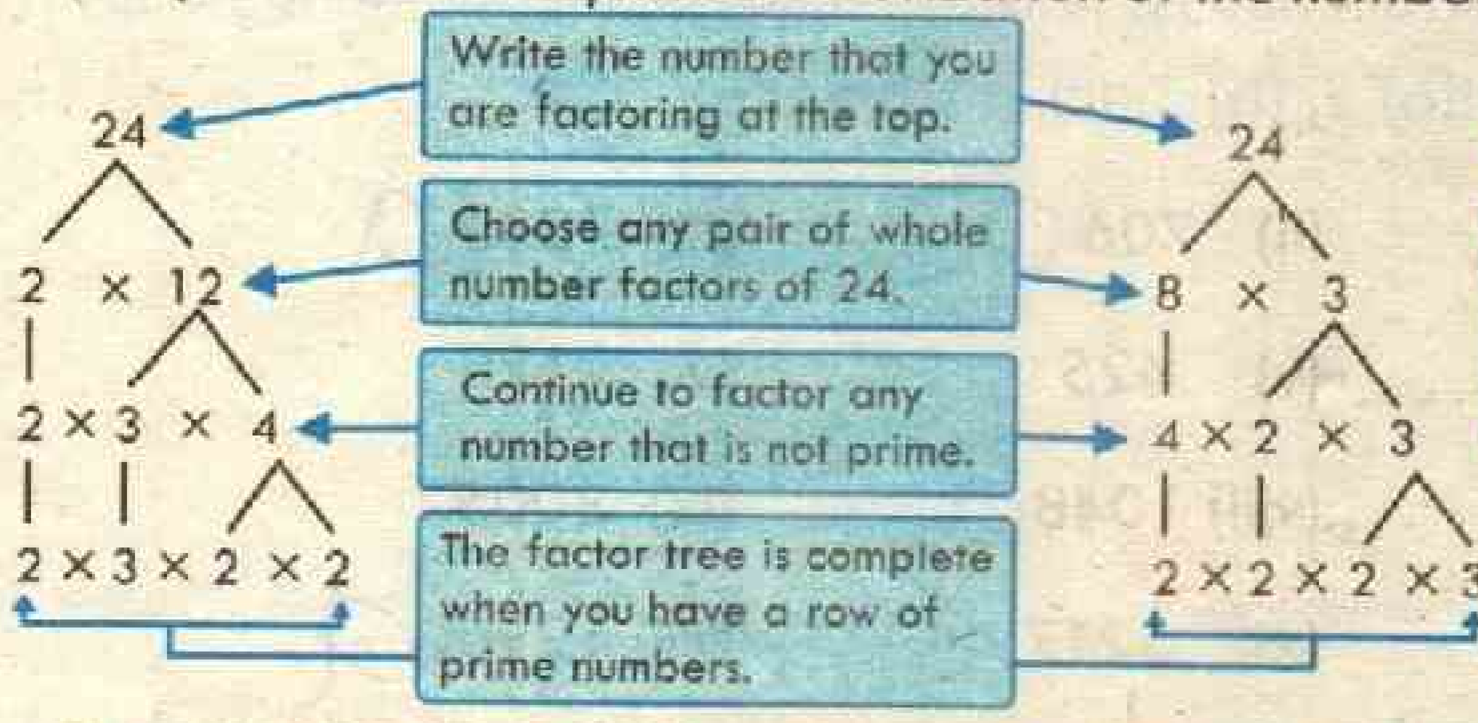
3. Using the tests of divisibility, find out which of the following numbers are divisible by (a) 11, (b) 12, (c) 15, (d) 25:

(i) 756	(ii) 5820	(iii) 7400
(iv) 2384	(v) 6500	(vi) 8235
(vii) 4128	(viii) 8775	(ix) 10064
(x) 26124	(xi) 13005	(xii) 14400

3.3 Prime Factorization

3.3.1 Definition of Prime Factorization

When a composite number is expressed as the product of prime factors, it is called the prime factorization of the number.



One way to find the prime factorization of a number is to use a factor tree.

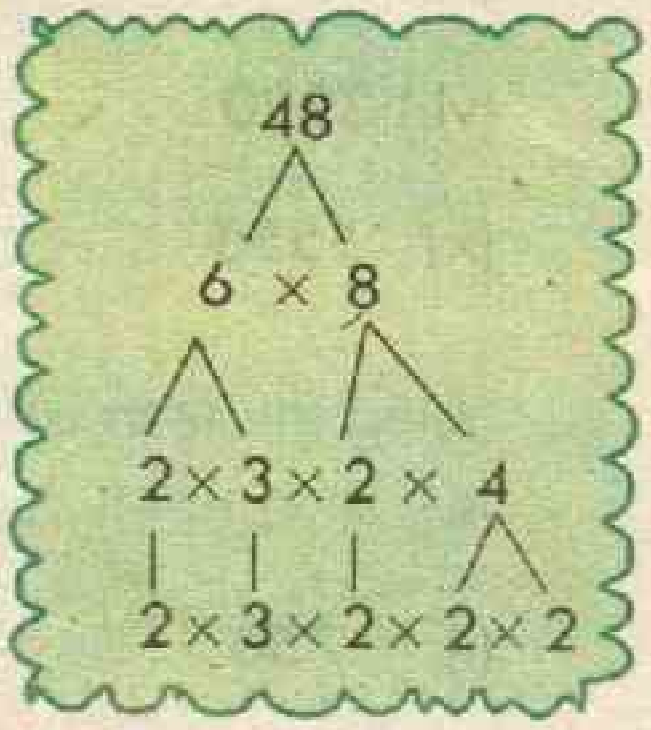
Both trees give the same prime factors, except in different orders. There is exactly one prime factorization of 24. The prime factorization of 24 is $2 \times 2 \times 2 \times 3$.

Example 1 factorization

Find the prime factorization of 48

Solution: Method I.

Factor tree



So $48 = 2 \times 2 \times 2 \times 2 \times 3$

Method II.

Repeated division method.

2	48
2	24
2	12
2	6
	3

So $48 = 2 \times 2 \times 2 \times 2 \times 3$

Practice Find the prime factorization of each number. Use exponents for repeated factors.

a) 18 b) 39 c) 50

3.3.2 Index Notation

We know that

$100 = 10 \times 10 = 10^2$ (Ten is multiplied two times)

$1000 = 10 \times 10 \times 10 = 10^3$ (Ten is multiplied three times)

Number of times a number is multiplied is called the index.

Example 1 Write Prime Factorization

Write the prime factorization of 36.

Solution:

The factorization is complete because 2 and 3 are prime numbers.

The prime factorization of 36 is $2 \times 2 \times 3 \times 3$ or $2^2 \times 3^2$

Method-I

36

6×6 $\quad 36 = 6 \times 6$

$2 \times 3 \times 2 \times 3$ $\quad 6 = 2 \times 3$

Method-II

2	36
2	18
3	9
	3

Exercise 3.3

1. Write down the prime factorization of:

(i) 6 (ii) 140 (iii) 450 (iv) 22050

2. Find the prime factorization in index form of the following:

(i) 144 (ii) 3600 (iii) 1250 (iv) 7056

Project

Twin primes are prime numbers that differ by 2, such as 3 and 5. List all the twin primes that are less than 50.

3.4 Highest Common Factor (HCF)

The highest common factor (HCF) of two or more numbers is the product of common factors.

3.4.1 Finding HCF by Prime Factorization

Example 1 Find the H.C.F.

Find the H.C.F. of 30 and 24.

Solution:

First, factor each number completely. Then circle the common factors.

30

2×15

$2 \times 3 \times 5$

24

2×12

$2 \times 2 \times 6$

$2 \times 2 \times 2 \times 3$

$30 = \boxed{2} \times \boxed{3} \times 5$

$24 = \boxed{2} \times 2 \times 2 \times \boxed{3}$

(The common prime factors are 2 and 3)

The HCF of 30 and 24 is 2×3 or 6.

Practice

Find the HCF of the numbers 63 and 42.

Example 2 Find the H.C.F.

Find the HCF of 72 and 60 by prime factorization.

Solution:

2	72
2	36
2	18
3	9
	3

2	60
2	30
3	15
	5

Here $72 = 2\times2\times2\times3\times3 = 2^3\times3^2$ (i)

$60 = 2\times2\times5\times3 = 2^2\times5\times3$ (ii)

Hence HCF $= 2\times2\times3 = 12$

Example 3 Find the H.C.F.

Find the HCF of 28, 42 and 70 by prime factorization.

Solution: Here $28 = 2\times2\times7 = 2^2\times7$ (i)

$42 = 2\times3\times7$ (ii)

$70 = 2\times5\times7$ (iii)

The product of common factors in (i), (ii) and (iii) is $2\times7 = 14$

Hence HCF of 28, 42 and 70 is 14.

Exercise 3.4

1. Write the following numbers in index form.

(i) $2\times3\times7\times5\times7$ (ii) $2\times3\times5\times7\times2\times5\times7\times11$

(iii) $2\times2\times2\times5\times5\times2\times2\times5\times5\times5$

2. Find HCF of the following numbers by prime factorization.

(i) 30,45 (ii) 45,108 (iii) 12,18,30 (iv) 36,54,72,90

3.4.2 Finding HCF by Division Method

To find the HCF of two or more numbers, division method can also be used, which is illustrated with the help of following examples.

Example 1 Find the H.C.F.

Find the HCF of 36 and 60 by division method.

Solution:

```
36) 60 (1
   -36
    24) 36 (1
       -24
        12) 24 (2
           -24
             0
```

Therefore, HCF of 36 and 60 is 12.

Example 2 Find the H.C.F.

Find the HCF of 48, 64 and 120.

Solution:

Here we first find the HCF of any two numbers. We first find the HCF of 120 and 48.

```
48) 120 (2
   - 96
     24) 48 (2
        -48
          0
```

```
24) 64 (2
   -48
    16) 24 (1
       -16
         8) 16 (2
           -16
             0
```

∴ H.C.F of 48 and 120 is 24.

∴ H.C.F of 24 and 64.

Therefore HCF of 48, 64 and 120 is 8.

Example 3 Find the H.C.F.

Find the HCF of 420, 840, 1155 and 1190.

Solution:

We form pairs of the given numbers. Let the pairs be (420, 1155) and (840, 1190). First we find the HCF of 420 and 1155.

```
420) 1155 (2
    -840
    315) 420 (1
        -315
        105) 315 (3
            -315
               0
```

```
840) 1190 (1
    -840
    350) 840 (1
        -700
        140) 350 (2
            -280
             70) 140 (2
                -140
                   0
```

HCF of 420 and 1155 is 105. HCF of 840 and 1190 is 70.

Now we find the HCF of 105, 70

```
70) 105 (1
    -70
    35) 70 (2
       -70
         0
```

Hence HCF of 420, 840, 1155 and 1190 is 35.

Exercise 3.5

1. **Find the HCF of the following numbers by division method.**

i) 36, 96 ii) 90, 54 iii) 325, 175
iv) 350, 420, 560 v) 735,840, 1050 vi) 189, 315, 420
vii)150, 315, 435, 675 viii) 135, 675, 540, 765

3.5 Least Common Multiple (L.C.M)

A multiple of a number is a product of that number and a whole number. Sometimes numbers have some of the same multiples. These are called common multiples.

The least number which is the multiple of two or more than two given numbers is called their **Least Common Multiple** (L.C.M).

Example 1 Find the L.C.M

Find the L.C.M of 4 and 6.

Solution:

Some common multiples of 4 and 6 are, 12 and 24.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, ...

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, ...

The least of the nonzero common multiples is called the least common multiple (LCM). So, the LCM of 4 and 6 is 12.

3.5.1 To find L.C.M by Prime Factorization

The method is illustrated with the help of following examples.

Example 1 Find the L.C.M

Find the LCM of 24 and 36 by the method of prime factorization.

Solution:

Prime factorization of 24 and 36 are as follows:

$24 = 2\times 2\times 2\times 3 = 2^3\times 3$ (i)

$36 = 2\times 2\times 3\times 3 = 2^2\times 3^2$ (ii)

Both the numbers contain 2 and 3 in their prime factorization.

The greatest exponent of 2 is 3 and of 3 is 2 in these factorizations.

$\therefore$ LCM of 24 and 36 $= 2^3\times 3^2$

$= 8\times 9 = 72.$

Example 2 Find the L.C.M

Find the LCM of 8, 12 and 18 by prime factorization.

Solution: Prime factorization of 8, 12 and 18 are:

$8 = 2 \times 2 \times 2 = 2^3$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

Factors include the numbers 2 and 3.

The greatest exponents of 2 and 3 used in any of these three factorizations are 3 and 2 respectively.

Therefore, LCM $= 2^3 \times 3^2$

$= 8 \times 9 = 72$

Example 3 Find the L.C.M

Find the LCM of 21, 28, 35 and 77 by prime factorization.

Solution: Prime factorizations of 21, 28, 35 and 77 are.

$21 = 3 \times 7$

$28 = 2 \times 2 \times 7$

$= 2^2 \times 7$

$35 = 5 \times 7$

$77 = 7 \times 11$

Factors in these prime factorization include the numbers 2,3,5,7 and 11. The greatest exponent of these factors in any factorization is 1 except for 2 which is 2.

Therefore, LCM $= 2^2 \times 3 \times 5 \times 7 \times 11$

$= 4620$

Exercise 3.6

1. Find the LCM of the following numbers by prime factorization.

i) 6, 8	ii) 15, 20	iii) 16, 20
iv) 8, 12, 20	v) 18, 30, 42	vi) 15, 30 50
vii) 70, 98, 175	viii) 32, 48, 72	ix) 6, 8, 12, 18, 24

3.5.2 To find LCM by Division Method

This is a short version of the method of finding LCM by prime factorization.

Procedure

The numbers are divided by such prime factors, which are common in at least two numbers and the remaining numbers are written as they are. When no common factor exists in at least two numbers, all the prime factors are multiplied together to get the L.C.M.

Example 1 Find the L.C.M

Find the LCM of 24, 32, 36 and 48 by division method.

Solution:

2	24 – 32 – 36 – 48
2	12 – 16 – 18 – 24
2	6 – 8 – 9 – 12
2	3 – 4 – 9 – 6
3	3 – 2 – 9 – 3
	1 – 2 – 3 – 1

$$\text{LCM} = 2^5 \times 3^2$$
$$= 32 \times 9$$
$$= 288$$

Explanation:
In the first two steps, 2 is a common factor of all the four numbers.
In the third step, 2 is a common factor of three numbers 6, 8 and 12 while 9 is written as it is.
In the fourth step, 2 is a common factor of 4 and 6 while 3 and 9 are not changed.
In fifth step, 3 is a common factor of 3, 9 and 3 while 2 is written as it is.

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 3$$
$$= 2^5 \times 3^2 = 32 \times 9 = 288$$

Exercise 3.7

1. Find the LCM by division method.

i) 16, 20, 24, 30 ii) 18, 27, 36, 45 iii) 35, 40, 21, 28

iv) 42, 56, 70, 84 v) 63, 175, 50, 45 vi) 15, 32, 45, 60

vii) 30, 35, 42, 63 viii) 16, 32, 24, 36 ix) 48, 72, 96, 144, 168

3.6 Real Life Problems Related to HCF and LCM

Example 1 Find the HCF

Find the greatest capacity of a measuring jar that can exactly measure the quantities: 165 litres, 175 litres and 200 litres.

Solution: We have to find the HCF of 165, 175 and 200.

$165 = 5 \times 31$

$175 = 5 \times 5 \times 7$

$200 = 5 \times 5 \times 2 \times 2 \times 2$

$\text{HCF} = 5$

Therefore the greatest capacity of the required jar is liters.

Practice

Find the greatest number of students in a room to accommodate 135, 180 and 225 students completely.

Example 2 Find the L.C.M

Find the least number of labourers that can be sent to workplaces in groups of 16, 20, 24 or 32 labourers.

Solution:

We have to find the LCM of 16, 20, 24 and 32.

2	16 – 20 – 24 – 32
2	8 – 10 – 12 – 16
2	4 – 5 – 6 – 8
2	2 – 5 – 3 – 4
	1 – 5 – 3 – 2

$\therefore \quad \text{LCM} = 2 \times 2 \times 2 \times 2 \times 5 \times 3 \times 2$

$= 2^5 \times 3 \times 5 = 32 \times 15 = 480$

So required number of labourers = 480

Practice

Find the least number of plants that can be planted in rows containing 12, 15, 18 or 21 plants in each row.

Exercise 3.8

1. Find the greatest length of a measuring tape that can measure distances of 405cm, 315cm and 390cm completely.

2. Find the greatest length of a piece of bamboo that can be used to completely measure four bamboos having length 45m, 60m, 75m and 90m.

3. Tahira guessed that she can distribute the amount of Zakat exactly among needy persons if she gives Rs. 25, Rs.30, Rs.45, Rs.50 or Rs.90 to each person. Find the least amount of Zakat Tahira has to pay.

4. Find the least number of students in school that can be sent exactly in groups of 15, 18, 27, 36 and 45 to visit the zoo

5. Find the greatest length of a string that can measure completely the strings having lengths 910cm, 945cm and 980cm respectively.

6. Find the least number of oranges that can be packed is crates having capacity of 45, 40 or 60 oranges per crate.

7. Find the least number of candies that can be packed in packets of 12, 24, 60 or 144 candies per packet.

8. Find the greatest measure of a container that can measure completely the quantities of kerosene oil measuring 210 litres, 350 litres and 490 litres.

9. Find the least number that is exactly divisible by 28, 98 and 105.

10. Find the greatest number that can completely divide the numbers 252, 315, 441 and 504.

Review Exercise 3

Select the correct answer from the three suggested answers for the following questions:

1. Which of the following numbers is not divisible by 3 ?
 a. 1512 b. 1645 c. 433

2. Which of the following numbers is divisible by 15 and 25?
 a. 7400 b. 2525 c. 6150

3. HCF of 18, 24 and 30 is:
 a. 360 b. 30 c. 6

4. LCM of 6, 8 and 12 is:
 a. 24 b. 48 c. 96

5. Which of the following is not a prime number.
 a. 7 b. 5 c. 1

6. Which of the following is a prime number?
 a. 8 b. 4 c. 2

7. Prime factorization of 56 is:
 a. 14×2^2 b. 7×2^3 c. 28×2

8. Which of the following numbers is not an even number?
 a. 2100 b. 59372 c. 48629

9. Which of the following numbers is divisible by 9?
 a. 482763 b. 596340 c. 70539

10. Which of the following number is divisible by 11?
 a. 7897593 b. 8478551 c. 323266

11. HCF of 14, 35 and 98 is:
 a. 7 b. 98 c. 70

Summary

- **Factor**

  If a number divides the dividend completely having no remainder, then it is said to be factor of the dividend.

- **Multiple**

  A multiple of a number is a product of that number and a whole number.

- **Even number and Odd number**

  If a number is divisible exactly by 2, it is called an even number, otherwise it is said to be an odd number.

- **Prime number**

  Prime numbers have only two factors i.e. 1 and the number itself.

- '1' is neither prime nor composite.

- **Composite Numbers**

  Numbers that have more than two factors are called composite numbers.

- **Prime factorization**

  Process of factorizing a number into prime factors is called prime factorization.

- **HCF**

  The highest number, which is a common factor of the given numbers, is called their HCF .

- **LCM**

  The least number, which is a multiple of given numbers is called their LCM.

UNIT 04 INTEGERS

What You'll Learn

- ❖ Know that
  - the natural numbers 1,2,3,......, are also called positive integers and the corresponding negative numbers $-1, -2, -3, ...$, are called negative integers,
  - '0' is an integer which is neither positive nor negative.
- ❖ Recognize integers.
- ❖ Represent integers on number line.
- ❖ Know that on the number line any number lying
  - to the right of zero is positive,
  - to the left of zero is negative,
  - to the right of another number is greater,
  - to the left of another number is smaller.
- ❖ Know that every positive integer is greater than a negative integer.
- ❖ Know that every negative integer is less than a positive integer.
- ❖ Arrange a given list of integers in ascending and descending order.
- ❖ Define absolute or numerical value of a number as its distance from zero on the number line and is always positive.
- ❖ Arrange the absolute or numerical values of the given integers in ascending and descending order.
- ❖ Use number line to display:
  - sum of two or more given negative integers,
  - difference of two given positive integers,
  - sum of two given integers.
- ❖ Add two integers (with like signs) in the following three steps:
  - Take absolute values of given integers,
  - Add the absolute values,
  - Give the result the common sign.

- ❖ Add two integers (with unlike signs) in the following three steps:
  - Take absolute values of given integers,
  - Subtract the smaller absolute value from the larger,
  - Give the result the sign of the integer with the larger absolute value.
- ❖ Recognize subtraction as the inverse process of addition.
- ❖ Subtract one integer from the other by changing the sign of the integer being subtracted and adding according to the rules for addition of integers.
- ❖ Recognize that
  - the product of two integers of like signs is a positive integer,
  - the product of two integers of unlike signs is a negative integer.
- ❖ Recognize that division is the inverse process of multiplication.
- ❖ Recognize that on dividing one integer by another
  - if both the integers have like signs the quotient is positive,
  - if both the integers have unlike signs the quotient is negative.
- ❖ Know that division of an integer by '0' is not possible.

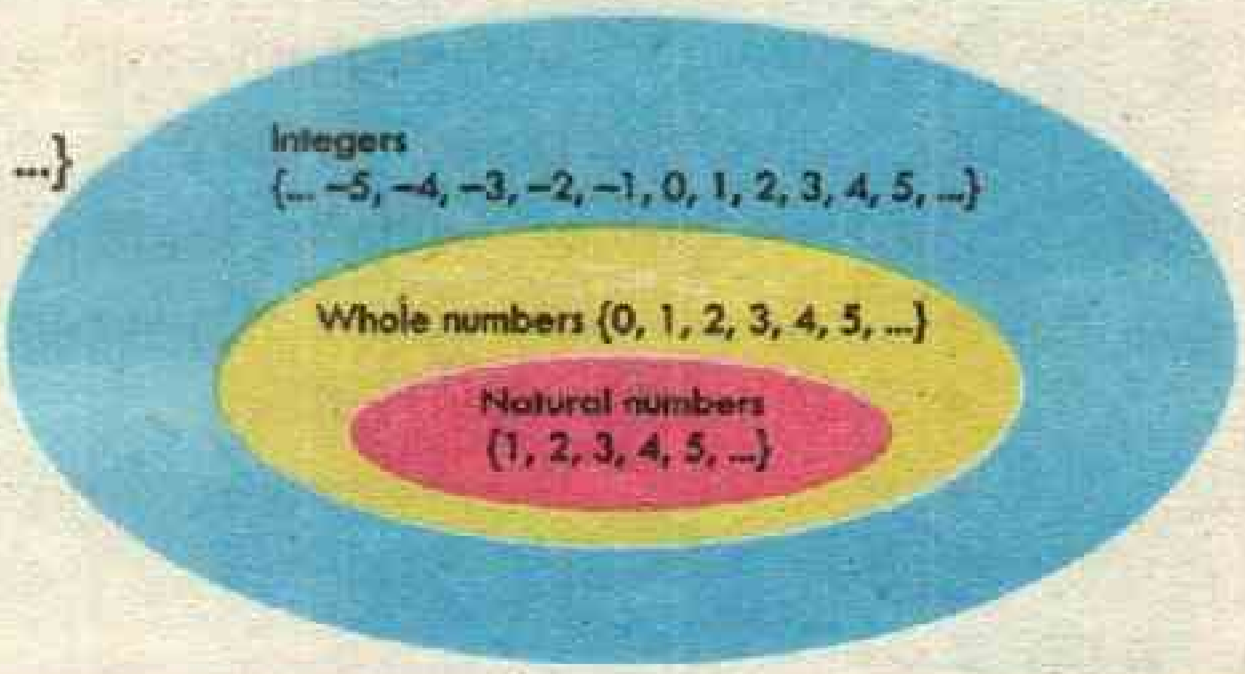
Why it's important

In both mathematics and everyday life, there are many situations where integers are used. Some examples include temperatures, sports such as golf and measuring the depth below sea level.

Integers
{... –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}

Whole numbers
{0, 1, 2, 3, 4, 5, ...}

Natural numbers
{1, 2, 3, 4, 5, ...}

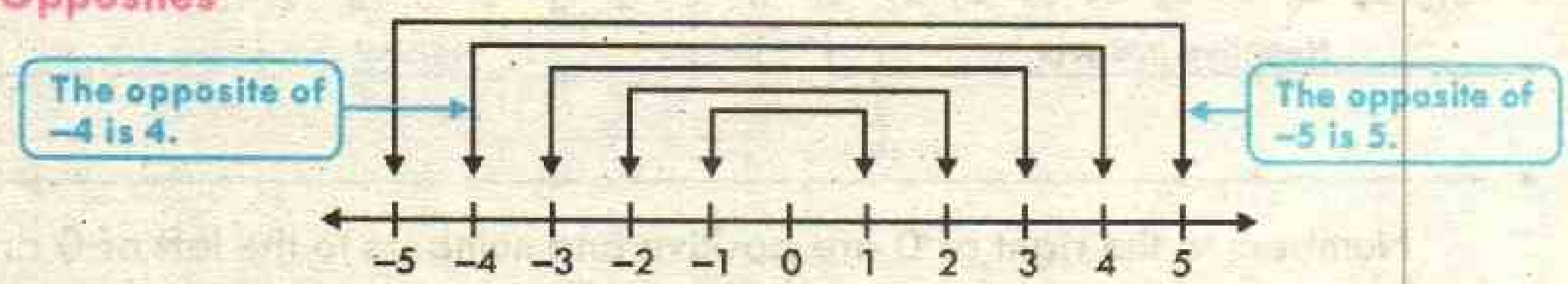


4.1 Integers

4.1.1 Positive Integers

We know that 1, 2, 3, ... are natural numbers. They are also called positive integers.

Opposites



The corresponding opposite numbers are called negative integers. While 0 is an integer which is neither positive nor negative.

A number and its opposite are additive inverses of each other.

Remember

Additive Inverse

Since $0 + 0 = 0$, zero is its own additive inverse.

The following numbers are called integers
..., −3, −2, −1, 0, 1, 2, 3, ...

Example 1 Identify numbers

Describe the following numbers as natural, whole or integer. There may be more than one answer for each.

a) 1 b) 0 c) −5

Solution:

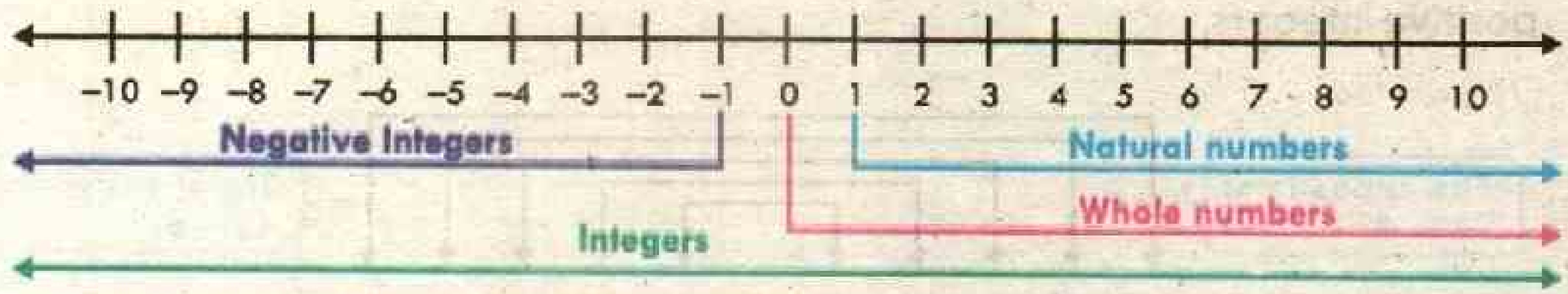
a) 1 is greater than zero so it a natural number.
All natural numbers are also whole numbers and integers.

b) 0 is not a natural number, but it is a whole number.
As it is a whole number it is also an integer.

c) −5 is less than zero so it is not a natural number and not a whole number but it is an integer.

4.1.2 Representation of Integers on number line

The number line is a useful diagram that shows how numbers are ordered. Integers can be represented by on number line as follows:

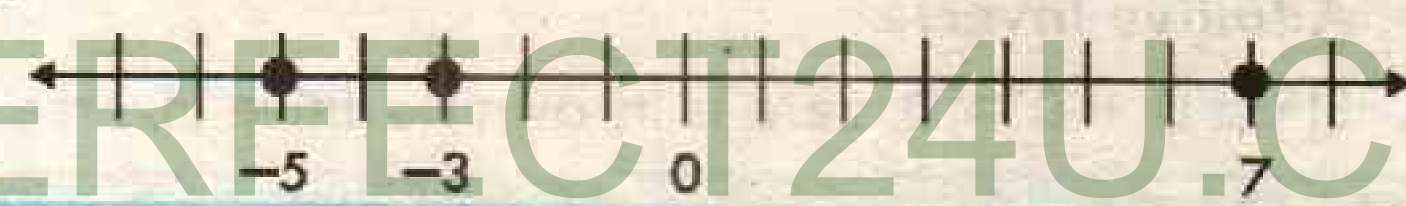


Numbers to the right of 0 are positive and numbers to the left of 0 are negative 1, 2, 3, 4,... are positive because they lie to the right of 0 and integers ... , −4, −3, − 2, −1 are negative because they lie to the left of 0.

Example 1 Arrange on number line

Arrange the numbers −3, −5, 3 and 7 on the number line.

Solution:



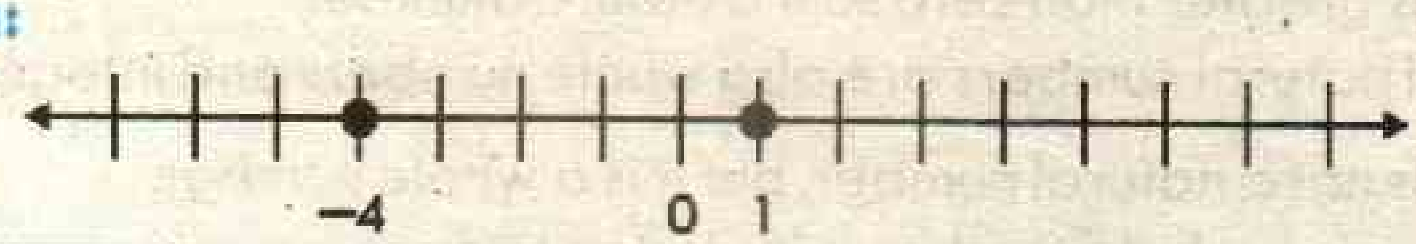
4.2 Comparing and Ordering Integers

If two numbers are plotted on the number line, then the number to the left is less than the number to the right. You could also say that the number to the right is greater than the number to the left.

Example 1 Compare on number line

By placing them on the number line, show that 1 is greater than −4.

Solution:



Practice

By placing them on the number line, show that −1 is less than 4.

4.2.1 Comparing of a Negative Integer with a Positive Integer

Since a negative integer is less than zero and a positive integer is greater than zero. So a positive integer is always greater than a negative integer.

Example 1 Compare

Compare -1 with $+1$.

Solution:

-1 is less than '0' and $+1$ is greater than "0" because -1 lies to the left and $+1$ lies to the right of '0'. So -1 is less than $+1$ and $+1$ is greater than -1. In symbol it can be written as $-1 < +1$ or $+1 > -1$ where $-1 < 0$ and $0 < +1$.

Example 2 Fill

Fill in the with < or >.

i) 0 1 ii) -45 -44 iii) 2 -22

Solution:

i) $0 < 1$ ii) $-45 < -44$ iii) $2 > -22$

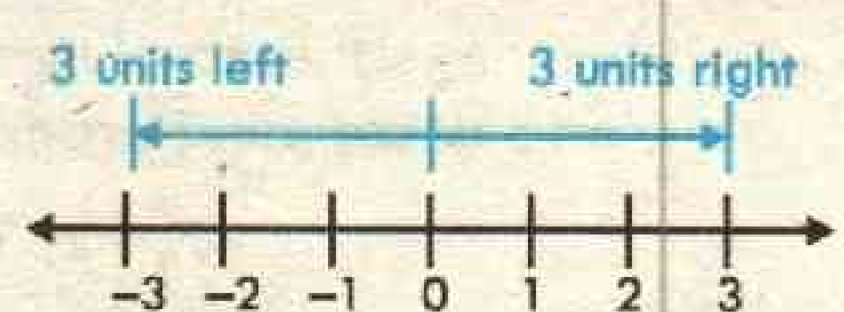
Example 3 Find the opposite

Write the opposite of each integer by using a number line.

i) $+3$ ii) -4

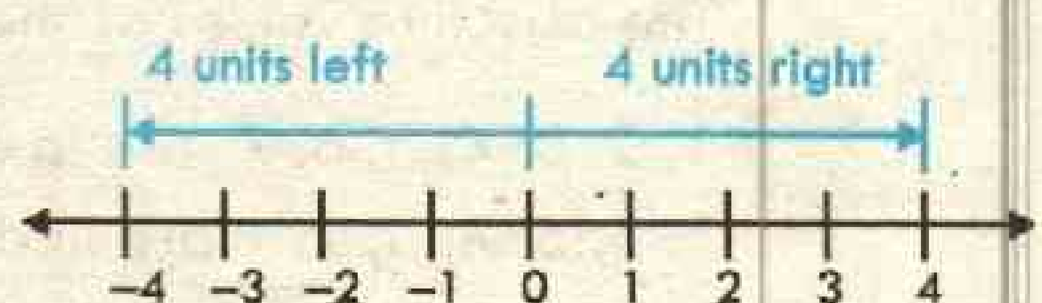
Solution:

i) $+3$



The opposite of $+3$ is -3.

ii) -4



The opposite of -4 is $+4$.

Exercise 4.1

1. Draw a number line from −9 to 9 and show the following integers on it.
   −2, −4, −6, 4, 6, 8

2. Write an integer to describe each situation.
   (i) A loss of Rs.10. (ii) A gain of Rs.50.
   (iii) 3 feet to the right. (iv) 5 feet to the left.
   (v) The temperature rise 7 degrees. (vi) 5 degrees below zero.

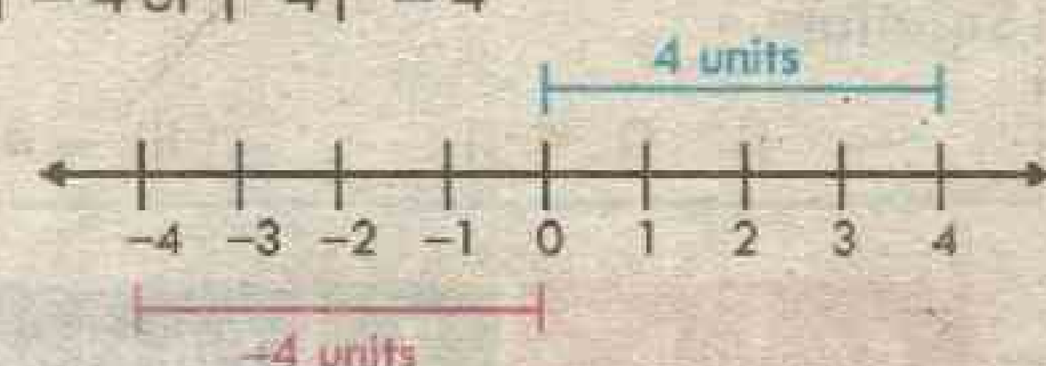
3. Write the opposite of each integer.
   (i) 32 (ii) −3 (iii) −21 (iv) 16

4.3 Absolute or numerical value of an integer

Absolute value of a number a, denoted by $|a|$ is the distance between "a" and "0" on the number line.

Since 4 is 4 units to the right of "0" and −4 is 4 units to the left of "0" both have an absolute value 4. Thus $|4| = 4$ or $|-4| = 4$

$$|4| = 4 - 0 = 4$$
$$|-4| = 0 - (-4) = 4$$



Example 1 Arrange Numbers

Arrange the absolute values of the following integers in ascending and descending order.

−7, −2, 3, −5, −4, 6, −1

Solution:

The absolute values of the numbers are 7, 2, 3, 5, 4, 6, 1.

1, 2, 3, 4, 5, 6, 7 are in ascending order.

7, 6, 5, 4, 3, 2, 1 are in descending order.

4.3.1 Display Sum of Two Integers

Example 1 Add on number line

Display the sum of +3 and −5 on a numbe line.

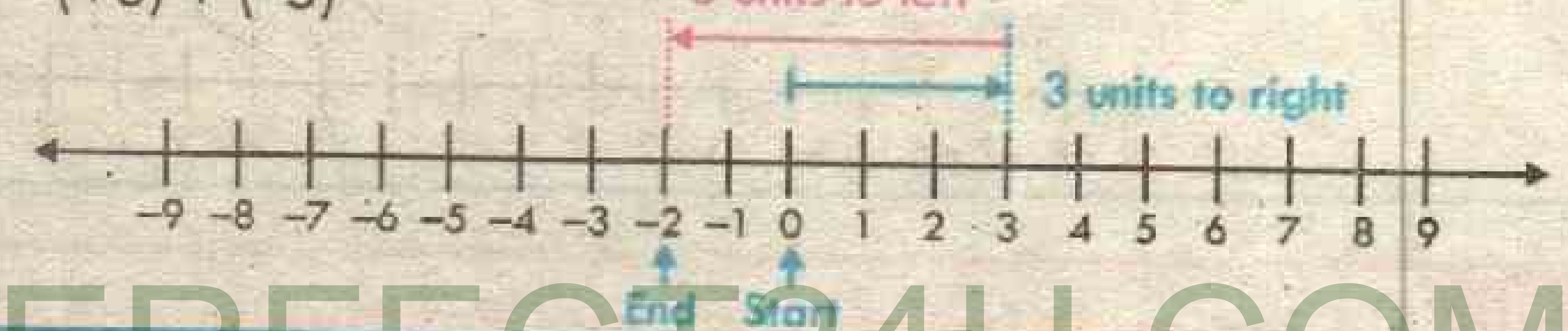
Solution:

Step 1 Consider "0" as the starting point. Move from '0' to +3 by means of an arrow in which the tip coincides on +3.

Step 2 Turn back 5 units from +3.

Step 3 The second arrow ends at −2. So the sum is −2.

$(+3) + (-5)$



Example 2 Add on number line

Display the sum of −4 and +7 on a number line.

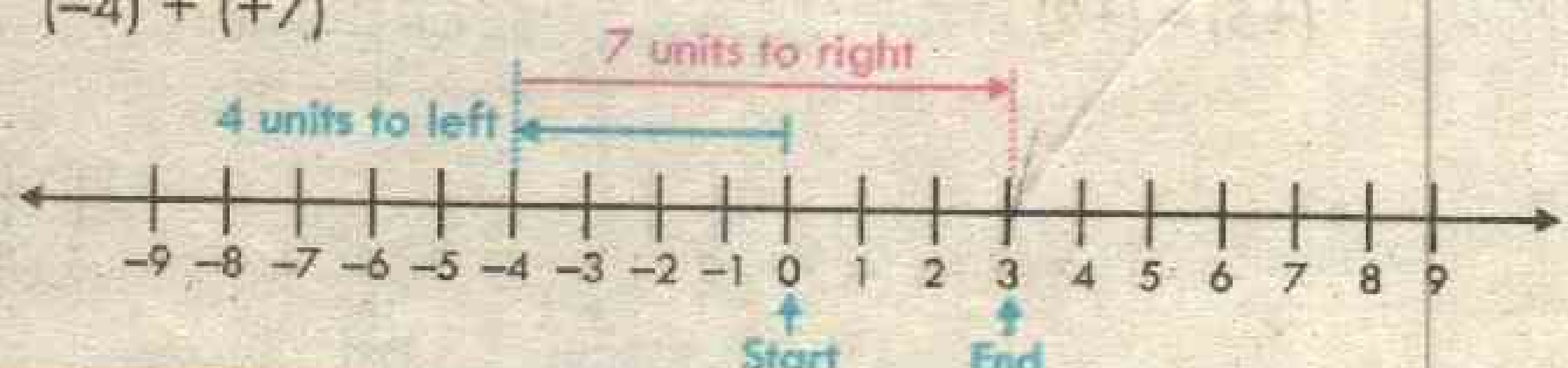
Solution:

Step 1 Consider "0" as the starting point. Move from '0' to −4 by means of an arrow in which the tip coincides on −4.

Step 2 Turn back 7 units from −4.

Step 3 The second arrow ends at +3. So the sum is +3.

$(-4) + (+7)$



4.3.2 Display Sum of Two Negative Integers

Example 1 Add on number line

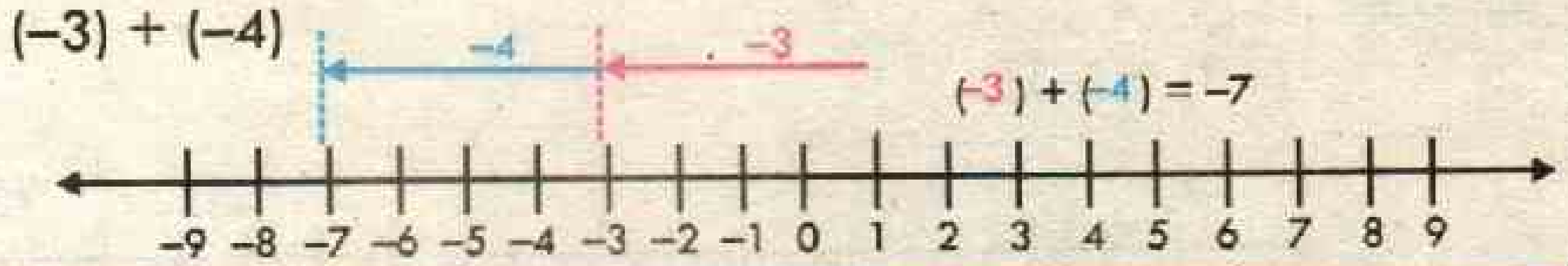
By using number line find (–3) + (–4).

Solution:

Step 1 Draw an arrow from 0 to –3.

Step 2 Then draw a second arrow 4 units to the left to represent adding –4.

Step 3 The second arrow ends at the sum –7. So, –3 +(–4) = –7



4.3.3 Display Difference of Two Positive Integers

Example 1 Subtract

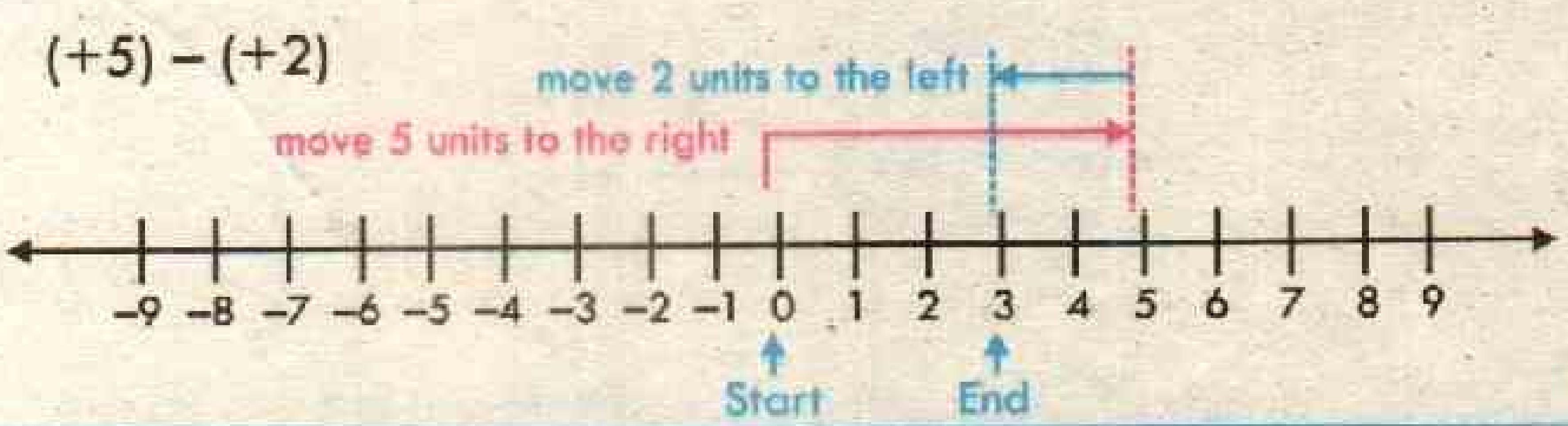
Find (+5) – (+2).

Solution:

Step 1 Draw an arrow from 0 to 5.

Step 2 Then draw a second arrow 2 units to the left to represent subtracting +2.

Step 3 The second arrow ends at the difference 3. So, (+5) – (+2)=+3.



4.3.4 Addition of two positive or two negative integers in three Steps:

Step 1 Take absolute value of given integers.

Step 2 Add the absolute values.

Step 3 Give the result with the common sign.

Example 1 Addition of two positive integers

Add +5 and +2.

Solution:

Step 1 $|+5| = 5$ and $|+2| = 2$

Step 2 $(+5) + (+2) = |+5| + |+2| = 5+2$

Step 3 $= +7$

Example 2 Addition of two negative integers

Add –4 and –3

Solution:

Step 1 $|-4| = 4$ and $|-3| = 3$

Step 2 $(-4) + (-3) = -(|-4| + |-3|)$

Step 3 $= -(4 + 3) = -7$

Practice

i) Add +8 and +3.

ii) Add –9 and –5.

4.3.5 Addition of a positive and a negative integers in three Steps:

Step 1 Take absolute value of given integers.

Step 2 Subtract the absolute values.

Step 3 Give the result the sign of the integer with the larger absolute value.

Example 1 Addition of positive & negative integers

Add –8 and +3.

Solution:

Step 1 $|-8| = 8$ and $|+3| = 3$

Step 2 $(-8) + (+3) = -(|-8| - |+3|) = -(8-3)$

Step 3 $= -5$

Exercise 4.2

1. Display the sum of the following integers on a number line.
   (i) $(+3) + (+5)$ (ii) $4 + (-6)$ (iii) $(-3) + (-5)$
2. Display the differences of the following integers on a number line.
   (i) $(+8) - (+3)$ (ii) $(+5) - (+6)$ (iii) $(+7) - (+2)$
3. By using number line, display the sum of the following integers.
   (i) $-1, -2$ (ii) $-3, -4$ (iii) $-5, -6$
4. Find each sum.
   (i) $7 + (-5)$ (ii) $(-6) + (-4)$ (iii) $5 + (-9)$
5. By three steps method, find each sum.
   (i) $(+5) + (-9)$ (ii) $(+3) + (+2)$ (iii) $(-8) + (+2)$

4.3.6 Subtraction of Integers

There are four different cases:

Example 1 Subtract

Subtract (+5) from +8.

Solution:

$(+8) - (+5) = 8 + (-5)$ (The opposite of +5 is −5)

$= 8 - 5$

$= 3$

Example 2 Subtract

Subtract (−3) from (−5).

Solution:

$(-5) - (-3) = -5 + 3$

$= -2$

Example 3 Subtract

Subtract (−4) from (+6)

Solution:

$(+6) - (-4) = 6 + 4$

$= 10$

Example 4 Subtract

Subtract (+5) from (−8).

Solution:

$-8 - (+5) = -8 + (-5)$

$= -13$

Practice

i) Subtract +7 from −9
ii) Subtract +4 from +8
iii) Subtract −6 from −2
vi) Subtract −3 from +1

4.3.7 Multiplication of Integers

Example 1 Multiply

Find the following products.

(i) $(+5) \times (+4)$ (ii) $(-2) \times (-5)$

(iii) $(+6) \times (-3)$ (iv) $(-7) \times (+4)$

Solution:

(i) $(+5) \times (+4)$

$$\begin{array}{r} +5 \\ \times +4 \\ \hline +20 \end{array}$$

(ii) $(-2) \times (-5)$

$$\begin{array}{r} -2 \\ \times -5 \\ \hline +10 \end{array}$$

Remember

Rules for multiplication

$(+) \times (+) = +$
$(+) \times (-) = -$
$(-) \times (+) = -$
$(-) \times (-) = +$

(iii) $(+6) \times (-3)$

$$\begin{array}{r} +6 \\ \times -3 \\ \hline -18 \end{array}$$

(iv) $(-7) \times (+4)$

$$\begin{array}{r} -7 \\ \times +4 \\ \hline -28 \end{array}$$

Practice

Find the following products.

(i) $(+7) \times (+2)$ (ii) $(-3) \times (-6)$

(iii) $(+5) \times (-4)$ (iv) $(-1) \times (+2)$

4.3.8 Division of Integers

Division is the inverse process of multiplication, because:

$8 \times 4 = 32$ so, $32 \div 4 = 8$

$6 \times (-2) = -12$ so, $(-12) \div (-2) = 6$

$(-5) \times (-3) = 15$ so, $(15) \div (-3) = -5$

Example 1 Division

Find the quotient in the following.

(i) $16 \div 4$ (ii) $(-8) \div 2$

(iii) $(-14) \div (-7)$ (iv) $(+20) \div (-5)$

Solution:

(i) $16 \div 4$

$$\begin{array}{r} 4 \\ 4\overline{)16} \\ \pm 16 \\ \hline 0 \end{array}$$

$\therefore$ Quotient $= 4$

Since both numbers are positive, so the quotient is also positive.

Remember

Rules for division

- $(+) \div (+) = +$
- $(-) \div (-) = +$
- $(+) \div (-) = -$
- $(-) \div (+) = -$

(ii) $(-8) \div 2$

$$\begin{array}{r} 4 \\ 2\overline{)8} \\ \pm 8 \\ \hline 0 \end{array}$$

$\therefore$ Quotient $= -4$

Since one number is positive and the other is negative, the quotient is negative.

(iii) $(-14) \div (-7)$

$$\begin{array}{r} 2 \\ 7\overline{)14} \\ \pm 14 \\ \hline 0 \end{array}$$

$\therefore$ Quotient = 2

Both number are negative, the quotient is positive.

(iv) $(+20) \div (-5)$

$$\begin{array}{r} 4 \\ 5\overline{)20} \\ \pm 20 \\ \hline 0 \end{array}$$

$\therefore$ Quotient = -4

Since the numbers have opposite signs, the quotient is negative.

Exercise 4.3

1. **Solve the following:**
   (i) $(+20) - (+10)$ (ii) $(+45) - (60)$ (iii) $(-40) - (60)$
   (iv) $(+60) - (-50)$ (v) $(-23) - (+47)$

2. **Find each product:**
   (i) $5 \times (-7)$ (ii) $(-7) \times (-7)$ (iii) $(-5) \times (-1)$
   (iv) $2 \times (3)$ (v) $-8 \times (-9)$

3. **Find each quotient:**
   (i) $45 \div (-3)$ (ii) $(63) \div (-9)$ (iii) $75 \div (-25)$
   (iv) $-81 \div (9)$ (v) $(-288) \div (36)$

Review Exercise 4

1. **Fill in the blank space in each question by putting a suitable number/word.**

   (i) -1 is negative integer

   (ii) '0' is neither positive nor negative

   (iii) -2 Greater than -3

   (iv) $|-4| =$ 4

2. **Encircle T for true statement and F for false statement in each of the following.**

   (i) $-1 < 0$ T – F

   (ii) $|-2| = -2$ T – F

   (iii) $(-1) + (-2) = -3$ T – F

   (iv) $(+2) \div (-1) = +2$ T – F

3. **Choose the correct answer.**

   (i) $(+3) \times (-2) =$ 6

   a) 6 b) -6 c) 1

   (ii) $|-4| =$

   a) 4 b) -4 c) ± 4

   (iii) $-1 \square 0$

   a) = b) < c) >

   (iv) $(+12) \div (-3) =$

   a) $+4$ b) -4 c) 9

4. **Use =, > & < sign to indicate the relationship between each of the following pairs of numbers.**

   (i) $-6 \Delta -7$ (ii) $2 \Delta -5$ (iii) $-7 \Delta |-7|$

   (iv) $-|9| \Delta -(-9)$ (v) $|-3| \Delta 3$ (vi) $0 \Delta -2$

Summary

- **Positive integers** are number greater than zero, lie to the right of zero on number line.
- **Negative integers** numbers which are less than zero, lie to the left of zero an a number line.
- **Set of integers is** $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
- **The absolute value** of a number is the distance between zero and the number on a number line. For any number x, $|\pm x| = x$
- **To add two positive integers,** add their absolute values.

- **To add two negative integers,** add their absolute values and put a negative sign in front of the result.
- **To subtract two numbers** add the opposite of the number which is to be subtracted to the other number.
- **To multiply integers:** The product of two negative or positive integers is positive, while the product of two opposite integers is negative.
- **To divide two integers:** Divide the absolute values of one number by the other and place a negative sign in front of the quotient if the two integers are opposite, otherwise no sign is placed if the integers have the same signs.

UNIT 05 SIMPLIFICATIONS

What You'll Learn

- Know that the following four kinds of brackets
  - — vinculum,
  - ( ) parentheses or curved brackets or round brackets,
  - { } braces or curly brackets,
  - [ ] square brackets or box brackets,

  are used to group two or more numbers together with operations.
- Know the order of preference as, — , ( ), { } and [ ], to remove (simplify) them from an expression.
- Recognize BODMAS rule to follow the order in which the operations, to simplify mathematical expressions, are performed.
- Simplify mathematical expressions involving fractions and decimals grouped with brackets using BODMAS rule.
- Solve real life problems involving fractions and decimals.

Why It's important

Whenever a problem can be simplified, you should simplify it before doing other calculations. This will make your job a lot easier. The Order of Operations is very important when simplifying expressions and equations.

5.1 Brackets or Grouping Symbols

5.1.1 Brackets

Bracket are used to group two or more numbers together with other operations.

1. Vinculum or Bar ‘____’
2. Parenthesis or Curved Brackets or Circled Brackets ‘( )’
3. Braces or Curly Brackets ‘{ }’
4. Square Brackets or Box Brackets ‘[ ]’

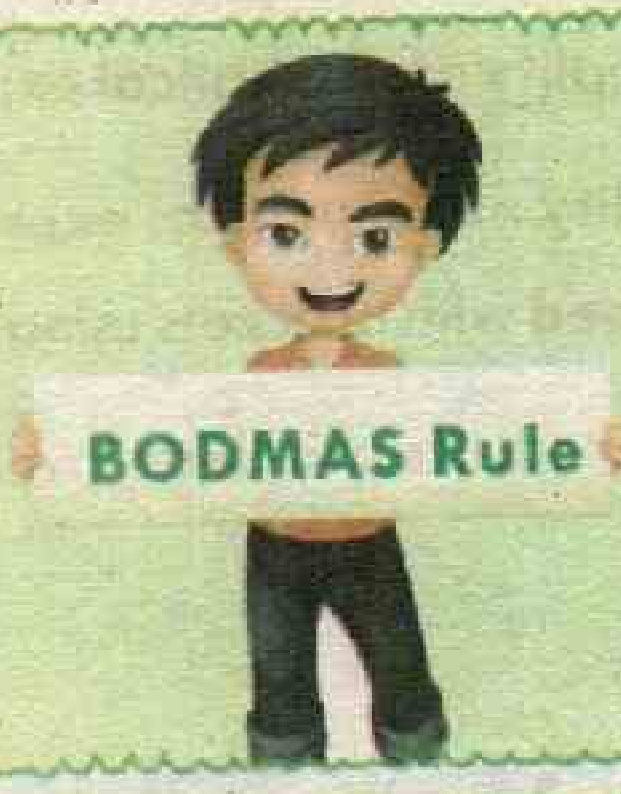
5.1.2 Order of Brackets

If more than one bracket is used in one question, then brackets will be opened in the above mentioned order that is ____ , ( ), { }, [ ].

5.1.3 BODMAS Rule

The word BODMAS is used for the order of fundamental operations.

Brackets,
Order,
Division,
Multiplication,
Addition,
Subtraction



Step 1 Evaluate expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

Example 1 Fraction Bar

Evaluate $\dfrac{6+4^2}{3^2\times 4}$

Solution:

$\dfrac{6+4^2}{3^2\times 4}$ means $(6+4^2)\div(3^2\times 4)$

$$\frac{6+4^2}{3^2\times 4} = \frac{6+16}{9\times 4}$$ (Evaluate the power in the numerator and denominator)

$$= \frac{22}{36}$$ (Add 6 and 16 in th numerator and multiply 9 and 4 in denominator)

$$= \frac{11}{18}$$ (Then simplify)

Example 2 Simplify

Simplify $6-2+[8+\{2\times 3+(4-\overline{5+2})\}]$

Solution:

$6-2+[8+\{2\times 3+(4-\overline{5+2})\}]$

$=6-2+[8+\{2\times 3+(4-7)\}]$ (Vinculum solved)

$=6-2+[8+\{2\times 3+(-3)\}]$ (Operation within parenthesis)

$=6-2+[8+\{2\times 3-3\}]$ (Parenthesis solved)

$=6-2+[8+\{6-3\}]$ (Operation within braces)

$=6-2+[8+3]$ (Braces solved)

$=6-2+11$ (Square bracket solved)

$=6+11-2$

$=17-2$

$=15$

5.1.4 Use of Brackets in Common Fractions

Example 1 Simplify

Simplify $\frac{1}{4}+\left(\frac{7}{8}-\frac{3}{4}\right)$

Solution:

$$\frac{1}{4}+\left(\frac{7}{8}-\frac{3}{4}\right)=\frac{1}{4}+\left(\frac{7-6}{8}\right)$$

$$=\frac{1}{4}+\left(\frac{1}{8}\right)=\frac{1}{4}+\frac{1}{8}$$

$$=\frac{2+1}{8}=\frac{3}{8}$$

Example 2 Simplify

Simplify $\frac{3}{2}\times\left(\frac{2}{3}-\overline{\frac{1}{2}+\frac{1}{3}}\right)$

Solution:

$$\frac{3}{2}\times\left(\frac{2}{3}-\overline{\frac{1}{2}+\frac{1}{3}}\right)$$

$$=\frac{3}{2}\times\left(\frac{2}{3}-\overline{\frac{3+2}{6}}\right)=\frac{3}{2}\times\left(\frac{2}{3}-\frac{5}{6}\right)$$

$$=\frac{3}{2}\times\left(\frac{4-5}{6}\right)=\frac{3}{2}\times\left(\frac{-1}{6}\right)$$

$$=-\frac{3\times1}{2\times6}=-\frac{1\times1}{2\times2}=-\frac{1}{4}$$

Practice

Simplify $4\frac{1}{2}-\left[5\frac{2}{3}-\left\{2\frac{1}{6}-\left(\frac{1}{3}-\overline{\frac{17}{18}\div\frac{34}{9}}\right)\right\}\right]$

Example 3 Simplify

Simplify $1\frac{3}{4} \div \left[9\frac{5}{8} \div \left\{\frac{2}{5} + \left(8\frac{2}{5} \times \overline{4\frac{1}{6} - 3\frac{1}{2}}\right)\right\}\right]$

Solution:

$$1\frac{3}{4} \div \left[9\frac{5}{8} \div \left\{\frac{2}{5} + \left(8\frac{2}{5} \times \overline{4\frac{1}{6} - 3\frac{1}{2}}\right)\right\}\right]$$

(Change compound fractions into improper fractions)

$$= \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2}{5} + \left(\frac{42}{5} \times \overline{\frac{25}{6} - \frac{7}{2}}\right)\right\}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2}{5} + \left(\frac{42}{5} \times \overline{\frac{25 \times 1 - 3 \times 7}{6}}\right)\right\}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2}{5} + \left(\frac{42}{5} \times \overline{\frac{25 - 21}{6}}\right)\right\}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2}{5} + \left(\frac{42}{5} \times \frac{4}{6}\right)\right\}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2}{5} + \left(\frac{7 \times 4}{5 \times 1}\right)\right\}\right] \quad = \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2}{5} + \frac{28}{5}\right\}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{8} \div \left\{\frac{2 + 28}{5}\right\}\right] \quad = \frac{7}{4} \div \left[\frac{77}{8} \div \frac{30}{5}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{8} \div 6\right] = \frac{7}{4} \div \left[\frac{77}{8} \times \frac{1}{6}\right]$$

$$= \frac{7}{4} \div \left[\frac{77}{48}\right] = \frac{7}{4} \div \frac{77}{48} = \frac{7}{4} \times \frac{48}{77} = \frac{12}{11} = 1\frac{1}{11}$$

Practice

Simplify $5\frac{1}{2} \div \left[\frac{3}{2} \div \left\{\left(\frac{1}{4} - \frac{1}{8}\right) \times \frac{3}{4}\right\}\right]$

Exercise 5.1

Simplify the following equations.

1. $8-\left(3-\overline{5+2}\right)$
2. $10-\left[8-\left\{5-\left(2-\overline{3+6}\right)\right\}\right]$
3. $30-2\left[5-\left\{15\div 3\times 4-\left(3-\overline{5-3}\right)\right\}\right]$
4. $\frac{5}{2}+\left(\frac{2}{3}-\frac{5}{6}\right)$
5. $3\frac{5}{8}-\left\{\frac{3}{8}-\left(\frac{3}{4}+\frac{5}{8}\right)\right\}$
6. $\frac{3}{2}+\left\{\frac{1}{2}+\left(\frac{3}{4}\div 2\frac{1}{4}\right)\right\}$
7. $2\frac{1}{4}-\left[\frac{3}{2}+\left\{\left(\frac{5}{2}-\frac{1}{4}\right)\times\frac{4}{3}\right\}\right]$
8. $2\frac{1}{3}\div\left[2\frac{5}{6}+\left\{1\frac{1}{2}+\left(3\frac{1}{2}\times\overline{2\frac{2}{3}-1\frac{1}{6}}\right)\right\}\right]$

Activity

Fill in the blanks.

B $\longrightarrow$ $(3+2)^2+\frac{40}{2}-3\times 2$

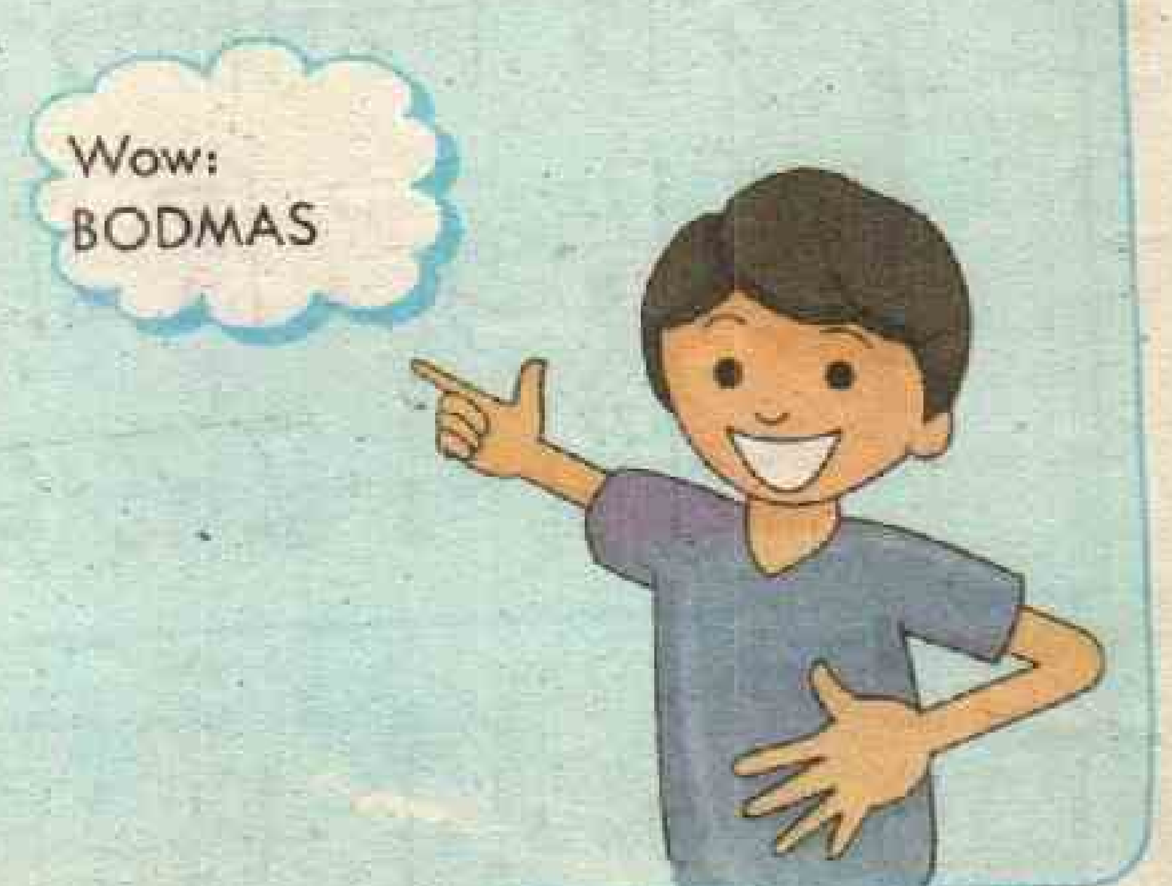
O $\longrightarrow$ $\square+\frac{40}{2}-3\times 2$

D $\longrightarrow$ $25+\square-3\times 2$

M $\longrightarrow$ $25+20-\square$

A $\longrightarrow$ $25+20-6$

S $\longrightarrow$ $45-6=39$



5.2 Solution of Expressions involving Decimal Fractions and Brackets

Decimal fractions with brackets can also be simplified by using BODMAS rule. These are solved in the same order as that of common fraction.

Example 1 Simplify

Simplify $1.5+\{0.7+(1.4-0.5)\}$

Solution:

$$1.5+\{0.7+(1.4-0.5)\}$$
$$=1.5+\{0.7+0.9\}$$
$$=1.5+1.6 \quad =3.1$$

Example 2 Simplify

Simplify $2.04+\left[1.56\div\left\{2.4-\left(1.8\times\overline{0.3+0.6}\right)\right\}\right]$

Solution:

$$2.04+\left[1.56\div\left\{2.4-\left(1.8\times\overline{0.3+0.6}\right)\right\}\right]$$
$$=2.04+\left[1.56\div\left\{2.4-(1.8\times0.9)\right\}\right]$$
$$=2.04+\left[1.56\div\{2.4-1.62\}\right]$$
$$=2.04+\left[1.56\div0.78\right]$$
$$=2.04+2 \quad =4.04$$

4.04

Exercise 5.2

Simplify the following equations.

1. $1.4+(1.6-0.3)$
2. $4.26+\left[1.35\div\left\{(0.5-0.3)\times1.5\right\}\right]$
3. $1.9\times\left[2.3\times\left\{3.6-(1.2+1.4)\right\}\right]$
4. $9.25\div\left[1.75+\left\{2.05-\left(1.5+\overline{2.5-2}\right)\right\}\right]$
5. $2.9-\left[1.05\times\left\{1.17+\left(2.5+\overline{1.9-0.4}\right)\right\}\right]$

5.3 Word Problems involving Common Fractions and Decimals

In our daily life we face many word problems. To solve these problems we must know:

- Which information is given?
- Which information is required?

Example 1 Subtract

Hamza has Rs.500. He gave one-fifth to Adil and $\frac{1}{2}$ of the remaining to Bilal. How much amount is left with him?

Solution:

Total amount that Hamza has $= \text{Rs.}500$

Amount given to Adil $= \frac{1}{5}$ of $500 = \frac{1}{5} \times 500 = 100$

Remaining amount $= 500 - 100 = \text{Rs.}400$

Amount given to Bilal $= \frac{1}{2}$ of $400 = \frac{1}{2} \times 400 = \text{Rs. } 200$

Remaining amount $= 400 - 200 = \text{Rs.}200$

Example 2 Divide

Amina purchased ribbon in Rs.96.50 at the rate of Rs.12.50 per metre. How much did she purchase the ribbon?

Solution:

Price of one metre $= \text{Rs.}12.50$

Total amount $= \text{Rs.}96.50$

Ribbon that she purchased $= 96.50 \div 12.50$

$= \frac{96.50}{12.50}$ metres

$= 7.72$metres

Exercise 5.3

1. A shopkeeper sold $5\frac{1}{2}$ kg sugar, $4\frac{1}{4}$ kg maize and $10\frac{3}{4}$ kg flour to three customers. How much did he sell in all?

2. A room is $5\frac{1}{2}$ metre long and $3\frac{1}{4}$ metre wide. How much square metre carpet is required for its floor?

3. Nasir's total income is Rs. 10, 000. He paid $\frac{1}{10}$ of his income as house rent and $\frac{1}{4}$ of the remaining on other necessities. What amount has been left with him?

4. Tasneem has 40 toffees. She gave $\frac{3}{5}$ of it to Ghazala. Ghazala gave $\frac{1}{6}$ of her own share to her younger brother. How much toffees Ghazala's brother got?

5. How much time Waseem will take to cover 50.5 kilometres distance with a speed of 12.25 kilometers per hour?

6. Shahab travelled 4.7 km distance on foot, 15.5 km distance by bicycle. How much distance traveled altogether? Also tell how much more distance he traveled by bicycle than on foot?

7. The price of one dozen eggs is Rs.60.50. Find the price of one egg? Ans $\boxed{6}$

8. A bamboo is 24 metres long. its 0.75 part is above the ground and remaining part is inside the ground. How many metre s bamboo inside the ground?

9. The total population of a village is 6000. Out of this 0.05 of the population is in service, 0.4 of it is in trade and 0.06 of it are labours. The rest of population is jobless. Tell how many are jobless?

$\frac{5+8}{10}$ $\frac{5+5}{10}$ $\frac{5+5}{10}$ $\frac{5+5}{10}$ $\frac{5+5}{10}$ $\frac{5+5}{10}$

Review Exercise 5

1. Choose the correct answer.

(i) ( ) is

a) Vinculum b) Parentheses
c) Braces d) Square brackets.

(ii) Word 'S' used in BODMAS is abbreviation of

a) Solution b) Subtraction
c) Square d) None

(iii) Order of brackets is

a. Braces, square bracket, vinculum and parenthesis
b. Vinculum, braces, parenthesis and square bracket.
c. Vinculum, parenthesis, braces and square brackets.
d. Parenthesis, vinculum, braces and square brackets.

(iv) 5.12 – 5.02 is equal to

a) 10.14 b) 0.14
c) 0.1 d) 0.01

(v) Brackets are of

a) 2 types b) 3 types
c) 4 types d) 5 types

2. Simplify the following.

(i) $\frac{1}{2}+\left\{\frac{5}{14}\times\left(\frac{25}{15}-\frac{21}{25}\right)\right\}$

(ii) $1.02+\left\{3.9\times\left(4.05-\overline{2.02-0.98}\right)\right\}$

(iii) Sum of two fractions is $10\frac{1}{2}$. If one fraction is $3\frac{6}{8}$, find the other.

(iv) What will be the fare for 30075 kilometre at the rate of Rs.2.25 per kilometre?

(v) In Peshawar the temperature on one day of June was 45.6°, on second day 43.3°, on the third day 40.5° and the fourth day it was 44.7°. Find the total temperature of four days.

Summary

- To simplify mathematical expressions there are four types of brackets
- Order of brackets is

  i. Vinculum ii. Parenthesis

  iii. Braces iv. Square brackets.
- Problems containing brackets and fundamental operations can be simplified according to BODMAS rule.
- Word BODMAS is abbreviation of

  B for Bracket

  O for Order

  D for Division

  M for Multiplication

  A for Addition

  S for Subtraction
- BODMAS rule is used in common fractions as well as in decimal fractions.

Activity

Make some nice charts showing BODMAS rule and put them on the wall of your class at some suitable place.

UNIT 06 RATIO AND PROPORTION

What You'll Learn

- Define ratio as a relation which one quantity bears to another quantity of the same kind with regard to their magnitudes.
- Know that of the two quantities forming a ratio, the first one is called antecedent and the second one consequent.
- Know that a ratio has no units.
- Calculate ratio of two numbers.
- Reduce given ratio into lowest (equivalent) form.
- Describe the relationship between ratio and fraction.
- Know that an equality of two ratios constitutes a proportion, e.g., $a:b::c:d$, where a, d are known as extremes and b, c are called the means.
- Find proportion (direct and inverse).
- Solve real life problems involving direct and inverse proportion.

Why It's important

Ratio and proportion are very important in problems of daily life. Their use is very frequent in banking, in inheritance of ancestral property, in business sharing, in land measuring, in mapping, in models of construction work, in distribution of prizes, in marks of the examination and what not.

Therefore special emphasis has been laid on learning of ratios and proportion to solve daily life problems.

How are ratios used in paint mixtures?

If a gallon of paint contains 2 parts blue paint and 4 parts yellow paint, then the ratio comparing the blue paint to the yellow paint can be written as follows.

2 to 4   2:4 $\frac{2}{4}$

6.1 Ratio

6.1.1 Definition of Ratio

A relation which one quantity bears to another quantity of the same kind with regard to their magnitude is called a **ratio.** Symbolically ratio between two same quantities a and b is written as $a : b$ or $\frac{a}{b}$. The symbol **':' means ratio.**

In the ratio $a : b$ (or $\frac{a}{b}$), a is called **antecedent** and b is called **consequent.**

Example 1 Write Ratios as Fractions

Express the ratio 9 goldfish out of 15 fish as a fraction in simplest form.

Solution:

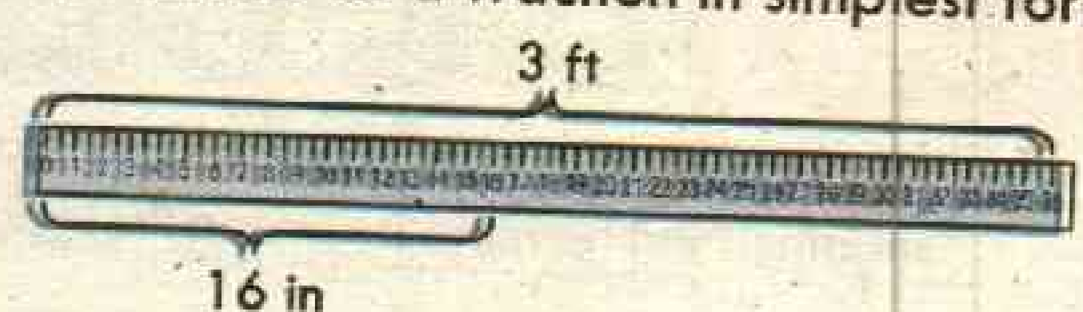
$$\overset{\div 3}{\underset{\div 3}{\frac{9}{15} = \frac{3}{5}}}$$ (Divide the numerator and denominator by the GCF, 3)

The ratio of goldfish to fish is 3 to 5. This means that for every 5 fish, 3 of them are goldfish.

Example 2 Write Ratios as Fractions

Express the ratio 3 feet to 16 inches as a fraction in simplest form.

Solution:



$$\frac{3 \text{ feet}}{16 \text{ inches}} = \frac{36 \text{ inches}}{16 \text{ inches}}$$ (Convert 3 feet to inches)

$$= \frac{9}{4}$$ (Divide the numerator and denominator by the HCF, 4)

Simplest form, the ratio is 9 to 4.

Example 3 Find ratio between distances

Find the ratio between 2 kilometers 5 hectometers and 7 kilometers.

> A ratio has no units, because it only tells the comparison.

Solution:

Since the measurements are not in the same unit, we change both the measurements to same unit hectometer.

Now 2 kilometers 5 hectometers = (2 × 10 + 5) hectometers
= 20 + 5
= 25 hectometers

7 kilometers = (7 × 10) hectometers
= 70 hectometers\ the required ratio
= 25 : 70
= 5 : 14

Example 4 Find ratio between rupees

Find the ratio between Rs.225 and Rs.150.

Solution:

225 : 150

= 45 : 30 (Dividing antecedent and consequent by 5)
= 3 : 2 (Dividing antecedent and consequent by 15)

Example 5 Find ratio between ages

The age of Ashraf is 15 years, 10 months and age of Akmal in 17 years, 5 months. Find he ratio between their ages.

Solution:

We change the age of both of them to months.

Age of Ashraf = 15 year, 10 months = 15 ×12 + 10 = 190 months.

Age of Akmal = 17 years, 5 months = 17 × 12 + 5 = 209 months.

Ratio between the age of Ashraf and of Akmal

= 190 : 209
= 10 : 11 (Dividing antecedent and consequent by 19)

6.1.2 Reduction of ratio to its lowest form

A ratio is unchanged where its antecedent and consequent are multiplied or divided by the same number.

For example:

$$\frac{2}{5} = \frac{4}{10} = \frac{8}{20} = \frac{12}{30} \text{ etc.}$$

or $\quad \frac{45}{30} = \frac{15}{10} = \frac{3}{2}$, or $3 : 5 = 6 : 10 = 18 : 30$ etc.

We also say that ratios $\frac{12}{5}, \frac{2}{10}, \frac{4}{30}$ are equivalent ratios.

Likewise $3 : 5$, $6 : 10$, $18 : 30$ are equivalent ratios.

In the above examples $\frac{2}{5}, \frac{3}{2}$ and $3 : 5$ are in lowest form respectively.

Example 1 Reduce

Reduce the ratio $\frac{420}{350}$ to its lowest form.

Solution:

$\frac{420}{350} = \frac{42}{35}$ (Dividing antecedent and consequent by 10)

$= \frac{6}{5}$ (Dividing antecedent and consequent by 7)

Hence $\frac{6}{5}$ is in the lowest form.

Practice

Reduce the ratio $\frac{675}{225}$ to its lowest f orm.

Exercise 6.1

1. Find the ratio between the following quantities and reduce them to the lowest form:
   - (i) Rs.450 and Rs.525
   - (ii) 364 cows and 455 cows
   - (iii) 594kg and 990kg.
   - (iv) 1 kilometer, 8 meters and 672 meters.
   - (v) 2 hours, 8 minute and 3 hours, 12 minutes.

2. Convert the ratios to their lowest form.
   - (i) 25 : 35 (ii) 28 : 84 (iii) 40 : 24
   - (iv) 200 : 150 (v) 0.45 : 0.75

3. The age of Uzair is 10 years, 8 months and the age of Usman is 13 years, 4 months. Find the ratio between their ages in the lowest form.

4. Weekly income of a labour is Rs.1500, and of a carpenter Rs.2000 per week. Find the ratio, between their weekly incomes in the lowest form.

5. Saeeda earns Rs.85,000 and spends Rs.51,000 in a year. Find the ratio of her income to expenditure.

6. A car covered 360 km in 4 hours, and another car covered 240 km in 4 hours. Find the ratio between the distances covered by both the cars.

7. In a 100 meter race Zahid Covered it in 5.8 minutes, while Iqbal covered it in 4.8 minutes. Find the ratio between their timing.

8. In an examination 112 students appeared. Out of them 16 failed. Find the ratio between the passed and failed students.

6.2 Proportion

Equality of two ratios is called a proportion.
A proportion is an equation stating that two ratios are equal.
$a : b$ $2 : 3 = 6 : 9$

Generally,

$a : b = c : d$ is written as $a : b :: c : d$

and it is read as a ratio b is the same as c ratio d," while the symbol '::' stands for proportion.

The numbers a, b and c, d are called terms of the proportions; a is the 1st term, b is 2nd term, c is the 3rd term and d is the 4th term. The 1st and 4th terms are called extremes and 2nd and 3rd terms are called the means.

Extremes

$a : b :: c : d$

Means

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times d = c \times b$$

Thus product of extremes = Product of means.

Example 1 Identify proportions

Is the given proportion true?

$$\frac{13}{7} = \frac{39}{21}$$

Solution:

The proportion $\frac{13}{7} = \frac{39}{21}$ can be written as $13 : 7 :: 39 : 21$.

Product of the extremes $= 13 \times 21 = 273$

Product of the means $= 7 \times 39 = 273$

As the product of extremes = the product of the means, hence the given proportion is true.

Example 2 Find unknown value

Find the value of x such that the following proportion is true.

$x : 12 :: 3 : 4$

Solution:

$x : 12 :: 3 : 4$

Product of extremes $= 4x$

Product of means $= 12 \times 3 = 36$

As the proportion is true, so

$4x = 36$

$x = \frac{36}{4} = 9$

Example 3 Find unknown value

If 25kg of rice costs Rs.250, find the cost of 45kg rice.

Solution:

Let the cost of 45kg be Rs. x.

As the cost of 45kg will be greater than Rs.250.

Therefore the proportion is

$25 : 45 :: 250 : x$

Product of extremes $= 25x$

Product of means $= 45 \times 250$

$\therefore 25x = 45 \times 250$

$x = \frac{45 \times 250}{25} =$ Rs.450

Practice

If 30kg of suger costs Rs.1250, find the cost of 35kg suger.

6.3 Direct Proportion

Two quantities are directly proportional to each other if one quantity increases the other quantity also increases in the same ratio. Similarly if one quantity decreases the other quantity also decreases in the same ratio.

Example 1 Identify direct proportions

Test whether the following relations are in direct proportion:

(i) $y = 2x$ (ii) $y = x$

Solution:

(i) $y = 2x$

We check the proportionality of the given relation by giving different values to x and finding the corresponding values of y.

(a).

x	1	2	3	4	...
$y = 2x$	2	4	6	8	...

From the above we see that as x increases y also increases.

(b).

x	8	7	6	5	...
$y = 2x$	16	14	12	10	...

From the above table we see that as x decreases y also decreases. Hence y and x both increase or decrease together keeping the same ratio. Therefore y and x are directly proportional to each other. Here 2 is the constant of proportionality.

(ii) $y = x$.

We check the proportionality of the relation through the following table:

x	1	2	3	4	...	8	7	6	5	...
y	1	2	3	4	...	8	7	6	5	...

From the above table we see that y and x increases and decreases together with the same ratio. Hence y is directly proportional to x. Here the constant of proportionality is 1.

6.4 Inverse Proportion

Two quantities y and x are said to be inversely proportional if x increases y decreases and if x decreases y increases in the same ratio.

We represent the inverse proportion as $y \propto \frac{1}{x}$

It follows that $y : \frac{1}{x} = k : 1$, where k is called the constant of proportionality.

Example 1 Identify Proportions

Show that the x and y are inversely proportional.

$$y = \frac{3}{x}$$

Solution:

We construct the following table by giving values to x.

x	2	3	4	7	6	5	...
$y = \frac{3}{x}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{7}$	$\frac{3}{6}$	$\frac{3}{5}$	...

From the above table:

We see that as x increases, y decreases and as x decreases, y increases.

$\therefore$ y is inversely proportional to x, where 3 is the constant of proportionality.

Example 2 Find price

If the price of 5 books is Rs.102.50, find the price of 12 such books.

Solution:

Let the price of 12 books be x, which will be more than Rs.102.50

Then $12 : 5 = x : 102.50$

$$12 \times 102.50 = 5x$$

$$x = \frac{12 \times 102.50}{5}$$

$$\therefore \quad x = 246$$

Exercise 6.2

1. Which of the following proportions are true?

   (i) $\frac{5}{3} = \frac{20}{12}$ (ii) $\frac{11}{13} = \frac{55}{65}$ (iii) $\frac{11}{13} = \frac{44}{65}$

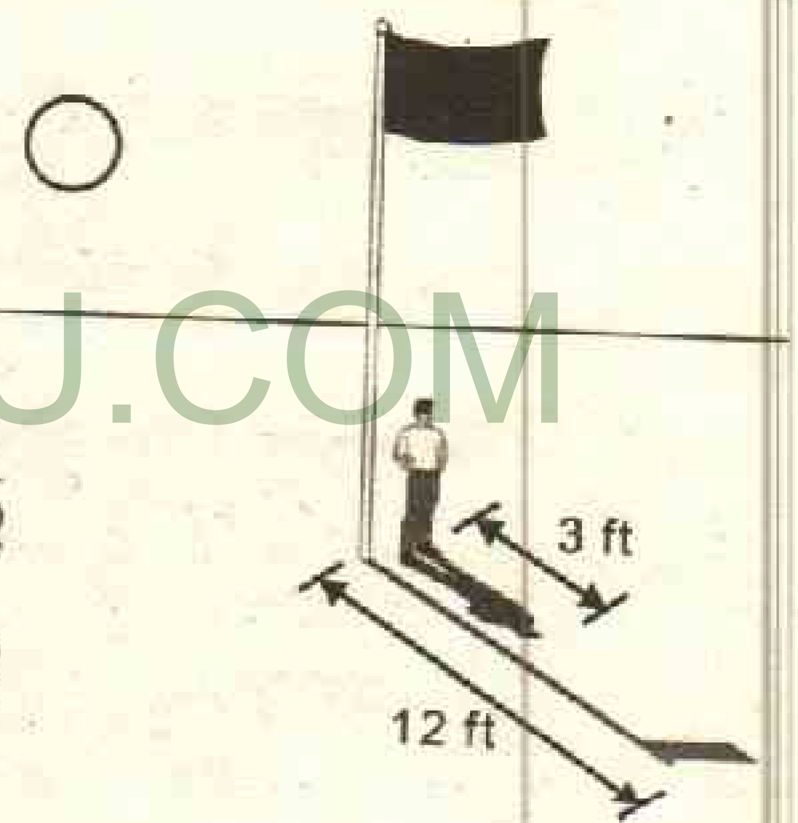
   (iv) 12 : 6 : : 24 : 12 (v) 6 : 2 : : 18 : 4

2. Find the value of $\square$ in the following proportions:

   (i) 5 : 3 : : $\square$: 12 (ii) $\square$: 13 : : 55 : 65

   (iii) 20 : 9 : : 40 : $\square$ (iv) 22 : $\square$: : 44 : 52

3. Anwar is standing next to a flagpole as shown at the right. Anwar is 6 feet tall. Which proportion could you use to find the height of the flagpole?



   (i) $\frac{3}{6} = \frac{x}{12}$ (ii) $\frac{x}{6} = \frac{3}{12}$

   (iii) $\frac{6}{3} = \frac{x}{12}$ (iv) $\frac{3}{x} = \frac{12}{6}$

4. Test which of the following relations are in direct proportion and which are inversely proportional?

   (i) $y = 3x$ (ii) $y = \frac{2}{3x}$ (iii) $y = 4x$ (iv) $y = \frac{7}{x}$

5. Price of 5 chairs is Rs.400. Find the price of 15 chairs.

6. Price of one dozen of bananas is Rs.30. Find the price of 100 bananas.

7. A shopkeeper earned a profit of Rs.27 on selling goods for Rs.360. Find the profit on selling goods for Rs.100.

8. Majida paid Zakat of Rs.25 on Rs.1000. How much Zakat Farida will pay on Rs.8000.

Review Exercise 6

1. Fill in the blanks.

i. The numbers 27 and 9 are in the ratio ____________

ii. The number 2.7 and 0.9 are in the ratio ____________

iii. The numbers $\frac{3}{2}$ and $\frac{7}{2}$ are in the ratio of ____________

iv. In 3 : 5, antecedent = ____________ and consequent = ____________

v. In 13.5 : 17.2, antecedent = ____________ and consequent = ____________

vi. The lowest form of $\frac{9}{12}$ = ____________

vii. The lowest form of $\frac{4.5}{1.5}$ = ____________

viii. The lowest form of 65 : 39 = ____________

ix. In 1 : 2 : : 2 : 4, extremes are ____________ and means are ____________

x. In 3.2 : 5 : : 6.4 : 10, extremes are ____________ and means are ____________

xi. $y = \frac{1}{2}x$ is in ____________ proportion

xii. $y = 7x$ is in ____________ proportion

2. The perimeter of a farm is 187.2 m. If its width is 39m, then find the ratio between its length and width.

3. Sum of the heights of Bushra and Sajida is 3.3m. If the height of Sajida is 1.5 m, find the ratio between their heights.

4. Price of 3 kg of apples is Rs.135. Find the price of 11kg of apples.

5. In a house of 15 persons food was sufficient for 30 days. 5 persons left the house. For how many days the food would be sufficient?

Summary

- **Ratio:** A relation which one quantity bears to another quantity of the same kind with regard to their magnitude is called ratio.
- **Ratio in the lowest form:** A ratio $a : b$ is said to be in the lowest form if a and b have no common factors.
- **Relationship between fraction and ratio:** A fraction $\frac{a}{b}$ can also be written as $a : b$.
- **Proportion:** Equality of two ratios is called a proportion.
- **Extremes of a proportion:** In $a : b : : c : d$, a and d are called extremes.
- **Means of a proportion:** In $a : b : : c : d$, b and c are means.

- **Property in a proportion:** If $a : b : : c : d$, then $a \times d = b \times c$
- **(Product of extremes = Product of means)**
- **Direct proportion:** Two quantities are directly proportional to each other if one quantity increases the other quantity also increases in the same ratio. Similarly if one quantity decreases the other quantity also decreases in the same ratio.
- **Inverse proportion:** Two quantities x and y are said to be inversely proportional if x increase y decreases and if x decrease y increases.

UNIT 07 FINANCIAL ARITHMETIC

What You'll Learn

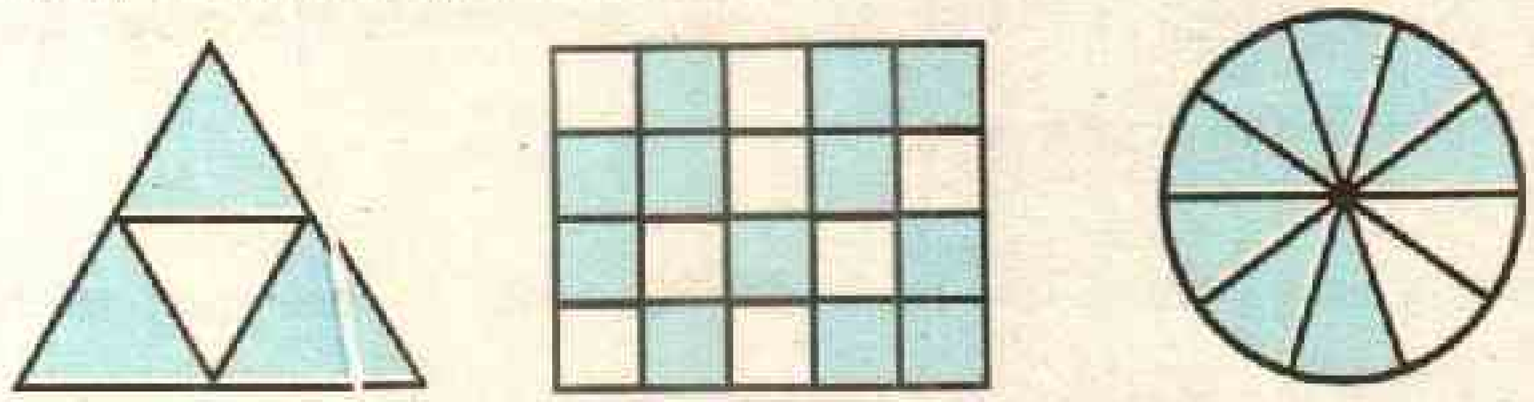
- Recognize percentage as a fraction with denominator of 100.
- Convert a percentage to a fraction by expressing it as a fraction with denominator 100 and then simplify.
- Convert a fraction to a percentage by multiplying it with 100%.
- Convert a percentage to a decimal by expressing it as a fraction with denominator 100 and then as a decimal.
- Convert a decimal to a percentage by expressing it as a fraction with denominator 100 then as a percentage.
- Solve real life problems involving percentage.
- Define
  - selling price and cost price,
  - profit, loss and discount,
  - profit percentage and loss percentage.
- Solve real life problems involving profit, loss and discount.

Why It's important

The word percent is commonly used in our daily life. It is used to refer to such things as taxes, discounts, commissions, tips, examination results (grading), profit, loss and many other fields of our real life.

How are percent related to fractions and decimals?

A portion of each figure is shaded.



75% or 0.75 or 3/4 portion of the triangle is shaded.
40% or 0.40 or 2/5 portion of the rectangle is shaded.
80% or 0.80 or 4/5 portion of the circle is shaded.

7.1 Percentage

Percent means "out of hundred". The symbol '%' is used for percent.

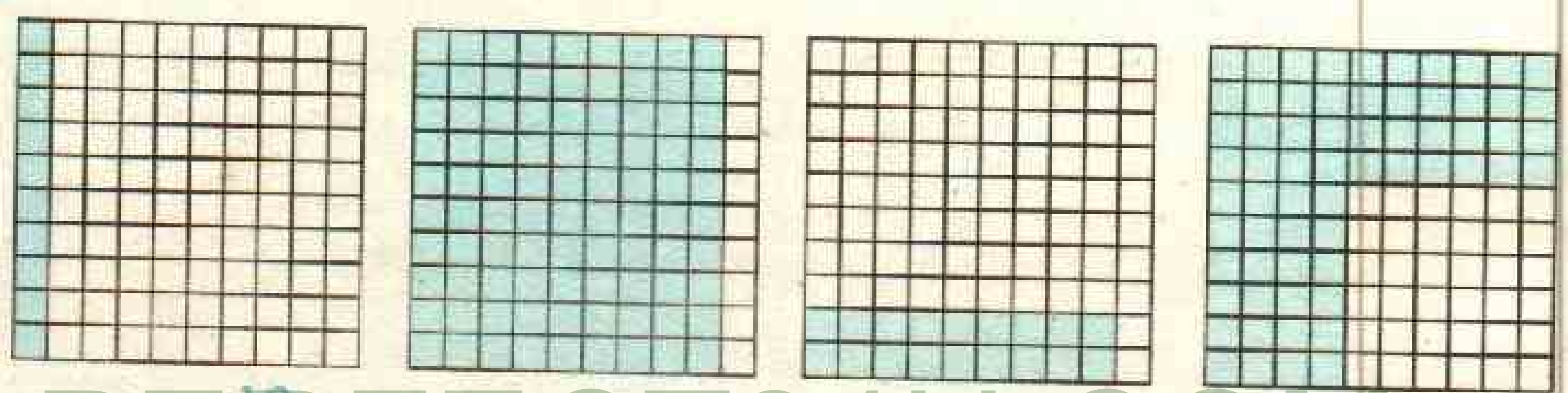
100% means $\frac{100}{100}$	50% means $\frac{50}{100}$	25% means $\frac{25}{100}$

In percent the denominator of a fraction must be 100.

Example 1 Percents

What part of each large square is shaded? Give each answer as a fraction with denominator 100 and also a number with a percent sign.

Solution:



Fraction: $\frac{10}{100}$	Fraction:	Fraction:	Fraction:
Percent: 10 %	Percent:	Percent:	Percent:

Example 2 Fraction as Percents

A wall constructed from 100 blocks of the same size 25 blocks painted black, 35 blocks painted red and 40 blocks painted blue. Show percentage of each block.

Solution:

In percentage these colour blocks can be expressed as:

Percentage of black blocks $= \frac{25}{100} = 25\%$

Percentage of red blocks $= \frac{35}{100} = 35\%$

Percentage of blue blocks $= \frac{40}{100} = 40\%$

7.1.1 Conversion of Percentage into Fraction

Step 1 Replace the percent sign (%) by $\frac{1}{100}$.

Step 2 Reduced it to its lowest form.

Example 1 Percentages into Fractions

Convert the following percentages into fractions:

$1\%,\ 4\%,\ 8\frac{1}{3}\%$

Solution:

$1\% = \frac{1}{100}$, $4\% = \frac{4}{100} = \frac{1}{25}$

$8\frac{1}{3}\% = \frac{25}{3} \times \frac{1}{100} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

($8\frac{1}{3}$ converted into improper fraction $\frac{25}{3}$ and then % is replaced by $\frac{1}{100}$ and reduced to lowest form)

Practice

Express as fraction in simplest form 25%, 12%.

7.1.2 Conversion of Fraction into Percentage

Step 1 Multiply and divide by 100.

Step 2 Replace $\frac{1}{100}$ by % and simplify the remaining fraction.

Example 1 Percentages into Fractions

Convert the following fractions into percentage.

$\frac{2}{5}, \frac{1}{4}$

Solution:

$$\frac{2}{5} = \frac{2}{5} \times \frac{100}{100}$$
$$= \left(\frac{2 \times 100}{5}\right) \times \left(\frac{1}{100}\right)$$
$$= (40) \times \left(\frac{1}{100}\right) = 40\%$$

$$\frac{1}{4} = \frac{1}{4} \times \frac{100}{100}$$
$$= \left(\frac{1}{4} \times 100\right) \times \left(\frac{1}{100}\right)$$
$$= (25) \times \left(\frac{1}{100}\right) = 25\%$$

7.1.3 Conversion of a Percentage into a Decimal Fraction

Step 1 Remove % sign.

Step 2 Move the decimal point two places to the left.

Example 1 Percentages into Decimal Fractions

Express each percent as a decimal.

a. 28% b. 375% c. 0.5%

Solution:

28%

$28\% = \frac{28}{100} = 0.28$

375%

$375\% = \frac{375}{100} = 3.75$

0.5%

$0.5\% = \frac{0.5}{100} = 0.005$

Practice

Express each percent as a decimal.

30%, 12%, 125%, 65%, 135%, 0.2%

7.1.4 Conversion of a Decimal Fraction into Percentage

Step 1 Replace decimal sign.

Step 2 Move the decimal point two places to the right and write percent sign.

Example 1 Decimal Fractions into Percents

Express 0.35, 0.07 and 1.49 as a percent.

a. 0.35 b. 0.07 c. 1.49

Solution:

0.35

$0.35 = \frac{35}{100} = 35\%$

0.07

$0.07 = \frac{7}{100} = 7\%$

1.49

$1.49 = \frac{149}{100} = 149\%$

Practice

Express each decimal as percent.

0.45, 1.3 and 0.008

Exercise 7.1

1. Convert each of the following percent into lowest common fraction.

(i) 80% (ii) 60% (iii) $12\frac{1}{2}\%$ (iv) $16\frac{2}{3}\%$

(v) 13% (vi) 10% (vii) $12\frac{2}{3}\%$

2. Convert the following as percent.

(i) $\frac{9}{15}$ (ii) $\frac{7}{30}$ (iii) $2\frac{1}{4}$ (iv) $\frac{1}{16}$ (v) $\frac{8}{25}$

3. Express each of the following as decimal fraction.

(i) 40% (ii) 75% (iii) 125% (iv) 0.4% (v) $33\frac{1}{3}\%$

4. Convert each of the following decimal fraction into percent.

(i) 0.5 (ii) 0.76 (iii) 2.45 (iv) 0.375 (v) 0.0125

7.1.5 World Problems Involving Percentage

Example 1 Find Percentage

Ahmad read 35% of a book on first day, 45% on the second day and the rest of the book he read on the third day. What percent of book did he read on the third day?

Solution:

Percentage of the book read on first day = 35%

Percentage of the book read on second day = 45%

Percentage of the book read in two days

= 35% + 45%

= 80%

So percentage of the book read on the third day

= 100% − 80%

= 20%

Example 2 Find Percentage

In a 50 students class, 30% received A-grade. How many students got A-grade?

Solution:

The problem can be solved by using a mathematical equation. Let x be the number of students getting A-grade. Then according to the statement of the problem we get,

x is 30% of 50 students

which can be written as

x = 30% of 50 students

$$=\left(\frac{30}{100}\right)\times 50 = 15 \text{ students}$$

Thus 15 students got A-grade

Practice

In a 60 students class, there are 45 boys and 15 girls. Find the percentage of girls in the class.

Example 3 Find Percentage

In a survey on motorway, 90 out of 300 drivers do not regularly use safety belts. What percent of the drivers do not regularly use safety belts?

Solution:

According to the problem, to determine 90 is what percent of 300.

Let x = unknown rate.

Then 90 is what percent of 300.

$$\text{or } 90 = x \times 300 \quad \text{or } 300x = 90$$

$$\frac{300x}{300} = \frac{90}{300} \qquad \text{(Dividing both sides by 300)}$$

$$x = \frac{3}{10}$$

$$= \frac{3}{10} \times 100 \times \frac{1}{100}$$

$$= 30\% \qquad \left(\frac{1}{100} \text{ replaced by } \%\right)$$

Thus 30% of drivers do not use safety belts.

Exercise 7.2

1. A squared box is divided into 100 equal circles of the same radii. 30 circles are black, 20 are blue and the remaining circles are red.

   What is the percentage of red circles in the box?

2. 25% of a wall painted blue, 30% painted green and the rest of the wall painted white. Find the percentage of the white part of the wall.

3. In a home examination Ali obtained 350 marks out of 500. Find the percentage of the marks.

4. Aslam got Rs.500 as pocket money in a month. He spent Rs.150 on cold drinks, Rs.100 on stationary and saved the rest amount. What percentage of the pocket money he spent on cold drinks?

5. 20 out of 32 members of a football team weigh over 100 kg. What percent of the team weighs over 100 kg?

7.2.1 Cost Price and Selling Price

The price, which is paid to purchase a good or commodity is called cost price or purchase price and the price by which a good is sold is called its selling price. Usually the cost price of a good is denoted by $(C.P)$ and its selling price is denoted by $(S.P)$.

7.2.2 Profit and Loss

Profit

If $S.P > C.P$, then profit is gained. The difference of sale price and cost price is called profit.

Profit $= S.P - C.P$

Loss

If $S.P < C.P$, then Loss is suffered. The difference of cost price and sale price is called loss.

Loss $= C.P - S.P$

Note

i) To encourage sales, some shops offer a reduction from the marked price for prompt payment.

ii) The reduction is called a discount.

iii) The discount is usually written as a percentage of the marked price.

Discount

The rebate (reduction) on the sale price is called discount.

Discount = Marked Price – Sale Price

Example 1 Find amount

Raheel bought a calculator for Rs.300 from an electronics shop. He got a discount of 20%. Find the amount that he paid.

Solution:

Price of calculator = Rs300

Discount rate = 20%

Discount $= 20\%$ of $300 = \frac{20}{100} \times 300 = \text{Rs}.60$

Thus the amount paid $= 300 - 60 = \text{Rs. } 240$

7.2.3 Profit and Loss Percent

Formula for computing profit and loss percent is given as:

$$\text{Profit percent} = \frac{Profit}{Cost\ Price} \times 100\%$$

$$= \left(\frac{S.P - C.P}{C.P}\right) \times 100\%$$

$$\text{Loss percent} = \frac{Loss}{Cost\ Price} \times 100\%$$

$$= \frac{C.P - S.P}{C.P} \times 100\%$$

Example 1 Find Loss or Profit

Ali bought a used computer for Rs.5000 and spent Rs.500 on its repair and then sold it for Rs.6500. Find his loss or profit.

Solution:

Purchased Price ($C.P$) = Rs.5000

Repair Cost = Rs.500

Total Cost Price = 5000 + 500 = Rs.5500

Sale Price ($S.P$) = Rs.6500

Since the sale price is more than the cost price. So Ali gained a profit.

Thus Profit $= S.P - C.P$

$= 6500 - 5500$

$=$ Rs.1000

Practice

Akram bought bicycle for Rs. 1500 and sold it for Rs.2000. Find the profit gained.

Example 2 Find Loss Percent

The cost price of a book was Rs.30. It is sold for Rs.20. Find the loss percent of the book.

Solution:

$C.P = \text{Rs.}30$

$S.P = \text{Rs.}20$

Since $S.P < C.P$

$\therefore$ Loss $= C.P - S.P$

$= 30 - 20$

$= \text{Rs.}10$

Loss percent $= \dfrac{Loss}{C\,P} \times 100\%$

$= \dfrac{10}{30} \times 100\% = 33\dfrac{1}{3}\%$

Example 3 Find Percent

Ahmad bought a T.V for Rs.4000 and sold it for Rs.5000. Find its profit percentage.

Solution:

$C.P = \text{Rs.}4000$

$S.P = \text{Rs.}5000$

Now Profit Percent $= \dfrac{Profit}{C.P} \times 100\%$

$= \left(\dfrac{SP - CP}{CP}\right) \times 100\%$

$= \left(\dfrac{5000 - 4000}{4000}\right) \times 100\%$

$= \dfrac{1000}{4000} \times 100\% = 25\%$

Exercise 7.3

1. Find the value of the discount and the amount actually paid for the following.

S#	Items	Price (Rs.)	Discount
(i)	Radio	500	10%
(ii)	Kettle	200	5%
(iii)	Record player	800	25%
(iv)	Refrigerator	25000	20%
(v)	Bicycle	6000	15%
(vi)	Washing machine	8000	10%

2. Aslam bought 5 dozens of eggs at the rate of Rs.30 per dozen and sold per dozen at the rate of Rs.40. Find the profit he gained.

3. Asim bought a bicycle for Rs.2500 and spent Rs.300 on its decoration. He then sold it for Rs.4000. Find profit percent.

4. A chair was purchaised for Rs.600 and sold for Rs.575. Find the profit or Loss.

5. Saima bought a pack of ten pencils for Rs.20 and sold per pencil for Rs.2.5. Find profit or loss percent.

6. A book seller bought 5 dozens note books at the rate of Rs.20 per note book and sold at the rate of Rs.300 per dozen. Find profit or loss.

7. Aqéel bought some books from a book stall costing Rs.1000. He got a discount of 10%. Find the amount that he paid.

8. If the written price of a toy is Rs.200 and its discounted price is Rs.150, find the rate of discount.

Review Exercise 7

1. If a statement is true then encircle 'T' and if the statement is False then encircle 'F' in each of the following questions.

(i) $\frac{1}{100} = 1\%$ T – F

(ii) 25% of 60 = 20 T – F

(iii) $\frac{2}{5} = 40\%$ T – F

(iv) 50% = 0.4 T – F

(v) Profit = Sale price – cost price T – F

2. Choose the correct answer.

(i) 25% = ______ (in friction form)

a. $\frac{1}{4}$ b. $\frac{1}{8}$ c. $\frac{1}{6}$

(ii) 0.2 = ______ (in percentage form)

a. 2% b. 20% c. 5%

(iii) 30% = ______ (in decimal form)

a. 0.03 b. 0.3 c. 30

3. Find the unknown in each of the following questions.

(i) What number is 50% of 150?

(ii) 8 is 5 % of what number?

(iii) 12 is 35% of what number?

(iv) 98 is what percent of 200?

(v) $8\frac{5}{9}$ is what percent of 11?

4. Twenty five out of 150 students failed mathematics in the annual examination. What percent of the whole class failed?

5. A car costing 240,000 rupees is reduced in price by 10%. Find the new price.

Summary

- **Percent (%)** means "out of hundred".
- Purchased price or cost price ($C.P$): **The price which is paid to purchase a good or commodity.**
- Sale Price ($S.P$): **The price at which a good is sold.**
- **Profit:** When $S.P > C.P$, profit is gained.
- **Loss:** When $S.P < C.P$, profit is suffered.
- **Discount:** Is a percentage deducted from the original price when an item goes an sale.
- **To c onvert a percent to a fraction:** The percent sign (%) is removed and 100 is put in the denominator of the given number and reduced to the lowest term.

- **To Convert a percentage to a decimal:** The percent sign (%) is removed and the decimal point is moved two places to the left.
- **To Convert a decimal to a percent:** The decimal point is moved two places to the right and the percent sign (%) is added.
- **To calculate profit and loss:** Profit $= S.P - C.P$

  L oss $= C.P - S.P$
- **To calculate profit percent and Loss percent:**

$$\text{Profit Percent} = \frac{Profit}{C.P} \times 100\%$$

$$\text{Loss Percent} = \frac{Loss}{C.P} \times 100\%$$

- **To calculate discount:** Discount = Rate of discount × Actual price

UNIT 08 INTRODUCTION TO ALGEBRA

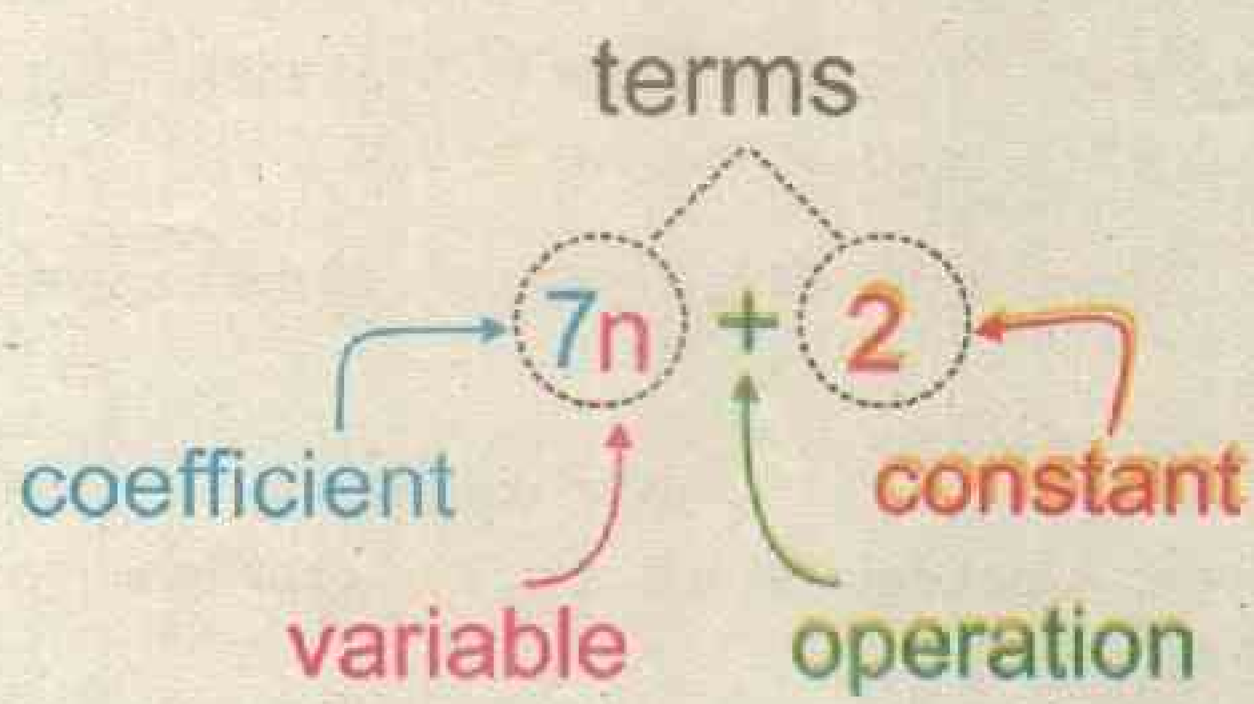
What You'll Learn

- Explain the term algebra as an extension of arithmetic in which letters replace the numbers.
- Know that
  - a sentence is a set of words making a complete grammatical structure and conveying full meaning.
  - sentences that are either true or false are known as statements.
  - a statement must be either true or false but not both.
  - a sentence that does not include enough information required to decide whether it is true or false is know as open statement (e.g., $\Delta + 2 = 9$).
  - a number that makes an open statement true is said to satisfy the statement (e.g. $\Delta = 7$ makes the statement $\Delta + 2 = 9$ true).
  - use English alphabet x in the open statement $\Delta + 2 = 9$ to modify it to $x + 2 = 9$.
- Define variables as letters used to denote numbers in algebra.
- Know that any numeral, variable or combination of numerals and variables connected by one or more of the symbols ' + ' and '–' is know as an algebraic expression (e.g., $x + 2y$).
- Know that x, $2y$ and 5 are called the terms of the expression $x + 2y + 5$.
- Know that the symbol or number appearing as multiple of a variable used in algebraic term is called its coefficient (e.g. in $2y$, 2 is the coefficient of y).

- Know that the number, appearing in algebraic expression, independent of a variable is called a constant term (e.g. in $x + 2y + 5$, number 5 is a constant term).
- Differentiate between like and unlike terms.
- Know that
  - like terms can be combined to give a single term,
  - addition or subtraction can not be performed with unlike terms.
- Add and subtract given algebraic expressions.
- Simplify algebraic expressions grouped with brackets.
- Evaluate and simplify an algebraic expression when the values of variables involved are given.

Why It's important

Algebra is that branch of Mathematics which deals with quantities using letters (a, b, c .... x, y, z) to symbolize them. Algebra is important because it is a generalization of arithmetic. e.g. the arithmetic's facts that $2+2+2=3\ \ 2$, $4+4+4=3\ \ 4$ etc are all the special cases of the (general) algebraic statement $x+x+x=3x$ when x is any number.



8.1 The Concepts of Constant and Variable

8.1.1 Definitions

Algebra is a language of symbols. One symbol that is frequently used is a variable. Variables are letters used to denote numbers in algebra.

Constant:

If the value of a quantity remains unchanged (fixed), then it is called a constant. The numbers $0, \pm1, \pm2, 3, \ldots$ are constants.

Variable:

The letters used to denote numbers in algebra are called variables. Variables are usually denoted by small letters of English alphabets $a, b, c, \ldots, x, y, z$.

Consider the examples $x+5$ and $y+3$.

In $x + 5$, the variable is x and constant is 5. In $y + 3$, the variable is y and constant is 3.

In these examples 5 and 3 are fixed numbers but the variables x and y can have any value.

8.1.2 Algebraic Expressions

Any numeral, variable or combination of numerals and variables connected by one or more of the symbols ' + ' and '–' is known as an algebraic expression. For example,

$$5x \qquad 3x-7 \qquad 4\frac{p}{q} \qquad m\times 5n \qquad 3ab : 5cd$$

are algebraic expressions.

8.1.3 Coefficient and Exponents

The symbol or number appearing as multiple of a variable used in algebraic term is called its coefficient.

In $5x, 2y, -z, 6a, 7b$ the coefficient are $5, 2, -1, 6$ and 7 as these are the multiples of the variables x, y, z and b.

8.1.4 Algebraic Sentences

A sentence is a set of words making a complete grammatical structure and conveying full meaning is called an algebraic sentence or sentence.

In algebraic sentences the symbols $=, \neq, >, <, \nless, \ngtr$ are commonly used. For example, $4+2=6$, $a \neq b$, $3>2$, $4x \nless 10$ etc are algebraic sentences.

8.1.5 Statements

Sentences that are either true or false are known as statements. For example,

$3+2=5$	True sentence
$7<4$	False sentence

Note

A statement must be either true or false but not both.

8.1.6 Open Statements

A sentence that does not include enough information required to decide whether it is true or false is known as open statement e.g. the sentence $\Delta+2=9$ is an open sentence because it cannot be decided whether it is true or false unless the exact value of Δ is known.

Solution of a Statement

A number that makes an open statement true is said to satisfy the statement and is called its solution e.g. $\Delta=7$ makes the statement $\Delta+2=9$ true. So $\Delta=7$ the solution.

Solving a Statement

A value for the variable that makes an equation true is called a solution. Similarly $x+7=19$, the solution is 12. The process of finding a solution is called solving a statement.

Example 1 Types of a Sentence

Separate true, false and open sentences in the following.

$6 < 4$, $6 \times 2 = 12$, $3x + 4x = 7x$

$7 > 5$, $x - 3 = 4$, $x < 4$

$7 + 5 = 13$, $x + 3 > 7$

Solution:

True sentences	False sentences	Open sentences.
$6 \times 2 = 12$	$6 < 4$	$x - 3 = 4$
$7 > 5$	$7 + 5 = 13$	$x + 3 > 7$
$3x + 4x = 7x$		$x < 4$

Exercise 8.1

1. **Separate the constants and variables in the following algebraic expressions.**

   (i) $x - 5$ (ii) $y - 1$ (iii) $a + 4$ (iv) $x + y + 2$

2. **Write the co-efficients and exponents in the following expressions.**

   (i) $-2x$ (ii) $3y^2$ (iii) $5z^3$ (iv) $-6x^2$

3. Identify true sentences, false sentences and open sentences in the following algebraic sentences.

   (i) $6 + 2 = 8$ (ii) $3 \times 5 = 16$ (iii) $7x - 5 = 10$

   (iv) $x < 3$ (v) $4 \ngtr 2$ (vi) $(11 \div 2) < 6$

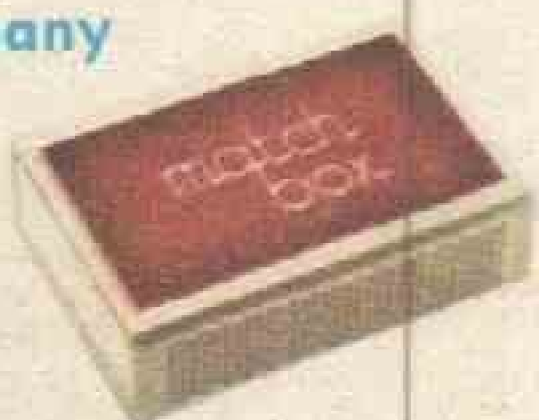
4. **Put "=", "<" or ">" in the blank spaces to make the sentences true.**

   (i) $5 + 3$ ____ 9 (ii) $6x - 2x$ ____ $4x$ (iii) $8 - 5$ ____ 2

5. **In a match box there are 50 matches. How many matches are there in**

   i) 2 boxes? ii) 3 boxes?

   iii) x boxes? iv) $4x$ boxes?



8.2 Terms of Algebraic Expression

8.2.1 Definition of Algebraic Expression

Different parts of an algebraic expression connected by Mathematical signs '+' or '–' are called terms of the algebraic expression.

This expression has four terms

$$2x + 8 + x + 3$$

2 is the coefficient x

1 is the coefficient of x because $x = 1x$

An algebraic expression consisting of one term e.g. $3x$ is called **monomial,** the algebraic expression consisting of two terms e.g. $7x + 9$ is called **binomial** and consisting of three terms e.g. $2x + 3y - 5$ is called **trinomial.**

8.2.2 Like and unlike terms

Terms that have the same variable parts are called like terms. Terms with different variable parts are called unlike terms.

Like terms

$$5y + 3 + 2y + 8y$$

Constant

Unlike terms

$$2x + 5y$$

8.2.3 Addition of like and unlike terms

In order to add two or more like terms the co–efficients of the terms are added and the variables remain unchanged, while addition of unlike terms cannot be performed. e.g.

$$3xy + 8xy = 11xy$$

(The coefficients 8 added to 3 and variable xy unchanged)

Practice

Add the following.

a. $4x + 5 + 3$ b. $x + 2 + 4x$ c. $3x + 5x + 2y$

Exercise 8.2

1. **Find the number of terms in the following algebraic expressions.**

(i) $6x$ 1 (ii) 2 ((iii) $2x+3$ 2 (iv) $4x-6y$ 2

(v) $a+b+c$ 3 (vi) $2xy$ 1 (vii) $7xy+9z$ 2

2. **In each of the following algebraic expressions find the like terms.**

(i) $a^2+ab+2ab+3a^2$ (ii) $x^2y+2xy^2+3x^2y+5xy^2$

(iii) $a^3-2a^2b+4a^2b+5a^3$ (iv) $xy+yz+2xy-3yz$

3. **Write the following sum as a single term.**

(i) $2x+3x$ (ii) $4a^2+5a^2$ (iii) $x+2x+3x$

(iv) $9ab+2ab$ (v) $2xyz+3xyz$ (vi) $4a^3+7a^3$

4. **Simplify the following expressions.**

(i) $3x+4x+6y$ (ii) $x^2y+2xy^2-3x^2y+5xy^2$

(iii) $ab+bc-2ab$ (iv) $ax+by+cx$

5. **Identify the terms, like terms, coefficients and constants in the expression.**

(i) $3x+4x+y-2$ (ii) $4x+3+5x+y$

6. **Match like terms.**

$-4y^2$	8
$17x^4y$	$7xy$
$6a^4$	13
$3xxx$	$3x^4y$
$10s$	$9a^4$
-8	$-7c$
$6yx$	$6y^2$

8.3 Addition and Subtraction of Algebraic Expressions

8.3.1 Two or more Algebraic Expressions

In order to add two or more algebraic expressions, the co–efficient of like terms in all expressions are added and reduced to a single expression.

Example 1 Add

Add $3x^2 + 5x$ and $6x^2 + 2x$

Solution:

Method-I

$$\begin{aligned}(3x^2 + 5x) + (6x^2 + 2x) &= 3x^2 + 5x + 6x^2 + 2x \\ &= 3x^2 + 6x^2 + 5x + 2x \quad \text{(Like terms combined)} \\ &= (3 + 6)x^2 + (5 + 2)x \quad \text{(Co-efficients added)} \\ &= 9x^2 + 7x\end{aligned}$$

Method-II

$$\begin{array}{rr} 3x^2 & + 5x \\ 6x^2 & + 2x \\ \hline 9x^2 & + 7x \end{array}$$

In case of subtraction the mathematical signs of the co–efficient of the like terms in the algebraic expressions which to be subtracted from the other change and then simplified.

Example 2 Subtract

Subtract $3x^2 + 4x + 5$ from $6x^2 - 7x + 7$.

Solution:

$$\begin{array}{rrr} 6x^2 & -7x & +7 \\ \pm 3x^2 & \pm 4x & \pm 5 \\ \hline 3x^2 & -11x & +2 \end{array}$$

Practice Subtract $9x^2 - 7x + 4$ from $5x^2 + 8x + 1$.

8.3.2 Simplifying Algebraic Expressions

Here we will show how to simplify algebraic expressions using a technique called "combining like terms".

Example 1 Simplifying Algebraic Expressions

Simplify $9x + 3 - 7x$

Solution:

$$
\begin{aligned}
9x + 3 - 7x &= 9x + 3 + (-7x) && \text{(Definition of Subtraction)}\\
&= 9x + (-7x) + 3 && \text{(Commutative Property)}\\
&= [9 + (-7)]\,x + 3 && \text{(Distributive Property)}\\
&= 2x + 3 && \text{(Simplified)}
\end{aligned}
$$

Exercise 8.3

1. **Add the following expressions:**
   (i) $x + y,\ \ 2x + 3y$ (ii) $2x + 4y,\ \ 3x + 5y,\ 6x + y$
   (iii) $x^2 + y^2 + z^2,\ 6x^2 + 2y^2 + 3z^2$ (iv) $x^2 + 2x + 3,\ \ 4x^2 + 6x - 5$

2. **Find the sums of the following expressions.**
   (i) $2x^2 + 3x + 2,\quad 4x^2 - 5x + 4,\quad 6x^2 + 8x - 1$
   (ii) $7a^2 - ab + b^2,\quad 8a^2 - 3ab - 2b^2,\quad 4a^2 + 8ab + 4b^2$
   (iii) $p^2 + 4pq + q^2,\quad 2p^2 - 3pq + 5q^2,\quad 3p^2 + pq - 2q^2$

3. **Subtract the first expression from the second in the following.**
   (i) $x + 3y$ from $4x - y$ (ii) $x^2 + 3xy + y^2$ from $3x^2 - 5xy - 2y^2$
   (iii) $a^2b + b^2c + ac^2$ from $4a^2b - 2b^2c + 6ac^2$

4. **Simplify:**
   (i) $(x + y) + (2x - y)$ (ii) $(a^2b - b^2) - (2a^2b + 3b^2)$
   (iii) $(x^2y + 2) - (2x^2y + 5y^2 + 6)$ (iv) $(6m^2 + 3n) - (2m^2 + 5n)$

8.4 Evaluation and simplification of an algebraic expression with values of the variables involved

Example 1 Evaluate Expressions

Evaluate $x + y - 9$, if $x = 15$ and $y = 26$.

Solution:

$x + y - 9 = 15 + 26 - 9$ (Replace x with 15 and y with 26)

$= 41 - 9$ (Add 15 and 26)

$= 32$ (Subtract 9 from 41)

Practice

Evaluate each expression if $a = 5$, $b = 12$ and $c = 4$

a) $b + 6$ b) $18 - 3c$ c) $\frac{2b}{8}$ d) $5a - (b - c)$

Exercise 8.4

1. **Find the value of the following if $x = 6$, $y = 5$ and $z = 2$.**

   (i) $x + y$ (ii) $2x$ (iii) $3x + y$

2. **If $x = 4$, evaluate the following.**

   (i) $4x^2$ (ii) $3x^2 - 4x$ (iii) $x + \frac{1}{x}$

3. **If $x = 1$, $y = 2$ find the value of the following algebraic expressions.**

   (i) $x^2y + 2$ (ii) $x^2 - 2x + 3y$ (iii) $\frac{1}{2}x + \frac{1}{3}y$

4. **If $x = 1$, $y = 2$ and $z = -1$ then find the value of the following expressions:**

   (i) $4xyz$ (ii) $x + 2y - 3z$ (iii) $(x + y)^2 - (y - z)^2$

5. Evaluate the following by taking $a = 1$, $b = 0$, $c = -1$.

   (i) $ab - c$ (ii) $\left[\frac{a - c}{a + b}\right] + ab$ (iii) $a^3 + b^3 + c^3$

Review Exercise 8

1. **Fill in the blanks.**
   (i) The number '5' is called ________.
   (ii) 'x' is called ________.
   (iii) In $a + b$ number of terms equal to ________.
   (iv) The coefficient in $3x$ is ________.

2. **Encircle "T" if the statement is correct and encircle "F" if it is false.**
   (i) The number of terms in $x^2 + 3x$ is two. T – F
   (ii) $7 < 16$ is true sentence. T – F
   (iii) $6x - 4x = 2x$ is true sentence. T – F
   (iv) x^2 and x are like terms. T – F

3. **Choose the correct answer.**

   (i) $x + y$ is an algebraic
   a) expression b) sentence c) equation

   (ii) $3 < 0$
   a) true sentence b) false sentence c) open sentence

   (iii) x^2y and xy^2 are
   a) like terms b) unlike terms c) constants

   (iv) Number of terms in the expression $2x^2 + 3x + 4$ is/are
   a) one b) two c) three

4. **Simplify each of the following algebraic expressions by combining like terms.**
   (i) $5x - 8x$ (ii) $5y - 2 - 8y - 6$
   (iii) $2(3a - 6) - 4(4a + 6)$ (iv) $(6 - t) - 4(4t - 7)$

5. **Evaluate each of the following algebraic expressions.**
   (i) $3x - 4y + z$ when $x = 2$, $y = -3$, $z = -5$
   (ii) $8(x - 6y)$ when $x = 5$ and $y = 3$

Summary

- **Algebra** is a generalization of arithmetic.
- **A constant** is a quantity that remains unchanged and fixed.
- **A variable** is a letter of English alphabets that represents an unknown quantity.
- **The coefficient** of a variable is the number that is multiplied by the variable.
- **The terms** are the parts of algebraic expression.
- **Monomial** is an algebraic expression consisting of one term.
- **Binomial** is an algebraic expression consisting of two terms.
- **Trinomial** is an algebraic expression consisting of three terms.
- **Like terms** are terms which have the same variables and same exponents.
- **Unlike terms** are terms which have different variables or the same variables of different exponents.
- **To combine like terms:** The coefficients of the like terms are added or subtracted.

UNIT 09

LINEAR EQUATIONS

What You'll Learn

- ❖ Define an algebraic equation.
- ❖ Differentiate between equation and an expression.
- ❖ Define linear equation in one variable.
- ❖ Construct linear expression and linear equation in one variable.
- ❖ Solve simple linear equations involving fractional and decimal coefficients like $\frac{1}{2}x + 5 = x - \frac{1}{3}$.
- ❖ Solve real life problems involving linear equations.

Why It's important

As you continue to study algebra, you will learn how to describe quantitative relationships using variables and equations. For example, the equation $s=vt$ shows the relationship between the variables s (distance), v (rate or speed) and t (time). You will solve a problem about ballooning this lesson.

9.1 Algebraic Equation

Two algebraic expressions form an equation when they are related with each other by the symbol of equality ('=').

Left hand side

Right hand side

$$\underbrace{x + 13 = 3}_{\text{Equation}}$$

Example 1 Find

Find the value of x. if $2x - 6 = 0$.

Solution:

In the equation $2x - 6 = 0$... (1)

We try the following values of $x : x = 1, 2, 3$.

When $x = 1$, equation (1) becomes $2 \times 1 - 6 = 0$, $2 - 6 = 0$ or $-4 = 0$

which is false.

When $x = 2$, equation (1) becomes

$2 \times 2 - 6 = 0$, or $-2 = 0$, which is false.

When $x = 3$, equation (1) becomes,

$2 \times 3 - 6 = 0$, or $6 - 6 = 0$, or $0 = 0$ $\therefore x = 3$

9.1.1 Difference Between An Equation And An Expression

We know that combination of numerals and variables connected by one or more of the symbols '+' or '−' is known as an algebraic expression and when algebraic expressions are connected with the sign of equality "=" it becomes an equation. For example,

$x - 1, x + 2, 2x - 1, x - 2, 2x + 1$ are all algebraic expressions.

$x - 1 = x + 2$ is an equation.

9.1.2 Linear Equation

Consider the following equations:

(a) $x - 1 = 2x + 3$ (b) $\ell x + m = 0$

In equation (a), the variable is x and its exponent is 1.

In equation (b), the variable is x and its exponent is 1, while l and m are certain constants.

The equations given in (a) and (b) are all cases of linear equations in one variable.

> An equation in one variable, where the exponent of the variable is 1, is called a linear equation in one variable.

(i) Construct Linear Expressions In One Variable

An expression in one variable in which the exponent of the variable is 1 is called a linear expression in one variable. For example, the exponent of a variable x is 1 and the constant term is 5, then we can construct the following linear expressions in one variable x as $x+5$ and $x-5$.

The following are some other examples of linear expressions in one variable:

(i) $3x+7$ (ii) $\frac{5}{2}y-9$ (iii) $\frac{-7}{3}z+3$ (iv) $\ell x+m$

(ii) To Construct Linear Equation In One Variable

Let us consider the following different linear expressions in one variable x:

$$3x+2,\ x-3$$

Then we can construct the following linear equation in one variable x as:

$$3x+2=x-3$$

The following are some other examples of linear equations in one variable:

(i) $x+7=0$ (ii) $\frac{3}{2}x+3=x-2$

Exercise 9.1

1. Solve the following equations to show that they are linear equations in one variable:

   (i) $4x+13=0$ (ii) $\frac{5}{7}x-\frac{7}{3}=0$ (iii) $\frac{3}{2}x-5=2x+4$

2. What should be the values of x which balance the following equations:

   (i) $x=0$ (ii) $x-5=0$ (iii) $2x+1=3$

3. Which of the following equations are true or false for the given values.

   (i) $x+5=9$ when $x=3$ (ii) $y-7=0$ when $y=7$

9.2 Rules To Simplify An Equation

i) The same number is added to both sides.

ii) The same number is subtracted from both sides.

iii) The same non-zero number is multiplied to both the sides.

iv) The same non-zero number is divided to both the sides.

By the solution of an equation we mean to find out the value of the variable for which the equation is true. That value of the variable which makes the equation true (or satisfies the equation) is called the **root** or **solution** of the equation.

Example 1 Solve an Equation

Which value is the solution of $2x + 1 = 7$?

i) 6 ii) 5 iii) 4 iv) 3

Solution:

The solution is the value that makes the equation true.

Test each value.

i) $2x + 1 = 7$
$2(6) + 1 = 7$
$13 \neq 7$

ii) $2x + 1 = 7$
$2(5) + 1 = 7$
$11 \neq 7$

iii) $2x + 1 = 7$
$2(4) + 1 = 7$
$9 \neq 7$

iv) $2x + 1 = 7$
$2(3) + 1 = 7$
$7 = 7$

Since 3 makes the equation true, the answer is iv).

Practice

Find the value of k that makes $6 = \dfrac{48}{k}$ true.

i) 6 ii) 7 iii) 8 iv) 12

Example 2 Involving fractional coefficients

Solve the equation $\frac{1}{2}x + 5 = x - \frac{1}{3}$

Solution:

Given equation is $\frac{1}{2}x + 5 = x - \frac{1}{3}$

$\frac{1}{2}x - \frac{1}{2}x + 5 = x - \frac{1}{2}x - \frac{1}{3}$ (Subtracting $\frac{1}{2}x$ to both sides)

$5 = \frac{1}{2}x - \frac{1}{3}$ (Simplifying)

$5 + \frac{1}{3} = \frac{1}{2}x - \frac{1}{3} + \frac{1}{3}$ (Adding $\frac{1}{3}$ to both sides)

$\frac{16}{3} = \frac{1}{2}x$ (Simplifying)

$2 \times \frac{16}{3} = 2 \times \frac{1}{2}x$ (Multiplying both sides by 2)

$\frac{32}{3} = x$ is the required solution. (Simplified)

Check:

Put $x = \frac{32}{3}$ in the given equation $\frac{1}{2}x + 5 = x - \frac{1}{3}$

$$\frac{1}{2}\left(\frac{32}{3}\right) + 5 = \frac{32}{3} - \frac{1}{3}$$

$$\frac{16}{3} + 5 = \frac{31}{3}$$

$$\frac{31}{3} = \frac{31}{3}$$

Hence $x = \frac{32}{3}$ satisfies the given equation.

Example 3 involving fractional decimal coefficients

Solve the equation $0.5x + 5 = x - 0.75$.

Solution:

The give equation is $0.5x + 5 = x - 0.75$

$-0.5x + 0.5x + 5 = -0.5x + x - 0.75$ (Subtracting $0.5x$ from both sides)

$5 = 0.5x - 0.75$

$5 + 0.75 = 0.5x - 0.75 + 0.75$ (Adding 0.75 to both sides)

$\frac{5.75}{0.5} = \frac{0.5x}{0.5}$ (Dividing both sides by 0.5)

$x = \frac{5.75}{0.5}$

$x = \frac{57.5}{5}$

$x = 11.5$ (Simplified)

Check: $x = 11.5$, which is the required solution.

Put $x = 11.5$ in both sides of the given equation

$0.5(11.5) + 5 = 11.5 - 0.75$

$5.75 + 5 = 10.75$

$10.75 = 10.75$

Hence $x = 11.5$ is the solution of the given equation.

9.3 Problems On Equations

We come across many real life problems, which can be solved by the help of linear equations. This is explained in the following examples.

Example 1 Solve

Seven less than three times a number is -22. Find the number.

Solution:

$3n - 7 = -22$ (Write the equation)

$3n - 7 + 7 = -22 + 7$ (Add 7 to each side)

$3n = -15$ (Simplify)

$n = -5$ (Mentally divide each side by 3)

Example 2 Translate and Solve an Equation

Seven more than three times a number is 31. Find the number.

Solution:

$3n + 7 = 31$ (Write the equation)

$3n + 7 - 7 = 31 - 7$ (Subtract 7 to each side)

$3n = 24$ (Simplify)

$n = 8$ (Mentally divide each side by 3)

Therefore, the number is 8.

Example 3 Solve an age problem

Saima is 6 years older than Fatima. Find the age of Fatima if Saima is three times older than her.

Solution:

Let age of Fatima $= x$

Then age of Saima $= 3x$ .... (1)

On the other hand Saima is 6 years older than Fatima

$\therefore$ age of Saima $= x + 6$ years .... (2)

Hence according to given condition, from equation (1) and (2), we get

$3x = x + 6$

$3x - x = -x + x + 6$ (Adding $-x$ to both sides)

$2x = 6$

$x = 3$ (Dividing both sides by 2)

$\therefore$ age of Fatima = 3 years

and age of Saima $= 3 \times 3 = 9$ years

$\times \frac{97}{2}$ 96

Exercise 9.2

1. Write any two equivalent equations of the following equations:
   (i) $x + 3 = 0$ (ii) $2x - 5 = 0$
   (iii) $3x + 1 = 2x$ (iv) $4x - 7 = x + 5$

2. Solve the following equations and find the values of the variables.
   (i) $x - 5 = 2$ (ii) $x + 9 = 11$ (iii) $x - 4 = -9$
   (iv) $3x + 1 = 7$ (v) $2x - 7 = -8$ (vi) $2x - 3 = x + 1$
   (vii) $\frac{2}{3}x + 5 = x - \frac{1}{3}$ (viii) $\frac{2}{3}x - \frac{5}{2} = -\frac{1}{3}x + 1$
   (ix) $5 + 11y = 13y + 1$ (x) $\frac{5}{7}y + 3 = y + 2$
   (xi) $0.4x + 0.3 = 0.8x - 0.7$ (xii) $0.7x + 1.2 = 1.8x - 2.1$

3. If 5 is added to a number then the number becomes 12. Find the number.
4. If 9 is added to a number then the number becomes 25. Find the number.
5. If 5 is subtracted from a number it becomes 13, find the number.
6. Find the number whose 3 times when subtracted from 9 becomes 12.
7. If 5 is subtracted from 3 times a number the result is 16. Find the number.
8. The length of a rectangular farm is twice its breadth. If the perimeter of the rectangular farm is 321 meters, find the length and breadth of the farm.
9. Age of a father is twice the age of his son. If father is 16 year older than his son, find the age of father and his son.

Review Exercise 9

1. Find the value of the variable, which balance the following Equations.
   (i) $3x + 5 = 8$ (ii) $0.4y - 2.5 = 5.5$

2. Evaluate the following expressions.
   (i) $2x - 5$ when $x = 2$ (ii) $2x^3 + 3x^2 + 5x - 3$ when $x = 1$

3. Evaluate the following expressions for given values of the variables:
   (i) $3(x+y)$, when $x = 5, y = -2$ (ii) $8xy$, when $x = \frac{-7}{2}$ and $y = \frac{3}{2}$

4. What should be the value of t which balance the following equations:
   (i) $0.4t + 0.5 = 0.3t$ (ii) $\frac{5}{2}t - 3 = 1$

5. Determine whether the following equations are true or false for given values of the variables.
   (i) $y - 3 = 7$, when $y = 10$ (ii) $\frac{3}{2} + \frac{5}{2}t = 1$, when $t = -\frac{1}{5}$

6. Solve the following equations and find their roots.
   (i) $\frac{3}{2}x - \frac{5}{2} = \frac{1}{2}$ (ii) $2(x - 3) = 4$
   (iii) $4(x + 2) = 3(x - 1)$ (iv) $0.37 - 0.5 = 0.2y$

7. The sum of a number and 9 is −2. Find the number.

8. The sum of −5 and a number is −15. Find the number.

9. The difference of a number and 3 is − 6. Find the number.

10. When 5 is subtracted from a number, the result is 16. Find the number.

11. Nadia is five years older than her brother Najam. The sum of their ages is 37. How old is Nadia?

Summary

- **Algebraic equation:** Two algebraic expressions form an equation when they are related with each other by the symbol of equality i.e '='.
- **Difference between an equation and an expression:** Combination of numerals and variables connected by one or more of the symbols '+' or '–' is known as an algebraic expression and when algebraic expressions are connected with the sign of equality "=" it becomes an equation.

- **Linear equation in one variable:** An equation having one variable, where power of the variable is 1, is called a linear equation in one variable.
- **Linear expression in one variable:** An expression in one variable where the power of the variable is 1.
- **Equivalent equations:** Equations with the same solution are called equivalent equations.

UNIT 10 GEOMETRY

What You'll Learn

- Add measures of two or more line segments.
- Subtract measure of a line segment from a longer one.
- Draw a right bisector of a given line segment using compasses.
- Draw a perpendicular to a given line from a point on it using compasses.
- Draw a perpendicular to a given line, from a point outside the line, using compasses.
- Use compasses to:
  - construct an angle equal in measure of a given angle,
  - construct an angle twice in measure of a given angle,
  - bisect a given angle,
  - divide a given angle into four equal angles,
  - construct the following angles:

  $60^\circ, 30^\circ, 15^\circ, 90^\circ, 45^\circ, (22\frac{1}{2})^\circ, 75^\circ, (67\frac{1}{2})^\circ, 120^\circ, 150^\circ, 165^\circ, 135^\circ, 105^\circ$
- Construct a triangle when three sides (SSS) are given.
  - **Caution:** Sum of two sides should be greater than the third side.
- Construct a triangle when two sides and their included angle (SAS) are given.
- Construct a triangle when two angles and the included side (ASA) are given.
- Construct a triangle when hypotenuse and one side (RHS) for a right angled triangle are given.

Why It's important

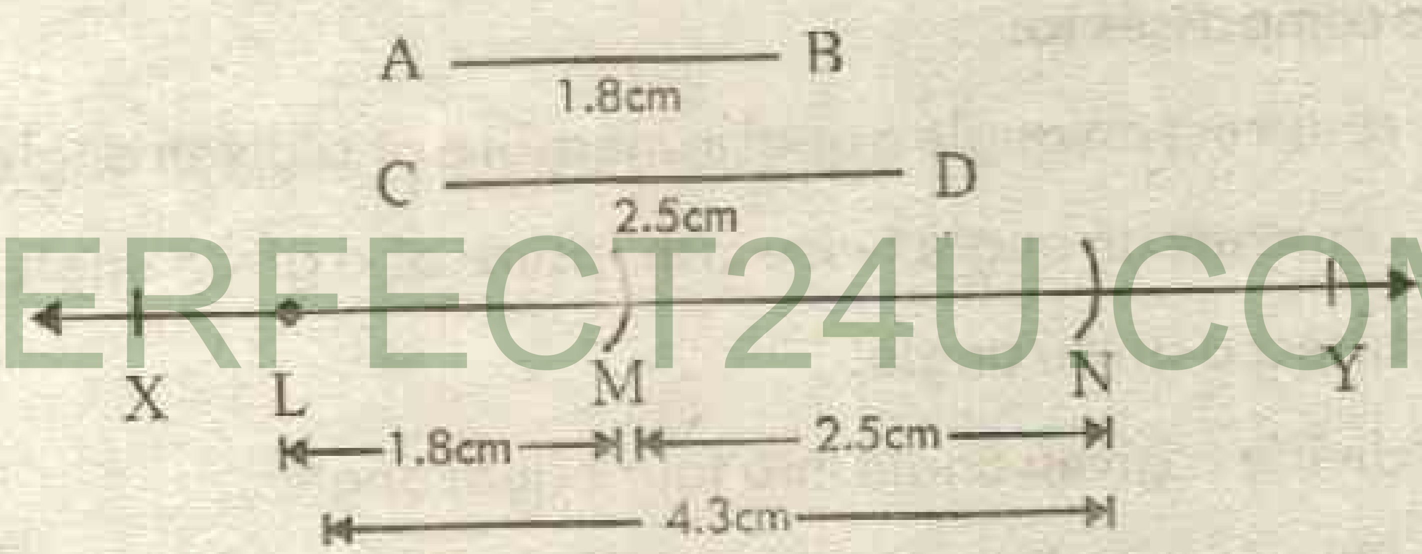
Engineers, architects and builders make use of geometry before they build buildings, bridges and other structures. Geometric awareness, such as the use of shapes, lines and figures, is necessary to design different projects. The knowledge of basic constructions such as construction of triangles, rectangles, squares is very useful in everyday life.

10.1 Add Measures of Two or More Line Segments

Example 1 Add

Two line segments measuring 1.8cm and 2.5cm are given. Draw a line segment whose length is equal to the sum of the lengths of the given line segments.

Solution:



Given: Two line segments $\overline{AB}$ and $\overline{CD}$ measuring 1.8cm and 2.5cm respectively.

Required: To draw a line segment whose measure is equal to the sum of the measures of $\overline{AB}$ and $\overline{CD}$.

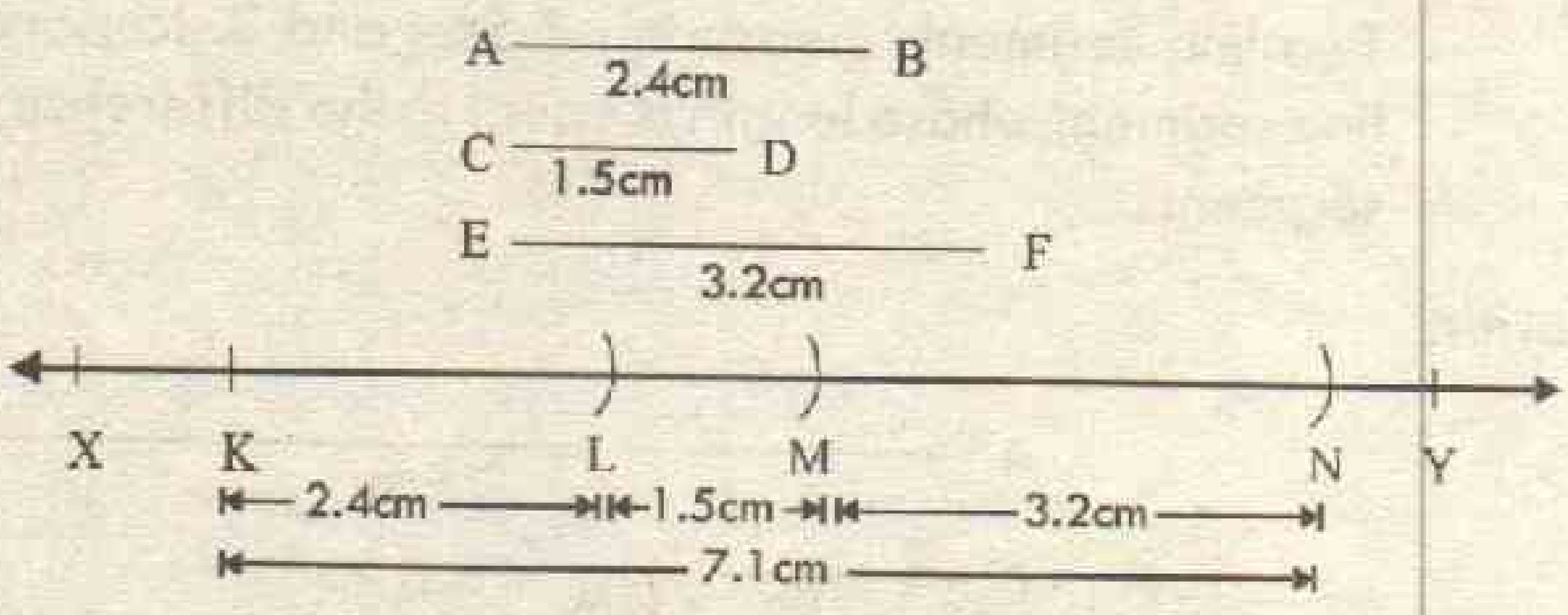
Steps of Construction

1. Draw a line $\overleftrightarrow{XY}$
2. Take a point L on $\overleftrightarrow{XY}$.
3. With L as centre draw an arc of $m\overline{AB}$ which cuts $\overleftrightarrow{XY}$ at M.
4. With M as centre draw another arc of $m\overline{CD}$ which cuts $\overleftrightarrow{XY}$ at N.
5. $\overline{LN}$ is a line segment such that $m\,\overline{LN} = m\,\overline{AB} + m\,\overline{CD}$.
6. With the help of ruler we see that $m\,\overline{LN}$ = 4.3cm.

Example 2 Draw a line

Draw a line segment whose length is equal to the sum of measures of three line segments $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ where $m\overline{AB} = 2.4cm$, $m\overline{CD} = 1.5cm$ and $m\overline{EF} = 3.2cm$.

Solution:



Given: $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ are three line segments whose measures are 2.4cm, 1.5cm and 3.2cm respectively.

Required: To draw line segment whose length is equal to the sum of measures of $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$.

Steps of Construction

1. Draw a line $\overleftrightarrow{XY}$.
2. Take a point K on $\overleftrightarrow{XY}$.
3. With K as centre draw an arc of radius $m\overline{AB}$ which cuts $\overleftrightarrow{XY}$ at L.
4. With L as centre draw an arc of radius $m\overline{CD}$ which cuts $\overleftrightarrow{XY}$ at M.
5. With M as centre draw an arc of radius $m\overline{EF}$ which cuts $\overleftrightarrow{XY}$ at N.
6. $\overline{KN}$ is the required line segment. Hence $m\overline{KN} = m\overline{KL} + m\overline{LM} + m\overline{MN}$.
7. With the help of ruler we see that $m\overline{KN} = 7.1cm$.

Practice

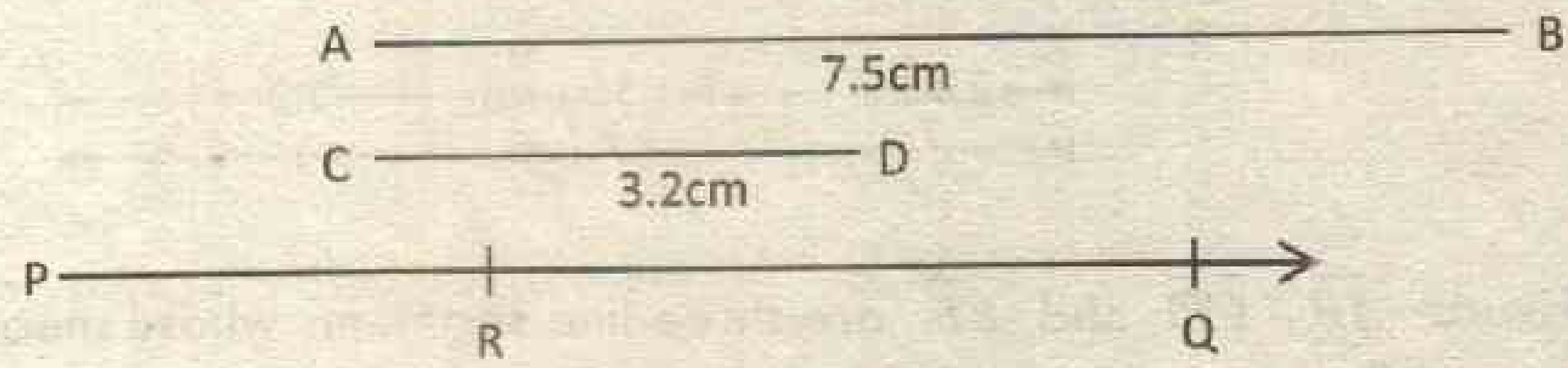
Draw a line segment whose length is equal to the sum of measures of three line segments $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ where $m\overline{AB} = 2.6cm$, $m\overline{CD} = 1.7cm$ and $m\overline{EF} = 2.5cm$.

10.1.1 Subtract Measure of a Line Segment from a Longer One

Example 1 Draw lines

Two line segments measuring 7.5cm and 3.2cm are given. Draw a line segment whose length is equal to the difference of the given line segments.

Solution:



Given: Two line segments $\overline{AB}$ and $\overline{CD}$ measuring 7.5cm and 3.2cm respectively.

Required: To draw a line segment whose measure is equal to the difference of the measures of $\overline{AB}$ and $\overline{CD}$.

Steps of Construction

1. Draw $\overline{PQ}$ equal in measure to $\overline{AB}$.
2. With P as center draw an arc of radius equal to the measure of $\overline{CD}$ i.e. 3.2cm which cuts $\overline{PQ}$ at R.
3. $\overline{RQ}$ is the required line segment whose measure is equal to the difference of $\overline{AB}$ and $\overline{CD}$ i.e. $m\ \overline{RQ} = m\ \overline{AB} - m\overline{CD}$.
4. With the help of ruler we see that $m\ \overline{RQ}$ = 4.3cm.

Practice

Two line segments measuring 8.2cm and 4.6cm are given. Draw a line segment whose length is equal to the difference of the given line segments.

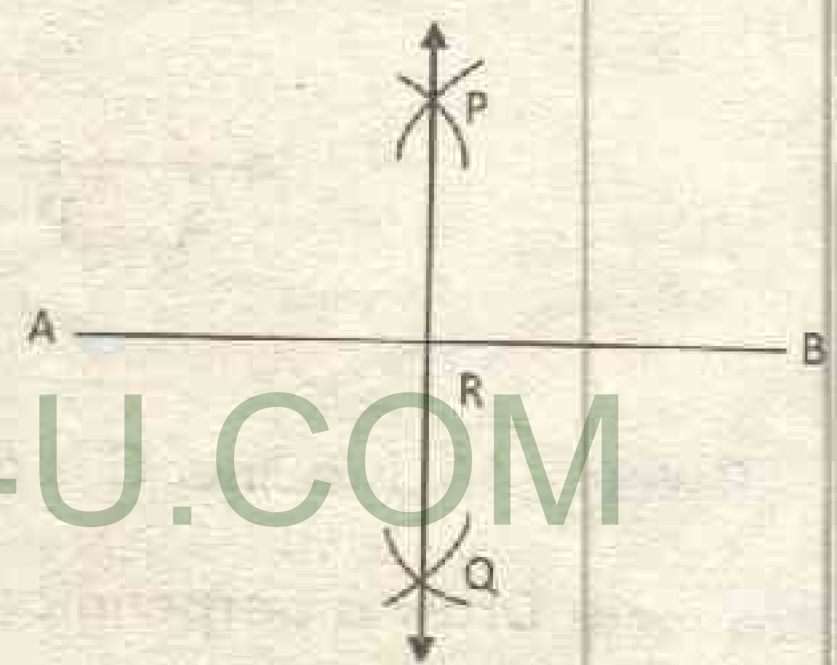
10.1.2 Draw right bisector of a given line segment by using Compass

A line which divides a line segment into two equal parts and also makes an angle of 90° with the given line segment, is called its Right bisector.

Example 1 Draw Perpendicular

A line segment is 5.6cm long. Draw its right bisector with the help of compasses.

Solution:



Given: $\overline{AB}$ is line segment measuring 5.6cm.

Required: Draw the right bisector of $\overline{AB}$ with the help of compasses.

Steps of Construction

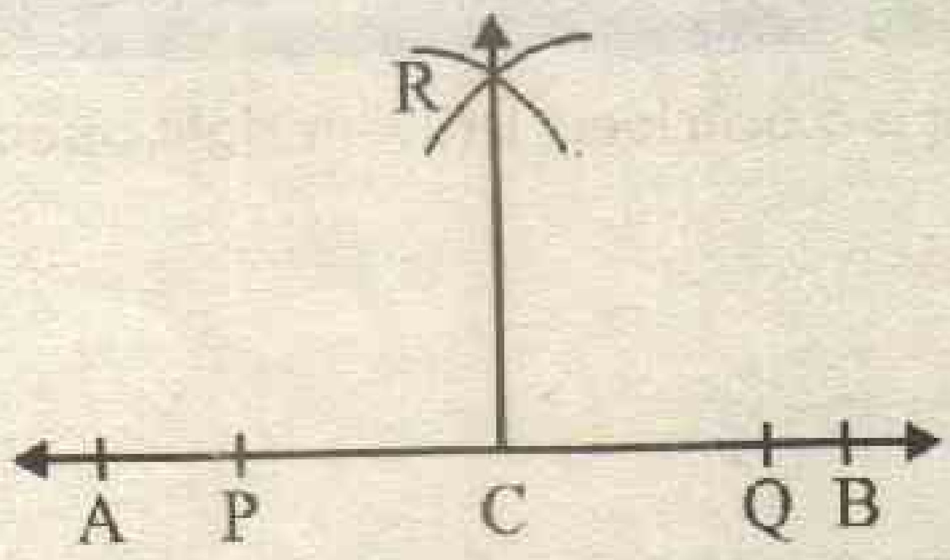
1. Draw $\overline{AB}$ 5.6cm long.
2. With A as center draw arcs of the same radius but more than half of m $\overline{AB}$ above and below $\overline{AB}$.
3. Without changing the radius and taking B as centre draw arcs that cuts side of $\overline{AB}$ cutting the arcs already drawn arcs at P and Q.
4. Draw line $\overleftrightarrow{PQ}$ which intersects $\overline{AB}$ at R.
5. $\overleftrightarrow{PQ}$ is the right bisector of $\overline{AB}$ i.e. makes an angle of 90° at R, and divides $\overline{AB}$ into two equal parts $\overline{AR}$ and $\overline{BR}$.

10.1.3 Draw a perpendicular to a given line from a point on it using Compasses

Example 1 Draw Perpendicular

Draw a line $\overleftrightarrow{AB}$. Take a point C on it and draw a perpendicular to $\overleftrightarrow{AB}$ at point C. Use compasses and ruler only.

Solution:



Given: $\overleftrightarrow{AB}$ is a line. C is a point on $\overleftrightarrow{AB}$.

Required: Draw a perpendicular to $\overleftrightarrow{AB}$ at C.

Steps of Construction

1. Draw $\overleftrightarrow{AB}$ and take a point C on it.
2. With C as centre draw arcs of equal radius to cut $\overleftrightarrow{AB}$ at P and Q.
3. With centres P and Q and radius greater than half of PQ, draw two arcs to cut each other at R.
4. Join C to R.
5. $\overleftrightarrow{CR}$ is perpendicular to $\overleftrightarrow{AB}$ at C.

Practice

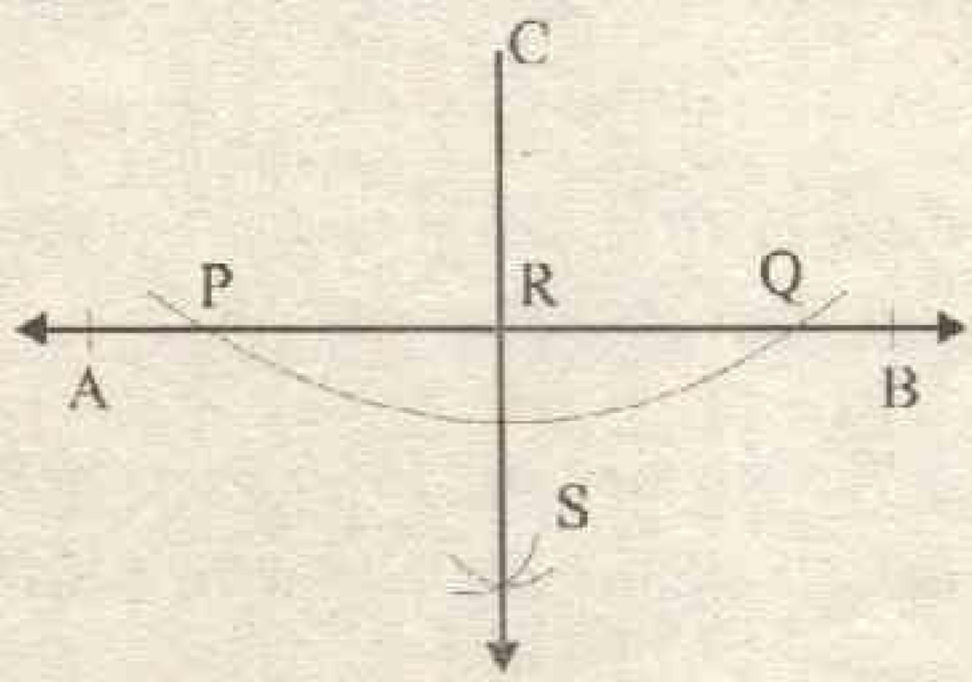
Draw a line $\overleftrightarrow{XY}$. Take a point P on it and draw a perpendicular to $\overleftrightarrow{XY}$ at point P. Use compasses and ruler only.

10.1.4 Draw a perpendicular to a given line, from a point outside the line, using Compasses

Example 1 Draw Perpendicular

C is a point outside a line $\overleftrightarrow{AB}$. Draw perpendicular with the help of compasses, from C to $\overleftrightarrow{AB}$.

Solution:



Given: $\overleftrightarrow{AB}$ is a line and C is a point outside $\overleftrightarrow{AB}$.

Required: Draw perpendicular with the help of compasses, from C to $\overleftrightarrow{AB}$.

Steps of Construction

1. Draw $\overleftrightarrow{AB}$ and take C, a point outside $\overleftrightarrow{AB}$.
2. With C as centre, draw an arc of suitable radius to intersect $\overleftrightarrow{AB}$ at P and Q.
3. With P and Q draw arcs of equal radius to intersect each other at S.
4. Draw $\overrightarrow{CS}$ intersecting $\overleftrightarrow{AB}$ at R.
5. $\overrightarrow{CR}$ is the required perpendicular.

Practice

P is a point outside a line $\overleftrightarrow{XY}$. Draw perpendicular with the help of compasses, from P to $\overleftrightarrow{XY}$.

Exercise 10.1

1. Draw two line segments 3.8cm and 2.6cm long. Draw a line segment whose length is equal to the sum of the lengths of these two line segments.

2. Draw three line segments 2.8cm, 3.4cm and 4.1cm long. Draw a line segment whose length is equal to the sum of the lengths of these three line segments.

3. Draw $\overline{AB}$ and $\overline{CD}$ where m$\overline{AB}$ = 4.5cm and m$\overline{CD}$ = 3.5cm. Draw $\overline{KL}$ such that m$\overline{KL}$ = m $\overline{AB}$ + m$\overline{CD}$.

4. Draw $\overline{KL}$, $\overline{PQ}$ and $\overline{MN}$ such that m$\overline{KL}$ = 2.9cm, m$\overline{PQ}$ = 3.2cm and m$\overline{MN}$ = 4.1cm. Draw $\overline{AB}$ where m$\overline{AB}$ = m$\overline{KL}$ + m$\overline{PQ}$ + m$\overline{MN}$.

5. Draw a line segment 5.2cm long. Cut off a segment 2.3cm long from it. Find the measure of the remaining part of the given segment.

6. Draw $\overline{AB}$ such that m$\overline{AB}$ = 6.5cm. Cut off $\overline{AC}$ such that m$\overline{AC}$ = 2cm. What is the length of $\overline{CB}$?

7. Draw line segments of the following lengths:

   (i) 4.8cm (ii) 5.4cm (iii) 6.6cm

   Draw their right bisectors by using compasses and ruler.

8. Draw line segments of the following lengths.

   (i) 3.5cm (ii) 4.4cm (iii) 5.5cm

   Take a point on each segment and draw perpendiculars to the given line segments. (Use ruler and compasses only)

9. Draw line segments of the following lengths:

   (i) 4.5cm (ii) 5.2cm (iii) 3.8cm

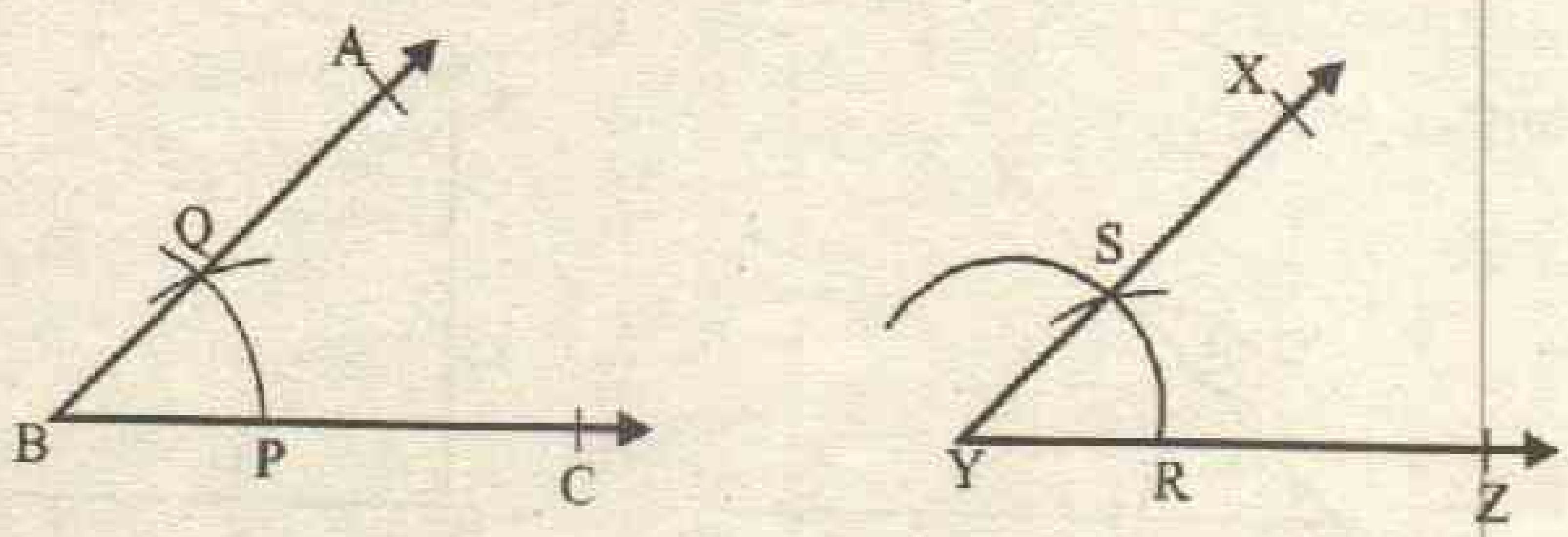
   Take a point outside each segment and draw perpendicular to the given segments from that point. (Use ruler and compasses only).

10.2 Construct an angle equal in measure of a given angle by using compasses

Example 1 Construct an angle

$\angle ABC$ is a given angle. Draw $\angle XYZ$ equal to $\angle ABC$.

Solution:



Given: $\angle ABC$ is the given angle.

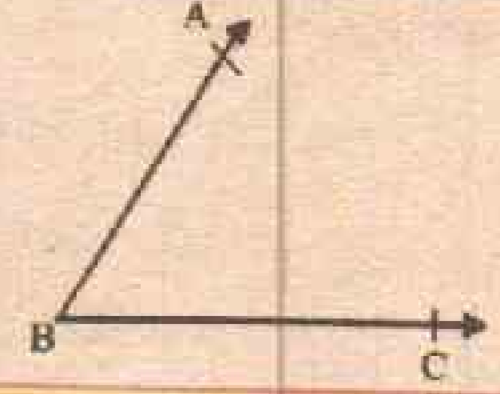
Required: Construct $\angle XYZ$ so that $m\angle XYZ = m\angle ABC$.

Steps of Construction

1. With Centre B and a suitable radius, draw an arc, cutting $\overrightarrow{BC}$ at P and $\overrightarrow{BA}$ at Q.
2. Take a ray $\overrightarrow{YZ}$. With centre Y and without changing the radius, draw an arc to cut $\overrightarrow{YZ}$ at R.
3. With R as centre and radius equal to $\overline{PQ}$, draw an arc to cut the former arc at S.
4. Through S, draw $\overrightarrow{YX}$.
5. XYZ is the required angle.

Practice

$\angle ABC$ is a given angle. Draw $\angle XYZ$ equal to $\angle ABC$.

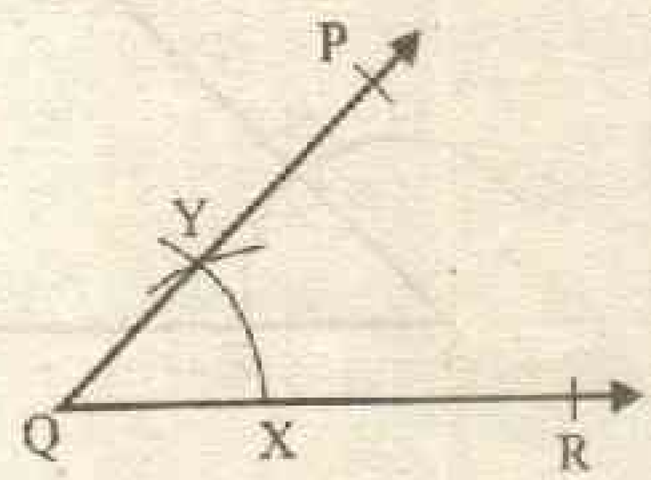


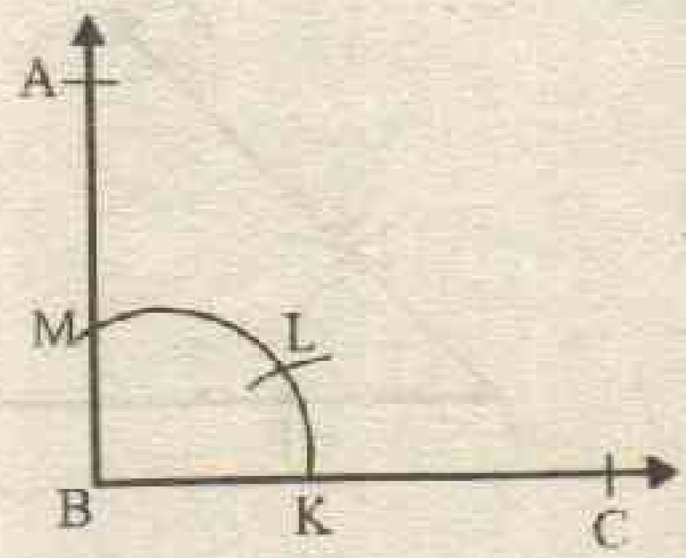
10.2.1 Construct an angle twice in measure of a given angle by using compasses

Example 1 Construct an angle

PQR is a given angle. Construct an angle at point B on $\overrightarrow{BC}$ which is double in measure of $\angle PQR$.

Solution:





Given: $\angle PQR$ is the given angle. $\overrightarrow{BC}$ is a ray with B as its end point.

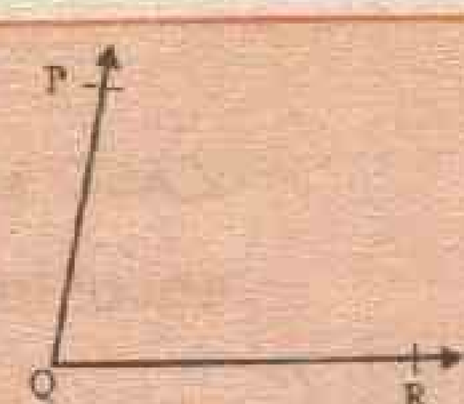
Required: Construct on $\overrightarrow{BC}$ at point B, an angle twice in measure to $\angle PQR$.

Steps of Construction

1. With centre Q, draw an arc of suitable radius to cut $\overrightarrow{QR}$ and $\overrightarrow{QP}$ at X and Y.
2. With centre B and without changing the radius, draw an arc, which cuts $\overrightarrow{BC}$ at K.
3. With K as centre and radius equal to $\overline{XY}$, draw two consecutive arcs to cut the former arc at L and M.
4. Through M, draw $\overrightarrow{BA}$.
5. ABC is the required angle.

Practice

PQR is a given angle. Construct an angle at point B on $\overrightarrow{BC}$ which is double in measure of $\angle PQR$.

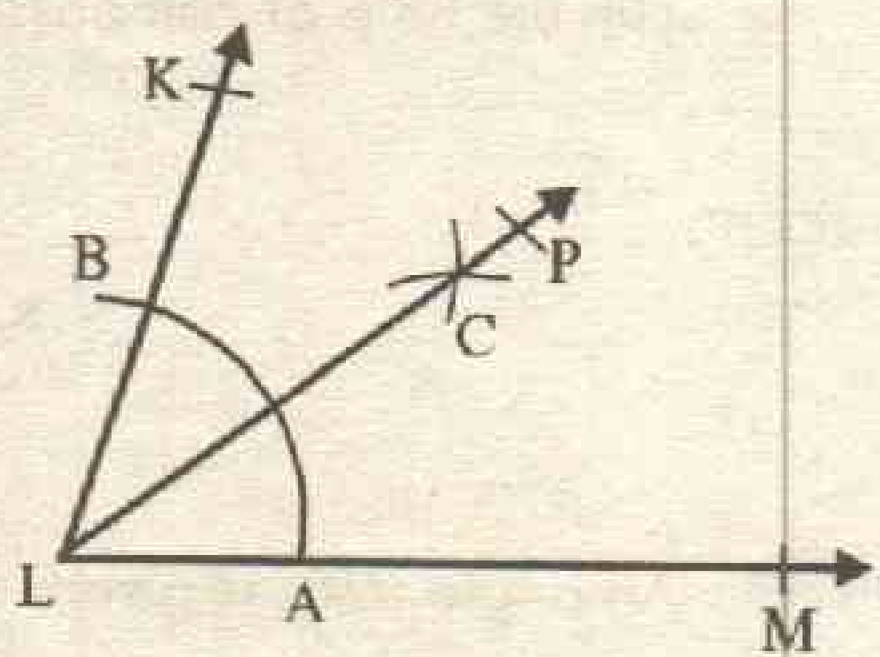


10.2.2 Bisect a given angle by using compasses

Example 1 Bisect an angle

KLM is the given angle. Bisect $\angle KLM$ with the help of compasses.

Solution:



Given: $\angle KLM$.

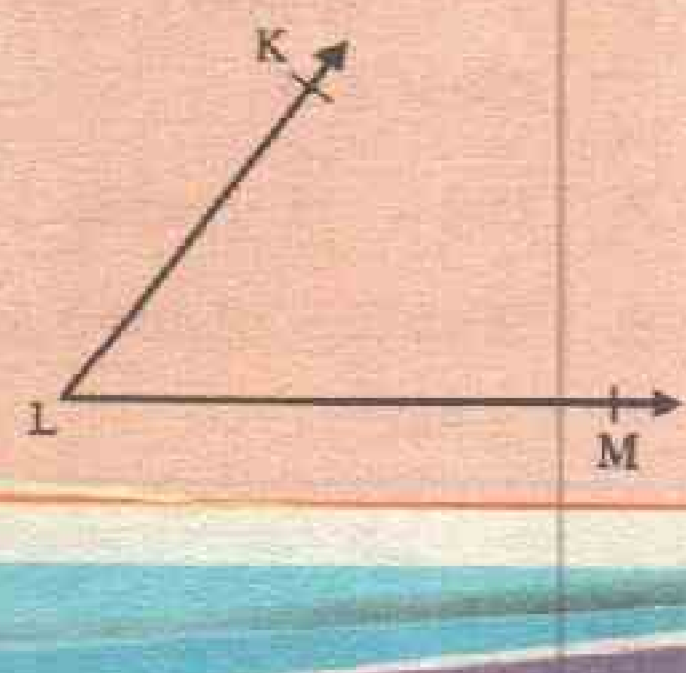
Required: Bisect angle *KLM*.

Steps of Construction

1. With *L* as centre draw an arc of suitable radius to cut $\overrightarrow{LM}$ and $\overrightarrow{LK}$ at *A* and *B* respectively.
2. With *A* as centre draw an arc with radius more than half of $\overline{AB}$.
3. With B as centre draw an arc with the same radius to cut the former arc at C.
4. Through C, draw $\overrightarrow{LP}$.
5. $\overrightarrow{LP}$ is the required bisector of $\angle KLM$ i.e. m $\angle KLP$ = m $\angle PLM$.

Practice

KLM is the given angle. Bisect $\angle KLM$ with the help of compasses.

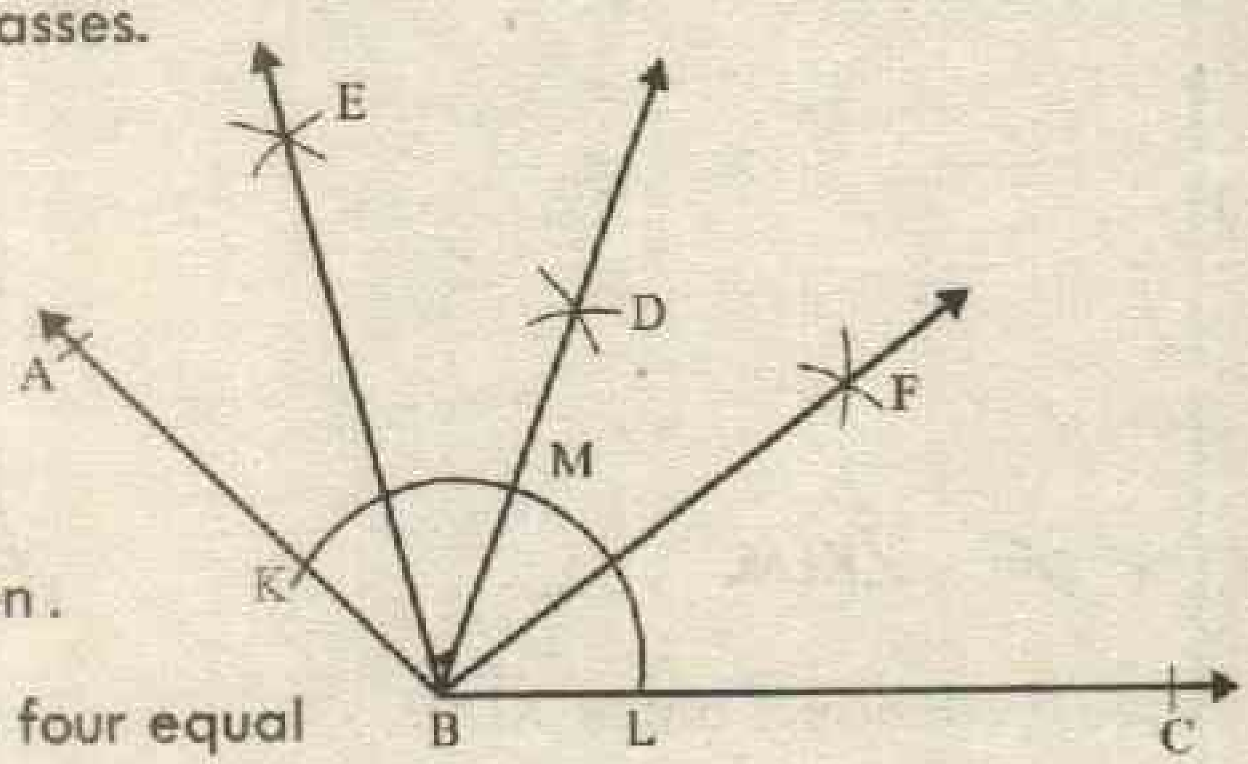


10.2.3 Divide a given angle into four equal angles by using compasses

Example 1 Divide an angle

ABC is a given angle. Divide angle ABC into four equal angles with the help of compasses.

Solution:



Given: An angle ABC is given.

Required: To divide $\angle ABC$ into four equal angles with the help of compasses.

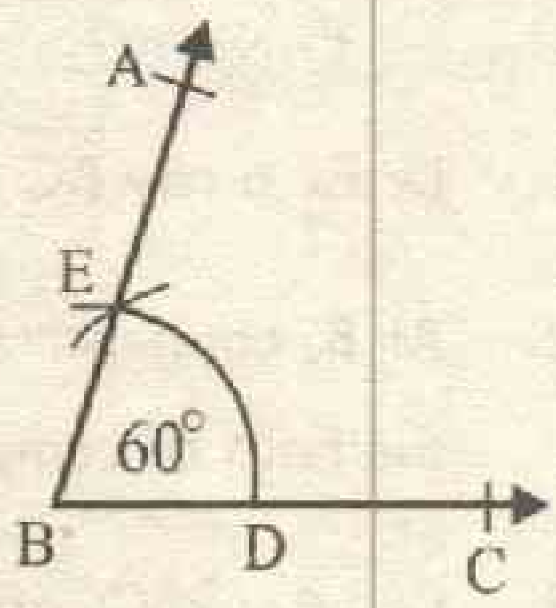
Steps of Construction

1. With B as centre, draw an arc of suitable radius to cut $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at K and L respectively.
2. With centre L, draw an arc with radius more than half of $\overline{LK}$.
3. With centre K, and without changing the radius, draw an arc to cut the former arc at D.
4. Draw $\overrightarrow{BD}$, which cuts the arc KL at M.
5. With L and M as centres draw arcs of suitable radius to cut each other at F.
6. With M and K as centres draw arcs of suitable radius to cut each other at E.
7. Draw $\overrightarrow{BF}$ and $\overrightarrow{BE}$.
8. $\angle ABC$ is divided into four equal angles i.e. $\angle ABE$, $\angle EBD$, $\angle DBF$ and $\angle FBC$.

10.2.4 To construct angles with the help of compasses

a) Construction of an angle of 60° by using compasses

Steps of Construction

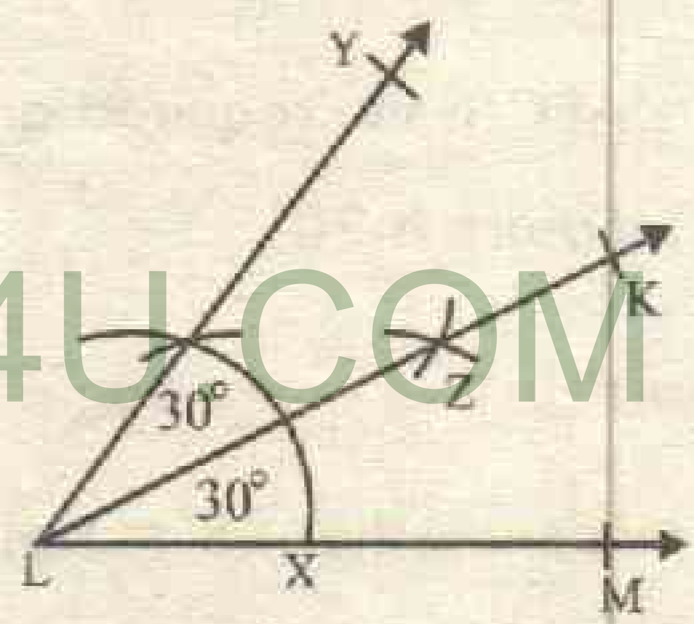


1. Take a ray $\overrightarrow{BC}$
2. With B as centre draw an arc of suitable radius, to intersect $\overrightarrow{BC}$ at D.
3. With D as centre and without changing the radius, draw an arc to cut the former arc at E.
4. Through E, draw $\overrightarrow{BA}$
5. ABC is the required angle i.e. m $\angle ABC = 60^\circ$.

b) Construction of an angle of 30°, by using compasses

As 30° is half of 60°, therefore we shall draw an angle of 60° and then bisect it, to get an angle of 30°.

Steps of Construction

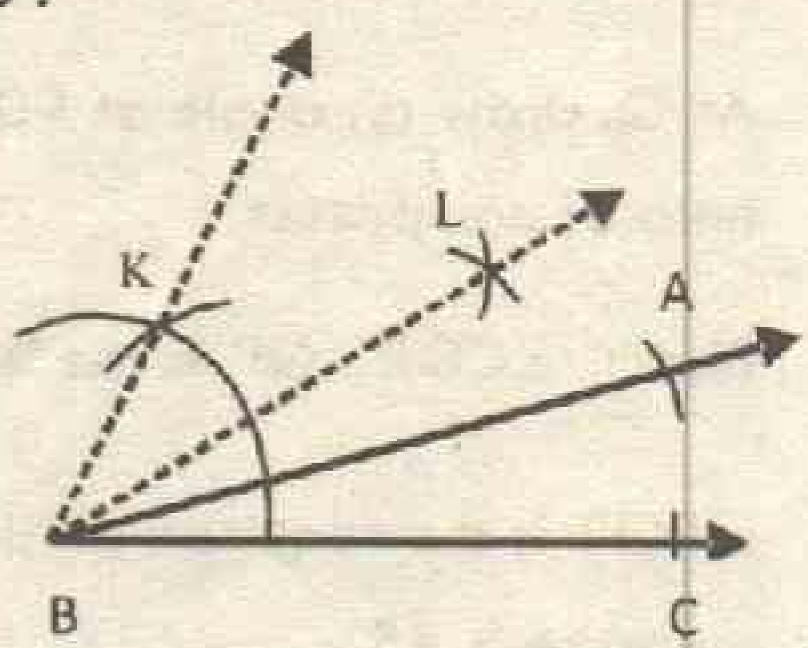


1. Take any ray $\overrightarrow{LM}$.
2. At L, construct an angle YLM measuring 60°.
3. Bisect $\angle YLM$ and draw $\overrightarrow{LK}$ the bisector of $\angle YLM$.
4. $\angle KLM$ is the required angle whose measure is 30°.

c) Construction of an angle of 15° by using compasses

To draw an angle of 15° we shall first construct an angle of 60° and then find its one fourth to get an angle measuring 15°.

Steps of Construction

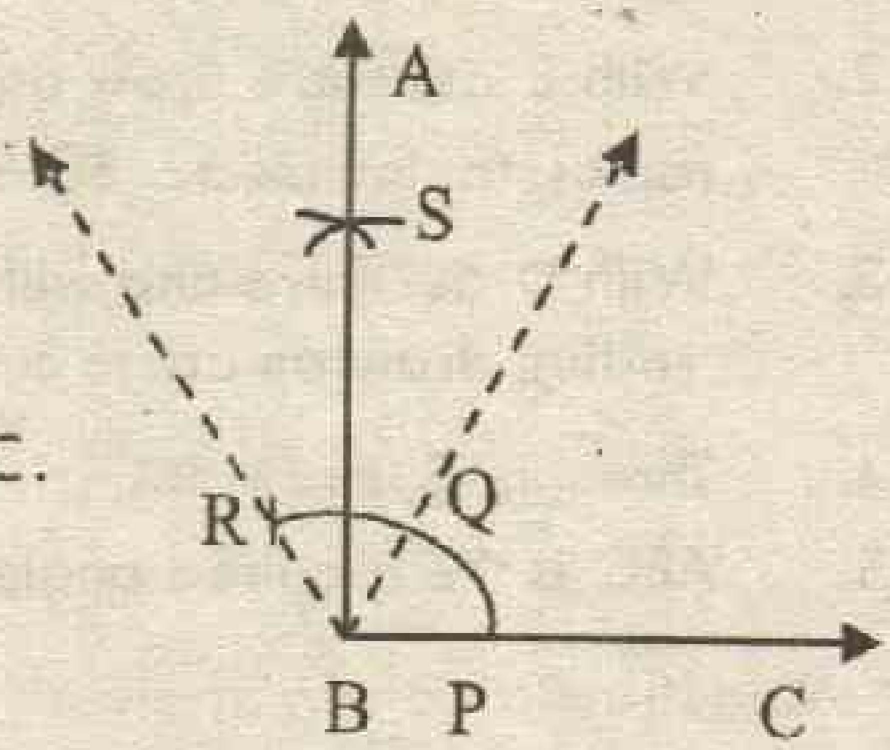


1. Draw $\overrightarrow{BC}$.
2. At B draw an angleKBC whose measure is 600.
3. Draw $\overrightarrow{BL}$, the bisector of $\angle KBC$ i.e. m $\angle LBC = 30^\circ$.
4. Draw $\overrightarrow{BA}$, the bisector of $\angle LBC$.
5. $\angle ABC$ is the required angle whose measure is 15°.

d) Construction of an angle of 90° by using compasses

We know that $90^\circ = 60^\circ + 30^\circ = 60^\circ +$ half of 60°, therefore we shall proceed as follow:

Steps of Construction

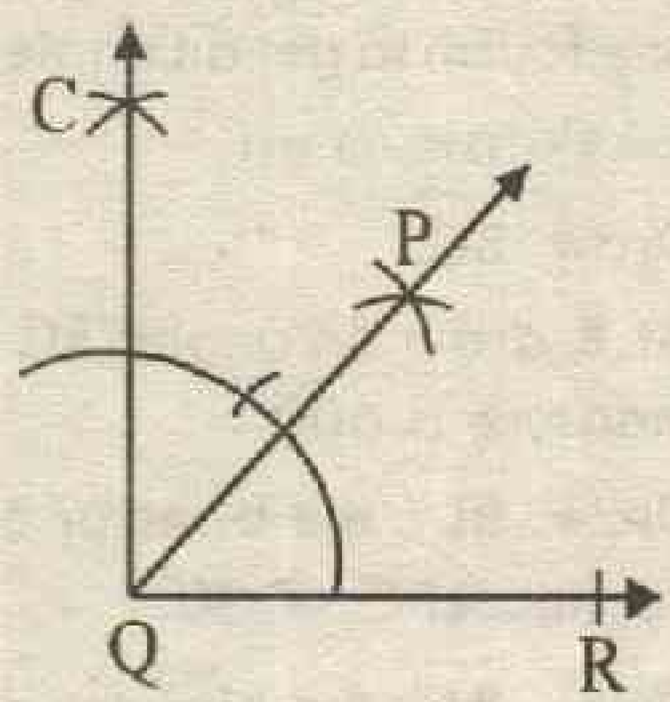


1. Draw a ray $\overrightarrow{BC}$.
2. At B, construct $\angle QBC$ of 60° with the help of compasses.
3. Draw $\angle RBQ$ of 60° adjacent to $\angle QBC$.
4. Bisect $\angle RBQ$. $\overrightarrow{BS}$ is its bisector.
5. Th rough S, draw $\overrightarrow{BA}$.
6. $\angle ABC$ is the required angle whose measure is 90°.

e) Construction of an angle of 45° by using compasses

We know that $45^\circ =$ Half of 90°, therefore we shall draw an angle of 90° and then bisect it to get an angle of 45°.

Steps of Construction



1. Take a ray $\overrightarrow{QR}$.
2. At Q, draw an angle of 90° with the help of compasses.
3. Bisect $\angle CQR$. $\overrightarrow{QP}$ is the bisector.
4. $\angle PQR$ is the required angle whose measure is 45°.

f) Construction of an angle of $22\frac{1}{2}^\circ$ by using compasses.

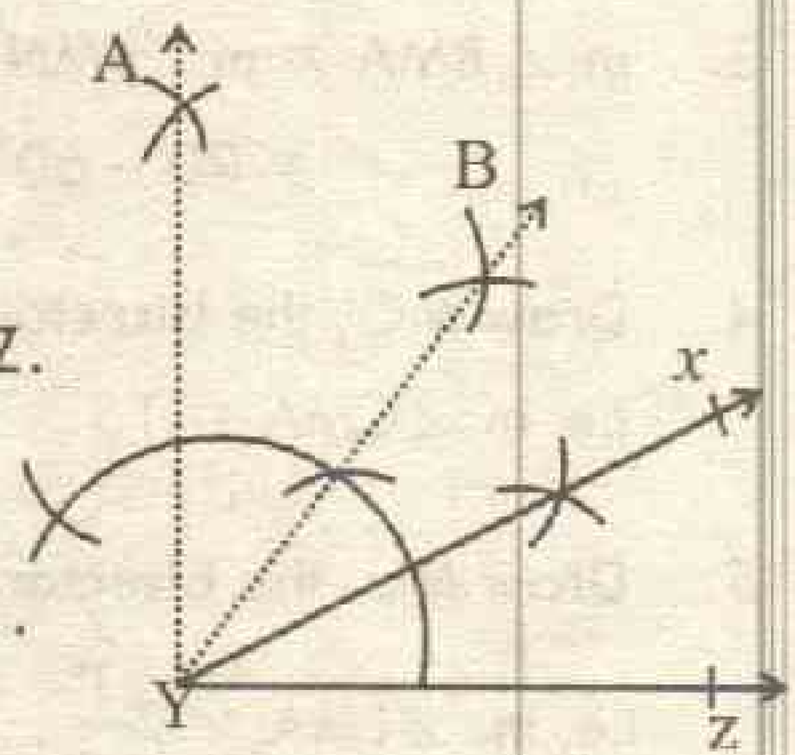
We know that $22\frac{1}{2}^\circ$ = half of 45°. Therefore to draw an angle of $22\frac{1}{2}^\circ$, we will have to bisect the angle of 45°.

Steps of Construction

1. Draw $\overrightarrow{YZ}$.
2. At Y, draw an angle of 90° by using compasses $m\angle AYZ = 90^\circ$.
3. Bisect $\angle AYZ$. $\overrightarrow{YB}$ is the bisector of $\angle AYZ$.

   $m\angle BYZ = \frac{1}{2}(90^\circ) = 45^\circ$.
4. Bisect $\angle BYZ$. $\overrightarrow{YX}$ is the bisector of $\angle BYZ$.

   $m\angle XYZ = \frac{1}{2}(45^\circ) = 22\frac{1^\circ}{2}$.
5. XYZ is the required angle measuring $22\frac{1^\circ}{2}$.



g) Construction of an angle of 75° by using compasses

Steps of Construction

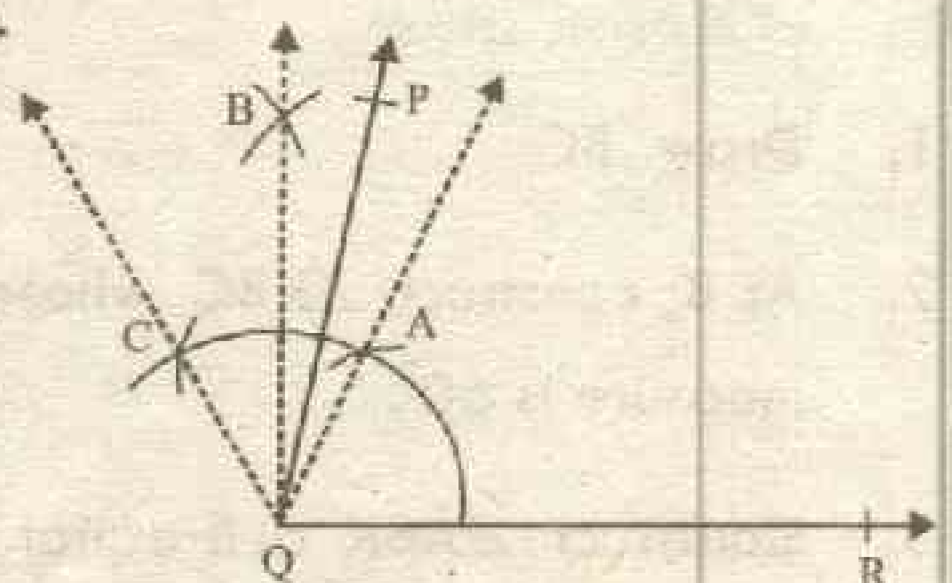
1. Draw $\overrightarrow{QR}$
2. At Q, draw angle $\angle AQR$ and $\angle BQR$ measuring 60° and 90° respectively.
3. $m\angle BQA = m\angle BQR - m\angle AQR$

   $= 90^\circ - 60^\circ = 30^\circ$
4. Bisect $\angle BQA$. $\overrightarrow{QP}$ is the bisector.

   $m\angle PQA = \frac{1}{2}(30^\circ) = 15^\circ$.
5. $\angle PQR$ is the required angle measuring 75°

   $(\because m\angle PQR = m\angle PQA + m\angle AQR$

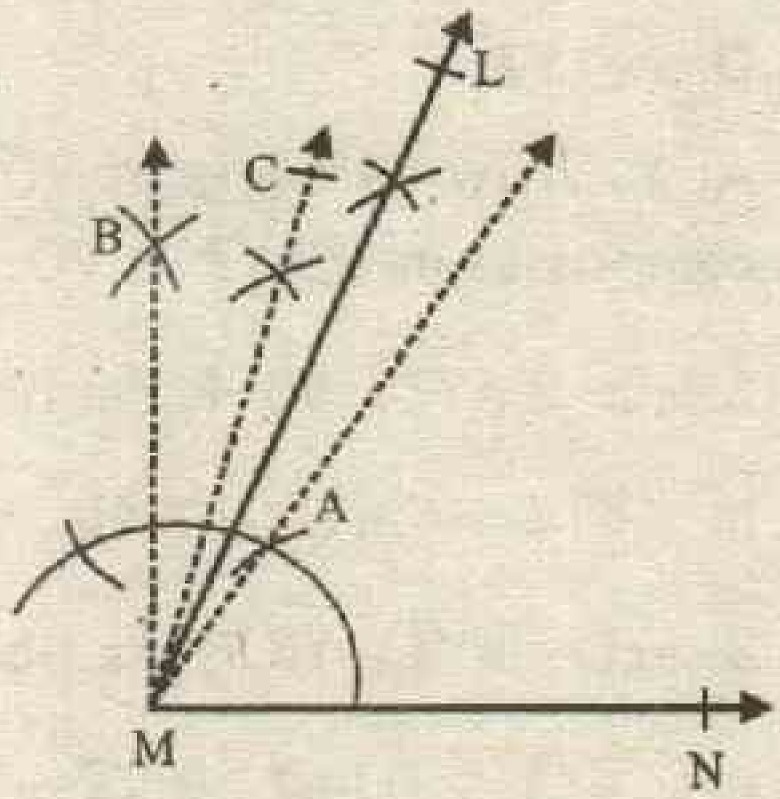
   $= 15^\circ + 60^\circ = 75^\circ)$



h) Construction of an angle of $67\frac{1^\circ}{2}$ by using compasses

Steps of Construction

1. Draw $\overrightarrow{MN}$.
2. At M, construct $\angle AMN$ measuring 60° and $\angle BMN$ measuring 90°.
3. $m\angle BMA = m\angle BMN - m\angle AMN$
   $= 90^\circ - 60^\circ = 30^\circ$



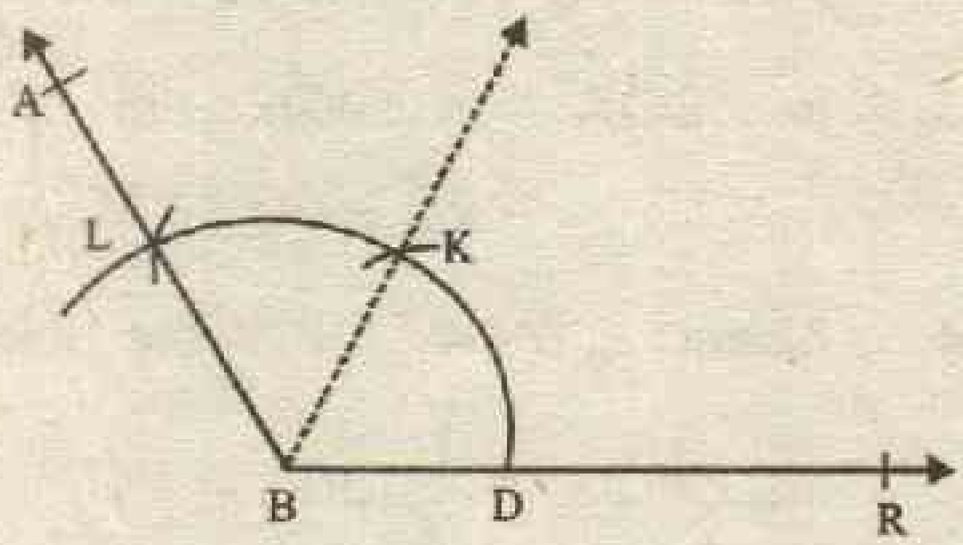
4. Draw $\overrightarrow{MC}$, the bisector of $\angle BMA$,
   i.e. $m\angle CMA = 15^\circ$.
5. Draw $\overrightarrow{ML}$, the bisector of $\angle CMA$,
   i.e. $m\angle LMA = 7\frac{1^\circ}{2}$.
6. $\angle LMN$ is the required angle measuring $67\frac{1^\circ}{2}$.

   $(\because m\angle LMN = m\angle LMA + m\angle AMN$
   $= 7\frac{1^\circ}{2} + 60^\circ = 67\frac{1^\circ}{2})$

i) Construction of an angle of 120° by using compasses

Steps of Construction

1. Draw $\overrightarrow{BC}$
2. At B, construct $\angle KBC$ whose measure is 60°.



3. Construct $\angle ABK$ at B equal in measure to $\angle KBC$ and adjacent to it.
4. Measure of $\angle ABC$ is twice the measure of $\angle KBC$.
5. ABC is the required angle whose measure is 120°.

Exercise 10.2

1. Construct angle of the following measures with the help of protractor:
   (i) 65° (ii) 73° (iii) 55°

   Construct angles equal in measure to these angles by using compasses and ruler.

2. Construct angles of the following measures with the help of protractor:
   (i) 35° (ii) 42° (iii) 70°

   Construct angles twice in measure of the given angles by using compasses and ruler.

3. Construct angles of the following measures with the help of protractor:
   (i) 76° (ii) 68° (iii) 98°

   Bisect these angles with the help of compasses. Measure each part.

4. Construct angles of the following measures with the help of protractor:
   (i) 96° (ii) 112° (iii) 128°

   Divide these angles into four equal angles with the help of compasses.

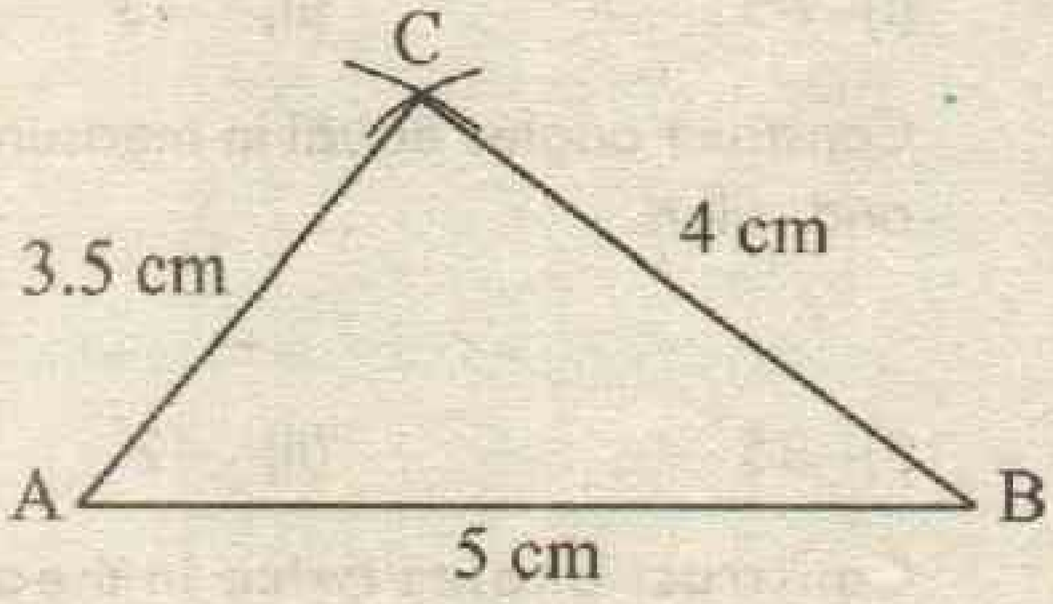
5. Construct angles of the following measures by using compasses and ruler only:
   (i) 150° (ii) 165° (iii) 135° (iv) 105°

10.3 Construction of a triangle when measures of all the three sides (SSS) are given

Example 1 Construct triangle

Construct a triangle ABC when m $\overline{AB}$ = 5cm, m$\overline{BC}$ = 4cm and m$\overline{CA}$ = 3.5cm.

Solution:



Steps of Construction

1. Draw a line segment $\overline{AB}$ measuring 5cm.
2. With B as centre draw an arc of radius equal in measure to $\overline{BC}$ i.e. 4cm.
3. With C as centre draw another arc of radius equal in measure to $\overline{CA}$ i.e. 3.5cm to cut the pervious arc at C.
4. Draw $\overline{BC}$ and $\overline{CA}$.
5. ABC is the required triangle.

Note

(i) Sum of the measures of any two sides of a triangle is always greater than the measure of its third side. This result can be verified by measuring the sides of different triangles. Formal proof is beyond the mental level of 6th class students. This means that we cannot construct a triangle having sides 4.5cm, 2cm and 1..5cm long, because sum of the lengths of two of its sides is 2 + 1.5 = 3.5cm which is less than the measuring of its third side i.e. 4.5cm.

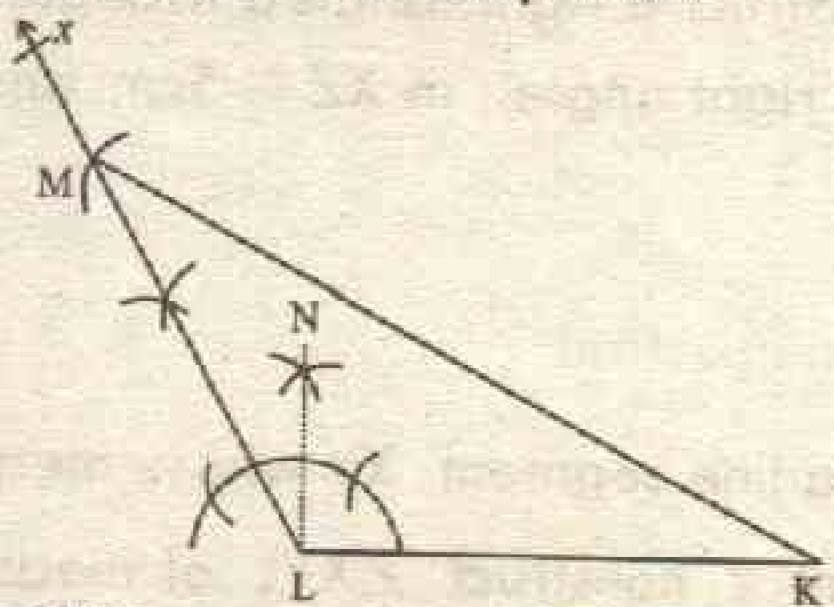
(ii) Similarly we cannot construct a triangle when the sum of the measures of two of its sides is equal to the measure of is third side e.g. 5cm 3cm and 2cm.

10.3.1 Construction of a triangle when measures of its two sides (SAS) and their included angle are given

Example 1 Construct triangle

Construct triangle KLM when m $\overline{KL}$ = 5.5cm, m $\overline{LM}$ = 4cm and m $\angle L = 105^\circ$

Solution:



Steps of Construction

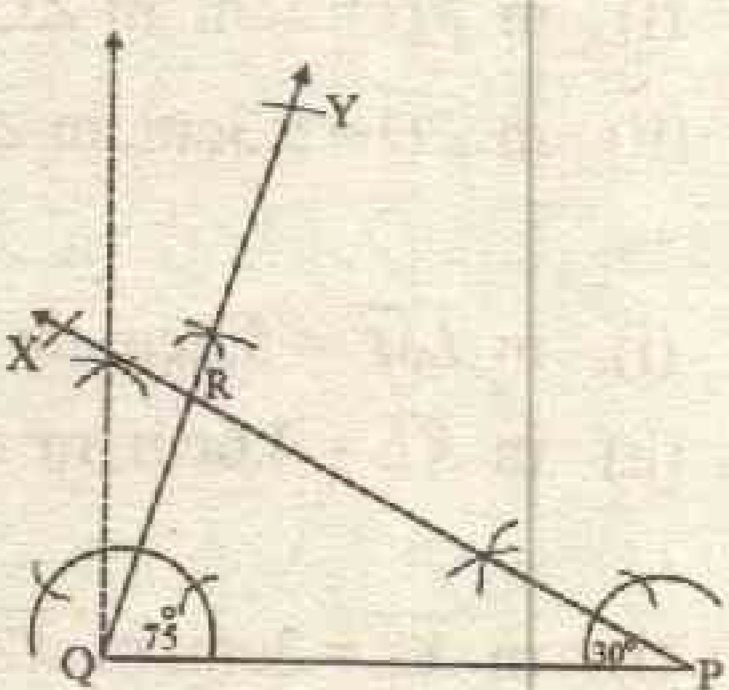
1. Draw a line segment $\overline{KL}$ whose measure is 5.5cm.
2. At point L, construct an angle KLX measuring 105°.
3. With L as centre, draw an arc of radius 4cm to cut $\overrightarrow{LX}$ at M.
4. Join M to K.
5. KLM is the required triangle.

10.3.2 Construction of a triangle when measures of two of its angles and the included side (ASA) are given

Example 1 Construct triangle

Construct Δ PQR when m PQ = 5cm, m $\angle P = 30^\circ$ and m $\angle Q = 75^\circ$.

Steps of Construction

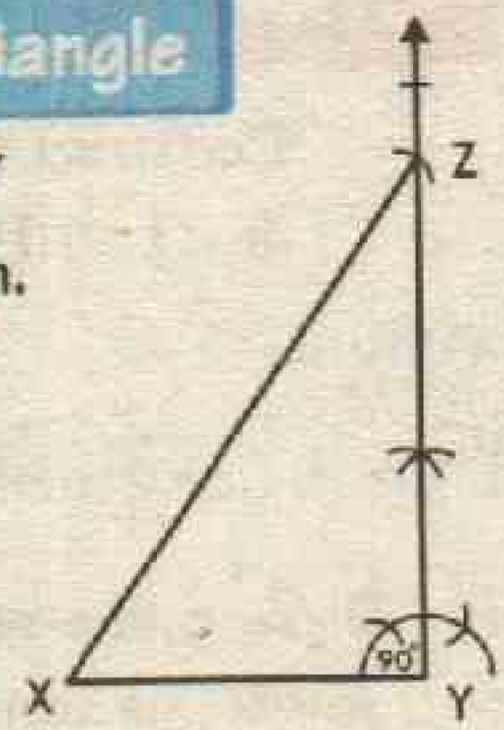


1. Draw a line segment $\overline{PQ}$ whose measure is 5cm.
2. At P, construct $\angle QPX$ measuring 30°.
3. At Q, construct $\angle PQY$ whose measure is 75°.
4. $\overrightarrow{PX}$ and $\overrightarrow{QY}$ intersect each other at R.
5. PQR is the required triangle.

10.3.3 Construction of a right-angled triangle when the measures of its hypotenuse and one of its sides (RHS) are given

Example 1 Construct right angle triangle

Construct a right-angled triangle XYZ in which Y is a right angle, m $\overline{XZ}$ = 5cm and m $\overline{XY}$ = 3.5cm.



Solution:

Steps of Construction

1. Draw a line segment $\overline{XY}$ whose measure is 3.5cm.
2. At point Y, construct $\angle XYL$ of measure 90°.
3. With X as centre draw an arc of radius 5cm to intersect $\overrightarrow{YL}$ at Z.
4. Join Z to X.
5. Δ XYZ is the required triangle.

Exercise 10.3

1. Construct ΔABC, when:
   (i) m $\overline{AB}$ = 7cm, m $\overline{BC}$ = 3cm and m $\overline{CA}$ = 6cm
   (ii) m $\overline{AB}$ = 4.5cm, m $\overline{BC}$ = 3.5cm and m $\overline{CA}$ = 2.5cm
2. Construct ΔPQR, when:
   (i) m $\overline{PQ}$ = 5cm, m $\angle Q = 45^\circ$ and m $\overline{QR}$ = 4cm
   (ii) m $\overline{PQ}$ = 5.3cm, m $\angle P = 60^\circ$ and m $\overline{PR}$ = 4.2cm
3. Construct ΔKLM, when:
   (i) m $\overline{LM}$ = 5.2cm, m $\angle L = 45^\circ$ and m $\angle M = 30^\circ$
   (ii) m $\overline{KL}$ = 4.6cm, m $\angle K = 90^\circ$ and m $\angle L = 60^\circ$
4. Construct ΔXYZ, when:
   (i) m $\overline{XY}$ = 3cm, m $\overline{XZ}$ = 5cm and m $\angle Y = 90^\circ$
   (ii) m $\overline{YZ}$ = 6.5cm, m $\overline{XY}$ = 2.5cm and m $\angle X = 90^\circ$

Review Exercise 10

1. Draw $\overline{AB}$ and $\overline{CD}$ such that $m\overline{AB} = 3.6cm$ and $m\overline{CD} = 1.9cm$. Draw $\overline{KL}$ where $m\overline{KL} = m\overline{AB} + m\overline{CD}$. Measure the length of $\overline{KL}$.

2. Draw $\overline{XY}$ 5.8cm long. Cut off $\overline{XA}$ where $m\overline{XA} = 2.6cm$. Measure the length of $\overline{AY}$.

3. Draw $\overline{KL}$ such that $m\overline{KL} = 6.4cm$. Cut off $\overline{KT}$ where $m\overline{KT} = 3cm$. Measure the length of $\overline{TL}$.

4. Draw $\overline{AB}$ 5.8cm long. Draw its right bisector with the help of ruler and compasses only.

5. Draw $\overline{AB}$ 4.8cm long. Take a point C on $\overline{AB}$ such that $m\overline{AC} = 1.6cm$. At C, draw a perpendicular to $\overline{AB}$ by using ruler and compasses only.

6. Draw $\overline{AB}$ 3.9 cm long. Take a point C outside $\overline{AB}$ and draw perpendicular to $\overline{AB}$ from C. (use ruler and compass only).

7. With the help of protractor, draw $\angle ABC$ such that $m\angle ABC = 80°$. Using ruler and compasses only

   (i) Construct an angle equal in measure to $\angle ABC$.

   (ii) Construct an angle twice in measure of $\angle ABC$.

   (iii) Draw the bisector of $\angle ABC$.

   (iv) Divide $\angle ABC$ into four equal angles.

   Verify the measures with the help of protractor.

8. Using ruler and compasses only, construct ΔABC, where

   (i) $m\overline{AB} = 6.5cm$, $m\overline{BC} = 4cm$ and $m\overline{CA} = 3.5cm$

   (ii) $m\overline{BC} = 4.4cm$, $m\angle B = 22\frac{1}{2}°$ and $m\angle C = 67\frac{1}{2}°$

   (iii) $m\overline{AB} = 4.5cm$, $m\angle B = 90°$ and $m\overline{AC} = 5.9cm$

Summary

- With the help of ruler and a pair of compasses.

  (i) A perpendicular can be drawn to a line from a point on the line as well as from a point outside the line.

  (ii) Right bisector of a given line segment can be drawn.

  (iii) Angles where measures are the multiples of $7\frac{1}{2}^\circ$ can be constructed.

  (iv) An angle can be constructed whose measure is
  (a) equal to
  (b) two times
  (c) half of
  (d) one fourth of

  a given angle.

- A triangle can not be constructed when the sum of the measures of any two of its sides is less than or equal to the measure of its third side.

- A triangle can be constructed when:

  (i) measures of its three sides are given.
  (ii) measures of its two sides and their included angle are given.
  (iii) measures of two of its angles and the included side are given.
  (iv) measures of its hypotenuse and one of its sides are given.

UNIT 11 PERIMETER AND AREA

What You'll Learn

- Find perimeter and area of a square and a rectangle.
- Find area of path (inside or outside) of a rectangle or square.
- Solve real life problems related to perimeter and area of a square and rectangle.
- Recognize altitude of a geometric figure as the measure of the shortest distance between the base and its top.
- Find area of a parallelogram when altitude and base are given.
- Define trapezium and find its area when altitude and measures of the parallel sides are given.
- Find area of a triangle when measures of the altitude and base are given.

Why It's important

Area and perimeter are two important and fundamental parts of mathematics. The unit and topics of area and perimeter are important to mathematics because they are the physical aspects of mathematics. They are the foundation for understanding other aspects of geometry such as volume and mathematical theorems that help us understand algebra, trigonometry, and calculus.

11.1 Perimeter of a Rectangle

11.1.1 Definition of Perimeter of a Rectangle

The perimeter of a rectangle is twice the sum of the length and width.

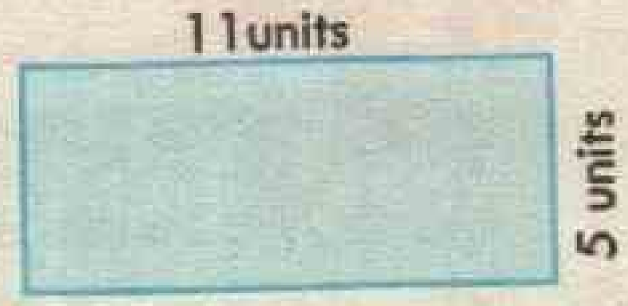
$P = l + l + w + w$

$P = 2l + 2w \quad \text{or } 2(l + w)$

Distance around a figure is called its Perimeter.

Example 1 Find the Perimeter of a Rectangle

Find the perimeter of the rectangle.



Solution:

$P = 2\,(l + w)$ Write the formula.

$P = 2\,(11 + 5)$ Replace l with 11 and w with 5.

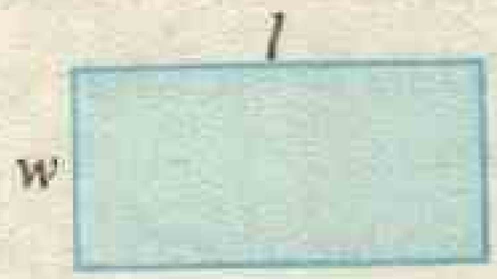
$P = 2\,(16)$ Add 11 and 5.

$P = 32$ units Simplify.

11.1.2 Area of a Rectangle

The area of rectangle is the product of the length and width.

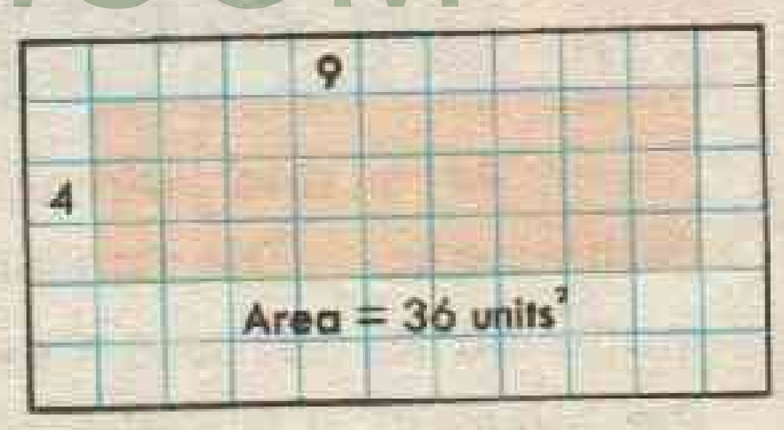
$A = l\,w$



The measure of the surface enclosed by a figure is its area.

Example 1 Find the Area of a Rectangle

Find the area of the rectangle shown.



Solution:

The rectangle has an area of 9×4 or 36 square units.

Example 2 Find the Area of a Rectangle

Find the area of the rectangle with length 15 metres and width 7 metres.

Solution:

$A = l\,w$ Write the formula.

$A = 15 \times 7$ Replace l with 15 and w with 7.

$A = 105\text{m}^2$ Simplify.

Example 3 Find a Missing Width

The area of a rectangle is 16 square feet. Its length is 8 feet. Find its width.

Solution:

$A = l\,w$	Write the formula.
$16 = 8w$	Replace A with 16 and l with 8.
$2 = w$	Mentally divide each side by 8.

Thus the width is 2 feet.

Practice:

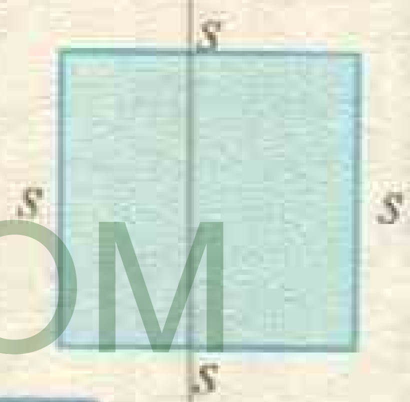
Find the perimeter and area of a 14-meter by 6-meter rectangle.

11.2 Perimeter of a Square

The perimeter of a square = 4 × measure of one side.

$$P = s + s + s + s$$

$$P = 4s$$



Example 1 Find the Area of a Square

Find the area of a square having its one side 36m long.

Solution:

Length of one side $= 36\text{m}$

Area of the square $= (36 \times 36)\text{m}^2$

$= 1296\,\text{m}^2$

Example 2 Find the Area of a Square

Find the area of the square shown.

Solution:

The square has an area of 3×3 or 9 square units.

Example 3 Find Perimeter

A garden is square shaped. If the length of one of its sides is 5.2 meter, find its perimeter.

Solution:

Length of one side of the garden = 5.2m

Perimeter = 4×5.2

= 20.8m

Example 4 Find Perimeter

Four square shaped flower beds having one side 2.5m long were formed in a rectangular shaped garden measuring 18×5m. Find the perimeter of the garden. Also find the perimeter of four flower beds.

Solution:

Length of the garden = 18 m

Breadth of the garden = 5 m

Perimeter of the garden = 2 (length + breadth)

= 2 (18 + 5)m

= 2 (23) m

= 46 m.

Length of one side of the flower bed = 2.5 m

Perimeter of the flower bed= $4 \times$ length of side

= 4×2.5 m

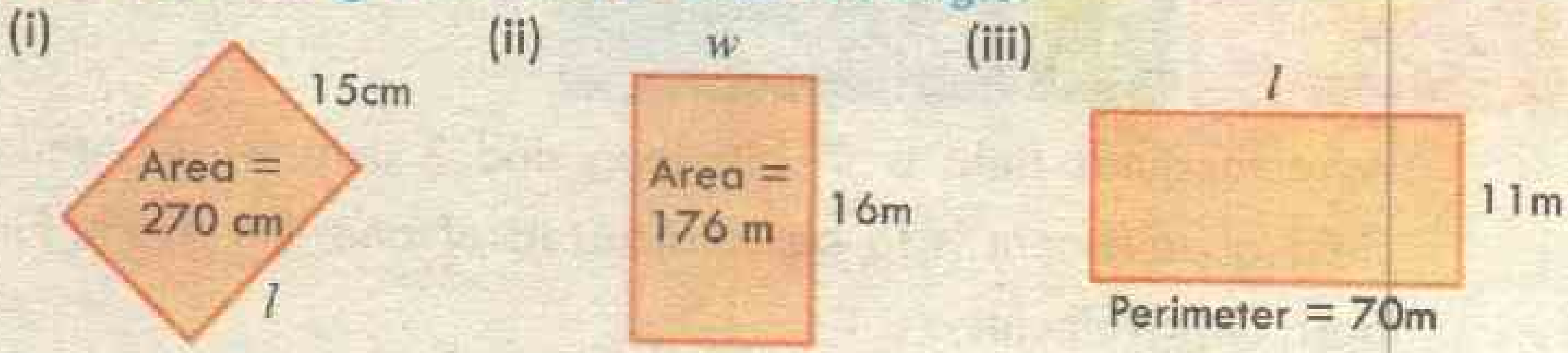
= 10 m

Perimeter of 4 flower beds = 10×4

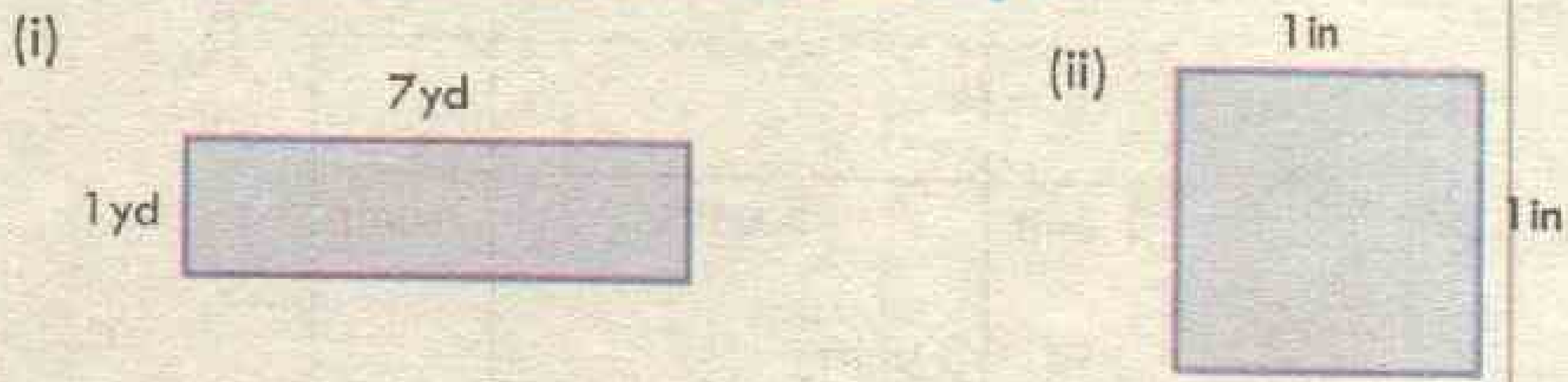
= 40 m

Exercise 11.1

1. Find the missing dimension in each rectangle.



2. Find the perimeter and area of each figure.



3. Find the perimeter and area of squares having one side equal to:
   (i) 20cm (ii) 3.6m (iii) 59cm

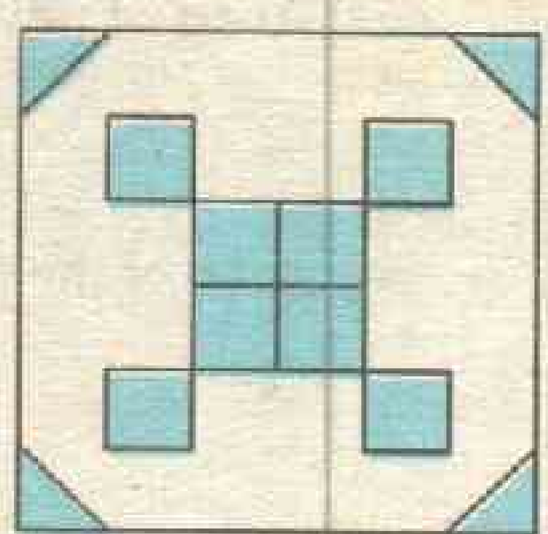
4. Find the perimeter and area of rectangles having:
   (i) Length = 9 cm, Breadth = 5 cm
   (ii) Length = 40.2 m, Breadth = 27.8 m

5. Assembly ground of a school is 150m long and 100m wide. Find the cost of its brick flooring if the rate is Rs.1500 per $100m^2$.

6. A room having 5m length and 3.5m breadth is to be carpeted. Find the total cost, if the rate is 545 rupees per square meter.

7. A gardener made 6 square shaped flower beds in a rectangular garden having length 20m and breadth 6m. If the side of each flower bed is 2.5m, find the remaining area of the garden.

8. The illustration shows a table mat which has a decorative pattern.
   If each coloured square has an area of $1cm^2$, can you find:
   a) the total area which is coloured?
   b) the area of the whole mat?



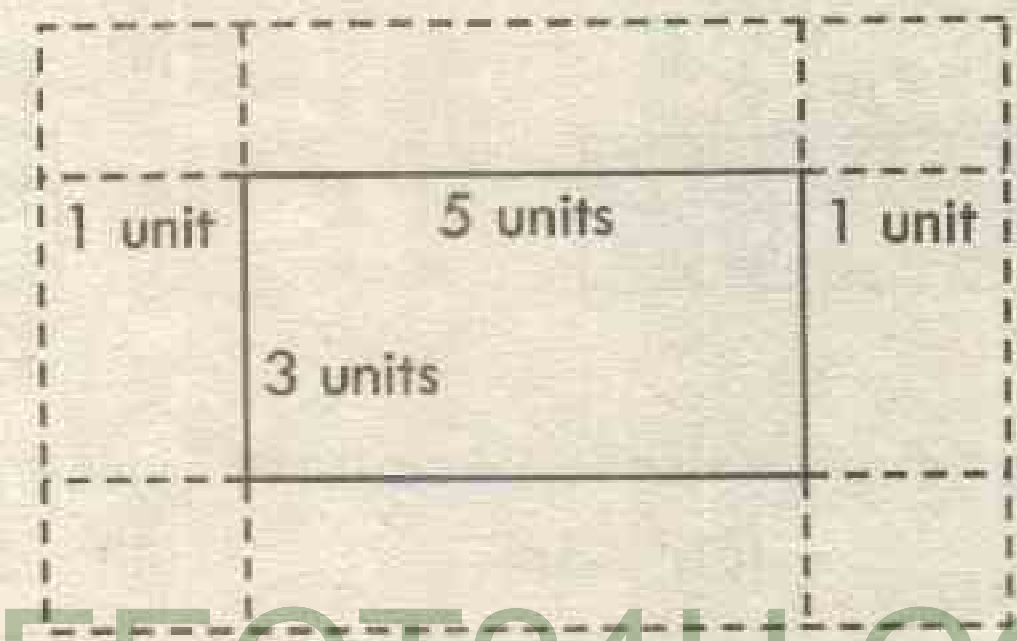
11.3 Area of Path

To find area of path (inside or outside) of a rectangle and a square:

Example 1 Find Area

A rectangular field is 5 units long and 3 units wide. Around this field a pavement is to be constructed of one unit width. Find the area of pavement.

Solution:



Area of the given field $= 5 \times 3$ square units

$= 15$ square units

By constructing a pavement around the field we get another rectangle whose length and width are 2 units more than those of the given rectangular field.

i.e. Length of new rectangle $= 5 + 1 + 1 = 7$ units.

Width of new rectangle $= 3 + 1 + 1 = 5$ units.

Area of new rectangle = Length x width

$= 7 \times 5 = 35$ square units

Area of the pavement = Area of new rectangle – Area of given rectangle

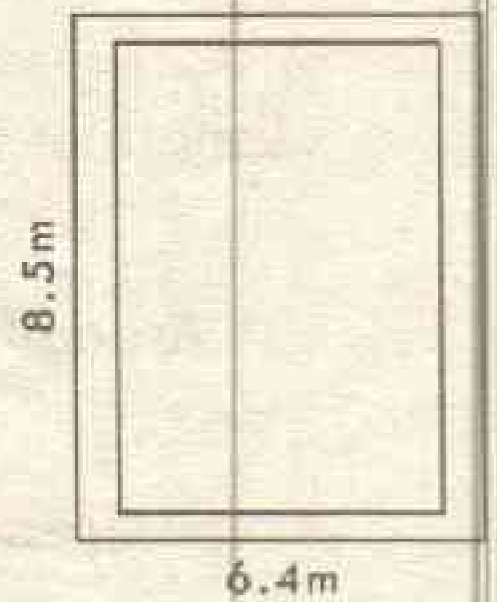
$= 35 - 15$

$= 20$ square units

Example 2 Find Area

A conference hall is 8.5m long and 6.4m wide. A carpet is laid in the Hall. Length and breadth of the carpet are 8m and 5.9m respectively. Find the area of the uncovered place of the conference Hall.

Solution:



Area of the conference hall $= 8.5 \times 6.4$

$= 54.4m^2$

Area of the carpet $= 8 \times 5.9$

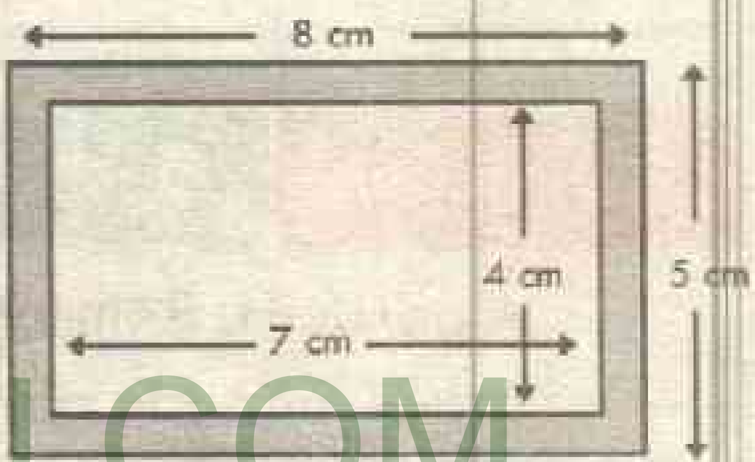
$= 47.2m^2$

Area of the uncovered place $= 54.4 - 47.2$

$= 7.2m^2$

Practice:

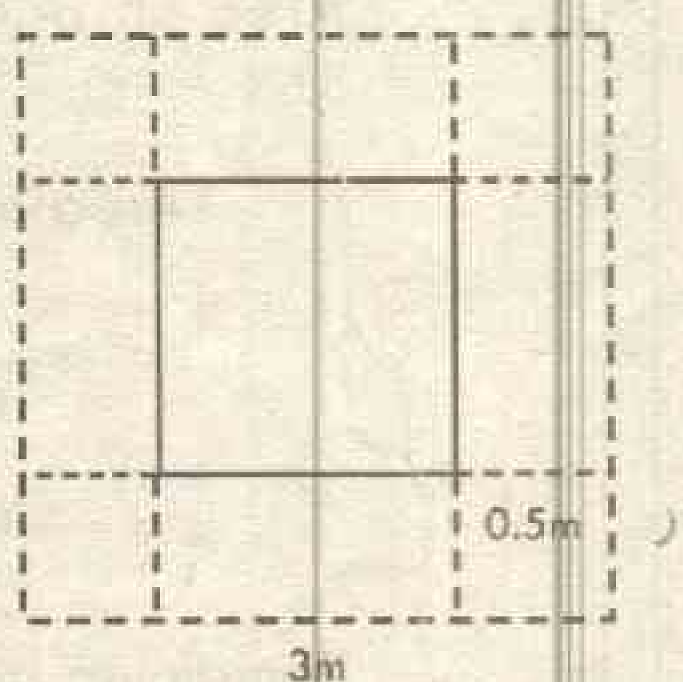
Find the area of the shaded part of each shape.



Example 3 Find Area

A square flower bed has one of its sides 3m long. A strip 0.5m wide is earmarked for planting roses inside the bed along with the four sides of the square flower bed. Find the area of the strip.

Solution:



Area of the outer square $= 3 \times 3$

$= 9m^2$

Area of the inner square $= 2.5 \times 2.5$

$= 6.25m^2$

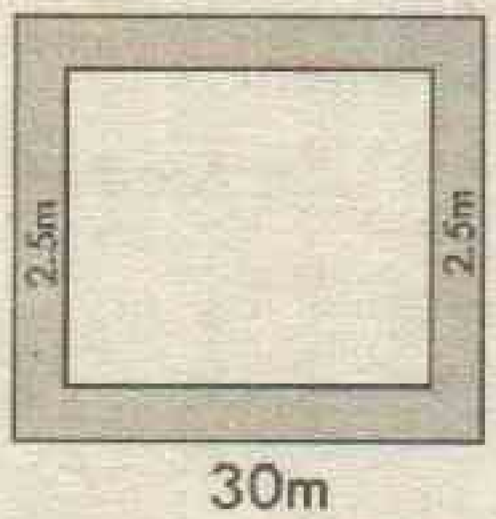
$\therefore$ Area of the strip $= 9 - 6.25$

$= 2.75m^2$

Example 4 Find Area

A square shaped garden is 30 m long. A pavement 2.5m wide is made all around it. Find the area of the pavement.

Solution:



Area of the garden $= 30 \times 30$
$= 900\ m^2$

Length of the outer square $= 30 + 2.5 + 2.5$
$= 35m$

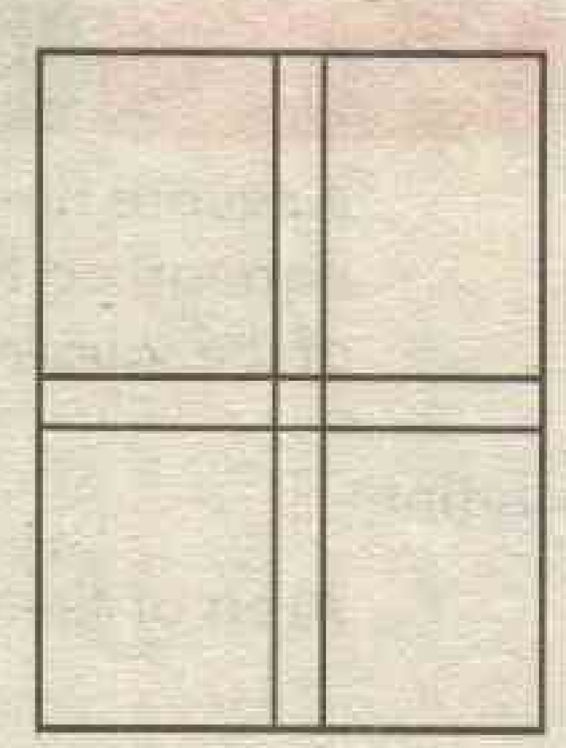
Area of the outer square $= 35 \times 35$
$= 1225m^2$

Area of the pavement $= 1225 - 900$
$= 325\ m^2$

Example 5 Find Area

A garden is 84m long and 68m wide. In the middle of the garden two roads 5m wide parallel to its sides are constructed. Find the area of the roads. Also find the area of the remaining part of the garden.

Solution:



Area of the garden $= 84 \times 68$
$= 5712\ m^2$

Area of road parallel to length $= 84 \times 5$
$= 420\ m^2$

Area of road parallel to width $= 68 \times 5$
$= 340\ m^2$

Area of the inner square $= 5 \times 5$
$= 25\ m^2$

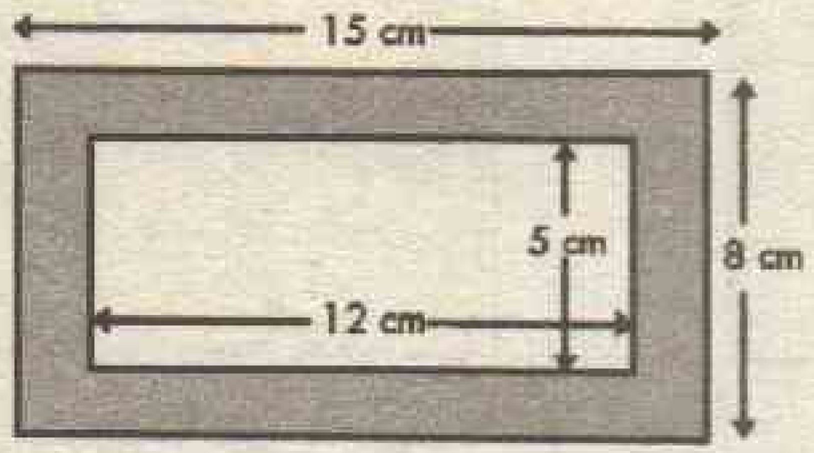
Area of the roads $= 420 + 340 - 25$
$= 760 - 25$
$= 735\ m^2$

Remaining area of the garden $= 5712 - 735$
$= 4977\ m^2$

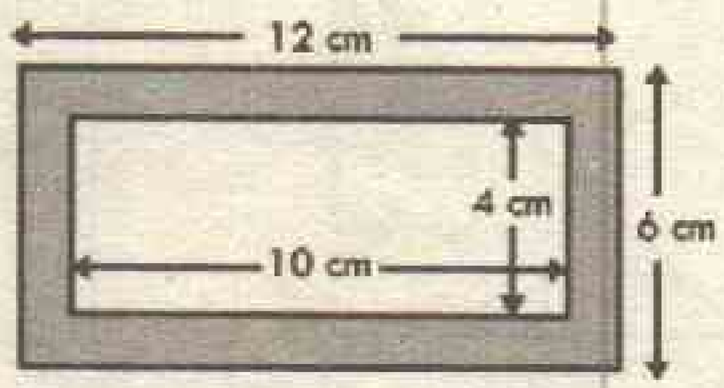
Exercise 11.2

1. Find the area of the shaded part of each shape.

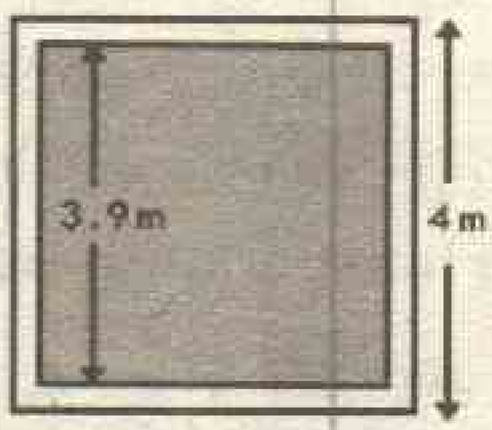
i)



ii)



2. A square carpet of side length 3.9 m is laid in a room measuring 4 m by 4 m. Find the area of floor left uncovered.



3. A square shaped park is 170 m long. A road 6 m wide has been constructed along the outer side of the boundary. Find the area of the road.

4. A park is of square shape and one side of the park is 275 m long. A road 9m wide has been constructed along the inner side of the boundary. Find the area of the road.

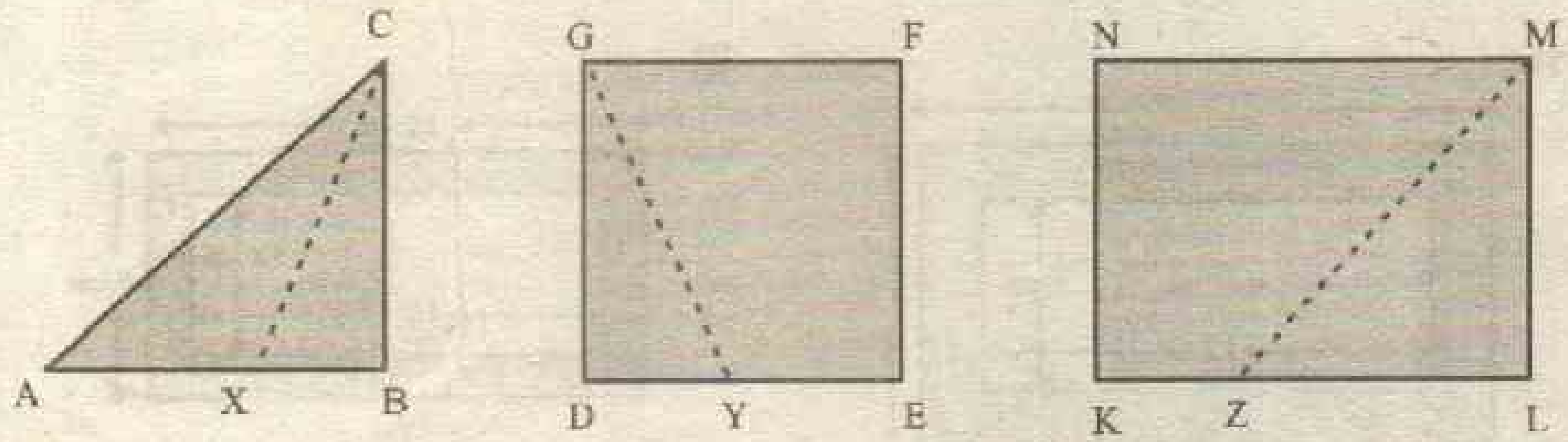
5. A rectangular park is 190 m long and 160 m wide. A road 8m wide has been constructed along the outer side of the boundary. Find the area of the road.

6. A pavement 4 m wide is constructed along the outer side of the boundary of a square shaped ground having its side of 13 m length. Find the cost of levelling the pavement at the rate of Rs.1.25 per square meter.

7. A park is 110 m long and 96 m wide. In the middle of the park two roads 4 m wide parallel to its sides are constructed. Find the area of the roads.

11.4 Altitude of a geometric figure

Altitude of a geometric figure is the measure of the shortest distance between the base and its opposite vertex. Draw following figures on a paper.



Measure $\overline{CB}, \overline{GD}$ and $\overline{ML}$

Take any point x on $\overline{AB}$, Y on $\overline{DE}$ and Z on $\overline{KL}$. Measure $\overline{CX}$, $\overline{GY}$ and $\overline{MZ}$.

What is the relation between the measures of ($\overline{CB}$ and $\overline{CX}$), ($\overline{GD}$ and $\overline{GY}$) and ($\overline{ML}$ and $\overline{MZ}$)?

Have you noticed that

$$m\,\overline{CX} > m\,\overline{CB},$$
$$m\,\overline{GY} > m\,\overline{GD}, \quad \text{and}$$
$$m\,\overline{MZ} > m\,\overline{ML}$$

Check by taking other points on $\overline{AB}$, $\overline{DE}$ and $\overline{KL}$ and joining these points to C, G or F, and N or M respectively. You will see that in each case m $\overline{CB}$, m $\overline{GD}$ or m $\overline{FE}$ and m $\overline{NK}$ or m $\overline{ML}$ is the shortest.

What is common in all these line segments?

These line segments are perpendiculars to the bases from the vertices of the geometric figures.

Measure of the shortest distance between the bases and its vertex is called '**Altitude**'.

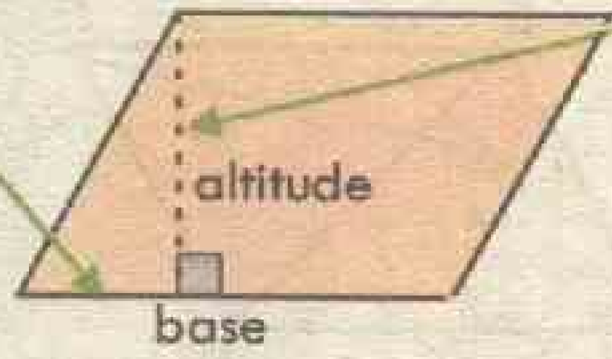
In the above figures $\overline{CB}$, $\overline{GD}$, $\overline{ML}$, $\overline{KR}$, $\overline{FQ}$, $\overline{SD}$, $\overline{PX}$ and $\overline{OY}$ are the altitudes.

So

> Altitude of a geometric figure is the measure of the shortest distance between the base and its opposite vertex

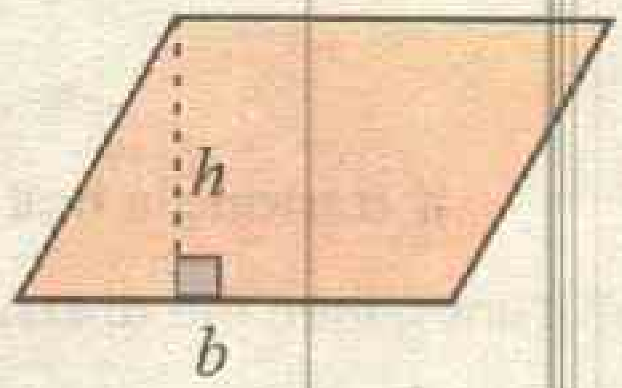
11.4.1 To find the area of a parallelogram when altitude and base are given:

The base can be any side of the parallelogram.



The height is the length of altitude, to the opposite side the base.

If a parallelogram has a base of base of b units and a height of h units then the area A is bh square units.



$$A = bh$$

Example 1 Find Area

Find the area of the following parallelograms.

Solution:

(a)

(b)

a) area = base × height
$= 15 \times 5$
$= 60\text{ cm}^2$

b) area = base × height
$= 2.5 \times 1.2$
$= 3\text{ cm}^2$

Example 2 Find Area

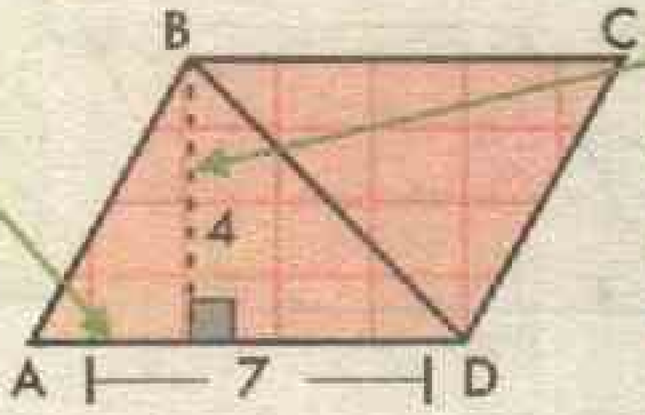
Find the area of a parallelogram with base 675 m and altitude 250 m.

Solution:

Base of the parallelogram = 675m
Altitude of the parallelogram = 250m
∴ Area of the pardlelogram = $(675 \times 250)\text{ m}^2$
$= 168750\text{ m}^2$

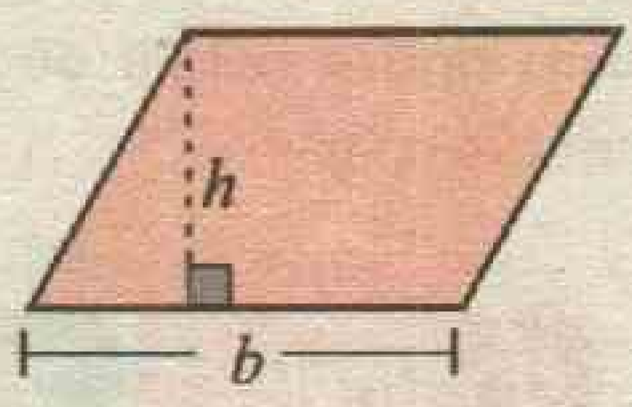
11.4.2 To find the area of a triangle when measures of the altitude and base are given:

The area of parallelogram ABCD



The area of triangle ABD is $\frac{1}{2} \times 28$ or 14 square units.

If a triangle has a base of b units and a height of h units, then the area A is $\frac{1}{2}bh$ square units.



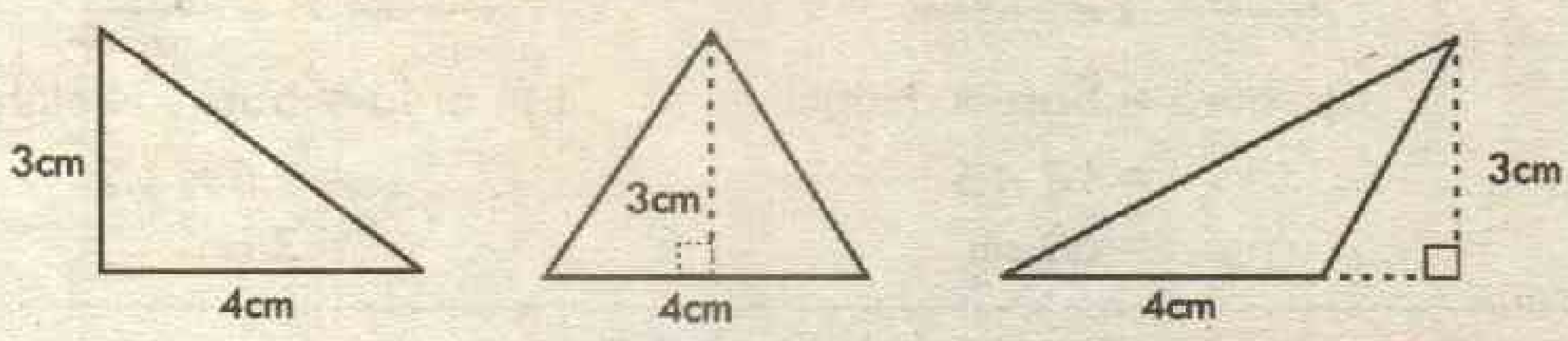
$$A = \frac{1}{2}bh$$

Example 1 Find Area

Find the area of each of the following triangles.

Solution:

The area of each of the three triangles below is $6cm^2$.

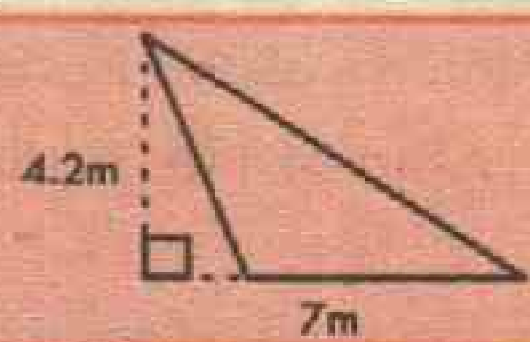


$$\text{Area (of a triangle)} = \frac{1}{2} \times (\text{length of base}) \times (\text{height})$$

$$= \frac{1}{2} \times 4 \times 3 = 6cm^2$$

Practice

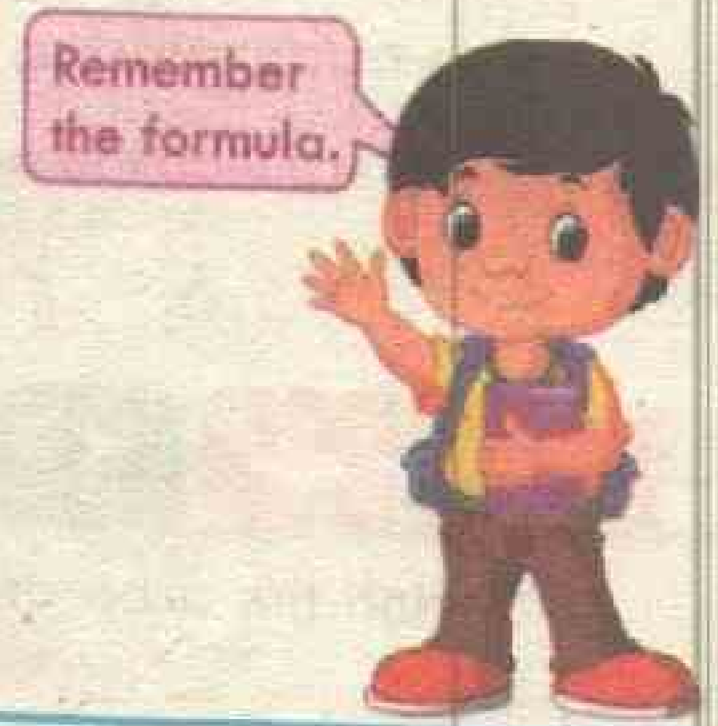
Find area of the triangle.



Example 2 Find Area

Find the area of a triangle whose base is 6.4cm and altitude is 3.2cm.

Solution:



Area of triangle $= \frac{1}{2}$ (base × altitude)

$= \frac{1}{2}(6.4 \times 3.2)$

$= 10.24\text{cm}^2$

Example 3 Find Area

The signal flag shown represents the number five. Five the area of the blue region. To find the area of the blue region, subtract the areas of the triangles from the area of the square.

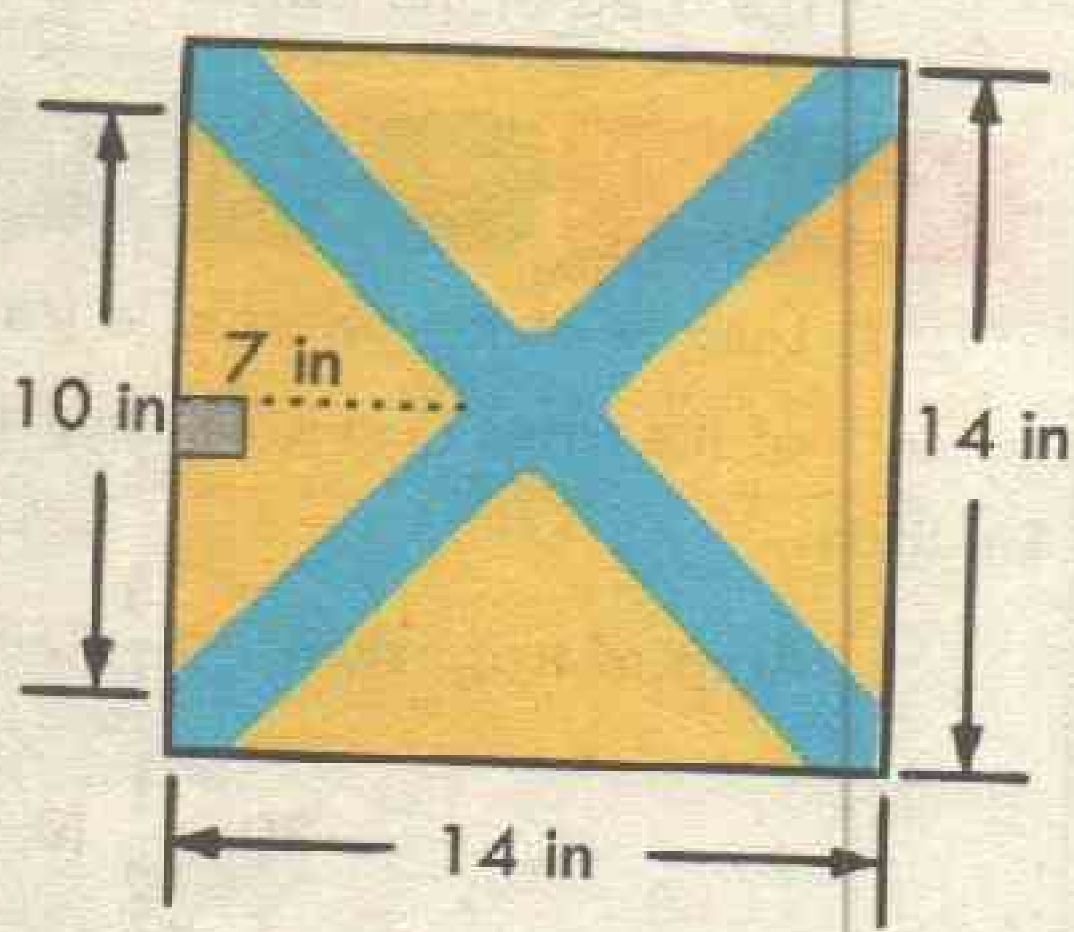
Solution:

Area of the square

$A = bh$

$A = 14 \times 14$

$A = 196 \text{ in}^2.$



Area of the triangle

$A = \frac{1}{2} bh$

$A = \frac{1}{2} \times 10 \times 7$

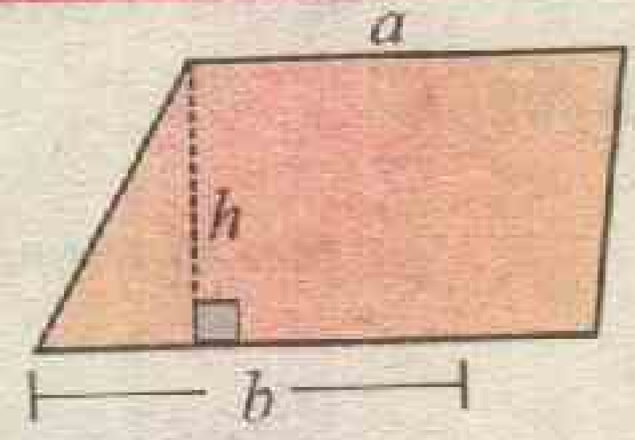
$A = 35 \text{ in}^2.$

The total area of the triangles is 4(35) or 140 square inches. So, the area of the blue region is 196 – 140 or 56 square inches.

11.4.3 To find the area of a trapezium when altitude and measures of its parallel sides are given:

If a trapezoid has a bases of b units and a height of h units, then the area A is $\frac{1}{2}h(a+b)$ square units.

$$A = \frac{1}{2}h(a+b)$$

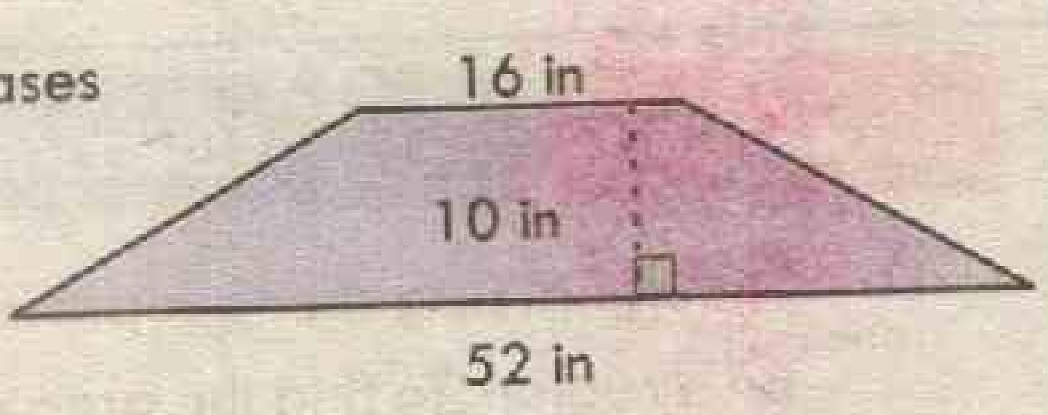


Example 1 Find Area of a Trapezoid

Find the area of the trapezoid.

Solution:

The height is 10 inches. The bases are 16 inches and 52 inches.



$$A = \frac{1}{2}h(b_1 + b_2)$$

$$= \frac{1}{2}(10)(16+52)$$

$$= 5(68) = 340$$

The area of the trapezoid is 340 square inches.

Example 2 Find Altitude of a Trapezoid

Lengths of the bases of a trapezium are 9cm and 4cm. If the area of the trapezium is 39 cm², find its altitude.

Solution:

Area of trapezium $= \frac{1}{2}$ (Sum of bases) × Altitude

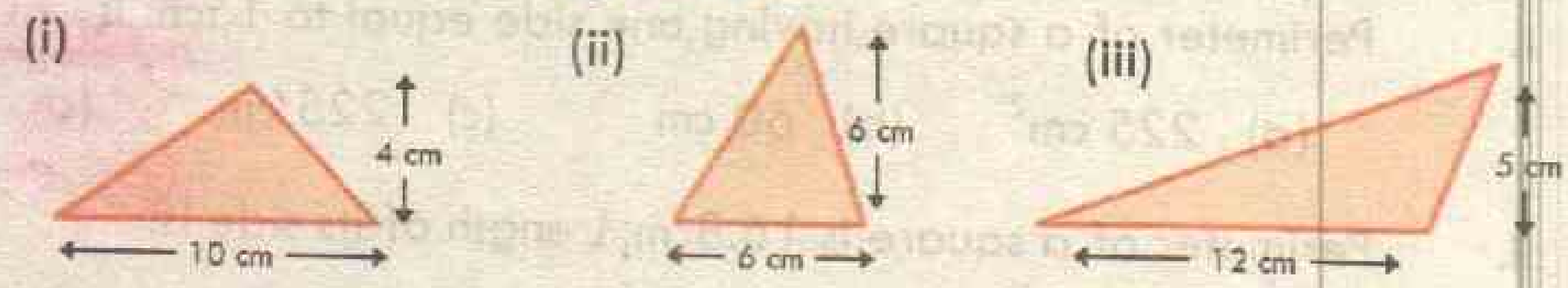
or, $39 = \frac{1}{2}(9+4) \times \text{Altitude} = \frac{1}{2} \times 13 \times \text{Altitude}$

or, $\frac{13}{2} \times \text{Altitude} = 39$

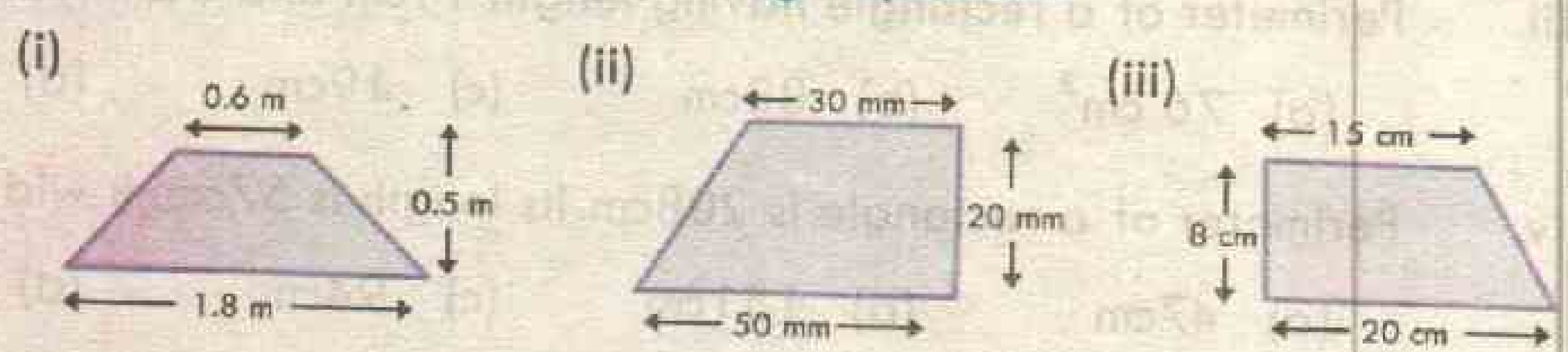
or, $\text{Altitude} = 39 \times \frac{2}{13} = 6 \text{ cm}$

Exercise 11.3

1. Find the area of the following triangles:



2. Find the area of each of the following trapezium.



3. Find the area of a parallelogram, when:
   (i) Length of base = 6cm, and altitude = 4cm.
   (ii) Length of base = 3.8cm and altitude = 2.2cm.

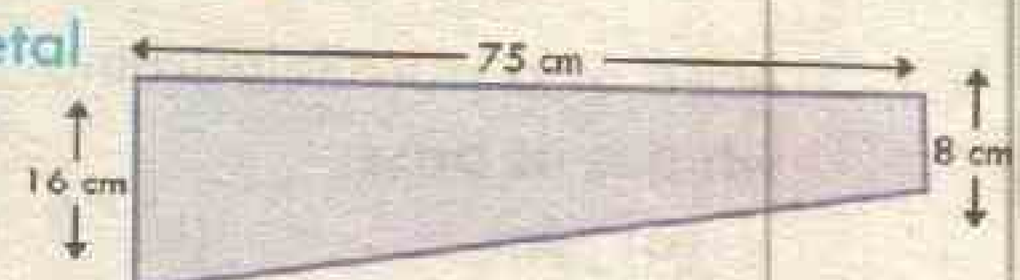
4. Find the area of a triangle whose:
   (i) Base = 8cm and altitude = 5cm
   (ii) Base = 7.5cm and altitude = 4cm

5. Find the area of a trapezium when:
   (i) Length of base 1 = 6cm, Length of base 2 = 4cm, and altitude = 3cm.
   (ii) Length of base 1 = 9.2cm, length of base 2 = 6.8cm, and altitude = 4cm

6. Aslam prepared parallelogram piece of land having base 18m and altitude 8m. Find the expenditure on planting grass when the rate of plantation is 0.35 rupees per square meter.

7. Base and altitude of a triangular farm are 570m and 295m respectively. If the rate of wheat production per square meter is 6kg, find the total yield of wheat the farm would give.

8. The diagram shows a piece of metal which is to be used for making a saw blade. Find its area.



Review Exercise 11

1) Choose the correct answer.

i. Perimeter of a square having one side equal to 15cm is:

(a) 225 cm^2 (b) 60 cm (c) 225 cm (d) 60 cm^2

ii. Perimeter of a square is 16.8cm. L ength of its side is:

(a) 4.2cm (b) 0.42cm (c) 8.4cm (d) 67.2cm

iii. Perimeter of a rectangle having length 11cm and width 8cm is:

(a) 76 cm^2 (b) 88cm (c) 19cm (d) 38cm

iv. Perimeter of a rectangle is 208cm. Its length is 57cm. It width is:

(a) 47cm (b) 151cm (c) 94cm (d) 52cm

v. Area of a square having one side 15cm is:

(a) 60 cm^2 (b) 225cm^2 (c) 11.5 cm^2 (d) 155 cm^2

vi. Area of a rectangle having length 0.9m and width 0.5m is:

(a) 4.5m^2 (b) 2.8m^2 (c) 0.045 m^2 (d) 0.45m^2

vii. Base of a parallelogram is 10cm. What will be the area of the parallelogram if the length of its altitude is 7cm?

(a) 34 cm^2 (b) 68 cm^2 (c) 70 cm^2 (d) 35 cm^2

viii. Area of a triangle having its base 9cm and altitude 10cm is:

(a) 90 cm^2 (b) 45 cm^2 (c) 76 cm^2 (d) 19cm^2

ix. Bases of a trapezium are 9cm and 6cm. If its altitude is 4cm, its area is:

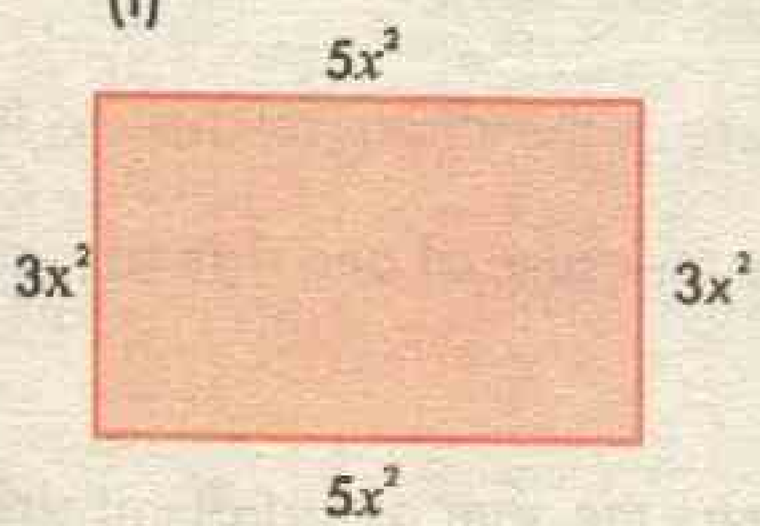
(a) 30 cm^2 (b) 60 cm^2 (c) 36 cm^2 (d) 24 cm^2

x. Area of a parallelogram is 36m^2. If the length of its attitude is 4m, what is its base:

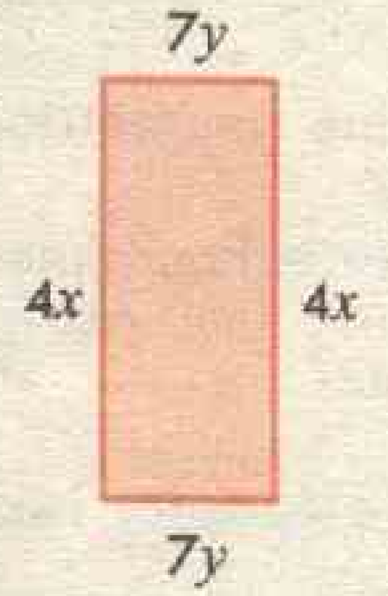
(a) 6m (b) 32m^2 (c) 18m (d) 9m

2. Write an expression for the area and the perimeter of each figure. Simplify each expression.

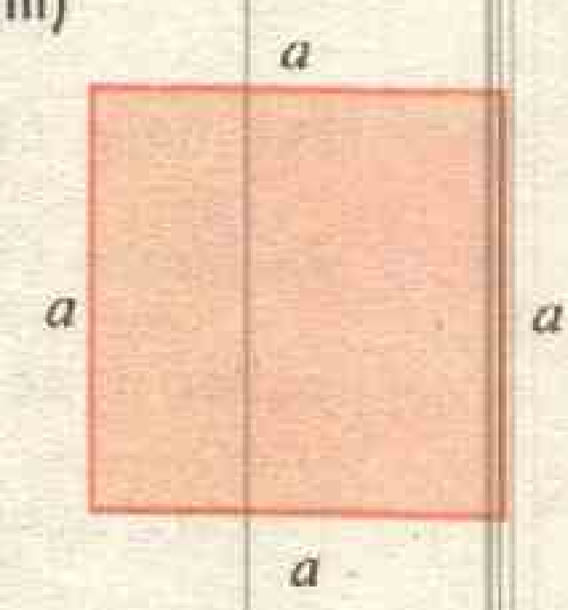
(i)



(ii)

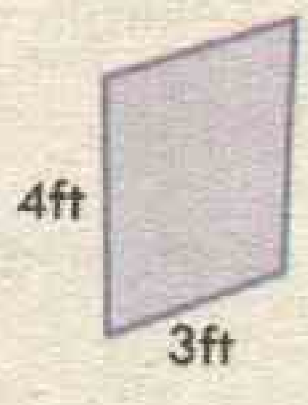


(iii)

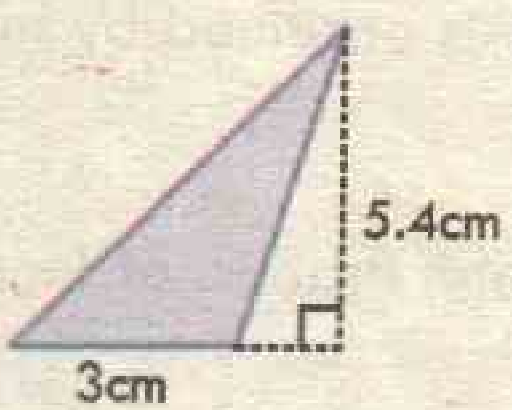


3. Find the area of each figure.

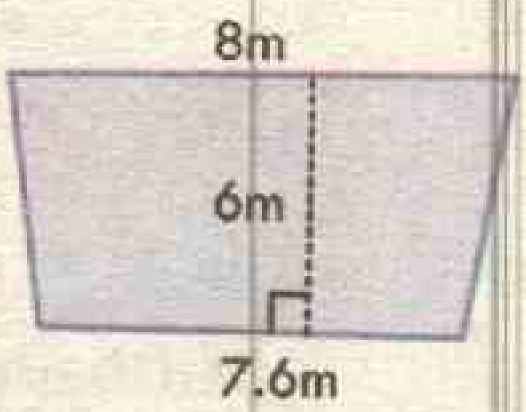
(i)



(ii)



(iii)



4. Area of a right-angled triangle is 2 1bc square centimetre. If the area is equal to 15 sq. cm, and $b = 5$, then find c.

5. If the perimeter of an equilateral triangle is 15 cm, find the length of each side.

6. Perimeter of a rectangle is 125 cm. If the breadth of the rectangle is 25cm, find its length.

7. Perimeter of a rectangle is 50 metre. If the length of the rectangle is 15 metre, find its breadth.

8. Area of a rectangular field is 90 square metre. If its length is 12 metre, find its breadth.

9. Area of a square field is 225 sq. metre. Find the length of its sides.

10. Perimeter of a square is 14 cm. Find length of its side.

Summary

- **Perimeter** of a plane geometric figure is the distance around it.
- **Perimeter of a square:** Four times the measure of one side of a square is called its perimeter.
- **Perimeter of the rectangle:** Two times the sum of adjacent sides of a rectangle.
- **Area of a plane** geometric figure is always shown in square units.
- **Area of a square** is obtained by multiplying the length of a side by itself.
- **Area of a rectangle:** Product of length and breadth of a rectangle is called its area.
- **Altitude:** Measure of the shortest distance between the base of a geometric figure and its opposite vertex.
- **Parallegram :** A quadrilateral having opposite sides parallel.
- **Area of Parallegram:** Product of the measure of base of a parallelogram and its altitude.
- **Area of a triangle:** Half the product of the measures of base of a triangle and its altitude is called area of the triangle.
- **Trapezium:** A quadrilateral in which two sides are parallel and two are not parallel, is called a trapezium. Parallel sides are called the bases of the trapezium.
- **Area of trapezium:** Product of half of the sum of the bases of a trapezium and its altitude is called the area of a trapezium.

UNIT 12 THREE DIMENSIONAL SOLIDS

What You'll Learn

- Identify 3D figure (cube, cuboid, sphere, cylinder and cone) with respect to their faces, edges and vertices.
- Define and recognize units of surface area and volume.
- Find surface area and volume of cube and cuboid.
- Solve real life problems involving volume and surface area.

Why It's important

The space that we live in has three dimensions: length, width, and height. Three-dimensional geometry is used to describe the buildings we live and work in, the tools we work with, and the objects we create. Geometry is everywhere.

12.1 3–Dimensional Solids

Many well-known objects are solids with mathematical names:

A dice is a cube. 

A match box is a cuboid. 

A football is a sphere. 

An ice cream is a cone. 

A tin of ghee is a cylinder. 

12.1.1 Cube

A solid body bounded by six equal square faces is called a cube as shown in the figure. Detail of the cube is given below:

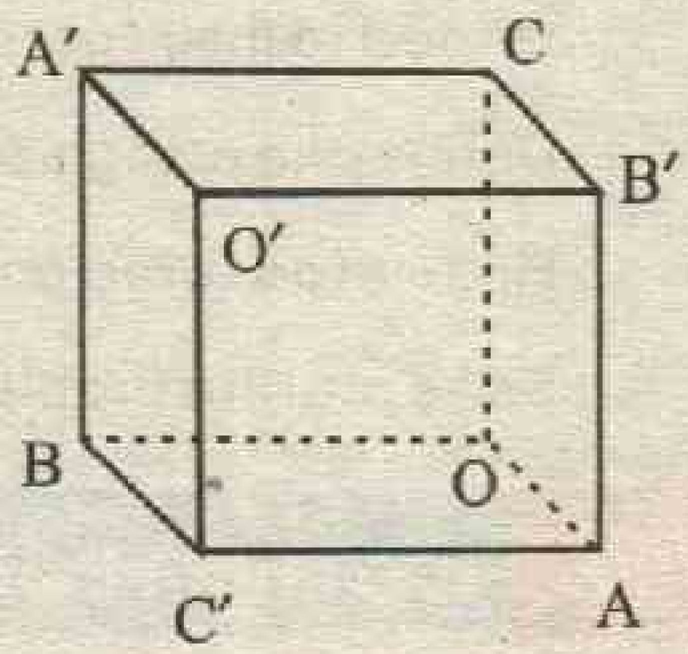
Square faces

Six square faces of the cube are:

$OAC'B$, $O'B'CA'$ (opposite faces)

$OAB'C$, $C'O'A'B$ (opposite faces)

and $C'AB'O'$, $OBA'C$ (opposites faces)



Edges

12 edges of the cube are:

OA, AC', $C'B$, OB, $O'B'$, $B'C$, CA', OC, $O'C'$, $A'B$, $O'A'$, AB'.

Vertices

8 vertices of the cube are:

12.1.2 Cuboid

A solid body bounded by six rectangular faces is called a cuboid as shown in the figure.

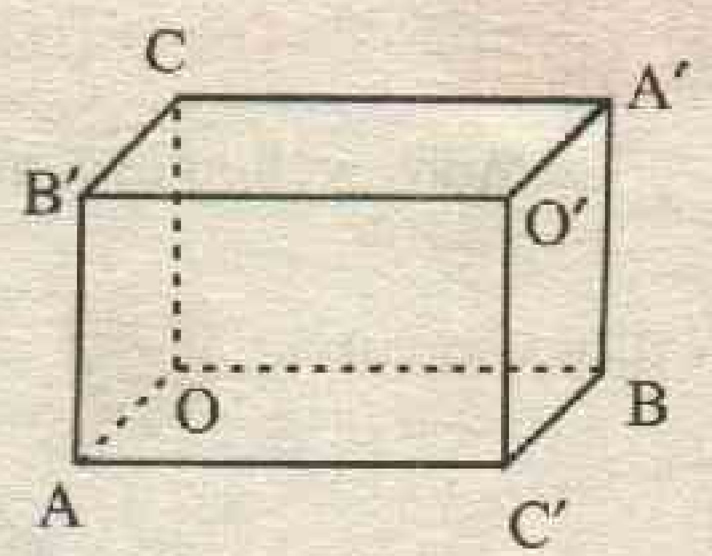
Rectangular Faces

Six rectangular faces of the cuboid are:

$OAC'B$, $O'A'CB'$ (opposite faces)

$OAB'C$, $C'O'A'B$ (opposite faces)

$C'AB'O'$, $OBA'C$ (opposite faces)



Edges

12 edges of the cuboid are:

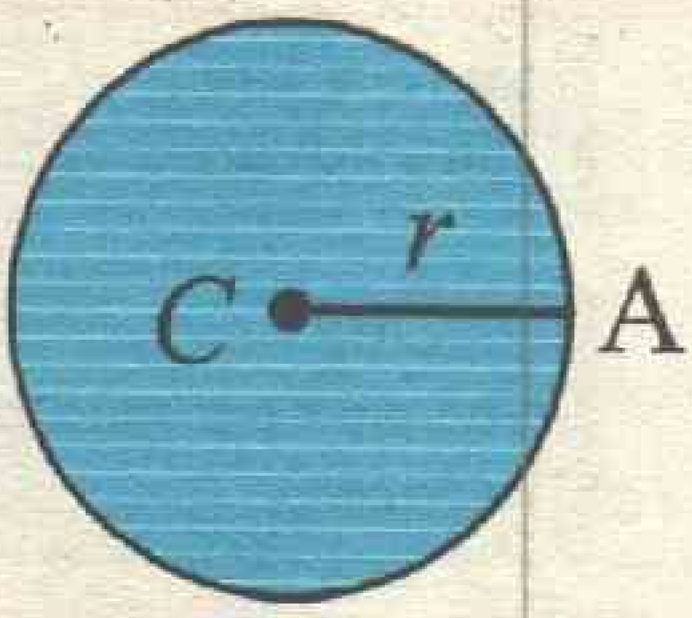
OA, AC', $C'B$, OB, $O'B'$, $B'C$, CA', OC, $O'C'$, $A'B$, $O'A'$, AB'.

Vertices

8 vertices of the cuboid are:

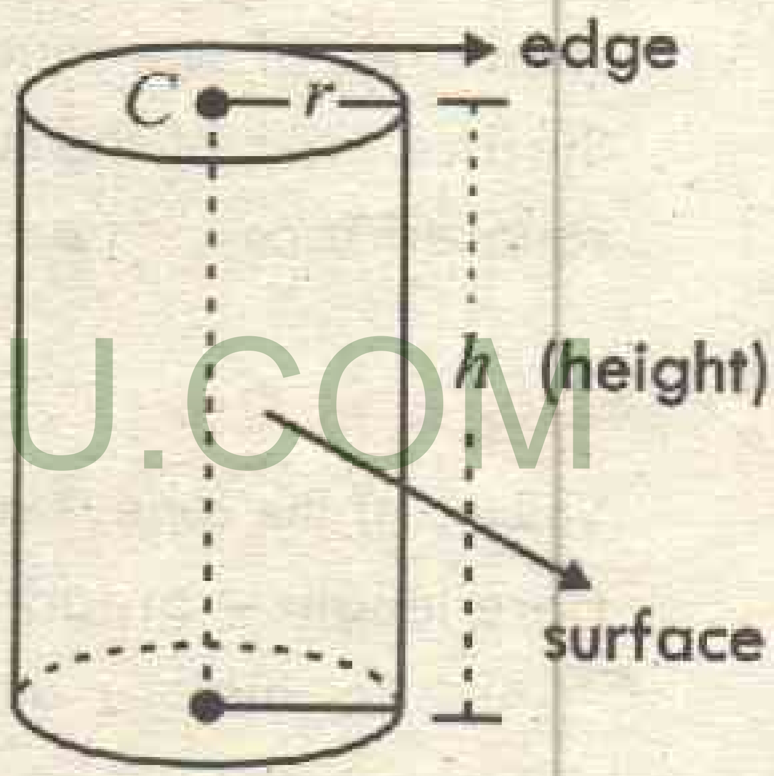
12.1.3 Sphere

A round evenly solid body is called a sphere as shown in the figure. Hockey ball, cricket ball, foot ball are some examples of a sphere. A sphere has a center C and a radius r ($r = CA$). Curved part of the sphere is called its surface.



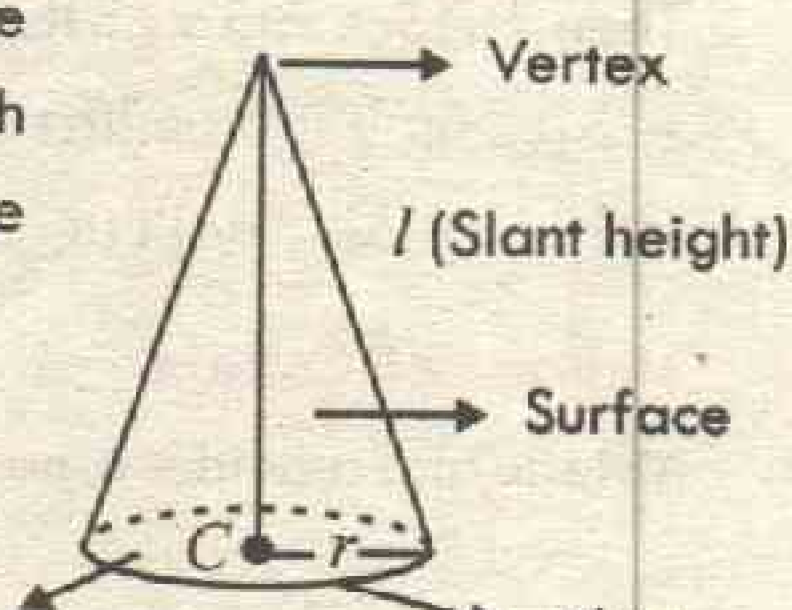
12.1.4 Cylinder

A cylinder has two circular faces of equal radius and one curved surface between them as shown in the figure. It has 2 edges, 2 plane circular faces and a curved surface. For example a pencil, an empty tin of ghee, and a jar represent a cylinder. In the figure of the cylinder h is its height, r is the radius of the circular faces, two edges and one curved surface.



12.1.5 Cone

A cone is a solid figure, which has one circular face and a slanting surface, which converges to a point above the circular surface as shown in the figure.



12.2 Surface Areas and Volumes

Surface Area
The total area of the surface of a three-dimensional object.

Volume
Volume of a solid body is the amount of space bounded by its faces.

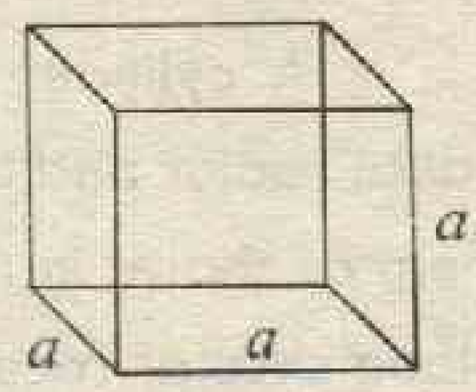
12.3 Surface Area and Volume of Cube and Cuboid

a) Surface Area of Cube

Let a cm be the side of the squared face.

Then area of one squared face $= a \times a = a^2$ cm^2.

Surface area of the cube = Sum of areas of six squared faces $= 6a^2$ cm^2

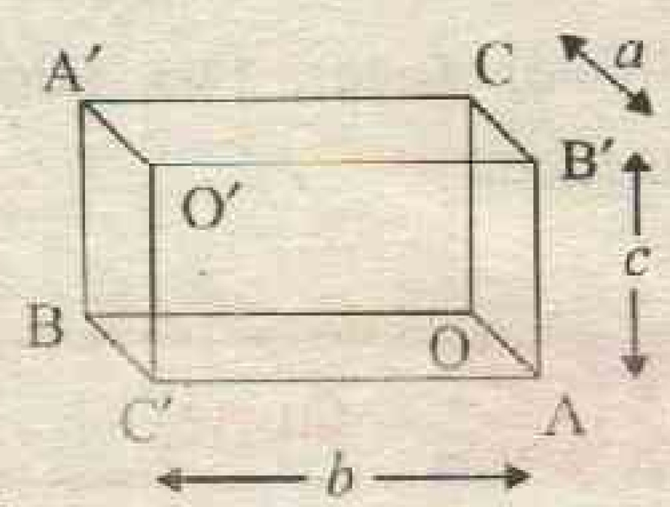


b) Volume of Cube

Volume of the cube = area of base × height = length × breadth × height

Here length = breadth = height $= a$ cm

Volume $= a\ \ a\ \ a = a^3$ cm^3.



c) Surface area of Cuboid

Let a cm, b cm and c cm be respectively the length, breadth and height of the cuboid.

Then surface area of the cuboid $= (2ab + 2bc + 2ca)$ cm^2.

= 2 [(length × breadth) + (breadth × height) + (height × length)]

$= 2\,(ab + bc + ca)$ cm^2

d) Volume of Cuboid

Volume of a cuboid = area of the base × height = length × breadth × height

$= a \times b \times c$

$= abc$ cm^3

Example 1 Find volume of Cube

Find the surface area of a cube of side 3 cm.

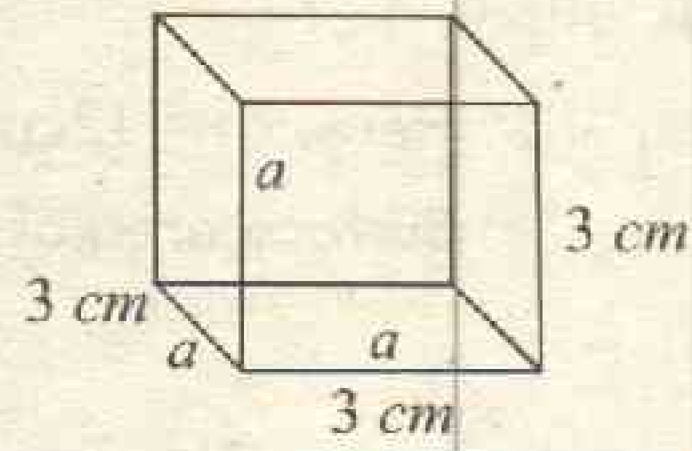
Solution:

Here $a = 3\text{cm}$

Surface area $= 6 \times a^2$

$= 6 \times 3^2$

$= 54\ \text{cm}^2$



Example 2 Find volume of Cube

Find the volume of a cubical room of side 12 m.

Solution:

Length of the cubical room $= 12$ m

Breadth of the cubical room $= 12$ m

Height of the cubical room $= 12$ m

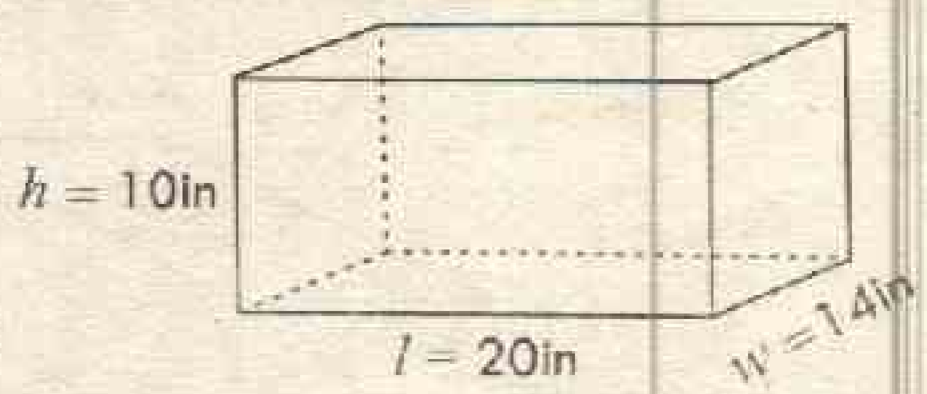
$\therefore$ Volume of the cubical room $= 12 \times 12 \times 12\ \text{m}^3$

$= 1728\ \text{m}^3$

Example 3 Surface Area of a Cuboid

Find the surface area of the cuboid.

Solution:



$S = 2lw + 2lh + 2wh$

$S = 2(20)(14) + 2(20)(10) + 2(14)(10)$

$S = 1240$

The surface area of the cuboid is 1240 square inches.

Example 4 Find surface area of Cuboid

Find the surface area of a cuboid with length 12cm, breadth 8cm and height 6 cm.

Solution:

Here $a = 12\text{cm}, b = 8\text{ cm and } c = 6\text{cm}.$

Surface area of a cuboid $= 2ab + 2bc + 2ca$

$= 2(12 \times 8) + 2(8 \times 6) + 2(6 \times 12)\text{ cm}^2$

$= 2 \times 96 + 2 \times 48 + 2 \times 72\text{ cm}^2$

$= 192 + 96 + 144\text{ cm}^2$

$= 432\text{ cm}^2$

Example 5 Find volume of Cuboid

Find the volume of a cuboid whose length, breadth and height are respectively equal to 7cm, 4cm, and 8cm.

Solution:

Here $a = 7\text{cm}, b = 4\text{cm and } c = 8\text{cm}$

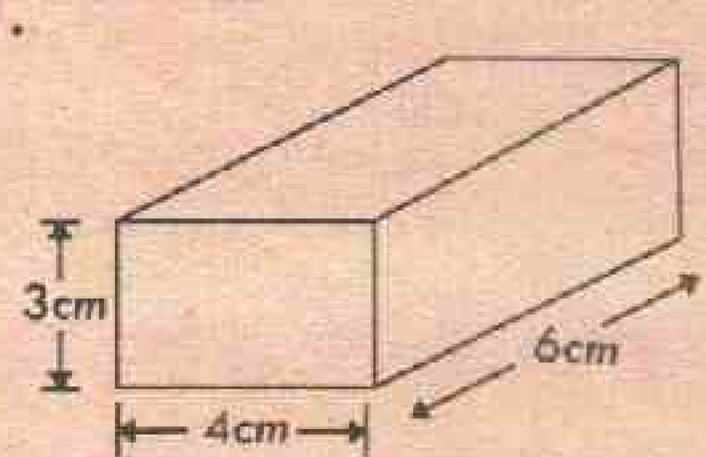
Volume of the cuboid $= a \times b \times c = 7 \times 4 \times 8\text{ cm}^3$

$= 224\text{cm}^3$

Practice

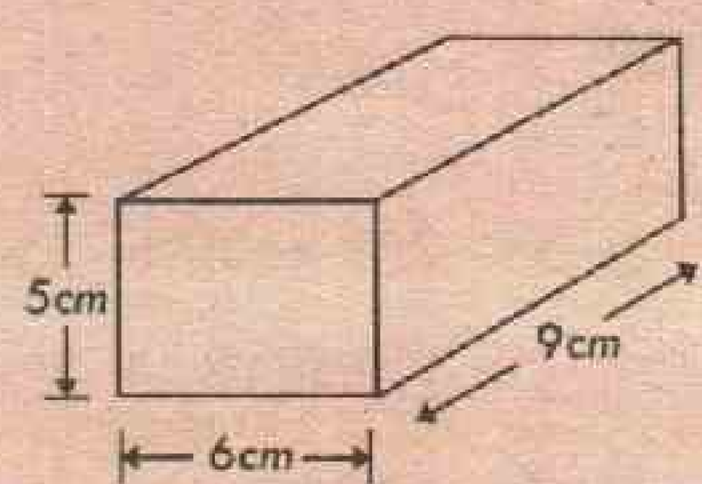
Find the surface area and volume of the cuboid.

1.



width = 4cm, height = 3cm, length = 6cm

2.



width = 6cm, height = 5cm, length = 9cm

Exercise 12

1. Find the surface area of a cube whose one side is:
   (i) 5cm (ii) 4.5cm (iii) 7.8cm (iv) 6.9cm

2. Find the surface areas of the following cuboids.

S#	Length	Breadth	Height
(i)	4cm	3cm	2cm
(ii)	5cm	4cm	9cm
(iii)	7.5m	3.4m	2m

3. Find volume of the cube whose one side is
   (i) 1cm (ii) 6cm (iii) 4.5m (iv) 5.4m

4. Find volume of the cuboids whose length, breadth, and height are given in question 2 above.

5. The side of a cubical pool is 5 meters. Find its
   (i) Surface area (ii) Volume

Note

Since the pool is open, its surface area will be sum of surface areas of 5 faces.

6. A rectangular room is 4.5 m long, 3.5 m wide and 5m deep. Find its
   (i) Surface area (ii) Volume

7. Volume of a box is 60 cm^3. If its base area is 20cm^2. Find its depth.

8. Give 2 examples of a sphere and draw their figures.

9. Give 2 examples of a cylinder and draw their figures

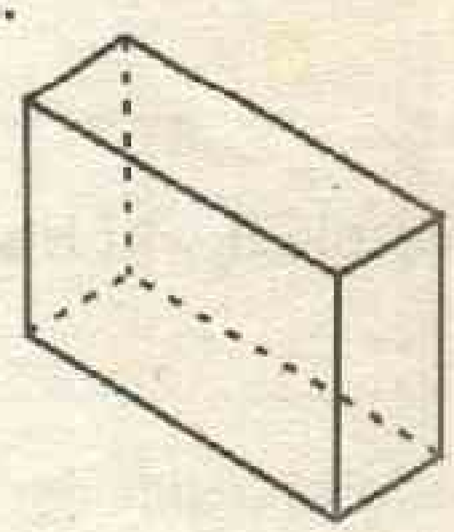
10. Give 2 examples of a cone and draw their figures.

Review Exercise 12

1. A triangle lies in __________ dimension.
2. Figures which lie in plane are called _________ dimensional.
3.  is a __________ dimensional figure.
4. A cuboid is a _____________ D-figure.
5. Surface area of cube of a cm side = _____________.
6. Volume of a cube of a cm side = __________.
7. Surface area of cuboid of sides a cm, b cm and c cm = __________.
8. Volume of cube of side 1m = ___________.
9. Volume of cuboid of sides 3 cm, 2cm, 1cm = __________.

10. Which figure does not have the same dimensions as the other figures?

A.



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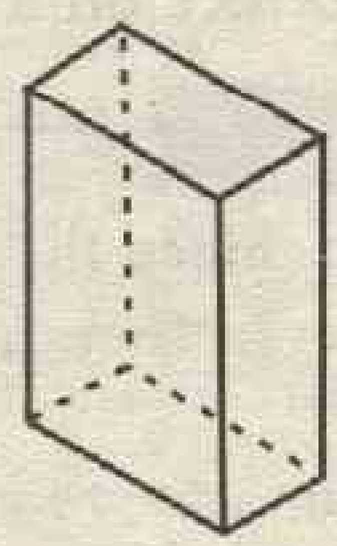
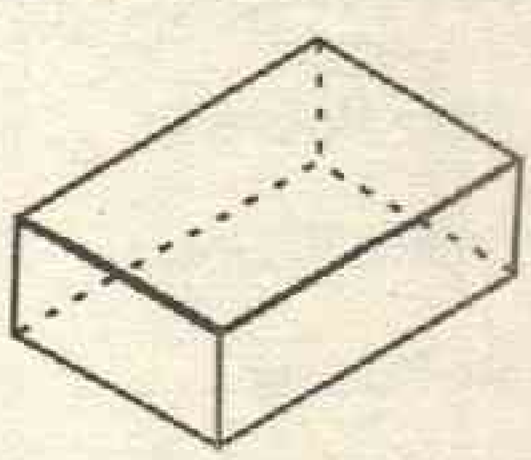
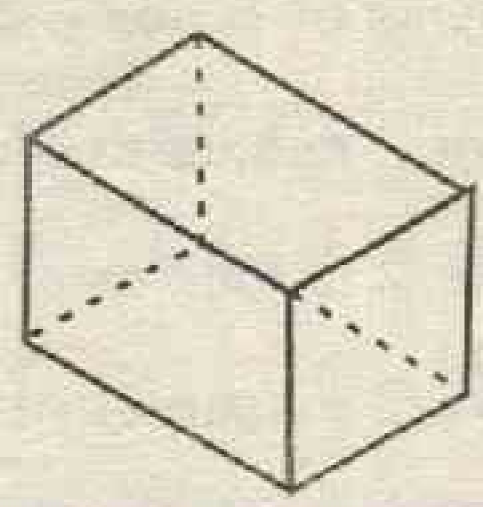


C.



D.



Summary

❖ **Cube:**

A solid body bounded by six square faces is called a **cube.**

❖ **Cubiod:**

A solid body bounded by six rectangular faces is called a **cuboid.**

❖ **Sphere :**

An evenly round solid body is called a sphere.

❖ **Cylinder:**

A cylinder has two circular faces and one curved surface between them.

❖ **Cone:**

A cone has one circular face and a curved surface having slant heights which meet in a point.

❖ **Surface area of cube:**

Surface area of a cube of side a cm = $6a^2$ cm^2

❖ **Volume of cube:**

Volume of cube of side a cm = a^3 cm^3

❖ **Surface area of cuboid:**

Surface area of a cuboid of sides a cm, b cm and c cm = $2ab + 2bc + 2\,ca$ cm^2

❖ **Volume of cuboid:**

Volume of a cuboid of sides a cm, b cm and c cm = abc cm^3

UNIT 13 INFORMATION HANDLING

What You'll Learn

- Define data and data collection.
- Distinguish between grouped and ungrouped data.
- Draw horizontal and vertical bar graphs.
- Read a pie graph.

Why It's important

For the development of country or to improve any field of life, we have to think many plans. For this purpose, we must need quantitative information, and these are obtained by counting or by measuring. Hence to know about something is called information and to represent them in a manageable way to obtain useful results are called information handling.

13.1 Data

13.1.1 Definition of Data

The numerical facts obtained from the observations and measurements or experiments is called data. The results derived from it are very useful for future planning. For example; The performance of a student can be improved by comparing the results of his previous tests of Mathematics with present tests.

13.1.2 Data Collection

Data can be collected by the following ways.

(i) It may be collected directly from concerned source.

(ii) It may be collected through media (News papers, T.V, computer, net, magazines etc).

(iii) It may be collected through questionnaire.

(iv) It can be collected from office records, published papers or from the people related to that concerned field.

13.1.3 Types of Data

There are two types of data:

a. Ungrouped data

Numerical facts which are obtained as such, are known as ungrouped data. 5 students of class 6th obtained 95, 94, 93, 92 and 91 marks out of 100 in Mathematics.

b. Grouped data:

Raw data when put in some order or arranged in different groups is called grouped data.

Example 1 Grouped data

50 students of class 6th may be divided into 5 groups according to their scores out of 100 in the subject of Mathematics as given below.

Solution:

Scores	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of students	5	10	15	5	15

Example 2 Grouped data

Given below the marks out of 100 obtained by 15 students of class 10th in a test of physics 70, 72, 75, 76, 77, 78, 76, 70, 83, 77, 79, 80, 73, 74, 73.

Solution:

The above data may be divided into following groups.

Marks obtained	No. of students
70 – 72	3
73 – 75	4
76 – 78	5
79 – 81	2
82 – 84	1
Total	15

Exercise 13.1

1. Define grouped data and give one example.

2. Define ungrouped data and give one example.

3. The number of patients of kidney problems admitted in hospital during a week according to their ages given below.

   40, 36, 17, 25, 12, 13, 31, 36, 47, 32, 26, 10, 15, 6, 45, 20, 9, 60, 19, 4, 14.

   Identify the type of this data.

4. Given below are the marks out of 50 obtained by 40 students of class 6 in a test of mathematics.

Qasim Shah

Marks obtained	No. of students
15 – 20	10
21 – 26	7
27 – 32	6
33 – 38	8
39 – 44	6
45 – 50	3
Total	40

Yasmin

Qasim Shah thinks it as a grouped data while Yasmin thinks it as an ungrouped data.
Who is correct?

13.2 Bar Graph

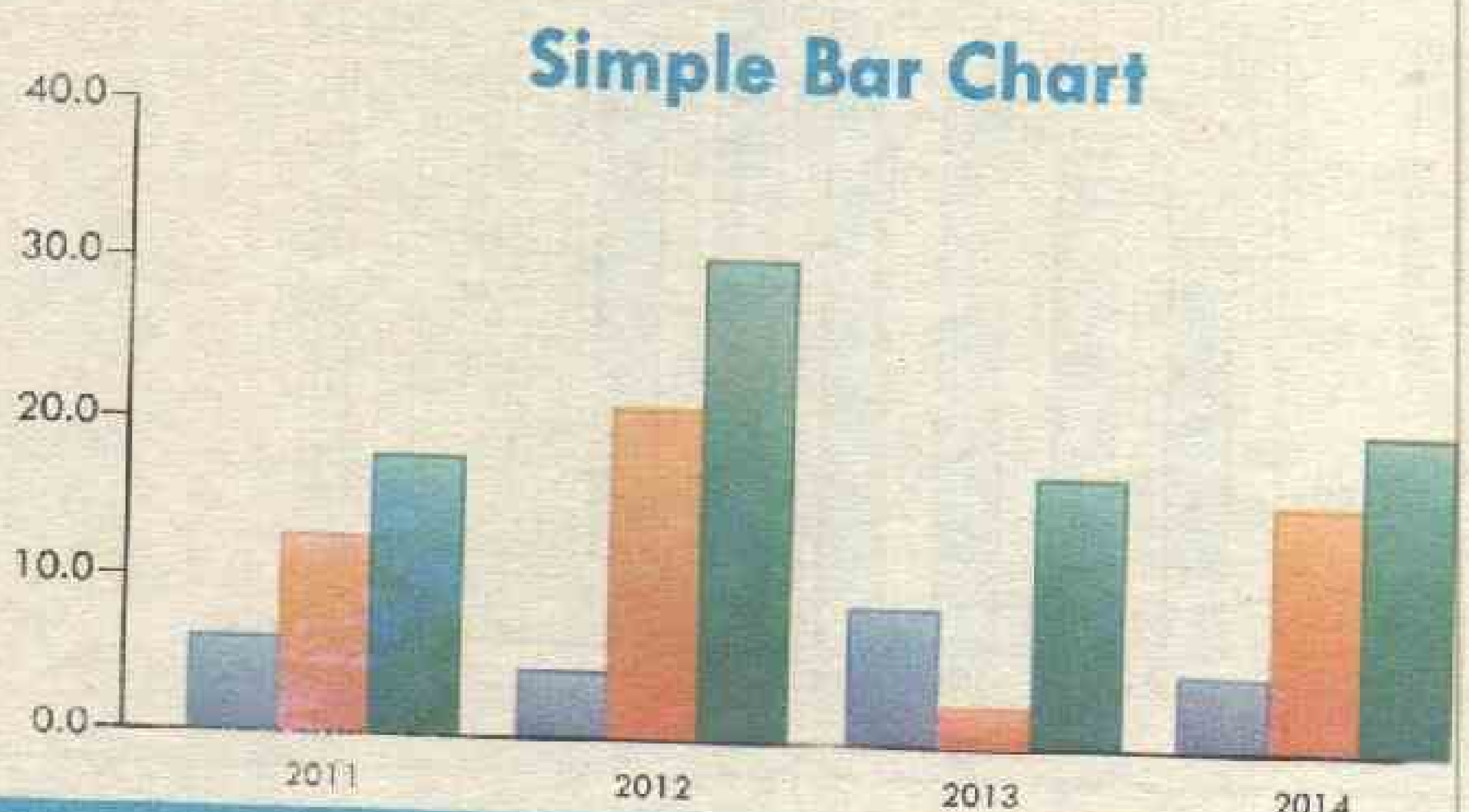
Bar graph is that type of graph in which data is represented by vertical and horizontal bars, which have equal width.

To draw a bar graph following procedure should be followed.

- Draw two mutually perpendicular rays, one is horizontal ray $\overrightarrow{OX}$ and other is vertical ray $\overrightarrow{OY}$.
- $\overrightarrow{OX}$ and $\overrightarrow{OY}$ meet at point 'O'.
- Select a suitable scale along $\overrightarrow{OY}$ or $\overrightarrow{OX}$ according to given data.
- The given quantities are shown along $\overrightarrow{OX}$ and $\overrightarrow{OY}$ respectively.
- Draw rectangles (bars) according to the scale.

(i) Width of rectangles (bars) should be the same.

(ii) Space between rectangles (bars) should be equal.



Example 1 Draw Vertical Bar Graph

Draw a vertical bar graph to represent population of Peshawar city for six years.

Year	1998	1999	2000	2001	2002	2003
Population (in lacs)	25	30	35	40	45	50

Solution:

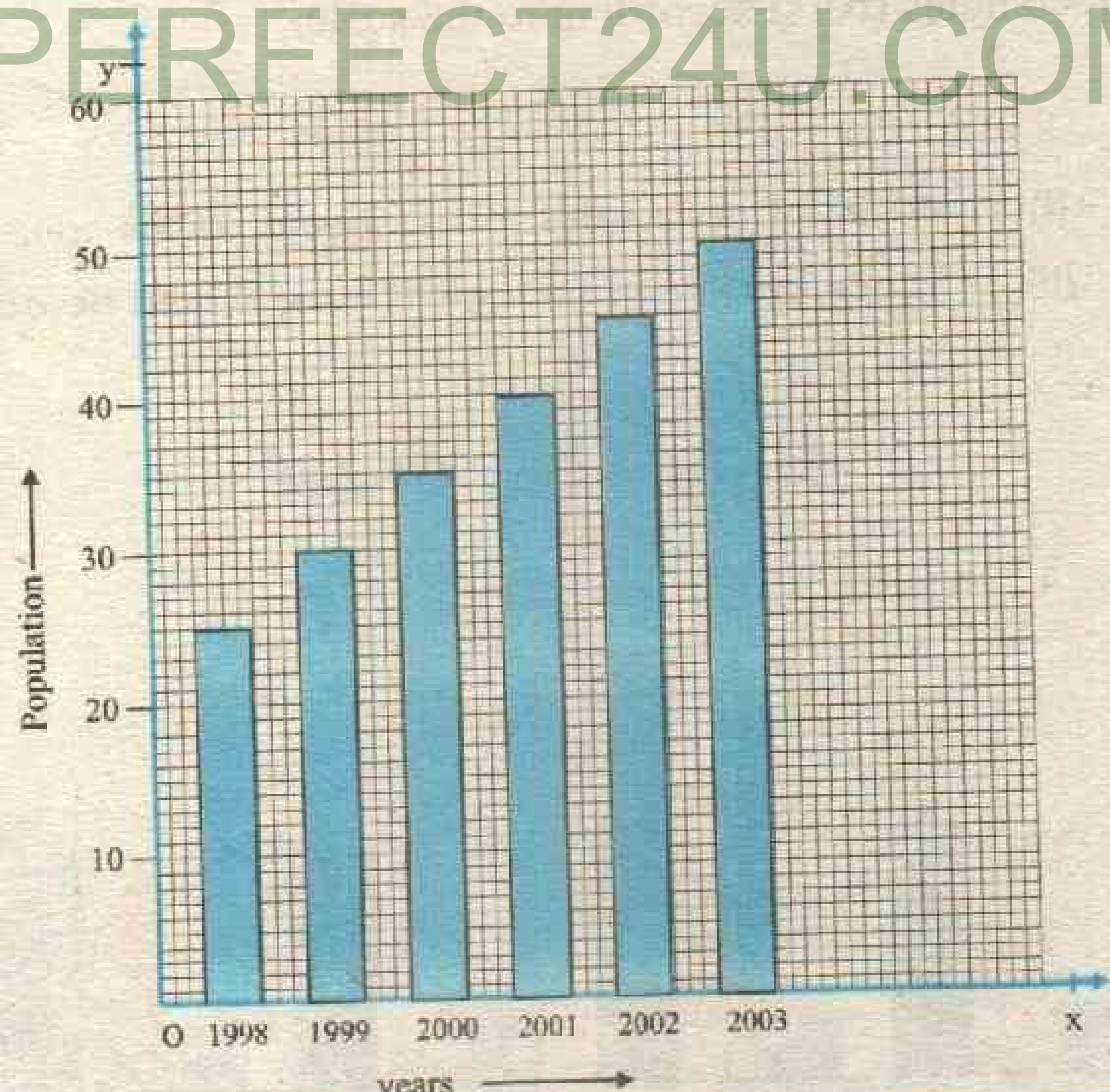
Draw two mutually perpendicular rays $\overrightarrow{OX}$ and $\overrightarrow{OY}$. Select a suitable scale.

Scale: One big square along $\overrightarrow{OY}$ represents 10 lac population.

Years are shown along $\overrightarrow{OX}$.

Note

To draw vertical bar graph, scale should by taken along Y–axis and to draw horizontal bar graph, scale should be taken along X–axis.



Example 2 Draw Horizontal Bar Graph

Draw horizontal bar graph to show the marks obtained by Hamza in second term examination given below.

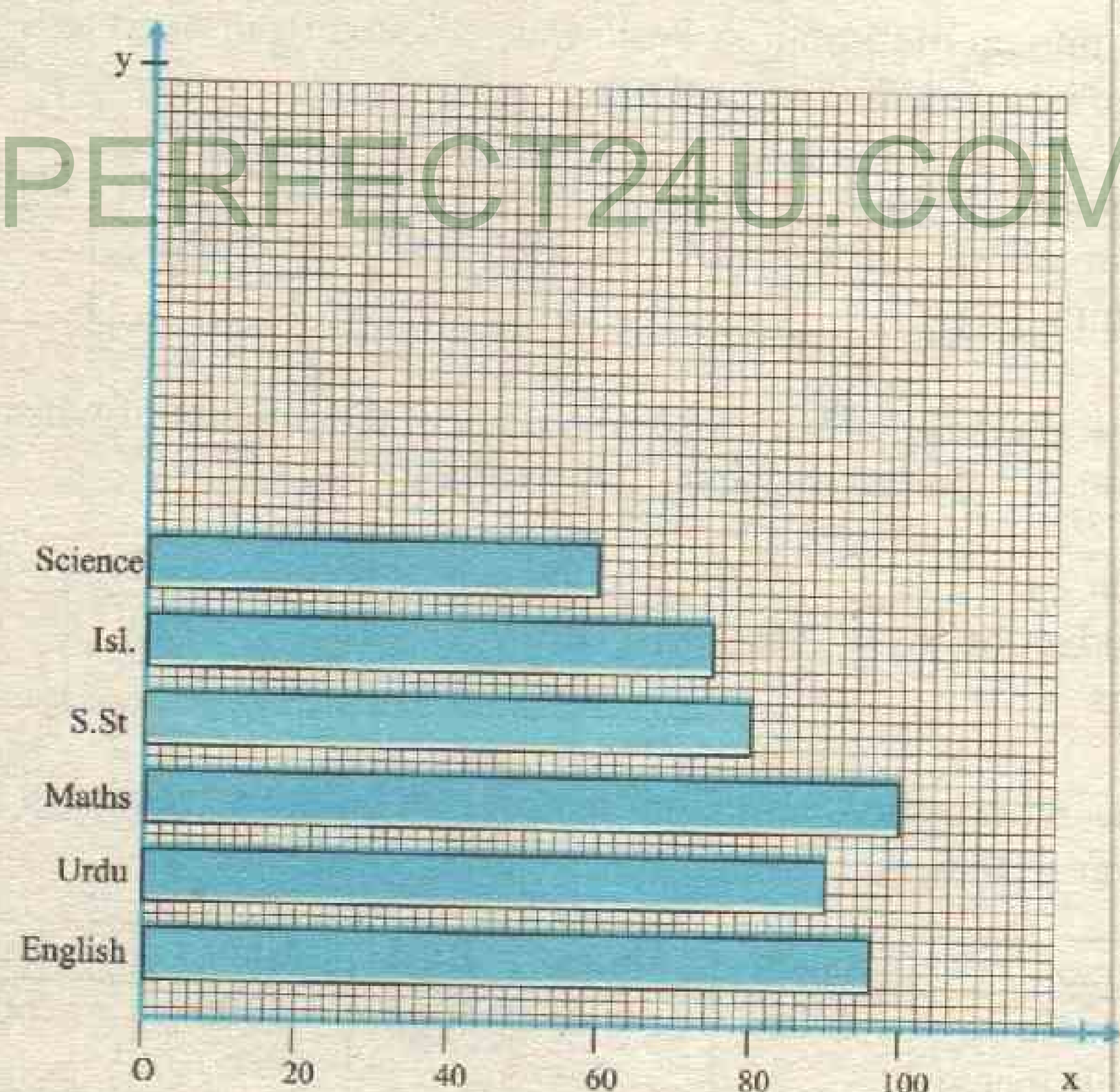
Subject	English	Urdu	Maths	S.St	Isl.	Science
Marks obtained	96	90	100	80	75	60

Solution:

Select a suitable scale.

Scale: One big square along $\overrightarrow{OX}$ represent 20 marks.

Subjects are shown along $\overrightarrow{OY}$.



Exercise 13.2

1. Draw a vertical bar graph to represent speed of a car in first four hours given below.

Time	1st hour	2nd hour	3rd hour	4th hour
Speed	50 km	100 km	150 km	200 km

2 The amount of rainfall in Peshawar recorded from April to September is given below. Draw vertical bar graph.

Months	April	May	June	July	August	September
Rain fall in mm	15	18	20	25	28	30

3. Draw a horizontal bar graph to represent the product of wheat in Charsadda during the year 1990 to 1994.

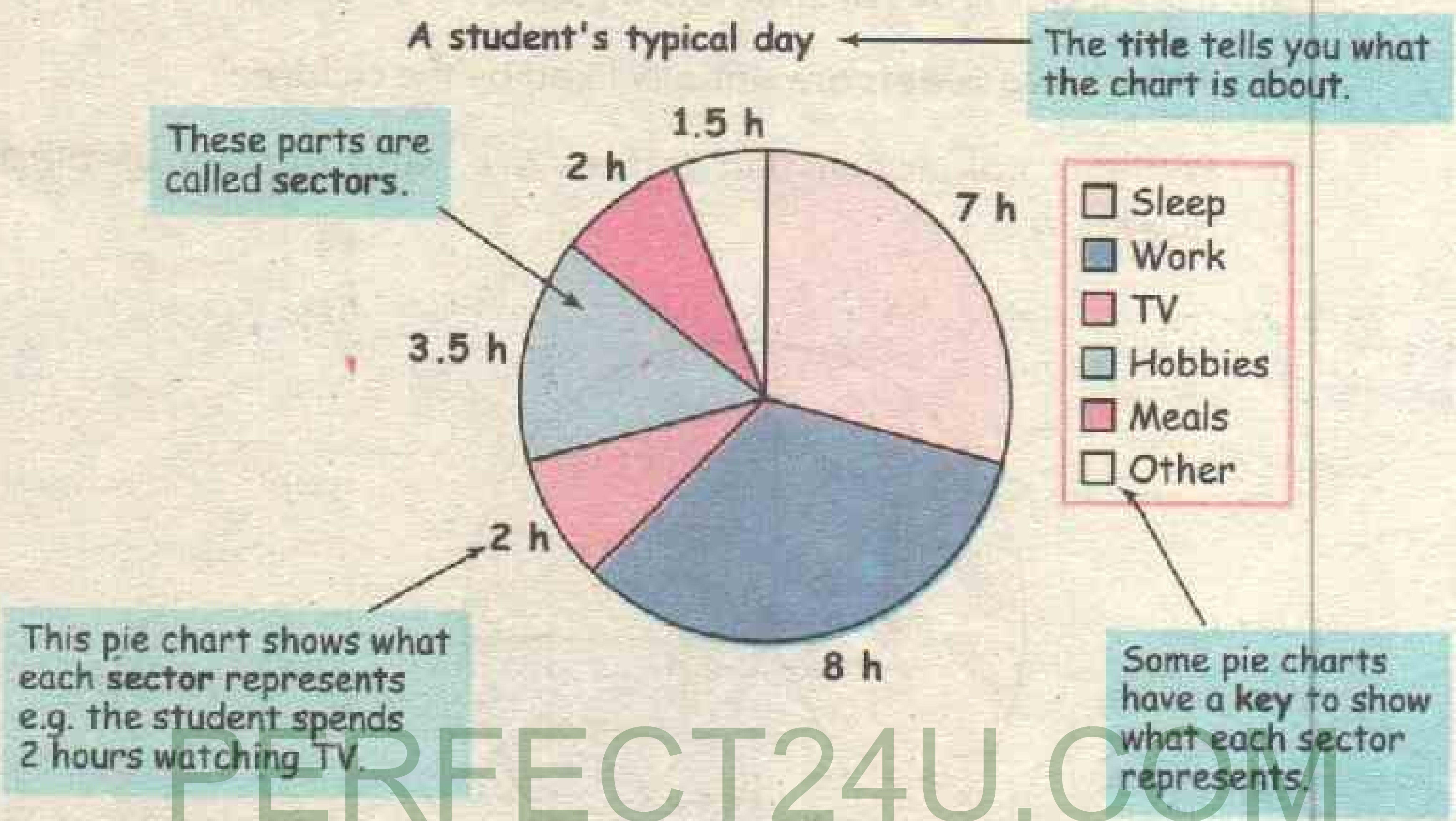
Years	1990	1991	1992	1993	1994
Production in tons	90	100	115	120	110

4. Draw a horizontal bar graph to represent monthly income of Aslam, Akram, Bilal, Ahmed and Hamza given below.

Person	Aslam	Akram	Bilal	Ahmad	Hamza
Income (in Rupees)	2000	3000	4000	5000	6000

13.3 Pie Graph

In pie graph, data is represented by the sectors of a circle. The sum of measure of the angles subtended at the centre of the circle is equal to 360°. The angle subtended by an arc of a represents an element of the data.



1 Use the pie chart above to answer these questions.

a) Which activity took the most time?

b) How much longer did the student spend on hobbies than on watching TV?

c) What is the total time shown on the pie chart? Why?

Some pie charts give information as percentages.

2 This pie chart shows the results of a survey about how people travel to work.

a) Which is

i) the most popular way of getting to work

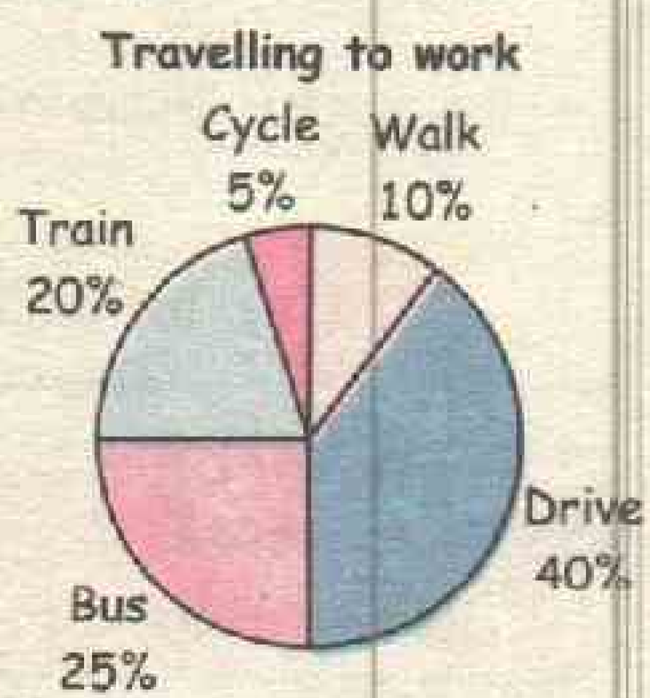
ii) the least popular way of getting to work?

b) What fraction of the people

i) travel to work by bus ii) walk to work

Find how many travelled by each method.

c) Altogether 400 people took part in the survey.



Example 1 Read Pie Graph

A pack consists of 4 types of sweets in the pie-graph according to the taste of the children.

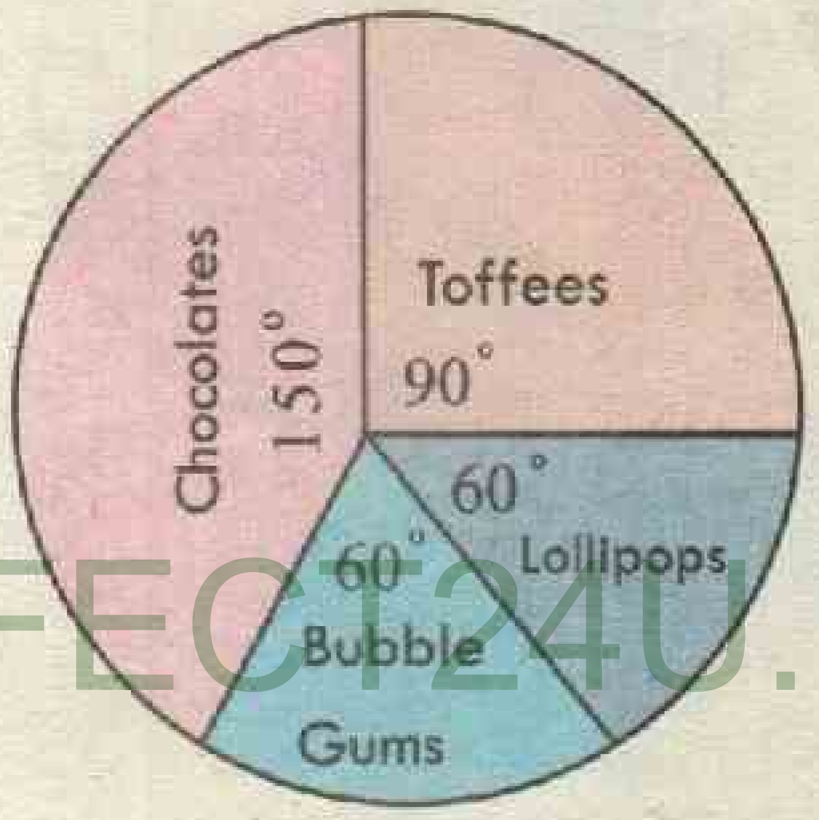
(i) Which is the favourite sweets?

(ii) How many types of sweets are there?

(iii) Which two sweets are equally liked by the children?

(iv) Without calculation find toffees are what percent of all the sweets.

Solution:



(i) As pie graph shows that chocolates make an angle of measure 150°, which is the largest angle as compared to other sweets. This shows that chocolates is favourite sweet.

(ii) There are four types of sweets.

(iii) Bubble Gums and Lollipops.

(iv) Toffies are 25% of all the sweets.

Remember

Pie-graph is also called Circular graph.

Example 2 Read Pie Graph

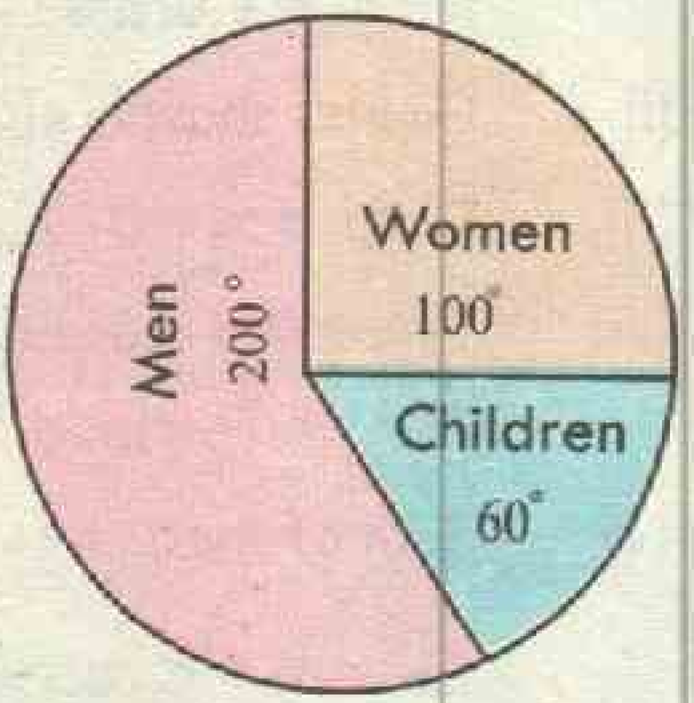
The following pie graph shows categorization of Hajis on a plane.

(i) Which category is the greatest in number?

(ii) Which category is the least in number?

(iii) The Pie-graph is divided into how many categories?

(iv) Women are half of men, true or false?



Solution:

(i) Men are greatest in number.

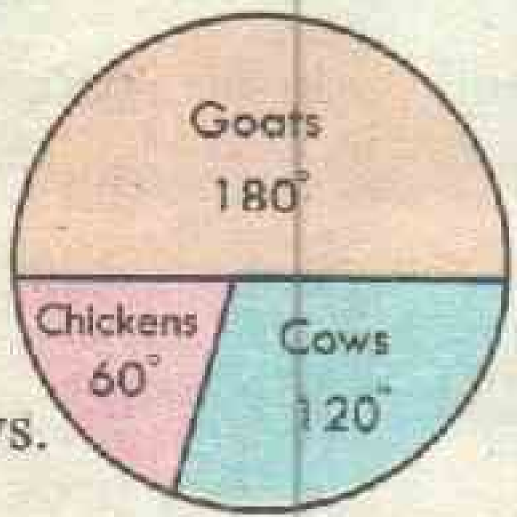
(ii) Children are least in number

(iii) The Pie-graph has been divided into three categories.

(iv) True

Exercise 13.3

1. Ahmad has three types of animals as shown in pie graph, read the graph and answer the following questions.

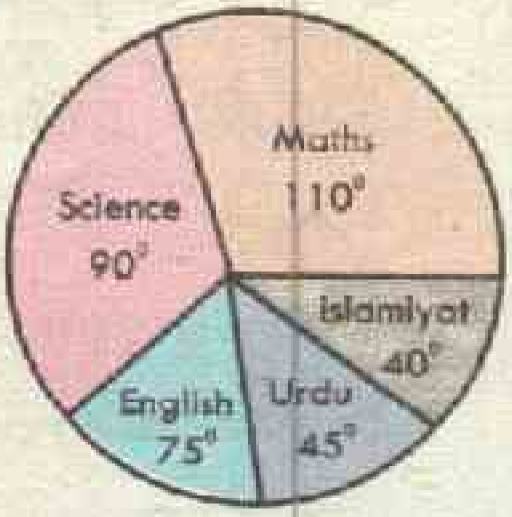


(i) Which type of animal is greatest in number?

(ii) Which type of animal is smallest in number?

(iii) Without calculation, find the ratio of goats and cows.

2. Aslam obtained marks in different subjects as given in the pie graph. Read the graph and answer the following questions.



(i) In which subject did he obtain maximum marks?

(ii) In which subject did he obtain minimum marks?

(iii) Without calculation, find ratio between the marks he obtained in Science and Urdu.

Review Exercise 13

1. Fill in the blanks.

(I) Graph shows relationship between two different ________ .

(ii) Data is set of ________ facts.

(iii) There are ________ types of data

(iv) The whole circle for pie graphs equals ________ percent.

(v) Sum of measures of angles in pie graph = ________ .

2. Choose the correct answer.

(i) Numerical facts obtained from observation and experiments are called.

a. Figure b. Data
c. Frequency d. Classes

(ii) Data is of

a. One type b. Two types
c. Three types d. Four types

(iii) In bar graph width of bars is

a. Equal b. Greater than first bar
c. Less than first bar d. Unequal

(iv) Pie graph is also known as

a. Bar graph b. Circular graph
c. Rectangular graph d. Line graph

(v) Sum of measures of angles in pie-graph is

a. 90° b. 180°
c. 270° d. 360°

(vi) Pie graph is divided into

a. Sectors b. Rectangles
c. Chords d. Squares

3. Following are the ages (years) of 30 students of a school.
16, 13, 12, 14, 8, 9, 15, 4, 7, 6, 11, 10, 7, 5, 6, 14, 13, 10, 9, 8, 7, 5, 4, 16, 13, 12, 9, 6, 5, 7
Identify the type of data.

4. Draw a vertical bar graph to represent number of admission in a school in different classes in different years. Detail is given below.

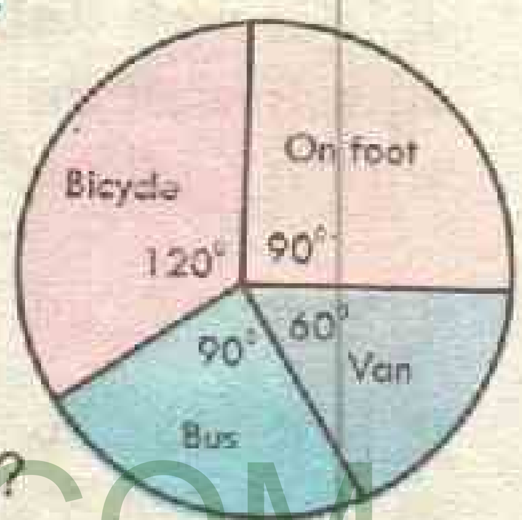
Years	2000	2001	2002	2003	2004	2005
No. of admission	50	60	70	80	90	100

5. The means of different students to go to school are shown in pie graph, read the graph and answer the following questions.

(i) In how many sectors pie graph has been divided?

(ii) Which means is mostly used for going to school?

(iii) Which means is less used for going to school?



Summary

- **Bar Graph:** Bar graph shows relationship between two different quantities in the form of vertical or horizontal bars.
- **Data:** The numerical facts obtained from observations, measurements or experiments is called data.
- **Ungrouped Data:** Numerical information which is obtained as such is called ungrouped data.
- **Grouped data:** When data is arranged in different groups of classes it is called grouped data.
- **Pie graph:** Pie graph is that type of graph in which data is represented by the sectors of a circle.

ANSWERS

Exercise 1.1

1. i) It is not well defined because some problems can be difficult for one student but easy for others. So it is not a set.
   ii) It is well defined because some difficult problems of the book can be identified for Amjid. So it is a set.
   iii) It is well defined and hence it is a set.
   iv) It is not well defined because class is not mentioned. So it is not a set.
   v) It is well defined and hence it is a set.
   vi) It is well defined and hence it is a set.
   vii) It is well defined and hence it is a set.

2. i) Wrong ii) Correct iii) Correct iv) Wrong v) Correct

3. i) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
   ii) {1,2,3,4,5,6,7,8,9,10} iii) {a,e,i,o,u}
   iv) {Muharram, Safar, Rabi-ul-Awal, Rabbi-ul-Sani, Jamadi-ul-Awal, Jamadi-ul-Sani}
   v) {January, February, March, April, May, June, July, August, September, October, November, December}
   vi) {2,3,5,7, ...} vii) {Summer, Autumn, Winter, Spring}

Exercise 1.2

1. vi
2. i) {1,2,3,4,5} ii) {a,b,c,d,e,f} iii) {1,3,5,7}
   iv) {1,3,4,6,12} v) {M, A, T, H, E, I, C, S}
   vi) {September, October, November, December}
3. i) Empty Set ii) Singleton iii) Singleton
4. $A \simeq B$, $C \simeq D$.
5. $A \backsimeq B$, $C \backsimeq D$.

Review Exercise 1

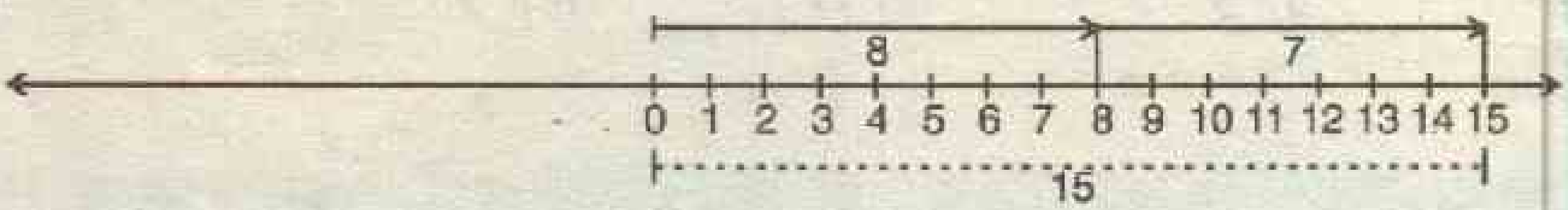
1. i) Tabular ii) Singleton iii) Three iv) Finite v) Infinite
   vi) Equal vii) Equivalent
2. i) $\subseteq$ ii) $\subseteq$ iii) $\not\subseteq$ iv) $\subseteq$ v) $\subseteq$ vi) $\not\subseteq$
3. No, because A is an infinite set and B is finite set.
4. i) G ii) $\varnothing$, H iii) $\varnothing$, {6}, {7}, {I}
   iv) $\varnothing$, {6}, {7}, {8}, {6,7}, {6,8}, {7,8}, J
   v) $\varnothing$, {6}, {7}, {8}, {9}, {6,7}, {6,8}, {6,9}, {7,8}, {7,9}, {8,9}, {6,7,8}, {6,7,9}, {6,8,9}, {7,8,9}, K
5. No. of subsets of L are $2^5 = 32$.
6. $\varnothing$, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, S
7. i) 4 ii) 0 iii) 50

Exercise 2.1

1. i) > ii) > iii) <
2. i) < ii) > iii) <
3. i) > ii) < iii) >
4. i) True ii) False iii) False
5. i)

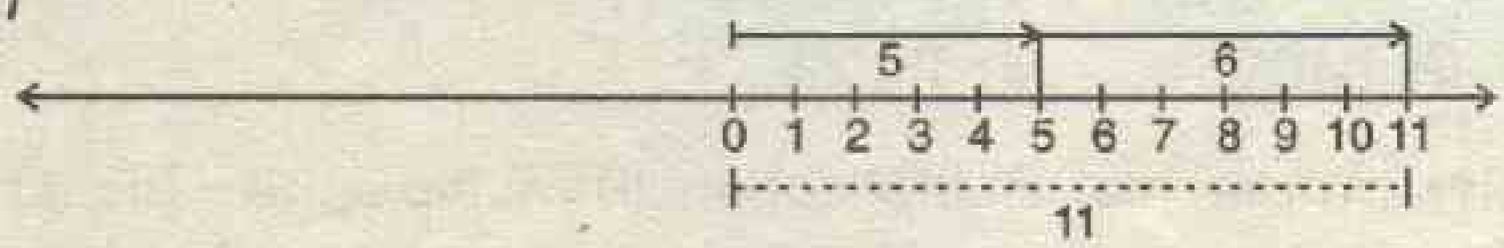
$10 + 5 = 15$

ii)



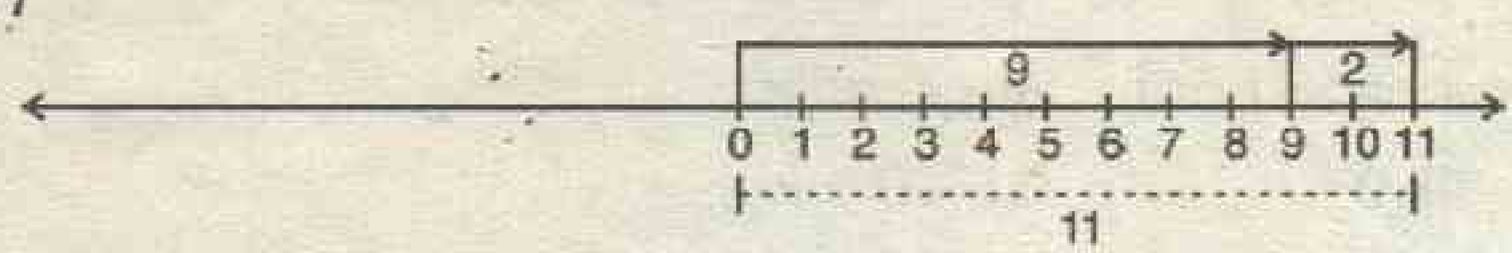
$8 + 7 = 15$

iii)



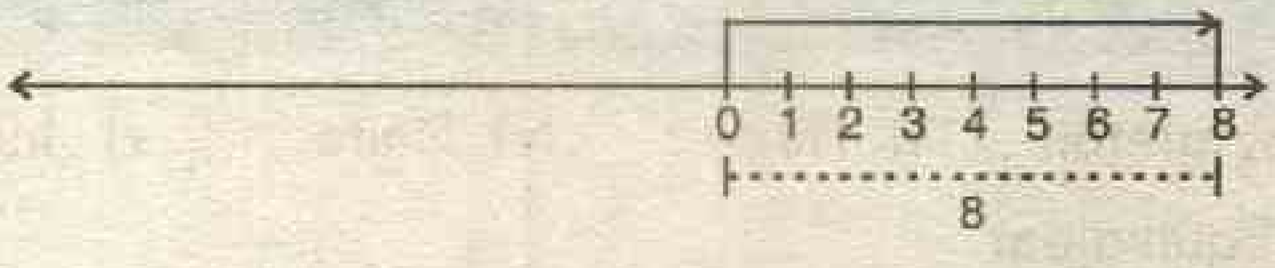
$5 + 6 = 11$

iv)



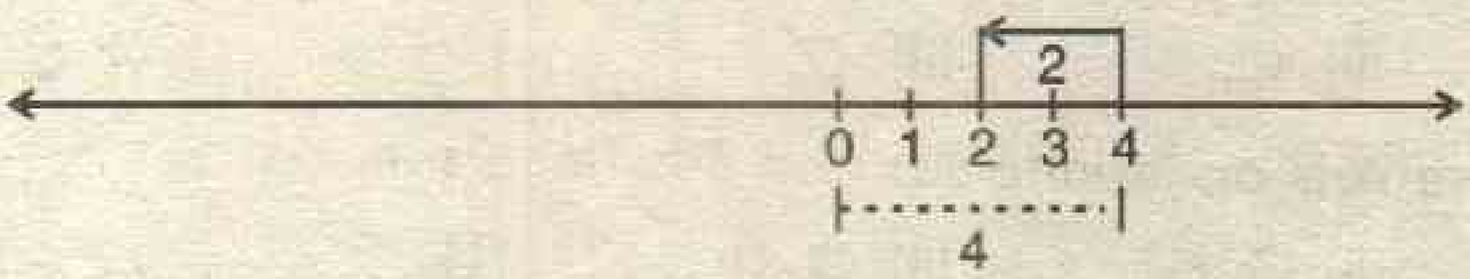
$9 + 2 = 11$

v)



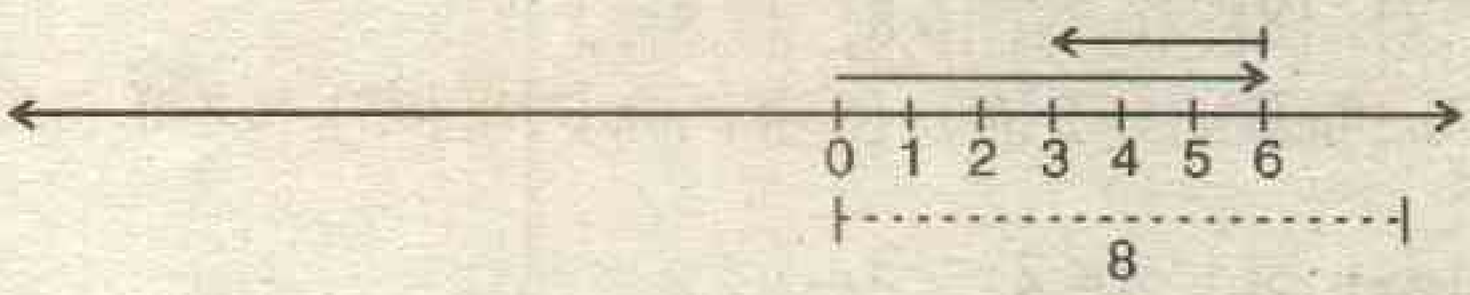
$0 + 8 = 8$

6. i)



$4 - 2 = 2$

ii)

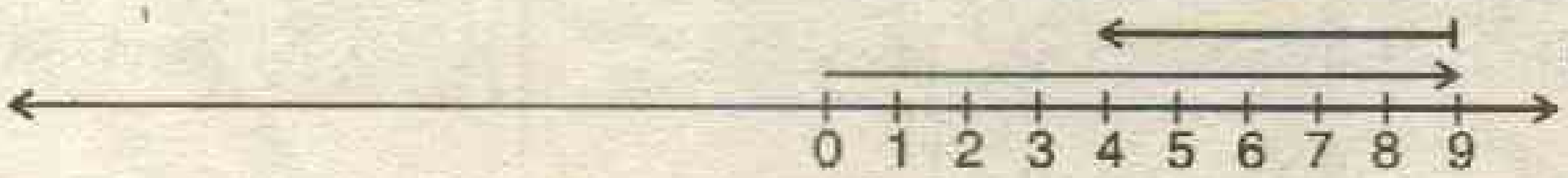


$6 - 3 = 3$

iii)

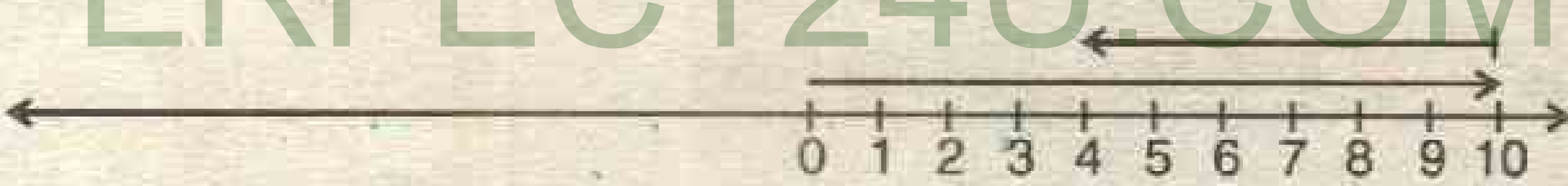
$7 - 4 = 3$

iv)



$9 - 5 = 4$

v)



$10 - 6 = 4$

Exercise 2.2

1. i) 36 ii) 22 iii) 29

2. i) 2 ii) 5 iii) 0 iv) 2

3. i) True ii) False iii) True

Exercise 2.3

1. i) 240 ii) 14098 iii) 7493

3. i) Commutative property of multiplication ii) Multiplicative identity

iii) Distributive property of multiplication over addition

iv) Associative property of multiplication

4. i) 55 ii) 0 iii) 40

Review Exercise 2

1. i) Whole ii) Less iii) 8 iv) (9+6)+3 v) $(7+3) \times x$
2. i) True ii) True iii) False iv) True v) True
3. i) a ii) b iii) b iv) a
4. b
5. i) Commutative of property of multiplication
   ii) Associative property of addition
   iii) Distributive property of multiplication over addition
   iv) Distributive property of multiplication over subtraction
6. i) 100 ii) 270 iii) 45 iv) 100
8. Amjid

Exercise 3.1

1. Prime numbers: (i), (iv), (vi), (viii), (x)
   Composite numbers: (ii), (iii), (v), (vii), (ix)
2. 23, 29, 31, 37
3. 42, 44, 45, 46, 48, 49
4. i) 1, 2, 4, 5, 10, 20
   ii) 1, 2, 13, 26
   iii) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
   iv) 1, 2, 3, 5, 6, 10, 15, 30
   v) 1, 2, 3, 4, 6, 7, 14, 21, 42
   vi) 1, 2, 3, 4, 8, 16
   vii) 1, 2, 3, 4, 8, 16, 32
   viii) 1, 7, 49

5. i) 7, 14, 21 ii) 11, 22, 33 iii) 17, 34, 51 iv) 19,38,57
   v) 12, 24, 36 vi) 15, 30, 45 vii) 18, 36, 54 viii) 20, 40, 60
6. 52, 54, 56, 58, 60, 62, 64, 66, 68
7. 81, 83, 85, 87, 89, 91, 93, 95, 97, 99

Exercise 3.2

1. a. (i), (ii), (iii), (vi), (viii), (x), (xii)
   b. (i), (iv), (vii), (xi), (xii)
   c. (ii), (vi), (viii), (x), (xii)
   d. (iii), (v), (ix), (xii)
2. a. (ii), (iv), (v), (xi), (xii), (xiii)
   b. (i), (iv), (vi), (x), (xii)
   c. (ii), (iv), (v), (vii), (xi), (xii)
   d. (ii), (vii)
3. a. (iii), (v), (vi), (ix), (xii)
   b. (i), (ii), (vii), (x), (xii)
   c. (ii), (vi), (viii), (xi), (xii)
   d. (iii), (v), (viii), (xii)

Exercise 3.3

1. i) 2×3 ii) $2\times2\times5\times7$ iii) $2\times3\times3\times5\times5$ iv) $2\times3\times3\times5\times5\times7\times7$
2. i) $2^4 \times 3^2$ ii) $2^4 \times 3^2 \times 5^2$ iii) 2×5^4 iv) $2^4 \times 3^2 \times 7^2$

Exercise 3.4

1. i) $2 \times 3 \times 5 \times 7^2$ ii) $2^2 \times 3 \times 5^2 \times 7^2 \times 11$ iii) $2^5 \times 5^5$
2. i) 15 ii) 9 iii) 6 iv) 18

Exercise 3.5

i) 12 ii) 18 iii) 25 iv) 70 v) 105
vi) 21 vii) 15 viii) 45

Exercise 3.6

i) 24 ii) 60 iii) 80 iv) 120 v) 630
vi) 150 vii) 2450 viii) 288 ix) 72

Exercise 3.7

i) 240 ii) 540 iii) 840 iv) 840 v) 3150
vi) 1440 vii) 630 viii) 288 ix) 2016

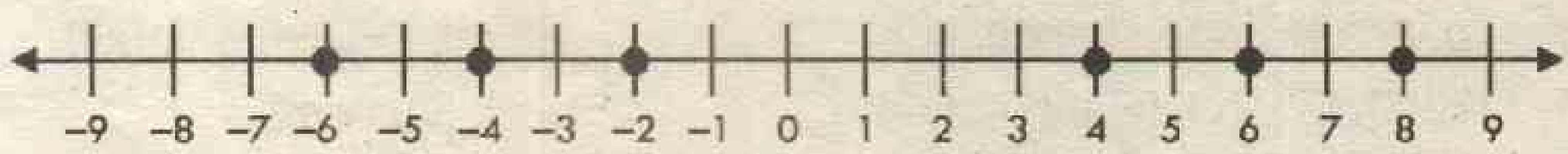
Exercise 3.8

1. 15cm 2. 15 3. 450 4. 540 5. 35cm
6. 360 7. 720 8. 70 Liter 9. 2940 10. 63

Review Exercise 3

1. b 2. c 3. c 4. a 5. c
6. c 7. b 8. c 9. b 10. a 11. a

Exercise 4.1

1. 

2. i) –10 ii) 50 iii) 3 iv) –5 v) 7 vi) –5
3. i) –32 ii) 3 iii) 21 iv) –16

Exercise 4.2

1. i) 8 ii) −2 iii) −8
2. i) 5 ii) −1 iii) 5
3. i) −3 ii) −7 iii) −11
4. i) 2 ii) −10 iii) −4
5. i) −4 ii) 5 iii) −6

Exercise 4.3

1. i) 10 ii) −15 iii) −100 iv) 110 v) −70
2. i) −35 ii) 49 iii) 5 iv) 6 v) 72
3. i) −15 ii) −7 iii) −3 iv) −9 v) −8

Review Exercise 4

1. i) Negative ii) Negative iii) Greater than iv) 4
2. i) True ii) False iii) True iv) False
3. i) b ii) a iii) b iv) b
4. i) > ii) > iii) < iv) < v) = vi) >

Exercise 5.1

1) 12 2) 14 3) 58 4) $2\frac{1}{3}$ 5) $4\frac{5}{8}$

6) $2\frac{1}{3}$ 7) $1\frac{3}{4}$ 8) $5\frac{19}{34}$

Exercise 5.2

1. 2.7 2. 8.76 3. 4.37 4. 5.14 5. −2.5285

Exercise 5.3

1. $20\frac{1}{2}$ kg 2. $17\frac{7}{8}$ m^2 3. Rs.6750 4. 4 Toffees 5. 4.122 hour

6. 20.2 km, 10.8 km 7. Rs. 5.041 8. 6m 9. 2940

Review Exercise 5

1. i) b ii) b iii) c iv) c v) c
2. i) $\frac{167}{210}$ ii) 12.759 iii) $6\frac{3}{4}$ iv) Rs. 676.6875
   v) 174.1°

Exercise 6.1

1. i) 6:7 ii) 4:5 iii) 3:5 iv) 3:2 v) 2:3
2. i) 5:7 ii) 1:3 iii) 5:3 iv) 4:3 v) 3:5
3. 4:5
4. 3:4
5. 5:3
6. 3:2
7. 29:24
8. 6:1

Exercise 6.2

1. i) True ii) True iii) False iv) True v) False
2. i) 20 ii) 11 iii) 18 iv) 26
3. iii
4. i) Direct ii) Inverse iii) Direct iv) Inverse
5. Rs. 1200
6. Rs. 250
7. Rs. 7.5
8. Rs. 200

Review Exercise 6

1. i) 3:1 ii) 3:1 iii) 3:7
   iv) Antecedent = 3, Consequent = 5
   v) Antecedent = 13.5, Consequent = 17.2 vi) $\frac{3}{4}$
   vii) $\frac{3}{1}$ viii) 5:3 ix) Extremes: 1, 4 , Means: 2, 2
   x) Extremes: 3.2, 10 ., Means: 5, 6.4 xi) Direct xii) Direct
2. 7:5
3. 6:5
4. Rs. 495
5. 45 days

Exercise 7.1

1. i) $\frac{4}{5}$ ii) $\frac{3}{5}$ iii) $\frac{1}{8}$ iv) $\frac{1}{6}$ v) $\frac{13}{100}$
   vi) $\frac{1}{10}$ vii) $\frac{19}{150}$

2. i) 60% ii) $23\frac{1}{3}$ % iii) 225% iv) $6\frac{1}{4}$ % v) 32%
3. i) 0.4 ii) 0.75 iii) 1.25 iv) 0.004 v) 0.33
4. i) 50% ii) 76% iii) 245% iv) 37.5% v) 1.25%

Exercise 7.2

1. 50% 2. 45% 3. 70% 4. 30% 5. 62.5%

Exercise 7.3

1. i) 50, 450 ii) 10, 190 iii) 200, 600
   iv) 5000, 20000 v) 900, 5100 vi) 800, 7200
2. Rs. 50 3. 42.9% 4. Loss Rs. 25 5. 25% profit
6. Profit Rs. 300 7. Rs. 900 8. 25%

Review Exercise 7

1. i) True ii) False iii) True iv) False v) True
2. i) a ii) b iii) b

3. i) 75 ii) 160 iii) 34.28 iv) 49% v) 77.77%
4. 16.6% 5. 216000

Exercise 8.1

1. i) Constant : –5, Variable : x ii) Constant : –1, Variable : y
   iii) Constant : 4, Variable : a iv) Constant : 2, Variable : x, y
2. i) –2, 1 ii) 3, 2 iii) 5, 3 iv) –6, 2
3. i) True ii) False iii) Open iv) Open v) False
   vi) True
4. i) $<$ ii) $=$ iii) $>$
5. i) 100 ii) 150 iii) $50x$ iv) $200x$

Exercise 8.2

1. i) One ii) One iii) Two iv) Two v) Three
   vi) One vii) Two

2. i) a^2 and $3a^2$, ab and $2ab$ ii) x^2y and $3x^2y$, $2xy^2$ and $5xy^2$
iii) a^3 and $5a^3$, $-2a^2b$ and $4a^2b$ iv) xy and $2xy$, yz and $-3yz$

3. i) $5x$ ii) $9a^2$ iii) $6x$
iv) $11ab$ v) $5xyz$ vi) $11a^3$

4. i) $7x+6y$ ii) $-2x^2y+7xy^2$
iii) $-ab+bc$ iv) $(a+c)x+by$

Exercise 8.3

1. i) $3x+4y$ ii) $11x+10y$ iii) $7x^2+3y^2+4z^2$ iv) $5x^2+8x-2$
2. i) $12x^2+6x+5$ ii) $19a^2+4ab+3b^2$ iii) $6p^2+2pq+4q^2$
3. i) $3x-4y$ ii) $2x^2-8xy-3y^2$
iii) $3a^2b-3b^2c+5ac^2$
4. i) $3x$ ii) $-a^2b-4b^2$ iii) $-x^2y-5y^2-4$ iv) $4m^2-2n$

Exercise 8.4

1. i) 11 ii) 12 iii) 23
2. i) 64 ii) 32 iii) $\frac{17}{4}$
3. i) 4 ii) 5 iii) $\frac{7}{6}$
4. i) -8 ii) 8 iii) zero
5. i) 1 ii) 2 iii) zero

Review Exercise 8

1. i) Constant ii) Variable iii) Two iv) 3
2. i) True ii) True iii) True iv) False
3. i) a ii) b iii) b iv) c
4. i) $-3x$ ii) $-3x-8$ iii) $-10a-36$ iv) $-17t+34$
5. i) 9 ii) -104

Exercise 9.1

1. i) $\frac{-13}{4}$ ii) $\frac{49}{15}$ iii) -18
2. i) 0 ii) 5 iii) 1
3. i) False ii) True

Exercise 9.2

1. i) $2x + 6 = 0, 3x + 9 = 0$ ii) $x - \frac{5}{2} = 0, 4x - 10 = 0$
   iii) $6x + 2 = 4x, 9x + 3 = 6x$
   iv) $8x - 14 = 2x + 10, 12x - 21 = 3x + 15$
2. i) $x = 7$ ii) $x = 2$ iii) $x = -5$ iv) $x = 2$ v) $x = -\frac{1}{2}$
   vi) $x = 4$ vii) $x = 16$ viii) $x = \frac{7}{2}$ ix) $y = 2$ x) $y = -\frac{7}{2}$
   xi) $x = 2.5$ xii) $x = 3$
3. 7 4. 16 5. 18 6. 7 7. 7
8. 107 meter, 53.5 meter 9. 32 years, 16 years

Review Exercise 9

1. i) $x = 1$ ii) $y = 20$ 2. i) -1 ii) 7
3. i) 9 ii) -42 4. i) -5 ii) $\frac{8}{5}$
5. i) True ii) True
6. i) $x = 2$ ii) $x = 5$ iii) $x = -11$ iv) $y = -0.65$
7. -11 8. -10 9. -3 10. 21 11. 21

Exercise 11.1

1. i) 18cm ii) 11m iii) 24m
2. i) $7yd^2$, $16yd$ ii) $1 sq.in$, $4in$
3. i) 80cm, $400cm^2$ ii) 14.4cm, $12.96cm^2$
   iii) 236cm, $3481cm^2$

4. i) 28cm, 45cm^2 ii) 191.6m , 117.56m^2 5) Rs.225,000 6) Rs.9537.50

7. 82.5m^2 8. 10cm^2, 36cm^2

Exercise 11.2

1. i) 60cm^2 ii) 32m^2 2. 0.79m^2 3. 4224m^2
4. 9576m^2 5. 5856m^2 6. Rs. 340 7. 808m^2

Exercise 11.3

1. i) 20cm^2 ii) 18cm^2 iii) 30cm^2
2. i) 6m^2 ii) 800mm^2 iii) 140cm^2
3. i) 24cm^2 ii) 8.36cm^2
4. i) 20cm^2 ii) 15cm^2
5. i) 15cm^2 ii) 32cm^2
6. Rs. 50.40 7. 504450Kg 8. 900cm^2

Review Exercise 11

1. i) b ii) a iii) d iv) a v) b
   vi) d vii) c viii) b ix) a x) d
2. i) $15x^4$, $16x^2$ ii) $28xy$, $22xy$ iii) a^2, $4a$
3. i) $12ft^2$ ii) $8.1cm^2$ iii) $46.8m^2$
4. $\frac{1}{7}$ 5. 5 6. 37.5cm 7. 10m 8. 7.5m
9. 15m 10. 3.5cm

Exercise 12

1. i) $150cm^2$ ii) $121.5cm^2$ iii) $365.04cm^2$ iv) $285.66cm^2$
2. i) $52cm^2$ ii) $202cm^2$ iii) $94.6m^2$
3. i) $1cm^3$ ii) $216cm^3$ iii) $91.125m^3$ iv) $157.464m^2$
4. i) $24cm^3$ ii) $180cm^3$ iii) $51cm^3$
5. i) $150cm^2$ ii) $125cm^3$ 6. i) $111.5m^2$ ii) $78.75m^3$ 7. 3cm

Review Exercise 12

1. 2
2. 2
3. 2
4. 3
5. $6a^2cm^2$
6. a^3cm^3
7. $2ab + 2bc + 2ca\ cm^2$
8. $1m^3$
9. $6cm^3$
10. D

Exercise 13.1

1.

Groups	No. of Patients
1 – 10	6
11 – 20	11
21 – 30	9
31 – 40	12
41 – 50	3
Total	41

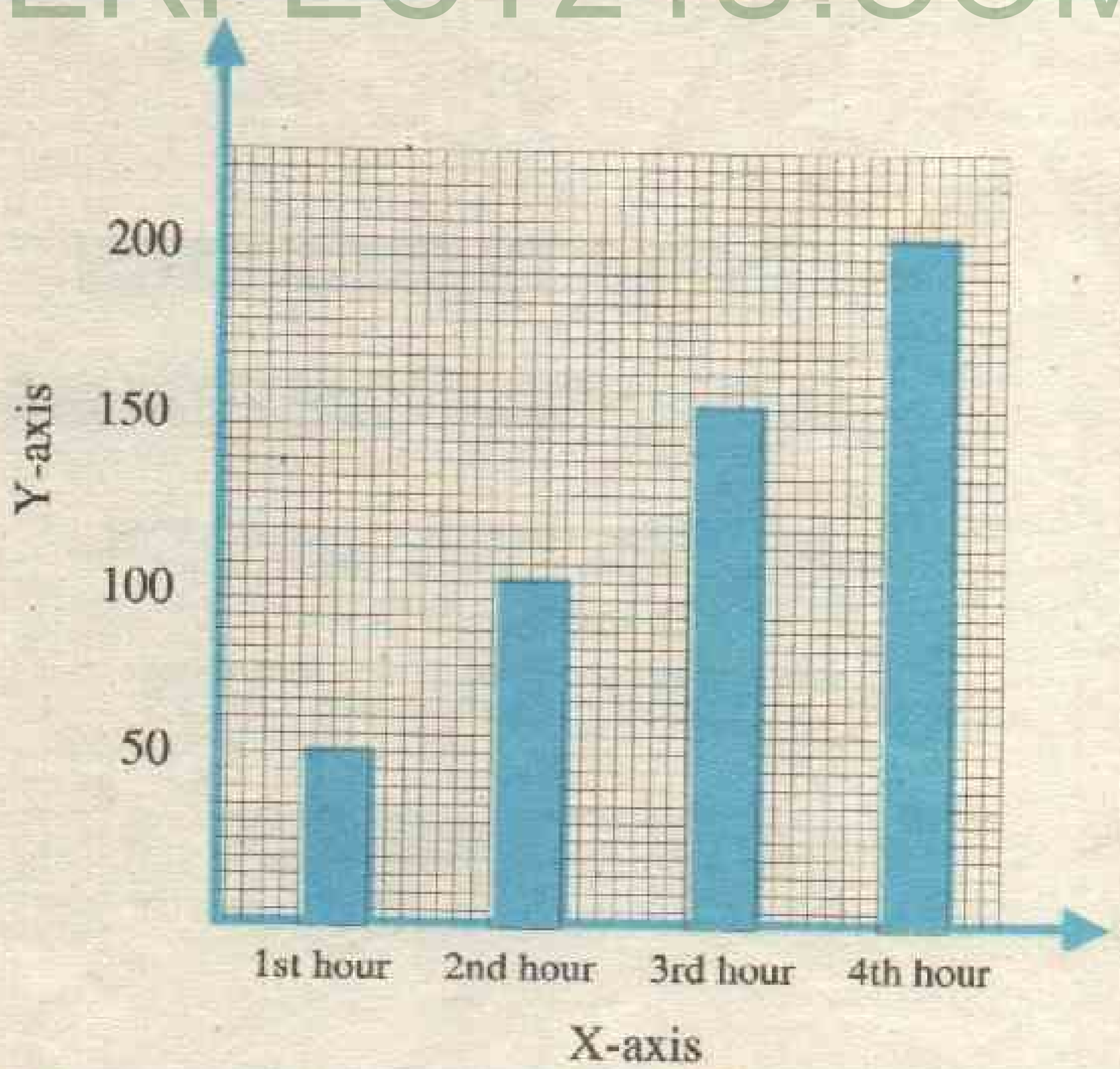
2. The following are the marks scored in a math test by students of the 6th grade 45, 30, 25, 36, 42, 27, 31, 43, 49 and 50.

3. Ungrouped data

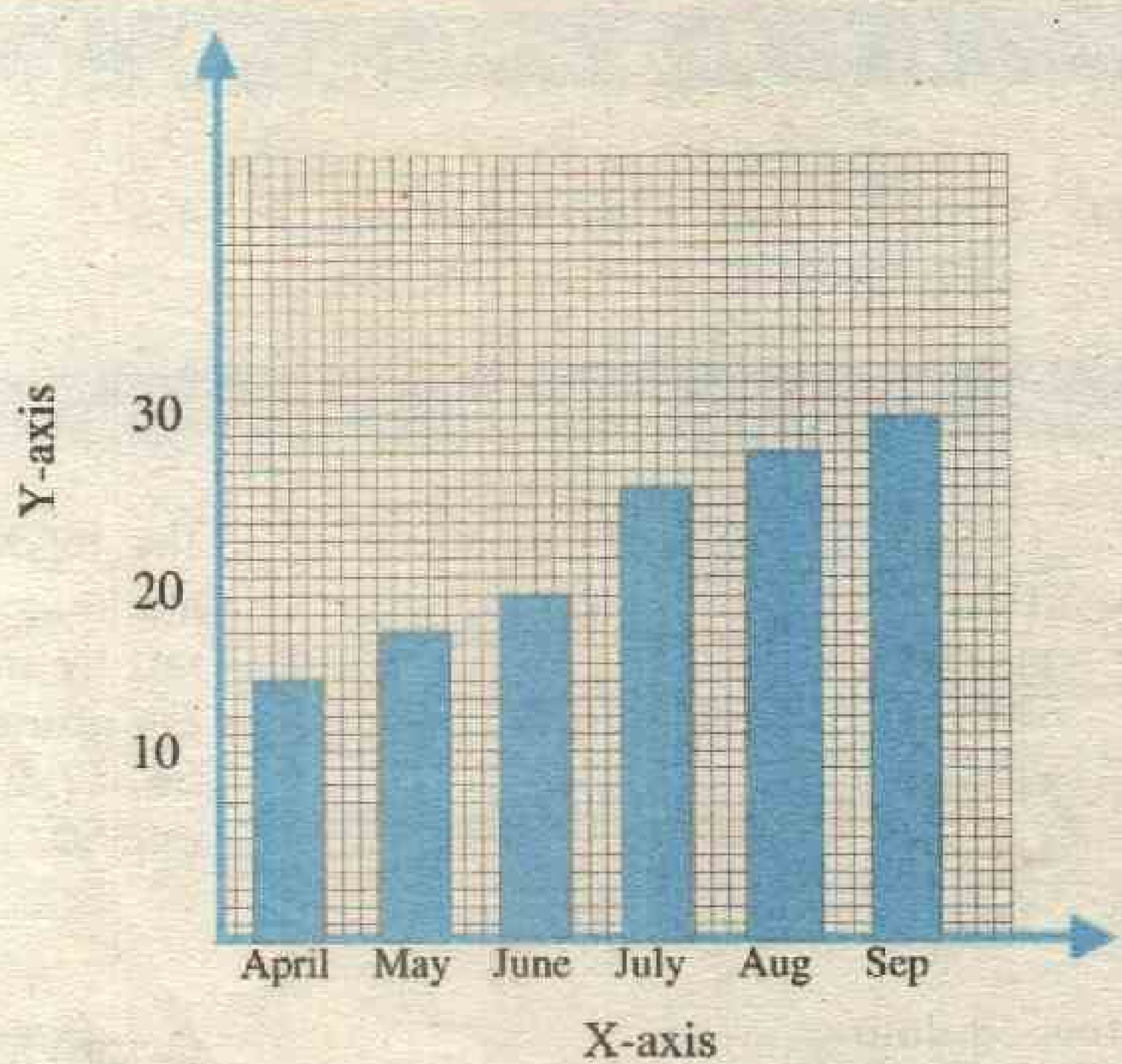
4. Qasim Shah

Exercise 13.2

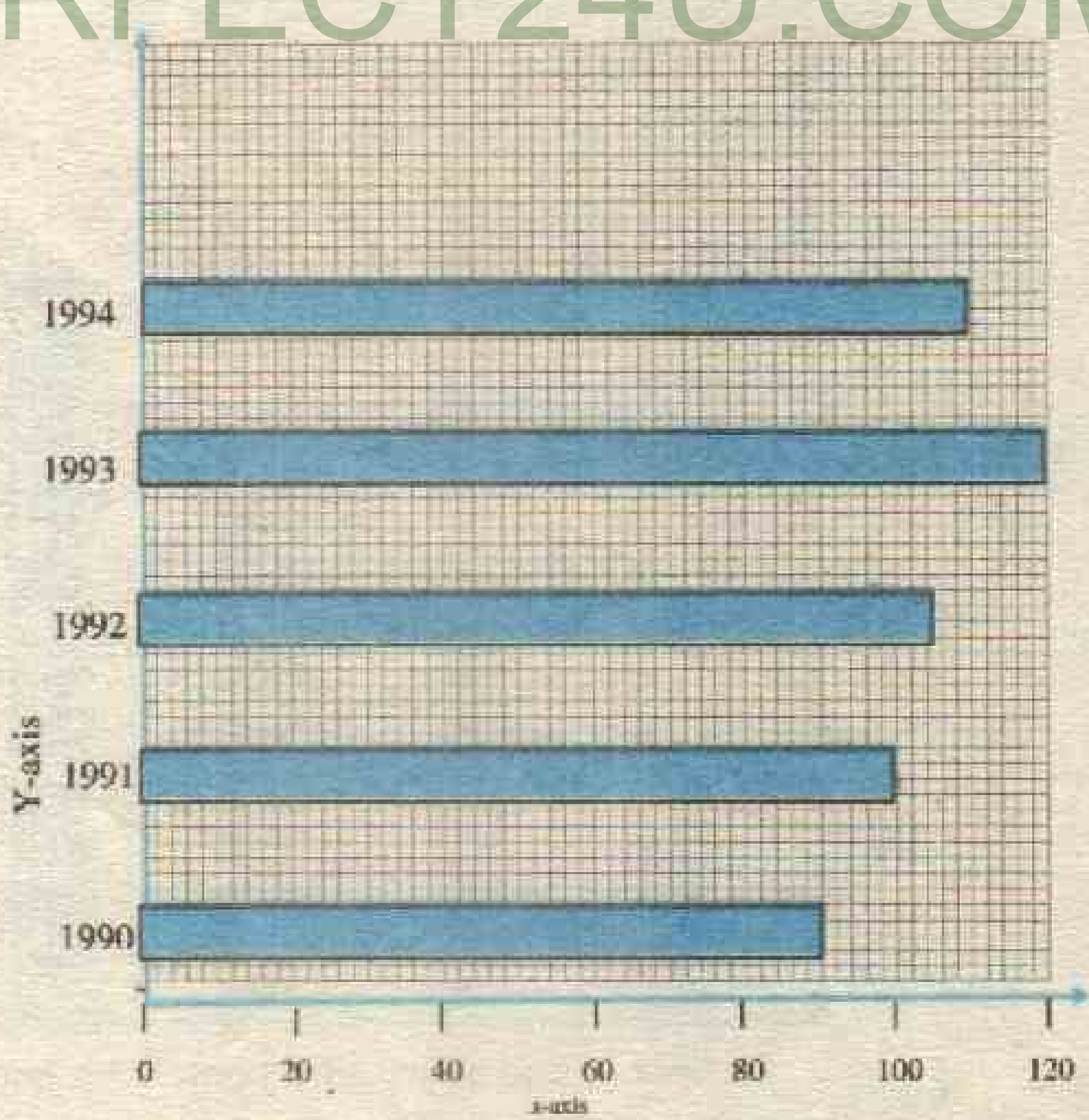
1.



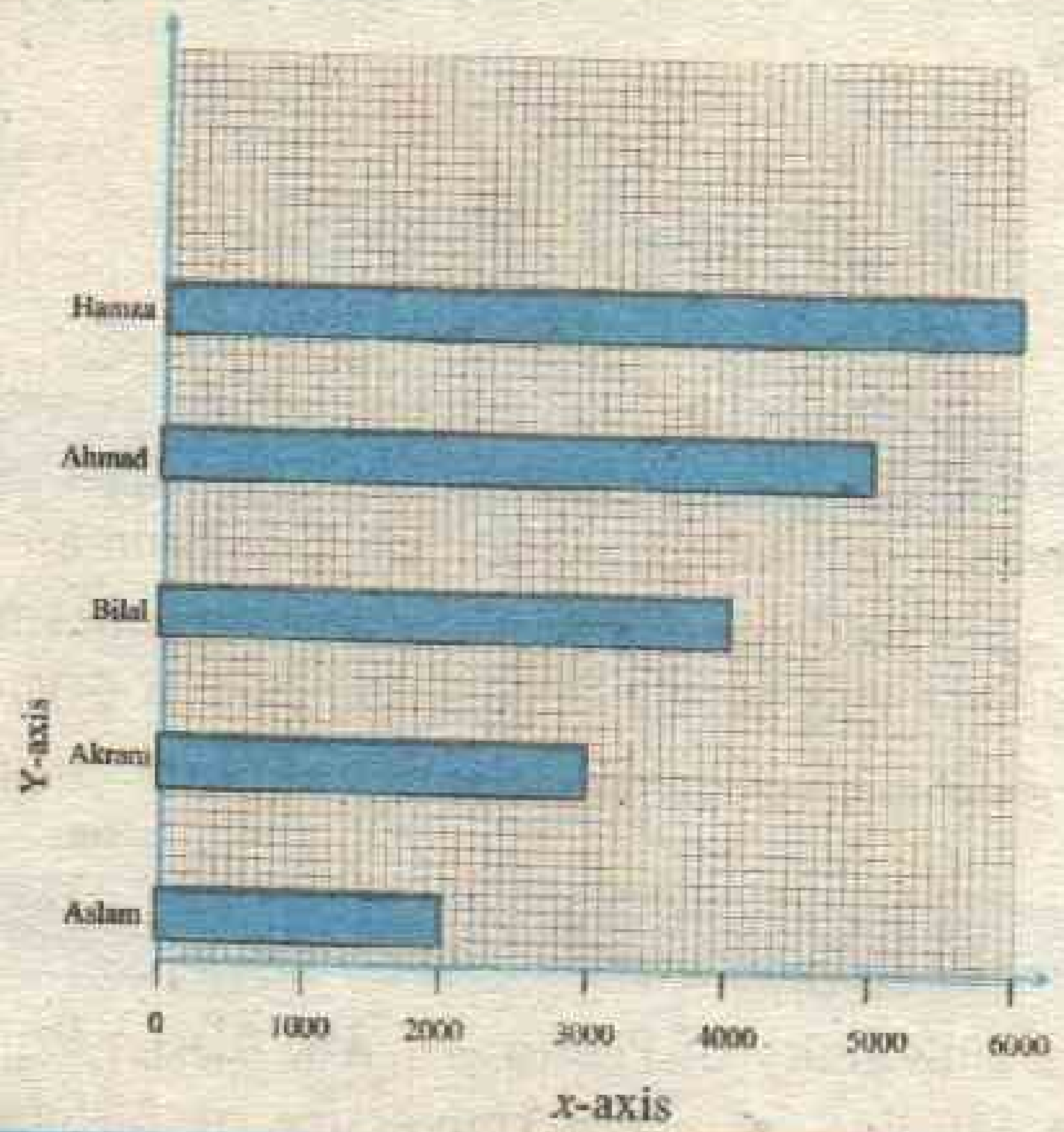
2.


Y-axis
30
20
10
April
May
June
July
Aug
Sep
X-axis

3.


1994
1993
1992
1991
1990
Y-axis
0
20
40
60
80
100
120
x-axis

4.

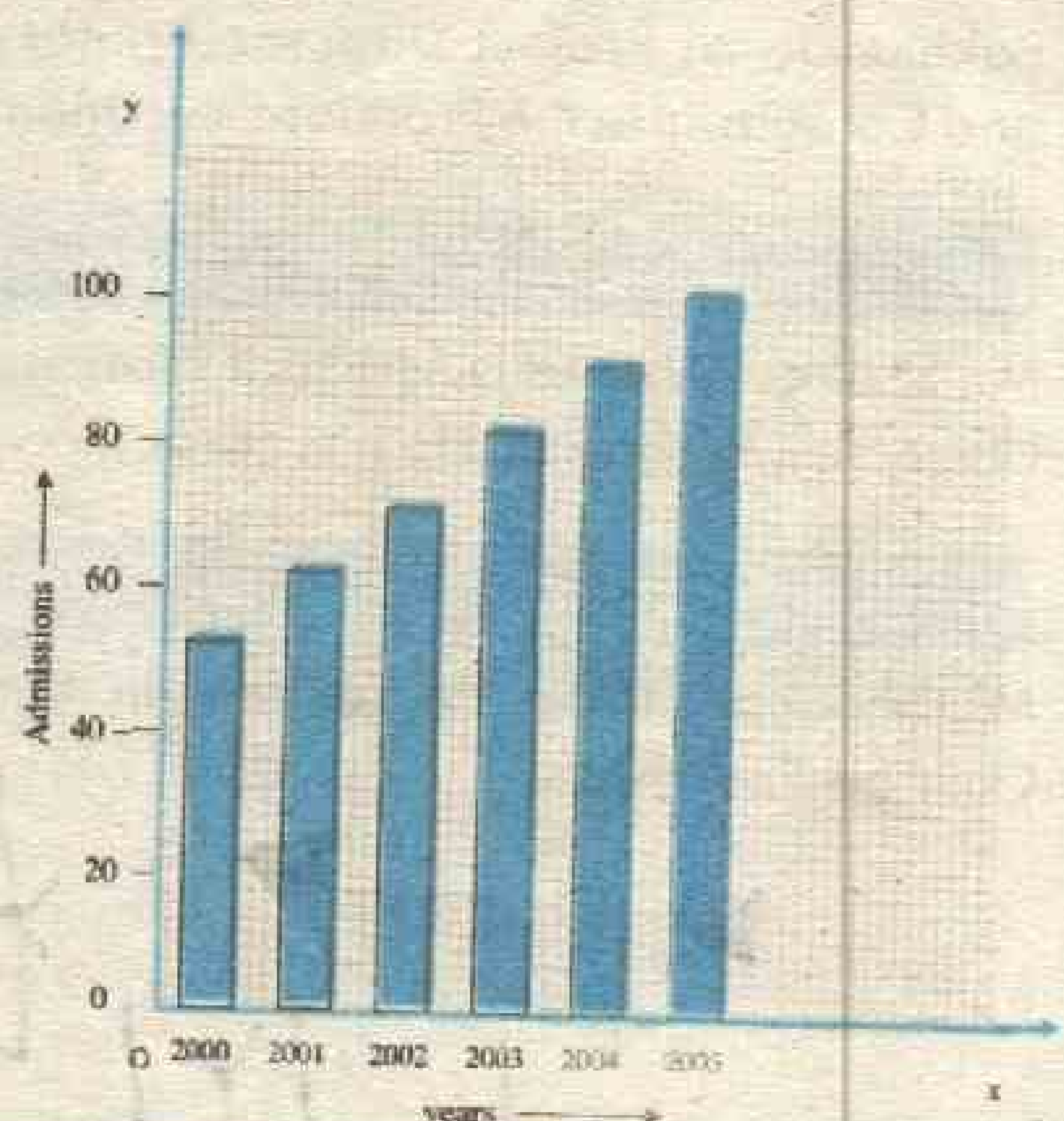


Exercise 13.3

1. i) Goats ii) chickens iii) 3:2
2. i) Maths ii) Islamiat iii) 2:1

Review Exercise 13

1. i) Quantities ii) Numerical iii) Two iv) 100 v) 360°
2. i) b ii) b iii) c iv) a v) b vi) a
3. Ungrouped data
4.



5. i) 4 sectors
   ii) Bicycle
   iii) 25%

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