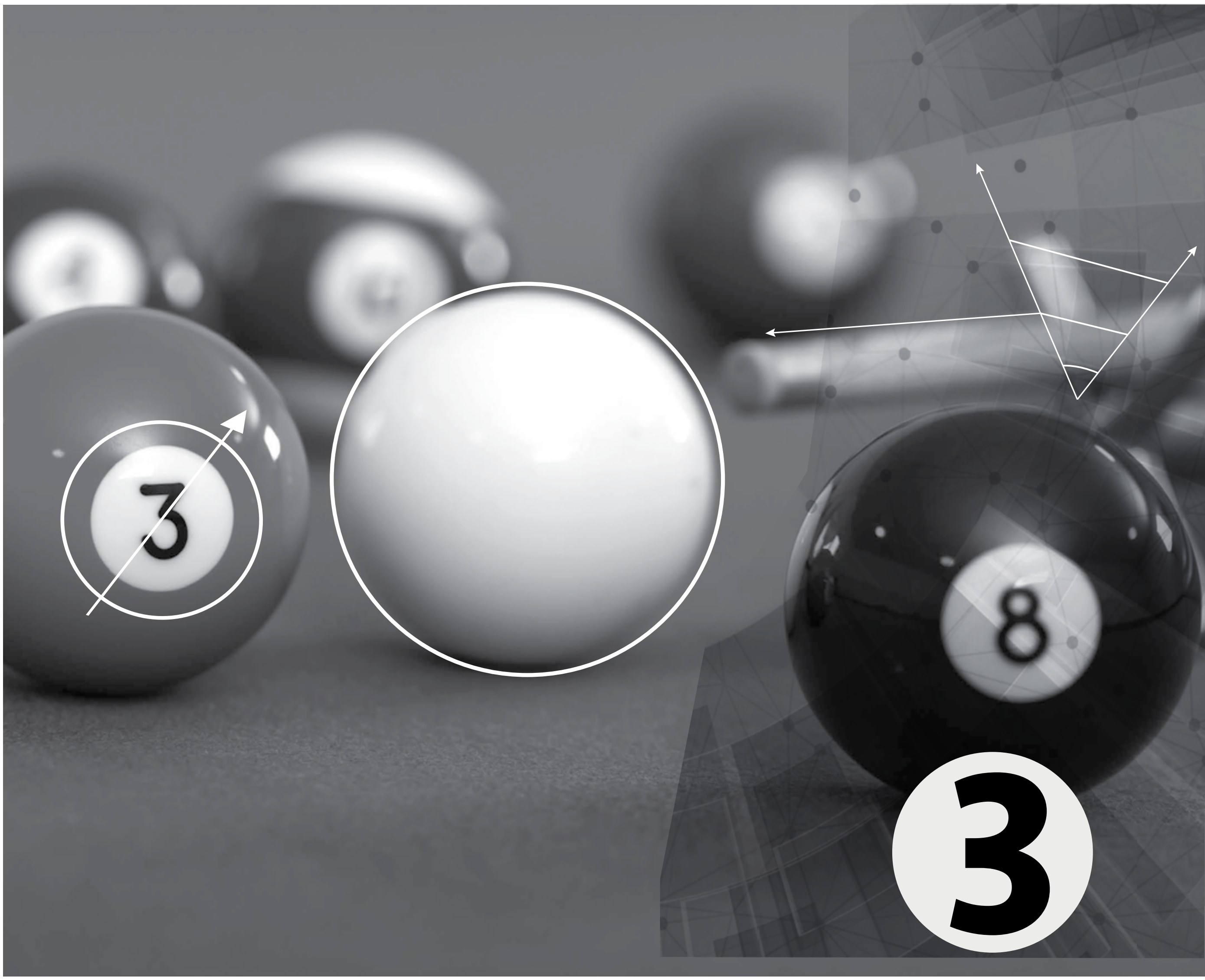
7th EDITION

NEW SYLLABUS MATHEMATICS
TEACHER'S RESOURCE BOOK



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CONTENTS

Syllabus Matching Grid

Cambridge O Level Mathematics (Syllabus D) 4024/4029. Syllabus for examination in 2018, 2019 and 2020.

Theme or Topic	Subject Content	Reference
1. Number	Identify and use: • Natural numbers • Integers (positive, negative and zero) • Prime numbers • Square numbers • Cube numbers • Common factors and common multiples • Rational and irrational numbers (e.g. π, $\sqrt{2}$) • Real numbers	Book 1: Chapter 1 Chapter 2
2. Set language and notation	• Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets • Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \leqslant x \leqslant b\}$, $D = \{a, b, c, \ldots\}$	Book 2: Chapter 14 Book 4: Chapter 2
2. Squares, square roots, cubes and cube roots	Calculate • Squares • Square roots • Cubes and cube roots of numbers	Book 1: Chapter 1 Chapter 2
4. Directed numbers	• Use directed numbers in practical situations	Book 1: Chapter 2
5. Vulgar and decimal fractions and percentages	• Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts • Recognise equivalence and convert between these forms	Book 1: Chapter 2
6. Ordering	• Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, <, >, \leqslant, \geqslant$.	Book 1: Chapter 2 Chapter 5
7. Standard form	• Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leqslant A < 10$.	Book 3: Chapter 4
8. The four operations	Use the four operations for calculations with: • Whole numbers • Decimals • Vulgar (and mixed) fractions including correct ordering of operations and use of brackets.	Book 1: Chapter 2
9. Estimation	• Make estimates of numbers, quantities and lengths • Give approximations to specified numbers of significant figures and decimal places • Round off answers to reasonable accuracy in the context of a given problem	Book 1: Chapter 3
10 Limits of accuracy	• Give appropriate upper and lower bounds for data given to a specified accuracy • Obtain appropriate upper and lower bounds to solutions of simple problems given to a specified accuracy	Book 3: Chapter 3

11. Ratio, proportion, rate	• Demonstrate an understanding of ratio and proportion • Increase and decrease a quantity by a given ratio • Use common measures of rate • Solve problems involving average speed	Book 1: Chapter 9 Book 2: Chapter 1
12. Percentages	• Calculate a given percentage of a quantity • Express one quantity as a percentage of another • Calculate percentage increase or decrease • Carry out calculations involving reverse percentages	Book 1: Chapter 8 Book 3: Chapter 5
13. Use of an electronic calculator	• Use an electronic calculator efficiently • Apply appropriate checks of accuracy • Enter a range of measures including 'time' • Interpret the calculator display appropriately	Book 1: Chapter 2 Chapter 4 Book 2: Chapter 11 Book 3: Chapter 10 Book 4: Chapter 4
14. Time	• Calculate times in terms of the 24-hour and 12-hour clock • Read clocks, dials and timetables	Book 1: Chapter 9
15. Money	• Solve problems involving money and convert from one currency to another	Book 3: Chapter 5
16. Personal and small business finance	• Use given data to solve problems on personal and small business finance involving earnings, simple interest and compound interest • Extract data from tables and charts	Book 3: Chapter 5
17. Algebraic representation and formulae	• Use letters to express generalised numbers and express arithmetic processes algebraically • Substitute numbers for words and letters in formulae • Construct and transform formulae and equations	Book 1: Chapter 4 Chapter 5 Book 2: Chapter 2 Book 3: Chapter 1
18. Algebraic manipulation	• Manipulate directed numbers • Use brackets and extract common factors • Expand product of algebraic expressions • Factorise where possible expressions of the form: $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$ $a^2 + 2ab + b^2$ $ax^2 + bx + c$ • Manipulate algebraic fractions • Factorise and simplify rational expressions	Book 1: Chapter 4 Book 2: Chapter 3 Chapter 4 Chapter 6
19. Indices	• Understand and use the rules of indices • Use and interpret positive, negative, fractional and zero indices	Book 3: Chapter 4

20. Solutions of equations and inequalities	• Solve simple linear equations in one unknown • Solve fractional equations with numerical and linear algebraic denominators • Solve simultaneous linear equations in two unknowns • Solve quadratic equations by factorisation, completing the square or by use of the formula • Solve simple linear inequalities	Book 1: Chapter 5 Book 2: Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 3
21. Graphical representation of inequalities	• Represent linear inequalities graphically	Book 4: Chapter 1
22. Sequences	• Continue a given number sequence • Recognise patterns in sequences and relationships between different sequences • Generalise sequences as simple algebraic statements	Book 1: Chapter 7
23. Variation	• Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities	Book 2: Chapter 1
24. Graphs in practical situations	• Interpret and use graphs in practical situations including travel graphs and conversion graphs • Draw graphs from given data • Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and deceleration • Calculate distance travelled as area under a linear speed-time graph	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 7
25. Graphs in practical situations	• Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer • Interpret graphs of linear, quadratic, cubic, reciprocal and exponential functions • Solve associated equations approximately by graphical methods • Estimate gradients of curve by drawing tangents	Book 1: Chapter 6 Book 2: Chapter 1 Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 7
26. Function notation	• Use function notation, e.g. $f(x) = 3x - 5$; $f : x \mapsto 3x - 5$, to describe simple functions • Find inverse functions $f^{-1}(x)$	Book 2: Chapter 7 Book 3: Chapter 2
27. Coordinate geometry	• Demonstrate familiarity with Cartesian coordinates in two dimensions • Find the gradient of a straight line • Calculate the gradient of a straight line from the coordinates of two points on it • Calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points • Interpret and obtain the equation of a straight line graph in the form $y = mx + c$ • Determine the equation of a straight line parallel to a given line • Find the gradient of parallel and perpendicular lines	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 6

28. Geometrical terms	• Use and interpret the geometrical terms: point; line; plane; parallel; perpendicular; bearing; right angle, acute, obtuse and reflex angles; interior and exterior angles; similarity and congruence • Use and interpret vocabulary of triangles, special quadrilaterals, circles, polygons and simple solid figures • Understand and use the terms: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment	Book 1: Chapter 10 Chapter 11 Book 2: Chapter 8 Book 3: Chapter 9 to Chapter 13
29. Geometrical constructions	• Measure lines and angles • Construct a triangle, given the three sides, using a ruler and a pair of compasses only • Construct other simple geometrical figures from given data, using a ruler and protractor as necessary • Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary • Read and make scale drawings • Use and interpret nets	Book 1: Chapter 12 Chapter 14 Book 2: Chapter 8 Book 4: Chapter 8
30. Similarity and congruence	• Solve problems and give simple explanations involving similarity and congruence • Calculate lengths of similar figures • Use the relationships between areas of similar triangles, with corresponding results for similar figures, and extension to volumes and surface areas of similar solids	Book 2: Chapter 8 Book 3: Chapter 11 Chapter 12
31. Symmetry	• Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions • Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone) • Use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length	Book 2: Chapter 13 Book 3: Chapter 13
32. Angles	• Calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines (c) angles formed within parallel lines (d) angle properties of triangles and quadrilaterals (e) angle properties of regular and irregular polygons (f) angle in a semi-circle (g) angle between tangent and radius of a circle (h) angle at the centre of a circle is twice the angle at the circumference (i) angles in the same segment are equal (j) angles in opposite segments are supplementary	Book 1: Chapter 10 Chapter 11 Book 3: Chapter 13
33. Loci	• Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point (b) at a given distance from a given straight line (c) equidistant from two given points (d) equidistant from two given intersecting straight line	Book 4: Chapter 8
34. Measures	• Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units	Book 1: Chapter 13 Chapter 14

35. Mensuration	• Solve problems involving: (a) the perimeter and area of a rectangle and triangle (b) the perimeter and area of a parallelogram and a trapezium (c) the circumference and area of a circle (d) arc length and sector area as fractions of the circumference and area of a circle (e) the surface area and volume of a cuboid, cylinder, prism, sphere, pyramid and cone (f) the areas and volumes of compound shapes	Book 1: Chapter 13 Chapter 14 Book 2: Chapter 12 Book 3: Chapter 10
36. Trigonometry	• Interpret and use three-figure bearings • Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or an angle of a right-angled triangles • Solve trigonometrical problems in two dimensions involving angles of elevation and depression • Extend sine and cosine functions to angles between 90° and 180° • Solve problems using the sine and cosine rules for any triangle and the formula area of triangle = $\frac{1}{2}$ $ab \sin C$ • Solve simple trigonometrical problems in three dimensions	Book 2: Chapter 10 Chapter 11 Book 3: Chapter 8 Chapter 9
37. Vectors in two dimensions	• Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or $\mathbf{a}$ • Add and subtract vectors • Multiple a vector by a scalar • Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • Represent vectors by directed line segments • Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors • Use position vectors	Book 4: Chapter 7
38. Matrices	• Display information in the form of a matrix of any order • Solve problems involving the calculation of the sum and product (where appropriate) of two matrices, and interpret the results • Calculate the product of a matrix and a scalar quantity • Use the algebra of 2×2 matrices including the zero and identity 2×2 matrices • Calculate the determinant $\lvert\mathbf{A}\rvert$ and inverse $\mathbf{A}^{-1}$ of a non-singular matrix $\mathbf{A}$	Book 4: Chapter 5
39. Transformations	• Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations • Identify and give precise descriptions of transformations connecting given figures • Describe transformations using coordinates and matrices	Book 2: Chapter 9 Book 4: Chapter 6
40. Probability	• Calculate the probability of a single event as either a fraction or a decimal • Understand that the probability of an event occurring = 1 – the probability of the event not occurring • Understand relative frequency as an estimate of probability • Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate	Book 2: Chapter 15 Book 4: Chapter 3

41. Categorical, numerical and grouped data	• Collect, classify and tabulate statistical data • Read, interpret and draw simple inferences from tables and statistical diagrams • Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used • Calculate an estimate of the mean for grouped and continuous data • Identify the modal class from a grouped frequency distribution	Book 1: Chapter 15 Book 2: Chapter 17 Book 4: Chapter 4
42. Statistical diagrams	• Construct and interpret bar charts, pie charts, pictograms, simple frequency distributions, frequency polygons, histograms with equal and unequal intervals and scatter diagrams • Construct and use cumulative frequency diagrams • Estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams • Calculate with frequency density • Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram • Draw a straight line of best fit by eye	Book 1: Chapter 15 Book 2: Chapter 16 Book 4: Chapter 4

Secondary 3 Mathematics Scheme of Work

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1	**1 Quadratic Equations and Functions**	**1.1 Solving Quadratic Equations by Completing the Square** (pp. 3 – 9)	• Solve quadratic equations in one variable by completing the square for equations of the form $x^2 + px + q = 0$	Solve quadratic equations by completing the square	Investigation – Completing the Square for Quadratic Expressions of the Form $x^2 + px$ (p. 6)			Investigation – Completing the Square for Quadratic Expressions of the Form $x^2 + px$ (p. 6)
1		**1.2 Solving Quadratic Equations by using Formula** (pp. 10 – 12)	• Solve quadratic equations in one variable by use of formula	Solve quadratic equations by the use of formula	Class Discussion – Solutions to Quadratic Equations (p. 11)			Class Discussion – Solutions to Quadratic Equations (p. 11)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
2		**1.3 Solving Quadratic Equations by Graphical Method** (pp. 12 – 17)	• Solve quadratic equations in one variable by graphical method	Draw graphs for functions of the form ax^n where a is a rational constant, $n = 2$	Thinking Time (p. 15) Journal Writing (p. 15)			Thinking Time (p. 15)
2		**1.4 Solving Fractional Equations that can be reduced to Quadratic Equations** (pp. 17 – 20)	• Solve fractional equations that can be reduced to quadratic equations		Thinking Time (p. 20)			Thinking Time (p. 20)
3		**1.5 Applications of Quadratic Equations in Real-World Contexts** (pp. 20 – 27)	• Formulate a quadratic equation in one variable to solve problems	Construct and transform formulae and equations				Ex 1D Q 15 (p. 27)
4		**1.6 Graphs of Quadratic Functions** (pp. 27 – 35)	• Sketch the graphs of quadratic equations of the form $y = (x - h)(x - k)$, $y = -(x - h)(x - k)$, $y = (x - p)^2 + q$ and $y = -(x - p)^2 + q$		Thinking Time (p. 28)			Thinking Time (p. 28)

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
					Investigation – Graphs of $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$ (p. 29) Investigation – Graphs of $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$ (p. 31) Thinking Time (p. 33) Class Discussion – Matching Quadratic Graphs with the Corresponding Functions (p. 34)	Investigation – Graphs of $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$ (p. 29) Investigation – Graphs of $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$ (p. 31)		Investigation – Graphs of $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$ (p. 29) Investigation – Graphs of $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$ (p. 31) Thinking Time (p. 33) Class Discussion – Matching Quadratic Graphs with the Corresponding Functions (p. 34)
4		**Miscellaneous**					Solutions for Challenge Yourself	
4	**2 Further Functions**	**2.1 Functions Involving Higher Order Expressions** (pp. 43 – 44)	• Find the value of the image of a function involving quadratic expressions					
5		**2.2 Inverse Functions** (pp. 45 – 49)	• Determine the inverse of a given function	Find inverse functions $f^{-1}(x)$	Worked Example 4 (p. 46)			Attention (p. 45)
5		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
6	**3 Linear Inequalities**	**3.1 Inequalities** (pp. 53 – 59)	• Solve linear inequalities in one variable and represent the solution on a number line	Solve simple linear inequalities	Investigation Properties of Inequalities (p. 53) Investigation – Inequalities (p. 55) Journal Writing (p. 56) Thinking Time (p. 57)			Journal Writing (p. 56) Thinking Time (p. 57)
6		**3.2 Problem Solving involving Inequalities** (pp. 60 – 61)	• Apply linear inequalities to solve word problems					
7		**3.3 Solving Simultaneous Linear Inequalities** (pp. 62 – 65)	• Solve linear inequalities in one variable and represent the solution on a number line		Performance Task (p. 63)			Performance Task (p. 63)
7		**3.4 Limits of Accuracy** (pp. 65 – 69)		• Give appropriate upper and lower bounds for data given to a specified accuracy • Obtain appropriate upper and lower bounds to solutions of simple problems given to a specified accuracy				EX 3C Q12 (p. 69)
7		**Miscellaneous**					Solutions for Challenge Yourself	

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
8	**4** **Indices and Standard Form**	**4.1 Indices** (pp. 75 – 76)	• State and apply the 5 laws of indices	Understand and use the rules of indices Use and interpret positive, negative, fractional and zero indices	Investigation – Indices (p. 75) Class Discussion – Comparing Numbers written in Index Form (p. 76)			Class Discussion – Comparing Numbers written in Index Form (p. 76)
8		**4.2 Laws of Indices** (pp. 76 – 85)	• State and apply the 5 laws of indices		Investigation – Law 1 of Indices (pp. 76 – 77) Investigation – Law 2 of Indices (p. 78) Investigation – Law 3 of Indices (p. 79)			Class Discussion – Simplification using the Law of Indices (p. 81)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
					Investigation – Law 4 of Indices (p. 80) Class Discussion – Simplification using the Law of Indices (p. 81) Investigation – Law 5 of Indices (p. 82) Journal Writing (p. 83) Class Discussion – Is $(a + b)^n = a^n + b^n$? Is $(a - b)^n = a^n - b^n$? (p. 84)			Journal Writing (p. 83) Class Discussion – Is $(a + b)^n = a^n + b^n$? Is $(a - b)^n = a^n - b^n$? (p. 84)
8		**4.3 Zero and Negative Indices** (pp. 85 – 90)	• State and use the definitions of zero, negative and rational indices		Investigation – Zero Index (pp. 85 – 86) Thinking Time (p. 87)			Investigation – Zero Index (pp. 85 – 86) Thinking Time (p. 87)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
					Investigation – Negative Indices (p. 87) Thinking Time (p. 89)			Investigation – Negative Indices (p. 87) Thinking Time (p. 89)
7		**4.4 Rational Indices** (pp. 90 – 97)	• State and use the definitions of zero, negative and rational indices		Class Discussion – Rational Indices (p. 92) Thinking Time (p. 92) Investigation – Rational Indices (p. 93) Thinking Time (p. 94)			Class Discussion – Rational Indices (p. 92) Thinking Time (p. 92) Thinking Time (p. 94)
9		**4.5 Standard Form** (pp. 98 – 105)	• Use the standard form to represent very large or very small numbers	Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leq A < 10$.	Class Discussion – Standard Form (p. 98) Performance Task (p. 102) Thinking Time (p. 104)	Information (p. 100) Internet Resources (p. 100) Performance Task (p. 102)		Class Discussion – Standard Form (pp. 98) Performance Task (p. 102)
9		**Miscellaneous**					Solutions for Challenge Yourself	

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
10	**5 Application of Mathematics in Practical Situations**	**5.1 Profit and Loss** (pp. 113 – 115)	• Solve problems involving profit and loss	Express one quantity as a percentage of another Calculate percentage increase or decrease Use given data to solve problems on personal and small business finance, involving earnings				
10		**5.2 Discount, Taxation and Commission** (pp. 115 – 124)	• Solve problems involving discount, taxation and commission	Carry out calculations involving reverse percentages Extract data from tables and charts	Investigation – Discount, Service Charge and GST (p. 118) Investigation – Percentage Point (p. 120)			Investigation – Discount, Service Charge and GST (p. 118) Investigation – Percentage Point (p. 120)
11		**5.3 Simple Interest and Compound Interest** (pp. 125 – 131)	• Solve problems involving simple interest, compound interest and hire purchase • Explain what percentage point is	Use given data to solve problems on personal and small business finance, involving simple interest and compound interest	Class Discussion – Body Mass Index (p. 125) Performance Task (p. 127) Investigation – Simple Interest and Compound Interest (pp. 127 – 128)			Performance Task (p. 127) Investigation – Simple Interest and Compound Interest (pp. 127 – 128)
11		**5.4 Hire Purchase** (pp. 132 – 134)						
12		**5.5 Money Exchange** (pp. 135 – 137)	• Convert one currency to another	Solve problems involving money and convert from one currency to another	Journal Writing (p. 136)			Journal Writing (p. 136)
12		**Miscellaneous**						

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
12	**6 Coordinate Geometry**	**6.1 Gradient of a Straight Line** (pp. 143 – 149)	• Find the gradient of a straight line given the coordinates of two points on it	Calculate the gradient of a straight line from the coordinates of two points on it	Class Discussion – Finding the Gradient of a Straight Line (p. 144) Thinking Time (p. 145)			Class Discussion – Finding the Gradient of a Straight Line (p. 144) Thinking Time (p. 145)
					Investigation – Gradient of a Straight Line (pp. 145 – 146)	Investigation – Gradient of a Straight Line (pp. 146 – 146)		Investigation – Gradient of a Straight Line (pp. 145 – 146)
13		**6.2 Length of a Line Segment** (pp. 149 – 155)	• Find the length of a line segment given the coordinates of its end points	Calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points.	Thinking Time (p. 150)			Thinking Time (p. 150)
13		**6.3 Equation of a Straight Line** (pp. 155 – 160)	• Interpret and find the equation of a straight line graph in the form $y = mx + c$ • Solve geometry problems involving the use of coordinates	Interpret and obtain the equation of a straight line graph in the form $y = mx + c$	Journal Writing (p. 158)			Journal Writing (p. 158)
14		**6.4 Parallel and Perpendicular Lines** (pp. 161 – 167)	• Find the gradients of parallel lines and the gradients of perpendicular lines	Determine the equation of a straight line parallel to a given line Find the gradient of parallel and perpendicular lines	Thinking Time (p. 161) Class Discussion – Parallel and Perpendicular Lines (pp. 161 – 162) Class Discussion – Perpendicular Lines (p. 164)			Thinking Time (p. 161) Class Discussion – Parallel and Perpendicular Lines (pp. 161 – 162)
14		**Miscellaneous**					Solutions for Challenge Yourself	

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Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
15	**7** **Graphs of Functions and Graphical Solution**	**7.1 Graphs of Cubic Functions** (pp. 173 – 176)	• Draw the graphs of simple sums of power functions $y = ax^n$, where $n = 3, 2, 1, 0, -1$ and -2	Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these Interpret graphs of cubic functions	Investigation – Graphs of Cubic Functions (p. 173)	Investigation – Graphs of Cubic Functions (p. 173)		
15		**7.2 Graphs of Reciprocal Functions** (pp. 176 – 182)		Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these Interpret graphs of reciprocal functions	Investigation – Graphs of $y = \frac{a}{x}$ (p. 176) Thinking Time (p. 177) Investigation – Graphs of $y = \frac{a}{x^2}$ (p. 179)	Investigation – Graphs of $y = \frac{a}{x}$ (p. 176) Investigation – Graphs of $y = \frac{a}{x^2}$ (p. 179)		Investigation – Graphs of $y = \frac{a}{x}$ (p. 176) Main Text (p. 177) Thinking Time (p. 177) Investigation – Graphs of $y = \frac{a}{x^2}$ (p. 179) Main Text (p. 179)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
15		**7.3 Graphs of Exponential Functions** (pp. 183 – 186)	• Draw the graphs of exponential functions $y = ka^x$, where a is a positive integer	Construct tables of values and draw graphs for functions of the form ka^x where a is a positive integer Interpret graphs of exponential functions	Investigation – Graphs of $y = a^x$ and $y = ka^x$ (p. 183) Class Discussion – Matching Graphs of Power Functions with the Corresponding Functions (p. 186)	Investigation – Graphs of $y = a^x$ and $y = ka^x$ (p. 135)		Investigation – Graphs of $y = a^x$ and $y = ka^x$ (p. 183) Class Discussion – Matching Graphs of Power Functions with the Corresponding Functions (p. 186)
					Journal Writing (p. 184)	Journal Writing (p. 184)		Journal Writing (p. 184)
16		**7.4 Gradient of a Curve** (pp. 187 – 192)	• Estimate the gradient of a curve by drawing a tangent	Estimate gradients of curve by drawing tangents				Ex 5B Q 9(b) (p. 192)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
16		**7.5 Applications of Graphs in Real-World Contexts** (pp. 192 – 209)	• Interpret and analyse data from tables and graphs, including distance-time and speed-time graphs	Interpret and use graphs in practical situations including travel graphs and conversion graphs Apply the idea of rate of change to easy kinematics involving distance-time and speed-tine graphs, acceleration and deceleration Calculate distance travelled as area under a linear speed-time graph Solve associated equations approximately by graphical methods	Class Discussion – Linear Distance-Time Graphs (p. 193) Thinking Time (p. 194) Thinking Time (p. 197)			Class Discussion – Linear Distance-Time Graphs (p. 193) Thinking Time (p. 194) Thinking Time (p. 197)
16		**Miscellaneous**					Solutions for Challenge Yourself	
17	**8 Further Trigonometry**	**8.1 Sine and Cosine of Obtuse Angles** (pp. 221 – 229)	• Determine the trigonometric values of obtuse angles	Extend sine and cosine functions to angles between 90° and 180°	Investigation – Relationship between Trigonometric Ratios of Acute and Obtuse Angles (p. 223)			Main Text (pp. 221 – 222) Investigation – Relationship between Trigonometric Ratios of Acute and Obtuse Angles (p. 223)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
17		**8.2 Area of Triangle** (pp. 229 – 234)	• Use of formula $\frac{1}{2}ab \sin C$ for the area of a triangle	Solve problems involving the sine and cosine rules for any triangle and the formula: area of triangle $= \frac{1}{2}ab \sin C$	Thinking Time (p. 232)			Thinking Time (p. 232)
18		**8.3 Sine Rule** (pp. 235 – 245)	• Find unknown sides/ angles of a triangle, given two sides and one angle, or two angles and one side, or three sides		Investigation – Sine Rule (pp. 235 – 236) Journal Writing (p. 242)	Investigation – Sine Rule (pp. 235 – 236)		Investigation – Sine Rule (pp. 235 – 236) Journal Writing (p. 242)
18		**8.4 Cosine Rule** (pp. 245 – 253)	• Find unknown sides/ angles of a triangle, given two sides and one angle, or two angles and one side, or three sides		Investigation – Cosine Rule (pp. 246 – 247) Thinking Time (p. 248)	Investigation – Cosine Rule (pp. 246 – 247)		Main Text (p. 245) Investigation – Cosine Rule (pp. 246 – 247) Thinking Time (p. 248)
18		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
19	**9** **Applications of Trigonometry**	**9.1 Angles of Elevation and Depression** (pp. 261 – 266)	• Solve simple practical problems in two and three dimensions including those involving angles of elevation and depression and bearings	Solve trigonometrical problems in two dimensions involving angles of elevation and depression				
19		**9.2 Bearings** (pp. 267 – 275)		Use and interpret the geometrical term: bearing Interpret and use three-figure bearings				Main Text (p. 219)
20		**9.3 Three-Dimensional Problems** (pp. 275 – 288)			Investigation – Visualising 3D Solids (pp. 275 – 277)			Investigation – Visualising 3D Solids (pp. 275 – 277) Just For Fun (p. 278) Just For Fun (p. 280)
20		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
21	**10 Arc Length, Area of Sector and Radian Measure**	**10.1 Length of Arc** (pp. 295 – 305)	• Find the arc length of a circle by expressing the arc length as a fraction of the circumference of the circle	Understand and use the terms: centre, radius, diameter, circumference, arc, sector Solve problems involving arc length as a fraction of the circumference of a circle	Investigation – Arc Length (pp. 296 – 297)	Investigation – Arc Length (pp. 296 – 297)		Investigation – Arc Length (pp. 296 – 297)
21		**10.2 Area of Sector** (pp. 306 – 312)	• Find the area of the sector of a circle by expressing the area of a sector as a fraction of the area of the circle • Find the area of a segment of a circle	Understand and use the terms: centre, radius, diameter, circumference, arc, sector Solve problems involving sector area as a fraction of the area of a circle	Investigation – Area of Sector (pp. 306 – 307)	Investigation – Area of Sector (pp. 306 – 307)		Investigation – Area of Sector (pp. 306 – 307)
22		**10.3 Radian Measure** (pp. 313 – 321)	• Convert angular measure from radians to degrees and vice versa	Use an electronic calculator efficiently Interpret the calculator display appropriately	Investigation – Visualise the Size of an Angle of 1 radian (p. 314) Class Discussion – Estimate the Size of Angles in Radians (p. 315)			Investigation – Visualise the Size of an Angle of 1 radian (p. 314) Class Discussion – Estimate the Size of Angles in Radians (p. 315)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
22		**10.4 Arc Length and Area of Sector using Radian Measure** (pp. 322 – 332)	• Use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ to solve problems involving arc length, area of a sector and area of a segment of a circle					
22		**Miscellaneous**					Solutions for Challenge Yourself	
23	**11 Congruence and Similarity Tests**	**11.1 Congruence Tests** (pp. 343 – 359)	• Apply the four congruence tests to determine whether two or more triangles are congruent	Solve problems and give simple explanations involving similarity and congruence	Investigation – SSS Congruence Test (p. 344) Investigation – SAS Congruence Test (pp. 346 – 347) Investigation – AAS Congruence Test (pp. 350 – 351) Investigation – RHS Congruence Test (p. 352) Class Discussion – Consolidation for Congruence Tests (p. 354)			Investigation – SSS Congruence Test (p. 344) Investigation – SAS Congruence Test (pp. 346 – 347) Investigation – AAS Congruence Test (pp. 350 – 351) Investigation – RHS Congruence Test (p. 352) Class Discussion – Consolidation for Congruence Tests (p. 354)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
23		**11.2 Similarity Tests** (pp. 359 – 373)	• Apply the three similarity tests to determine whether two or more triangles are similar		Investigation – AA Similarity Test (p. 360) Thinking Time (p. 360) Investigation – SSS Similarity Test (p. 363) Thinking Time (p. 364) Investigation – SAS Similarity Test (pp. 365 – 366) Thinking Time (p. 366)			Investigation – AA Similarity Test (p. 360) Thinking Time (p. 360) Investigation – SSS Similarity Test (p. 363) Thinking Time (p. 364) Investigation – SAS Similarity Test (pp. 365 – 366) Thinking Time (p. 366)
24		**11.3 Applications of Congruent and Similar Triangles** (pp. 374 – 378)	• Solve problems involving congruent and/or similar triangles					
24		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
24	**12 Area and Volume of Similar Figures and Solids**	**12.1 Area of Similar Figures** (pp. 387 – 395)	• Solve problems using the relationship between areas of similar figures	Use the relationships between areas of similar triangles, with corresponding results for similar figures, and extension to volumes and surface areas of similar solids	Investigation – Areas of Similar Figures (pp. 387 – 389)			Investigation – Areas of Similar Figures (pp. 387 – 389)
25		**12.2 Volume of Similar Solids** (pp. 396 – 404)	• Solve problems using the relationship between volumes of different solids		Investigation – Volume and Mass of Similar Solids (pp. 396 – 397) Thinking Time (p. 402) Journal Writing (p. 402)	Journal Writing (p. 402)		Investigation – Volume and Mass of Similar Solids (pp. 396 – 397) Thinking Time (p. 402) Journal Writing (p. 402)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
25	**13 Geometrical Properties of Circles**	**13.1 Symmetric Properties of Circles** (pp. 411 – 429)	• Apply the symmetry properties of circles	Understand and use the term: chord, tangent Use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length	Investigation – Circle Symmetric Property 1 (pp. 411 – 413) Thinking Time (p. 414) Class Discussion – Application of Circle Symmetric Property 1 (p. 416) Investigation – Circle Symmetric Property 2 (pp. 417 – 418) Investigation – Circle Symmetric Property 3 (p. 422) Investigation – Circle Symmetric Property 4 (p. 425)	Investigation – Circle Symmetric Property 1 (pp. 411 – 413) Investigation – Circle Symmetric Property 2 (pp. 417 – 418) Investigation – Circle Symmetric Property 3 (p. 422) Investigation – Circle Symmetric Property 4 (p. 425)		Investigation – Circle Symmetric Property 1 (pp. 411 – 413) Thinking Time (p. 414) Investigation – Circle Symmetric Property 2 (pp. 417 – 418) Investigation – Circle Symmetric Property 3 (p. 425) Investigation – Circle Symmetric Property 4 (p. 425)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
26		**13.2 Angle Properties of Circles** (pp. 430 – 454)	• Apply the angle properties of circles	Calculate unknown angles and give simple explanations using the geometrical properties of angle in a semi-circle and angle between tangent and radius of circle	Class Discussion – Identifying Angles at the Centre and at the Circumference (p. 431) Investigation – Circle Angle Property 1 (p. 432) Thinking Time (p. 434) Investigation – Circle Angle Property 2 (p. 436) Thinking Time (p. 438)	Investigation – Circle Angle Property 1 (p. 432) Investigation – Circle Angle Property 2 (p. 436)		Class Discussion – Identifying Angles at the Centre and at the Circumference (p. 431) Investigation – Circle Angle Property 1 (p. 432) Thinking Time (p. 434) Investigation – Circle Angle Property 2 (p. 436) Thinking Time (p. 438)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
					Class Discussion – Angles in Same or Opposite Segments (p. 439) Investigation – Circle Angle Property 3 (p. 440) Investigation – Circle Angle Property 4 (pp. 442 – 443) Investigation – Circle Angle Property 5 (p. 445) Thinking Time (p. 446) Class Discussion – Proof of Tangent-Chord Theorem (p. 446)	Investigation – Circle Angle Property 3 (p. 440) Investigation – Circle Angle Property 4 (pp. 442 – 443) Investigation – Circle Angle Property 5 (p. 445)		Class Discussion – Angles in Same or Opposite Segments (p. 439) Investigation – Circle Angle Property 3 (p. 440) Investigation – Circle Angle Property 4 (pp. 442 – 443) Investigation – Circle Angle Property 5 (p. 445) Class Discussion – Proof of Tangent-Chord Theorem (p. 446)
26		**Miscellaneous**					Solutions for Challenge Yourself	

Chapter 1 Quadratic Equations and Functions

TEACHING NOTES

Suggested Approach

In Book 2, students have learnt how to solve quadratic equations by factorisation. Teachers may want to begin this chapter by doing a recap and building up on what students have learnt so far. Once students have learnt the methods of solving quadratic equations, teachers may get the students to do a reflection on the methods of solving quadratic equations (see Journal Writing on page 15 of the textbook). Teachers should give students the opportunity to use a graphing software to explore the characteristics of the graphs of the form $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$ and $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$.

Section 1.1: Solving Quadratic Equations by Completing the Square

Teachers can first get students to discuss with one another on what they have learnt about quadratic equations in Book 2. Students should be reminded that the right hand side of the equation should be zero before they factorise the equation. Teachers can ask students to solve a quadratic equation that cannot be easily solved by factorisation and highlight to them that such an equation can be rearranged into the form $(x + a)^2 = b$, where a and b are constants. The equation can then be solved easily by taking the square roots on both sides of the equation to obtain the solution.

Since students are familiar with algebra discs which they have come across in Book 2, teachers can first teach students how to complete the square for a quadratic expression with the use of the discs. When the students have understood how to complete the square, teachers can then teach them how to use a multiplication frame instead (see Investigation: Completing the Square for Quadratic Expressions of the Form $x^2 + px$).

Section 1.2: Solving Quadratic Equations Using Formula

To give students a better understanding of how the general formula is derived, teachers can get the students to work in small groups and use the method of completing the square for the general form of the quadratic equation. Teachers may guide the students along when they have difficulty deriving the formula.

Teachers can raise the question to the class on how they can tell if a quadratic equation has no real solutions, one real solution or two real solutions (see Class Discussion: Solutions to Quadratic Equations).

Section 1.3: Solving Quadratic Equations by Graphical Method

The graphical method is another method of solving quadratic equations in which the solutions of the quadratic equation $ax^2 + bx + c = 0$ are the x-coordinates of the points of intersection of the graph $y = ax^2 + bx + c$ with the x-axis. However, students need to note that the solutions obtained by the graphical method can only be an approximation. Teachers should highlight to students that the answers obtained by the graphical method can only be accurate up to half of a small square grid. Teachers should give students more examples of the different types of quadratic graphs cutting the x-axis so that the students can better understand and identify the number of points of intersection between the graph and the x-axis.

Section 1.4: Solving Fractional Equations that can be reduced to Quadratic Equations

In Book 2, students have learnt how to solve equations involving algebraic fractions using the method of changing the subject of a formula. In this section, they will learn to solve equations which have one or more algebraic fractions, known as fractional equations which can be reduced to quadratic equations. Also, teachers should highlight the common mistakes that students tend to make when solving fractional equations, especially when students cancel the common factors out (see Thinking Time on page 20 of the textbook).

Section 1.5: Applications of Quadratic Equations and Functions in Real-World Contexts

Teachers should go through the problems given in the textbook and take a look at mathematics and real-life problems that involve the graphs of quadratic equations and functions. Students should use Polya's 4-step Problem Solving Model to help them understand the problem and form an equation to solve the problem (see pages 21 and 22 of the textbook).

Section 1.6: Graphs of Quadratic Functions

In Book 2, students have learnt the properties of a quadratic graph of the form $y = ax^2 + bx + c$. Teachers should revise with them on what they have learnt (see Thinking Time on page 28 of the textbook) before teaching them to sketch the graphs of the form $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$ and $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$.

For graphs of the form $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$, teachers should guide the students along and help them to recognise that the graphs cut the x-axis at $(h, 0)$ and $(k, 0)$ and are symmetrical about the vertical line that passes through the minimum or maximum point (see Investigation: Graphs of $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$). Teachers should also highlight to students that the line of symmetry is halfway between the x-intercepts and how this can be used to find the coordinates of the minimum/maximum point.

For graphs of the form $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$, teachers should guide the students along and help them to recognise that the minimum or maximum point is (p, q) and the line of symmetry is $x = p$ (see Investigation: Graphs of $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$).

Teachers can check on students' level of understanding of the graphs of the form $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$ and $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$ by getting them to match graphs with their respective functions and justify their answers (see Class Discussion: Matching Quadratic Graphs with the Corresponding Functions).

Challenge Yourself

For Question 1, guide the students to use the information on 'sum of its digits is 6' to represent the other digit in terms of x. Also, teachers may go through numerical examples on the meaning of 2-digit numbers which are represented by the tens and ones digits.

For Question 2, advise the students to use the quadratic formula to attempt this question.

WORKED SOLUTIONS

Investigation (Completing the Square for Quadratic Expressions of the Form $x^2 + px$)

<table>
<tr><th></th><th>Quadratic Expression
$x^2 + px$</th><th>Number that must be added to complete the square,
b</th><th>$\frac{1}{2} \times$ coefficient of x, $\frac{p}{2}$</th><th>Quadratic expression of the form $(x + a)^2 - b$</th></tr>
<tr><td>(a)</td><td>$x^2 + 4x$
$\begin{array}{c|cc} \times & x & 2 \\ \hline x & x^2 & 2x \\ 2 & 2x & \end{array}$</td><td>$2^2 = 4$</td><td>$\frac{4}{2} = 2$</td><td>$x^2 + 4x$
$= x^2 + 4x + 2^2 - 2^2$
$= (x + 2)^2 - 4$
$\begin{array}{c|ccc} \times & x & 2 & \\ \hline x & x^2 & 2x & \\ 2 & 2x & 4 & -4 \end{array}$</td></tr>
<tr><td>(b)</td><td>$x^2 + 6x$
$\begin{array}{c|cc} \times & x & 3 \\ \hline x & x^2 & 3x \\ 3 & 3x & \end{array}$</td><td>$3^2 = 9$</td><td>$\frac{6}{2} = 3$</td><td>$x^2 + 6x$
$= x^2 + 6x + 3^2 - 3^2$
$= (x + 3)^2 - 9$
$\begin{array}{c|ccc} \times & x & 3 & \\ \hline x & x^2 & 3x & \\ 3 & 3x & 9 & -9 \end{array}$</td></tr>
<tr><td>(c)</td><td>$x^2 + 8x$
$\begin{array}{c|cc} \times & x & 4 \\ \hline x & x^2 & 4x \\ 4 & 4x & \end{array}$</td><td>$4^2 = 16$</td><td>$\frac{8}{2} = 4$</td><td>$x^2 + 8x$
$= x^2 + 8x + 4^2 - 4^2$
$= (x + 4)^2 - 16$
$\begin{array}{c|ccc} \times & x & 4 & \\ \hline x & x^2 & 4x & \\ 4 & 4x & 16 & -16 \end{array}$</td></tr>
<tr><td>(d)</td><td>$x^2 + 10x$
$\begin{array}{c|cc} \times & x & 5 \\ \hline x & x^2 & 5x \\ 5 & 5x & \end{array}$</td><td>$5^2 = 25$</td><td>$\frac{10}{2} = 5$</td><td>$x^2 + 10x$
$= x^2 + 10x + 5^2 - 5^2$
$= (x + 5)^2 - 25$
$\begin{array}{c|ccc} \times & x & 5 & \\ \hline x & x^2 & 5x & \\ 5 & 5x & 25 & -25 \end{array}$</td></tr>
</table>

1. The number that must be added to each of the quadratic expressions $x^2 + px$ to complete the square is the square of the coefficient of x of the quadratic expression.
2. $$\begin{aligned} x^2 + px &= x^2 + px + \left(\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 \\ &= \left(x + \frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 \\ &= (x + a)^2 - b \text{ where } a = \frac{p}{2} \text{ and } b = \left(\frac{p}{2}\right)^2 \end{aligned}$$

Class Discussion (Solutions to Quadratic Equations)

(a) $4x^2 - 12x + 9 = 0$
Comparing $4x^2 - 12x + 9 = 0$ with $ax^2 + bx + c = 0$,
we have $a = 4$, $b = -12$ and $c = 9$.

1. $b^2 - 4ac = (-12)^2 - 4(4)(9) = 0$
2. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-12) \pm \sqrt{0}}{2(4)} = 1.5$
3. When $b^2 - 4ac = 0$, the equation has one real solution.

(b) $2x^2 + 5x + 8 = 0$

Comparing $2x^2 + 5x + 8 = 0$ with $ax^2 + bx + c = 0$,
we have $a = 2$, $b = 5$ and $c = 8$.

1. $b^2 - 4ac = 5^2 - 4(2)(8) = -39$

2. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-5 \pm \sqrt{-39}}{2(5)}$ (N.A.)

3. When $b^2 - 4ac < 0$, the equation has no real solutions.

(c) $3x^2 + 5x - 4 = 0$

Comparing $3x^2 + 5x - 4 = 0$ with $ax^2 + bx + c = 0$,
we have $a = 3$, $b = 5$ and $c = -4$.

1. $b^2 - 4ac = 5^2 - 4(3)(-4) = 73$

2. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-5 \pm \sqrt{73}}{2(3)}$

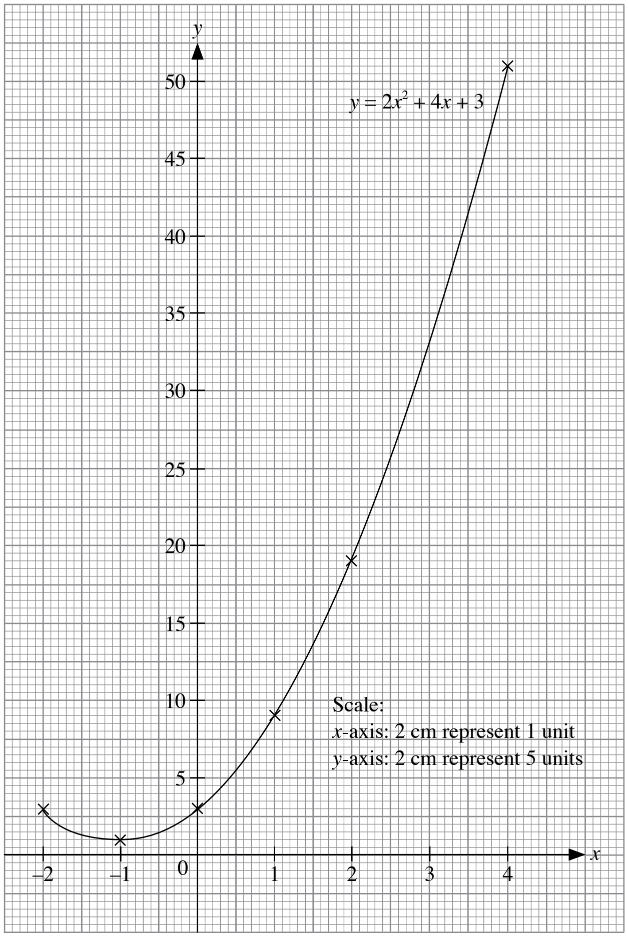
$= -2.26$ (to 3 s.f.) or 0.591 (to 3 s.f.)

3. When $b^2 - 4ac > 0$, the equation has two real solutions.

Thinking Time (Page 15)

$y = 2x^2 + 4x + 3$

x	-2	-1	0	1	2	4
y	3	1	3	9	19	51



(i) From the graph, there are no points of intersection between the graph and the x-axis.

(ii) There are no real solutions to the equation $2x^2 + 4x + 3 = 0$. There are no points of intersection between the graph and the x-axis.

(iii) Comparing $2x^2 + 4x + 3 = 0$ with $ax^2 + bx + c = 0$,
we have $a = 2$, $b = 4$ and $c = 3$.
$b^2 - 4ac = 4^2 - 4(2)(3) = -8$

(iv) When $b^2 - 4ac < 0$, the equation has no real solutions. As such, there are no points of intersection between the graph and the x-axis.

Journal Writing (Page 15)

(i) Factorisation

Advantage: Can be solved easily by factorising the quadratic expression.

Disadvantage: Can only be used when the quadratic expression can be factorised.

(ii) Completing the square

Disadvantages: This method is complex and hence the chances of errors are higher.
This method always works and it gives some insight into how algebra works more generally.

(iii) Use of the quadratic formula

Advantage: This method always works and is straightforward.

Disadvantage: This formula provides no insight and can become a rote technique.

(iv) Graphical method

Advantage: The answer can be observed from the graph, given that the graph has been drawn accurately.

Disadvantage: The exact value cannot be obtained and it is a more tedious method.

Thinking Time (Page 20)

No, the solution $x = 3$ is not valid since the terms $\dfrac{7}{x-3}$ and $\dfrac{21}{x(x-3)}$ of the original equation will be invalid.

Thinking Time (Page 28)

$y = x^2 + 4x - 5$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5$ or $x = 1$

x-intercepts $= -5, 1$

When $x = 0$,

$y = (0)^2 + 4(0) - 5$

$= -5$

y-intercept $= -5$

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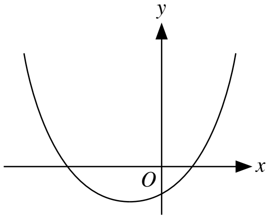


Graph 4

$\therefore$ Graph 4 corresponds to the quadratic function $y = x^2 + 4x - 5$.

$y = -x^2 - 4x + 5$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

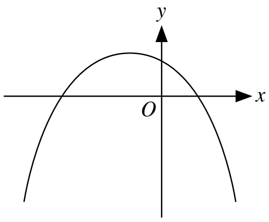
$$\begin{aligned} -x^2 - 4x + 5 &= 0 \\ x^2 + 4x - 5 &= 0 \\ (x + 5)(x - 1) &= 0 \\ x &= -5 \text{ or } x = 1 \end{aligned}$$

x-intercepts $= -5, 1$

When $x = 0$,

$$\begin{aligned} y &= (0)^2 - 4(0) + 5 \\ &= 5 \end{aligned}$$

y-intercept $= 5$



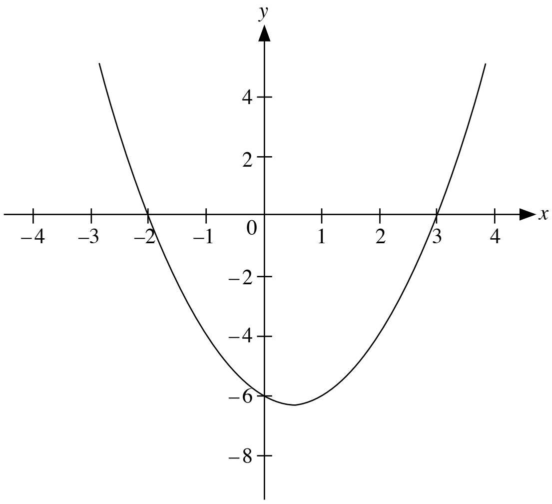
Graph 8

$\therefore$ Graph 8 corresponds to the quadratic function $y = -x^2 - 4x + 5$.

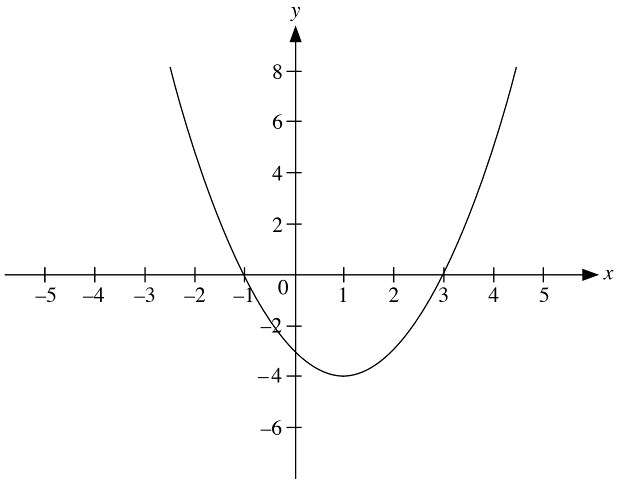
Investigation (Graphs of $y = (x - h)(x - k)$ or $y = -(x - h)(x - k)$)

1. $y = (x - 3)(x - k)$

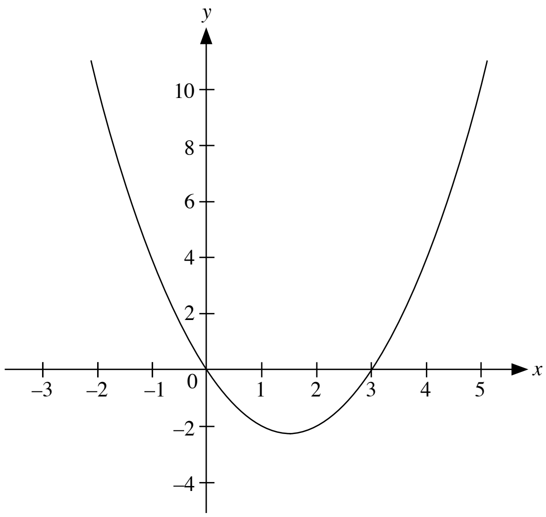
For $k = -2$, $y = (x - 3)(x + 2)$



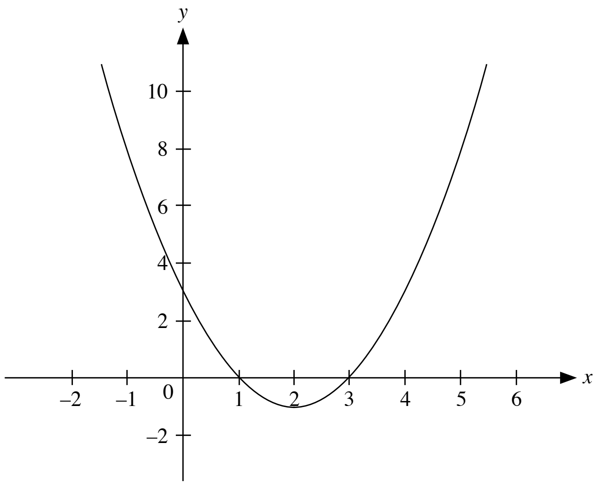
For $k = -1$, $y = (x - 3)(x + 1)$



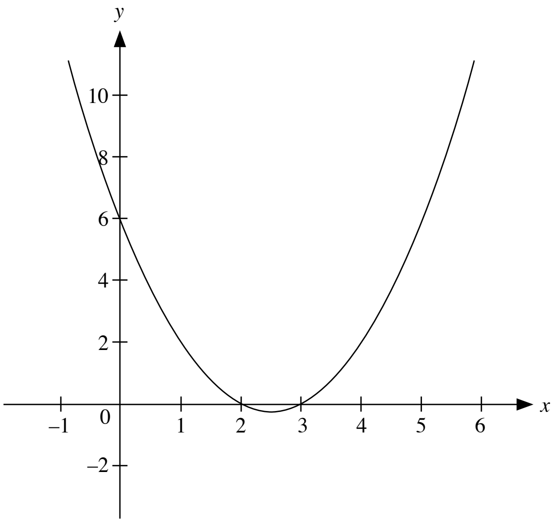
For $k = 0$, $y = x(x - 3)$



For $k = 1$, $y = (x - 3)(x - 1)$



For $k = 2$, $y = (x - 3)(x - 2)$



2. For $k = -2$, $y = (x - 3)(x + 2)$

(a) The graph opens upwards.

(b) $(-2, 0)$ and $(3, 0)$

(c) $(0, -6)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = \frac{1}{2}$

(f) minimum point $\left(\frac{1}{2}, -6\frac{1}{4}\right)$

For $k = -1$, $y = (x - 3)(x + 1)$

(a) The graph opens upwards.

(b) $(-1, 0)$ and $(3, 0)$

(c) $(0, -3)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 1$

(f) minimum point $(1, -4)$

For $k = 0$, $y = x(x - 3)$

(a) The graph opens upwards.

(b) $(0, 0)$ and $(3, 0)$

(c) $(0, 0)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 1\frac{1}{2}$

(f) minimum point $\left(1\frac{1}{2}, -2\frac{1}{4}\right)$

For $k = 1$, $y = (x - 3)(x - 1)$

(a) The graph opens upwards.

(b) $(1, 0)$ and $(3, 0)$

(c) $(0, 3)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2$

(f) minimum point $(2, -1)$

For $k = 2$, $y = (x - 3)(x - 2)$

(a) The graph opens upwards.

(b) $(2, 0)$ and $(3, 0)$

(c) $(0, 6)$

(d) The line of symmetry is halfway between the x-intercepts.

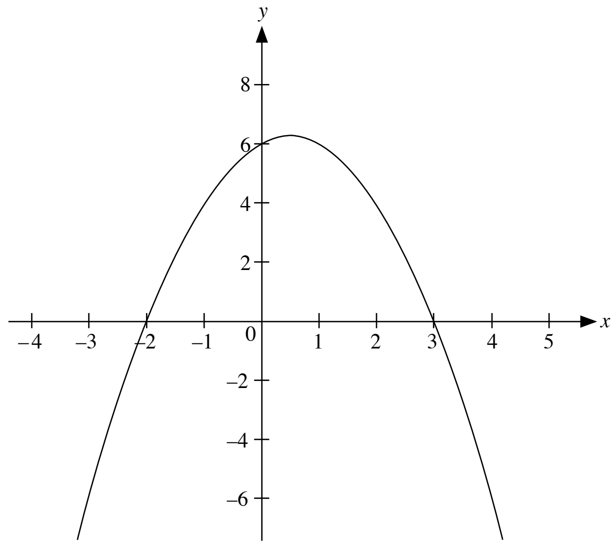
(e) $x = 2\frac{1}{2}$

(f) minimum point $\left(2\frac{1}{2}, -\frac{1}{4}\right)$

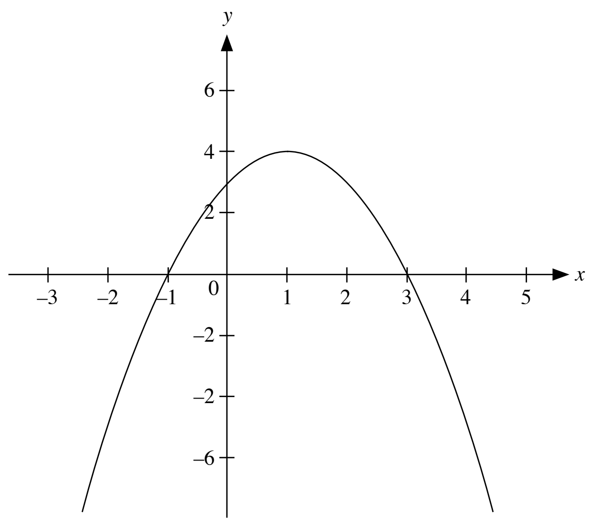
3. **Step 1:**

$y = -(x - 3)(x - k)$

For $k = -2$, $y = -(x - 3)(x + 2)$



For $k = -1$, $y = -(x - 3)(x + 1)$



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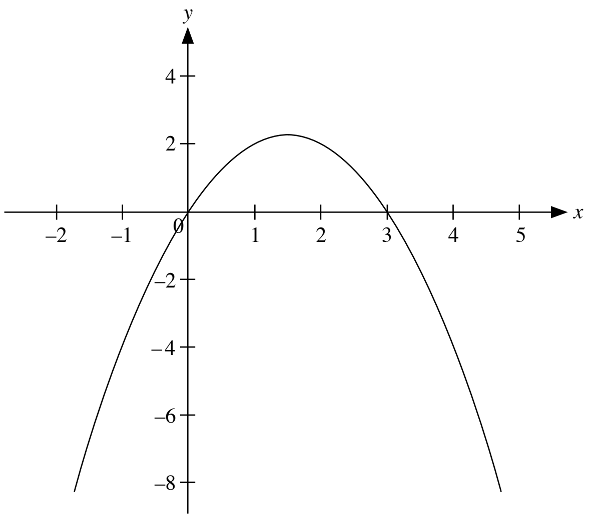
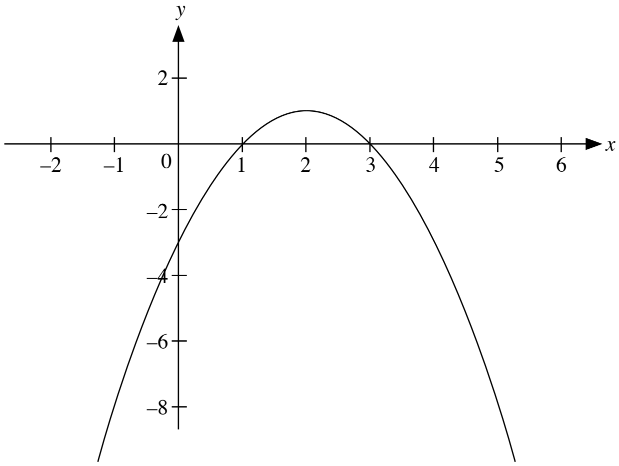
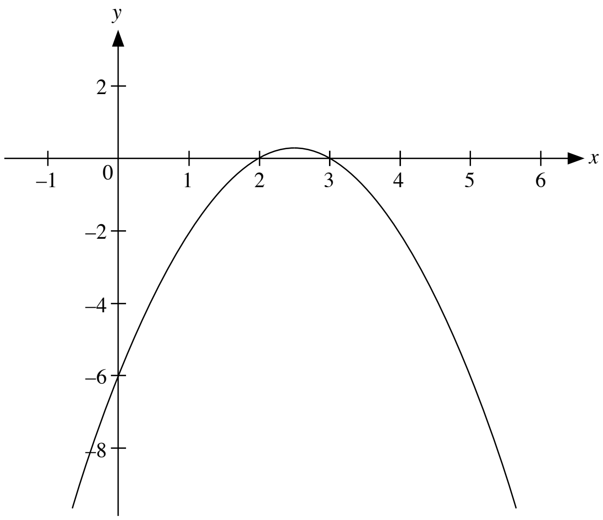
For $k = 0$, $y = -x(x - 3)$



For $k = 1$, $y = -(x - 3)(x - 1)$

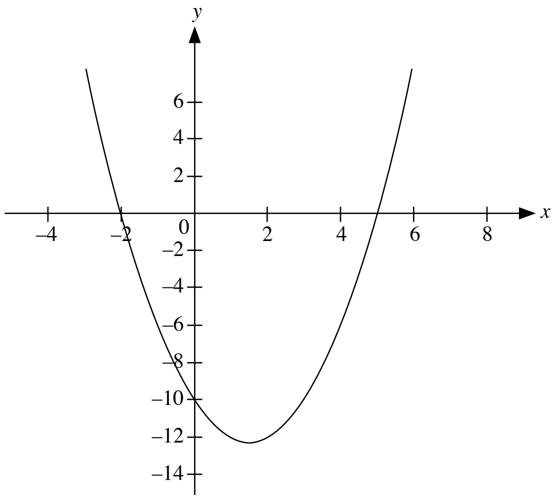


For $k = 2$, $y = -(x - 3)(x - 2)$

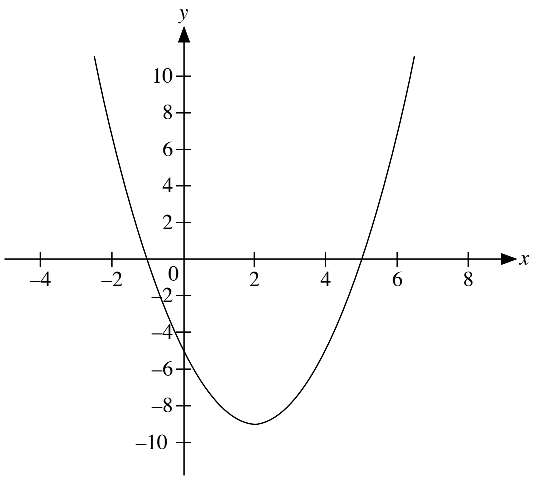


$y = (x - 5)(x - k)$

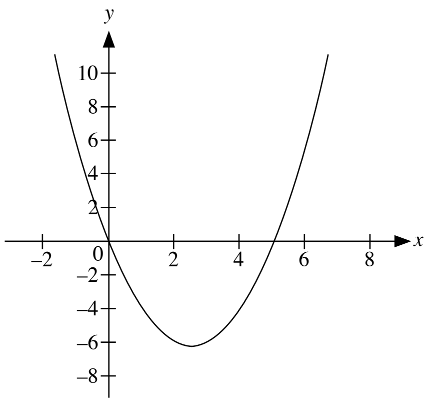
For $k = -2$, $y = (x - 5)(x + 2)$



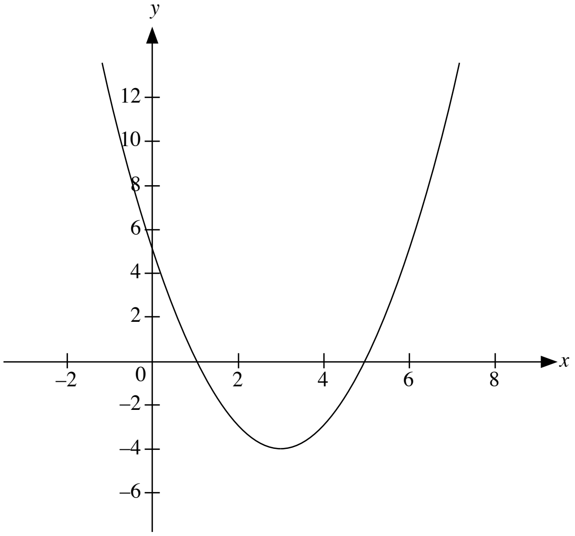
For $k = -1$, $y = (x - 5)(x + 1)$



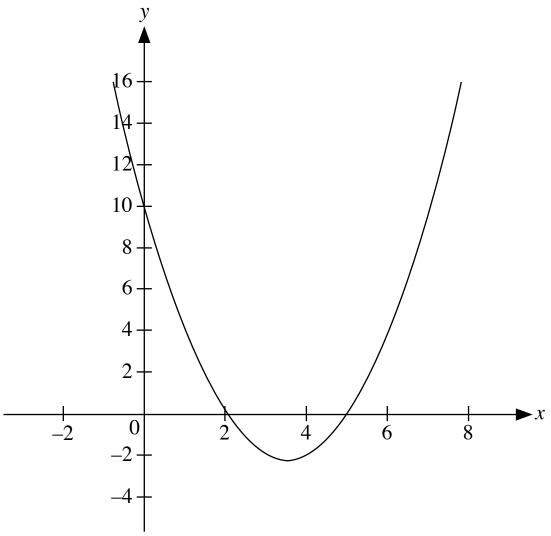
For $k = 0$, $y = x(x - 5)$



For $k = 1$, $y = (x - 5)(x - 1)$

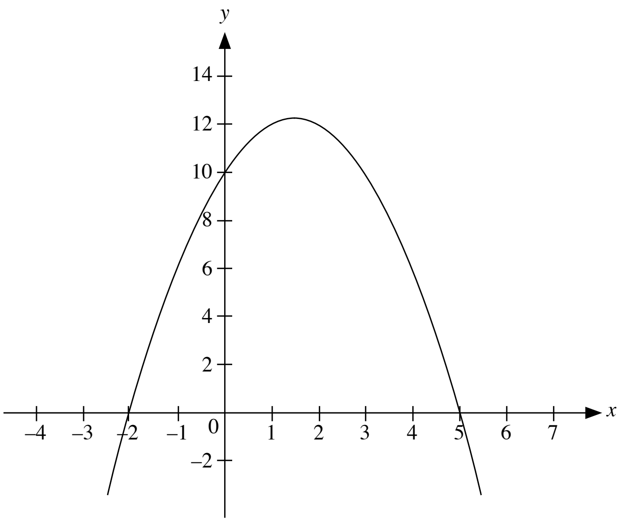


For $k = 2$, $y = (x - 5)(x - 2)$

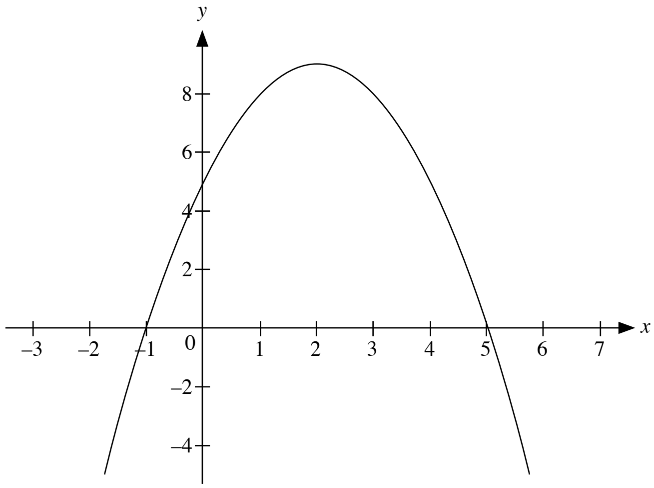


$y = -(x - 5)(x - k)$

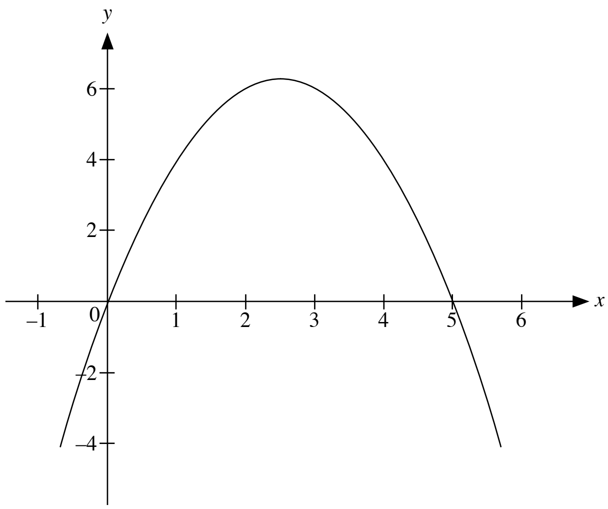
For $k = -2$, $y = -(x - 5)(x + 2)$



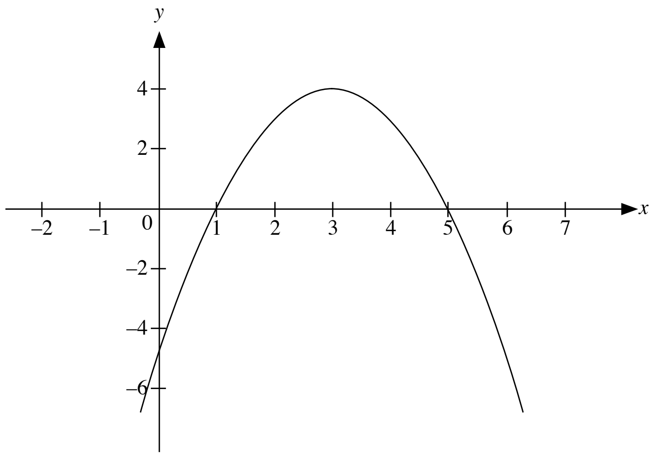
For $k = -1$, $y = -(x - 5)(x + 1)$



For $k = 0$, $y = -x(x - 5)$



For $k = 1$, $y = -(x - 5)(x - 1)$



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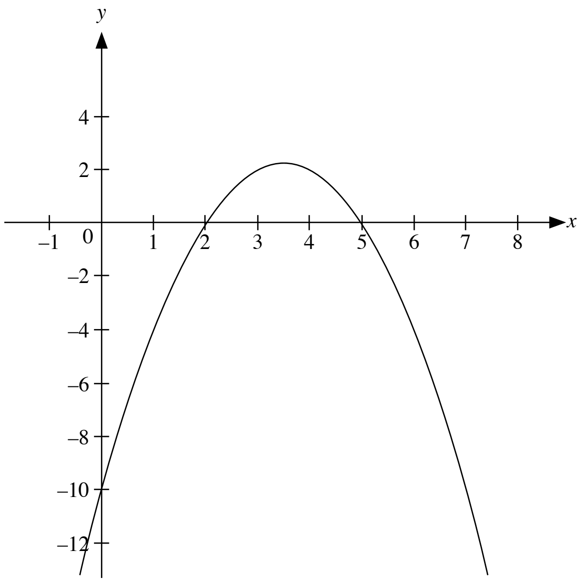
For $k = 2$, $y = -(x - 5)(x - 2)$



Step 2:

$y = -(x - 3)(x - k)$

For $k = -2$, $y = -(x - 3)(x + 2)$

(a) The graph opens downwards.

(b) $(-2, 0)$ and $(3, 0)$

(c) $(0, 6)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = \frac{1}{2}$

(f) maximum point $\left(\frac{1}{2}, 6\frac{1}{4}\right)$

For $k = -1$, $y = -(x - 3)(x + 1)$

(a) The graph opens downwards.

(b) $(-1, 0)$ and $(3, 0)$

(c) $(0, 3)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 1$

(f) maximum point $(1, 4)$

For $k = 0$, $y = -x(x - 3)$

(a) The graph opens downwards.

(b) $(0, 0)$ and $(3, 0)$

(c) $(0, 0)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 1\frac{1}{2}$

(f) maximum point $\left(1\frac{1}{2}, 2\frac{1}{4}\right)$

For $k = 1$, $y = -(x - 3)(x - 1)$

(a) The graph opens downwards.

(b) $(1, 0)$ and $(3, 0)$

(c) $(0, -3)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2$

(f) maximum point $(2, 1)$

For $k = 2$, $y = -(x - 3)(x - 2)$

(a) The graph opens downwards.

(b) $(2, 0)$ and $(3, 0)$

(c) $(0, -6)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2\frac{1}{2}$

(f) maximum point $\left(2\frac{1}{2}, \frac{1}{4}\right)$

$y = (x - 5)(x - k)$

For $k = -2$, $y = (x - 5)(x + 2)$

(a) The graph open upwards.

(b) $(-2, 0)$ and $(5, 0)$

(c) $(0, -10)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 1\frac{1}{2}$

(f) minimum point $\left(1\frac{1}{2}, -12\frac{1}{4}\right)$

For $k = -1$, $y = (x - 5)(x + 1)$

(a) The graph opens upwards.

(b) $(-1, 0)$ and $(5, 0)$

(c) $(0, -5)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2$

(f) minimum point $(2, -9)$

For $k = 0$, $y = x(x - 5)$

(a) The graph opens upwards.

(b) $(0, 0)$ and $(5, 0)$

(c) $(0, 0)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2\frac{1}{2}$

(f) minimum point $\left(2\frac{1}{2}, -6\frac{1}{4}\right)$

For $k = 1$, $y = (x - 5)(x - 1)$

(a) The graph opens upwards.

(b) $(1, 0)$ and $(5, 0)$

(c) $(0, 5)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 3$

(f) minimum point $(3, -4)$

For $k = 2$, $y = (x - 5)(x - 2)$

(a) The graph opens upwards.

(b) $(2, 0)$ and $(5, 0)$

(c) $(0, 10)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 3\dfrac{1}{2}$

(f) minimum point $\left(3\dfrac{1}{2}, -2\dfrac{1}{4}\right)$

$y = -(x - 5)(x - k)$

For $k = -2$, $y = -(x - 5)(x + 2)$

(a) The graph opens downwards.

(b) $(-2, 0)$ and $(5, 0)$

(c) $(0, 10)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 1\dfrac{1}{2}$

(f) maximum point $\left(1\dfrac{1}{2}, 12\dfrac{1}{4}\right)$

For $k = -1$, $y = -(x - 5)(x + 1)$

(a) The graph opens downwards.

(b) $(-1, 0)$ and $(5, 0)$

(c) $(0, 5)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2$

(f) maximum point $(2, 9)$

For $k = 0$, $y = -x(x - 5)$

(a) The graph opens downwards.

(b) $(0, 0)$ and $(5, 0)$

(c) $(0, 0)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 2\dfrac{1}{2}$

(f) maximum point $\left(2\dfrac{1}{2}, 6\dfrac{1}{4}\right)$

For $k = 1$, $y = -(x - 5)(x - 1)$

(a) The graph opens downwards.

(b) $(1, 0)$ and $(5, 0)$

(c) $(0, -5)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 3$

(f) maximum point $(3, 4)$

For $k = 2$, $y = -(x - 5)(x - 2)$

(a) The graph opens downwards.

(b) $(2, 0)$ and $(5, 0)$

(c) $(0, -10)$

(d) The line of symmetry is halfway between the x-intercepts.

(e) $x = 3\dfrac{1}{2}$

(f) maximum point $\left(3\dfrac{1}{2}, 2\dfrac{1}{4}\right)$

4. For the equation $y = (x - 3)(x - k)$, the graph opens upwards while for the equation $y = -(x - 3)(x - k)$, the graph opens downwards.

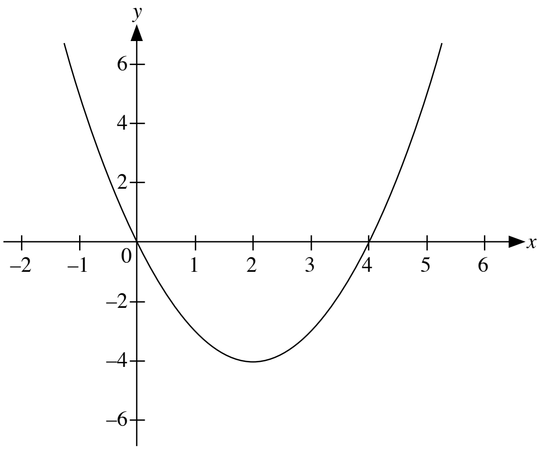
5. For the equation $y = (x - 3)(x - k)$, the graph cuts the x-axis at $(3, 0)$ and $(k, 0)$. For the equation $y = -(x - 3)(x - k)$, the graph cuts the x-axis at $(3, 0)$ and $(k, 0)$.

6. For the equation $y = (x - 3)(x - k)$, the graph is symmetrical about the vertical line that passes through the minimum point. For the equation $y = -(x - 3)(x - k)$, the graph is symmetrical about the vertical line that passes through the maximum point.

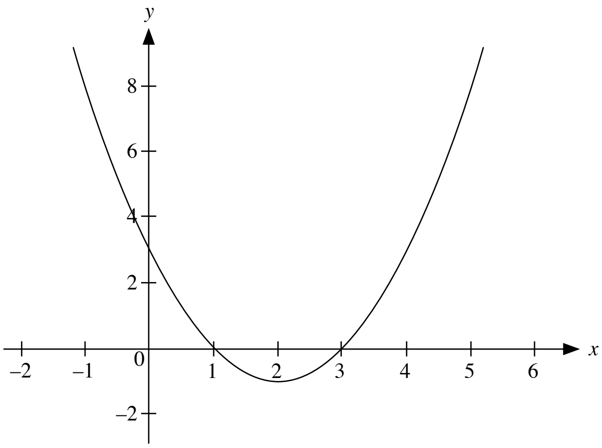
Investigation (Graphs of $y = (x - p)^2 + q$ or $y = -(x - p)^2 + q$)

1. $y = (x - 2)^2 + q$

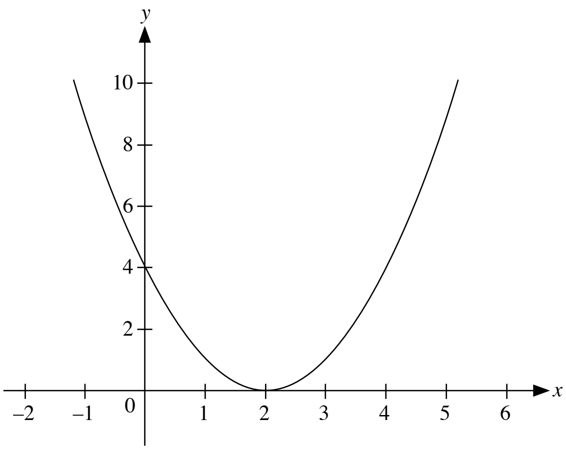
For $q = -4$, $y = (x - 2)^2 - 4$



For $q = -1$, $y = (x - 2)^2 - 1$



For $q = 0$, $y = (x - 2)^2$



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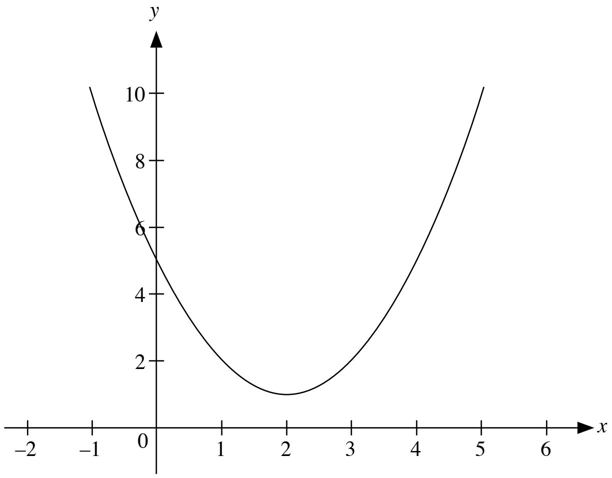
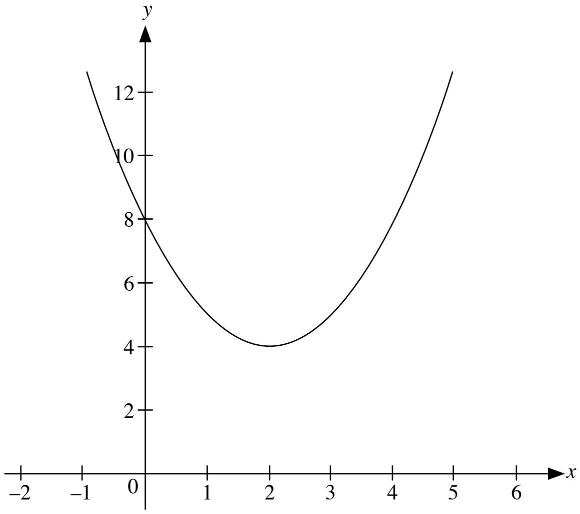
For $q = 1$, $y = (x - 2)^2 + 1$



For $q = 4$, $y = (x - 2)^2 + 4$



2. $y = (x - 2)^2 + q$

For $q = -4$, $y = (x - 2)^2 - 4$

(a) The graph opens upwards.

(b) $(0, 0)$ and $(4, 0)$

(c) $(0, 0)$

(d) $x = 2$

(e) minimum point $(2, -4)$

For $q = -1$, $y = (x - 2)^2 - 1$

(a) The graph opens upwards.

(b) $(1, 0)$ and $(3, 0)$

(c) $(0, 3)$

(d) $x = 2$

(e) minimum point $(2, -1)$

For $q = 0$, $y = (x - 2)^2$

(a) The graph opens upwards.

(b) $(2, 0)$

(c) $(0, 4)$

(d) $x = 2$

(e) minimum point $(2, 0)$

For $q = 1$, $y = (x - 2)^2 + 1$

(a) The graph opens upwards.

(b) No x-intercepts

(c) $(0, 5)$

(d) $x = 2$

(e) minimum point $(2, 1)$

For $q = 4$, $y = (x - 2)^2 + 4$

(a) The graph opens upwards.

(b) No x-intercepts

(c) $(0, 8)$

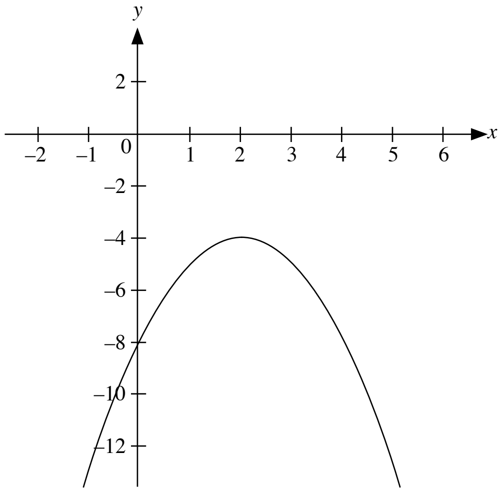
(d) $x = 2$

(e) minimum point $(2, 4)$

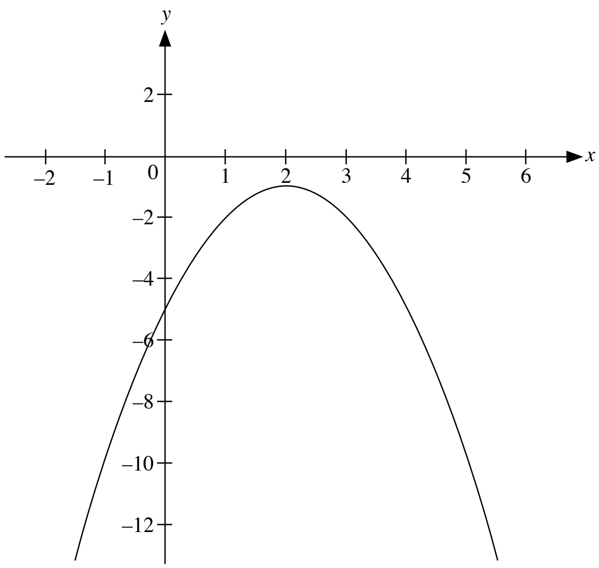
3. Step 1:

$y = -(x - 2)^2 + q$

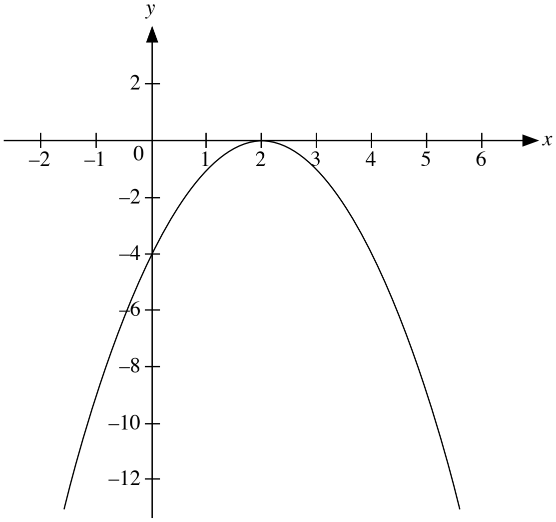
For $q = -4$, $y = -(x - 2)^2 - 4$



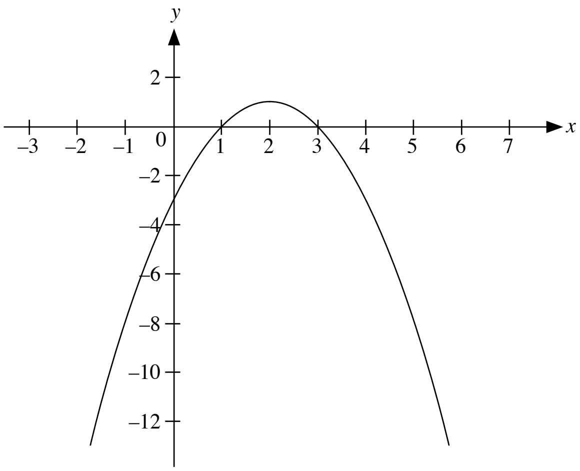
For $q = -1$, $y = -(x - 2)^2 - 1$



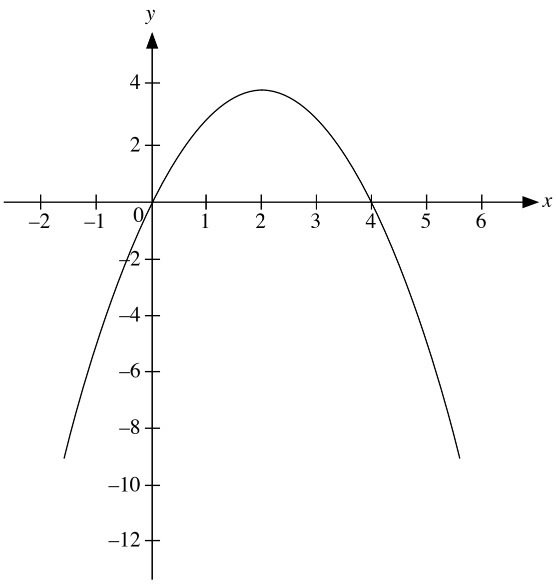
For $q = 0$, $y = -(x-2)^2$



For $q = 1$, $y = -(x-2)^2 + 1$

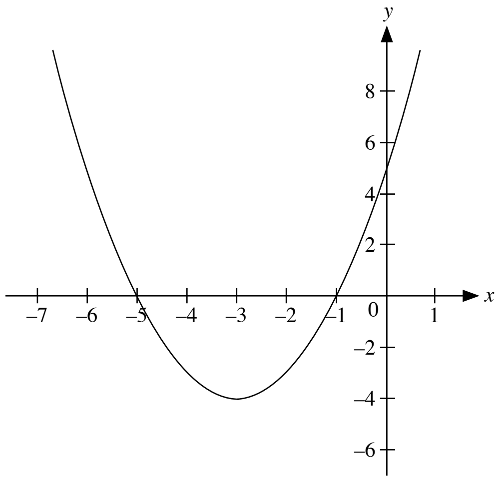


For $q = 4$, $y = -(x-2)^2 + 4$

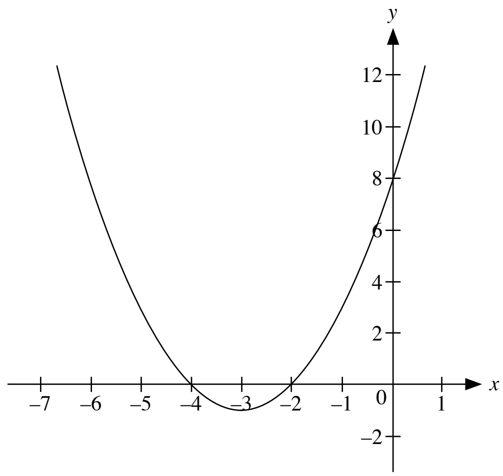


$y = (x+3)^2 + q$

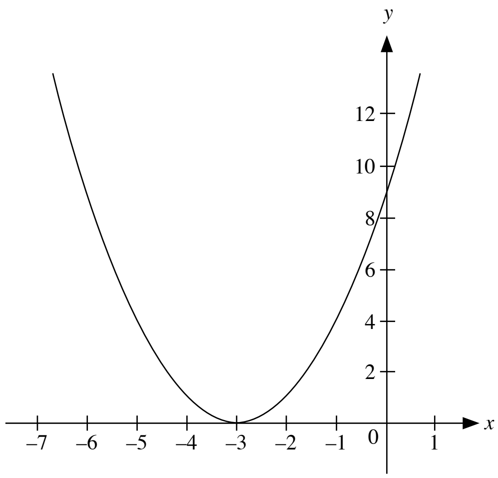
For $q = -4$, $y = (x+3)^2 - 4$



For $q = -1$, $y = (x+3)^2 - 1$



For $q = 0$, $y = (x+3)^2$



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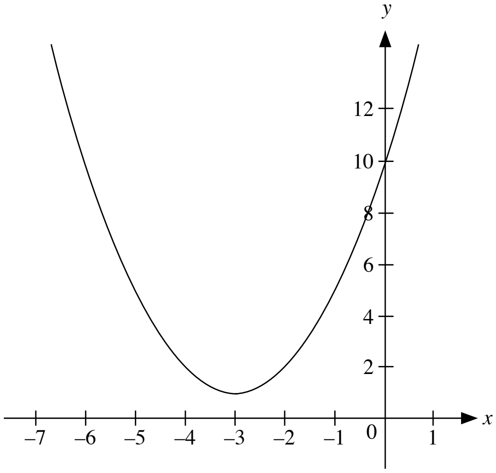
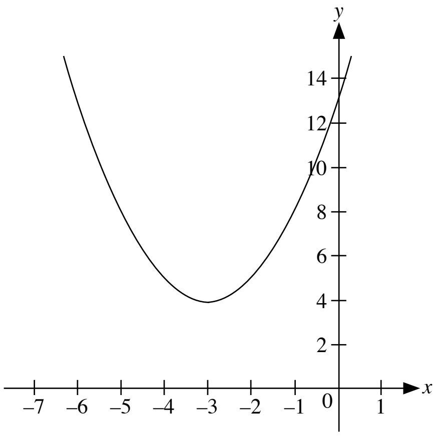
For $q = 1$, $y = (x + 3)^2 + 1$

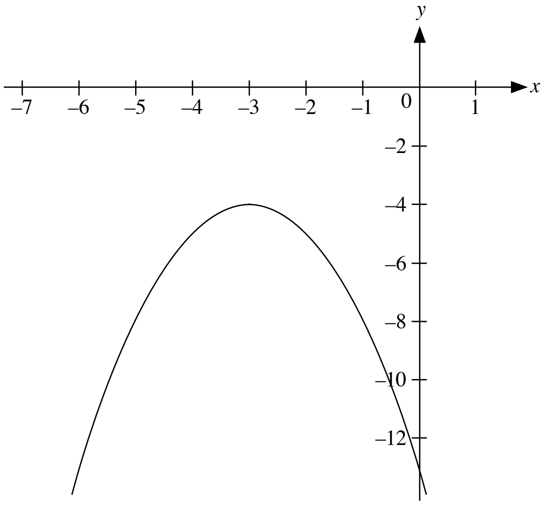


For $q = 4$, $y = (x + 3)^2 + 4$

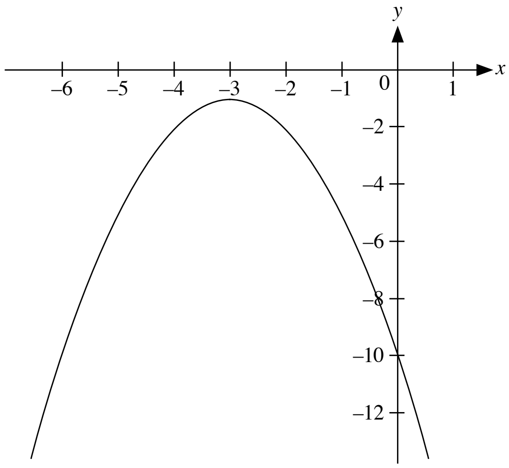


$y = -(x + 3)^2 + q$

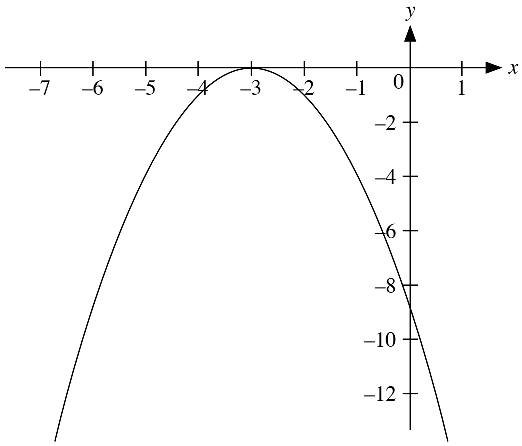
For $q = -4$, $y = -(x + 3)^2 - 4$



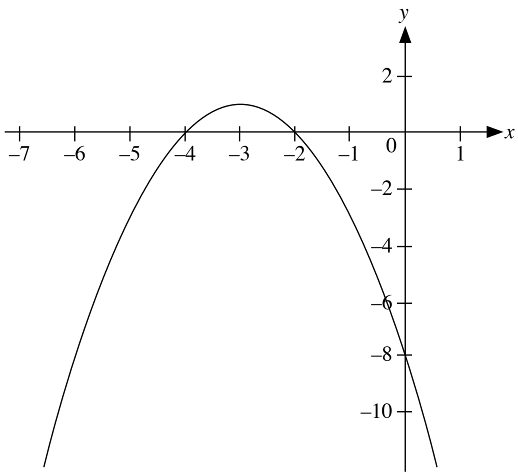
For $q = -1$, $y = -(x + 3)^2 - 1$



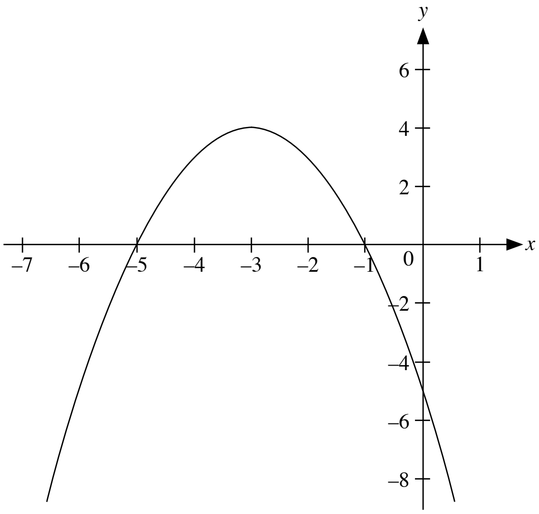
For $q = 0$, $y = -(x + 3)^2$



For $q = 1$, $y = -(x + 3)^2 + 1$



For $q = 4$, $y = -(x + 3)^2 + 4$



Step 2:

$y = -(x - 2)^2 + q$

For $q = -4$, $y = -(x - 2)^2 - 4$

(a) The graph opens downwards.
(b) No x-intercepts
(c) $(0, -8)$
(d) $x = 2$
(e) maximum point $(2, -4)$

For $q = -1$, $y = -(x - 2)^2 - 1$

(a) The graph opens downwards.
(b) No x-intercepts
(c) $(0, -5)$
(d) $x = 2$
(e) maximum point $(2, -1)$

For $q = 0$, $y = -(x - 2)^2$

(a) The graph opens downwards.
(b) $(2, 0)$
(c) $(0, -4)$
(d) $x = 2$
(e) maximum point $(2, 0)$

For $q = 1$, $y = -(x - 2)^2 + 1$

(a) The graph opens downwards.
(b) $(1, 0), (3, 0)$
(c) $(0, -3)$
(d) $x = 2$
(e) maximum point $(2, 1)$

For $q = 4$, $y = -(x - 2)^2 + 4$

(a) The graph opens downwards.
(b) $(0, 0), (4, 0)$
(c) $(0, 0)$
(d) $x = 2$
(e) maximum point $(2, 4)$

$y = (x + 3)^2 + q$

For $q = -4$, $y = (x + 3)^2 - 4$

(a) The graph opens upwards.
(b) $(-5, 0), (-1, 0)$
(c) $(0, 5)$
(d) $x = -3$
(e) minimum point $(-3, -4)$

For $q = -1$, $y = (x + 3)^2 - 1$

(a) The graph opens upwards.
(b) $(-4, 0), (-2, 0)$
(c) $(0, 8)$
(d) $x = -3$
(e) minimum point $(-3, -1)$

For $q = 0$, $y = (x + 3)^2$

(a) The graph opens upwards.
(b) $(-3, 0)$
(c) $(0, 9)$
(d) $x = -3$
(e) minimum point $(-3, 0)$

For $q = 1$ $y = (x + 3)^2 + 1$

(a) The graph opens upwards.
(b) No x-intercepts
(c) $(0, 10)$
(d) $x = -3$
(e) minimum point $(-3, 1)$

For $q = 4$, $y = (x + 3)^2 + 4$

(a) The graph opens upwards.
(b) No x-intercepts
(c) $(0, 13)$
(d) $x = -3$
(e) minimum point $(-3, 4)$

$y = -(x + 3)^2 + q$

For $q = -4$, $y = -(x + 3)^2 - 4$

(a) The graph opens downwards.
(b) No x-intercepts
(c) $(0, -13)$
(d) $x = -3$
(e) maximum point $(-3, -4)$

For $q = -1$, $y = -(x + 3)^2 - 1$

(a) The graph opens downwards.
(b) No x-intercepts
(c) $(0, -10)$
(d) $x = -3$
(e) maximum point $(-3, -1)$

For $q = 0$, $y = -(x + 3)^2$

(a) The graph opens downwards.
(b) $(-3, 0)$
(c) $(0, -9)$
(d) $x = -3$
(e) maximum point $(-3, 0)$

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For $q = 1$, $y = -(x + 3)^2 + 1$

(a) The graph opens downwards.

(b) $(-4, 0)$, $(-2, 0)$

(c) $(0, -8)$

(d) $x = -3$

(e) maximum point $(-3, 1)$

For $q = 4$, $y = -(x + 3)^2 + 4$

(a) The graph opens downwards.

(b) $(-5, 0)$, $(-1, 0)$

(c) $(0, -5)$

(d) $x = -3$

(e) maximum point $(-3, 4)$

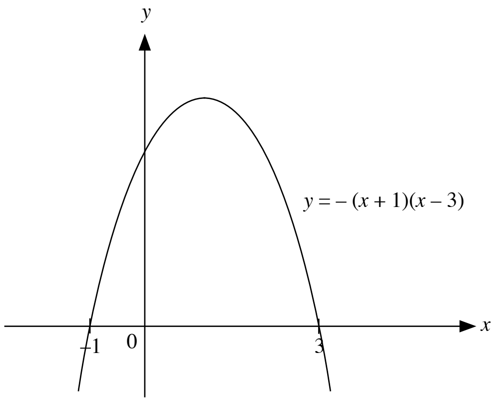
4. For the equation $y = (x - p)^2 + q$ for $q = -4, -1, 0, 1$ and 4, the graph opens upwards while for the equation $y = -(x - p)^2 + q$ for $q = -4, -1, 0, 1$ and 4, the graph opens downwards.

5. For the equation $y = (x - p)^2 + q$ for $q = -4, -1, 0, 1$ and 4, the coordinates of the minimum point of the graph are (p, q).
For the equation $y = -(x - p)^2 + q$ for $q = -4, -1, 0, 1$ and 4, the coordinates of the maximum point of the graph are (p, q).

6. The line of symmetry of each graph is about the line $x = p$.

Thinking Time (Page 33)

1. $y = -(x - 1)^2 + 4$
Since the graph cuts the x-axis at $(3, 0)$ and $(-1, 0)$,
$y = -(x + 1)(x - 3)$



2. $y = (x - a)^2 + b$
For the equation $y = (x - a)^2 + b$, the graph opens upwards. The coordinates of the minimum point of the graph are (a, b) and the graph is symmetrical about the line $x = a$.
$\therefore$ Since the minimum point is at $(2, -1)$, the equation of the curve can be expressed as $y = (x - 2)^2 - 1$.

Class Discussion (Matching Quadratic Graphs with the Corresponding Functions)

For Graphs 1, 2, 4 and 8, the graphs open upwards and so, the function is of the form $y = (x - h)(x - k)$, where (h, k) is the minimum point.
For Graphs 3, 5, 6 and 7, the graphs open downwards and so, the function is of the form $y = -(x - h)(x - k)$, where (h, k) is the maximum point.

Graph 1 **C:** $y = (x - 1)(x + 6)$
Graph 2 $y = (x - 1)(x - 6)$
E: $y = x^2 - 7x + 6$
Graph 3 **D:** $y = -(x - 1)(x - 6)$
Graph 4 $y = (x + 1)(x - 6)$
G: $y = x^2 - 5x - 6$
Graph 5 $y = -(x - 1)(x + 6)$
F: $y = -x^2 - 5x + 6$
Graph 6 **A:** $y = -(x + 1)(x + 6)$
Graph 7 $y = -(x + 1)(x - 6)$
H: $y = -x^2 + 5x + 6$
Graph 8 **B:** $y = (x + 1)(x + 6)$

Practise Now 1

(a)
$$\begin{aligned} x^2 + 7x - 8 &= 0 \\ (x + 8)(x - 1) &= 0 \\ x + 8 = 0 \quad &\text{or} \quad x - 1 = 0 \\ x = -8 \quad & \quad\quad x = 1 \end{aligned}$$
$\therefore x = -8$ or $x = 1$

(b)
$$\begin{aligned} 6y^2 + 7y - 20 &= 0 \\ (3y - 4)(2y + 5) &= 0 \\ 3y - 4 = 0 \quad &\text{or} \quad 2y + 5 = 0 \\ 3y = 4 \quad & \quad\quad 2y = -5 \\ y = 1\tfrac{1}{3} \quad & \quad\quad y = -2\tfrac{1}{2} \end{aligned}$$
$\therefore y = 1\frac{1}{3}$ or $y = -2\frac{1}{2}$

Practise Now 2

(a) $(x + 7)^2 = 100$
$$\begin{aligned} x + 7 &= \pm\sqrt{100} \\ x + 7 &= \pm 10 \\ x + 7 = 10 \quad &\text{or} \quad x + 7 = -10 \\ x = 3 \quad & \quad\quad x = -17 \end{aligned}$$
$\therefore x = 3$ or $x = -17$

(b) $(y - 5)^2 = 11$
$$\begin{aligned} y - 5 &= \pm\sqrt{11} \\ y - 5 = \sqrt{11} \quad &\text{or} \quad y - 5 = -\sqrt{11} \\ y = \sqrt{11} + 5 \quad & \quad\quad y = -\sqrt{11} + 5 \\ = 8.32 \text{ (to 3 s.f.)} \quad & \quad\quad = 1.68 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore y = 8.32$ or $y = 1.68$

Practise Now 3

(a) The coefficient of x is 20. Half of this is 10.
$$\begin{aligned} \therefore x^2 + 20x &= [x^2 + 20x + 10^2] - 10^2 \\ &= (x + 10)^2 - 100 \end{aligned}$$

(b) The coefficient of x is –7. Half of this is $-\frac{7}{2}$.

$$\therefore x^2 - 7x = \left[x^2 - 7x + \left(-\frac{7}{2}\right)^2\right] - \left(-\frac{7}{2}\right)^2$$
$$= \left(x - \frac{7}{2}\right)^2 - \frac{49}{4}$$

(c) The coefficient of x is $\frac{1}{5}$. Half of this is $\frac{1}{10}$.

$$\therefore x^2 + \frac{1}{5}x = \left[x^2 + \frac{1}{5}x + \left(\frac{1}{10}\right)^2\right] - \left(\frac{1}{10}\right)^2$$
$$= \left(x + \frac{1}{10}\right)^2 - \frac{1}{100}$$

(d) $x^2 + 6x - 9 = (x^2 + 6x) - 9$

The coefficient of x is 6. Half of this is 3.

$\therefore x^2 + 6x - 9 = [x^2 + 6x + 3^2] - 3^2 - 9$

$= (x + 3)^2 - 18$

Practise Now 4

1. (a)

$$\begin{aligned} x^2 + 6x - 4 &= 0 \\ x^2 + 6x &= 4 \\ x^2 + 6x + \left(\frac{6}{2}\right)^2 &= 4 + \left(\frac{6}{2}\right)^2 \\ x^2 + 6x + 3^2 &= 4 + 3^2 \\ (x + 3)^2 &= 13 \\ x + 3 &= \pm\sqrt{13} \end{aligned}$$

$x + 3 = \sqrt{13}$ or $x + 3 = -\sqrt{13}$

$x = \sqrt{13} - 3$ $\qquad$ $x = -\sqrt{13} - 3$

$= 0.61$ (to 2 d.p.) $\qquad$ $= -6.61$ (to 2 d.p)

$\therefore x = 0.61$ or $x = -6.61$

(b)

$$\begin{aligned} x^2 + 7x + 5 &= 0 \\ x^2 + 7x &= -5 \\ x^2 + 7x + \left(\frac{7}{2}\right)^2 &= -5 + \left(\frac{7}{2}\right)^2 \\ \left(x + \frac{7}{2}\right)^2 &= -5 + \frac{49}{4} \\ x + \frac{7}{2} &= \pm\sqrt{\frac{29}{4}} \end{aligned}$$

$x + \frac{7}{2} = \sqrt{\frac{29}{4}}$ or $x + \frac{7}{2} = -\sqrt{\frac{29}{4}}$

$x = \sqrt{\frac{29}{4}} - \frac{7}{2}$ $\qquad$ $x = -\sqrt{\frac{29}{4}} - \frac{7}{2}$

$= -0.81$ (to 2 d.p.) $\qquad$ $= -6.19$ (to 2 d.p.)

$\therefore x = -0.81$ or $x = -6.19$

(c)

$$\begin{aligned} x^2 - x - 1 &= 0 \\ x^2 - x &= 1 \\ x^2 - x + \left(-\frac{1}{2}\right)^2 &= 1 + \left(-\frac{1}{2}\right)^2 \\ \left(x - \frac{1}{2}\right)^2 &= 1 + \frac{1}{4} \\ x - \frac{1}{2} &= \pm\sqrt{\frac{5}{4}} \end{aligned}$$

$x - \frac{1}{2} = \sqrt{\frac{5}{4}}$ or $x - \frac{1}{2} = -\sqrt{\frac{5}{4}}$

$x = \sqrt{\frac{5}{4}} + \frac{1}{2}$ $\qquad$ $x = -\sqrt{\frac{5}{4}} + \frac{1}{2}$

$= 1.62$ (to 2 d.p.) $\qquad$ $= -0.62$ (to 2 d.p.)

$\therefore x = 1.62$ or $x = -0.62$

2.

$$\begin{aligned} (x + 4)(x - 3) &= 15 \\ x^2 + x - 12 &= 15 \\ x^2 + x &= 27 \\ x^2 + x + \left(\frac{1}{2}\right)^2 &= 27 + \left(\frac{1}{2}\right)^2 \\ \left(x + \frac{1}{2}\right)^2 &= 27 + \frac{1}{4} \\ x + \frac{1}{2} &= \pm\sqrt{\frac{109}{4}} \end{aligned}$$

$x + \frac{1}{2} = \sqrt{\frac{109}{4}}$ or $x + \frac{1}{2} = -\sqrt{\frac{109}{4}}$

$x = \sqrt{\frac{109}{4}} - \frac{1}{2}$ $\qquad$ $x = -\sqrt{\frac{109}{4}} - \frac{1}{2}$

$= 4.72$ (to 3 s.f.) $\qquad$ $= -5.72$ (to 3 s.f.)

$\therefore x = 4.72$ or $x = -5.72$

Practise Now 5

(a) Comparing $2x^2 + 3x - 7 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = 3$ and $c = -7$.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{65}}{4}$$

$= 1.27$ (to 3 s.f.), -2.77 (to 3 s.f.)

$\therefore x = 1.27$ or $x = -2.77$

(b) Comparing $5x^2 - 8x - 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 5$, $b = -8$ and $c = -1$.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(-1)}}{2(5)}$$
$$= \frac{8 \pm \sqrt{84}}{10}$$

$= 1.72$ (to 3 s.f.), -0.117 (to 3 s.f.)

$\therefore x = 1.72$ or $x = -0.117$

(c) $(x-1)^2 = 4x - 5$

$x^2 - 2x + 1 = 4x - 5$

$x^2 - 6x + 6 = 0$

Comparing $x^2 - 6x + 6 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -6$ and $c = 6$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$= 4.73$ (to 3 s.f.), 1.27 (to 3 s.f.)

$\therefore x = 4.73$ or $x = 1.27$

(d) $(x+3)(x-1) = 8x - 7$

$x^2 + 2x - 3 = 8x - 7$

$x^2 - 6x + 4 = 0$

Comparing $x^2 - 6x + 4 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -6$ and $c = 4$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{12}}{2}$$

$= 5.24$ (to 3 s.f.), 0.764 (to 3 s.f.)

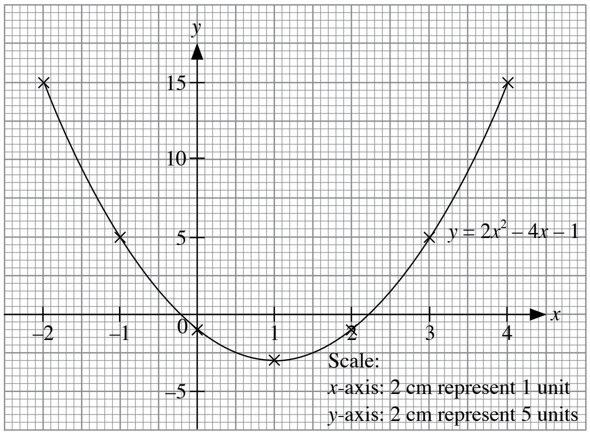
$\therefore x = 5.24$ or $x = 0.764$

Practise Now 6

1. **(i)** $y = 2x^2 - 4x - 1$

x	-2	-1	0	1	2	3	4
y	15	5	-1	-3	-1	5	15

(ii)

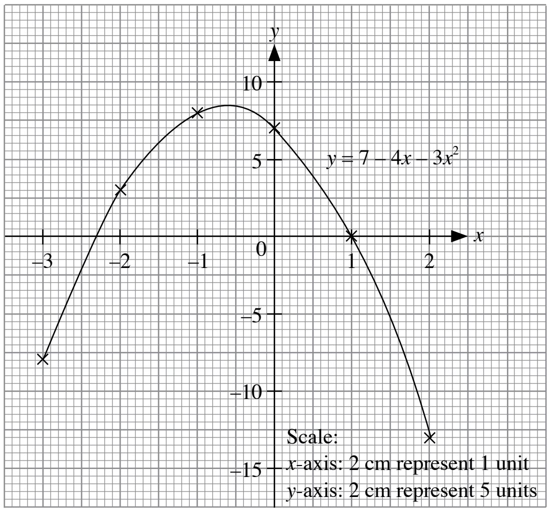


(iii) From the graph, the x-coordinates of the points of intersection of $y = 2x^2 - 4x - 1$ and the x-axis are $x = -0.2$ and $x = 2.2$.

$\therefore$ The solutions of the equation $2x^2 - 4x - 1 = 0$ are $x = -0.2$ and $x = 2.2$.

2. $y = 7 - 4x - 3x^2$

x	-3	-2	-1	0	1	2
y	-8	3	8	7	0	-13



From the graph, the x-coordinates of the points of intersection of $y = 7 - 4x - 3x^2$ and the x-axis are $x = -2.3$ and $x = 1$.

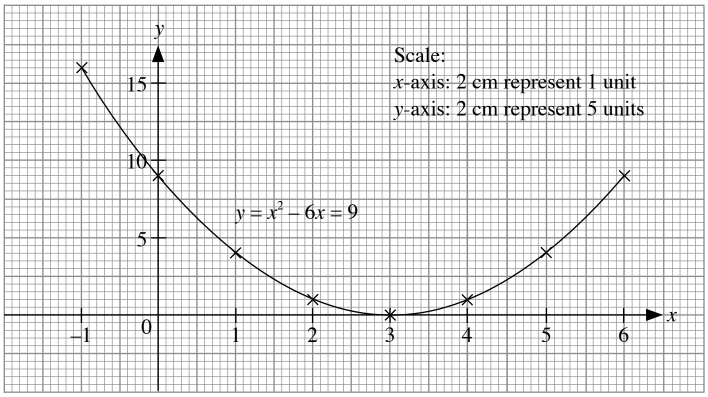
$\therefore$ The solutions of the equation $7 - 4x - 3x^2 = 0$ are $x = -2.3$ and $x = 1$.

Practise Now 7

1. **(i)** $y = x^2 - 6x + 9$

x	-1	0	1	2	3	4	5	6
y	16	9	4	1	0	1	4	9

(ii)

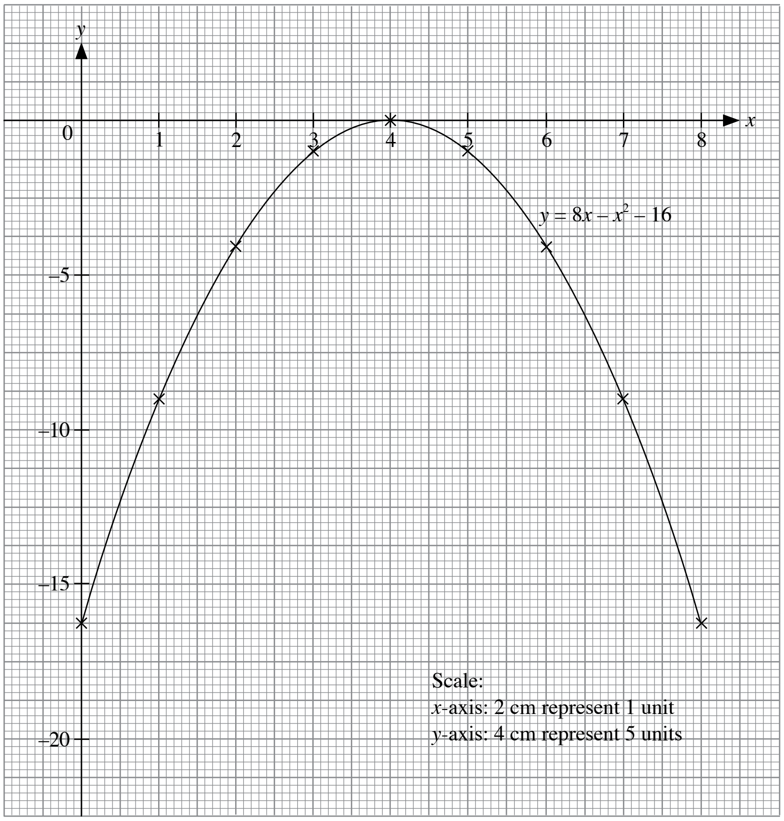


(iii) From the graph, the x-coordinate of the point of intersection of $y = x^2 - 6x + 9$ and the x-axis is $x = 3$.

$\therefore$ The solution of the equation $x^2 - 6x + 9 = 0$ is $x = 3$.

2. $y = 8x - x^2 - 16$

x	0	1	2	3	4	5	6	7	8
y	–16	–9	–4	–1	0	–1	–4	–9	–16



From the graph, the x-coordinate of the point of intersection of $y = 8x - x^2 - 16$ and the x-axis is $x = 4$.

$\therefore$ The solution of the equation $8x - x^2 - 16 = 0$ is $x = 4$.

Practise Now 8

1. (a)

$$\begin{aligned}
\frac{6}{x+4} &= x + 3 \\
\frac{6}{x+4} \times (x+4) &= (x+3) \times (x+4) \\
6 &= (x+3)(x+4) \\
6 &= x^2 + 7x + 12 \\
0 &= x^2 + 7x + 6 \\
x^2 + 7x + 6 &= 0 \\
(x+1)(x+6) &= 0 \\
x + 1 = 0 \quad &\text{or} \quad x + 6 = 0 \\
x = -1 \quad & \quad\quad x = -6
\end{aligned}$$

$\therefore x = -1$ or $x = -6$

(b)

$$\begin{aligned}
\frac{3}{x+2} &= 3x - 1 \\
\frac{3}{x+2} \times (x+2) &= (3x-1) \times (x+2) \\
3 &= (3x-1)(x+2) \\
3 &= 3x^2 + 5x - 2 \\
0 &= 3x^2 + 5x - 5 \\
3x^2 + 5x - 5 &= 0
\end{aligned}$$

Comparing $3x^2 + 5x - 5 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = 5$ and $c = -5$.

$$\begin{aligned}
x &= \frac{-5 \pm \sqrt{5^2 - 4(3)(-5)}}{2(3)} \\
&= \frac{-5 \pm \sqrt{85}}{6} \\
&= 0.703 \text{ (to 3 s.f.)}, -2.37 \text{ (to 3 s.f.)}
\end{aligned}$$

$\therefore x = 0.703$ or $x = -2.37$

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2.
$$\frac{4}{x} = 2x - 3$$
$$\frac{4}{x} \times x = (2x - 3) \times x$$
$$4 = x(2x - 3)$$
$$4 = 2x^2 - 3x$$
$$0 = 2x^2 - 3x - 4$$
$2x^2 - 3x - 4 = 0$

Comparing $2x^2 - 3x - 4 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -3$ and $c = -4$.
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{41}}{4}$$
$= 2.35$ (to 3 s.f.), -0.851 (to 3 s.f.)

$\therefore x = 2.35$ or $x = -0.851$

Practise Now 9

1. (a)
$$\frac{1}{x-2} + \frac{2}{x-3} = 5$$
$$\left[\frac{1}{x-2} + \frac{2}{x-3}\right] \times (x-2)(x-3) = 5(x-2)(x-3)$$
$$\frac{1}{x-2} \times (x-2)(x-3) + \frac{2}{x-3} \times (x-2)(x-3) = 5(x-2)(x-3)$$
$$(x-3) + 2(x-2) = 5(x-2)(x-3)$$
$$x - 3 + 2x - 4 = 5(x^2 - 5x + 6)$$
$$3x - 7 = 5x^2 - 25x + 30$$
$$0 = 5x^2 - 28x + 37$$
$$5x^2 - 28x + 37 = 0$$
Comparing $5x^2 - 28x + 37 = 0$ with $ax^2 + bx + c = 0$, we have $a = 5$, $b = -28$ and $c = 37$.
$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(5)(37)}}{2(5)}$$
$$= \frac{28 \pm \sqrt{44}}{10}$$
$= 3.46$ (to 3 s.f.), 2.14 (to 3 s.f.)

$\therefore x = 3.46$ or $x = 2.14$

(b)
$$\frac{5}{x-3} - \frac{x-1}{x-2} = 7$$
$$\left[\frac{5}{x-3} - \frac{x-1}{x-2}\right] \times (x-2)(x-3) = 7(x-2)(x-3)$$
$$\frac{5}{x-3} \times (x-2)(x-3) - \frac{x-1}{x-2} \times (x-2)(x-3) = 7(x-2)(x-3)$$
$$5(x-2) - (x-1)(x-3) = 7(x-2)(x-3)$$
$$5x - 10 - (x^2 - 4x + 3) = 7(x^2 - 5x + 6)$$
$$5x - 10 - x^2 + 4x - 3 = 7x^2 - 35x + 42$$
$$9x - 13 - x^2 = 7x^2 - 35x + 42$$
$$0 = 8x^2 - 44x + 55$$
$$8x^2 - 44x + 55 = 0$$
Comparing $8x^2 - 44x + 55 = 0$ with $ax^2 + bx + c = 0$, we have $a = 8$, $b = -44$ and $c = 55$.
$$x = \frac{-(-44) \pm \sqrt{(-44)^2 - 4(8)(55)}}{2(8)}$$
$$= \frac{44 \pm \sqrt{176}}{16}$$
$= 3.58$ (to 3 s.f.), 1.92 (to 3 s.f.)

$\therefore x = 3.58$ or $x = 1.92$

2.
$$\frac{3}{x-2} - \frac{1}{(x-2)^2} = 2$$
$$\left[\frac{3}{x-2} - \frac{1}{(x-2)^2}\right] \times (x-2)^2 = 2 \times (x-2)^2$$
$$\frac{3}{x-2} \times (x-2)^2 - \frac{1}{(x-2)^2} \times (x-2)^2 = 2 \times (x-2)^2$$
$$3(x-2) - 1 = 2(x-2)^2$$
$$3x - 6 - 1 = 2(x^2 - 4x + 4)$$
$$3x - 7 = 2x^2 - 8x + 8$$
$$0 = 2x^2 - 11x + 15$$
$$2x^2 - 11x + 15 = 0$$
$$(x-3)(2x-5) = 0$$
$x - 3 = 0$ or $2x - 5 = 0$

$x = 3$ $\quad$ $2x = 5$

$x = 2.5$

$\therefore x = 3$ or $x = 2.5$

Practise Now 10

(i) $AB = 17 - 8 - x$

$= (9 - x)$ cm

(ii) By Pythagoras' Theorem,
$$AC^2 = AB^2 + BC^2$$
$$8^2 = (9 - x)^2 + x^2$$
$$64 = 81 - 18x + x^2 + x^2$$
$$0 = 17 - 18x + 2x^2$$
$2x^2 - 18x + 17 = 0$ (shown)

(iii) $2x^2 - 18x + 17 = 0$

Comparing $2x^2 - 18x + 17 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -18$ and $c = 17$.
$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(2)(17)}}{2(2)}$$
$$= \frac{18 \pm \sqrt{188}}{4}$$
$= 7.928$ (to 3 d.p.), 1.072 (to 3 d.p.)

$\therefore x = 7.928$ or $x = 1.072$

(iv) $BC = 1.072$ cm

$AB = 9 - 1.072 = 7.928$ cm

Area of triangle $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 7.928 \times 1.072$

$= 4.25\ \text{cm}^2$ (to 3 s.f.)

Practise Now 11

(i)
$$\frac{600}{x} - \frac{600}{x+7} = \frac{15}{60}$$
$$\frac{600}{x} - \frac{600}{x+7} = \frac{1}{4}$$
$$\frac{600}{x} \times x(x+7) - \frac{600}{x+7} \times x(x+7) = \frac{1}{4} \times x(x+7)$$
$$600(x+7) - 600x = \frac{1}{4}x(x+7)$$
$$600x + 4200 - 600x = \frac{1}{4}x^2 + \frac{7}{4}x$$
$$4200 = \frac{1}{4}x^2 + \frac{7}{4}x$$
$$\frac{1}{4}x^2 + \frac{7}{4}x - 4200 = 0$$
$$x^2 + 7x - 16\,800 = 0 \text{ (shown)}$$

(ii) $x^2 + 7x - 16\,800 = 0$

Comparing $x^2 + 7x - 16\,800 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 7$ and $c = -16\,800$.

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-16\,800)}}{2(1)}$$
$$= \frac{-7 \pm \sqrt{67\,249}}{2}$$
$= 126.16$ (to 2 d.p.), -133.16 (to 2 d.p.)

$\therefore$ $x = 126.16$ or $x = -133.16$

(iii) $x = 126.16$ or $x = -133.16$ (rejected, since $x > 0$)

Time taken for the return journey = $\dfrac{600}{126.16 + 7}$ = 4.51 h (to 3 s.f.)

Practise Now 12

(a) $y = (x - 2)(x - 6)$

Since the coefficient of x^2 is 1, the graph opens upwards.

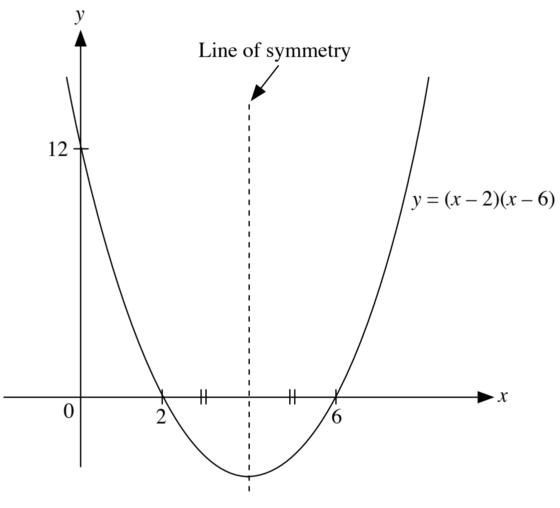
When $y = 0$,

$(x - 2)(x - 6) = 0$

$x - 2 = 0$ or $x - 6 = 0$

$x = 2$ $\quad$ $x = 6$

$\therefore$ The graph cuts the y-axis at (0, 12).



(b) $y = -(x - 3)(x + 1)$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

$-(x - 3)(x + 1) = 0$

$x - 3 = 0$ or $x + 1) = 0$

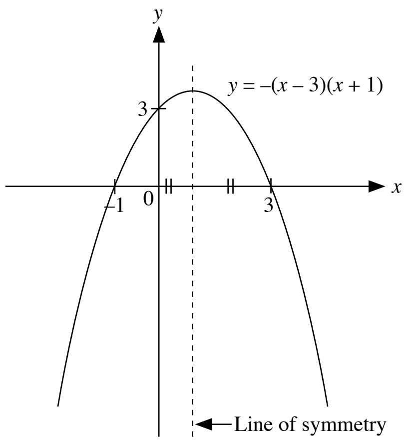
$x = 3$ $\quad$ $x = -1$

$\therefore$ The graph cuts the x-axis at $(-1, 0)$ and $(3, 0)$.

When $x = 0$,

$y = -(-3)(1) = 3$

$\therefore$ The graph cuts the y-axis at (0, 3).



(c) $y = (3 - x)(x + 5)$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

$(3 - x)(x + 5) = 0$

$3 - x = 0$ or $x + 5 = 0$

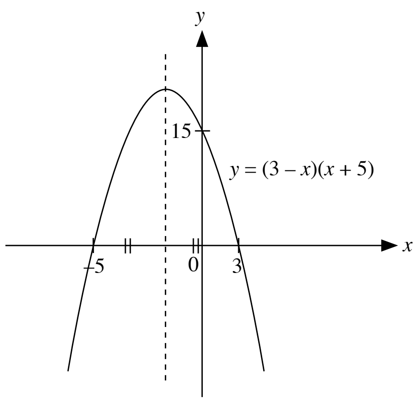
$x = 3$ $\quad$ $x = -5$

$\therefore$ The graph cuts the x-axis at $(-5, 0)$ and $(3, 0)$.

When $x = 0$,

$y = (3)(5) = 15$

$\therefore$ The graph cuts the y-axis at (0, 15).



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Practise Now 13

1. **(i)** $y = -(x - 2)^2 + 9$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

$$-(x - 2)^2 + 9 = 0$$
$$-(x - 2)^2 = -9$$
$$(x - 2)^2 = 9$$
$$x - 2 = 3 \quad \text{or} \quad x - 2 = -3$$
$$x = 5 \qquad\qquad x = -1$$

$\therefore$ The graph cuts the x-axis at $(5, 0)$ and $(-1, 0)$.

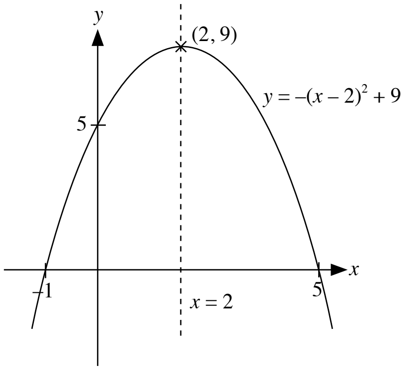
When $x = 0$,

$y = -(-2)^2 + 9 = 5$

$\therefore$ The graph cuts the y-axis at $(0, 5)$.

(ii) The coordinates of the maximum point are $(2, 9)$.

(iii)



(iv) The equation of the line of symmetry is $x = 2$.

2. **(i)** $y = (x + 1)^2 - 1$

Since the coefficient of x^2 is 1, the graph opens upwards.

When y 0,

$$(x + 1)^2 - 1 = 0$$
$$(x + 1)^2 = 1$$
$$x + 1 = 1 \quad \text{or} \quad x + 1 = -1$$
$$x = 0 \qquad\qquad x = -2$$

$\therefore$ The graph cuts the x-axis at $(0, 0)$ and $(-2, 0)$.

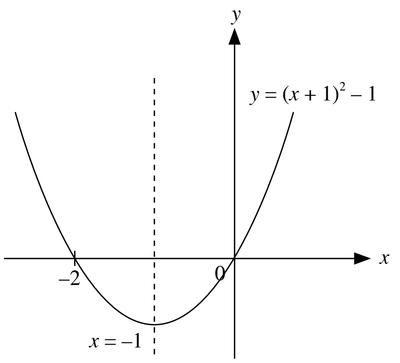
When $x = 0$,

$y = (1)^2 - 1 = 0$

$\therefore$ The graph cuts the y-axis at $(0, 0)$.

(ii) The coordinates of the minimum point are $(-1, -1)$.

(iii)



(iv) The equation of the line of symmetry is $x = -1$.

Practise Now 14

1. **(i)** $x^2 - 6x + 6 = \left[x^2 - 6x + \left(-\frac{6}{2}\right)^2\right] - \left(-\frac{6}{2}\right)^2 + 6$

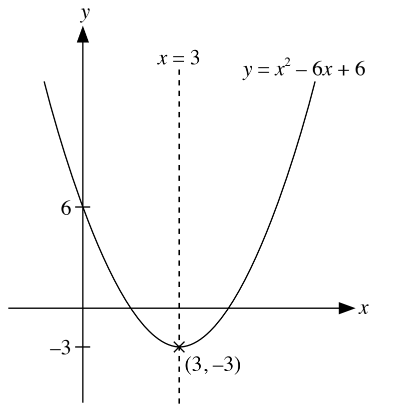
$\qquad = (x - 3)^2 - 3$

(ii) The coordinates of the minimum point are $(3, -3)$.

(iii) When $x = 0$,

$y = (0)^2 - 6(0) + 6 = 0$

$\therefore$ The graph cuts the y-axis at $(0, 6)$.



(iv) The equation of the line of symmetry is $x = 3$.

2. **(i)** $x^2 + x + 1 = \left[x^2 + x + \left(\frac{1}{2}\right)^2\right] - \left(\frac{1}{2}\right)^2 + 1$

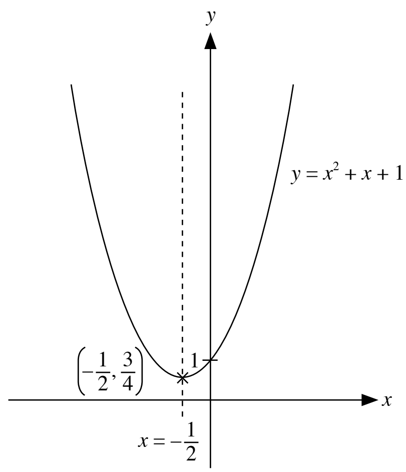
$\qquad = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

(ii) The coordinates of the minimum point are $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

(iii) When $x = 0$,

$y = (0)^2 + 0 + 1 = 1$

$\therefore$ The graph cuts the y-axis at $(0, 1)$.



(iv) The equation of the line of symmetry is $x = -\frac{1}{2}$.

Exercise 1A

1. (a)
$$\begin{aligned} 2x^2+5x-7&=0\\ (x-1)(2x+7)&=0 \end{aligned}$$
$x-1=0$ or $2x+7=0$

$x=1$ $\quad$ $2x=-7$

$x=-3\frac{1}{2}$

$\therefore x=1$ or $x=-3\frac{1}{2}$

(b)
$$\begin{aligned} 4x^2-5x-6&=0\\ (x-2)(4x+3)&=0 \end{aligned}$$
$x-2=0$ or $4x+3=0$

$x=2$ $\quad$ $4x=-3$

$x=-\frac{3}{4}$

$\therefore x=2$ or $x=-\frac{3}{4}$

(c)
$$\begin{aligned} 7x+x^2-18&=0\\ (x-2)(x+9)&=0 \end{aligned}$$
$x-2=0$ or $x+9=0$

$x=2$ $\quad$ $x=-9$

$\therefore x=2$ or $x=-9$

(d)
$$\begin{aligned} 4-3x-x^2&=0\\ (1-x)(x+4)&=0 \end{aligned}$$
$1-x=0$ or $x+4=0$

$x=1$ $\quad$ $x=-4$

$\therefore x=1$ or $x=-4$

(e)
$$\begin{aligned} x(3x-1)&=2\\ 3x^2-x&=2\\ 3x^2-x-2&=0\\ (x-1)(3x+2)&=0 \end{aligned}$$
$x-1=0$ or $3x+2=0$

$x=1$ $\quad$ $3x=-2$

$x=-\frac{2}{3}$

$\therefore x=1$ or $x=-\frac{2}{3}$

(f)
$$\begin{aligned} (7-3x)(x+2)&=4\\ 7x+14-3x^2-6x&=4\\ x+14-3x^2&=4\\ x+10-3x^2&=0\\ (2-x)(3x+5)&=0 \end{aligned}$$
$2-x=0$ or $3x+5=0$

$x=2$ $\quad$ $3x=-5$

$x=-1\frac{2}{3}$

$\therefore x=2$ or $x=-1\frac{2}{3}$

2. (a) $(x+1)^2=9$
$$\begin{aligned} x+1&=\pm\sqrt{9}\\ x+1&=\pm3 \end{aligned}$$
$x+1=3$ or $x+1=-3$

$x=2$ $\quad$ $x=-4$

$\therefore x=2$ or $x=-4$

(b) $(2x+1)^2=16$
$$\begin{aligned} 2x+1&=\pm\sqrt{16}\\ 2x+1&=\pm4 \end{aligned}$$
$2x+1=4$ or $2x+1=-4$

$2x=3$ $\quad$ $2x=-5$

$x=1\frac{1}{2}$ $\quad$ $x=-2\frac{1}{2}$

$\therefore x=1\frac{1}{2}$ or $x=-2\frac{1}{2}$

(c) $(5x-4)^2=81$
$$\begin{aligned} 5x-4&=\pm\sqrt{81}\\ 5x-4&=\pm9 \end{aligned}$$
$5x-4=9$ or $5x-4=-9$

$5x=13$ $\quad$ $5x=-5$

$x=2\frac{3}{5}$ $\quad$ $x=-1$

$\therefore x=2\frac{3}{5}$ or $x=-1$

(d) $(7-3x)^2=\frac{9}{16}$
$$\begin{aligned} 7-3x&=\pm\sqrt{\frac{9}{16}}\\ 7-3x&=\pm\frac{3}{4} \end{aligned}$$
$7-3x=\frac{3}{4}$ or $7-3x=-\frac{3}{4}$

$3x=6\frac{1}{4}$ $\quad$ $3x=7\frac{3}{4}$

$x=2\frac{1}{12}$ $\quad$ $x=2\frac{7}{12}$

$\therefore x=2\frac{7}{12}$ or $x=2\frac{1}{12}$

(e) $(x+3)^2=11$

$x+3=\pm\sqrt{11}$

$x+3=\sqrt{11}$ or $x+3=-\sqrt{11}$

$x=\sqrt{11}-3$ $\quad$ $x=-\sqrt{11}-3$

$=0.32$ (to 2 d.p.) $\quad$ $=-6.32$ (to 2 d.p.)

$\therefore x=0.32$ or $x=-6.32$

(f) $(2x-3)^2=23$

$2x-3=\pm\sqrt{23}$

$2x-3=\sqrt{23}$ or $2x-3=-\sqrt{23}$

$2x=\sqrt{23}+3$ $\quad$ $2x=-\sqrt{23}+3$

$x=3.90$ (to 2 d.p.) $\quad$ $x=-0.90$ (to 2 d.p.)

$\therefore x=3.90$ or $x=-0.90$

(g) $(5-x)^2 = 7$

$$5 - x = \pm\sqrt{7}$$

$$5 - x = \sqrt{7} \quad \text{or} \quad 5 - x = -\sqrt{7}$$

$$x = 5 - \sqrt{7} \qquad x = 5 + \sqrt{7}$$

$$= 2.35 \text{ (to 2 d.p.)} \qquad = 7.65 \text{ (to 2 d.p.)}$$

$\therefore x = 2.35$ or $x = 7.65$

(h) $\left(\frac{1}{2} - x\right)^2 = 10$

$$\frac{1}{2} - x = \pm\sqrt{10}$$

$$\frac{1}{2} - x = \sqrt{10} \quad \text{or} \quad \frac{1}{2} - x = -\sqrt{10}$$

$$x = \frac{1}{2} - \sqrt{10} \qquad x = \frac{1}{2} + \sqrt{10}$$

$$= -2.66 \text{ (to 2 d.p.)} \qquad = 3.66 \text{ (to 2 d.p.)}$$

$\therefore x = -2.66$ or $x = 3.66$

3. **(a)** The coefficient of x is 12. Half of this is 6.

$$\therefore x^2 + 12x = [x^2 + 12x + 6^2] - 6^2$$
$$= (x+6)^2 - 36$$

(b) The coefficient of x is –6. Half of this is –3.

$$\therefore x^2 - 6x + 1 = [x^2 - 6x + (-3)^2] - (-3)^2 + 1$$
$$= (x-3)^2 - 8$$

(c) The coefficient of x is 3. Half of this is $\frac{3}{2}$.

$$\therefore x^2 + 3x - 2 = \left[x^2 + 3x + \left(\frac{3}{2}\right)^2\right] - \left(\frac{3}{2}\right)^2 - 2$$
$$= \left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$$

(d) The coefficient of x is 9. Half of this is $\frac{9}{2}$.

$$\therefore x^2 + 9x - 1 = \left[x^2 + 9x + \left(\frac{9}{2}\right)^2\right] - \left(\frac{9}{2}\right)^2 - 1$$
$$= \left(x + \frac{9}{2}\right)^2 - \frac{85}{4}$$

(e) The coefficient of x is $\frac{1}{2}$. Half of this is $\frac{1}{4}$.

$$\therefore x^2 + \frac{1}{2}x = \left[x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2\right] - \left(\frac{1}{4}\right)^2$$
$$= \left(x + \frac{1}{4}\right)^2 - \frac{1}{16}$$

(f) The coefficient of x is $-\frac{2}{9}$. Half of this is $-\frac{1}{9}$.

$$\therefore x^2 - \frac{2}{9}x = \left[x^2 - \frac{2}{9}x + \left(-\frac{1}{9}\right)^2\right] - \left(-\frac{1}{9}\right)^2$$
$$= \left(x - \frac{1}{9}\right)^2 - \frac{1}{81}$$

(g) The coefficient of x is 0.2. Half of this is 0.1.

$$\therefore x^2 + 0.2x = [x^2 + 0.2x + 0.1^2] - 0.1^2$$
$$= (x + 0.1)^2 - 0.01$$

(h) The coefficient of x is –1.4. Half of this is –0.7.

$$\therefore x^2 - 1.4x = [x^2 - 1.4 + (-0.7)^2] - (-0.7)^2$$
$$= (x - 0.7)^2 - 0.49$$

4. **(a)** $x^2 + 2x - 5 = 0$

$$x^2 + 2x = 5$$
$$x^2 + 2x + 1^2 = 5 + 1^2$$
$$(x+1)^2 = 6$$
$$x + 1 = \pm\sqrt{6}$$
$$x + 1 = \sqrt{6} \quad \text{or} \quad x + 1 = -\sqrt{6}$$
$$x = \sqrt{6} - 1 \qquad x = -\sqrt{6} - 1$$
$$= 1.45 \text{ (to 2 d.p.)} \qquad = -3.45 \text{ (to 2 d.p.)}$$

$\therefore x = 1.45$ or $x = -3.45$

(b) $x^2 + 17x - 30 = 0$

$$x^2 + 17x = 30$$
$$x^2 + 17x + \left(\frac{17}{2}\right)^2 = 30 + \left(\frac{17}{2}\right)^2$$
$$\left(x + \frac{17}{2}\right)^2 = 30 + \frac{289}{4}$$
$$x + \frac{17}{2} = \pm\sqrt{\frac{409}{4}}$$
$$x + \frac{17}{2} = \sqrt{\frac{409}{4}} \quad \text{or} \quad x + \frac{17}{2} = -\sqrt{\frac{409}{4}}$$
$$x = \sqrt{\frac{409}{4}} - \frac{17}{2} \qquad x = -\sqrt{\frac{409}{4}} - \frac{17}{2}$$
$$= 1.61 \text{ (to 2 d.p.)} \qquad = -18.61 \text{ (to 2 d.p.)}$$

$\therefore x = 1.61$ or $x = -18.61$

(c) $x^2 - 12x + 9 = 0$

$$x^2 - 12x = -9$$
$$x^2 - x + (-6)^2 = -9 + (-6)^2$$
$$(x-6)^2 = 27$$
$$x - 6 = \pm\sqrt{27}$$
$$x - 6 = \sqrt{27} \quad \text{or} \quad x - 6 = -\sqrt{27}$$
$$x = \sqrt{27} + 6 \qquad x = -\sqrt{27} + 6$$
$$= 11.20 \text{ (to 2 d.p.)} \qquad = 0.80 \text{ (to 2 d.p.)}$$

$\therefore x = 11.20$ or $x = 0.80$

(d) $x^2 - 5x - 5 = 0$

$$x^2 - 5x = 5$$
$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = 5 + \left(-\frac{5}{2}\right)^2$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{45}{4}$$
$$x - \frac{5}{2} = \pm\sqrt{\frac{45}{4}}$$
$$x - \frac{5}{2} = \sqrt{\frac{45}{4}} \quad \text{or} \quad x - \frac{5}{2} = -\sqrt{\frac{45}{4}}$$
$$x = \sqrt{\frac{45}{4}} + \frac{5}{2} \qquad x = -\sqrt{\frac{45}{4}} + \frac{5}{2}$$
$$= 5.85 \text{ (to 2 d.p.)} \qquad = -0.85 \text{ (to 2 d.p.)}$$

$\therefore x = 5.85$ or $x = -0.85$

(e) $x^2 + \frac{1}{4}x - 3 = 0$

$$x^2 + \frac{1}{4}x = 3$$

$$x^2 + \frac{1}{4}x + \left(\frac{1}{8}\right)^2 = 3 + \left(\frac{1}{8}\right)^2$$

$$\left(x + \frac{1}{8}\right)^2 = \frac{193}{64}$$

$$x + \frac{1}{8} = \pm\sqrt{\frac{193}{64}}$$

$$x + \frac{1}{8} = \sqrt{\frac{193}{64}} \quad \text{or} \quad x + \frac{1}{8} = -\sqrt{\frac{193}{64}}$$

$$x = \sqrt{\frac{193}{64}} - \frac{1}{8} \qquad x = -\sqrt{\frac{193}{64}} - \frac{1}{8}$$

$$= 1.61 \text{ (to 2 d.p.)} \qquad = -1.86 \text{ (to 2 d.p.)}$$

$\therefore x = 1.61$ or $x = -1.86$

(f) $x^2 - \frac{6}{7}x + \frac{2}{49} = 0$

$$x^2 - \frac{6}{7}x = -\frac{2}{49}$$

$$x^2 - \frac{6}{7}x + \left(-\frac{3}{7}\right)^2 = -\frac{2}{49} + \left(-\frac{3}{7}\right)^2$$

$$\left(x - \frac{3}{7}\right)^2 = \frac{1}{7}$$

$$x - \frac{3}{7} = \pm\sqrt{\frac{1}{7}}$$

$$x - \frac{3}{7} = \sqrt{\frac{1}{7}} \quad \text{or} \quad x - \frac{3}{7} = -\sqrt{\frac{1}{7}}$$

$$x = \sqrt{\frac{1}{7}} + \frac{3}{7} \qquad x = -\sqrt{\frac{1}{7}} + \frac{3}{7}$$

$$= 0.81 \text{ (to 2 d.p.)} \qquad = 0.05 \text{ (to 2 d.p.)}$$

$\therefore x = 0.81$ or $x = 0.05$

(g) $x^2 + 0.6x - 1 = 0$

$$x^2 + 0.6x = 1$$

$$x^2 + 0.6x + 0.3^2 = 1 + 0.3^2$$

$$(x + 0.3)^2 = 1.09$$

$$x + 0.3 = \pm\sqrt{1.09}$$

$$x + 0.3 = \sqrt{1.09} \quad \text{or} \quad x + 0.3 = -\sqrt{1.09}$$

$$x = \sqrt{1.09} - 0.3 \qquad x = -\sqrt{1.09} - 0.3$$

$$= 0.74 \text{ (to 2 d.p.)} \qquad = -1.34 \text{ (to 2 d.p.)}$$

$\therefore x = 0.74$ or $x = -1.34$

(h) $x^2 - 4.8x + 2 = 0$

$$x^2 - 4.8x = -2$$

$$x^2 - 4.8x + (-2.4)^2 = -2 + (-2.4)^2$$

$$(x - 2.4)^2 = 3.76$$

$$x - 2.4 = \pm\sqrt{3.76}$$

$$x - 2.4 = \sqrt{3.76} \quad \text{or} \quad x - 2.4 = -\sqrt{3.76}$$

$$x = \sqrt{3.76} + 2.4 \qquad x = -\sqrt{3.76} + 2.4$$

$$= 4.34 \text{ (to 2 d.p.)} \qquad = 0.46 \text{ (to 2 d.p.)}$$

$\therefore x = 4.34$ or $x = 0.46$

5. (a) $x(x - 3) = 5x + 1$

$$x^2 - 3x = 5x + 1$$

$$x^2 - 8x = 1$$

$$x^2 - 8x + (-4)^2 = 1 + (-4)^2$$

$$(x - 4)^2 = 17$$

$$x - 4 = \pm\sqrt{17}$$

$$x - 4 = \sqrt{17} \quad \text{or} \quad x - 4 = -\sqrt{17}$$

$$x = \sqrt{17} + 4 \qquad x = -\sqrt{17} + 4$$

$$= 8.12 \text{ (to 2 d.p.)} \qquad = -0.12 \text{ (to 2 d.p.)}$$

$\therefore x = 8.12$ or $x = -0.12$

(b) $(x + 1)^2 = 7x$

$$x^2 + 2x + 1 = 7x$$

$$x^2 - 5x = -1$$

$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = -1 + \left(-\frac{5}{2}\right)^2$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{21}{4}$$

$$x - \frac{5}{2} = \pm\sqrt{\frac{21}{4}}$$

$$x - \frac{5}{2} = \sqrt{\frac{21}{4}} \quad \text{or} \quad x - \frac{5}{2} = -\sqrt{\frac{21}{4}}$$

$$x = \sqrt{\frac{21}{4}} + \frac{5}{2} \qquad x = -\sqrt{\frac{21}{4}} + \frac{5}{2}$$

$$= 4.79 \text{ (to 2 d.p.)} \qquad = 0.21 \text{ (to 2 d.p.)}$$

$\therefore x = 4.79$ or $x = 0.21$

(c) $(x + 2)(x - 5) = 4x$

$$x^2 - 3x - 10 = 4x$$

$$x^2 - 7x = 10$$

$$x^2 - 7x + \left(-\frac{7}{2}\right)^2 = 10 + \left(-\frac{7}{2}\right)^2$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{89}{4}$$

$$x - \frac{7}{2} = \pm\sqrt{\frac{89}{4}}$$

$$x - \frac{7}{2} = \sqrt{\frac{89}{4}} \quad \text{or} \quad x - \frac{7}{2} = -\sqrt{\frac{89}{4}}$$

$$x = \sqrt{\frac{89}{4}} + \frac{7}{2} \qquad x = -\sqrt{\frac{89}{4}} + \frac{7}{2}$$

$$= 8.22 \text{ (to 2 d.p.)} \qquad = -1.22 \text{ (to 2 d.p.)}$$

$\therefore x = 8.22$ or $x = -1.22$

(d) $x(x - 4) = 2(x + 7)$

$$x^2 - 4x = 2x + 14$$

$$x^2 - 6x = 14$$

$$x^2 - 6x + (-3)^2 = 14 + (-3)^2$$

$$(x - 3)^2 = 23$$

$$x - 3 = \pm\sqrt{23}$$

$$x - 3 = \sqrt{23} \quad \text{or} \quad x - 3 = -\sqrt{23}$$

$$x = \sqrt{23} + 3 \qquad x = -\sqrt{23} + 3$$

$$= 7.80 \text{ (to 2 d.p.)} \qquad = -1.80 \text{ (to 2 d.p.)}$$

$\therefore x = 7.80$ or $x = -1.80$

6.
$$\begin{aligned} y^2 - ay - 6 &= 0 \\ y - ay &= 6 \\ y^2 - ay + \left(-\frac{a}{2}\right)^2 &= 6 + \left(-\frac{a}{2}\right)^2 \\ \left(y - \frac{a}{2}\right)^2 &= 6 + \frac{a^2}{4} \\ y - \frac{a}{2} &= \pm\sqrt{6 + \frac{a^2}{4}} \\ y - \frac{a}{2} &= \pm\sqrt{\frac{a^2 + 24}{4}} \\ y - \frac{a}{2} &= \pm\sqrt{\frac{a^2 + 24}{4}} \\ y &= \frac{a}{2} \pm \sqrt{\frac{a^2 + 24}{4}} \\ y &= \frac{a \pm \sqrt{a^2 + 24}}{2} \end{aligned}$$

Exercise 1B

1. **(a)** Comparing $x^2 + 4x + 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 4$ and $c = 1$.

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{12}}{2} \\ &= -0.268 \text{ (to 3 s.f.)}, -3.73 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = -0.268$ or $x = -3.73$

(b) Comparing $3x^2 + 6x - 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = 6$ and $c = -1$.

$$\begin{aligned} x &= \frac{-6 \pm \sqrt{6^2 - 4(3)(-1)}}{2(3)} \\ &= \frac{-6 \pm \sqrt{48}}{6} \\ &= 0.155 \text{ (to 3 s.f.)}, -2.15 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 0.155$ or $x = -2.15$

(c) Comparing $2x^2 - 7x + 2 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -7$ and $c = 2$.

$$\begin{aligned} x &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)} \\ &= \frac{7 \pm \sqrt{33}}{4} \\ &= 3.19 \text{ (to 3 s.f.)}, 0.314 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 3.19$ or $x = 0.314$

(d) Comparing $3x^2 - 5x - 17 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = -5$ and $c = -17$.

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-17)}}{2(3)} \\ &= \frac{5 \pm \sqrt{229}}{6} \\ &= 3.36 \text{ (to 3 s.f.)}, -1.69 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 3.36$ or $x = -1.69$

(e) Comparing $-3x^2 - 7x + 9 = 0$ with $ax^2 + bx + c = 0$, we have $a = -3$, $b = -7$ and $c = 9$.

$$\begin{aligned} x &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-3)(9)}}{2(-3)} \\ &= \frac{7 \pm \sqrt{157}}{-6} \\ &= -3.25 \text{ (to 3 s.f.)}, 0.922 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = -3.25$ or $x = 0.922$

(f) Comparing $-5x^2 + 10x - 2 = 0$ with $ax^2 + bx + c = 0$, we have $a = -5$, $b = 10$ and $c = -2$.

$$\begin{aligned} x &= \frac{-10 \pm \sqrt{10^2 - 4(-5)(-2)}}{2(-5)} \\ &= \frac{-10 \pm \sqrt{60}}{-10} \\ &= 0.225 \text{ (to 3 s.f.)}, 1.77 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 0.225$ or $x = 1.77$

2. **(a)**
$$\begin{aligned} x^2 + 5x &= 21 \\ x^2 + 5x - 21 &= 0 \end{aligned}$$

Comparing $x^2 + 5x - 21 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 5$ and $c = -21$.

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4(1)(-21)}}{2(1)} \\ &= \frac{-5 \pm \sqrt{109}}{2} \\ &= 2.72 \text{ (to 3 s.f.)}, -7.72 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 2.72$ or $x = -7.72$

(b)
$$\begin{aligned} 10x^2 - 12x &= 15 \\ 10x^2 - 12x - 15 &= 0 \end{aligned}$$

Comparing $10x^2 - 12x - 15 = 0$ with $ax^2 + bx + c = 0$, we have $a = 10$, $b = -12$ and $c = -15$.

$$\begin{aligned} x &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(10)(-15)}}{2(10)} \\ &= \frac{12 \pm \sqrt{744}}{20} \\ &= 1.96 \text{ (to 3 s.f.)}, -0.764 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 1.96$ or $x = -0.764$

(c)
$$\begin{aligned} 8x^2 &= 3x + 6 \\ 8x^2 - 3x - 6 &= 0 \end{aligned}$$

Comparing $8x^2 - 3x - 6 = 0$ with $ax^2 + bx + c = 0$, we have $a = 8$, $b = -3$ and $c = -6$.

$$\begin{aligned} x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(8)(-6)}}{2(8)} \\ &= \frac{3 \pm \sqrt{201}}{16} \\ &= 1.07 \text{ (to 3 s.f.)}, -0.699 \text{ (to 3 s.f.)} \end{aligned}$$

$\therefore x = 1.07$ or $x = -0.699$

(d) $4x^2 - 7 = 2x$

$4x^2 - 2x - 7 = 0$

Comparing $4x^2 - 2x - 7 = 0$ with $ax^2 + bx + c = 0$, we have $a = 4$, $b = -2$ and $c = -7$.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-7)}}{2(4)}$$

$$= \frac{2 \pm \sqrt{116}}{8}$$

$= 1.60$ (to 3 s.f.), -1.10 (to 3 s.f.)

$\therefore x = 1.60$ or $x = -1.10$

(e) $9 - 5x^2 = -3x$

$5x^2 - 3x - 9 = 0$

Comparing $5x^2 - 3x - 9 = 0$ with $ax^2 + bx + c = 0$, we have $a = 5$, $b = -3$ and $c = -9$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-9)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{189}}{10}$$

$= 1.67$ (to 3 s.f.), -1.07 (to 3 s.f.)

$\therefore x = 1.67$ or $x = -1.07$

(f) $16x - 61 = x^2$

$-x^2 + 16x - 61 = 0$

Comparing $-x^2 + 16x - 61 = 0$ with $ax^2 + bx + c = 0$, we have $a = -1$, $b = 16$ and $c = -61$.

$$x = \frac{-16 \pm \sqrt{16^2 - 4(-1)(-61)}}{2(-1)}$$

$$= \frac{-16 \pm \sqrt{12}}{-2}$$

$= 6.27$ (to 3 s.f.), 9.73 (to 3 s.f.)

$\therefore x = 6.27$ or $x = 9.73$

3. **(a)** $x(x + 1) = 1$

$x^2 + x = 1$

$x^2 + x - 1 = 0$

Comparing $x^2 + x - 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 1$ and $c = -1$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$= 0.618$ (to 3 s.f.), -1.62 (to 3 s.f.)

$\therefore x = 0.618$ or $x = -1.62$

(b) $3(x + 1)(x - 1) = 7x$

$3(x^2 - 1) = 7x$

$3x^2 - 3 = 7x$

$3x^2 - 7x - 3 = 0$

Comparing $3x^2 - 7x - 3 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = -7$ and $c = -3$.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-3)}}{2(3)}$$

$$= \frac{7 \pm \sqrt{85}}{6}$$

$= 2.70$ (to 3 s.f.), -0.370 (to 3 s.f.)

$\therefore x = 2.70$ or $x = -0.370$

(c) $(x - 1)^2 - 2x = 0$

$x^2 - 2x + 1 - 2x = 0$

$x^2 - 4x + 1 = 0$

Comparing $x^2 - 4x + 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -4$ and $c = 1$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$= 3.73$ (to 3 s.f.), 0.268 (to 3 s.f.)

$\therefore x = 3.73$ or $x = 0.268$

(d) $x(x - 5) = 7 - 2x$

$x^2 - 5x = 7 - 2x$

$x^2 - 3x - 7 = 0$

Comparing $x^2 - 3x - 7 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -3$ and $c = -7$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{37}}{2}$$

$= 4.54$ (to 3 s.f.), -1.54 (to 3 s.f.)

$\therefore x = 4.54$ or $x = -1.54$

(e) $(2x + 3)(x - 1) - x(x + 2) = 0$

$2x^2 - 2x + 3x - 3 - x^2 - 2x = 0$

$x^2 - x - 3 = 0$

Comparing $x^2 - x - 3 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -1$ and $c = -3$.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{13}}{2}$$

$= 2.30$ (to 3 s.f.), -1.30 (to 3 s.f.)

$\therefore x = 2.30$ or $x = -1.30$

(f) $(4x - 3)^2 + (4x + 3)^2 = 25$

$16x^2 - 24x + 9 + 16x^2 + 24x + 9 = 25$

$32x^2 + 18 = 25$

$32x^2 - 7 = 0$

Comparing $32x^2 - 7 = 0$ with $ax^2 + bx + c = 0$, we have $a = 32$, $b = 0$ and $c = -7$.

$$x = \frac{0 \pm \sqrt{0^2 - 4(32)(-7)}}{2(32)}$$

$$= \frac{0 \pm \sqrt{896}}{64}$$

$= 0.468$ (to 3 s.f.), -0.468 (to 3 s.f.)

$\therefore x = 0.468$ or $x = -0.468$

4. (a) $0.5(x^2 + 1) = x$

$0.5x^2 + 0.5 = x$

$0.5x^2 - x + 0.5 = 0$

Comparing $0.5x^2 - x + 0.5 = 0$ with $ax^2 + bx + c = 0$, we have $a = 0.5$, $b = -1$ and $c = 0.5$.

$b^2 - 4ac = (-1)^2 - 4(0.5)(0.5)$

$= 0$

Since $b^2 - 4ac = 0$, the equation has one real solution.

$$x = \frac{-(-1) \pm \sqrt{0}}{2(0.5)}$$

$$= \frac{1}{1}$$

$$= 1$$

$\therefore x = 1$

(b) Comparing $\frac{3}{4}x^2 + 2x - \frac{1}{2} = 0$ with $ax^2 + bx + c = 0$, we have $a = \frac{3}{4}$, $b = 2$ and $c = -\frac{1}{2}$.

$b^2 - 4ac = 2^2 - 4\left(\frac{3}{4}\right)\left(-\frac{1}{2}\right)$

$= 5\frac{1}{2}$

Since $b^2 - 4ac > 0$, the equation has two real solutions.

$$x = \frac{-2 \pm \sqrt{5\frac{1}{2}}}{2\left(\frac{3}{4}\right)}$$

$$= \frac{-2 \pm \sqrt{5\frac{1}{2}}}{\frac{3}{2}}$$

$= 0.230$ (to 3 s.f.), -2.90 (to 3 s.f.)

$\therefore x = 0.230$ or $x = -2.90$

(c) $5x - 7 = x^2$

$x^2 - 5x + 7 = 0$

Comparing $x^2 - 5x + 7 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -5$ and $c = 7$.

$b^2 - 4ac = (-5)^2 - 4(1)(7)$

$= -3$

Since $b^2 - 4ac < 0$, the equation has no real solutions.

(d) $3x - 4 = (4x - 3)^2$

$3x - 4 = 16x^2 - 24x + 9$

$16x^2 - 27x + 13 = 0$

Comparing $16x^2 - 27x + 13 = 0$ with $ax^2 + bx + c = 0$, we have $a = 16$, $b = -27$ and $c = 13$.

$b^2 - 4ac = (-27)^2 - 4(16)(13)$

$= -103$

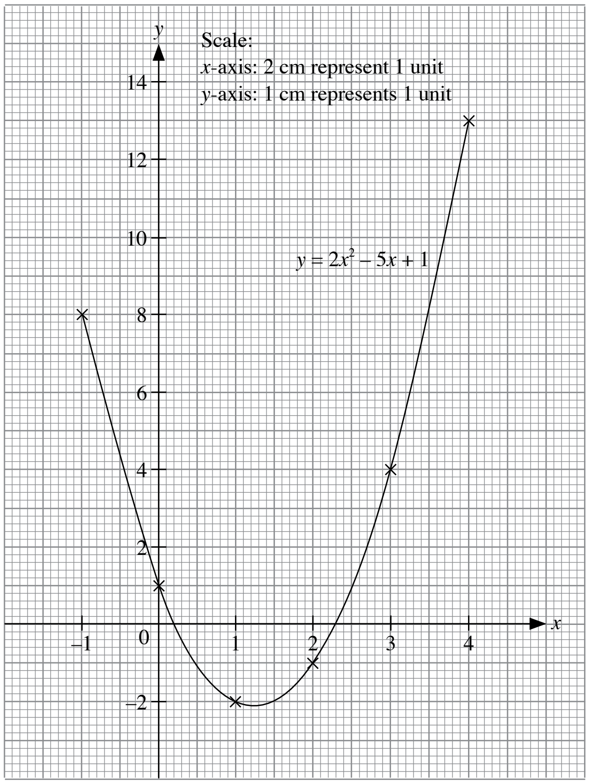
Since $b^2 - 4ac < 0$, the equation has no real solutions.

Exercise 1C

1. (i) $y = 2x^2 - 5x + 1$

x	–1	0	1	2	3	4
y	8	1	–2	–1	4	13

(ii)



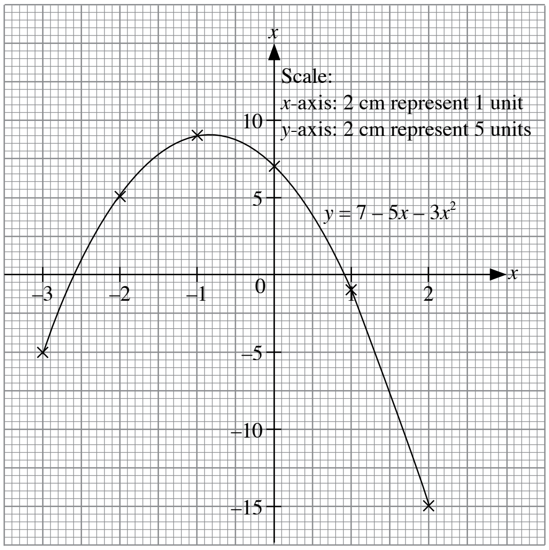
(iii) From the graph, the x-coordinates of the points of intersection of $y = 2x^2 - 5x + 1$ and the x-axis are $x = 2.3$ and $x = 0.20$.

$\therefore$ The solutions of the equation $2x^2 - 5x + 1 = 0$ are $x = 2.3$ and $x = 0.20$.

2. (i) $y = 7 - 5x - 3x^2$

x	–3	–2	–1	0	1	2
y	–5	5	9	7	–1	–15

(ii)

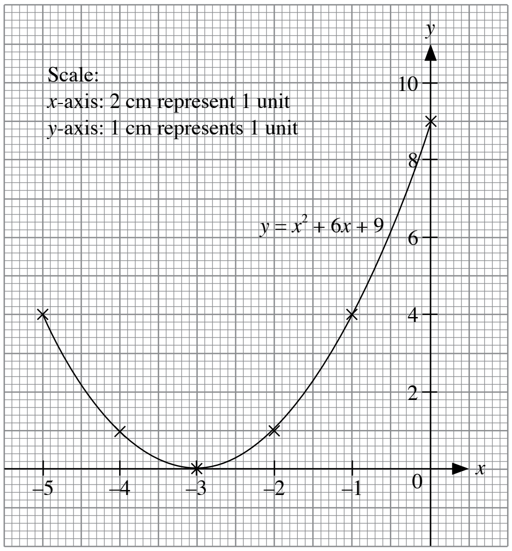


(iii) From the graph, the x-coordinates of the points of intersection of $y = 7 - 5x - 3x^2$ and the x-axis are $x = -2.55$ and $x = 0.90$.
$\therefore$ The solutions of the equation $7 - 5x - 3x^2 = 0$ are $x = -2.55$ and $x = 0.90$.

3. **(i)** $y = x^2 + 6x + 9$

x	–5	–4	–3	–2	–1	0
y	4	1	0	1	4	9

(ii)

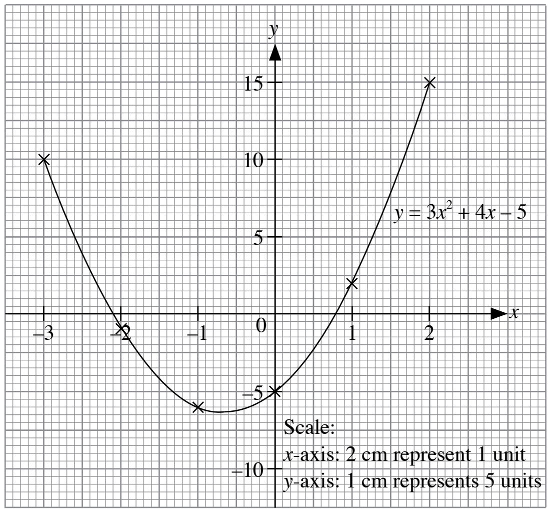


(iii) From the graph, the x-coordinate of the point of intersection of $y = x^2 + 6x + 9$ and the x-axis is $x = -3$.
$\therefore$ The solution of the equation $x^2 + 6x + 9 = 0$ is $x = -3$.

4. **(i)** $y = 3x^2 + 4x - 5$

x	–3	–2	–1	0	1	2
y	10	–1	–6	–5	2	15

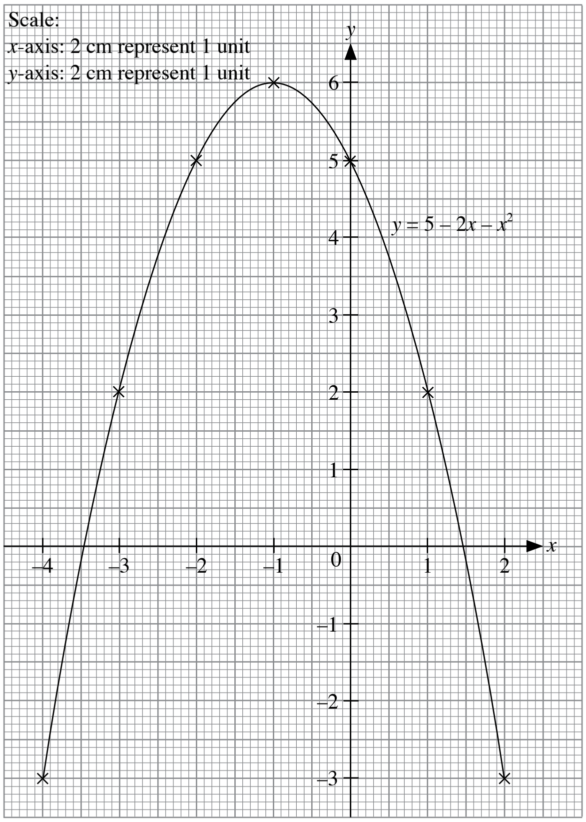
(ii)



(iii) From the graph, the x-coordinates of the points of intersection of $y = 3x^2 + 6x - 5$ and the x-axis are $x = -2.1$ and $x = 0.8$.
$\therefore$ The solutions of the equation $3x^2 + 4x - 5 = 0$ are $x = -2.1$ and $x = 0.8$.

5. $y = 5 - 2x - x^2$

x	–4	–3	–2	–1	0	1	2
y	–3	2	5	6	5	2	–3



From the graph, the x-coordinates of the points of intersection of $y = 5 - 2x - x^2$ and the x-axis are $x = -3.45$ and $x = 1.45$.
$\therefore$ The solutions of the equation $5 - 2x - x^2 = 0$ are $x = -3.45$ and $x = 1.45$.

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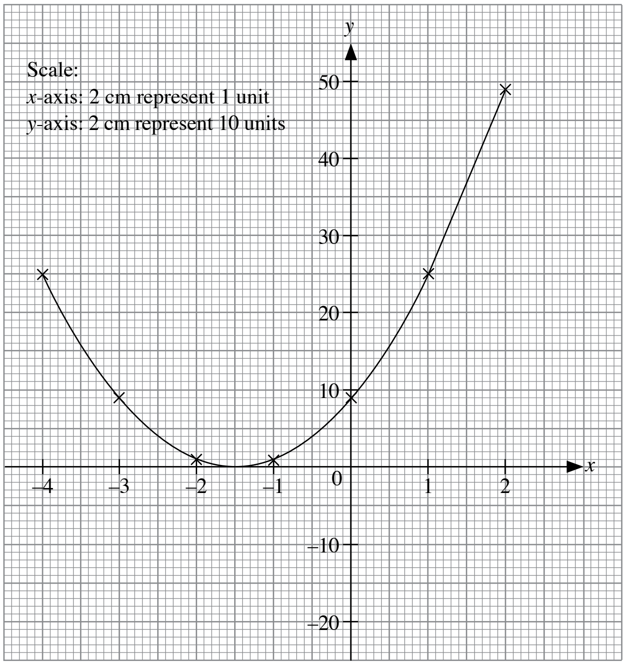
6. **(i)** $y = 4x^2 + 12x + 9$

x	–4	–3	–2	–1	0	1	2
y	25	9	1	1	9	25	49

(ii)

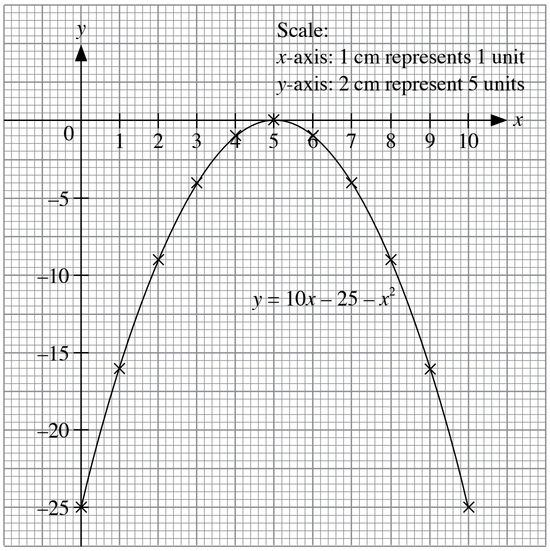


From the graph, the x-coordinate of the point of intersection of $y = 4x^2 + 12x + 9$ and the x-axis is $x = -1.5$.

$\therefore$ The solution of the equation $4x^2 + 12x + 9 = 0$ is $x = -1.5$.

7. $y = 10x - 25 - x^2$

x	0	1	2	3	4	5	6	7	8	9	10
y	–25	–16	–9	–4	–1	0	–1	–4	–9	–16	–25

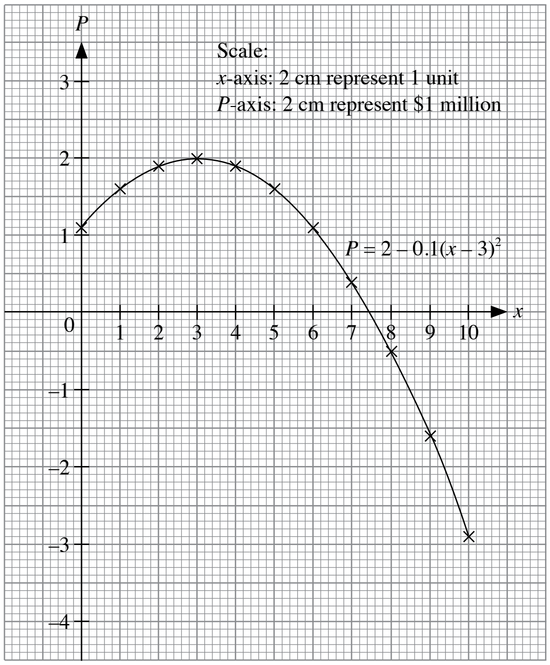


From the graph, the x-coordinate of the point of intersection of $y = 10x - 25 - x^2$ and the x-axis is $x = 5$.

$\therefore$ The solution of the equation $10x - 25 - x^2 = 0$ is $x = 5$.

8. **(a)** $P = 2 - 0.1(x - 3)^2$

x	0	1	2	3	4	5	6	7	8	9	10
P	1.1	1.6	1.9	2.0	1.9	1.6	1.1	0.4	–0.5	–1.6	–2.9

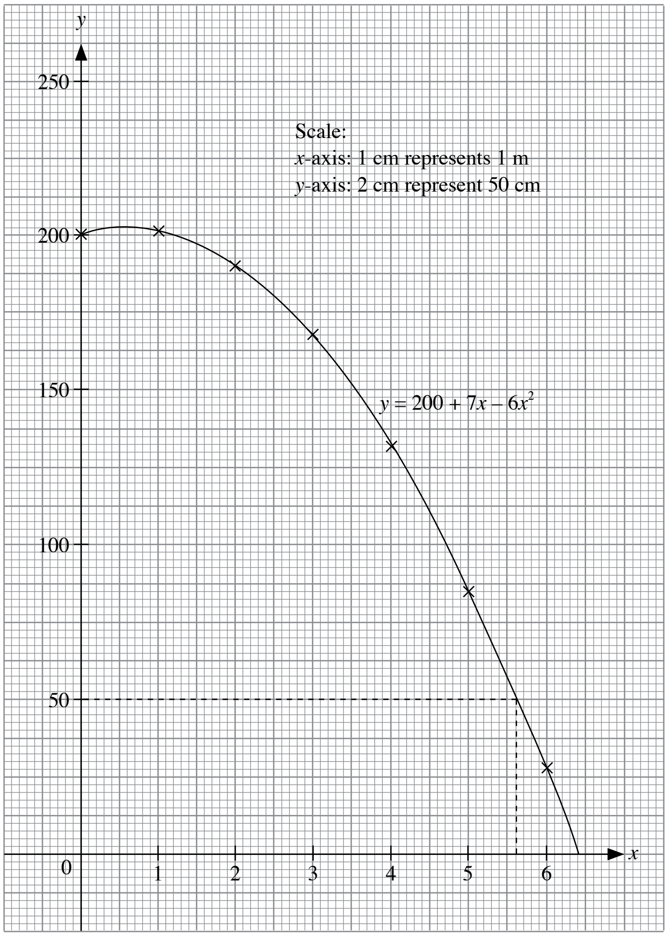


(b) When the profit of the company is zero, $P = 0$.

From the graph, the x-coordinate of the point of intersection of $P = 2 - 0.1(x - 3)^2$ and the x-axis is $x = 7.5$.

$\therefore$ When the profit of the company is zero, $x = 7.5$.

9. **(a)** $y = 200 + 7x - 6x^2$



(b) **(i)** From the graph, the x-coordinate of the point of intersection of $y = 200 + 7x - 6x^2$ and the x-axis is $x = 6.4$.

$\therefore$ The solution of the equation $200 + 7x - 6x^2 = 0$ is $x = 6.4$.

(ii) From the graph, when the balloon is 50 cm above the ground, $y = 50$, $x = 5.6$.

$\therefore$ The horizontal distance from the foot of the platform when the balloon is 50 cm above the ground is 5.6 m.

(c) $t = 6.4$. After $t = 6.4$, the flight of the ball will be below ground level which is not valid in this case.

Exercise 1D

1. **(a)** $\frac{8}{x} = 2x + 1$

$8 = x(2x + 1)$

$8 = 2x^2 + x$

$0 = 2x^2 + x - 8$

$2x^2 + x - 8 = 0$

Comparing $2x^2 + x - 8 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = 1$ and $c = -8$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-8)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{65}}{4}$$

$= 1.77$ (to 3 s.f.), -2.27 (to 3 s.f.)

$\therefore x = 1.77$ or $x = -2.27$

(b) $3x - 1 = \frac{7}{x + 4}$

$(3x - 1) \times (x + 4) = \frac{7}{x + 4} \times (x + 4)$

$(3x - 1)(x + 4) = 7$

$3x^2 + 12x - x - 4 = 7$

$3x^2 + 11x - 11 = 0$

Comparing $3x^2 + 11x - 11 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = 11$ and $c = -11$.

$$x = \frac{-11 \pm \sqrt{11^2 - 4(3)(-11)}}{2(3)}$$

$$= \frac{-11 \pm \sqrt{253}}{6}$$

$= 0.818$ (to 3 s.f.), -4.48 (to 3 s.f.)

$\therefore x = 0.818$ or $x = -4.48$

(c) $\frac{x + 1}{5 - x} = x$

$\frac{x + 1}{5 - x} \times (5 - x) = x \times (5 - x)$

$x + 1 = 5x - x^2$

$0 = -x^2 + 4x - 1$

$-x^2 + 4x - 1 = 0$

Comparing $-x^2 + 4x - 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = -1$, $b = 4$ and $c = -1$.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-1)}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{12}}{-2}$$

$= 0.268$ (to 3 s.f.), 3.73 (to 3 s.f.)

$\therefore x = 0.268$ or $x = 3.73$

(d) $x + \frac{7}{x} = 9$

$\left(x + \frac{7}{x}\right) \times x = 9 \times x$

$x^2 + 7 = 9x$

$x^2 - 9x + 7 = 0$

Comparing $x^2 - 9x + 7 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -9$ and $c = 7$.

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{53}}{2}$$

$= 8.14$ (to 3 s.f.), 0.860 (to 3 s.f.)

$\therefore x = 8.14$ or $x = 0.860$

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(e) $$2x + 1 = \frac{x+1}{x-5}$$

$$(2x + 1) \times (x - 5) = \frac{x+1}{x-5} \times (x - 5)$$
$$(2x + 1)(x - 5) = x + 1$$
$$2x^2 - 10x + x - 5 = x + 1$$
$$2x^2 - 9x - 5 = x + 1$$
$$2x^2 - 10x - 6 = 0$$
$$x^2 - 5x - 3 = 0$$

Comparing $x^2 - 5x - 3 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -5$ and $c = -3$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{37}}{2}$$
$= 5.54$ (to 3 s.f.), -0.541 (to 3 s.f.)

$\therefore x = 5.54$ or $x = -0.541$

(f) $$\frac{5x}{x+4} = 3x + 1$$

$$\frac{5x}{x+4} \times (x + 4) = (3x + 1) \times (x + 4)$$
$$5x = (3x + 1)(x + 4)$$
$$5x = 3x^2 + 12x + x + 4$$
$$5x = 3x^2 + 13x + 4$$
$$0 = 3x^2 + 8x + 4$$
$$3x^2 + 8x + 4 = 0$$
$$(3x + 2)(x + 2) = 0$$

$3x + 2 = 0$ or $x + 2 = 0$

$3x = -2$ $\quad$ $x = -2$

$x = -\frac{2}{3}$

$\therefore x = -\frac{2}{3}$ or $x = -2$

2. (i) $$\frac{12}{x} - \frac{12}{x+1} = 1$$

$$\frac{12}{x} \times x(x + 1) - \frac{12}{x+1} \times x(x + 1) = 1 \times x(x + 1)$$
$$12(x + 1) - 12x = x(x + 1)$$
$$12x + 12 - 12x = x^2 + x$$
$$12 = x^2 + x$$
$$x^2 + x - 12 = 0 \text{ (shown)}$$

(ii) $$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$

$x + 4 = 0$ or $x - 3 = 0$

$x = -4$ $\quad$ $x = 3$

$\therefore x = -4$ or $x = 3$

(iii) Since the numbers are positive, $x = 3$.

$$\frac{12}{3+1} = 3$$
$$\frac{12}{3} = 4$$

$\therefore$ The two numbers are 3 and 4.

3. (a) $$\frac{2}{x+1} = \frac{5x}{3-x}$$

$$\frac{2}{x+1} \times (x + 1)(3 - x) = \frac{5x}{3-x} \times (x + 1)(3 - x)$$
$$2(3 - x) = 5x(x + 1)$$
$$6 - 2x = 5x^2 + 5x$$
$$0 = 5x^2 + 7x - 6$$
$$(5x - 3)(x + 2) = 0$$

$5x - 3 = 0$ or $x + 2 = 0$

$5x = 3$ $\quad$ $x = -2$

$x = \frac{3}{5}$

$\therefore x = \frac{3}{5}$ or $x = -2$

(b) $$\frac{(x-2)(x-3)}{(x-1)(x+2)} = \frac{2}{3}$$

$$\frac{(x-2)(x-3)}{(x-1)(x+2)} \times (x - 1)(x + 2) = \frac{2}{3} \times (x - 1)(x + 2)$$
$$(x - 2)(x - 3) = \frac{2}{3}(x - 1)(x + 2)$$
$$x^2 - 3x - 2x + 6 = \frac{2}{3}(x^2 + 2x - x - 2)$$
$$x^2 - 5x + 6 = \frac{2}{3}(x^2 + x - 2)$$
$$x^2 - 5x + 6 = \frac{2}{3}x^2 + \frac{2}{3}x - \frac{4}{3}$$
$$\frac{1}{3}x^2 - \frac{17}{3}x + \frac{22}{3} = 0$$
$$x^2 - 17x + 22 = 0$$

Comparing $x^2 - 17x + 22 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -17$ and $c = 22$.

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(1)(22)}}{2(1)}$$
$$= \frac{17 \pm \sqrt{201}}{2}$$
$= 15.6$ (to 3 s.f.), 1.41 (to 3 s.f.)

$\therefore x = 15.6$ or $x = 1.41$

4.
$$\frac{x(x-3)}{(x+1)^2} = \frac{3}{5}$$
$$\frac{x(x-3)}{(x+1)^2} \times (x+1)^2 = \frac{3}{5} \times (x+1)^2$$
$$x(x-3) = \frac{3}{5}(x+1)^2$$
$$x^2 - 3x = \frac{3}{5}(x^2 + 2x + 1)$$
$$x^2 - 3x = \frac{3}{5}x^2 + \frac{6}{5}x + \frac{3}{5}$$
$$\frac{2}{5}x^2 - \frac{21}{5}x - \frac{3}{5} = 0$$
$$2x^2 - 21x - 3 = 0$$

Comparing $2x^2 - 21x - 3 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -21$ and $c = -3$.

$$x = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(2)(-3)}}{2(2)}$$
$$= \frac{21 \pm \sqrt{465}}{4}$$

$= 10.6$ (to 3 s.f.), -0.141 (to 3 s.f.)

$\therefore x = 10.6$ or $x = -0.141$

5. (a)
$$\frac{x}{2} = \frac{4}{x} - 1$$
$$\frac{x}{2} \times x = \left(\frac{4}{x} - 1\right) \times x$$
$$\frac{x^2}{2} = x\left(\frac{4}{x} - 1\right)$$
$$\frac{x^2}{2} = 4 - x$$
$$\frac{x^2}{2} + x - 4 = 0$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2$$

$\therefore x = -4$ or $x = 2$

(b)
$$\frac{2}{x+5} = 1 - \frac{x+1}{5}$$
$$\frac{2}{x+5} + \frac{x+1}{5} = 1$$
$$\left[\frac{2}{x+5} + \frac{x+1}{5}\right] \times 5(x+5) = 1 \times 5(x+5)$$
$$\frac{2}{x+5} \times 5(x+5) + \frac{x+1}{5} \times 5(x+5) = 1 \times 5(x+5)$$
$$10 + (x+1)(x+5) = 5(x+5)$$
$$10 + x^2 + 5x + x + 5 = 5x + 25$$
$$x^2 + 6x + 15 = 5x + 25$$
$$x^2 + x - 10 = 0$$

Comparing $x^2 + x - 10 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 1$ and $c = -10$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-10)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{41}}{2}$$

$= 2.70$ (to 3 s.f.), -3.70 (to 3 s.f.)

$\therefore x = 2.70$ or $x = -3.70$

(c)
$$\frac{x-2}{5} + \frac{1}{2x-3} = 1$$
$$\left[\frac{x-2}{5} + \frac{1}{2x-3}\right] \times 5(2x-3) = 1 \times 5(2x-3)$$
$$\frac{x-2}{5} \times 5(2x-3) + \frac{1}{2x-3} \times 5(2x-3) = 1 \times 5(2x-3)$$
$$(x-2)(2x-3) + 5 = 5(2x-3)$$
$$2x^2 - 3x - 4x + 6 + 5 = 10x - 15$$
$$2x^2 - 7x + 11 = 10x - 15$$
$$2x^2 - 17x + 26 = 0$$
$$(2x-13)(x-2) = 0$$
$$2x - 13 = 0 \quad \text{or} \quad x - 2 = 0$$
$$2x = 13 \qquad\qquad x = 2$$
$$x = 6.5$$

$\therefore x = 6.5$ or $x = 2$

(d)
$$\frac{3}{x} + \frac{2}{x+1} = 5$$
$$\left[\frac{3}{x} + \frac{2}{x+1}\right] \times x(x+1) = 5 \times x(x+1)$$
$$\frac{3}{x} \times x(x+1) + \frac{2}{x+1} \times x(x+1) = 5x(x+1)$$
$$3(x+1) + 2x = 5x(x+1)$$
$$3x + 3 + 2x = 5x^2 + 5x$$
$$5x + 3 = 5x^2 + 5x$$
$$5x^2 - 3 = 0$$
$$5x^2 = 3$$
$$x^2 = \frac{3}{5}$$
$$x = \pm\sqrt{\frac{3}{5}}$$

$= 0.775$ (to 3 s.f.), -0.775 (to 3 s.f.)

$\therefore x = 0.775$ or $x = -0.775$

(e)
$$\frac{1}{x+2} + \frac{1}{x-2} = \frac{3}{8}$$
$$\left[\frac{1}{x+2}+\frac{1}{x-2}\right]\times(x+2)(x-2) = \frac{3}{8}\times(x+2)(x-2)$$
$$\frac{1}{x+2}\times(x+2)(x-2) + \frac{1}{x-2}\times(x+2)(x-2) = \frac{3}{8}(x+2)(x-2)$$
$$(x-2)+(x+2) = \frac{3}{8}(x^2-4)$$
$$2x = \frac{3}{8}x^2 - \frac{3}{2}$$
$$0 = \frac{3}{8}x^2 - 2x - \frac{3}{2}$$
$$\frac{3}{8}x^2 - 2x - \frac{3}{2} = 0$$
$$3x^2 - 16x - 12 = 0$$
Comparing $3x^2 - 16x - 12 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = -16$ and $c = -12$.
$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(-12)}}{2(3)}$$
$$= \frac{16 \pm \sqrt{400}}{6}$$
$$= 6, -\frac{2}{3}$$
$\therefore x = 6$ or $x = -\frac{2}{3}$

(f)
$$\frac{7}{x-1} - \frac{x+1}{x+3} = \frac{1}{2}$$
$$\left[\frac{7}{x-1}-\frac{x+1}{x+3}\right]\times(x-1)(x+3) = \frac{1}{2}\times(x-1)(x+3)$$
$$\frac{7}{x-1}\times(x-1)(x+3) - \frac{x+1}{x+3}\times(x-1)(x+3) = \frac{1}{2}(x-1)(x+3)$$
$$7(x+3) - (x+1)(x-1) = \frac{1}{2}(x-1)(x+3)$$
$$7x + 21 - (x^2-1) = \frac{1}{2}(x^2+3x-x-3)$$
$$7x + 21 - x^2 + 1 = \frac{1}{2}(x^2+2x-3)$$
$$7x + 22 - x^2 = \frac{1}{2}x^2 + x - \frac{3}{2}$$
$$0 = \frac{3}{2}x^2 - 6x - \frac{47}{2}$$
$$\frac{3}{2}x^2 - 6x - \frac{47}{2} = 0$$
$$3x^2 - 12x - 47 = 0$$
Comparing $3x^2 - 12x - 47 = 0$ with $ax^2 + bx + c = 0$, we have $a = 3$, $b = -12$ and $c = -47$.
$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-47)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{708}}{6}$$
$= 6.43$ (to 3 s.f.), -2.43 (to 3 s.f.)

$\therefore x = 6.43$ or $x = -2.43$

(g)
$$\frac{5}{x-2} = 2 - \frac{4}{(x-2)^2}$$
$$\frac{5}{x-2} + \frac{4}{(x-2)^2} = 2$$
$$\left[\frac{5}{x-2}+\frac{4}{(x-2)^2}\right]\times(x-2)^2 = 2\times(x-2)^2$$
$$\frac{5}{x-2}\times(x-2)^2 + \frac{4}{(x-2)^2}\times(x-2)^2 = 2(x-2)^2$$
$$5(x-2) + 4 = 2(x-2)^2$$
$$5x - 10 + 4 = 2(x^2 - 4x + 4)$$
$$5x - 6 = 2x^2 - 8x + 8$$
$$0 = 2x^2 - 13x + 14$$
$$2x^2 - 13x + 14 = 0$$
Comparing $2x^2 - 13x + 14 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -13$ and $c = 14$.
$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(14)}}{2(2)}$$
$$= \frac{13 \pm \sqrt{57}}{4}$$
$= 5.14$ (to 3 s.f.), 1.36 (to 3 s.f.)

$\therefore x = 5.14$ or $x = 1.36$

(h)
$$\frac{5}{x-1} + \frac{x}{(x-1)^2} = 1$$
$$\left[\frac{5}{x-1}+\frac{x}{(x-1)^2}\right]\times(x-1)^2 = 1\times(x-1)^2$$
$$\frac{5}{x-1}\times(x-1)^2 + \frac{x}{(x-1)^2}\times(x-1)^2 = (x-1)^2$$
$$5(x-1) + x = (x-1)^2$$
$$5x - 5 + x = x^2 - 2x + 1$$
$$6x - 5 = x^2 - 2x + 1$$
$$0 = x^2 - 8x + 6$$
$$x^2 - 8x + 6 = 0$$
Comparing $x^2 - 8x + 6 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -8$ and $c = 6$.
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(6)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{40}}{2}$$
$= 7.16$ (to 3 s.f.), 0.838 (to 3 s.f.)

$\therefore x = 7.16$ or $x = 0.838$

6. (i) Length of rectangle $= \frac{112 - 2x}{2}$

$= (56 - x)$ cm

(ii) Area of rectangle = 597 cm^2
$$x(56 - x) = 597$$
$$56x - x^2 = 597$$
$$56x - x^2 - 597 = 0$$
$$x^2 - 56x + 597 = 0 \text{ (shown)}$$

(iii) $x^2 - 56x + 597 = 0$

Comparing $x^2 - 56x + 597 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -56$ and $c = 597$.

$$x = \frac{-(-56) \pm \sqrt{(-56)^2 - 4(1)(597)}}{2(1)}$$

$$= \frac{56 \pm \sqrt{748}}{2}$$

$= 41.67$ (to 2 d.p.), 14.33 (to 2 d.p.)

$\therefore x = 41.67$ or $x = 14.33$

(iv) Since the length of a rectangle usually refers to the longer side,

When $x = 14.33$,

Breadth $= 14.33$ cm

Length $= 56 - 14.33 = 41.67$ cm

By Pythagoras' Theorem,

$$(\text{Length of diagonal})^2 = \text{Length}^2 + \text{Breadth}^2 = 41.67^2 + 14.33^2$$

$$\text{Length of the diagonal} = \sqrt{41.67^2 + 14.33^2} = 44.1 \text{ cm (to 3 s.f.)}$$

7. **(i)** Given $\dfrac{AP}{AB} = \dfrac{PQ}{BC}$,

$$\frac{6x}{3x+5} = \frac{x}{1}$$

$$\frac{6x}{3x+5} = x$$

$$\frac{6x}{3x+5} \times (3x+5) = x \times (3x+5)$$

$$6x = x(3x+5)$$

$$6x = 3x^2 + 5x$$

$$0 = 3x^2 - x$$

$3x^2 - x = 0$ (shown)

(ii)
$$3x^2 - x = 0$$

$$x(3x - 1) = 0$$

$x = 0$ or $3x - 1 = 0$

$3x = 1$

$x = \dfrac{1}{3}$

$\therefore x = 0$ or $x = \dfrac{1}{3}$

(iii) Since $x > 0$, $x = \dfrac{1}{3}$

$PB = AB - AP$

$$= \left[\left(3 \times \frac{1}{3}\right) + 5\right] - 6\left(\frac{1}{3}\right)$$

$= 6 - 2$

$= 4$ cm

8. Let the length of the smaller square be x.

Area of smaller square $= x^2$

Area of larger square $= 9x^2$

Length of square $= \pm\sqrt{9x^2}$

$= 3x$ or $-3x$ (reject, $x > 0$)

Total perimeter of both squares $= 4x + 4(3x)$

$= 200$ cm

$16x = 200$

$\therefore x = 12.5$

Perimeter of larger square $= 12(12.5)$

$= 150$ cm

9. Let the length of one square be x and the length of the other square be y.

Since total length of the wire is 100 cm,

$4x + 4y = 100$

$4y = 100 - 4x$

$y = 25 - x$ — (1)

Since total area of the squares is 425 cm^2,

$x^2 + y^2 = 425$ — (2)

Subst. (1) into (2):

$$x^2 + (25 - x)^2 = 425$$

$$x^2 + (625 - 50x + x^2) = 425$$

$$2x^2 - 50x + 200 = 0$$

$$x^2 - 25x + 100 = 0$$

$$(x - 20)(x - 5) = 0$$

$x - 20 = 0$ or $x - 5 = 0$

$x = 20$ $\qquad$ $x = 5$

From (1), when $x = 20$, $y = 5$;

when $x = 5$, $y = 20$.

$\therefore$ The lengths of the sides of the two squares are 5 cm and 20 cm.

10. Let the original number of students be x.

Amount each student pays at first $= \$\,\dfrac{120}{x}$

Amount each student pays if 2 join $= \$\left(\dfrac{120}{x} - 2\right) = \$\left(\dfrac{120}{x+2}\right)$

$$\frac{120}{x} - 2 = \frac{120}{x+2}$$

$$\frac{120 - 2x}{x} = \frac{120}{x+2}$$

$$(120 - 2x)(x + 2) = 120x$$

$$120x + 240 - 2x^2 - 4x = 120x$$

$$2x^2 + 4x - 240 = 0$$

$$x^2 + 2x - 120 = 0$$

$$(x - 10)(x + 12) = 0$$

$x - 10 = 0$ or $x + 12 = 0$

$x = 10$ $\qquad$ $x = -12$

(rejected, since $x > 0$)

$\therefore$ There were originally 10 students in the group.

11. Let the number of members in 2016 be x.

Subscription fee of each member in 2016 = $\dfrac{4200}{x}$ cents

Subscription fee of each member in 2017 = $\left(\dfrac{4200}{x} - 10\right)$ cents

$= \dfrac{4500}{x+20}$ cents

$$\begin{aligned}
\frac{4200}{x} - 10 &= \frac{4500}{x+20} \\
\frac{4200-10x}{x} &= \frac{4500}{x+20} \\
(4200-10x)(x+20) &= 4500x \\
4200x + 84\,000 - 10x^2 - 200x &= 4500x \\
10x^2 + 500x - 84\,000 &= 0 \\
x^2 + 50x - 8400 &= 0 \\
(x-70)(x+120) &= 0
\end{aligned}$$

$x - 70 = 0$ or $x + 120$

$x = 70$ $\quad x = -120$ (rejected, since $x > 0$)

$\therefore$ There were 70 members in 2016.

12. (i) No. of pages printed by Printer A in 1 minute = $\dfrac{60}{x}$

(ii) No. of pages printed by Printer B in 1 minute = $\dfrac{60}{x+2}$

(iii)
$$\begin{aligned}
\frac{60}{x} + \frac{60}{x+2} &= 144 \\
\frac{60(x+2)}{x(x+2)} + \frac{60x}{x(x+2)} &= 144 \\
\frac{60(x+2)+60x}{x(x+2)} &= 144 \\
60(x+2) + 60x &= 144x(x+2) \\
60x + 120 + 60x &= 144x^2 + 288x \\
144x^2 + 168x - 120 &= 0 \\
6x^2 + 7x - 5 &= 0 \text{ (Shown)}
\end{aligned}$$

(iv)
$$\begin{aligned}
6x^2 + 7x - 5 &= 0 \\
(2x-1)(3x+5) &= 0
\end{aligned}$$

$2x - 1 = 0$ or $3x + 5 = 0$

$2x = 1$ $\quad 3x = -5$

$x = \dfrac{1}{2}$ $\quad x = -1\dfrac{2}{3}$

$\therefore x = \dfrac{1}{2}$ or $x = -1\dfrac{2}{3}$

(v) Since $x > 0$, $x = \dfrac{1}{2}$

No. of pages printed by Printer B in 1 minute = $\dfrac{60}{x+2}$

$= \dfrac{60}{\left(\frac{1}{2}+2\right)}$

$= \dfrac{60}{2\frac{1}{2}}$

$= 24$

Time taken by Printer B to print 144 pages = $\dfrac{144}{24}$

$= 6$ minutes

13. (i) Av. amount of rice ordered in Jan 2009 = $\dfrac{350}{x}$ kg

(ii) Av. amount of rice ordered in Jan 2012 = $\dfrac{350}{x+0.15}$ kg

(iii)
$$\begin{aligned}
\frac{350}{x} - \frac{350}{x+0.15} &= 30 \\
\frac{350(x+0.15)}{x(x+0.15)} - \frac{350x}{x(x+0.15)} &= 30 \\
\frac{350(x+0.15)-350x}{x(x+0.15)} &= 30 \\
350(x+0.15) - 350x &= 30x(x+0.15) \\
350x + 52.5 - 350x &= 30x^2 + 4.5x \\
30x^2 + 4.5x - 52.5 &= 0 \\
60x^2 + 9x - 105 &= 0 \\
20x^2 + 3x - 35 &= 0 \text{ (Shown)}
\end{aligned}$$

(iv)
$$\begin{aligned}
20x^2 + 3x - 35 &= 0 \\
(4x-5)(5x+7) &= 0
\end{aligned}$$

$4x - 5 = 0$ or $5x + 7 = 0$

$4x = 5$ $\quad 5x = -7$

$x = 1.25$ $\quad x = -1\dfrac{2}{5}$ (rejected since $x > 0$)

$\therefore$ Price per kilogram of rice in Jan 2012 = 1.25 + 0.15

= \$1.40

14. (i) Time taken by Rui Feng = $\left(\dfrac{2}{x} + \dfrac{8}{x+1}\right)$ h

(ii) Time taken by Jun Wei = $\dfrac{10}{x}$ h

$$\begin{aligned}
\frac{10}{x} - \left(\frac{2}{x} + \frac{8}{x+1}\right) &= \frac{40}{60} \\
\frac{10}{x} - \frac{2}{x} - \frac{8}{x+1} &= \frac{2}{3} \\
\frac{8}{x} - \frac{8}{x+1} &= \frac{2}{3} \\
\left[\frac{8}{x} - \frac{8}{x+1}\right] \times x(x+1) &= \frac{2}{3} \times x(x+1) \\
\frac{8}{x} \times x(x+1) - \frac{8}{x+1} \times x(x+1) &= \frac{2}{3}x(x+1) \\
8(x+1) - 8x &= \frac{2}{3}x^2 + \frac{2}{3}x \\
8x + 8 - 8x &= \frac{2}{3}x^2 + \frac{2}{3}x \\
8 &= \frac{2}{3}x^2 + \frac{2}{3}x \\
0 &= \frac{2}{3}x^2 + \frac{2}{3}x - 8 \\
\frac{2}{3}x^2 + \frac{2}{3}x - 8 &= 0 \\
2x^2 + 2x - 24 &= 0 \\
x^2 + x - 12 &= 0 \text{ (Shown)}
\end{aligned}$$

(iii)

$$\begin{aligned} x^2 + x - 12 &= 0 \\ (x+4)(x-3) &= 0 \\ x + 4 = 0 \quad &\text{or} \quad x - 3 = 0 \\ x = -4 \quad & \quad\quad x = 3 \end{aligned}$$

$\therefore$ $x = -4$ or $x = 3$

$\therefore$ Since the running speed cannot be a negative value, $x = -4$ is rejected.

(iv) Time taken by Rui Feng $= \left(\frac{2}{3} + \frac{8}{3+1}\right)$

$$\begin{aligned} &= \left(\frac{2}{3} + \frac{8}{4}\right) \\ &= \left(\frac{2}{3} + 2\right) \text{ h} \\ &= 2 \text{ h } 40 \text{ minutes} \end{aligned}$$

15. (i)

$$\begin{aligned} \frac{700}{x} + \frac{700}{x-30} &= 20 \\ \left(\frac{700}{x} + \frac{700}{x-30}\right) \times x(x-30) &= 20 \times x(x-30) \\ \frac{700}{x} \times x(x-30) + \frac{700}{x-30} \times x(x-30) &= 20x(x-30) \\ 700(x-30) + 700x &= 20x^2 - 600x \\ 700x - 21\,000 + 700x &= 20x^2 - 600x \\ 1400x - 21\,000 &= 20x^2 - 600x \\ 0 &= 20x^2 - 2000x + 21\,000 \\ 20x^2 - 2000x + 21\,000 &= 0 \\ x^2 - 100x + 1050 &= 0 \text{ (Shown)} \end{aligned}$$

(ii) $x^2 - 100x + 1050 = 0$

Comparing $x^2 - 100x + 1050 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -100$ and $c = 1050$.

$$\begin{aligned} x &= \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(1050)}}{2(1)} \\ &= \frac{100 \pm \sqrt{5800}}{2} \\ &= 88.08 \text{ (to 2 d.p.)}, 11.92 \text{ (to 2 d.p.)} \end{aligned}$$

$\therefore$ $x = 88.08$ or $x = 11.92$

(iii) Since average speed of the car > 30 km/h, $x = 88.08$

$$\begin{aligned} \text{Time taken for return journey} &= \frac{700}{88.08} \\ &= 7.95 \text{ hours (to 3 s.f.)} \end{aligned}$$

16. (i) No. of minutes taken by Pump $A = \frac{1500}{x}$

(ii) No. of minutes taken by Pump $B = \frac{1500}{x+50}$

(iii)

$$\begin{aligned} \frac{1500}{x} - \frac{1500}{x+50} &= \frac{30}{60} \\ \left[\frac{1500}{x} - \frac{1500}{x+50}\right] \times x(x+50) &= \frac{1}{2} \times x(x+50) \\ \frac{1500}{x} \times x(x+50) - \frac{1500}{x+50} \times x(x+50) &= \frac{1}{2} \times x(x+50) \\ 1500(x+50) - 1500x &= \frac{1}{2}x(x+50) \\ 1500x + 75\,000 - 1500x &= \frac{1}{2}x^2 + 25x \\ 75\,000 &= \frac{1}{2}x^2 + 25x \\ 0 &= \frac{1}{2}x^2 + 25x - 75\,000 \\ \frac{1}{2}x^2 + 25x - 75\,000 &= 0 \\ x^2 + 50x - 150\,000 &= 0 \text{ (Shown)} \end{aligned}$$

(iv) $x^2 + 50x - 150\,000 = 0$

Comparing $x^2 + 50x - 150\,000 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 50$ and $c = -150\,000$.

$$\begin{aligned} x &= \frac{-50 \pm \sqrt{50^2 - 4(1)(-150\,000)}}{2(1)} \\ &= \frac{-50 - \sqrt{602\,500}}{2} \\ &= 363.10 \text{ (to 2 d.p.)}, -413.10 \text{ (to 2 d.p.)} \end{aligned}$$

$\therefore$ $x = 363.10$ or $x = -413.10$

(v) Since $x > 0$, $x = 363.10$

Time taken by Pump B

$$\begin{aligned} &= \frac{1500}{363.10 + 50} \\ &= 3.63 \text{ mins (to 3 s.f.)} \\ &= 3 \text{ minutes } 38 \text{ seconds (to the nearest second)} \end{aligned}$$

17. (i) At Samy's Money Exchange,

$$\begin{aligned} \text{S\$}x &= \text{US\$}1 \\ \text{S\$}2000 &= \text{US\$}\left(\frac{1}{x} \times 2000\right) \\ &= \text{US\$}\frac{2000}{x} \end{aligned}$$

(ii) At Chan's Money Exchange,

$$\begin{aligned} \text{S\$}(x + 0.05) &= \text{US\$}1 \\ \text{S\$}1000 &= \text{US\$}\left(\frac{1}{x+0.05} \times 1000\right) \\ &= \text{US\$}\frac{1000}{x+0.05} \end{aligned}$$

(iii)
$$\frac{2000}{x} + \frac{1000}{x+0.05} = 2370$$
$$\left[\frac{2000}{x} + \frac{1000}{x+0.05}\right] \times x(x+0.05) = 2370 \times x(x+0.05)$$
$$\frac{2000}{x} \times x(x+0.05) + \frac{1000}{x+0.05} \times x(x+0.05) = 2370x(x+0.05)$$
$$2000(x+0.05) + 1000x = 2370x(x+0.05)$$
$$2000x + 100 + 1000x = 2370x^2 + 118.5x$$
$$3000x + 100 = 2370x^2 + 118.5x$$
$$0 = 2370x^2 - 2881.5x - 100$$
$$2370x^2 - 2881.5x - 100 = 0$$
$$237x^2 - 288.15x - 10 = 0 \text{ (Shown)}$$

(iv) $237x^2 - 288.15x - 10 = 0$

Comparing $237x^2 - 288.15x - 10 = 0$ with $ax^2 + bx + c = 0$, we have $a = 237$, $b = -288.15$ and $c = -10$.

$$x = \frac{-(-288.15) \pm \sqrt{(-288.15)^2 - 4(237)(-10)}}{2(237)}$$
$$= \frac{288.15 \pm \sqrt{92\,510.4225}}{474}$$

$= 1.25$ (to 2 d.p.), -0.03 (to 2 d.p.)

$\therefore x = 1.25$ or $x = -0.03$

(v) Since $x > 0$, $x = 1.25$

At Chan's Money Exchange,

US\$1 = S\$(1.25 + 0.05)

= S\$1.30

18. (a)
$$\frac{4}{x-1} = \frac{x}{2x^2 + 3x - 5}$$
$$\frac{4}{x-1} = \frac{x}{(x-1)(2x+5)}$$
$$\frac{4}{x-1} - \frac{x}{(x-1)(2x+5)} = 0$$
$$\left[\frac{4}{x-1} - \frac{x}{(x-1)(2x+5)}\right] \times (x-1)(2x+5) = 0 \times (x-1)(2x+5)$$
$$\frac{4}{x-1} \times (x-1)(2x+5) - \frac{x}{(x-1)(2x+5)} \times (x-1)(2x+5) = 0$$
$$4(2x+5) - x = 0$$
$$8x + 20 - x = 0$$
$$7x + 20 = 0$$
$$7x = -20$$
$$x = -2\frac{6}{7}$$

(b)
$$\frac{1}{x} + \frac{2}{x-1} + \frac{3}{x+1} = 0$$
$$\left[\frac{1}{x} + \frac{2}{x-1} + \frac{3}{x+1}\right] \times x(x-1)(x+1) = 0 \times x(x-1)(x+1)$$
$$\frac{1}{x} \times x(x-1)(x+1) + \frac{2}{x-1} \times x(x-1)(x+1) + \frac{3}{x+1} \times x(x-1)(x+1) = 0$$
$$(x-1)(x+1) + 2x(x+1) + 3x(x-1) = 0$$
$$x^2 - 1 + 2x^2 + 2x + 3x^2 - 3x = 0$$
$$6x^2 - x - 1 = 0$$
$$(2x-1)(3x+1) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad 3x + 1 = 0$$
$$2x = 1 \qquad 3x = -1$$
$$x = \frac{1}{2} \qquad x = -\frac{1}{3}$$

$\therefore x = \frac{1}{2}$ or $x = -\frac{1}{3}$

(c)
$$\frac{1}{x^2 - 9} - \frac{2}{3-x} = 1$$
$$\frac{1}{(x+3)(x-3)} - \frac{2}{-(x-3)} = 1$$
$$\frac{1}{(x+3)(x-3)} + \frac{2}{x-3} = 1$$
$$\left[\frac{1}{(x+3)(x-3)} + \frac{2}{x-3}\right] \times (x+3)(x-3) = 1 \times (x+3)(x-3)$$
$$\frac{1}{(x+3)(x-3)} \times (x+3)(x-3) + \frac{2}{x-3} \times (x+3)(x-3) = (x+3)(x-3)$$
$$1 + 2(x+3) = (x+3)(x-3)$$
$$1 + 2x + 6 = x^2 - 9$$
$$7 + 2x = x^2 - 9$$
$$x^2 - 2x - 16 = 0$$

Comparing $x^2 - 2x - 16 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -2$ and $c = -16$.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-16)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{68}}{2}$$

$= 5.12$ (to 3 s.f.), -3.12 (to 3 s.f.)

$\therefore x = 5.12$ or $x = -3.12$

(d)
$$\frac{3}{x-3} + \frac{x+1}{x^2-5x+6} = 1$$
$$\frac{3}{x-3} + \frac{x+1}{(x-2)(x-3)} = 1$$
$$\left[\frac{3}{x-3} + \frac{x+1}{(x-2)(x-3)}\right] \times (x-2)(x-3) = 1 \times (x-2)(x-3)$$
$$\frac{3}{x-3} \times (x-2)(x-3) + \frac{x+1}{(x-2)(x-3)} \times (x-2)(x-3) = (x-2)(x-3)$$
$$3(x-2) + (x+1) = (x-2)(x-3)$$
$$3x - 6 + x + 1 = x^2 - 3x - 2x + 6$$
$$4x - 5 = x^2 - 5x + 6$$
$$x^2 - 9x + 11 = 0$$

Comparing $x^2 - 9x + 11 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -9$ and $c = 11$.

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(11)}}{2(1)}$$
$$= \frac{9 \pm \sqrt{37}}{2}$$
$= 7.54$ (to 3 s.f.), 1.46 (to 3 s.f.)

$\therefore x = 7.54$ or $x = 1.46$

19. Assumptions: The aircraft is flying in the same direction as the wind from Sandy Land to White City and against the wind from White City to Sandy Land.

Let the speed of the wind be x km/h.
$$\frac{450}{165+x} + \frac{450}{165-x} = 5\frac{1}{2}$$
$$\left[\frac{450}{165+x} + \frac{450}{165-x}\right] \times (165+x)(165-x) = \frac{11}{2} \times (165+x)(165-x)$$
$$\frac{450}{165+x} \times (165+x)(165-x) + \frac{450}{165-x} \times (165+x)(165-x) = \frac{11}{2}(165+x)(165-x)$$
$$450(165-x) + 450(165+x) = \frac{11}{2}(165+x)(165-x)$$
$$74\,250 - 450x + 74\,250 + 450x = \frac{11}{2}(27\,225 - x^2)$$
$$148\,500 = 149\,737.5 - 5.5x^2$$
$$5.5x^2 = 1237.5$$
$$x^2 = 225$$
$$x = \pm\sqrt{225}$$
$$= \pm 15$$

Since the speed cannot be a negative value, $x = 15$.

$\therefore$ The speed of the wind is 15 km/h.

Exercise 1E

1. (a) $y = (x + 1)(x + 3)$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

$(x + 1)(x + 3) = 0$

$x + 1 = 0$ or $x + 3 = 0$

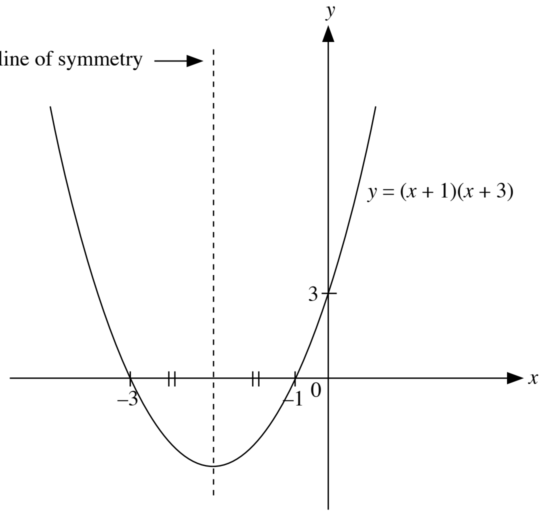
$x = -1$ $\quad$ $x = -3$

$\therefore$ The graph cuts the x-axis at $(-1, 0)$ and $(-3, 0)$.

When $x = 0$,

$y = (1)(3) = 3$

$\therefore$ The graph cuts the y-axis at $(0, 3)$.



(b) $y = (x - 2)(x + 4)$

Since the coefficent of x^2 is 1, the graph opens upwards.

When $y = 0$.

$(x - 2)(x + 4) = 0$

$x - 2 = 0$ or $x + 4 = 0$

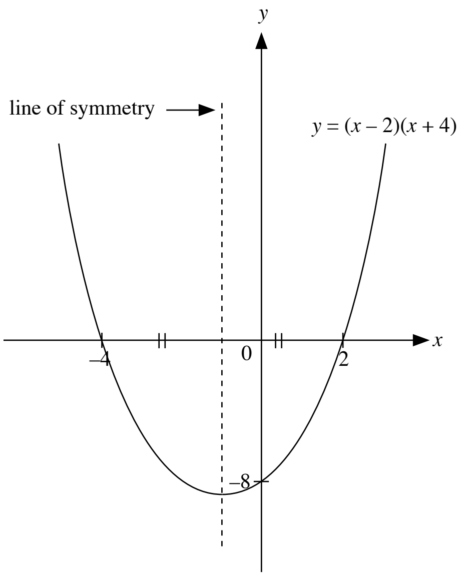
$x = 2$ $\quad$ $x = -4$

$\therefore$ The graph cuts the x-axis at $(2, 0)$ and $(-4, 0)$.

When $x = 0$,

$y = (-2)(4) = -8$

$\therefore$ The graph cuts the y-axis at $(0, -8)$.



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(c) $y = -(x + 1)(x - 5)$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

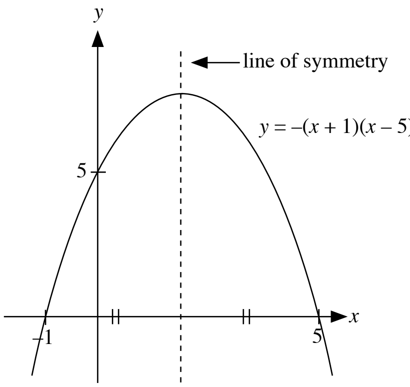
$$\begin{aligned} -(x+1)(x-5) &= 0 \\ x + 1 = 0 \quad &\text{or} \quad x - 5 = 0 \\ x = -1 \quad &\quad\quad\; x = 5 \end{aligned}$$

$\therefore$ The graph cuts the x-axis at $(-1, 0)$ and $(5, 0)$.

When $x = 0$,

$y = -(1)(-5) = 5$

$\therefore$ The graph cuts the y-axis at $(0, 5)$.



(d) $y = -(x - 1)(x + 6)$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

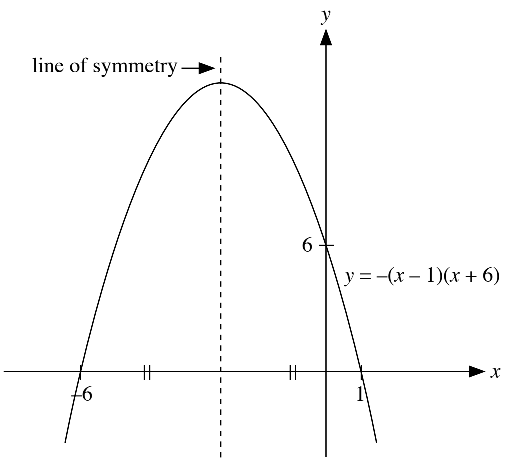
$$\begin{aligned} -(x-1)(x+6) &= 0 \\ x - 1 = 0 \quad &\text{or} \quad x + 6 = 0 \\ x = 1 \quad &\quad\quad\; x = -6 \end{aligned}$$

$\therefore$ The graph cuts the x-axis at $(1, 0)$ annd $(-6, 0)$.

When $x = 0$,

$y = -(-1)(6) = 6$

$\therefore$ The graph cuts the y-axis at $(0, 6)$.



(e) $y = (3 - x)(x + 2)$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

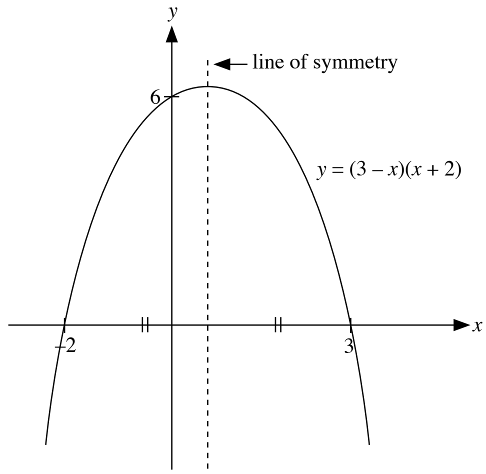
$$\begin{aligned} (3-x)(x+2) &= 0 \\ 3 - x = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = 3 \quad &\quad\quad\; x = -2 \end{aligned}$$

$\therefore$ The graph cuts the x-axis at $(3, 0)$ and $(-2, 0)$.

When $x = 0$,

$y = (3)(2) = 6$

$\therefore$ The graph cuts the y-axis at $(0, 6)$.



(f) $y = (2 - x)(4 - x)$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

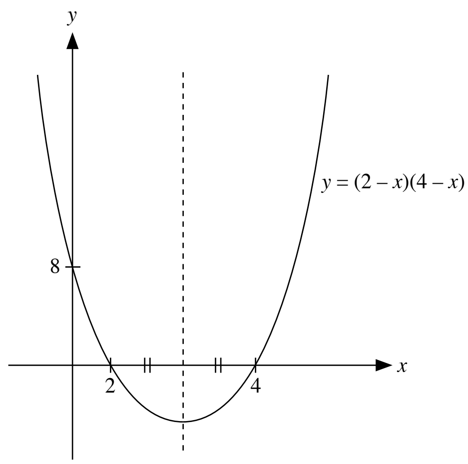
$$\begin{aligned} (2-x)(4-x) &= 0 \\ 2 - x = 0 \quad &\text{or} \quad 4 - x = 0 \\ x = 2 \quad &\quad\quad\; x = 4 \end{aligned}$$

$\therefore$ The graph cuts the x-axis at $(2, 0)$ and $(4, 0)$.

When $x = 0$,

$y = (2)(4) = 8$

$\therefore$ The graph cuts the y-axis at $(0, 8)$.



2. **(a)** $y = x^2 + 2$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

$x^2 + 2 = 0$

$x^2 = -2$

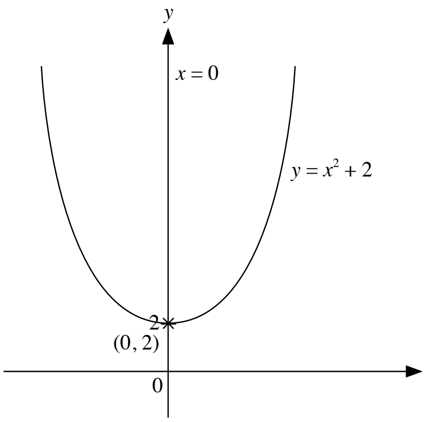
$\therefore$ The graph does not cut the x-axis.

When $x = 0$,

$y = 0^2 + 2 = 2$

$\therefore$ The graph cuts the y-axis at $(0, 2)$.

The equation of the line of symmetry is $x = 0$.



(b) $y = -x^2 - 6$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

$-x^2 - 6 = 0$

$x^2 = -6$

$\therefore$ The graph does not cut the x-axis.

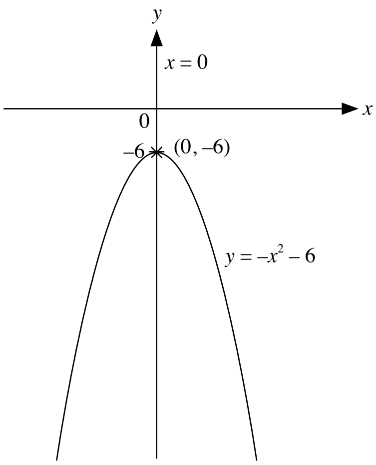
When $x = 0$,

$y = -(0)^2 - 6 = -6$

$\therefore$ The graph cuts the y-axis at $(0, -6)$.

The coordinates of the maximum point are $(0, -6)$.

The equation of the line of symmetry is $x = 0$.



(c) $y = (x - 3)^2 + 1$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

$(x - 3)^2 + 1 = 0$

$(x - 3)^2 = -1$

$\therefore$ The graph does not cut the x-axis.

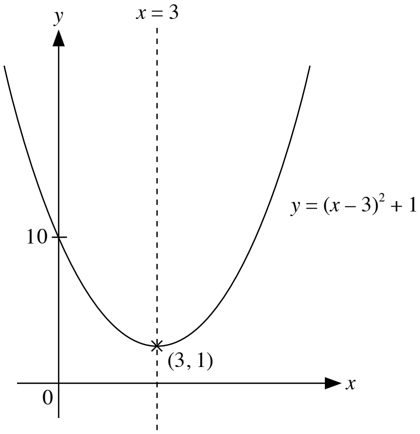
When $x = 0$,

$y = (0 - 3)^2 + 1 = 10$

$\therefore$ The graph cuts the y-axis at $(0, 10)$.

The coordinates of the minimum point are $(3, 1)$.

The equation of the line of symmetry is $x = 3$.



(d) $y = (x + 1)^2 - 3$

Since the coefficient of x^2 is 1, the graph opens upwards.

The coordinates of the minimum point are $(-1, -3)$.

The equation of the line of symmetry is $x = -1$.

When $y = 0$,

$$
\begin{aligned}
(x + 1)^2 - 3 &= 0 \\
(x + 1)^2 &= 3 \\
x + 1 &= \pm\sqrt{3}
\end{aligned}
$$

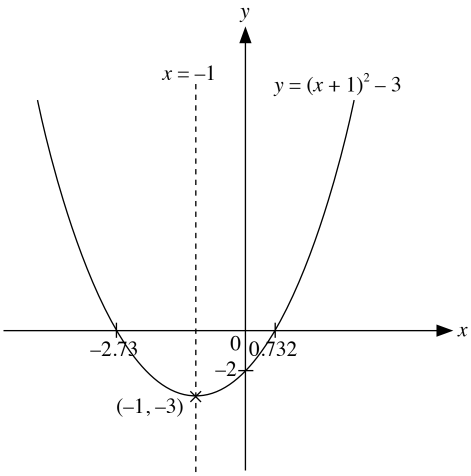
$$
\begin{aligned}
x + 1 &= \sqrt{3} & \text{or} \quad x + 1 &= -\sqrt{3} \\
x &= \sqrt{3} - 1 & x &= -\sqrt{3} - 1 \\
&= 0.732 \text{ (to 3 s.f.)} & &= -2.73 \text{ (to 3 s.f.)}
\end{aligned}
$$

$\therefore$ The graph cut the x-axis at $(0.732, 0)$ and $(-2.73, 0)$.

When $x = 0$,

$y = (0 + 1)^2 - 3 = -2$

$\therefore$ The graph cuts the y-axis at $(0, -2)$.



(e) $y = -(x + 2)^2 + 3$

Since the coefficient of x^2 is -1, the graph opens downwards.

The coordinates of the minimum point are $(-2, 3)$.

The equation of the line of symmetry is $x = -2$.

When $y = 0$,

$-(x + 2)^2 + 3 = 0$

$-(x + 2)^2 = -3$

$(x + 2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x + 2 = \sqrt{3}$ or $x + 2 = -\sqrt{3}$

$x = \sqrt{3} - 2$ $\quad$ $x = -\sqrt{3} - 2$

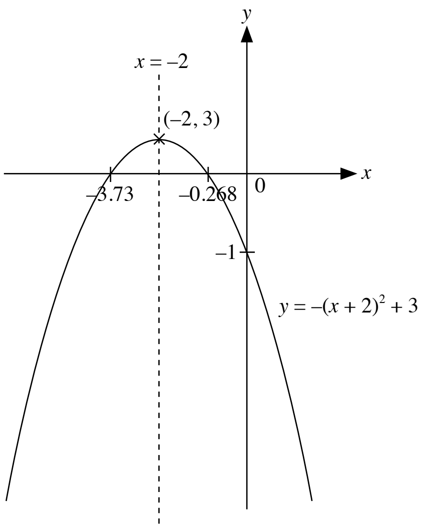
$= -0.268$ (to 3 s.f.) $\quad$ $= -3.73$ (to 3 s.f.)

$\therefore$ The graph cut the x-axis at $(-0.268, 0)$ and $(-3.73, 0)$.

When $x = 0$,

$y = -(0 + 2)^2 + 3 = -1$

$\therefore$ The graph cuts the y-axis at $(0, -1)$.



(f) $y = -(x - 4)^2 - 1$

Since the coefficient of x^2 is -1, the graph opens downwards.

The coordinates of the minimum point are $(4, -1)$.

The equation of the line of symmetry is $x = 4$.

When $y = 0$,

$-(x - 4)^2 - 1 = 0$

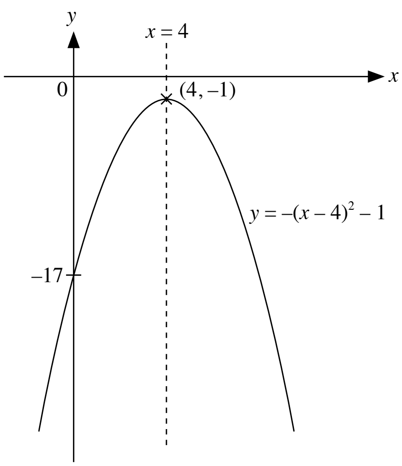
$(x - 4)^2 = -1$

$\therefore$ The graph does not cut the x-axis.

When $x = 0$,

$y = -(0 - 4)^2 - 1 = -17$

$\therefore$ The graph cuts the y-axis at $(0, -17)$.



3. **(i)** $x^2 + \frac{3}{4}x = x\left(x + \frac{3}{4}\right)$

(ii) $y = x^2 + \frac{3}{4}x$

$= x\left(x + \frac{3}{4}\right)$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

$x\left(x + \frac{3}{4}\right) = 0$

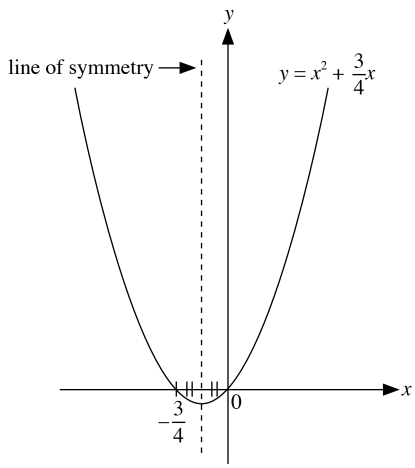
$x = 0$ or $x = -\frac{3}{4}$

$\therefore$ The graph cuts the x-axis at $(0, 0)$ and $\left(-\frac{3}{4}, 0\right)$.

When $x = 0$,

$y = 0\left(0 + \frac{3}{4}\right) = 0$

$\therefore$ The graph cuts the y-axis at $(0, 0)$.



4. $y = -(x^2 - x)$

Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

$-(x^2 - x) = 0$

$-x(x - 1) = 0$

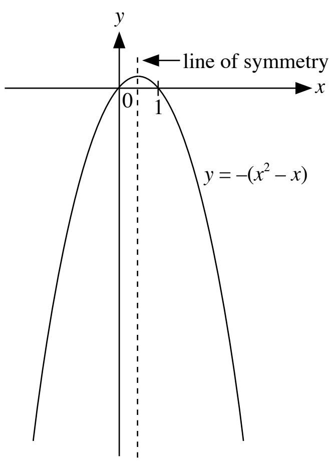
$x = 0$ or $x = 1$

$\therefore$ The graph cuts the x-axis at $(0, 0)$ and $(1, 0)$.

When $x = 0$,

$y = -(0^2 - 0) = 0$

$\therefore$ The graph cuts the y-axis at $(0, 0)$.



5. **(i)** $x^2 + x - 6 = (x + 3)(x - 2)$

(ii) $y = x^2 + x - 6$
$= (x + 3)(x - 2)$

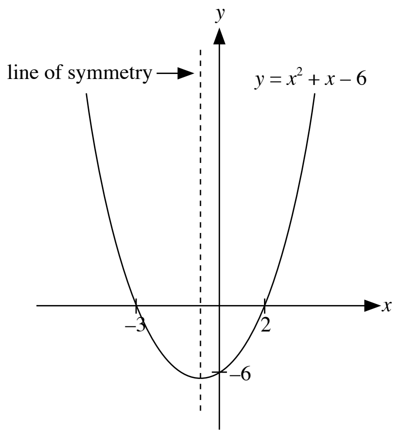
Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,
$(x + 3)(x - 2) = 0$
$x = -3$ or $x = 2$

$\therefore$ The graph cuts the x-axis at $(-3, 0)$ and $(2, 0)$.

When $x = 0$,
$y = (0 + 3)(0 - 2) = -6$

$\therefore$ The graph cuts the y-axis at $(0, -6)$.



6. $y = x^2 - 4x + 3$
$= (x - 1)(x - 3)$

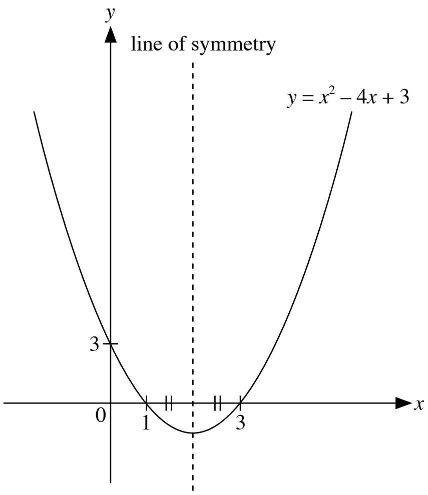
Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,
$(x - 1)(x - 3) = 0$
$x - 1 = 0$ or $x - 3 = 0$
$x = 1$ $\quad x = 3$

$\therefore$ The graph cuts the x-axis at $(1, 0)$ and $(3, 0)$.

When $x = 0$,
$y = (-1)(-3) = 3$

$\therefore$ The graph cuts the y-axis at $(0, 3)$.



7. **(i)** $x^2 - 8x + 5 = \left[x^2 - 8x + \left(-\frac{8}{2}\right)^2\right] - \left(-\frac{8}{2}\right)^2 + 5$
$= (x - 4)^2 - 11$

(ii) $y = x^2 - 8x + 5$
$= (x - 4)^2 - 11$

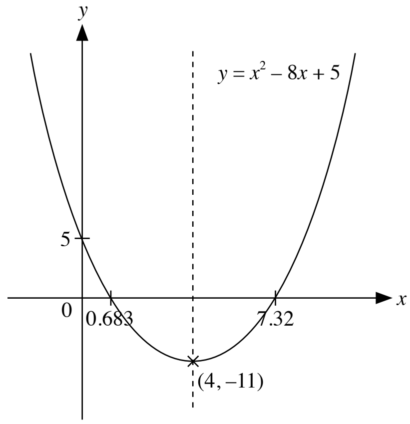
Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,
$(x - 4)^2 - 11 = 0$
$(x - 4)^2 = 11$
$x - 4 = \pm\sqrt{11}$
$x - 4 = \sqrt{11}$ or $x - 4 = -\sqrt{11}$
$x = \sqrt{11} + 4$ $\quad x = -\sqrt{11} + 4$
$= 7.32$ (to 3 s.f.) $\quad = 0.683$ (to 3 s.f.)

$\therefore$ The graph cuts the x-axis at $(7.32, 0)$ and $(0.683, 0)$.

When $x = 0$,
$y = (0 - 4)^2 - 11 = 5$

$\therefore$ The graph cuts the y-axis at $(0, 5)$.



(iii) The coordinates of the minimum point are $(4, -11)$.

(iv) The equation of the line of symmetry is $x = 4$.

8. $y = x^2 + 3x + 1$
$= \left[x^2 + 3x + \left(\frac{3}{2}\right)^2\right] - \left(\frac{3}{2}\right)^2 + 1$
$= \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,
$\left(x - \frac{3}{2}\right)^2 - \frac{5}{4} = 0$
$\left(x - \frac{3}{2}\right)^2 = \frac{5}{4}$
$x - \frac{3}{2} = \pm\sqrt{\frac{5}{4}}$
$x - \frac{3}{2} = \sqrt{\frac{5}{4}}$ or $x - \frac{3}{2} = -\sqrt{\frac{5}{4}}$
$x = \sqrt{\frac{5}{4}} + \frac{3}{2}$ $\quad x = -\sqrt{\frac{5}{4}} + \frac{3}{2}$
$= 2.62$ (to 3 s.f.) $\quad = 0.382$ (to 3 s.f.)

$\therefore$ The graph cuts the x-axis at $(2.62, 0)$ and $(0.382, 0)$.

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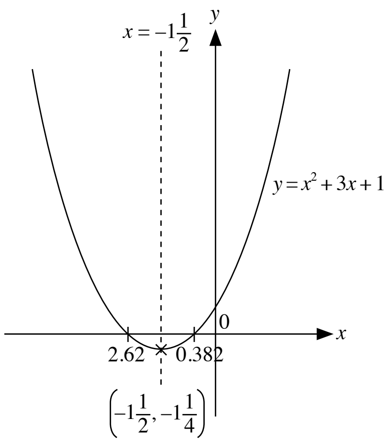
When $x = 0$,

$$y = \left(0 - \frac{3}{2}\right)^2 - \frac{5}{4} = 1$$

$\therefore$ The graph cuts the y-axis at $(0, 1)$.

The coordinates of the minimum point are $\left(-1\frac{1}{2}, -1\frac{1}{4}\right)$.

The equation of the line of symmetry is $x = -1\frac{1}{2}$.



9. **(i)** Since the minimum point is $\left(-\frac{1}{2}, \frac{3}{4}\right)$,

$$y = \left[x - \left(-\frac{1}{2}\right)\right]^2 + \frac{3}{4}$$
$$= (x - h)^2 + k$$

$\therefore$ $h = -\frac{1}{2}$, $k = \frac{3}{4}$

(ii) $y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

Since the coefficient of x^2 is 1, the graph opens upwards.

The coordinates of the minimum point are $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

When $y = 0$,

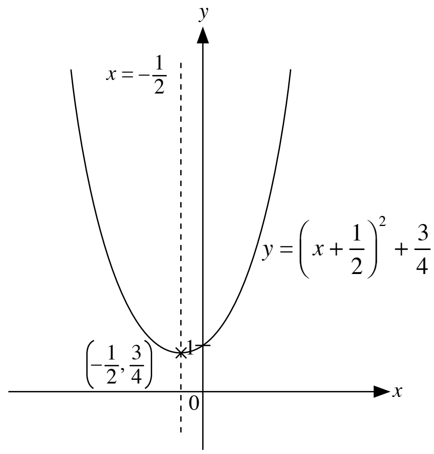
$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = 0$$
$$\left(x + \frac{1}{2}\right)^2 = -\frac{3}{4}$$

$\therefore$ The graph does not cuts the x-axis.

When $x = 0$,

$$y = \left(0 + \frac{1}{2}\right)^2 + \frac{3}{4} = 1$$

$\therefore$ The graph cuts the y-axis at $(0, 1)$.



10. $-x^2 + 10x - 4 = -(x^2 - 10x + 4)$

$$= -\left(\left[x^2 - 10x + \left(-\frac{10}{2}\right)^2\right] - \left(-\frac{10}{2}\right)^2 + 4\right)$$
$$= -(x - 5)^2 + 21$$
$$= -(x - p)^2 + q \text{ where } p = 5 \text{ and } q = 21$$

$y = -(x - 5)^2 + 21$

Since the coefficient of x^2 is -1, the graph opens downwards.

The coordinates of the maximum point are $(5, 21)$.

When $y = 0$,

$$-(x - 5)^2 + 21 = 0$$
$$(x - 5)^2 = 21$$
$$x - 5 = \pm\sqrt{21}$$

$x - 5 = \sqrt{21}$ or $x - 5 = -\sqrt{21}$

$x = \sqrt{21} + 5$ $\quad$ $x = -\sqrt{21} + 5$

$= 9.58$ (to 3 s.f.) $\quad$ $= 0.417$ (to 3 s.f.)

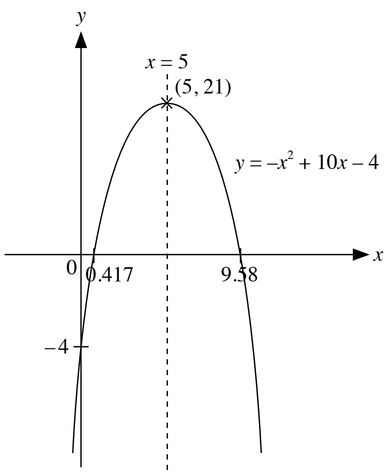
$\therefore$ The graph cuts the x-axis at $(9.58, 0)$ and $(0.417, 0)$.

When $x = 0$,

$y = -(0 - 5)^2 + 21 = -4$

$\therefore$ The graph cuts the y-axis at $(0, -4)$.

The equation of the line of symmetry is $x = 5$.



Review Exercise 1

1. (a)
$$\begin{aligned} x^2+8x+5 &= 0 \\ x^2+8x &= -5 \\ x^2+8x+\left(\frac{8}{2}\right)^2 &= -5+\left(\frac{8}{2}\right)^2 \\ x^2+8x+4^2 &= -5+4^2 \\ (x+4)^2 &= 11 \\ x+4 &= \pm\sqrt{11} \end{aligned}$$

$x+4=\sqrt{11}$ or $x+4=-\sqrt{11}$

$x=\sqrt{11}-4$ $\quad$ $x=-\sqrt{11}-4$

$=-0.683$ (to 3 s.f.) $\quad$ $=-7.32$ (to 3 s.f.)

$\therefore x=-0.683$ or $x=-7.32$

(b)
$$\begin{aligned} x^2+7x-3 &= 0 \\ x^2+7x &= 3 \\ x^2+7x+\left(\frac{7}{2}\right)^2 &= 3+\left(\frac{7}{2}\right)^2 \\ \left(x+\frac{7}{2}\right)^2 &= \frac{61}{4} \\ x+\frac{7}{2} &= \pm\sqrt{\frac{61}{4}} \end{aligned}$$

$x+\frac{7}{2}=\sqrt{\frac{61}{4}}$ or $x+\frac{7}{2}=-\sqrt{\frac{61}{4}}$

$x=\sqrt{\frac{61}{4}}-\frac{7}{2}$ $\quad$ $x=-\sqrt{\frac{61}{4}}-\frac{7}{2}$

$=0.405$ (to 3 s.f.) $\quad$ $=-7.41$ (to 3 s.f.)

$\therefore x=0.405$ or $x=-7.41$

(c)
$$\begin{aligned} x^2-11x-7 &= 0 \\ x^2-11x &= 7 \\ x^2-11x+\left(-\frac{11}{2}\right)^2 &= 7+\left(-\frac{11}{2}\right)^2 \\ \left(x-\frac{11}{2}\right)^2 &= \frac{149}{4} \\ x-\frac{11}{2} &= \pm\sqrt{\frac{149}{4}} \end{aligned}$$

$x-\frac{11}{2}=\sqrt{\frac{149}{4}}$ or $x-\frac{11}{2}=-\sqrt{\frac{149}{4}}$

$x=\sqrt{\frac{149}{4}}+\frac{11}{2}$ $\quad$ $x=-\sqrt{\frac{149}{4}}+\frac{11}{2}$

$=11.6$ (to 3 s.f.) $\quad$ $=-0.603$ (to 3 s.f.)

$\therefore x=11.6$ or $x=-0.603$

(d)
$$\begin{aligned} x^2+1.2x &= 1 \\ x^2+1.2x+\left(\frac{1.2}{2}\right)^2 &= 1+\left(\frac{1.2}{2}\right)^2 \\ (x+0.6)^2 &= 1.36 \\ x+0.6 &= \pm\sqrt{1.36} \end{aligned}$$

$x+0.6=\sqrt{1.36}$ or $x+0.6=-\sqrt{1.36}$

$x=\sqrt{1.36}-0.6$ $\quad$ $x=-\sqrt{1.36}-0.6$

$=0.566$ (to 3 s.f.) $\quad$ $=-1.77$ (to 3 s.f.)

$\therefore x=0.566$ or $x=-1.77$

2. (a) $2x^2+6x+1=0$

Comparing $2x^2+6x+1=0$ with $ax^2+bx+c=0$, we have $a=2$, $b=6$ and $c=1$.

$$\begin{aligned} x &= \frac{-6\pm\sqrt{6^2-4(2)(1)}}{2(2)} \\ &= \frac{-6\pm\sqrt{28}}{4} \end{aligned}$$

$=-0.177$ (to 3 s.f.), -2.82 (to 3 s.f.)

$\therefore x=-0.177$ or $x=-2.82$

(b) $3x^2-7x-2=0$

Comparing $3x^2-7x-2=0$ with $ax^2+bx+c=0$, we have $a=3$, $b=-7$ and $c=-2$.

$$\begin{aligned} x &= \frac{-(-7)\pm\sqrt{(-7)^2-4(3)(-2)}}{2(3)} \\ &= \frac{7\pm\sqrt{73}}{6} \end{aligned}$$

$=2.59$ (to 3 s.f.), -0.257 (to 3 s.f.)

$\therefore x=2.59$ or $x=-0.257$

(c) $-4x^2+x+5=0$

Comparing $-4x^2+x+5=0$ with $ax^2+bx+c=0$, we have $a=-4$, $b=1$ and $c=5$.

$$\begin{aligned} x &= \frac{-1\pm\sqrt{1^2-4(-4)(5)}}{2(-4)} \\ &= \frac{-1\pm\sqrt{81}}{-8} \\ &= -1 \text{ or } 1\frac{1}{4} \end{aligned}$$

$\therefore x=-1$ or $x=1\frac{1}{4}$

(d)
$$\begin{aligned} 3x^2 &= 5x+1 \\ 3x^2-5x-1 &= 0 \end{aligned}$$

Comparing $3x^2-5x-1=0$ with $ax^2+bx+c=0$, we have $a=3$, $b=-5$ and $c=-1$.

$$\begin{aligned} x &= \frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-1)}}{2(3)} \\ &= \frac{5\pm\sqrt{37}}{6} \end{aligned}$$

$=1.85$ (to 3 s.f.), -0.180 (to 3 s.f.)

$\therefore x=1.85$ or $x=-0.180$

3. (a)
$$\begin{aligned} (x-3)^2 &= \frac{4}{25} \\ x-3 &= \pm\sqrt{\frac{4}{25}} \\ x-3 &= \pm\frac{2}{5} \end{aligned}$$

$x-3=\frac{2}{5}$ or $x-3=-\frac{2}{5}$

$x=\frac{2}{5}+3$ $\quad$ $x=-\frac{2}{5}+3$

$=3\frac{2}{5}$ $\quad$ $=2\frac{3}{5}$

$\therefore x=3\frac{2}{5}$ or $x=2\frac{3}{5}$

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(b) $(4 - x)^2 = 12$

$4 - x = \pm\sqrt{12}$

$4 - x = \sqrt{12}$ or $4 - x = -\sqrt{12}$

$x = 4 - \sqrt{12}$ $\quad$ $x = 4 + \sqrt{12}$

$= 0.536$ (to 3 s.f.) $\quad$ $= 7.46$ (to 3 s.f.)

$\therefore x = 0.536$ or $x = 7.46$

(c) $(x - 1)(x + 3) = 9$

$x^2 + 3x - x - 3 = 9$

$x^2 + 2x - 12 = 0$

Comparing $x^2 + 2x - 12 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 2$ and $c = -12$.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{52}}{2}$$

$= 2.61$ (to 3 s.f.), -4.61 (to 3 s.f.)

$\therefore x = 2.61$ or $x = -4.61$

(d) $x(x + 4) = 17$

$x^2 + 4x = 17$

$x^2 + 4x - 17 = 0$

Comparing $x^2 + 4x - 17 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 4$ and $c = -17$.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-17)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{84}}{2}$$

$= 2.58$ (to 3 s.f.), -6.58 (to 3 s.f.)

$\therefore x = 2.58$ or $x = -6.58$

4. **(i)** $2x^2 - 7x + 4 = 0$

Comparing $2x^2 - 7x + 4 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -7$ and $c = 4$.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{17}}{4}$$

$= 2.78$ (to 2 d.p.), 0.72 (to 2 d.p)

$\therefore x = 2.78$ or $x = 0.72$

(ii) $2(y - 1)^2 - 7(y - 1) + 4 = 0$

Let $(y - 1)$ be x.

$2x^2 - 7x + 4 = 0$

Since $x = 2.78$ (to 2 d.p.), $0{,}72$ (to 2 d.p.),

$\therefore y - 1 = 2.78$ or $y - 1 = 0.72$

$y = 3.78$ $\quad$ $y = 1.72$

5. **(a)** $(x - 2)\left(x - \frac{6}{7}\right) = 0$

$x^2 - \frac{6}{7}x - 2x + \frac{12}{7} = 0$

$x^2 - \frac{20}{7}x + \frac{12}{7} = 0$

$7x^2 - 20x + 12 = 0$, where $a = 7$, $b = -20$ and $c = 12$.

(b) $\left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right) = 0$

$x^2 + \frac{2}{3}x + \frac{1}{2} + \frac{1}{3} = 0$

$x + \frac{7}{6}x + \frac{1}{3} = 0$

$6x^2 + 7x + 2 = 0$, where $a = 6$, $b = 7$ and $c = 2$.

6. **(a)** $x - 1 = \frac{5}{x + 7}$

$(x - 1) \times (x + 7) = \frac{5}{x + 7} \times (x + 7)$

$(x - 1)(x + 7) = 5$

$x^2 + 7x - x - 7 = 5$

$x^2 + 6x - 12 = 0$

Comparing $x^2 + 6x - 12 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 6$ and $c = -12$.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{84}}{2}$$

$= 1.58$ (to 3 s.f.), -7.58 (to 3 s.f.)

$\therefore x = 1.58$ or $x = -7.58$

(b) $\frac{x - 1}{x + 4} = \frac{2x}{x - 3}$

$\frac{x - 1}{x + 4} - \frac{2x}{x - 3} = 0$

$\left[\frac{x - 1}{x + 4} - \frac{2x}{x - 3}\right] \times (x + 4)(x - 3) = 0 \times (x + 4)(x - 3)$

$\frac{x - 1}{x + 4} \times (x + 4)(x - 3) - \frac{2x}{x - 3} \times (x + 4)(x - 3) = 0 \times (x + 4)(x - 3)$

$(x - 1)(x - 3) - 2x(x + 4) = 0$

$x^2 - 3x - x + 3 - 2x^2 - 8x = 0$

$-x^2 - 12x + 3 = 0$

Comparing $-x^2 - 12x + 3 = 0$ with $ax^2 + bx + c = 0$, we have $a = -1$, $b = -12$ and $c = 3$.

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(-1)(3)}}{2(-1)}$$

$$= \frac{12 \pm \sqrt{156}}{-2}$$

$= -12.2$ (to 3 s.f.), 0.245 (to 3 s.f.)

$\therefore x = -12.2$ or $x = 0.245$

(c)
$$\frac{1}{x} - 5x = 5$$
$$\left(\frac{1}{x} - 5x\right) \times x = 5 \times x$$
$$1 - 5x^2 = 5x$$
$$5x^2 + 5x - 1 = 0$$
Comparing $5x^2 + 5x - 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 5$, $b = 5$ and $c = -1$.
$$x = \frac{-5 \pm \sqrt{5^2 - 4(5)(-1)}}{2(5)}$$
$$= \frac{-5 \pm \sqrt{45}}{10}$$
$= 0.171$ (to 3 s.f.), -1.17 (to 3 s.f.)

$\therefore x = 0.171$ or $x = -1.17$

(d)
$$\frac{5}{x} = 3 - \frac{x}{x-3}$$
$$\frac{5}{x} + \frac{x}{x-3} = 3$$
$$\left(\frac{5}{x} + \frac{x}{x-3}\right) \times x(x-3) = 3 \times x(x-3)$$
$$\frac{5}{x} \times x(x-3) + \frac{x}{x-3} \times x(x-3) = 3x(x-3)$$
$$5(x-3) + x(x) = 3x(x-3)$$
$$5x - 15 + x^2 = 3x^2 - 9x$$
$$2x^2 - 14x + 15 = 0$$
Comparing $2x^2 - 14x + 15 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -14$ and $c = 15$.
$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(2)(15)}}{2(2)}$$
$$= \frac{14 \pm \sqrt{76}}{4}$$
$= 5.68$ (to 3 s.f.), 1.32 (to 3 s.f.)

$\therefore x = 5.68$ or $x = 1.32$

(e)
$$\frac{2}{x+1} + \frac{1}{x-3} = 5$$
$$\left(\frac{2}{x+1} + \frac{1}{x-3}\right) \times (x+1)(x-3) = 5 \times (x+1)(x-3)$$
$$\frac{2}{x+1} \times (x+1)(x-3) + \frac{1}{x-3} \times (x+1)(x-3) = 5(x+1)(x-3)$$
$$2(x-3) + (x+1) = 5(x+1)(x-3)$$
$$2x - 6 + x + 1 = 5(x^2 - 3x + x - 3)$$
$$3x - 5 = 5(x^2 - 2x - 3)$$
$$3x - 5 = 5x^2 - 10x - 15$$
$$5x^2 - 13x - 10 = 0$$
Comparing $5x^2 - 13x - 10 = 0$ with $ax^2 + bx + c = 0$, we have $a = 5$, $b = -13$ and $c = -10$.
$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(5)(-10)}}{2(5)}$$
$$= \frac{13 \pm \sqrt{369}}{10}$$
$= 3.22$ (to 3 s.f.), -0.621 (to 3 s.f.)

$\therefore x = 3.22$ or $x = -0.621$

(f)
$$\frac{x}{x+1} + \frac{1}{5} = \frac{3}{x-2}$$
$$\frac{3}{x-2} - \frac{x}{x+1} = \frac{1}{5}$$
$$\left(\frac{3}{x-2} - \frac{x}{x+1}\right) \times (x-2)(x+1) = \frac{1}{5}(x-2)(x+1)$$
$$\frac{3}{x-2} \times (x-2)(x+1) - \frac{x}{x+1} \times (x-2)(x+1) = \frac{1}{5}(x-2)(x+1)$$
$$3(x+1) - x(x-2) = \frac{1}{5}(x-2)(x+1)$$
$$3x + 3 - x^2 + 2x = \frac{1}{5}(x^2 + x - 2x - 2)$$
$$5x + 3 - x^2 = \frac{1}{5}(x^2 - x - 2)$$
$$5x + 3 - x^2 = \frac{1}{5}x^2 - \frac{1}{5}x - \frac{2}{5}$$
$$\frac{6}{5}x^2 - \frac{26}{5}x - \frac{17}{5} = 0$$
$$6x^2 - 26x - 17 = 0$$
Comparing $6x^2 - 26x - 17 = 0$ with $ax^2 + bx + c = 0$, we have $a = 6$, $b = -26$ and $c = -17$.
$$x = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(6)(-17)}}{2(6)}$$
$$= \frac{26 \pm \sqrt{1084}}{12}$$
$= 4.91$ (to 3 s.f.), -0.577 (to 3 s.f.)

$\therefore x = 4.91$ or $x = -0.577$

(g)
$$\frac{5}{x-2} - \frac{3}{x^2-4} = \frac{2}{7}$$
$$\frac{5}{x-2} - \frac{3}{(x-2)(x+2)} = \frac{2}{7}$$
$$\left[\frac{5}{x-2} - \frac{3}{(x-2)(x+2)}\right] \times (x-2)(x+2) = \frac{2}{7} \times (x-2)(x+2)$$
$$\frac{5}{x-2} \times (x-2)(x+2) - \frac{3}{(x-2)(x+2)} \times (x-2)(x+2)$$
$$= \frac{2}{7}(x-2)(x+2)$$
$$5(x+2) - 3 = \frac{2}{7}(x^2 - 4)$$
$$5x + 10 - 3 = \frac{2}{7}x^2 - \frac{8}{7}$$
$$5x + 7 = \frac{2}{7}x^2 - \frac{8}{7}$$
$$\frac{2}{7}x^2 - 5x - \frac{57}{7} = 0$$
$$2x^2 - 35x - 57 = 0$$
$$(2x+3)(x-19) = 0$$
$$2x + 3 = 0 \quad \text{or} \quad x - 19 = 0$$
$$2x = -3 \qquad x = 19$$
$$x = -1\frac{1}{2}$$

$\therefore x = -1\frac{1}{2}$ or $x = 19$

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(h) $$\frac{1}{2x+1} + \frac{x+3}{2x^2-5x-3} = 2$$
$$\frac{1}{2x+1} + \frac{x+3}{(2x+1)(x-3)} = 2$$
$$\left[\frac{1}{2x+1} + \frac{x+3}{(2x+1)(x-3)}\right] \times (2x+1)(x-3) = 2 \times (2x+1)(x-3)$$
$$\frac{1}{2x+1} \times (2x+1)(x-3) + \frac{x+3}{(2x+1)(x-3)} \times (2x+1)(x-3) = 2(2x+1)(x-3)$$
$$\begin{aligned} (x-3)+(x+3) &= 2(2x+1)(x-3) \\ x-3+x+3 &= 2(2x^2-6x+x-3) \\ 2x &= 2(2x^2-5x-3) \\ 2x &= 4x^2-10x-6 \\ 4x^2-12x-6 &= 0 \\ 2x^2-6x-3 &= 0 \end{aligned}$$

Comparing $2x^2 - 6x - 3 = 0$ with $ax^2 + bx + c = 0$, we have $a = -2$, $b = -6$ and $c = -3$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-3)}}{2(2)}$$
$$= \frac{6 \pm \sqrt{60}}{4}$$
$= 3.44$ (to 3 s.f.), -0.436 (to 3 s.f.)

$\therefore x = 3.44$ or $x = -0.436$

7. (i) $y = x^2 - 7x + 12$

$y = (x-3)(x-4)$

(ii) $y = (x-3)(x-4)$

Since the coefficient of x^2 is 1, the graph opens upwards.

When $y = 0$,

$(x-3)(x-4) = 0$

$x - 3 = 0$ or $x - 4 = 0$

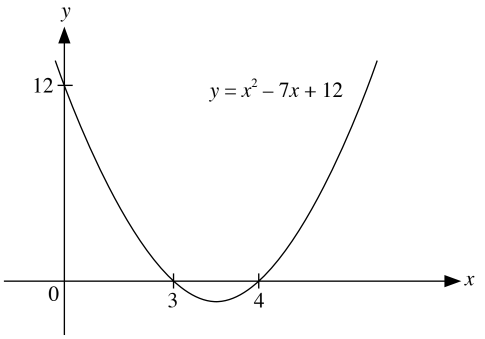
$x = 3$ $\qquad$ $x = 4$

$\therefore$ The graph cuts the x-axis at $(3, 0)$ and $(4, 0)$.

When $x = 0$,

$y = (-3)(-4) = 12$

$\therefore$ The graph cuts the y-axis at $(0, 12)$.



8. (i) $y = -x^2 + 5x - 4$

$y = -(x^2 - 5x + 4)$

The coefficient of x is -5. Half of this is $-\frac{5}{2}$.

$$\begin{aligned} y &= -(x^2-5x+4) \\ &= -\left[\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2\right] - \left(-\frac{5}{2}\right)^2 + 4\right] \\ &= -\left[\left(x-\frac{5}{2}\right)^2 - \frac{25}{4} + 4\right] \\ &= -\left[\left(x-\frac{5}{2}\right)^2 - \frac{9}{4}\right] \\ &= -\left(x-\frac{5}{2}\right)^2 + \frac{9}{4} \end{aligned}$$

(ii) Since the coefficient of x^2 is -1, the graph opens downwards.

When $y = 0$,

$$-\left(x-\frac{5}{2}\right)^2 + \frac{9}{4} = 0$$
$$\left(x-\frac{5}{2}\right)^2 = \frac{9}{4}$$

$x - \frac{5}{2} = \frac{3}{2}$ or $x - \frac{5}{2} = -\frac{3}{2}$

$x = \frac{3}{2} + \frac{5}{2}$ $\qquad$ $x = -\frac{3}{2} + \frac{5}{2}$

$= 4$ $\qquad$ $= 1$

$\therefore$ The graph cuts the x-axis at $(4, 0)$ and $(1, 0)$.

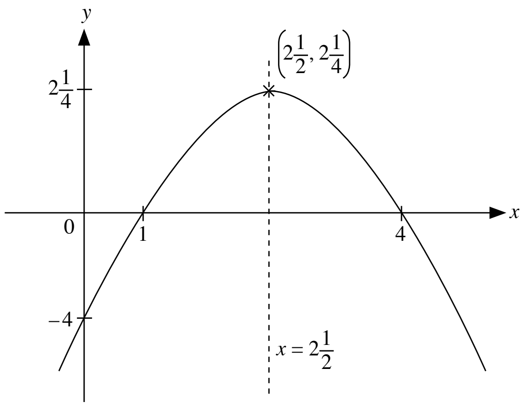
When $y = 0$,

$$y = -\left(0-\frac{5}{2}\right)^2 + \frac{9}{4} = -4$$

$\therefore$ The graph cuts the y-axis at $(0, -4)$.

The coordinates of the maximum point are $\left(2\frac{1}{2}, 2\frac{1}{4}\right)$.

The equation of the line of symmetry is $x = 2\frac{1}{2}$



9. Let the smaller positive integer be x, then the bigger positive integer is $x + 1$.

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{12}$$
$$\left(\frac{1}{x} - \frac{1}{x+1}\right) \times x(x+1) = \frac{1}{12} \times x(x+1)$$
$$\frac{1}{x} \times x(x+1) - \frac{1}{x+1} \times x(x+1) = \frac{1}{12} \times x(x+1)$$
$$(x+1) - x = \frac{1}{12}x(x+1)$$
$$x + 1 - x = \frac{1}{12}x^2 + \frac{1}{12}x$$
$$1 = \frac{1}{12}x^2 + \frac{1}{12}x$$
$$\frac{1}{12}x^2 + \frac{1}{12}x - 1 = 0$$
$$x^2 + x - 12 = 0$$
$$(x-3)(x+4) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = 3 \qquad x = -4 \text{ (rejected since } x \text{ is a positive integer)}$$

$x + 1 = 3 + 1 = 4$

$\therefore$ The two numbers are 3 and 4.

10. (i) In November 2013,

$$S\$x = A\$100$$
$$S\$650 = A\$\left(\frac{100}{x} \times 650\right)$$
$$= A\$\frac{65\,000}{x}$$

In December 2013,

$$S\$(x-5) = A\$100$$
$$S\$650 = A\$\left(\frac{100}{x-5} \times 650\right)$$
$$= A\$\frac{65\,000}{x-5}$$
$$\frac{65\,000}{x-5} - \frac{65\,000}{x} = 20$$

(ii)

$$\frac{65\,000}{x-5} - \frac{65\,000}{x} = 20$$
$$\frac{6500}{x-5} - \frac{6500}{x} = 2$$
$$\frac{6500}{x-5} \times x(x-5) - \frac{6500}{x} \times x(x-5) = 2 \times x(x-5)$$
$$6500x - 6500(x-5) = 2x(x-5)$$
$$6500x - 6500x + 32\,500 = 2x^2 - 10x$$
$$2x^2 - 10x - 32\,500 = 0$$
$$x^2 - 5x - 16\,250 = 0$$

Comparing $x^2 - 5x - 16\,250 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -5$ and $c = -16\,250$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-16\,250)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{65\,025}}{2}$$
$$= 130 \text{ or } -125 \text{ (rejected since } x > 0)$$

In November 2013,

$$A\$100 = S\$130$$
$$A\$1250 = S\$\left(\frac{130}{100} \times 1250\right)$$
$$= S\$1625$$

11. (i) Time taken by Farhan $= \frac{40}{x}$ h

(ii) Time taken by Khairul $= \frac{40}{x-30}$ h

(iii)

$$\frac{40}{x-30} - \frac{40}{x} = \frac{10}{60}$$
$$\frac{40}{x-30} - \frac{40}{x} = \frac{1}{6}$$
$$\left(\frac{40}{x-30} - \frac{40}{x}\right) \times x(x-30) = \frac{1}{6} \times x(x-30)$$
$$\frac{40}{x-30} \times x(x-30) - \frac{40}{x} \times x(x-30) = \frac{1}{6} \times x(x-30)$$
$$40x - 40(x-30) = \frac{1}{6}x(x-30)$$
$$40x - 40x + 1200 = \frac{1}{6}x^2 - 5x$$
$$1200 = \frac{1}{6}x^2 - 5x$$
$$\frac{1}{6}x^2 - 5x - 1200 = 0$$
$$x^2 - 30x - 7200 = 0 \text{ (Shown)}$$

(iv) $x^2 - 30 - 7200 = 0$

Comparing $x^2 - 30x - 7200 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -30$ and $c = -7200$.

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(-7200)}}{2(1)}$$
$$= \frac{30 \pm \sqrt{29\,700}}{2}$$
$$= 101.17 \text{ (to 2 d.p.) or } -71.17 \text{ (to 2 d.p.)}$$

(v) Since $x > 0$, $x = 101.17$

$$\text{Time taken by Khairul} = \frac{40}{101.17 - 30}$$
$$= 0.562 \text{ h}$$
$$= 33.7 \text{ minutes (to 3 s.f.)}$$

12. (i) In November 2013,

No. of litres of petrol that could be bought $= \frac{6000}{x}$

(ii) In December 2013,

No. of litres of petrol that could be bought $= \frac{6000}{x+10}$

(iii)
$$\frac{6000}{x} - \frac{6000}{x+10} = 1\frac{3}{7}$$
$$\frac{6000}{x} - \frac{6000}{x+10} = \frac{10}{7}$$
$$\left(\frac{6000}{x} - \frac{6000}{x+10}\right) \times x(x+10) = \frac{10}{7} \times x(x+10)$$
$$\frac{6000}{x} \times x(x+10) - \frac{6000}{x+10} \times x(x+10) = \frac{10}{7}x(x+10)$$
$$6000(x+10) - 6000x = \frac{10}{7}x(x+10)$$
$$6000x + 60\,000 - 6000x = \frac{10}{7}x^2 + \frac{100}{7}x$$
$$60\,000 = \frac{10}{7}x^2 + \frac{100}{7}x$$
$$\frac{10}{7}x^2 + \frac{100}{7}x - 60\,000 = 0$$
$$10x^2 + 100x - 420\,000 = 0$$
$$x^2 + 10x - 42\,000 = 0 \text{ (Shown)}$$

(iv) $x^2 + 10x - 42\,000 = 0$

Comparing $x^2 + 10x - 42\,000 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 10$ and $c = -42\,000$.

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(-42000)}}{2(1)}$$
$$= \frac{-10 \pm \sqrt{168\,100}}{2}$$
$$= 200 \text{ or } -210$$

(v) Since $x > 0$, $x = 200$

In December 2013,

No. of litres of petrol that could be bought with \$60

$$= \frac{6000}{200+10}$$
$$= 28.57 \text{ (to 4 s.f.)}$$

No. of litres of petrol that could be bought with \$34

$$= \frac{28.57}{60} \times 34$$
$$= 16.2 \text{ (to 3 s.f.)}$$

13. (i) Length of the floor covered by the tiles = $(35 - 2x)$ m

Breadth of the floor covered by the tiles = $(22 - 2x)$ m

(ii) Area of the floor covered by the tiles = 400 m^2

$$(35 - 2x)(22 - 2x) = 400$$
$$770 - 70x - 44x + 4x^2 = 400$$
$$4x^2 - 114x + 770 = 400$$
$$4x^2 - 114x + 370 = 0$$
$$2x^2 - 57x + 185 = 0 \text{ (Shown)}$$

(iii) $2x^2 - 57x + 185 = 0$

Comparing $2x^2 - 57x + 185 = 0$ with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -57$ and $c = 185$.

$$x = \frac{-(-57) \pm \sqrt{(-57)^2 - 4(2)(185)}}{2(2)}$$
$$= \frac{57 \pm \sqrt{1769}}{4}$$
$$= 24.76 \text{ (to 2 d.p.) or } 3.74 \text{ (to 2 d.p.)}$$

(iv) Width of the floor not covered by the tiles = 3.74 m

14. (a) (i) $60 + 25x - x^2 = 0$

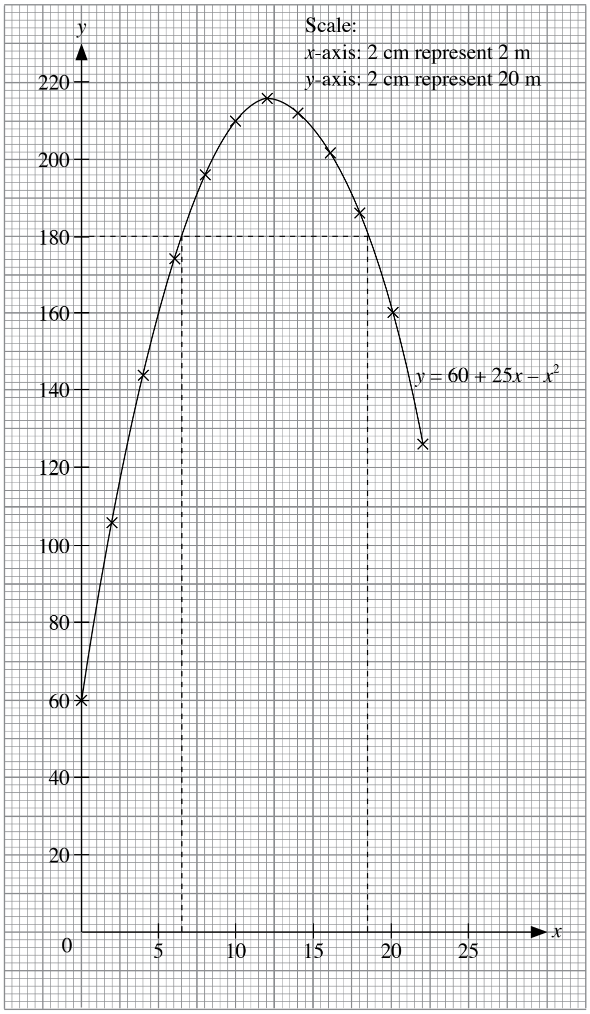
$-x^2 + 25x + 60 = 0$

Comparing $-x^2 + 25x + 60 = 0$ with $ax^2 + bx + c = 0$, we have $a = -1$, $b = 25$ and $c = 60$.

$$x = \frac{-25 \pm \sqrt{25^2 - 4(-1)(60)}}{2(-1)}$$
$$= \frac{-25 \pm \sqrt{865}}{-2}$$
$$= -2.2 \text{ (to 1 d.p.) or } 27.2 \text{ (to 1 d.p.)}$$

(ii) The positive solution in **(a)(i)** represents the horizontal distance of the stone from the foot of the tower.

(b)



(c) (i) From the graph,

The greatest height reached by the stone = 216 m

(ii) From the graph,

When the stone is 180 m above sea level, $y = 180$,

The horizontal distance from the foot of the tower = 6.5 m or 18.5 m

15. Let the time taken by the larger pipe to fill the tank be x minutes. Then the time taken by the smaller pipe to fill the tank is $(x + 5)$ minutes.

In 1 minute, the smaller pipe can fill $\frac{1}{x+5}$ of the tank while the bigger pipe can fill $\frac{1}{x}$ of the tank.

$$\begin{aligned}
\frac{1}{x} + \frac{1}{x+5} &= \frac{1}{11\frac{1}{9}} \\
\frac{1}{x} + \frac{1}{x+5} &= \frac{9}{100} \\
\left(\frac{1}{x} + \frac{1}{x+5}\right) \times x(x+5) &= \frac{9}{100} \times x(x+5) \\
\frac{1}{x} \times x(x+5) + \frac{1}{x+5} \times x(x+5) &= \frac{9}{100} x(x+5) \\
(x+5) + x &= \frac{9}{100} x(x+5) \\
2x + 5 &= \frac{9}{100} x^2 + \frac{9}{20} x \\
\frac{9}{100} x^2 - \frac{31}{20} x - 5 &= 0 \\
9x^2 - 155x - 500 &= 0
\end{aligned}$$

Comparing $9x^2 - 155x - 500 = 0$ with $ax^2 + bx + c = 0$, we have $a = 9$, $b = -155$ and $c = -500$.

$$\begin{aligned}
x &= \frac{-(-155) \pm \sqrt{(-155)^2 - 4(9)(-500)}}{2(9)} \\
&= \frac{155 \pm \sqrt{42\,025}}{18} \\
&= 20 \text{ or } -2\frac{7}{9} \text{ (rejected since time cannot be negative)}
\end{aligned}$$

$\therefore$ The time taken by the smaller pipe to fill the tank is 20 minutes while by the larger pipe is 25 minutes.

16. When the boat travels upstream, it moves against the current, hence the speed is $(x - 5)$ km/h.

When the boat travels back (downstream), it moves with the current, hence the speed is $(x + 5)$ km/h.

$$\begin{aligned}
\frac{12}{x-5} + \frac{12}{x+5} &= 1\frac{1}{2} \\
\frac{12}{x-5} + \frac{12}{x+5} &= \frac{3}{2} \\
\left(\frac{12}{x-5} + \frac{12}{x+5}\right) \times (x-5)(x+5) &= \frac{3}{2} \times (x-5)(x+5) \\
\frac{12}{x-5} \times (x-5)(x+5) + \frac{12}{x+5} \times (x-5)(x+5) &= \frac{3}{2}(x-5)(x+5) \\
12(x+5) + 12(x-5) &= \frac{3}{2}(x-5)(x+5) \\
12x + 60 + 12x - 60 &= \frac{3}{2}(x^2 - 25) \\
24x &= \frac{3}{2}x^2 - \frac{75}{2} \\
\frac{3}{2}x^2 - 24x - \frac{75}{2} &= 0 \\
3x^2 - 48x - 75 &= 0 \\
x^2 - 16x - 25 &= 0
\end{aligned}$$

Comparing $x^2 - 16x - 25 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -16$ and $c = -25$.

$$\begin{aligned}
x &= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(-25)}}{2(1)} \\
&= \frac{16 \pm \sqrt{356}}{2}
\end{aligned}$$

$= 17.4$ (to 3 s.f.) or -1.43 (to 3 s.f.) (rejected since speed is positive)

$\therefore$ Speed of boat in still water = 17.4 km/h

Challenge Yourself

1. Let x be the digit in the tens place and $(6 - x)$ be the digit in the ones place.

$$\begin{aligned}
x(6 - x) &= \frac{1}{3}(x \times 10 + 6 - x) \\
6x - x^2 &= \frac{1}{3}(9x + 6) \\
6x - x^2 &= 3x + 2 \\
x^2 - 3x + 2 &= 0 \\
(x-1)(x-2) &= 0 \\
x &= 1 \text{ or } x = 2
\end{aligned}$$

$\therefore$ The original number is 15 or 24.

2. Since $x = h$ and $x = k$ are the roots of the equation $ax^2 + bx + c = 0$,

$$\begin{aligned}
ax^2 + bx + c &= (x-h)(x-k) \\
ax^2 + bx + c &= x^2 - (h+k)x + hk \\
x^2 + \frac{b}{a}x + \frac{c}{a} &= x^2 - (h+k)x + hk
\end{aligned}$$

By comparing the coefficients,

$$\begin{aligned}
\frac{b}{a} &= -(h+k) \\
-\frac{b}{a} &= h + k \\
\frac{c}{a} &= hk
\end{aligned}$$

$\therefore h + k = -\frac{b}{a}$ and $hk = \frac{c}{a}$

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Chapter 2 Further Functions

TEACHING NOTES

Suggested Approach

This is an extension of the topic of functions that students have learnt previously in Book 2. Teachers should ensure that students are able to grasp the important concept of how to substitute a given domain into the equations. For example, for the function $f(x) = x^2 + 2x - 3$, a domain k would give: $f(k) = k^2 + 2k - 3$.

Section 2.1: Functions Involving Higher Order Expressions

Teachers should guide students with learning the technique to correctly substitute variables and form equations in order to find unknowns. As the pre-requisite for students to manipulate functions is the ability to solve simultaneous equations, teachers could revise the methods of solving a pair of simultaneous linear equations.

Section 2.2: Inverse Functions

Students may find it confusing to identify the relation that if $f(x) = y$, then $f^{-1}(y) = x$. Teachers should break it down and explain that in order to find the inverse function, x needs to be expressed in terms of y, i.e. make x the subject. Teachers could first revise the process of expressing one variable in terms of another and give some practice questions to the students. For example, for the equation $y = 3x + 2$, expressing x in terms of y would give $x = \frac{y-2}{3}$.

Practise Now (Page 43)

$f(3) = 6(3)^2 + 3(3) + 1$
$= 64$

Practise Now 1

(i) $f(3x) = 4(3x)^2 - 5(3x) + 2$
$= 36x^2 - 15x + 2$

(ii) $f(2x + 3) = 4(2x + 3)^2 - 5(2x + 3) + 2$
$= 4(4x^2 + 12x + 9) - 10x - 15 + 2$
$= 16x^2 + 48x + 36 - 10x - 13$
$= 16x^2 + 38x + 23$

(iii) $f(x^2 - 3) = 4(x^2 - 3)^2 - 5(x^2 - 3) + 2$
$= 4(x^4 - 6x^2 + 9) - 5x^2 + 15 + 2$
$= 4x^4 - 24x^2 + 36 - 5x^2 + 17$
$= 4x^4 - 29x^2 + 53$

Practise Now 2

$f(3) = a(3)^2 + 3b$
$= 9a + 3b$
$= 15 \quad \text{— (1)}$

$f(-2) = a(-2)^2 + b(-2)$
$= 4a - 2b$
$= 8$
$2b = 4a - 8$
$b = 2a - 4 \quad \text{— (2)}$

Subst. (2) into (1)
$9a + 3(2a - 4) = 15$
$9a + 6a - 12 = 15$
$15a = 27$
$a = 1\frac{4}{5}$

From (2), $b = 2\left(1\frac{4}{5}\right) - 4$
$= -\frac{2}{5}$

$\therefore a = 1\frac{4}{5}, b = -\frac{2}{5}$

$f(1) = 1\frac{4}{5}(1)^2 - \frac{2}{5}(1)$
$= 1\frac{2}{5}$

$f(-5) = 1\frac{4}{5}(-5)^2 - \frac{2}{5}(-5)$
$= 47$

Practise Now 3

$f(x) = 8x + 3$
Let $y = 8x + 3$
$f(x) = y$ and $f^{-1}(y) = x$
$x = \frac{1}{8}(y - 3)$
$\therefore f^{-1}(y) = \frac{1}{8}(y - 3)$
$f^{-1}(x) = \frac{1}{8}(x - 3)$

Practise Now 4

$f(x) = 7x - 4$
Let $y = 7x - 4$
$f(x) = y$ and $f^{-1}(y) = x$
$x = \frac{1}{7}(y + 4)$
$\therefore f^{-1}(y) = \frac{1}{7}(y + 4)$
Hence $f^{-1}(x) = \frac{1}{7}(x + 4)$

$f^{-1}(10) = \frac{1}{7}(10 + 4)$
$= 2$

$f^{-1}(-4) = \frac{1}{7}(-4 + 4)$
$= 0$

$f^{-1}\left(\frac{1}{7}\right) = \frac{1}{7}\left(\frac{1}{7} + 4\right)$
$= \frac{29}{49}$

Practise Now 5

$f(x) = \frac{2}{x - 5}$
Let $y = \frac{2}{x - 5}$
$f(x) = y$ and $f^{-1}(y) = x$
$y(x - 5) = 2$
$xy - 5y = 2$
$x = \frac{2 + 5y}{y}$
$= \frac{2}{y} + 5$
$\therefore f^{-1}(y) = \frac{2}{y} + 5$
Hence $f^{-1}(x) = \frac{2}{x} + 5$
f^{-1} is not defined when $x = 0$.

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$f^{-1}(6) = \frac{2}{6} + 5$

$= 5\frac{1}{3}$

$f^{-1}(-3) = \frac{2}{-3} + 5$

$= 4\frac{1}{3}$

$f^{-1}\left(\frac{1}{4}\right) = \frac{2}{\frac{1}{4}} + 5$

$= 13$

Practise Now 6

$f(x) = px + q$

$f^{-1}(3) = 6 \Rightarrow f(6) = 3$

Thus, $f(3) = 15$ and $f(6) = 3$.

$3p + q = 15$ — (1)

$6p + q = 3$ — (2)

(2) – (1): $3p = -12$

$p = -4$

Subst. $p = -4$ into (1): $3(-4) + q = 15$

$q = 15 + 12$

$= 27$

Exercise 2A

1. $g(x) = x^2 + 5$

(i) $g(a) = a^2 + 5$

(ii) $g(a + 1) = (a + 1)^2 + 5$

$= a^2 + 2a + 1 + 5$

$= a^2 + 2a + 6$

(iii) $g(a + 1) - g(a - 1)$

$= (a^2 + 2a + 6) - [(a - 1)^2 + 5]$

$= a^2 + 2a + 6 - (a^2 - 2a + 1 + 5)$

$= 4a$

2. $F(x) = \frac{1}{2}x(x + 1)$

(i) $F(1) = \frac{1}{2}(1)(1 + 1)$

$= 1$

(ii) $F(2) = \frac{1}{2}(2)(2 + 1)$

$= 3$

(iii) $F(3) = \frac{1}{2}(3)(3 + 1)$

$= 6$

(iv) $F(x - 1) = \frac{1}{2}(x - 1)[(x - 1) + 1]$

$= \frac{1}{2}x(x - 1)$

(v) $F(5) = \frac{1}{2}(5)(5 + 1)$

$= 15$

$F(4) = \frac{1}{2}(4)(4 + 1)$

$= 10$

$F(5) - F(4) = 15 - 10$

$= 5$

(vi) $F(7) = \frac{1}{2}(7)(7 + 1)$

$= 28$

$F(6) = \frac{1}{2}(6)(6 + 1)$

$= 21$

$F(7) - F(6) = 28 - 21$

$= 7$

(vii) $F(x + 1) = \frac{1}{2}(x + 1)[(x + 1) + 1]$

$= \frac{1}{2}(x + 1)(x + 2)$

(viii) $F(x - 1) = \frac{1}{2}x(x - 1)$

$F(x) - F(x - 1) = \frac{1}{2}x(x + 1) - \frac{1}{2}x(x - 1)$

$= \frac{1}{2}x[(x + 1) - (x - 1)]$

$= \frac{1}{2}x(2)$

$= x$

(ix) $F(x^2) = \frac{1}{2}x^2(x^2 + 1)$

3. $h(x) = x^2 - 5x + 4$

(i) $h(2a) = (2a)^2 - 5(2a) + 4$

$= 4a^2 - 10a + 4$

$h(a) = a^2 - 5a + 4$

$h(2a) - h(a)$

$= (4a^2 - 10a + 4) - (a^2 - 5a + 4)$

$= 3a^2 - 5a$

(ii) $a^2 - 5a + 4 = 0$

$(a - 4)(a - 1) = 0$

$a - 4 = 0$ or $a - 1 = 0$

$a = 4$ $a = 1$

(iii) $h(a^2) = (a^2)^2 - 5(a^2) + 4$

$= a^4 - 5a^2 + 4$

$h(a) = a^2 - 5a + 4$

$h(a^2) + h(a) = (a^4 - 5a^2 + 4) + (a^2 - 5a + 4)$

$= a^4 - 4a^2 - 5a + 8$

4. $g(x) = mx + c$

$g(1) = m + c = 5 \quad — (1)$

$g(5) = 5m + c = -4 \quad — (2)$

$(2) - (1)$: $4m = -9$

$m = -2\frac{1}{4}$

Subst. $m = -2\frac{1}{4}$ into (1): $-2\frac{1}{4} + c = 5$

$c = 5 + 2\frac{1}{4}$

$= 7\frac{1}{4}$

$\therefore m = -2\frac{1}{4}, c = 7\frac{1}{4}$

$g(3) = -2\frac{1}{4}(3) + 7\frac{1}{4}$

$= \frac{1}{2}$

$g(-4) = -2\frac{1}{4}(-4) + 7\frac{1}{4}$

$= 16\frac{1}{4}$

5. $h(x) = px^2 + qx + 2$

$h(2) = p(2)^2 + q(2) + 2$

$= 4p + 2q + 2$

$= 34$

$2p + q = 16$

$q = 16 - 2p \quad — (1)$

$h(-3) = p(-3)^2 + q(-3) + 2$

$= 9p - 3q + 2$

$= 29$

$9p - 3q = 27 \quad — (2)$

Subst. (1) into (2): $9p - 3(16 - 2p) = 27$

$9p + 6p - 48 = 27$

$15p = 75$

$p = 5$

From (1), $q = 16 - 2(5)$

$= 6$

$\therefore p = 5, q = 6$

$h(4) = 5(4)^2 + 6(4) + 2$

$= 106$

$h(-2) = 5(-2)^2 + 6(-2) + 2$

$= 10$

Exercise 2B

1. $f(x) = \frac{1}{4}x - 3$

Let $y = f(x)$

$f(x) = y$ and $f^{-1}(y) = x$

$y = \frac{1}{4}x - 3$

$x = 4y + 12$

$= 4(y + 3)$

$\therefore f^{-1}(y) = 4(y + 3)$

Hence $f^{-1}(x) = 4(x + 3)$

2. $f(x) = x - 7$

Let $y = f(x)$

$f(x) = y$ and $f^{-1}(y) = x$

$y = x - 7$

$x = y + 7$

$\therefore f^{-1}(y) = y + 7$

Hence $f^{-1} : x \mapsto x + 7$

$f^{-1}(3) = 3 + 7$

$= 10$

$f^{-1}(7) = 7 + 7$

$= 14$

$f^{-1}(-5) = -5 + 7$

$= 2$

$f^{-1}\left(\frac{1}{3}\right) = \frac{1}{3} + 7$

$= 7\frac{1}{3}$

3. $g(x) = 3x + 4$

Let $y = g(x)$

$g(x) = y$ and $g^{-1}(y) = x$

$y = 3x + 4$

$x = \frac{y-4}{3}$

$\therefore g^{-1}(y) = \frac{y-4}{3}$

Hence $g^{-1} : x \mapsto \frac{x-4}{3}$

$g^{-1}(3) = \frac{1}{3}(3 - 4)$

$= -\frac{1}{3}$

$g^{-1}(-4) = \frac{1}{3}(-4 - 4)$

$= -2\frac{2}{3}$

$g^{-1}\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{2} - 4\right)$

$= -1\frac{1}{6}$

$g^{-1}\left(-\frac{3}{4}\right) = \frac{1}{3}\left(-\frac{3}{4} - 4\right)$

$= -1\frac{7}{12}$

4. $h(x) = 5x + 6$

Let $y = h(x)$

$h(x) = y$ and $h^{-1}(y) = x$

$y = 5x + 6$

$x = \frac{y-6}{5}$

$\therefore h^{-1}(y) = \frac{y-6}{5}$

Hence $h^{-1} : x \mapsto \frac{x-6}{5}$

$h^{-1}(6) = \frac{6-6}{5}$

$= 0$

$h^{-1}(10) = \frac{10-6}{5}$

$= \frac{4}{5}$

$h^{-1}\left(-\frac{2}{5}\right) = \frac{-\frac{2}{5}-6}{5}$

$= -\frac{32}{25}$

$= -1\frac{7}{25}$

$h^{-1}\left(12\frac{1}{2}\right) = \frac{12\frac{1}{2}-6}{5}$

$= \frac{13}{10}$

$= 1\frac{3}{10}$

5. $f(x) = 8 - 3x$

Let $y = f(x)$

$f(x) = y$ and $f^{-1}(y) = x$

$y = 8 - 3x$

$x = \frac{8-y}{3}$

$\therefore f^{-1}(y) = \frac{8-y}{3}$

Hence $f^{-1}(x) = \frac{8-x}{3}$

$f^{-1}(9) = \frac{8-9}{3}$

$= -\frac{1}{3}$

$f^{-1}(-12) = \frac{8-(-12)}{3}$

$= 6\frac{2}{3}$

$f^{-1}\left(3\frac{1}{3}\right) = \frac{8-\left(3\frac{1}{2}\right)}{3}$

$= 1\frac{5}{9}$

$f^{-1}\left(-\frac{3}{16}\right) = \frac{8-\left(-\frac{3}{16}\right)}{3}$

$= 2\frac{35}{48}$

6. $g(x) = 6x - 8$

For $g(x) = 10$, $6x - 8 = 10$

$6x = 18$

$x = 3$

For $g(x) = 40$, $6x - 8 = 40$

$6x = 48$

$x = 8$

For $g(x) = -4$, $6x - 8 = -4$

$6x = 4$

$x = \frac{2}{3}$

For $g(x) = -6$, $6x - 8 = -6$

$6x = 2$

$x = \frac{1}{3}$

7. $f(x) = 7 - \frac{3}{5}x$

Let $y_1 = f(x)$

$f(x) = y_1$ and $f^{-1}(y_1) = x$

$y_1 = 7 - \frac{3}{5}x$

$x = \frac{7-y_1}{\frac{3}{5}}$

$= \frac{5(7-y_1)}{3}$

$\therefore f^{-1}(y_1) = \frac{5(7-y_1)}{3}$

Hence $f^{-1}(x) = \frac{5(7-x)}{3}$

$g(x) = \frac{1}{4}x - 6$

Let $y_2 = g(x)$

$g(x) = y_2$ and $g^{-1}(y_2) = x$

$y_2 = \frac{1}{4}x - 6$

$x = \frac{y_2+6}{\frac{1}{4}}$

$= 4(y_2 + 6)$

$\therefore g^{-1}(y_2) = 4(y_2 + 6)$

Hence $g^{-1}(x) = 4(x + 6)$

(i) $f^{-1}(3) = \frac{5(7-3)}{3}$

$= 6\frac{2}{3}$

(ii) $f^{-1}(-17) = \frac{5[7-(-17)]}{3}$

$= 40$

(iii) $g^{-1}(5) = 4(5 + 6)$

$= 44$

(iv) $g^{-1}(-6) = 4(-6 + 6)$

$= 0$

(v) $f^{-1}(2) = \dfrac{5(7-2)}{3}$

$= 8\dfrac{1}{3}$

$g^{-1}(1) = 4(1+6)$

$= 28$

$f^{-1}(2) + g^{-1}(1) = 8\dfrac{1}{3} + 28$

$= 36\dfrac{1}{3}$

(vi) $f^{-1}(4) = \dfrac{5(7-4)}{3}$

$= 5$

$g^{-1}(4) = 4(4+6)$

$= 40$

$f^{-1}(4) - g^{-1}(4) = 5 - 40$

$= -35$

8. $f(x) = ax^2 + bx$

$f(-2) = a(-2)^2 + b(-2)$

$= 4a - 2b$

$= 20$ — (1)

$f(4) = a(4)^2 + b(4)$

$= 16a + 4b$

$= 32$ — (2)

$(1) \times 2$: $8a - 4b = 40$ — (3)

$(2) + (3)$: $24a = 72$

$a = 3$

Subst. $a = 3$ into (1): $4(3) - 2b = 20$

$2b = -8$

$b = -4$

9. $h(x) = px^2 + qx$

$h(1) = p + q = 2$ — (1)

$h^{-1}(36) = 3$

$h(3) = p(3)^2 + 3q$

$= 9p + 3q$

$= 36$

$3p + q = 12$ — (2)

$(2) - (1)$: $2p = 10$

$p = 5$

Subst. $p = 5$ into (1)

$5 + q = 2$

$q = -3$

$\therefore p = 5, q = -3$

$h(-1) = 5(-1)^2 - 3(-1)$

$= 5 + 3$

$= 8$

$h(2) = 5(2)^2 - 3(2)$

$= 20 - 6$

$= 14$

10. $f(x) = \dfrac{5x}{2-4x}$

Let $y = f(x)$

$f(x) = y$ and $f^{-1}(y) = x$

$y = \dfrac{5x}{2-4x}$

$y(2-4x) = 5x$

$2y - 4xy = 5x$

$5x + 4xy = 2y$

$x(5+4y) = 2y$

$x = \dfrac{2y}{5+4y}$

$\therefore f^{-1}(y) = \dfrac{2y}{5+4y}$

Hence $f^{-1}(x) = \dfrac{2x}{5+4x}$

$f^{-1}(x)$ is not defined when $5 + 4x = 0$ i.e. $x = -\dfrac{5}{4}$

$f^{-1}(4) = \dfrac{2(4)}{5+4(4)}$

$= \dfrac{8}{21}$

$f^{-1}(-6) = \dfrac{2(-6)}{5+4(-6)}$

$= \dfrac{-12}{-19}$

$= \dfrac{12}{19}$

11. $f(x) = \dfrac{3x-1}{x-2}$

Let $y = f(x)$

$f(x) = y$ and $f^{-1}(y) = x$

$y = \dfrac{3x-1}{x-2}$

$y(x-2) = 3x - 1$

$xy - 2y - 3x = -1$

$x(y-3) = 2y - 1$

$x = \dfrac{2y-1}{y-3}$

$\therefore f^{-1}(y) = \dfrac{2y-1}{y-3}$

Hence $f^{-1}(x) = \dfrac{2x-1}{x-3}$

$f^{-1}(x)$ is not defined when $x - 3 = 0$ i.e. $x = 3$.

$f^{-1}(5) = \dfrac{2(5)-1}{5-3}$

$= \dfrac{9}{2}$

$= 4\dfrac{1}{2}$

$f^{-1}(7) = \dfrac{2(7)-1}{7-3}$

$= \dfrac{13}{4}$

$= 3\dfrac{1}{4}$

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12. $f(x) = ax + b$

$f(1) = a + b = 1 \quad — (1)$

$f^{-1}(5) = 2$

$f(2) = 2a + b = 5 \quad — (2)$

$(2) - (1)$: $a = 4$

Subst. $a = 4$ into (1):

$4 + b = 1$

$b = -3$

$\therefore a = 4, b = -3$

$f(x) = 4x - 3$

Let $f(x)$ be y

$f(x) = y$ and $f^{-1}(y) = x$

$y = 4x - 3$

$x = \dfrac{y+3}{4}$

$\therefore f^{-1}(y) = \dfrac{y+3}{4}$

Hence $f^{-1}(x) = \dfrac{x+3}{4}$

$f^{-1}(7) = \dfrac{7+3}{4}$

$= 2\dfrac{1}{2}$

$f^{-1}\left(-5\dfrac{1}{2}\right) = \dfrac{-5\frac{1}{2}+3}{4}$

$= -\dfrac{5}{8}$

13. $f(x) = ax + b$

$f(1) = a + b = 3 \quad — (1)$

$f^{-1}(7) = 5$

$f(5) = 5a + b = 7 \quad — (2)$

$(2) - (1)$: $4a = 4$

$a = 1$

Subst. $a = 1$ into (1):

$1 + b = 3$

$b = 2$

$\therefore a = 1, b = 2$

$f(x) = x + 2$

Let $f(x)$ be y

$f(x) = y$ and $f^{-1}(y) = x$

$y = x + 2$

$x = y - 2$

$\therefore f^{-1}(y) = y - 2$

Hence $f^{-1}(x) = x - 2$

14. $f(x) = px + q$

$f(1) = p + q = -5 \quad — (1)$

$f(-2) = -2p + q = -10 \quad — (2)$

$(1) - (2)$: $3p = 5$

$p = 1\dfrac{2}{3}$

Subst. $p = 1\dfrac{2}{3}$ into (1):

$1\dfrac{2}{3} + q = -5$

$q = -5 - 1\dfrac{2}{3}$

$= -6\dfrac{2}{3}$

$\therefore p = 1\dfrac{2}{3}, q = -6\dfrac{2}{3}$

$f(x) = 1\dfrac{2}{3}x - 6\dfrac{2}{3}$

Let $f(x)$ be y

$f(x) = y$ and $f^{-1}(y) = x$

$y = 1\dfrac{2}{3}x - 6\dfrac{2}{3}$

$x = \dfrac{y + 6\frac{2}{3}}{1\frac{2}{3}}$

$= \dfrac{3}{5}\left(y + \dfrac{2}{3}\right)$

$= \dfrac{3}{5}y + 4$

$\therefore f^{-1}(y) = \dfrac{3}{5}y + 4$

Hence $f^{-1}(x) = \dfrac{3}{5}x + 4$

15. $g(x) = mx + c$

$g^{-1}(-3) = 0$

$g(0) = c = -3$

$g^{-1}(1) = 2$

$g(2) = 2m + c = 1 \quad — (1)$

Subst. $c = -3$ into (1):

$2m - 3 = 1$

$2m = 4$

$m = 2$

$\therefore m = 2, c = -3$

$g(x) = 2x - 3$

Let $g(x)$ be y

$g(x) = y$ and $g^{-1}(y) = x$

$y = 2x - 3$

$x = \dfrac{y+3}{2}$

$\therefore g^{-1}(y) = \dfrac{y+3}{2}$

Hence $g^{-1}(x) = \dfrac{x+3}{2}$

$g(5) = 2(5) - 3$

$= 7$

$g^{-1}(4) = \dfrac{4+3}{2}$

$= 3\dfrac{1}{2}$

Review Exercise 2

1. $f(x) = 6x - 9$

(i) $f(3) = 6(3) - 9$
$= 9$
$f(-4) = 6(-4) - 9$
$= -33$

(ii) Let $f(x)$ be y
$f(x) = y$ and $f^{-1}(y) = x$
$y = 6x - 9$
$x = \dfrac{y+9}{6}$
$\therefore f^{-1}(y) = \dfrac{y+9}{6}$
Hence $f^{-1} : x \mapsto \dfrac{x+9}{6}$

(iii) $f^{-1}(3) = \dfrac{3+9}{6}$
$= 2$
$f^{-1}(-8) = \dfrac{-8+9}{6}$
$= \dfrac{1}{6}$

2. **(i)** $f(x) = 2px + 3q$
$f(2) = 4p + 3q = 17$ — (1)
$f^{-1}(5) = -1$
$f(-1) = -2p + 3q = 5$ — (2)
(1) – (2): $6p = 12$
$p = 2$
Subst. $p = 2$ into (1):
$4(2) + 3q = 17$
$3q = 17 - 8$
$= 9$
$q = 3$
$\therefore p = 2, q = 3$
$f(x) = 4x + 9$
$f(4) = 4(4) + 9$
$= 25$
$f(-5) = 4(-5) + 9$
$= -11$

(ii) Let $f(x)$ be y
$f(x) = y$ and $f^{-1}(y) = x$
$y = 4x + 9$
$x = \dfrac{y-9}{4}$
$\therefore f^{-1}(y) = \dfrac{y-9}{4}$
Hence $f^{-1}(x) = \dfrac{x-9}{4}$
$f^{-1}(10) = \dfrac{10-9}{4}$
$= \dfrac{1}{4}$
$f^{-1}(-16) = \dfrac{-16-9}{4}$
$= -6\dfrac{1}{4}$

3. $f(x) = x^2 - 3x + 5$

(i) $f(a) = a^2 - 3a + 5$

(ii) $f(2a) = (2a)^2 - 3(2a) + 5$
$= 4a^2 - 6a + 5$

(iii) $f(a+3) = (a+3)^2 - 3(a+3) + 5$
$= a^2 + 6a + 9 - 3a - 9 + 5$
$= a^2 + 3a + 5$

(iv) $f(a^2) = (a^2)^2 - 3(a^2) + 5$
$= a^4 - 3a^2 + 5$

(v) $f(a^2 + 2) = (a^2 + 2)^2 - 3(a^2 + 2) + 5$
$= a^4 + 4a^2 + 4 - 3a^2 - 6 + 5$
$= a^4 + a^2 + 3$

4. $f(x) = 10x - 7$
$g(x) = 3 - \dfrac{1}{4}x$

(i) **(a)** $f(2) = 10(2) - 7$
$= 13$
$g(4) = 3 - \dfrac{1}{4}(4)$
$= 2$
$f(2) + g(4) = 13 + 2$
$= 15$

(b) $f(-1) = 10(-1) - 7$
$= -17$
$g(8) = 3 - \dfrac{1}{4}(8)$
$= 1$
$f(-1) - g(8) = -17 - 1$
$= -18$

(c) $3f(3) = 3[10(3) - 7]$
$= 3(23)$
$= 69$
$2g(3) = 2\left[3 - \dfrac{1}{4}(3)\right]$
$= 2\left(2\dfrac{1}{4}\right)$
$= 4\dfrac{1}{2}$
$3f(3) - 2g(3) = 69 - 4\dfrac{1}{2}$
$= 64\dfrac{1}{2}$

(ii) For $f(x) = 2g(x)$,
$10x - 7 = 2\left[3 - \dfrac{1}{4}x\right]$
$10x - 7 = 6 - \dfrac{1}{2}x$
$10\dfrac{1}{2}x = 13$
$x = 1\dfrac{5}{21}$

(iii) For $f(2p) = g(4p)$,

$$10(2p) - 7 = 3 - \frac{1}{4}(4p)$$
$$20p - 7 = 3 - p$$
$$21p = 10$$
$$p = \frac{10}{21}$$

(iv) Let $y_1 = f(x)$

$f(x) = y_1$ and $f^{-1}(y_1) = x$

$y_1 = 10x - 7$

$$x = \frac{y_1 + 7}{10}$$

$$\therefore f^{-1}(y_1) = \frac{y_1 + 7}{10}$$

Hence $f^{-1}(x) = \dfrac{x + 7}{10}$

Let $y_2 = g(x)$

$g(x) = y_2$ and $g^{-1}(y_2) = x$

$$y_2 = 3 - \frac{1}{4}x$$

$$x = \frac{3 - y_2}{\frac{1}{4}}$$
$$= 4(3 - y_2)$$
$$= 12 - 4y_2$$

$\therefore g^{-1}(y_2) = 12 - 4y_2$

Hence $g^{-1}(x) = 12 - 4x$

(v) $f^{-1}(x) = \dfrac{x + 7}{10}$

$$f^{-1}(3) = \frac{3 + 7}{10}$$
$$= 1$$

$g^{-1}(x) = 12 - 4x$

$$g^{-1}(4) = 12 - 4(4)$$
$$= -4$$

$$f^{-1}(3) - g^{-1}(4) = 1 - (-4)$$
$$= 5$$

5. $g(x) = px + q$

$g(5) = 5p + q = 4$ — (1)

$g^{-1}(3) = 17$

$g(17) = 17p + q = 3$ — (2)

(2) – (1): $12p = -1$

$$p = -\frac{1}{12}$$

Subst. $p = -\dfrac{1}{12}$ into (1):

$$5\left(-\frac{1}{12}\right) + q = 4$$
$$q = 4\frac{5}{12}$$

$\therefore p = -\dfrac{1}{12}, q = 4\dfrac{5}{12}$

$$g(x) = -\frac{1}{12}x + 4\frac{5}{12}$$

Let $g(x)$ be y

$g(x) = y$ and $g^{-1}(y) = x$

$$y = -\frac{1}{12}x + 4\frac{5}{12}$$

$$x = \frac{4\frac{5}{12} - y}{\frac{1}{12}}$$
$$= 53 - 12y$$

$\therefore g^{-1}(y) = 53 - 12y$

Hence $g^{-1}(x) = 53 - 12x$

6. **(i)** $h(x) = x^2 - 7x + 6$

$$h(a^2) = (a^2)^2 - 7(a^2) + 6$$
$$= a^4 - 7a^2 + 6$$
$$h(a) = a^2 - 7a + 6$$

$h(a^2) - h(a)$
$= (a^4 - 7a^2 + 6) - (a^2 - 7a + 6)$
$= a^4 - 8a^2 + 7a$

(ii) For $h(k) = 0$, $k^2 - 7k + 6 = 0$

$(k - 6)(k - 1) = 0$

$k - 6 = 0$ or $k - 1 = 0$

$k = 6$ $\quad$ $k = 1$

$\therefore k = 6$ or $k = 1$

(iii) $h(2a) = (2a)^2 - 7(2a) + 6$

$$= 4a^2 - 14a + 6$$

$$h(a + 1) = (a + 1)^2 - 7(a + 1) + 6$$
$$= a^2 + 2a + 1 - 7a - 7 + 6$$
$$= a^2 - 5a$$

$h(2a) - h(a + 1)$
$= (4a^2 - 14a + 6) - (a^2 - 5a)$
$= 3a^2 - 9a + 6$

7. $h(x) = 5x^2 + 2x + 1$

(i) $h(2x) = 5(2x)^2 + 2(2x) + 1$

$$= 5(4x^2) + 4x + 1$$
$$= 20x^2 + 4x + 1$$

(ii) $h(x + 1) = 5(x + 1)^2 + 2(x + 1) + 1$

$$= 5(x^2 + 2x + 1) + 2x + 2 + 1$$
$$= 5x^2 + 10x + 5 + 2x + 2 + 1$$
$$= 5x^2 + 12x + 8$$

(iii) $h(x + 1) = 5x^2 + 12x + 8$

$$h(x - 1) = 5(x - 1)^2 + 2(x - 1) + 1$$
$$= 5(x^2 - 2x + 1) + 2x - 2 + 1$$
$$= 5x^2 - 10x + 5 + 2x - 2 + 1$$
$$= 5x^2 - 8x + 4$$

$h(x + 1) - h(x - 1)$
$= (5x^2 + 12x + 8) - (5x^2 - 8x + 4)$
$= 20x + 4$

Challenge Yourself

Domain of inverse function = range of function
$= \{2, 6, 7\}$
Range of inverse function = Domain of function
$= \{-3, 1, 3, 5\}$

Since the element 2 in the domain has two images, –3 and 5, the inverse is not a function.

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Chapter 3 Linear Inequalities

TEACHING NOTES

Suggested Approach

Teachers can begin this chapter by linking to students' prior knowledge of solving simple linear equations in one variable. By replacing the equality sign with inequality signs, teachers can emphasise that the variable can take more than one value.

Since students have learnt simple inequalities in the form $ax > b$, $ax \geqslant b$, $ax < b$ and $ax \leqslant b$, where a and b are integers, teachers can remind students that "solving an inequality" involves finding all the solutions that satisfy the inequality, which is akin to solving a simple linear equation. To help students better understand linear inequalities and see how it can be applied in our daily lives, teachers may get students to give some real-life examples where inequalities are involved.

Section 3.1: Inequalities

Teachers should recap with students how to solve simple linear inequalities and to represent the solution on a number line. The use of number lines will help students to visualise and understand the meanings of $<$, $>$, $\leqslant$ and $\geqslant$ (see Investigation: Inequalities). Teachers should guide students when solving linear inequalities that involve reversing the inequality signs when multiplying or dividing the inequalities by a negative number as this may be confusing to them. Teachers can use actual numbers to explain how the signs will change when multiplying and dividing by a negative number. Teachers can get students to explore the relationship between the solution of an inequality and that of the corresponding linear equation (see Thinking Time on page 57 of the textbook).

Section 3.2: Problem Solving involving Inequalities

In problem solving involving inequalities, students must work on their mathematical process of interpretation and thinking skills. Teachers can guide students to understand terms such as 'at most', 'at least', 'not more than' and 'not lesser than' and how to form an inequality and solve it to find the answer to the problem.

Section 3.3: Solving Simultaneous Linear Inequalities

In solving simultaneous linear inequalities, teachers should guide students on how to solve two linear inequalities separately and to consider only the common solutions of the inequalities after representing both inequalities on a number line. The use of number lines in this section is essential to help students visualise the solving of simultaneous linear inequalities. Teachers should highlight to students that there may not always be a solution to the simultaneous linear inequalities (see Worked Example 8 on page 63 of the textbook).

Section 3.4: Limits of Accuracy

Teachers should highlight to students that the concept of limits of accuracy is somewhat an application of rounding off. Students should be familiar with rounding up a number with the digit '5', for instance 6.75 is often taken to be 6.8 (correct to the nearest 1 decimal place). In theory, rounding up 6.75 to 6.8 has an error of 0.5. Rounding it down to 6.7 would also give an error of 0.5. Teachers should reinforce that for the case of limits of accuracy, the upper bound should be taken as the first number that would be rounded up. For instance, the upper and lower bound for 20 cm (correct to the nearest cm) would be 20.5 cm and 19.5 cm respectively. Teachers can impress upon students that the upper bound is not 20.4 cm, as numbers such as 20.45 and 20.488 will not be accounted for.

Challenge Yourself

Since $z = \frac{x}{y}$, teachers can ask students to find the greatest and least possible values of $\frac{x}{y}$ to find the limits in which z must lie.

Teachers can get the students to first factorise the denominator, $x^2 - 14x + 49$ and observe that value of the denominator is more than zero.

WORKED SOLUTIONS

Investigation (Properties of Inequalities)

1.

Multiplication by a *negative* number on both sides of the inequality $10 > 6$	LHS $= 10 \times (-5)$ $= -50$ RHS $= 6 \times (-5)$ $= -30$	$-50 < -30$	Yes	If $x > y$ and $d < 0$, then $dx < dy$.
Division by a *negative* number on both sides of the inequality $10 > 6$	LHS $= 10 \div (-5)$ $= -2$ RHS $= 6 \div (-5)$ $= -1.2$	$-2 < -1.2$	Yes	If $x > y$ and $d < 0$, then $\frac{x}{d} < \frac{y}{d}$.

Table 3.1

2. Yes, the conclusions drawn from Table 3.1 apply to $10 \geqslant 6$.
   The following conclusions hold for $x \geqslant y$:
   - If $x \geqslant y$ and $d < 0$, then $dx \leqslant dy$ and $\frac{x}{d} \leqslant \frac{y}{d}$.

   The following conclusions hold for $x < y$:
   - If $x < y$ and $d < 0$, then $dx > dy$ and $\frac{x}{d} > \frac{y}{d}$.

   The following conclusions hold for $x \leqslant y$:
   - If $x \leqslant y$ and $d < 0$, then $dx \geqslant dy$ and $\frac{x}{d} \geqslant \frac{y}{d}$.

Investigation (Inequalities)

1. **(a)** **(ii)** $6 + 2 = 8 < 12 + 2 = 14$
   (iii) $6 - 4 = 2 < 12 - 4 = 8$
   (b) If $6 < 12$ and a is a real number, then $6 + a < 12 + a$ and $6 - a < 12 - a$.
   (c) If $12 > 6$ and a is a real number, then $12 + a > 6 + a$ and $12 - a > 6 - a$.
2. **(a)** **(i)** $-6 < 12$
   (ii) $-6 + 2 = -4 < 12 + 2 = 14$
   (iii) $-6 - 4 = -10 < 12 - 4 = 8$
   (b) If $-6 < 12$ and a is a real number, then $-6 + a < 12 + a$ and $-6 - a < 12 - a$.
   (c) If $12 > -6$ and a is a real number, then $12 + a > -6 + a$ and $12 - a > -6 - a$.
3. **(a)** **(i)** $6 > -12$
   (ii) $6 + 2 = 8 > -12 + 2 = -10$
   (iii) $6 - 4 = 2 > -12 - 4 = -16$
   (b) The addition or subtraction of a positive number does not change the inequality sign.
4. Yes, the conclusion applies.

Journal Writing (Page 56)

Other real life applications of inequalities:
BMI, grades, credit limits, text messaging, travel times, weight limits, financial planning, wages and taxes, temperature limits, height limits for vehicles, etc.
Teachers should note that the list is not exhaustive.

Thinking Time (Page 57)

1. $ax + b = c$, where a, b and c are constants and $a > 0$
   Step 1: Arrange the terms such that the constants are all on one side of the equation, i.e. $ax = c - b$.
   Step 2: Divide both sides by a to solve for x. The equality sign remains.
   The steps will not change if $a < 0$.
2. $ax + b > c$, where a, b and c are constants and $a > 0$
   Step 1: Arrange the terms such that the constants are all on one side of the inequality, i.e. $ax > c - b$.
   Step 2: Divide both sides by a to solve for x. The inequality sign ($>$) remains.
   The steps will change if $a < 0$ such that the inequality sign will change to $<$.
3. $ax + b \geqslant c$, where a, b and c are constants and $a > 0$
   Step 1: Arrange the terms such that the constants are all on one side of the inequality, i.e. $ax \geqslant c - b$.
   Step 2: Divide both sides by a to solve for x. The inequality sign ($\geqslant$) remains.
   The step will change if $a < 0$ such that the inequality sign will change to $\leqslant$.
4. The solutions of $ax + b > c$ and $ax + b < c$ do not include the solution of its corresponding linear equation $ax + b = c$.
   The solutions of $ax + b \geqslant c$ and $ax + b \leqslant c$ include the solution of its corresponding linear equation $ax + b = c$.

Performance Task (Page 63)

Postage rates for parcels to Thailand

Mass (m g)	Postage (cents)
$0 < m \leqslant 20$	65
$20 < m \leqslant 30$	90
$30 < m \leqslant 40$	115
$40 < m \leqslant 50$	140
$50 < m \leqslant 60$	165

Postage rates for parcels to New Zealand and United Kingdom

Mass (m g)	Postage (cents)
$0 < m \leqslant 20$	110
$20 < m \leqslant 30$	145
$30 < m \leqslant 40$	180
$40 < m \leqslant 50$	215
$50 < m \leqslant 60$	250

The information is taken from
http://www.singpost.com/send-documentparcel/postage-rates.html#Airmail

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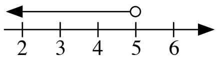
Practise Now 1

1. **(a)** $-6x > -30$

$6x < 30$

$x < \dfrac{30}{6}$

$\therefore x < 5$

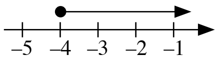


(b) $-8x \leqslant 32$

$8x \geqslant -32$

$x \geqslant -\dfrac{32}{8}$

$\therefore x \geqslant -4$



2. $-3x < -13$

$3x > 13$

$x > \dfrac{13}{3}$

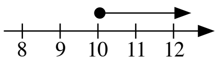
$x > 4\dfrac{1}{3}$

$\therefore$ The smallest integer value of x is 5.

Practise Now 2

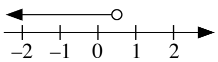
(a)

$$\begin{aligned} x - 3 &\geqslant 7 \\ x - 3 + 3 &\geqslant 7 + 3 \\ x &\geqslant 10 \end{aligned}$$



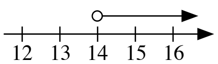
(b)

$$\begin{aligned} -2y + 4 &> 3 \\ -2y + 4 - 4 &> 3 - 4 \\ -2y &> -1 \\ 2y &< 1 \\ \frac{2y}{2} &< \frac{1}{2} \\ y &< \frac{1}{2} \end{aligned}$$



Practise Now 3

$$\begin{aligned} 5 - x &< -9 \\ 5 - 5 - x &< -9 - 5 \\ -x &< -14 \\ x &> 14 \end{aligned}$$



(i) Smallest prime value of x is 17

(ii) Smallest perfect cube value of x is $27 = 3^3$

Practise Now 4

1. **(a)**

$$\begin{aligned} 15x + 1 &< 5(3 + x) \\ 15x + 1 &< 15 + 5x \\ 15x + 1 - 5x &< 15 + 5x - 5x \\ 10x + 1 &< 15 \\ 10x + 1 - 1 &< 15 - 1 \\ 10x &< 14 \\ x &< \frac{14}{10} \\ x &< 1\frac{2}{5} \end{aligned}$$

(b)

$$\begin{aligned} \frac{16y}{3} &\geqslant \frac{y+1}{2} \\ 3 \times 2 \times \frac{16y}{3} &\geqslant 3 \times 2 \times \frac{y+1}{2} \\ 32y &\geqslant 3(y + 1) \\ 32y &\geqslant 3y + 3 \\ 32y - 3y &\geqslant 3y + 3 - 3y \\ 29y &\geqslant 3 \\ y &\geqslant \frac{3}{29} \end{aligned}$$

(c)

$$\begin{aligned} \frac{1}{2}(z - 4) &\leqslant \frac{1}{3}(z + 1) + 2 \\ 2 \times 3 \times \frac{1}{2}(z - 4) &\leqslant 2 \times 3 \times \left[\frac{1}{3}(z+1) + 2\right] \\ 3(z - 4) &\leqslant 2(z + 1) + 6(2) \\ 3z - 12 &\leqslant 2z + 2 + 12 \\ 3z - 12 &\leqslant 2z + 14 \\ 3z - 12 - 2z &\leqslant 2z + 14 - 2z \\ z - 12 &\leqslant 14 \\ z - 12 + 12 &\leqslant 14 + 12 \\ z &\leqslant 26 \end{aligned}$$

2.

$$\begin{aligned} \frac{3}{4}(p - 2) + \frac{1}{2} &> \frac{1}{2}(p - 1) \\ 4 \times \left[\frac{3}{4}(p - 2) + \frac{1}{2}\right] &> 4 \times \frac{1}{2}(p - 1) \\ 3(p - 2) + 2 &> 2(p - 1) \\ 3p - 6 + 2 &> 2p - 2 \\ 3p - 4 &> 2p - 2 \\ 3p - 4 - 2p &> 2p - 2 - 2p \\ p - 4 &> -2 \\ p - 4 + 4 &> -2 + 4 \\ p &> 2 \end{aligned}$$

Smallest perfect square value of p is $4 = 2^2$

Practise Now 5

Let x be the marks scored by Priya in her first quiz.

$$\begin{aligned}
\frac{x+76+89}{3} &\geqslant 75\\
3\times\frac{x+76+89}{3} &\geqslant 3\times 75\\
x+76+89 &\geqslant 225\\
x+165 &\geqslant 225\\
x+165-165 &\geqslant 225-165\\
x &\geqslant 60
\end{aligned}$$

$\therefore$ Priya must have scored at least 60 marks for her first quiz.

Practise Now 6

Let x and y be the number of \$10 notes and \$5 notes respectively.

$x + y = 12$ — (1)

$x \times 10 + y \times 5 < 95$

i.e. $10x + 5y < 95$ — (2)

From (1),

$y = 12 - x$ — (3)

Substitute (3) into (2):

$$\begin{aligned}
10x+5(12-x) &< 95\\
10x+60-5x &< 95\\
5x+60 &< 95\\
5x+60-60 &< 95-60\\
5x &< 35\\
x &< \frac{35}{5}\\
x &< 7
\end{aligned}$$

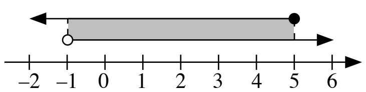
$\therefore$ The maximum number of \$10 notes that Vishal has is 6.

Practise Now 7

Solving the two linear inequalities separately,

$$\begin{aligned}
2x-3 &\leqslant 7 & &\text{and} & 2x+1 &> -3x-4\\
2x-3+3 &\leqslant 7+3 & & & 2x+1+3x &> -3x-4+3x\\
2x &\leqslant 10 & & & 5x+1 &> -4\\
x &\leqslant \frac{10}{2} & & & 5x+1-1 &> -4-1\\
x &\leqslant 5 & & & 5x &> -5\\
 & & & & x &> \frac{-5}{5}\\
 & & & & x &> -1
\end{aligned}$$

Representing $x \leqslant 5$ and $x > -1$ on a number line,



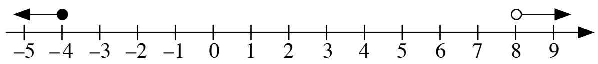
$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-1 < x \leqslant 5$.

Practise Now 8

1. Solving the two linear inequalities separately,

$$\begin{aligned}
8x+13 &\leqslant 4x-3 & &\text{and} & 4x-3 &< 5x-11\\
8x+13-4x &\leqslant 4x-3-4x & & & 4x-3-5x &< 5x-11-5x\\
4x+13 &\leqslant -3 & & & -x-3 &< -11\\
4x+13-13 &\leqslant -3-13 & & & -x-3+3 &< -11+3\\
4x &\leqslant -16 & & & -x &< -8\\
x &\leqslant \frac{-16}{4} & & & x &> 8\\
x &\leqslant -4
\end{aligned}$$

Representing $x \leqslant -4$ and $x > 8$ on a number line,

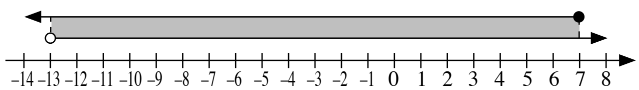


$\therefore$ The simultaneous linear inequalities have no solution.

2. Solving the two linear inequalities separately,

$$\begin{aligned}
\frac{y-2}{3} &< \frac{2y+1}{5} & &\text{and} & \frac{2y+1}{5} &\leqslant 3\\
3\times5\times\frac{y-2}{3} &< 3\times5\times\frac{2y+1}{5} & & & 5\times\frac{2y+1}{5} &\leqslant 5\times3\\
5(y-2) &< 3(2y+1) & & & 2y+1 &\leqslant 15\\
5y-10 &< 6y+3 & & & 2y+1-1 &\leqslant 15-1\\
5y-10-6y &< 6y+3-6y & & & 2y &\leqslant 14\\
-y-10 &< 3 & & & y &\leqslant \frac{14}{2}\\
-y-10+10 &< 3+10 & & & y &\leqslant 7\\
-y &< 13\\
y &> -13
\end{aligned}$$

Representing $y > -13$ and $y \leqslant 7$ on a number line,



$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-13 < x \leqslant 7$.

Practise Now (Page 65)

(a) 375, 325

(b) 1800.5, 1799.5

(c) 85, 75

Practise Now 9

Upper bound limits of 5 m = 5.5

$\therefore$ Upper bound of perimeter $= 4(5.5)$

$= 22$ m

Practise Now 10

Maximum length of reel = 25.5 m
Minimum length of reel = 24.5 m
Maximum length used for scarf = 6.5 m
Minimum length used for scarf = 5.5 m
Maximum amount of wool remaining
= maximum length of reel – minimum length used
= 25.5 – 5.5
= 20 m
Minimum amount of wool remaining
= minimum length of reel – maximum length used
= 24.5 – 6.5
= 18 m

Practise Now 11

Maximum value of width = 40.5 m
Minimum value of width = 39.5 m
Maximum value of length = 60.5 m
Minimum value of length = 59.5 m

$$\text{Maximum area} = 40.5 \times 60.5 = 2450.25 \text{ m}^2$$

$$\text{Minimum area} = 39.5 \times 59.5 = 2350.25 \text{ m}^2$$

$\therefore$ $2350.25 \text{ m}^2 \leqslant \text{area} < 2450.25 \text{ m}^2$

Practise Now 12

For 1 litre of water,
Upper bound area = 20.5 m^2
Lower bound area = 19.5 m^2
For the farm,
Upper bound area = 60.5 m^2
Lower bound area = 59.5 m^2

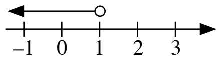
$$\therefore \text{Maximum amount of water needed} = \frac{60.5}{19.5} = 3.10 \text{ litres (to 3 s.f.)}$$

$$\text{Minimum amount of water needed} = \frac{59.5}{20.5} = 2.90 \text{ litres (to 3 s.f.)}$$

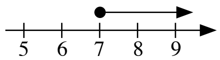
Exercise 3A

1. **(a)** If $x > y$, then $-6x < -6y$
   (b) If $x < y$, then $\frac{x}{-30} > \frac{y}{-30}$
   (c) If $x \geqslant y$, then $-4x \leqslant -4y$
   (d) If $x \leqslant y$, then $\frac{x}{-10} \geqslant \frac{y}{-10}$
   (e) $5 + h < 7 + h$
   (f) $5 - k < 7 - k$

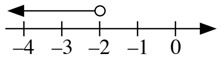
2. **(a)**
$$a + 2 < 3$$
$$a + 2 - 2 < 3 - 2$$
$$a < 1$$



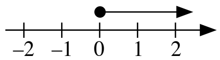
(b)
$$b - 3 \geqslant 4$$
$$b - 3 + 3 \geqslant 4 + 3$$
$$b \geqslant 7$$



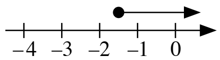
(c)
$$-c + 3 > 5$$
$$-c + 3 - 3 > 5 - 3$$
$$-c > 2$$
$$c < -2$$



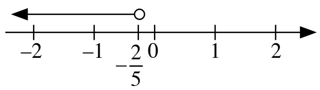
(d)
$$4 - d \leqslant 4$$
$$4 - d - 4 \leqslant 4 - 4$$
$$-d \leqslant 0$$
$$d \geqslant 0$$



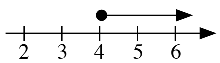
(e)
$$-2e - 1 \leqslant 2$$
$$-2e - 1 + 1 \leqslant 2 + 1$$
$$-2e \leqslant 3$$
$$e \geqslant \frac{3}{-2}$$
$$e \geqslant -1\frac{1}{2}$$



(f)
$$2 + 5f < 0$$
$$2 + 5f - 2 < 0 - 2$$
$$5f < -2$$
$$f < -\frac{2}{5}$$



(g)
$$g - 7 \geqslant 1 - g$$
$$g - 7 + g \geqslant 1 - g + g$$
$$2g - 7 \geqslant 1$$
$$2g - 7 + 7 \geqslant 1 + 7$$
$$2g \geqslant 8$$
$$g \geqslant \frac{8}{2}$$
$$g \geqslant 4$$

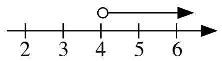


(h) $5h > 4(h + 1)$

$5h > 4h + 4$

$5h - 4h > 4h + 4 - 4h$

$h > 4$



(i) $8j + 3 < 2(7 - j)$

$8j + 3 < 14 - 2j$

$8j + 3 + 2j < 14 - 2j + 2j$

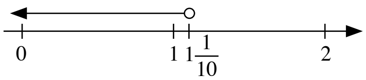
$10j + 3 < 14$

$10j + 3 - 3 < 14 - 3$

$10j < 11$

$j < \frac{11}{10}$

$j < 1\frac{1}{10}$



(j) $4k + 5 \geqslant 2(-2k)$

$4k + 5 \geqslant -4k$

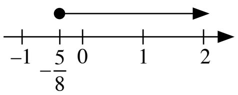
$4k + 5 + 4k \geqslant -4k + 4k$

$8k + 5 \geqslant 0$

$8k + 5 - 5 \geqslant 0 - 5$

$8k \geqslant -5$

$k \geqslant -\frac{5}{8}$



(k) $2(m - 5) \leqslant 2 - m$

$2m - 10 \leqslant 2 - m$

$2m - 10 + m \leqslant 2 - m + m$

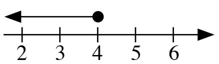
$3m - 10 \leqslant 2$

$3m - 10 + 10 \leqslant 2 + 10$

$3m \leqslant 12$

$m \leqslant \frac{12}{3}$

$m \leqslant 4$



(l) $3(1 - 4n) > 8 - 7n$

$3 - 12n > 8 - 7n$

$3 - 12n + 7n > 8 - 7n + 7n$

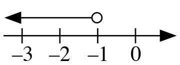
$3 - 5n > 8$

$3 - 5n - 3 > 8 - 3$

$-5n > 5$

$n < \frac{5}{-5}$

$n < -1$



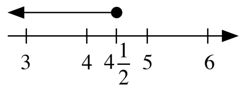
3. $7 + 2x \leqslant 16$

$7 + 2x - 7 \leqslant 16 - 7$

$2x \leqslant 9$

$x \leqslant \frac{9}{2}$

$x \leqslant 4\frac{1}{2}$



(i) Largest integer value of x is 4

(ii) Largest perfect square value of x is $4 = 2^2$

4. $3 - 4x > 3x - 18$

$3 - 4x - 3 > 3x - 18 - 3$

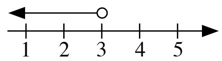
$-4x > 3x - 21$

$-4x - 3x > 3x - 21 - 3x$

$-7x > -21$

$x < \frac{-21}{-7}$

$x < 3$



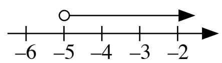
(i) Prime value of x is 2

(ii) Yes, $x = 0$ is less than 3.

5. (a) $-5x < 25$

$x > \frac{25}{-5}$

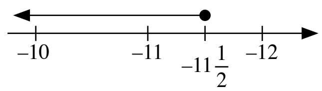
$x > -5$



(b) $-12x \geqslant 138$

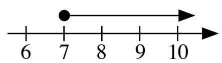
$x \leqslant \frac{138}{-12}$

$x \leqslant -11\frac{1}{2}$



(c) $-y \leqslant -7$

$y \geqslant 7$

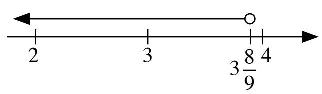


(d) $-9y > -35$

$9y < 35$

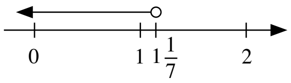
$y < \frac{35}{9}$

$y < 3\frac{8}{9}$



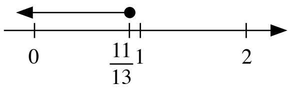
(e)

$$\begin{aligned}
4(p+1) &< -3(p-4)\\
4p+4 &< -3p+12\\
4p+4+3p &< -3p+12+3p\\
7p+4 &< 12\\
7p+4-4 &< 12-4\\
7p &< 8\\
p &< \frac{8}{7}\\
p &< 1\frac{1}{7}
\end{aligned}$$



(f)

$$\begin{aligned}
6-(1-2q) &\geqslant 3(5q-2)\\
6-1+2q &\geqslant 15q-6\\
5+2q &\geqslant 15q-6\\
5+2q-15q &\geqslant 15q-6-15q\\
5-13q &\geqslant -6\\
5-13q-5 &\geqslant -6-5\\
-13q &\geqslant -11\\
q &\leqslant \frac{-11}{-13}\\
q &\leqslant \frac{11}{13}
\end{aligned}$$



6. (a)

$$\begin{aligned}
\frac{4a}{3} &\geqslant 2\\
3\times\frac{4a}{3} &\geqslant 3\times 2\\
4a &\geqslant 6\\
a &\geqslant \frac{6}{4}\\
a &\geqslant 1\frac{1}{2}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{2b-1}{3} &> \frac{3b}{5}\\
3\times 5\times\frac{2b-1}{3} &> 3\times 5\times\frac{3b}{5}\\
5(2b-1) &> 3(3b)\\
10b-5 &> 9b\\
10b-5-9b &> 9b-9b\\
b-5 &> 0\\
b-5+5 &> 0+5\\
b &> 5
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{c+4}{4} &> \frac{c+1}{3}\\
4\times 3\times\frac{c+4}{4} &> 4\times 3\times\frac{c+1}{3}\\
3(c+4) &> 4(c+1)\\
3c+12 &> 4c+4\\
3c+12-4c &> 4c+4-4c\\
-c+12 &> 4\\
-c+12-12 &> 4-12\\
-c &> -8\\
c &< 8
\end{aligned}$$

(d)

$$\begin{aligned}
\frac{2-d}{2} &< \frac{3-d}{4}+\frac{1}{2}\\
4\times\frac{2-d}{2} &< 4\times\left(\frac{3-d}{4}+\frac{1}{2}\right)\\
2(2-d) &< (3-d)+2\\
4-2d &< 5-d\\
4-2d+d &< 5-d+d\\
4-d &< 5\\
4-d-4 &< 5-4\\
-d &< 1\\
d &> -1
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{1}{4}(e-2)+\frac{2}{3} &< \frac{1}{6}(e-4)\\
12\times\left[\frac{1}{4}(e-2)+\frac{2}{3}\right] &< 12\times\frac{1}{6}(e-4)\\
3(e-2)+8 &< 2(e-4)\\
3e-6+8 &< 2e-8\\
3e+2 &< 2e-8\\
3e+2-2e &< 2e-8-2e\\
e+2 &< -8\\
e+2-2 &< -8-2\\
e &< -10
\end{aligned}$$

(f)

$$\begin{aligned}
\frac{f+1}{2}+\frac{3f+1}{4} &\leqslant \frac{3f-1}{4}+2\\
4\times\left(\frac{f+1}{2}+\frac{3f+1}{4}\right) &\leqslant 4\times\left(\frac{3f-1}{4}+2\right)\\
2(f+1)+(3f+1) &\leqslant (3f-1)+8\\
2f+2+3f+1 &\leqslant 3f-1+8\\
5f+3 &\leqslant 3f+7\\
5f+3-3f &\leqslant 3f+7-3f\\
2f+3-3 &\leqslant 7-3\\
2f &\leqslant 4\\
f &\leqslant \frac{4}{2}\\
f &\leqslant 2
\end{aligned}$$

(g)
$$\frac{1}{5}(3g+4)-\frac{1}{3}(g+1) \geqslant 1-\frac{1}{3}(g+5)$$
$$5\times3\times\left[\frac{1}{5}(3g+4)-\frac{1}{3}(g+1)\right] \geqslant 5\times3\times\left[1-\frac{1}{3}(g+5)\right]$$
$$\begin{aligned}
3(3g+4)-5(g+1) &\geqslant 15-5(g+5)\\
9g+12-5g-5 &\geqslant 15-5g-25\\
4g+7 &\geqslant -10-5g\\
4g+7+5g &\geqslant -10-5g+5g\\
9g+7 &\geqslant -10\\
9g+7-7 &\geqslant -10-7\\
9g &\geqslant -17\\
g &\geqslant \frac{-17}{9}\\
g &\geqslant -1\frac{8}{9}
\end{aligned}$$

(h)
$$\begin{aligned}
4\left(\frac{h}{3}+\frac{3}{4}\right) &< 3\left(\frac{h}{2}-5\right)\\
\frac{4}{3}h+\frac{12}{4} &< \frac{3}{2}h-15\\
\frac{4}{3}h+3-\frac{3}{2}h &< \frac{3}{2}h-15-\frac{3}{2}h\\
-\frac{1}{6}h+3 &< -15\\
-\frac{1}{6}h+3-3 &< -15-3\\
-\frac{1}{6}h &< -18\\
6\times-\frac{1}{6}h &< 6\times-18\\
-h &< -108\\
h &> 108
\end{aligned}$$

7.
$$\frac{1}{6}(2-p)-3 \geqslant \frac{p}{10}$$
$$6\times10\times\left[\frac{1}{6}(2-p)-3\right] \geqslant 6\times10\times\frac{p}{10}$$
$$\begin{aligned}
10(2-p)-180 &\geqslant 6p\\
20-10p-180 &\geqslant 6p\\
-10p-160 &\geqslant 6p\\
-10p-160-6p &\geqslant 6p-6p\\
-16p-160 &\geqslant 0\\
-16p-160+160 &\geqslant 0+160\\
-16p &\geqslant 160\\
p &\leqslant \frac{160}{-16}\\
p &\leqslant -10
\end{aligned}$$
$\therefore$ The largest possible value of p is -10.

8. (i)
$$\begin{aligned}
\frac{1}{3}(2x-7) &\leqslant \frac{3x+2}{2}\\
3\times2\times\frac{1}{3}(2x-7) &\leqslant 3\times2\times\frac{3x+2}{2}\\
2(2x-7) &\leqslant 3(3x+2)\\
4x-14 &\leqslant 9x+6\\
4x-14-9x &\leqslant 9x+6-9x\\
-5x-14 &\leqslant 6\\
-5x-14+14 &\leqslant 6+14\\
-5x &\leqslant 20\\
x &\geqslant \frac{20}{-5}\\
x &\geqslant -4
\end{aligned}$$

(ii) Smallest value of x^2 is $(0)^2 = 0$

Exercise 3B

1. Let the number of tickets Kate can buy be x.

$$\begin{aligned}
10.50x &\leqslant 205\\
x &\leqslant \frac{205}{10.50}\\
x &\leqslant 19\frac{11}{21}
\end{aligned}$$

$\therefore$ The number of tickets Kate can buy is 19.

2. Since the area of rectangle = length × breadth,

$$x\times y = 24$$

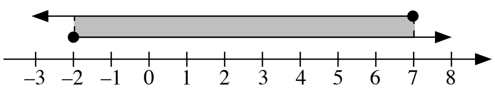
Factors of 24: 1×24, 2×12, 3×8, 4×6

Hence, where $x > y$, the possible pairs of integer values of x and y are $x = 24, y = 1$; $x = 12, y = 2$; $x = 8, y = 3$; $x = 6, y = 4$.

3. (a) Solving the two linear inequalities separately,

$x-4 \leqslant 3$	and	$3x \geqslant -6$
$x-4+4 \leqslant 3+4$		$x \geqslant \frac{-6}{3}$
$x \leqslant 7$		$x \geqslant -2$

Representing $x \leqslant 7$ and $x \geqslant -2$ on a number line,

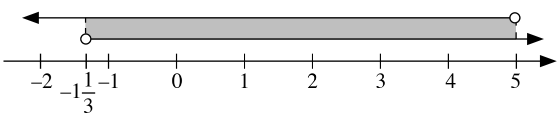


$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-2 \leqslant x \leqslant 7$.

(b) Solving the two linear inequalities separately,

$2x+5 < 15$	and	$3x-2 > -6$
$2x+5-5 < 15-5$		$3x-2+2 > -6+2$
$2x < 10$		$3x > -4$
$x < 5$		$x > \frac{-4}{3}$
		$x > -1\frac{1}{3}$

Representing $x < 5$ and $x > -1\frac{1}{3}$ on a number line,



$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-1\frac{1}{3} < x < 5$.

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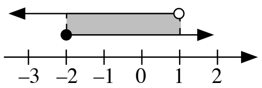
4. **(a)** Solving the two linear inequalities separately,

$$\begin{array}{rcl} 5x - 1 < 4 & \text{and} & 3x + 5 \geqslant x + 1 \\ 5x - 1 + 1 < 4 + 1 & & 3x + 5 - x \geqslant x + 1 - x \\ 5x < 5 & & 2x + 5 \geqslant 1 \\ x < 1 & & 2x + 5 - 5 \geqslant 1 - 5 \\ & & 2x \geqslant -4 \\ & & x \geqslant -2 \end{array}$$

Representing $x < 1$ and $x \geqslant -2$ on a number line,



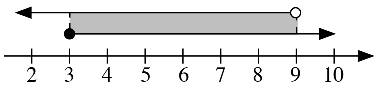
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-2 \leqslant x < 1$.

$\therefore$ The integer values of x which satisfy both inequalities are -2, -1 and 0.

(b) Solving the two linear inequalities separately,

$$\begin{array}{rcl} 2x - 5 \geqslant 1 & \text{and} & 3x - 1 < 26 \\ 2x - 5 + 5 \geqslant 1 + 5 & & 3x - 1 + 1 < 26 + 1 \\ 2x \geqslant 6 & & 3x < 27 \\ x \geqslant 3 & & x < 9 \end{array}$$

Representing $x \geqslant 3$ and $x < 9$ on a number line,



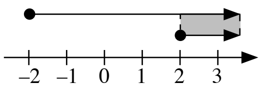
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $3 \leqslant x < 9$.

$\therefore$ The integer values of x which satisfy both inequalities are 3, 4, 5, 6, 7 and 8.

5. **(a)** Solving the two linear inequalities separately,

$$\begin{array}{rcl} -4 \leqslant 2x & \text{and} & 2x \leqslant 3x - 2 \\ 2x \geqslant -4 & & 2x - 3x \leqslant 3x - 2 - 3x \\ x \geqslant -2 & & -x \leqslant -2 \\ & & x \geqslant \dfrac{-2}{-1} \\ & & x \geqslant 2 \end{array}$$

Representing $x \geqslant -2$ and $x \geqslant 2$ on a number line,

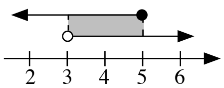


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $x \geqslant 2$.

(b) Solving the two linear inequalities separately,

$$\begin{array}{rcl} 1 - x < -2 & \text{and} & -2 \leqslant 3 - x \\ 1 - x - 1 < -2 - 1 & & -2 + x \leqslant 3 - x + x \\ -x < -3 & & -2 + x \leqslant 3 \\ x > \dfrac{-3}{-1} & & -2 + x + 2 \leqslant 3 + 2 \\ x > 3 & & x \leqslant 5 \end{array}$$

Representing $x > 3$ and $x \leqslant 5$ on a number line,

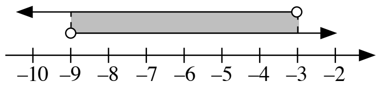


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $3 < x \leqslant 5$.

(c) Solving the two linear inequalities separately,

$$\begin{array}{rcl} 3x - 3 < x - 9 & \text{and} & x - 9 < 2x \\ 3x - 3 - x < x - 9 - x & & x - 9 - 2x < 2x - 2x \\ 2x - 3 < -9 & & -x - 9 < 0 \\ 2x - 3 + 3 < -9 + 3 & & -x - 9 + 9 < 0 + 9 \\ 2x < -6 & & -x < 9 \\ x < -3 & & x > -9 \end{array}$$

Representing $x < -3$ and $x > -9$ on a number line,

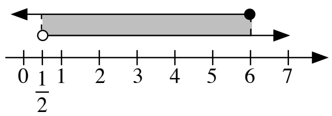


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-9 < x < -3$.

(d) Solving the two linear inequalities separately,

$$\begin{array}{rcl} 2x \leqslant x + 6 & \text{and} & x + 6 < 3x + 5 \\ 2x - x \leqslant x + 6 - x & & x + 6 - 3x < 3x + 5 - 3x \\ x \leqslant 6 & & -2x + 6 < 5 \\ & & -2x + 6 - 6 < 5 - 6 \\ & & -2x < -1 \\ & & x > \dfrac{-1}{-2} \\ & & x > \dfrac{1}{2} \end{array}$$

Representing $x \leqslant 6$ and $x > \dfrac{1}{2}$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\dfrac{1}{2} < x \leqslant 6$.

6. Let the number of days be x.

$$\begin{aligned} 8x + 20 &\leqslant 100 \\ 8x + 20 - 20 &\leqslant 100 - 20 \\ 8x &\leqslant 80 \\ x &\leqslant 10 \end{aligned}$$

$\therefore$ The maximum number of days is 10.

7. Let the three consecutive integers be x, $(x + 1)$ and $(x + 2)$.

$$\begin{aligned} x + (x + 1) + (x + 2) &< 75 \\ 3x + 3 &< 75 \\ 3x + 3 - 3 &< 75 - 3 \\ 3x &< 72 \\ x &< 24 \end{aligned}$$

Largest possible integer value of $x = 23$

Largest possible integer $= 23 + 2 = 25$

$\therefore$ Cube of the largest possible integer $= 25^3 = 15\,625$

8. Let the number of correct answers obtained be x.

$$\begin{aligned} 5x - 2(30 - x) &\leqslant 66 \\ 5x - 60 + 2x &\leqslant 66 \\ 7x - 60 &\leqslant 66 \\ 7x - 60 + 60 &\leqslant 66 + 60 \\ 7x &\leqslant 126 \\ x &\leqslant 18 \end{aligned}$$

$\therefore$ The maximum number of correct answers obtained is 18.

9. Let x and y be the number of \$5 notes and \$2 notes respectively.

$x + y = 50 \qquad —\ (1)$

$x \times 5 + y \times 2 > 132$

i.e. $5x + 2y > 132 \qquad —\ (2)$

From (1),

$y = 50 - x \qquad —\ (3)$

Substitute (3) into (2):

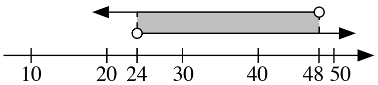
$$\begin{aligned} 5x + 2(50 - x) &> 132 \\ 5x + 100 - 2x &> 132 \\ 3x + 100 &> 132 \\ 3x + 100 - 100 &> 132 - 100 \\ 3x &> 32 \\ x &> \frac{32}{3} \\ x &> 10\frac{2}{3} \end{aligned}$$

$\therefore$ The minimum number of \$5 notes that he has is 11.

10. Solving the two linear inequalities separately,

$$\begin{aligned} \frac{1}{2}x - 4 &> \frac{1}{3}x \\ \frac{1}{2}x - 4 - \frac{1}{3}x &> \frac{1}{3}x - \frac{1}{3}x \\ \frac{1}{6}x - 4 &> 0 \\ \frac{1}{6}x - 4 + 4 &> 0 + 4 \\ \frac{1}{6}x &> 4 \\ 6 \times \frac{1}{6}x &> 6 \times 4 \\ x &> 24 \end{aligned} \qquad \text{and} \qquad \begin{aligned} \frac{1}{6}x + 1 &< \frac{1}{8}x + 3 \\ \frac{1}{6}x + 1 - \frac{1}{8}x &< \frac{1}{8}x + 3 - \frac{1}{8}x \\ \frac{1}{24}x + 1 &< 3 \\ \frac{1}{24}x + 1 - 1 &< 3 - 1 \\ \frac{1}{24}x &< 2 \\ 24 \times \frac{1}{24}x &< 24 \times 2 \\ x &< 48 \end{aligned}$$

Representing $x > 24$ and $x < 48$ on a number line,



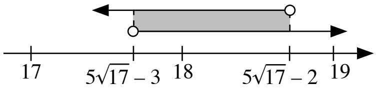
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $24 < x < 48$.

$\therefore$ The prime values of x which satisfy both inequalities are 29, 31, 37, 41, 43 and 47.

11. Solving the two linear inequalities separately,

$$\begin{aligned} x + 2 &< 5\sqrt{17} \\ x + 2 - 2 &< 5\sqrt{17} - 2 \\ x &< 5\sqrt{17} - 2 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 5\sqrt{17} &< x + 3 \\ 5\sqrt{17} - x &< x + 3 - x \\ 5\sqrt{17} - x &< 3 \\ 5\sqrt{17} - x - 5\sqrt{17} &< 3 - 5\sqrt{17} \\ -x &< 3 - 5\sqrt{17} \\ x &> -(3 - 5\sqrt{17}) \\ x &> 5\sqrt{17} - 3 \end{aligned}$$

Representing $x < 5\sqrt{17} - 2$ and $x > 5\sqrt{17} - 3$ on a number line,



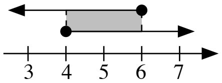
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $5\sqrt{17} - 3 < x < 5\sqrt{17} - 2$.

$\therefore$ The integer value of x is 18.

12. Solving the two linear inequalities separately,

$$\begin{aligned} 3x - 2 &\geqslant 10 \\ 3x - 2 + 2 &\geqslant 10 + 2 \\ 3x &\geqslant 12 \\ x &\geqslant 4 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 10 &\geqslant x + 4 \\ 10 - x &\geqslant x + 4 - x \\ 10 - x &\geqslant 4 \\ 10 - x - 10 &\geqslant 4 - 10 \\ -x &\geqslant -6 \\ x &\leqslant 6 \end{aligned}$$

Representing $x \geqslant 4$ and $x \leqslant 6$ on a number line,



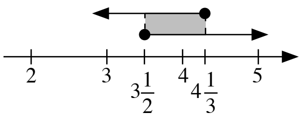
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $4 \leqslant x \leqslant 6$.

$\therefore$ The prime value of x is 5.

13. **(a)** Solving the two linear inequalities separately,

$$\begin{aligned} 3 - a &\leqslant a - 4 \\ 3 - a - a &\leqslant a - 4 - a \\ 3 - 2a &\leqslant -4 \\ 3 - 2a - 3 &\leqslant -4 - 3 \\ -2a &\leqslant -7 \\ a &\geqslant \frac{-7}{-2} \\ a &\geqslant 3\frac{1}{2} \end{aligned} \qquad \text{and} \qquad \begin{aligned} a - 4 &\leqslant 9 - 2a \\ a - 4 + 2a &\leqslant 9 - 2a + 2a \\ 3a - 4 &\leqslant 9 \\ 3a - 4 + 4 &\leqslant 9 + 4 \\ 3a &\leqslant 13 \\ a &\leqslant \frac{13}{3} \\ a &\leqslant 4\frac{1}{3} \end{aligned}$$

Representing $a \geqslant 3\frac{1}{2}$ and $a \leqslant 4\frac{1}{3}$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $3\frac{1}{2} \leqslant a \leqslant 4\frac{1}{3}$.

(b) Solving the two linear inequalities separately,

$$\begin{aligned} 1 - b &< b - 1 \\ 1 - b - b &< b - 1 - b \\ 1 - 2b &< -1 \\ 1 - 2b - 1 &< -1 - 1 \\ -2b &< -2 \\ b &> \frac{-2}{-2} \\ b &> 1 \end{aligned} \qquad \text{and} \qquad \begin{aligned} b - 1 &< 11 - 2b \\ b - 1 + 2b &< 11 - 2b + 2b \\ 3b - 1 &< 11 \\ 3b - 1 + 1 &< 11 + 1 \\ 3b &< 12 \\ b &< \frac{12}{3} \\ b &< 4 \end{aligned}$$

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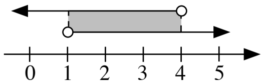
Representing $b > 1$ and $b < 4$ on a number line,

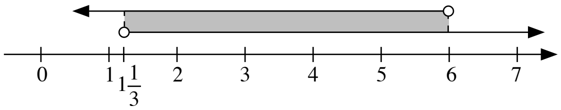


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $1 < b < 4$.

(c) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 3 - c &< 2c - 1 & \text{and} & 2c - 1 &< 5 + c \\ 3 - c - 2c &< 2c - 1 - 2c & & 2c - 1 - c &< 5 + c - c \\ 3 - 3c &< -1 & & c - 1 &< 5 \\ 3 - 3c - 3 &< -1 - 3 & & c - 1 + 1 &< 5 + 1 \\ -3c &< -4 & & c &< 6 \\ c &> \dfrac{-4}{-3} & & & \\ c &> 1\dfrac{1}{3} & & & \end{array}$$

Representing $c > 1\dfrac{1}{3}$ and $c < 6$ on a number line,

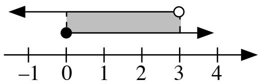


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $1\dfrac{1}{3} < c < 6$.

(d) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 3d - 5 &< d + 1 & \text{and} & d + 1 &\leqslant 2d + 1 \\ 3d - 5 - d &< d + 1 - d & & d + 1 - 2d &\leqslant 2d + 1 - 2d \\ 2d - 5 &< 1 & & -d + 1 &\leqslant 1 \\ 2d - 5 + 5 &< 1 + 5 & & -d + 1 - 1 &\leqslant 1 - 1 \\ 2d &< 6 & & -d &\leqslant 0 \\ d &< 3 & & d &\geqslant 0 \end{array}$$

Representing $d < 3$ and $d \geqslant 0$ on a number line,

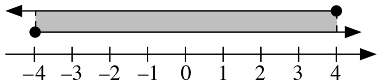


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $0 \leqslant d < 3$.

14. (a) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} \dfrac{a}{4} + 3 &\leqslant 4 & \text{and} & 4 &\leqslant \dfrac{a}{2} + 6 \\ \dfrac{a}{4} + 3 - 3 &\leqslant 4 - 3 & & 4 - \dfrac{a}{2} &\leqslant \dfrac{a}{2} + 6 - \dfrac{a}{2} \\ \dfrac{a}{4} &\leqslant 1 & & 4 - \dfrac{a}{2} &\leqslant 6 \\ 4 \times \dfrac{a}{4} &\leqslant 4 \times 1 & & 4 - \dfrac{a}{2} - 4 &\geqslant 6 - 4 \\ a &\leqslant 4 & & -\dfrac{a}{2} &\leqslant 2 \\ & & & 2 \times -\dfrac{a}{2} &\leqslant 2 \times 2 \\ & & & -a &\leqslant 4 \\ & & & a &\geqslant -4 \end{array}$$

Representing $a \leqslant 4$ and $a \geqslant -4$ on a number line,

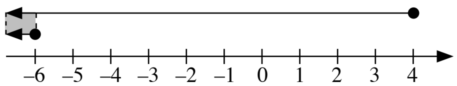


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-4 \leqslant a \leqslant 4$.

(b) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} \dfrac{b}{3} &\geqslant \dfrac{b}{2} + 1 & \text{and} & \dfrac{b}{2} + 1 &\geqslant b - 1 \\ \dfrac{b}{3} - \dfrac{b}{2} &\geqslant \dfrac{b}{2} + 1 - \dfrac{b}{2} & & \dfrac{b}{2} + 1 - b &\geqslant b - 1 - b \\ -\dfrac{b}{6} &\geqslant 1 & & -\dfrac{b}{2} + 1 &\geqslant -1 \\ 6 \times -\dfrac{b}{6} &\geqslant 6 \times 1 & & -\dfrac{b}{2} + 1 - 1 &\geqslant -1 - 1 \\ -b &\geqslant 6 & & -\dfrac{b}{2} &\geqslant -2 \\ b &\leqslant -6 & & 2 \times -\dfrac{b}{2} &\geqslant 2 \times -2 \\ & & & -b &\geqslant -4 \\ & & & b &\leqslant \dfrac{-4}{-1} \\ & & & b &\leqslant 4 \end{array}$$

Representing $b \leqslant -6$ and $b \leqslant 4$ on a number line,

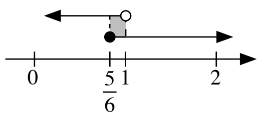


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $b \leqslant -6$.

(c) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 2(1 - c) &> c - 1 & \text{and} & c - 1 &\geqslant \dfrac{c - 2}{7} \\ 2 - 2c &> c - 1 & & 7 \times (c - 1) &\geqslant 7 \times \dfrac{c - 2}{7} \\ 2 - 2c - c &> c - 1 - c & & 7(c - 1) &\geqslant c - 2 \\ 2 - 3c &> -1 & & 7c - 7 &\geqslant c - 2 \\ 2 - 3c - 2 &> -1 - 2 & & 7c - 7 - c &\geqslant c - 2 - c \\ -3c &> -3 & & 6c - 7 &\geqslant -2 \\ c &< \dfrac{-3}{-3} & & 6c - 7 + 7 &\geqslant -2 + 7 \\ c &< 1 & & 6c &\geqslant 5 \\ & & & c &\geqslant \dfrac{5}{6} \end{array}$$

Representing $c < 1$ and $c \geqslant \dfrac{5}{6}$ on a number line,

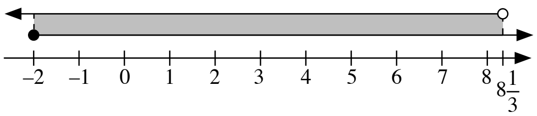


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\dfrac{5}{6} \leqslant c < 1$.

(d) Solving the two linear inequalities separately,

$$\begin{array}{rclcrcl}
d-5 &<& \dfrac{2d}{5} & \text{and} & \dfrac{2d}{5} &\leqslant& \dfrac{d}{2}+\dfrac{1}{5} \\
5\times(d-5) &<& 5\times\dfrac{2d}{5} & & 5\times\dfrac{2d}{5} &\leqslant& 5\times\left(\dfrac{d}{2}+\dfrac{1}{5}\right) \\
5(d-5) &<& 2d & & 2d &\leqslant& 5\left(\dfrac{d}{2}+\dfrac{1}{5}\right) \\
5d-25 &<& 2d & & 2d &\leqslant& \dfrac{5d}{2}+1 \\
5d-25-2d &<& 2d-2d & & 2d-\dfrac{5d}{2} &\leqslant& \dfrac{5d}{2}+1-\dfrac{5d}{2} \\
3d-25 &<& 0 & & -\dfrac{d}{2} &\leqslant& 1 \\
3d-25+25 &<& 0+25 & & 2\times-\dfrac{d}{2} &\leqslant& 2\times 1 \\
3d &<& 25 & & -d &\leqslant& 2 \\
d &<& \dfrac{25}{3} & & d &\geqslant& -2 \\
d &<& 8\dfrac{1}{3} & & & &
\end{array}$$

Representing $d < 8\frac{1}{3}$ and $d \geqslant -2$ on a number line,

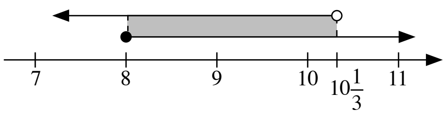


The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-2 \leqslant d < 8\frac{1}{3}$.

15. (a) Solving the two linear inequalities separately,

$$\begin{array}{rclcrcl}
3x-5 &<& 26 & \text{and} & 26 &\leqslant& 4x-6 \\
3x-5+5 &<& 26+5 & & 26-4x &\geqslant& 4x-6-4x \\
3x &<& 31 & & 26-4x &\leqslant& -6 \\
x &<& \dfrac{31}{3} & & 26-4x-26 &\leqslant& -6-26 \\
x &<& 10\dfrac{1}{3} & & -4x &\leqslant& -32 \\
& & & & x &\geqslant& \dfrac{-32}{-4} \\
& & & & x &\geqslant& 8
\end{array}$$

Representing $x < 10\frac{1}{3}$ and $x \geqslant 8$ on a number line,



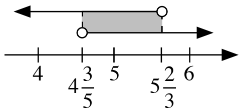
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $8 \leqslant x < 10\frac{1}{3}$.

$\therefore$ The integer values of x which satisfy both inequalities are 8, 9 and 10.

(b) Solving the two linear inequalities separately,

$$\begin{array}{rclcrcl}
3x+2 &<& 19 & \text{and} & 19 &<& 5x-4 \\
3x+2-2 &<& 19-2 & & 19-5x &<& 5x-4-5x \\
3x &<& 17 & & 19-5x &<& -4 \\
x &<& \dfrac{17}{3} & & 19-5x-19 &<& -4-19 \\
x &<& 5\dfrac{2}{3} & & -5x &<& -23 \\
& & & & x &>& \dfrac{-23}{-5} \\
& & & & x &>& 4\dfrac{3}{5}
\end{array}$$

Representing $x < 5\frac{2}{3}$ and $x > 4\frac{3}{5}$ on a number line,



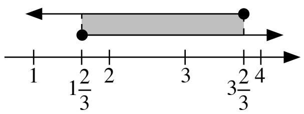
The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $4\frac{3}{5} < x < 5\frac{2}{3}$.

$\therefore$ The integer value of x which satisfies both inequalities is 5.

(c) Solving the two linear inequalities separately,

$$\begin{array}{rclcrcl}
-4 &\leqslant& 7-3x & \text{and} & 7-3x &\leqslant& 2 \\
-4+3x &\leqslant& 7-3x+3x & & 7-3x-7 &\leqslant& 2-7 \\
-4+3x &\leqslant& 7 & & -3x &\leqslant& -5 \\
-4+3x+4 &\leqslant& 7+4 & & x &\geqslant& \dfrac{-5}{-3} \\
3x &\leqslant& 11 & & x &\geqslant& 1\dfrac{2}{3} \\
x &\leqslant& \dfrac{11}{3} & & & & \\
x &\leqslant& 3\dfrac{2}{3} & & & &
\end{array}$$

Representing $x \leqslant 3\frac{2}{3}$ and $x \geqslant 1\frac{2}{3}$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $1\frac{2}{3} \leqslant x \leqslant 3\frac{2}{3}$.

$\therefore$ The integer values of x which satisfy both inequalities are 2 and 3.

(d) Solving the two linear inequalities separately,

$$\begin{array}{rclcrcl}
-10 &<& 7-2x & \text{and} & 7-2x &\leqslant& -1 \\
-10+2x &<& 7-2x+2x & & 7-2x-7 &\leqslant& -1-7 \\
-10+2x &<& 7 & & -2x &\leqslant& -8 \\
-10+2x+10 &<& 7+10 & & x &\geqslant& \dfrac{-8}{-2} \\
2x &<& 17 & & x &\geqslant& 4 \\
x &<& \dfrac{17}{2} & & & & \\
x &<& 8\dfrac{1}{2} & & & &
\end{array}$$

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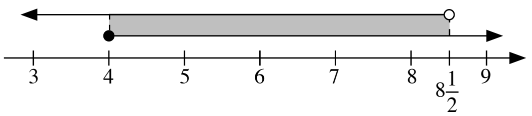
Representing $x < 8\frac{1}{2}$ and $x \geqslant 4$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $4 \leqslant x < 8\frac{1}{2}$.

$\therefore$ The integer values of x which satisfy both inequalities are 4, 5, 6, 7 and 8.

16. Given that $0 \leqslant x \leqslant 7$ and $1 \leqslant y \leqslant 5$,

(a) Greatest value of $x + y = 7 + 5$
$= 12$

(b) Least value of $x - y = 0 - 5$
$= -5$

(c) Largest value of $xy = (7)(5)$
$= 35$

(d) Smallest value of $\frac{x}{y} = \frac{0}{5}$
$= 0$

(e) Least value of $x^2 = 0^2$
$= 0$

Greatest value of $x^2 = 7^2$
$= 49$

17. Given that $-4 \leqslant a \leqslant -1$ and $-6 \leqslant b \leqslant -2$,

(a) Least value of $a + b = -4 + (-6)$
$= -4 - 6$
$= -10$

(b) Greatest value of $a - b = -1 - (-6)$
$= -1 + 6$
$= 5$

(c) Smallest value of $ab = (-1)(-2)$
$= 2$

(d) Largest value of $\frac{a}{b} = \frac{-4}{-2}$
$= 2$

(e) Least value of $a^2 = (-1)^2$
$= 1$

Greatest value of $a^2 = (-4)^2$
$= 16$

(f) Largest value of $b^2 - a = (-6)^2 - (-4)$
$= 36 + 4$
$= 40$

18. **(a)** False. If $a = -1$ and $b = -2$, then $\frac{-1}{-2} = \frac{1}{2}$ which is less than 1.

(b) True

(c) True

Exercise 3C

1. **(i)** **(a)** 4.55, 4.65 **(b)** $4.55 \leqslant x < 4.65$

(ii) **(a)** 0.725, 0.735 **(b)** $0.725 \leqslant x < 0.735$

(iii) **(a)** 675, 685 **(b)** $675 \leqslant x < 685$

(iv) **(a)** 995, 1050 **(b)** $995 \leqslant x < 1050$

(v) **(a)** 0.0345, 0.0355 **(b)** $0.0345 \leqslant x < 0.0355$

(vi) **(a)** 71 500, 72 500 **(b)** $71\,5000 \leqslant x < 72\,500$

2. $0.6\text{m}^3 = 0.6 \times 10^6 = 600\,000 \text{ cm}^3$
Upper bound = 600 000.5
Lower bound = 599 999.5

3. Upper bound = 0.0045 s
Lower bound = 0.0035 s
$\therefore 0.0035 \leqslant x < 0.0045$

4. **(a)** 775, 725
(b) 35, 25
(c) 758.5, 757.5
(d) 1500.5, 1499.5
(e) 497.5, 492.5

5. Upper bounds of the lengths (m) = 6.5, 8.5, 10.5
Lower bounds of the lengths (m) = 5.5, 7.5, 9.5
Upper bound perimeter = 6.5 + 8.5 + 10.5
= 25.5 m
Lower bound perimeter = 5.5 + 7.5 + 9.5
= 22.5 m

6. Maximum number of students = 649
Maximum number of girls = 649 – 297
= 352

7. Maximum length of original string = 50.05 cm
Minimum length of original string = 49.95 cm
Maximum length of cut string = 25.05 cm
Minimum length of cut string = 24.95 cm
Maximum length of string left = 50.05 – 24.95
= 25.1 cm
Minimum length of string left = 49.95 – 25.05
= 24.9 cm
$\therefore 24.9 \leqslant l < 25.1$

8. Radius of trunk = 20 cm = 200 mm
Minimum radius = 198.5 mm = 19.85 cm
Minimum circumference = $2\pi(19.85)$
= 125 cm (to 3 s.f.)

9. Maximum speed = 65 km/h
Minimum speed = 55 km/h
Maximum time taken = 3.5 h
Minimum time taken = 2.5 h
Maximum distance = 65×3.5
= 227.5 km
Minimum distance = 55×2.5
= 137.5 km
$\therefore 137.5 \leqslant d < 227.5$

10. Maximum mass = 35 g
Minimum mass = 25 g
Maximum volume = $\frac{35}{2.4}$
= 14.6 cm^3 (to 3 s.f.)
Minimum volume = $\frac{25}{2.4}$
= 10.4 cm^3 (to 3 s.f.)

11. For 1 litre of paint,

Upper bound area = 3.5 m^2

Lower bound area = 2.5 m^2

For the wall,

Upper bound area = 700.5 m^2

Lower bound area = 699.5 m^2

$\therefore$ Maximum amount of paint needed $= \dfrac{700.5}{2.5}$

$= 280.2$ litres

12. For 1 bottle,

Upper bound capacity = 1.5 l

Lower bound capacity = 0.5 l

For the tank,

Upper bound capacity = 55 l

Lower bound capacity = 45 l

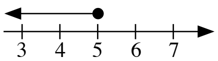
Maximum number of bottles of water drawn $= \dfrac{55}{0.5}$

$= 110$

Review Exercise 3

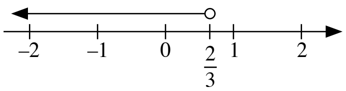
1. **(a)**

$$\begin{aligned} a - 2 &\leqslant 3 \\ a - 2 + 2 &\leqslant 3 + 2 \\ a &\leqslant 5 \end{aligned}$$



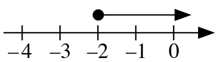
(b)

$$\begin{aligned} 2b + 1 &< 5 - 4b \\ 2b + 1 + 4b &< 5 - 4b + 4b \\ 6b + 1 &< 5 \\ 6b + 1 - 1 &< 5 - 1 \\ 6b &< 4 \\ b &< \frac{4}{6} \\ b &< \frac{2}{3} \end{aligned}$$



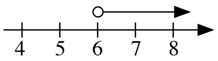
(c)

$$\begin{aligned} c &\geqslant \frac{1}{2}c - 1 \\ c - \frac{1}{2}c &\geqslant \frac{1}{2}c - 1 - \frac{1}{2}c \\ \frac{1}{2}c &\geqslant -1 \\ 2 \times \frac{1}{2}c &\geqslant 2 \times -1 \\ c &\geqslant -2 \end{aligned}$$



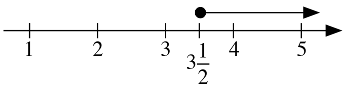
(d)

$$\begin{aligned} \frac{1}{2}d &> 1 + \frac{1}{3}d \\ \frac{1}{2}d - \frac{1}{3}d &> 1 + \frac{1}{3}d - \frac{1}{3}d \\ \frac{1}{6}d &> 1 \\ 6 \times \frac{1}{6}d &> 6 \times 1 \\ d &> 6 \end{aligned}$$



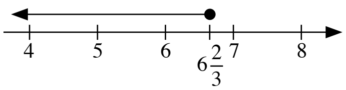
(e)

$$\begin{aligned} 2(e - 3) &\geqslant 1 \\ 2e - 6 &\geqslant 1 \\ 2e - 6 + 6 &\geqslant 1 + 6 \\ 2e &\geqslant 7 \\ e &\geqslant \frac{7}{2} \\ e &\geqslant 3\frac{1}{2} \end{aligned}$$



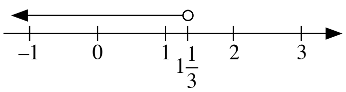
(f)

$$\begin{aligned} 5(f - 4) &\leqslant 2f \\ 5f - 20 &\leqslant 2f \\ 5f - 20 - 2f &\leqslant 2f - 2f \\ 3f - 20 &\leqslant 0 \\ 3f - 20 + 20 &\leqslant 0 + 20 \\ 3f &\leqslant 20 \\ f &\leqslant \frac{20}{3} \\ f &\leqslant 6\frac{2}{3} \end{aligned}$$



(g)

$$\begin{aligned} -3 - g &> 2g - 7 \\ -3 - g - 2g &> 2g - 7 - 2g \\ -3 - 3g &> -7 \\ -3 - 3g + 3 &> -7 + 3 \\ -3g &> -4 \\ g &< \frac{-4}{-3} \\ g &< 1\frac{1}{3} \end{aligned}$$



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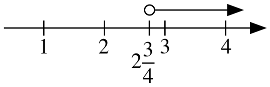
(h)

$$\begin{aligned} 18 - 3h &< 5h - 4 \\ 18 - 3h - 5h &< 5h - 4 - 5h \\ 18 - 8h &< -4 \\ 18 - 8h - 18 &< -4 - 18 \\ -8h &< -22 \\ h &> \frac{-22}{-8} \\ h &> 2\frac{3}{4} \end{aligned}$$



2. (a)

$$\begin{aligned} 3 + \frac{a}{4} &> 5 + \frac{a}{3} \\ 4 \times 3 \times \left(3 + \frac{a}{4}\right) &> 4 \times 3 \times \left(5 + \frac{a}{3}\right) \\ 36 + 3a &> 60 + 4a \\ 36 + 3a - 4a &> 60 + 4a - 4a \\ 36 - a &> 60 \\ 36 - a - 36 &> 60 - 36 \\ -a &> 24 \\ a &< -24 \end{aligned}$$

(b)

$$\begin{aligned} \frac{4b}{9} - 5 &< 3 - \frac{2b}{3} \\ 9 \times \left(\frac{4b}{9} - 5\right) &< 9 \times \left(3 - \frac{2b}{3}\right) \\ 4b - 45 &< 27 - 6b \\ 4b - 45 + 6b &< 27 - 6b + 6b \\ 10b - 45 &< 27 \\ 10b - 45 + 45 &< 27 + 45 \\ 10b &< 72 \\ b &< 7\frac{1}{5} \end{aligned}$$

(c)

$$\begin{aligned} \frac{4c}{9} - \frac{3}{4} &\geqslant c - \frac{1}{2} \\ 9 \times 4 \times \left(\frac{4c}{9} - \frac{3}{4}\right) &\geqslant 9 \times 4 \times \left(c - \frac{1}{2}\right) \\ 16c - 27 &\geqslant 36c - 18 \\ 16c - 27 - 36c &\geqslant 36c - 18 - 36c \\ -20c - 27 &\geqslant -18 \\ -20c - 27 + 27 &\geqslant -18 + 27 \\ -20c &\geqslant 9 \\ c &\leqslant -\frac{9}{20} \end{aligned}$$

(d)

$$\begin{aligned} \frac{d-2}{3} &< \frac{2d+3}{5} + \frac{5}{8} \\ \frac{d-2}{3} - \frac{2d+3}{5} &< \frac{5}{8} \\ 3 \times 5 \times \left(\frac{d-2}{3} - \frac{2d+3}{5}\right) &< 3 \times 5 \times \frac{5}{8} \\ 5(d-2) - 3(2d+3) &< \frac{75}{8} \\ 5d - 10 - 6d - 9 &< \frac{75}{8} \\ -d - 19 &< \frac{75}{8} \\ -d - 19 + 19 &< \frac{75}{8} + 19 \\ -d &< 28\frac{3}{8} \\ d &> -28\frac{3}{8} \end{aligned}$$

(e)

$$\begin{aligned} \frac{1}{3}(e+2) &\geqslant \frac{2}{3} + \frac{1}{4}(e-1) \\ \frac{1}{3}(e+2) - \frac{1}{4}(e-1) &\geqslant \frac{2}{3} \\ 3 \times 4 \times \left[\frac{1}{3}(e+2) - \frac{1}{4}(e-1)\right] &\geqslant 3 \times 4 \times \frac{2}{3} \\ 4(e+2) - 3(e-1) &\geqslant 8 \\ 4e + 8 - 3e + 3 &\geqslant 8 \\ e + 11 &\geqslant 8 \\ e + 11 - 11 &\geqslant 8 - 11 \\ e &\geqslant -3 \end{aligned}$$

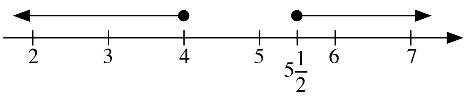
(f)

$$\begin{aligned} 5 - \frac{2f-5}{6} &\leqslant \frac{f+3}{2} + \frac{2(f+1)}{3} \\ 6 \times \left(5 - \frac{2f-5}{6}\right) &\leqslant 6 \times \left[\frac{f+3}{2} + \frac{2(f+1)}{3}\right] \\ 30 - (2f-5) &\leqslant 3(f+3) + 4(f+1) \\ 30 - 2f + 5 &\leqslant 3f + 9 + 4f + 4 \\ 35 - 2f &\leqslant 7f + 13 \\ 35 - 2f - 7f &\leqslant 7f + 13 - 7f \\ 35 - 9f &\leqslant 13 \\ 35 - 9f - 35 &\leqslant 13 - 35 \\ -9f &\leqslant -22 \\ f &\geqslant \frac{-22}{-9} \\ f &\geqslant 2\frac{4}{9} \end{aligned}$$

3. (a) Solving the two linear inequalities separately,

$$\begin{aligned} 5 - a &\leqslant a - 6 & &\text{and} & a - 6 &\leqslant 10 - 3a \\ 5 - a - a &\leqslant a - 6 - a & & & a - 6 + 3a &\leqslant 10 - 3a + 3a \\ 5 - 2a &\leqslant -6 & & & 4a - 6 &\leqslant 10 \\ 5 - 2a - 5 &\leqslant -6 - 5 & & & 4a - 6 + 6 &\leqslant 10 + 6 \\ -2a &\leqslant -11 & & & 4a &\leqslant 16 \\ a &\geqslant \frac{-11}{-2} & & & a &\leqslant 4 \\ a &\geqslant 5\frac{1}{2} & & & & \end{aligned}$$

Representing $a \geqslant 5\frac{1}{2}$ and $a \leqslant 4$ on a number line,

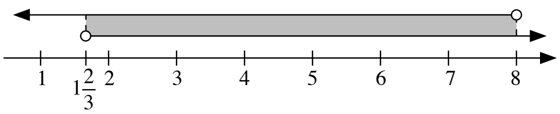


$\therefore$ The simultaneous linear inequalities have no solution.

(b) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 4 - b &< 2b - 1 & \text{and} & 2b - 1 &< 7 + b \\ 4 - b - 2b &< 2b - 1 - 2b & & 2b - 1 - b &< 7 + b - b \\ 4 - 3b &< -1 & & b - 1 &< 7 \\ 4 - 3b - 4 &< -1 - 4 & & b - 1 + 1 &< 7 + 1 \\ -3b &< -5 & & b &< 8 \\ b &> \frac{-5}{-3} & & & \\ b &> 1\frac{2}{3} & & & \end{array}$$

Representing $b > 1\frac{2}{3}$ and $b < 8$ on a number line,

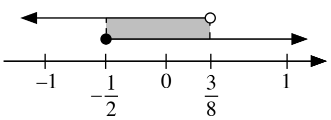


$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $1\frac{2}{3} < b < 8$.

(c) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 4c - 1 &< \frac{1}{2} & \text{and} & \frac{1}{2} &\leqslant 3c + 2 \\ 4c - 1 + 1 &< \frac{1}{2} + 1 & & \frac{1}{2} - 3c &\leqslant 3c + 2 - 3c \\ 4c &< 1\frac{1}{2} & & \frac{1}{2} - 3c &\leqslant 2 \\ c &< \frac{1\frac{1}{2}}{4} & & \frac{1}{2} - 3c - \frac{1}{2} &\leqslant 2 - \frac{1}{2} \\ c &< \frac{3}{8} & & -3c &\leqslant 1\frac{1}{2} \\ & & & c &\geqslant \frac{1\frac{1}{2}}{-3} \\ & & & c &\geqslant -\frac{1}{2} \end{array}$$

Representing $c < \frac{3}{8}$ and $c \geqslant -\frac{1}{2}$ on a number line,

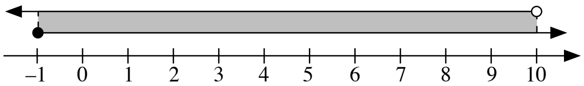


$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-\frac{1}{2} \leqslant c < \frac{3}{8}$.

(d) Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 2d + 1 &\geqslant d & \text{and} & d &> 3d - 20 \\ 2d + 1 - d &\geqslant d - d & & d - 3d &> 3d - 20 - 3d \\ d + 1 &\geqslant 0 & & -2d &> -20 \\ d + 1 - 1 &\geqslant 0 - 1 & & d &< \frac{-20}{-2} \\ d &\geqslant -1 & & d &< 10 \end{array}$$

Representing $d \geqslant -1$ and $d < 10$ on a number line,



$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-1 \leqslant d < 10$.

4. **(a)** Largest integer value of $x = 14$

(b) Largest prime value of $x = 13$

(c) Largest rational value of $x = 14\frac{1}{2}$

5.

$$\begin{aligned} 27 - 2x &\leqslant 8 \\ 27 - 2x - 27 &\leqslant 8 - 27 \\ -2x &\leqslant -19 \\ x &\geqslant \frac{-19}{-2} \\ x &\geqslant 9\frac{1}{2} \end{aligned}$$

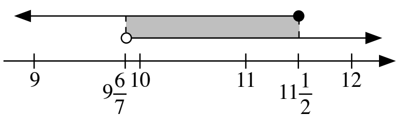
(a) Least value of $x = 9\frac{1}{2}$

(b) Least integer value of $x = 10$

6. **(a)** Solving the two linear inequalities separately,

$$\begin{array}{rlcrl} 5x &> 69 - 2x & \text{and} & 27 - 2x &\geqslant 4 \\ 5x + 2x &> 69 - 2x + 2x & & 27 - 2x - 27 &\geqslant 4 - 27 \\ 7x &> 69 & & -2x &\geqslant -23 \\ x &> \frac{69}{7} & & x &\leqslant \frac{-23}{-2} \\ x &> 9\frac{6}{7} & & x &\leqslant 11\frac{1}{2} \end{array}$$

Representing $x > 9\frac{6}{7}$ and $x \leqslant 11\frac{1}{2}$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $9\frac{6}{7} < x \leqslant 11\frac{1}{2}$.

$\therefore$ The integer values of x which satisfying both inequalities are 10 and 11.

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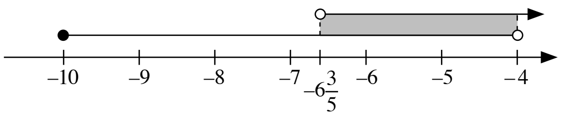
(b) Solving the two linear inequalities separately,

$$-10 \leqslant x < -4 \quad \text{and} \quad 2 - 5x < 35$$
$$2 - 5x - 2 < 35 - 2$$
$$-5x < 33$$
$$x > \frac{33}{-5}$$
$$x > -6\frac{3}{5}$$

Representing $-10 \leqslant x < -4$ and $x > -6\frac{3}{5}$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-6\frac{3}{5} < x < -4$.

$\therefore$ The integer values of x which satisfy both inequalities are -6 and -5.

7. Given that $-1 \leqslant x \leqslant 5$ and $2 \leqslant y \leqslant 6$,

(a) Greatest value of $y - x = 6 - (-1)$
$= 6 + 1$
$= 7$

Least value of $y - x = 2 - 5$
$= -3$

(b) Greatest value of $\frac{x}{y} = \frac{5}{2}$
$= 2\frac{1}{2}$

Least value of $\frac{x}{y} = \frac{-1}{2}$
$= -\frac{1}{2}$

(c) Greatest value of $\frac{x^2}{y} = \frac{5^2}{2}$
$= \frac{25}{2}$
$= 12\frac{1}{2}$

Least value of $\frac{x^2}{y} = \frac{0}{6}$
$= 0$

8. Given that $-3 \leqslant x \leqslant 7$ and $4 \leqslant y \leqslant 10$,

(a) Smallest value of $x - y = -3 - 10$
$= -13$

(b) Largest value of $\frac{x}{y} = \frac{7}{4}$
$= 1\frac{3}{4}$

(c) Largest value of $x^2 - y^2 = 7^2 - 4^2$
$= 49 - 16$
$= 33$

(d) Smallest value of $x^3 + y^3 = (-3)^3 + 4^3$
$= -27 + 64$
$= 37$

9. Let the side of a square be x cm.

$$4x \leqslant 81$$
$$x \leqslant \frac{81}{4}$$
$$x \leqslant 20\frac{1}{4}$$

Greatest possible value of $x = 20\frac{1}{4}$

$\therefore$ Greatest possible area of the square $= 20\frac{1}{4} \times 20\frac{1}{4}$
$= 410.1 \text{ cm}^2$ (to 4 s.f.)

10.
$$x \times 3 + 5 \leqslant 20$$
$$3x + 5 \leqslant 20$$
$$3x + 5 - 5 \leqslant 20 - 5$$
$$3x \leqslant 15$$
$$x \leqslant 5$$

$\therefore$ The maximum number of sheets of writing paper that he can use is 5.

11. Let the age of Rui Feng now be x years. Then the age of Lixin is $(x - 3)$ years.

$$x + (x - 3) \leqslant 50$$
$$2x - 3 \leqslant 50$$
$$2x - 3 + 3 \leqslant 50 + 3$$
$$2x \leqslant 53$$
$$x \leqslant \frac{53}{2}$$
$$x \leqslant 26\frac{1}{2}$$

The maximum possible age of Rui Feng now is 26 years.
The maximum possible age of Lixin now is 23 years.
$\therefore$ The maximum possible age of Lixin 5 years ago is 18 years.

12. Let x be the number of 20-cent coins.

$$16 \times 1 + x \times 0.20 \leqslant 22$$
$$16 + 0.20x \leqslant 22$$
$$16 + 0.20x - 16 \leqslant 22 - 16$$
$$0.20x \leqslant 6$$
$$x \leqslant \frac{6}{0.20}$$
$$x \leqslant 30$$

$\therefore$ The maximum number of 20-cent coins that he has is 30.

13. Let x be the number of questions Amirah answered wrongly.

$$2 \times (15 - x) - 1 \times x > 24$$
$$2(15 - x) - x > 24$$
$$30 - 2x - x > 24$$
$$30 - 3x > 24$$
$$30 - 3x - 30 > 24 - 30$$
$$-3x > -6$$
$$x < \frac{-6}{-3}$$
$$x < 2$$

$\therefore$ The maximum number of questions Amirah answered wrongly is 1.

15. Upper bound mass of 20 cabbages = 92.5 kg
Lower bound mass of 20 cabbages = 91.5 kg
Upper bound average mass of one cabbage
$= \dfrac{92.5}{20}$
= 4.625 kg
Lower bound average mass of one cabbage
$= \dfrac{91.5}{20}$
= 4.575 kg

16. **(i)** Least possible individual masses
= 35 kg, 35 kg, 45 kg, 55 kg, 55 kg, 65 kg
Least possible total mass
= 35 + 35 + 45 + 55 + 55 + 65
= 290 kg

(ii) Greatest possible individual masses
= 45 kg, 45 kg, 55 kg, 65 kg, 65 kg, 75 kg
Greatest possible total mass
= 45 + 45 + 55 + 65 + 65 + 75
= 350 kg

17. Maximum dimensions = 20.5 cm, 30.5 cm, 15.5 cm
Maximum volume = $20.5 \times 30.5 \times 15.5$
= 9691.375 cm^3

18. Least possible lengths of sides = 5.5 cm, 6.5 cm, 7.5 cm
Least possible perimeter = 5.5 + 6.5 + 7.5
= 19.5 cm

19. Upper bound of radius = 9.55 cm
Lower bound of radius = 9.45 cm

(a) Upper bound circumference = $2\pi(9.55)$
= 60.00 cm (to 2 d.p.)
Lower bound circumference = $2\pi(9.45)$
= 59.38 cm (to 2 d.p.)

(b) Upper bound area = $\pi(9.55)^2$
= 286.52 cm^2 (to 2 d.p.)
Lower bound area = $\pi(9.45)^2$
= 280.55 cm^2 (to 2 d.p.)

Challenge Yourself

1. $z = \dfrac{x}{y}$

Greatest $\dfrac{x}{y} = \dfrac{8}{0.2} = 40$

Least $\dfrac{x}{y} = \dfrac{6}{0.5} = 12$

$\therefore 12 \leqslant z \leqslant 40$

2. $\dfrac{3x-5}{x^2-14x+49} = \dfrac{3x-5}{(x-7)^2}$

$\dfrac{3x-5}{x^2-14x+49} > 0$

$\dfrac{3x-5}{(x-7)^2} > 0$

Since $(x-7)^2$ is positive, $3x - 5$ must be more than 0 for $\dfrac{3x-5}{(x-7)^2}$ to be more than 0.

$3x - 5 > 0$
$3x > 5$
$x > \dfrac{5}{3}$
$x > 1\dfrac{2}{3}$

Chapter 4 Indices and Standard Form

TEACHING NOTES

Suggested Approach

In Book 1, the students have been introduced to writing numbers in index notation. In this chapter, they will learn the laws of indices, zero and negative indices and rational indices.

Teacher should consider using the Investigation activities provided in the textbook to allow students to explore the laws of indices for numbers before moving on to variables. It is not advisable to state all the laws of indices to the students when teaching this chapter. After the students are familiar with laws of indices introduced in Section 4.2, where all the indices are positive integers, teachers can extend it to Section 4.3: Zero and Negative Indices and Section 4.4: Rational Indices.

Teacher should also conduct more discussions on how compound interest and standard form are used in real life.

Section 4.1: Indices

This section gives students a better understanding on the meanings of the base and the index represented in an index notation. Teachers may start on this chapter by giving scenarios where indices are involved and ask the students to represent their answers in index notation, like what they have learnt in Book 1 (see Investigation: Indices). Teachers should guide students along as they learn how to describe and compare numbers written in index form (see Class Discussion: Comparing Numbers written in Index Form).

Section 4.2: Laws of Indices

Teachers should provide simple numerical examples to illustrate each law of indices. Ample examples should be given to the students to master each law first before moving on to the next law (see Investigation: Law 1 of Indices, Investigation: Law 2 of Indices, Investigation: Law 3 of Indices, Investigation: Law 4 of Indices and Investigation: Law 5 of Indices).

Teachers should clarify any common misconceptions students may have or difficulties they may encounter when working on questions involving the use of a few laws of indices (see Journal writing on page 83 of the textbook).

Section 4.3: Zero and Negative Indices

Teachers may ask the students to explore the meaning of zero and negative indices through activities instead of only asking them to state the definition of such indices (see Investigation: Zero Index and see Investigation: Negative Indices).

It is important to emphasise to the students the meaning of 'evaluate' and 'leaving your answer in positive index form. Teachers should also highlight the importance of recognising where the brackets are placed in a question (see Thinking Time on page 87 of the textbook).

Section 4.4: Rational Indices

In Book 1, students have learnt about the square root and cube root of a number. Teacher may wish to extend on this by introducing the meaning of positive n^{th} root and radical expression.

Teachers should highlight to students to consider the need for the base to be positive in rational indices (see Thinking Time on page 92 of the textbook).

Section 4.5: Standard Form

Teacher may begin this section by getting students to explore how standard forms are being expressed by giving them some examples of very large and small numbers for them to express these numbers in standard form (see Class Discussion: Standard Form). Teachers should highlight the difference between numbers expressed in standard form and numbers not expressed in standard form so that students can better identify which expressions are in standard form.

For the introduction of common prefixes used in our daily lives, teachers may use the range of prefixes used in our daily lives (see page 100 of the textbook) to get the students to give more examples of prefixes that they encounter in their daily lives and to practise reading prefixes.

Some students may find it difficult to manipulate numbers in standard form using a calculator. Teachers should give them time and guide them through some examples on using the calculator to evaluate numbers represented in standard form.

WORKED SOLUTIONS

Investigation (Indices)

Amount of allowance on the 31st day of the month $= 2^{31}$

$= 2\,147\,483\,548$ cents

$\approx \$21.5$ million

Class Discussion (Comparing Numbers written in Index Form)

1. 2^{10} means 2 multiplied by itself 10 times, while 10^2 means 10 multiplied by itself.
2. $2^{10} = (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2)$
   $= 32^2 > 10^2$
3. $3^7 = 3(3)^6 = 3(9)^3 > 7^3$
4.

Value of a^b	Value of b^a
$2^3 = 8$	$3^2 = 9$
$2^4 = 16$	$4^2 = 16$
$3^4 = 81$	$4^3 = 64$
$3^5 = 243$	$5^3 = 125$
$4^5 = 1024$	$5^4 = 625$
$4^6 = 4096$	$6^4 = 1296$

(i) If a and b are positive integers such that $b > a$, $a^b = b^a$ when $a = 2$ and $b = 4$.
It is not easy to prove that this is the only solution; students are only expected to use guess and check to find a solution.

(ii) If a and b are positive integers such that $b > a$, $a^b < b^a$ when $a = 1$, i.e. $1^2 < 2^1$, $1^3 < 3^1$, $1^4 < 4^1$, etc and when $a = 2$ and $b = 3$, $a^b < b^a$, i.e. $2^3 < 3^2$.

5. In general, if a and b are positive integers such that $b > a$, then $a^b > b^a$, with some exceptions when $a = 2$ and $b \leqslant 4$ and when $a = 1$.

Investigation (Law 1 of Indices)

1. $7^2 \times 7^3 = (7 \times 7) \times (7 \times 7 \times 7)$
   $= 7 \times 7 \times \ldots \times 7$
   $= 7^5$
   $= 7^{2+3}$
2. $6^4 \times 6^5 = (6 \times 6 \times 6 \times 6) \times (6 \times 6 \times 6 \times 6 \times 6)$
   $= 6 \times 6 \times \ldots \times 6$
   $= 6^9$
   $= 6^{4+5}$
3. $a^3 \times a^4 = (a \times a \times a) \times (a \times a \times a \times a)$
   $= a \times a \times \ldots \times a$
   $= a^7$
   $= a^{3+4}$
4. $a^m \times a^n = \underbrace{(a \times a \times \ldots \times \ldots \times a)}_{m \text{ times}} \times \underbrace{(a \times a \times \ldots \times a)}_{n \text{ times}}$
   $= \underbrace{a \times a \times \ldots \times a}_{m+n \text{ times}}$
   $= a^{m+n}$

Investigation (Law 2 of Indices)

1. $3^5 \div 3^2 = \dfrac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$
   $= 3^3$
   $= 3^{5-2}$
2. $\dfrac{10^6}{10^4} = \dfrac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10}$
   $= 10^2$
   $= 10^{6-4}$
3. $a^7 \div a^3 = \dfrac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a}$
   $= a^4$
   $= a^{7-3}$
4. $a^m \div a^n = \dfrac{\overbrace{a \times a \times \ldots \times \ldots \times a}^{m \text{ times}}}{\underbrace{a \times a \times \ldots \times a}_{n \text{ times}}}$
   $= \underbrace{a \times a \times \ldots \times a}_{m-n \text{ times}}$
   $= a^{m-n}$

Investigation (Law 3 of Indices)

1. $(2^5)^2 = 2^5 \times 2^5$
   $= 2^{5+5}$ (using Law 1 of indices)
   $= 2^{5 \times 2}$
2. $(10^4)^3 = 10^4 \times 10^4 \times 10^4$
   $= 10^{4+4+4}$ (using Law 1 of indices)
   $= 10^{4 \times 3}$
3. $(a^m)^n = \underbrace{(a^m \times a^m \times \ldots \times a^m)}_{n \text{ times}}$
   $= a^{\overbrace{m+m+\ldots+m}^{n \text{ times}}}$
   $= a^{m \times n}$

Investigation (Law 4 of Indices)

1. $2^3 \times 7^3 = (2 \times 2 \times 2) \times (7 \times 7 \times 7)$
   $= (2 \times 7) \times (2 \times 7) \times (2 \times 7)$
   $= (2 \times 7)^3$
2. $(-3)^2 \times (-4)^2 = (-3) \times (-3) \times (-4) \times (-4)$
   $= [(-3) \times (-4)] \times [(-3) \times (-4)]$
   $= [(-3) \times (-4)]^2$
3. $a^n \times b^n = \underbrace{(a \times a \times \ldots \times a)}_{n \text{ times}} \times \underbrace{(b \times b \times \ldots \times b)}_{n \text{ times}}$
   $= \underbrace{(a \times b) \times (a \times b) \times \ldots \times (a \times b)}_{n \text{ times}}$
   $= (a \times b)^n$

Class Discussion (Simplification using the Laws of Indices)

$$(xy^2)^4 \times (3x^2y)^4 = (x^4y^{2\times4}) \times (3^4x^{2\times4}y^4) \quad \text{(Law 4 and Law 3)}$$
$$= (x^4y^8) \times (81x^8y^4)$$
$$= 81x^{4+8}y^{8+4} \quad \text{(Law 1)}$$
$$= 81x^{12}y^{12}$$

$$(xy^2)^4 \times (3x^2y)^4 = [(xy^2) \times (3x^2y)]^4 \quad \text{(Law 4)}$$
$$= (3x^{1+2}y^{2+1})^4 \quad \text{(Law 1)}$$
$$= (3x^3y^3)^4$$
$$= 3^4x^{3\times4}y^{3\times4} \quad \text{(Law 3)}$$
$$= 81x^{12}y^{12}$$

Investigation (Law 5 of Indices)

1. $8^3 \div 5^3 = \dfrac{8^3}{5^3}$

$$= \frac{8\times8\times8}{5\times5\times5}$$
$$= \frac{8}{5}\times\frac{8}{5}\times\frac{8}{5}$$
$$= \left(\frac{8}{5}\right)^3$$

2. $(-12)^4 \div (-7)^4 = \dfrac{(-12)^4}{(-7)^4}$

$$= \frac{(-12)\times(-12)\times(-12)\times(-12)}{(-7)\times(-7)\times(-7)\times(-7)}$$
$$= \frac{(-12)}{(-7)}\times\frac{(-12)}{(-7)}\times\frac{(-12)}{(-7)}\times\frac{(-12)}{(-7)}$$
$$= \left[\frac{(-12)}{(-7)}\right]^4$$

3. $a^n \div b^n = \dfrac{\overbrace{a\times a\times\ldots\times a}^{n \text{ times}}}{\underbrace{b\times b\times\ldots\times b}_{n \text{ times}}}$

$$= \underbrace{\frac{a}{b}\times\frac{a}{b}\times\ldots\times\frac{a}{b}}_{n \text{ times}}$$
$$= \left(\frac{a}{b}\right)^n$$

Journal Writing (Page 83)

1. **(i)** To simplify $\left(\dfrac{2x^2}{y}\right)^3$, Law 3 of indices must be applied to the entire expression.

Nora applied Law 3 of Indices to the algebraic terms x and y but she did not apply the same law to the number.

Farhan applied Law 3 of indices to the number correctly but he applied Law 1 of Indices to the algebraic terms x and y which is the wrong law.

(ii) $\left(\dfrac{2x^2}{y}\right)^3 = \dfrac{2^3\times x^{2\times3}}{y^3} = \dfrac{8x^6}{y^3}$

Class Discussion (Is $(a+b)^n = a^n + b^n$? Is $(a-b)^n = a^n - b^n$?)

$(a+b)^n \neq a^n + b^n$

Example: $a = 3$, $b = 2$, $n = 4$

$(3+2)^4 = 5^4 = 625$

$3^4 + 2^4 = 81 + 16 = 97$

$(a-b)^n \neq a^n - b^n$

Example: $a = 3$, $b = 2$, $n = 4$

$(3-2)^4 = 1^4 = 1$

$3^4 - 2^4 = 81 - 16 = 65$

Investigation (Zero Index)

1.

Index Form	Value
3^4	81
3^3	27
3^2	9
3^1	3
3^0	1

Table 4.1

2. 81 (i.e. 3^4) must be divided by 3 to get 27 (i.e. 3^3).

3. 27 (i.e. 3^3) must be divided by 3 to get the value of 3^2.

4. 3^2 must be divided by 3 to get the value of 3^1.

5. **(a)** 3^1 must be divided by 3 to get the value of 3^0.

6.

Index Form	Value
$(-2)^4$	16
$(-2)^3$	-8
$(-2)^2$	4
$(-2)^1$	-2
$(-2)^0$	1

Table 4.2

7. No. Any number which is divided by zero is undefined.

Thinking Time (Page 87)

LHS $= -5^0 = -(5^0) = -1$

RHS $= (-5)^0 = 1$

$\therefore -5^0 \neq (-5)^0$

Investigation (Negative Indices)

1.

Index Form	Value
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{9}$

Table 4.3

2.

Index Form	Value
$(-2)^2$	4
$(-2)^1$	-2
$(-2)^0$	1
$(-2)^{-1}$	$-\frac{1}{2}$
$(-2)^{-2}$	$\frac{1}{4}$

Table 4.4

3. Undefined. Any number which is divided by zero is undefined.

Thinking Time (Page 89)

1. If a and b are real numbers, and m and n are *integers*, then

Law 1 of Indices: $a^m \times a^n = \underline{a^{m+n}}$ if $a \neq 0$

Law 2 of Indices: $a^m \div a^n = \underline{a^{m-n}}$ if $a \neq 0$

Law 3 of Indices: $(a^m)^n = \underline{a^{mn}}$ if $\underline{a \neq 0}$

Law 4 of Indices: $a^n \times b^n = \underline{(a \times b)^n}$ if $a, b \neq 0$

Law 5 of Indices: $a^n \div b^n = \left(\frac{a}{b}\right)^n$ if $\underline{b \neq 0}$

2. **(i)** In Law 1, it is necessary for $a \neq 0$ because if $0^{-2} \times 0^{-1}$, it is undefined.

(ii) In Law 4, it is necessary for $a, b \neq 0$ because if $0^{-2} \times 0^{-2}$, it is undefined.

3. **(i)** If $m = n$ in Law 2, then LHS $= a^n \div a^n = 1$ and
RHS $= a^{n-n} = a^0 = 1$.
So a^0 is a special case of Law 2.

(ii) If $m = 0$, then RHS $= a^{0-n} = \frac{1}{a^n}$ and

LHS $= a^0 \div a^n = 1 \div a^n = \frac{1}{a^n}$

So $a^{-n} = \frac{1}{a^n}$ is a special case of Law 2.

Class Discussion (Rational Indices)

Let $p = 5^{\frac{1}{3}}$. Then $p^3 = (5^{\frac{1}{3}})^3$

$= 5^{\frac{1}{3}\times 3}$ (Using Law 3 of Indices)

$= 5^1$

$= 5$

$\therefore p = \sqrt[3]{5}$

In this case, there is only one possible value of p.

Hence, $5^{\frac{1}{3}} = \sqrt[3]{5}$.

Thinking Time (Page 92)

1. If $a < 0$, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ is undefined.

2. If $a = 0$, then $0^{\frac{1}{n}} = \sqrt[n]{0}$ is still undefined when n is a positive integer.

Investigation (Rational Indices)

(a) $5^{\frac{2}{3}} = 5^{2\times\frac{1}{3}}$

$= (5^2)^{\frac{1}{3}}$

$= \sqrt[3]{5^2}$

(b) $5^{\frac{2}{3}} = 5^{\frac{1}{3}\times 2}$

$= (5^{\frac{1}{3}})^2$

$= (\sqrt[3]{5})^2$

Thinking Time (Page 94)

1. If a and b are real numbers, and m and n are *rational numbers*, then

Law 1 of Indices: $a^m \times a^n = \underline{a^{m+n}}$ if $a > 0$

Law 2 of Indices: $a^m \div a^n = \underline{a^{m-n}}$ if $a > 0$

Law 3 of Indices: $(a^m)^n = \underline{a^{mn}}$ if $\underline{a > 0}$

Law 4 of Indices: $a^n \times b^n = \underline{(a \times b)^n}$ if $a, b > 0$

Law 5 of Indices: $a^n \div b^n = \underline{\left(\frac{a}{b}\right)^n}$ if $\underline{b > 0}$

2. **(i)** In Law 1, it is necessary for $a > 0$ otherwise a^m or a^n is not defined.

(ii) In Law 4, it is necessary for $a, b > 0$ otherwise a^n or b^n is not defined.

3. $\sqrt{(-1)\times(-1)}$ is undefined. In Law 4, both $a, b > 0$ but in this case, $a = b = -1 < 0$.

Class Discussion (Standard Form)

1. The powers of 10 are all positive integers.

2. The powers of 10 are all negative integers.

3. **(v)** 2.9×10^4

(vi) 3×10^8 m/s

(vii) 3.8×10^{-5} cm

(viii) 2.99×10^{-23} g

Performance Task (Page 102)

1 GB = 1 073 741 824 bytes

All computer data is stored in a binary format as either a one or a zero. Hence each level is an increment of 2 to the 10^{th} power or 1024. As such, 1 GB = 2^{30}.

2^7 = 128 MB

2^8 = 256 MB

2^9 = 512 MB

Thinking Time (Page 104)

1. 57 910 000 km = 5.791×10^7 km

5 945 900 000 km = 5.9459×10^9 km

2. 3 683 000 m/h = $\frac{3683\text{ km}}{1\text{ h}}$ = 3683 km/h = 3.683×10^3 km/h

3. $0.000\,000\,0004\text{ m} = 4 \times 10^{-10}\text{ m}$

$500\,000\,000\,000\,000\,000\,000\,000\,000 = 5 \times 10^{26}$

$$\text{Total volume of air molecules} = 5 \times 10^{26} \times \pi \times \left(\frac{4 \times 10^{-10}}{2}\right)^2$$
$$= 6.28 \times 10^7 \text{ m}^3 \text{ (to 3 s.f.)}$$

4. $100 \text{ trillion} = 100 \times 10^{12} = 1 \times 10^{14}$

$1 \times 10^6 = 1 \text{ million} \Rightarrow 2 \times 10^9 = 2 \times 10^6 \times 10^3 = 2000 \text{ million}$

$42\,000\,000 = 4.2 \times 10^7$

Practise Now 1

(a) $4^7 \times 4^5 = 4^{7+5}$
$= 4^{12}$

(b) $(-3)^6 \times (-3) = (-3)^{6+1}$
$= (-3)^7$

(c) $a^{12} \times a^8 = a^{12+8}$
$= a^{20}$

(d) $2xy^4 \times 3x^5y^3 = 6x^{1+5}y^{4+3}$
$= 6x^6y^7$

Practise Now 2

(a) $9^7 \div 9^3 = 9^{7-3}$
$= 9^4$

(b) $(-4)^8 \div (-4) = (-4)^{8-1}$
$= (-4)^7$

(c) $a^{10} \div a^6 = a^{10-6}$
$= a^4$

(d) $27x^9y^4 \div 9x^6y^3 = \dfrac{27x^9y^4}{9x^6y^3}$
$= 3x^{9-6}y^{4-3}$
$= 3x^3y$

Practise Now 3

1. **(a)** $(6^3)^4 = 6^{3\times 4}$
$= 6^{12}$

(b) $(k^5)^9 = k^{5\times 9}$
$= k^{45}$

(c) $(3^q)^6 \times (3^4)^q = 3^{6q} \times 3^{4q}$
$= 3^{6q+4q}$
$= 3^{10q}$

2. $x^8 \times (x^3)^n \div (x^n)^2 = x^{10}$
$x^8 \times x^{3n} \div x^{2n} = x^{10}$
$x^{8+3n-2n} = x^{10}$
$8 + n = 10$
$n = 2$

Practise Now 4

(a) $3^7 \times 8^7 = 24^7$

(b) $(5b^4)^3 = 5^3 \times b^{4\times 3}$
$= 125 \times b^{12}$
$= 125b^{12}$

(c) $(-2c^2d^5)^5 = (-2)^5 \times c^{2\times 5} \times d^{5\times 5}$
$= -32 \times c^{10} \times d^{25}$
$= -32c^{10}d^{25}$

(d) $(m^2n)^4 \times (m^4n^3)^5 = (m^{2\times 4}n^4) \times (m^{4\times 5}n^{3\times 5})$
$= (m^8n^4) \times (m^{20}n^{15})$
$= m^{8+20}n^{4+15}$
$= m^{28}n^{19}$

(e) $(-p^7q^5)^2 \div (3p^3q^2)^3 = \dfrac{(-p^7q^5)^2}{(3p^3q^2)^3}$
$= \dfrac{(-1)^2 \times p^{7\times 2} \times q^{5\times 2}}{3^3 \times p^{3\times 3} \times q^{2\times 3}}$
$= \dfrac{p^{14}q^{10}}{27p^9q^6}$
$= \dfrac{p^{14-9}q^{10-6}}{27}$
$= \dfrac{p^5q^4}{27}$

Practise Now 5

(a) $21^3 \div 7^3 = \dfrac{21^3}{7^3}$
$= \left(\dfrac{21}{7}\right)^3$
$= 3^3$

(b) $(26^5)^3 \div 13^{15} = \dfrac{26^{15}}{13^{15}}$
$= \left(\dfrac{26}{13}\right)^{15}$
$= 2^{15}$

(c) $\left(\dfrac{p^2}{q}\right)^3 \div \dfrac{q^7}{p^5} = \dfrac{p^6}{q^3} \div \dfrac{q^7}{p^5}$
$= \dfrac{p^6}{q^3} \times \dfrac{q^7}{p^5}$
$= \dfrac{p^{6+5}}{q^{3+7}}$
$= \dfrac{p^{11}}{q^{10}}$

(d) $\left(\dfrac{3x^2}{x^3}\right)^3 \div \dfrac{27x^7}{x^{21}} = \dfrac{27x^6}{x^9} \div \dfrac{27x^7}{x^{21}}$
$= \dfrac{27x^6}{x^9} \times \dfrac{x^{21}}{27x^7}$
$= \dfrac{27x^{6+21}}{27x^{9+7}}$
$= \dfrac{x^{27}}{x^{16}}$
$= x^{27-16}$
$= x^{11}$

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Practise Now 6

1. **(a)** $2015^0 = 1$

(b) $(-7)^0 = 1$

(c) $3y^0 = 3(1)$
$= 3$

(d) $(3y)^0 = 3^0 \times y^0$
$= 1 \times 1$
$= 1$

2. **(a)** $3^0 \times 3^3 \div 3^2 = 3^{0+3-2}$
$= 3^1$
$= 3$

(b) $3^0 + 3^2 = 1 + 9$
$= 10$

Practise Now 7

(a) $6^{-2} = \dfrac{1}{6^2}$
$= \dfrac{1}{36}$

(b) $(-8)^{-1} = \dfrac{1}{(-8)^1}$
$= -\dfrac{1}{8}$

(c) $\left(\dfrac{4}{5}\right)^{-3} = \dfrac{1}{\left(\dfrac{4}{5}\right)^3}$
$= \dfrac{1}{\left(\dfrac{64}{125}\right)}$
$= 1 \div \dfrac{64}{125}$
$= 1 \times \dfrac{125}{64}$
$= 1\dfrac{61}{64}$

(d) $\left(\dfrac{1}{9}\right)^{-1} = \dfrac{1}{\left(\dfrac{1}{9}\right)^{-1}}$
$= 1 \div \dfrac{1}{9}$
$= 1 \times 9$
$= 9$

Practise Now 8

(a) $a^{-1} \times a^3 \div a^{-2} = a^{-1+3-(-2)}$
$= a^4$

(b) $(b^{-5}c^2)^{-3} = b^{-5\times-3}c^{2\times-3}$
$= b^{15}c^{-6}$
$= \dfrac{b^{15}}{c^6}$

(c) $\dfrac{16d^{-2}e}{(2d^{-1}e)^3} = \dfrac{16d^{-2}e}{2^3 \times d^{-1\times3} \times e^3}$
$= \dfrac{16d^{-2}e}{8 \times d^{-3} \times e^3}$
$= \dfrac{2d^{-2+3}}{e^{3-1}}$
$= \dfrac{2d}{e^2}$

(d) $5f^0 \div 3(f^{-2})^2 = \dfrac{5f^0}{3(f^{-2})^2}$
$= \dfrac{5}{3f^{-4}}$
$= \dfrac{5f^4}{3}$

(e) $18g^{-6} \div 3(g^{-2})^2 = \dfrac{18g^{-6}}{3(g^{-2})^2}$
$= \dfrac{18g^{-6}}{3g^{-4}}$
$= \dfrac{6}{g^{-4+6}}$
$= \dfrac{6}{g^2}$

(f) $6h^2 \div 2h^{-2} - h \times h^3 - \dfrac{4}{h^{-4}} = \dfrac{6h^2}{2h^{-2}} - h \times h^3 - \dfrac{4}{h^{-4}}$
$= 3h^{2+2} - h^{1+3} - 4h^4$
$= 3h^4 - h^4 - 4h^4$
$= -2h^4$

Practise Now 9

(a) By prime factorisation, $256 = 4 \times 4 \times 4 \times 4 = 4^4$.
$\therefore \sqrt[4]{256} = \sqrt[4]{4 \times 4 \times 4 \times 4}$
$= 4$

(b) By prime factorisation, $1024 = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$.
$\therefore \sqrt[5]{1024} = \sqrt[5]{4 \times 4 \times 4 \times 4 \times 4}$
$= 4$

(c) By prime factorisation, $8 = 2 \times 2 \times 2 = 2^3$ and $27 = 3 \times 3 \times 3 = 3^3$.
$\therefore \sqrt[3]{\dfrac{8}{27}} = \sqrt[3]{\dfrac{2 \times 2 \times 2}{3 \times 3 \times 3}}$
$= \dfrac{2}{3}$

Practise Now 10

(a) $36^{\frac{1}{2}} = \sqrt{36}$
$= 6$

(b) $8^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{1}{8}}$

$= \frac{1}{2}$

(c) $(-125)^{\frac{1}{3}} = \left(\frac{1}{-125}\right)^{\frac{1}{3}}$

$= \frac{1}{\sqrt[3]{-125}}$

$= -\frac{1}{5}$

Practise Now 11

1. **(a)** $64^{\frac{2}{3}} = (\sqrt[3]{64})^2$

$= 4^2$

$= 16$

(b) $32^{-\frac{3}{5}} = \left(\frac{1}{32}\right)^{\frac{3}{5}}$

$= \left(\sqrt[5]{\frac{1}{32}}\right)^3$

$= \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$

(c) $100^{1.5} = 100^{\frac{3}{2}}$

$= (\sqrt{100})^3$

$= (10)^3$

$= 1000$

2. **(a)** $\sqrt[3]{a^n} = (a^n)^{\frac{1}{3}}$

$= a^{\frac{n}{3}}$

(b) $\frac{1}{\sqrt[5]{x^2}} = \frac{1}{(x^2)^{\frac{1}{5}}}$

$= \frac{1}{x^{\frac{2}{5}}}$

$= x^{-\frac{2}{5}}$

Practise Now 12

(a) $(m^2)^{\frac{5}{6}} \times m^{\frac{1}{3}} = m^{2\times\frac{5}{6}} \times m^{\frac{1}{3}}$

$= m^{\frac{5}{3}} \times m^{\frac{1}{3}}$

$= m^{\frac{5}{3}+\frac{1}{3}}$

$= m^2$

(b) $\sqrt[5]{m} \div \sqrt[3]{m^2} = \frac{\sqrt[5]{m}}{\sqrt[3]{m^2}}$

$= \frac{m^{\frac{1}{5}}}{(m^2)^{\frac{1}{3}}}$

$= \frac{m^{\frac{1}{5}}}{m^{\frac{2}{3}}}$

$= m^{\frac{1}{5}-\frac{2}{3}}$

$= m^{-\frac{7}{15}}$

$= \frac{1}{m^{\frac{7}{15}}}$

(c) $(m^{-3}n^5)^{-\frac{1}{3}} = m^{-3\times-\frac{1}{3}}n^{5\times-\frac{1}{3}}$

$= mn^{-\frac{5}{3}}$

$= \frac{m}{n^{\frac{5}{3}}}$

(d) $\frac{m^{-\frac{1}{3}}n^{-\frac{1}{4}}}{(m^2n^{-\frac{1}{3}})^{-2}} = \frac{m^{-\frac{1}{3}}n^{-\frac{1}{4}}}{m^{2\times-2}n^{-\frac{1}{3}\times-2}}$

$= \frac{m^{-\frac{1}{3}}n^{-\frac{1}{4}}}{m^{-4}n^{\frac{2}{3}}}$

$= m^{-\frac{1}{3}-(-4)}n^{-\frac{1}{4}-\frac{2}{3}}$

$= m^{\frac{11}{3}}n^{-\frac{11}{12}}$

$= \frac{m^{\frac{11}{3}}}{n^{\frac{11}{12}}}$

(e) $(25m^2n^{-4})^{\frac{1}{2}}(m^3n^{-\frac{2}{5}})^2 = (25^{\frac{1}{2}} \times m^{2\times\frac{1}{2}} \times n^{-4\times\frac{1}{2}})(m^{3\times2} \times n^{-\frac{2}{5}\times2})$

$= (5mn^{-2})(m^6n^{-\frac{4}{5}})$

$= 5m^{1+6}n^{-2-\frac{4}{5}}$

$= 5m^7n^{-\frac{14}{5}}$

$= \frac{5m^7}{n^{\frac{14}{5}}}$

(f) $(m^2n^{-\frac{1}{7}}) \times \sqrt[5]{(m^5n^{-5})} = (m^2n^{-\frac{1}{7}}) \times (m^5n^{-5})^{\frac{1}{5}}$

$= (m^2n^{-\frac{1}{7}}) \times (m^{5\times\frac{1}{5}}n^{-5\times\frac{1}{5}})$

$= (m^2n^{-\frac{1}{7}}) \times (mn^{-1})$

$= m^{2+1}n^{-\frac{1}{7}-1}$

$= m^3n^{-\frac{8}{7}}$

$= \frac{m^3}{n^{\frac{8}{7}}}$

Practise Now 13

(a) $5^x = 125$
$5^x = 5^3$
$x = 3$

(b) $7^y = \frac{1}{49}$
$7^y = \frac{1}{7^2}$
$7^y = 7^{-2}$
$y = -2$

(c) $8^z = 16$
$(2^3)^z = 2^4$
$2^{3z} = 2^4$
$3z = 4$
$z = 1\frac{1}{3}$

Practise Now 14

1. **(a)** $5\ 300\ 000 = 5.3 \times 10^6$
   (b) $600\ 000\ 000 = 6 \times 10^8$
   (c) $0.000\ 048 = 4.8 \times 10^{-5}$
   (d) $0.000\ 000\ 000\ 21 = 2.1 \times 10^{-10}$
2. **(a)** $1.325 \times 10^6 = 1\ 325\ 000$
   (b) $4.4 \times 10^{-3} = 0.0044$

Practise Now 15

(a) 1 micrometre $= 10^{-6}$ metres
25.4 micrometres $= 25.4 \times 10^{-6}$ metres
$= 2.54 \times 10^{-5}$ metres

(b) 10 mm = 1 cm
2340 mm $= \frac{1}{10} \times 2340$ cm
$= 234$ cm
$= 2.34 \times 10^2$ cm

(c) 1 terabyte $= 10^{12}$ bytes
4.0 terabytes $= 4.0 \times 10^{12}$ bytes

Practise Now 16

(a) $(1.14 \times 10^5) \times (4.56 \times 10^4)$
$= \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{1}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{)}$
$\boxed{\times}\ \boxed{(}\ \boxed{4}\ \boxed{.}\ \boxed{5}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}\ \boxed{=}$
$= 5.20 \times 10^9$ (to 3 s.f.)

(b) $(4.2 \times 10^{-4}) \times (2.6 \times 10^2)$
$= \boxed{(}\ \boxed{4}\ \boxed{.}\ \boxed{2}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}$
$\boxed{\times}\ \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{2}\ \boxed{)}\ \boxed{=}$
$= 0.1092$
$= 1.09 \times 10^{-1}$ (to 3 s.f.)

(c) $(2.4 \times 10^8) \div (6 \times 10^4)$
$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{8}\ \boxed{)}\ \boxed{\div}$
$\boxed{(}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}\ \boxed{=}$
$= 4000$
$= 4 \times 10^3$

(d) $\frac{3.5 \times 10^{-5}}{1.4 \times 10^8}$
$= \boxed{(}\ \boxed{3}\ \boxed{.}\ \boxed{5}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{5}\ \boxed{)}$
$\boxed{\div}\ \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{8}\ \boxed{)}\ \boxed{=}$
$= 2.5 \times 10^{-13}$

(e) $1.14 \times 10^5 + 4.56 \times 10^4$
$= \boxed{1}\ \boxed{.}\ \boxed{1}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{\div}$
$\boxed{4}\ \boxed{.}\ \boxed{5}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{=}$
$= 159\ 600$
$= 1.60 \times 10^5$ (to 3 s.f.)

(f) $4 \times 10^4 - 2.6 \times 10^6$
$= \boxed{4}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{-}\ \boxed{2}\ \boxed{.}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{6}\ \boxed{=}$
$= -2\ 560\ 000$
$= -2.56 \times 10^6$

(g) $\frac{2.37 \times 10^{-3} + 3.25 \times 10^{-4}}{4.1 \times 10^5}$
$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{3}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{3}\ \boxed{+}\ \boxed{3}\ \boxed{.}\ \boxed{2}\ \boxed{5}$
$\boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}\ \boxed{\div}\ \boxed{4}\ \boxed{.}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{=}$
$= 6.57 \times 10^{-9}$ (to 3 s.f.)

(h) $\frac{6.3 \times 10^6}{1.5 \times 10^2 - 3 \times 10^{-1}}$
$= \boxed{6}\ \boxed{.}\ \boxed{3}\ \boxed{\times 10^x}\ \boxed{6}\ \boxed{\div}\ \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{5}$
$\boxed{\times 10^x}\ \boxed{2}\ \boxed{-}\ \boxed{3}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{1}\ \boxed{)}\ \boxed{=}$
$= 4.21 \times 10^4$ (to 3 s.f.)

Practise Now 17

1 MB $= 10^6$ bytes
512 MB $= 512 \times 10^6$ bytes
$= 5.12 \times 10^8$ bytes

No. of photographs that can be stored $= \frac{5.12 \times 10^8}{640 \times 10^3}$
$= 800$

Exercise 4A

1. **(a)** $2^3 \times 2^7 = 2^{3+7}$
   $= 2^{10}$
   (b) $(-4)^6 \times (-4)^5 = (-4)^{6+5}$
   $= (-4)^{11}$
   (c) $x^8 \times x^3 = x^{8+3}$
   $= x^{11}$
   (d) $(3y^2) \times (8y^7) = 24y^{2+7}$
   $= 24y^9$

2. **(a)** $5^8 \div 5^5 = 5^{8-5}$
$= 5^3$

(b) $(-7)^{11} \div (-7)^4 = (-7)^{11-4}$
$= (-7)^7$

(c) $6x^7 \div x^3 = 6x^{7-3}$
$= 6x^4$

(d) $(-15y^9) \div 5y^4 = \dfrac{-15y^9}{5y^4}$
$= -3y^{9-4}$
$= -3y^5$

3. **(a)** $(9^2)^4 = 9^{2\times 4}$
$= 9^8$

(b) $(h^2)^5 = h^{2\times 5}$
$= h^{10}$

(c) $3^{14} \times (5^2)^7 = 3^{14} \times 5^{2\times 7}$
$= 3^{14} \times 5^{14}$
$= (3 \times 5)^{14}$
$= 15^{14}$

(d) $2^3 \times 9^3 = (2 \times 9)^3$
$= 18^3$

(e) $(2k^6)^3 = 2^3 \times k^{6\times 3}$
$= 8 \times k^{18}$
$= 8k^{18}$

(f) $(-3x^6y^2)^4 = (-3)^4 \times x^{6\times 4} \times y^{2\times 4}$
$= 81 \times x^{24} \times y^8$
$= 81x^{24}y^8$

4. **(a)** $14^{13} \div 7^{13} = \dfrac{14^{13}}{7^{13}}$
$= 2^{13}$

(b) $(9^5)^4 \div 3^{20} = \dfrac{9^{5\times 4}}{3^{20}}$
$= \dfrac{(3^2)^{20}}{3^{20}}$
$= \dfrac{3^{2\times 20}}{3^{20}}$
$= \dfrac{3^{40}}{3^{20}}$
$= 3^{40-20}$
$= 3^{20}$

(c) $\left(\dfrac{m}{2}\right)^5 = \dfrac{m^5}{2^5}$
$= \dfrac{m^5}{32}$

(d) $\left(\dfrac{3}{n^2}\right)^3 = \dfrac{3^3}{n^{2\times 3}}$
$= \dfrac{27}{n^6}$

(e) $\left(\dfrac{p^4}{q}\right)^6 = \dfrac{p^{4\times 6}}{q^6}$
$= \dfrac{p^{24}}{q^6}$

(f) $\left(-\dfrac{x}{y^2}\right)^4 = \left(-1 \times \dfrac{x}{y^2}\right)^4$
$= (-1)^4 \times \dfrac{x^4}{y^{2\times 4}}$
$= 1 \times \dfrac{x^4}{y^8}$
$= \dfrac{x^4}{y^8}$

5. **(a)** $h^2k \times h^{11}k^9 = h^{2+11} \times k^{1+9}$
$= h^{13}k^{10}$

(b) $(-m^7n^3) \times 4m^{11}n^9 = -1 \times 4 \times m^{7+11} \times n^{3+9}$
$= -4 \times m^{18} \times n^{12}$
$= -4m^{18}n^{12}$

(c) $11p^6q^7 \times 2p^3q^{10} = 11 \times p^{6+3} \times 2 \times q^{7+10}$
$= 22 \times p^9 \times q^{17}$
$= 22p^9q^{17}$

(d) $h^9k^6 \div h^5k^4 = \dfrac{h^9k^6}{h^5k^4}$
$= h^{9-5}k^{6-4}$
$= h^4k^2$

(e) $15m^8n^7 \div 3m^2n = \dfrac{15m^8n^7}{3m^2n}$
$= 5m^{8-2}n^{7-1}$
$= 5m^6n^6$

(f) $(-10x^5y^6) \div (-2xy^3) = \dfrac{-10x^5y^6}{-2xy^3}$
$= 5x^{5-1}y^{6-3}$
$= 5x^4y^3$

6. **(a)** $(a^2)^3 \times a^5 = a^{2\times 3} \times a^5$
$= a^6 \times a^5$
$= a^{6+5}$
$= a^{11}$

(b) $(b^3)^7 \times (b^4)^5 = b^{3\times 7} \times b^{4\times 5}$
$= b^{21} \times b^{20}$
$= b^{21+20}$
$= b^{41}$

(c) $(c^6)^5 \div (-c^2) = c^{6\times 5} \div (-c^2)$
$= \dfrac{c^{30}}{-c^2}$
$= \dfrac{c^{30-2}}{-1}$
$= -c^{28}$

(d) $(-3d^3)^2 \div (2d)^3 = \dfrac{(-3d^3)^2}{(2d)^3}$
$= \dfrac{(-3)^2 \times d^{3\times 2}}{2^3 \times d^3}$
$= \dfrac{9 \times d^6}{8 \times d^3}$
$= \dfrac{9}{8} \times d^{6-3}$
$= \dfrac{9d^3}{8}$

(e) $(e^3)^5 \div (-e^2)^4 = \dfrac{(e^3)^5}{(-e^2)^4}$

$= \dfrac{e^{3\times5}}{(-1)^4 \times e^{2\times4}}$

$= \dfrac{e^{15}}{1\times e^8}$

$= e^{15-8}$

$= e^7$

(f) $(4f^6)^3 \div (-2f^3)^3 = \dfrac{(4f^6)^3}{(-2f^3)^3}$

$= \dfrac{4^3\times f^{6\times3}}{(-2)^3\times f^{3\times3}}$

$= \dfrac{64\times f^{18}}{-8\times f^9}$

$= -8\times f^{18-9}$

$= -8f^9$

7. **(a)** $(ab^2)^3 \times (2a^2b)^3 = a^3\times b^{2\times3}\times 2^3\times a^{2\times3}\times b^3$

$= a^3\times b^6\times 8\times a^6\times b^3$

$= 8\times a^{3+6}\times b^{6+3}$

$= 8\times a^9\times b^9$

$= 8a^9b^9$

(b) $c^2d^2\times(-5c^3d^3)^2 = c^2d^2\times(-5)^2\times c^{3\times2}\times d^{3\times2}$

$= c^2\times d^2\times 25\times c^6\times d^6$

$= 25\times c^{2+6}\times d^{2+6}$

$= 25\times c^8\times d^8$

$= 25c^8d^8$

(c) $(8e^5f^3)^2 \div (e^3f)^3 = \dfrac{(8e^5f^3)^2}{(e^3f)^3}$

$= \dfrac{8^2\times e^{5\times2}\times f^{3\times2}}{e^{3\times3}\times f^3}$

$= \dfrac{64\times e^{10}\times f^6}{e^9\times f^3}$

$= 64\times e^{10-9}\times f^{6-3}$

$= 64\times e\times f^3$

$= 64ef^3$

(d) $16g^8h^7 \div (-2g^3h^2)^3 = \dfrac{16g^8h^7}{(-2g^3h^2)^3}$

$= \dfrac{16g^8h^7}{(-2)^3\times g^{3\times3}\times h^{2\times3}}$

$= \dfrac{16g^8h^7}{-8\times g^9\times h^6}$

$= -2g^{8-9}h^{7-6}$

$= -2g^{-1}h$

$= -\dfrac{2h}{g}$

8. **(a)** $\dfrac{2a^2}{b}\times\left(\dfrac{a}{b^2}\right)^2 = \dfrac{2a^2}{b}\times\dfrac{a^2}{b^{2\times2}}$

$= \dfrac{2a^2}{b}\times\dfrac{a^2}{b^4}$

$= \dfrac{2a^{2+2}}{b^{1+4}}$

$= \dfrac{2a^4}{b^5}$

(b) $\left(\dfrac{c}{d^2}\right)^3\times\left(\dfrac{c^3}{2d}\right)^2 = \dfrac{c^3}{d^{2\times3}}\times\dfrac{c^{3\times2}}{2^2\times d^2}$

$= \dfrac{c^3}{d^6}\times\dfrac{c^6}{4\times d^2}$

$= \dfrac{c^{3+6}}{4\times d^{6+2}}$

$= \dfrac{c^9}{4d^8}$

(c) $\left(\dfrac{3e^3}{f^2}\right)^4 \div \dfrac{27e^9}{f^{11}} = \dfrac{3^4\times e^{3\times4}}{f^{2\times4}} \div \dfrac{27e^9}{f^{11}}$

$= \dfrac{81\times e^{12}}{f^8}\div\dfrac{27e^9}{f^{11}}$

$= \dfrac{81\times e^{12}}{f^8}\times\dfrac{f^{11}}{27\times e^9}$

$= \dfrac{81}{27}\,e^{12-9}f^{11-8}$

$= 3e^3f^3$

(d) $\left(\dfrac{g^2}{h^3}\right)^6 \div \left(\dfrac{-3g^5}{2h^2}\right)^3 = \dfrac{g^{2\times6}}{h^{3\times6}} \div \dfrac{(-3)^3\times g^{5\times3}}{2^3\times h^{2\times3}}$

$= \dfrac{g^{12}}{h^{18}}\div\dfrac{-27\times g^{15}}{8\times h^6}$

$= \dfrac{g^{12}}{h^{18}}\times\dfrac{8\times h^6}{-27\times g^{15}}$

$= -\dfrac{8}{27}\,g^{12-15}h^{6-18}$

$= -\dfrac{8}{27}\,g^{-3}h^{-12}$

$= -\dfrac{8}{27g^3h^{12}}$

9. **(a)** $\dfrac{(2x^2y)^3}{(10xy^3)^2}\times\dfrac{(5xy^4)^3}{4xy} = \dfrac{2^3\times x^{2\times3}\times y^3}{10^2\times x^2\times y^{3\times2}}\times\dfrac{5^3\times x^3\times y^{4\times3}}{4xy}$

$= \dfrac{8\times x^6\times y^3}{100\times x^2\times y^6}\times\dfrac{125\times x^3\times y^{12}}{4xy}$

$= \dfrac{1000\times x^{6+3}\times y^{3+12}}{400\times x^{2+1}\times y^{6+1}}$

$= \dfrac{5\times x^9\times y^{15}}{2\times x^3\times y^7}$

$= \dfrac{5}{2}\times x^{9-3}\times y^{15-7}$

$= \dfrac{5x^6y^8}{2}$

(b) $\dfrac{8x^8y^4}{(2xy^2)^2} \times \dfrac{(4x^2y^2)^2}{(3xy)^2} = \dfrac{8x^8y^4}{2^2 \times x^2 \times y^{2\times 2}} \times \dfrac{4^2 \times x^{2\times 2} \times y^{2\times 2}}{3^2 \times x^2 \times y^2}$

$= \dfrac{8x^8y^4}{4 \times x^2 \times y^4} \times \dfrac{16 \times x^4 \times y^4}{9 \times x^2 \times y^2}$

$= \dfrac{128 \times x^{8+4} \times y^{4+4}}{36 \times x^{2+2} \times y^{4+2}}$

$= \dfrac{32 \times x^{12} \times y^8}{9 \times x^4 \times y^6}$

$= \dfrac{32}{9} \times x^{12-4} \times y^{8-6}$

$= \dfrac{32x^8y^2}{9}$

(c) $\dfrac{(2xy^2)^5}{(4x^2y)^2(xy^3)} = \dfrac{2^5 \times x^5 \times y^{2\times 5}}{4^2 \times x^{2\times 2} \times y^2 \times xy^3}$

$= \dfrac{32 \times x^5 \times y^{10}}{16 \times x^4 \times y^2 \times xy^3}$

$= \dfrac{32 \times x^5 \times y^{10}}{16 \times x^{4+1} \times y^{2+3}}$

$= \dfrac{32 \times x^5 \times y^{10}}{16 \times x^5 \times y^5}$

$= 2 \times x^{5-5} \times y^{10-5}$

$= 2 \times x^0 \times y^5$

$= 2y^5$

(d) $\dfrac{4x^2y^4 \times 8x^4y^2}{(4x^2y^2)^2} = \dfrac{32 \times x^{2+4} \times y^{4+2}}{4^2 \times x^{2\times 2} \times y^{2\times 2}}$

$= \dfrac{32 \times x^6 \times y^6}{16 \times x^4 \times y^4}$

$= 2 \times x^{6-4} \times y^{6-4}$

$= 2 \times x^2 \times y^2$

$= 2x^2y^2$

10. $\dfrac{(2p^3q^4)^4}{(-3q^5)^2} \div \dfrac{(4p^2q)^2}{9} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{2^4 \times p^{3\times 4} \times q^{4\times 4}}{(-3)^2 \times q^{5\times 2}} \div \dfrac{4^2 \times p^{2\times 2} \times q^2}{9} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{16 \times p^{12} \times q^{16}}{9 \times q^{10}} \div \dfrac{16 \times p^4 \times q^2}{9} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{16 \times p^{12} \times q^{16}}{9 \times q^{10}} \times \dfrac{9}{16 \times p^4 \times q^2} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{p^{12} \times q^{16}}{q^{10}} \times \dfrac{1}{p^4 \times q^2} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{p^{12} \times q^{16}}{p^4 \times q^{10+2}} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{p^{12-4}}{q^{12-16}} = \dfrac{p^{a+b}}{q^{a-b}}$

$\dfrac{p^8}{q^{-4}} = \dfrac{p^{a+b}}{q^{a-b}}$

$a + b = 8$

$a = 8 - b$ — (1)

$a - b = -4$ — (2)

Substitute (1) into (2):

$(8 - b) - b = -4$

$8 - 2b = -4$

$2b = 12$

$b = 6$

$a = 8 - 6$

$= 2$

$\therefore a = 2, b = 6$

Exercise 4B

1. **(a)** $17^0 = 1$

(b) $\left(-\dfrac{2}{7}\right)^0 = 1$

(c) $4a^0 = 4(1)$

$= 4$

(d) $-8b^0 = -8(1)$

$= -8$

(e) $(72cd^2)^0 = 1$

(f) $7(e^8)^0 = 7(1)$

$= 7$

2. **(a)** $2^0 \times 2^4 = 2^{0+4}$

$= 2^4$

$= 16$

(b) $7^2 \times 7^0 \div 7 = \dfrac{7^2 \times 7^0}{7}$

$= 7^{2+0-1}$

$= 7$

(c) $8^0 - 8^2 = 1 - 64$

$= -63$

(d) $6^3 + 6^0 - 6 = 216 + 1 - 6$

$= 211$

3. **(a)** $7^{-3} = \dfrac{1}{7^3}$

$= \dfrac{1}{343}$

(b) $(-5)^{-1} = \dfrac{1}{(-5)^1}$

$= -\dfrac{1}{5}$

(c) $\left(\dfrac{3}{4}\right)^{-2} = \dfrac{1}{\left(\dfrac{3}{4}\right)^2}$

$= \dfrac{1}{\left(\dfrac{9}{16}\right)}$

$= 1 \div \dfrac{9}{16}$

$= 1 \times \dfrac{16}{9}$

$= 1\dfrac{7}{9}$

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(d) $\left(\frac{5}{3}\right)^{-1} = \frac{1}{\left(\frac{5}{3}\right)}$

$= 1 \div \frac{5}{3}$

$= 1 \times \frac{3}{5}$

$= \frac{3}{5}$

4. **(a)** $(7^2)^{-2} \div 7^{-4} = \frac{(7^2)^{-2}}{7^{-4}}$

$= \frac{7^{-4}}{7^{-4}}$

$= 1$

(b) $5^0 - 5^{-2} = 5^0 - \frac{1}{5^2}$

$= 1 - \frac{1}{25}$

$= \frac{24}{25}$

(c) $(2^{15})^0 + \left(\frac{3}{5}\right)^{-1} = 1 + \frac{5}{3}$

$= 2\frac{2}{3}$

(d) $\left(\frac{3}{4}\right)^{-2} \times 3^2 \times 2015^0 = \left(\frac{4}{3}\right)^2 \times 9 \times 1$

$= \frac{16}{9} \times 9$

$= 16$

5. **(a)** By prime factorisation, $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$.

$\therefore \sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7}$

$= 14$

(b) By prime factorisation, $125 = 5 \times 5 \times 5 = 5^3$.

$\therefore \sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5}$

$= 5$

(c) By prime factorisation, $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

$\therefore \sqrt[5]{\frac{1}{32}} = \sqrt[5]{\frac{1}{2 \times 2 \times 2 \times 2 \times 2}}$

$= \frac{1}{2}$

(d) By prime factorisation, $16 = 2 \times 2 \times 2 \times 2 = 2^4$ and $81 = 3 \times 3 \times 3 \times 3 = 3^4$.

$\therefore \sqrt[4]{\frac{16}{81}} = \sqrt[4]{\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}}$

$= \frac{2}{3}$

6. **(a)** $81^{\frac{1}{2}} = \sqrt{81}$

$= 9$

(b) $(-27)^{\frac{1}{3}} = \sqrt[3]{-27}$

$= -3$

(c) $(16)^{-\frac{1}{4}} = \left(\frac{1}{16}\right)^{\frac{1}{4}}$

$= \frac{1}{\sqrt[4]{16}}$

$= \frac{1}{2}$

(d) $4^{1.5} = 4^{\frac{3}{2}}$

$= (\sqrt{4})^3$

$= 2^3$

$= 8$

(e) $8^{-\frac{5}{3}} = \left(\frac{1}{8}\right)^{\frac{5}{3}}$

$= \frac{1}{(\sqrt[3]{8})^5}$

$= \frac{1}{2^5}$

$= \frac{1}{32}$

(f) $(-1000)^{\frac{2}{3}} = (\sqrt[3]{-1000})^2$

$= (-10)^2$

$= 100$

7. **(a)** $\sqrt[4]{a} = a^{\frac{1}{4}}$

(b) $\sqrt[3]{b^2} = (b^2)^{\frac{1}{3}}$

$= b^{2 \times \frac{1}{3}}$

$= b^{\frac{2}{3}}$

(c) $(\sqrt[5]{c})^4 = (c^{\frac{1}{5}})^4$

$= c^{\frac{1}{5} \times 4}$

$= c^{\frac{4}{5}}$

(d) $\frac{1}{\sqrt[6]{d}} = \frac{1}{d^{\frac{1}{6}}}$

$= d^{-\frac{1}{6}}$

(e) $\frac{1}{\sqrt[8]{e^4}} = \frac{1}{e^{\frac{4}{8}}}$

$= \frac{1}{e^{\frac{1}{2}}}$

$= e^{-\frac{1}{2}}$

(f) $\frac{1}{(\sqrt[3]{f})^5} = \frac{1}{f^{\frac{5}{3}}}$

$= f^{-\frac{5}{3}}$

8. **(a)** $11^a = 1331$

$11^a = 11^3$

$a = 3$

(b) $2^b = \frac{1}{128}$

$2^b = \frac{1}{2^7}$

$2^b = 2^{-7}$

$b = -7$

(c) $9^c = 3^5$

$3^{2c} = 3^5$

$2c = 5$

$c = 2\frac{1}{2}$

(d) $10^d = 0.01$

$10^d = \frac{1}{100}$

$10^d = 10^{-2}$

$d = -2$

9. (a) $5a^4 \times 3a^2 \div a^{-3} = \frac{5a^4 \times 3a^2}{a^{-3}}$

$= \frac{5 \times 3 \times a^{4+2}}{a^{-3}}$

$= \frac{15 \times a^6}{a^{-3}}$

$= 15 \times a^6 \times a^3$

$= 15 \times a^{6+3}$

$= 15a^9$

(b) $-24b^{-6} \div (3b^{-3})^2 = \frac{-24b^{-6}}{(3b^{-3})^2}$

$= \frac{-24b^{-6}}{3^2 \times b^{-3\times 2}}$

$= \frac{-24b^{-6}}{9 \times b^{-6}}$

$= -2\frac{2}{3}$

(c) $(3c)^0 \div (c^{-3}d^5)^{-2} = \frac{(3c)^0}{(c^{-3}d^5)^{-2}}$

$= \frac{1}{c^{-3\times -2}d^{5\times -2}}$

$= \frac{1}{c^6 d^{-10}}$

$= \frac{d^{10}}{c^6}$

(d) $\frac{(4e^{-6}f^3)^2}{8e^{12}f^6} = \frac{4^2 \times e^{-6\times 2} \times f^{3\times 2}}{8e^{12}f^6}$

$= \frac{16 \times e^{-12} \times f^6}{8e^{12}f^6}$

$= \frac{2 \times f^{6-6}}{e^{12+12}}$

$= \frac{2 \times f^0}{e^{24}}$

$= \frac{2}{e^{24}}$

(e) $(3g^{-3}h^{-1})^2 \times (-4g^3h^{-2})^2$

$= (3^2 \times g^{-3\times 2} \times h^{-1\times 2}) \times ((-4)^2 \times g^{3\times 2} \times h^{-2\times 2})$

$= (9 \times g^{-6} \times h^{-2}) \times (16 \times g^6 \times h^{-4})$

$= 9 \times 16 \times g^{-6+6} \times h^{-2-4}$

$= 144 \times g^0 \times h^{-6}$

$= \frac{144}{h^6}$

(f) $(j^2k^{-1})^{-3} \times \left(\frac{j^2}{k^3}\right)^{-3} = j^{2\times -3} \times k^{-1\times -3} \times \frac{j^{2\times -3}}{k^{3\times -3}}$

$= j^{-6} \times k^3 \times \frac{j^{-6}}{k^{-9}}$

$= j^{-6-6} \times k^{3+9}$

$= j^{-12} \times k^{12}$

$= \frac{k^{12}}{j^{12}}$

(g) $\frac{(m^5n^3) \times (m^2)^{-2}}{(m^{-1}n)^2} = \frac{m^5n^3 \times m^{2\times -2}}{m^{-1\times 2} \times n^2}$

$= \frac{m^5n^3 \times m^{-4}}{m^{-2} \times n^2}$

$= \frac{m^{5-4} \times n^3}{m^{-2} \times n^2}$

$= \frac{m \times n^3}{m^{-2} \times n^2}$

$= m^{1+2} \times n^{3-2}$

$= m^3n$

(h) $(5p)^3 - 10p \times 7p^2 + \frac{6}{p^{-3}} = 125p^3 - 10p \times 7p^2 + \frac{6}{p^{-3}}$

$= 125p^3 - 70p^3 + 6p^3$

$= 61p^3$

10. (a) $\sqrt{a} \times \sqrt[3]{a} = a^{\frac{1}{2}} \times a^{\frac{1}{3}}$

$= a^{\frac{1}{2}+\frac{1}{3}}$

$= a^{\frac{3}{6}+\frac{2}{6}}$

$= a^{\frac{5}{6}}$

(b) $\sqrt[3]{b^2} \div \sqrt[6]{b} = (b^2)^{\frac{1}{3}} \div b^{\frac{1}{6}}$

$= \frac{b^{2\times\frac{1}{3}}}{b^{\frac{1}{6}}}$

$= \frac{b^{\frac{2}{3}}}{b^{\frac{1}{6}}}$

$= b^{\frac{2}{3}-\frac{1}{6}}$

$= b^{\frac{4}{6}-\frac{1}{6}}$

$= b^{\frac{3}{6}}$

$= b^{\frac{1}{2}}$

(c) $c^{\frac{4}{5}} \times c^{\frac{1}{2}} \div c^{-\frac{2}{5}} = \frac{c^{\frac{4}{5}+\frac{1}{2}}}{c^{-\frac{2}{5}}}$

$= c^{\frac{4}{5}+\frac{1}{2}+\frac{2}{5}}$

$= c^{\frac{8}{10}+\frac{5}{10}+\frac{4}{10}}$

$= c^{\frac{17}{10}}$

(d) $d^{\frac{1}{10}} \div d^{-\frac{1}{5}} \times d^{-\frac{3}{2}} = \dfrac{d^{\frac{1}{10}} \times d^{-\frac{3}{2}}}{d^{-\frac{1}{5}}}$

$= d^{\frac{1}{10}-\frac{3}{2}-\left(-\frac{1}{5}\right)}$

$= d^{\frac{1}{10}-\frac{15}{10}+\frac{2}{10}}$

$= d^{-\frac{6}{5}}$

$= \dfrac{1}{d^{\frac{6}{5}}}$

(e) $(e^{-3}f^4)^{-\frac{1}{2}} = \dfrac{1}{(e^{-3}f^4)^{\frac{1}{2}}}$

$= \dfrac{1}{e^{-3\times\frac{1}{2}} \times f^{4\times\frac{1}{2}}}$

$= \dfrac{1}{e^{-\frac{3}{2}} \times f^2}$

$= \dfrac{e^{\frac{3}{2}}}{f^2}$

(f) $(g^{\frac{2}{3}}h^{-\frac{4}{5}})^{\frac{3}{2}} = g^{\frac{2}{3}\times\frac{3}{2}}h^{-\frac{4}{5}\times\frac{3}{2}}$

$= gh^{-\frac{6}{5}}$

$= \dfrac{g}{h^{\frac{6}{5}}}$

11. (a) $(a^{-2}b^3)^{\frac{1}{3}} \times (a^4b^{-5})^{\frac{1}{2}} = (a^{-2\times\frac{1}{3}} \times b^{3\times\frac{1}{3}}) \times (a^{4\times\frac{1}{2}} \times b^{-5\times\frac{1}{2}})$

$= (a^{-\frac{2}{3}} \times b) \times (a^2 \times b^{-\frac{5}{2}})$

$= a^{-\frac{2}{3}+2} \times b^{1-\frac{5}{2}}$

$= a^{\frac{4}{3}} \times b^{-\frac{3}{2}}$

$= \dfrac{a^{\frac{4}{3}}}{b^{\frac{3}{2}}}$

(b) $(c^{-3}d^{\frac{3}{5}})^{-2} \times (c^{-3}d^{\frac{3}{5}})^{-2} = (c^{-3\times-2} \times d^{\frac{3}{5}\times-2}) \times (c^{\frac{4}{5}\times 5} d^{-\frac{2}{5}\times 5})$

$= (c^6 \times d^{-\frac{6}{5}}) \times (c^4 \times d^{-2})$

$= c^{6+4} \times d^{-\frac{6}{5}-2}$

$= c^{10} \times d^{-\frac{16}{5}}$

$= \dfrac{c^{10}}{d^{\frac{16}{5}}}$

(c) $\dfrac{e^{-\frac{1}{3}}f^{-\frac{1}{4}}}{(e^2f^{-\frac{1}{3}})^{-2}} = \dfrac{e^{-\frac{1}{3}}f^{-\frac{1}{4}}}{e^{2\times-2} \times f^{-\frac{1}{3}\times-2}}$

$= \dfrac{e^{-\frac{1}{3}}f^{-\frac{1}{4}}}{e^{-4} \times f^{\frac{2}{3}}}$

$= e^{-\frac{1}{3}+4}f^{-\frac{1}{4}-\frac{2}{3}}$

$= e^{\frac{11}{3}}f^{-\frac{11}{12}}$

$= \dfrac{e^{\frac{11}{3}}}{f^{\frac{11}{12}}}$

(d) $\left(\dfrac{g^{-2}h^2}{25}\right)^{-\frac{1}{2}} = \left(\dfrac{25}{g^{-2}h^2}\right)^{\frac{1}{2}}$

$= \dfrac{25^{\frac{1}{2}}}{g^{-2\times\frac{1}{2}} \times h^{2\times\frac{1}{2}}}$

$= \dfrac{5}{g^{-1} \times h}$

$= \dfrac{5g}{h}$

(e) $(4j^4k)^{\frac{1}{2}} \div 2h^3k^{-\frac{1}{2}} = \dfrac{(4j^4k)^{\frac{1}{2}}}{2h^3k^{-\frac{1}{2}}}$

$= \dfrac{4^{\frac{1}{2}} \times j^{4\times\frac{1}{2}} \times k^{\frac{1}{2}}}{2h^3k^{-\frac{1}{2}}}$

$= \dfrac{2 \times j^2 \times k^{\frac{1}{2}}}{2h^3k^{-\frac{1}{2}}}$

$= \dfrac{j^2 \times k^{\frac{1}{2}+\frac{1}{2}}}{h^3}$

$= \dfrac{j^2k}{h^3}$

(f) $(m^3n^{-\frac{1}{4}})^4 \div \sqrt[5]{32m^4n^{-8}} = \dfrac{(m^3n^{-\frac{1}{4}})^4}{\sqrt[5]{32m^4n^{-8}}}$

$= \dfrac{m^{3\times4} \times n^{-\frac{1}{4}\times4}}{(32m^4n^{-8})^{\frac{1}{5}}}$

$= \dfrac{m^{12} \times n^{-1}}{32^{\frac{1}{5}} \times m^{4\times\frac{1}{5}} \times n^{-8\times\frac{1}{5}}}$

$= \dfrac{m^{12} \times n^{-1}}{2 \times m^{\frac{4}{5}} \times n^{-\frac{8}{5}}}$

$= \dfrac{m^{12-\frac{4}{5}} \times n^{-1+\frac{8}{5}}}{2}$

$= \dfrac{m^{\frac{56}{5}} \times n^{\frac{3}{5}}}{2}$

$= \dfrac{m^{\frac{56}{5}}n^{\frac{3}{5}}}{2}$

12. (a) $\left(\dfrac{x^{-4}y^7z^{-6}}{x^3y^{-1}z^3}\right)^3 \times \left(\dfrac{x^5y^2z^{-6}}{x^{-3}y^{-5}z^4}\right)^{-4}$

$= \dfrac{x^{-4\times3} \times y^{7\times3} \times z^{-6\times3}}{x^{3\times3} \times y^{-1\times3} \times z^{3\times3}} \times \dfrac{x^{5\times-4} \times y^{2\times-4} \times z^{-6\times-4}}{x^{-3\times-4} \times y^{-5\times-4} \times z^{4\times-4}}$

$= \dfrac{x^{-12} \times y^{21} \times z^{-18}}{x^9 \times y^{-3} \times z^9} \times \dfrac{x^{-20} \times y^{-8} \times z^{24}}{x^{12} \times y^{20} \times z^{-16}}$

$= \dfrac{x^{-12-20} \times y^{21-8} \times z^{-18+24}}{x^{9+12} \times y^{-3+20} \times z^{9-16}}$

$= \dfrac{x^{-32} \times y^{13} \times z^6}{x^{21} \times y^{17} \times z^{-7}}$

$= x^{-32-21} \times y^{13-17} \times z^{6+7}$

$= x^{-53} \times y^{-4} \times z^{13}$

$= \dfrac{z^{13}}{x^{53}y^4}$

(b) $\left(\dfrac{x^3y^{-4}z^7}{x^{-5}y^2}\right)^3 \div \left(\dfrac{x^{-4}yz^{-5}}{x^7y^{-3}}\right)^{-2}$

$$= \frac{x^{3\times3}\times y^{-4\times3}\times z^{7\times3}}{x^{-5\times3}\times y^{2\times3}} \div \frac{x^{-4\times-2}\times y^{-2}\times z^{-5\times-2}}{x^{7\times-2}\times y^{-3\times-2}}$$

$$= \frac{x^9\times y^{-12}\times z^{21}}{x^{-15}\times y^6} \div \frac{x^8\times y^{-2}\times z^{10}}{x^{-14}\times y^6}$$

$$= \frac{x^9\times y^{-12}\times z^{21}}{x^{-15}\times y^6} \times \frac{x^{-14}\times y^6}{x^8\times y^{-2}\times z^{10}}$$

$$= \frac{x^{9-14}\times y^{-12+6}\times z^{21}}{x^{-15+8}\times y^{6-2}\times z^{10}}$$

$$= \frac{x^{-5}\times y^{-6}\times z^{21}}{x^{-7}\times y^4\times z^{10}}$$

$$= x^{-5+7}\times y^{-6-4}\times z^{21-10}$$

$$= x^2\times y^{-10}\times z^{11}$$

$$= \frac{x^2z^{11}}{y^{10}}$$

(c) $\dfrac{ab^n}{bc}\times\dfrac{c^nd}{cd}\div\dfrac{b^{n+2}}{c^{n+3}} = \dfrac{ab^n}{bc}\times\dfrac{c^nd}{cd}\times\dfrac{c^{n+3}}{b^{n+2}}$

$$= \frac{a\times b^n\times c^{n+n+3}\times d}{b^{1+n+2}\times c^2\times d}$$

$$= \frac{a\times b^n\times c^{2n+3}\times d}{b^{n+3}\times c^2\times d}$$

$$= a\times b^{n-(n+3)}\times c^{2n+3-2}$$

$$= a\times b^{-3}\times c^{2n+1}$$

$$= \frac{ac^{2n+1}}{b^3}$$

(d) $\dfrac{(a+b)^n}{bc^2}\div\dfrac{(a+b)^{n+3}}{abc} = \dfrac{(a+b)^n}{bc^2}\times\dfrac{abc}{(a+b)^{n+3}}$

$$= \frac{(a+b)^n\times abc}{(a+b)^{n+3}\times bc^2}$$

$$= \frac{(a+b)^n\times a}{(a+b)^n\times(a+b)^3\times c}$$

$$= \frac{a}{c(a+b)^3}$$

Exercise 4C

1. **(a)** $85\,300 = 8.53\times10^4$

(b) $52\,700\,000 = 5.27\times10^7$

(c) $0.000\,23 = 2.3\times10^{-4}$

(d) $0.000\,000\,094 = 9.4\times10^{-8}$

2. **(a)** $9.6\times10^3 = 9600$

(b) $4\times10^5 = 400\,000$

(c) $2.8\times10^{-4} = 0.000\,28$

(d) $1\times10^{-6} = 0.000\,001$

3. **(i)** $300\,000\,000$ Hz $= 3\times10^8$ Hz

$= 3\times10^2\times10^6$ Hz

$= 3\times10^2$ MHz

(ii) 300 GHz $= 300\times10^9$ Hz

$= 3\times10^{11}$ Hz

$= 3\times10^5\times10^6$ Hz

$= 3\times10^5$ MHz

4. **(i)** a pm $= 70\times10^{-12}$ m

$= 7\times10^{-11}$ m

(ii) b nm $= 0.074\times10^{-9}$ m

$= 7.4\times10^{-1}$ m

(iii) $a : b = 7.0\times10^{-11} : 7.4\times10^{-11}$

$= 35 : 37$

5. c Mm $= 1500\times10^6$ m

$= 1.5\times10^9$ m

d Tm $= 5.91\times10^{12}$ m

$$\frac{d}{c}\times100\% = \frac{5.91\times10^{12}}{1.5\times10^9}\times100\%$$

$= 394\,000\%$

$= 3.94\times10^5\ \%$

6. **(a)** $(2.34\times10^5)\times(7.12\times10^{-4})$

$= \boxed{(}\,\boxed{2}\,\boxed{.}\,\boxed{3}\,\boxed{4}\,\boxed{\times10^x}\,\boxed{5}\,\boxed{)}$

$\boxed{\times}\,\boxed{(}\,\boxed{7}\,\boxed{.}\,\boxed{1}\,\boxed{2}\,\boxed{\times10^x}\,\boxed{-}\,\boxed{4}\,\boxed{)}\,\boxed{=}$

$= 166.608$

$= 1.67\times10^2$ (to 3 s.f.)

(b) $(5.1\times10^{-7})\times(2.76\times10^{-3})$

$= \boxed{(}\,\boxed{5}\,\boxed{.}\,\boxed{1}\,\boxed{\times10^x}\,\boxed{-}\,\boxed{7}\,\boxed{)}$

$\boxed{\times}\,\boxed{(}\,\boxed{2}\,\boxed{.}\,\boxed{7}\,\boxed{6}\,\boxed{\times10^x}\,\boxed{-}\,\boxed{3}\,\boxed{)}\,\boxed{=}$

$= 1.4076\times10^{-9}$

$= 1.41\times10^{-9}$ (to 3 s.f.)

(c) $(13.4\times10^4)\div(4\times10^5)$

$= \boxed{(}\,\boxed{1}\,\boxed{3}\,\boxed{.}\,\boxed{4}\,\boxed{\times10^x}\,\boxed{4}\,\boxed{)}$

$\boxed{\div}\,\boxed{(}\,\boxed{4}\,\boxed{\times10^x}\,\boxed{5}\,\boxed{)}\,\boxed{=}$

$= 0.335$

$= 3.35\times10^{-1}$

(d) $\dfrac{3\times10^{-4}}{9\times10^{-8}}$

$= \boxed{(}\,\boxed{3}\,\boxed{\times10^x}\,\boxed{-}\,\boxed{4}\,\boxed{)}\,\boxed{\div}\,\boxed{(}\,\boxed{9}\,\boxed{\times10^x}$

$\boxed{-}\,\boxed{8}\,\boxed{)}\,\boxed{=}$

$= 3330$ (to 3 s.f.)

$= 3.33\times10^3$

(e) $2.54\times10^3 + 3.11\times10^4$

$= \boxed{2}\,\boxed{.}\,\boxed{5}\,\boxed{4}\,\boxed{\times10^x}\,\boxed{3}$

$\boxed{+}\,\boxed{3}\,\boxed{.}\,\boxed{1}\,\boxed{1}\,\boxed{\times10^x}\,\boxed{4}\,\boxed{=}$

$= 33\,640$

$= 3.36\times10^4$ (to 3 s.f.)

(f) $6\times10^5 - 3.1\times10^7$

$= \boxed{6}\,\boxed{\times10^x}\,\boxed{5}\,\boxed{-}\,\boxed{3}\,\boxed{.}\,\boxed{1}\,\boxed{\times10^x}\,\boxed{7}\,\boxed{=}$

$= -30\,400\,000$

$= -3.04\times10^7$

(g) $\dfrac{4.37 \times 10^{-4} + 2.16 \times 10^{-5}}{3 \times 10^{-3}}$

= (4 . 3 7 $\times 10^x$ – 4 + 2 . 1 6 $\times 10^x$ – 5) ÷ 3 $\times 10^x$ – 3 =

= 0.153 (to 3 s.f.)

= 1.53×10^{-1}

(h) $\dfrac{2.4 \times 10^{-10}}{7.2 \times 10^{-6} - 3.5 \times 10^{-8}}$

= 2 . 4 $\times 10^x$ – 1 0 ÷ (7 . 2 $\times 10^x$ – 6 – 3 . 5 $\times 10^x$ – 8) =

= 3.35×10^{-5} (to 3 s.f.)

7. (a) $(1.35 \times 10^{-4})^3$

= (1 . 3 5 $\times 10^x$ – 4) x^y 3 =

= 2.46×10^{-12} (to 3 s.f.)

(b) $6(3.4 \times 10^3)^2$

= 6 × (3 . 4 $\times 10^x$ 3) x^y 2 =

= 69 360 000

= 6.94×10^7 (to 3 s.f.)

(c) $\sqrt{1.21 \times 10^8}$

= $\sqrt{\ }$ (1 . 2 1 $\times 10^x$ 8) =

= 1.1×10^4

(d) $\sqrt[3]{9.261 \times 10^6}$

= shift $\sqrt[3]{\ }$ (9 . 2 6 1 $\times 10^x$ 6) =

= 210

= 2.1×10^2

(e) $\dfrac{2.3 \times 10^{-2} \times 4.7 \times 10^3}{2 \times 10^3}$

= (2 . 3 $\times 10^x$ – 2 × 4 . 7 $\times 10^x$ 3) ÷ 2 $\times 10^x$ 3 =

= 0.05405

= 5.41×10^{-2} (to 3 s.f.)

(f) $\dfrac{8 \times 10^2 + 2.5 \times 10^3}{2 \times 10^{-2} - 3.4 \times 10^{-3}}$

= (8 $\times 10^x$ 2 + 2 . 5 $\times 10^x$ 3) ÷ (2 $\times 10^x$ – 2 – 3 . 4 $\times 10^x$ – 3) =

= 199 000 (to 3 s.f.)

= 1.99×10^5

8. (a) $2P \times 4Q = 2(7.5 \times 10^3) \times 4(5.25 \times 10^4)$

$= 3.15 \times 10^9$

(b) $Q - P = (5.25 \times 10^4) - (7.5 \times 10^3)$

$= 4.5 \times 10^4$

9. $x + 8y = (2 \times 10^{-3}) + 8(7 \times 10^{-4})$

$= 7.6 \times 10^{-3}$

10. (a) $MN = (3.2 \times 10^6) \times (5.0 \times 10^7)$

$= 1.6 \times 10^{14}$

(b) $\dfrac{M}{N} = \dfrac{3.2 \times 10^6}{5.0 \times 10^7}$

$= 6.4 \times 10^{-2}$

11. $R = \dfrac{M}{EI}$

$= \dfrac{6 \times 10^4}{(4.5 \times 10^8) \times (4 \times 10^2)}$

$= 3.33 \times 10^{-7}$ (to 3 s.f.)

12. (i) 300 000 000 m/s = 3×10^8 m/s

(ii) 778.5 million km = 778.5×10^6 km

$= 778.5 \times 10^6 \times 10^3$ m

$= 7.785 \times 10^{11}$ m

Time taken = $\dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{7.785 \times 10^{11}}{3 \times 10^8}$

= 2595 seconds

= 43 minutes 15 seconds

13. (i) Distance travelled by rockets in 4 days = 4.8×10^5 km

Distance travelled by rockets in 12 days = $\dfrac{4.8 \times 10^5}{4} \times 12$

$= 1.44 \times 10^6$ km

(ii) Speed = $\dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{4.8 \times 10^5}{4}$

$= 1.2 \times 10^5$ km/day

Time taken = $\dfrac{4.8 \times 10^7}{1.2 \times 10^5}$

= 400 days

14. (i) Increase in population = $5.45 \times 10^8 - 4.20 \times 10^8$

$= 1.25 \times 10^8$

(ii) $\dfrac{1.17 \times 10^9}{5.45 \times 10^8} = 2.15$ (to 3 s.f.)

(iii) $\dfrac{1.23 \times 10^9}{7.28 \times 10^8} = 1.69$ (to 3 s.f.)

Review Exercise 4

1. (a) $(a^3b) \times (a^4b^3) = a^{3+4}b^{1+3}$
$= a^7b^4$

(b) $(6a^5b^4) \div (2a^3b^2) = \dfrac{6a^5b^4}{2a^3b^2}$
$= 3a^{5-3}b^{4-2}$
$= 3a^2b^2$

(c) $(-3a^3b^5)^3 = (-3)^3 \times a^{3\times 3} \times b^{5\times 3}$
$= -27a^9b^{15}$

(d) $\left(\dfrac{2a^2b}{b^3}\right)^3 \div \left(\dfrac{16a^5}{ab^7}\right) = \left(\dfrac{8a^{2\times 3}b^3}{b^{3\times 3}}\right) \div \left(\dfrac{16a^5}{ab^7}\right)$
$= \left(\dfrac{8a^6b^3}{b^9}\right) \times \left(\dfrac{ab^7}{16a^5}\right)$
$= \left(\dfrac{8a^{6+1}b^{3+7}}{16a^5b^9}\right)$
$= \left(\dfrac{8a^7b^{10}}{16a^5b^9}\right)$
$= \left(\dfrac{a^{7-5}b^{10-9}}{2}\right)$
$= \dfrac{a^2b}{2}$

2. (a) $5^{24} \div 5^8 = 5^{24-8}$
$= 5^{16}$

(b) $\dfrac{1}{125} = \dfrac{1}{5^3} = 5^{-3}$

(c) $\sqrt[5]{5} = 5^{\frac{1}{5}}$

3. (a) $5^2 \div 5^{-1} \times 5^0 = 5^{2-(-1)+0}$
$= 5^3$
$= 125$

(b) $2^{-2} - 3^{-2} = \dfrac{1}{2^2} - \dfrac{1}{3^2}$
$= \dfrac{1}{4} - \dfrac{1}{9}$
$= \dfrac{9}{36} - \dfrac{4}{36}$
$= \dfrac{5}{36}$

(c) $3^{-2} + \left(\dfrac{1}{3}\right)^{-1} - (-3)^0 = \dfrac{1}{3^2} + 3 - 1$
$= \dfrac{1}{9} + 3 - 1$
$= 2\dfrac{1}{9}$

(d) $\left(\dfrac{2}{5}\right)^3 \div \left(\dfrac{9}{2}\right)^{-2} = \left(\dfrac{2^3}{5^3}\right) \div \left(\dfrac{2}{9}\right)^2$
$= \left(\dfrac{8}{125}\right) \div \left(\dfrac{2^2}{9^2}\right)$
$= \left(\dfrac{8}{125}\right) \div \left(\dfrac{4}{81}\right)$
$= \dfrac{8}{125} \times \dfrac{81}{4}$
$= 1\dfrac{37}{125}$

4. (a) By prime factorisation, $81 = 3 \times 3 \times 3 \times 3 = 3^4$.
$\sqrt[4]{81} = \sqrt[4]{3 \times 3 \times 3 \times 3}$
$= 3$

(b) By prime factorisation, $27 = 3 \times 3 \times 3 = 3^3$
and $125 = 5 \times 5 \times 5 = 5^3$.
$\sqrt[3]{\dfrac{27}{125}} = \sqrt[3]{\dfrac{3\times 3\times 3}{5\times 5\times 5}}$
$= \dfrac{3}{5}$

(c) $16^{1.5} = 16^{\frac{3}{2}}$
$= (\sqrt{16})^3$
$= 4^3$
$= 64$

(d) $1024^{-\frac{3}{5}} = \left(\dfrac{1}{1024}\right)^{\frac{3}{5}}$
$= \left(\dfrac{1}{\sqrt[5]{1024}}\right)^3$
$= \left(\dfrac{1}{4}\right)^3$
$= \dfrac{1}{64}$

5. (a) $\left(\dfrac{3}{x}\right)^{-4} = \left(\dfrac{x}{3}\right)^4 = \dfrac{x^4}{81}$

(b) $3 \div x^{-3} = 3 \div \dfrac{1}{x^3} = 3 \times x^3 = 3x^3$

6. (a) $(x^3y^{-2}) \times (x^{-3}y^5) = x^{3-3} \times y^{-2+5}$
$= x^0 \times y^3$
$= y^3$

(b) $(5x^2y^3)^0 \div (-2x^{-3}y^5)^{-2} = \dfrac{(5x^2y^3)^0}{(-2x^{-3}y^5)^{-2}}$
$= \dfrac{1}{(-2x^{-3}y^5)^{-2}}$
$= (-2x^{-3}y^5)^2$
$= (-2)^2 \times x^{-3\times 2} \times y^{5\times 2}$
$= 4x^{-6}y^{10}$
$= \dfrac{4y^{10}}{x^6}$

(c) $\left(\dfrac{x^2}{y^{-3}}\right)^4 \div \left(\dfrac{x^5}{y^7}\right)^3 = \dfrac{x^8}{y^{-12}} \div \dfrac{x^{15}}{y^{21}}$

$= \dfrac{x^8}{y^{-12}} \times \dfrac{y^{21}}{x^{15}}$

$= x^{8-15}y^{21-(-12)}$

$= x^{-7}y^{33}$

$= \dfrac{y^{33}}{x^7}$

(d) $\dfrac{(3x^{-2}y^5)^2 \times (-2x^3y^{-2})^2}{9x^4y^6} = \dfrac{9x^{-4}y^{10} \times 4x^6y^{-4}}{9x^4y^6}$

$= \dfrac{36x^{-4+6}y^{10-4}}{9x^4y^6}$

$= \dfrac{36x^2y^6}{9x^4y^6}$

$= \dfrac{4}{x^2}$

7. (a) $\sqrt[5]{p^3} \times \sqrt[3]{8p} = p^{\frac{3}{5}} \times (8p)^{\frac{1}{3}}$

$= p^{\frac{3}{5}} \times 2p^{\frac{1}{3}}$

$= 2p^{\frac{3}{5}+\frac{1}{3}}$

$= 2p^{\frac{14}{15}}$

(b) $\left(p^{-3}q^{\frac{3}{5}}\right)^{-\frac{2}{3}} \times \left(p^{\frac{4}{5}}q^{-\frac{2}{5}}\right)^3 = \left(p^{-3\times-\frac{2}{3}}q^{\frac{3}{5}\times-\frac{2}{3}}\right) \times \left(p^{\frac{4}{5}\times 3}q^{-\frac{2}{3}\times 3}\right)$

$= \left(p^2q^{-\frac{2}{5}}\right) \times \left(p^{\frac{12}{5}}q^{-2}\right)$

$= p^{2+\frac{12}{5}}q^{-\frac{2}{5}-2}$

$= p^{\frac{22}{5}}q^{-\frac{12}{5}}$

$= \dfrac{p^{\frac{22}{5}}}{q^{\frac{12}{5}}}$

(c) $\dfrac{p^{\frac{2}{3}}q^{-\frac{2}{5}}}{\left(p^2q^{-\frac{1}{5}}\right)^{-3}} = \dfrac{p^{\frac{2}{3}}q^{-\frac{2}{5}}}{p^{-6}q^{\frac{3}{5}}}$

$= \dfrac{p^{\frac{2}{3}+6}}{q^{\frac{3}{5}+\frac{2}{5}}}$

$= \dfrac{p^{\frac{20}{3}}}{q}$

(d) $\left(p^{-\frac{1}{3}}q^2\right)^5 \times \sqrt[3]{27(p^{-3}q^2)} = p^{-\frac{5}{3}}q^{10} \times (27(p^{-3}q^2))^{\frac{1}{3}}$

$= p^{-\frac{5}{3}}q^{10} \times 3\left(p^{-3\times\frac{1}{3}}q^{2\times\frac{1}{3}}\right)$

$= p^{-\frac{5}{3}}q^{10} \times 3p^{-1}q^{\frac{2}{3}}$

$= 3p^{-\frac{5}{3}-1}q^{10+\frac{2}{3}}$

$= 3p^{-\frac{8}{3}}q^{\frac{32}{3}}$

$= \dfrac{3q^{\frac{32}{3}}}{p^{\frac{8}{3}}}$

8. (a) $4^{-6} \times 4^x = 1$

$4^{-6+x} = 4^0$

$-6 + x = 0$

$x = 6$

(b) $x^{-3} = 7$

$\dfrac{1}{x^3} = 7$

$7x^3 = 1$

$x^3 = \dfrac{1}{7}$

(c) $5^{12} \times 5^{-2} \div 5^x = 25$

$5^{12-2-x} = 5^2$

$5^{10-x} = 5^2$

$10 - x = 2$

$x = 8$

9. (a) $16^a = 8$

$(2^4)^a = 2^3$

$2^{4a} = 2^3$

$4a = 3$

$a = \dfrac{3}{4}$

(b) $2015^b = 2015^0$

$b = 0$

(c) $\dfrac{10^c}{10} = 0.01$

$10^{c-1} = \dfrac{1}{100}$

$10^{c-1} = \dfrac{1}{10^2}$

$10^{c-1} = 10^{-2}$

$c - 1 = -2$

$c = -1$

(d) $\dfrac{2^{d-6}}{2} = 2^9$

$2^{d-6-1} = 2^9$

$2^{d-7} = 2^9$

$d - 7 = 9$

$d = 16$

10. (a) $(6.4 \times 10^6) \times (5.1 \times 10^{-3})$

$= \boxed{(}\ \boxed{6}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{6}\ \boxed{)}\ \boxed{\times}\ \boxed{(}\ \boxed{5}\ \boxed{.}\ \boxed{1}$
$\boxed{\times 10^x}\ \boxed{-}\ \boxed{3}\ \boxed{)}\ \boxed{=}$

$= 32\,640$

$= 3.26 \times 10^4$ (to 3 s.f.)

(b) $(2.17 \times 10^{-5}) \div (7 \times 10^4)$

$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{1}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{5}\ \boxed{)}$
$\boxed{\div}\ \boxed{(}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

$= 3.1 \times 10^{-10}$

(c) $(3.17 \times 10^4) + (2.26 \times 10^5)$

$= \boxed{(}\ \boxed{3}\ \boxed{.}\ \boxed{1}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}$

$\boxed{+}\ \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{2}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{)}\ \boxed{=}$

$= 257\ 700$

$= 2.58 \times 10^5$ (to 3 s.f.)

(d) $(4.15 \times 10^{-3}) - (5.12 \times 10^{-4})$

$= \boxed{(}\ \boxed{4}\ \boxed{.}\ \boxed{1}\ \boxed{5}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{3}\ \boxed{)}$

$\boxed{-}\ \boxed{(}\ \boxed{5}\ \boxed{.}\ \boxed{1}\ \boxed{2}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

$= 3.64 \times 10^{-3}$ (to 3 s.f.)

(e) $\dfrac{5.1 \times 10^{-6} - 2.34 \times 10^5}{4.87 \times 10^{-3} + 9 \times 10^{-2}}$

$= \boxed{(}\ \boxed{5}\ \boxed{.}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{6}\ \boxed{-}\ \boxed{2}\ \boxed{.}\ \boxed{3}\ \boxed{4}$

$\boxed{\times 10^x}\ \boxed{5}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{4}\ \boxed{.}\ \boxed{8}\ \boxed{7}\ \boxed{\times 10^x}$

$\boxed{-}\ \boxed{3}\ \boxed{+}\ \boxed{9}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{2}\ \boxed{)}\ \boxed{=}$

$= -2.47 \times 10^6$ (to 3 s.f.)

(f) $\dfrac{8.43 \times 10^7 + 6.8 \times 10^8}{(1.01 \times 10^4)^3}$

$= \boxed{(}\ \boxed{8}\ \boxed{.}\ \boxed{4}\ \boxed{3}\ \boxed{\times 10^x}\ \boxed{7}\ \boxed{+}\ \boxed{6}\ \boxed{.}\ \boxed{8}$

$\boxed{\times 10^x}\ \boxed{8}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{0}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}$

$\boxed{\times x^3}\ \boxed{=}$

$= 7.42 \times 10^{-4}$ (to 3 s.f.)

11. (a) $a - b = 110\ 000\ 000 - 12\ 100\ 000$

$= 1.1 \times 10^8 - 1.21 \times 10^7$

$= 9.79 \times 10^7$

(b) $\sqrt[3]{ab} = \sqrt[3]{(1.1 \times 10^8) \times (1.21 \times 10^7)}$

$= \sqrt[3]{1.331 \times 10^{15}}$

$= 1.1 \times 10^5$

(c) $6c^2 = 6(0.000\ 007)^2$

$= 6(7 \times 10^{-6})^2$

$= 2.94 \times 10^{-10}$

(d) $\dfrac{ac}{b} = \dfrac{(1.1 \times 10^8) \times (7 \times 10^{-6})}{1.21 \times 10^7}$

$= 6.36 \times 10^{-5}$ (to 3 s.f.)

12. (i) $1 \text{ nm} = 10^{-9} \text{ m}$

$7 \text{ nm} = 7 \times 10^{-9} \text{ m}$

Circumference $= \pi d$

$= 3.142(7 \times 10^{-9})$

$= 2.20 \times 10^{-8}$ m (to 3 s.f.)

(ii) Area $= \pi r^2$

$= 3.142\left(\dfrac{7 \times 10^{-9}}{2}\right)^2$

$= 3.85 \times 10^{-17} \text{ m}^2$ (to 3 s.f.)

13. (i) $149\ 597\ 870\ 700 \text{ nm} = 1.496 \times 10^{11}$ m (to 4 s.f.)

(ii) Time $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{1.496 \times 10^{11}}{3 \times 10^8 \text{ m/s}}$

$= 499$ s (to 3 s.f.)

14. (i) $1 \text{ Mm} = 10^6 \text{ m}$

$240 \text{ Mm} = 240 \times 10^6 \text{ m}$

$= 2.4 \times 10^8 \text{ m}$

(ii) Speed of rocket $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{1 \text{ m}}{8000 \text{ ns}}$

$= \dfrac{1 \text{ m}}{8000 \times 10^{-9} \text{ s}}$

$= 1.25 \times 10^5 \text{ m/s}$

Time taken $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{2.4 \times 10^8 \text{ m}}{1.25 \times 10^5 \text{ m/s}}$

$= 1920$ s

15. (i) Mass of water molecule $= 2(1.66 \times 10^{-24}) + (2.66 \times 10^{-23})$

$= 2.99 \times 10^{-23}$ g (to 3 s.f.)

(ii) Approx. no. of water molecules $= \dfrac{280}{2.99 \times 10^{-23}}$

$= 9.36 \times 10^{24}$ g (to 3 s.f.)

Challenge Yourself

1. $2^{34} = (2^{(3^4)} = 2^{81}$

$2^{43} = 2^{(4^3)} = 2^{64}$

So, $2^{34} < 2^{43} < 2^{43} = 2^{64} < 2^{34} = 2^{81}$.

$3^{24} = 3^{(2^4)} = 3^{16}$

$3^{42} = 3^{(4^2)} = 3^{16}$

So, $3^{16} < 3^{24} < 3^{42}$.

$4^{32} = 4^{(3^2)} = 4^9$

$4^{23} = 4^{(2^3)} = 4^8$

So, $4^8 < 4^9 < 4^{23} < 4^{30}$.

$3^{16} < 4^{30} = 2^{60} < 2^{34} = 2^{81}$

Hence 2^{34} is the largest.

2. $3^1, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, \ldots$

$3, 9, 27, 81, 243, 729, 2187, \ldots$

We can observe that the last digit of 3^n are in the sequence:

$3, 9, 7, 1, 3, 9, 7, 1, \ldots$

$\therefore$ Since $2015 \div 4$ gives a remainder of 3, hence the largest digit of 3^{2015} is 7.

3. Let $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$.

$$\text{Then } x^2 = \left(\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}\right)^2$$

$$x^2 = 2 + \sqrt{2 + \sqrt{2 + \ldots}}$$

$$x^2 = 2 + x$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Since $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}} > 0$, hence $x = 2$.

Revision Exercise A1

1. **(a)** $\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^{3} = 8$

(b) $0.04^{-1.5} = 0.04^{-\frac{3}{2}}$

$= \left(\frac{1}{0.04}\right)^{\frac{3}{2}}$

$= (\sqrt{25}\,)^3$

$= 5^3$

$= 125$

(c) $\left(1\frac{9}{16}\right)^{-\frac{1}{2}} = \left(\frac{25}{16}\right)^{-\frac{1}{2}}$

$= \left(\frac{16}{25}\right)^{\frac{1}{2}}$

$= \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$

(d) $9^{2.5} \div 27^{1\frac{1}{3}} = 9^{\frac{5}{2}} \div 27^{\frac{4}{3}}$

$= (\sqrt{9}\,)^5 \div (\sqrt[3]{27}\,)^4$

$= 3^5 \div 3^4$

$= 3^{5-4}$

$= 3$

(e) $9^{\frac{1}{2}} - 0.36^{-\frac{1}{2}} = \sqrt{9} - \sqrt{\frac{1}{0.36}}$

$= 3 - \frac{1}{0.6}$

$= 1\frac{1}{3}$

2. **(a)** $a^5 \div a^{-2} = a^{5-(-2)}$

$= a^{5+2}$

$= a^7$

(b) $b^4 \div \sqrt{b} \times b^{-7} = b^4 \div b^{\frac{1}{2}} \times b^{-7}$

$= b^{4-\frac{1}{2}+(-7)}$

$= b^{-3.5}$

$= \frac{1}{b^{3.5}}$

(c) $\left(\frac{c^{-3}d}{c^2d^{-2}}\right)^{-5} = \left(\frac{c^2d^{-2}}{c^{-3}d}\right)^{5}$

$= (c^{2-(-3)}d^{-2-1})^5$

$= (c^5d^{-3})^5$

$= c^{25}d^{-15}$

$= \frac{c^{25}}{d^{15}}$

3. **(a)** $7^x = 2^4 \div 4^2$

$7^x = 2^4 \div (2^2)^2$

$7^x = 2^4 \div 2^4$

$7^x = 1$

$7^x = 7^0$

$x = 0$

(b) $(2y+3)^{\frac{1}{2}} = 5$

$\left[(2y+3)^{\frac{1}{2}}\right]^2 = 5^2$

$2y + 3 = 5^2$

$2y = 25 - 3$

$2y = 22$

$y = 11$

4. **(i)** $3 - 5p > 17$

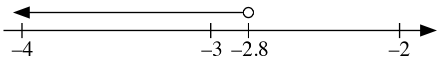
$3 - 5p - 3 > 17 - 3$

$-5p > 14$

$p < \frac{14}{-5}$

$p < -2.8$

(ii) $p < -2.8$



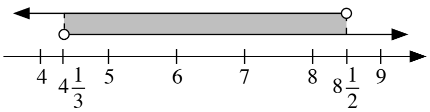
$\therefore$ The greatest integer value of p is –3.

5. Let x be the number.

Solving the two linear inequalities separately,

$2x - 5 < 12$	and	$3x - 1 > 12$
$2x - 5 + 5 < 12 + 5$		$3x - 1 + 1 > 12 + 1$
$2x < 17$		$3x > 13$
$x < \frac{17}{2}$		$x > \frac{13}{3}$
$x < 8\frac{1}{2}$		$x > 4\frac{1}{3}$

Representing $x < 8\frac{1}{2}$ and $x > 4\frac{1}{3}$ on a number line,



The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $4\frac{1}{3} < x < 8\frac{1}{2}$.

$\therefore$ The possible integer values of the number are 5, 6, 7 and 8.

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6. $f(x) = 10x - 3$

Let $y = 10x - 3$

$f(x) = y$ and $f^{-1}(y) = x$

$x = \frac{y+3}{10}$

$\therefore f^{-1}(y) = \frac{y+3}{10}$

Hence $f^{-1}(x) = \frac{x+3}{10}$

$f^{-1}(9) = \frac{9+3}{10}$

$= 1\frac{1}{5}$

$f^{-1}(-3) = \frac{(-3)+3}{10}$

$= 0$

$f^{-1}\left(-\frac{1}{2}\right) = \frac{\left(-\frac{1}{2}\right)+3}{10}$

$= \frac{1}{4}$

7. (i) 1 nanometre $= 10^{-9}$ metres

8.8 nanometres $= 8.8 \times 10^{-9}$ metres

Circumference $= \pi d$

$= 3.142(8.8 \times 10^{-9})$

$= 2.76 \times 10^{-8}$ m (to 3 s.f.)

(ii) Area $= \pi r^2$

$= 3.142\left(\frac{8.8 \times 10^{-9}}{2}\right)^2$

$= 6.08 \times 10^{-17}$ m^2 (to 3 s.f.)

8. Let the side of the larger square be x cm and that of the smaller square be y cm.

$4x - 4y = 100$

$x - y = 25$

$x = y + 25$ — (1)

$x^2 - 3y^2 = 325$ — (2)

Substitute (1) into (2):

$(y^2 + 25)^2 - 3y^2 = 325$

$y^2 + 50y + 625 - 3y^2 = 325$

$50y + 300 - 2y^2 = 0$

$25y + 150 - y^2 = 0$

$y^2 - 25y - 150 = 0$

$(y - 30)(y + 5) = 0$

$y - 30 = 0$ or $y + 5 = 0$

$y = 30$ $\quad$ $y = -5$ (rejected since $y > 0$)

When $y = 30$,

$x = 30 + 25 = 55$

$\therefore$ The lengths of a side of each of the squares are 30 cm and 55 cm.

9. Time taken for Train A to travel from P to $Q = \frac{330}{x}$ h

Time taken for Train B to travel from Q to $P = \frac{330}{x-5}$ h

$\frac{330}{x-5} - \frac{330}{x} = \frac{1}{2}$

$x(x-5) \times \left(\frac{330}{x-5} - \frac{330}{x}\right) = x(x-5) \times \frac{1}{2}$

$x(330) - 330(x-5) = \frac{1}{2}x(x-5)$

$330x - 330x + 1650 = \frac{1}{2}x^2 - \frac{5}{2}x$

$1650 = \frac{1}{2}x^2 - \frac{5}{2}x$

$\frac{1}{2}x^2 - \frac{5}{2}x - 1650 = 0$

$x^2 - 5x - 3300 = 0$

$(x - 60)(x + 55) = 0$

$x - 60 = 0$ or $x + 55 = 0$

$x = 60$ $\quad$ $x = -55$

(rejected since $x > 0$)

$\therefore$ Time taken for Train A to travel from P to $Q = \frac{330}{60} = 5.5$ h

Time taken for Train B to travel from Q to $P = \frac{330}{60-5} = 6$ h

10. (i) By Pythagoras' Theorem,

$AS^2 = PA^2 + PS^2$

$= y^2 + 4^2$

$= y^2 + 16$

By Pythagoras' Theorem,

$AB^2 = AQ^2 + BQ^2$

$= (8 - y)^2 + 3^2$

$= y^2 - 16y + 64 + 9$

$= y^2 - 16y + 73$

(ii) By Pythagoras' Theorem,

$BS^2 = BR^2 + RS^2$

$BS^2 = (4 - 3)^2 + 8^2$

$BS^2 = 1^2 + 8^2$

$BS^2 = 65$ — (1)

By Pythagoras' Theorem,

$BS^2 = AS^2 + AB^2$

$BS^2 = (y^2 + 16) + (y^2 - 16y + 73)$

$BS^2 = 2y^2 - 16y + 89$ — (2)

Substitute (1) into (2):

$2y^2 - 16y + 89 = 65$

$2y^2 - 16y + 24 = 0$

$y^2 - 8y + 12 = 0$ (Shown)

(iii) $y^2 - 8y + 12 = 0$

$(y-2)(y-6) = 0$

$y - 2 = 0$ or $y - 6 = 0$

$y = 2$ $\quad$ $y = 6$

$\therefore$ $y = 2$ or $y = 6$

When $y = 2$,

$AB^2 = 2^2 - 16(2) + 73 = 45$

$AB = \sqrt{45}$ cm

$AS^2 = 2^2 + 16 = 20$

$AS = \sqrt{20}$ cm

Area of $\triangle ABS = \frac{5}{2} \times AB \times AS$

$= \frac{5}{2} \times \sqrt{45} \times \sqrt{20}$

$= 15$ cm^2

When $y = 6$,

$AB^2 = 6^2 - 16(6) + 73 = 13$

$AB = \sqrt{13}$ cm

$AS^2 = 6^2 + 16 = 52$

$AS = \sqrt{52}$ cm

Area of $\triangle ABS = \frac{5}{2} \times AB \times AS$

$= \frac{5}{2} \times \sqrt{13} \times \sqrt{52}$

$= 13$ cm^2

$\therefore$ The possible values of the area of $\triangle ABS$ are 15 cm^2 or 13 cm^2.

11. (i) Area of picture $= 160$ cm^2

$(16 - 2x) \times (20 - 2x) = 160$

$320 - 32x - 40x + 4x^2 = 160$

$320 - 72x + 4x^2 = 160$

$160 - 72x + 4x^2 = 0$

$4x^2 - 72x + 160 = 0$

$x^2 - 18x + 40 = 0$ (shown)

(ii) $x^2 - 18x + 40 = 0$

Comparing $x^2 - 18x + 40 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -18$ and $c = 40$.

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(40)}}{2(1)}$$

$$= \frac{18 \pm \sqrt{164}}{2}$$

$= 15$ (to 2 s.f.), 2.6 (to 2 s.f.)

$\therefore$ $x = 15$ or $x = 2.6$

(iii) Width of border = 2.6 cm

Revision Exercise A2

1. **(a)** $(2ab^2)^3 = 2^3 \times a^3 \times b^{2\times 3} = 8a^3b^6$

(b) $c^3 \times c^{-2} \div c^0 = c^{3+(-2)-0}$
$$= c^{3-2}$$
$$= c$$

(c) $\left(\frac{2}{d}\right)^{-2} = \left(\frac{d}{2}\right)^2$
$$= \frac{d^2}{4}$$

(d) $2 \div 4e^{-3} = \frac{2}{4e^{-3}}$
$$= \frac{2}{2^2 \times e^{-3}}$$
$$= \frac{2^{1-2}}{e^{-3}}$$
$$= \frac{2^{-1}}{e^{-3}}$$
$$= \frac{e^3}{2}$$
$$= \frac{5}{2}e^3$$

2. $5^{-6} \div 5^p = 125^0$
$$5^{-6-p} = 1$$
$$5^{-6-p} = 5^0$$
$$-6 - p = 0$$
$$p = -6$$
$$\frac{1}{2^p} = \frac{1}{2^{-6}} = 2^6 = 64$$

3. **(a)** $10^{2x+3} = 0.001$
$$10^{2x+3} = \frac{1}{1000}$$
$$10^{2x+3} = \frac{1}{10^3}$$
$$10^{2x+3} = 10^{-3}$$
$$2x + 3 = -3$$
$$2x = -6$$
$$x = -3$$

(b) $y^{-2} = \frac{1}{81}$
$$\frac{1}{y^2} = \frac{1}{81}$$
$$y^2 = 81$$
$$y = \pm\sqrt{81}$$
$$= \pm 9$$

(c) $(2z-1)^{\frac{1}{3}} = 2$
$$\left[(2z-1)^{\frac{1}{3}}\right]^3 = 2^3$$
$$2z - 1 = 8$$
$$2z = 9$$
$$z = 4.5$$

4. **(a)** Largest possible value of $x^2 - y = 3^2 - 3 = 6$

(b) Smallest possible value of $\frac{x}{y} = \frac{1}{6}$

5. Solving the two linear inequalities separately,

$-3 \leqslant 2q + 7$ and $2q + 7 < 23$

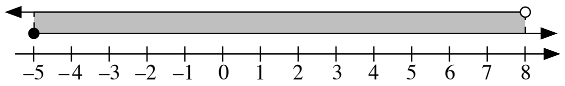
$2q + 7 \geqslant -3$ $\qquad$ $2q + 7 - 7 < 23 - 7$

$2q + 7 - 7 \geqslant -3 - 7$ $\qquad$ $2q < 16$

$2q \geqslant -10$ $\qquad$ $q < 8$

$q \geqslant -5$

Representing $q \geqslant -5$ and $q < 8$ on a number line,



$\therefore$ The solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-5 \leqslant q < 8$.

6. $\frac{4x}{5} - \frac{3}{10} \leqslant x - 2\frac{1}{4}$
$$\frac{4x}{5} - \frac{3}{10} + \frac{3}{10} \leqslant x - 2\frac{1}{4} + \frac{3}{10}$$
$$\frac{4x}{5} \leqslant x - 1\frac{19}{20}$$
$$\frac{4x}{5} - x \leqslant x - 1\frac{19}{20} - x$$
$$-\frac{1}{5}x \leqslant -1\frac{19}{20}$$
$$x \geqslant \frac{\left(-1\frac{19}{20}\right)}{\left(-\frac{1}{5}\right)}$$
$$x \geqslant 9\frac{3}{4}$$

(a) Smallest prime value of $x = 11$

(b) Smallest integer value of $x = 10$

(c) Smallest rational value of $x = 9\frac{3}{4}$

7. $f(x) = \frac{2}{3x-2}$

Let $y = f(x)$

$f(x) = y$ and $f^{-1}(y) = x$
$$y = \frac{2}{3x-2}$$
$$y(3x - 2) = 2$$
$$3xy - 2y = 2$$
$$3xy = 2 + 2y$$
$$x = \frac{2+2y}{3y}$$
$$\therefore f^{-1}(y) = \frac{2+2y}{3y}$$
$$= \frac{2}{3}\left(\frac{1+y}{y}\right)$$
$$= \frac{2}{3}\left(\frac{1}{y}+1\right)$$

Hence $f^{-1}(x) = \frac{2}{3}\left(\frac{1}{x}+1\right)$

$f^{-1}(x)$ is not defined when $x = 0$
$$f^{-1}(6) = \frac{2}{3}\left(\frac{1}{6}+1\right)$$
$$= \frac{7}{9}$$

8. **(i)** Area of the trapezium $PQRS = \frac{1}{2} \times RQ \times (SR + PQ)$

$$= \frac{1}{2} \times (a+1) \times (2a+1+3a+2)$$

$$= \frac{1}{2}(a+1)(5a+3) \text{ cm}^2$$

(ii) $\frac{1}{2}(a+1)(5a+3) = 9$

$$(a+1)(5a+3) = 18$$
$$5a^2 + 3a + 5a + 3 = 18$$
$$5a^2 + 8a - 15 = 0 \text{ (shown)}$$

(iii) $5a^2 + 8a - 15 = 0$

Comparing $5a^2 + 8a - 15 = 0$ with $ax^2 + bx + c = 0$, we have $a = 5$, $b = 8$ and $c = -15$.

$$x = \frac{-8 \pm \sqrt{8^2 - 4(5)(-15)}}{2(5)}$$

$$= \frac{-8 \pm \sqrt{364}}{10}$$

$= 1.11$ (to 2 d.p.), -2.71 (to 2 d.p.)

$\therefore x = 1.11$ or $x = -2.71$

(iv) $PQ = 3(1.108) + 2$

$= 5.32$ cm (to 3 s.f.)

9. Let the time taken by the larger pipe to fill the tank be x h.

Then the time taken by the smaller pipe to fill the tank is $(x + 2)$ h.

In 1 minute, the smaller pipe can fill $\frac{1}{x+2}$ of the tank while the larger pipe can fill $\frac{1}{x}$ of the tank.

$$\frac{1}{x} + \frac{1}{x+2} = \frac{1}{1\frac{20}{60}}$$

$$\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}$$

$$\left[\frac{1}{x} + \frac{1}{x+2}\right] \times x(x+2) = \frac{3}{4} \times x(x+2)$$

$$\frac{1}{x} \times x(x+2) + \frac{1}{x+2} \times x(x+2) = \frac{3}{4}x(x+2)$$

$$(x+2) + x = \frac{3}{4}x(x+2)$$

$$2x + 2 = \frac{3}{4}x^2 + \frac{3}{2}x$$

$$\frac{3}{4}x^2 - \frac{1}{2}x - 2 = 0$$

$$3x^2 - 2x - 8 = 0$$

$$(x-2)(3x+4) = 0$$

$x - 2 = 0$ or $3x + 4 = 0$

$x = 2$ $\quad\quad 3x = -4$

$x = -1\frac{1}{3}$

(rejected since $x > 0$)

$\therefore$ The time taken by the larger pipe is 2 hours while by the smaller pipe to fill the tank is 4 hours.

10. **(i)** By Pythagoras' Theorem,

$$XY^2 = OX^2 + OY^2$$
$$9^2 = (r+5)^2 + (4+r)^2$$
$$81 = r^2 + 10r + 25 + r^2 + 8r + 16$$
$$81 = 2r^2 + 18r + 41$$
$$2r^2 + 18r - 40 = 0$$
$$r^2 + 9r - 20 = 0 \text{ (shown)}$$

(ii) $r^2 + 9r - 20 = 0$

Comparing $r^2 + 9r - 20 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 9$ and $c = -20$.

$$r = \frac{-9 \pm \sqrt{9^2 - 4(1)(-20)}}{2(1)}$$

$$= \frac{-9 \pm \sqrt{161}}{2}$$

$= 1.84$ (to 3 s.f.), -10.8 (to 3 s.f.)

$\therefore OX = 1.84 + 5 = 6.84$ cm

$OY = 4 + 1.84 = 5.84$ cm

11. Time taken for the first 40 km $= \frac{40}{x}$ h

Time taken for the last 10 km $= \frac{10}{x-5}$ h

$$\frac{40}{x} + \frac{10}{x-5} = 2\frac{40}{60}$$

$$\frac{40}{x} + \frac{10}{x-5} = \frac{8}{3}$$

$$x(x-5) \times \left(\frac{40}{x} + \frac{10}{x-5}\right) = x(x-5) \times \frac{8}{3}$$

$$40(x-5) + 10x = \frac{8}{3}x(x-5)$$

$$40x - 200 + 10x = \frac{8}{3}x^2 - \frac{40}{3}x$$

$$50x - 200 = \frac{8}{3}x^2 - \frac{40}{3}x$$

$$\frac{8}{3}x^2 - \frac{190}{3}x + 200 = 0$$

$$8x^2 - 190x + 600 = 0$$

$$4x^2 - 95x + 300 = 0$$

$$(x-20)(4x-15) = 0$$

$x - 20 = 0$ or $4x - 15 = 0$

$x = 20$ $\quad\quad 4x = 15$

$x = 3\frac{3}{4}$ (rejected)

$\therefore$ Michael's average speed for the first 40 km of the journey is 20 km/h.

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Chapter 5 Application of Mathematics in Practical Situations

TEACHING NOTES

Suggested Approach

Teachers can get students to discuss examples of percentages, which are used in everyday life. Although the concepts covered in this chapter are applicable to the real world, students might not have encountered the need to be familiar with them and hence might not identify with the situations easily. Teachers should prepare more relatable material, such as advertisements on discounted products, to allow students to appreciate the application of mathematics in practical situations.

Section 5.1: Profit and Loss

The definitions of profit and loss should be made clear to students, whereby:

$$\text{Profit} = \text{Selling price} - \text{Cost price}$$

$$\text{Loss} = \text{Cost price} - \text{Selling price}.$$

Teachers should also emphasise the difference between the expression of profit and loss as a percentage of the cost price and the calculation of percentage gain or loss in terms of the selling price, that may occur in some business transactions.

$$\frac{\text{Profit}}{\text{Cost price}} \times 100\% \qquad\qquad \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

$$\text{Percentage gain} = \frac{\text{Profit}}{\text{Selling price}} \times 100\% \qquad\qquad \text{Percentage loss} = \frac{\text{Loss}}{\text{Selling price}} \times 100\%$$

Thus, teachers should remind students to read the questions carefully in order to ascertain the correct percentage to report.

Section 5.2: Discount, Taxation and Commission

These real-world concepts would be useful for students when they start to work and plan their finances. However, teachers should note that students may not encounter terms such as discount, GST and commission often, and thus should explain the terms clearly before going through the topic.

The examples and figures used in the textbook are those applicable in Singapore, but the theory involved would be relevant for use in other countries. Teachers may supplement the questions with current figures and tax brackets in Pakistan, for instance, and further discuss with students the implications and importance of taxes in a country.

Section 5.3: Simple Interest and Compound Interest

Teachers may apply the prevailing interest rate in the region to an investment example using both simple interest and compound interest, in order to illustrate the effect of the significant difference in the final amount. Teachers should highlight to students that the computation of interest would be different depending on whether a simple or compounded interest is charged, and hence students need to be careful when faced with such questions.

Section 5.4: Hire Purchase

To assert the real-world context of this section, teachers may show students some advertisements on posters or other promotional material that feature the availability of a hire purchase alternative. Teachers can also suggest to students to think about whether Shop *A* having a cheaper interest rate for the hire purchase of the exact same item as compared to Shop *B* implies that a buyer should get the item from Shop *A*. There may be other hidden terms and conditions that make Shop *B*'s item more attractive, such as a longer period of warranty for instance.

Section 5.5: Money Exchange

Teachers may wish to conduct a class exercise by asking students to find out the current exchange rates of the local currency against prominent currencies such as the US Dollar, Euro, Sterling Pound, Japanese Yen etc. compared to five to ten years ago. Based on the the trend, students can try to predict which currency would be a good investment to make. Teachers can highlight to students that when exchanging money, the money changer would offer both a buying and selling rate, and ensure that students are clear about the difference. Teachers can then explain why the exchange of one currency to another, and back to the previous currency, will usually result in a loss.

WORKED SOLUTIONS

Investigation (Discount, Service Charge and GST)

1. GST as calculated by Kate $= \frac{7}{100} \times \$14.40$

$= \$1.01$ (to the nearest cent)

GST as calculated by the restaurant

$= \frac{7}{100} \times \left[\$14.40 + \left(\frac{10}{100} \times \$14.40\right)\right]$

$= \frac{7}{100} \times (\$14.40 + \$1.44)$

$= \frac{7}{100} \times \$15.84$

$= \$1.11$ (to the nearest cent)

2. GST is an acronym for Goods and Services Tax, thus the tax is also imposed on the service charge, which is 10% of the subtotal.

3. If the discount is given before the service charge and GST are taken into account, the bill received will be as follows:

Yummy Restaurant	
Fish and Chips:	$ 8.50
Chicken Chop:	$ 9.50
Subtotal:	$ 18.00
Discount:	– $ 3.60
Subtotal:	$ 14.40
Service Charge 10%:	$ 1.44
GST 7%:	$ 1.1088
Total:	**$ 16.9488**

If the discount is given after the service charge and GST have been taken into account, the bill received will be as follows:

Yummy Restaurant	
Fish and Chips:	$ 8.50
Chicken Chop:	$ 9.50
Subtotal:	$ 18.00
Service Charge 10%:	$ 1.80
GST 7%:	$ 1.386
Subtotal:	$ 21.186
Discount:	– $ 4.2372
Total:	**$ 16.9488**

4. If the discount is given before the service charge and GST are taken into account, both the service charge and GST will be calculated based on a smaller amount, i.e. $14.40, and thus the service charge and GST will have already been discounted.

If the discount is given after the service charge and GST have been taken into account, both the service charge and GST will be calculated based on a greater amount, i.e. $18, and thus the discount will be given on the service charge and GST as well.

Hence, it makes no difference whether the discount is given before or after the service charge and GST are taken into account as the total bill will still be the same.

Investigation (Percentage Point)

1. Increase from 5% to 7% = 7% – 5%

= 2%

Percentage increase from 5% to 7% $= \frac{2}{5} \times 100\%$

= 40%

2. Yes, Jun Wei is right in saying that the increase from 5% to 7% is an increase of 40%. Please see solution to Question 1.

3. While the percentage increase from 5% to 7% is 40%, the difference between 5% and 7% is 2%. The term 'percentage point' is used to describe the difference between two percentages.

Class Discussion (Body Mass index)

1. –

2. Medical practitioners in Singapore make use of the BMI to determine which risk category you belong to as shown in the Table 5.2 of the textbook.

With this information, they will outline patients' health risks with increasing obesity and provide the necessary advices such as to start to eat more healthily and increase the activity level to lose weight.

3. Other real-life applications of rates include rate of flow of tap water, mobile phone charges and housing loan rate.

Performance Task (Page 127)

Teachers may wish to ask the students to search on the internet to find out the different interest rates as well as charges offered by the different credit card companies such as DBS, UOB, OCBC, Citibank, Standard Chartered and etc. Students will then present the findings to the class.

Investigation (Simple Interest and Compound Interest)

1. Interest $= \frac{PRT}{100}$

$= \frac{1000 \times 2 \times 3}{100}$

$= \$60$

Total amount after 3 years = $1000 + $60

= $1060

2. **1st year:** Principal $P_1 = \$1000$

Interest $I_1 = \$1000 \times 2\%$

$= \underline{\$20}$

Total amount at the end of the 1st year,

$A_1 = P_1 + I_1$

$= \$1000 + \underline{\$20}$

$= \$1020$

2nd year: Principal $P_2 = A_1 = \$1020$

Interest $I_2 = \underline{\$1020} \times 2\%$

$= \underline{\$20.40}$

Total amount at the end of the 2nd year,

$A_2 = P_2 + I_2$

$= \$1020 + \underline{\$20.40}$

$= \underline{\$1040.40}$

3rd year: Principal $P_3 = A_2 = \underline{\$1040.40}$

Interest $I_3 = \underline{\$1040.40} \times 2\%$

$= \underline{\$20.808}$

Total amount at the end of the 3rd year,

$A_3 = P_3 + I_3$

$= \$1040.40 + \underline{\$20.808}$

$= \underline{\$1061.21}$ (to the nearest cent)

3. Interest offered by Bank B = \$1061.21 – \$1000

= \$61.21

Difference in amount of interest offered by Bank A and Bank B

= \$61.21 – \$60

= \$1.21

$\therefore$ Bank B offers a higher interest of \$1.21.

Journal Writing (Page 136)

2. 'Buying rate' means the rate at which the money changer will buy the foreign currency and 'selling rate' means the rate at which it will sell the currency.

3. No, both 'buying rate' and 'selling rate' are independent. It depends on how much the money changer wants to earn from the deal.

Practise Now 1

1. (a) Required percentage $= \dfrac{\$240 - \$180}{\$180} \times 100\%$

$= \dfrac{\$60}{\$180} \times 100\%$

$= 33\dfrac{1}{3}\%$

(b) Required percentage $= \dfrac{\$6000 - \$5000}{\$5000} \times 100\%$

$= \dfrac{\$1000}{\$5000} \times 100\%$

$= 20\%$

2. (a) Selling price of gold chain $= \dfrac{127}{100} \times \$500$

$= \$635$

(b) Selling price of car $= \dfrac{94}{100} \times \$78\,400$

$= \$73\,696$

Practise Now 2

1. 135% of cost price = \$1282.50

1% of cost price $= \dfrac{\$1282.50}{135}$

100% of cost price $= \dfrac{\$1282.50}{135} \times 100$

$= \$950$

The cost price of the smartphone is \$950.

2. 88% of cost price = \$16.50

1% of cost price $= \dfrac{\$16.50}{88}$

100% of cost price $= \dfrac{\$16.50}{88} \times 100$

$= \$18.75$

The cost price of the book is \$18.75.

Practise Now 3

Cost price of 1800 eggs $= \dfrac{1800}{12} \times \$1.20$

$= \$180$

Total selling price of eggs so as to earn a 33% profit on the cost price

$= \dfrac{133}{100} \times \$180$

$= \$239.40$

Number of eggs that the shopkeeper can sell $= \dfrac{95}{100} \times 1800$

$= 1710$

Selling price of each egg $= \dfrac{\$239.40}{1710}$

$= \$0.14$

Practise Now 4

1. Percentage discount $= \dfrac{\$100 - \$88}{\$100} \times 100\%$

$= \dfrac{\$12}{\$100} \times 100\%$

$= 12\%$

2. Sale price of washing machine $= \dfrac{94}{100} \times \$600$

$= \$564$

Practise Now 5

(i) 91% of marked price = \$1274

1% of marked price $= \dfrac{\$1274}{91}$

100% of marked price $= \dfrac{\$1274}{91} \times 100$

$= \$1400$

The marked price of the laptop is \$1400.

(ii) Sale price of laptop after a 5% discount $= \dfrac{95}{100} \times \$1400$

$= \$1330$

Sale price of laptop after a further discount of 4% $= \dfrac{96}{100} \times \$1330$

$= \$1276.80$

No, the sale price would not be \$1274.

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Practise Now 6

1. GST payable $= \frac{7}{100} \times \$85$

$= \$5.95$

Total amount of money the man has to pay for article

$= \$85 + \$5.95$

$= \$90.95$

2. 107% of marked price = $642

1% of marked price $= \frac{\$642}{107}$

100% of marked price $= \frac{\$642}{107} \times 100$

$= \$600$

The marked price of the printer is $600.

Practise Now 7

1. Discount $= \frac{15}{100} \times \$6.90$

$= \$1.035$

Service charge = 10% × (marked price – discount)

$= \frac{10}{100} \times (\$6.90 - \$1.035)$

$= \frac{10}{100} \times \$5.865$

$= \$0.5865$

GST payable = 7% × (marked price – discount + service charge)

$= \frac{7}{100} \times (\$6.90 - \$1.035 + \$0.5865)$

$= \frac{7}{100} \times \$6.4515$

$= \$0.451\ 605$

Total amount payable

= marked price – discount + service charge + GST payable

= $6.90 – $1.035 + $0.5865 + $0.451 605

= $6.90 (to the nearest cent)

2. 117.7% of price after discount = $23.54

1% of price after discount $= \frac{\$23.54}{117.7}$

100% of price after discount $= \frac{\$23.54}{117.7} \times 100$

$= \$20$

The price of the set meal after discount is $20.

80% of marked price = $20

1% of marked price $= \frac{\$20}{80}$

100% of marked price $= \frac{\$20}{80} \times 100$

$= \$25$

The marked price of the set meal is $25.

Practise Now 8

On the first $40 000, tax payable = $550

On the next $2000, tax payable = 7% of $2000

= $140

∴ Total tax payable = $690

Practise Now 9

Total reliefs = $3000 + 2($4000) + $28 500 + $3500

= $43 000

Taxable income = $284 000 – $43 000

= $241 000

		Tax
$241 000 →	First $200 000	: $20 750
$241 000 →	Next $41 000 at 18%	: $\frac{18}{100} \times \$41\ 000 = \$7380$

∴ Income tax payable = $20 750 + $7380

= $28 130

Practise Now 10

1. Amount of commission the agent receives $= \frac{2}{100} \times \$528\ 000$

$= \$10\ 560$

2. 3.5% of selling price = $25 375

1% of selling price $= \frac{\$25\ 375}{3.5}$

100% of selling price $= \frac{\$25\ 375}{3.5} \times 100$

$= \$725\ 000$

The selling price of the piece of property is $725 000.

Practice Now 11

(a) Amount of interest the man has to pay at the end of 1 year

$= \$150\ 000 \times \frac{5.5}{100}$

$= \$8250$

Amount of interest the man has to pay at the end of 3 years

$= \$8250 \times 3$

$= \$24\ 750$

Total amount he owes the bank

= $150 000 + $24 750

= $174 750

(b) Total amount of interest Shirley earns

= $6720 – $6000

= $720

Amount of interest Shirley earns per last year

$= \$6000 \times \frac{3}{100}$

$= \$180$

Time taken for her investment to grow to $6720

$= \frac{\$720}{\$180}$

= 4 years

Practise Now 12

1. $P = \$3000, R = 5, n = 4$

At the end of 4 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 3000\left(1 + \frac{5}{100}\right)^4$$

$= \$3646.52$ (to the nearest cent)

Total interest, $I = A - P$

$= \$3646.52 - \$3000$

$= \$646.51$

2. **(a)** $P = \$1500, R = 2, n = 2$

At the end of 2 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 1500\left(1 + \frac{2}{100}\right)^2$$

$= \$1560.60$ (to the nearest cent)

Total interest, $I = A - P$

$= \$1560.60 - \$1500$

$= \$60.60$

(b) Since interest is calculated monthly,

$P = \$1500, R = \frac{2}{12} = \frac{1}{6}, n = 2 \times 12 = 24$

At the end of 2 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 1500\left(1 + \frac{\left(\frac{1}{6}\right)}{100}\right)^{24}$$

$= \$1561.16$ (to the nearest cent)

Total interest, $I = A - P$

$= \$1561.16 - \$1500$

$= \$61.16$

3. Since interest is calculated yearly,

$P = \$4000, A = 4243.60, n = 2$

At the end of 2 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$\$4243.60 = \$4000\left(1 + \frac{R}{100}\right)^2$$

$$\frac{4243.60}{4000} = \left(1 + \frac{R}{100}\right)^2$$

$$1 + \frac{R}{100} = \sqrt{1.0609}$$

$$\frac{R}{100} = \sqrt{1.0609} - 1$$

$R = 100(\sqrt{1.0609} - 1)$

$= 3$

$\therefore$ The interest rate is 3%.

Practise Now 13

(i) Downpayment $= \frac{20}{100} \times \$900$

$= \$180$

Remaining amount $= \$900 - \$180$

$= \$720$

Amount of interest Amirah owes at the end of 1 year

$= \$720 \times \frac{10}{100}$

$= \$72$

Amount of interest Amirah owes at the end of 4 years

$= \$72 \times 4$

$= \$288$

Total amount to be paid in monthly instalments

$= \$720 + \$288$

$= \$1008$

Monthly instalment $= \frac{\$1008}{48}$ (4 years = 48 months)

$= \$21$

(ii) Total amount Amirah pays for the air conditioner

$= \$1008 + \$180$

$= \$1188$

(iii) She has to pay $\$(1188 - 900) = \$288$ more for buying the air conditioner on hire purchase.

Practise Now 14

1. **(a)** **(i)** NZ\$1 = S\$0.9451

NZ\$2360 = S\$0.9451 $\times$ 2360

= S\$2230.44 (to the nearest cent)

(ii) ₱100 = S\$2.8542

₱1 = S\$ $\frac{2.8542}{100}$

₱25 600 = S\$ $\frac{2.8542}{100} \times 25\,600$

= S\$730.68 (to the nearest cent)

(b) **(i)** S\$1.7624 = €1

S\$1 = € $\frac{1}{1.7624}$

S\$5690 = € $\frac{1}{1.7624} \times 5690$

= €3229 (to the nearest €)

(ii) S\$4.2136 = 100 baht

S\$1 = $\frac{100}{4.2136}$ baht

S\$7460 = $\frac{100}{4.2136} \times 7460$

= 177 046 (to the nearest baht)

2. HK\$100 = S\$16.235

HK\$1 = S\$ $\frac{16.235}{100}$

HK\$35 000 = S\$ $\frac{16.235}{100} \times 35\,000$

= S\$5682.25

Amount of remaining Singapore dollars
= S$(5682.25 – 3500)
= S$2182.25
S$16.242 = HK$100

S$1 = HK$ $\frac{100}{16.242}$

Amount of Hong Kong dollars they receive

= HK$ $\frac{100}{16.242} \times 2182.25$

= HK$13 436 (to the nearest dollar)

Exercise 5A

1. **(a)** Profit = \$45 – \$40
= \$5

Required percentage = $\frac{\$5}{\$40} \times 100\%$
= 12.5%

(b) Loss = \$600 – \$480
= \$120

Required percentage = $\frac{\$120}{\$600} \times 100\%$
= 20%

(c) Selling price = $\frac{104}{100} \times \$88\,000$
= \$91 520

Profit = \$91 520 – \$88 000
= \$3520

(d) Selling price = $\frac{77.5}{100} \times \$5680$
= \$4402

Loss = \$5680 – \$4402
= \$1278

(e) $117\frac{1}{4}\%$ of cost price = \$28.14

1% of cost price = $\frac{\$28.14}{117\frac{1}{4}}$

100% of cost price = $\frac{\$28.14}{117\frac{1}{4}} \times 100$

= \$24

Cost price = \$24
Profit = \$28.14 – \$24
= \$4.14

(f) 93% of cost price = \$506.85

1% of cost price = $\frac{\$506.85}{93}$

100% of cost price = $\frac{\$506.85}{93} \times 100$
= \$545

Cost price = \$545
Loss = \$545 – \$506.85
= \$38.15

2. **(i)** 35% of cost price = \$280

1% of cost price = $\frac{\$280}{35}$

100% of cost price = $\frac{\$280}{35} \times 100$
= \$800

The cost price of the refrigerator is \$800.

(ii) Selling price of refrigerator = \$800 + \$280
= \$1080

3. Cost price of 5 kg of mixture = 2 × \$8 + 3 × \$6
= \$16 + \$18
= \$34

Selling price of 5 kg of mixture = 20 × \$2.55
= \$51

Required percentage = $\frac{\$51 - \$34}{\$51} \times 100\%$

= $\frac{\$17}{\$51} \times 100\%$

= $33\frac{1}{3}\%$

4. Selling price of one dozen of roses = 12 × \$1.20
= \$14.40

Required percentage = $\frac{\$18 - \$14.40}{\$14.40} \times 100\%$

= $\frac{\$3.60}{\$14.40} \times 100\%$

= 25%

5. 75% of price Devi buys from Shirley = \$360

1% of price Devi buys from Shirley = $\frac{\$360}{75}$

100% of price Devi buys from Shirley = $\frac{\$360}{75} \times 100$
= \$480

Devi buys the fax machine from Shirley at \$480.
125% of cost price = \$480

1% of cost price = $\frac{\$480}{125}$

100% of cost price = $\frac{\$480}{125} \times 100$
= \$384

Shirley paid \$384 for the fax machine.

6. Total number of apples Raj buys = 200 × 60
= 12 000

Cost price of 12 000 apples = 200 × \$28
= \$5600

Total selling price of apples so as to earn a 80% profit on the cost price

= $\frac{180}{100} \times \$5600$

= \$10 080

Number of apples that Raj can sell = $\frac{85}{100} \times 12\,000$
= 10 200

Selling price per apple = $\frac{\$10\,080}{10\,200}$
= \$0.99 (to the nearest cent)

7. Cost price of each article $= \dfrac{\$1500}{300}$

$= \$5$

Selling price of each of the 260 articles $= \dfrac{120}{100} \times \$5$

$= \$6$

Selling price of each of the remaining 40 articles $= \dfrac{50}{100} \times \$6$

$= \$3$

Selling price of articles $= 260 \times \$6 + 40 \times \$3$

$= \$1560 + \$120$

$= \$1680$

Required percentage $= \dfrac{\$1680 - \$1500}{\$1500} \times 100\%$

$= \dfrac{\$180}{\$1500} \times 100\%$

$= 12\%$

Exercise 5B

1. Percentage discount $= \dfrac{\$580 - \$464}{\$580} \times 100\%$

$= \dfrac{\$116}{\$580} \times 100\%$

$= 20\%$

2. Sale price of folding table $= \dfrac{88}{100} \times \$45$

$= \$39.60$

3. **(i)** 7% of marked price $= \$49$

1% of marked price $= \dfrac{\$49}{7}$

100% of marked price $= \dfrac{\$49}{7} \times 100$

$= \$700$

The marked price of the television set is \$700.

(ii) Sale price of television set $= \$700 - \$49$

$= \$651$

4. GST payable $= \dfrac{7}{100} \times \$270$

$= \$18.90$

Total amount of money Ethan has to pay for microwave oven

$= \$270 + \$18.90$

$= \$288.90$

5. 107% of marked price $= \$1391$

1% of marked price $= \dfrac{\$1391}{107}$

100% of marked price $= \dfrac{\$1391}{107} \times 100$

$= \$1300$

The marked price of the electronic gadget is \$1300.

6. **(a)** On the first \$20 000, tax payable = \$0

On the next \$5400, tax payable = 2% of \$5400

$= \$108$

$\therefore$ Total tax payable = \$108

(b) On the first \$30 000, tax payable = \$200

On the next \$8400, tax payable = 3.50% of \$8400

$= \$294$

$\therefore$ Total tax payable = \$494

(c) On the first \$40 000, tax payable = \$550

On the next \$32 000, tax payable = 7% of \$32 000

$= \$2240$

$\therefore$ Total tax payable = \$2790

(d) On the first \$80 000, tax payable = \$3350

On the next \$25 000, tax payable = 11.5% of \$25 000

$= \$2875$

$\therefore$ Total tax payable = \$6225

7. **(a)** Amount of commission the agent receives $= \dfrac{2.5}{100} \times \$650\,000$

$= \$16\,250$

(b) 2.5% of selling price $= \$12\,000$

1% of selling price $= \dfrac{\$12\,000}{2.5}$

100% of selling price $= \dfrac{\$12\,000}{2.5} \times 100$

$= \$480\,000$

The selling price of the house is \$480 000.

8. **(i)** 87.5% of marked price $= \$700$

1% of marked price $= \dfrac{\$700}{87.5}$

100% of marked price $= \dfrac{\$700}{87.5} \times 100$

$= \$800$

The marked price of the air conditioner is \$800.

(ii) Sale price of air conditioner after a 10% discount

$= \dfrac{90}{100} \times \$800$

$= \$720$

Sale price of air conditioner after a further discount of 2.5%

$= \dfrac{97.5}{100} \times \$720$

$= \$702$

No, the sale price would not be \$700.

9. Price of seafood fried rice after discount $= \dfrac{75}{100} \times \$9.50$

$= \$7.125$

Total amount payable $= \dfrac{117.7}{100} \times \$7.125$

$= \$8.39$ (to the nearest cent)

10. Total reliefs $= \$3000 + \$4000 + 2(\$5000) + \$18\ 600$
$= \$35\ 600$

Taxable income $= \$185\ 000 - \$35\ 600$
$= \$149\ 400$

Tax

$149 400 → First $120 000 : $7950

$149 400 → Next $29 400 at 15% : $\frac{15}{100} \times \$29\ 400 = \$4410$

$\therefore$ Income tax payable $= \$7950 + \$4410$
$= \$12\ 360$

11. Property tax payable yearly

$= \$28\ 800 \times \frac{10}{100}$

$= \$2880$

Property tax payable for 6 months

$= \frac{\$2880}{2}$

$= \$1440$

12. Amount of commission Lixin receives $= \$1220 - \$500$
$= \$720$

4% of Lixin's sales $= \$720$

1% of Lixin's sales $= \frac{\$720}{4}$

100% of Lixin's sales $= \frac{\$720}{4} \times 100$
$= \$18\ 000$

Lixin's sales for that month are $18 000.

13. Let the marked price of the sofa set be $\$x$.

Sale price of sofa set $= 90\% \times 80\% \times 75\% \times \$x$
$= 90\% \times 75\% \times 80\% \times \$x$
$= 80\% \times 90\% \times 75\% \times \$x$
$= 80\% \times 75\% \times 90\% \times \$x$
$= 75\% \times 90\% \times 80\% \times \$x$
$= 75\% \times 80\% \times 90\% \times \$x$
$= \$0.54x$

Thus the sale price of the sofa set is the same regardless of the order Khairul chooses to arrange the 3 discounts.

14. 117.7% of price after discount $= \$10.13$

1% of price after discount $= \frac{\$10.13}{117.7}$

100% of price after discount $= \frac{\$10.13}{117.7} \times 100$
$= \$8.61$ (to the nearest cent)

The price of the ramen after discount is $8.61.

82% of marked price $= \$8.61$

1% of marked price $= \frac{\$8.61}{82}$

100% of marked price $= \frac{\$8.61}{82} \times 100$
$= \$10.50$ (to the nearest cent)

The marked price of the ramen is $10.50.

15. Let the amount of money be taxable at 7% be $\$x$.

Amount of tax payable for $\$x$ at 7%
$= \$1474 - \$550$
$= \$924$

Hence,

$\frac{7}{100} \times \$x = \$924$

$0.07x = 924$

$x = 13\ 200$

Total reliefs
$= \$3000 + \$2000 + \$4(4000) + 2(\$5000) + \$15\ 600 + \$200$
$= \$46\ 800$

Gross annual income $= \$46\ 800 + \$40\ 000 + \$13\ 200$
$= \$100\ 000$

Exercise 5C

1.

	Principal	Interest rate	Time	Simple Interest	Amount
(a)	$12 000	8%	7 years	$6720	$18 720
(b)	$500	11%	4 years	$220	$720
(c)	$300	9%	4 years	$108	$408
(d)	$3000	4%	10 years	$1200	$4200
(e)	$3600	5%	2 years	$360	$3960
(f)	$1800	7%	18 months	$189	$1989
(g)	$4500	6%	2 years	$540	$5040
(h)	$1200	5%	18 months	$90	$1290

2. Amount of interest paid $= \$55$

Let the sum of money borrowed be $\$x$.

$\$55 = 12\% \times \frac{5}{12} \times \$x$

$x = 55 \div \frac{12}{100} \div \frac{5}{12}$
$= 1100$

$\therefore$ The sum of money was $1100.

3. Total interest $= \frac{2.25}{100} \times 25 \times \$6400$
$= \$3600$

4. Annual interest on $800 investment $= \frac{6}{100} \times \$800$
$= \$48$

Annual interest on $1200 investment $= \frac{7}{100} \times \$1200$
$= \$84$

Total annual interest $= \$48 + \$84$
$= \$132$

5. Amount of interest earned per year $= \$1250 \times \frac{6}{100}$
$= \$75$

Time taken for interest to grow to $750 $= \frac{\$750}{\$75}$
$= 10$ years

6. Interest rate $= \dfrac{\$119}{\$4800} \times 100\% \div \dfrac{7}{12}$

$= 4\dfrac{1}{4}\ \%$

7. Amount of interest Rui Feng has to pay at the end of 1 year

$= \$48\ 000 \times \dfrac{6}{100}$

$= \$2880$

Amount of interest Rui Feng has to pay at the end of 2 years

$= \$2880 \times 2$

$= \$5760$

Total amount of money he has to pay at the end of 2 years

$= \$48\ 000 + \$5760$

$= \$53\ 760$

8. Total amount of interest the man earns

$= \$18\ 900 - \$16\ 800$

$= \$2100$

Amount of interest the man earns per last year

$= \$16\ 800 \times \dfrac{5}{100}$

$= \$840$

Time taken for his investment to grow to \$16 800

$= \dfrac{\$2100}{\$840}$

$= 2\dfrac{1}{2}$ years

9. $P = \$5000, R = 8, n = 3$

At the end of 3 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 5000\left(1 + \dfrac{8}{100}\right)^3$

$= \$6298.56$

Total interest, $I = A - P$

$= \$6298.56 - \$5000$

$= \$1298.56$

10. (a) $P = \$450, R = 10, n = 2$

At the end of 2 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 450\left(1 + \dfrac{10}{100}\right)^2$

$= \$544.50$

Total interest, $I = A - P$

$= \$544.50 - \$450$

$= \$94.50$

(b) $P = \$700, R = 11, n = 3$

At the end of 3 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 700\left(1 + \dfrac{11}{100}\right)^3$

$= \$957.34$ (to the nearest cent)

Total interest, $I = A - P$

$= \$957.34 - \$700$

$= \$257.34$

(c) $P = \$5000, R = 11\dfrac{3}{4}, n = 2$

At the end of 2 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 5000\left(1 + \dfrac{\left(11\frac{3}{4}\right)}{100}\right)^2$

$= \$6244.03$ (to the nearest cent)

Total interest, $I = A - P$

$= \$6244.03 - \$5000$

$= \$1244.03$

(d) $P = \$1200, R = 4, n = 3$

At the end of 3 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 1200\left(1 + \dfrac{4}{100}\right)^3$

$= \$1349.84$ (to the nearest cent)

Total interest, $I = A - P$

$= \$1349.84 - \$1200$

$= \$149.84$

(e) $P = \$10\ 000, R = 7\dfrac{1}{2}, n = 2$

At the end of 2 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 10\ 000\left(1 + \dfrac{\left(7\frac{1}{2}\right)}{100}\right)^2$

$= \$11\ 556.25$

Total interest, $I = A - P$

$= \$11\ 556.25 - \$10\ 000$

$= \$1556.25$

11. $P = \$5000, R = 5\dfrac{1}{4}, n = 3$

At the end of 3 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 5000\left(1 + \dfrac{\left(5\frac{1}{4}\right)}{100}\right)^3$

$= \$5829.57$ (to the nearest cent)

12. Amount of interest from end March to June

$= \frac{3}{100} \times \frac{3}{12} \times \$600$

$= \$4.50$

Amount of interest from end June to December

$= \frac{3}{100} \times \frac{6}{12} \times \$(600 + 400)$

$= \$15$

Total amount in the bank at the end of the year

$= \$600 + \$4.50 + \$400 + \$15$

$= \$1019.50$

13. Initially, at 3.5% interest rate, interest received

$= \$6400 \times \frac{3.5}{100} \times \frac{1}{2}$

$= \$112$

At new 4% interest rate, interest received

$= \$6400 \times \frac{4}{100} \times \frac{1}{2}$

$= \$128$

Difference in amount of interest $= \$128 - \$112$

$= \$16$

14. Interest received during first 2 years

$= \$4000 \times \frac{7\frac{1}{4}}{100} \times 2$

$= \$580$

Interest received during next 5 years

$= \$4000 \times \frac{7.6}{100} \times 5$

$= \$1520$

Total amount at the end of 7 years

$= \$4000 + \$580 + \$1520$

$= \$6100$

15. Let the sum of money deposited by Daniel be $\$x$.

$$\frac{3\frac{3}{4}}{100}x - \frac{3\frac{1}{2}}{100}x = 50$$

$$\frac{\frac{1}{4}}{100}x = 50$$

$$x = 20\ 000$$

$\therefore$ The sum of money Daniel deposits is \$20 000.

16. Interest received at 2.75% interest rate

$= \frac{2.75}{100} \times \$20\ 000$

$= \$550$

Interest received at x% interest rate $= \$550 - \$50$

$= \$500$

New simple interest, $x\% = \frac{\$500}{\$20\ 000} \times 100\%$

$= 2.5\%$

$\therefore x = 2.5$

17. (a) Since interest is calculated monthly,

$P = \$15\ 000, R = \frac{5.68}{12}, n = 6 \times 12 = 72$

At the end of 6 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 15\ 000\left(1 + \frac{\left(\frac{5.68}{12}\right)}{100}\right)^{72}$$

$= \$21\ 074.13$ (to the nearest cent)

(b) Since interest is calculated half-yearly,

$P = \$15\ 000, R = \frac{5.68}{2} = 2.84, n = 6 \times 2 = 12$

At the end of 6 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 15\ 000\left(1 + \frac{2.84}{100}\right)^{12}$$

$= \$20\ 991.14$ (to the nearest cent)

18. Since interest is calculated yearly,

$P = \$5000, A = \$5800, n = 5$

At the end of 5 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$\$5800 = \$5000\left(1 + \frac{R}{100}\right)^5$$

$$\frac{5800}{5000} = \left(1 + \frac{R}{100}\right)^5$$

$$1 + \frac{R}{100} = \sqrt[5]{1.16}$$

$$\frac{R}{100} = \sqrt[5]{1.16} - 1$$

$$R = 100(\sqrt[5]{1.16} - 1)$$

$= 3.01$ (to 3 s.f.)

$\therefore$ The interest rate is 3.01%.

19. Since interest is calculated quarterly,

$P = \$96.60, R = \frac{4.2}{4} = 1.05, n = 1 \times 4 = 4$

At the end of the first years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$P + \$96.60 = P\left(1 + \frac{1.05}{100}\right)^4$$

$$P + \$96.60 = 1.0105^4P$$

$$(1.0105^4 - 1)P = \$96.60$$

$$P = \$\frac{96.60}{1.0105^4 - 1}$$

$= \$2264.09$ (to the nearest cent)

20. Since interest is calculated monthly,

$P = \$800,\ R = \dfrac{12\frac{1}{2}}{12} = \dfrac{25}{24},\ n = 12$

At the end of 1 year, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 800\left(1 + \dfrac{\left(\frac{25}{24}\right)}{100}\right)^2$

= \$905.93 (to the nearest cent)

Total interest, $I = A - P$

= \$905.93 – \$800

= \$105.93

21. Since interest is calculated daily,

$P = \$9000,\ R = \dfrac{2}{365},\ n = 3$

At the end of 3 days, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 9000\left(1 + \dfrac{\left(\frac{2}{365}\right)}{100}\right)^3$

= \$9001.48 (to the nearest cent)

22. **(i)** Kate should invest in Company B since the interest earned is higher.

(ii) For Company A,

$I = \dfrac{PRT}{100}$

$= \dfrac{8000 \times 4.9 \times 4}{100}$

= \$1568

For Company B,

Since interest is calculated half-yearly,

$P = \$8000,\ R = \dfrac{4.8}{2} = 2.4,\ n = 4 \times 2 = 8$

At the end of 6 years, total amount accumulated is

$A = P\left(1 + \dfrac{R}{100}\right)^n$

$= 8000\left(1 + \dfrac{2.4}{100}\right)^8$

= \$9671.41 (to the nearest cent)

Total interest, $I = A - P$

= \$9671.41 – \$8000

= \$1671.41

Difference in interest earned = \$1671.41 – \$1568

= \$103.41

Exercise 5D

1. **(a)** **(i)** Amount paid by hire purchase

= \$(40 × 10) + \$50

= \$450

Additional amount paid = \$450 – \$360

= \$90

(ii) Percentage of cash price = $\dfrac{\$90}{\$360} \times 100\%$

= 25%

(b) **(i)** Amount paid by hire purchase

= \$(75 × 12) + \$150

= \$1050

Additional amount paid = \$1050 – \$900

= \$150

(ii) Percentage of cash price = $\dfrac{\$150}{\$900} \times 100\%$

$= 16\frac{2}{3}\ \%$

(c) **(i)** Amount paid by hire purchase

= \$(500 × 36) + \$10 000

= \$28 000

Additional amount paid = \$28 000 – \$25 000

= \$3000

Percentage of cash price = $\dfrac{\$3000}{\$25\,000} \times 100\%$

= 12%

2. **(a)** **(i)** Amount paid by hire purchase

$= \$(9 \times 24) + \$\left(\dfrac{10}{100} \times 200\right)$

= \$216 + \$20

= \$236

(ii) Additional amount = \$236 – \$200

= \$36

Percentage saved by paying cash

$= \dfrac{36}{200} \times 100\%$

= 18%

(b) **(i)** Amount paid by hire purchase

$= \$(18 \times 20) + \$\left(\dfrac{15}{100} \times 350\right)$

= \$360 + \$52.50

= \$412.50

(ii) Additional amount = \$412.50 – \$350

= \$62.50

Percentage saved by paying cash

$= \dfrac{62.50}{350} \times 100\%$

= 17.9% (to 3 s.f.)

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(c) **(i)** Amount paid by hire purchase

$= \$(52 \times 30) + \$\left(\frac{25}{100} \times 1600\right)$

$= \$1560 + \$400$

$= \$1960$

(ii) Additional amount $= \$1960 - \$1600$

$= \$360$

Percentage saved by paying cash

$= \frac{360}{1600} \times 100\%$

$= 22.5\%$

3. **(a)** **(i)** Downpayment = \$100

Remaining amount $= \$800 - \$100$

$= \$700$

Amount of interest owed at the end of 1 year

$= \$700 \times \frac{8}{100}$

$= \$56$

Total amount to be paid in monthly instalments

$= \$700 + \$56$

$= \$756$

Monthly instalment $= \frac{\$756}{12}$

$= \$63$

(ii) Difference as percentage of cash price

$= \frac{56}{800} \times 100\%$

$= 7\%$

(b) **(i)** Downpayment = \$3200

Remaining amount $= \$8000 - \$3200$

$= \$4800$

Amount of interest owed at the end of 1 year

$= \$4800 \times \frac{10}{100}$

$= \$480$

Amount of interest owed at the end of $2\frac{1}{2}$ years

$= \$480 \times 2\frac{1}{2}$

$= \$1200$

Total amount to be paid in monthly instalments

$= \$4800 + \$1200$

$= \$6000$

Monthly instalment $= \$\frac{\$6000}{30}$

$= \$200$

(ii) Difference as percentage of cash price

$= \frac{1200}{8000} \times 100\%$

$= 15\%$

(c) **(i)** Downpayment = \$200

Remaining amount $= \$1200 - \$200$

$= \$1000$

Amount of interest owed at the end of 1 year

$= \$1000 \times \frac{15}{100}$

$= \$150$

Amount of interest owed at the end of $1\frac{1}{3}$ years

$= \$150 \times 1\frac{1}{3}$

$= \$200$

Total amount to be paid in monthly instalments

$= \$1000 + \$200$

$= \$1200$

Monthly instalment $= \$\frac{\$1200}{16}$

$= \$75$ (to the nearest cent)

(ii) Difference as percentage of cash price

$= \frac{200}{1200} \times 100\%$

$= 16\frac{2}{3}\%$

4. **(a)** Percentage discount $= \frac{\$(2198 - 1798)}{\$2198} \times 100\%$

$= 18.2\%$ (to 1 d.p.)

(b) Hire purchase price $= \$55 \times 38$

$= \$2090$

Difference $= \$2198 - \$2090$

$= \$108$

(c) Total amount of interest $= \$2090 - \$1798$

$= \$292$

Amount of interest at the end of 1 year

$= \frac{\$292}{38} \times 12$

$= \$92.21$ (to the nearest cent)

Let the rate of simple interest be $x\%$.

$\$92.21 = \$1798 \times \frac{x}{100}$

$x = \frac{92.21(100)}{1798}$

$= 5.13$ (to 3 s.f.)

$\therefore$ The rate of simple interest charged for hire purchase is 5.13%.

5. **(i)** Downpayment

$= \frac{15}{100} \times \$3200$

$= \$480$

Remaining amount

$= \$3200 - \$480$

$= \$2720$

Amount of interest the man owes at the end of 1 year

$= \$2720 \times \frac{9.5}{100}$

$= \$258.40$

Amount of interest the man owes at the end of 2 years

$= \$258.40 \times 2$

$= \$516.80$

Total amount to be paid in monthly instalments

$= \$2720 + \$516.80$

$= \$3236.80$

Monthly instalment

$= \frac{\$3236.80}{24}$ (2 years = 24 months)

$= \$134.87$ (to the nearest cent)

(ii) Total amount the man pays for the computer system

$= \$3236.80 + \$480$

$= \$3716.80$

(iii) He has to pay $\$(3716.80 - 3200) = \$516.80$ more for buying the computer system on hire purchase.

6. Downpayment $= \frac{25}{100} \times \$x$

$= \$0.25x$

Remaining amount $= \$x - \$0.25x$

$= \$0.75x$

Amount of interest the man owes at the end of 30 months

$= \$0.75x \times \frac{12}{100} \times \frac{30}{12}$

$= \$0.975x$

Total amount to be paid in monthly instalments

$= \$52 \times 30$

$= \$1560$

Hence $0.975x = 1560$

$x = 1600$

Exercise 5E

1. **(a)** US\$1 = 34.77B

US\$150 = 150 × 34.77B

= 5215.50B

(b) 34.77B = US\$1

$1\text{B} = \text{US\$}\frac{1}{34.77}$

$1617\text{B} = \text{US\$}\frac{1}{34.77} \times 1617$

= US\$46.51 (to the nearest cent)

2. **(a)** £1 = €1.19

£320 = 320 × €1.19

= €380.80

(b) €1.19 = £1

$€1 = £\frac{1}{1.19}$

$€956 = £\frac{1}{1.19} \times 956$

= £803 (to the nearest £)

3. **(a)** S\$1.70 = US\$1

$\text{S\$}1 = \text{US\$}\frac{1}{1.70}$

$\text{S\$}8500 = \text{US\$}\frac{1}{1.70} \times 8500$

= US\$5000

(b) After one year, amount in the bank

$= \frac{104}{100} \times \text{US\$}5000$

= US\$5200

US\$1 = S\$1.65

US\$5200 = 5200 × S\$1.65

= S\$8580

Amount made from investment

= S\$(8580 – 8500)

= S\$80

4. **(a)** US\$1 = S\$1.4103

US\$765 = 765 × S\$1.4103

= S\$1078.88 (to the nearest cent)

(b) Rp 100 = S\$0.0111

$\text{Rp } 1 = \text{S\$}\frac{0.0111}{100}$

$\text{Rp } 2\,560\,000 = \text{S\$}\frac{0.0111}{100} \times 2\,560\,000$

= S\$284.16

5. **(a)** S\$1.0373 = A\$1

$\text{S\$}1 = \text{A\$}\frac{1}{1.0373}$

$\text{S\$}350 = \text{A\$}\frac{1}{1.0373} \times 350$

= A\$337 (to the nearest A\$)

(b) S\$2.8250 = ₱100

$\text{S\$}1 = ₱\frac{100}{2.8250}$

$\text{S\$}5670 = ₱\frac{100}{2.8250} \times 5670$

= ₱200 708 (to the nearest ₱)

6. NZ\$100 = S\$94.85

$NZ\$1 = S\$\frac{94.85}{100}$

$NZ\$3200 = S\$\frac{94.85}{100} \times 3200$

$= S\$3035.20$

Amount of remaining Singapore dollars

= S\$(3035.20 – 2560.20)

= S\$475

S\$97.65 = NZ\$100

$S\$1 = NZ\$\frac{100}{97.65}$

Amount of New Zealand dollars they receive

$= NZ\$\frac{100}{97.65} \times 475$

= NZ\$486.43 (to the nearest cent)

Review Exercise 5

1. Profit the man earns $= 12 \times \$36$

$= \$432$

Selling price of cameras = \$1800 + \$432

= \$2232

$\text{Required percentage} = \frac{\$432}{\$2232} \times 100\%$

$= 19\frac{11}{31}\%$

2. 125% of cost price of Item A = \$48

$1\% \text{ of cost price of Item } A = \frac{\$48}{125}$

$100\% \text{ of cost price of Item } A = \frac{\$48}{125} \times 100$

$= \$38.40$

The cost price of Item A is \$38.40.

80% of cost price of Item B = \$48

$1\% \text{ of cost price of Item } B = \frac{\$48}{80}$

$100\% \text{ of cost price of Item } B = \frac{\$48}{80} \times 100$

$= \$60$

The cost price of Item B is \$60.

Total cost price of the two items = \$38.40 + \$60

= \$98.40

Total selling price of the two items $= 2 \times \$48$

= \$96

Net loss = \$98.40 – \$96

= \$2.40

$\text{Required percentage} = \frac{\$2.40}{\$98.40} \times 100\%$

$= 2\frac{18}{41}\%$

3. $\text{Percentage discount} = \frac{\$60\,000 - \$57\,000}{\$60\,000} \times 100\%$

$= \frac{\$3000}{\$60\,000} \times 100\%$

$= 5\%$

4. Marked price of 200 textbooks $= 200 \times \$20$

= \$4000

$\text{Amount of money the school has to pay} = \frac{93.5}{100} \times \$4000$

$= \$3740$

5. **(a)** $\text{Value-added tax payable} = \frac{15}{100} \times \$20$

$= \$3$

Amount of profit the retailer makes = \$26 – (\$20 + \$3)

= \$26 – \$23

= \$3

(b) **(i)** Price the retailer buys each bread toaster at from the manufacturer

$= \frac{120}{100} \times \$20$

$= \$24$

$\text{Value-added tax payable} = \frac{25}{100} \times \$24$

$= \$6$

Price a customer has to pay for a bread toaster

= \$24 + \$6 + \$3

= \$33

(ii) $\text{Selling price of a bread toaster} = \frac{130}{100} \times \$33$

$= \$42.90$

Amount of profit the retailer makes on a bread toaster

= \$42.90 – \$24 – \$6

= \$12.90

Amount of profit the retailer makes on 25 bread toasters

$= 25 \times \$12.90$

= \$322.50

6. 117.7% of price after discount = \$7.94

$1\% \text{ of price after discount} = \frac{\$7.94}{117.7}$

$100\% \text{ of price after discount} = \frac{\$7.94}{117.7} \times 100$

= \$6.75 (to the nearest cent)

The price of the noodles after discount is \$6.75.

90% of marked price = \$6.75

$1\% \text{ of marked price} = \frac{\$6.75}{90}$

$100\% \text{ of marked price} = \frac{\$6.75}{90} \times 100$

= \$7.50 (to the nearest cent)

The marked price of the noodles is \$7.50.

7. Total reliefs = \$3000 + 2(\$5000) + \$16 000 + \$750
= \$29 750
Taxable income = \$80 000 – \$29 750
= \$50 250

			Tax
\$80 000	→	First \$40 000	: \$550
	→	Next \$10 250 at 7%	: $\frac{7}{100} \times \$10\,250 = \$717.50$

∴ Income tax payable = \$550 + \$717.50
= \$1267.50

8. Amount of commission the agent receives

$= \frac{5}{100} \times \$50\,000 + \frac{2.25}{100} \times (\$240\,000 - \$50\,000)$

$= \frac{5}{100} \times \$50\,000 + \frac{2.25}{100} \times \$190\,000$

= \$2500 + \$4275
= \$6775

9. Amount of interest earned for \$600 at the end of 2011

$= \frac{3}{100} \times \$600$

= \$18

Amount of interest earned for \$1000 at the end of 2012

$= \frac{3}{100} \times \$1000$

= \$30

Total amount Khairul has in the bank at the end of 2013
= \$1000 + \$18 + \$30 + \$30
= \$1078

10. **(a)** Since interest is calculated monthly,

$P = \$15\,000, R = \frac{4.12}{12}, n = 3 \times 12 = 36$

At the end of 3 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 15\,000\left(1 + \frac{\left(\frac{4.12}{2}\right)}{100}\right)^{36}$$

= \$16 969.85 (to the nearest cent)

(b) Since interest is calculated half-yearly,

$P = \$15\,000, R = \frac{4.12}{2} = 2.06, n = 3 \times 2 = 6$

At the end of 3 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 15\,000\left(1 + \frac{2.06}{100}\right)^6$$

= \$16 952.14 (to the nearest cent)

11. Since interest is calculated half-yearly,

$I = \$5800, R = \frac{4}{2} = 2, n = 3 \times 2 = 6$

At the end of the first year, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$P + \$5800 = P\left(1 + \frac{2}{100}\right)^6$$

$$P + \$5800 = 1.02^6 P$$

$$(1.02^6 - 1)P = \$5800$$

$$P = \$\frac{5800}{1.02^6 - 1}$$

= \$45 972 (to the nearest dollar)

12. **(i)** Downpayment

$= \frac{15}{100} \times \$450$

= \$67.50

Remaining amount
= \$450 – \$67.50
= \$382.50

Amount of interest owed at the end of 1 year

$= \$382.50 \times \frac{12}{100}$

= \$45.90

Amount of interest owed at the end of 18 months

$= \$45.90 \times \frac{18}{12}$

= \$68.85

Total amount to be paid in monthly instalments
= \$382.50 + \$68.85
= \$451.35

Monthly instalment

$= \frac{\$451.35}{18}$

= \$25.08 (to the nearest cent)

(ii) Total amount the man pays for the printer
= \$450 + \$68.85
= \$518.85

13. Number of litres of petrol that the car used

$= \frac{1850}{12}$

$= 154\frac{1}{6}$ litres

Amount of euros spent on petrol

$= 154\frac{1}{6} \times$ €0.80

= €123.33

£100 = €116.50

£1 = € $\frac{116.50}{100}$

Amount of remaining euros

$= €\,\frac{116.50}{100} \times 75$

$= €87.375$

Total amount they spent for the short trip

$= €3500 + €123.33 + €87.375$

$= €3711$

$€115 = £100$

$€1 = £\,\frac{100}{115}$

Amount of Sterling pounds that they exchanged

$= £\,\frac{100}{115} \times 3711$

$= £3227$ (to the nearest pound)

Challenge Yourself

(i) GST paid by Lixin $= \frac{7}{100} \times \$500$

$= \$35$

(ii) GST paid by Jun Wei $= \frac{7}{107} \times \$500$

$= \$32.71$ (to the nearest cent)

(iii) GST on \$500 is \$35 which is the same answer as in **(i)**.
The shopkeeper is not complaining about it because he rather pays a GST of \$32.71 than a GST of \$35 to the government.

(iv) The amount paid by each customer at Shops *B* and *C* is \$500. As far as the government is concerned, this amount must be inclusive of GST. Another way of looking at this is to ask how the government can keep track of the shops which absorb GST and charge them a different GST amount. All the shops will tell the government that the final transacted amount is inclusive of GST because they can pay a *lower* amount for GST, so this agrees with why the final transacted amount is inclusive of GST regardless of whether the shops charge or absorb GST.

(v) Yes, it makes a difference. The difference is the *original* selling price of the TV *before* the government announces that they will charge GST. Shop *C* has been selling the TV for \$500 and decides to absorb GST after the announcement, so it still sells the TV for \$500 (inclusive of GST). If Shop *C* decides not to absorb GST, they will sell the TV for \$500 (before GST) or \$535 (inclusive of GST), just like what Shop *A* does. Since Shop *B* has been selling the TV for *about* \$467.29 and decides to charge GST after the announcement, it sells the TV for \$500 (inclusive of GST) *now*.

Chapter 6 Coordinate Geometry

TEACHING NOTES

Suggested Approach

Teachers should revise with the students on what they have learnt about functions and linear graphs in Book 1 before teaching this chapter. As students may have difficulty distinguishing equations parallel to the x- and y- axes, or may incorrectly refer to the equation of the x-axis as $x = 0$ and the equation of the y-axis as $y = 0$, teachers should highlight these common mistakes to the students and give them ample practice so that they can better grasp the concepts in coordinate geometry.

Section 6.1: Gradient of a Straight Line

Students have learnt in Book 2 that the gradient of a straight line is the ratio of the vertical change to the horizontal change. Teachers can get the students to recall this and build upon what they have learnt to find the formula for gradient given two points on a line (see Class Discussion: Finding the Gradient of a Straight Line).

In this section, the students will learn that the gradient of a straight line can be positive, negative, zero or undefined (see Investigation: Gradient of a Straight Line). Teachers should not only teach students how to find the gradient given two points, but also how to use the gradient to determine the coordinates of a point on the line.

Section 6.2: Length of Line Segment

Teachers should recall Pythagoras' Theorem with the students before showing them how the formula for the length of any line segment PQ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is proven. The formula is used to evaluate the distance between two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$. While students might ask whether the length of any line segment PQ can also be given by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, teachers can get the students to check if that is true (see Thinking Time on page 150 of the textbook).

Section 6.3: Equation of a Straight Line

Students have learnt in Book 1 how the graph of a straight line in the form $y = mx + c$ changes when either m or c varies. Teachers should recap the equations of a vertical line and a horizontal line and introduce the gradient-intercept form of the equation of a straight line, $y = mx + c$.

Teachers can guide the students along and show them the different cases and ways of finding the equation of a straight line (see Journal Writing on page 158 of the textbook).

Section 6.4: Parallel and Perpendicular Lines

Teachers should demonstrate to students how the gradients of parallel and perpendicular lines are related, and prove this with the use of trigonometric ratios (see Class Discussion: Parallel and Perpendicular Lines). Once students have understood the properties of parallel and perpendicular lines, they should be able to see that obtaining the equation of a line is possible when given the gradient of either a parallel or perpendicular line as well as the coordinates of a point on the line.

Challenge Yourself

Students should observe that the question states that AB is parallel to OC. Hence, they should equate the gradients of AB and OC together to find the proof.

WORKED SOLUTIONS

Class Discussion (Finding the Gradient of a Straight Line)

1. (i) In Fig. 6.3**(a)**,

$$\text{Gradient of } AB = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{6}{3} = 2$$

In Fig. 6.3**(b)**,

$$\text{Gradient of } AB = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-3}{1} = -3$$

(ii) The gradient of each line segment is the same. This is because the points lie on the same line segment.

2. Gradient of $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{y_1 - y_2}{x_1 - x_2}$

3. (a) Gradient $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-4}{3-(-1)} = \dfrac{3}{4}$

(b) Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-11-(-3)}{2-(-4)} = \dfrac{-8}{6} = -\dfrac{4}{3}$

(c) Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-3}{-4-6} = \dfrac{0}{-10} = 0$

(d) Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-(-1)}{2-2} = \dfrac{9}{0} =$ undefined

Thinking Time (Page 145)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-(-y_2 + y_1)}{-(-x_2 + x_1)} = \frac{y_1 - y_2}{x_1 - x_2}$$

Yes, they are equal.

Investigation (Gradient of a Straight Line)

	Coordinates of End-points	Gradient of Line segment	Sign of Gradient	$y_2 - y_1$	$x_2 - x_1$
(a)	$A(-2, 1)$ and $B(0, 5)$	$\frac{4}{2} = 2$	positive	$5 - 1$ $= 4$	$0 - (-2)$ $= 2$
(b)	$C(7, 5)$ and $D(4, 8)$	$\frac{3}{-3} = -1$	negative	$8 - 5$ $= 3$	$4 - 7$ $= -3$
(c)	$E(-2, 6)$ and $F(-4, 3)$	$\frac{-3}{-2} = \frac{3}{2}$	positive	$3 - 6$ $= -3$	$-4 - (-2)$ $= -2$
(d)	$G(1, 1)$ and $H(3, 1)$	$\frac{0}{2} = 0$	zero	$1 - 1$ $= 0$	$3 - 1 = 2$
(e)	$I(-4, 3)$ and $J(-4, 6)$	$\frac{0}{3}$ = undefined	undefined	$6 - 3$ $= 3$	$-4 - (-4)$ $= 0$

Table 6.1

5. (a) When $y_2 - y_1 > 0$ and $x_2 - x_1 < 0$, the sign of the gradient is negative.

(b) When $y_2 - y_1 < 0$ and $x_2 - x_1 > 0$, the sign of the gradient is negative.

(c) When the signs of $y_2 - y_1$ and $x_2 - x_1$ are the same, the sign of the gradient is positve.

(d) When $y_2 - y_1 = 0$, i.e. $y_1 = y_2$, the gradient of the line is zero.

(e) When $x_2 - x_1 = 0$, i.e. $x_1 = x_2$, the gradient of the line is undefined.

Thinking Time (Page 150)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[-(-x_2 + x_1)]^2 + [-(-y_2 + y_1)]^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Yes, they are equal.

Journal Writing (Page 158)

Case 1: Given the gradient m and the y-intercept c, substitute m and c into $y = mx + c$ to find the equation of the line.

Case 2: Given the gradient m and the coordinates of a point (a, b), substitute the coordinates of the point and the gradient into $y = mx + c$ to find the y-intercept. Then, substitute m and c into $y = mx + c$ to find the equation of the line.

Case 3: Given the coordinates of two points (a, b) and (c, d), find the gradient of the line using the coordinates of the two points. Substitute the coordinates of the point and the gradient into $y = mx + c$ to find the y-intercept. Then, substitute m and c into $y = mx + c$ to find the equation of the line.

Thinking Time (Page 161)

1. $$\text{Gradient of } AB = \frac{5-3}{3-2} = \frac{2}{1} = 2$$

$$\text{Gradient of } BC = \frac{9-5}{5-3} = \frac{4}{2} = 2$$

$$\text{Gradient of } AC = \frac{9-3}{5-2} = \frac{6}{3} = 2$$

The three points are collinear since the gradients are equal.

2. The points are collinear.

Class Discussion (Parallel and Perpendicular Lines)

1. **Case 1:** **(i)** θ_1 and θ_2 are equal (corresponding $\angle$s); the gradients of these lines are equal, i.e. $m_1 = m_2$.

(ii) Gradients are equal, thus l_1 and l_2 are parallel.

2. **Case 2:** **(i)** $m_1 = \tan \alpha$

$$= \frac{SR}{PS}$$

(ii) $m_2 = \tan \beta$

$$= \frac{\sin\beta}{\cos\beta}$$

$$= \frac{\sin(180° - \beta)}{-\cos(180° - \beta)}$$

$$= -\tan(180° - \beta)$$

$$= -\frac{PS}{SR}$$

$$m_1 m_2 = \frac{SR}{PS} \times \left(-\frac{PS}{SR}\right)$$

$$= -1$$

$$\therefore m_1 = -\frac{1}{m_1} \text{ (shown)}$$

(iii) If l_1 and l_2 are perpendicular, $m_1 m_2 = -1$.

(iv) If $m_1 m_2 = -1$, l_1 and l_2 are perpendicular.

Class Discussion (Perpendicular Lines)

Yes. Note that $AB^2 = 2^2 + 6^2$, $BC^2 = 12^2 + (k - 1)^2$ and $AC^2 = 10^2 + (k + 5)$. Thus, $10^2 + (k + 5)^2 = 4 + 46 + 144 + (k - 1)^2$. Solving this equation would give $k = 5$.

Practise Now 1

(a) Gradient of CD $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3 - 1}{6 - 3}$$

$$= \frac{2}{3}$$

(b) Gradient of HK $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2 - (-7)}{0 - 5}$$

$$= \frac{5}{-5}$$

$$= -1$$

(c) Gradient of MN $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{1 - 1}{16 - (-4)}$$

$$= \frac{0}{20}$$

$$= 0$$

Practise Now 2

Gradient of line, $\dfrac{y_2 - y_1}{x_2 - x_1} = -3$

$$\frac{h - (-9)}{-3 - 4} = -3$$

$$\frac{h + 9}{-7} = -3$$

$$h + 9 = -3(-7)$$

$$h + 9 = 21$$

$$\therefore h = 12$$

Practise Now 3

(a) Length of line segment CD $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(3 - 6)^2 + (-2 - 2)^2}$$

$$= \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$= 5$ units

(b) Length of line segment MN $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{[6 - (-1)]^2 + (-4 - 5)^2}$$

$$= \sqrt{7^2 + (-9)^2}$$

$$= \sqrt{49 + 81}$$

$$= \sqrt{130}$$

$= 11.4$ units (to 3 s.f.)

(c) Length of line segment PQ $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(8 - 2)^2 + (7 - 7)^2}$$

$$= \sqrt{6^2 + 0^2}$$

$$= \sqrt{36 + 0}$$

$$= \sqrt{36}$$

$= 6$ units

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Practise Now 4

(a) Let the coordinates of the point E be $(0, k)$.

$$\begin{aligned} CE &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0-4)^2 + [k - (-1)]^2} \\ &= \sqrt{(-4)^2 + (k+1)^2} \\ &= \sqrt{16 + (k+1)^2} \end{aligned}$$

$$\begin{aligned} DE &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[0 - (-2)]^2 + (k-7)^2} \\ &= \sqrt{2^2 + (k-7)^2} \\ &= \sqrt{4 + (k-7)^2} \end{aligned}$$

Since $CE = DE$,

$$\begin{aligned} \sqrt{16 + (k+1)^2} &= \sqrt{4 + (k-7)^2} \\ \left(\sqrt{16 + (k+1)^2}\right)^2 &= \left(\sqrt{4 + (k-7)^2}\right)^2 \\ 16 + (k+1)^2 &= 4 + (k-7)^2 \\ 16 + k^2 + 2k + 1 &= 4 + k^2 - 14k + 49 \\ k^2 + 2k + 17 &= k^2 - 14k + 53 \\ 2k + 14k &= 53 - 17 \\ 16k &= 36 \\ k &= \frac{36}{16} \\ k &= 2\frac{1}{4} \end{aligned}$$

$\therefore$ Coordinates of E are $\left(0, 2\frac{1}{4}\right)$

(b) Let the coordinates of the point F be $(p, 0)$.

$$\begin{aligned} CF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(p-4)^2 + [0 - (-1)]^2} \\ &= \sqrt{(p-4)^2 + 1^2} \\ &= \sqrt{(p-4)^2 + 1} \end{aligned}$$

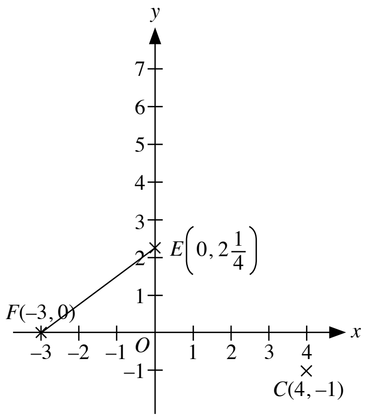
$$\begin{aligned} DF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[p - (-2)]^2 + (0-7)^2} \\ &= \sqrt{(p+2)^2 + (-7)^2} \\ &= \sqrt{(p+2)^2 + 49} \end{aligned}$$

Since $CF = DF$,

$$\begin{aligned} \sqrt{(p-4)^2 + 1} &= \sqrt{(p+2)^2 + 49} \\ \left(\sqrt{(p-4)^2 + 1}\right)^2 &= \left(\sqrt{(p+2)^2 + 49}\right)^2 \\ (p-4)^2 + 1 &= (p+2)^2 + 49 \\ p^2 - 8k + 16 + 1 &= p^2 + 4k + 4 + 49 \\ p^2 - 8k + 17 &= p^2 + 4k + 53 \\ -8k - 4k &= 53 - 17 \\ -12k &= 36 \\ k &= -3 \end{aligned}$$

$\therefore$ Coordinates of F are $(-3, 0)$

$$\begin{aligned} \text{Area of } \triangle OEF &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times OF \times OE \\ &= \frac{1}{2} \times 3 \times 2\frac{1}{4} \\ &= 3\frac{3}{8} \text{ units}^2 \end{aligned}$$



Practise Now 5

1.
$$\begin{aligned} DE^2 &= (2-6)^2 + (3-1)^2 \\ &= (-4)^2 + 2^2 \\ &= 16 + 4 \\ &= 20 \end{aligned}$$

$$\begin{aligned} EF^2 &= (-1-2)^2 + (-3-3)^2 \\ &= (-3)^2 + (-6)^2 \\ &= 9 + 36 \\ &= 45 \end{aligned}$$

$$\begin{aligned} DF^2 &= (-1-6)^2 + (-3-1)^2 \\ &= (-7)^2 + (-4)^2 \\ &= 49 + 16 \\ &= 65 \end{aligned}$$

Since $DE^2 + EF^2 = 20 + 45$
$= 65$
$= DF^2$,

the triangle is a right-angled triangle with $\angle DEF = 90°$. (Converse of Pythagoras' Theorem)

2. $PQ^2 = [6 - (-3)]^2 + (3 - 1)^2$
$= 9^2 + 2^2$
$= 81 + 4$
$= 85$
$QR^2 = (1 - 6)^2 + (8 - 3)^2$
$= (-5)^2 + 5^2$
$= 25 + 25$
$= 50$
$PR^2 = [1 - (-3)^2 + (8 - 1)^2$
$= 4^2 + 7^2$
$= 49 + 16$
$= 65$
Since $QR^2 + PR^2 = 50 + 65$
$= 115$
$\neq PQ^2$,
the triangle is not a right-angled triangle.

Practise Now 6

1. Since $(-1, 2)$ lies on the line $y = 5x + a$, the coordinates $(-1, 2)$ must satisfy the equation,
i.e. $2 = 5(-1) + c$
$= -5 + c$
$\therefore c = 7$

2. Since $(6, 8)$ lies on the line $y = -4x + b$, the coordinates $(6, 8)$ must satisfy the equation,
i.e. $8 = -4(6) + b$
$= -24 + b$
$\therefore b = 32$

Practise Now 7

(a) Gradient of $AB = \dfrac{3 - 1}{5 - (-2)}$
$= \dfrac{2}{7}$

Equation of AB is in the form $y = \dfrac{2}{7}x + c$

Since $(-2, 1)$ lies on the line,
$1 = \dfrac{2}{7}(-2) + c$
$1 = -\dfrac{4}{7} + c$
$c = \dfrac{11}{7}$
$\therefore$ Equation of AB is $y = \dfrac{2}{7}x + \dfrac{11}{7}$

(b) $C(6, 4)$ and $D(-4, 4)$ have the same y-coordinate of value 4.
$\therefore$ CD is a horizontal line with equation $y = 4$.

(c) $E(-3, 5)$ and $F(-3, 8)$ have the same x-coordinate of value -3.
$\therefore$ EF is a vertical line with equation $x = -3$.

Practise Now 8

(a) If A, B and C are collinear, they lie on the same straight line,
i.e. gradient of AB = gradient of AC

$$\frac{9 - (k + 4)}{0 - (k + 1)} = \frac{9 - (k + 3)}{0 - 2k}$$
$$\frac{5 - k}{-k - 1} = \frac{6 - k}{-2k}$$
$$-10k + 2k^2 = k^2 - 5k - 6$$
$$k^2 - 5k + 6 = 0$$
$$(k - 3)(k - 2) = 0$$
$k - 3 = 0$ or $k - 2 = 0$
$k = 3$ $\quad\quad k = 2$
$\therefore k = 2$ or 3

(b) If AB is parallel to CD,
gradient of AB = gradient of CD

$$\frac{9 - (k + 4)}{0 - (k + 1)} = \frac{(k + 6) - (k + 3)}{(2k + 2) - 2k}$$
$$\frac{5 - k}{-k - 1} = \frac{3}{2}$$
$$10 - 2k = -3k - 3$$
$$k = -13$$

Practise Now 9

Since $ABC = 90$, gradient of $AB \times$ gradient of $BC = -1$
$$\frac{-2 - 1}{-1 - 0} \times \frac{k - (-2)}{2 - (-1)} = -1$$
i.e.
$$3 \times \frac{k + 2}{3} = -1$$
$$k + 2 = -1$$
$$k = -3$$

Practise Now 10

Gradient of DC = gradient of AB
$= \dfrac{8 - 2}{16 - 4}$
$= \dfrac{6}{12}$
$= \dfrac{1}{2}$

Since BC is perpendicular to AB,
gradient of $BC = -2$

Equation of DC is $y - 10 = \dfrac{1}{2}(x - 5)$
$2y - 20 = x - 5$
$2y = x + 15$
Equation of BC is $y - 8 = -2(x - 16)$
$y = -2x + 32 + 8$
$y = -2x + 40$

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Exercise 6A

1. **(a)** Gradient $= \dfrac{1-0}{-2-0}$

$= -\dfrac{1}{2}$

(b) Gradient $= \dfrac{7-(-3)}{1-2}$

$= \dfrac{10}{-1}$

$= -10$

(c) Gradient $= \dfrac{8-4}{-5-(-2)}$

$= \dfrac{4}{-5+2}$

$= -\dfrac{4}{3}$

(d) Gradient $= \dfrac{-8-7}{1-(-4)}$

$= \dfrac{-15}{5}$

$= -3$

(e) Gradient $= \dfrac{6-(-5)}{2-(-2)}$

$= \dfrac{11}{4}$

(f) Gradient $= \dfrac{9-9}{6-(-7)}$

$= \dfrac{0}{13}$

$= 0$

2. Gradient of $AB = \dfrac{1-1}{7-0}$

$= \dfrac{0}{7}$

$= 0$

Gradient of $AE = \dfrac{4-1}{6-0}$

$= \dfrac{3}{6}$

$= \dfrac{1}{2}$

Gradient of $DC = \dfrac{0-5}{6-0}$

$= -\dfrac{5}{6}$

Gradient of $DE = \dfrac{4-5}{6-0}$

$= -\dfrac{1}{6}$

3. Gradient of line $= \dfrac{3}{5}$

$\dfrac{p-(-7)}{4-(-3)} = \dfrac{3}{5}$

$\dfrac{p+7}{7} = \dfrac{3}{5}$

$5(p+7) = 21$

$5p + 35 = 21$

$5p = -14$

$p = -2\dfrac{4}{5}$

4. Gradient of $AB = 3$

$\dfrac{-3-8}{k-3k} = 3$

$\dfrac{-11}{-2k} = 3$

$-11 = -6k$

$k = 1\dfrac{5}{6}$

5. Gradient $= \dfrac{2}{a}$

$\dfrac{1-a}{2a-9} = \dfrac{2}{a}$

$a(1-a) = 2(2a-9)$

$a - a^2 = 4a - 18$

$a^2 + 3a - 18 = 0$

$(a+6)(a-3) = 0$

$a + 6 = 0$ or $a - 3 = 0$

$\therefore a = -6$ or $a = 3$

6. Gradient of PQ = Gradient of PR

$\dfrac{-9-(-11)}{k-6} = \dfrac{-3-(-11)}{2k-6}$

$\dfrac{2}{k-6} = \dfrac{8}{2k-6}$

$2(2k-6) = 8(k-6)$

$4k - 12 = 8k - 48$

$4k = 36$

$k = 9$

7. Since the points are collinear, i.e. they lie on a straight line,

Gradient of PQ = Gradient of PR

$\dfrac{-2-(-3)}{3-2} = \dfrac{z-(-3)}{8-2}$

$\dfrac{1}{1} = \dfrac{z+3}{6}$

$6 = z + 3$

$z = 3$

8. Gradient of $AB = 2$

$$\frac{2t^2 + 7 - t}{7 - 2} = 2$$

$$\frac{2t^2 + 7 - t}{5} = 2$$

$$2t^2 + 7 - t = 10$$

$$2t^2 - 3 - t = 0$$

$$(2t - 3)(t + 1) = 0$$

$$2t - 3 = 0 \quad \text{or} \quad t + 1 = 0$$

$$2t = 3 \qquad t = -1$$

$$t = 1\frac{1}{2}$$

$\therefore t = 1\frac{1}{2}$ or $t = -1$

9. **(i)** Gradient of $AB = \frac{1-6}{2-0}$

$$= \frac{-5}{2}$$

$$= -\frac{5}{2}$$

Gradient of $BC = \frac{3-1}{7-2}$

$$= \frac{2}{5}$$

Gradient of $CD = \frac{8-3}{5-7}$

$$= \frac{5}{-2}$$

$$= -\frac{5}{2}$$

Gradient of $AD = \frac{8-6}{5-0}$

$$= \frac{2}{5}$$

(ii) Gradient of AB = Gradient of $CD = -\frac{5}{2}$

Gradient of BC = Gradient of $AD = \frac{2}{5}$

They are equal.

Exercise 6B

1. **(a)** $AB = \sqrt{(9-2)^2 + (7-3)^2}$

$$= \sqrt{7^2 + 4^2}$$

$$= \sqrt{65}$$

$= 8.06$ units (to 3 s.f.)

(b) $CD = \sqrt{(-5-3)^2 + (9-6)^2}$

$$= \sqrt{(-8)^2 + 3^2}$$

$$= \sqrt{73}$$

$= 8.54$ units (to 3 s.f.)

(c) $EF = \sqrt{(8-(-1))^2 + (-3-4)^2}$

$$= \sqrt{9^2 + (-7)^2}$$

$$= \sqrt{130}$$

$= 11.4$ units (to 3 s.f.)

(d) $GH = \sqrt{(-4-(-10))^2 + (-7-2)^2}$

$$= \sqrt{6^2 + (-9)^2}$$

$$= \sqrt{117}$$

$= 10.8$ units (to 3 s.f.)

2. $AB = 10$ units

$$\sqrt{(p-0)^2 + (0-p)^2} = 10$$

$$\sqrt{p^2 + p^2} = 10$$

$$\sqrt{2p^2} = 10$$

$$2p^2 = 100$$

$$p^2 = 50$$

$$p = \pm\sqrt{50}$$

$= \pm 7.07$ units (3 s.f.)

3. **(a)** Since R lies on the y-axis, x-coordinate of $R = 0$.

Let the coordinates of point R be $(0, y)$.

$$PR = QR$$

$$\sqrt{(0-(-2))^2 + (y-6)^2} = \sqrt{(0-9)^2 + (y-3)^2}$$

$$\sqrt{(2)^2 + (y-6)^2} = \sqrt{(-9)^2 + (y-3)^2}$$

$$4 + (y-6)^2 = 81 + (y-3)^2$$

$$4 + y^2 - 12y + 36 = 81 + y^2 - 6y + 9$$

$$40 - 12y = 90 - 6y$$

$$6y = -50$$

$$y = -8\frac{1}{3}$$

$\therefore$ Coordinates of R are $\left(0, -8\frac{1}{3}\right)$

(b) Since S lies on the x-axis, y-coordinate of $S = 0$.

Let the coordinates of point S be $(x, 0)$.

$$PS = QS$$

$$\sqrt{(x-(-2))^2 + (0-6)^2} = \sqrt{(x-9)^2 + (0-3)^2}$$

$$\sqrt{(x+2)^2 + (-6)^2} = \sqrt{(x-9)^2 + (-3)^2}$$

$$(x+2)^2 + 36 = (x-9)^2 + 9$$

$$x^2 + 4x + 4 + 36 = x^2 - 18x + 81 + 9$$

$$4x + 40 = 90 - 18x$$

$$22x = 50$$

$$x = 2\frac{3}{11}$$

$\therefore$ Coordinates of S are $\left(2\frac{3}{11}, 0\right)$

4. Since W lies on the y-axis, x-coordinate of $W = 0$.
Let the coordinates of point W be $(0, y)$.
Since W is equidistant from M and from N.

$$\begin{aligned} MW &= NW \\ \sqrt{(0-3)^2 + (y-7)^2} &= \sqrt{(0-11)^2 + [y-(-6)]^2} \\ \sqrt{(-3)^2 + (y-7)^2} &= \sqrt{(-11)^2 + (y+6)^2} \\ 9 + (y-7)^2 &= 121 + (y+6)^2 \\ 9 + y^2 - 14y + 49 &= 121 + y^2 + 12y + 36 \\ y^2 - 14y + 58 &= y^2 + 12y + 157 \\ -14y - 12y &= 157 - 58 \\ -26y &= 99 \\ y &= -3\frac{21}{26} \end{aligned}$$

$\therefore$ Coordinates of W are $\left(0, -3\frac{21}{26}\right)$

5. **(i)** $AB = 12$ units

$$\begin{aligned} BC &= \sqrt{(2-8)^2 + (6-(-2))^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(2-(-4))^2 + (6-(-2))^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

$$\begin{aligned} \text{Perimeter of } \triangle ABC &= 12 + 10 + 10 \\ &= 32 \text{ units} \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 12 \times 8 \\ &= 48 \text{ units}^2 \end{aligned}$$

(ii) Let the length of the perpendicular from A to BC be x units.

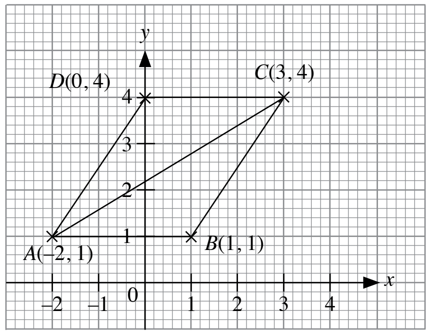
$$\begin{aligned} \text{Area of } \triangle ABC &= 48 \text{ units}^2 \\ \frac{1}{2} \times \text{base} \times \text{height} &= 48 \\ \frac{1}{2} \times BC \times x &= 48 \\ \frac{1}{2} \times 10 \times x &= 48 \\ 5x &= 48 \\ x &= 9.6 \end{aligned}$$

$\therefore$ The length of the perpendicular from A to BC is 9.6 units.

6. **(i)**
$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 3 \times 3 \\ &= 4.5 \text{ units}^2 \end{aligned}$$

(ii)
$$\begin{aligned} AC &= \sqrt{[3-(-2)]^2 + (4-1)^2} \\ &= \sqrt{5^2 + 3^2} \\ &= \sqrt{34} \\ &= 5.83 \text{ units (to 2 d.p.)} \end{aligned}$$

(iii)



$\therefore$ From the graph, the coordinates of D are $(0, 4)$.

(iv) Length of perpendicular from B to $CK = 3$ units

$$\begin{aligned} \text{Area of } \triangle BCK &= 12 \text{ units}^2 \\ \frac{1}{2} \times \text{base} \times \text{height} &= 12 \\ \frac{1}{2} \times CK \times 3 &= 48 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \times (t-3) \times 3 &= 12 \\ 3(t-3) &= 24 \\ 3t - 9 &= 24 \\ 3t &= 33 \\ t &= 11 \end{aligned}$$

or

$$\begin{aligned} \frac{1}{2} \times (3-t) \times 3 &= 12 \\ 3(3-t) &= 24 \\ 9 - 3t &= 24 \\ 3t &= -15 \\ t &= -5 \end{aligned}$$

$\therefore t = 11$ or -5

7. **(i)** $AB = 3$ units

$$\begin{aligned} BC &= \sqrt{[3-(-1)]^2 + [-1-(-2)]^2} \\ &= \sqrt{4^2 + 1^2} \\ &= \sqrt{17} \\ &= 4.12 \text{ units (to 3 s.f.)} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{[3-(-1)]^2 + (-1-1)^2} \\ &= \sqrt{4^2 + (-2)^2} \\ &= \sqrt{20} \\ &= 4.47 \text{ units (to 3 s.f.)} \end{aligned}$$

(ii) Length of the perpendicular from C to $AB = 4$ units

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times AB \times 4 \\ &= \frac{1}{2} \times 3 \times 4 \\ &= 6 \text{ units}^2 \end{aligned}$$

(iii) Length of the perpendicular from B to CE = 4 units

$$
\begin{aligned}
\text{Area of } \triangle BCE &= 14 \text{ units}^2\\
\frac{1}{2} \times \text{base} \times \text{height} &= 14\\
\frac{1}{2} \times CE \times 4 &= 14\\
2CE &= 14\\
CE &= 7 \text{ units}
\end{aligned}
$$

$\therefore$ Possible values of $k = 6$ or -8

8.
$$
\begin{aligned}
\sqrt{[1-(1-t)]^2 + (2t-1)^2} &= \sqrt{11-9t}\\
\left(\sqrt{[1-(1-t)]^2 + (2t-1)^2}\right)^2 &= \left(\sqrt{11-9t}\right)^2\\
[1-(1-t)]^2 + (2t-1)^2 &= 11-9t\\
t^2 + 4t^2 - 4t + 1 &= 11-9t\\
5t^2 - 4t + 1 &= 11-9t\\
5t^2 + 5t - 10 &= 0\\
t^2 + t - 2 &= 0\\
(t+2)(t-1) &= 0\\
t+2 = 0 \quad &\text{or} \quad t-1 = 0\\
t = -2 \quad & \quad\;\; t = 1
\end{aligned}
$$

$\therefore t = -2$ or $t = 1$

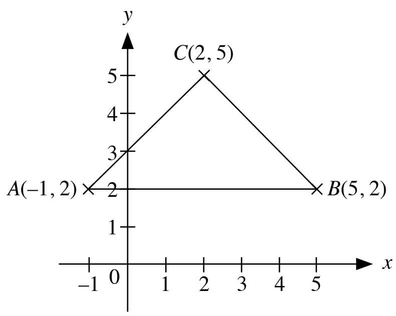
9. (i) $AB = 6$ units

$$
\begin{aligned}
BC &= \sqrt{(2-5)^2 + (5-2)^2}\\
&= \sqrt{(-3)^2 + 3^2}\\
&= \sqrt{18}\\
AC &= \sqrt{[2-(-1)]^2 + (5-2)^2}\\
&= \sqrt{3^2 + 3^2}\\
&= \sqrt{18}
\end{aligned}
$$

Since $BC = AC$, the points A, B and C are the vertices of an isosceles triangle.

(ii) Length of the perpendicular from C to AB = 3 units

$$
\begin{aligned}
\text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height}\\
&= \frac{1}{2} \times 6 \times 3\\
&= 9 \text{ units}^2
\end{aligned}
$$



10.
$$
\begin{aligned}
PQ^2 &= (3-3)^2 + (1-4)^2\\
&= 0^2 + (-3)^2\\
&= 9\\
QR^2 &= (8-3)^2 + (4-1)^2\\
&= 5^2 + 3^2\\
&= 25 + 9\\
&= 34\\
PR^2 &= (8-3)^2 + (4-4)^2\\
&= 5^2 + 0^2\\
&= 25
\end{aligned}
$$

$$
\begin{aligned}
\text{Since } PQ^2 + PR^2 &= 9 + 25\\
&= 34\\
&= QR^2,
\end{aligned}
$$

the triangle is a right-angled triangle with $\angle QPR = 90°$. (Converse of Pythagoras' Theorem)

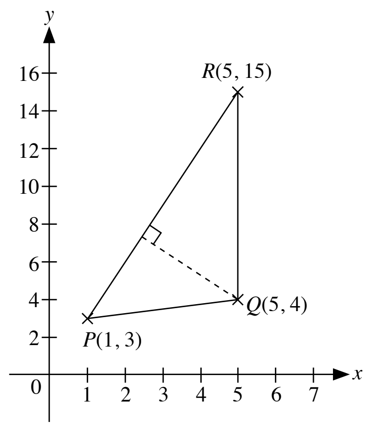
$$
\begin{aligned}
\text{Area of } \triangle PQR &= \frac{1}{2} \times PQ \times PR\\
&= \frac{1}{2} \times 3 \times 5\\
&= 7.5 \text{ units}^2
\end{aligned}
$$

Let the length of the perpendicular from P to QR be x units.

$$
\begin{aligned}
\text{Area of } \triangle PQR &= 7.5 \text{ units}^2\\
\frac{1}{2} \times \text{base} \times \text{height} &= 7.5\\
\frac{1}{2} \times QR \times x &= 7.5\\
\frac{1}{2} \times \sqrt{34} \times x &= 7.5\\
x &= 2.57 \text{ (to 3 s.f.)}
\end{aligned}
$$

$\therefore$ The length of the perpendicular from P to QR is 2.57 units.

11.



$$
\begin{aligned}
PR &= \sqrt{(5-1)^2 + (15-3)^2}\\
&= \sqrt{4^2 + 12^2}\\
&= \sqrt{160} \text{ units}
\end{aligned}
$$

Length of the perpendicular from P to QR = 4 units

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$$\text{Area of } \triangle PQR = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times QR \times 4$$
$$= \frac{1}{2} \times 11 \times 4$$
$$= 22 \text{ units}^2$$

Let the length of the perpendicular from Q to PR be h units.

$$\text{Area of } \triangle PQR = 22 \text{ units}^2$$
$$\frac{1}{2} \times \text{base} \times \text{height} = 22$$
$$\frac{1}{2} \times PR \times h = 22$$
$$\frac{1}{2} \times \sqrt{160} \times h = 22$$
$$h = 3.48 \text{ (to 3 s.f.)}$$

$\therefore$ The length of the perpendicular from Q to PR is 3.48 units.

Exercise 6C

1. Since the point (1, 2) passes through the line $y = -x + c$,

$2 = -1 + c$

$c = 3$

2. Since the point (–3, 3) passes through the line $y = 4x + c$,

$3 = 4(-3) + c$

$3 = -12 + c$

$c = 15$

3. **(a)** Gradient of $AB = = \frac{-1-0}{1-0} = \frac{-1}{1} = -1$

y-intercept = 0

Equation of AB is $y = -x$

(b) Gradient of $CD = \frac{5-3}{2-1} = \frac{2}{1} = 2$

Equation of CD is in the form $y = 2x + c$

Since (1, 3) lies on the line,

$3 = 2(1) + c$

$3 = 2 + c$

$c = 1$

$\therefore y = 2x + 1$

(c) Gradient of $EF = \frac{3-4}{-2-2} = \frac{-1}{-4} = \frac{1}{4}$

Equation of EF is in the form $y = \frac{1}{4}x + c$

Since (2, 4) lies on the line,

$4 = \frac{1}{4}(2) + c$

$4 = \frac{1}{2} + c$

$c = \frac{7}{2}$

$\therefore y = \frac{1}{4}x + \frac{7}{2}$

(d) Gradient of $GH = \frac{4-(-5)}{4-(-6)} = \frac{9}{10}$

Equation of GH is in the form $y = \frac{9}{10}x + c$

Since (–6, –5) lies on the line,

$-5 = \frac{9}{10}(-6) + c$

$-5 = -5\frac{2}{5} + c$

$c = \frac{2}{5}$

$\therefore y = \frac{9}{10}x + \frac{2}{5}$

(e) Gradient of $IJ = \frac{-7-(-4)}{1-(-2)} = \frac{-3}{3} = -1$

Equation of IJ is in the form $y = -x + c$

Since (–2, –4) lies on the line,

$-4 = (-1)(-2) + c$

$-4 = 2 + c$

$c = -6$

$\therefore y = -x - 6$

(f) Gradient of $KL = \frac{-1-(-5)}{-1-(-7)} = \frac{4}{6} = \frac{2}{3}$

Equation of KL is in the form $y = \frac{2}{3}x + c$

Since (–1, –1) lies on the line,

$-1 = \frac{2}{3}(-1) + c$

$-1 = -\frac{2}{3} + c$

$c = -\frac{1}{3}$

$\therefore y = \frac{2}{3}x - \frac{1}{3}$

(g) $M(8, 0)$ and $N(-9, 0)$ have the same y-coordinate of value 0.

$\therefore$ MN is a horizontal line with equation $y = 0$.

(h) $O(0, 0)$ and $P(0, 7)$ have the same x-coordinate of value 0.

$\therefore$ OP is a horizontal line with equation $x = 0$.

4. **(a)** $y = mx + c$

$0 = \frac{1}{3}(0) + c$

$c = 0$

$\therefore y = \frac{1}{3}x$

(b) $y = mx + c$

$1 = 3(1) + c$

$1 = 3 + c$

$c = -2$

$\therefore y = 3x - 2$

(c) $y = mx + c$

$-5 = -3(2) + c$

$-5 = -6 + c$

$c = 1$

$\therefore y = -3x + 1$

(d) $y = mx + c$

$7 = \left(-\frac{1}{2}\right)(5) + c$

$7 = -\frac{5}{2} + c$

$c = \frac{19}{2}$

$\therefore y = -\frac{1}{2}x + \frac{19}{2}$

(e) $y = mx + c$

$4 = (0)(5) + c$

$c = 4$

$\therefore y = 4$

(f) $y = mx + c$

$a = a(0) + c$

$c = a$

$\therefore y = ax + a$

5. $y = mx + c$

Gradient of line = 2

$c = 0$

Equation of line is in the form $y = 2x + 0$

$\therefore$ Equation of line is $y = 2x$.

6. (a) Since the line is a horizontal line, the gradient is 0 and the y-intercept = 1.

$\therefore$ Equation of line is $y = 1$.

(b) Since the line is a vertical line, the gradient is undefined and there is no y-intercept.

$\therefore$ Equation of line is $x = 1.5$.

(c) Since $(0, -1)$ and $(1, 0)$ lie on the line,

Gradient of line = $\frac{0-(-1)}{1-0} = \frac{1}{1} = 1$

$c = -1$

$\therefore$ Equation of the line is $y = x - 1$.

(d) Since $(0, 1)$ and $(2, 0)$ lie on the line,

Gradient of line = $\frac{0-1}{2-0} = -\frac{1}{2}$

$c = 1$

$\therefore$ Equation of the line is $y = -\frac{1}{2}x + 1$.

7. (i) Length of perpendicular from C to AB = 4 units

Area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2 \times 4$

$= 4 \text{ units}^2$

(ii) Gradient of line passing through B and $C = \frac{5-3}{5-1} = \frac{2}{4} = \frac{1}{2}$

(iii) Gradient of line passing through A and $C = \frac{5-1}{5-1} = \frac{4}{4} = 1$

Equation of AC is in the form $y = x + c$

Since $(1, 1)$ lies on the line,

$1 = (1)(1) + c$

$1 = 1 + c$

$c = 0$

$\therefore y = x$

8. For the line $2x - 5 = ky$,

$ky = 2x - 5$

$y = \frac{2}{k}x - \frac{5}{k}$

Gradient of the line $2x - 5 = ky$ is $\frac{2}{k}$.

For the line $(k + 1)x = 6y - 3$,

$6y = (k + 1)x + 3$

$y = \frac{k+1}{6}x - \frac{3}{6}$

$y = \frac{k+1}{6}x - \frac{1}{2}$

Gradient of the line $(k + 1)x = 6y - 3$ is $\frac{k+1}{6}$.

Since both lines have the same gradient,

$\frac{2}{k} = \frac{k+1}{6}$

$12 = k(k + 1)$

$12 = k^2 + k$

$k^2 + k - 12 = 0$

$(k + 4)(k - 3) = 0$

$k + 4 = 0$ or $k - 3 = 0$

$\therefore k = -4$ or $k = 3$

9. (i) $\frac{x}{3} + \frac{y}{2} = 1$

$6 \times \left(\frac{x}{3} + \frac{y}{2}\right) = 6 \times 1$

$2x + 3y = 6$

$3y = 6 - 2x$

$y = \frac{6-2x}{3}$

$y = \frac{6}{3} - \frac{2x}{3}$

$y = -\frac{2}{3}x + 2$

(ii) $y = -\frac{2}{3}x + 2$

$\therefore$ The gradient of the line is $-\frac{2}{3}$.

(iii) $y = -\frac{2}{3}x + 2$

Where the line cuts the x-axis, $y = 0$.

$0 = -\frac{2}{3}x + 2$

$\frac{2}{3}x = 2$

$2x = 6$

$x = 3$

$\therefore$ The coordinates of the point is $(3, 0)$.

10. **(i)** Given the gradient of line $= -\frac{2}{3}$,

Equation of the line is in the form $y = -\frac{2}{3}x + c$

Since (–3, 5) lies on the line,

$$5 = -3\left(-\frac{2}{3}\right) + c$$
$$5 = 2 + c$$
$$c = 3$$

$\therefore$ Equation of the line is $y = -\frac{2}{3}x + 3$.

(ii) Given that $(p, 3)$ passes through the line $y = -\frac{2}{3}x + 3$,

$$3 = -\frac{2}{3}p + 3$$
$$\frac{2}{3}p = 0$$
$$p = 0$$

11. Given that the line has the same gradient as the line $2y = 5x + 7$,

$$2y = 5x + 7$$
$$y = \frac{5x+7}{2}$$
$$y = \frac{5}{2}x + \frac{7}{2}$$

Gradient of the line $= \frac{5}{2}$

Since (3, –2) lies on the line,

$$-2 = \frac{5}{2}(3) + c$$
$$-2 = \frac{15}{2} + c$$
$$c = -\frac{19}{2}$$

$\therefore$ Equation of the line is $y = \frac{5}{2}x - \frac{19}{2}$.

12. **(i)**
$$y = mx + c$$
$$1 = 3(3) + c$$
$$1 = 9 + c$$
$$c = -8$$
$$\therefore y = 3x - 8$$

(ii) $y = 3x - 8$ — (1)

$y = x$ — (2)

Substitute (1) into (2):

$$3x - 8 = x$$
$$2x = 8$$
$$x = 4$$
$$y = 4$$

The coordinates of the point of intersection are (4, 4).

13. **(i)** Where l crosses the x-axis, $y = 0$

$$5x + 6y + 30 = 0$$
$$5x + 6(0) + 30 = 0$$
$$5x + 30 = 0$$
$$5x = -30$$
$$x = -6$$

$\therefore$ The coordinates of the point are (–6, 0).

(ii) Where l intersects the line $x = 2$

$$5x + 6y + 30 = 0$$
$$5(2) + 6y + 30 = 0$$
$$10 + 6y + 30 = 0$$
$$6y + 40 = 0$$
$$6y = -40$$
$$y = -6\frac{2}{3}$$

$\therefore$ The coordinates of the point of intersection are $\left(2, -6\frac{2}{3}\right)$.

(iii) Since the line has the same gradient as l,

$$5x + 6y + 30 = 0$$
$$6y = -5x - 30$$
$$y = -\frac{5}{6}x - \frac{30}{6}$$
$$y = -\frac{5}{6}x - 5$$

Gradient of the line $= -\frac{5}{6}$

Since (3, –1) lies on the line,

$$-1 = \left(-\frac{5}{6}\right)(3) + c$$
$$-1 = -\frac{5}{2} + c$$
$$c = \frac{3}{2}$$

$\therefore$ Equation of the line is $y = -\frac{5}{6}x + \frac{3}{2}$.

(iv) Since (3, –1) lies on the line and the gradient is 0,

$$-1 = 0(3) + c$$
$$c = -1$$

$\therefore$ Equation of the line is $y = -1$.

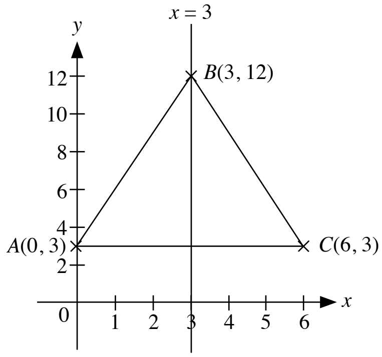
14. **(a)** **(i)** Gradient of line $l = \frac{12-3}{3-0} = \frac{9}{3} = 3$

(ii) Equation of the line is in the form $y = 3x + c$

Since (0, 3) lies on the line,

$$3 = 3(0) + c$$
$$c = 3$$
$$\therefore y = 3x + 3$$

(b)



$\therefore$ The coordinates of the point of intersection are (6, 3).

15. Since the line has the same gradient as the x-axis, the gradient is 0.

$$mx = ny + 2$$
$$ny = mx - 2$$
$$y = \frac{m}{n}x - \frac{2}{n}$$
$$\frac{m}{n} = 0$$
$$m = 0$$

For the line to be parallel to the y-axis, $n = 0$.

16. (i) $3x + 4y = 24$

Where l crosses the x-axis at the point A, $y = 0$

$$3x + 4y = 24$$
$$3x + 4(0) = 24$$
$$3x = 24$$
$$x = 8$$

$\therefore$ The coordinates of the point A are $(8, 0)$.

Where l crosses the y-axis at the point B, $x = 0$

$$3x + 4y = 24$$
$$3(0) + 4y = 24$$
$$4y = 24$$
$$y = 6$$

$\therefore$ The coordinates of the point B are $(0, 6)$.

(ii) $AB = \sqrt{(0-8)^2 + (6-0)^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{100}$

$= 10$ units

(iii) Let the coordinates of C be (k, k).

$$3k + 4k = 24$$
$$7k = 24$$
$$k = \frac{24}{7}$$
$$k = 3\frac{3}{7}$$

$\therefore$ The coordinates of the point C are $\left(3\frac{3}{7}, 3\frac{3}{7}\right)$.

(iv) Gradient of line $OC = \dfrac{3\frac{3}{7} - 0}{3\frac{3}{7} - 0} = 1$

Since $(0, 0)$ lies on the line,

$0 = 0(1) + c$

$c = 0$

$\therefore$ Equation of the line is $y = x$.

17. (i) Gradient of the line $= \dfrac{5-3}{9-2} = \dfrac{2}{7}$

Since $(2, 3)$ lies on the line,

$$3 = \frac{2}{7}(2) + c$$
$$3 = \frac{4}{7} + c$$
$$c = \frac{17}{7}$$

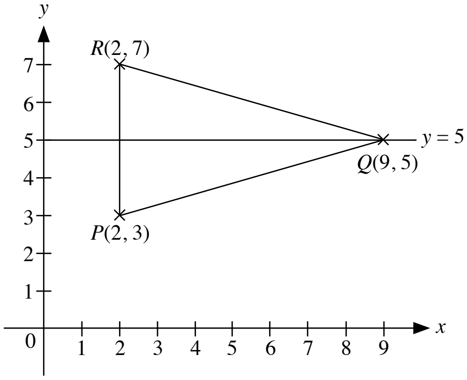
$\therefore$ Equation of the line is $y = \dfrac{2}{7}x + \dfrac{17}{7}$.

Where the line intersects the x-axis, $y = 0$

$$y = \frac{2}{7}x + \frac{17}{7}$$
$$0 = \frac{2}{7}x + \frac{17}{7}$$
$$\frac{2}{7}x = -\frac{17}{7}$$
$$2x = -17$$
$$x = -8\frac{1}{2}$$

$\therefore$ The coordinates of the point are $\left(-8\frac{1}{2}, 0\right)$.

(ii)



$\therefore$ The coordinates of R are $(2, 7)$.

(iii) $PQ = \sqrt{(9-2)^2 + (5-3)^2}$

$= \sqrt{7^2 + 2^2}$

$= \sqrt{53}$

$= 7.28$ units (to 3 s.f.)

(iv) Length of perpendicular from Q to $RP = 7$ units

Area of $\triangle PQR = \dfrac{1}{2} \times \text{base} \times \text{height}$

$= \dfrac{1}{2} \times 4 \times 7$

$= 14$ units^2

Let the length of perpendicular from R to PQ be h units.

Area of $\triangle PQR = 14$ units^2

$$\frac{1}{2} \times PQ \times h = 14$$
$$\frac{1}{2} \times \sqrt{53} \times h = 14$$
$$h = 3.85 \text{ units (to 3 s.f.)}$$

$\therefore$ The length of perpendicular from R to PQ is 3.85 units.

Exercise 6D

1. (a) Gradient = 8

(b) Gradient $= -\dfrac{1}{2}$

2. (a) Gradient $= \dfrac{-1}{3}$

$= -\dfrac{1}{3}$

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(b) Gradient $= \dfrac{-1}{\frac{3}{4}}$

$= \dfrac{4}{3}$

3. **(a)** Since the points O, A and B are collinear,
gradient of OA = gradient of OB

$$\frac{3k-0}{2-0} = \frac{6-0}{4k-0}$$
$$\frac{3k}{2} = \frac{6}{4k}$$
$$12k^2 = 12$$
$$k^2 = 1$$
$$k = \pm 1$$

(b) Since OA is parallel to BC,
gradient of OA = gradient of BC

$$\frac{3k-0}{2-0} = \frac{7-6}{10k-4k}$$
$$\frac{3k}{2} = \frac{1}{6k}$$
$$18k^2 = 2$$
$$k^2 = \frac{2}{18}$$
$$= \frac{1}{9}$$
$$k = \pm\frac{1}{3}$$

4. Gradient of $AB \times$ gradient of $BC = -1$

Gradient of $AB = \dfrac{4-1}{-1-1}$

$= -\dfrac{3}{2}$

Gradient of $BC = \dfrac{2}{3}$

$$\frac{k-4}{6-(-1)} = \frac{2}{3}$$
$$3k - 12 = 12 + 2$$
$$3k = 26$$
$$k = \frac{26}{3}$$

5. **(a)** $3y + 7x = 29$

$$3y = -7x + 29$$
$$y = -\frac{7}{3}x + \frac{29}{3}$$

$\therefore$ Gradient of line $= -\dfrac{7}{3}$

Equation of line is $y - 5 = -\dfrac{7}{3}[x - (-2)]$

$$y = -\frac{7}{3}x - \frac{14}{3} + 5$$
$$3y = -7x + 1$$

(b) $42x - 7y = 5$

$$7y = 42x - 5$$
$$y = 6x - \frac{5}{7}$$

$\therefore$ Gradient of line $= -\dfrac{1}{6}$

Equation of line is $y - (-6) = -\dfrac{1}{6}[x - (-1)]$

$$y + 6 = -\frac{1}{6}x - \frac{1}{6}$$
$$y = -\frac{1}{6}x - \frac{37}{6}$$
$$6y = -x - 37$$

(c) $3x + y = 17$

$$y = -3x + 17$$

$\therefore$ Gradient of line $= -3$

Equation of line is $y - 8 = -3(x - 4)$

$$y = -3x + 12 + 8$$
$$y = -3x + 20$$

(d) $y + 2x = 13$

$$y = -2x + 13$$

$\therefore$ Gradient $= \dfrac{1}{2}$

Equation of line is $y - (-3) = \dfrac{1}{2}(x - 2)$

$$y + 3 = \frac{1}{2}x - 1$$
$$2y = x - 8$$

6. Gradient of $AB = \dfrac{3-(-3)}{2-(-1)}$

$= \dfrac{6}{3}$

$= 2$

Gradient of $BC = -\dfrac{1}{2}$

$$\frac{k-3}{(k+5)-2} = -\frac{1}{2}$$
$$2k - 6 = -k - 3$$
$$3k = 3$$
$$k = 1$$

7. $PR = 12$ units

$PQ = \sqrt{(-1-6)^2 + (3-8)^2}$

$= \sqrt{74}$ units

$QR = \sqrt{(6-11)^2 + (8-1)^2}$

$= \sqrt{74}$ units

Since $PQ = QR$, the points P, Q and R are vertices of an isosceles triangle. (shown)

Gradient of $PQ = \frac{8-3}{6-(-1)}$

$= \frac{5}{7}$

Gradient of $QR = \frac{1-8}{11-6}$

$= -\frac{7}{8}$

Since gradient of $PQ \times$ gradient of $QR = -1$, PQ is perpendicular to QR and PQR is a right angle.

8. **(i)** Let point C be $(k, 0)$.

$AB = \sqrt{(6-0)^2 + (6-4)^2}$

$= \sqrt{40}$ units

$BC = \sqrt{(6-k)^2 + (6-0)^2}$

$= \sqrt{(6-k)^2 + 36}$ units

$$\sqrt{(6-k)^2 + 36} = \sqrt{40}$$
$$(6-k)^2 + 36 = 40$$
$$36 - 12k + k^2 + 36 - 40 = 0$$
$$k^2 - 12k + 32 = 0$$
$$(k-4)(k-8) = 0$$
$$k - 4 = 0 \quad \text{or} \quad k - 8 = 0$$
$$k = 4 \quad \text{or} \quad k = 8$$

$\therefore$ Coordinates of C are $(4, 0)$ or $(8, 0)$.

(ii) Let point D be the point $(0, p)$.

Gradient of $AB = \frac{6-4}{6-0}$

$= \frac{2}{6}$

$= \frac{1}{3}$

$\therefore$ Gradient of $DB = -3$

$$\frac{6-p}{6-0} = -3$$
$$6 - p = -18$$
$$p = 24$$

$\therefore$ Coordinates of D are $(0, 24)$.

9. **(i)** Let the point P be $(2m, m)$.

$$RP = SP$$
$$\sqrt{(-2-2m)^2 + (4-m)^2} = \sqrt{(6-2m)^2 + (-4-m)^2}$$
$$\sqrt{(4+8m+4m^2)+(16-8m+m^2)} =$$
$$\sqrt{(36-24m+4m^2)+(16+8m+m^2)}$$
$$\sqrt{20+5m^2} = \sqrt{52-16m+5m^2}$$
$$20 + 5m^2 = 52 - 16m + 5m^2$$
$$16m = 32$$
$$m = 2$$

$\therefore$ Coordinates of P are $(4, 2)$.

(ii) Gradient of $RP = \frac{2-4}{4-(-2)}$

$= -\frac{2}{6}$

$= -\frac{1}{3}$

Gradient of $RS = \frac{-4-4}{6-(-2)}$

$= -\frac{8}{8}$

$= -1$

Since the gradients of RP and RS are not equal, points P, R and S are not collinear.

10. **(a)** Gradient of $AB = \frac{-12-(-6)}{3-(-1)}$

$= -\frac{6}{4}$

$= -\frac{3}{2}$

Gradient of $AC = \frac{6-(-6)}{k-(-1)}$

$= \frac{12}{k+1}$

Since A, B and C are collinear,

$$\frac{12}{k+1} = -\frac{3}{2}$$
$$24 = -3k - 3$$
$$3k = -27$$
$$k = -9$$

(b) If AB is perpendicular to AC,

Gradient of $AC = \frac{2}{3}$

$$\frac{12}{k+1} = \frac{2}{3}$$
$$36 = 2k + 2$$
$$2k = 34$$
$$k = 17$$

11. **(i)** Gradient of $AB = \frac{3-(-3)}{2-(-1)}$

$= \frac{6}{3}$

$= 2$

Gradient of $BC = \frac{k-3}{6-2}$

Since AB is perpendicular to BC,

$$\frac{k-3}{6-2} = -\frac{1}{2}$$
$$2k - 6 = -6 + 2$$
$$2k = 2$$
$$k = 1$$

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(ii) Gradient of $AC = \frac{1-(-3)}{6-(-1)}$

$= \frac{4}{7}$

12. Let the points be $A(2, 1)$, $B(1, 5)$, $C(-2, -1)$ and $D(-1, -5)$.

Gradient of $AB = \frac{5-1}{1-2}$

$= -4$

Gradient of $CD = \frac{-5-(-1)}{-1-(-2)}$

$= -4$

Since gradient of AB = gradient of CD, $AB \parallel CD$.

Gradient of $BC = \frac{-5-(-1)}{1-(-2)}$

$= \frac{6}{3}$

$= 2$

Gradient of $AD = \frac{-5-1}{-1-2}$

$= \frac{-6}{-3}$

$= 2$

Since gradient of BC = gradient of AD, $BC \parallel AD$,

$\therefore$ the 4 points are the vertices of a parallelogram.

13. Let the points be $A(5, 8)$, $B(3, 5)$, $C(5, 2)$ and $D(7, 5)$.

Gradient of $AB = \frac{8-5}{5-3}$

$= \frac{3}{2}$

Gradient of $CD = \frac{5-2}{7-5}$

$= \frac{3}{2}$

Since gradient of AB = gradient of CD, $AB \parallel CD$.

Gradient of $BC = \frac{5-2}{3-5}$

$= -\frac{3}{2}$

Gradient of $AD = \frac{8-5}{5-7}$

$= -\frac{3}{2}$

Since gradient of BC = gradient of AD, $BC \parallel AD$.

Length of $AB = \sqrt{(5-3)^2 + (8-5)^2}$

$= \sqrt{(2)^2 + (3)^2}$

$= \sqrt{13}$ units

Length of $BC = \sqrt{(5-3)^2 + (2-5)^2}$

$= \sqrt{(2)^2 + (-3)^2}$

$= \sqrt{13}$ units

Since $AB = BC$, $AB \parallel CD$ and $BC \parallel AD$,

$\therefore$ the 4 points are vertices of a rhombus.

14. (a) Gradient of $AB = \frac{12-(-6)}{3-(-1)}$

$= \frac{18}{4}$

$= \frac{9}{2}$

Gradient of $AC = \frac{6-(-6)}{k-(-1)}$

$= \frac{12}{k+1}$

Since A, B and C are collinear,

$$\frac{12}{k+1} = \frac{9}{2}$$
$$24 = 9k + 9$$
$$9k = 15$$
$$k = \frac{15}{9}$$
$$= 1\frac{2}{3}$$

(b) Since AB is perpendicular to AC,

$$\frac{12}{k+1} = -\frac{2}{9}$$
$$108 = -2k - 2$$
$$2k = -110$$
$$k = -55$$

(c) Gradient of $BC = \frac{12-6}{3-k}$

Since BC is perpendicular to AC,

$$\frac{12-6}{3-k} \times \frac{12}{k+1} = -1$$
$$\frac{144-72}{(3-k)(k+1)} = -1$$
$$\frac{72}{3k+3-k^2-k} = -1$$
$$72 = k^2 - 2k - 3$$
$$k^2 - 2k - 75 = 0$$
$$k^2 - 2k + 1 = 76$$
$$(k-1)^2 = 76$$
$$k - 1 = \pm\sqrt{76}$$
$$k = 1 \pm 2\sqrt{19}$$

15. (a) Gradient of $AB = \frac{7-2}{6-1}$

$= \frac{5}{5}$

$= 1$

Gradient of perpendicular bisector $= -1$

Midpoint of $AB = \left(\frac{1+6}{2}, \frac{2+7}{2}\right)$

$= \left(\frac{7}{2}, \frac{9}{2}\right)$

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Equation of perpendicular bisector is

$$y - \frac{9}{2} = -1\left(x - \frac{7}{2}\right)$$

$$y = -x + \frac{7}{2} + \frac{9}{2}$$

$$y + x = 8$$

(b) Gradient of $BC = \frac{7-2}{6-7}$

$= -5$

Gradient of perpendicular bisector $= \frac{1}{5}$

Midpoint of $BC = \left(\frac{6+7}{2}, \frac{7+2}{2}\right)$

$= \left(\frac{13}{2}, \frac{9}{2}\right)$

Equation of perpendicular bisector is

$$y - \frac{9}{2} = \frac{1}{5}\left(x - \frac{13}{2}\right)$$

$$y = \frac{1}{5}x - \frac{13}{10} + \frac{9}{2}$$

$$y = \frac{1}{5}x + \frac{16}{5}$$

$$5y = x + 16$$

16. (i) Gradient of $AC = \frac{8-0}{-2-4}$

$= -\frac{8}{6}$

$= -\frac{4}{3}$

(ii) $AB = \sqrt{[4-(-3)]^2 + (-1)^2}$

$= \sqrt{(7)^2 + (-1)^2}$

$= \sqrt{50}$ units

$BC = \sqrt{[-3-(-2)^2 + (1-8)^2}$

$= \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{50}$ units

$\therefore AB = BC$

Gradient of $AB = \frac{1-0}{-3-4}$

$= -\frac{1}{7}$

Gradient of $BC = \frac{8-1}{-2-(-3)}$

$= 7$

Since gradient of $AB \times$ gradient of $BC = -1$,
AB is perpendicular to BC and $\angle ABC = 90°$.

(iii) Length of one side $= \sqrt{50}$ units

Area of square $= (\sqrt{50})^2$

$= 50$ units^2

Review Exercise 6

1. (i)

$$y = mx + c$$
$$-3 = 2(0) + c$$
$$c = -3$$
$$\therefore y = 2x - 3$$

(ii) Given that the line passes through the point $(4, k)$,

$k = 2(4) - 3$

$k = 8 - 3$

$k = 5$

2. (i) $6x + 2y = 7$

$$2y = -6x + 7$$
$$y = \frac{-6x+7}{2}$$
$$y = -\frac{6}{2}x + \frac{7}{2}$$
$$y = -3x + \frac{7}{2}$$

$\therefore$ Gradient of the line $= -3$

(ii) Given that the line passes through $(3, 5)$ and the gradient $= -3$,

$y = mx + c$

$5 = -3(3) + c$

$5 = -9 + c$

$c = 14$

3. Gradient of $AB = \frac{-3-5}{2-1} = \frac{-8}{1} = -8$

Given that the line passes through the origin, $(0, 0)$,

$y = mx + c$

$0 = -8(0) + c$

$c = 0$

$\therefore y = -8x$

4. (i) $3x - 4y = 24$

$$4y = 3x - 24$$
$$y = \frac{3x-24}{4}$$
$$y = \frac{3}{4}x - \frac{24}{4}$$
$$y = \frac{3}{4}x - 6$$

$\therefore$ Gradient of $l = \frac{3}{4}$

(ii) $3x - 4y = 24$

Where l crosses the x-axis at the point A, $y = 0$

$$3x - 4y = 24$$
$$3x - 4(0) = 24$$
$$3x = 24$$
$$x = 8$$

$\therefore$ The coordinates of the point A are $(8, 0)$.

Where l crosses the y-axis at the point B, $x = 0$

$$3x - 4y = 24$$
$$3(0) - 4y = 24$$
$$-4y = 24$$
$$y = -6$$

$\therefore$ The coordinates of the point B are $(0, -6)$.

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$$AB = \sqrt{(0-8)^2 + (-6-0)^2}$$
$$= \sqrt{8^2 + 6^2}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

(iii) Gradient of $OM = \dfrac{-3-0}{4-0} = -\dfrac{3}{4}$

Given that the line passes through $B(0, -6)$,

$$y = mx + c$$
$$-6 = -\frac{3}{4}(0) + c$$
$$c = -6$$
$$\therefore y = -\frac{3}{4}x - 6$$

5. **(i)** Gradient of $AB = \dfrac{0-6}{8-0} = -\dfrac{3}{4}$

Given that the line passes through $A(0, 6)$,

$$y = mx + c$$
$$6 = -\frac{3}{4}(0) + c$$
$$c = 6$$
$$\therefore y = -\frac{3}{4}x + 6$$

(ii) $y = -\dfrac{3}{4}x + 6$ — (1)

$y = x + 1$ — (2)

Substitute (1) into (2):

$$-\frac{3}{4}x + 6 = x + 1$$
$$-\frac{3}{4}x - x = 1 - 6$$
$$-1\frac{3}{4}x = -5$$
$$x = 2\frac{6}{7}$$
$$y = 2\frac{6}{7} + 1$$
$$= 3\frac{6}{7}$$

$\therefore$ The coordinates of M are $\left(2\frac{6}{7}, 3\frac{6}{7}\right)$.

(iii) Given that the line passes through $M\left(2\frac{6}{7}, 3\frac{6}{7}\right)$ and is parallel to the x-axis,

Gradient of the line = 0

$$y = mx + c$$
$$3\frac{6}{7} = 0\left(2\frac{6}{7}\right) + c$$
$$c = 3\frac{6}{7}$$
$$\therefore y = 3\frac{6}{7}$$

(iv) Given that the line passes through $M\left(2\frac{6}{7}, 3\frac{6}{7}\right)$ and is parallel to the y-axis,

Gradient of the line = undefined

$$\therefore x = 2\frac{6}{7}$$

6. **(i)** Gradient of $l = \dfrac{5-1}{5-(-1)} = \dfrac{4}{6} = \dfrac{2}{3}$

(ii) Given that the line passes through $B(5, 5)$,

$$y = mx + c$$
$$5 = \frac{2}{3}(5) + c$$
$$5 = 3\frac{1}{3} + c$$
$$c = \frac{5}{3}$$
$$\therefore y = \frac{2}{3}x + \frac{5}{3}$$

(iii) Length of perpendicular from B to AC = 4 units

Area of $\triangle ABC = \dfrac{1}{2} \times 5 \times 4$

$= 10 \text{ units}^2$

(iv)
$$BC = \sqrt{(4-5)^2 + (1-5)^2}$$
$$= \sqrt{(-1)^2 + (-4)^2}$$
$$= \sqrt{17}$$
$$= 4.12 \text{ units (to 2 d.p.)}$$

7. **(i)** $3y = 2x + 8$

When l crosses the x-axis at the point A, $y = 0$

$$3(0) = 2x + 8$$
$$2x = -8$$
$$x = -4$$

$\therefore$ The coordinates of the point A are $(-4, 0)$.

(ii) Let the x-coordinate be t, then the y-coordinate is $\dfrac{2t+8}{3}$.

Since the y-coordinate is twice the x-coordinate,

$$2t = \frac{2t+8}{3}$$
$$6t = 2t + 8$$
$$4t = 8$$
$$t = 2$$
$$2t = 4$$

$\therefore$ The coordinates of the point K are $(2, 4)$.

(iii) Length of perpendicular from K to AH = 4 units

Area of $\triangle ABC = \dfrac{1}{2} \times 8 \times 4$

$= 16 \text{ units}^2$

8. **(i)** Gradient of the line $= \dfrac{3-0}{0-(-1)} = \dfrac{3}{1} = 3$

Given that the line passes through $B(0, 3)$,

$$y = mx + c$$
$$3 = 3(0) + c$$
$$c = 3$$
$$\therefore y = 3x + 3$$

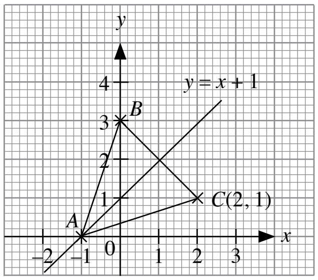
(ii) $AB = \sqrt{h}$

$$\sqrt{[0-(-1)]^2 + (3-0)^2} = \sqrt{h}$$
$$\sqrt{1^2 + 3^2} = \sqrt{h}$$
$$\sqrt{10} = \sqrt{h}$$
$$(\sqrt{10})^2 = (\sqrt{h})^2$$
$$10 = h$$
$$\therefore h = 10$$

(iii) Since the point $(-5, k)$ lies on BA produced,

$$y = 3x + 3$$
$$k = 3(-5) + 3$$
$$k = -12$$

(iv)



$\therefore$ The coordinates of the point C are $(2, 1)$.

9. **(i)** $5y + 12x - 60 = 0$

Where l_1 crosses the x-axis at the point P, $y = 0$

$$5y + 12x - 60 = 0$$
$$5(0) + 12x - 60 = 0$$
$$12x = 60$$
$$x = 5$$

$\therefore$ The coordinates of the point P are $(5, 0)$.

Where l_1 crosses the y-axis at the point Q, $x = 0$

$$5y + 12x - 60 = 0$$
$$5y + 12(0) - 60 = 0$$
$$5y = 60$$
$$y = 12$$

$\therefore$ The coordinates of the point Q are $(0, 12)$.

(ii) $PQ = \sqrt{(0-5)^2 + (12-0)^2}$

$$= \sqrt{(-5)^2 + 12^2}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

(iii) Given that l_2 has the same gradient as l_1,

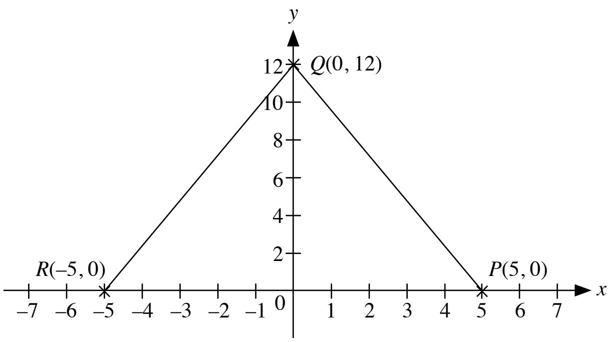
$$5y + 12x - 60 = 0$$
$$5y = -12x + 60$$
$$y = -\frac{12}{5}x + \frac{60}{5}$$
$$y = -\frac{12}{5}x + 12$$

Gradient of $l_2 = -\dfrac{12}{5}$

Given that the line passes through $(0, -2)$,

$$y = mx + c$$
$$-2 = -\frac{12}{5}(0) + c$$
$$c = -2$$
$$\therefore y = -\frac{12}{5}x - 2$$

(iv)



$\therefore$ The coordinates of the point R are $(-5, 0)$.

10. $AB = h + 11$

$DC = 2 + t$

Perpendicular height of trapezium $ABCD$ = 6 units

Area of trapezium $ABCD$ = 84 units^2

$$\frac{1}{2} \times (AB + DC) \times 6 = 84$$
$$\frac{1}{2} \times (h + 11 + 2 + t) \times 6 = 84$$
$$3(h + 13 + t) = 84$$
$$h + 13 + t = 28$$
$$t = 15 - h$$

11. **(i)** Since the line through C at B is a horizontal line, the equation of the line $y = -4$.

(ii) Since A is a point on the y-axis, $x = 0$

$$2y - 5x = 4$$
$$2y - 5(0) = 4$$
$$2y = 4$$
$$y = 2$$

$\therefore$ The coordinates of the point A are $(0, 2)$.

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Since B lies on line AB and its y-coordinate = –4,

$$
\begin{aligned}
2(-4) - 5x &= 4 \\
-8 - 5x &= 4 \\
5x &= -12 \\
x &= -2\frac{2}{5}
\end{aligned}
$$

$\therefore$ The coordinates of the point B are $\left(-2\frac{2}{5}, -4\right)$.

$$
\begin{aligned}
\text{Area of } \triangle ABC &= \frac{1}{2} \times 2\frac{2}{5} \times 6 \\
&= 7.2 \text{ units}^2
\end{aligned}
$$

(iii)

$$
\begin{aligned}
AB &= \sqrt{l} \\
\sqrt{\left(-2\frac{2}{5} - 0\right)^2 + (-4-2)^2} &= \sqrt{l} \\
\sqrt{\left(-2\frac{2}{5}\right)^2 + (-6)^2} &= \sqrt{l} \\
\sqrt{41.76} &= \sqrt{l} \\
(\sqrt{41.76})^2 &= (\sqrt{l})^2 \\
41.76 &= l \\
\therefore l &= 41.76
\end{aligned}
$$

(iv) Let the length of the perpendicular from C to AB be h units.

Area of $\triangle ABC$ = 7.2 units^2

$$
\begin{aligned}
\frac{1}{2} \times AB \times h &= 7.2 \\
\frac{1}{2} \times \sqrt{41.76} \times h &= 7.2 \\
h &= 2.23 \text{ units (to 3 s.f.)}
\end{aligned}
$$

(v) Gradient of $AD = \dfrac{2-2}{0-2.4} = 0$

Gradient of $BC = 0$

Since AD = 2.4 units = BC,

hence $ABCD$ is a parallelogram (proven).

12. (i) Gradient $= \dfrac{12-15}{20-50} = \dfrac{-3}{-30} = 0.1$

Given that (20, 12) lies on the line,

$$
\begin{aligned}
12 &= 0.1(20) + c \\
12 &= 2 + c \\
c &= 10 \\
\therefore y &= 0.1x + 10
\end{aligned}
$$

(ii) The value of the y-intercept represents the length of the spring when it is suspended freely without a mass attached to it.

13. $3x + 5y = 13$

$$
\begin{aligned}
5y &= -3x + 13 \\
y &= -\frac{3}{5}x + \frac{13}{5}
\end{aligned}
$$

$$
\begin{aligned}
\text{Gradient of line} &= \frac{-1}{-\frac{3}{5}} \\
&= \frac{5}{3}
\end{aligned}
$$

Equation of the line is $y - \dfrac{3}{11} = \dfrac{5}{3}\left(x - \dfrac{10}{11}\right)$

$$
\begin{aligned}
y &= x\frac{5}{3} - \frac{50}{33} + \frac{3}{11} \\
&= \frac{5}{3}x - \frac{41}{33} \\
33y &= 55x - 41
\end{aligned}
$$

14. Gradient of line = 2

Equation of the line is $y - 3 = 2(x - 2)$

$$
\begin{aligned}
y &= 2x - 4 + 3 \\
&= 2x - 1
\end{aligned}
$$

Challenge Yourself

Gradient of $AB = \dfrac{a^2 - b^2}{a - b} = \dfrac{(a+b)(a-b)}{a-b} = a + b$

Gradient of $OC = \dfrac{c^2}{c} = c$

Since AB is parallel to OC,

Gradient of AB = Gradient of OC

$\therefore a + b = c$ (proven)

Chapter 7 Graphs of Functions and Graphical Solution

TEACHING NOTES

Suggested Approach

This chapter serves as an introduction to the important concepts of relations and functions. Before plotting graphs of functions, revise the choice of scales and labelling of scales on both axes. Students are often weak in some of these areas. Teachers should encourage the students to draw the curves free hand, as well as to use curved rules to assist them.

It will be worthwhile to ask students to remember the general shapes of quadratic graphs, the "U" shape and the upside down "U" shape. Also, students should remember the general shapes of cubic, reciprocal and exponential graphs. This will help them to identify and rectify errors when they sketch or plot a few points wrongly and if the shapes of their graphs look odd.

Section 7.1: Graphs of Cubic Functions

Teachers should help students to see how the coefficient of x^3 affects the shape of the graph. Teachers can ask students who are confident and have grasped the concept behind graphs of cubic functions to explain to the class (see Investigation: Graphs of Cubic Functions). Teachers will facilitate the class through the activity as the students lead and learn together.

Section 7.2: Graphs of Reciprocal Functions

When introducing graphs of reciprocal functions, teachers should bring the students' attention to the marginal notes on the four quadrants on the Cartesian plane and the order of rotational symmetry about a particular point (see Information and Recall on page 176). Students need to know why the graphs of $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ do not intersect with both axes and the values obtained when any real number is divided by zero $\left(\text{see Investigation: Graphs of } y = \frac{a}{x} \text{ and Investigation: Graphs of } \frac{a}{x^2}\right)$.

When the students are able to understand the shapes of the graphs of reciprocal functions, they can go on to further describe the graphs in the cases where $a < 0$ and where $a > 0$ (see Thinking Time on page 177 of the textbook).

Section 7.3: Graphs of Exponential Functions

Teachers should help students to see how the values of a and k affect the shape of the graphs of $y = a^x$ and $y = ka^x$. Teachers can ask students who are confident and have grasped the concept behind graphs of exponential functions to explain to the class (see Investigation: Graphs of $y = a^x$ and $y = ka^x$) while the teachers facilitate.

Now that the students have learnt the graphs of power and exponential functions, teachers can get them to match such graphs with their corresponding functions and see whether they are able to differentiate between the different types of graphs (see Class Discussion: Matching Graphs of Power Functions with the Corresponding Functions).

Section 7.4: Gradient of a Curve

Teachers can first revise what was covered in Chapter 6, where the gradient of a straight line is the ratio of vertical change to horizontal change. Students may have difficulty in drawing the tangent to the curve. They may draw the line too close to the curve or out of the point. Teachers can guide them along and suggest to them to pick two points, say A and B, each with equal distance and near the point of contact, and then draw the tangent parallel to line AB at the point of contact.

Section 7.5: Applications of Graphs in Real-World Contexts

Students tend to make mistakes with the quantities "distance", "time" and "speed". As such, teachers should revise the formula for speed as the distance per unit time. Teachers should highlight to the students the difference between finding the instantaneous speed and the average speed. The total distance travelled by an object has to include the distance travelled in the returning journey when there is such a case, while the total time taken has to include the time taken when the object is at rest or stationary.

Teachers can introduce the many different applications of graphs in real-world contexts through the worked examples and activities (see Class Discussion: Linear Distance-Time Graphs and Thinking Time on pages 193 and 194 of the textbook).

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For graphs involving the rate of flow of liquid in various shapes and sizes of containers, teachers should guide the students along and teach them how to deduce the rate of increase of depth when the shape of the container changes (see Thinking Time on page 197 of the textbook).

Some common mistakes that students make are that they may write 1 hour 15 minutes as 1.15 h instead of 1.25 h. Students are also prone to mistakes when different units are given in a problem. For example, when a problem gives the speed as v km/h and the time as t minutes, teachers can highlight to the students to convert the units where needed. Students are encouraged to search on the internet for more real life examples related to graphs so that they can familiarise themselves with the units used in the real world.

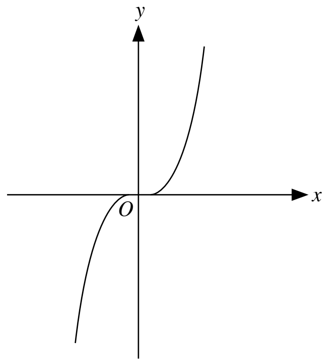
Challenge Yourself

From the acceleration-time graph, students should observe the change in acceleration in three different time periods, i.e. between $t = 1$ and $t = 2$, between $t = 2$ and $t = 4$ and between $t = 4$ and $t = 6$. To deduce how the motion of the object is and sketch a possible speed-time graph for that, students need to know that the gradient of a speed-time graph gives the acceleration.

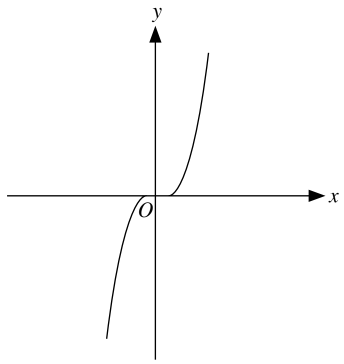
WORKED SOLUTIONS

Investigation (Graphs of Cubic Functions)

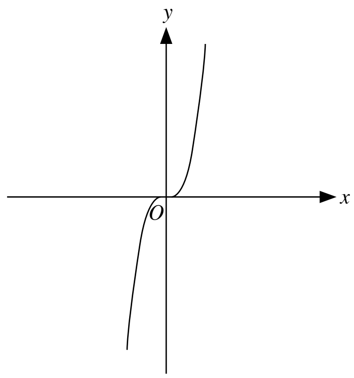
1. **(a)** $y = x^3$



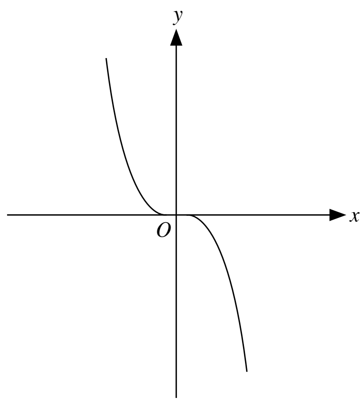
(b) $y = 2x^3$



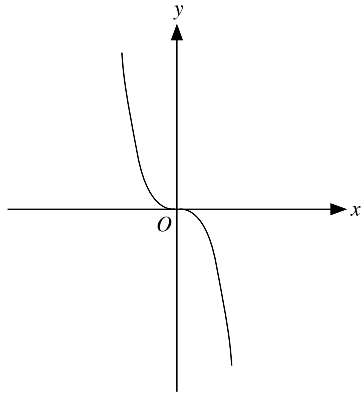
(c) $y = 5x^3$



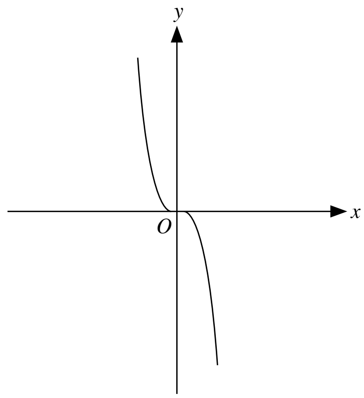
(d) $y = -x^3$



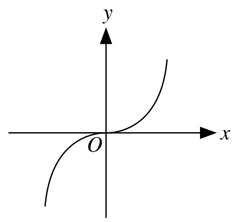
(e) $y = -2x^3$



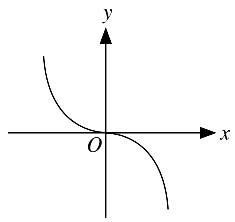
(f) $y = -5x^3$



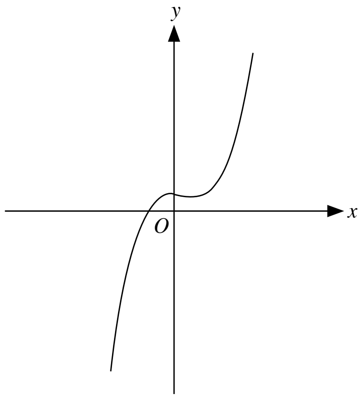
2. Where $a > 0$, the graph takes the shape



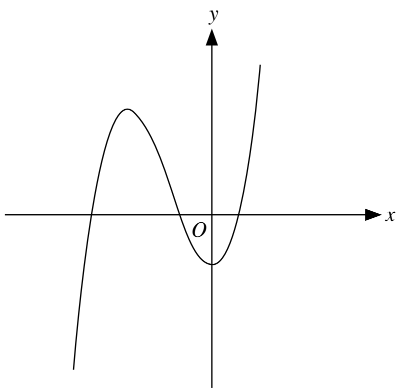
Where $a > 0$, the graph takes the shape



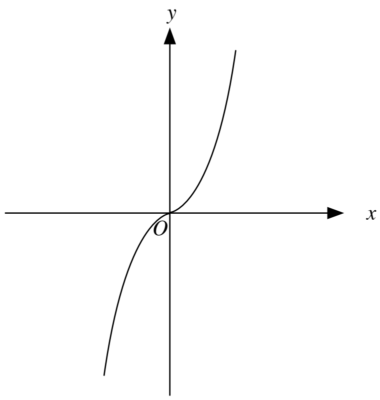
3. **(a)** $y = x^3 - x^2 + 1$



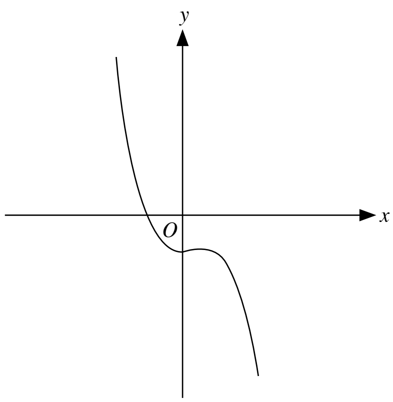
(b) $y = x^3 + 4x^2 - 3$



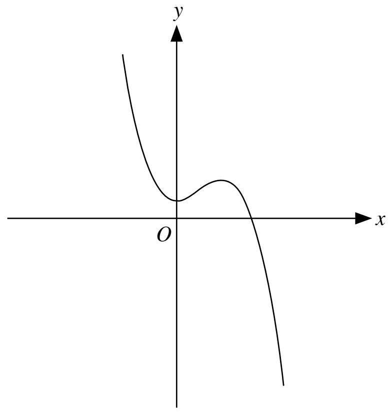
(c) $y = x^3 + x$



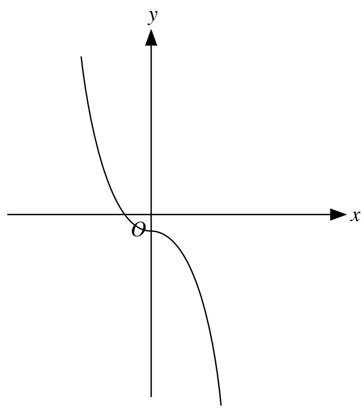
(d) $y = -x^3 + x^2 - 2$



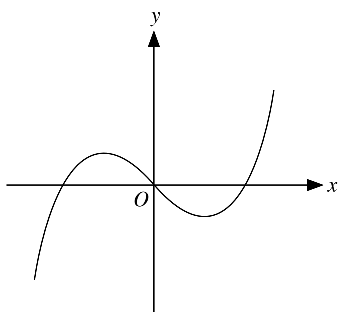
(e) $y = -x^3 + 2x^2 + 1$



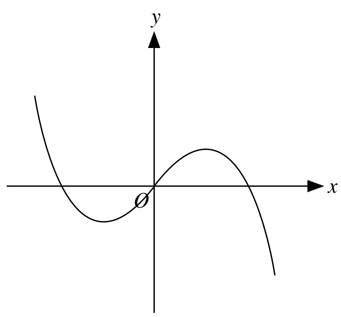
(f) $y = -x^3 - 0.5x - 1$



4. When the coefficient of x^3 is positive, the graph takes the shape

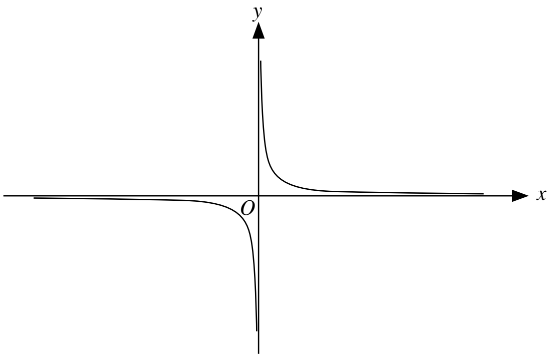


When the coefficient of x^3 is negative, the graph takes the shape

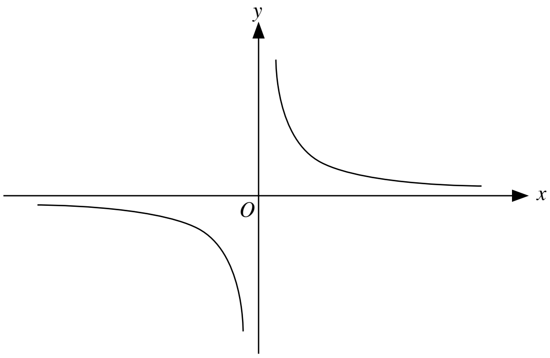


Investigation $\left(\textbf{Graphs of } y = \dfrac{a}{x}\right)$

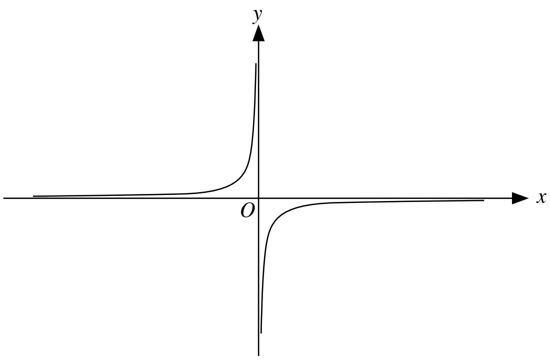
$y = \dfrac{1}{x}$



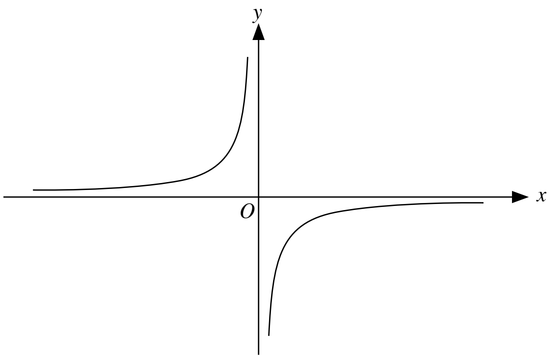
$y = \dfrac{5}{x}$



$y = -\dfrac{1}{x}$



$y = -\dfrac{3}{x}$



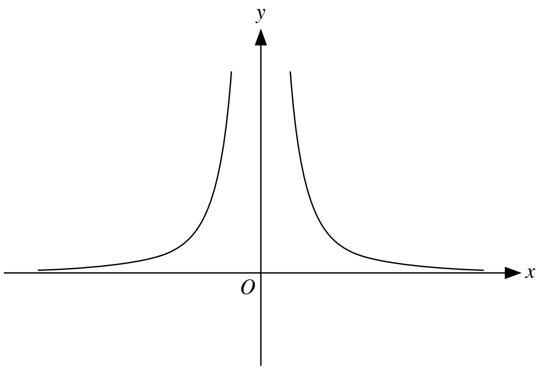
1. **(i)** For $a > 0$, the graph consists of two parts that lie in the 1st and 3rd quadrants.
   (ii) For $a < 0$, the graph consists of two parts that lie in the 2nd and 4th quadrants.
2. There is rotational symmetry of order 2 about the origin, i.e. it maps onto itself twice by rotation in 360°.
3. No. The curves get very close to the x-axis and y-axis but will never touch them.

Thinking Time (Page 177)

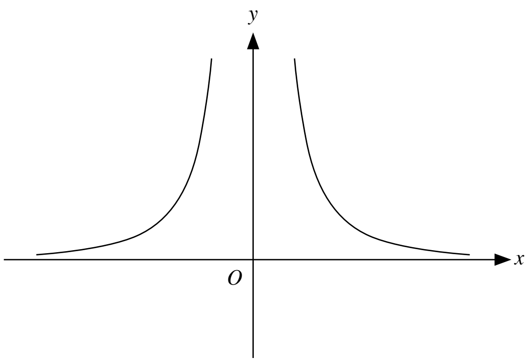
(a) When $a > 0$, the equation of the line of symmetry is $y = x$.
(b) When $a < 0$, the equation of the line of symmetry is $y = -x$.

Investigation $\left(\textbf{Graphs of } y = \dfrac{a}{x^2}\right)$

$y = \dfrac{2}{x^2}$



$y = \dfrac{4}{x^2}$



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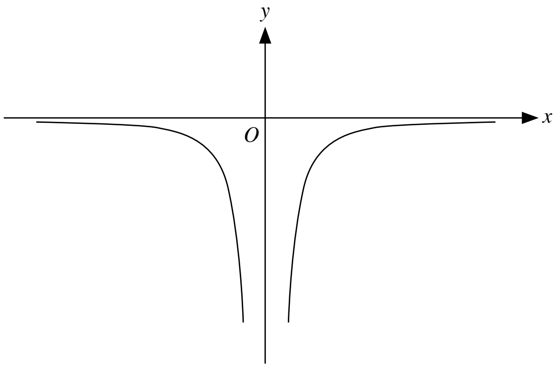
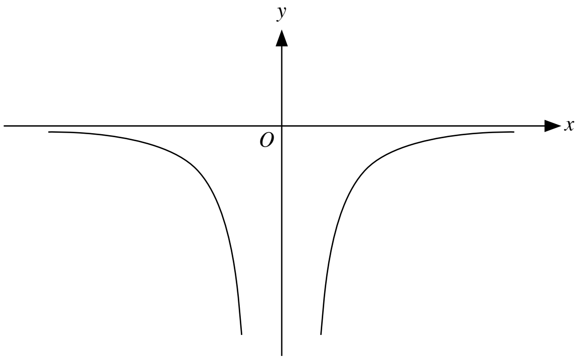
$y = -\dfrac{1}{x^2}$



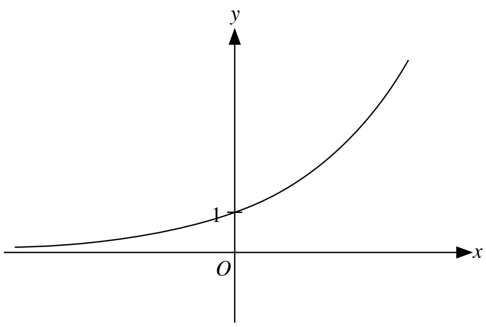
$y = -\dfrac{3}{x^2}$



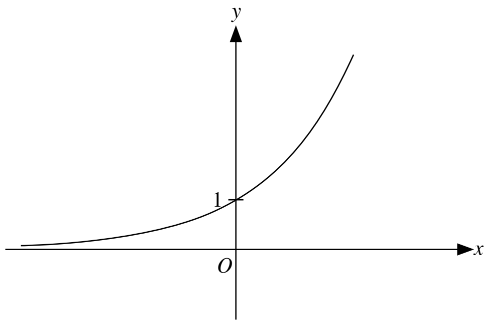
1. **(i)** For $a > 0$, the graph consists of two parts that lie in the 1st and 2nd quadrants.
   (ii) For $a < 0$, the graph consists of two parts that lie in the 3rd and 4th quadrants.
2. The y-axis is the line of symmetry, i.e. the graph is symmetrical about the y-axis.
3. No. The curves get very close to the x-axis and y-axis but will never touch them.

Investigation (Graphs of $y = a^x$ and $y = ka^x$)

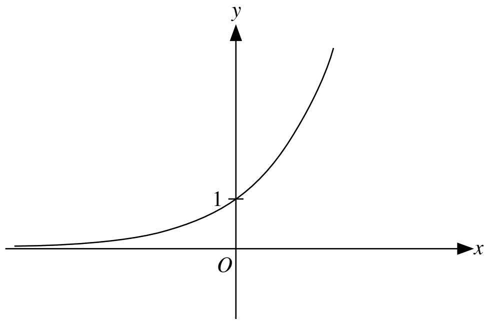
1. **(a)** $y = 2^x$



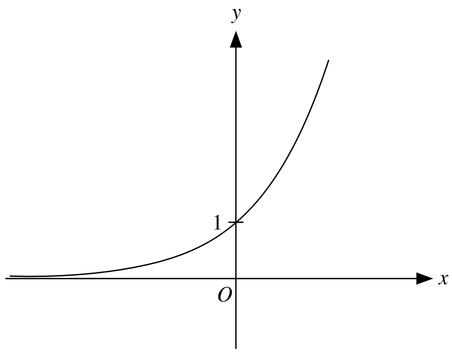
(b) $y = 3^x$



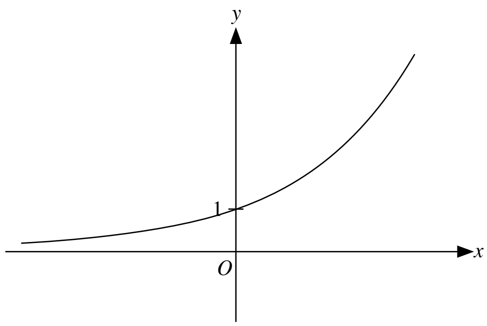
(c) $y = 4^x$



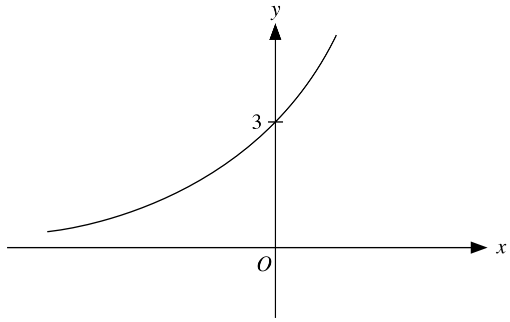
(d) $y = 5^x$



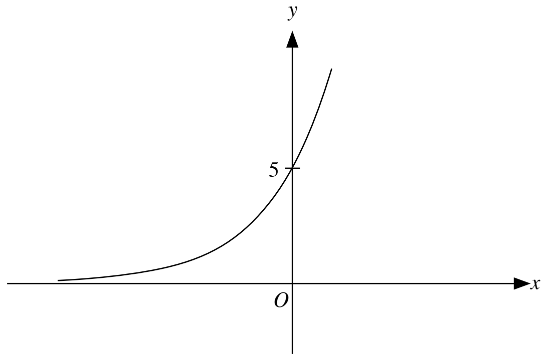
2. **(a)** (0, 1)
   (b) As x increases, y increases very rapidly and approaches infinity.
   (c) No, it does not.
3. As a increases, the steepness of the graph increases.
4. **(a)** $y = 2^x$



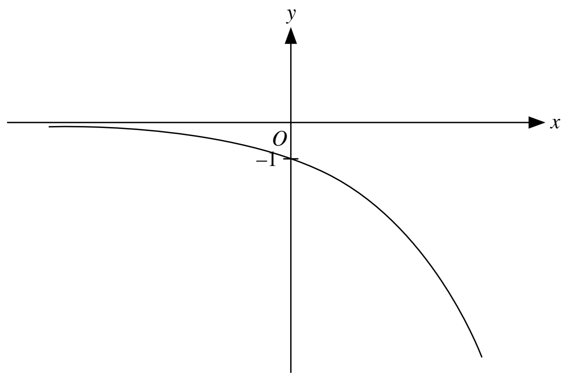
(b) $y = 3(2^x)$



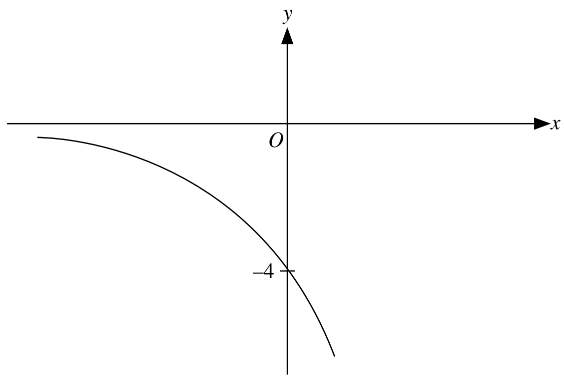
(c) $y = 5(2^x)$



(d) $y = -2^x$



(e) $y = -4(2^x)$



5. (a) The coordinates of the point where the graph intersects the y-axis in **4(a)** are $(0, 1)$.
The coordinates of the point where the graph intersects the y-axis in **4(b)** are $(0, 3)$.
The coordinates of the point where the graph intersects the y-axis in **4(c)** are $(0, 5)$.
The coordinates of the point where the graph intersects the y-axis in **4(d)** are $(0, -1)$.
The coordinates of the point where the graph intersects the y-axis in **4(e)** are $(0, -4)$.

(b) In graphs **4(a)**, **4(b)** and **4(c)**, as x increases and tends to the right of the graph, the value of y increases very rapidly and approaches infinity, while in graphs **4(d)** and **4(e)**, as x increases and tends to the right of the graph, the value of y decreases very rapidly and approaches negative infinity.

(c) No, the graphs do not intersect the x-axis.

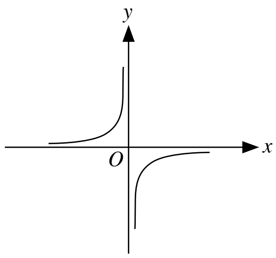
6. If $k > 0$, the values of y are always positive, i.e. the graph lies entirely above the x-axis.
If $k < 0$, the values of y are always negative, i.e. the graph lies entirely below the x-axis.

Journal Writing (Page 184)

(i) As the number of months x increases, the number of members y increases dramatically. The curve tends towards the vertical as x increases.

(ii) More real-life applications of exponential graphs include bacterial decay, population growth, money investment, half-lives of unstable, radioactive atoms.
Teachers should take note that the list is not exhaustive.

Class Discussion (Matching Graphs of Power Functions with the Corresponding Functions)



Graph 1

Graph 1 is a graph of $y = \dfrac{a}{x}$, where $a < 0$.

$\therefore$ The function of Graph 1 is **B**: of $y = -\dfrac{6}{x}$.

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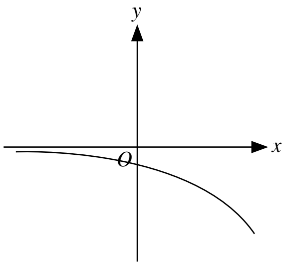
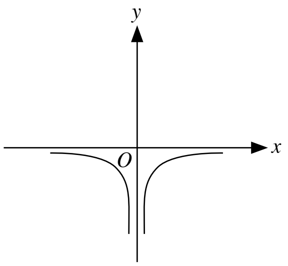


Graph 2

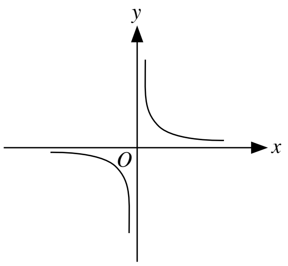
Graph 2 is a graph of $y = ka^x$, where $k < 0$.
$\therefore$ The function of Graph 2 is **F**: of $y = -2(6^x)$.



Graph 3

Graph 3 is a graph of $y = \dfrac{a}{x^2}$, where $a < 0$.

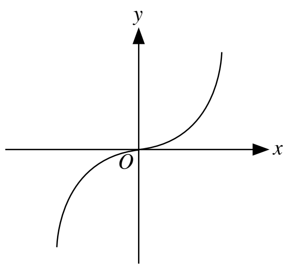
$\therefore$ The function of Graph 3 is **E**: of $y = -\dfrac{3}{x^2}$.



Graph 4

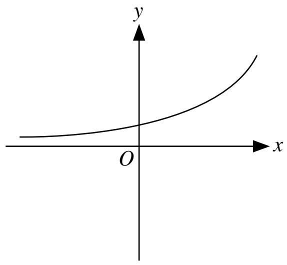
Graph 4 is a graph of $y = \dfrac{a}{x}$, where $a > 0$.

$\therefore$ The function of Graph 4 is **G**: of $y = \dfrac{1}{2x}$.



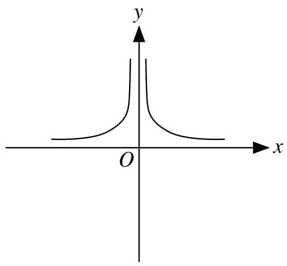
Graph 5

Graph 5 is a graph of $y = ax^3$, where $a > 0$.
$\therefore$ The function of Graph 5 is **A**: of $y = 2x^3$.



Graph 6

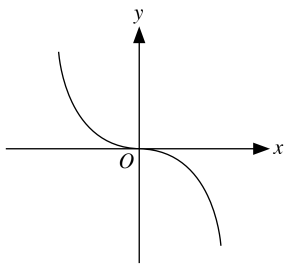
Graph 6 is a graph of $y = ka^x$, where $k > 0$.
$\therefore$ The function of Graph 6 is **D**: of $y = 5^x$.



Graph 7

Graph 7 is a graph of $y = \dfrac{a}{x^2}$, where $a > 0$.

$\therefore$ The function of Graph 7 is **C**: of $y = \dfrac{5}{2x^2}$.



Graph 8

Graph 8 is a graph of $y = ax^3$, where $a < 0$.
$\therefore$ The function of Graph 8 is **H**: of $y = -3x^3$.

Class Discussion (Linear Distance-Time Graphs)

1. From 0900 to 0930, since the graph is a horizontal line, gradient = 0. This gradient represents the speed of the cyclist which is 0 km/h. The cyclist is resting from 0900 to 0930.

2. From 0930 to 1030, gradient = $\dfrac{50 - 20}{1} = 30$ km/h, the cyclist travels at a constant speed of 30 km/h.

3. From 1030 to 1200, gradient = $\dfrac{-50}{1.5} = -33\dfrac{1}{3}$ km/h, the cyclist travels at a constant speed of $33\dfrac{1}{3}$ km/h and returns to his starting point.

4. The average speed is defined as $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$.

$\therefore$ $\dfrac{20 + 0 + 30 + 50}{4}$ is wrong.

Hence the average speed of the cyclist $= \dfrac{50 + 50}{12 - 8} = \dfrac{100}{4} = 25$ km/h.

Thinking Time (Page 194)

A: For an exchange rate between Singapore dollars and Hong Kong dollars where S\$1 = HK\$6, S\$10 can be exchanged for HK\$60. The graph will be a linear graph with the point (10, 60).
∴ Graph 5 matches scenario A.

B: Since the height of water in the water catchment increased at a constant rate from 10 cm to 60 cm, the graph is a linear line that begins from 10 cm and ends at 60 cm.
∴ Graph 2 matches scenario B.

C: When Mr Neo drove at a constant speed of 60 km/h, the speed-time graph will be a horizontal line initially. When he suddenly applied the brakes and came to a stop, his speed decreases until he comes to a stop when his speed reaches 0 km/h.
∴ Graph 6 matches scenario C.

D: The battery level in a smartphone decreased non-uniformly from 60% to 15%. The graph is non-uniform and it will begin at 60 and end at 15.
∴ Graph 4 matches scenario D.

E: The temperature of a substance in a freezer decreased from 60 °C to 15 °C in 20 minutes. The graph will begin at 60 °C and end at 15 °C at the 20-minute mark.
∴ Graph 1 matches scenario E.

F: The plant grew slowly to a height of 15 cm when it was kept indoors for 4 weeks, then grew more quickly to a height of 60 cm when it was placed outdoors for the next 4 weeks. The graph will consist of two lines, of which the second line is steeper since the plant grew more quickly.
∴ Graph 3 matches scenario F.

Thinking Time (Page 197)

1. The graph of Fig. 7.9**(a)** is a straight line because the container has a uniform cross-sectional area.

2.

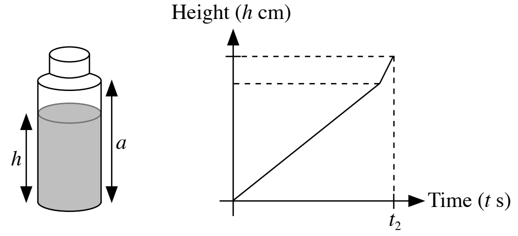


Fig. 7.9(b)

The graph of Fig. 7.9**(b)** consists of two straight lines, of which the second line is steeper because the container is narrower at the top.

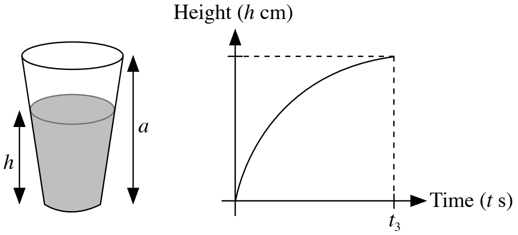
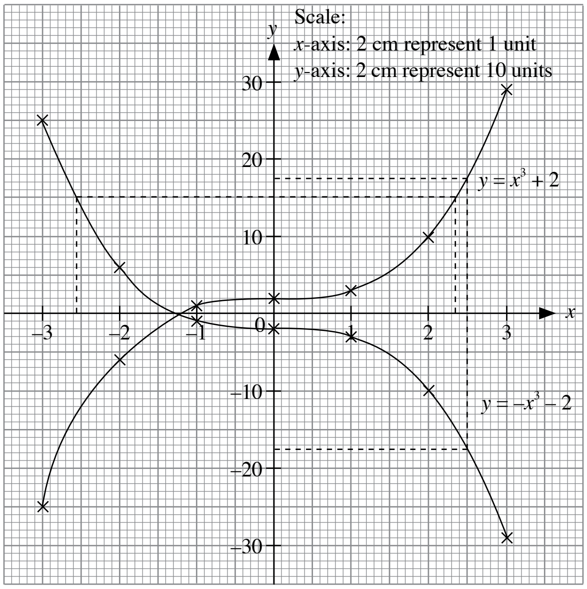


Fig. 7.9(c)

Since the cross-sectional area of the container is non-uniform, the graph of Fig. 7.9**(c)** consists of a curve that becomes less and less steep as the container gets wider at the top.

Practise Now 1

x	–3	–2	–1	0	1	2	3
$y = x^3 + 2$	–25	–6	1	2	3	10	29
$y = -x^3 - 2$	25	6	–1	–2	–3	–10	–29

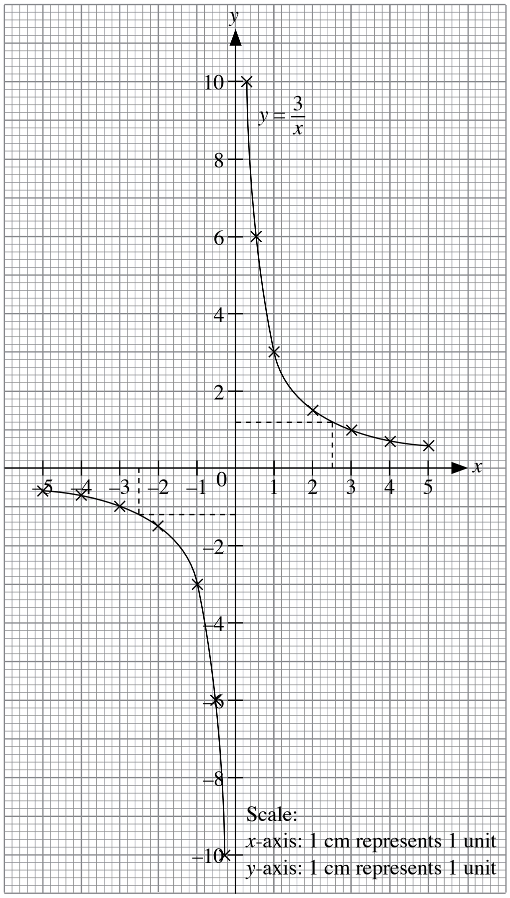


(i) Consider $y = x^3 + 2$.
From the graph, when $x = 2.5$, $y = 17.5$
Consider $y = -x^3 - 2$.
From the graph, when $x = 2.5$, $y = -17.5$

(ii) Consider $y = x^3 + 2$.
From the graph, when $y = 15$, $x = 2.35$
Consider $y = -x^3 - 2$.
From the graph, when $y = 15$, $x = -2.55$

Practise Now 2

x	–5	–4	–3	–2	–1	–0.5	–0.3	0.3	0.5	1	2	3	4	5
y	–0.6	–0.75	1	–1.5	–3	–6	–10	10	6	3	1.5	1	0.75	0.6

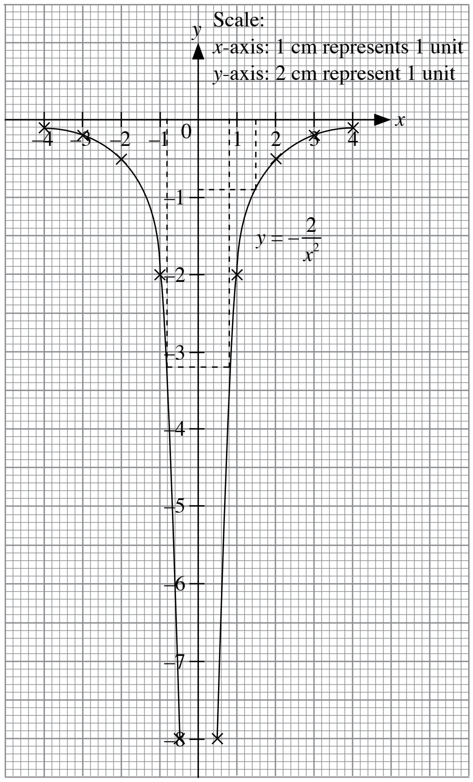


(i) From the graph, when $x = 2.5$, $y = 1.2$.

(ii) From the graph, when $y = –1.2$, $x = –2.5$.

Practise Now 3

x	–4	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–0.13	–0.22	–0.5	–2	–8	–8	–2	–0.5	–0.22	–0.13



(i) From the graph, when $x = 1.5$, $y = –0.9$.

(ii) From the graph, when $y = –3.2$, $x = 0.8$ or $–0.8$.

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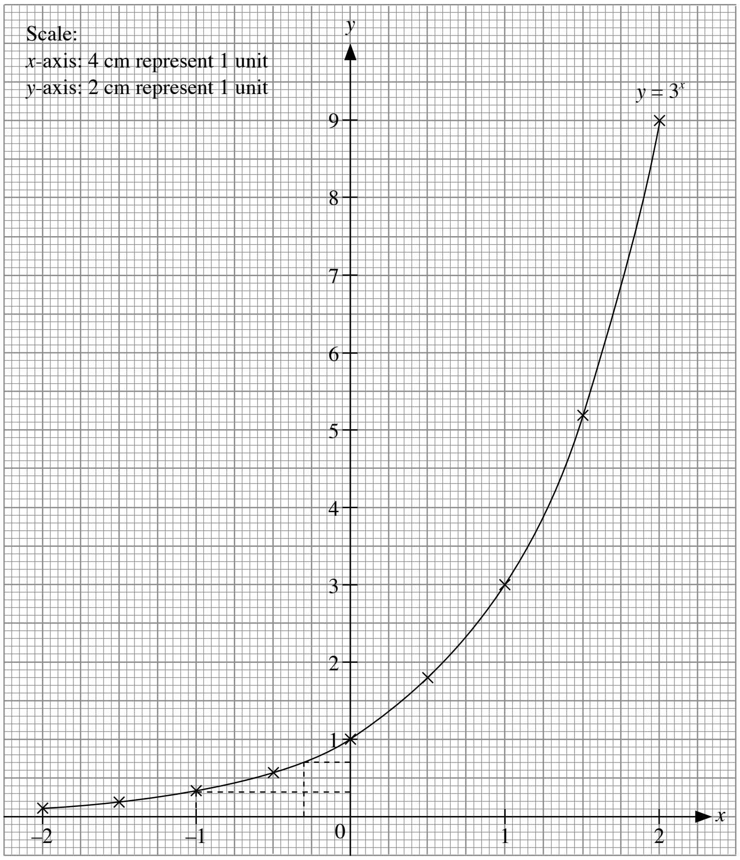
Practise Now 4

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
y	0.11	0.19	0.33	0.58	1.0	1.73	3.0	5.20	9.0



(i) From the graph, when $x = –1$, $y = 0.3$.

(ii) From the graph, when $y = 0.7$, $x = –0.3$.

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Practise Now 5

(a) When $x = -1$,

$y = (-1)^2 - 4(-1) = 5$

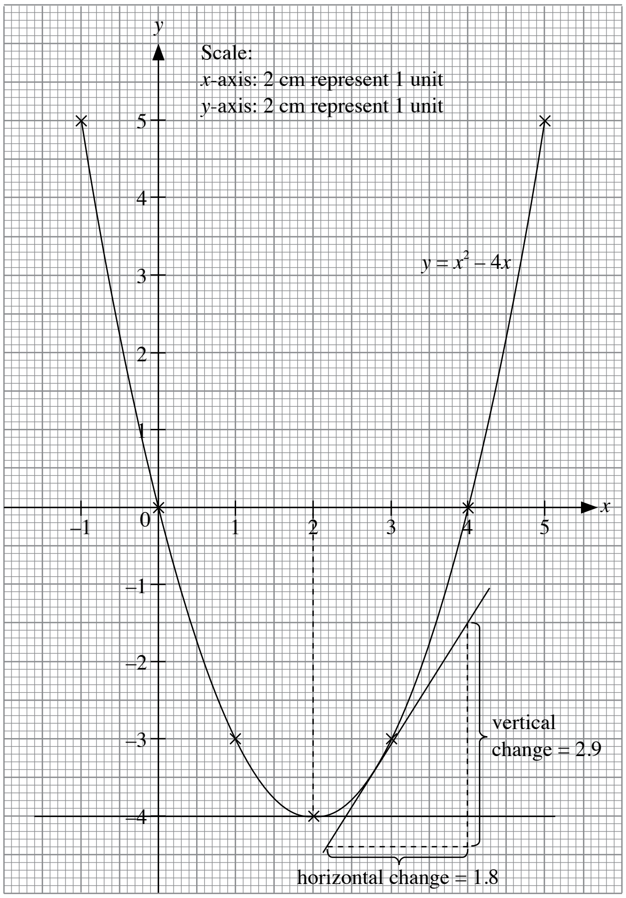
$\therefore a = 5$

When $x = 3$,

$y = 3^2 - 4(3) = -3$

$\therefore b = -3$

(b)



(c) A tangent is drawn to the curve at the point where $x = 2.8$.

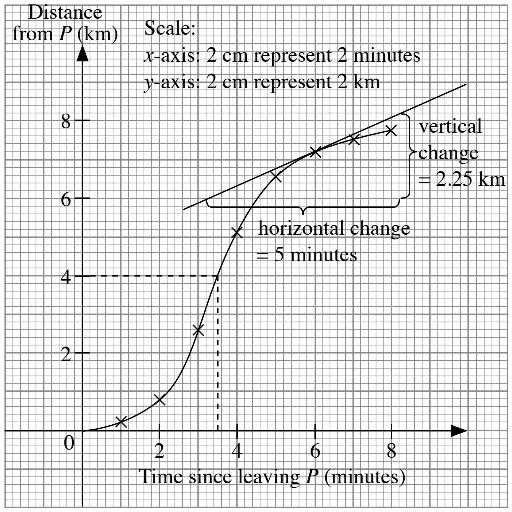
From the graph,

$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{2.9}{1.8} = 1.61 \text{ (to 3 s.f.)}$$

(d) **(ii)** A line parallel to the x-axis at the minimum point of the curve has a gradient equal to zero. From the graph and table, $h = 2$, $k = -4$.

Practise Now 6

(a)



(b) From the graph, the train takes approximately 3.5 minutes to travel the first 4 km.

(c) The gradient of the tangent at the point 6 minutes after it left station P gives the speed at that particular point. It is called the instantaneous speed. A tangent is drawn to the curve at the point 6 minutes after it has left station P.

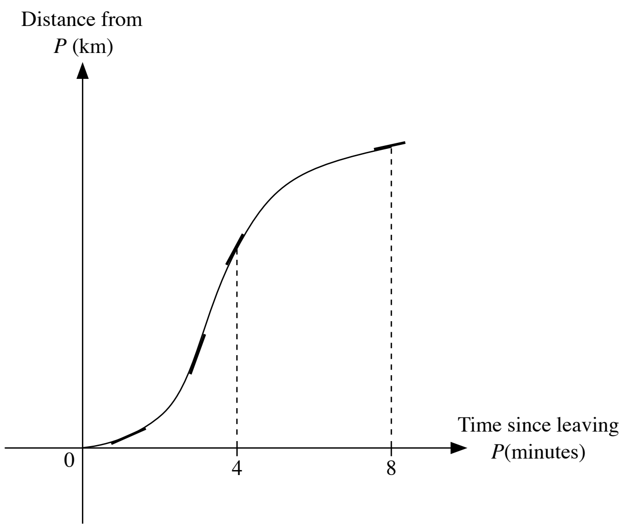
From the graph,

$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{2.25 \text{ km}}{5 \text{ minutes}} = \frac{2.25 \text{ km}}{\frac{5}{60} \text{ h}} = 27 \text{ km/h}$$

$\therefore$ The speed of the train 6 minutes after it has left station P is approximately 27 km/h.

(d) During the first 4 minutes, the speed of the train increases as the gradient of the curve increases.

During the last 4 minutes, the speed of the train decreases as the gradient of the curve decreases.

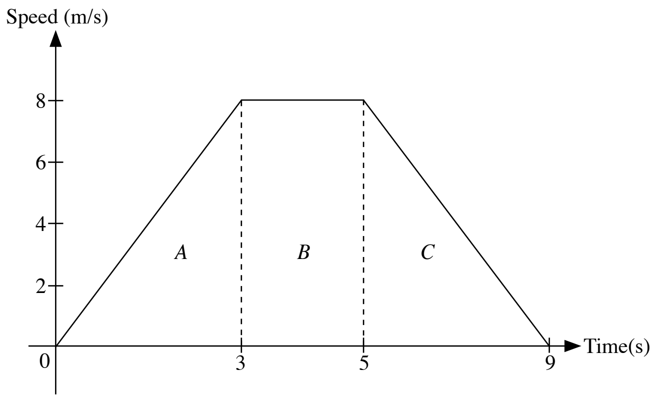


Practise Now 7

(i) Acceleration $= \dfrac{8 \text{ m/s}}{3 \text{ s}}$

$= 2\dfrac{2}{3}$ m/s^2

(ii)



Total distance = area under graph

= Area of $(A + B + C)$

$= \left(\dfrac{1}{2} \times 3 \times 8\right) + (2 + 8) + \left(\dfrac{1}{2} \times 4 \times 8\right)$

$= 12 + 16 + 16$

$= 44$ m

Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{44}{9}$

$= 4.89$ m/s (to 3 s.f.)

(iii) Deceleration in the last 3 seconds = deceleration in the last 4 seconds

$= \dfrac{8 - 0}{9 - 5}$

$= 2$ m/s^2

Practise Now 8

(a) When $t = 2$,

$v = 2(2)^2 - 9(2) + 12 = 2$

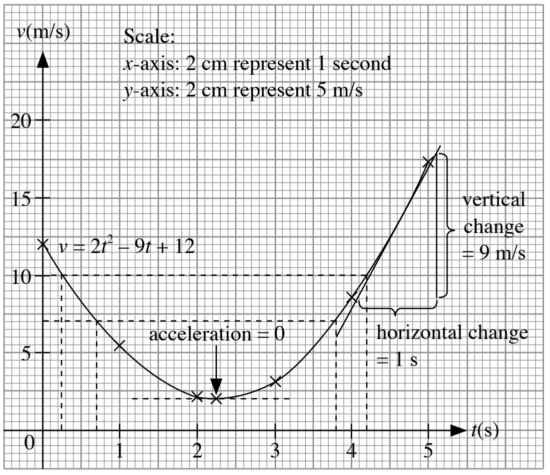
$\therefore a = 2$

When $t = 5$,

$v = 2(5)^2 - 9(5) + 12 = 17$

$\therefore b = 17$

(b)



(c) **(i)** From the graph, when $v = 7$,

$t = 0.7$ or $t = 3.8$.

(ii) The acceleration is zero when the gradient of the curve is zero.

From the graph, the acceleration is zero at $t = 2.25$.

(iii) A tangent is drawn to the curve at the point $t = 4.5$.

From the graph,

Gradient $= \dfrac{\text{vertical change}}{\text{horizontal change}}$

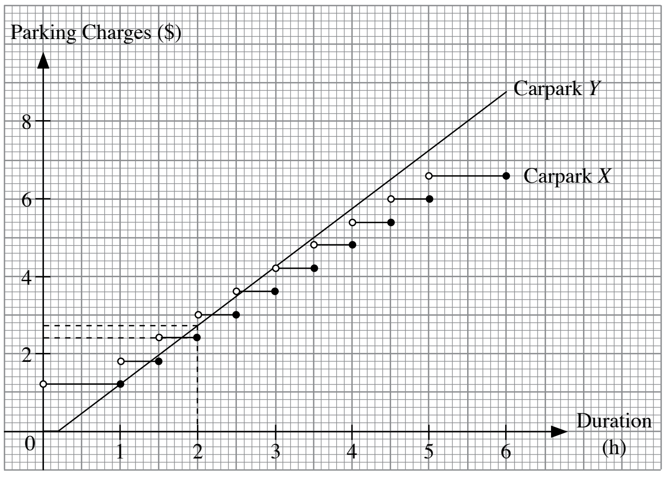
$= \dfrac{9}{1}$

$= 9$

$\therefore$ This acceleration of the particle at $t = 4.5$ is 9 m/s^2.

(iv) From the graph, when $v < 10$, $0.25 < t < 4.2$.

Practise Now 9



Carpark Y:

For the first 12 minutes, the parking charge is \$0.

When the duration is 3 h, the parking charge is

$0.025 \times (3 \times 60 - 12) = \$4.20$

When the duration is 6 h, the parking charge is

$0.025 \times (6 \times 60 - 12) = \$8.70$

From the graph, when the duration is 2 h,

Carpark X charges \$2.40 while Carpark Y charges \$2.70.

$\therefore$ Mr Wong should park in Carpark X.

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Practise Now 10

(a) From the graph, the resting heart rate is 60 beats/minute.

(b) To find the rate of increase in his heart rate as he brisk walks for the first time, we need to calculate the gradient of the line from the 10^{th} minute to the 20^{th} minute.

$$\text{Gradient} = \frac{120 - 60}{20 - 10}$$
$$= 6 \text{ beats/minute}^2$$

$\therefore$ The rate of increase in his heart rate as he brisk walks for the first time is 6 beats/minute^2.

(c) To find the rate of decrease in his heart rate as he slows down in the last 20 minutes, we need to calculate the gradient of the line from the 40^{th} minute to the 60^{th} minute.

$$\text{Gradient} = \frac{140 - 120}{60 - 40}$$
$$= 1 \text{ beat/minute}^2$$

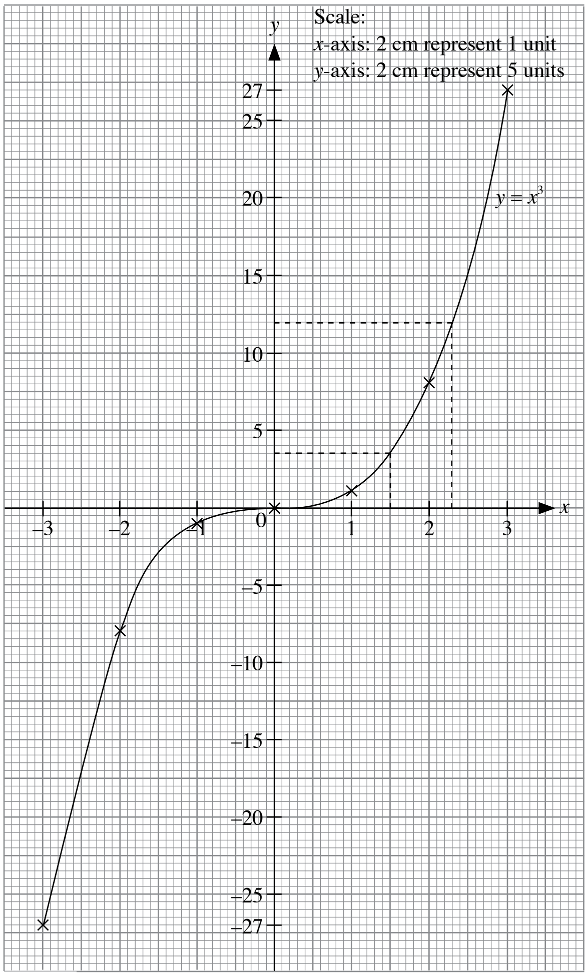
$\therefore$ The rate of decrease in his heart rate as he slows down in the last 20 minutes is 1 beat/minute^2.

Exercise 7A

1. **(a)** $y = x^3$

x	–3	–2	–1	0	1	2	3
y	–27	–8	–1	0	1	8	27

(b)



(c) **(i)** From the graph, when $x = 1.5$, $y = 3.5$.

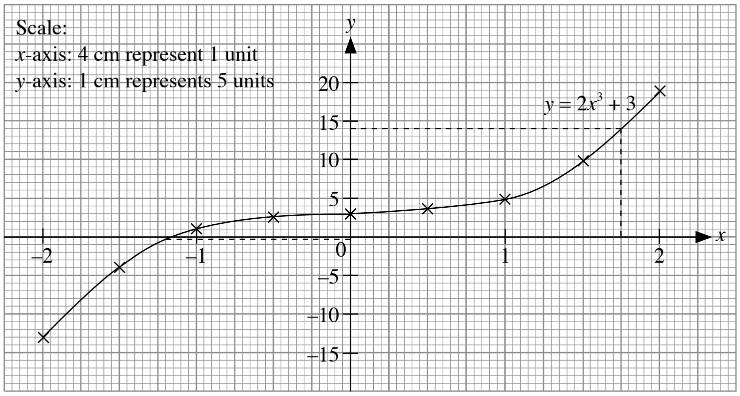
(ii) From the graph, when $y = 12$, $x = 2.3$.

2. **(a)** When $x = –0.5$,

$y = 2(–0.5)^2 + 0.5 = 2.75$

$\therefore p = 2.75$

(b)



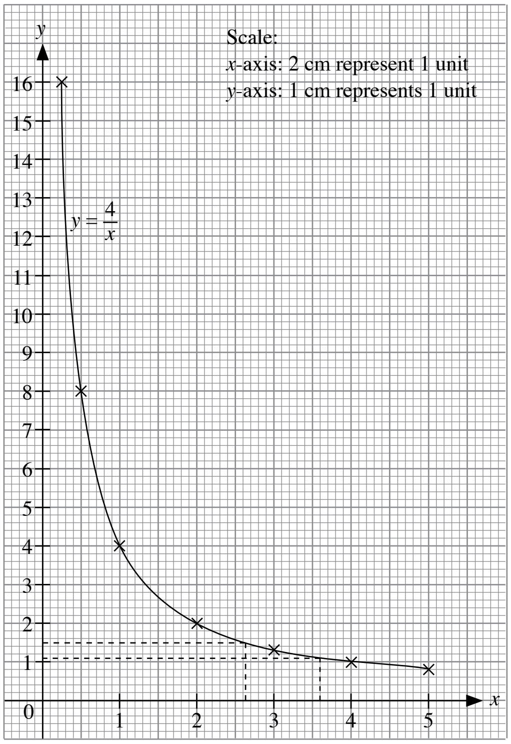
(c) **(i)** From the graph, when $x = –1.2$, $y = –0.5$.

(ii) From the graph, when $y = 14$, $x = 1.75$.

3. **(a)** $y = \dfrac{4}{x}$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	5
y	16	8	4	2	1.3	1	0.8

(b)



(c) **(i)** From the graph, when $x = 3.6$, $y = 1.1$.

(ii) From the graph, when $y = 1.5$, $x = 2.65$.

4. **(a)** When $x = 3$,

$y = \dfrac{10}{3^2} = 1.1$ (to 1 d.p.)

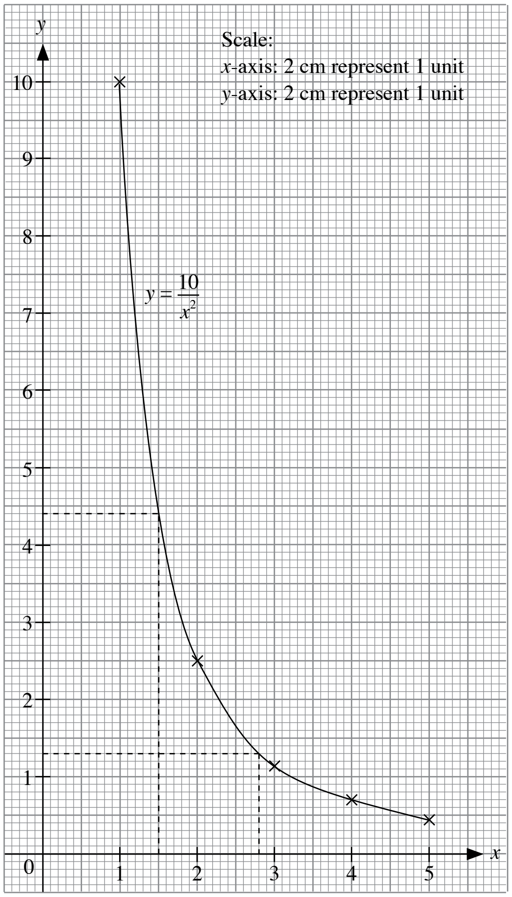
$\therefore a = 1.1$

When $x = 5$,

$y = \dfrac{10}{5^2} = 0.4$

$\therefore b = 0.4$

(b)

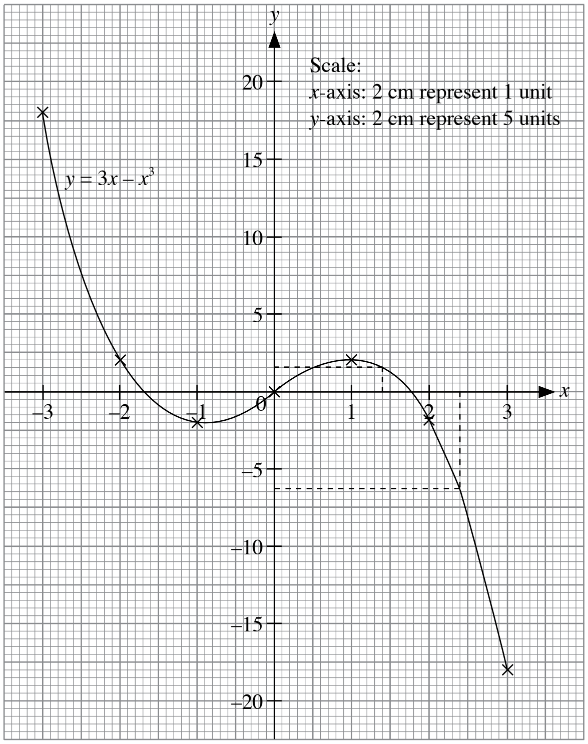


(c) **(i)** From the graph, when $x = 2.8$, $y = 1.3$.

(ii) From the graph, when $y = 4.4$, $x = 1.5$.

5. $y = 3x - x^3$

x	–3	–2	–1	0	1	2	3
y	18	2	–2	0	2	–2	–18



(i) From the graph, when $x = 1.4$, $y = 1.5$.

(ii) From the graph, when $y = -6.6$, $x = 2.4$.

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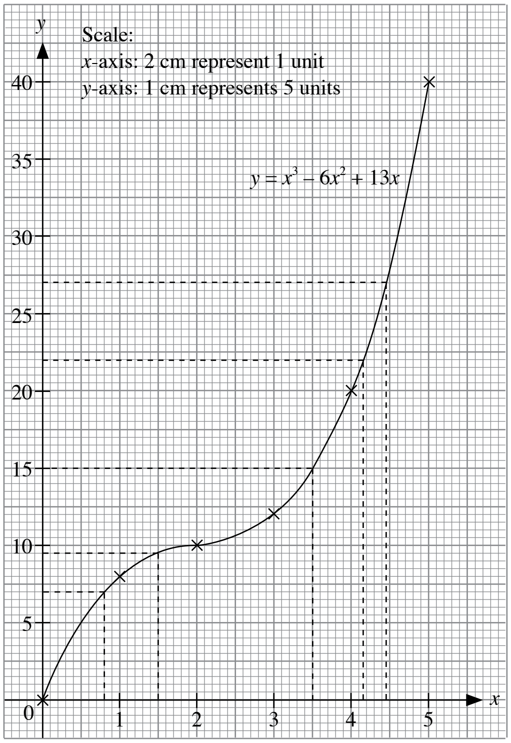
6. $y = x^3 - 6x^2 + 13x$

x	0	1	2	3	4	5
y	0	8	10	12	20	40



(a) **(i)** From the graph, when $x = 1.5$, $y = 9.5$.

(ii) From the graph, when $x = 3.5$, $y = 15$.

(iii) From the graph, when $x = 4.45$, $y = 27$.

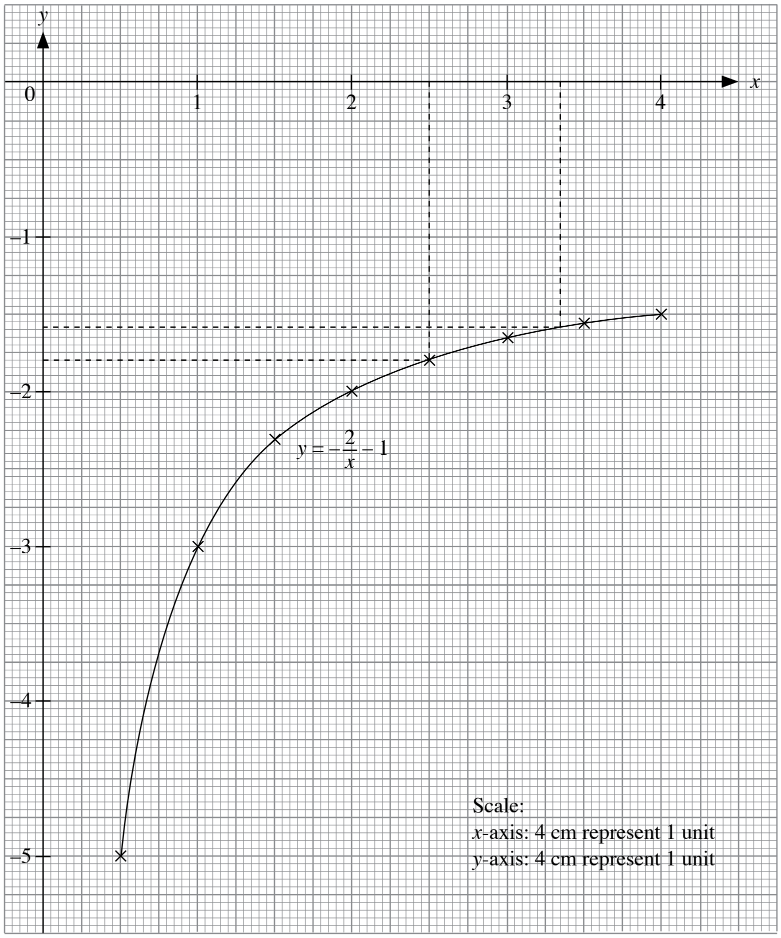
(b) **(i)** From the graph, when $y = 7$, $x = 0.8$.

(ii) From the graph, when $y = 15$, $x = 3.5$.

(iii) From the graph, when $y = 22$, $x = 4.15$.

7. $y = -\frac{2}{x} - 1$

x	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
y	–5	–3	–2.33	–2	–1.8	–1.67	–1.57	–1.5



(i) From the graph, when $x = 2.5$, $y = -1.8$.

(ii) From the graph, when $y = -1.6$, $x = 3.35$.

8. **(a)** When $x = 3$,

$y = 3 - \frac{3}{3} = 2$

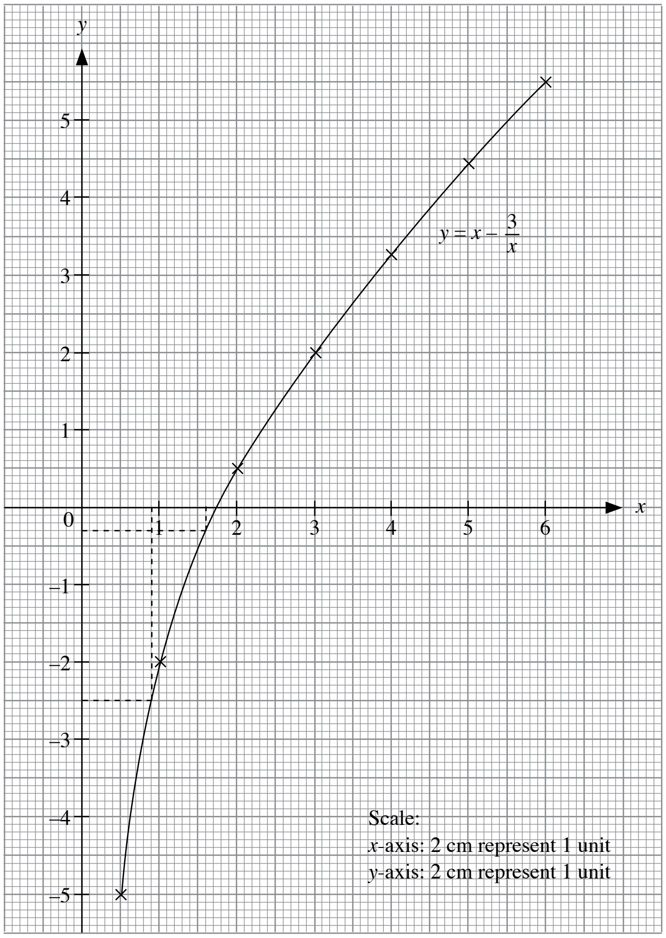
$\therefore h = 2$

When $x = 3$,

$y = 6 - \frac{3}{6} = 5.5$

$\therefore k = 5.5$

(b)

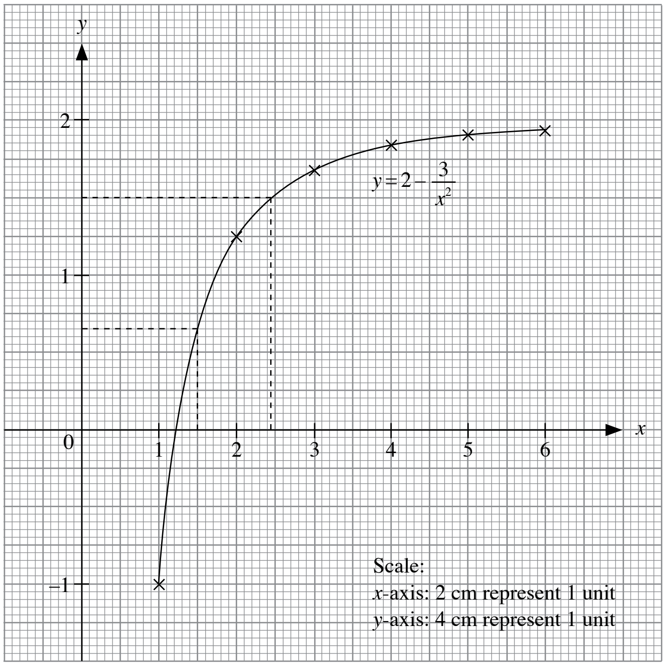


(c) **(i)** From the graph, when $x = 1.6$, $y = -0.3$.

(ii) From the graph, when $y = -2.5$, $x = 0.9$.

9. $y = 2 - \frac{3}{x^2}$

x	1	2	3	4	5	6
y	–1	1.25	1.67	1.81	1.88	1.92

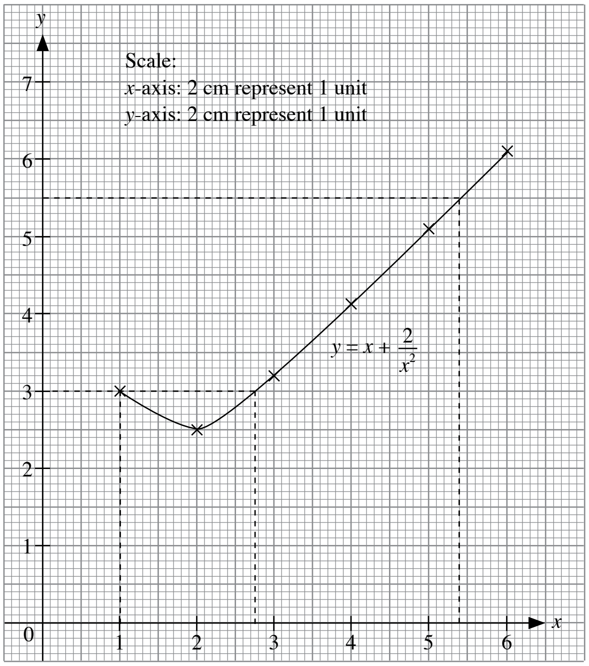


(i) From the graph, when $x = 1.5$, $y = 0.65$.

(ii) From the graph, when $y = 1.5$, $x = 2.45$.

10. $y = x + \dfrac{2}{x^2}$

x	1	2	3	4	5	6
y	3	2.5	3.2	4.1	5.1	6.1

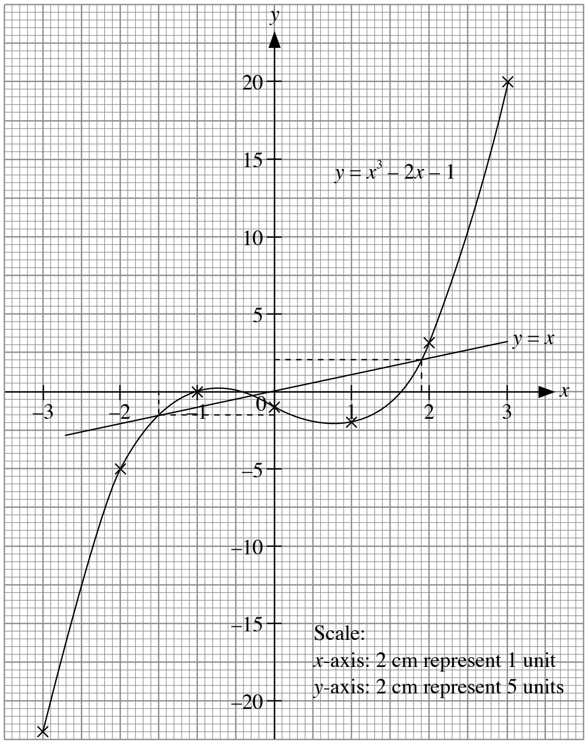


(i) From the graph, when $x = 5.4$, $y = 5.5$.

(ii) From the graph, when $y = 3$, $x = 1$ or 2.75.

11. $y = x^3 - 2x - 1$

x	–3	–2	–1	0	1	2	3
y	–22	–5	0	–1	–2	3	20



(a) From the graph, the x-coordinates of the points of intersection of the curve with the x-axis are -1, -0.6 and 1.6.

(b) (i) $y = x$

x	–3	–2	–1	0	1	2	3
y	–3	–2	–1	0	1	2	3

From the graph, the x-coordinates of the points at which the line meets the curve are -1.5, -0.35, and 1.9.

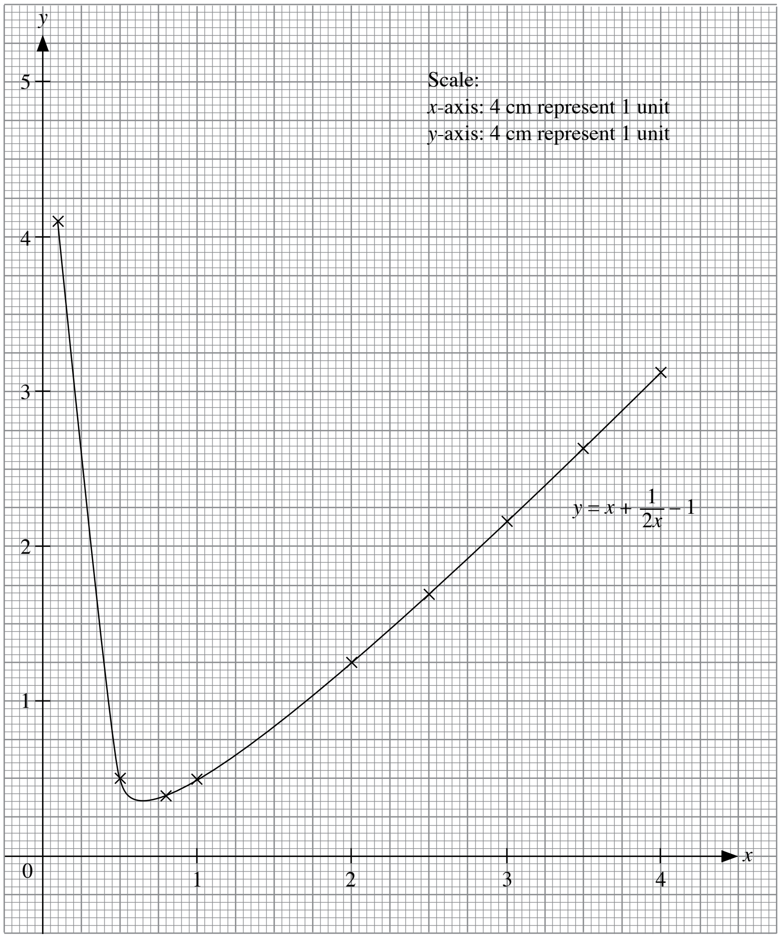
(ii) Hence, the solutions of the equation are -1.5, -0.35 and 1.9.

12. **(a)** When $x = 2.5$,

$$y = 2.5 + \frac{1}{2(2.5)} - 1 = 1.7$$

$\therefore p = 1.7$

(b)



(c)

$$x + \frac{1}{2x} = 1$$

$$x + \frac{1}{2x} - 1 = 0$$

$$y = 0$$

From the graph, in the range $0.1 \leqslant x \leqslant 4$, there are no solutions.

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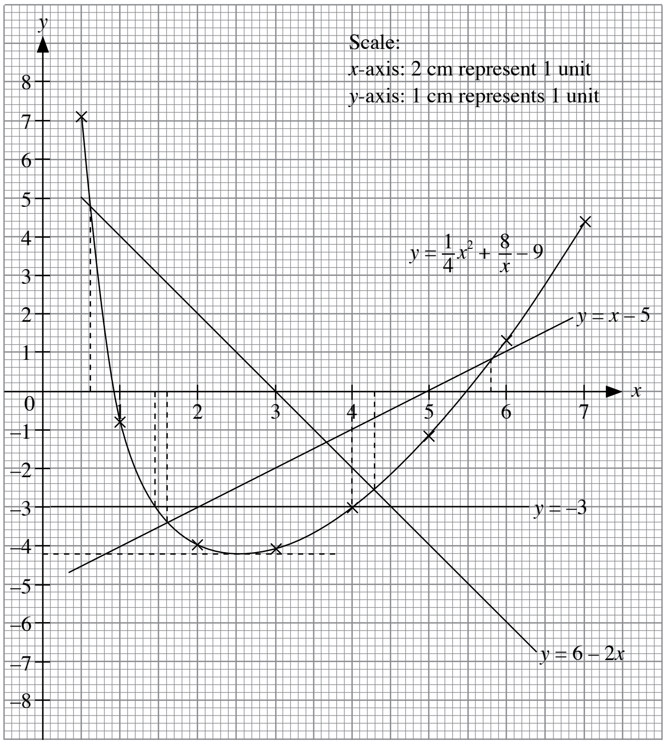
13. $y = \frac{1}{4}x^2 + \frac{8}{x} - 9$

x	0.5	1	2	3	4	5	6	7
y	7.1	–0.8	–4	–4.1	–3	–1.2	–1.3	4.4



(a) From the graph,
The minimum value of y in the given range $0.5 \leqslant x \leqslant 7$ is –4.2.

(b) (i)

$$\frac{1}{4}x^2 + \frac{8}{x} = 6$$
$$\frac{1}{4}x^2 + \frac{8}{x} - 9 = 6 - 9$$
$$\frac{1}{4}x^2 + \frac{8}{x} - 9 = -3$$
$$y = -3$$

A line $y = -3$ is drawn.
From the graph, the solution is $x = 1.5$ (to 1 d.p.) or 4.

(ii)

$$\frac{1}{4}x^2 + \frac{8}{x} = x + 4$$
$$\frac{1}{4}x^2 + \frac{8}{x} - 9 = x + 4 - 9$$
$$\frac{1}{4}x^2 + \frac{8}{x} - 9 = x - 5$$
$$y = x - 5$$

x	0.5	1	2	3	4	5	6	7
y	–4.5	–4	–3	–2	–1	0	1	2

A line $y = x - 5$ is drawn.
From the graph, the solution is $x = 1.6$ or 5.8.

(iii)

$$\frac{1}{4}x^2 + 2x = 15 - \frac{8}{x}$$
$$\frac{1}{4}x^2 + 2x + \frac{8}{x} = 15 - \frac{8}{x} + \frac{8}{x}$$
$$\frac{1}{4}x^2 + 2x + \frac{8}{x} = 15$$
$$\frac{1}{4}x^2 + 2x + \frac{8}{x} - 2x = 15 - 2x$$
$$\frac{1}{4}x^2 + \frac{8}{x} = 15 - 2x$$
$$\frac{1}{4}x^2 + \frac{8}{x} - 9 = 15 - 2x - 9$$
$$\frac{1}{4}x^2 + \frac{8}{x} - 9 = 6 - 2x$$
$$y = 6 - 2x$$

x	0	1	2	3	4	5
y	0	8	10	12	20	40

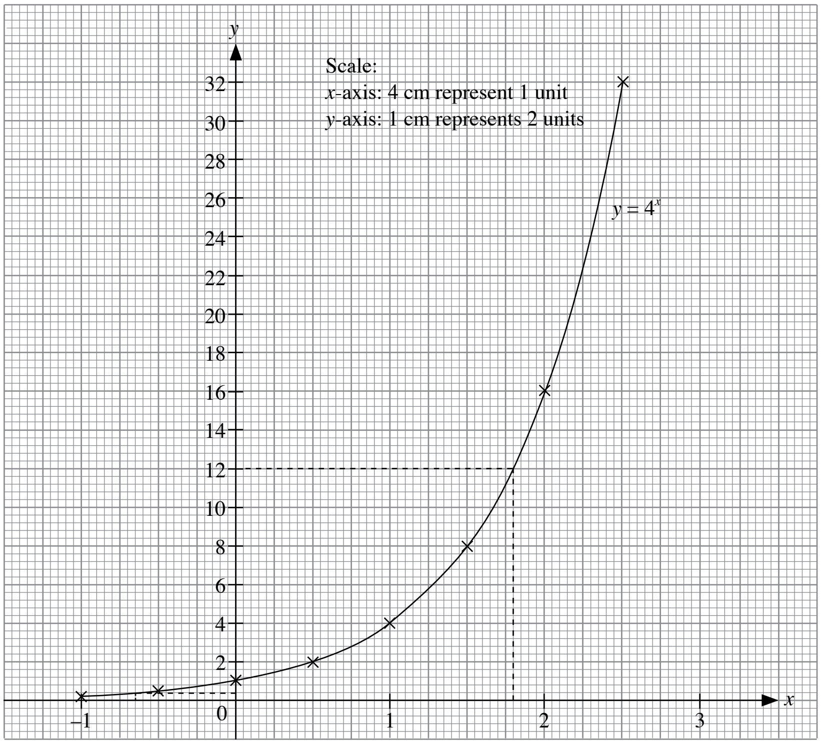
A line $y = 6 - 2x$ is drawn.
From the graph, the solution is $x = 0.6$ or 4.3.

Exercise 7B

1. **(a)** $y = 4^x$

x	–1	–0.5	0	0.5	1	1.5	2	2.5
y	0.25	0.5	1	2	4	8	16	32

(b)



(c) **(i)** From the graph, when $x = 1.8$, $y = 12$.

(ii) From the graph, when $y = 0.4$, $x = –0.65$.

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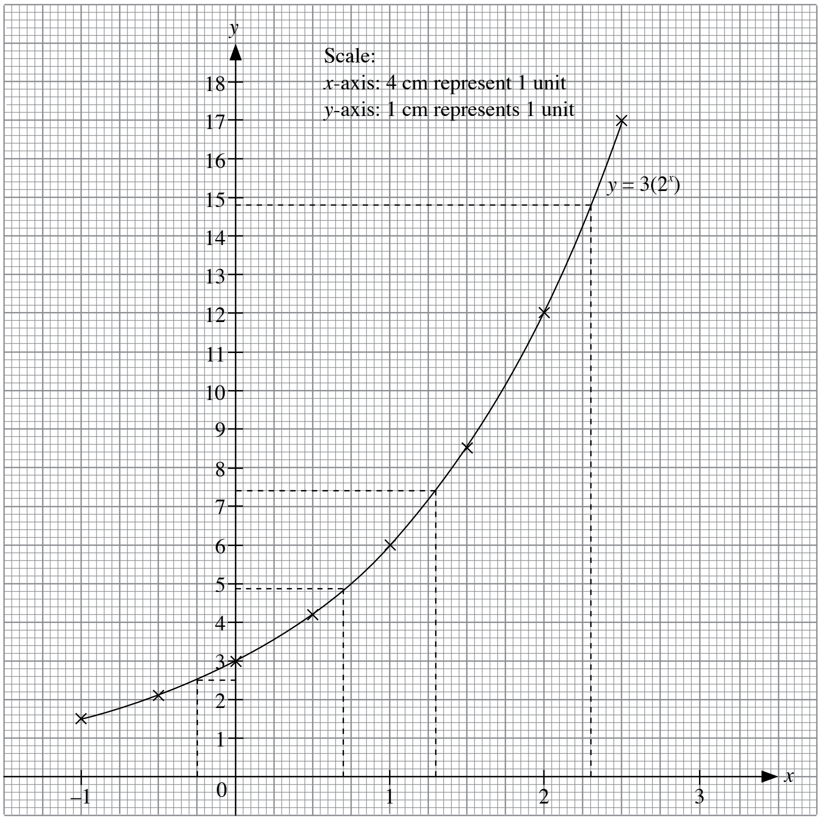
2. **(a)** $y = 3(2^x)$

x	–1	–0.5	0	0.5	1	1.5	2	2.5
y	1.5	2.1	3	4.2	6	8.5	12	17.0

(b)



(c) **(i)** From the graph,

When $x = 0.7$, $y = 4.9$.

When $x = 2.3$, $y = 14.8$.

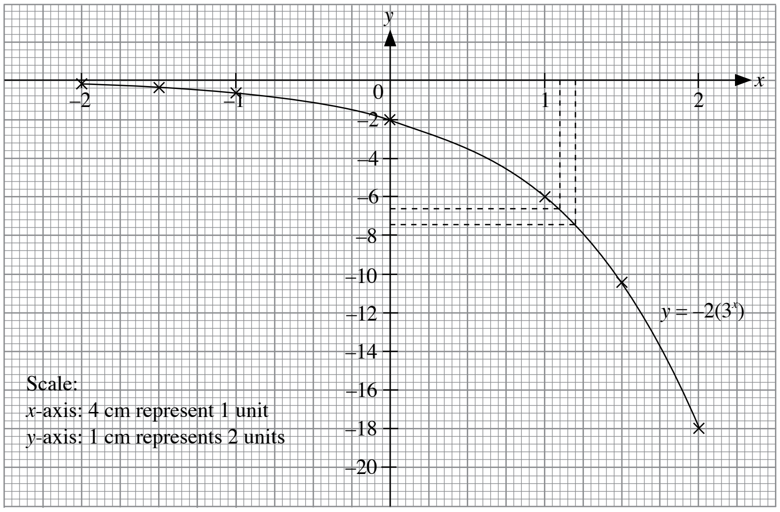
(ii) From the graph,

When $y = 2.5$, $x = –0.25$.

When $y = 7.4$, $x = 1.30$.

3. $y = -2(3^x)$

x	–2	–1.5	–1	0	1	1.5	2
y	–0.2	–0.4	–0.7	–2	–6	–10.4	–18



(i) From the graph, when $x = 1.2$, $y = -7.4$,

(ii) From the graph, when $y = -6.7$, $x = 1.1$.

4. **(a)** When $x = -1$,

$y = 2 + 2^{-1} = 2.5$

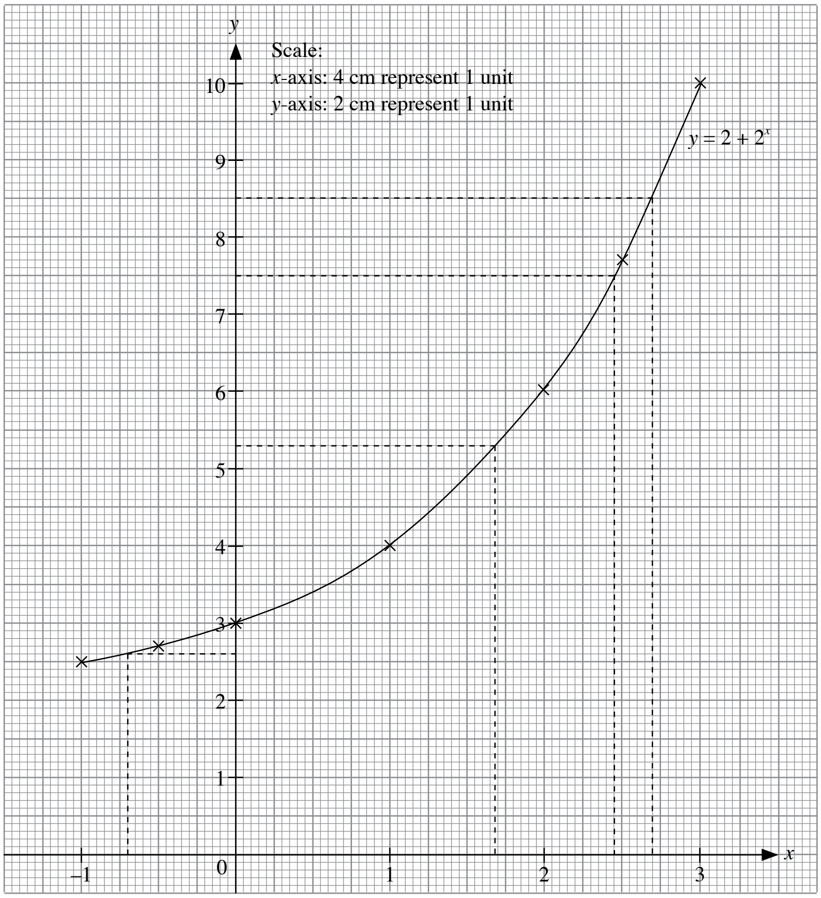
$\therefore a = 2.5$

When $x = 2.5$,

$y = 2 + 2^{2.5} = 7.7$ (to 1 d.p.)

$\therefore b = 7.7$

(b)



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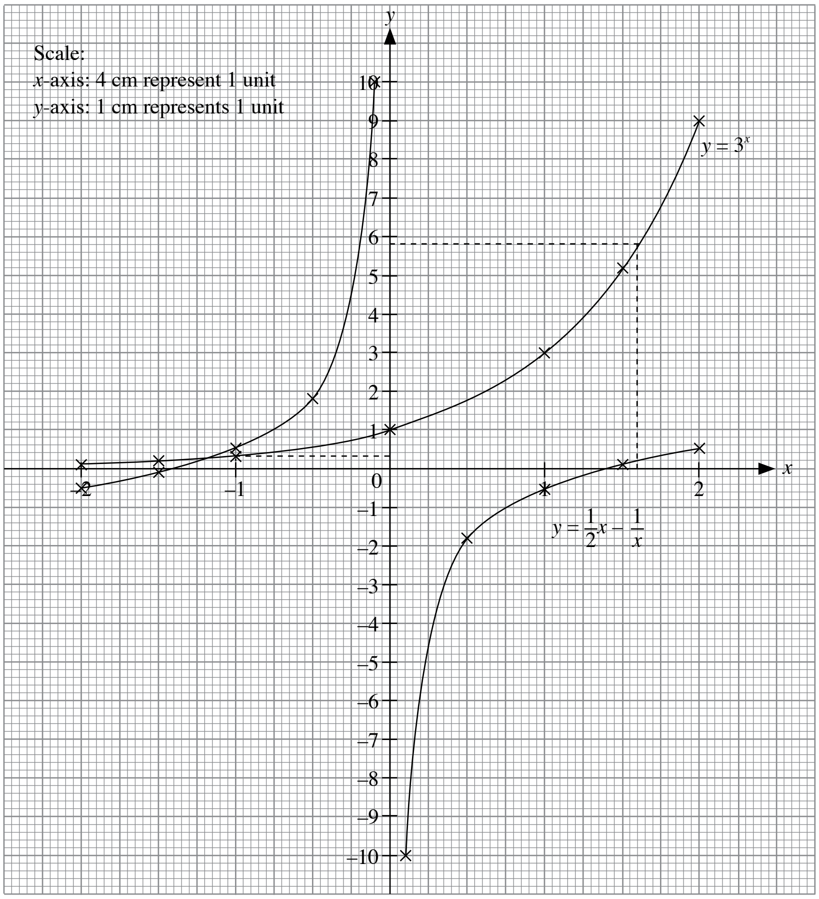
(c) **(i)** From the graph,
When $x = -0.7$, $y = 2.6$,
When $x = 2.7$, $y = 8.5$.

(ii) From the graph,
When $y = 5.3$, $x = 1.725$,
When $y = 7.5$, $x = 2.45$.

5. $y = 3^x$

x	–2	–1.5	–1	0	1	1.5	2
y	0.1	0.2	0.3	1	3	5.2	9



(a) From the graph, when $y = 5.8$, $x = 1.6$,

(b) $y = \frac{1}{2}x - \frac{1}{x}$

x	–2	–1.5	–1	–0.5	–0.1	0.1	0.5	1	1.5	2
y	–0.5	–0.08	0.5	1.75	9.95	–9.95	–1.75	–0.5	0.08	0.5

(i) From the graph, the coordinates of the point are (–1.15, 0.3).

(ii) $3^x + \frac{1}{x} - \frac{1}{2}x = 0$

$3^x = \frac{1}{2}x - \frac{1}{x}$

Hence, the solution of the equation is –1.15.

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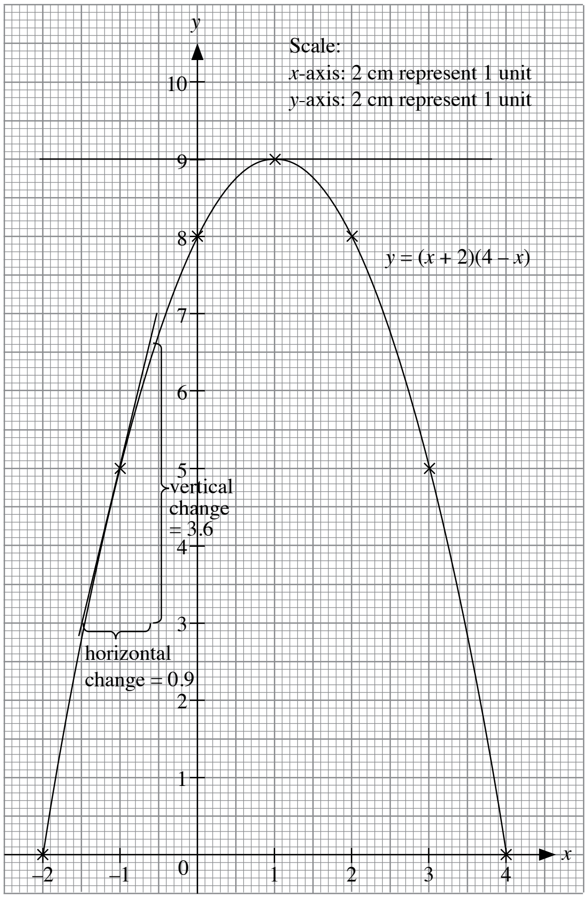
6. $y = ka^x$
$y = 3a^x$
$k = 3$

7. **(a)** $y = (x + 2)(4 - x)$

x	–2	–1	0	1	2	3	4
y	0	5	8	9	8	5	0

(b)



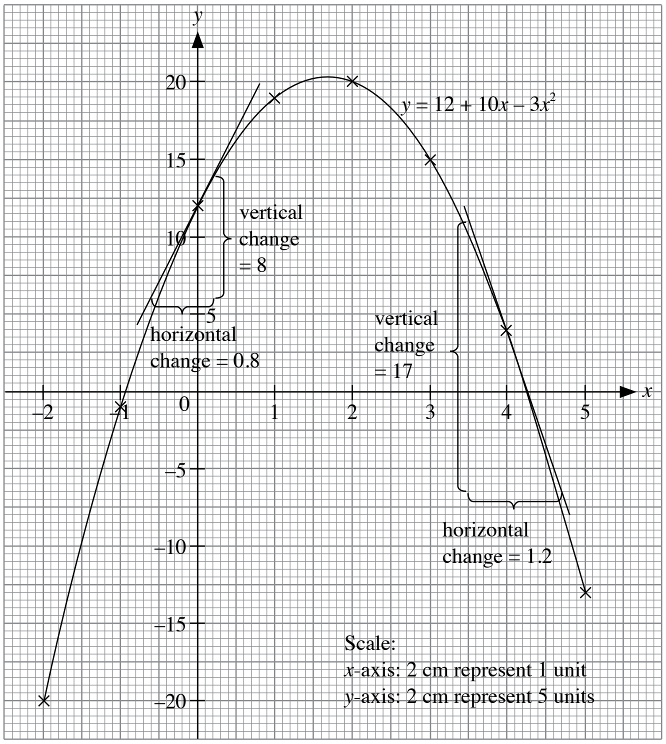
(c) A tangent is drawn to the curve at the point where $x = -1$.
From the graph,

Gradient = $\dfrac{3.6}{0.9} = 4$

(d) **(i)** A line parallel to the x-axis the maximum point of the curve has a gradient equal to zero.
(ii) From the graph and table, $h = 1$, $k = 9$.

8. **(a)** $y = 12 + 10x - 3x^2$

x	–2	–1	0	1	2	3	4	5
y	–20	–1	12	19	20	15	4	–13



(b) A tangent is drawn to the curven at the point when $x = 4$.
From the graph,

Gradient = $\dfrac{-17}{1.2} = -14$ (to 2 s.f.)

(c) A tangent is drawn to the curve at the point where the curve intersects the y-axis.
From the graph,

Gradient = $\dfrac{8}{0.8} = 10$

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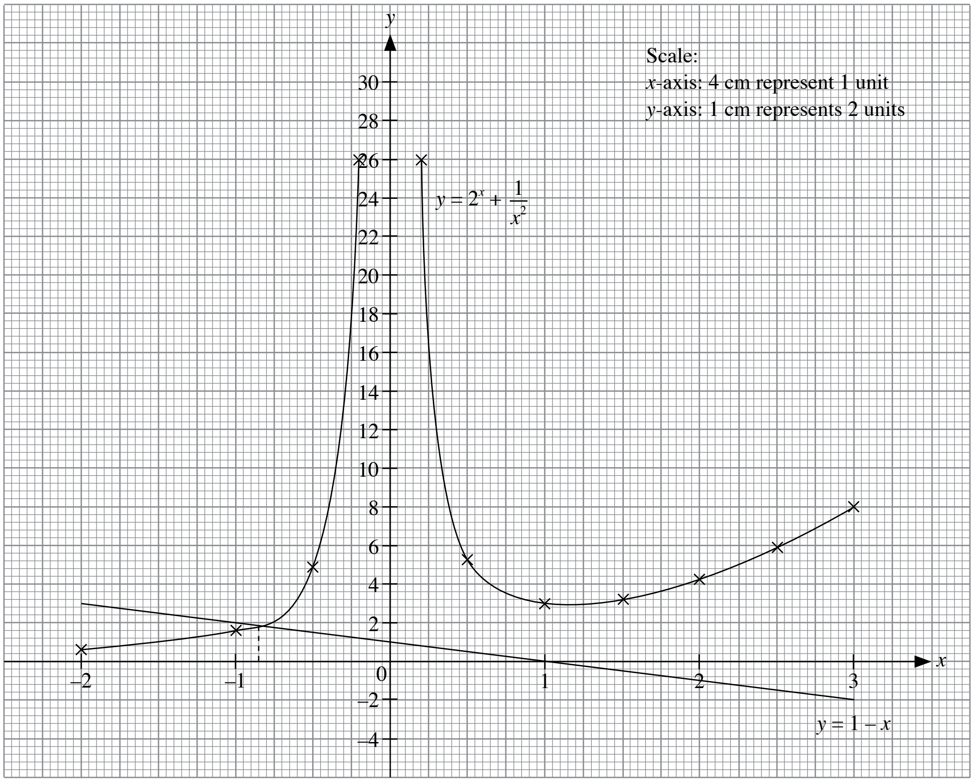
9. $y = 2^x + \dfrac{1}{x^2}$

x	–2	–1	–0.5	–0.2	0.2	0.5	1	1.5	2	2.5	3
y	0.5	1.5	4.7	25.9	26.1	5.4	3	3.3	4.3	5.8	8.1



(a) **(i)** $y = 1 - x$

x	–2	0	2	3
y	3	1	–1	–2

(ii) $2^x + \dfrac{1}{x^2} - 1 + x = 0$

$$2^x + \frac{1}{x^2} = 1 - x$$

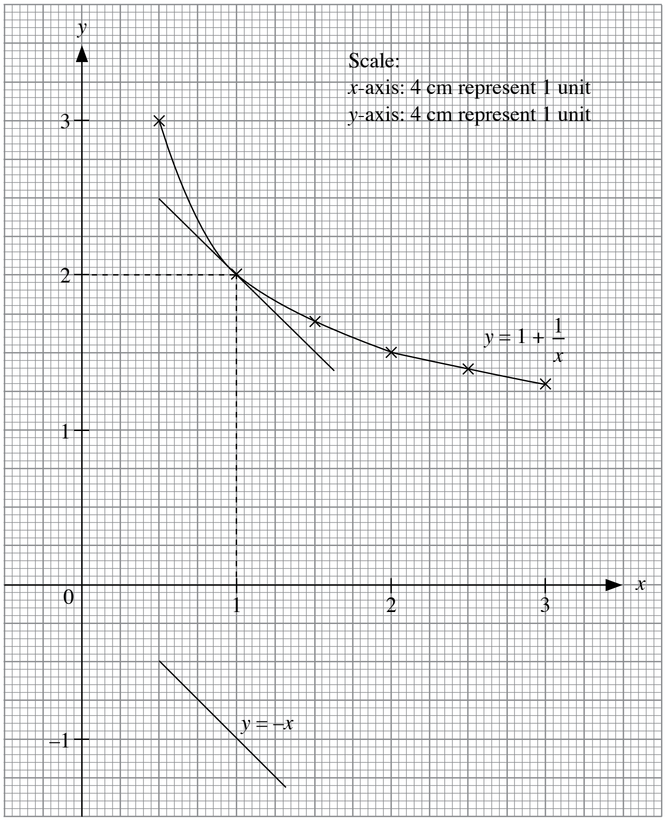
From the graph, the solution is –0.85.

(b) For all real values of x, 2^x and $\dfrac{1}{x^2}$ are always positive, hence y is always positive.

Hence the graph will not lie below the x-axis.

10. (a) $y = 1 + \frac{1}{x}$

x	0.5	1	1.5	2	2.5	3
y	3	2	1.7	1.5	1.4	1.3



(b) $y = -x$

x	0.5	1	1.5
y	–0.5	–1	–1.5

(c) A line parallel to the line $y = -x$ at a point of the curve has a gradient equal to –1.

From the graph, the coordinates are (1, 2).

Exercise 7C

1. (i) Distance travelled between 0900 and 1030 = 15×1.5

= 22.5 km

Distance travelled after the rest = 40 – 22.5

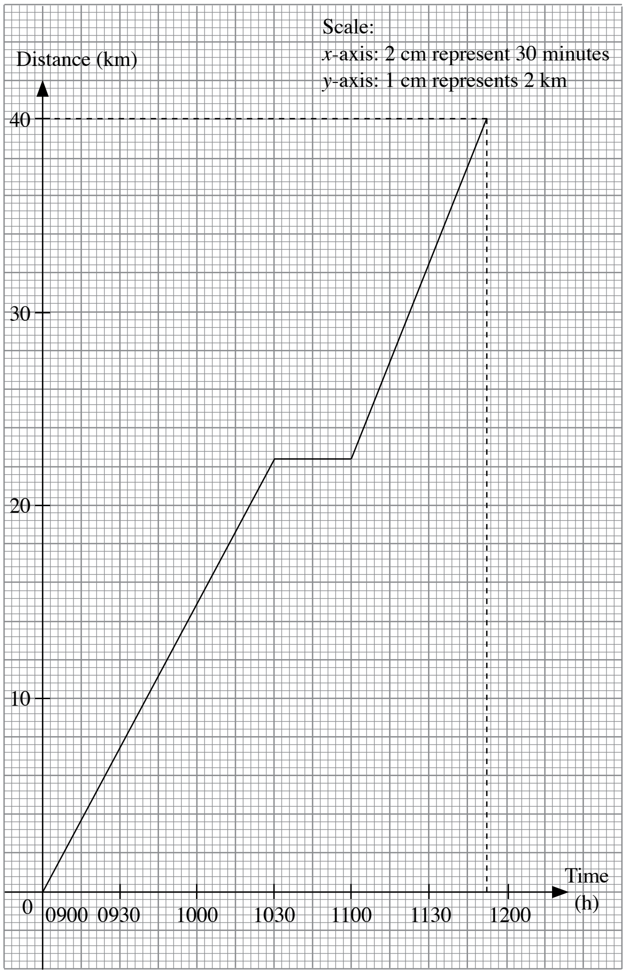
= 17.5 km

Time taken for the distance travelled after the rest

$= \frac{17.5}{20}$

= 0.875 h

= 52.5 minutes



(ii) Hence, from the graph, the time at which the cyclist reached his destination is 1153.

2. (i) Distance travelled between 0900 and 0945 = $20 \times \frac{45}{60}$

= 15 km

(ii) Distance travelled after the rest = 30 – 15

= 15 km

Time taken for the distance travelled after the rest = $\frac{15}{30}$

= 0.5 h

Time at which he continued on this journey after rest = 1050

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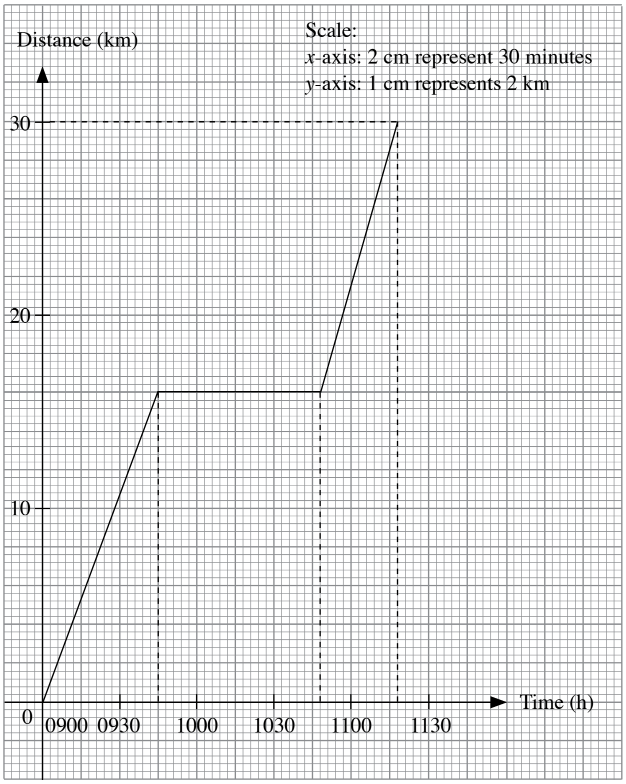


(iii) Hence, from the graph, the duration of his stop is 65 minutes.

3. (i) From the graph, the car is not moving between $t = 1.5$ and $t = 2.5$.

(ii) Average speed of car in first 2 hours $= \dfrac{\text{Total distance}}{\text{Total time}}$

$$= \frac{60}{2}$$

$$= 30 \text{ km/h}$$

(iii) Average speed of car for the whole journey

$$= \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{120}{3.5}$$

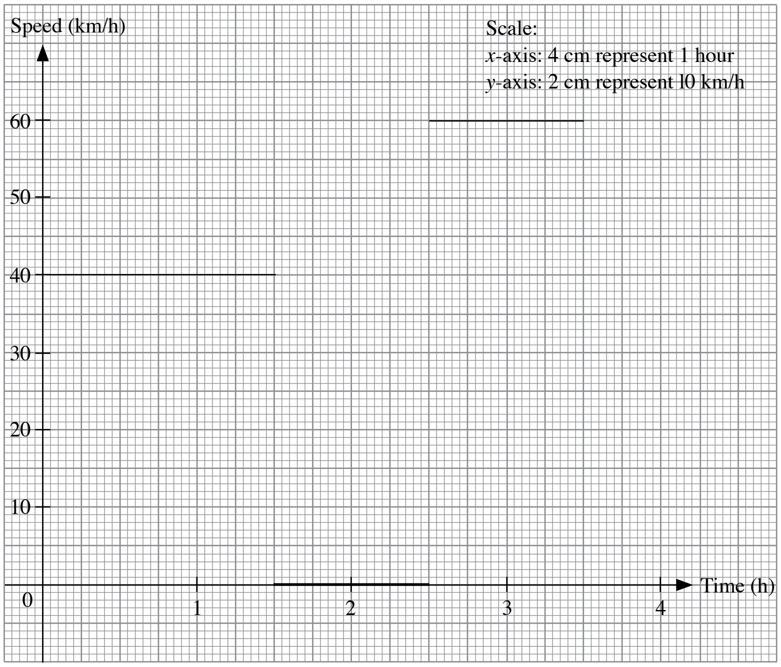
$$= 34.3 \text{ km/h (to 3 s.f.)}$$

(iv) Speed of car between $t = 0$ and $t = 1.5 = \dfrac{60}{1.5}$

$$= 40 \text{ km/h}$$

Speed of car between $t = 2.5$ and $t = 3.5 = \dfrac{120 - 60}{1}$

$$= 60 \text{ km/h}$$



4. **(i)** Acceleration $= \dfrac{10 \text{ m/s}}{2 \text{ s}}$

$= 5 \text{ m/s}^2$

(ii) Total distance = area under graph

$= \left(\dfrac{1}{2} \times 2 \times 10\right) + (3 \times 10) + \left(\dfrac{1}{2} \times 1 \times 10\right)$

$= 10 + 30 + 5$

$= 45 \text{ m}$

Average speed during the whole journey $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{45}{6}$

$= 7.5 \text{ m/s}$

5. **(i)** Gradient of $OA = \dfrac{\text{m/s}}{\text{s}}$

$= \text{m/s}^2$

= Acceleration between $t = 0$ and $t = 30$

(ii) Acceleration between $t = 0$ and $t = 30 = \dfrac{14 \text{ m/s}}{30 \text{ s}}$

$= \dfrac{7}{15} \text{ m/s}^2$

Let the speed of the car when $t = 15$ be x m/s.

Acceleration between $t = 0$ and $t = 15$

= Acceleration between $t = 0$ and $t = 30$

$= \dfrac{7}{15}$

$\dfrac{x}{15} = \dfrac{7}{15}$

$x = 7$

$\therefore$ The speed of the car when $t = 15$ is 7 m/s.

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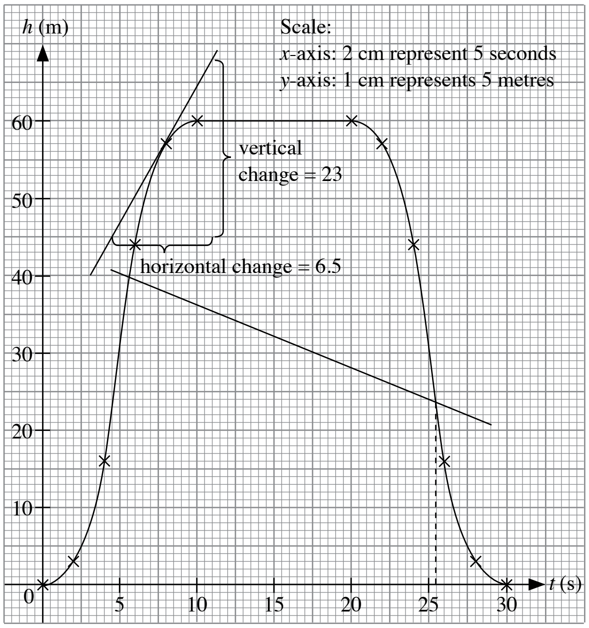
6. **(i)**



(ii) From the graph, at $t = 8$,

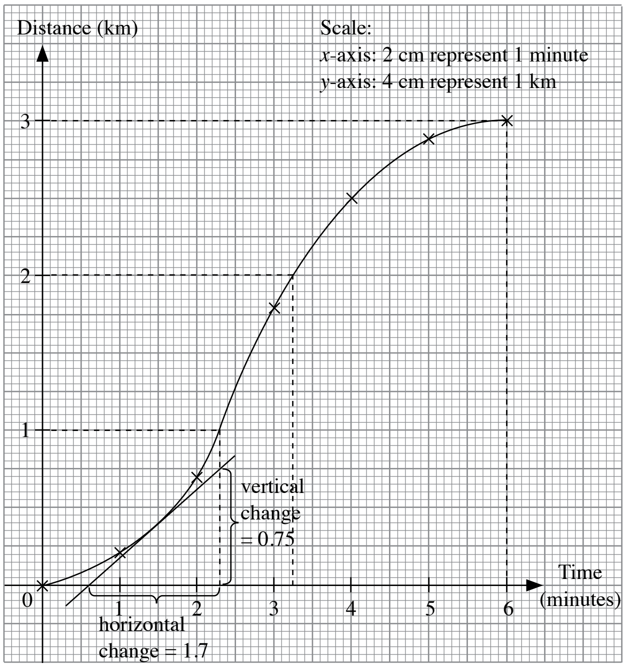
Gradient = $\dfrac{23 \text{ m}}{6.5 \text{ s}}$ = 3.54 m/s (to 3 s.f.)

The gradient of the tangent at $t = 8$ represents the speed at that particular point. It is called the instantaneous speed.

(iii) After 25 s, the worker will have descended $0.8 \times 25 = 20$ m. Hence at $t = 30$ s, the worker is at the 20-metre level. The distance-time curve of the worker is drawn onto the graph.

From the graph, the worker and the lift are at the same height at $t = 25.5$.

7. **(a)**



(b) **(i)** From the graph,

The approximate time taken to travel the first 1 km = 2.3 minutes

(ii) From the graph, at $t = 1\frac{1}{2}$,

Gradient = $\frac{0.75}{1.7}$ = 0.44 km/min (to 3 s.f.)

The gradient of the tangent at $t = 1\frac{1}{2}$ represents the speed at that particular point.

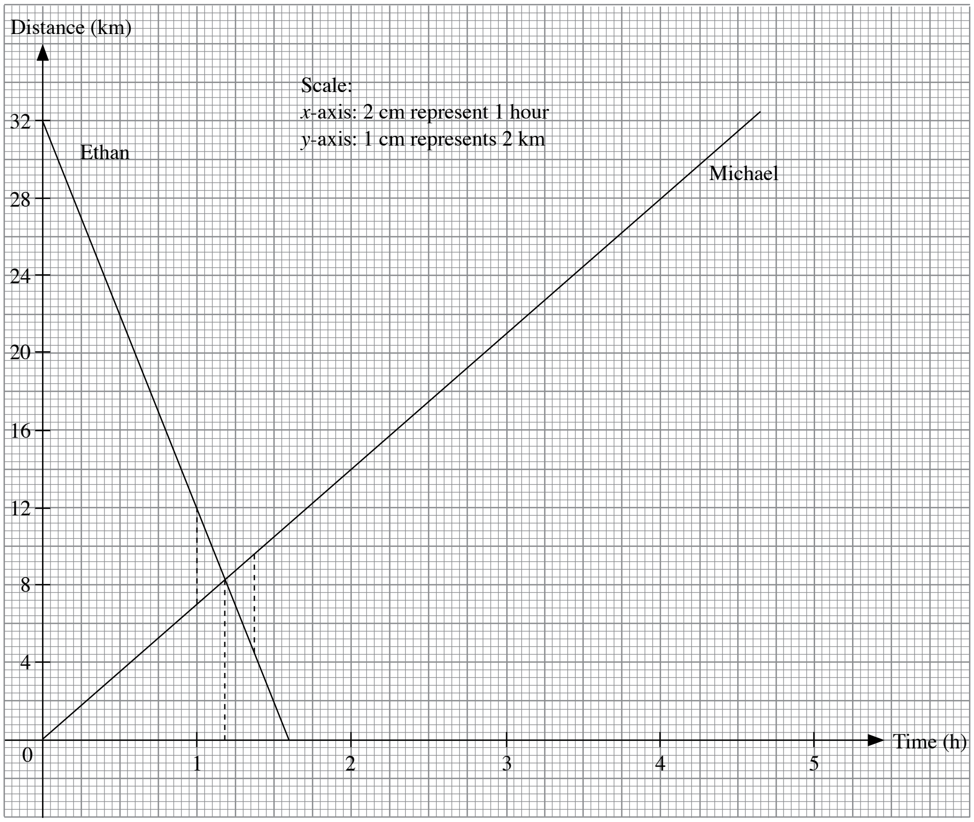
(iii) From the graph,

Time taken to travel the last 1 km = 6 – 3.25

= 2.75 minutes

8. **(a)** Time taken for Ethan = $\frac{32}{20}$ = 1.6 h

Time taken for Michael = $\frac{32}{7}$ = 4.6 h (to 1 d.p)



(b) **(i)** From the graph,

Time taken for Ethan and Michael to pass each other = 1 h 11 min

(ii) From the graph,

Times when Ethan and Michael will be 5 km apart = 1 h and 1 h 22 min

9. **(a)** Distance travelled by Shirley for the first half hour = $18 \times \frac{1}{2}$ = 9 km

Time taken for Shirley to travel the remaining 11 km = $\frac{11}{8}$ = 1.375 h

Time taken for Kate to travel 20 km = $\frac{20}{7}$ = 2.9 h (to 1 d.p.)

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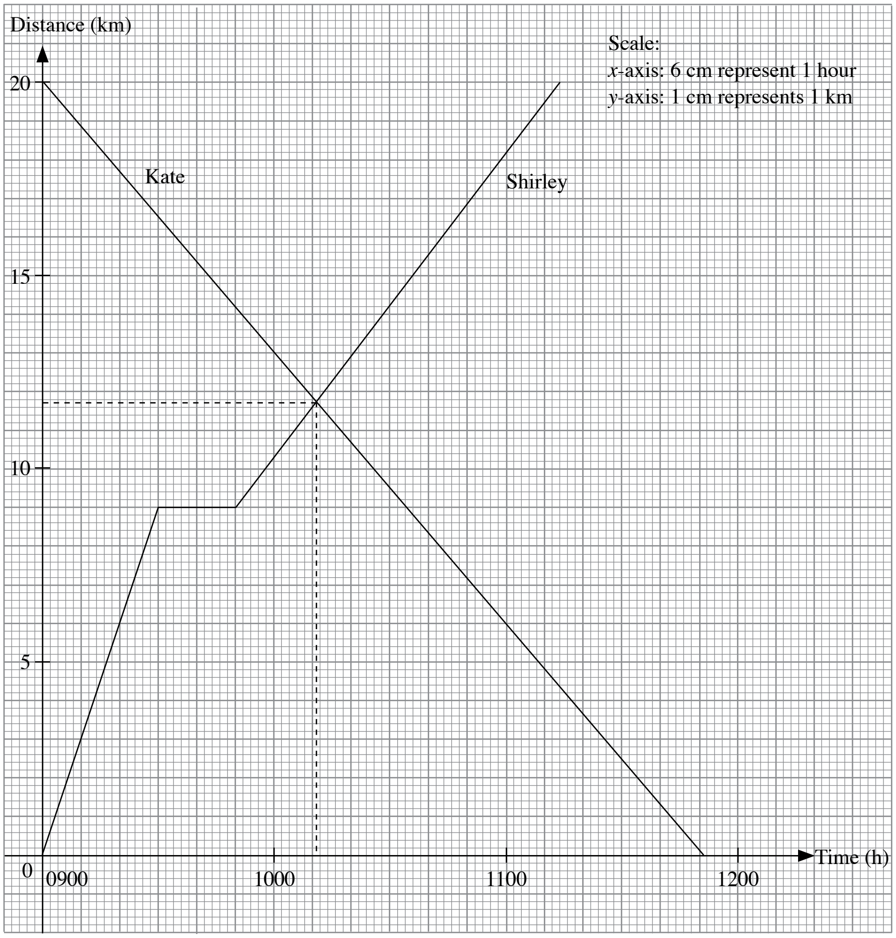


(b) (i) From the graph, the time at which Shirley and Kate meet is 1011.

(ii) From the graph, the distance away from Kate's home when they meet is 8.3 km.

10. Given that it takes 20 seconds to fill each container at a constant rate,

Rate of liquid filling each container = $\frac{10\,000}{20}$ = 500 cm^3/s

Volume of each container = 10 000 cm^3

For container B,

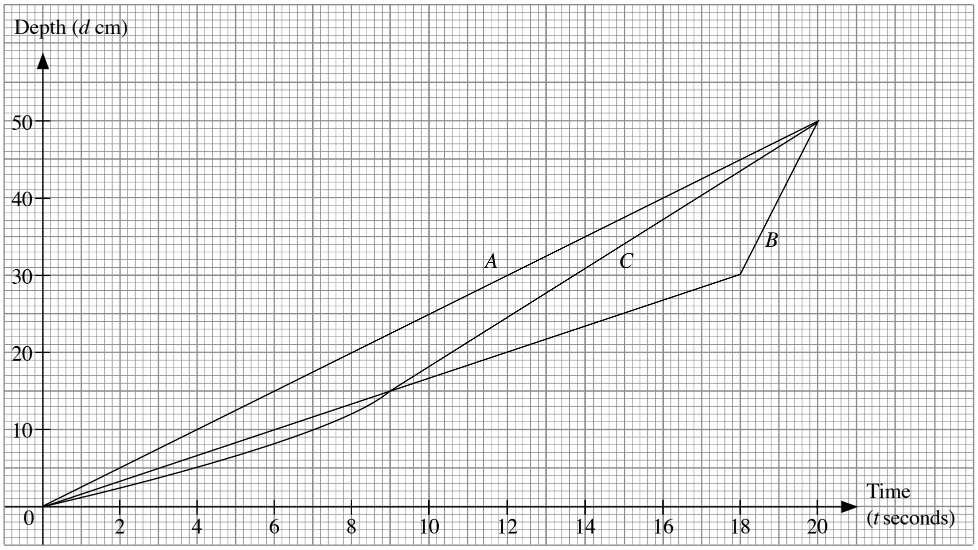
Time taken to fill the first 30 cm = $\frac{30 \times 30 \times 10}{500}$ = 18 s

Volume of container B = 30 × 30 × 10 + 5 × 20 × 10 = 10 000 cm^3

For container C,

Time taken to fill the first 10 cm = $\dfrac{10\,000 - \frac{1}{2} \times 32 \times 10 \times 40}{500}$ = 7.2 s

Time taken to fill the first 30 cm = $\dfrac{10\,000 - \frac{1}{2} \times 16 \times 10 \times 40}{500}$ = 13.6 s



11. **(i)** Given that the object slows down at a rate of 12 m/s^2,

Deceleration = 12 m/s^2

$$\frac{36-0}{t-6} = 12$$
$$36 = 12(t-6)$$
$$36 = 12t - 72$$
$$12t = 108$$
$$t = 9$$

(ii) Given that the distance travelled when the object is slowing down is 54 m,

Total distance travelled $= 36 \times 6 + 54$
$= 270$ m

Average speed for the whole journey $= \frac{270}{9}$
$= 30$ m/s

12. **(i)** Acceleration during the first 20 s $= \frac{30-0}{20}$
$= 1.5$ m/s^2

(ii) Given that the train decelerates at a rate of 0.75 m/s^2,

Deceleration = 0.75 m/s^2

$$\frac{30-0}{t-60} = 0.75$$
$$30 = 0.75(t-60)$$
$$30 = 0.75t - 45$$
$$0.75t = 75$$
$$t = 100$$

$\therefore$ The time taken for the whole journey is 100 s.

13. (a) When $t = 3$,

$v = 3^2 - 7(3) + 16 = 4$

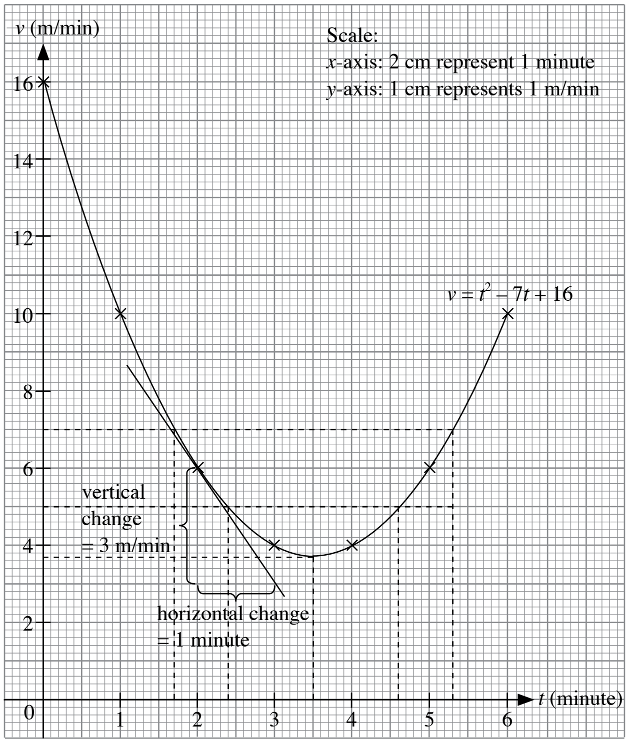
$\therefore a = 4$

When $t = 6$,

$v = 6^2 - 7(6) + 16 = 10$

$\therefore b = 10$

(b)



(c) (i) From the graph,

When $v = 7$, $t = 1.7, 5.3$.

(ii) From the graph,

The time at which the speed is a minimum is at 3.5 s.

(iii) A tangent is drawn to the curve at $t = 2$.

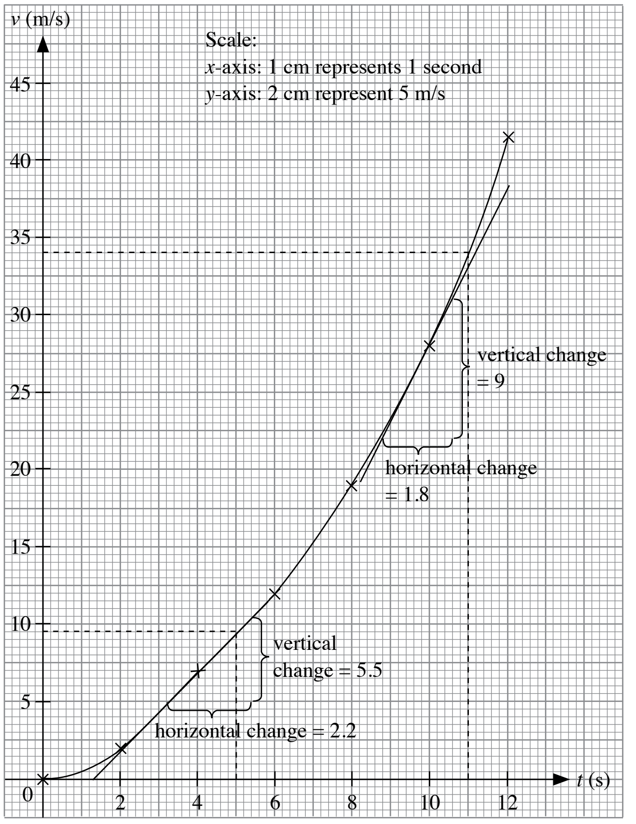
From the graph,

Gradient $= \dfrac{-3 \text{ m/min}}{1 \text{ min}} = -3 \text{ m/min}^2$

The gradient of the tangent at $t = 2$ represents the deceleration at that particular point.

(iv) From the graph, the time interval when the speed is not more than 5 m/min is $2.4 < t < 4.6$.

14. (i)



(ii) From the graph,

Speed when $t = 5$ is 9.5 m/s

Speed when $t = 11$ is 34 m/s

(iii) A tangent is drawn to the curve at $t = 4$.

From the graph,

Gradient $= \dfrac{5.5 \text{ m/s}}{2.2 \text{ s}} = 2.5 \text{ m/s}^2$

$\therefore$ Acceleration at $t = 4 = 2.5 \text{ m/s}^2$

A tangent is drawn to the curve at $t = 10$.

From the graph

Gradient $= \dfrac{9 \text{ m/s}}{1.8 \text{ s}} = 5 \text{ m/s}^2$

$\therefore$ Acceleration at $t = 10 = 5 \text{ m/s}^2$

15. (a) When $t = 2$,

$v = 3(2)^2 - 17(2) + 30 = 8$

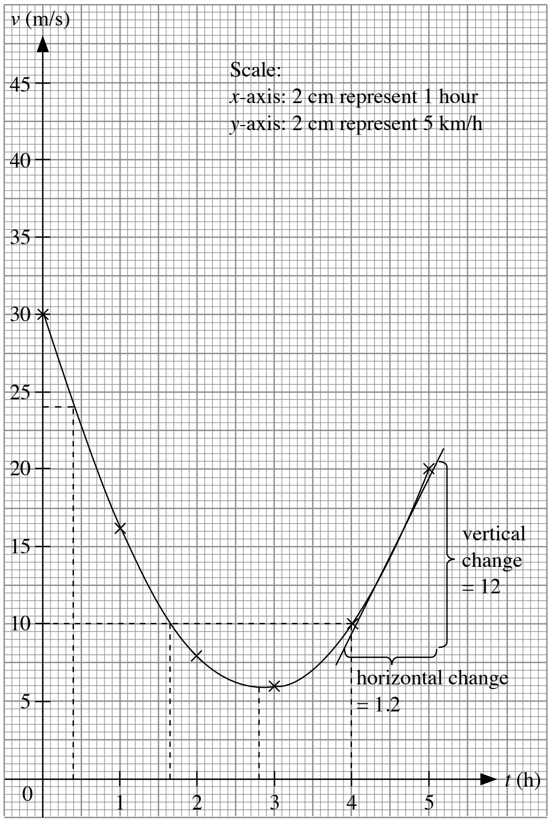
$\therefore h = 8$

When $t = 4$,

$v = 3(4)^2 - 17(4) + 30 = 10$

$\therefore k = 10$

(b)



(c) **(i)** From the graph, the time at which the speed is a minimum is at $t = 2.8$ s.

(ii) A tangent is drawn to the curve at $t = 4.5$.

From the graph,

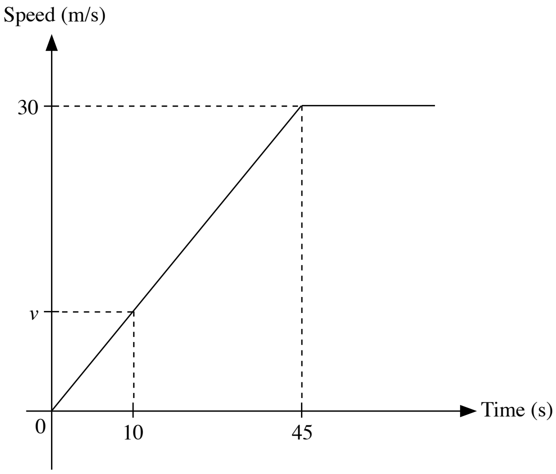
Gradient = $\dfrac{12 \text{ km/h}}{1.2 \text{ h}} = 10 \text{ km/h}^2$

The gradient of the tangent at $t = 4.5$ represents the acceleration at that particular point.

(iii) From the graph, the time interval when the speed does not exceed 10 km/h is $1.65 < t < 4$.

(d) From the graph, the value of t at which both objects have the same speed is 0.4.

16.



Since the gradient of a speed-time graph gives the acceleration,

$$\frac{v}{10} = \frac{30}{45}$$

$$v = 10 \times \frac{30}{45}$$

$$= 6\frac{2}{3} \text{ m/s}$$

Hence the speed after 10 seconds is $6\frac{2}{3}$ m/s.

17. **(a)** From the graph,

Postage to mail a letter with a mass of 50 g = 55 cents

(b) From the graph,

For Company *A*,

Postage to mail a letter with a mass of 220 g = 280 cents

= \$2.80

For Company *B*,

Postage to mail a letter with a mass of 220 g

= \$1 + \$0.01 × (220 – 80)

= \$2.40

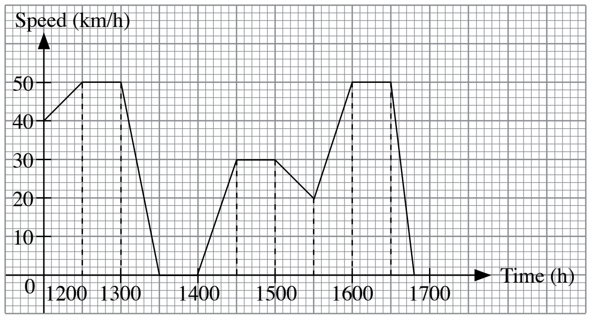
∴ Company *B* offers a lower postage.

18. **(a)** From the graph, the coach stopped between 1330 and 1400.

Duration of stop = 30 minutes

(b) Initial acceleration of the coach = $\dfrac{50 - 40}{0.5} = 20 \text{ km/h}^2$

(c)



Distance travelled

= area under the graph

$= \frac{1}{2} \times (40 + 50) \times \frac{1}{2} + 50 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times 50$

$+ \frac{1}{2} \times \frac{1}{2} \times 30 + \frac{1}{2} \times 30 + \frac{1}{2} \times (30 + 20) \times \frac{1}{2} + \frac{1}{2}$

$\times (20 + 50) \times \frac{1}{2} + \frac{1}{2} \times 50 + \frac{1}{2} \times \frac{18}{60} \times 50$

= 22.5 + 25 + 12.5 + 7.5 + 15 + 12.5 + 17.5 + 25 + 7.5

= 145 km

∴ The distance between Blue Town and Summer City is less than 250 km.

(d) From the graph, the time when coach reached Summer City is at 1648.

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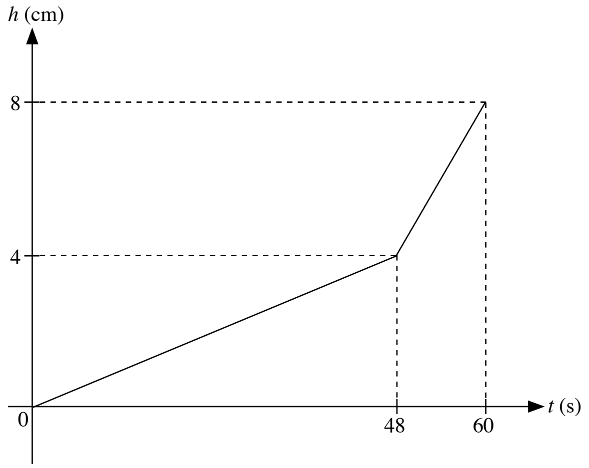
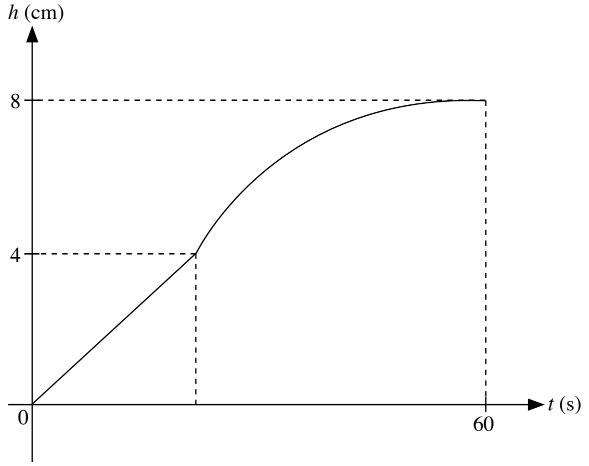
19. Given that it takes 20 seconds to fill each container at a constant rate,

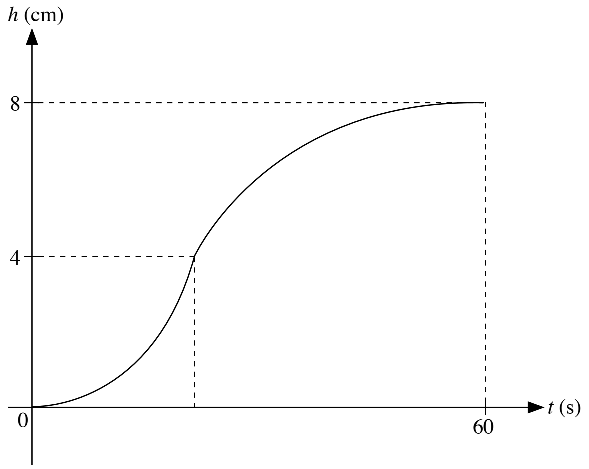
For the first container,



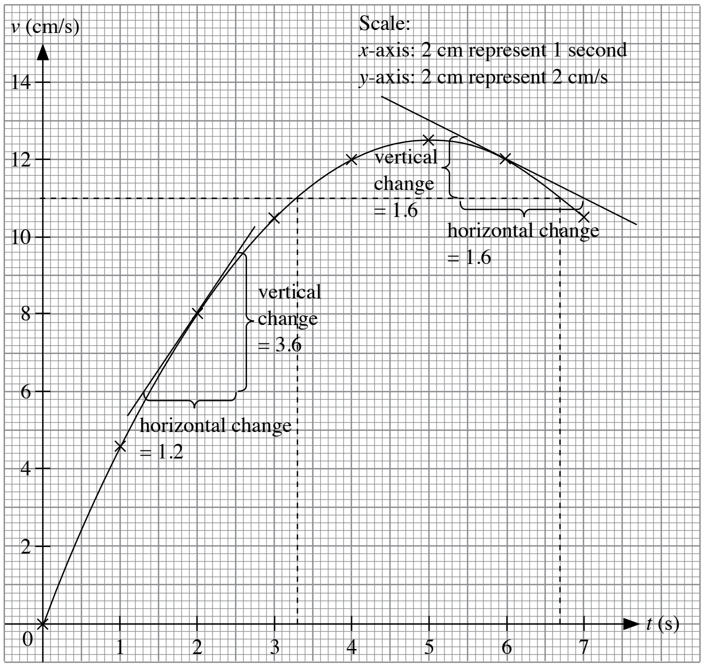
For the second container,



For the third container,



20. (a)



(b) (i) A tangent is drawn to the curve at $t = 2$.

From the graph,

$$\text{Gradient} = \frac{3.6 \text{ cm/s}}{1.2 \text{ s}} = 3 \text{ cm/s}^2$$

$\therefore$ Acceleration at $t = 2 = 3$ cm/s^2

A tangent is drawn to the curve at $t = 6$

From the graph,

$$\text{Gradient} = \frac{-1.6 \text{ cm/s}}{1.6 \text{ s}} = -1 \text{ cm/s}^2$$

$\therefore$ Acceleration at $t = 6 = -1$ cm/s^2

(ii) From the graph, the time interval when the speed is greater than 11 m/s is $3.3 < t < 6.7$.

(c) $v = at^2 + bt + c$

When $t = 0$, $v = 0$, $c = 0$.

When $t = 1$, $v = 4.5$,

$4.5 = a + b$

$a = 4.5 - b \quad$ —(1)

When $t = 2$, $v = 8$,

$8 = a(2)^2 + 2b$

$4a + 2b = 8$

$2a + b = 4 \quad$ —(2)

Substitute (1) into (2):

$2(4.5 - b) + b = 4$

$9 - 2b + b = 4$

$9 - b = 4$

$b = 5$

$a = 4.5 - 5$

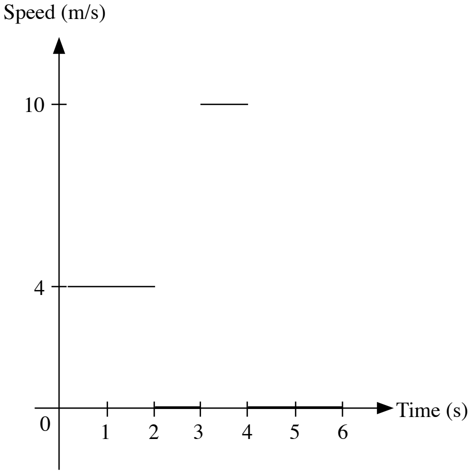
$= -0.5$

$v = -0.5t^2 + 5t$

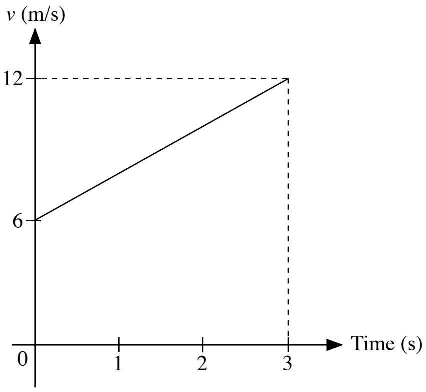
$\therefore a = -0.5, b = 5, c = 0$

21. Between $t = 0$ and $t = 2$, speed $= \dfrac{8}{2} = 4$ m/s

Between $t = 3$ and $t = 4$, speed $= \dfrac{18 - 8}{1} = 10$ m/s

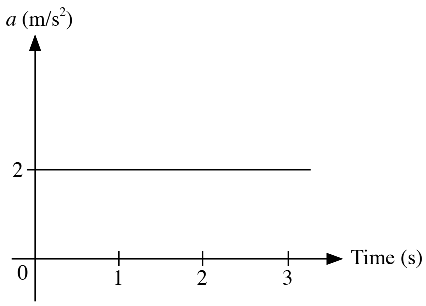


22. **(a)** $v = 6 + 2t$



(b) When $t = 3$, $v = 6 + 2(3) = 12$ m/s

(c) Acceleration $= \dfrac{12 - 6}{3} = 2$ m/s^2

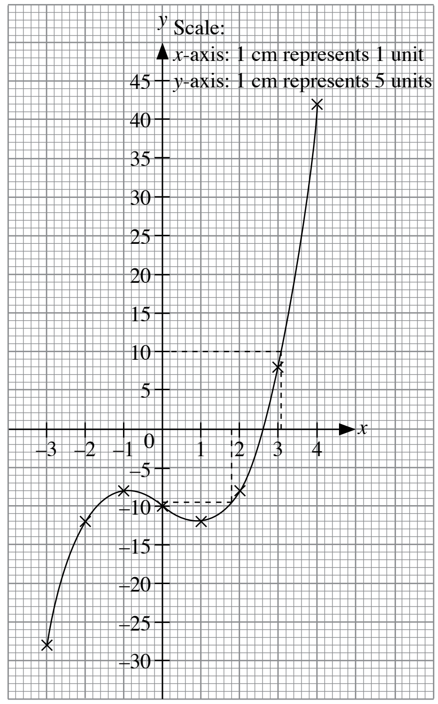


Review Exercise 7

1. **(a)** $y = x^3 - 3x - 10$

x	–3	–2	–1	0	1	2	3	4
y	–28	–12	–8	–10	–12	–8	8	42

(b)



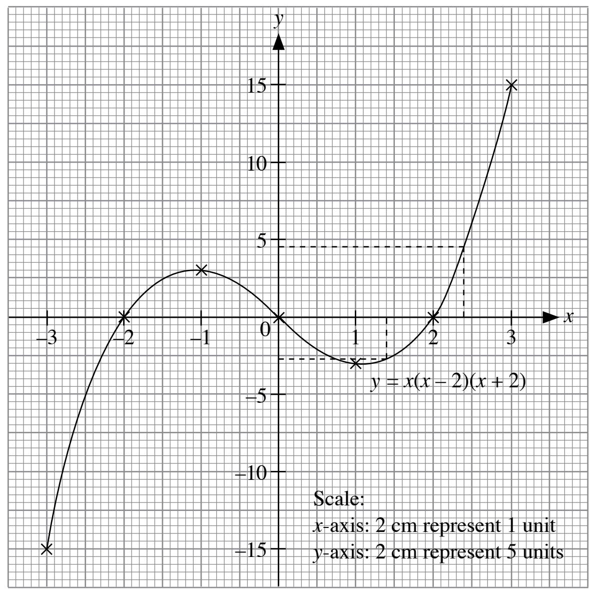
(c) **(i)** From the graph, when $x = 1.8$, $y = -9.5$.

(ii) From the graph, when $y = 10$, $x = 3.1$.

2. **(a)** $y = x(x - 2)(x + 2)$

x	–3	–2	–1	0	1	2	3
y	–15	0	3	0	–3	0	15

(b)



(c) **(i)** From the graph, when $x = 1.4$, $y = -2.75$.

(ii) From the graph, when $y = 4.5$, $x = 2.4$.

(iii) From the graph, the solutions to the equation $x(x - 2)(x + 2) = 0$ are –2, 0 and 2.

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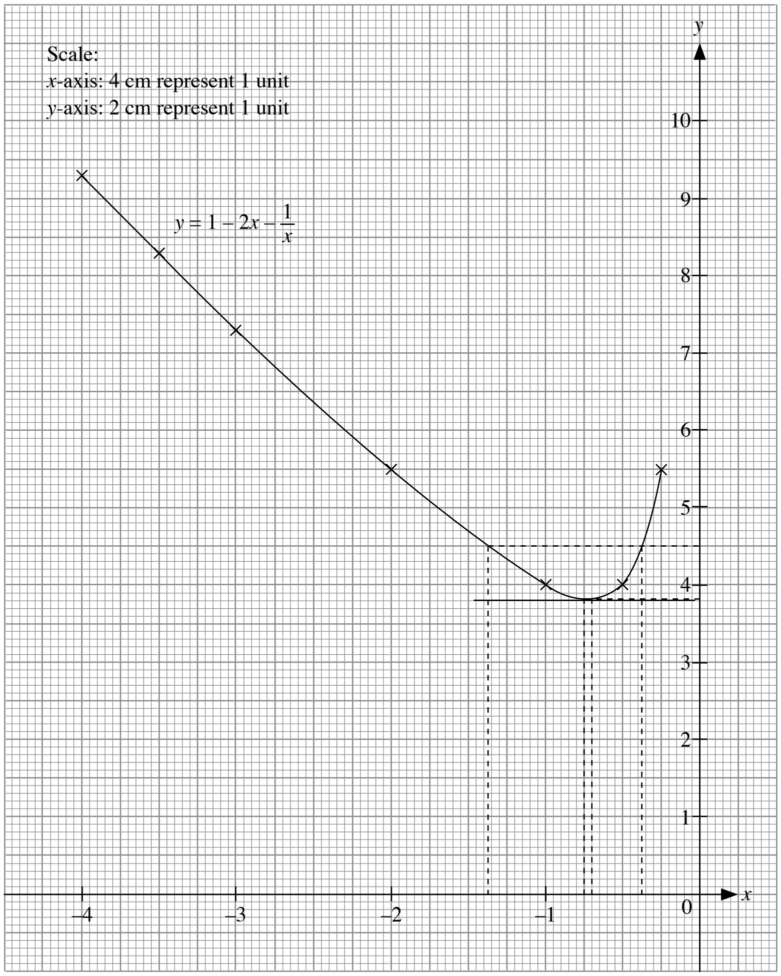
3. $y = 1 - 2x - \dfrac{1}{x}$

x	–4	–3.5	–3	–2	–1	–0.5	–0.25
y	9.3	8.3	7.3	5.5	4	4	5.5



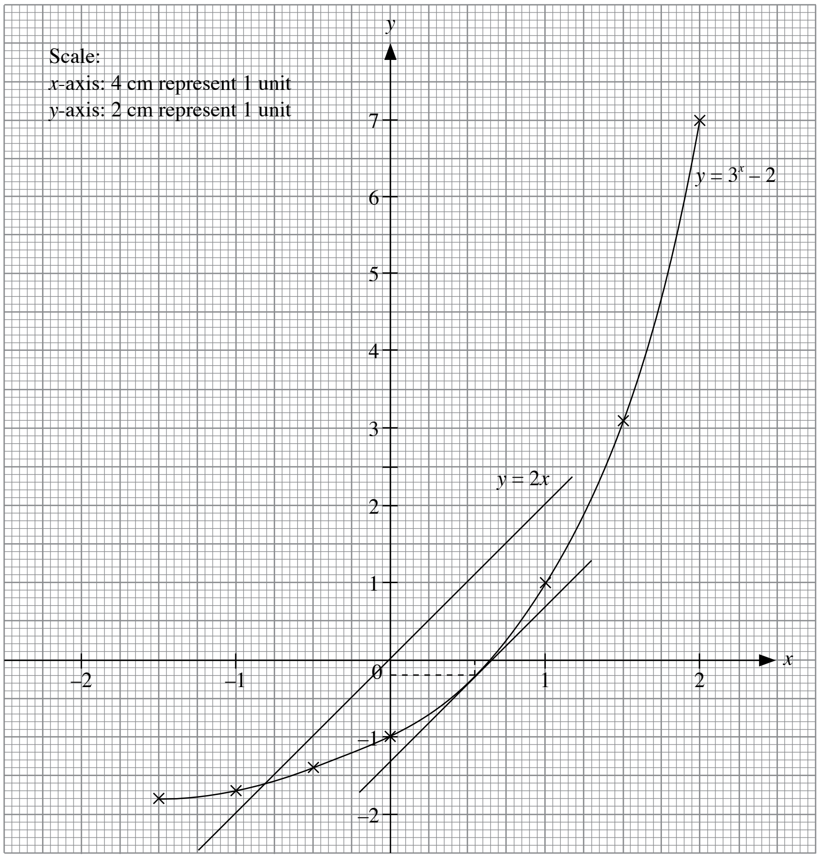
(a) **(i)** From the graph, when $x = –0.75$, $y = 3.85$.

(ii) From the graph, when $y = 4.5$, $x = –0.375$ or $–1.375$.

(b) From the graph, the coordinates of the point on the curve where the tangent to the curve is a horizontal line are $(–0.7, 3.8)$.

4. $y = 3^x - 2$

x	–1.5	–1	–0.5	0	1	1.5	2
y	–1.8	–1.7	–1.4	–1	1	3.1	7



(i) $3^x = 2$

$3^x - 2 = 0$

$y = 0$

From the graph, when $y = 0$, $x = 0.625$.

(ii) $y = 2x$

x	–1	0	1
y	–2	0	–1

A tangent parallel to the line $y = 2x$ is drawn to the curve.

From the graph, the coordinates of the point on the graph of $y = 3^x - 2$ where the gradient of tangent is 2 are (0.55, –0.2).

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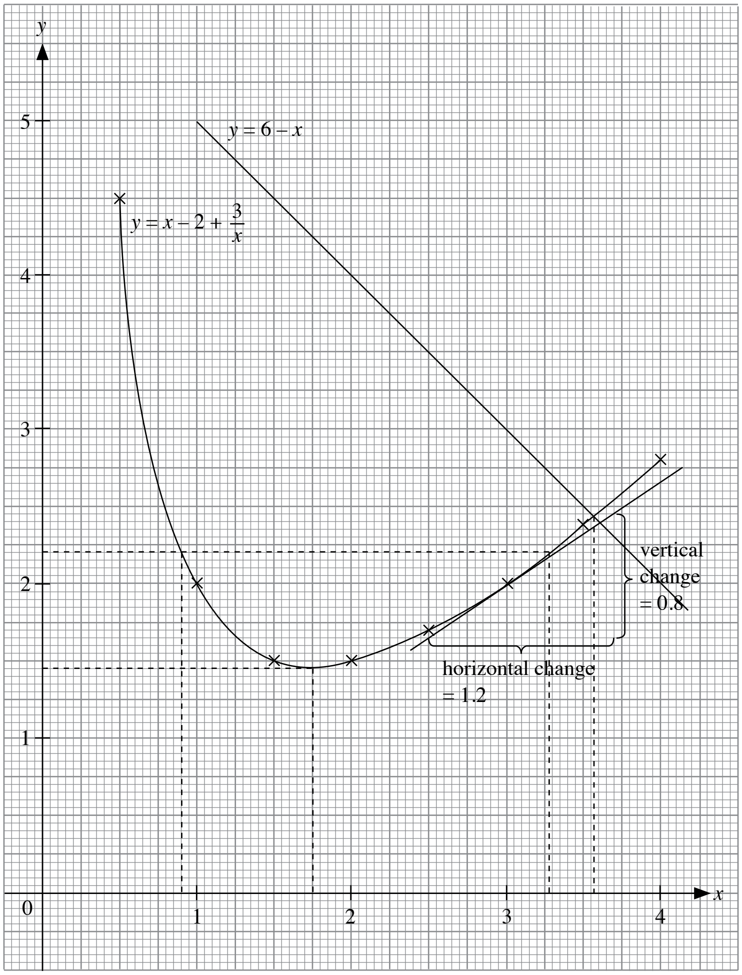
5. $y = x - 2 + \dfrac{3}{x}$

x	0.5	1	1.5	2	2.5	3	3.5	4
y	4.5	2	1.5	1.5	1.7	2	2.4	2.8



(i) From the graph, the minimum value of $y = 1.45$ where $x = 1.75$.

(ii) From the graph, the range of values of x for which $y < 2.2$ is $0.9 < x < 3.275$.

(iii) A tangent is drawn to the curve at the point where $x = 3$.

From the graph,

$$\text{Gradient} = \frac{0.8}{1.2} = 0.67$$

(iv)
$$2x + \frac{3}{x} = 8$$
$$2x + \frac{3}{x} - x = 8 - x$$
$$x + \frac{3}{x} = 8 - x$$
$$x + \frac{3}{x} - 2 = 8 - x - 2$$
$$x - 2 + \frac{3}{x} = 6 - x$$
$$y = 6 - x$$

x	1	2	3	4
y	5	4	3	2

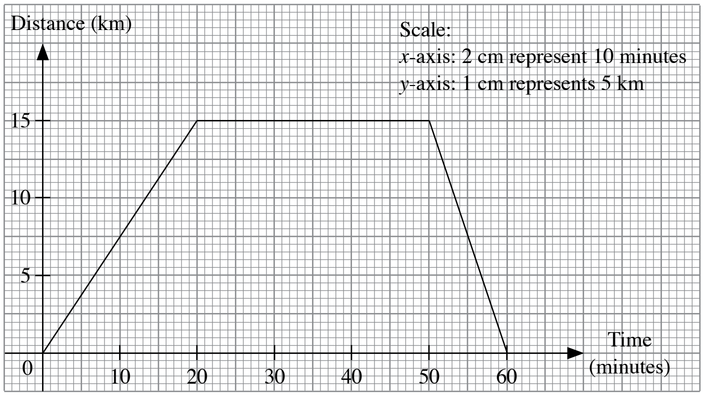
From the graph, the value of $x = 3.575$.

6. **(i)** Distance travelled by coach from airport to hotel $= 45 \times \frac{20}{60}$

$$= 15 \text{ km}$$

Time taken by coach from hotel to airport $= \frac{15}{60}$

$$= \frac{1}{4} \text{ h}$$



(ii) Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{(15 + 15) \text{ km}}{\frac{65}{60} \text{ km}}$$

$= 27.7$ km/h (to 3 s.f.)

7. **(a)** **(i)** From the graph, the time interval during which the vehicle stopped to unload goods is $1 \leqslant t \leqslant 2$.

(ii) Speed when $t = 3 = \frac{60 - 40}{3.5 - 2}$

$$= 13\frac{1}{3} \text{ km/h}$$

(iii) Maximum speed during journey = Speed between $t = 0$ and $t = 1$

$$= \frac{40}{1}$$

$$= 40 \text{ km/h}$$

(iv) Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{60 \text{ km}}{3.5 \text{ h}}$$

$= 17.1$ km/h (to 3 s.f.)

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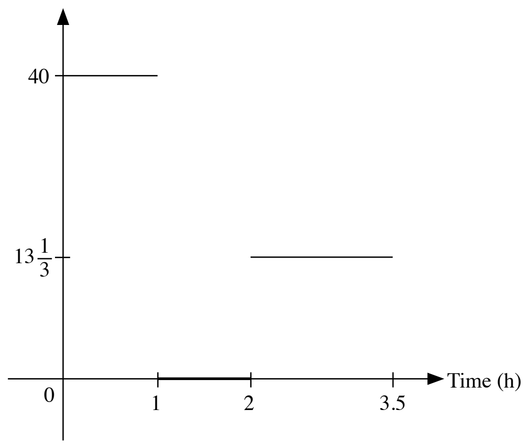
(b)



8. **(i)** Acceleration $= \dfrac{4-0}{2}$

$= 2$ m/s^2

(ii) Greatest acceleration = acceleration from 4 s to 6 s

$= \dfrac{10-4}{2}$

$= 3$ m/s^2

(iii) Total distance moved

= area under the graph

$= \dfrac{1}{2} \times 2 \times 4 + 2 \times 4 + 2 \times 4 + \dfrac{1}{2} \times 2 \times 6$

$= 4 + 8 + 8 + 6$

$= 26$ m

(iv) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{26}{6}$

$= 4\dfrac{1}{4}$ m/s

9. **(i)** Total distance travelled

= area under the graph

$= \dfrac{1}{2} \times \dfrac{10}{60} \times 60 + 60 \times \dfrac{10}{60} + 60 \times \dfrac{10}{60} + 60 \times 5 + \dfrac{1}{2} \times \dfrac{5}{60} \times 60$

$= 5 + 10 + 10 + 5 + 2.5$

$= 32.5$ km

(ii) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{32.5}{\frac{25}{60}}$

$= 78$ km/h

10. **(i)** $I = \dfrac{k}{D^2}$

When $D = 20, I = 800$,

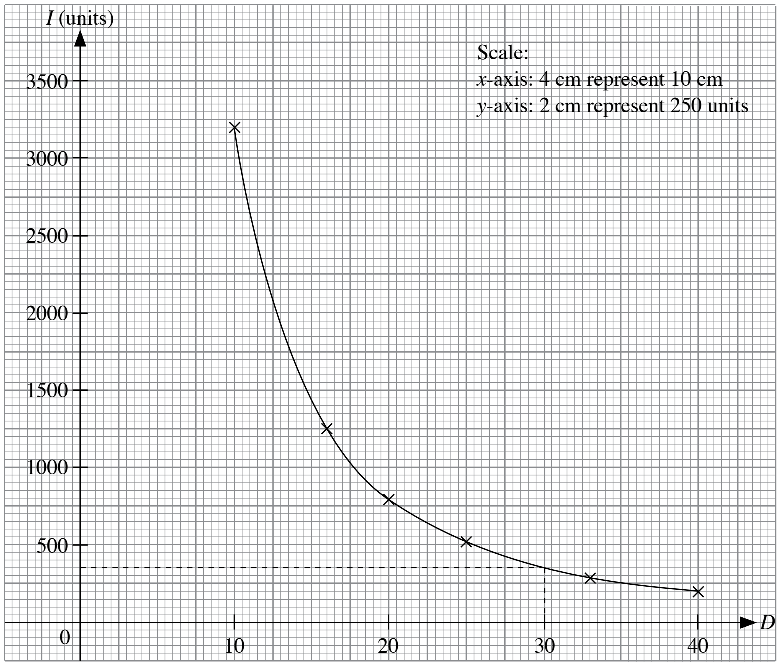
$800 = \dfrac{k}{20^2}$

$k = 800 \times 20^2$

$= 320\ 000$

(ii) $I = \dfrac{320\ 000}{D^2}$

D	10	16	20	25	33	40
I	3200	1250	800	512	294	200



(iii) From the graph, when $D = 30$ cm, $I = 350$ units.

(iv) I is inversely proportional to D^2.

11. (a) Rate of increase in temperature in the first 10 minutes

$= \dfrac{(70 - 20)°\text{C}}{10 \text{ minutes}}$

= 5°C/minute

(b) The fresh vegetables from the refrigerator caused the drop in temperature between the 10^{th} and 11^{th} minute when they were added into the soup.

(c) Rate of decrease in temperature in the first 10 minutes

$= \dfrac{(90 - 80)°\text{C}}{5 \text{ minutes}}$

= 2°C/minute

Challenge Yourself

Since the gradient of a speed-time graph gives the acceleration,

Between $t = 0$ and $t = 2$:

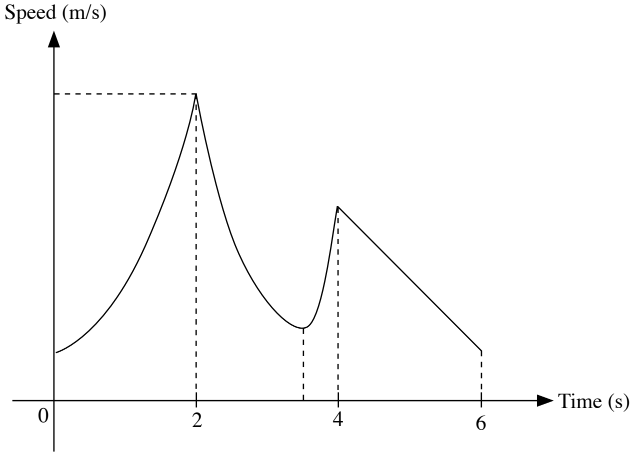
The object is undergoing an increasing acceleration, as such the gradient of the speed-time graph increases at an increasing rate during this period.

Between $t = 2$ and $t = 4$:
Where the graph is below the x-axis, the object is undergoing decreasing deceleration until it reaches zero acceleration. The gradient of the speed-time graph decreases at a decreasing rate during this period.

Where the graph is above the x-axis, the object is undergoing increasing acceleration. The gradient of the speed-time graph increases at an increasing rate during this period. The gradient is steeper than that between $t = 0$ and $t = 2$.

Between $t = 4$ and $t = 6$:

The object is undergoing constant deceleration, as such the gradient is decreasing at a constant rate.



Teachers should note that this is one possible speed-time graph for the motion of the object. As long as the students are able to give the correct shape for each time period of the speed-time graph, that is a possible speed-time graph.

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Revision Exercise B1

1. 100% of cost price = \$160

$$1\% \text{ of cost price} = \frac{\$160}{100}$$

$$145\% \text{ of cost price} = \frac{\$160}{100} \times 145 = \$232$$

The article was sold to the dealer at \$232.

$$\text{Dealer's profit} = \frac{(\$266.80 - 232)}{\$232} \times 100\% = 15\%$$

2. Cost of 3 kg of Coffee A = $3 \times \$9$ = \$27

Cost of 1 kg of Coffee B = \$13

Total cost of 4 kg of mixture = \$27 + \$13 = \$40

$$\text{Total cost of 100 g mixture} = \frac{\$40}{4000} \times 100 = \$1$$

$$\text{Profit} = \frac{\$(1.25 - 1)}{\$1} \times 100\% = 25\%$$

3. $$\text{Gradient of } AB = \frac{12-7}{8-5} = \frac{5}{3}$$

Equation of AB is in the form $y = \frac{5}{3}x + c$

Since (5, 7) lies on the line,

$$7 = \frac{5}{3}(5) + c$$

$$7 = \frac{25}{3} + c$$

$$c = -\frac{4}{3}$$

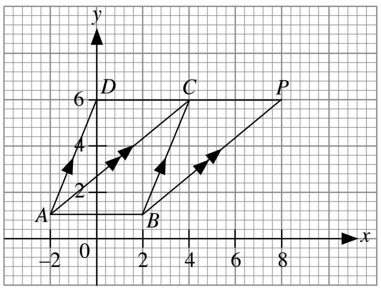
$$\therefore y = \frac{5}{3}x - \frac{4}{3}$$

$$3y = 5x - 4$$

4. **(i)** Length of perpendicular from C to AB = 5 units

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 4 \times 5 = 10 \text{ units}^2$$

(ii), (iii)



From graph, $D(0, 6)$ and $P(8, 6)$.

5. **(i)** $3x + 4y = 24$

Where l crosses the x-axis at the point A, $y = 0$

$$3x + 4y = 24$$
$$3x + 4(0) = 24$$
$$3x = 24$$
$$x = 8$$

$\therefore$ The coordinates of the point A are $(8, 0)$.

Where l crosses the y-axis at the point B, $x = 0$

$$3x + 4y = 24$$
$$3(0) + 4y = 24$$
$$4y = 24$$
$$y = 6$$

$\therefore$ The coordinates of the point B are $(0, 6)$.

(ii) Length of perpendicular from B to AC = 13 units

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 13 \times 6 = 39 \text{ units}^2$$

(iii) $$\text{Gradient of } BC = \frac{0-6}{-5-0} = 1.2$$

(iv) $$AB = \sqrt{(8-0)^2 + (0-6)^2} = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10 \text{ units}$$

Let the length of the perpendicular from C to AB be x units.

Area of $\triangle ABC$ = 39 units^2

$$\frac{1}{2} \times x \times 10 = 39$$
$$5x = 39$$
$$x = 7.8$$

$\therefore$ The length of the perpendicular from C to AB is 7.8 units.

6. **(i)** $$\text{Jun Wei's average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{40}{3} = 13\frac{1}{3} \text{ km/h}$$

(ii) From the graph, the time when Rui Feng and Jun Wei meet is 1030 and they are 20 km from the airport when they met.

(iii) From the graph, Rui Feng took a rest between 1000 and 1100. Hence, the time interval during which Rui Feng took a rest is 1 h.

(iv) From the graph, when Jun Wei reaches the train station, Rui Feng is 10 km away from the airport.

7. **(i)** Acceleration during the first 8 seconds = 1.5 m/s^2

$$\frac{V-4}{8} = 1.5$$
$$V - 4 = 12$$
$$V = 16$$

(ii) Acceleration between 8 s and t s $= -1$

$$\frac{0 - 16}{t - 8} = -1$$
$$-16 = -t + 8$$
$$t = 24$$

8. **(a)** When $x = 1.5$,

$y = \frac{12}{1.5} + 1.5 - 6 = 3.5$

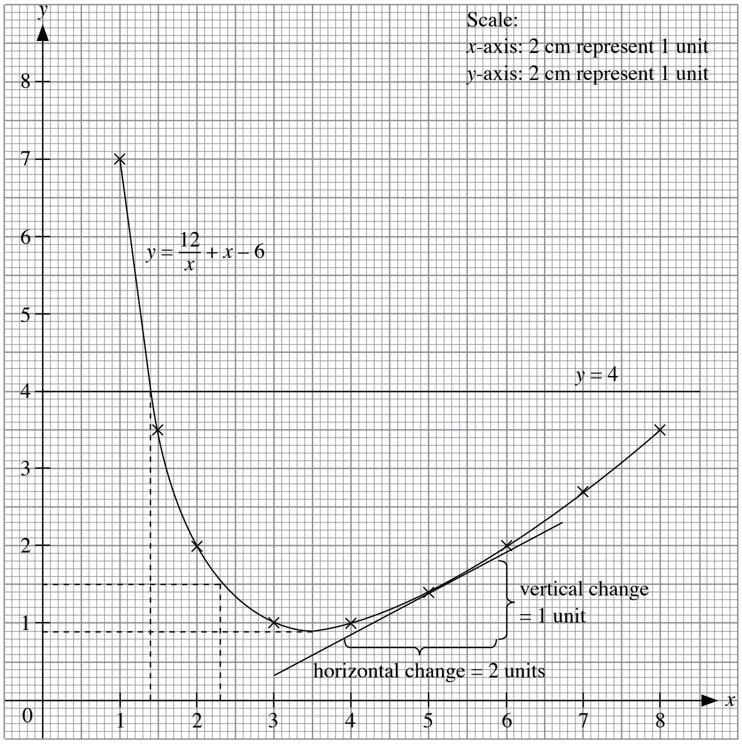
$\therefore h = 3.5$

When $x = 7$,

$y = \frac{12}{7} + 7 - 6 = 2.7$ (to 1 d.p.)

$\therefore k = 2.7$

(b)



(c) **(i)** From the graph, when $x = 2.3$, $y = 1.5$.

(ii) From the graph, the minimum value of $y = 0.9$.

(d) A tangent is drawn to the curve at the point where $x = 5$.

From the graph,

Gradient $= \frac{1}{2} = 0.5$

(e)
$$x^2 + 12 = 10x$$
$$x + \frac{12}{x} = 10$$
$$\frac{12}{x} + x - 6 = 10 - 6$$
$$y = 4$$

From the graph, the solution of the equation $x^2 + 12 = 10x$ is 1.4.

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Revision Exercise B2

1. **(i)** 96% of cost price = \$408

$$1\% \text{ of cost price} = \frac{\$408}{96}$$

$$100\% \text{ of cost price} = \frac{\$408}{96} \times 100$$

$$= \$425$$

$\therefore$ The cost price of the vase is \$425.

(ii) $$\text{Percentage gain} = \frac{\$(510 - 425)}{\$425} \times 100\%$$

$$= 20\%$$

2. $$\text{Commission for the month} = \frac{25}{100} \times \$5264$$

$$= \$1316$$

Total income for the month = \$520 + \$1316

= \$1836

3. $5x + 7y = 46$

$$7y = -5x + 46$$

$$y = \frac{-5x + 46}{7}$$

$$y = -\frac{5}{7}x + \frac{46}{7}$$

$\therefore$ Gradient of line $= -\frac{5}{7}$

Given that the line passes through $(2, -5)$ and the gradient $= -\frac{5}{7}$,

$$y = mx + c$$

$$-5 = -\frac{5}{7}(2) + c$$

$$-5 = -\frac{10}{7} + c$$

$$c = -\frac{25}{7}$$

$$\therefore y = -\frac{5}{7}x - \frac{25}{7}$$

$$7y = -5x - 25$$

$$5x + 7y + 25 = 0$$

4. **(i)** $x - 2y = -4$

Where the line crosses the x-axis at the point P, $y = 0$

$$x - 2(0) = -4$$

$$x = -4$$

$\therefore$ The coordinates of the point P are $(-4, 0)$.

Where the line crosses the y-axis at the point Q, $x = 0$

$$0 - 2y = -4$$

$$-2y = -4$$

$$y = 2$$

$\therefore$ The coordinates of the point Q are $(0, 2)$.

(ii) For M to be equidistant from the coordinate axes,

Vertical distance of M from the x-axis = Vertical distance of M from the y-axis

$$x = -y \quad \text{— (1)}$$

$$x - 2y = -4 \quad \text{— (2)}$$

Substitute (1) into (2):

$$(-y) - 2y = -4$$

$$-3y = -4$$

$$y = 1\frac{1}{3}$$

$$x = -1\frac{1}{3}$$

$\therefore$ The coordinates of the point M are $\left(-1\frac{1}{3}, 1\frac{1}{3}\right)$.

(iii) $$\text{Area of } \triangle PMO = \frac{1}{2} \times PO \times h$$

$$= \frac{1}{2} \times 4 \times 1\frac{1}{3}$$

$$= 2\frac{2}{3} \text{ units}^2$$

5. **(i)** $$DC = \sqrt{13} \text{ units}$$

$$\sqrt{(0 - a)^2 + (6 - 4)^2} = \sqrt{13}$$

$$(-a)^2 + 2^2 = 13$$

$$a^2 + 4 = 13$$

$$a^2 = 9$$

$$a = 3 \text{ (since } a > 0)$$

$\therefore$ The coordinates of the point C are $(3, 4)$.

(ii) Let the coordinates of the point A be $(0, y)$.

$$\text{Gradient of } AB = \frac{2}{3}$$

$$\frac{-1 - y}{3 - 0} = \frac{2}{3}$$

$$\frac{-1 - y}{3} = \frac{2}{3}$$

$$-1 - y = 2$$

$$y = -3$$

$\therefore$ The coordinates of the point A are $(0, -3)$.

(iii) $$\text{Area of trapezium } ABCD = \frac{1}{2} \times (AD + BC) \times h$$

$$= \frac{1}{2} \times (9 + 5) \times 3$$

$$= 21 \text{ units}^2$$

(iv) $$AB = \sqrt{(3 - 0)^2 + [-1 - (-3)]^2}$$

$$= \sqrt{3^2 + 2^2}$$

$$= \sqrt{13}$$

$$= 3.61 \text{ units (to 3 s.f.)}$$

(v) $$\text{Gradient of } AB = \frac{2}{3}$$

y-intercept $= -3$

$$\therefore y = \frac{2}{3} - 3$$

$$3y = 2x - 9$$

6. **(a)** Time taken by the coach to travel from Watertown to Sandcity

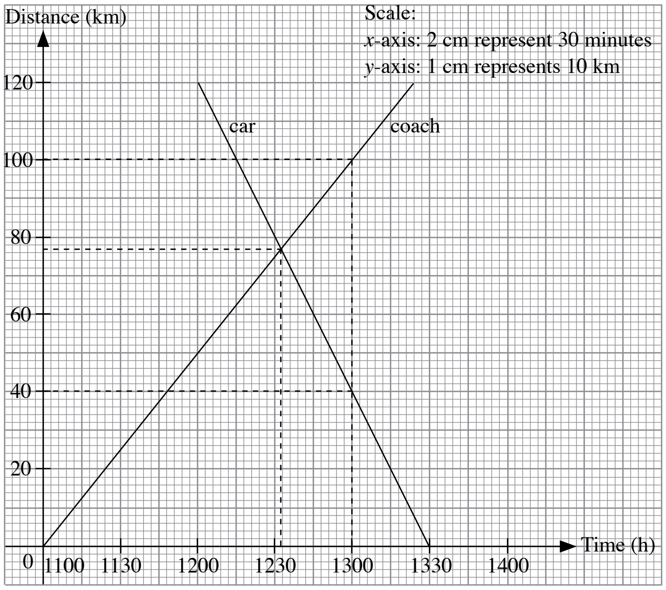
$= \dfrac{120}{50}$

$= 2.4$ h

Time taken by the car to travel from Sandcity to Watertown

$= \dfrac{120}{80}$

$= 1.5$ h



(b) **(i)** From the graph, the time when the car meets the coach is 1233 and the distance from Watertown at this instant is 77 km.

(ii) From the graph, the distance between the coach and the car at 1300 is 60 km.

7. **(i)** Let the total time taken for the journey be t s.

Deceleration = 0.6 m/s^2

$$\frac{27 - 0}{t - 50} = 0.6$$

$$27 = 0.6(t - 50)$$

$$27 = 0.6t - 30$$

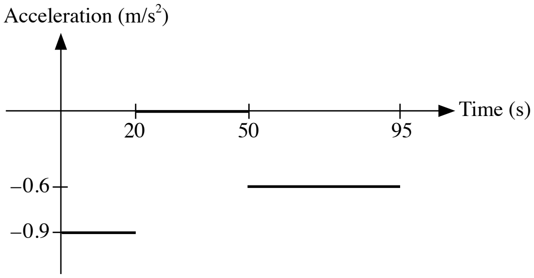
$$0.6t = 57$$

$$t = 95 \text{ s}$$

$$= 1 \text{ minute } 35 \text{ seconds}$$

$\therefore$ The total time taken for the journey is 1 minute 35 seconds.

(ii) Acceleration during the first 20 s $= \dfrac{27 - 45}{20} = -0.9$ m/s^2

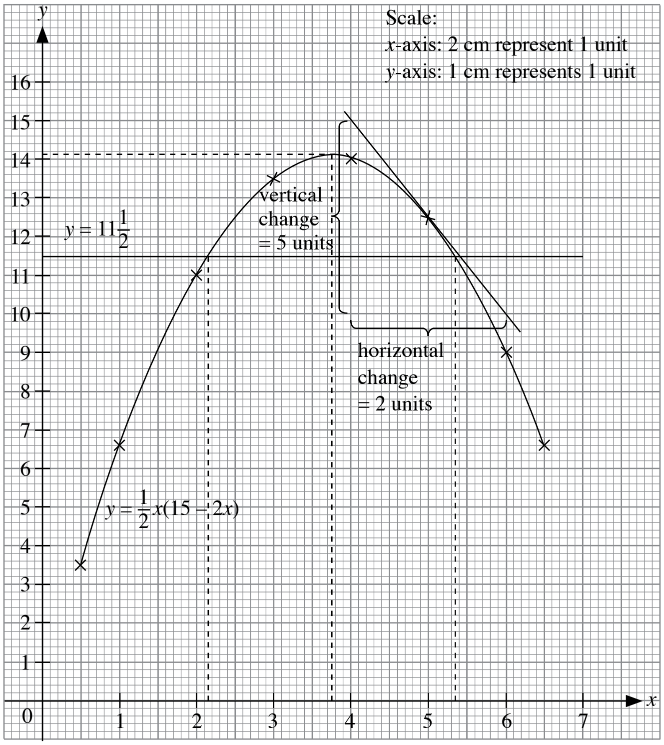


8. **(i)** When $x = 6.5$,

$$y = \frac{1}{2}(6.5)[15 - 2(6.5)] = 6.5$$

$\therefore p = 6.5$

(ii)



(iii) $x(15 - 2x) = 23$

$$\frac{1}{2}x(15 - 2x) = \frac{1}{2}(23)$$

$$\frac{1}{2}x(15 - 2x) = 11\frac{1}{2}$$

$$y = 11\frac{1}{2}$$

From the graph, the solutions of the equation $x(15 - 2x) = 23$ are 2.15 and 5.35.

(iv) A tangent is drawn to the curve at the point (5, 12.5).

From the graph,

Gradient $= -\dfrac{5}{2} = -2.5$

(v) From the graph, the maximum value of $y = 14.1$ where $x = 3.75$.

(vi) Hence, when the area is a maximum, the area is 14.1 cm^2 and the length of one side of the rectangle is 3.75 cm.

Length of other side of the rectangle $\dfrac{14.1}{3.75} = 3.76$ cm

$\therefore$ The dimensions of the rectangle are 3.75 cm by 3.76 cm.

The rectangle is a square.

Teachers may wish to highlight to the students that although the dimensions differ by 0.01 cm, based on calculations instead of using the graph, the rectangle is a square when the area is a maximum.

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Chapter 8 Further Trigonometry

TEACHING NOTES

Suggested Approach

Teachers may want to introduce this topic by asking students how measurements are obtained for towers and buildings in real life. Teachers can mention that such measurements can be easily obtained using trigonometry before introducing Sine Rule and Cosine Rule to them. This chapter exposes students to problems involving triangles which can be solved using trigonometric ratios of acute and obtuse angles.

Section8.1: Sine and Cosine of Obtuse Angles

Teachers may wish to recap with the students the trigonometric ratios of an acute angle before introducing the trigonometric ratios of the sine and cosine of obtuse angles. Before the students move on to solving simple trigonometric equations, teachers can guide them along in discovering the relationship between trigonometric ratios of acute and obtuse angles (see Investigation: Relationship between Trigonometric Ratios of Acute and Obtuse Angles).

Section 8.2: Area of Triangle

In primary school, students are taught to find the area of a triangle by the formula, $\frac{1}{2} \times \text{base} \times \text{height}$. Teachers can build upon this and ask the students to derive a formula with what they have learnt so far, such that the area of a triangle can still be calculated if the height of the triangle is not given. Some guidance will be needed while the students work among themselves to derive the formula.

Section 8.3: Sine Rule

Teachers can start off with an activity to introduce the Sine Rule to the students (see Investigation: Sine Rule).

In using a calculator, it is important to remind students to check and see that the MODE is set as DEG. Also, the examination requirements state that students are to give their answers correct to 3 significant figures and angles in degrees correct to 1 decimal place. Therefore, students should have developed the habit of working with 4 or 5 significant figures and angles in degrees to 2 decimal places and give their final answers correct to the required accuracy since Secondary 2.

Worked Example 8 provides an example of an ambiguous case of Sine Rule. Teachers should carefully go through this example with the students and highlight to them that for ambiguous cases, two sets of solutions will be obtained.

Section 8.4: Cosine Rule

Similar to Sine Rule, teachers can start off with an activity to introduce the Cosine Rule to the students (see Investigation: Cosine Rule). Teachers can show students how the Cosine Rule is derived when the angle is an acute angle, and then challenge the students' thinking further by asking them to prove the Cosine Rule where the angle is an obtuse angle (see Thinking Time on page 248 of the textbook).

Challenge Yourself

Students should recall the formula for finding the value of angles in a polygon to calculate some of the angles needed to find the lengths of HK and GK and then to find the area of $\triangle FGK$. Both Sine Rule and Cosine Rule, and the formula for the area of triangle which they have learnt in this chapter, will have to applied when solving this problem.

WORKED SOLUTIONS

Investigation (Relationship between Trigonometric Ratios of Acute and Obtuse Angles)

	A	$180° - A$	$\sin A$	$\sin (180° - A)$	$\cos A$	$\cos (180° - A)$
(a)	30°	150°	0.5	0.5	0.866 (to 3 s.f.)	–0.866 (to 3 s.f.)
(b)	76°	104°	0.970 (to 3 s.f.)	0.970 (to 3 s.f.)	0.242 (to 3 s.f.)	–0.242 (to 3 s.f.)
(c)	111°	69°	0.934 (to 3 s.f.)	0.934 (to 3 s.f.)	–0.358 (to 3 s.f.)	0.358 (to 3 s.f.)
(d)	167°	13°	0.225 (to 3 s.f.)	0.225 (to 3 s.f.)	–0.974 (to 3 s.f.)	0.974 (to 3 s.f.)

Table 8.1

1. $\sin A = \sin (180° - A)$
2. $\cos A = -\cos (180° - A)$

Thinking Time (Page 232)

(a) Since $6^2 + 8^2 = 10^2$, $\angle B = 90°$

Area of $\triangle ABC = \frac{1}{2} \times 6 \times 8 = 24$ cm^2

$s = \frac{1}{2}(6 + 8 + 10) = 12$

Area of $\triangle ABC = \sqrt{12(12-6)(12-8)(12-10)}$
$= 24$ cm^2

(b) $\angle A = \cos^{-1} \frac{10^2 + 9^2 - 8^2}{2(10)(9)} = 49.46°$ (to 2 d.p.)

Area of $\triangle ABC = \frac{1}{2} \times 10 \times 9 \times \sin 49.46°$
$= 34.2$ cm^2 (to 3 s.f.)

$s = \frac{1}{2}(8 + 9 + 10) = 13.5$

Area of $\triangle ABC = \sqrt{13.5(13.5-8)(13.5-9)(13.5-10)}$
$= 34.2$ cm^2 (to 3 s.f.)

(c) $\angle B = \cos^{-1} \frac{7^2 + 5^2 - 3^2}{2(7)(5)} = 21.79°$ (to 2 d.p.)

Area of $\triangle ABC = \frac{1}{2} \times 7 \times 5 \times \sin 21.79°$
$= 6.50$ cm^2 (to 3 s.f.)

$s = \frac{1}{2}(5 + 3 + 7) = 7.5$

Area of $\triangle ABC = \sqrt{7.5(7.5-3)(7.5-5)(7.5-7)}$
$= 6.50$ cm^2 (to 3 s.f.)

Investigation (Sine Rule)

1. **(b)** The length of the triangle opposite vertex B is labelled <u>b</u>.
   (c) The length of the triangle opposite vertex C is labelled <u>c</u>.
2. *Teachers may guide students to fill in the necessary information in the table.*
3. The 3 quantities are equal; or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
4. The 3 quantities are equal; or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
5. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
6. Yes. We can manipulate $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to obtain $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.
7. The lengths of the sides of a triangle are <u>proportional</u> to the sine of the angles opposite the sides.

Journal Writing (Page 242)

Sine rule can be used to solve a triangle when
Case 1: two angles and a side are given,
Case 2: two sides and an angle opposite a given side is given.

Investigation (Cosine Rule)

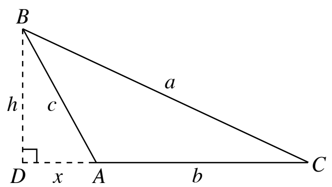
1. **(b)** The length of the triangle opposite vertex B is labelled <u>b</u>.
   (c) The length of the triangle opposite vertex C is labelled <u>c</u>.
2. *Teachers may guide students to fill in the necessary information in the table.*
3. The values in the 5th column are equal to the values in the 6th column.
   The values in the 7th column are equal to the values in the 8th column.
   The values in the 9th column are equal to the values in the 10th column.
4. The values in the 5th column are equal to the values in the 6th column.
   The values in the 7th column are equal to the values in the 8th column.
   The values in the 9th column are equal to the values in the 10th column.
5. $a^2 = b^2 + c^2 - 2bc \cos A$
   $b^2 = a^2 + c^2 - 2ac \cos B$
   $c^2 = a^2 + b^2 - 2ab \cos C$
6. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

   $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

   $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Thinking Time (Page 248)

1. When $\angle A$ is an obtuse angle,



In $\triangle BCD$,
$a^2 = h^2 + (b + x)^2$ (Pythagoras' Theorem)
$= h^2 + b^2 + 2bx + x^2$
$= b^2 + (h^2 + x^2) + 2bx \quad$ — (1)

In $\triangle BAD$,
$c^2 = h^2 + x^2$ (Pythagoras' Theorem) — (2)

and $\cos A = -\cos(180° - \angle BAD) = -\frac{x}{c}$,

i.e. $x = -c \cos A$ — (3)

Substituting (2) and (3) into (1),

$a^2 = b^2 + (h^2 + x^2) + 2bx$
$= b^2 + c^2 - 2bc \cos A$ (proven)

2. **(a)** If $A = 90°$,
$a^2 = b^2 = c^2 - 2bc \cos 90°$
$a^2 = b^2 + c^2 - 2bc(0)$
$a^2 = b^2 + c^2$

(b) Yes, this is because $\cos 90° = 0$.

3. Pythagoras' Theorem is a special case of the Cosine Rule.

Practise Now 1

1. **(a)** $\sin 96° = \sin(180° - 96°)$
$= \sin 84°$
$= 0.995$

(b) $\cos 51° = -\cos(180° - 51°)$
$= -\cos 129°$
$= -(-0.629)$
$= 0.629$

2. $\sin 8° - \cos 140° = \sin(180° - 8°) - [-\cos(180° - 140°)]$
$= \sin 172° - (-\cos 40°)$
$= \sin 172° + \cos 40°$
$= 0.139 + 0.766$
$= 0.905$

Practise Now 2

1. **(a)** $\sin \angle ACD = \sin(180° - \angle ACD)$
$= \sin \angle ACB$
$= \frac{\text{opp}}{\text{hyp}}$
$= \frac{AB}{AC}$
$= \frac{3}{5}$

(b) $\cos \angle ACD = -\cos(180° - \angle ACD)$
$= -\cos \angle ACB$
$= -\frac{\text{adj}}{\text{hyp}}$
$= -\frac{BC}{AC}$
$= -\frac{4}{5}$

(c) $\tan \angle BAC = \frac{\text{opp}}{\text{adj}}$
$= \frac{BC}{AB}$
$= \frac{4}{3}$
$= 1\frac{1}{3}$

2. **(a)** $BC = \sqrt{[8-(-4)]^2 + (1-6)^2}$
$= \sqrt{12^2 + (-5)^2}$
$= \sqrt{169}$
$= 13$ units

(b) **(i)** $\sin \angle HBC = \sin(180° - \angle HBC)$
$= \sin \angle ABC$
$= \frac{\text{opp}}{\text{hyp}}$
$= \frac{AC}{BC}$
$= \frac{12}{13}$

(ii) $\cos \angle BCK = -\cos(180° - \angle BCK)$
$= -\cos \angle ACB$
$= -\frac{\text{adj}}{\text{hyp}}$
$= -\frac{BC}{AC}$
$= -\frac{4}{5}$

(ii) $\tan \angle ABC = \frac{\text{opp}}{\text{adj}}$
$= \frac{AC}{AB}$
$= \frac{12}{5}$
$= 2\frac{2}{5}$

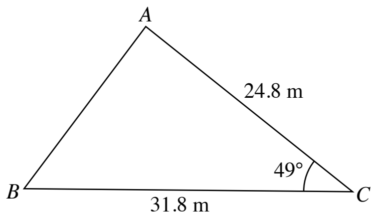
Practise Now 3

(a) Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle.
$\sin x = 0.415$
$x = \sin^{-1} 0.415 = 24.5°$ (to 1 d.p.)
or $180° - 24.5° = 155.5°$ (to 1 d.p.)
$\therefore x = 24.5°$ or $155.5°$

(b) Since $\cos x$ is negative, x is an obtuse angle.
$\cos x = -0.234$
$x = \cos^{-1}(-0.234) = 103.5°$ (to 1 d.p.)

(c) Since $\cos x$ is positive, x is an acute angle.
$\cos x = 0.104$
$x = \cos^{-1}(0.104) = 84.0°$ (to 1 d.p.)

Practise Now 4



We have $a = 31.8$, $b = 24.8$ and $C = 49°$.

$$\text{Area of } \triangle ABC = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 31.8 \times 24.8 \times \sin 49°$$
$$= 298 \text{ m}^2$$

Practise Now 5

1. We have $q = 2x$, $r = x$ and $P = 104°$.

$$\text{Area of } \triangle PQR = \frac{1}{2}qr\sin P$$
$$12.5 = \frac{1}{2} \times 2x \times x \times \sin 104°$$
$$= x^2 \sin 104°$$
$$x^2 = \frac{12.5}{\sin 104°}$$
$$x = \sqrt{\frac{12.5}{\sin 104°}} \text{ (since } x \text{ is positive)}$$
$$= 3.59 \text{ (to 3 s.f.)}$$

2. $$\text{Area of } \triangle XYZ = \frac{1}{2}xz\sin \angle XYZ$$
$$12 = \frac{1}{2} \times 6 \times 5 \times \sin \angle XYZ$$
$$12 = 15 \sin \angle XYZ$$
$$\sin \angle XYZ = \frac{12}{15}$$
$$\angle XYZ = \sin^{-1}\left(\frac{12}{15}\right)$$
$$= 53.1° \text{ (to 1 d.p.)}$$

Practise Now 6

(i) $\angle A = 180° - 58.3° - 39.4°$ ($\angle$ sum of a $\triangle$)
$= 82.3°$

(ii) Using sine rule,

$$\frac{c}{\sin \angle C} = \frac{a}{\sin \angle A}$$
$$\frac{c}{\sin 39.4°} = \frac{12.5}{\sin 82.3°}$$
$$c = \frac{12.5 \sin 39.4°}{\sin 82.3°}$$
$$= 8.01 \text{ cm (to 3 s.f.)}$$
$\therefore AB = 8.01$ cm

(iii) Using sine rule,

$$\frac{b}{\sin \angle B} = \frac{a}{\sin \angle A}$$
$$\frac{b}{\sin 58.3°} = \frac{12.5}{\sin 82.3°}$$
$$b = \frac{12.5 \sin 58.3°}{\sin 82.3}$$
$$= 10.7 \text{ cm (to 3 s.f.)}$$
$\therefore AC = 10.7$ cm

Practise Now 7

1. (i) Using sine rule,

$$\frac{\sin \angle R}{r} = \frac{\sin \angle Q}{q}$$
$$\frac{\sin \angle R}{10.2} = \frac{\sin 42°}{12}$$
$$\sin \angle R = \frac{10.2 \sin 42°}{12}$$
$$= 0.5688 \text{ (to 4 s.f.)}$$
$\angle R = \sin^{-1} 0.5688 = 34.67°$ (to 2 d.p.)
or $180° - 34.67° = 145.33°$ (to 2 d.p.)
Since $r < q$, $\angle R < \angle Q$, hence $\angle R$ cannot be 145.33°.
$\therefore \angle R = 34.7°$ (to 1 d.p.)

(ii) $\angle P = 180° - 42° - 34.67°$ ($\angle$ sum of a $\triangle$)
$= 103.3°$ (to 1 d.p.)

(iii) Using sine rule,

$$\frac{p}{\sin \angle P} = \frac{q}{\sin \angle Q}$$
$$\frac{p}{\sin 103.33°} = \frac{12}{\sin 42°}$$
$$p = \frac{12 \sin 103.33°}{\sin 42°}$$
$$= 17.5 \text{ cm (to 3 s.f.)}$$
$QR = 17.5$ cm

2. (i) Using sine rule,

$$\frac{\sin \angle B}{b} = \frac{\sin \angle A}{a}$$
$$\frac{\sin \angle B}{12.4} = \frac{\sin 96.8°}{15.6}$$
$$\sin \angle B = \frac{12.4 \sin 96.8°}{15.6}$$
$$= 0.7893 \text{ (to 4 s.f.)}$$
$\angle B = \sin^{-1} 0.7893 = 52.12°$ (to 2 d.p.)
or $180° - 52.12° = 127.88°$ (to 2 d.p.)
Since $b < c$, $\angle B < \angle C$, hence $\angle B$ cannot be 127.88°.
$\therefore \angle ABC = 52.1°$ (to 1 d.p.)

(ii) $\angle BCA = 180° - 96.8° - 51.12°$ ($\angle$ sum of a $\triangle$)
$= 31.1°$ (to 1 d.p.)

(iii) Using sine rule,

$$\frac{c}{\sin \angle C} = \frac{a}{\sin \angle A}$$

$$\frac{c}{\sin 31.08°} = \frac{15.6}{\sin 96.8°}$$

$$c = \frac{15.6 \sin 31.08°}{\sin 96.8°}$$

$= 8.11$ cm (to 3 s.f.)

$AB = 8.11$ cm

Practise Now 8

Using sine rule,

$$\frac{\sin \angle ACB}{9.8} = \frac{\sin 46°}{7.1}$$

$$\sin \angle ACB = \frac{9.8 \sin 46°}{7.1}$$

$= 0.9929$ (to 4 s.f.)

$\angle ACB = \sin^{-1} 0.9929 = 83.17°$ (to 2 d.p.)

or $180° - 83.17° = 96.83°$ (to 2 d.p.) (Since $c > b$, $\angle C < \angle B$, i.e. $\angle ACB > 46°$, hence both answers are possible.)

When $\angle ACB$ is $83.17°$, $\angle BAC = 180° - 46° - 83.17°$

$= 50.83°$

When $\angle ACB$ is $96.83°$, $\angle BAC = 180° - 46° - 96.83°$

$= 37.17°$

Case 1: when $\angle ACB = 83.17°$ and $\angle BAC = 50.83°$

Using sine rule,

$$\frac{a}{\sin 50.83°} = \frac{9.8}{\sin 83.17°}$$

$$a = \frac{9.8 \sin 50.83°}{\sin 83.17°}$$

$= 7.65$ cm (to 3 s.f.)

$\therefore ACB = 83.2°$, $\angle BAC = 50.8°$ and $BC = 7.65$ cm

Case 2: when $\angle ACB = 96.82°$ and $\angle BAC = 37.17°$

Using sine rule,

$$\frac{a}{\sin 37.17°} = \frac{9.8}{\sin 96.83°}$$

$$a = \frac{9.8 \sin 37.17°}{\sin 96.83°}$$

$= 5.96$ cm (to 3 s.f.)

$\therefore \angle ACB = 96.8°$, $\angle BAC = 37.2°$ and $BC = 5.96$ cm

Practise Now 9

(i) Using cosine rule,

$q^2 = p^2 + r^2 - 2pr \cos Q$

$PR^2 = 15.9^2 + 10.8^2 - 2 \times 15.9 \times 10.8 \times \cos 71°$

$= 257.6$ (to 4 s.f.)

$\therefore PR = \sqrt{257.6}$

$= 16.1$ cm (to 3 s.f.)

(ii) Using sine rule,

$$\frac{\sin \angle QPR}{QR} = \frac{\sin \angle PQR}{PR}$$

$$\frac{\sin \angle QPR}{15.9} = \frac{\sin 71°}{16.05}$$

$$\sin \angle QPR = \frac{15.9 \sin 71°}{16.05}$$

$= 0.9367$ (to 4 s.f.)

$\angle QPR = \sin^{-1} 0.9367 = 69.50°$ (to 2 d.p.)

or $180° - 69.50° = 110.5°$ (to 2 d.p.)

Since $QR < PR$, $\angle P < \angle Q$, hence $\angle P$ cannot be $110.5°$.

$\therefore \angle QPR = 69.5°$ (to 1 d.p.)

(iii) $\angle PRQ = 180° - 71° - 69.50°$

$= 39.5°$ (to 1 d.p.)

Practise Now 10

The largest angle is the angle opposite the longest side, i.e. $\angle QPR$.

Using cosine rule,

$$\cos P = \frac{q^2 + r^2 - p^2}{2qr}$$

$$= \frac{11^2 + 13^2 - 18^2}{2 \times 11 \times 13}$$

$$= -\frac{17}{143}$$

$$\angle P = \cos^{-1}\left(-\frac{17}{143}\right)$$

$= 96.8°$ (to 1 d.p.)

$\therefore$ The largest angle is $96.8°$.

Exercise 8A

1. **(a)** $\sin 110° = \sin (180° - 110°)$
$= \sin 70°$

(b) $\sin 176° = \sin (180° - 176°)$
$= \sin 4°$

(c) $\sin 98° = \sin (180° - 98°)$
$= \sin 82°$

(d) $\cos 99° = -\cos (180° - 99°)$
$= -\cos 81°$

(e) $\cos 107° = -\cos (180° - 107°)$
$= -\cos 73°$

(f) $\cos 175° = -\cos (180° - 175°)$
$= -\cos 5°$

2. **(a)** $\sin 148° = \sin (180° - 148°)$
$= \sin 32°$
$= 0.530$

(b) $\cos 35° = -\cos (180° - 35°)$
$= -\cos 145°$
$= -(-0.819)$
$= 0.819$

3. **(a)** $2 \cos 45° + 3 \sin 135° = 2 \cos 45° + 3 \sin (180° - 135°)$
$= 2 \cos 45° + 3 \sin 45°$
$= 2(0.707) + 3(0.707)$
$= 1.414 + 2.121$
$= 3.535$

(b) $3 \cos 135° + 4 \sin 135°$
$= 3 \times -\cos (180° - 135°) + 4 \sin (180° - 135°)$
$= -3 \cos 45° + 4 \sin 45°$
$= -3(0.707) + 4(0.707)$
$= -2.121 + 2.828$
$= 0.707$

(c) $\cos 135° - 2 \sin 45° = -\cos (180° - 135°) - 2 \sin 45°$
$= -\cos 45° - 2 \sin 45°$
$= -0.707 - 2(0.707)$
$= -0.707 - 1.414$
$= -2.121$

4. **(a)** $\sin \angle ABD = \sin (180° - \angle ABD)$
$= \sin \angle CBD$
$= \dfrac{CD}{BD}$
$= \dfrac{8}{10}$
$= \dfrac{4}{5}$

(b) $\cos \angle DBA = -\cos (180° - \angle DBA)$
$= -\cos \angle CBD$
$= -\dfrac{BC}{BD}$
$= -\dfrac{6}{10}$
$= -\dfrac{3}{5}$

(c) $\tan \angle CBD = \dfrac{CD}{BC} = \dfrac{8}{6} = \dfrac{4}{3} = 1\dfrac{1}{3}$

5. **(a)** By Pythagoras' Theorem,
$PR^2 = PQ^2 + QR^2$
$41^2 = x^2 + 40^2$
$x^2 = 41^2 - 40^2$
$= 81$
$x = \sqrt{81}$ (since $x > 0$)
$= 9$

(b) **(i)** $\sin \angle PRS = \sin (180° - \angle PRS)$
$= \sin \angle PRQ$
$= \dfrac{PQ}{PR}$
$= \dfrac{9}{41}$

(ii) $\cos \angle PRS = -\cos (180° - \angle PRS)$
$= -\cos \angle PRQ$
$= -\dfrac{QR}{PR}$
$= -\dfrac{40}{41}$

(iii) $\tan \angle PRQ = \dfrac{PQ}{QR} = \dfrac{9}{40}$

6. **(a)** Since $\sin x$ is positive and it is an acute angle,
$\sin x = 0.52$
$x = \sin^{-1} 0.52 = 31.3°$ (to 1 d.p.)
$\therefore x = 31.3°$

(b) Since $\sin x$ is positive and it is an acute angle,
$\sin x = 0.75$
$x = \sin^{-1} 0.75 = 48.6°$ (to 1 d.p.)
$\therefore x = 48.6°$

(c) Since $\sin x$ is positive and it is an acute angle,
$\sin x = 0.875$
$x = \sin^{-1} 0.875 = 61.0°$ (to 1 d.p.)
$\therefore x = 61.0°$

(d) Since $\sin x$ is positive and it is an acute angle,
$\sin x = 0.3456$
$x = \sin^{-1} 0.3456 = 20.2°$ (to 1 d.p.)
$\therefore x = 20.2°$

7. **(a)** Since $\sin x$ is positive and it is an acute angle,
$\sin x = 0.52$
$x = \sin^{-1} 0.52 = 31.33°$ (to 2 d.p.)
$180° - 31.33° = 148.7°$ (to 1 d.p.)
$\therefore x = 148.7°$

(b) Since $\sin x$ is positive and it is an obtuse angle,
$\sin x = 0.75$
$x = \sin^{-1} 0.75 = 48.59°$ (to 2 d.p.)
$180° - 48.59° = 131.4°$ (to 1 d.p.)
$\therefore x = 131.4°$

(c) Since $\sin x$ is positive and it is an obtuse angle,
$\sin x = 0.875$
$x = \sin^{-1} 0.875 = 61.04°$ (to 2 d.p.)
$180° - 61.04° = 119.0°$ (to 1 d.p.)
$\therefore x = 119.0°$

(d) Since $\sin x$ is positive and it is an obtuse angle,
$\sin x = 0.3456$
$x = \sin^{-1} 0.3456 = 20.22°$ (to 2 d.p.)
$180° - 20.22° = 159.8°$ (to 1 d.p.)
$\therefore x = 159.8°$

8. **(a)** Since $\cos x$ is positive and it is an acute angle,
$\cos x = 0.67$
$x = \cos^{-1} 0.67$
$= 47.9°$ (to 1 d.p.)

(b) Since $\cos x$ is positive and it is an acute angle,
$\cos x = 0.756$
$x = \cos^{-1} 0.756$
$= 40.9°$ (to 1 d.p.)

(c) Since $\cos x$ is positive and it is an acute angle,
$\cos x = 0.5$
$x = \cos^{-1} 0.5$
$= 60°$

(d) Since $\cos x$ is positive and it is an acute angle,
$\cos x = 0.985$
$x = \cos^{-1} 0.985$
$= 9.9°$ (to 1 d.p.)

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9. **(a)** Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle,

$\sin x = 0.753$

$x = \sin^{-1} 0.753 = 48.9°$ (to 1 d.p.)

or $180° - 48.9° = 131.1°$ (to 1 d.p.)

$\therefore x = 48.9°$ or $131.1°$

(b) Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle,

$\sin x = 0.952$

$x = \sin^{-1} 0.952 = 72.2°$ (to 1 d.p.)

or $180° - 72.2° = 107.8°$ (to 1 d.p.)

$\therefore x = 72.2°$ or $107.8°$

(c) Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle,

$\sin x = 0.4714$

$x = \sin^{-1} 0.4714 = 28.1°$ (to 1 d.p.)

or $180° - 28.1° = 151.9°$ (to 1 d.p.)

$\therefore x = 28.1°$ or $151.9°$

(d) Since $\cos x$ is negative, x is an obtuse angle.

$\cos x = -0.238$

$x = \cos^{-1} (-0.238)$

$= 103.8°$ (to 1 d.p.)

(e) Since $\cos x$ is negative, x is an obtuse angle.

$\cos x = -0.783$

$x = \cos^{-1} (-0.783)$

$= 141.5°$ (to 1 d.p.)

(f) Since $\cos x$ is negative, x is an obtuse angle.

$\cos x = 0.524$

$x = \cos^{-1} 0.524$

$= 58.4°$ (to 1 d.p.)

10. **(a)** By Pythagoras' Theorem,

$PR^2 = PQ^2 + QR^2$

$PR^2 = 8^2 + 15^2$

$= 64 + 225$

$= 289$

$PR = \sqrt{289}$ (since $PR > 0$)

$= 17$

$\sin \angle PRS = \sin (180° - \angle PRS)$

$= \sin \angle PRQ$

$= \dfrac{PQ}{PR}$

$= \dfrac{8}{17}$

(b) $\cos \angle SRP = -\cos (180° - \angle SRP)$

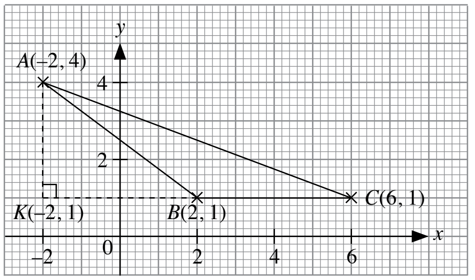
$= -\cos \angle PRQ$

$= -\dfrac{RQ}{PR}$

$= -\dfrac{15}{17}$

(c) $\tan \angle PRQ = \dfrac{PQ}{QR} = \dfrac{8}{15}$

11.



K is the point $(-2, 1)$.

$AK = 3$ units, $BK = 4$ units

By Pythagoras' Theorem,

$AB^2 = AK^2 + BK^2$

$= 3^2 + 4^2$

$= 25$

$AB = \sqrt{25}$

$= 5$ units

(a) $\sin \angle ABC = \sin (180° - \angle ABC)$

$= \sin \angle ABK$

$= \dfrac{AK}{AB}$

$= \dfrac{3}{5}$

(b) $\cos \angle ABC = -\cos (180° - \angle ABC)$

$= -\cos \angle ABK$

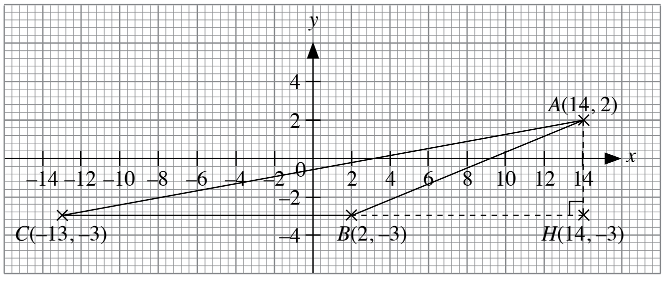
$= -\dfrac{BK}{AB}$

$= -\dfrac{4}{5}$

(c) $CK = 8$ units

$\tan \angle ACB = \dfrac{AK}{CK} = \dfrac{3}{8}$

12.



H is the point $(14, -3)$.

$AH = 5$ units, $BH = 12$ units

By Pythagoras' Theorem,

$AB^2 = AH^2 + BH^2$

$= 5^2 + 12^2$

$= 169$

$AB = \sqrt{169}$

$= 13$ units

(a) $\sin \angle ABC = \sin (180° - \angle ABC)$
$= \sin \angle ABH$
$= \dfrac{AH}{AB}$
$= \dfrac{5}{13}$

(b) $\cos \angle ABC = -\cos (180° - \angle ABC)$
$= -\cos \angle ABH$
$= -\dfrac{BH}{AB}$
$= -\dfrac{12}{13}$

(c) $CH = 27$ units
$\tan \angle ACB = \dfrac{AH}{CH} = \dfrac{5}{27}$

13. $\sin x° = \sin 27° = 0.4540$ (to 4 s.f.)
Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle.
$x = 27°$
or $180° - 27° = 153°$
$\therefore x = 27°$ or $153°$

14. (a) Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle.
$\sin (x + 10°) = 0.47$
$x + 10° = \sin^{-1} 0.47 = 28.0°$ (to 1 d.p.)
$x = 28.0° - 10° = 18.0°$
or $x + 10° = 180° - 28.0° = 152.0°$ (to 1 d.p.)
$x = 152.0° - 10° = 142.0°$
$\therefore x = 18.0°$ or $142.0°$

(b) Since $\cos x$ is negative, x is an obtuse angle.
$\cos (x - 10°) = -0.56$
$x - 10° = \cos^{-1} (-0.56) = 124.1°$ (to 1 d.p.)
$x = 124.1° + 10° = 134.1°$

Exercise 8B

1. (a) Area of $\triangle ABC = \dfrac{1}{2} bc \sin A$
$= \dfrac{1}{2} \times 9 \times 8 \times \sin 72°$
$= 34.2 \text{ cm}^2$ (to 3 s.f.)

(b) Area of $\triangle DEF = \dfrac{1}{2} df \sin E$
$= \dfrac{1}{2} \times 9 \times 7 \times \sin 111°$
$= 29.4 \text{ cm}^2$ (to 3 s.f.)

(c) $\angle H = 180° - 62° - 57°$ ($\angle$sum of $\triangle$)
$= 61°$
Area of $\triangle GHI = \dfrac{1}{2} gi \sin H$
$= \dfrac{1}{2} \times 10 \times 9.5 \times \sin 61°$
$= 41.5 \text{ m}^2$ (to 3 s.f.)

(d) $\angle K = 180° - 105° - 28°$ ($\angle$sum of $\triangle$)
$= 47°$
Area of $\triangle JKL = \dfrac{1}{2} jl \sin K$
$= \dfrac{1}{2} \times 13.35 \times 6.5 \times \sin 47°$
$= 31.7 \text{ m}^2$ (to 3 s.f.)

(e) Area of $MNOP = 2 \times$ Area of $\triangle PMN$
$= 2\left(\dfrac{1}{2} \times PM \times MN \times \sin \angle PMN\right)$
$= 2\left(\dfrac{1}{2} \times 5.3 \times 5.8 \times \sin 117°\right)$
$= 27.4 \text{ cm}^2$ (to 3 s.f.)

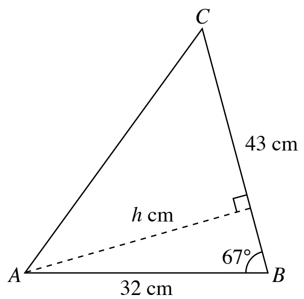
(f) Area of $QRST = 2 \times$ Area of $\triangle TQR$
$= 2\left(\dfrac{1}{2} \times TQ \times QR \times \sin \angle TQR\right)$
$= 2\left(\dfrac{1}{2} \times 8.5 \times 8.5 \times \sin 78°\right)$
$= 70.7 \text{ m}^2$ (to 3 s.f.)

2. Area of $\triangle ABC = \dfrac{1}{2} \times AB \times AC \times \sin \angle BAC$
$= \dfrac{1}{2} \times 22 \times 15 \times \sin 45°$
$= 117 \text{ cm}^2$ (to 3 s.f.)

3. Area of $\triangle PQR = \dfrac{1}{2} qr \sin \angle P$
$= \dfrac{1}{2} \times 152 \times 125 \times \sin 72°$
$= 9040 \text{ cm}^2$ (to 3 s.f.)

4. (i) Area of $\triangle ABC = \dfrac{1}{2} \times AB \times BC \times \sin \angle ABC$
$= \dfrac{1}{2} \times 32 \times 43 \times \sin 67°$
$= 633 \text{ cm}^2$ (to 3 s.f.)

(ii) Let the perpendicular distance from A to BC be h cm.



Area of $\triangle ABC = 633.3 \text{ cm}^2$
$\dfrac{1}{2} \times BC \times h = 633.3$
$\dfrac{1}{2} \times 43 \times h = 633.3$
$21.5h = 633.3$
$h = 29.5$ (to 3 s.f.)
$\therefore$ The perpendicular distance from A to BC is 29.5 cm.

5. Total area $= \frac{1}{2} \times 112 \times 202 \times \sin 30° + \frac{1}{2} \times 202 \times 197 \times \sin 60.5°$
$= 5646 + 17\ 317$
$= 23\ 000\ \text{m}^2$ (to 3 s.f.)

6. (i) $\sin \angle ACD = \frac{3.7}{8.0}$

$\angle ACD = \sin^{-1} \frac{3.7}{8.0}$
$= 27.5°$ (to 1 d.p.)

(ii) $\cos \angle BAC = \frac{AC}{AB}$

$\cos 40.4° = \frac{8.0}{AB}$

$AB = \frac{8.0}{\cos 40.4°}$
$= 10.5$ cm (to 3 s.f.)

(iii) Area of $\triangle AED = \frac{1}{2} \times AE \times AD \times \sin \angle DAE$
$= \frac{1}{2} \times 4.1 \times 3.7 \times \sin 55.1°$
$= 6.22\ \text{cm}^2$ (to 3 s.f.)

7. Area of $\triangle ABC = 97\ \text{cm}^2$

$\frac{1}{2} \times AB \times AC \times \sin \angle BAC = 97$

$\frac{1}{2} \times 5x \times 4x \times \sin 68° = 97$

$10 \sin 68° \times x^2 = 97$

$x^2 = \frac{97}{10 \sin 68°}$

$x = \sqrt{\frac{97}{10 \sin 68°}}$ (since $x > 0$)

$= 3.23$ (to 3 s.f.)

8. (i) $\angle ABD = 180° - \angle ACB - \angle BAC$ ($\angle$ sum of $\triangle ABC$)
$= 180° - 30° - 90°$
$= 60°$

$\angle BAD = 180° - \angle ABD - \angle ADB$ ($\angle$sum of $\triangle ABD$)
$= 180° - 60° - 90°$
$= 30°$

(ii) $\sin \angle BAD = \frac{BD}{AB}$

$\sin 30° = \frac{BD}{20}$

$BD = 20 \sin 30°$
$= 10$ cm

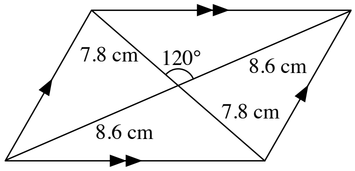
(iii) $\sin \angle ACB = \frac{AB}{BC}$

$\sin 30° = \frac{20}{BC}$

$BC = \frac{20}{\sin 30°}$
$= 40$ cm

Area of $\triangle ABC = \frac{1}{2} \times AB \times BC \times \sin \angle ABD$
$= \frac{1}{2} \times 20 \times 40 \times \sin 60°$
$= 346\ \text{cm}^2$ (to 3 s.f.)

9.



Area of parallelogram $= 4\left(\frac{1}{2} \times 7.8 \times 8.6 \times \sin 120°\right)$
$= 116\ \text{cm}^2$ (to 3 s.f.)

10. Let the length of QR be x cm.

$QR = x$ cm
$3QR = 4PR$
$3x = 4PR$
$PR = \frac{3}{4}x$ cm

Area of $\triangle PQR = 158\ \text{cm}^2$

$\frac{1}{2} \times PR \times QR \times \sin PRQ = 158$

$\frac{1}{2} \times \frac{3}{4} x \times x \times \sin 55° = 158$

$x^2 \times \frac{3}{8} \sin 55° = 158$

$x^2 \times 3 \sin 55° = 1264$

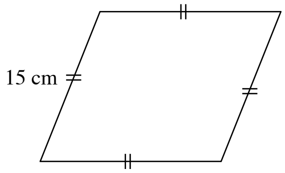
$x^2 = \frac{1264}{3 \sin 55°}$

$x = \sqrt{\frac{1264}{3 \sin 55°}}$ (since $x > 0$)

$= 22.7$ (to 3 s.f.)

$\therefore$ The length of QR is 22.7 cm.

11. Let the angle of one side of the rhombus be $a°$.



Area of rhombus $= 40\ \text{cm}^2$

$2\left(\frac{1}{2} \times 15 \times 15 \times \sin a°\right) = 40$

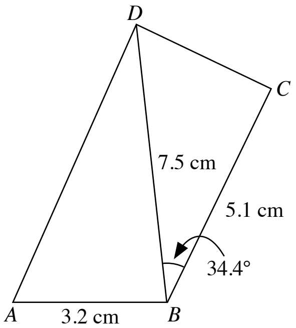
$225 \sin a° = 40$

$\sin a° = \frac{40}{225}$

$a° = \sin^{-1}\left(\frac{40}{225}\right)$
$= 10.2°$ (to 1 d.p.)

Angles of rhombus $= 10.2°$ and $180° - 10.2° = 169.8°$

12. (i)



$$\text{Area of } \triangle BCD = \frac{1}{2} \times BC \times BD \times \sin \angle CBD$$
$$= \frac{1}{2} \times 5.1 \times 7.5 \times \sin 34.4°$$
$$= 10.8 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(ii)
$$\text{Area of } \triangle ABD = 11.62 \text{ cm}^2$$
$$\frac{1}{2} \times AB \times BD \times \sin \angle ABD = 11.62$$
$$\frac{1}{2} \times 3.2 \times 7.5 \times \sin \angle ABD = 11.62$$
$$12 \sin \angle ABD = 11.26$$
$$\sin \angle ABD = \frac{11.62}{12}$$
$$= 0.9683 \text{ (to 4 s.f.)}$$

$\angle ABD = \sin^{-1} 0.9683 = 75.5°$ (to 1 d.p.)

or $\angle ABD = 180° - 75.5° = 104.5°$ (to 1 d.p.)

Given that $\angle ABD$ is obtuse,

$\therefore \angle ABD = 104.5°$

Exercise 8C

1. (a) $\angle C = 180° - 76° - 42°$ ($\angle$ sum of $\triangle$)

$= 62°$

Using sine rule,

$$\frac{b}{\sin 76°} = \frac{7.4}{\sin 42°}$$
$$b = \frac{7.4 \sin 76°}{\sin 42°}$$
$$= 10.7 \text{ cm (to 3 s.f.)}$$

Using sine rule,

$$\frac{c}{\sin 62°} = \frac{7.4}{\sin 42°}$$
$$c = \frac{7.4 \sin 62°}{\sin 42°}$$
$$= 9.76 \text{ cm (to 3 s.f.)}$$

$\therefore \angle C = 62°, b = 10.7$ cm, $c = 9.76$ cm

(b) $\angle F = 180° - 62° - 38.7°$ ($\angle$ sum of $\triangle$)

$= 79.3°$

Using sine rule,

$$\frac{d}{\sin 38.7°} = \frac{6.25}{\sin 62°}$$
$$d = \frac{6.25 \sin 38.7°}{\sin 62°}$$
$$= 4.43 \text{ m (to 3 s.f.)}$$

Using sine rule,

$$\frac{f}{\sin 79.3°} = \frac{6.25}{\sin 62°}$$
$$f = \frac{6.25 \sin 79.3°}{\sin 62°}$$
$$= 6.96 \text{ m (to 3 s.f.)}$$

$\therefore \angle F = 79.3°, d = 4.43$ m, $f = 6.96$ m

(c) $\angle H = 180° - 188° - 24°$ ($\angle$ sum of $\triangle$)

$= 38°$

Using sine rule,

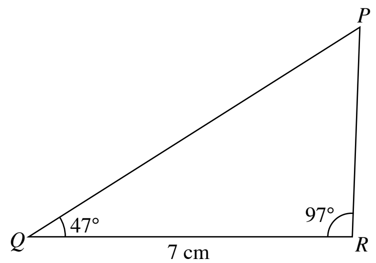
$$\frac{g}{\sin 118°} = \frac{8}{\sin 38°}$$
$$g = \frac{8 \sin 118°}{\sin 38°}$$
$$= 11.5 \text{ mm (to 3 s.f.)}$$

Using sine rule,

$$\frac{i}{\sin 24°} = \frac{8}{\sin 38°}$$
$$i = \frac{8 \sin 24°}{\sin 38°}$$
$$= 5.29 \text{ mm (to 3 s.f.)}$$

$\therefore \angle H = 38°, g = 11.5$ mm, $t = 5.29$ mm

2.



$\angle QPR = 180° - 97° - 47°$ ($\angle$ sum of $\triangle$)

$= 36°$

Using sine rule,

$$\frac{PQ}{\sin 97°} = \frac{7}{\sin 36°}$$
$$PQ = \frac{7 \sin 97°}{\sin 36°}$$
$$= 11.8 \text{ cm (to 3 s.f.)}$$

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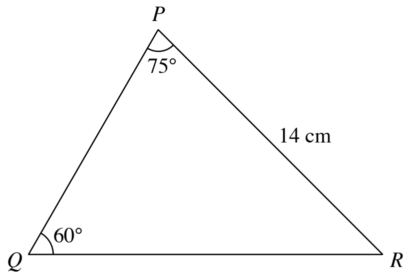
3.



$\angle R = 180° - 60° - 75°$ ($\angle$sum of $\triangle$)

$= 45°$

The longest side is the side opposite the largest angle, i.e. QR.

Using sine rule,

$$\frac{QR}{\sin 75°} = \frac{14}{\sin 60°}$$

$$QR = \frac{14 \sin 75°}{\sin 60°}$$

$= 15.6$ cm (to 3 s.f.)

4. **(a)** Using sine rule,

$$\frac{\sin \angle B}{b} = \frac{\sin \angle A}{a}$$

$$\frac{\sin \angle B}{6.93} = \frac{\sin 92.0°}{15.3}$$

$$\sin \angle B = \frac{6.93 \sin 92.0°}{15.3}$$

$= 0.4527$ (to 4 s.f.)

$\angle B = \sin^{-1} 0.4527 = 26.92°$ (to 2 d.p.)

Or $180° - 26.92° = 153.08°$ (to 2 d.p.)

Since $a > b$, then $\angle A > \angle B$, i.e. $\angle B < 92.0°$, hence $\angle B$ cannot be 153.08°,

$\therefore \angle B = 26.9°$ (to 1 d.p.)

$\angle C = 180° - 92.0° - 26.92°$

$= 61.1°$ (to 1 d.p.)

Using sine rule,

$$\frac{c}{\sin 61.08°} = \frac{15.3}{\sin 92.0°}$$

$$c = \frac{15.3 \sin 61.08°}{\sin 92.0°}$$

$= 13.4$ cm (to 3 s.f.)

$\therefore \angle B = 26.9°, \angle C = 61.1°, c = 13.4$ cm

(b) Using sine rule,

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b}$$

$$\frac{\sin \angle A}{14.5} = \frac{\sin 98.0°}{17.4}$$

$$\sin \angle A = \frac{14.5 \sin 98.0°}{17.4}$$

$= 0.8252$ (to 4 s.f.)

$\angle A = \sin^{-1} 0.8252 = 55.61°$ (to 2 d.p.)

Or $180° - 55.61° = 124.39°$ (to d.p.)

Since $a < b$, then $\angle A < \angle B$, i.e. $\angle A < 98.0°$, hence $\angle A$ cannot be 124.39°.

$\therefore \angle A = 55.6°$ (to 1 d.p.)

$\angle C = 180° - 98.0° - 55.61°$

$= 26.4°$ (to 1 d.p.)

Using sine rule,

$$\frac{c}{\sin 26.39°} = \frac{17.4}{\sin 98.0°}$$

$$c = \frac{17.4 \sin 26.39°}{\sin 98.0°}$$

$= 7.81$ m (3 s.f.)

$\therefore \angle A = 55.6°, \angle C = 26.4°, c = 7.81$ m

(c) Using sine rule,

$$\frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

$$\frac{\sin \angle B}{8.7} = \frac{\sin 35.0°}{9.5}$$

$$\sin \angle B = \frac{8.7 \sin 35.0°}{9.5}$$

$= 0.5253$ (to 4 s.f.)

$\angle B = \sin^{-1} 0.5253 = 31.69°$ (to 2 d.p.)

Or $180° - 31.69° = 148.31°$ (to 2 d.p.)

Since $b < c$, then $\angle B > \angle C$. i.e. $\angle B < 35.0°$, hence $\angle B$ cannot be 148.31°.

$\therefore \angle B = 31.7°$ (to 1 d.p.)

$\angle A = 180° - 35.0° - 31.69°$

$= 113.3°$ (to 1 d.p.)

Using sine rule,

$$\frac{a}{\sin 113.3°} = \frac{9.5}{\sin 35.0°}$$

$$a = \frac{9.5 \sin 113.3°}{\sin 35.0°}$$

$= 15.2$ cm (to 3 s.f.)

$\therefore \angle A = 113.3°, \angle B = 31.7°, a = 15.2$ cm

5. **(i)** Using sine rule,

$$\frac{13.4}{\sin \angle R} = \frac{20.8}{\sin 101°}$$

$$\sin \angle R = \frac{13.4 \sin 101°}{20.8}$$

$= 0.6324$ (to 4 s.f.)

$\angle R = \sin^{-1} 39.23° = 140.77°$ (to 2 d.p.)

Since $PQ < RQ$, then $\angle R < \angle P$, i.e. $\angle R < 101°$, hence $\angle R$ cannot be 140.77°.

$\therefore \angle R = 39.2°$ (to 1 d.p.)

(ii) $\angle Q = 180° - 101° - 39.23°$

$= 39.8°$ (to 1 d.p.)

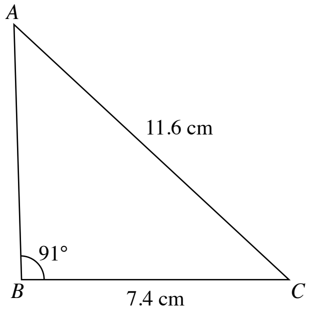
(iii) Using sine rule,

$$\frac{PR}{\sin 39.77°} = \frac{20.8}{\sin 101°}$$

$$PR = \frac{20.8 \sin 39.77°}{\sin 101°}$$

$= 13.6$ cm (to 3 s.f.)

6. **(i)**



Using sine rule,

$$\frac{\sin \angle BAC}{7.4} = \frac{\sin 91°}{11.6}$$

$$\sin \angle BAC = \frac{7.4 \sin 91°}{11.6}$$

$$= 0.6378 \text{ (to 4 s.f.)}$$

$\angle BAC = \sin^{-1} 0.6378 = 39.63°$ (to 2 d.p.)

Or $180° - 39.63° = 140.37°$ (to 2 d.p.)

Since $BC < AC$, then $\angle BAC < \angle ABC$, i.e. $\angle BAC < 91°$, hence $\angle BAC$ cannot be $140.37°$.

$\therefore \angle BAC = 39.6°$ (to 1 d.p.)

(ii) $\angle ACB = 180° - 91° - 39.63°$

$= 49.4°$ (to 1 d.p.)

(iii) Using sine rule,

$$\frac{AB}{\sin 49.37°} = \frac{11.6}{\sin 91°}$$

$$AB = \frac{11.6 \sin 49.37°}{\sin 91°}$$

$$= 8.80 \text{ cm (to 3 s.f.)}$$

7. **(i)** $\sin 25° = \dfrac{5.3}{AB}$

$$AB = \frac{5.3}{\sin 25°}$$

$$= 12.5 \text{ m (to 3 s.f.)}$$

(ii) $\cos \angle DBN = \dfrac{5.3}{7.1}$

$$\angle DBN = \cos^{-1}\left(\frac{5.3}{7.1}\right)$$

$$= 41.7° \text{ (to 1 d.p.)}$$

(iii) $\angle BCD = 180° - 103° - 46°$ ($\angle$ sum of $\triangle$)

$= 31°$

Using sine rule,

$$\frac{CD}{\sin 103°} = \frac{7.1}{\sin 31°}$$

$$CD = \frac{7.1 \sin 103°}{\sin 31°}$$

$$= 13.4 \text{ m (to 3 s.f.)}$$

8. **(i)** $\angle BAC = 180° - 62° - 68°$ ($\angle$ sum of $\triangle$)

$= 50°$

Using sine rule,

$$\frac{AC}{\sin 62°} = \frac{6}{\sin 50°}$$

$$AC = \frac{6 \sin 62°}{\sin 50°}$$

$$= 6.92 \text{ m (to 3 s.f.)}$$

(ii) $\angle BCD = 180° - 68°$ (adj. $\angle$s on a str. line)

$= 112°$

Area of region

$= \dfrac{1}{2} \times AC \times BC \times \sin \angle ACB + \dfrac{1}{2} \times BC \times CD \times \sin \angle BCD$

$= \dfrac{1}{2} \times 6.916 \times 6 \times \sin 68° + \dfrac{1}{2} \times 6 \times 7.5 \times \sin 112°$

$= 19.24 + 20.86$

$= 40.1 \text{ m}^2$ (to 3 s.f.)

9. **(i)** $\angle ADB = 180° - 30° - 80°$ ($\angle$ sum of $\triangle$)

$= 70°$

Using sine rule,

$$\frac{AB}{\sin 70°} = \frac{5}{\sin 30°}$$

$$AB = \frac{5 \sin 70°}{\sin 30°}$$

$$= 9.40 \text{ cm (to 3 s.f.)}$$

(ii) $\angle BDC = 80° - 40°$ (ext. $\angle$ of $\triangle$)

$= 40°$

Since $\angle BDC = \angle BCD$, $\triangle BDC$ is an isosceles triangle.

$BC = BD = 5$ cm

(iii) $\angle CBD = 180° - 80°$ (adj. $\angle$s on a str. line)

$= 100°$

Let the vertical distance between the mass and the horizontal bar be h cm.

Area of $\triangle ADC$

$=$ Area of $\triangle ABD$ + Area of $\triangle BCD$

$= \dfrac{1}{2} \times 9.397 \times 5 \times \sin 80° + \dfrac{1}{2} \times 5 \times 5 \times \sin 100°$

$= 23.14 + 12.31$

$= 35.45 \text{ cm}^2$ (to 4 s.f.)

$$\frac{1}{2} \times AC \times h = 35.45$$

$$\frac{1}{2} \times (AB + BC) \times h = 35.45$$

$$\frac{1}{2} \times (9.397 + 5) \times h = 35.45$$

$$7.1985h = 35.45$$

$$h = 4.92 \text{ (to 3 s.f.)}$$

$\therefore$ The vertical distance between the mass and the horizontal bar is 4.92 cm.

10. **(i)** $\tan \angle PTS = \dfrac{PS}{ST} = \dfrac{4.3}{5.7}$

$$\angle PTS = \tan^{-1} \frac{4.3}{5.7}$$

$$= 37.03° \text{ (to 2 d.p.)}$$

$\angle QSR = 180° - 90° - 74°$ ($\angle$s on a straight line)

$= 16°$

Since $\angle PTS \neq \angle QSR$, QS is not parallel to PT.

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(ii) $\cos \angle SPR = \dfrac{PS}{PR}$

$\cos 63° = \dfrac{4.3}{PR}$

$PR = \dfrac{4.3}{\cos 63°}$

$= 9.47$ cm (to 3 s.f.)

(iii) $\angle PQS = 180° - 63° - 74°$ ($\angle$ sum of $\triangle$)

$= 43°$

Using sine rule,

$\dfrac{QS}{\sin 63°} = \dfrac{4.3}{\sin 43°}$

$QS = \dfrac{4.3 \sin 63°}{\sin 43°}$

$= 5.62$ cm (to 3 s.f.)

11. $\cos 73° = \dfrac{QR}{5.7}$

$QR = 5.7 \cos 73°$

$= 1.667$ km (to 4 s.f.)

$\angle QPS = 180° - 48° - 55°$ ($\angle$ sum of $\triangle$)

$= 77°$

$\dfrac{PQ}{\sin 55°} = \dfrac{5.7}{\sin 77°}$

$PQ = \dfrac{5.7 \sin 55°}{\sin 77°}$

$= 4.792$ km (to 4 s.f.)

Area of nature reserve

= Area of $\triangle QRS$ + Area of $\triangle PQS$

$= \dfrac{1}{2} \times 1.667 \times 5.7 \times \sin 73° + \dfrac{1}{2} \times 5.7 \times 4.792 \times \sin 48°$

$= 4.543 + 10.149$

$= 14.7$ km^2 (to 3 s.f.)

12. (i) $\sin 27.6° = \dfrac{QR}{5.7}$

$QR = 5.7 \sin 27.6$

$= 2.64$ cm (to 3 s.f.)

(ii) $\cos \angle SPR = \dfrac{3.2}{5.7}$

$\angle SPR = \cos^{-1} \dfrac{3.2}{5.7}$

$= 55.8°$ (to 1 d.p.)

(iii) Using sine rule,

$\dfrac{\sin \angle PST}{2.7} = \dfrac{\sin 64.2°}{3.2}$

$\sin \angle PST = \dfrac{2.7 \sin 64.2°}{3.2}$

$= 0.7596$ (to 4 s.f.)

$\angle PST = \sin^{-1} 0.7596$

$= 49.4°$ (to 1 d.p.)

13. (a) Using sine rule,

$\dfrac{\sin \angle B}{7.5} = \dfrac{\sin 91°}{8.5}$

$\sin \angle B = \dfrac{7.5 \sin 91°}{8.5}$

$= 0.8822$ (to 4 s.f.)

$\angle B = \sin^{-1} 0.8822 = 61.91°$ (to 2 d.p.)

Or $180° - 61.91° = 118.09°$ (to 2 d.p.)

Since $b < a$, then $\angle B < \angle A$, i.e. $\angle B < 92°$, hence $\angle B$ cannot be 118.09°.

$\therefore \angle B = 61.9°$ (to 1 d.p.)

$\angle C = 180° - 61.91° = 118.1°$ (to 1 d.p.)

$\therefore$ The triangle exists and it is not an ambiguous case.

(b) Using sine rule,

$\dfrac{\sin \angle E}{80} = \dfrac{\sin 47°}{75}$

$\sin \angle E = \dfrac{80 \sin 47°}{75}$

$= 0.7801$ (to 4 s.f.)

$\angle E = \sin^{-1} 0.7801 = 51.27°$ (to 2 d.p.)

Or $180° - 51.27° = 128.73°$ (to 2 d.p.) (Since $d < e$, then $\angle D < \angle E$, i.e. $\angle E > 47°$, hence both answers are possible.)

$\therefore$ when $\angle E = 51.3°$ (to 1 d.p.),

$\angle F = 180° - 47° - 51.2° = 81.7°$ (to 1 d.p.)

$\therefore$ when $\angle E = 128.7°$ (to 1 d.p.)

$\angle F = 180° - 47° - 128.73° = 4.3°$ (to 1 d.p.)

$\therefore$ The triangle exists and it is an ambiguous case.

(c) Using sine rule,

$\dfrac{\sin \angle H}{37} = \dfrac{\sin 58°}{37}$

$\sin \angle H = \dfrac{37 \sin 58°}{37}$

$= 0.8480$ (to 4 s.f.)

$\angle H = \sin^{-1} 0.8480 = 57.99°$ (to 2 d.p.)

Or $180° - 57.99° = 122.01°$ (to 2 d.p.)

Since $g = h$, then $\angle G = \angle H$, i.e. $\angle G = 58°$, hence $\angle H$ cannot be 122.01°.

$\therefore \angle H = 58.0°$ (to 1 d.p.)

$\angle I = 180° - 58° - 57.99° = 64.0°$ (to 1 d.p.)

$\therefore$ The triangle exists and it is not an ambiguous case.

(d) Using sine rule,

$\dfrac{\sin \angle J}{19} = \dfrac{\sin 39°}{15}$

$\sin \angle J = \dfrac{19 \sin 39°}{15}$

$= 0.7971$ (to 4 s.f.)

$\angle J = \sin^{-1} 0.7971 = 52.85°$ (to 2 d.p.)

or $180° - 52.85° = 127.15°$ (to 2 d.p.) (Since $k < j$, then $\angle K < \angle J$, i.e. $\angle J > 39°$, hence both answers are possible.)

$\therefore$ when $\angle J = 52.9°$ (to 1 d.p.),

$\angle L = 180° - 39° - 52.85° = 88.1°$ (to 1 d.p.)

$\therefore$ when $\angle J = 127.1°$ (to 1 d.p.),

$\angle L = 180° - 39° - 127.15° = 13.9°$ (to 1 d.p.)

$\therefore$ The triangle exists and it is an ambiguous case.

(e) Using sine rule,

$$\frac{\sin \angle N}{80} = \frac{\sin 43°}{67}$$

$$\sin \angle N = \frac{80 \sin 43°}{67}$$

$$= 0.8143 \text{ (to 4 s.f.)}$$

$\angle N = \sin^{-1} 0.8143 = 54.52°$ (to 2 d.p.)

or $180° - 54.52° = 125.48°$ (to 2 d.p.) (Since $o < n$, then $\angle O < \angle N$, i.e. $\angle N > 43°$, hence both answers are possible.)

$\therefore$ when $\angle N = 54.5°$ (to 1 d.p.),

$\angle M = 180° - 43° - 54.52° = 82.5°$ (to 1 d.p.)

$\therefore$ when $\angle N = 125.5°$ (to 1 d.p.),

$\angle M = 180° - 43° - 125.48° = 11.5°$ (to 1 d.p.)

$\therefore$ The triangle exists and it is an ambiguous case.

(f) Using sine rule,

$$\frac{\sin \angle P}{19} = \frac{\sin 52°}{25}$$

$$\sin \angle P = \frac{19 \sin 52°}{25}$$

$$= 0.5989 \text{ (to 4 s.f.)}$$

$\angle P = \sin^{-1} 0.5989 = 36.79°$ (to 2 d.p.)

Or $180° - 36.79° = 142.21°$ (to 2 d.p.)

Since $p < q$, then $\angle P < \angle Q$, i.e. $\angle P < 52°$, hence $\angle P$ cannot be 143.21°.

$\therefore \angle P = 36.8°$ (to 1 d.p.)

$\angle R = 180° - 52° - 36.79° = 91.2°$ (to 1 d.p.)

$\therefore$ The triangle exists and it is not an ambiguous case.

14. (a) $\angle BAC = 180° - 90° - 35°$ ($\angle$ sum of $\triangle$)

$= 55°$

$$\frac{\sin \angle BAC}{BC} = \frac{\sin 55°}{8} = 0.1024 \text{ (to 4 s.f.)}$$

$$\frac{\sin \angle ACB}{AB} = \frac{\sin 35°}{6} = 0.09560 \text{ (to 4 s.f.)}$$

Since $\frac{\sin \angle BAC}{BC} \neq \frac{\sin \angle ACB}{AB}$,

$\therefore$ It is not possible to construct $\triangle ABC$.

(b) $$\frac{\sin \angle PQR}{PR} = \frac{\sin 30°}{5} = 0.1$$

$$\frac{\sin \angle PRQ}{PQ} = \frac{\sin 36.9°}{6} = 0.1$$

Since $\frac{\sin \angle PQR}{PR} = \frac{\sin \angle PRQ}{PQ} = 0.1$,

$\therefore$ It is possible to construct $\triangle PQR$.

(c) $$\frac{\sin \angle LMN}{LN} = \frac{\sin 42°}{7.8} = 0.08579 \text{ (to 4 s.f.)}$$

$$\frac{\sin \angle LNM}{LM} = \frac{\sin 57°}{6.9} = 0.1215 \text{ (to 4 s.f.)}$$

Since $\frac{\sin \angle LMN}{LN} \neq \frac{\sin \angle LNM}{LM}$,

$\therefore$ It is not possible to construct $\triangle LMN$.

(d) $\angle GKH = 180° - 90° - 60°$ ($\angle$ sum of $\triangle$)

$= 50°$

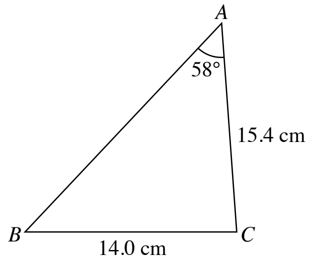
$$\frac{\sin \angle GHK}{GK} = \frac{\sin 90°}{12.8} = 0.07813 \text{ (to 4 s.f.)}$$

$$\frac{\sin \angle GKH}{GH} = \frac{\sin 30°}{6.4} = 0.0.7813 \text{ (to 4 s.f.)}$$

Since $\frac{\sin \angle GHK}{GK} = \frac{\sin \angle GKH}{GH}$,

$\therefore$ It is possible to construct $\triangle GHK$.

15.



$$\frac{\sin \angle ABC}{15.4} = \frac{\sin 58°}{14.0}$$

$$\sin \angle ABC = \frac{15.4 \sin 58°}{14.0}$$

$$= 0.9329 \text{ (to 4 s.f.)}$$

$\angle ABC = \sin^{-1} 0.9329 = 68.89°$ (to 2 d.p.)

Or $180° - 68.89° = 111.11°$ (to 2 d.p.)

Since $BC < AC$, then $\angle BAC < \angle ABC$, i.e. $\angle ABC > 58°$, hence both answers are possible.

When $\angle ABC = 68.9°$ (to 1 d.p.),

$\angle ACB = 180° - 58° - 68.89° = 53.1°$ (to 1 d.p.)

Using sine rule,

$$\frac{AB}{\sin 53.11} = \frac{14.0}{\sin 58°}$$

$$AB = \frac{14.0 \sin 53.11°}{\sin 58°}$$

$$= 13.2 \text{ cm (to 3 s.f.)}$$

When $\angle ABC = 111.1°$ (to 1 d.p.),

$\angle ACB = 180° - 58° - 111.11° = 10.9°$ (to 1 d.p.)

Using sine rule,

$$\frac{AB}{\sin 10.89°} = \frac{14.0}{\sin 58°}$$

$$AB = \frac{14.0 \sin 10.89°}{\sin 58°}$$

$$= 3.12 \text{ cm (to 3 s.f.)}$$

$\therefore \angle ABC = 68.9°, \angle ACB = 53.1°, AB = 13.2$ cm

or $\angle ABC = 111.1°, \angle ACB = 10.9°, AB = 3.12$ cm

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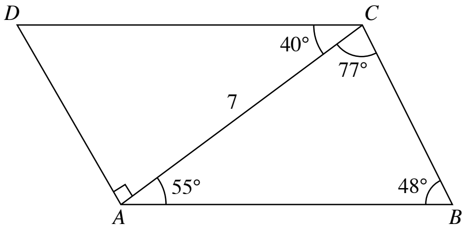
16. 

(i) Using sine rule,

$$\frac{AB}{\sin 77^\circ} = \frac{7}{\sin 48^\circ}$$

$$AB = \frac{7 \sin 77^\circ}{\sin 48^\circ}$$

$$= 9.18 \text{ cm (to 3 s.f.)}$$

(ii) $\tan 40^\circ = \dfrac{AD}{7}$

$$AD = 7 \tan 40^\circ$$

$$= 5.874 \text{ cm (to 4 s.f.)}$$

Map		**Actual**
8 cm	represents	1 km
1 cm	represents	$\frac{1}{8} = 0.125$ km
5.874 cm	represents	(5.874×0.125) km
		$= 0.734$ km (to 3 s.f.)

$\therefore$ The length represented by AD is 0.734 km.

(iii)

Map		**Actual**
8 cm	represents	1 km
1 cm	represents	$\frac{1}{8} = 0.125$ km
7 cm	represents	(7×0.125) km $= 0.875$ km

Area of $\triangle ADC = \dfrac{1}{2} \times 0.734 \times 0.875$

$$= 0.321 \text{ km}^2 \text{ (to 3 s.f.)}$$

17. (i) $\sin \angle B = \dfrac{4}{3} \sin \angle A$

$$= \frac{4}{3} \sin 35^\circ$$

$$= 0.7648 \text{ (to 4 s.f.)}$$

$\angle B = \sin^{-1} 0.7648 = 49.89^\circ$ (to 2 d.p.)

or $180^\circ - 49.89^\circ = 130.11^\circ$ (to 2 d.p.)

$\therefore \angle B = 49.9^\circ$ (to 1 d.p.), 130.1° (to 1 d.p.)

(ii) $\sin \angle B = \dfrac{4}{3} \sin \angle A$

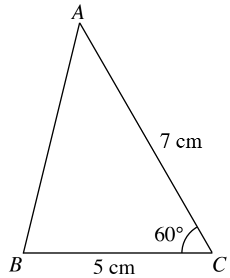
$$\frac{\sin \angle B}{\sin \angle A} = \frac{4}{3}$$

$$\frac{b}{\sin \angle B} = \frac{a}{\sin \angle A} = \frac{5}{\sin 35^\circ}$$

$$b = \frac{5 \sin \angle B}{\sin \angle A} = 5\left(\frac{4}{3}\right) = 6\frac{2}{3} \text{ cm}$$

$$\therefore AC = 6\frac{2}{3} \text{ cm}$$

Exercise 8D

1. 

Using cosine rule,

$c^2 = a^2 + b^2 - 2ab \cos C$

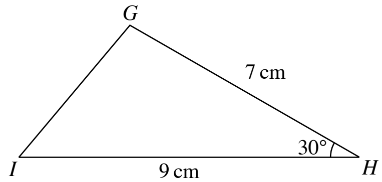
$c^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \cos 60^\circ$

$c^2 = 74 - 70 \cos 60^\circ$

$= 39$

$c = \sqrt{39}$

$= 6.24$ cm (to 3 s.f.)

2. 

Using cosine rule,

$h^2 = g^2 + i^2 - 2gi \cos H$

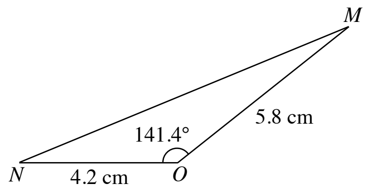
$= 9^2 + 7^2 - 2 \times 9 \times 7 \times \cos 30^\circ$

$= 130 - 126 \cos 30^\circ$

$= 20.88$ (to 4 s.f.)

$h = \sqrt{20.88}$

$= 4.57$ cm (to 3 s.f.)

3. 

Using cosine rule,

$o^2 = m^2 + n^2 - 2mn \cos O$

$= 4.2^2 + 5.8^2 - 2 \times 4.2 \times 5.8 \times \cos 141.4^\circ$

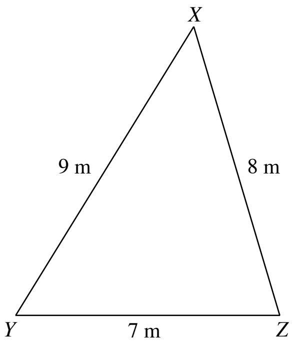
$= 51.28 - 48.72 \cos 141.4^\circ$

$= 89.36$ (to 4 s.f.)

$o = \sqrt{89.36}$

$= 9.45$ cm (to 3 s.f.)

4.



Using cosine rule,

$$\begin{aligned} x^2 &= y^2 + z^2 - 2yz \cos X \\ 7^2 &= 8^2 + 9^2 - 2 \times 8 \times 9 \cos X \\ 49 &= 145 - 144 \cos X \\ 144 \cos X &= 96 \\ \cos X &= \frac{96}{144} \\ \angle X &= \cos^{-1} \frac{96}{144} \\ &= 48.2° \text{ (to 1 d.p.)} \end{aligned}$$

Using cosine rule,

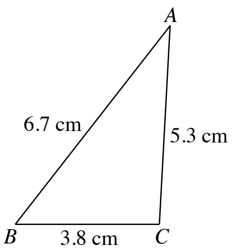
$$\begin{aligned} y^2 &= x^2 + z^2 - 2xz \cos Y \\ 8^2 &= 7^2 + 9^2 - 2 \times 7 \times 9 \times \cos Y \\ 64 &= 130 - 126 \cos Y \\ 126 \cos Y &= 66 \\ \cos Y &= \frac{66}{126} \\ \angle Y &= \cos^{-1} \frac{66}{126} \\ &= 58.4° \text{ (to 1 d.p.)} \end{aligned}$$

$\angle Z = 180° - 48.19° - 58.14°$ ($\angle$ sum of $\triangle$)

$= 73.4°$ (to 1 d.p.)

$\therefore$ $\angle X = 48.2°$, $\angle Y = 58.4°$ and $\angle Z = 73.4°$

5.



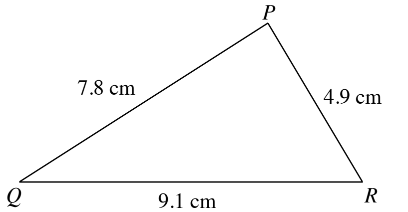
The smallest angle is the angle opposite the shortest side, i.e. $\angle A$.

Using cosine rule,

$$\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{ab} \\ &= \frac{5.3^2 + 6.7^2 - 3.8^2}{2 \times 5.3 \times 6.7} \\ &= 0.8243 \text{ (to 4 s.f.)} \\ \angle A &= \cos^{-1} 0.8243 \\ &= 34.5° \text{ (to 1 d.p.)} \end{aligned}$$

$\therefore$ The smallest angle is 34.5°.

6.



The largest angle is the angle opposite the longest side, i.e. $\angle P$.

Using cosine rule,

$$\begin{aligned} \cos P &= \frac{q^2 + r^2 - p^2}{2qr} \\ &= \frac{4.9^2 + 7.8^2 - 9.1^2}{2 \times 4.9 \times 7.8} \\ &= 0.02669 \text{ (to 4 s.f.)} \\ \angle P &= \cos^{-1} 0.02669 \\ &= 88.5° \text{ (to 1 d.p.)} \end{aligned}$$

$\therefore$ The largest angle is 88.5°.

7. **(i)** $\angle DBC = 180° - 125°$ (adj. $\angle$s on a str. line)

$= 55°$

Since $\angle CBD = \angle BCD$,

$BD = CD = 9$ m (isos. $\triangle$)

$\therefore$ $CD = 9$ m

(ii) Using cosine rule,

$$\begin{aligned} AD^2 &= AB^2 + BD^2 - 2 \times AB \times BD \times \cos \angle ABD \\ &= 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 125° \\ &= 145 - 144 \cos 125° \\ &= 227.6 \text{ (to 4 s.f.)} \\ AD &= \sqrt{227.6} \\ &= 15.1 \text{ m (to 3 s.f.)} \end{aligned}$$

8. **(i)** $\angle APB = 180° - 60°$ (adj. $\angle$s on a str. line)

$= 120°$

$$\begin{aligned} \frac{AB}{\sin 120°} &= \frac{5}{\sin 45°} \\ AB &= \frac{5 \sin 120°}{\sin 45°} \\ &= 6.12 \text{ m (to 3 s.f.)} \end{aligned}$$

(ii) Using cosine rule,

$$\begin{aligned} AC^2 &= AP^2 + PC^2 - 2 \times AP \times PC \times \cos \angle APC \\ &= 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 60° \\ &= 89 - 80 \cos 60° \\ &= 49 \\ AC &= \sqrt{49} \\ &= 7 \text{ m} \end{aligned}$$

9. **(i)** $MC = MB = 2$ cm

$\angle AMC = 180° - 120°$ (adj. $\angle$s on a str. line)

$= 60°$

Using cosine rule,

$$\begin{aligned} AC^2 &= AM^2 + MC^2 - 2 \times AM \times MC \times \cos \angle AMC \\ &= 4^2 + 2^2 - 2 \times 4 \times 2 \times \cos 60° \\ &= 20 - 16 \cos 60° \\ &= 12 \\ AC &= \sqrt{12} \\ &= 3.46 \text{ cm (to 3 s.f.)} \end{aligned}$$

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(ii) Using cosine rule,

$AB^2 = AM^2 + BM^2 - 2 \times AM \times BM \times \cos \angle AMB$

$= 4^2 + 2 - 2 \times 4 \times 2 \times \cos 120°$

$= 20 - 16 \cos 120°$

$= 28$

$AB = \sqrt{28}$

$= 5.29$ cm (to 3 s.f.)

(iii) Using sine rule,

$$\frac{\sin \angle ACB}{4} = \frac{\sin 60°}{3.464}$$

$$\sin \angle ACB = \frac{4 \sin 60°}{3.464}$$

$= 1.000$ (to 4 s.f.)

$\angle ACB = \sin^{-1} 1.000 = 90°$

10. (i) $\tan \angle BAD = \frac{5}{12}$

$\angle BAD = \tan^{-1} \frac{5}{12}$

$= 22.6°$ (to 1 d.p.)

(ii) $\angle APD = 180° - 50° - 22.62°$ ($\angle$ sum of $\triangle$)

$= 107.38°$ (to 2 d.p.)

Using sine rule,

$$\frac{PD}{\sin 22.62°} = \frac{12}{\sin 107.38°}$$

$$PD = \frac{12 \sin 22.62°}{\sin 107.38°}$$

$= 4.84$ m (to 3 s.f.)

(iii) $\angle BCD = \angle BAD = 22.62°$

Using cosine rule,

$DQ^2 = CD^2 + CQ^2 - 2 \times CD \times CQ \times \cos \angle BCD$

$= 12^2 + 6^2 - 2 \times 12 \times 6 \times \cos 22.62°$

$= 180 - 144 \cos 22.62°$

$= 47.08$ (to 4 s.f.)

$DQ = \sqrt{47.08}$

$= 6.86$ m (to 3 s.f.)

11. (i) Using cosine rule,

$a^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos 92°$

$= 61 - 60 \cos 92°$

$= 63.09$ (to 4 s.f.)

$a = \sqrt{63.09}$

$= 7.94$ (to 3 s.f.)

(ii) Using cosine rule,

$$\cos \theta = \frac{7^2 + 5^2 - 7.943^2}{2 \times 7 \times 5}$$

$= 0.1558$ (to 4 s.f.)

$\theta = \cos^{-1} 0.1558$

$= 81.0°$ (to 1 d.p.)

12. (i) Using cosine rule,

$$\cos \angle ADC = \frac{2^2 + 3.5^2 - 4.5^2}{2 \times 2 \times 3.5}$$

$= -0.2857$ (to 4 s.f.)

$\angle ADC = \cos^{-1} (-0.2857)$

$= 106.60°$ (to 2 d.p.)

$\angle ADB = 180° - 106.60°$ (adj. $\angle$s on a str. line)

$= 73.4°$ (to 1 d.p.)

(ii) Area of $\triangle ACD = \frac{1}{2} \times 2 \times 3.5 \times \sin 106.60°$

$= 3.354 \text{ cm}^2$ (to 4 s.f.)

Let the shortest distance from A to CB be h cm.

$\frac{1}{2} \times 3.5 \times h = 3.354$

$h = 1.92$ (to 3 s.f.)

$\therefore$ The shortest distance from A to CB is 1.92 cm.

(iii) $\angle BAD = 180° - 50° - 73.40°$ ($\angle$ sum of $\triangle$)

$= 56.60°$ (to 2 d.p.)

Using sine rule,

$$\frac{BD}{\sin 56.60°} = \frac{2}{\sin 50°}$$

$$BD = \frac{2 \sin 56.60°}{\sin 50°}$$

$= 2.18$ cm (to 3 s.f.)

13. (i)

Map		**Actual**
2 cm	represents	5 km
1 cm	represents	$\frac{5}{2}$ km = 2.5 km
8 cm	represents	(2.5×8) km = 20 km

$XZ = 20$ km

(ii) Using cosine rule,

$$\cos \angle YXZ = \frac{8^2 + 9^2 - 12^2}{2 \times 8 \times 9}$$

$$= \frac{1}{144}$$

$$\angle YXZ = \cos^{-1} \left(\frac{1}{144}\right)$$

$= 89.6°$ (to 1 d.p.)

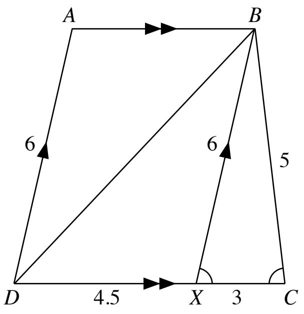
(iii) Area of $\triangle XYZ = \frac{1}{2} \times 8 \times 9 \times \sin 89.60°$

$= 36.00 \text{ cm}^2$ (to 4 s.f.)

Map		**Actual**
1 cm	represents	$\frac{5}{2}$ km = 2.5 km
1 cm^2	represents	$(2.5 \text{ km})^2 = 6.25 \text{ km}^2$
36.00 cm^2	represents	$(6.25 \times 36.00) \text{ km}^2 = 225 \text{ km}^2$

Area of $\triangle XYZ = 225 \text{ km}^2$

14.



$ABXD$ becomes a parallelogram.

$CD = 3$ cm, $BX = 6$ cm

$$\cos \angle BCX = \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5}$$
$$= -\frac{1}{15}$$
$$\angle BCX = \cos^{-1}\left(-\frac{1}{15}\right)$$
$$= 93.8° \text{ (1 d.p.)}$$
$$BD^2 = 5^2 + 7.5^2 - 2 \times 5 \times 7.5 \times \cos \angle BCX$$
$$BD^2 = 81.25 - 75\left(-\frac{1}{15}\right)$$
$$BD = \sqrt{86.25}$$
$$= 9.29 \text{ cm (to 3 s.f.)}$$

15. **(i)** $\dfrac{AD}{AB} = \dfrac{3+2}{3} = \dfrac{5}{3}$

$\dfrac{AE}{AC} = \dfrac{6+5}{6} = \dfrac{11}{6}$

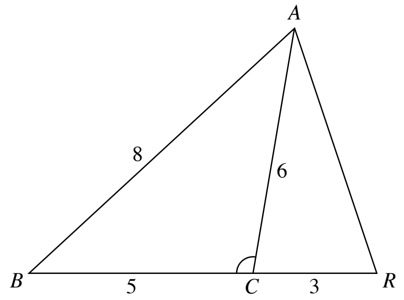
$\dfrac{AD}{AB} \neq \dfrac{AE}{AC}$

$\therefore$ ADE is not an enlargement of $\triangle ABC$.

(ii) $\cos \theta = \dfrac{6^2 + 3^2 - 3.5^2}{2 \times 6 \times 3} = \dfrac{32.75}{36} = \dfrac{131}{144}$

(iii)
$$\frac{131}{144} = \frac{11^2 + 5^2 - x^2}{2 \times 11 \times 5}$$
$$\frac{131}{144} = \frac{146 - x^2}{110}$$
$$110 \times \frac{131}{144} = 146 - x^2$$
$$x^2 = 146 - 100\frac{5}{72}$$
$$x^2 = \frac{3307}{72}$$
$$x = \sqrt{\frac{3307}{72}}$$
$$= 6.78 \text{ (to 3 s.f.)}$$

16.



(i) $\cos \angle BCA = \dfrac{5^2 + 6^2 - 8^2}{2 \times 5 \times 6} = -\dfrac{1}{20}$

(ii) $\cos \angle RCA = –\cos (180° – \angle BCA)$
$$= \cos \angle RCA$$
$$= -\left(-\frac{1}{20}\right)$$
$$= \frac{1}{20}$$
$$AR^2 = 3^2 + 6^2 - 2 \times 3 \times 6 \times \cos \angle RCA$$
$$AR^2 = 45 - 36\left(\frac{1}{20}\right)$$
$$AR = \sqrt{\frac{216}{5}}$$
$$= 6.57 \text{ cm (to 3 s.f.)}$$

17. In $\triangle ABC$,
$$\cos A = \frac{8^2 + 13^2 - 14^2}{2 \times 8 \times 13} = \frac{37}{208}$$
$$PQ^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos A$$
$$PQ^2 = 61 - 60\left(\frac{37}{208}\right)$$
$$PQ = \sqrt{\frac{2617}{52}}$$
$$= 7.09 \text{ cm (to 3 s.f.)}$$

Review Exercise 8

1. **(a)** By Pythagoras' Theorem,
$$AC^2 = 7^2 + 24^2$$
$$= 625$$
$$AC = \sqrt{625}$$
$$= 25 \text{ cm}$$

(b) **(i)** $\tan \angle ACB = \dfrac{AB}{BC} = \dfrac{7}{24}$

(ii) $\cos \angle ACD = –\cos (180° – \angle ACD)$
$$= -\cos \angle ACB$$
$$= -\frac{BC}{AC}$$
$$= -\frac{24}{25}$$

(iii) $\sin \angle TAC = \sin (180° – \angle TAC)$
$$= \sin \angle BAC$$
$$= \frac{BC}{AC}$$
$$= \frac{24}{25}$$

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2. **(a)** By Pythagoras' Theorem,

$QR^2 = 12^2 + 35^2$

$= 1369$

$QR = \sqrt{1369}$

$= 37$ cm

(b) **(i)** $\sin \angle PQT = \sin (180° - \angle PQT)$

$= \sin \angle RQT$

$= \dfrac{RT}{QR}$

$= \dfrac{12}{37}$

(ii) $\cos \angle PQT = -\cos (180° - \angle PQT)$

$= -\cos \angle RQT$

$= -\dfrac{QT}{QR}$

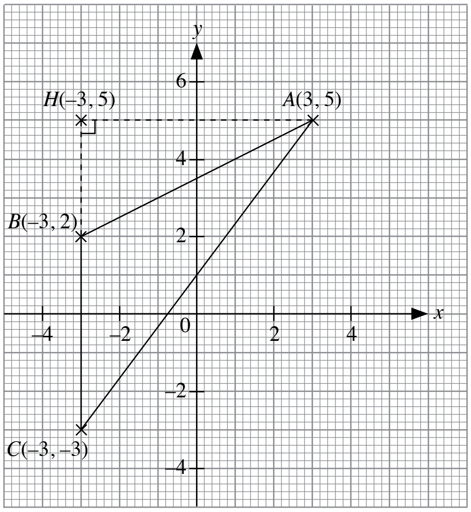
$= -\dfrac{35}{37}$

(ii) $\tan \angle TQR + \tan \angle TRQ = \dfrac{RT}{QT} + \dfrac{QT}{RT}$

$= \dfrac{12}{35} + \dfrac{35}{12}$

$= 3\dfrac{109}{420}$

3.



H is the point (–3, 5).

$AH = 6$ units, $BH = 3$ units

By Pythagoras' Theorem,

$AB^2 = AH^2 + BH^2$

$= 6^2 + 3^2$

$= 45$

$AB = \sqrt{45}$ units

(i) $\cos \angle ABC = -\cos (180° - \angle ABC)$

$= -\cos \angle ABH$

$= -\dfrac{BH}{AB}$

$= -\dfrac{3}{\sqrt{45}}$

$= -0.4472$ (to 4 s.f.)

(ii) $\sin \angle ABC = \sin (180° - \angle ABC)$

$= \sin \angle ABH$

$= \dfrac{AH}{AB}$

$= \dfrac{6}{\sqrt{45}}$

$= 0.8944$ (to 4 s.f.)

(iii) $CH = 8$ units

$\tan \angle ACB = \dfrac{AH}{CH} = \dfrac{6}{8} = \dfrac{3}{4}$

4. **(i)** $BC = \sqrt{[9 - (-3)]^2 + [4 - (-1)]^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{169}$

$= 13$ units

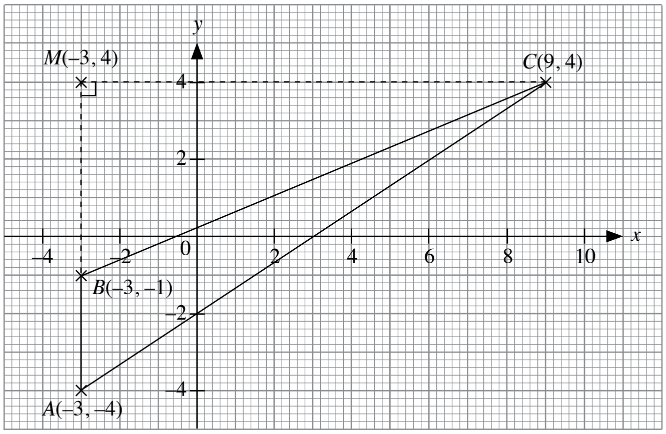
$AC = \sqrt{[9 - (-3)]^2 + [4 - (-4)]^2}$

$= \sqrt{12^2 + 8^2}$

$= \sqrt{208}$

$= 14.4$ units (to 3 s.f.)

(ii)



M is the point (–3, 4).

$BM = 5$ units, $CM = 12$ units

$\sin \angle ABC = \sin (180° - \angle ABC)$

$= \sin \angle CBM$

$= \dfrac{CM}{BC}$

$= \dfrac{12}{13}$

$\cos \angle ABC = -\cos (180° - \angle ABC)$

$= -\cos \angle CBM$

$= -\dfrac{BM}{BC}$

$= -\dfrac{5}{13}$

(iii) Area of $\triangle ABC = \dfrac{1}{2} \times 3 \times 12$

$= 18$ units^2

(iv) Let the length of the perpendicular from B to AC be h cm.

Area of $\triangle ABC = 18$

$$\frac{1}{2} \times AC \times h = 18$$

$$\frac{1}{2} \times 14.42 \times h = 8$$

$$7.21h = 18$$

$$h = 2.50 \text{ units (to 3 s.f.)}$$

5. **(a)** Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle.

$\sin x = 0.419$

$x = \sin^{-1} 0.419 = 24.8°$ (to 1 d.p.)

or $180° - 24.8° = 155.2°$ (to 1 d.p.)

$\therefore x = 24.8°$ or $155.2°$

(b) Since $\cos x$ is positive, x is an acute angle.

$\cos x = 0.932$

$x = \cos^{-1} (0.932)$

$= 21.3°$ (to 1 d.p.)

(c) Since $\tan x$ is positive, x is an acute angle.

$\tan x = 0.503$

$x = \tan^{-1} 0.503$

$= 26.7°$ (to 1 d.p.)

(d) Since $\cos x$ is negative, x is an obtuse angle.

$\cos x = -0.318$

$x = \cos^{-1} (-0.318)$

$= 108.5°$ (to 1 d.p.)

6. **(a)** $PQ^2 = 12^2 = 144$

$SQ^2 = 35^2 = 1225$

$PR^2 = 37^2 = 1369$

$PQ^2 + SQ^2 = 144 + 1225$

$= 1369$

$= PR^2$

Since $PQ^2 + SQ^2 = PR^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is a right-angled triangle with $\angle PQR = 90°$.

(b) **(i)** $\cos \angle PRS = -\cos (180° - \angle PRS)$

$= -\cos \angle PRQ$

$$= -\frac{QR}{PR}$$

$$= -\frac{35}{37}$$

$\sin \angle PRS = \sin (180° - \angle PRS)$

$= \sin \angle PRQ$

$$= \frac{PQ}{PR}$$

$$= \frac{12}{37}$$

(ii) $$\tan \angle QPS = \frac{SQ}{PQ} = \frac{16+35}{12} = \frac{51}{12}$$

$$\angle QPS = \tan^{-1} \frac{51}{12} = 76.76° \text{ (to 2 d.p.)}$$

By cosine rule,

$$\cos \angle QPR = \frac{PQ^2 + PR^2 - QR^2}{2 \times PQ \times PR}$$

$$= \frac{12^2 + 37^2 - 35^2}{2 \times 12 \times 37}$$

$= 0.3242$ (to 4 s.f.)

$\angle QPR = \cos^{-1} 0.3243 = 71.08°$ (to 2 d.p.)

$\angle RPS = \angle QPS - \angle QPR$

$= 76.76° - 71.08°$

$= 5.6°$ (to 1 d.p.)

7. **(i)** $\sin \angle CBA = \sin (180° - \angle CBA)$

$= \sin \angle CBD$

$$= \frac{3}{4}$$

Using sine rule,

$$\frac{\sin \angle ACB}{AB} = \frac{\sin \angle CBA}{CA}$$

$$\frac{\sin \angle ACB}{7} = \frac{\left(\frac{3}{4}\right)}{11}$$

$$\sin \angle ACB = 7 \times \frac{\left(\frac{3}{4}\right)}{11}$$

$$= \frac{21}{44}$$

(ii) $$\angle ACB = \sin^{-1} \frac{21}{44} = 28.51° \text{ (to 2 d.p.)}$$

$$\angle CBD = \sin^{-1} \frac{3}{4} = 48.59° \text{ (to 2 d.p.)}$$

$\angle BAC = 48.59° - 28.51° = 20.1°$ (to 1 d.p.) (ext. $\angle$ of $\triangle$)

(iii) Area of $\triangle ABC = \frac{1}{2} \times 7 \times 11 \times \sin 20.08°$

$= 13.2$ cm^2 (to 3 s.f.)

(iv) By cosine rule,

$BC^2 = 7^2 + 11 - 2 \times 7 \times 11 \times \cos 20.08°$

$= 25.36$ (to 4 s.f.)

$BC = \sqrt{25.36}$

$= 5.04$ cm (to 3 s.f.)

8. **(i)** $$\cos \angle CBQ = \frac{BC}{BQ}$$

$$\cos 52° = \frac{BC}{4.8}$$

$BC = 4.8 \cos 52°$

$= 2.96$ m (3 s.f.)

(ii) $\angle ABP = 90° - 52° - 26° = 12°$

$$\sin \angle ABP = \frac{AP}{BP}$$

$$\sin 12° = \frac{AP}{5.7}$$

$AP = 5.7 \sin 12°$

$= 1.19$ m (to 3 s.f.)

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(iii) Area of $\triangle PBQ = \frac{1}{2} \times 4.8 \times 5.7 \times \sin 26°$

$= 6.00 \text{ m}^2$ (to 3 s.f.)

(iv) By cosine rule,

$PQ^2 = 4.8^2 + 5.7^2 - 2 \times 4.8 \times 5.7 \times \cos 26°$

$= 6.348$ (to 4 s.f.)

$PQ = \sqrt{6.348}$

$= 2.52$ m (to 3 s.f.)

(v) Using sine rule,

$$\frac{\sin \angle BPQ}{BQ} = \frac{\sin \angle PBQ}{PQ}$$

$$\frac{\sin \angle BPQ}{4.8} = \frac{\sin 26°}{2.519}$$

$$\sin \angle BPQ = 4.8 \times \frac{\sin 26°}{2.519}$$

$= 0.8353$ (to 4 s.f.)

$\angle BPQ = \sin^{-1} 0.8353 = 56.6°$ (to 1 d.p.)

9. **(i)** $\angle PQS = 180° - 55° - 38°$ ($\angle$ sum of $\triangle$)

$= 87°$

Using sine rule,

$$\frac{QS}{\sin \angle QPS} = \frac{PS}{\sin \angle PQS}$$

$$\frac{QS}{\sin 55°} = \frac{460}{\sin 87°}$$

$$QS = \frac{460 \sin 55°}{\sin 87°}$$

$= 377$ m (to 3 s.f.)

(ii) By cosine rule,

$RS^2 = 325^2 + 377.3^2 - 2 \times 325 \times 377.3 \times \cos 32°$

$= 40\,000.7$ (to 6 s.f.)

$RS = \sqrt{40\,000.7}$

$= 200$ m (to 3 s.f.)

(iii) Area of $\triangle PQS = \frac{1}{2} \times 460 \times 377.3 \times \sin 38°$

$= 53\,430 \text{ m}^2$ (to 4 s.f.)

Let the shortest distance between Q and PS be h m.

Area of $\triangle PQS = 53\,430 \text{ m}^2$

$\frac{1}{2} \times 460 \times h = 53\,430$

$230h = 53\,430$

$h = 232$ (to 3 s.f.)

$\therefore$ The shortest distance between Q and PS is 232 m.

(iv) Area of $\triangle QRS = \frac{1}{2} \times 325 \times 377.3 \times \sin 32°$

$= 32\,490 \text{ m}^2$ (to 4 s.f.)

Area occupied by the park $= 32\,490 + 53\,430$

$= 85\,900 \text{ m}^2$ (to 3 s.f.)

10. **(i)** $\angle ACB = 180° - 64° - 42°$ ($\angle$ sum of $\triangle$)

$= 74°$

Using sine rule,

$$\frac{AC}{\sin \angle ABC} = \frac{AB}{\sin \angle ACB}$$

$$\frac{AC}{\sin 64°} = \frac{168 \sin 64°}{\sin 74°}$$

$$AC = \frac{168 \sin 64°}{\sin 74°}$$

$= 157$ cm (to 3 s.f.)

(ii) By cosine rule,

$$\cos \angle ADC = \frac{AD^2 + CD^2 - AC^2}{2 \times AD \times CD}$$

$$= \frac{210^2 + 192^2 - 157.1^2}{2 \times 210 \times 192}$$

$= 0.6980$ (to 4 s.f.)

$\angle ADC = \cos^{-1} 0.6980 = 45.7°$ (to 1 d.p.)

(iii) Area of $ABCD$

= Area of $\triangle ABC$ + Area of $\triangle ACD$

$= \frac{1}{2} \times 1.68 \times 1.571 \times \sin 42° + \frac{1}{2} \times 1.92 \times 2.10 \times \sin 45.73°$

$= 2.33 \text{ m}^2$ (to 3 s.f.)

11. **(i)** Using sine rule,

$$\frac{QS}{\sin \angle QRS} = \frac{RS}{\sin \angle RQS}$$

$$\frac{QS}{\sin 71°} = \frac{68}{\sin 62°}$$

$$QS = \frac{68 \sin 71°}{\sin 62°}$$

$= 72.82$ m (to 4 s.f.)

$\angle QSR = 180° - 71° - 62°$ ($\angle$ sum of $\triangle$)

$= 47°$

Area of $\triangle SQR = \frac{1}{2} \times 68 \times 72.82 \times \sin 47°$

$= 1810 \text{ m}^2$ (to 3 s.f.)

(ii) By cosine rule,

$PS^2 = 72.82^2 + 98^2 - 2 \times 72.82 \times 98 \times \cos 43°$

$= 4468$ (to 4 s.f.)

$PS = \sqrt{4468}$

$= 66.8$ m (to 3 s.f.)

12. **(i)** Area of the park $= \frac{1}{2} \times 320 \times 470 \times \sin 35°$

$= 43\,100 \text{ m}^2$ (to 3 s.f.)

(ii) By cosine rule,

$BC^2 = 320^2 + 470^2 - 2 \times 320 \times 470 \times \cos 35°$

$= 76\,900$ (to 4 s.f.)

$BC = \sqrt{76\,900}$

$= 277$ m (to 3 s.f.)

(iii) $\angle PBC = 180° - 118° - 24°$ ($\angle$ sum of $\triangle$)

$= 38°$

Using sine rule,

$$\frac{PC}{\sin \angle PBC} = \frac{BC}{\sin \angle BPC}$$

$$\frac{PC}{\sin 38°} = \frac{277.3}{\sin 118°}$$

$$PC = \frac{277.3 \sin 38°}{\sin 118°}$$

$= 193$ m (to 3 s.f.)

Challenge Yourself

(a) $\angle KAH = \dfrac{180°(5-2)}{5} = 108°$

$HK = \sqrt{8^2 + 8^2 - 2(8)(8)\cos 108°}$

$= 12.944$ cm (to 5 s.f.)

$= 12.9$ cm (to 3 s.f.)

$\angle GHA = \dfrac{180°(8-2)}{8} = 135°$

$\angle AHK = \dfrac{180° - 108°}{2} = 36°$

$\angle GHK = 135° + 36° = 171°$

$GK = \sqrt{12.944^2 + 8^2 - 2(8)(12.944)\cos 171°}$

$= 20.883$ cm (to 5 s.f.)

$= 20.9$ cm (to 3 s.f.)

(b) $\dfrac{\sin \angle HGK}{12.944} = \dfrac{\sin 171°}{20.883}$

$\sin \angle HGK = \dfrac{12.944 \sin 171°}{20.883}$

$\angle HGK = 5.56°$ (to 2 d.p.)

$\angle FGK = 135° - 5.56° = 129.44°$ (to 2 d.p.)

Area of $\triangle FGK = \dfrac{1}{2} \times 20.883 \times 8 \times \sin 129.44°$

$= 64.5 \text{ cm}^2$ (to 3 s.f.)

Chapter 9 Applications of Trigonometry

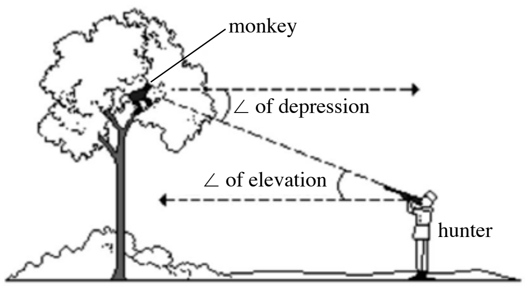
TEACHING NOTES

Suggested Approach

In Book 2, students have learnt how to use the clinometer to measure angles and find the heights of buildings, flagpoles, etc. This chapter introduces students to angles of elevation and depression. Teachers can use various illustrations to help students with their learning and understanding of angles of elevation and depression and how they may be used to solve simple problems.

Section 9.1: Angles of Elevation and Depression

Teachers may use the following illustration to show students the angle of elevation and the angle of depression.



Using the board or a visualiser, draw a picture of a monkey up in a tree and a hunter with a gun, looking for game. When the hunter spots the monkey in the tree, his gun, which was initially down, is now raised through an angle towards the monkey. Teachers may illustrate this as the angle of elevation.

When the monkey notices that a hunter is pointing a gun at it, it looks down at an angle and teachers may illustrate this as the angle of depression.

Section 9.2: Bearings

To introduce the concept of true bearing, teachers may revisit the general compass bearing that students would have learnt in Geography. Thus true North is equivalent to 000° or 360°and East is equivalent to 090° while North-West will be represented by 315°, etc. From this explanation, students could understand better how the concept of bearing is used in the different disciplines.

Section 9.3: Three-Dimensional Problems

Teachers can start off with an activity to introduce three-dimensional solids so that students are better able to visualise these solids when solving three-dimensional problems (see Investigation: Visualising 3D Solids). Teachers should highlight to the students that the basic technique used in solving three-dimensional problems is to reduce it to a problem in a plane. Students may encounter difficulties in this area, as such, much practice and guidance will have to be given to them.

Challenge Yourself

For Questions 1 and 2, students should be able to visualise and reduce the three-dimensional problem to a problem in a plane. Students should approach the problem by applying trigonometric ratios to find the unknown sides and angles in triangles.

WORKED SOLUTIONS

Investigation (Visualising 3D Solids)

1. Yes.

2. 90° for both angles.

4. Yes, $\angle NOB$ is a 90° angle. An object on a flat surface may make a right angle look smaller or larger than 90°.

5. Method 1: Find a rectangle

(a) Yes, the plane *EFGH* is a rectangle because it is a face of the cuboid.

(b) Since $\angle EFG$ is a right angle of the rectangle *EFGH*, then $\angle EFG = 90°$ and $\triangle EFG$ is a right-angled triangle.

(c) $\triangle CGH$ lies on the plane *DCGH*. The plane *DCGH* is a rectangle because it is a face of the cuboid. Since $\angle CGH$ is a right angle of the rectangle *DCGH*, then $\angle CGH = 90°$ and so $\triangle CGH$ is a right-angled triangle.

Method 2: Find a normal to a plane

(a) Since the plane *EFGH* is horizontal and the line *CG* is vertical, then *CG* is a normal to the plane *EFGH*.

(b) Yes, the line *GE* is a line on the plane *EFGH*.

(c) Since *CG* is a normal to the plane *EFGH*, and *GE* is a line on the plane *EFGH*, then $\angle CGE = 90°$ because a normal to a plane is perpendicular to every line on the plane. Thus $\triangle CGE$ is a right-angled triangle.

(d) Since the plane *DCGH* is vertical and the line *EH* is horizontal, then *EH* is a normal to the plane *DCGH*. Since the line *HC* lies on the plane *DCGH*, then the normal *EH* is perpendicular to the line *HC*, i.e. $\angle CHE = 90°$. Therefore, $\triangle CHE$ is a right-angled triangle.

Practise Now 1

1. Let x m be the distance of the point on the ground from the foot of the office tower.

$$\tan 34° = \frac{43}{x}$$

$$x \tan 34° = 43$$

$$x = \frac{43}{\tan 34°} = 63.8 \text{ (to 3 s.f.)}$$

$\therefore$ The distance of the point on the ground from the foot of the office tower is 63.8 m.

2.

$$\tan 48° = \frac{50}{LA}$$

$$LA \tan 48° = 50$$

$$LA = \frac{50}{\tan 48°} = 45.02 \text{ (to 4 s.f.)}$$

$$\tan 38° = \frac{50}{LB}$$

$$LB \tan 38° = 50$$

$$LB = \frac{50}{\tan 38°} = 64.00 \text{ (to 4 s.f.)}$$

$$\begin{aligned}\text{Distance between boats } A \text{ and } B &= 64.00 - 45.02 \\ &= 19.0 \text{ m (to 3 s.f.)}\end{aligned}$$

Practise Now 2

1. **(i)** $\angle ACB = 25°$ (alt. $\angle$s)

$$\tan 25° = \frac{165}{BC}$$

$$BC = \frac{165}{\tan 25°} = 354 \text{ m (to 3 s.f.)}$$

$\therefore$ The distance of the sports car from the point *B* is 354 m.

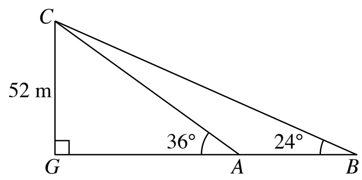
(ii)

$$\tan \theta = \frac{165 - 75}{353.84}$$

$$\begin{aligned}\theta &= \tan^{-1} \frac{90}{353.84} \\ &= 14.3° \text{ (to 1 d.p.)}\end{aligned}$$

$\therefore$ The angle of depression of the sports car from the centre of the wheels is 14.3°.

2.



$$\tan 36° = \frac{52}{GA}$$

$$\begin{aligned}GA &= \frac{52}{\tan 36°} \\ &= 71.57 \text{ (to 4 s.f.)}\end{aligned}$$

$$\tan 24° = \frac{52}{GA + AB}$$

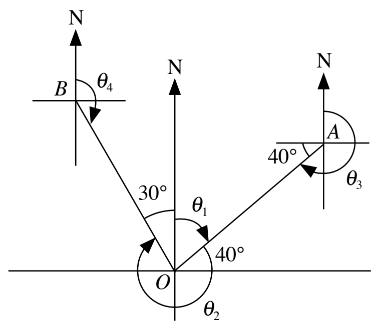
$$\tan 24° \times (71.57 + AB) = 52$$

$$AB = \frac{52}{\tan 24°} - 71.57 = 45.2 \text{ m (to 3 s.f.)}$$

$\therefore$ The distance between the two ships is 45.2 m.

Practise Now 3

1.



(a) The bearing of *A* from *O* is equal to (90° – 40°).

$\therefore$ Bearing of *A* from *O* is 050°.

(b) The bearing of *B* from *O* is given by the reflex angle θ_2, which is (360° – 30°).

$\therefore$ Bearing of *B* from *O* is 330°.

(c) The bearing of *O* from *A* is given by the reflex angle θ_3, which is (270° – 40°).

$\therefore$ Bearing of *O* from *A* is 230°.

(d) The bearing of *O* from *B* is given by the obtuse angle θ_4, which (180° – 30°).

$\therefore$ Bearing of *O* from *B* is 150°.

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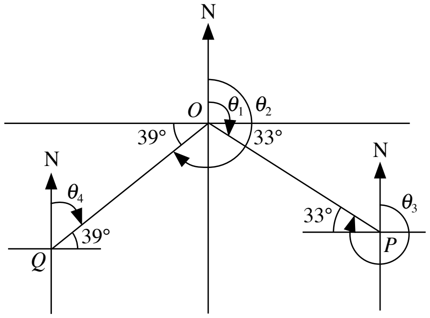
2.



(a) The bearing of P from O is equal to (90° + 33°).
$\therefore$ Bearing of P from O is 123°.

(b) The bearing of Q from O is given by the reflex angle θ_2, which is (270° – 39°).
$\therefore$ Bearing of Q from O is 231°.

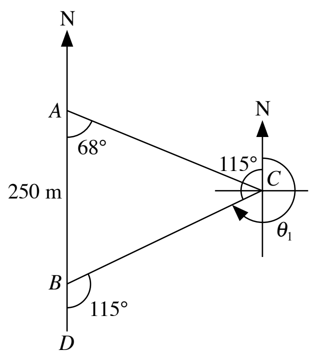
(c) The bearing of O from P is given by the reflex angle θ_3, which is (270° + 33°).
$\therefore$ Bearing of O from P is 303°.

(d) The bearing of O from Q is given by the acute angle θ_4, which is (90° – 39°).
$\therefore$ Bearing of O from Q is 051°.

Practise Now 4

1. Since the bearing of R from Q is 118°,
$\angle PQR = 180° - 118° = 62°$
i.e. $\angle PRQ = 180° - 62° - 44°$ ($\angle$ sum of a $\triangle$)
$= 74°$
Using sine rule,
$$\frac{PQ}{\sin 74°} = \frac{150}{\sin 44°}$$
$$PQ = \frac{150 \sin 74°}{\sin 44°}$$
$= 208$ m (to 3 s.f.)

2. (i)



The bearing of B from C is given by the reflex angle θ_1, which is (360° – 115°).
$\therefore$ Bearing of B from C is 245°.

(ii) $\angle ABC = 180° - 115° = 65°$ (adj. $\angle$s on a str. line)
i.e. $\angle ACB = 180° - 65° - 68°$ ($\angle$ sum of a $\triangle$)
$= 47°$
Using sine rule,
$$\frac{AC}{\sin 65°} = \frac{250}{\sin 47°}$$
$$AC = \frac{250 \sin 65°}{\sin 47°}$$
$= 310$ m (to 3 s.f.)
Using sine rule,
$$\frac{BC}{\sin 68°} = \frac{250}{\sin 47°}$$
$$BC = \frac{250 \sin 68°}{\sin 47°}$$
$= 317$ m (to 3 s.f.)

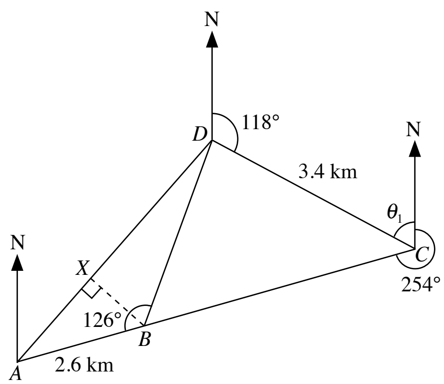
Practise Now 5

(i) $\angle ABC = 360° - 238° - 60° = 62°$ ($\angle$s at a point)
Using cosine rule,
$AC^2 = 35^2 + 55^2 - 2 \times 35 \times 55 \times \cos 62°$
$= 4250 - 3850 \cos 62°$
$= 2443$ (to 4 s.f.)
$AC = \sqrt{2443}$
$= 49.4$ km (to 3 s.f.)

(ii) Using sine rule,
$$\frac{\sin BAC}{55} = \frac{\sin 62°}{49.42}$$
$$\sin BAC = \frac{55 \sin 62°}{49.42}$$
$$\angle BAC = \sin^{-1} \frac{55 \sin 62°}{49.42}$$
$= 79.31$ (to 2 d.p.)
$120° + 79.31° = 199.3°$ (to 1 d.p.)
$\therefore$ Bearing of town C from town A is 199.3°.

Practise Now 6

(i)



$\theta_1 = 180° - 118° - 60°$ (int. $\angle$s)
$= 62°$
i.e. $\angle BCD = 360° - 254° - 62°$ ($\angle$s at a point)
$= 44°$
$\angle CBD = 180° - 126°$ (adj. $\angle$s on a str. line)
$= 54°$
$\angle BDC = 180° - 54° - 44°$ ($\angle$ sum of a $\triangle$)
$= 82°$
$118° + 82° = 200°$
$\therefore$ Bearing of B from D is 200°.

(ii) Using sine rule,

$$\frac{BD}{\sin 44°} = \frac{3.4}{\sin 54°}$$

$$BD = \frac{3.4 \sin 44°}{\sin 54°}$$

$$= 2.92 \text{ km (to 3 s.f.)}$$

(iii) Using cosine rule,

$$AD^2 = 2.6^2 + 2.919^2 - 2 \times 2.6 \times 2.919 \times \cos 126°$$

$$= 24.20 \text{ (to 4 s.f.)}$$

$$AD = \sqrt{24.20}$$

$$= 4.92 \text{ km (to 3 s.f.)}$$

(iv) The shortest distance from B to AD is BX, where BX is perpendicular to AD.

Using sine rule,

$$\frac{\sin \angle BAD}{2.919} = \frac{\sin 126°}{4.919}$$

$$\angle BAD = \sin^{-1} \frac{2.919 \sin 126°}{4.919}$$

$$= 28.69° \text{ (to 2 d.p.)}$$

In $\triangle ABX$,

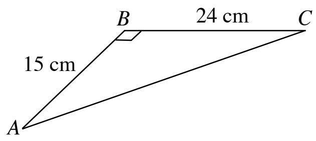
$$\sin \angle BAX = \frac{BX}{2.6}$$

$$BX = 2.6 \sin 28.69°$$

$$= 1.25 \text{ km (to 3 s.f.)}$$

Practise Now 7

1. (i)



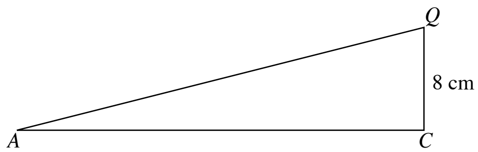
In $\triangle ABC$, $\angle ABC = 90°$.

$$\tan \angle BAC = \frac{24}{15}$$

$$\angle BAC = \tan^{-1} \frac{24}{15}$$

$$= 58.0° \text{ (to 1 d.p.)}$$

(ii)



Using Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 15^2 + 24^2$$

$$= 225 + 576$$

$$= 801$$

$$AC = \sqrt{801}$$

$$= 28.30 \text{ cm (to 4 s.f.)}$$

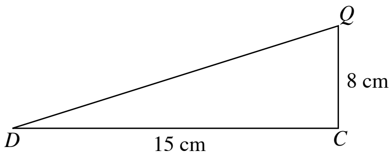
In $\triangle ACQ$, $\angle ACQ = 90°$.

$$\tan \angle AQC = \frac{28.30}{8}$$

$$\angle AQC = \tan^{-1} \frac{28.30}{8}$$

$$= 74.2° \text{ (to 1 d.p.)}$$

(iii)



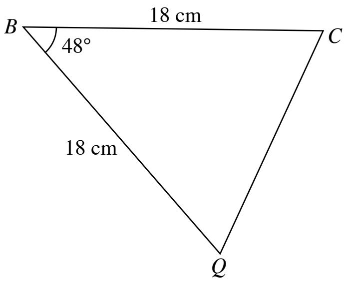
$\triangle DCQ$, $\angle DCQ = 90°$.

$$\tan \angle CDQ = \frac{8}{15}$$

$$\angle CDQ = \tan^{-1} \frac{8}{15}$$

$$= 28.1° \text{ (to 1 d.p.)}$$

2. (i)



Using cosine rule,

$$CQ^2 = 18^2 + 18^2 - 2 \times 18 \times 18 \times \cos 48°$$

$$= 648 - 648 \cos 48°$$

$$= 214.4 \text{ (to 4 s.f.)}$$

$$CQ = \sqrt{214.4}$$

$$= 14.6 \text{ cm (to 3 s.f.)}$$

(iii) Using Pythagoras' Theorem,

$$AQ^2 = AB^2 + BQ^2$$

$$= 24^2 + 18^2$$

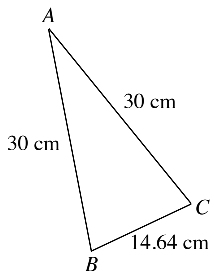
$$= 576 + 324$$

$$= 900$$

$$AQ = \sqrt{900}$$

$$= 30 \text{ cm}$$

$$CA = AQ = 30 \text{ cm}$$



Using cosine rule,

$$\cos \angle CAQ = \frac{30^2 + 30^2 - 14.64^2}{2 \times 30 \times 30}$$

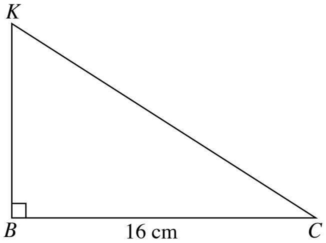
$$= 0.8809 \text{ (to 4 s.f.)}$$

$$\angle CAQ = \cos^{-1} 0.8809$$

$$= 28.3 \text{ (to 1 d.p.)}$$

Practise Now 8

1. **(i)**



In $\triangle BCK$,

$\angle CBK = 90°$

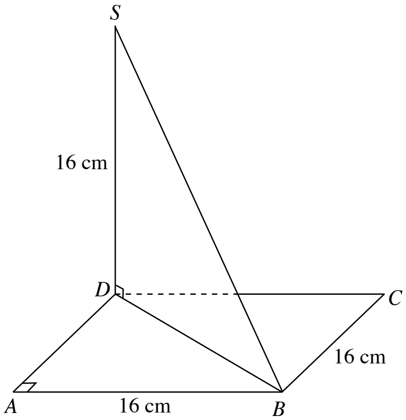
$BK = 16 - 6 = 10$ cm

$\tan \angle BCK = \dfrac{10}{16}$

$\angle BCK = \tan^{-1} \dfrac{10}{16}$

$= 32.0°$ (to 1 d.p.)

(ii)



In $\triangle ABD$,

Using Pythagoras' Theorem,

$DB^2 = AD^2 + AB^2$

$= 16^2 + 16^2$

$= 256 + 256$

$= 512$

$DB = \sqrt{512}$

$= 22.63$ cm (to 4 s.f.)

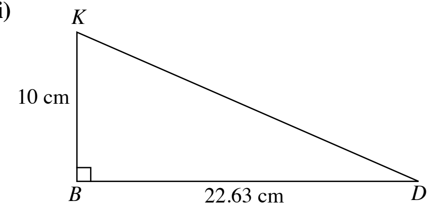
In $\triangle BDS$, $\angle BDS = 90°$.

$\tan \angle SBD = \dfrac{16}{22.63}$

$\angle SBD = \tan^{-1} \dfrac{16}{22.63}$

$= 35.3°$ (to 1 d.p.)

(iii)



In $\triangle BDK$,

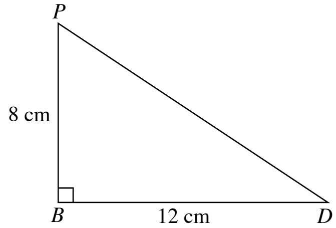
$\angle DBK = 90°$

$\tan \angle BDK = \dfrac{10}{22.63}$

$\angle BDK = \tan^{-1} \dfrac{10}{22.63}$

$= 23.8°$ (to 1 d.p.)

2. **(i)**



In $\triangle ABP$,

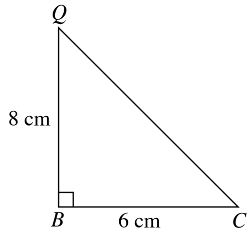
$\angle BAP = 90°$

$\tan \angle ABP = \dfrac{8}{12}$

$\angle ABP = \tan^{-1} \dfrac{8}{12}$

$= 33.7°$ (to 1 d.p.)

(ii)



In $\triangle BCQ$,

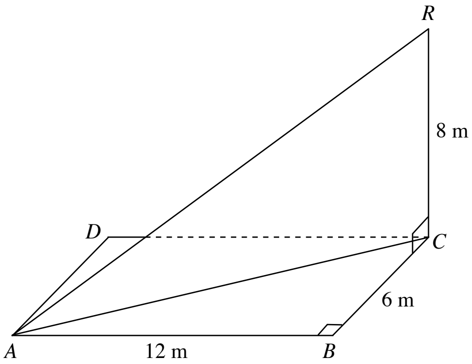
$\angle CBQ = 90°$

$\tan \angle BCQ = \dfrac{8}{6}$

$\angle BCQ = \tan^{-1} \dfrac{8}{6}$

$= 53.1°$ (to 1 d.p.)

(iii)



In $\triangle ABC$,

Using Pythagoras' Theorem,

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 12^2 + 6^2 \\ &= 144 + 36 \\ &= 180 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{180} \\ &= 13.42 \text{ cm (to 4 s.f.)} \end{aligned}$$

In $\triangle ACR$,

$\angle ACR = 90°$

$$\tan \angle CAR = \frac{8}{13.42}$$

$$\begin{aligned} \angle CAR &= \tan^{-1} \frac{8}{13.42} \\ &= 30.8° \text{ (to 1 d.p.)} \end{aligned}$$

Practise Now 9

(i) In $\triangle ABC$,

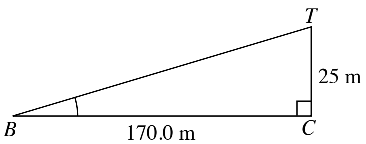
$$\begin{aligned} \angle ACB &= 180° - 105° - 32° \ (\angle \text{ sum of a } \triangle) \\ &= 43° \end{aligned}$$

Using sine rule,

$$\frac{BC}{\sin 105°} = \frac{120}{\sin 43°}$$

$$\begin{aligned} BC &= \frac{120 \sin 105°}{\sin 43°} \\ &= 170 \text{ m (to 3 s.f.)} \end{aligned}$$

(ii)



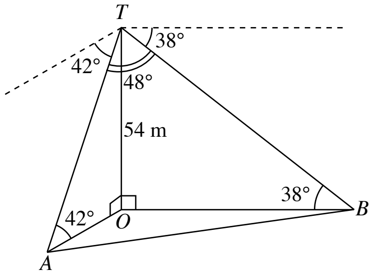
In $\triangle BCT$,

$$\tan \angle TBC = \frac{25}{170.0}$$

$$\begin{aligned} \angle TBC &= \tan^{-1} \frac{25}{170.0} \\ &= 8.4° \text{ (to 1 d.p.)} \end{aligned}$$

$\therefore$ The angle of elevation of T from B is 8.4°.

Practise Now 10



In $\triangle ATO$,

$\angle AOT = 90°$

$$\sin 42° = \frac{54}{AT}$$

$$\begin{aligned} AT &= \frac{54}{\sin 42°} \\ &= 80.70 \text{ m (to 4 s.f.)} \end{aligned}$$

In $\triangle BTO$,

$\angle BOT = 90°$

$$\sin 38° = \frac{54}{BT}$$

$$\begin{aligned} BT &= \frac{54}{\sin 38°} \\ &= 87.71 \text{ m (to 4 s.f.)} \end{aligned}$$

Using cosine rule,

$$\begin{aligned} AB^2 &= AT^2 + BT^2 - 2 \times AT \times BT \times \cos \angle ATB \\ &= 80.70^2 + 87.71^2 - 2 \times 80.70 \times 87.71 \times \cos 48° \\ &= 4733 \text{ (to 4 s.f.)} \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{4733} \\ &= 68.8 \text{ m (to 3 s.f.)} \end{aligned}$$

$\therefore$ The distance between A and B is 68.8 m.

Exercise 9A

1. Let h m be the height of the kite above Lixin's hand.

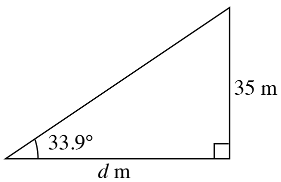
$$\sin 58° = \frac{h}{140}$$

$$\begin{aligned} h &= 140 \sin 58° \\ &= 119 \text{ m (to 3 s.f.)} \end{aligned}$$

$\therefore$ The height of the kite above Lixin's hand is 119 m.

2. Let d m be the distance between the two buildings.

Height difference between the two buildings $= 120 - 85$
$= 35$ m



$$\tan 33.9° = \frac{35}{d}$$

$$\begin{aligned} d &= \frac{35}{\tan 33.9°} \\ &= 52.1 \text{ m (to 3 s.f.)} \end{aligned}$$

$\therefore$ The distance between the two buildings is 52.1 m.

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3. Let $x°$ be the angle of elevation of the top of the spire from the point R.

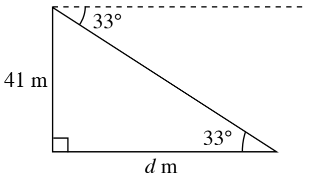
$\tan x° = \dfrac{44}{84}$

$x° = \tan^{-1} \dfrac{44}{84}$

$= 27.6$ (to 1 d.p.)

$\therefore$ The angle of elevation of the top of the spire from the point R is 27.6°.

4. Let d m be the distance between the fire hydrant and the foot of the building.



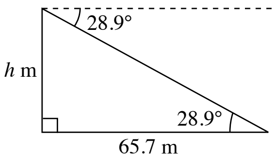
$\tan 33° = \dfrac{41}{d}$

$d = \dfrac{41}{\tan 33°}$

$= 63.1$ m (to 3 s.f.)

$\therefore$ The distance between the fire hydrant and the foot of the building is 63.1 m.

5. Let h m be the height of the cliff.



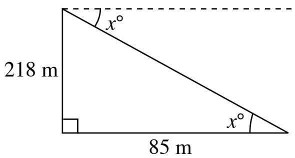
$\tan 28.9° = \dfrac{h}{65.7}$

$h = 65.7 \tan 28.9°$

$= 36.3$ m (to 3 s.f.)

$\therefore$ The height of the cliff is 36.3 m.

6. Let $x°$ be the angle of depression of the bird from the top of the castle.



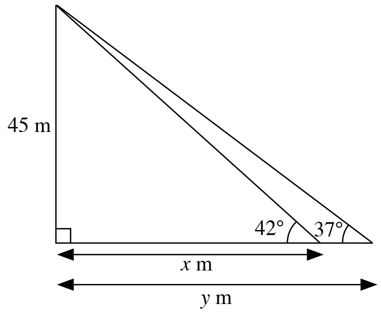
$\tan x° = \dfrac{218}{85}$

$x° = \tan^{-1} \dfrac{218}{85}$

$= 68.7$ (to 1 d.p.)

$\therefore$ The angle of depression of the bird from the top of the castle is 68.7°.

7.



$\tan 42° = \dfrac{45}{x}$

$x = \dfrac{45}{\tan 42°}$

$= 49.98$ (to 4 s.f.)

$\tan 37° = \dfrac{45}{y}$

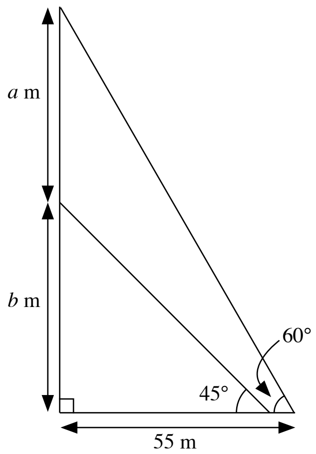
$y = \dfrac{45}{\tan 37°}$

$= 59.72$ (to 4 s.f.)

Distance between the two points $= 59.72 - 49.98$

$= 9.74$ m (to 3 s.f.)

8. Let a m be the height of the castle and b m be the height of the mountain.



$\tan 45° = \dfrac{b}{55}$

$b = 55 \tan 45°$

$= 55$

$\tan 60° = \dfrac{a + 55}{55}$

$a + 55 = 55 \tan 60°$

$a = 55 \tan 60° - 55$

$= 40.3$ (to 3 s.f.)

$\therefore$ The height of the castle is 40.3 m.

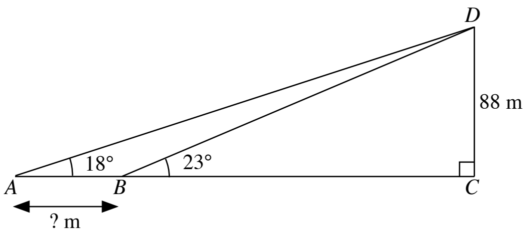
9. Let the distance of Q from the bottom of the bridge by y m and the distance of P from the bottom of the bridge be z m.

$$\tan 23° = \frac{5.5}{y}$$
$$y = \frac{5.5}{\tan 23°}$$
$$= 12.96 \text{ (to 4 s.f.)}$$
$$z = 12.96 - 5.1 = 7.86$$
$$\tan x° = \frac{5.5}{7.86}$$
$$x° = \tan^{-1}\frac{5.5}{7.86}$$
$$= 35.0 \text{ (to 1 d.p.)}$$

10. $\tan 58° = \frac{320}{BC}$

$$BC = \frac{320}{\tan 58°}$$
$$= 200.0 \text{ m (to 4 s.f.)}$$
$$\tan 38° = \frac{320}{AC}$$
$$AC = \frac{320}{\tan 38°}$$
$$= 410.0 \text{ m (to 4 s.f.)}$$
$$\therefore x = 410.0 - 200.0$$
$$= 200 \text{ (to 3 s.f.)}$$

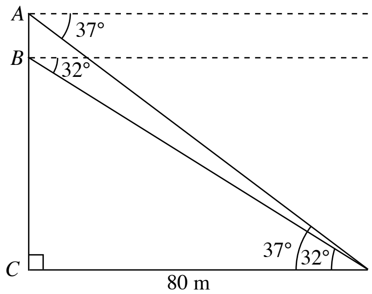
11.



$$\tan 23° = \frac{88}{BC}$$
$$BC = \frac{88}{\tan 23°}$$
$$= 207.3 \text{ m (to 4 s.f.)}$$
$$\tan 18° = \frac{88}{AC}$$
$$AC = \frac{88}{\tan 18°}$$
$$= 270.8 \text{ m (to 4 s.f.)}$$
$$AB = 270.8 - 207.3$$
$$= 63.5 \text{ m (to 3 s.f.)}$$

$\therefore$ The distance between the two boats is 63.5 m.

12.



$$\tan 32° = \frac{BC}{80}$$
$$BC = 80 \tan 32°$$
$$= 49.99 \text{ m (to 4 s.f.)}$$
$$\tan 37° = \frac{AC}{80}$$
$$AC = 80 \tan 37°$$
$$= 60.28 \text{ m (to 4 s.f.)}$$
$$AB = 60.28 - 49.99$$
$$= 10.3 \text{ m (to 3 s.f.)}$$

$\therefore$ The height of the satellite dish is 10.3 m.

13. Let d m be the distance between the bottom of the building and point T on level ground.

$$\tan 26° = \frac{h}{d}$$
$$d = \frac{h}{\tan 26°} \qquad \text{— (1)}$$
$$\tan 35° = \frac{h + 12.2}{d}$$
$$d = \frac{h + 12.2}{\tan 35°} \qquad \text{— (2)}$$

Substitute (1) into (2):

$$\frac{h}{\tan 26°} = \frac{h + 12.2}{\tan 35°}$$
$$h \tan 35° = \tan 26° (h + 12.2)$$
$$h \tan 35° = h \tan 26° + 12.2 \tan 26°$$
$$h \tan 35° - h \tan 26° = 12.2 \tan 26°$$
$$h(\tan 35° - \tan 26°) = 12.2 \tan 26°$$
$$h = \frac{12.2 \tan 26°}{(\tan 35° - \tan 26°)}$$
$$= 28.0 \text{ (to 3 s.f.)}$$

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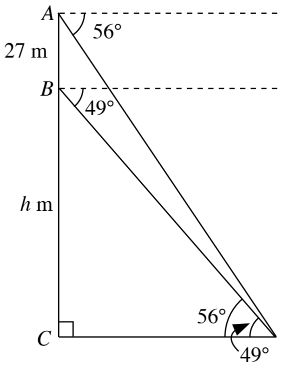
14. Let h m be the height of the cliff.



(i) $\tan 49° = \dfrac{h}{CD}$

$h = CD \tan 49° \quad — (1)$

$\tan 56° = \dfrac{h + 27}{CD}$

$h + 27 = CD \tan 56°$

$h = CD \tan 56° - 27 \quad — (2)$

Substitute (1) into (2):

$CD \tan 49° = CD \tan 56° - 27$

$CD \tan 56° - CD \tan 49° = 27$

$CD(\tan 56° - \tan 49°) = 27$

$CD = \dfrac{27}{\tan 56° - \tan 49°}$

$= 81.3$ m (to 3 s.f.)

∴ The distance between the base of the cliff and the guard house is 81.3 m.

(ii) $h = 81.28 \tan 49°$

$= 93.5$ m (to 3 s.f.)

∴ The height of the cliff is 93.5 m.

Exercise 9B

1. (a) ∴ Bearing of A from O is 033°.

(b) The bearing of B from O is (90° + 28°).
∴ Bearing of B from O is 118°.

(c) The bearing of C from O is (270° – 44°).
∴ Bearing of C from O is 226°.

(d) The bearing of D from O is (360° – 39°).
∴ Bearing of D from O is 321°.

2. (a) The bearing of A from P is equal to (90° – 35°).
∴ Bearing of A from P is 055°.

(b) The bearing of B from P is (180° – 15°).
∴ Bearing of B from P is 165°.

(c) The bearing of C from P is (270° + 47°).
∴ Bearing of C from P is 317°.

(d) The bearing of P from A is (270° – 35°).
∴ Bearing of P from A is 235°.

(e) The bearing of P from B is (360° – 15°).
∴ Bearing of P from B is 345°.

(f) The bearing of P from C is (90° + 47°).
∴ Bearing of P from C is 137°.

3. (a) Bearing of Q from P is 036°.

(b) The bearing of P from Q is (180°+ 36°).
∴ Bearing of P from Q is 216°.

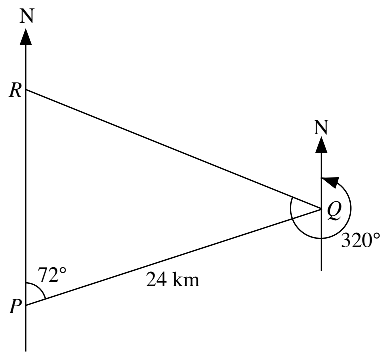
(c) The bearing of R from P is (36° + 37°).
∴ Bearing of R from P is 073°.

(d) 36° + 37° = 73°
The bearing of P from R is (180° + 73°)
∴ Bearing of P from R is 253°.

(e) 100° – 36° = 64°
The bearing of Q from R is (360° – 64°)
∴ Bearing of Q from R is 296°.

(f) 100° – 36° = 64°
The bearing of R from Q is (180° – 64°)
∴ Bearing of R from Q is 116°.

4.



(a) $\angle PQR = 320° - 180° - 72°$

$= 68°$

$\angle PRQ = 180° - 68° - 72°$ (∠ sum of a △)

$= 40°$

Using sine rule,

$\dfrac{PR}{\sin 68°} = \dfrac{24}{\sin 40°}$

$PR = \dfrac{24 \sin 68°}{\sin 40°}$

$= 34.6$ km (to 3 s.f.)

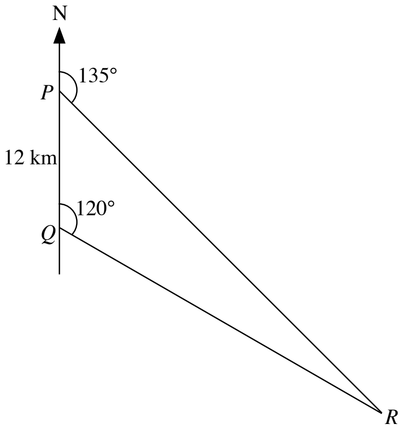
(b) Using sine rule,

$\dfrac{QR}{\sin 72°} = \dfrac{24}{\sin 40°}$

$QR = \dfrac{24 \sin 72°}{\sin 40°}$

$= 35.5$ km (to 3 s.f.)

5.



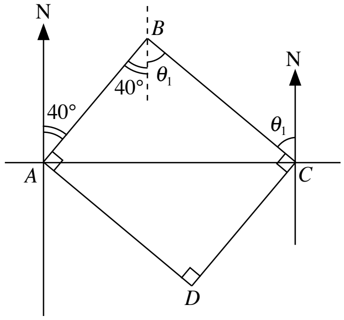
$\angle RPQ = 180° - 135°$ (adj. $\angle$s on a str. line)
$= 45°$
$\angle PRQ = 180° - 120° - 45°$ ($\angle$ sum of a $\triangle$)
$= 15°$
Using sine rule,

$$\frac{PR}{\sin 120°} = \frac{12}{\sin 15°}$$

$$PR = \frac{12 \sin 120°}{\sin 15°}$$

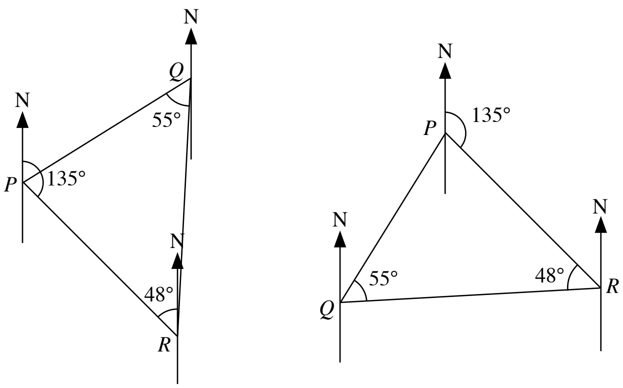
$= 40.2$ km (to 3 s.f.)

6.



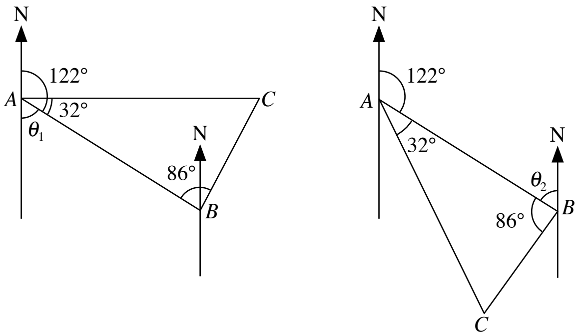
(a) $\theta_1 = 90° - 40° = 50°$
The bearing of B from C is equal to $(360° - 50°)$
$\therefore$ Bearing of B from C is 310°.
(b) Bearing of A from C is 270°.
(c) The bearing of D from C is $(360° - 90° - 50°)$.
$\therefore$ Bearing of D from C is 220°.

7.



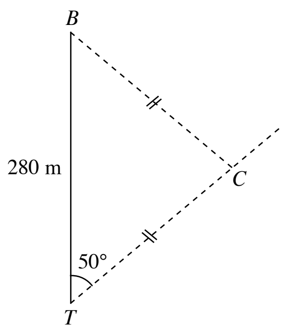
(a) $180° - 135° = 45°$
The bearing of P from R is $(360° - 45°)$.
$\therefore$ Bearing of P from R is 315°.
(b) The bearing of Q from R is $(48° - 45°)$.
$\therefore$ Bearing of Q from R is 003°.
Or
The bearing of Q from R is $(360° - 45° - 48°)$.
$\therefore$ Bearing of Q from R is 267°.
(c) $\angle QPR = 180° - 55° - 48°$ ($\angle$ sum of a $\triangle$)
$= 77°$
$135° - 77° = 58°$
$180° - 58° = 122°$
The bearing of P from Q is $(360° - 122°)$.
$\therefore$ Bearing of P from Q is 238°.
Or
The bearing of P from Q is $(77° - 45°)$.
$\therefore$ Bearing of P from Q is 032°.

8.



$\theta_1 = 180° - 122°$ (adj. $\angle$s on a str. line)
$= 58°$
The bearing of C from B is $(86° - 58°)$.
$\therefore$ Bearing of C from B is 028°.
Or
$\theta_2 = 180° - 122°$ (alt. $\angle$s)
$= 58°$
The bearing of C from B is $(360° - 86° - 58°)$.
$\therefore$ Bearing of C from B is 216°.

9. **(a)**



$\angle CBT = 50°$ (isos. $\triangle$)
$\angle BCT = 180° - 50° - 50°$ ($\angle$ sum of a $\triangle$)
$= 80°$
Using sine rule,

$$\frac{CT}{\sin 50°} = \frac{280}{\sin 80°}$$

$$CT = \frac{280 \sin 50°}{\sin 80°}$$

$= 218$ m (to 3 s.f.)

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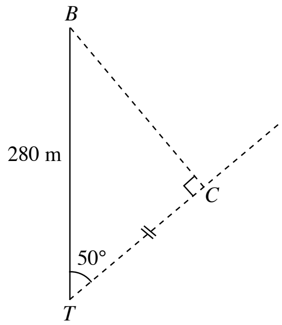
(b)

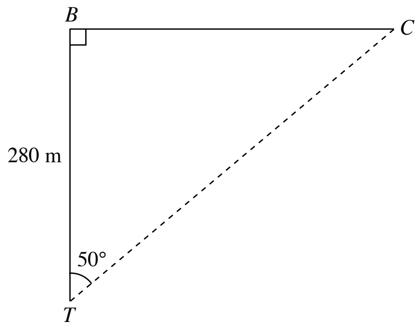


$$\cos 50° = \frac{CT}{280}$$

$$CT = 280 \cos 50°$$

$$= 180 \text{ m (to 3 s.f.)}$$

(c)



$$\cos 50° = \frac{280}{CT}$$

$$CT = \frac{280}{\cos 50°}$$

$$= 436 \text{ m (to 3 s.f.)}$$

10. $\theta_1 = 180° - 128°$ (int. $\angle$s)

$= 52°$

$\angle PQR = 360° - 295° - 52°$ ($\angle$s at a point)

$= 13°$

Using cosine rule,

$PR^2 = 25° + 30^2 - 2(25)(30) \cos 13°$

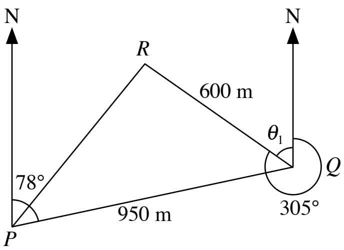
$= 1525 - 1500 \cos 13°$

$= 63.44$ (to 4 s.f.)

$PR = \sqrt{63.44} = 7.97$ km (to 3 s.f.)

$\therefore$ The distance between P and R is 7.97 km.

11.



(i) $\theta_1 = 360° - 305°$ ($\angle$s at a point)

$= 55°$

$\angle PQR = 180° - 78° - 55°$ (int. $\angle$s)

$= 47°$

Using cosine rule,

$PR^2 = 600^2 + 950^2 - 2(600)(950) \cos 47°$

$= 1\,262\,500 - 1\,140\,000 \cos 47°$

$= 485\,000$ (to 4 s.f.)

$PR = \sqrt{485\,000} = 696$ m (to 3 s.f.)

(ii) Using sine rule,

$$\frac{\sin \angle QPR}{600} = \frac{\sin 47°}{696.4}$$

$$\sin \angle QPR = \frac{600 \sin 47°}{696.4}$$

$= 0.6301$ (to 4 s.f.)

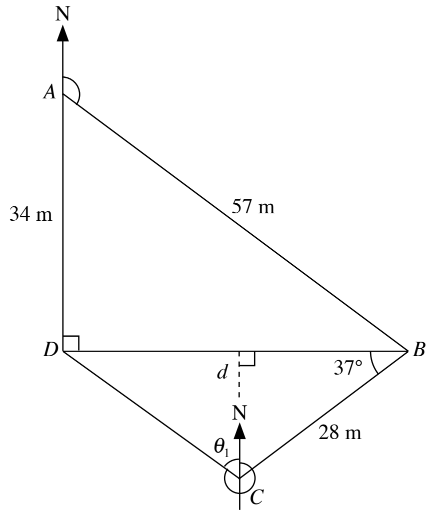
$\angle QPR = \sin^{-1} 0.6301$

$= 39.06°$ (to 2 d.p.)

The bearing of R from P is $(78° - 39.06°)$.

$\therefore$ Bearing of R from P is 038.9° (to 1 d.p.).

12.



(i) $\cos \angle BAD = \dfrac{34}{57}$

$\angle BAD = \cos^{-1} \dfrac{34}{57}$

$= 53.38$ (to 2 d.p.)

The bearing of B from A is $(180° - 53.38°)$.

$\therefore$ Bearing of B from A is 126.6° (to 1 d.p.).

(ii) Let d m be the shortest distance from C to BD.

$\sin \angle CBD = \dfrac{d}{28}$

$\sin 37° = \dfrac{d}{28}$

$d = 28 \sin 37°$

$= 16.9$ m (to 3 s.f.)

(iii) By Pythagoras' Theorem,

$DB^2 = 57^2 - 34^2$

$= 2093$

$DB = \sqrt{2093}$

$= 45.75$ m (to 4 s.f.)

Using cosine rule,

$CD^2 = 28^2 + 45.75^2 - 2(28)(45.75) \cos 37°$

$= 831.0$ (to 4 s.f.)

$CD = \sqrt{831.0} = 28.83$ m (to 4 s.f.)

$\cos \theta = \dfrac{16.85}{28.83}$

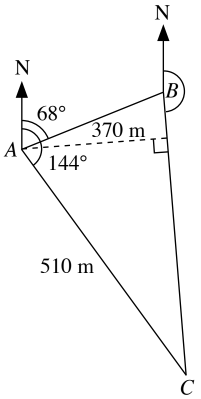
$\theta = \cos^{-1} \dfrac{16.85}{28.83}$

$= 54.24°$ (to 2 d.p.)

The bearing of D from C is $(360° - 54.24°)$.

$\therefore$ Bearing of D from C is 305.8° (to 1 d.p.).

13.



(i) Using cosine rule,

$BC^2 = 370° + 510^2 - 2(370)(510) \cos 76°$

$= 397\ 000 - 377\ 400 \cos 76°$

$= 305\ 700$ (to 4 s.f.)

$BC = \sqrt{305\ 700} = 553$ m (to 3 s.f.)

(ii) Using sine rule,

$\dfrac{\sin \angle ACB}{370} = \dfrac{\sin 76°}{552.9}$

$\sin \angle ACB = \dfrac{370 \sin 76°}{552.9}$

$\angle ACB = \sin^{-1} \dfrac{370 \sin 76°}{552.9}$

$= 40.5°$ (to 1 d.p.)

(iii) $180° - 68° = 112°$ (int. $\angle$s)

$\angle ABC = 180° - 76° - 40.49°$ ($\angle$ sum of a $\triangle$)

$= 63.51°$ (to 2 d.p.)

The bearing of C from B is $(360° - 112° - 63.51°)$.

$\therefore$ Bearing of C from B is 184.5°.

(iv) Let the shortest distance from A to BC be h m.

Area of $\triangle ABC = \dfrac{1}{2} \times 370 \times 510 \times \sin 76°$

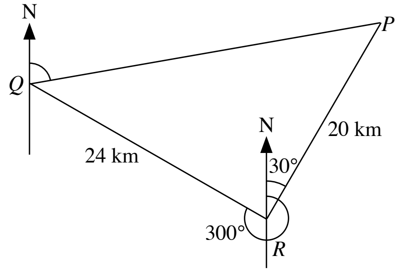
$= 94\ 350 \sin 76°$

$\dfrac{1}{2} \times 552.9 \times h = 94\ 350 \sin 76°$

$h = 331$ m (to 3 s.f.)

$\therefore$ The shortest distance from A to BC is 331 m.

14.



After 2 hours,

Distance travelled by $P = 10 \times 2 = 20$ km

Distance travelled by $Q = 12 \times 2 = 24$ km

$360° - 300° = 60°$ ($\angle$s at a point)

$\angle PRQ = 60° + 30° = 90°$

$\therefore \triangle PRQ$ is a right-angled triangle.

By Pythagoras' Theorem,

$PQ^2 = PR^2 + QR^2$

$= 20^2 + 24^2$

$= 976$

$PQ = \sqrt{976}$

$= 31.2$ km (to 3 s.f.)

$\tan \angle PQR = \dfrac{20}{24}$

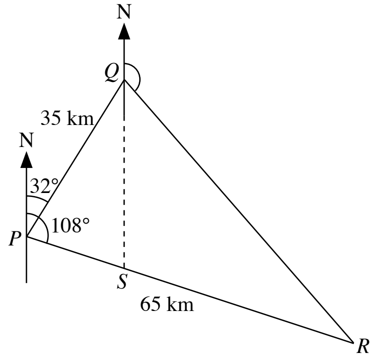
$\angle PQR = \tan^{-1} \dfrac{20}{24}$

$= 39.81°$ (to 2 d.p.)

The bearing of P from Q is $(180° - 60° - 39.81°)$.

$\therefore$ Bearing of P from Q is 080.2°.

15.



(a) (i) $\angle QPR = 108° - 32° = 76°$

Using cosine rule,

$QR^2 = 35^2 + 65^2 - 2 \times 35 \times 65 \times \cos 76°$

$= 4349$ (to 4 s.f.)

$QR = 65.9$ km (to 3 s.f.)

(ii) Using sine rule,

$\dfrac{\sin \angle PQR}{65} = \dfrac{\sin 76°}{65.95}$

$\angle PQR = \sin^{-1} \dfrac{65 \sin 76°}{65.95}$

$= 73.00°$ (to 2 d.p.)

$180° - 32° = 148°$

The bearing of R from Q is $(360° - 148° - 73.00°)$.

$\therefore$ Bearing of R from Q is 139.0° (to 1 d.p.).

(b) $\angle PQS = 32°$ (alt. $\angle$s)

$\angle ABC = 180° - 76° - 32°$ ($\angle$ sum of a $\triangle$)

$= 72°$

$$\frac{PS}{\sin 32°} = \frac{35}{\sin 72°}$$

$$PS = \frac{35 \sin 32°}{\sin 72°}$$

$= 19.50$ km

Time taken $= \frac{19.50}{30} = 0.65$ h $= 39$ minutes

$\therefore$ The time when the ship reaches S is 1009.

Exercise 9C

1. **(i)** By Pythagoras' Theorem,

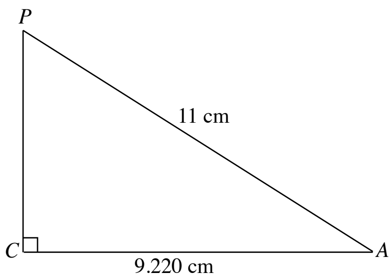
$AC^2 = 6^2 + 7^2$

$= 85$

$AC = \sqrt{85}$

$= 9.22$ cm (to 3 s.f.)

(ii)



By Pythagoras' Theorem,

$PC^2 = 11^2 - 9.220^2$

$= 35.99$ (to 4 s.f.)

$PC = \sqrt{35.99}$

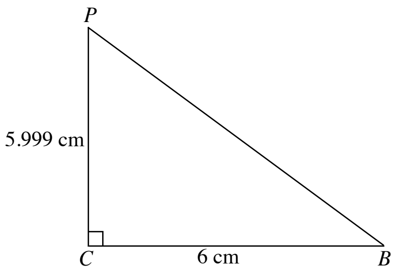
$= 6.00$ cm (to 3 s.f.)

(iii) $\cos \angle PAC = \frac{9.220}{11}$

$\angle PAC = \cos^{-1} \frac{9.220}{11}$

$= 33.1°$ (to 1 d.p.)

(iv)



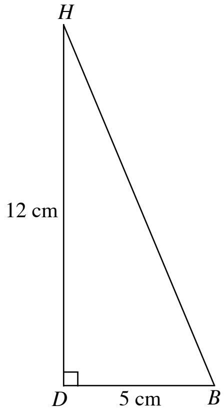
$\tan \angle PBC = \frac{5.999}{6}$

$\angle PBC = \tan^{-1} \frac{5.999}{6}$

$= 45.0°$ (to 1 d.p.)

$\therefore$ The angle of elevation of P from B is 45.0°.

2. **(i)**



By Pythagoras' Theorem,

$BH^2 = DH^2 + DB^2$

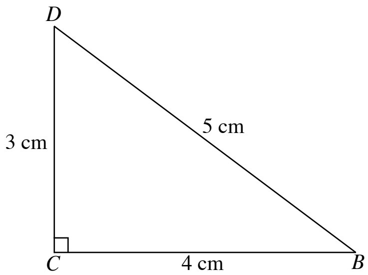
$= 12^2 + 5^2$

$= 169$

$BH = \sqrt{169}$

$= 13$ cm

(ii)



$\tan \angle BDC = \frac{4}{3}$

$\angle BDC = \tan^{-1} \frac{4}{3}$

$= 53.1°$ (to 1 d.p.)

(iii) $\tan \angle HBD = \frac{12}{5}$

$\angle HBD = \tan^{-1} \frac{12}{5}$

$= 67.4°$ (to 1 d.p.)

3. **(i)** $\tan \angle DGH = \frac{4}{6}$

$\angle DGH = \tan^{-1} \frac{4}{6}$

$= 33.7°$ (to 1 d.p.)

(ii) By Pythagoras' Theorem,

$HF^2 = GH^2 + GF^2$

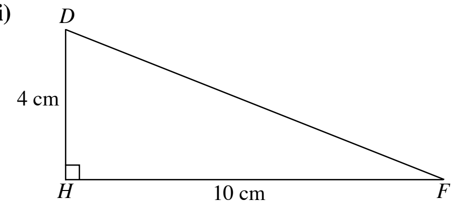
$= 6^2 + 8^2$

$= 100$

$HF = \sqrt{100}$

$= 10$ cm

(iii)



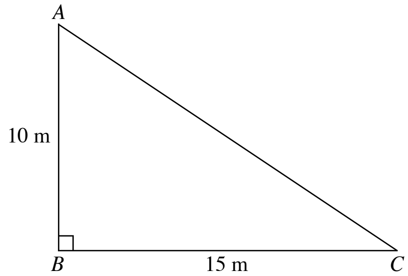
$$\tan \angle DFH = \frac{4}{10}$$

$$\angle DFH = \tan^{-1} \frac{4}{10}$$

$$= 21.8° \text{ (to 1 d.p.)}$$

$\therefore$ The angle of elevation of D from F is 21.8°.

4. **(i)**

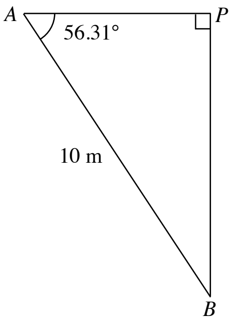


$$\tan \angle BAC = \frac{15}{10}$$

$$\angle BAC = \tan^{-1} \frac{15}{10}$$

$$= 56.3° \text{ (to 1 d.p.)}$$

(ii)



$$\cos 56.31° = \frac{AP}{10}$$

$$AP = 10 \cos 56.31°$$

$$= 5.55 \text{ m (to 3 s.f.)}$$

(iii) By Pythagoras' Theorem,

$$OP^2 = AO^2 - AP^2$$

$$= 12^2 - 5.547^2$$

$$= 113.2 \text{ (to 4 s.f.)}$$

$$OP = \sqrt{113.2}$$

$$= 10.6 \text{ cm (to 3 s.f.)}$$

(iv) By Pythagoras' Theorem,

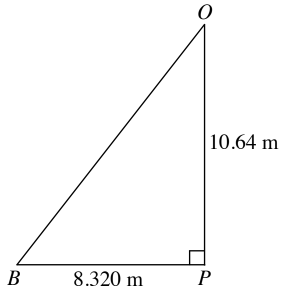
$$BP^2 = AB^2 - AP^2$$

$$= 10^2 - 5.547^2$$

$$= 69.23 \text{ (to 4 s.f.)}$$

$$BP = \sqrt{69.23}$$

$$= 8.320 \text{ cm (to 4 s.f.)}$$



$$\tan \angle OBP = \frac{10.64}{8.320}$$

$$\angle OBP = \tan^{-1} \frac{10.64}{8.320}$$

$$= 52.0° \text{ (to 1 d.p.)}$$

5. $\angle AQB = 180° - 75° - 45°$ ($\angle$ sum of a $\triangle$)

$$= 60°$$

Using sine rule,

$$\frac{AQ}{\sin 75°} = \frac{60}{\sin 60°}$$

$$AQ = \frac{60 \sin 75°}{\sin 60°}$$

$$= 66.92 \text{ m (to 4 s.f.)}$$

$$\tan 30° = \frac{PQ}{66.92}$$

$$PQ = 38.6 \text{ m (to 3 s.f.)}$$

$\therefore$ The height of the tower is 38.6 m.

6. **(i)** By Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 20^2 + 20^2$$

$$= 800$$

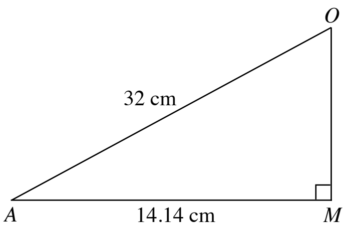
$$AC = \sqrt{800}$$

$$= 28.28 \text{ cm (to 4 s.f.)}$$

$$AM = \frac{28.28}{2}$$

$$= 14.1 \text{ cm (to 2 s.f.)}$$

(ii)



By Pythagoras' Theorem,

$$OM^2 = AO^2 - AM^2$$

$$= 32^2 - 14.14^2$$

$$= 824.1 \text{ (to 4 s.f.)}$$

$$OM = \sqrt{824.1}$$

$$= 28.7 \text{ cm (to 3 s.f.)}$$

(iii) $\cos \angle OAM = \dfrac{14.14}{32}$

$$\angle OAM = \cos^{-1} \frac{14.14}{32}$$

$$= 63.8° \text{ (to 1 d.p.)}$$

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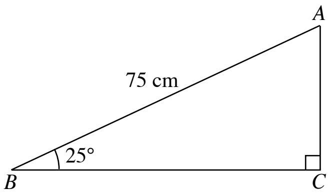
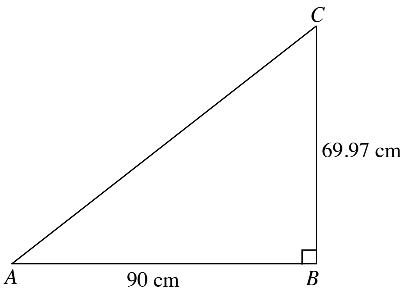
7. **(i)**



$$\cos 25° = \frac{BC}{75}$$

$$BC = 75 \cos 25°$$

$$= 67.97 \text{ cm (to 4 s.f.)}$$



By Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 90^2 + 69.97^2$$

$$= 13\ 000 \text{ (to 4 s.f.)}$$

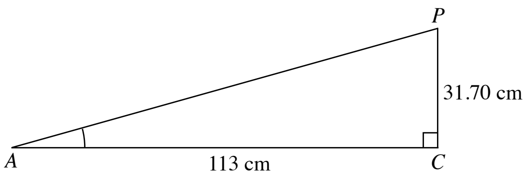
$$AC = \sqrt{13\ 000}$$

$$= 113 \text{ cm (to 3 s.f.)}$$

(ii) $\sin 25° = \dfrac{PC}{75}$

$$PC = 75 \sin 25°$$

$$= 31.70 \text{ cm (to 4 s.f.)}$$

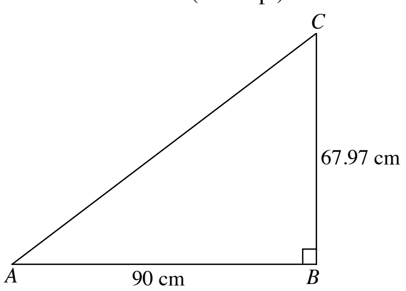


$$\tan \angle PAC = \frac{31.70}{113}$$

$$\angle PAC = \tan^{-1} \frac{31.70}{113}$$

$$= 15.7° \text{ (to 1 d.p.)}$$

(iii)



$$\tan \angle CAB = \frac{67.97}{90}$$

$$\angle CAB = \tan^{-1} \frac{67.97}{90}$$

$$= 37.1° \text{ (to 1 d.p.)}$$

8. **(i)** By Pythagoras' Theorem,

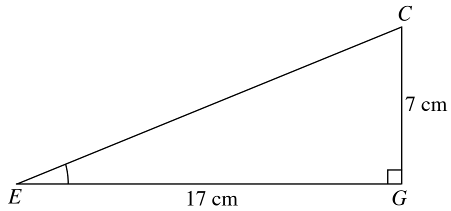
$$EG^2 = EH^2 + HG^2$$

$$= 8^2 + 15^2$$

$$= 289$$

$$EG = \sqrt{289}$$

$$= 17 \text{ cm}$$



$$\tan \angle CEG = \frac{7}{17}$$

$$\angle CEG = \tan^{-1} \frac{7}{17}$$

$$= 22.4° \text{ (to 1 d.p.)}$$

(ii) By Pythagoras' Theorem,

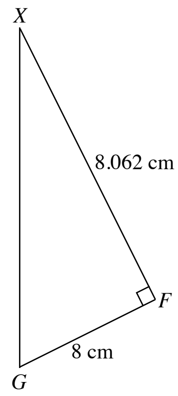
$$XF^2 = XB^2 + BF^2$$

$$= 4^2 + 7^2$$

$$= 65$$

$$XF = \sqrt{65}$$

$$= 8.062 \text{ cm (to 4 s.f.)}$$



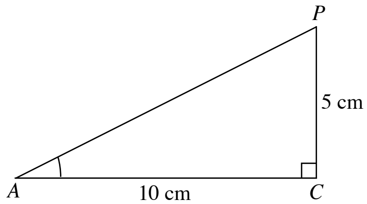
$$\tan \angle GXF = \frac{8}{8.062}$$

$$\angle GXF = \tan^{-1} \frac{8}{8.062}$$

$$= 44.8° \text{ (to 1 d.p.)}$$

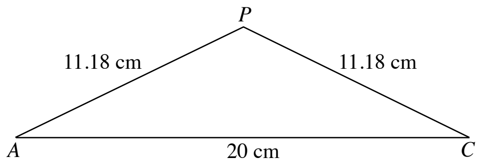
9. **(i)** By Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$= 12^2 + 16^2$
$= 400$
$AC = \sqrt{400}$
$= 20$ cm



By Pythagoras' Theorem,

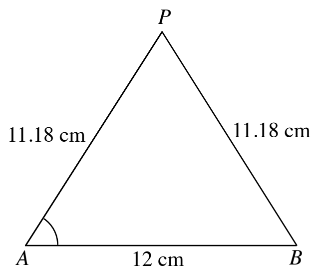
$AP^2 = 5^2 + 10^2$
$= 125$
$AP = \sqrt{125}$
$= 11.18$ cm (to 4 s.f.)



Using cosine rule,

$$\cos \angle PAC = \frac{11.18^2 + 20^2 - 11.18^2}{2 \times 11.18 \times 20}$$
$= 0.8945$ (to 4 s.f.)
$\angle PAC = \cos^{-1} 0.8945$
$= 26.6°$ (to 1 d.p.)

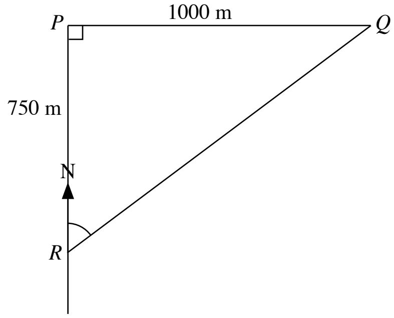
(ii)



Using cosine rule,

$$\cos \angle PAB = \frac{11.18^2 + 12^2 - 11.18^2}{2 \times 11.18 \times 12}$$
$= 0.5367$ (to 4 s.f.)
$\angle PAB = \cos^{-1} 0.5367$
$= 57.5°$ (to 1 d.p.)

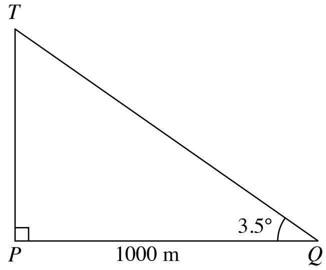
10. **(i)**



$$\tan \angle PRQ = \frac{1000}{750}$$
$$\angle PRQ = \tan^{-1} \frac{1000}{750}$$
$= 53.1°$ (to 1 d.p.)

$\therefore$ Bearing of Q from R is 053.1°.

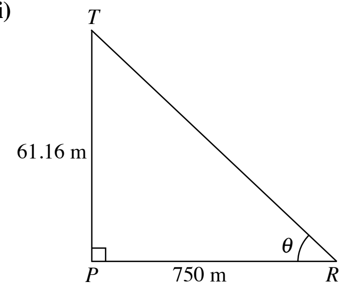
(ii)



$$\tan 3.5° = \frac{TP}{1000}$$
$TP = 1000 \tan 3.5°$
$= 61.2$ m (to 3 s.f.)

$\therefore$ The height of the mast is 61.2 m.

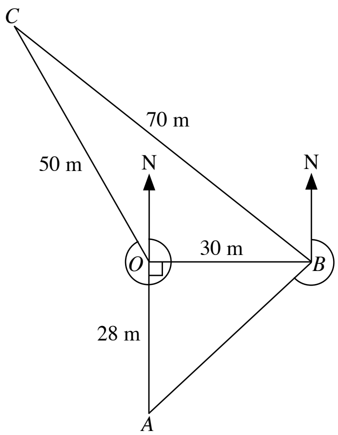
(iii)



$$\tan \theta = \frac{61.16}{750}$$
$$\theta = \tan^{-1} \frac{61.16}{750}$$
$= 4.7°$ (to 1 d.p.)

$\therefore$ The angle of elevation of T from R is 4.7°.

11. (a)



(i) $\tan \angle OBA = \dfrac{28}{30}$

$\angle OBA = \tan^{-1} \dfrac{28}{30}$

$= 43.03°$ (to 2 d.p.)

The bearing of A from B is $(270° - 43.03°)$.

$\therefore$ Bearing of A from B is 227°.

(ii) Using cosine rule,

$\cos \angle COB = \dfrac{50^2 + 30^2 - 70^2}{2 \times 50 \times 30}$

$= -0.5$

$\angle COB = \cos^{-1}(-0.5)$

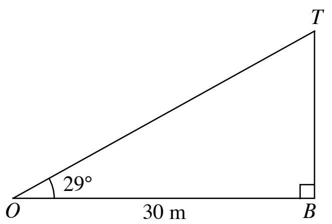
$= 120°$

(iii) $120° - 90° = 30°$

The bearing of C from O is $(360° - 30°)$.

$\therefore$ Bearing of C from O is 330°.

(b)



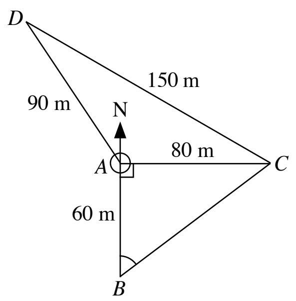
$\tan 29° = \dfrac{TB}{30}$

$TB = 30 \tan 29°$

$= 16.6$ m (to 3 s.f.)

$\therefore$ The height of the flagpole is 16.6 m.

12. (a)



(i) $\tan \angle ABC = \dfrac{80}{60}$

$\angle ABC = \tan^{-1} \dfrac{80}{60}$

$= 53.1°$ (to 1 d.p.)

$\therefore$ Bearing of C from B is 053.1°.

(ii) Using cosine rule,

$\cos \angle DAC = \dfrac{80^2 + 90^2 - 150^2}{2 \times 80 \times 90}$

$= -0.5556$ (to 4 s.f.)

$\angle DAC = \cos^{-1}(-0.5556)$

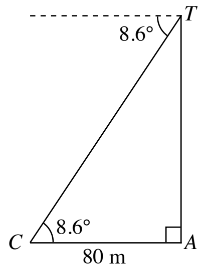
$= 123.75°$ (to 2 d.p.)

$123.75° - 90° = 33.75°$

The bearing of D from A is $(360° - 33.75°)$.

$\therefore$ Bearing of D from A is 326.3°

(b)

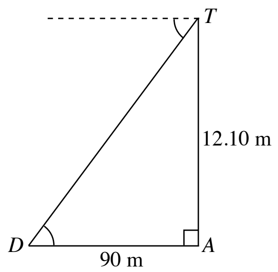


$\tan 8.6° = \dfrac{TA}{80}$

$TA = 80 \tan 8.6°$

$= 12.1$ m (to 3 s.f.)

$\therefore$ The height of the mast is 12.1 m.



$\tan \angle TDA = \dfrac{12.10}{90}$

$\angle TDA = \tan^{-1} \dfrac{12.10}{90}$

$= 7.7°$ (to 1 d.p.)

$\therefore$ The angle of depression of D from the top of the mast is 7.7°.

13. (a) (i) $\angle ACB = 180° - 94° - 47°$ ($\angle$ sum of a $\triangle$)

$= 39°$

Using sine rule,

$\dfrac{BC}{\sin 94°} = \dfrac{240}{\sin 39°}$

$BC = \dfrac{240 \sin 94°}{\sin 39°}$

$= 380$ m (to 3 s.f.)

(ii) Area of $\triangle ABC = \dfrac{1}{2} \times 240 \times 380.4 \times \sin 47°$

$= 33\ 400$ m^2 (to 3 s.f.)

(iii) Let the shortest distance from A to BC be h m.

Area of $\triangle ABC = 33\,400 \text{ m}^2$

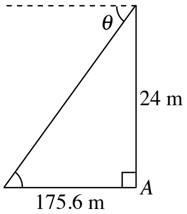
$$\frac{1}{2} \times BC \times h = 33\,400$$

$$\frac{1}{2} \times 380.4 \times h = 33\,400$$

$$h = 176 \text{ (to 3 s.f.)}$$

$\therefore$ The shortest distance from A to BC is 176 m.

(b)



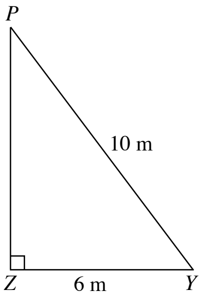
$$\tan\theta = \frac{24}{175.6}$$

$$\theta = \tan^{-1}\frac{24}{175.6}$$

$$= 7.8° \text{ (to 1 d.p.)}$$

$\therefore$ The greatest angle of depression of the boat from the top of the tree is 7.8°

14. (i)



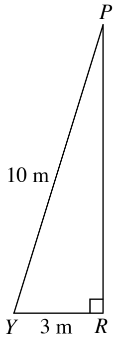
$$\cos\angle PYZ = \frac{6}{10}$$

$$\angle PYZ = \cos^{-1}\frac{6}{10}$$

$$= 53.1° \text{ (to 1 d.p.)}$$

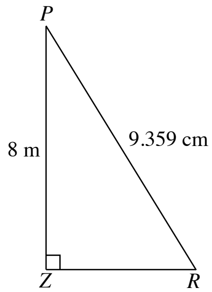
(ii) By Pythagoras' Theorem,

$$PZ^2 = PY^2 - ZY^2$$
$$= 10^2 - 6^2$$
$$= 64$$
$$PZ = \sqrt{64}$$
$$= 8 \text{ cm}$$



By Pythagoras' Theorem,

$$PR^2 = PY^2 - RY^2$$
$$= 10^2 - 3^2$$
$$= 91$$
$$PR = \sqrt{91}$$
$$= 9.539 \text{ cm (to 4 s.f.)}$$



$$\sin\angle PRZ = \frac{8}{9.359}$$

$$\angle PRZ = \sin^{-1}\frac{8}{9.359}$$

$$= 57.0° \text{ (to 1 d.p.)}$$

15. (i) By Pythagoras' Theorem,

$$AC^2 = 10^2 + 8^2$$
$$= 164$$
$$AC = \sqrt{164}$$
$$= 12.81 \text{ cm (to 4 s.f.)}$$

By Pythagoras' Theorem,

$$AB^2 = 8^2 + 6^2$$
$$= 100$$
$$AB = \sqrt{100}$$
$$= 10 \text{ cm}$$

By Pythagoras' Theorem,

$$BC^2 = 10^2 + 6^2$$
$$= 136$$
$$BC = \sqrt{136}$$
$$= 11.66 \text{ cm (to 4 s.f.)}$$

Using cosine rule,

$$\cos\angle ABC = \frac{10^2 + 11.66^2 - 12.81^2}{2 \times 10 \times 11.66}$$
$$= 0.3081 \text{ (to 4 s.f.)}$$
$$\angle ABC = \cos^{-1}(0.3081)$$
$$= 72.0° \text{ (to 1 d.p.)}$$

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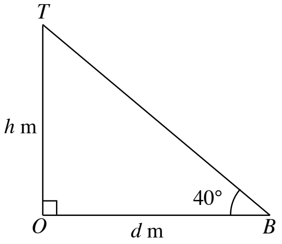
(ii) Area of surface to be coated with varnish

$= \frac{1}{2} \times 10 \times 11.66 \times \sin 72.02°$

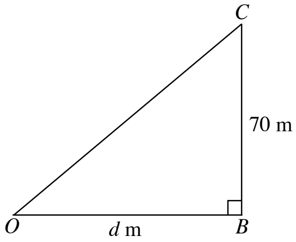
$= 55.5 \text{ cm}^2$ (to 3 s.f.)

16. Let h m be the height of the tower and d m be the distance OB.



$\tan 40° = \frac{h}{d}$

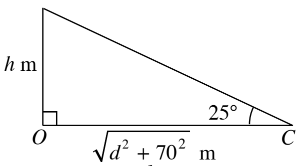
$d = \frac{h}{\tan 40°}$ — (1)



By Pythagoras' Theorem,

$OC^2 = d^2 + 70^2$

$OC = \sqrt{d^2 + 70^2}$ m



$\tan 25° = \frac{h}{\sqrt{d^2 + 70^2}}$

$h = \tan 25° \sqrt{d^2 + 70^2}$ — (2)

Substitute (1) into (2):

$$h = \tan 25° \sqrt{\left(\frac{h}{\tan 40°}\right)^2 + 70^2}$$

$$\frac{h}{\tan 25°} = \sqrt{\left(\frac{h}{\tan 40°}\right)^2 + 70^2}$$

$$\left(\frac{h}{\tan 25°}\right)^2 = \left(\frac{h}{\tan 40°}\right)^2 + 70^2$$

$4.5989h^2 = 1.420h^2 + 4900$

$3.1789h^2 = 4900$

$h^2 = 1541$

$h = \sqrt{1541}$

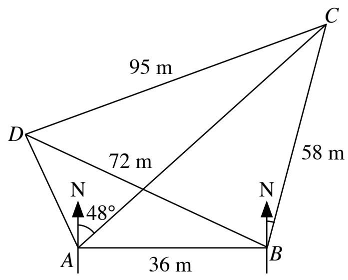
$= 39.3$ m (to 3 s.f.)

$d = \frac{39.26}{\tan 40°}$

$= 46.8$ m (to 3 s.f.)

$\therefore$ The height of the tower is 39.3 m and the distance OB is 46.8 m.

17. (i)



$\angle CAB = 90° - 48° = 42°$

Using sine rule,

$\frac{\sin \angle ACB}{36} = \frac{\sin 42°}{58}$

$\sin \angle ACB = \frac{36 \sin 42°}{58}$

$= 0.4153$ (to 4 s.f.)

$\angle ACB = \sin^{-1} 0.4153$

$= 24.54°$ (to 2 d.p.)

$\angle ABC = 180° - 42° - 24.54°$ ($\angle$ sum of a $\triangle$)

$= 113.46°$

The bearing of C from B is $(113.46° - 90°)$.

$\therefore$ Bearing of C from B is 023.5° (to 1 d.p.).

(ii) Let the shortest distance from B to DC be h m.

Using cosine rule,

$\cos \angle CBD = \frac{58^2 + 72^2 - 95^2}{2 \times 58 \times 72}$

$= -0.05711$ (to 4 s.f.)

$\angle CBD = \cos^{-1} (-0.05711)$

$= 93.27°$ (to 1 d.p.)

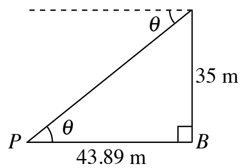
Area of $\triangle ABC = \frac{1}{2} \times 72 \times 58 \times \sin 93.27°$

$= 2085 \text{ m}^2$ (to 4 s.f.)

$\frac{1}{2} \times DC \times d = 2085$

$\frac{1}{2} \times 95 \times d = 2085$

$d = 43.89$ m (to 4 s.f.)



$\tan \theta = \frac{35}{43.89}$

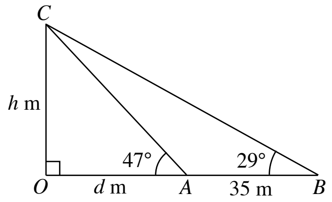
$\theta = \tan^{-1} \frac{35}{43.89}$

$= 38.6°$ (to 1 d.p.)

$\therefore$ The angle of depression of P from the top of the tower is 38.6°.

Review Exercise 9

1. Let d m be the distance between the castle and point A and h m be the height of the castle.



$$\tan 47° = \frac{h}{d}$$

$$d = \frac{h}{\tan 47°} \quad \text{— (1)}$$

$$\tan 29° = \frac{h}{d + 35}$$

$\tan 29° (d + 35) = h$ — (2)

Substitute (1) into (2):

$$\tan 29° \left(\frac{h}{\tan 47°} + 35 \right) = h$$

$$\frac{h \tan 29°}{\tan 47°} + 35 \tan 29° = h$$

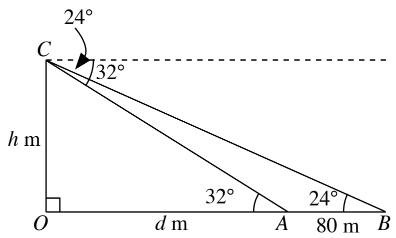
$$0.5169h + 35 \tan 29° = h$$

$$0.4831h = 35 \tan 29°$$

$$h = 40.2 \text{ m (to 3 s.f.)}$$

$\therefore$ The height of the castle is 40.2 m.

2.



$$\tan 32° = \frac{h}{d}$$

$$d = \frac{h}{\tan 32°} \quad \text{— (1)}$$

$$\tan 24° = \frac{h}{d + 80}$$

$\tan 24° (d + 80) = h$ — (2)

Substitute (1) into (2):

$$\tan 24° \left(\frac{h}{\tan 32°} + 80 \right) = h$$

$$\frac{h \tan 24°}{\tan 32°} + 80 \tan 24° = h$$

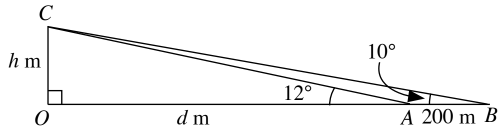
$$0.7125h + 80 \tan 24° = h$$

$$0.2875h = 80 \tan 24°$$

$$h = 124 \text{ m (to 3 s.f.)}$$

$\therefore$ The height of the cliff is 124 m.

3.



$$\tan 12° = \frac{h}{d}$$

$$d = \frac{h}{\tan 12°} \quad \text{— (1)}$$

$$\tan 10° = \frac{h}{d + 200}$$

$\tan 10° (d + 200) = h$ — (2)

Substitute (1) into (2):

$$\tan 10° \left(\frac{h}{\tan 12°} + 200 \right) = h$$

$$\frac{h \tan 10°}{\tan 12°} + 200 \tan 10° = h$$

$$0.8296h + 200 \tan 10° = h$$

$$0.1704h = 200 \tan 10°$$

$$h = 207 \text{ m (to 3 s.f.)}$$

$\therefore$ The height of the lighthouse is 207 m.

4. **(i)** $\sin 40° = \dfrac{AT}{23}$

$AT = 23 \sin 40°$

$= 14.8$ cm (to 3 s.f.)

(ii) $\tan \angle ABT = \dfrac{14.78}{14}$

$\angle ABT = \tan^{-1} \dfrac{14.78}{14}$

$= 46.6°$ (to 1 d.p.)

$\therefore$ The angle of elevation of T from B is 46.6°.

(iii) $\cos 40° = \dfrac{AC}{23}$

$AC = 23 \cos 40°$

$= 17.62$ cm (to 4 s.f.)

By Pythagoras' Theorem,

$BC^2 = AB^2 + AC^2$

$= 14^2 + 17.62^2$

$= 506.5$ (to 4 s.f.)

$BC = \sqrt{506.5}$

$= 22.5$ cm (to 3 s.f.)

5. **(i)** $\tan 35° = \dfrac{12}{AB}$

$AB = \dfrac{12}{\tan 35°}$

$= 17.1$ m (to 3 s.f.)

(ii) By Pythagoras' Theorem,

$BC^2 = AC^2 - AB^2$

$= 24 - 17.14^2$

$= 282.2$ (to 4 s.f.)

$BC = \sqrt{282.2}$

$= 16.8$ m (to 3 s.f.)

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(iii) $\tan \angle BCT = \dfrac{12}{16.80}$

$\angle BCT = \tan^{-1} \dfrac{12}{16.80}$

$= 35.5°$ (to 1 d.p.)

$\therefore$ The angle of depression of C from T is 35.5°.

6. **(i)** By Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$= 12^2 + 16^2$
$= 400$

$AC = \sqrt{400}$
$= 20$ m

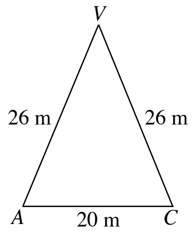
$AX = \dfrac{1}{2} \times 20 = 10$ m

By Pythagoras' Theorem,

$VX^2 = AV^2 - AX^2$
$= 26^2 - 10^2$
$= 576$

$VX = \sqrt{576}$
$= 24$ m

(ii)



Using cosine rule,

$\cos \angle AVC = \dfrac{26^2 + 26^2 - 20^2}{2 \times 26 \times 26}$

$= 0.7041$ (to 4 s.f.)

$\angle AVC = \cos^{-1} 0.7041$

$= 45.2°$ (to 1 d.p.)

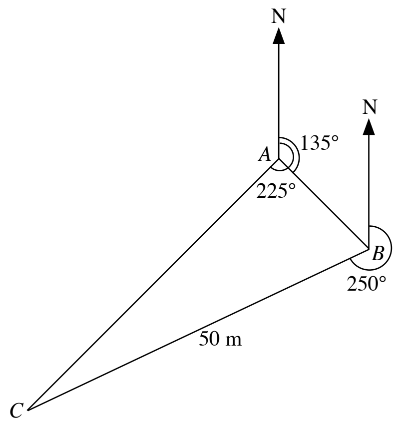
(iii) By Pythagoras' Theorem,

$VP^2 = VX^2 + XP^2$
$= 24^2 + 6^2$
$= 612$

$VP = \sqrt{612}$
$= 24.7$ m (to 3 s.f.)

$\therefore$ The length of the crack is 24.7 m.

7.



$\angle CAB = 225° - 135° = 90°$

$180° - 135° = 45°$ (int. $\angle$s)

$\angle ABC = 360° - 250° - 45°$ ($\angle$s at a point)
$= 65°$

$\cos 65° = \dfrac{AB}{50}$

$AB = 50 \cos 65° = 21.13$ m (to 4 s.f.)

$\tan 35° = \dfrac{AT}{21.13}$

$AT = 21.13 \tan 35°$
$= 14.8$ m (to 3 s.f.)

$\sin 65° = \dfrac{AC}{50}$

$AC = 50 \sin 65° = 45.32$ (to 4 s.f.)

$\tan \angle ACT = \dfrac{14.80}{45.32}$

$\angle ACT = \tan^{-1} \dfrac{14.80}{45.32} = 18.1°$ (to 1 d.p.)

$\therefore$ The height of of the tower is 14.8 m and the angle of elevation of T from C is 18.1°.

8. **(a)** **(i)** $180° - 125° = 55°$ (int. $\angle$s)

$\angle ABC = 90° - 55° = 35°$

Using cosine rule,

$AC^2 = 460^2 + 430^2 - 2 \times 460 \times 430 \times \cos 35°$
$= 72\,440$ (to 4 s.f.)

$AC = \sqrt{72\,440} = 269$ m (to 3 s.f.)

(ii) Using sine rule,

$\dfrac{\sin \angle ACB}{430} = \dfrac{\sin 35°}{269}$

$\angle ACB = \sin^{-1} \dfrac{430 \sin 35°}{269}$

$= 66.5°$ (to 1 d.p.)

(iii) Using sine rule,

$\dfrac{\sin \angle CAB}{460} = \dfrac{\sin 66.47°}{430}$

$\sin \angle CAB = \dfrac{460 \sin 66.47°}{430}$

$\angle CAB = \sin^{-1} \dfrac{460 \sin 66.47°}{430}$

$= 78.76°$ (to 2 d.p.)

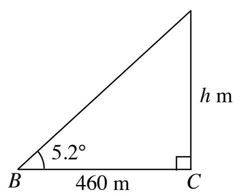
The bearing of C from A is $(90° - 78.76°)$.

$\therefore$ Bearing of C from A is 011.2°.

(iv) Area of $\triangle ABC = \dfrac{1}{2} \times 460 \times 269.1 \times \sin 66.47°$

$= 56\,700\ \text{m}^2$ (to 3 s.f.)

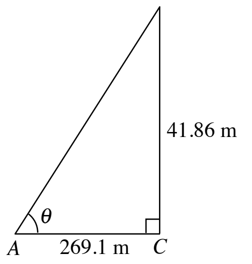
(b) Let the distance of the hot air balloon above C be h m.



$$\tan 5.2° = \frac{h}{460}$$

$$h = 460 \tan 5.2°$$

$$= 41.86 \text{ m (to 4 s.f.)}$$



$$\tan \theta = \frac{41.86}{269.1}$$

$$\theta = \tan^{-1} \frac{41.86}{269.1}$$

$$= 8.8° \text{ (to 1 d.p.)}$$

$\therefore$ The angle of elevation of the hot air balloon from A is 8.8°.

9. (a) (i) Using cosine rule,

$$\cos \angle CAD = \frac{650^2 + 520^2 - 900^2}{2 \times 650 \times 520}$$

$$= -0.1732 \text{ (to 4 s.f.)}$$

$$\angle CAD = \cos^{-1}(-0.1732)$$

$$= 100.0° \text{ (to 1 d.p.)}$$

(ii) The bearing of C from A is (226° – 100.0°).

$\therefore$ Bearing of C from A is 126.0°.

(iii) $\angle BAC = 226° - 90° - 100.0° = 36.0°$

Using sine rule,

$$\frac{BC}{\sin 36.0°} = \frac{520}{\sin 54°}$$

$$BC = \frac{520 \sin 36.0°}{\sin 54°}$$

$$= 378 \text{ m (to 3 s.f.)}$$

(iv) Let the shortest distance from A to CD be d m.

$$\text{Area of } \triangle ACD = \frac{1}{2} \times 520 \times 650 \times \sin 100.0°$$

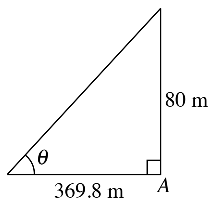
$$= 166\,400 \text{ m}^2 \text{ (to 4 s.f.)}$$

$$\frac{1}{2} \times 900 \times d = 166\,400$$

$$d = 370 \text{ m (to 3 s.f.)}$$

$\therefore$ The shortest distance from A to CD is 370 m.

(b)



$$\tan \theta = \frac{80}{369.8}$$

$$\theta = \tan^{-1} \frac{80}{369.8}$$

$$= 12.2° \text{ (to 1 d.p.)}$$

$\therefore$ The angle of elevation of the top of the tower from P is 12.2°.

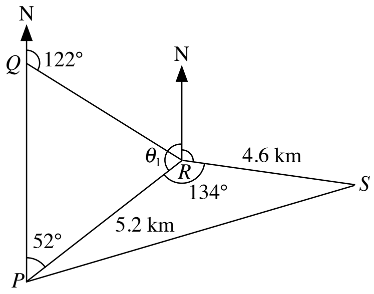
10. (i) Using cosine rule,

$$PS^2 = 4.6^2 + 5.2^2 - 2 \times 4.6 \times 5.2 \times \cos 134°$$

$$= 81.43 \text{ (to 4 s.f.)}$$

$$PS = \sqrt{81.43} = 9.02 \text{ km (to 3 s.f.)}$$

(ii)



$$\theta_1 = 180° - 52° \text{ (int. } \angle\text{s)}$$

$$= 128°$$

The bearing of S from R is (360° – 134° – 128°).

$\therefore$ Bearing of S from R is 098°.

(iii) $\angle PQR = 180° - 122°$ (adj. $\angle$s on a str. line)

$$= 58°$$

$\angle PRQ = 180° - 58° - 52°$ ($\angle$ sum of a $\triangle$)

$$= 70°$$

Using sine rule,

$$\frac{PQ}{\sin 70°} = \frac{5.2}{\sin 58°}$$

$$PQ = \frac{5.2 \sin 70°}{\sin 58°}$$

$$= 5.76 \text{ km (to 3 s.f.)}$$

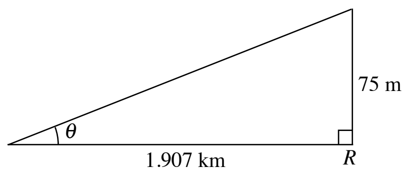
(iv) Let the shortest distance from R to PS be d km.

$$\text{Area of } \triangle PRS = \frac{1}{2} \times 5.2 \times 4.6 \times \sin 134°$$

$$= 8.603 \text{ km}^2 \text{ (to 4 s.f.)}$$

$$\frac{1}{2} \times 9.024 \times d = 8.603$$

$$d = 1.907 \text{ (to 4 s.f.)}$$



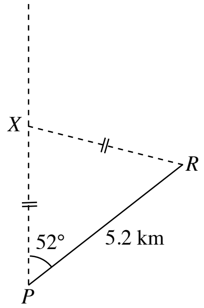
$$\tan \theta = \frac{0.075}{1.907}$$

$$\theta = \tan^{-1} \frac{0.075}{1.907}$$

$$= 2.3° \text{ (to 1 d.p.)}$$

$\therefore$ The greatest angle elevation of the top of the building as Huixian walks along PS is 2.3°.

(v) Let the point where the policeman is standing be X.



$\angle XRP = 52°$ (isos. $\triangle$)

$\angle PXR = 180° - 52° - 52°$ ($\angle$ sum of a $\triangle$)

$= 76°$

Using sine rule,

$$\frac{PX}{\sin 52°} = \frac{5.2}{\sin 76°}$$

$$PX = \frac{5.2 \sin 52°}{\sin 76°}$$

$= 4.22$ km (to 3 s.f.)

$\therefore$ The distance between the policeman and P is 4.22 km.

Challenge Yourself

1. In $\triangle BCP$,

$$\sin \alpha = \frac{PC}{BP}$$

$$PC = BP \sin \alpha$$

In $\triangle ABP$,

$$\cos \beta = \frac{BP}{AP}$$

$$AP = \frac{BP}{\cos \beta}$$

In $\triangle ACP$,

$$\sin \theta = \frac{PC}{AP} = \frac{BP \sin \alpha}{\frac{BP}{\cos \beta}}$$

$\sin \theta = \sin \theta \cos \beta$

2. In $\triangle ABC$,

$$\tan \alpha = \frac{BC}{h}$$

$$BC = h \tan \alpha$$

In $\triangle TBC$,

$$\cos \beta = \frac{BC}{TB}$$

$$TB = \frac{BC}{\cos \beta} = \frac{h \tan \alpha}{\cos \beta}$$

Chapter 10 Arc Length, Sector Area and Radian Measure

TEACHING NOTES

Suggested Approach

In this chapter, students will be introduced to circles and how to calculate the arc length and area of the sector of a circle. Teachers may begin the chapter by asking students to identify the different parts of a circle using real-life examples of arcs, sectors and segments of a circle.

Students are expected to know how to apply the formulas of trigonometry and Pythagoras' Theorem when solving problems involving the arc length, sector area and radian measure.

Section 10.1: Length of Arc

Teachers may begin the chapter by showing students a circle with centre O and highlight to students the minor arc, major arc, minor sector, major sector, minor segment and major segment of a circle. Once students are familiar with these terms and are able to identify the parts of a circle, teachers can proceed to guide students on how to derive the formula for the length of an arc of a circle (see Investigation: Arc Length).

Section 10.2: Area of Sector

Teachers can ask students to discover the formula for the area of sector on their own (see Investigation: Area of Sector). Teachers should take note that some students may need some guidance when finding the area of a shaded region involving sectors of circles. Teachers can suggest to students that when tackling such questions, they may need to draw additional lines in the figures given to help them better visualise and work out the solutions.

Section 10.3: Radian Measure

So far, students have been using the measurement of 360° to denote the angle for one complete revolution. In this section, students will learn how to use the measurement of 2π to denote the angle for one complete revolution (see Investigation: Visualise the Size of an Angle of 1 radian and Class Discussion: Estimate the Size of Angles in Radians).

Since the students have learnt how to find the arc length and area of sector in the previous sections, teachers may ask students to find the arc length and area of a sector, giving their answers both in degrees and radians, and ask them if the answers that they got for both are the same.

Teachers should remind students that when questions require the answers to be given in radians, students have to first set the calculator to the 'radian' mode.

Section 10.4: Arc Length and Area of Sector using Radian Measure

In this section, students will solve problems involving trigonometry and Pythagoras' Theorem. Students need to have grasped these concepts which they have learnt in Books 2 and 3 well in order to solve such problems. Teachers may guide the students along as they practise as many problems as possible.

Challenge Yourself

For Question 1, students need to make an observation from the perimeters of each figure. For Question 2, students need to use Pythagoras' Theorem to find the value of r and then be able to solve the remaining parts of the question. For Questions 3 and 4, students will need to add additional lines to each of the figures in both questions so that they can better visualise and find the respective areas of the shaded region.

WORKED SOLUTIONS

Investigation (Arc Length)

3. The third last column and the last column are equivalent.

4. The third last column and the last column are equivalent.

5. Arc length = $\frac{x°}{360°}$ × circumference, where $x°$ is the angle subtended by the arc at the centre of the circle of radius r.

Investigation (Area of Sector)

3. The third last column and the last column are equivalent.

4. The third last column and the last column are equivalent.

5. Area of a sector of a circle = $\frac{x°}{360°}$ × area of the circle, where $x°$ is the angle subtended by the arc at the centre of the circle of radius r.

Investigation (Visualise the Size of an Angle of 1 radian)

4. $\theta = \frac{5}{5} = 1$ radian

6. **(a)** **(i)** Size of the angle $\approx \frac{15}{5} \approx 3$ radians

(ii) Size of the angle $\approx \frac{30}{5} \approx 6$ radians

(b) The approximate size of $\angle AOB$ is 60°.

(c) Hence, an angle of 1 radian is approximately equal to 60°.

7. **(a)** $\theta = \frac{2\pi(5)}{5} = 2\pi$ radians

(b) 2π radians = 360°

$$1 \text{ radian} = \frac{360°}{2\pi} = \frac{180°}{\pi}$$

Class Discussion (Estimate the Size of Angles in Radians)

(i) Figure (c)
(ii) Figure (a)
(iii) Figure (f)
(iv) Figure (d)
(v) Figure (b)
(vi) Figure (e)

Practise Now 1

External radius of ring, $R = \frac{40}{2}$
$= 20$ mm

Internal radius of ring, $r = \frac{33}{2}$
$= 16.5$ mm

$$\begin{aligned}\text{Area of ring} &= \pi R^2 - \pi r^2 \\ &= \pi(20)^2 - \pi(16.5)^2 \\ &= \pi(20^2 - 16.5^2) \\ &= 401 \text{ mm}^2 \text{ (to 3 s.f.)}\end{aligned}$$

Practise Now 2

1. **(i)** Length of major arc $AYB = \frac{228°}{360°} \times 2\pi \times 25$
$= 99.5$ cm (to 3 s.f.)

(ii) Perimeter of minor sector = length of arc $AXB + OA + OB$
$= \frac{360° - 228°}{360°} \times 2\pi \times 25 + 25 + 25$
$= 108$ cm (to 3 s.f.)

2. Perimeter of shaded region = length of major arc $AOB + OA + OB$
$= \frac{360° - 150°}{360°} \times 2\pi \times 9 + 9 + 9$
$= \left(\frac{21}{2}\pi + 18\right)$ cm

3. Length of major arc $PXQ = 36$ cm

$$\begin{aligned}\frac{360° - 50°}{360°} \times 2\pi \times r &= 36 \\ 5.411r &= 36 \\ r &= 6.65\end{aligned}$$

Practise Now 3

1. $\angle OQR = 180° - 90° - 36.9°$ ($\angle$ sum of a $\triangle$)
$= 53.1°$

$\tan 36.9° = \frac{RQ}{8}$

$RQ = 8 \tan 36.9°$
$= 6.007$ m (to 4 s.f.)

Length of arc $RP = \frac{36.9°}{360°} \times 2\pi \times 8$
$= 1.64\pi$ m

By Pythagoras' Theorem,

$$\begin{aligned}OQ^2 &= OR^2 + RQ^2 \\ &= 8^2 + 6.007^2 \\ &= 100.1 \text{ (to 4 s.f.)} \\ OQ &= \sqrt{100.1} \\ &= 10.00 \text{ m (to 4 s.f.)} \\ PQ &= OQ - OP \\ &= 10.00 - 8 \\ &= 2.00 \text{ m}\end{aligned}$$

∴ Perimeter of shaded region PQR = $RQ + PQ$ + length of arc RP
$= 6.007 + 2.00 + 1.64\pi$
$= 13.2$ m

2. Perimeter of sector = $\frac{80°}{360°} \times 2\pi \times 10 + 10 + 10$
$= 34.0$ cm (to 3 s.f.)

Practise Now 4

(i) In $\triangle OQR$,

$\cos 36° = \frac{RQ}{35}$

$RQ = 35 \cos 36°$
$= 28.32$ cm (to 4 s.f.)

∴ $PQ = 2RQ = 2(28.32) = 56.6$ cm (to 3 s.f.)

(ii) $\angle ROQ = 180° - 90° - 36°$ ($\angle$ sum of a $\triangle$)
$= 54°$

$\angle POQ = 2\angle ROQ = 2(54°) = 108°$

Length of arc $PAQ = \frac{108°}{360°} \times 2\pi \times 35$
$= 65.97$ cm (to 4 s.f.)

Length of arc $PBQ = \frac{1}{2} \times \pi \times 56.63$
$= 88.95$ cm (to 4 s.f.)

$\therefore$ Perimeter of shaded region $= 65.97 + 88.95$
$= 15.5$ cm (to 3 s.f.)

Practise Now 5

(i) Since the length of the minor arc AQB is 33 cm,

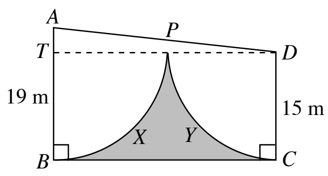
$$\frac{\angle AOB}{360°} \times 2\pi \times 15 = 33$$
$$\frac{\angle AOB}{360°} = \frac{33}{30\pi}$$
$$\angle AOB = \frac{33}{30\pi} \times 360° = 126.1° \text{ (to 1 d.p.)}$$

(ii) Reflex $\angle AOB = 360° - 126.05°$ ($\angle$s at a point)
$= 233.95°$

Area of major sector $OAPB = \frac{233.95°}{360°} \times \pi \times 15^2$
$= 459$ cm^2 (to 3 s.f.)

Practise Now 6

(i) Draw a line DT such that T lies on AB and DT is perpendicular to AB.



$AT = AB - TB$
$= 19 - 15$
$= 4$ m

$AD = AP + PD$
$= 19 + 15$
$= 34$ m

In $\triangle ATD$,

$\sin \angle ADT = \frac{AT}{AD} = \frac{4}{34}$

$\angle ADT = \sin^{-1} \frac{4}{34} = 6.76°$ (to 2 d.p.)

$\angle ADC = 6.76° + 90° = 96.8°$ (to 1 d.p.)

(ii) Using Pythagoras' Theorem,

$DT^2 = 34^2 - 4^2$

$DT = \sqrt{34^2 - 4^2}$
$= \sqrt{1140}$
$= 33.76$ m (to 4 s.f.)

$\angle DAT = 180° - 90° - 6.76°$ ($\angle$ sum of a $\triangle$)
$= 83.24°$

Area of shaded region
= Area of trapezium $ABCD$ – area of sector APB – area of sector DPC
$= \frac{1}{2}(15 + 19)(33.76) - \frac{83.24°}{360°} \times \pi \times 19^2 - \frac{96.76°}{360°} \times \pi \times 15^2$
$= 122$ m^2 (to 3 s.f.)

Practise Now 7

(a) Since π radians $= 180°$,
$$\frac{\pi}{15} \text{ rad} = \frac{180°}{15} = 12°$$

(b) Since π radians $= 180°$,
$$\frac{3\pi}{2} \text{ rad} = \frac{3 \times 180°}{2} = 270°$$

(c) Since π radians $= 180°$,
$$1 \text{ radian} = \frac{180°}{\pi}$$
$$3.04 \text{ radians} = 3.04 \times \frac{180°}{\pi} = 174.2° \text{ (to 1 d.p.)}$$

(d) Since π radians $= 180°$,
$$1 \text{ radian} = \frac{180°}{\pi}$$
$$8 \text{ radians} = 8 \times \frac{180°}{\pi} = 458.4° \text{ (to 1 d.p.)}$$

Practise Now 8

(a) Since $180° = \pi$ radians,
$$36° = \frac{\pi}{180°} \times 36° = 0.628 \text{ rad (to 3 s.f.)}$$

(b) Since $180° = \pi$ radians,
$$288° = \frac{\pi}{180°} \times 288° = 5.03 \text{ rad (to 3 s.f.)}$$

(c) Since $180° = \pi$ radians,
$$197.5° = \frac{\pi}{180°} \times 197.5° = 3.45 \text{ rad (to 3 s.f.)}$$

(d) Since $180° = \pi$ radians,
$$400° = \frac{\pi}{180°} \times 400° = 6.98 \text{ rad (to 3 s.f.)}$$

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Practise Now 9

(a) Press $\boxed{\sin}\ \boxed{0}\ \boxed{.}\ \boxed{6}\ \boxed{5}\ \boxed{=}$ and the display shows 0.605 186 405,
i.e. $\sin 0.65 = 0.605$ (to 3 s.f.)

(b) Press $\boxed{\cos}\ \boxed{0}\ \boxed{.}\ \boxed{2}\ \boxed{3}\ \boxed{5}\ \boxed{=}$ and the display shows 0.972 514 341,
i.e. $\cos 0.235 = 0.973$ (to 3 s.f.)

(c) Press $\boxed{\tan}\ \boxed{1}\ \boxed{.}\ \boxed{2}\ \boxed{3}\ \boxed{=}$ and the display shows 2.819 815 734,
i.e. $\tan 1.23 = 2.82$ (to 3 s.f.)

Practise Now 10

(a) For $\sin x = 0.87$, press $\boxed{\sin^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{8}\ \boxed{7}\ \boxed{=}$ to get 1.055 202 321,
i.e. when $\sin x = 0.87$,
$x = 1.06$ rad (to 3 s.f.)

(b) For $\cos x = 0.347$, press $\boxed{\cos^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{3}\ \boxed{4}\ \boxed{7}\ \boxed{=}$ to get 1.216 425 878,
i.e., when $\cos x = 0.347$,
$x = 1.22$ rad (to 3 s.f.)

(c) For $\tan x = 0.88$, press $\boxed{\tan^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{8}\ \boxed{8}\ \boxed{=}$ to get 0.721 654 85,
i.e. when $\tan x = 0.88$,
$x = 0.722$ rad (to 3 s.f.)

Practise Now 11

(i) $\tan 0.63 = \dfrac{QR}{9.6}$

$QR = 9.6 \tan 0.63$
$= 7.00$ cm (to 3 s.f.)

(ii) $\cos 0.63 = \dfrac{9.6}{PR}$

$PR = \dfrac{9.6}{\cos 0.63}$
$= 11.9$ cm (to 3 s.f.)

Practise Now 12

(i) $\sin \angle KAC = \dfrac{8.3}{11.9}$

$\angle KAC = \sin^{-1} \dfrac{8.3}{11.9}$
$= 0.772$ rad (to 3 s.f.)

(ii) $\tan 1.2 = \dfrac{8.3}{BC}$

$BC = \dfrac{8.3}{\tan 1.2}$
$= 3.23$ cm (to 3 s.f.)

(iii) $\tan 0.7719 = \dfrac{8.3}{AC}$

$AC = \dfrac{8.3}{\tan 0.7719}$
$= 8.527$ cm (to 4 s.f.)

$AB = AC - BC$
$= 8.527 - 3.227$
$= 5.30$ cm (to 3 s.f.)

Practise Now 13

$\angle AOB = 2\pi - \dfrac{7\pi}{4}$
$= \dfrac{\pi}{4}$ rad

Length of minor arc $APB = 6 \times \dfrac{\pi}{4}$
$= 4.71$ cm (to 3 s.f.)

Practise Now 14

1. (i) Given that the perimeter = 33 m,

$r + r + s = 33$
$12 + 12 + s = 33$
$24 + s = 33$
$s = 33 - 24$
$= 9$ m

Using $s = r\theta$,
$9 = 12\theta$
$\theta = \dfrac{9}{12} = 0.75$ rad
$\therefore \angle AOB = 0.75$ rad

(ii) Area of sector $OAB = \dfrac{1}{2} \times 12^2 \times 0.75$
$= 54\ \text{m}^2$

2. Given that the area = $30.25\ \text{cm}^2$,

$\dfrac{1}{2} \times r^2 \times \theta = 30.25$

$\dfrac{1}{2} \times 55^2 \times \theta = 30.25$

$15.125\theta = 30.25$
$\theta = 2$ rad

Length of arc $= r\theta$
$= 5.5 \times 2$
$= 11$ cm

Perimeter of sector $= r + r + s$
$= 5.5 + 5.5 + 11$
$= 22$ cm

Practise Now 15

(i) $\cos 0.98 = \dfrac{OP}{19}$

$OP = 19 \cos 0.98$
$= 10.6$ m (to 3 s.f.)

(ii) $\sin 0.98 = \dfrac{AP}{19}$

$AP = 19 \sin 0.98$
$= 15.8$ m (to 3 s.f.)

(iii) Area of sector $OAB = \frac{1}{2} \times 19^2 \times 0.98$

$= 176.89 \text{ m}^2$

Area of $\triangle OAP = \frac{1}{2} \times 10.6 \times 15.78$

$= 83.634 \text{ m}^2$

Area of shaded region $= 176.89 - 83.634$

$= 93.4 \text{ m}^2$ (to 3 s.f.)

Practise Now 16

(i) Length of arc $AB = 12 \times 1.08$

$= 12.96 \text{ m}$

(ii) Area of sector $OAB = \frac{1}{2} \times 12^2 \times 1.08$

$= 77.76 \text{ m}^2$

$OK = 12 - 5.5 = 6.5 \text{ cm}$

Area of $\triangle OAK = \frac{1}{2} \times 12 \times 6.5 \times \sin 1.08$

$= 34.40 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 77.76 - 34.40$

$= 43.4 \text{ m}^2$ (to 3 s.f.)

Practise Now 17

(i) Using cosine rule,

$AC^2 = 4.8^2 + 4.8^2 - 2 \times 4.8 \times 4.8 \times \cos 2.4$

$= 80.06$ (to 4 s.f.)

$AC = \sqrt{80.06}$

$= 8.95 \text{ cm}$ (to 3 s.f.)

(ii) Reflex $\angle AOC = (2\pi - 2.4)$ rad

Length of major arc $ABC = (4.8)(2\pi - 2.4) = (9.6\pi - 11.5) \text{ cm}$

(iii) Area of major sector $ABC = \frac{1}{2} \times 4.8^2 \times (2\pi - 2.4)$

$= 44.73 \text{ cm}^2$ (to 4 s.f.)

Area of $\triangle AOC = \frac{1}{2} \times 4.8 \times 4.8 \times \sin 2.4$

$= 7.781 \text{ cm}^2$ (to 4 s.f.)

Area of major segment $ABC = 44.73 + 7.781$

$= 52.5 \text{ cm}^2$ (to 3 s.f.)

Exercise 10A

1. **(a)** Area of circle, $\pi r^2 = 616 \text{ mm}^2$

$r^2 = \frac{616}{\pi}$

$r = \sqrt{\frac{616}{\pi}} \text{ mm}$

$\therefore$ Diameter of circle $= 2\sqrt{\frac{616}{\pi}}$

$= 28.0 \text{ mm}$ (to 3 s.f.)

(b) Area of circle, $\pi r^2 = 779\frac{5}{8} \text{ m}^2$

$r^2 = \frac{779\frac{5}{8}}{\pi}$

$r = \sqrt{\frac{779\frac{5}{8}}{\pi}} \text{ m}$

$\therefore$ Diameter of circle $= 2\sqrt{\frac{779\frac{5}{8}}{\pi}}$

$= 31.5 \text{ m}$ (to 3 s.f.)

(c) Area of circle, $\pi r^2 = 3850 \text{ cm}^2$

$r^2 = \frac{3850}{\pi}$

$r = \sqrt{\frac{3850}{\pi}} \text{ cm}$

$\therefore$ Diameter of circle $= 2\sqrt{\frac{3850}{\pi}}$

$= 70.0 \text{ cm}$ (to 3 s.f.)

2. **(a)** External radius of ring, $R = \frac{15}{2}$

$= 7.5 \text{ cm}$

Internal radius of ring, $r = \frac{13}{2}$

$= 6.5 \text{ cm}$

Area of ring $= \pi R^2 - \pi r^2$

$= \pi(7.5)^2 - \pi(6.5)^2$

$= \pi(7.5^2 - 6.5^2)$

$= 44.0 \text{ cm}^2$ (to 3 s.f.)

(b) External radius of ring, $R = \frac{1.2}{2}$

$= 0.6 \text{ m}$

Internal radius of ring, $r = \frac{0.9}{2}$

$= 0.45 \text{ m}$

Area of ring $= \pi R^2 - \pi r^2$

$= \pi(0.6)^2 - \pi(0.45)^2$

$= \pi(0.6^2 - 0.45^2)$

$= 0.495 \text{ m}^2$ (to 3 s.f.)

3. **(a)** **(i)** Area of big semicircle $= \frac{1}{2} \times \pi \times 14^2$

$= 98\pi \text{ cm}^2$

Area of two small semicircles

$= 2 \times \frac{1}{2} \times \pi \times \left(\frac{14}{2}\right)^2$

$= 49\pi \text{ cm}^2$

Area of shaded region $= 98\pi - 49\pi$

$= 49\pi$

$= 154 \text{ cm}^2$ (to 3 s.f.)

(ii) Length of arc of big semicircle

$= \frac{1}{2} \times \pi \times 28$

$= 14\pi$ cm

Length of arc of small semicircles

$= 2 \times \frac{1}{2} \times \pi \times 14$

$= 14\pi$ cm

Perimeter of shaded region

$= 14\pi + 14\pi$

$= 28\pi$

$= 88.0$ cm (to 3 s.f.)

(b) (i) Area of square $= 14^2$

$= 196 \text{ cm}^2$

Area of 4 quadrants $= 4 \times \frac{1}{4} \times \pi \times \left(\frac{14}{2}\right)^2$

$= 49\pi \text{ cm}^2$

Area of shaded region $= 196 - 49\pi$

$= 42.1 \text{ cm}^2$

(ii) Perimeter of shaded region

$= 4 \times$ arc length of quadrant

$= 4 \times \frac{1}{4} \times \pi \times 14$

$= 44.0$ cm (to 3 s.f.)

(c) (i) Area of shaded region $= \frac{1}{2} \times$ area of circle

$= \frac{1}{2} \times \pi \times 28^2$

$= 1230 \text{ cm}^2$ (to 3 s.f.)

(ii) Arc length of big semicircle

$= \frac{1}{2} \times \pi \times (28 \times 2)$

$= 28\pi$ cm

Arc length of small semicircle

$= \frac{1}{2} \times \pi \times 28$

$= 14\pi$ cm

Perimeter of shaded region

$= 28\pi + 14\pi + 14\pi$

$= 176$ cm (to 3 s.f.)

(d) (i) Area of square $= 10^2$

$= 100 \text{ cm}^2$

Area of shaded region

= Area of square – area of 4 quadrants

$= 100 - 4 \times \frac{1}{4} \times \pi \times 3.5^2$

$= 100 - 12.25\pi$

$= 61.5 \text{ cm}^2$ (to 3 s.f.)

(ii) Perimeter of shaded region

$= 4 \times \frac{1}{4} \times \pi \times (3.5 \times 2) + 4 \times (10 - 3.5 \times 2)$

$= 7\pi + 12$

$= 34.0$ cm (to 3 s.f.)

4. (a) Area of shaded region

= Area of square – area of 4 circles

$= 56^2 - 4 \times \pi \times \left(\frac{56}{4}\right)^2$

$= 3136 - 784\pi$

$= 673 \text{ cm}^2$ (to 3 s.f.)

(b) Area of unshaded region

$= 4 \times$ area of semicircles

$= 4 \times \frac{1}{2} \times \pi \times \left(\frac{3.5}{2}\right)^2$

$= 6.125\pi \text{ cm}^2$

Area of shaded region

= Area of circle – area of unshaded region

$= \pi \times 3.5^2 - 6.125\pi$

$= 19.2 \text{ cm}^2$ (to 3 s.f.)

(c) Area of shaded region

= Area of big circle – area of small circle

$= \pi \times 14^2 - \pi \times \left(\frac{14}{2}\right)^2$

$= \pi(14^2 - 7^2)$

$= 462 \text{ cm}^2$ (to 3 s.f.)

(d) Area of middle shaded region

$= 48 \times 14 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{14}{2}\right)^2$

$= (672 - 49\pi) \text{ cm}^2$

Area of shaded region

$= 2 \times \frac{1}{2} \times \pi \times \left(\frac{48}{2}\right)^2 + (672 - 49\pi)$

$= 576\pi + (672 - 49\pi)$

$= 2330 \text{ cm}^2$ (to 3 s.f.)

5. (a) Diameter of outer circular edge of grass

$= 12 \times 2 + 2 + 2$

$= 28$ m

Circumference of outer circular edge of grass

$= \pi \times 28$

$= 28\pi$ m

(b) Area of land in between $= \pi\left(\frac{28}{2}\right)^2 - \pi(12)^2$

$= \pi(14^2 - 12^2)$

$= 52\pi \text{ m}^2$

6. (a) Area of shaded region

= Area of rectangle + area of semicircle

$= 5 \times 4 + \frac{1}{2} \times \pi \times \left(\frac{4}{2}\right)^2$

$= 20 + 2\pi$

$= 26 \text{ m}^2$ (to 2 s.f.)

(b) Total length painted in black

= Circumference of circle + 5 + 4 + 5

$= \pi \times 4 + 14$

$= 27$ m (to 2 s.f.)

7. (a) Length of arc $AXB = \frac{82°}{360°} \times 2\pi \times 8$

$= 11.4$ cm (to 3 s.f.)

(b) Length of arc $AXB = \frac{134°}{360°} \times 2\pi \times 14$

$= 32.7$ cm (to 3 s.f.)

(c) Length of arc $AXB = \frac{214°}{360°} \times 2\pi \times 17$

$= 63.5$ cm (to 3 s.f.)

(d) Reflex $\angle AOB = 360° - 46° = 314°$

Length of arc $AXB = \frac{314°}{360°} \times 2\pi \times 9.8$

$= 53.7$ cm (to 3 s.f.)

8. (a) (i) Length of minor arc $AXB = \frac{76°}{360°} \times 2\pi \times 9$

$= 11.9$ cm (to 3 s.f.)

(ii) Reflex $\angle AOB = 360° - 76° = 284°$

Perimeter of major sector $OAYB$

= length of major arc $AYB + OA + OB$

$= \frac{284°}{360°} \times 2\pi \times 9 + 9 + 9$

$= 62.6$ cm (to 3 s.f.)

(b) (i) Length of minor arc $AXB = \frac{112°}{360°} \times 2\pi \times 16$

$= 31.3$ cm (to 3 s.f.)

(ii) Reflex $\angle AOB = 360° - 112° = 248°$

Perimeter of major sector $OAYB$

= length of major arc $AYB + OA + OB$

$= \frac{248°}{360°} \times 2\pi \times 16 + 16 + 16$

$= 101$ cm (to 3 s.f.)

(c) (i) $\angle AOB = 360° - 215° = 145°$

Length of minor arc $AXB = \frac{145°}{360°} \times 2\pi \times 17.6$

$= 44.5$ cm (to 3 s.f.)

(ii) Perimeter of major sector $OAYB$

= length of major arc $AYB + OA + OB$

$= \frac{215°}{360°} \times 2\pi \times 17.6 + 17.6 + 17.6$

$= 101$ cm (to 3 s.f.)

9. (a) Since the length of minor arc is 26.53 cm,

$\frac{95°}{360°} \times 2\pi \times r = 26.53$

$1.658r = 26.53$

$r = 16.0$ cm (to 3 s.f.)

(b) Since the length of major arc is 104.6 cm,

$\frac{214°}{360°} \times 2\pi \times r = 104.6$

$3.735r = 104.6$

$r = 28.0$ cm (to 3 s.f.)

10. (a) Since the length of arc is 12 m,

$\frac{\theta}{360°} \times 2\pi \times 14 = 12$

$0.2443\theta = 12$

$\theta = 49°$ (to the nearest degree)

(b) Since the length of arc is 19.5 m,

$\frac{\theta}{360°} \times 2\pi \times 14 = 19.5$

$0.2443\theta = 19.5$

$\theta = 80°$ (to the nearest degree)

(c) Since the length of arc is 64.2 m,

$\frac{\theta}{360°} \times 2\pi \times 14 = 64.2$

$0.2443\theta = 64.2$

$\theta = 263°$ (to the nearest degree)

(d) Since the length of arc is 84.6,

$\frac{\theta}{360°} \times 2\pi \times 14 = 84.6$

$0.2443\theta = 84.6$

$\theta = 346°$ (to the nearest degree)

11. Distance travelled by the tip of the hour hand $= \frac{45°}{360°} \times 2\pi \times 1.5$

$= 1.18$ m (to 3 s.f.)

12. Since the length of wire is 32 cm,

$\frac{\theta}{360°} \times 2\pi \times 6 + 6 + 6 = 32$

$0.1047\theta + 12 = 32$

$0.1047\theta = 20$

$\theta = 191.0°$ (to 1 d.p.)

13. (a) Since the perimeter of minor sector is 77.91 cm,

$\frac{148°}{360°} \times 2\pi \times r + r + r = 77.91$

$2.583r + 2r = 77.91$

$4.583r = 77.91$

$r = 17.0$ cm (to 3 s.f.)

(b) Reflex $\angle AOB = 360° - 44° = 316°$

Since the perimeter of major sector is 278.1 cm,

$\frac{316°}{360°} \times 2\pi \times r + r + r = 278.1$

$5.515r + 2r = 278.1$

$7.515r = 278.1$

$r = 37.0$ cm (to 3 s.f.)

14. Perimeter of arc $AOB = \frac{60°}{360°} \times 2\pi \times 8$

$= \frac{8\pi}{3}$ cm

Perimeter of arc $POQ = \frac{60°}{360°} \times 2\pi \times 17$

$= \frac{17\pi}{3}$ cm

$AP = BQ = 17 - 8 = 9$ cm

Perimeter of shaded region = arc AB + arc $PQ + AP + BQ$

$= \frac{8\pi}{3} + \frac{17\pi}{3} + 9 + 9$

$= \left(18 + \frac{25\pi}{3}\right)$ cm

15. (i) Length of minor arc $AOB = \left(\frac{\angle AOB}{360°} \times 2\pi r\right)$ cm

Circumference of circle = $(2\pi r)$ cm

Since the length of the minor arc is $\frac{7}{24}$ of the circumference of the circle,

$$\frac{\angle AOB}{360°} \times 2\pi r = \frac{7}{24} \times 2\pi r$$
$$\frac{\angle AOB}{360°} = \frac{7}{24}$$
$$\angle AOB = \frac{7}{24} \times 360°$$
$$= 105°$$

(ii) Radius of circle = $\frac{14}{2}$ = 7 cm

Length of minor arc = $\frac{105°}{360°} \times 2\pi \times 7$

= 12.8 cm (to 3 s.f.)

16. (i) $\tan \angle POA = \frac{14}{7.5}$

$\angle POA = \tan^{-1} \frac{14}{7.5}$

= 61.8° (to 1 d.p.)

(ii) Length of minor arc $OAB = \frac{61.82°}{360°} \times 2\pi \times 7.5$

= 8.092 cm (to 4 s.f.)

By Pythagoras' Theorem,

$OP^2 = 7.5^2 + 14^2$

= 252.25

$OP = \sqrt{252.25}$

= 15.88 cm (to 4 s.f.)

$BP = OP - OB = 15.88 - 7.5 = 8.38$ cm

Perimeter of shaded region = arc $OAB + BP + AP$

= 8.092 + 8.38 + 14

= 30.5 cm (to 3 s.f.)

17. (i) $\angle ROQ = \frac{138°}{2} = 69°$

$\tan 69° = \frac{QR}{26}$

$QR = 26 \tan 69°$

= 67.7 cm (to 3 s.f.)

(ii) Length of minor arc $OPQ = \frac{138°}{360°} \times 2\pi \times 26$

= 62.62 cm (to 4 s.f.)

Perimeter of shaded region = arc $OPQ + RP + QR$

= 62.62 + 67.73 + 67.73

= 198 cm (to 3 s.f.)

18. (i) Using cosine rule,

$\cos \angle AOB = \frac{13^2 + 13^2 - 22^2}{2 \times 13 \times 13}$

= –0.4320 (to 4 s.f.)

$\angle AOB = \cos^{-1}(-0.4320)$

= 115.6° (to 1 d.p.)

(ii) Length of arc = $\frac{115.59°}{360°} \times 2\pi \times 13$

= 26.23 cm (to 4 s.f.)

Perimeter of shaded region = 26.23 + 22

= 48.2 cm (to 3 s.f.)

19. (i) AP = 16 cm

$OP = OP$ = 9 cm

Using cosine rule,

$\cos \angle APB = \frac{9^2 + 16^2 - 9^2}{2 \times 9 \times 16}$

= 0.8889 (to 4 s.f.)

$\angle APB = \cos^{-1} 0.8889$

= 27.3° (to 1 d.p.)

(ii) $\angle PAO = \angle APB = 27.27°$

$\angle AOB = 2 \times 27.27°$ (ext. ∠ of △)

= 54.4° (to 1 d.p.)

(iii) Length of arc $ABD = \frac{54.54° \times 2}{360°} \times 2\pi \times 9$

= 17.13 cm (to 4 s.f.)

Legnth of arc $ACD = \frac{27.27° \times 2}{360°} \times 2\pi \times 16$

= 15.23 cm (to 4 s.f.)

Perimeter of shaded region = 17.13 + 15.23

= 32.4 cm (to 3 s.f.)

20. Let the radius of the circle be r cm.

$\angle OBA = \angle OAB = 30°$

$\angle AOB = 180° - 30° - 30°$ (∠ sum of a △)

= 120°

Using cosine rule,

$\left(\frac{15}{2}\sqrt{13}\right)^2 = r^2 + r^2 - 2 \times r \times r \times \cos 120°$

$168.75 = 2r^2 + r^2$

$168.75 = 3r^2$

$r^2 = 56.25$

$r = 7.5$

Length of arc = $\frac{360° - 120°}{360°} \times 2\pi \times 7.5$

= 31.42 cm (to 4 s.f.)

Perimeter of shaded region = $\frac{15}{2}\sqrt{3} + 31.42$

= 44.4 cm (to 3 s.f.)

21. $\sin 36° = \frac{OP}{14}$

$OP = 14 \sin 36°$

= 8.229 cm (to 4 s.f.)

$\angle POR = 36° + 90°$ (ext. ∠ of △)

= 126°

Length of arc $PR = \frac{126°}{360°} \times 2\pi \times 8.229$

= 18.10 cm (to 4 s.f.)

$OR = OP$ = 8.229 cm

$TR = 14 + 8.229 = 22.229$ cm

Length of arc $QR = \frac{36°}{360°} \times 2\pi \times 22.229$

$= 13.97$ cm (to 4 s.f.)

$\cos 36° = \frac{TP}{14}$

$TP = 14 \cos 36°$

$= 11.33$ cm (to 4 s.f.)

$PQ = 22.229 - 11.33$

$= 10.899$ cm

Perimeter of shaded region = arc PR + arc QR + PQ

$= 18.10 + 13.13.97 + 10.899$

$= 43.0$ cm (to 3 s.f.)

Exercise 10B

1. **(a)** Arc length $= \frac{72°}{360°} \times 2\pi \times 7$

$= 8.80$ cm (to 3 s.f.)

Area $= \frac{72°}{360°} \times \pi \times 7^2$

$= 30.8$ cm^2 (to 3 s.f.)

Perimeter $= 8.796 + 7 + 7$

$= 22.8$ cm (to 3 s.f.)

(b) Perimeter $= 136$

$s + 35 + 35 = 136$

$s + 70 = 136$

Arc length, $s = 66$ mm

$\frac{\theta}{360°} \times 2\pi \times 35 = 66$ mm

$\frac{7\pi\theta}{36} = 66$

Angle at centre, $\theta = \frac{66 \times 36}{7\pi}$

$= 108.0°$ (to 1 d.p.)

Area $= \frac{108.04°}{360°} \times \pi \times 35^2$

$= 1150$ mm^2 (to 3 s.f.)

(c) Area $= 1848$ mm^2

$\frac{270°}{360°} \times \pi \times r^2 = 1848$

$\frac{3}{4}\pi \times r^2 = 1848$

$r^2 = \frac{2464}{\pi}$

Radius, $r = 28.0$ mm (to 3 s.f.)

Arc length $= \frac{270°}{360°} \times 2\pi \times 28.00$

$= 132$ mm (to 3 s.f.)

Peterimeter $= 131.9 + 28.00 + 28.00$

$= 188$ mm (to 3 s.f.)

(d) Arc length $= 220$ cm

$\frac{150°}{360°} \times 2\pi \times r = 220$

$\frac{5}{6}\pi r = 220$

Radius, $r = 84.0$ cm (to 3 s.f.)

Area $= \frac{150°}{360°} \times \pi \times 84.03^2$

$= 9240$ cm^2 (to 3 s.f.)

Perimeter $= 220 + 84.03 + 84.03$

$= 388$ cm (to 3 s.f.)

(e) Arc length $= 55$ m

$\frac{\theta}{360°} \times 2\pi \times 14 = 55$

$\frac{7}{90}\pi\theta = 55$

Angle at centre, $\theta = \frac{55 \times 90}{7\pi}$

$= 225.1°$ (to 1 d.p.)

Area $= \frac{225.09°}{360°} \times \pi \times 14^2$

$= 385$ m^2 (to 3 s.f.)

Perimeter $= 55 + 14 + 14$

$= 83$ m

(f) Area $= 154$ cm^2

$\frac{75°}{360°} \times \pi \times r^2 = 154$

$\frac{5}{24}\pi \times r^2 = 154$

$r^2 = \frac{739.2}{\pi}$

$r = 15.3$ cm (to 3 s.f.)

Arc length $= \frac{75°}{360°} \times 2\pi \times 15.34$

$= 20.1$ cm (to 3 s.f.)

Perimeter $= 20.08 + 15.34 + 15.34$

$= 50.8$ cm (to 3 s.f.)

2. **(a)** **(i)** Perimeter $= \frac{30°}{360°} \times 2\pi \times 7 + 7 + 7$

$= 17.7$ cm (to 3 s.f.)

(ii) Area $= \frac{30°}{360°} \times \pi \times 7^2$

$= 12.8$ cm^2 (to 3 s.f.)

(b) **(i)** Perimeter $= \frac{360° - 340°}{360°} \times 2\pi \times 3.5 + 3.5 + 3.5$

$= 8.22$ cm (to 3 s.f.)

(ii) Area $= \frac{20°}{360°} \times \pi \times 3.5^2$

$= 2.14$ cm^2 (to 3 s.f.)

(c) **(i)** Perimeter $= \frac{140°}{360°} \times 2\pi \times 6 + 6 + 6$

$= 26.7$ cm (to 3 s.f.)

(ii) Area $= \frac{140°}{360°} \times \pi \times 6^2$

$= 44.0$ cm^2 (to 3 s.f.)

3. Circumference of circle = 88 cm

$$2\pi \times r = 88$$
$$r = 14.01 \text{ cm (to 4 s.f.)}$$

(a) Length of arc $ACB = \frac{60°}{360°} \times 2\pi \times 14.01$
$= 14.7$ cm (to 3 s.f.)

Area of sector $OACB = \frac{60°}{360°} \times \pi \times 14.01^2$
$= 103$ cm^2 (to 3 s.f.)

(b) Length of arc $ACB = \frac{99°}{360°} \times 2\pi \times 14.01$
$= 24.2$ cm (to 3 s.f.)

Area of sector $OACB = \frac{99°}{360°} \times \pi \times 14.01^2$
$= 169$ cm^2 (to 3 s.f.)

(c) Length of arc $ACB = \frac{126°}{360°} \times 2\pi \times 14.01$
$= 30.8$ cm (to 3 s.f.)

Area of sector $OACB = \frac{126°}{360°} \times \pi \times 14.01^2$
$= 216$ cm^2 (to 3 s.f.)

(d) Length of arc $ACB = \frac{216°}{360°} \times 2\pi \times 14.01$
$= 52.8$ cm (to 3 s.f.)

Area of sector $OACB = \frac{216°}{360°} \times \pi \times 14.01^2$
$= 370$ cm^2 (to 3 s.f.)

4. Area of circle = 3850 cm^2

$$\pi \times r^2 = 3850$$
$$r^2 = \frac{3850}{\pi}$$
$$r = 35.00 \text{ cm (to 4 s.f.)}$$

(a) Area of sector $OPSQ = \frac{36°}{360°} \times \pi \times 35.00^2$
$= 385$ cm^2 (to 3 s.f.)

Length of arc $PSQ = \frac{36°}{360°} \times 2\pi \times 35.00$
$= 22.0$ cm (to 3 s.f.)

(b) Area of sector $OPSQ = \frac{84°}{360°} \times \pi \times 35.00^2$
$= 898$ cm^2 (to 3 s.f.)

Length of arc $PSQ = \frac{84°}{360°} \times 2\pi \times 35.00$
$= 51.3$ cm (to 3 s.f.)

(c) Area of sector $OPSQ = \frac{108°}{360°} \times \pi \times 35.00^2$
$= 1150$ cm^2 (to 3 s.f.)

Length arc $PSQ = \frac{108°}{360°} \times 2\pi \times 35.00$
$= 66.0$ cm (to 3 s.f.)

(d) Area of sector $OPSQ = \frac{198°}{360°} \times \pi \times 35.00^2$
$= 2120$ cm^2 (to 3 s.f.)

Legnth of arc $= \frac{198°}{360°} \times 2\pi \times 35.00$
$= 121$ cm (to 3 s.f.)

5. **(a)** Area of minor sector = 114 cm^2

$$\frac{150°}{360°} \times \pi \times r^2 = 114$$
$$\frac{5}{12}\pi \times r^2 = 114$$
$$r^2 = \frac{273.6}{\pi}$$
$$r = 9.33 \text{ cm (to 3 s.f.)}$$

(b) Area of major sector = 369 cm^2

$$\frac{360° - 66°}{360°} \times \pi r^2 = 369$$
$$\frac{49}{60}\pi \times r^2 = 369$$
$$r^2 = \frac{22\,140}{\pi}$$
$$r = 12.0 \text{ cm (to 3 s.f.)}$$

6. Radius of circle $= \frac{18}{2} = 9$ cm

(a) Area of sector = 42.6 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 42.6$$
$$\frac{9}{40}\pi\theta = 42.6$$
$$\theta = 60.3° \text{ (to 1 d.p.)}$$

(b) Area of sector = 117.2 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 117.2$$
$$\frac{9}{40}\pi\theta = 117.2$$
$$\theta = 165.8° \text{ (to 1 d.p.)}$$

(c) Area of sector = 214.5 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 214.5$$
$$\frac{9}{40}\pi\theta = 214.5$$
$$\theta = 303.5° \text{ (to 1 d.p.)}$$

(d) Area of sector = 18.9 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 18.9$$
$$\frac{9}{40}\pi\theta = 18.9$$
$$\theta = 26.7° \text{ (to 1 d.p.)}$$

7. (i) Length of arc $AB = \frac{45°}{360°} \times 2\pi \times 10$
$= 7.854$ cm (to 4 s.f.)
Length of arc $CD = \frac{45°}{360°} \times 2\pi \times 20$
$= 15.71$ cm (to 4 s.f.)
$AD = BC = 20 - 10 = 10$ cm
Perimeter of shaded region $= 7.854 + 15.71 + 10 + 10$
$= 43.6$ cm (to 3 s.f.)
Area of sector $OAB = \frac{45°}{360°} \times \pi \times 10^2$
$= 39.27\text{ cm}^2$ (to 4 s.f.)
Area of sector $ODC = \frac{45°}{360°} \times \pi \times 20^2$
$= 157.1\text{ cm}^2$ (to 4 s.f.)
Area of shaded region $= 157.1 - 39.27$
$= 118\text{ cm}^2$ (to 3 s.f.)

(ii) Length of arc $AB = \frac{120°}{360°} \times 2\pi \times 5$
$= 10.47$ cm (to 4 s.f.)
Length of arc $CD = \frac{120°}{360°} \times 2\pi \times 8$
$= 16.76$ cm (to 4 s.f.)
$AD = BC = 8 - 5 = 3$ cm
Perimeter of shaded region $= 10.47 + 16.76 + 3 + 3$
$= 33.2$ cm (to 3 s.f.)
Area of sector $OAB = \frac{120°}{360°} \times \pi \times 5^2$
$= 26.18\text{ cm}^2$ (to 4 s.f.)
Area of sector $ODC = \frac{120°}{360°} \times \pi \times 8^2$
$= 67.02\text{ cm}^2$ (to 4 s.f.)
Area of shaded region $= 67.02 - 26.18$
$= 40.8\text{ cm}^2$ (to 3 s.f.)

(iii) Length of arc $AB = \frac{160°}{360°} \times 2\pi \times 35$
$= 97.74$ cm (to 4 s.f.)
Length of arc $CD = \frac{160°}{360°} \times 2\pi \times 49$
$= 136.8$ cm (to 4 s.f.)
$AD = BC = 49 - 35 = 14$ cm
Perimeter of shaded region $= 97.74 + 136.8 + 14 + 14$
$= 263$ cm (to 3 s.f.)
Area of sector $OAB = \frac{160°}{360°} \times \pi \times 35^2$
$= 1710\text{ cm}^2$ (to 4 s.f.)
Area of sector $ODC = \frac{160°}{360°} \times \pi \times 49^2$
$= 3352\text{ cm}^2$ (to 4 s.f.)
Area of shaded region $= 3352 - 1710$
$= 1640\text{ cm}^2$ (to 3 s.f.)

8. (i) Since the shaded area POQ is $\frac{5}{18}$ of the area of the whole circle,

$$\frac{\angle POQ}{360°} \times \pi r^2 = \frac{5}{18} \times \pi r^2$$
$$\frac{\angle POQ}{360°} = \frac{5}{18}$$
$$\angle POQ = \frac{5}{18} \times 360°$$
$$= 100°$$

(ii) Area of shaded sector $= 385\text{ cm}^2$

$$\frac{100°}{360°} \times \pi \times r^2 = 385$$
$$\frac{5}{18}\pi r^2 = 385$$
$$r^2 = \frac{1386}{\pi}$$
$$r = 21.00 \text{ cm (to 4 s.f.)}$$

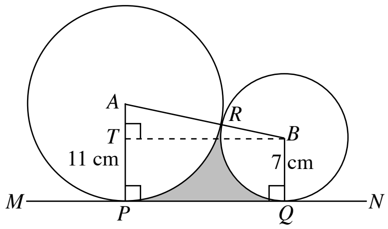
Diameter of circle $= 21.00 \times 2$
$= 42.0$ cm (to 3 s.f.)

9. Perimeter $= 38$ cm
Arc length $+ 12 + 12 = 38$
Arc length $= 38 - 12 - 12$
$= 14$ cm

$$\frac{\theta}{360°} \times 2\pi \times 12 = 14$$
$$\frac{\pi\theta}{15} = 14$$
$$\theta = 66.85° \text{ (to 2 d.p.)}$$

Area of paper used $= \frac{66.85°}{360°} \times \pi \times 12^2$
$= 84.0\text{ cm}^2$ (to 3 s.f.)

10. (i) Draw a line BT such that T lies on AP and BT is perpendicular to AP.



$AT = AP - TP$
$= 11 - 7$
$= 4$ cm
$AB = AR + RB$
$= 11 + 7$
$= 18$ cm
In $\triangle ATB$,
$\cos \angle BAT = \frac{AT}{AB} = \frac{4}{18}$
$\angle BAT = \cos^{-1} \frac{4}{18} = 77.2°$ (to 1 d.p.)
$\therefore \angle PAB = 77.2°$

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(ii) Using Pythagoras' Theorem,

$BT^2 = 18^2 - 4^2$

$BT = \sqrt{18^2 - 4^2}$

$= \sqrt{308}$

$= 17.55$ cm (to 4 s.f.)

$\angle ABT = 180° - 90° - 77.16°$ ($\angle$ sum of a $\triangle$)

$= 12.84°$

$\angle ABQ = 90° + 12.84° = 102.84°$

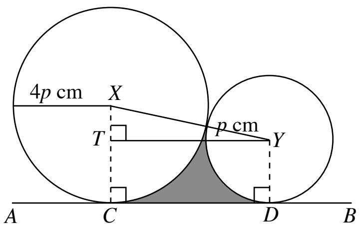
Area of shaded region

= Area of trapezium $ABPQ$ – area of sector APR – area of sector RBQ

$= \frac{1}{2}(11 + 7)(17.55) - \frac{77.16°}{360°} \times \pi \times 11^2 - \frac{102.84°}{360°} \times \pi \times 7^2$

$= 32.5$ cm^2 (to 3 s.f.)

11. Draw a line YT such that T lies on XC and YT is perpendicular to XC.



$XT = XC - TC$

$= 4p - p$

$= 3p$ cm

$XY = 4p + p$

$= 5p$ cm

In $\triangle XTY$,

$\cos \angle CXY = \frac{XT}{XY} = \frac{3p}{5p} = \frac{3}{5}$

$\angle CXY = \cos^{-1} \frac{3}{5} = 53.13°$ (to 2 d.p.)

Using Pythagoras' Theorem,

$YT^2 = XY^2 - XT^2$

$= (5p)^2 - (3p)^2$

$= 25p^2 - 9p^2$

$= 16p^2$

$YT = \sqrt{16p^2} = 4p$ cm

$\angle XYT = 180° - 90° - 53.13°$ ($\angle$ sum of a $\triangle$)

$= 36.87°$

$\angle XYD = 90° + 36.87° = 126.87°$

Area of enclosed region

$= \frac{1}{2}(4p + p)(4p) - \frac{53.13°}{360°} \times \pi \times (4p)^2 - \frac{126.87°}{360°} \times \pi \times p^2$

$= 10p^2 - 7.418p^2 - 1.107p^2$

$= 1.47p^2$ cm^2 (to 3 s.f.)

12. (i) Since $OQ = OA = 16$ cm,

$\angle OQA = \angle OAQ = 66°$

$\angle BOQ = 2 \times 66°$ (ext. $\angle$ of $\triangle$)

$= 132°$

(ii) $\angle AOQ = 180° - 132° = 48°$

Using sine rule,

$\frac{AQ}{\sin 48°} = \frac{16}{\sin 66°}$

$AQ = \frac{16 \sin 48°}{\sin 66°}$

$= 13.0$ cm (to 3 s.f.)

(iii) Length of arc $QB = \frac{132°}{360°} \times 2\pi \times 16$

$= 36.86$ cm (to 4 s.f.)

Length of arc $PYB = \frac{66°}{360°} \times 2\pi \times 32$

$= 36.86$ cm (to 4 s.f.)

$PQ = 32 - 13.02 = 18.98$ cm

Perimeter of shaded region $= 36.86 + 36.86 + 18.98$

$= 92.7$ cm (to 3 s.f.)

(iv) Area of sector $BOQ = \frac{132°}{360°} \times \pi \times 16^2$

$= 294.9$ cm^2 (to 4 s.f.)

Area of sector $APYB = \frac{66°}{360°} \times \pi \times 32^2$

$= 589.8$ cm^2 (to 4 s.f.)

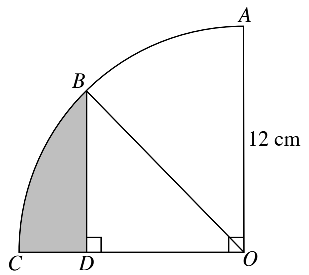
Area of $\triangle AOQ = \frac{1}{2} \times 16 \times 13.02 \times \sin 66°$

$= 95.15$ cm^2 (to 4 s.f.)

Area of shaded region $= 589.8 - 294.9 - 95.15$

$= 200$ cm^2 (to 3 s.f.)

13. (i)



Since B is the midpoint of arc AC,

$\angle BOC = 45°$

$OB = OA = 12$ cm

$\sin 45° = \frac{BD}{12}$

$BD = 12 \sin 45°$

$= 8.49$ cm (to 3 s.f.)

(ii) $\cos 45° = \frac{OD}{12}$

$OD = 12 \cos 45°$

$= 8.485$ cm (to 4 s.f.)

$CD = 12 - 8.485 = 3.515$ cm

Length of arc $CB = \frac{45°}{360°} \times 2\pi \times 12$

$= 9.425$ cm (to 4 s.f.)

Perimeter of shaded region $= 8.485 + 3.515 + 9.425$

$= 21.4$ cm (to 3 s.f.)

(iii) Area of sector $OBC = \dfrac{45°}{360°} \times \pi \times 12^2$

$= 56.55 \text{ cm}^2$ (to 4 s.f.)

Area of $\triangle BDO = \dfrac{1}{2} \times 8.485 \times 8.485$

$= 36.00 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 56.55 - 36.00$

$= 20.5 \text{ cm}^2$ (to 3 s.f.)

Exercise 10C

1. **(a)** Since π radians = 180°,

$\dfrac{5\pi}{6}$ rad $= \dfrac{5 \times 180°}{6}$

$= 150°$

(b) Since π radians = 180°,

$\dfrac{\pi}{7}$ rad $= \dfrac{180°}{7}$

$= 25.7°$ (to 1 d.p.)

(c) Since π radians = 180°,

1 radian $= \dfrac{180°}{\pi}$

3.2 radians $= 3.2 \times \dfrac{180°}{\pi}$

$= 183.3°$ (to 1 d.p.)

(d) Since π radians = 180°,

1 radian $= \dfrac{180°}{\pi}$

2.56 radians $= 2.56 \times \dfrac{180°}{\pi}$

$= 146.7°$ (to 1 d.p.)

2. **(a)** Since 180° = π radians,

$37.4° = \dfrac{\pi}{180°} \times 37.4°$

$= 0.653$ rad (to 3 s.f.)

(b) Since 180° = π radians,

$78.9° = \dfrac{\pi}{180°} \times 78.9°$

$= 1.38$ rad (to 3 s.f.)

(c) Since 180° = π radians,

$142° = \dfrac{\pi}{180°} \times 142°$

$= 2.48$ rad (to 3 s.f.)

(d) Since 180° = π radians,

$308° = \dfrac{\pi}{180°} \times 308°$

$= 5.38$ rad (to 3 s.f.)

3. **(a)** Since 180° = π radians,

$15° = \dfrac{\pi}{180°} \times 15°$

$= \dfrac{\pi}{12}$ rad

(b) Since 180° = π radians,

$18° = \dfrac{\pi}{180°} \times 18°$

$= \dfrac{\pi}{10}$ rad

(c) Since 180° = π radians,

$75° = \dfrac{\pi}{180°} \times 75°$

$= \dfrac{5\pi}{12}$ rad

(d) Since 180° = π radians,

$225° = \dfrac{\pi}{180°} \times 225°$

$= \dfrac{5\pi}{4}$ rad

4. **(a)** Press $\boxed{\sin}\ \boxed{0}\ \boxed{.}\ \boxed{8}\ \boxed{=}$ and the display shows 0717 356 09,

i.e. sin 0.8 = 0.717 (to 3 s.f.)

(b) Press $\boxed{\cos}\ \boxed{0}\ \boxed{.}\ \boxed{5}\ \boxed{4}\ \boxed{3}\ \boxed{=}$ and the display shows 0.856 162 416,

i.e. cos 0.543 = 0.856 (to 3 s.f.)

(c) Press $\boxed{\tan}\ \boxed{1}\ \boxed{.}\ \boxed{5}\ \boxed{=}$ and the display shows 14.101 419 95,

i.e. tan 1.5 = 14.1 (to 3 s.f.)

(d) Press $\boxed{\sin}\ \boxed{(}\ \boxed{\pi}\ \boxed{\div}\ \boxed{8}\ \boxed{)}\ \boxed{=}$ and the display shows 0.382 683 432,

i.e. $\sin \dfrac{\pi}{8} = 0.383$ (to 3 s.f.)

(e) Press $\boxed{\cos}\ \boxed{0}\ \boxed{.}\ \boxed{4}\ \boxed{5}\ \boxed{\pi}\ \boxed{=}$ and the display shows 0.156 434 465,

i.e. $\cos 0.45\pi = 0.156$ (to 3 s.f.)

(f) Press $\boxed{\tan}\ \boxed{(}\ \boxed{2}\ \boxed{\pi}\ \boxed{\div}\ \boxed{5}\ \boxed{)}\ \boxed{=}$ and the display shows 3.077 683 537,

i.e. $\tan \dfrac{2\pi}{5} = 3.08$ (to 3 s.f.)

5. **(a)** For $\sin x = 0.74$, press $\boxed{\sin^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{7}\ \boxed{4}\ \boxed{=}$ to get 0.833 070 358,

i.e. when $\sin x = 0.74$,

$x = 0.833$ rad (to 3 s.f.)

(b) For $\cos x = 0.17$, press $\boxed{\cos^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{1}\ \boxed{7}\ \boxed{=}$ to get 1.399 966 658,

i.e. when $\cos x = 0.17$,

$x = 1.40$ rad (to 3 s.f.)

(c) For $\tan x = 0.48$, press $\boxed{\tan^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{4}\ \boxed{8}\ \boxed{=}$ to get 0.447 519 975,

i.e. when $\tan x = 0.48$,

$x = 0.448$ rad (to 3 s.f.)

(d) For $\sin x = 0.147$, press $\boxed{\sin^{-1}}\ \boxed{0}\ \boxed{.}\ \boxed{1}\ \boxed{4}\ \boxed{7}\ \boxed{=}$ to get 0.147 534 635,

i.e. when $\sin x = 0.147$,

$x = 0.148$ rad (to 3 s.f.)

(e) For $\cos x = 0.769$, press $\boxed{\cos^{-1}}$ $\boxed{0}$ $\boxed{.}$ $\boxed{7}$ $\boxed{6}$ $\boxed{9}$ $\boxed{=}$ to get 0.693 520 986,

i.e. when $\cos x = 0.769$,

$x = 0.694$ rad (to 3 s.f.)

(f) For $\tan x = 1.256$, press $\boxed{\tan^{-1}}$ $\boxed{1}$ $\boxed{.}$ $\boxed{2}$ $\boxed{5}$ $\boxed{6}$ $\boxed{=}$ to get 0.898 390 01,

i.e. when $\tan x = 1.256$,

$x = 0.898$ rad (to 3 s.f.)

6. **(i)** $\tan 0.93 = \dfrac{BC}{8.7}$

$BC = 8.7 \tan 0.93$

$= 11.7$ cm (to 3 s.f.)

(ii) $\cos 0.93 = \dfrac{8.7}{AC}$

$AC = \dfrac{8.7}{\cos 0.93}$

$= 14.6$ cm (to 3 s.f.)

7. **(i)** $\tan 1.08 = \dfrac{12.7}{AB}$

$AB = \dfrac{12.7}{\tan 1.08}$

$= 6.79$ m (to 3 s.f.)

$\therefore$ The height of the tree is 6.79 m.

(ii) $\sin 1.08 = \dfrac{12.7}{AC}$

$AC = \dfrac{12.7}{\sin 1.08}$

$= 14.4$ m (to 3 s.f.)

8. **(i)** $\cos 0.98 = \dfrac{PQ}{16.8}$

$PQ = 16.8 \cos 0.98$

$= 9.36$ cm (to 3 s.f.)

$\therefore$ The length of cake to be sprinkled with chocolate rice is 9.36 cm.

(ii) $\sin 0.98 = \dfrac{QR}{16.8}$

$QR = 16.8 \sin 0.98$

$= 14.0$ cm (to 3 s.f.)

$\therefore$ The length of cake to be sprinkled with icing sugar is 14.0 cm.

9. **(i)** $\cos \angle PSR = \dfrac{SR}{PS} = \dfrac{7.4}{13.9}$

$\angle PSR = \cos^{-1} \dfrac{7.4}{13.9}$

$= 1.009$ rad (to 4 s.f.)

$\angle PSQ = 1.009 - 0.85$

$= 0.159$ rad (to 3 s.f.)

(ii) $\cos \angle QSR = \dfrac{SR}{SQ}$

$\cos 0.85 = \dfrac{7.4}{SQ}$

$SQ = \dfrac{7.4}{\cos 0.85}$

$= 11.2$ m (to 3 s.f.)

(iii) $\tan \angle QSR = \dfrac{QR}{SR}$

$\tan 0.85 = \dfrac{QR}{7.4}$

$QR = 7.4 \tan 0.85$

$= 8.424$ m (to 4 s.f.)

Using Pythagoras' Theorem,

$PR^2 = PS^2 - SR^2$

$= 13.9^2 - 7.4^2$

$= 138.45$

$PR = \sqrt{138.45}$

$= 11.77$ m (to 4 s.f.)

$PQ = 11.77 - 8.424$

$= 3.34$ m (to 3 s.f.)

10. Let h m be the height of the tree and d m be the distance between points A and Q.

$\tan \angle AQT = \dfrac{AT}{AQ}$

$\tan 1.3 = \dfrac{h}{d}$

$d = \dfrac{h}{\tan 1.3}$ — (1)

$\tan \angle APT = \dfrac{AT}{AP}$

$\tan 0.72 = \dfrac{h}{d + 11.9}$

$h = \tan 0.72 \times (d + 11.9)$ — (2)

Substitute (1) into (2):

$$h = \tan 0.72 \times \left(\frac{h}{\tan 1.3} + 11.9\right)$$

$$h = \frac{\tan 0.72}{\tan 1.3} h + 11.9 \tan 0.72$$

$$h - \frac{\tan 0.72}{\tan 1.3} h = 11.9 \tan 0{,}72$$

$0.7565h = 11.9 \tan 0.72$

$h = 13.8$ (to 3 s.f.)

$\therefore$ The height of the tree is 13.8 m.

Exercise 10D

1. **(a)** Arc length $= 6 \times 1.6 = 9.6$ cm

(b) Arc length $= 14 \times 0.25 = 3.5$ cm

(c) Arc length $= 25 \times 1.75 = 43.75$ m

(d) Arc length $= 12 \times \dfrac{3}{4} = 9$ mm

2. **(a)** Area of sector $= \dfrac{1}{2} \times 8^2 \times 2.2 = 70.4$ cm^2

(b) Area of sector $= \dfrac{1}{2} \times 17^2 \times 0.46 = 66.47$ cm^2

(c) Area of sector $= \dfrac{1}{2} \times 33^2 \times \dfrac{1}{5} = 108.9$ m^2

(d) Area of sector $= \frac{1}{2} \times 94^2 \times 0.6 = 26508 \text{ mm}^2$

3. (i) Perimeter = 50 cm,

$$r\theta + r + r = 50$$
$$16 \times \theta + 16 + 16 = 50$$
$$16\theta + 32 = 50$$
$$16\theta = 50 - 32$$
$$16\theta = 18$$
$$\theta = 1\frac{1}{8}$$
$$\therefore \angle AOB = 1\frac{1}{8} \text{ rad}$$

(ii) Area of sector $OAB = \frac{1}{2} \times 16^2 \times 1\frac{1}{8}$

$= 144 \text{ cm}^2$

4. (a) Arc length $= 4 \times 1.25 = 5$ cm

Area of sector $= \frac{1}{2} \times 4^2 \times 1.25$

$= 10 \text{ cm}^2$

(b) Arc length = 9 cm

$$6 \times \theta = 9$$
$$\theta = 1.5$$

$\therefore$ Angle at centre = 1.5 rad

Area of sector $= \frac{1}{2} \times 6^2 \times 1.5$

$= 27 \text{ cm}^2$

(c) Arc length = 9.6 m

$$r \times 0.8 = 9.6$$
$$r = 12$$

$\therefore$ Radius = 12 m

Area of sector $= \frac{1}{2} \times 12^2 \times 0.8$

$= 57.6 \text{ m}^2$

(d) Area of sector $= 60 \text{ m}^2$

$$\frac{1}{2} \times r^2 \times 1.2 = 60$$
$$0.6r^2 = 60$$
$$r^2 = 100$$
$$r = 10$$

$\therefore$ Radius = 10 m

Arc length $= 10 \times 1.2$

$= 12$ m

(e) Area of sector $= 64 \text{ mm}^2$

$$\frac{1}{2} \times 8^2 \times \theta = 64$$
$$32\theta = 64$$
$$\theta = 2$$

$\therefore$ Angle at centre = 2 rad

Arc length $= 8 \times 2$

$= 16$ mm

(f) Arc length = 6 mm

$$r\theta = 6$$
$$r = \frac{6}{\theta} \quad \text{— (1)}$$

Area of sector $= 27 \text{ mm}^2$

$$\frac{1}{2} r^2\theta = 27 \quad \text{— (2)}$$

Substitute (1) into (2):

$$\frac{1}{2}\left(\frac{6}{\theta}\right)^2 \theta = 27$$
$$\frac{1}{2} \times \frac{36}{\theta^2} \times \theta = 27$$
$$\frac{18}{\theta} = 27$$
$$27\theta = 18$$
$$\theta = \frac{2}{3}$$

$\therefore$ Angle at centre $= \frac{2}{3}$ rad

$$r = \frac{6}{\left(\frac{2}{3}\right)} = 9$$

$\therefore$ Radius = 9 mm

5. (i) Arc length $= 14 \times 3.8 = 53.2$ cm

Perimeter of major sector $OAB = 14 + 14 + 53.2$

$= 81.2$ cm

(ii) Area of minor sector $OAB = \frac{1}{2} \times 14^2 \times (2\pi - 3.8)$

$= 243 \text{ cm}^2$

6. Area of sector $= 60 \text{ cm}^2$

$$\frac{1}{2} \times 10^2 \times \theta = 60$$
$$50\theta = 60$$
$$\theta = 1.2$$

Perimeter of sector $= 10 \times 1.2 + 10 + 10$

$= 32$ cm

7. Area of sector $= 729 \text{ cm}^2$

$$\frac{1}{2} \times 18^2 \times \theta = 729$$
$$162\theta = 729$$
$$\theta = 4.5$$

Perimeter of sector $= 18 \times 4.5 + 18 + 18$

$= 117$ cm

8. (i) Area of minor sector $AOB = \frac{1}{2} \times 5^2 \times 1.8$

$= 22.5 \text{ cm}^2$

(ii) Reflex $\angle AOB = (2\pi - 1.8)$ rad

(iii) Length of major arc $ACB = 5 \times (2\pi - 1.8)$

$= (10\pi - 9)$ cm

9. **(i)** Perimeter of sector $= 18$ cm

$$r\theta + r + r = 18$$
$$r\theta + 2r = 18 \quad \text{— (1)}$$

Area of sector $= 8 \text{ cm}^2$

$$\frac{1}{2}r^2\theta = 8 \quad \text{— (2)}$$

(ii) From (2):

$$\frac{1}{2}r^2\theta = 8$$
$$r^2\theta = 16$$
$$\theta = \frac{16}{r^2} \quad \text{— (3)}$$

Substitute (3) into (1):

$$r\left(\frac{16}{r^2}\right) + 2r = 18$$
$$\frac{16}{r} + 2r = 18$$
$$16 + 2r^2 = 18r$$
$$2r^2 - 18r + 16 = 0$$
$$r^2 - 9r + 8 = 0$$
$$(r-1)(r-8) = 0$$

$r = 1$ or $r = 8$

$\theta = \dfrac{16}{1^2}$ or $\theta = \dfrac{16}{8^2}$

$= 16$ (rejected) $\qquad = \dfrac{1}{4}$

$\therefore r = 8, \theta = \dfrac{1}{4}$

10. **(i)** $\sin 1.05 = \dfrac{AC}{18}$

$AC = 18 \sin 1.05$

$= 15.6$ cm (to 3 s.f.)

(ii) Area of sector $AOB = \dfrac{1}{2} \times 18^2 \times 1.05$

$= 170.1 \text{ cm}^2$

Area of $\triangle AOC = \dfrac{1}{2} \times 18 \times 15.61 \times \sin\left(\pi - \dfrac{\pi}{2} - 1.05\right)$

$= 69.90 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 170.1 - 69.90$

$= 100 \text{ cm}^2$ (to 3 s.f.)

11. Arc length $= 31$ cm

$25 \times \theta = 31$

$\theta = 1.24$

Area of sector $AOB = \dfrac{1}{2} \times 25^2 \times 1.24$

$= 387.5 \text{ cm}^2$

$\cos 1.24 = \dfrac{OC}{25}$

$OC = 25 \cos 1.24$

$= 8.120$ cm (to 4 s.f.)

Area of $\triangle AOC = \dfrac{1}{2} \times 8.120 \times 25 \times \sin 1.24$

$= 96.00 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 387.5 - 96.00$

$= 292 \text{ cm}^2$ (to 3 s.f.)

12. **(i)** Given that $\triangle AOB$ is an equilateral triangle,

$\angle AOB = \angle OBA = \angle OAB = \dfrac{\pi}{3}$

$\therefore \theta = \dfrac{\pi}{3}$

(ii) Length of arc $APB = 12 \times \dfrac{\pi}{3} = 12.6$ cm (to 3 s.f.)

(iii) Area of sector segment APB

$=$ Area of sector $APBO$ – Area of $\triangle ABO$

$= \dfrac{1}{2} \times 12^2 \times \dfrac{\pi}{3} - \dfrac{1}{2} \times 12 \times 12 \times \sin \dfrac{\pi}{3}$

$= 13.04 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region

$=$ Area of semicircle ABQ – Area of segment APB

$= \dfrac{1}{2} \times \pi \times 6^2 - 13.04$

$= 43.5 \text{ cm}^2$ (to 3 s.f.)

13. **(i)** Length of arc $AB = 15 \times 1.2 = 18$ cm

(ii) $2OK = 3BK$

$\dfrac{OK}{BK} = \dfrac{3}{2}$

$OK = \dfrac{3}{5}OB = \dfrac{3}{5} \times 15 = 9$ cm

Area of shaded region = Area of sector OAB – Area of $\triangle AKO$

$= \dfrac{1}{2} \times 15^2 \times 1.2 - \dfrac{1}{2} \times 15 \times 9 \times \sin 1.2$

$= 72.1 \text{ cm}^2$ (to 3 s.f.)

14. **(i)** Length of arc $AB = 8 \times 0.8 = 6.4$ cm

(ii) $\dfrac{OP}{PA} = \dfrac{3}{2}$

$OP = \dfrac{3}{5}OA = \dfrac{3}{5} \times 8 = 4.8$ cm

$\dfrac{OQ}{QB} = \dfrac{3}{4}$

$OQ = \dfrac{3}{7}OB = \dfrac{3}{7} \times 8 = 3.429$ cm (to 4 s.f.)

Area of shaded region

$=$ Area of sector OAB – Area of $\triangle OPQ$

$= \dfrac{1}{2} \times 8^2 \times 0.8 - \dfrac{1}{2} \times 4.8 \times 3.429 \times \sin 0.8$

$= 19.7 \text{ cm}^2$ (to 3 s.f.)

15. **(i)** Using cosine rule,

$\cos \angle AOB = \dfrac{8^2 + 8^2 - 7^2}{2 \times 8 \times 8}$

$= 0.6172$ (to 4 s.f.)

$\angle AOB = \cos^{-1} 0.6172$

$= 0.906$ rad (to 3 s.f.)

Max. no. of slices that can be obtained $= \dfrac{2\pi}{0.906} \approx 6$

(ii) Area of shaded segment

$=$ Area of sector OAB – Area of $\triangle OPQ$

$= \dfrac{1}{2} \times 8^2 \times 0.9056 - \dfrac{1}{2} \times 8 \times 8 \times \sin 0.9056$

$= 3.80 \text{ cm}^2$ (to 3 s.f.)

16. (i) Area of $\triangle ABC = \dfrac{1}{2} \times 40 \times 80 \times \sin \dfrac{\pi}{6}$

$= 800 \text{ cm}^2$

(ii) Area of shaded region = Area of $\triangle ABC$ – Area of sector BXY

$= 800 - \dfrac{1}{2} \times 10^2 \times \dfrac{\pi}{6}$

$= 774 \text{ cm}^2$ (to 3 s.f.)

17. (i) Major arc length = 24 m

$6 \times \theta = 24$

$\theta = 4$

$\therefore$ Reflex $\angle AOB = 4$ rad

(ii) Using cosine rule,

$AB^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos 4$

$= 119.1$ (to 4 s.f.)

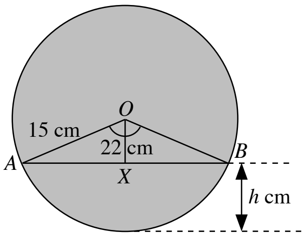
$AB = 10.9$ m (to 3 s.f.)

(iii) Area of major segment = Area of sector APB + Area of $\triangle AOB$

$= \dfrac{1}{2} \times 6^2 \times 4 + \dfrac{1}{2} \times 6 \times 6 \times \sin (2\pi - 4)$

$= 85.6 \text{ cm}^2$ (to 3 s.f.)

18. (i)



Using Pythagoras' Theorem,

$OX^2 = 15^2 - 11^2$

$= 104$

$OX = \sqrt{104}$

$= 10.20$ cm (to 4 s.f.)

$h = 15 - 10.20$

$= 4.80$ cm (to 3 s.f.)

(ii) Using cosine rule,

$\cos \angle AOB = \dfrac{15^2 + 15^2 - 22^2}{2 \times 15 \times 15}$

$= -0.07556$ (to 4 s.f.)

$\angle AOB = \cos^{-1} (-0.07556)$

$= 1.656$ rad (to 4 s.f.)

Area of cross section

= Area of sector AOB – Area of $\triangle AOB$

$= \dfrac{1}{2} \times 15^2 \times 1.646 - \dfrac{1}{2} \times 15 \times 15 \times \sin 1.646$

$= 73.0 \text{ cm}^2$ (to 3 s.f.)

19. (i) $\cos \theta = \dfrac{3}{6}$

$\theta = \cos^{-1} \dfrac{3}{6}$

$= 1.047$ (to 4 s.f.)

Obtuse $\angle POQ = 1.047 \times 2$

$= 2.09$ rad (to 3 s.f.)

(ii) Area of shaded segment

= Area of sector POQ – Area of $\triangle POQ$

$= \dfrac{1}{2} \times 6^2 \times 2.094 - \dfrac{1}{2} \times 6 \times 6 \times \sin 2.094$

$= 22.1 \text{ cm}^2$ (to 3 s.f.)

20. Surface area of cone $= \dfrac{1}{2} \times 81^2 \times 2$

$= 6561 \text{ cm}^2$

$\pi \times r \times 81 = 6561 \text{ cm}^2$

$r = 25.78$ cm (to 4 s.f.)

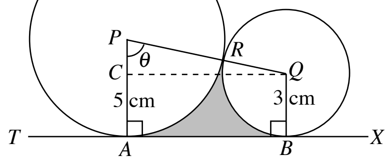
Using Pythagoras' Theorem,

$h^2 = 81^2 - 25.78^2$

$= 5.896$ (to 4 s.f.)

$h = 76.8$ cm (to 3 s.f.)

21. (i) Draw a line QC such that C lies on AP and CQ is perpendicular to AP.



$PC = PA - CA$

$= 5 - 3$

$= 2$ cm

$PQ = PR + RQ$

$= 5 + 3$

$= 8$ cm

In $\triangle PCQ$,

$\cos \angle CPQ = \dfrac{PC}{PQ} = \dfrac{2}{8}$

$\angle CPQ = \cos^{-1} \dfrac{2}{8} = 1.32$ rad (to 3 s.f.)

$\angle APQ = 1.32$ rad

$\therefore \theta = 1.32$

(ii) Using Pythagoras' Theorem,

$CQ^2 = 8^2 - 2^2$

$CQ = \sqrt{8^2 - 2^2}$

$= \sqrt{60}$

$= 7.746$ cm (to 4 s.f.)

$\angle PQC = \pi - \dfrac{\pi}{2} - 1.318$ ($\angle$ sum of a $\triangle$)

$= 0.2527$ rad (to 4 s.f.)

$\angle PQB = \dfrac{\pi}{2} + 0.2527 = 1.823$ rad (to 4 s.f.)

Area of shaded region

= Area of trapezium $ABPQ$ – area of sector APR

– area of sector RBQ

$= \dfrac{1}{2}(3 + 5)(7.746) - \dfrac{1}{2} \times 5^2 \times 1.318 - \dfrac{1}{2} \times 3^2 \times 1.823$

$= 6.30 \text{ cm}^2$ (to 3 s.f.)

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22. (i) $\tan\frac{\theta}{2} = \frac{OP}{PT} = \frac{8}{14}$

$\frac{\theta}{2} = \tan^{-1}\frac{8}{14} = 0.5191$ (to 4 s.f.)

$\therefore \theta = 0.5191 \times 2$

$= 1.04$ (to 3 s.f.)

(ii) $\angle POT = \pi - \frac{\pi}{2} - 0.5191$ ($\angle$sum of a $\triangle$)

$= 1.052$ rad (to 4 s.f.)

$\angle POQ = 1.052 \times 2$

$= 2.103$ rad (to 4 s.f.)

Area of sector $POQ = \frac{1}{2} \times 8^2 \times 2.103$

$= 67.31\text{ cm}^2$ (to 4 s.f.)

Area of $\triangle POT = \frac{1}{2} \times 14 \times 8$

$= 56\text{ cm}^2$

Area of shaded region $= 2 \times 56 - 67.31$

$= 44.7\text{ cm}^2$ (to 3 s.f.)

23. (i) $\cos\angle HAB = \frac{AB}{AH} = \frac{10}{12.5}$

$\angle HAB = \cos^{-1}\frac{10}{12.5} = 0.6435$ rad (to 4 s.f.)

$\therefore \angle HAK = \frac{\pi}{2} - 0.6435 - 0.6435$

$= 0.284$ rad (to 3 s.f.)

(ii) Area of $\triangle ABH$ = Area of $\triangle ADK$

$= \frac{1}{2} \times 10 \times 12.5 \times \sin 0.6435$

$= 37.50\text{ cm}^2$ (to 4 s.f.)

Area of sector $AKH = \frac{1}{2} \times 12.5^2 \times 0.2838$

$= 22.17\text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 10 \times 10 - 22.07 - 2 \times 37.50$

$= 2.83\text{ cm}^2$ (to 3 s.f.)

24. (i) Length of arc $PSQ = 12 \times 1.2 = 14.4$ cm

(ii) Area of segment $PSQ = \frac{1}{2} \times 12^2 \times 1.2 - \frac{1}{2} \times 12 \times 12 \times \sin 1.2$

$= 19.3\text{ cm}^2$ (to 3 s.f.)

(iii) Using cosine rule,

$PQ^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 1.2$

$= 183.6$ (to 4 s.f.)

$PQ = 13.6$ cm (to 3 s.f.)

(iv) $\angle OPQ = \frac{\pi - 1.2}{2}$ ($\angle$ sum of an isos. $\triangle$)

$= 0.9708$ rad (to 4 s.f.)

$\angle QPR = \frac{\pi}{2} - 0.9708$ ($\angle$ sum of an isos. $\triangle$)

$= 0.6$ rad

Area of $\triangle PQR = \frac{1}{2} \times 13.55 \times 14.4 \times \sin 0.6$

$= 55.09\text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 55.09 - 19.29$

$= 35.8\text{ cm}^2$ (to 3 s.f.)

25. (i) Length of arc $PQR = \frac{1}{4} \times 2 \times \pi \times 15$

$= 7.5\pi$ cm

$\frac{PQ}{QR} = \frac{4}{3}$

$QR = \frac{3}{7} \times 7.5\pi$

$= \frac{45}{14}\pi$ cm

$15 \times \theta = \frac{45}{14}\pi$

$\theta = \frac{3}{14}\pi$

(ii) $\angle POQ = \frac{\pi}{2} - \frac{45}{14}\pi = \frac{2}{7}\pi$ rad

Area of shaded region

= Area of quadrant – Area of sector OQR – Area of $\triangle OPQ$

$= \frac{1}{4} \times \pi \times 15^2 - \frac{1}{2} \times 15^2 \times \frac{3}{14}\pi - \frac{1}{2} \times 15 \times 15 \times \sin\frac{2}{7}\pi$

$= 164\text{ cm}^2$ (to 3 s.f.)

Review Exercise 10

1. (i) Length of arc $BPA = \frac{360° - 120°}{360°} \times 2\pi \times 12$

$= 5.03$ cm (to 3 s.f.)

(ii) Area of sector $OBPA = \frac{240°}{360°} \times \pi \times 12^2$

$= 302\text{ cm}^2$ (to 3 s.f.)

2. (i) Area of minor sector $POQ = \frac{1}{2} \times 8^2 \times 1.9$

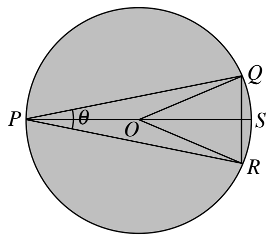
$= 60.8\text{ cm}^2$

(ii) Reflex $\angle POQ = (2\pi - 1.9)$ rad

(iii) Length of major arc $PRQ = 8 \times (2\pi - 1.9)$

$= (16\pi - 15.2)$ cm

3. (i)



In $\triangle POQ$,

$\angle SPQ = \frac{\theta}{2}$ rad

$\angle PQO = \frac{\theta}{2}$ rad (isos. $\triangle$)

Using cosine rule,

$\cos\angle QOR = \frac{8^2 + 8^2 - 7^2}{2 \times 8 \times 8}$

$= 0.6172$ (to 4 s.f.)

$\angle QOR = \cos^{-1} 0.6172$

$= 0.9056$ rad (to 4 s.f.)

Reflex $\angle QOR = 2\pi - 0.9056$
$= 5.378$ rad (to 4 s.f.)

$\angle POQ = \frac{5.378}{2} = 2.689$ rad (to 4 s.f.)

$\angle SPQ = \frac{2\pi - 2.689}{2} = 1.797$ rad (to 4 s.f.)

$\angle QPR = 0.2265 \times 2 = 0.453$ rad (to 3 s.f.)

$\therefore\ \theta = 0.453$

(ii) Area of minor segment
= Area of sector OQR – Area of $\triangle OQR$

$= \frac{1}{2} \times 8^2 \times 0.9056 - \frac{1}{2} \times 8 \times 8 \times \sin 0.9056$

$= 3.80$ cm^2 (to 3 s.f.)

4. **(i)** Using cosine rule,

$\cos \angle AOB = \frac{10^2 + 10^2 - 14^2}{2 \times 10 \times 10}$
$= 0.02$

$\angle AOB = \cos^{-1} 0.02$
$= 1.55$ rad (to 3 s.f.)

(ii) Cross-sectional area of cake sliced off
= Area of sector AOB – Area of $\triangle AOB$

$= \frac{1}{2} \times 10^2 \times 1.551 - \frac{1}{2} \times 10 \times 10 \times \sin 1.551$

$= 27.6$ cm^2 (to 3 s.f.)

5. Let the radius of the circle be r cm.

Area of circle $= \pi r^2$ cm^2

Area of segment = Area of sector POQ – Area of $\triangle POQ$

$= \frac{1}{2} \times r^2 \times 2.2 - \frac{1}{2} \times r \times r \times \sin 2.2$

$= 0.6958r^2$ cm^2 (to 4 s.f.)

Percentage $= \frac{0.6958r^2}{\pi r^2} \times 100\%$
$= 22\%$ (to the nearest integer)

6. **(i)** $\angle AOB = (2\pi - 5.8)$ rad

Area of $\triangle OAB = \frac{1}{2} \times 15^2 \times \sin (2\pi - 5.8)$
$= 52.3$ cm^2 (to 3 s.f.)

(ii) Area of unshaded region
= Area of sector AOB – Area of $\triangle AOB$

$= \frac{1}{2} \times 15^2 \times (2\pi - 5.8) - 52.3$

$= 2.06$ cm^2 (to 3 s.f.)

P(dart lands on unshaded area) $= \frac{2.06}{\pi \times 15^2}$
$= 2.91 \times 10^{-3}$ (to 3 s.f.)

7. **(i)** $\angle AOB = (\pi - 1.15)$ rad (adj. $\angle$s on a str. line)

Length of arc $AB = 12 \times (\pi - 1.15)$
$= 23.9$ cm (o 3 s.f.)

(ii) Area of shaded segment
= Area of sector AOB – Area of $\triangle AOB$

$= \frac{1}{2} \times 12^2 \times (\pi - 1.15) - \frac{1}{2} \times 12 \times 12 \times \sin (\pi - 1.15)$

$= 77.7$ cm^2 (to 3 s.f.)

(iii) $OC = 30 - 12 = 18$ cm

Area of $\triangle BOC = \frac{1}{2} \times 12 \times 18 \times \sin 1.15$
$= 98.6$ cm^2 (to 3 s.f.)

(iv) Using cosine rule,

$BC^2 = 12^2 + 18^2 - 2 \times 12 \times 18 \times \cos 1.15$
$= 291.5$ (to 4 s.f.)

$BC = 17.1$ cm (to 3 s.f.)

8. **(i)** Using sine rule,

$\frac{OC}{\sin 0.6} = \frac{12}{\sin 1.2}$

$OC = \frac{12 \sin 0.6}{\sin 1.2} = 7.270$ (to 4 s.f.)

$BC = 12 - 7.270 = 4.73$ cm (to 3 s.f.)

$\angle AOC = \pi - 1.2 - 0.6 = (\pi - 1.8)$ rad

Using sine rule,

$\frac{AC}{\sin (\pi - 1.8)} = \frac{12}{\sin 1.2}$

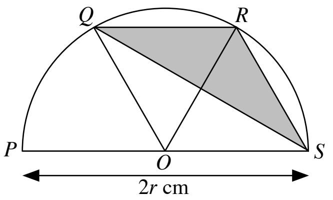
$AC = \frac{12 \sin (\pi - 1.8)}{\sin 1.2} = 12.5$ cm (to 3 s.f.)

(ii) Area of shaded region
= Area of sector AOB – Area of $\triangle AOC$

$= \frac{1}{2} \times 12^2 \times (\pi - 1.8) - \frac{1}{2} \times 12 \times 12.54 \times \sin 0.6$

$= 54.1$ cm^2 (to 3 s.f.)

9.



Since lengths of arcs $PQ = QR = RS$,

$\angle POQ = \angle QOR = \angle ROS = \frac{\pi}{3}$ rad

Using cosine rule,

$QR^2 = r^2 + r^2 - 2 \times r \times r \times \cos \frac{\pi}{3}$
$= 2r^2 - r^2$
$= r^2$

$QR = r$ cm

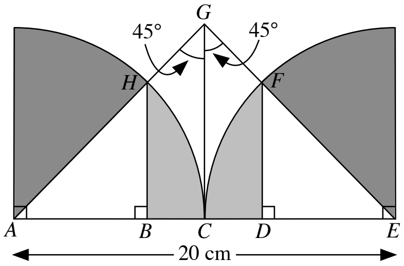
$RS^2 = r^2 + r^2 - 2 \times r \times r \times \cos \frac{\pi}{3}$
$= 2r^2 - r^2$
$= r^2$

$RS = r$ cm

$\angle QRS = \angle QRO + \angle ORS$
$= \frac{\pi}{3} + \frac{\pi}{3}$
$= \frac{2\pi}{3}$ rad

Area of shaded region $= \frac{1}{2} \times r \times r \times \sin \frac{2\pi}{3}$
$= 0.433r^2$ cm^2 (to 3 s.f.)

10.



$\angle HGF = 45° + 45° = 90°$

$\angle GEC = 180° - 90° - 45°$ ($\angle$ sum of a $\triangle$)

$= 45°$

Radius of quadrant = 10 cm

$EF = 10$ cm

$$\cos \angle DEF = \frac{DE}{EF}$$

$$\cos 45° = \frac{DE}{10}$$

$$DE = (10 \cos 45°) \text{ cm}$$

$$\text{Area of } \triangle DEF = \frac{1}{2} \times 10 \times 10 \cos 45° \times \sin 45° = 25 \text{ cm}^2$$

$$\text{Total area of shaded region} = 2\left[\frac{1}{4} \times \pi \times 10^2 - 25\right] = 2[25\pi - 25] = 50(\pi - 1) \text{ cm}^2$$

11. (a) (i) Using cosine rule,

$$PQ^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \times \cos 1.25 = 308.1 \text{ (to 4 s.f.)}$$

$$PQ = 17.6 \text{ cm (to 3 s.f.)}$$

(ii) $\text{Area of minor sector } APQ = \frac{1}{2} \times 15^2 \times 1.25 = 141 \text{ cm}^2 \text{ (to 3 s.f.)}$

(iii) $\text{Area of } \triangle PAQ = \frac{1}{2} \times 15 \times 15 \times \sin 1.25 = 107 \text{ cm}^2 \text{ (to 3 s.f.)}$

(b) Using cosine rule,

$$\cos \angle PBQ = \frac{12^2 + 12^2 - 17.55^2}{2 \times 12 \times 12} = -0.06979 \text{ (to 4 s.f.)}$$

$$\angle PBQ = \cos^{-1}(-0.06979) = 1.641 \text{ rad (to 4 s.f.)}$$

$$\text{Area of minor sector } BPQ = \frac{1}{2} \times 12^2 \times 1.641 = 118.1 \text{ cm}^2 \text{ (to 4 s.f.)}$$

$$\text{Area of } \triangle PBQ = \frac{1}{2} \times 12 \times 12 \times \sin 1.641 = 71.82 \text{ cm}^2 \text{ (to 4 s.f.)}$$

$$\text{Area of shaded region} = (140.6 - 106.8) + (118.1 - 71.82) = 80.0 \text{ cm}^2 \text{ (to 3 s.f.)}$$

Challenge Yourself

1. (a) Perimeter = arc $AB + AB$

$$= \pi \times \frac{14}{2} + 14 = (7\pi + 14) \text{ cm}$$

(b) $\text{Perimeter} = \left(\pi \times \frac{14}{2}\right) + \left(\pi \times \frac{14}{2}\right) + 14 = (7\pi + 14) \text{ cm}$

(c) $\text{Perimeter} = \left(\pi \times \frac{14}{6}\right) + \left(\pi \times \frac{14}{6}\right) + \left(\pi \times \frac{14}{6}\right) + 14 = (7\pi + 14) \text{ cm}$

(d) $\text{Perimeter} = \left(\pi \times \frac{14}{8}\right) + \left(\pi \times \frac{14}{8}\right) + \left(\pi \times \frac{14}{8}\right) + \left(\pi \times \frac{14}{8}\right) + 14 = (7\pi + 14) \text{ cm}$

All the perimeters are equal.

In general, it does not matter how many identical semicircles are on the line AB, all the perimeters are equal.

2. (i) By Pythagoras' Theorem,

$$AP^2 = PB^2 + AB^2$$
$$(12 + r)^2 = (12 - r)^2 + 12^2$$
$$144 + 24r + r^2 = r^2 - 24r + 144 + 144$$
$$144 + 24r + r^2 = r^2 - 24r + 288$$
$$24r = -24r + 144$$
$$48r = 144$$
$$r = 3$$

(ii) $AP = 12 + 3 = 15$ cm

$PB = 12 - 3 = 9$ cm

Using cosine rule,

$$\cos \angle PAB = \frac{15^2 + 12^2 - 9^2}{2 \times 15 \times 12} = 0.8$$

$$\angle PAB = \cos^{-1} 0.8 = 0.644 \text{ rad (to 3 s.f.)}$$

$\therefore \angle PAC = 0.644$ rad

(iii) $\angle PCA = \angle PAC = 0.6435$ rad

$\angle APC = \pi - 0.6435 - 0.6435$ ($\angle$ sum of a $\triangle$)

$= 1.855$ rad

$$\text{Area of minor sector } RPS = \frac{1}{2} \times 3^2 \times 1.855 = 8.346 \text{ cm}^2 \text{ (to 4 s.f.)}$$

$$\text{Area of minor sector } ARB = \frac{1}{2} \times 12^2 \times 0.6435 = 46.33 \text{ cm}^2 \text{ (to 4 s.f.)}$$

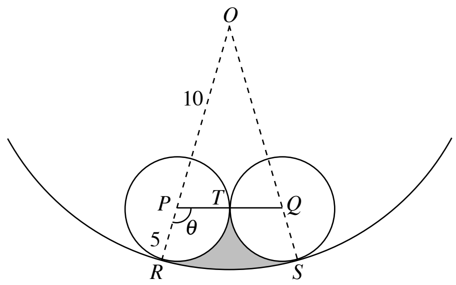
Area of minor sector BCS = Area of minor sector ARB

$= 46.33 \text{ cm}^2$

$$\text{Area of } \triangle APC = \frac{1}{2} \times 15 \times 15 \times \sin 1.855 = 108.0 \text{ cm}^2 \text{ (to 4 s.f.)}$$

$$\text{Area of shaded region} = 108.0 - 46.33 - 46.33 - 8.346 = 6.99 \text{ cm}^2 \text{ (to 3 s.f.)}$$

3. **(i)**



$PC = 2 \times 5 = 10$ cm

$\therefore \triangle OPQ$ is an equilateral triangle.

$\angle POQ = \dfrac{\pi}{3}$ rad

$\angle RPT = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$ rad

Length of arc $RT = 5 \times \dfrac{2\pi}{3} = \dfrac{10\pi}{3}$ cm

Length of arc $RS = 15 \times \dfrac{\pi}{3} = 5\pi$ cm

Perimeter of shaded region $= 2 \times \dfrac{10\pi}{3} + 5\pi$

$= \dfrac{35\pi}{3}$ cm

(ii) Area of sector $ORS = \dfrac{1}{2} \times 15^2 \times \dfrac{\pi}{3}$

$= 37\dfrac{1}{2}\pi$ cm^2

Area of $\triangle OPQ = \dfrac{1}{2} \times 10 \times 10 \times \sin \dfrac{\pi}{3}$

$= 25\sqrt{3}$ cm^2

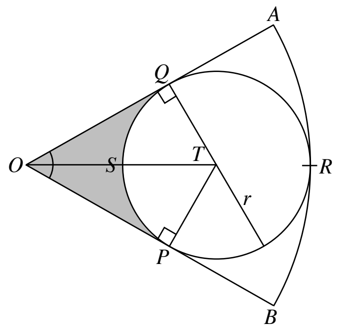
Area of sector $RPT = \dfrac{1}{2} \times 5^2 \times \dfrac{2\pi}{3}$

$= 8\dfrac{1}{3}\pi$ cm^2

Area of shaded region $= 37\dfrac{1}{2}\pi - 25\sqrt{3} - 2 \times 8\dfrac{1}{3}\pi$

$= 22.1$ cm^2 (to 3 s.f.)

4.



(i) $OT = (48 - r)$ cm

$PT = r$ cm

$\angle TOP = 30°$

$\sin 30° = \dfrac{r}{48 - r}$

$48 - r = 2r$

$3r = 48$

$r = 16$

(ii) $\angle OTP = 90° - 30° = 60°$

Area of $\triangle OTP = \dfrac{1}{2} \times 16 \times (48 - 16) \sin 60°$

$= 256\left(\dfrac{\sqrt{3}}{2}\right)$

$= 128\sqrt{3}$ cm^2

Area of sector $STP = \dfrac{60°}{360°} \times \pi \times 16^2$

$= \dfrac{128}{3}\pi$ cm^2

Ara of shaded region $= 2\left(128\sqrt{3} - \dfrac{128}{3}\pi\right)$

$= 175$ cm^2 (to 3 s.f.)

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Revision Exercise C1

1. Since $\sin x$ is positive, x can either be an acute angle or an obtuse angle.

$\sin x = 0.345$

$x = \sin^{-1} 0.345 = 0.352$ (to 3 s.f.)

or $\pi - 0.3522 = 2.79$ (to 3 s.f.)

$\therefore x = 0.352$ or 2.79

2. **(i)** Since the length of arc is 15.2 cm,

$8 \times \theta = 15.2$

$\theta = 1.9$

$\therefore \angle AOB = 1.9$ rad

(ii) Area of sector $= \frac{1}{2} \times 8^2 \times 1.9$

$= 60.8$ cm^2

3. **(i)** $\cos 0.6 = \frac{AP}{16}$

$AP = 16 \cos 0.6$

$= 13.2$ cm (to 3 s.f.)

(ii) Area of $\triangle PAB = \frac{1}{2} \times 13.21 \times 16 \times \sin 0.6$

$= 59.7$ cm^2 (to 3 s.f.)

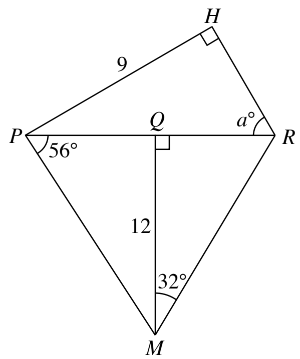
(iii) $\angle POB = 1.2$ rad

Area of shaded region = Area of sector POB – Area of $\triangle POB$

$= \frac{1}{2} \times 8^2 \times 1.2 - \frac{1}{2} \times 8 \times 8 \times \sin 1.2$

$= 8.57$ cm^2 (to 3 s.f.)

4.



In $\triangle QRM$,

$\tan 32° = \frac{QR}{12}$

$QR = 12 \tan 32°$

$= 7.498$ cm (to 4 s.f.)

In $\triangle PQM$,

$\tan 56° = \frac{12}{PQ}$

$PQ = \frac{12}{\tan 56°}$

$= 8.094$ cm (to 4 s.f.)

$PR = 7.498 + 8.094 = 15.592$ cm

$\sin a° = \frac{9}{15.592}$

$a = \sin^{-1} \frac{9}{15.592}$

$= 35.3$ (to 3 s.f.)

5. **(i)** $\sin 40° = \frac{CD}{8}$

$CD = 8 \sin 40°$

$= 5.14$ cm (to 3 s.f.)

(ii) $\angle PQD = 180° - 20° - 40° = 120°$

Using cosine rule,

$PD^2 = 4^2 + 8^2 - 2 \times 4 \times 8 \times \cos 120°$

$= 112$

$PD = \sqrt{112} = 10.58$ cm (to 4 s.f.)

In $\triangle PBQ$,

$\cos 70° = \frac{BP}{4}$

$BP = 4 \cos 70°$

$= 1.368$ cm (to 4 s.f.)

$AP = 5.142 - 1.368 = 3.774$ cm

$\cos \angle APD = \frac{3.774}{10.58}$

$\angle APD = \cos^{-1} \frac{3.774}{10.58}$

$= 69.1°$ (to 1 d.p.)

(iii) By Pythagoras' Theorem,

$AD^2 = PD^2 - AP^2$

$= 10.58^2 - 3.774^2$

$= 97.69$ (to 4 s.f.)

$AD = \sqrt{97.69}$

$= 9.89$ cm (to 3 s.f.)

6. **(i)** Using cosine rule,

$\cos \angle BAD = \frac{6^2 + 6^2 - 9^2}{2 \times 6 \times 6}$

$= -0.125$

$\angle BAD = \cos^{-1}(-0.125)$

$= 97.2°$ (to 1 d.p.)

(ii) Area of $\triangle ABD = \frac{1}{2} \times 6 \times 6 \times \sin 97.18°$

$= 17.9$ cm^2 (to 3 s.f.)

(iii) Area of $\triangle BCD = \frac{1}{2} \times 9 \times 12 \times \sin 60°$

$= 46.77$ cm^2 (to 4 s.f.)

Area of quadrilateral $ABCD = 17.86 + 46.77$

$= 64.6$ cm^2 (to 3 s.f.)

(iv) Using sine rule,

$\frac{\sin \angle ABD}{6} = \frac{\sin 97.18°}{9}$

$\angle ABD = \sin^{-1} \frac{\sin 97.18°}{9}$

$= 41.41°$ (to 2 d.p.)

$\angle ABC = 60° + 41.41°$

$= 101.4°$ (to 1 d.p.)

(v) Using cosine rule,

$DC^2 = 9^2 + 12^2 - 2 \times 9 \times 12 \times \cos 60°$
$= 117$
$DC = \sqrt{117}$
$= 10.8$ cm (to 3 s.f.)

7. (i) Using sine rule,

$\frac{\sin \theta}{12} = \frac{\sin 42°}{10}$

$\sin \theta = \frac{12 \sin 42°}{10}$
$= 0.8030$ (to 4 s.f.)

$\angle\theta = \sin^{-1} 0.8030 = 53.42°$ (to 2 d.p.)
or $180° - 53.42° = 126.58°$ (to 2 d.p.)

$\therefore\ \angle ACB = 53.42°$ and $\angle ADB = 126.58°$

$\angle BAC = 180° - 53.42° - 42°$ ($\angle$ sum of a $\triangle$)
$= 84.58°$

The bearing of C from A is $(90° - 84.58°)$.

$\therefore$ Bearing of C from A is 005.4°.

(ii) $\angle BAD = 180° - 42° - 126.58°$ ($\angle$ sum of a $\triangle$)
$= 11.42°$

The bearing of D from A is $(90° - 11.42°)$.

$\therefore$ Bearing of D from A is 078.6°.

(iii) $\angle CAD = 84.58° - 11.42° = 73.16°$

Using cosine rule,

$CD^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 73.16°$
$= 142.1$ (to 4 s.f.)
$CD = \sqrt{142.1}$
$= 11.9$ km (to 3 s.f.)

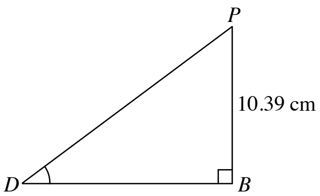
8. (i) $\cos \angle PAB = \frac{6}{12}$

$\angle PAB = \cos^{-1} \frac{6}{12}$
$= 60°$

(ii) By Pythagoras' Theorem,

$PB^2 = AP^2 - AB^2$
$= 12^2 - 6^2$
$= 108$
$PB = \sqrt{108}$
$= 10.4$ cm (to 3 s.f.)

(iii)



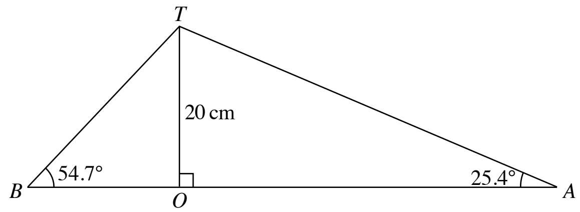
By Pythagoras' Theorem,

$BD^2 = AB^2 + AD^2$
$= 6^2 + 8^2$
$= 100$
$BD = 10$ cm

$\tan \angle PDB = \frac{10.39}{10}$

$\angle PDB = \tan^{-1} \frac{10.39}{10}$
$= 46.1°$ (to 1 d.p.)

9.



$\tan 54.7° = \frac{20}{OB}$

$OB = \frac{20}{\tan 54.7°}$
$= 14.16$ m (to 4 s.f.)

$\tan 25.4° = \frac{20}{OA}$

$OA = \frac{20}{\tan 25.4°}$
$= 42.12$ m (to 4 s.f.)

$AB = 14.16 + 42.12$
$= 56.3$ m (to 3 s.f.)

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Revision Exercise C2

1. Since $\cos x$ is negative, x is an obtuse angle.

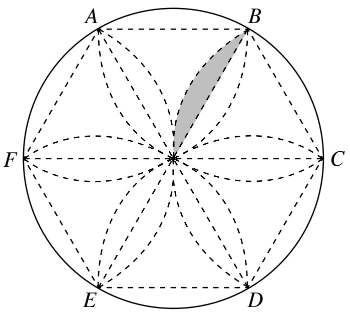
$\cos x = -0.5$

$x = \cos^{-1}(-0.5) = 2.09$ (to 3 s.f.)

2. **(a)** $\sin 140° = \sin (180° - 140°)$
$= \sin 40°$
$= 0.643$

(b) $\cos 165° = -\cos (180° - 165°)$
$= -\cos 15°$
$= -0.966$

3.



Area of half a petal $= \frac{60°}{360°} \times \pi \times 4^2 - \frac{1}{2} \times 4^2 \times \sin 60°$
$= 8.378 - 6.928$
$= 1.450\text{ cm}^2$ (to 4 s.f.)

Total shaded area $= 12(1.450)$
$= 17.4\text{ cm}^2$ (to 3 s.f.)

4. $180° - 50° - 70° = 60°$ ($\angle$ sum of a $\triangle$)

Using sine rule,

$$\frac{x}{\sin 60°} = \frac{8}{\sin 70°}$$

$$x = \frac{8 \sin 60°}{\sin 70°}$$

$= 7.37$ (to 3 s.f.)

Using sine rule,

$$\frac{y}{\sin 50°} = \frac{8}{\sin 70°}$$

$$y = \frac{8 \sin 50°}{\sin 70°}$$

$= 6.52$ (to 3 s.f.)

5. **(i)** $\tan 40° = \frac{AB}{10}$

$AB = 10 \tan 40°$
$= 8.391$ cm (to 4 s.f.)

By Pythagoras' Theorem,

$AM^2 = AB^2 + BM^2$
$= 8.391^2 + 5^2$
$= 95.41$ (to 4 s.f.)

$AM = \sqrt{95.41}$
$= 9.77$ cm (to 3 s.f.)

(ii) By Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$= 8.391^2 + 10^2$
$= 170.4$ (to 4 s.f.)

$AC = \sqrt{170.4}$
$= 13.1$ cm (to 3 s.f.)

(iii) $\tan \angle AMB = \frac{8.391}{5}$

$\angle AMB = \tan^{-1} \frac{8.391}{5}$
$= 59.2°$ (to 1 d.p.)

(iv) $\angle BAM = 180° - 90° - 59.21°$ ($\angle$ sum of a $\triangle$)
$= 30.79°$

$\angle BAC = 180° - 90° - 40°$ ($\angle$ sum of a $\triangle$)
$= 50°$

$\angle CAM = 50° - 30.79° = 19.2°$ (to 1 d.p.)

6. **(i)** Using cosine rule,

$AC^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos 120°$
$= 61$

$AC = \sqrt{61}$
$= 7.81$ cm (to 3 s.f.)

(ii) Using sine rule,

$$\frac{\sin \angle DAC}{5} = \frac{\sin 120°}{7.810}$$

$$\angle DAC = \sin^{-1} \frac{5 \sin 120°}{7.810}$$

$= 33.7°$ (to 1 d.p.)

(iii) Using sine rule,

$$\frac{\sin \angle ABC}{7.810} = \frac{\sin 60°}{10}$$

$$\angle ABC = \sin^{-1} \frac{7.810 \sin 60°}{10}$$

$= 42.6°$ (to 1 d.p.)

(iv) $\angle BAC = 180° - 60° - 42.56°$ ($\angle$ sum of a $\triangle$)
$= 77.44°$

$\angle BAD = 33.67° + 77.44°$
$= 111.11°$

Using cosine rule,

$BD^2 = 4^2 + 10^2 - 2 \times 4 \times 10 \times \cos 111.11°$
$= 144.8$ (to 4 s.f.)

$BD = \sqrt{144.8}$
$= 12.0$ cm (to 3 s.f.)

(v) Area of quadrilateral $ABCD$

$=$ Area of $\triangle ADC$ + Area of $\triangle ABC$

$= \frac{1}{2} \times 4 \times 5 \times \sin 120° + \frac{1}{2} \times 10 \times 78.10 \times \sin 77.44°$

$= 46.8\text{ cm}^3$ (to 3 s.f.)

7. **(a)** **(i)** $\angle ABC = \dfrac{(5-2)\times 180°}{5} = 108°$

The bearing of C from B is $(180° - 108°)$.

$\therefore$ Bearing of C from B is 072°.

(ii) $\angle BAE = 108°$

$\therefore$ Bearing of E from A is 108°.

(iii) The bearing of D from E is $(108° - 72°)$.

$\therefore$ Bearing of D from E is 036°.

(b) Using cosine rule,

$AC^2 = 800^2 + 800^2 - 2 \times 800 \times 800 \times \cos 108°$

$= 1\ 676\ 000$ (to 4 s.f.)

Area of $ABCDE$

$= \dfrac{1}{2} \times 800 \times 800 \times \sin 108° + \dfrac{1}{2} \times 1\ 676\ 000 \times \sin 36°$

$= 1.101 \times 10^6$ cm^3 (to 4 s.f.)

8. **(a)** **(i)** $MQ = \dfrac{1}{2} QR = 3$ cm

By Pythagoras' Theorem,

$PM^2 = 6^2 - 3^2 = 27$

$PM = \sqrt{27} = 5.196$ cm (to 4 s.f.)

Since $PQ = QR = PR$, $\triangle PQR$ is an equilateral triangle.

$\cos 30° = \dfrac{3}{PX}$

$PX = \dfrac{3}{\cos 30°} = 3.464$ cm (to 4 s.f.)

By Pythagoras' Theorem,

$OX^2 = 8^2 - 3.464^2 = 52.00$ (to 4 s.f.)

$OX = \sqrt{52.00} = 7.21$ cm (to 3 s.f.)

(ii) $\sin \angle OPX = \dfrac{7.211}{8}$

$\angle OPX = \sin^{-1} \dfrac{7.211}{8}$

$= 64.3°$ (to 1 d.p.)

(b) $MX = 5.196 - 3.464$

$= 1.732$ cm

$\tan \angle OMX = \dfrac{7.211}{1.732}$

$\angle OMX = \tan^{-1} \dfrac{7.211}{1.732}$

$= 76.5°$ (to 1 d.p.)

9. **(i)** $\tan 24° = \dfrac{VQ}{8}$

$VQ = 8 \tan 24°$

$= 3.56$ m (to 3 s.f.)

(ii) By Pythagoras' Theorem,

$SQ^2 = 6^2 + 8^2 = 100$

$SQ = \sqrt{100} = 10$ m

By Pythagoras' Theorem,

$VS^2 = 10^2 + 3.562^2 = 112.7$ (to 4 s.f.)

$VS = \sqrt{112.7} = 10.6$ m (to 3 s.f.)

(iii) $\cos \angle QVS = \dfrac{3.562}{10.62}$

$\angle QVS = \cos^{-1} \dfrac{3.562}{10.62}$

$= 70.4°$ (to 1 d.p.)

(iv) $\tan \angle VRQ = \dfrac{3.562}{6}$

$\angle VRQ = \tan^{-1} \dfrac{3.562}{6}$

$= 30.7°$ (to 1 d.p.)

$\therefore$ The angle of elevation of V from R is 30.7°.

Chapter 11 Congruence and Similarity Tests

TEACHING NOTES

Suggested Approach

Teachers may want to introduce this topic by asking students to recall what they have learnt in Book 2 on congruent and similar triangles. Then, teachers may get students to investigate whether all the conditions are necessary to prove whether two triangles are congruent or similar.

Section 11.1: Congruence Tests

Teachers may wish to recap with the students, that for congruent triangles, all the corresponding lengths and angles are equal.

For each of the 4 congruent tests that are covered in this section, teachers should ask the students to construct a triangle in as many ways as possible and see what conclusion they can make by comparing these triangles (see Investigation: SSS Congruence Test, Investigation: SAS Congruence Test, Investigation: AAS Congruence Test and RHS Congruence Test).

Teachers should teach students how to match the vertices of two triangles correctly, even if the two triangles are not congruent. Once students have learnt all the 4 congruent tests, they can learn to identify pairs of congruent triangles and prove the congruency (see Class Discussion: Consolidation for Congruence Tests).

Teachers should highlight to students that the 4 congruent tests covered in this chapter are not the only congruence tests.

Section 11.2: Similarity Tests

Teachers may wish to recap with the students, that for similar triangles, all the corresponding lengths and angles are proportional and equal, respectively.

For each of the 3 similarity tests that are covered in this section, teachers should ask the students to construct a triangle in as many ways as possible and see what conclusion they can make by comparing these triangles (see Investigation: AA Similarity Test, Investigation: SSS Similarity Test and Investigation: SAS Similarity Test).

Once students have learnt all the 4 congruent tests and 3 similarity tests, teachers may ask students to compare the congruent tests with the similarity tests (see Thinking Time on page 366). Teachers should highlight to students that the 3 similarity tests covered in this chapter are not the only similarity tests.

Section 11.3: Applications of Congruent and Similar Triangles

Now that students have learnt the congruent and similarity tests, they can apply the concepts to solve problems in mathematics and in real life. For Worked Example 10, teachers should recap with students the properties of a perpendicular bisector and an angle bisector before going through the problem.

Challenge Yourself

For Question 1, let the height of $\triangle PST$ from P to ST be h' units and use similar triangles to solve the problem. Students need to manipulate algebra properly, otherwise they may end up with a long and tedious working. For Question 2, students should identify a pair of similar triangles and let QU be x cm and VS be y cm. Then they can formulate a pair of simultaneous equations involving x and y and solve for x and y to find the length of QU. For Question 3, students should identify two pairs of similar triangles first.

WORKED SOLUTIONS

Investigation (SSS Congruence Test)

5. From this investigation, we can conclude that if the 3 sides of a triangle are equal to the 3 corresponding sides of another triangle, then the two triangles are congruent.

Investigation (SAS Congruence Test)

5. From part 1 of this investigation, if two sides and the included angle of a triangle are given, then only a unique triangle can be constructed.

9. From part 2 of this investigation, if two sides and an angle which is not the included angle of a triangle are given, then there is more than one way to construct the triangle.

Investigation (AAS Congruence Test)

3. From part 1 of this investigation, if two angles and the side of the triangle between the two angles are given, then only a unique triangle can be constructed.

6. From part 2 of this investigation, if two angles and the side of the triangle that is not between the two angles are given, then only a unique triangle can be constructed.

7. It does not matter. Given the values of two angles, we can find the value of the third angle in the triangle.

Investigation (RHS Congruence Test)

3. From the investigation, if the hypotenuse and one side of a right-angled triangle are given, then only a unique triangle can be constructed.

Class Discussion (Consolidation for Congruence Tests)

(a) $A \leftrightarrow A$
$B \leftrightarrow D$
$C \leftrightarrow C$
$AB = AD$ (given)
$BC = DC$ (given)
$AC = AC$ (common side)
$\therefore \triangle ABC \equiv ADC$ (SSS)

(b) $D \leftrightarrow D$
$E \leftrightarrow G$
$F \leftrightarrow F$
$D\hat{E}F = D\hat{G}F$ (given)
$D\hat{F}E = D\hat{F}G = 90°$
$DF = DF$ (common side)
$\therefore \triangle DEF \equiv \triangle DGF$ (AAS)

(c) $P \leftrightarrow P$
$Q \leftrightarrow S$
$R \leftrightarrow R$
$QR = SR$ (given)
$P\hat{R}Q = P\hat{R}S = 90°$
$PR = PR$ (common side)
$\therefore \triangle PQR \equiv \triangle PSR$ (RHS)

(d) $W \leftrightarrow W$
$X \leftrightarrow Z$
$Y \leftrightarrow Y$
$WX = WZ$ (given)
$W\hat{Y}X = W\hat{Y}Z = 90°$
$WY = WY$ (common side)
$\therefore \triangle WXY \equiv WZY$ (SAS)

(e) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$AB = CD$ (given)
$B\hat{A}C = D\hat{C}A$ (corr. $\angle$s, AB // DC)
$AC = CA$ (common side)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

(f) $E \leftrightarrow G$
$F \leftrightarrow H$
$G \leftrightarrow E$
$EF = GH$
$EF = HE$
$EG = GE$ (common side)
$\therefore \triangle EFG \equiv \triangle GHE$ (SSS)

(g) $I \leftrightarrow K$
$J \leftrightarrow L$
$K \leftrightarrow I$
$IJ = KL$ (given)
$JK = LI$ (given)
$IK = KI$ (common side)
$\therefore \triangle IJK \equiv \triangle KLI$ (SSS)

(h) $M \leftrightarrow O$
$N \leftrightarrow P$
$O \leftrightarrow M$
$ON = MP$ (given)
$M\hat{O}N = \mathrm{O}\hat{M}P$ (corr. $\angle$s, ON // PM)
$OM = MO$ (common side)
$\therefore \triangle MNO \equiv \triangle OPM$ (SAS)

Investigation (AA Similarity Test)

2. $\angle ACB = 180° - 50° - 30° = 100°$
$\angle XZY = 180° - 50° - 30° = 100°$
Yes, $\angle ACB = \angle XZY$

3. Yes, $\dfrac{AB}{XY} = \dfrac{BC}{YZ} = \dfrac{AC}{XZ}$

4. Yes, the two triangles are similar.

5. Yes, these given conditions are enough to prove that the two triangles are similar.

Thinking Time (Page 360)

1. If two angles of a triangle are given, the third unknown angle is a unique angle of the triangle. As such, the AAA Similarity Test is not necessary.

2. Yes, two congruent triangles satisfy the AA Similarity Test. Congruence is a special case of similarity.

Investigation (SSS Similarity Test)

3. $\dfrac{DE}{PQ} = \dfrac{2}{4} = \dfrac{1}{2}$

$\dfrac{EF}{QR} = \dfrac{3}{6} = \dfrac{1}{2}$

$\dfrac{DF}{PR} = \dfrac{4}{8} = \dfrac{1}{2}$

Yes, $\dfrac{DE}{PQ} = \dfrac{EF}{QR} = \dfrac{DF}{PR}$

4. $\angle EDF = \angle QPR$

$\angle DEF = \angle PQR$

$\angle DFE = \angle PRQ$

5. Yes, the two triangles are similar.

6. Yes, these given conditions are enough to prove that the two triangles are similar.

Thinking Time (Page 364)

For both the SSS Congruence Test and the SSS Similarity test, the 3 ratios of the corresponding sides of two triangles must be equal. However, for the SSS Congruence Test, the ratio of the corresponding sides of the two triangles must be equal to 1.

Investigation (SAS Similarity Test)

3. $\dfrac{PQ}{AB} = \dfrac{4.5}{3} = 1.5$

$\dfrac{QR}{BC} = \dfrac{7.5}{5} = 1.5$

Yes, $\dfrac{PQ}{AB} = \dfrac{QR}{BC}$

4. Yes, $\dfrac{PR}{AC} = \dfrac{PQ}{AB} = \dfrac{QR}{BC}$

5. $\angle BAC = \angle QPR$ and $\angle ACB = \angle PRQ$

6. Yes, the two triangles are similar.

7. Yes, these given conditions are enough to prove that the two triangles are similar.

Thinking Time (Page 366)

1. For both the SAS Congruence Test and the SAS Similarity test, the 2 ratios of the corresponding sides of two triangles must be equal and the pair of included angles must also be equal. However, for the SAS Congruence Test, the ratio of the corresponding sides of the two triangles must be equal to 1.

2. Since the given conditions for the AA Similarity Test is enough, there is no need for AAS Similarity Test.

3. Yes. For RHS Similarity Test, if the ratio of the hypotenuse and one side of a right-angled triangle is equal to the ratio of the hypotenuse and one side of another right-angled triangle, then the two triangles are similar. However, this test is not included in the syllabus.

Practise Now 1

1. $A \leftrightarrow \underline{E}$

$B \leftrightarrow \underline{F}$

$C \leftrightarrow \underline{D}$

$AB = \underline{EF} = 5$ m

$BC = \underline{FD} = \underline{11}$ m

$AC = \underline{ED}$ (given)

$\therefore \triangle ABC \equiv \triangle \underline{ABC}$ (<u>SSS</u>)

2. $W \leftrightarrow W$

$X \leftrightarrow Z$

$Y \leftrightarrow Y$

$WX = WZ$ (given)

$XY = ZY$ (given)

$WY = WY$

$\therefore \triangle WXY \equiv \triangle WZY$ (SSS)

Worked Example 2

$P \leftrightarrow \underline{G}$

$Q \leftrightarrow \underline{H}$

$R \leftrightarrow \underline{F}$

$PQ = \underline{GH} = 9$ mm

$Q\hat{P}R = \underline{H\hat{G}F} = \underline{40°}$

$PR = \underline{GF} = \underline{12}$ mm

$\therefore \triangle PQR \equiv \triangle \underline{GHF}$ (SAS)

Practise Now 2

1. $P \leftrightarrow S$

$Q \leftrightarrow P$

$R \leftrightarrow T$

$PQ = SP$ (given)

$P\hat{Q}R = S\hat{P}T$ (given)

$QR = PT$ (given)

$\therefore \triangle PQR \equiv \triangle SPT$ (SAS)

2. $A \leftrightarrow X$

$B \leftrightarrow Y$

$C \leftrightarrow Z$

$A\hat{B}C = X\hat{Y}Z = 36°$

$BC = YZ = 12$ cm

However, AB is not equal to XY.

$\therefore \triangle ABC$ is not congruent to $\triangle XYZ$.

Practise Now 3

1. **(i)** $A \leftrightarrow C$

$O \leftrightarrow O$

$B \leftrightarrow D$

$AB = CD$ (given)

$O\hat{A}B = O\hat{D}D = 25°$ (alt. $\angle$s)

$OA = OC$

$\therefore \triangle AOB \equiv \triangle COD$ (SAS)

(ii) Since $\triangle AOB \equiv \triangle COD$, then all corresponding angles are equal.

$B\hat{D}C = A\hat{B}O = 25°$

2. **(i)** $P \leftrightarrow R$
$Q \leftrightarrow S$
$S \leftrightarrow Q$
$PQ = RS$ (given)
$P\hat{Q}S = R\hat{S}Q$ (alt. $\angle$s)
$SQ = QS$ (common side)
$\therefore \triangle PQS \equiv \triangle RSQ$ (SAS)

(ii) Since $\triangle PQS \equiv \triangle RSQ$, then all the corresponding sides and angles are equal.
$QR = PS = 7$ cm
$Q\hat{P}S = S\hat{R}Q = 140°$

Worked Example 4

In $\triangle DEF$, $E\hat{F}D = 180° - 80° - 30°$ ($\angle$ sum of a $\triangle$)
$= 70°$
$A \leftrightarrow \underline{F}$
$B \leftrightarrow \underline{E}$
$C \leftrightarrow \underline{D}$
$A\hat{B}C = \underline{F\hat{E}D} = 80°$
$B\hat{A}C = \underline{E\hat{F}D} = \underline{70°}$
$BC = \underline{ED} = \underline{10}$ mm
$\therefore \triangle ABC \equiv \triangle \underline{FED}$ (AAS)

Practise Now 4

(a) $V \leftrightarrow Z$
$W \leftrightarrow Y$
$X \leftrightarrow X$
$V\hat{W}X = Z\hat{Y}X$ (alt. $\angle$s)
$W\hat{X}V = Y\hat{X}Z$ (vert. opp. $\angle$s)
$WX = YX$ (given)
$\therefore \triangle VWX \equiv \triangle ZYX$ (AAS)

(b) $A \leftrightarrow D$
$B \leftrightarrow C$
$C \leftrightarrow B$
$B\hat{A}C = C\hat{D}B = 35°$
$A\hat{C}B = D\hat{B}C$ (given)
$BC = CB$ (same side)
$\therefore \triangle ABC \equiv \triangle DCB$ (AAS)

Worked Example 5

By Pythagoras' Theorem,

$$ST = \sqrt{TU^2 + SU^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ cm}$$

$P \leftrightarrow \underline{S}$
$Q \leftrightarrow \underline{U}$
$R \leftrightarrow \underline{T}$
$P\hat{Q}R = \underline{S\hat{U}T} = \underline{90°}$
$PR = \underline{ST} = \underline{5}$ cm
$QR = \underline{TU} = \underline{3}$ cm
$\therefore \triangle PQR \equiv \triangle \underline{SUT}$ (RHS)

Practise Now 5

(a) $A \leftrightarrow E$
$B \leftrightarrow D$
$C \leftrightarrow C$
$A\hat{C}B = E\hat{C}D = 90°$
$AB = ED$ (given)
$BC = DC$ (given)
$\therefore \triangle ABC \equiv \triangle EDC$ (RHS)

(b) $X \leftrightarrow Z$
$W \leftrightarrow Y$
$Z \leftrightarrow X$
$X\hat{W}Z = Z\hat{Y}X = 90°$
$WZ = YX$ (given)
$XZ = ZX$ (same side)
$\therefore \triangle XWZ \equiv \triangle ZYX$ (RHS)

Worked Example 6

$A\hat{C}B = A\hat{B}C$ (base $\angle$s of isos. $\triangle$)
$= 70°$
$Y\hat{X}Z = Y\hat{Z}X = \dfrac{180° - 40°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 70°$
$A \leftrightarrow Y$
$B \leftrightarrow \underline{X}$
$C \leftrightarrow \underline{Z}$
$A\hat{B}C = \underline{Y\hat{X}Z} = 70°$
$A\hat{C}B = \underline{Y\hat{Z}X} = \underline{70°}$
$\therefore \triangle ABC$ is similar to $\triangle \underline{YXZ}$ (2 pairs of corr. $\angle$s equal).

Practise Now 6

1. **(a)** $A\hat{B}C = 180° - 60° - 45°$ ($\angle$ sum of a $\triangle$)
$= 75°$
$Y\hat{X}Z = 180° - 60° - 75°$ ($\angle$ sum of a $\triangle$)
$= 45°$
$A \leftrightarrow Y$
$B \leftrightarrow Z$
$C \leftrightarrow X$
$B\hat{A}C = Z\hat{Y}X = 60°$
$A\hat{C}B = Y\hat{X}Z = 45°$
$\therefore \triangle ABC$ is similar to $\triangle YZX$ (2 pairs of corr. $\angle$s equal).

(b) $D\hat{E}F = 180° - 90° - 30°$ (∠ sum of △)
$= 60°$
$Q\hat{P}R = 180° - 90° - 50°$ (∠ sum of a △)
$= 40°$
$D \leftrightarrow P$
$E \leftrightarrow Q$
$F \leftrightarrow R$
Since there are no corresponding pairs of angles that are equal, the two triangles are not similar.

(c) $R \leftrightarrow V$
$S \leftrightarrow U$
$T \leftrightarrow T$
$S\hat{T}R = U\hat{T}V$ (vert. opp. ∠s)
$R\hat{S}T = V\hat{U}T$ (alt. ∠s)
∴ △RST is similar to △VUT (2 pairs of corr. ∠s equal).

(d) $K \leftrightarrow K$
$L \leftrightarrow N$
$M \leftrightarrow P$
$L\hat{K}M = N\hat{K}P$ (common angle)
$K\hat{L}M = K\hat{N}P$ (corr. ∠s, LM // NP)
∴ △KLM is similar to △KNP (2 pairs of corr. ∠s equal).

2. **(i)** $A \leftrightarrow A$
$B \leftrightarrow D$
$C \leftrightarrow E$
$B\hat{A}C = D\hat{A}E$ (common angle)
$A\hat{B}C = A\hat{D}E$ (corr. ∠s, BC // DE)
∴ △ABC is similar to △ADE (2 pairs of corr. ∠s equal).

(ii) Since △ABC is similar to △ADE,
$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$
$$\frac{7}{DE} = \frac{8}{12}$$
$8DE = 84$
$DE = 10.5$ cm
$$\frac{6}{AD} = \frac{8}{12}$$
$8AD = 72$
$AD = 9$ cm
$BD = 9 - 6 = 3$ cm

(iii) $\frac{AB}{AD} = \frac{6}{3} = 2$
$$\frac{AC}{CE} = \frac{8}{12-8} = \frac{8}{4} = 2$$
$$\therefore \frac{AB}{BD} = \frac{AC}{CE}$$

Worked Example 7

$A \leftrightarrow T$
$B \leftrightarrow \underline{U}$
$C \leftrightarrow \underline{S}$
$$\frac{AB}{TU} = \frac{7.5}{3} = 2.5$$
$$\frac{AC}{TS} = \frac{10}{4} = \underline{2.5}$$
$$\frac{BC}{US} = \frac{15}{6} = \underline{2.5}$$
∴ △ABC is similar to △$\underline{TUS}$ (3 ratios of corr. ∠s equal).

Practise Now 7

(a) $A \leftrightarrow Z$
$B \leftrightarrow Y$
$C \leftrightarrow X$
$$\frac{AB}{ZY} = \frac{5}{7.5} = \frac{2}{3}$$
$$\frac{AC}{ZX} = \frac{6}{9} = \frac{2}{3}$$
$$\frac{BC}{YX} = \frac{8}{12} = \frac{2}{3}$$
∴ △ABC is similar to △ZYX (3 ratios of corr. ∠s equal).

(b) $P \leftrightarrow U$
$Q \leftrightarrow S$
$R \leftrightarrow T$
$$\frac{PQ}{US} = \frac{5}{1.5} = 3\frac{1}{3}$$
$$\frac{PR}{UT} = \frac{3}{1} = 3$$
$$\frac{QR}{ST} = \frac{6}{2} = 3$$
∴ Since the 3 ratios of corresponding angles are not equal, the two triangles are not similar.

Worked Example 8

$D \leftrightarrow \underline{D}$
$E \leftrightarrow \underline{G}$
$F \leftrightarrow \underline{H}$
$E\hat{D}F = \underline{G\hat{D}H}$ (common angle)
$$\frac{DE}{DG} = \frac{1}{2}$$
$$\frac{DF}{DH} = \frac{1}{\underline{2}}$$
$$\therefore \frac{DE}{DG} = \frac{DF}{DH}$$
∴ △DEF is similar to △$\underline{DGH}$ (2 ratios of corr. sides and included ∠ equal).

Practise Now 8

(a) $J \leftrightarrow N$

$K \leftrightarrow M$

$L \leftrightarrow L$

$J\hat{L}K = N\hat{L}M$ (vert. opp $\angle$s)

$\frac{JL}{NL} = \frac{3}{5.4} = \frac{5}{9}$

$\frac{KL}{ML} = \frac{4}{7.2} = \frac{5}{9}$

$\therefore \frac{JL}{NL} = \frac{KL}{ML}$

$\therefore \triangle JKL$ is similar to $\triangle NML$ (2 ratios of corr. sides and included $\angle$ equal).

(b) $A \leftrightarrow Z$

$B \leftrightarrow Y$

$C \leftrightarrow X$

$A\hat{C}B = Z\hat{X}Y = 75°$

$\frac{AC}{ZX} = \frac{15}{10} = 1.5$

$\frac{BC}{YX} = \frac{10}{5} = 2$

$\therefore$ Since the 2 ratios of corresponding sides are not equal, the two triangles are not similar.

Practise Now 9

1. (i) $A \leftrightarrow E$

$B \leftrightarrow C$

$C \leftrightarrow G$

$AC = EG$ (given)

$A\hat{C}B = E\hat{G}C$ (corr. $\angle$s, BC // EG)

$BC = CG$ (given)

$\therefore \triangle ABC \equiv \triangle ECG$ (SAS)

(ii) $A \leftrightarrow F$

$C \leftrightarrow F$

$D \leftrightarrow D$

$A\hat{D}C = E\hat{D}F$ (vert. opp. $\angle$s)

Since $\triangle ABC$ and $\triangle ECG$ are congruent,

then $B\hat{A}C = C\hat{E}G$,

i.e. $D\hat{A}C = D\hat{E}F$.

$\therefore \triangle ACD$ is similar to $\triangle EFD$ (2 pairs of corr. $\angle$s equal).

(iii) Since $AC = EF = 15$ cm, then $EF = 15 - 9 = 6$ cm

Since $\triangle ACD$ and $\triangle EFD$ are similar, then

$$\frac{DF}{DC} = \frac{EF}{AC}$$

i.e. $\frac{DF}{5} = \frac{6}{15}$

$$\therefore DF = \frac{6}{15} \times 5 = 2 \text{ cm}$$

2. $A \leftrightarrow X$

$X \leftrightarrow B$

$P \leftrightarrow Q$

$A\hat{X}P = X\hat{B}Q$ (corr. $\angle$s, XP // BC)

$A\hat{P}X = X\hat{Q}B$ (corr. $\angle$s, XQ // AC)

$\therefore \triangle AXP$ is similar to $\triangle XBQ$ (2 pairs of corr. $\angle$s equal).

Since $\triangle AXP$ and $\triangle XBQ$ are similar, then

$$\frac{BQ}{XP} = \frac{XB}{AX}$$

i.e. $\frac{BQ}{15} = \frac{4}{3}$

$$\therefore BQ = \frac{4}{3} \times 15 = 20 \text{ cm}$$

$A \leftrightarrow X$

$B \leftrightarrow B$

$C \leftrightarrow Q$

$A\hat{B}C = X\hat{B}Q$ (common angle)

$A\hat{C}B = X\hat{Q}B$ (corr. $\angle$s, XP // BC)

$\therefore \triangle ABC$ is similar to $\triangle XBQ$ (2 pairs of corr. $\angle$s equal)

Since $\triangle ABC$ and $\triangle XBQ$ are similar, then

$$\frac{AC}{XQ} = \frac{AB}{XB}$$

i.e. $\frac{AC}{16} = \frac{7}{4}$

$$\therefore AC = \frac{7}{4} \times 16 = 28 \text{ cm}$$

Practise Now 10

$P \leftrightarrow P$

$A \leftrightarrow B$

$Q \leftrightarrow Q$

$AP = BP$

$AQ = BQ$

$PQ = PQ$ (common side)

$\therefore \triangle PAQ$ and $\triangle PBQ$ are congruent (SSS Congruence Test).

$\therefore A\hat{P}Q = B\hat{P}Q$

i.e. $A\hat{P}R = B\hat{P}R$

Since $AP = BP$ and PR is a common side, $\triangle PAR$ and $\triangle PBR$ are congruent (SAS Congruence Test).

$\therefore AR = RB$

$$A\hat{R}P = B\hat{R}P = \frac{180°}{2} \text{ (adj. } \angle\text{s on a str. line)} = 90°$$

$\therefore PQ$ is the perpendicular bisector of AB.

Practise Now 11

1. $A \leftrightarrow A$
   $B \leftrightarrow D$
   $C \leftrightarrow E$
   $$\frac{BC}{DE} = \frac{AC}{AE}$$
   i.e. $\frac{BC}{11.2} = \frac{10}{2}$
   $\therefore BC = \frac{10}{2} \times 11.2$
   $= 56$ m

2. $C \leftrightarrow A$
   $D \leftrightarrow B$
   $E \leftrightarrow E$
   $$\frac{AB}{CD} = \frac{BE}{DE}$$
   i.e. $\frac{AB}{1.4} = \frac{18}{2.1}$
   $\therefore AB = \frac{18}{2.1} \times 1.4$
   $= 12$ m

Exercise 11A

1. **(a)** Comparing triangle **(ii)** and triangle **(vii)**,
   The 3 sides of triangle **(ii)** are equal to the 3 corresponding sides of triangle **(vii)**.
   $\therefore$ The two triangles are congruent (SSS).
   (b) Comparing triangle **(iii)** and triangle **(v)**,
   The 2 sides and the included angle of triangle **(iii)** are equal to the 2 corresponding sides and the corresponding included angle of triangle **(v)**.
   $\therefore$ The two triangles are congruent (SAS).
   (c) Comparing triangle **(i)** and triangle **(ix)**,
   The 2 angles and 1 side of triangle **(i)** are equal to the 2 corresponding angles and the corresponding side of triangle **(ix)**.
   $\therefore$ The two triangles are congruent (AAS).
   (d) Comparing triangle **(vi)** and triangle **(viii)**,
   The hypotenuse and 1 side of triangle **(vi)** are equal to the hypotenuse and 1 side of triangle **(viii)**.
   $\therefore$ The two triangles are congruent (RHS).

2. **(a)** $A \leftrightarrow \underline{P}$
   $B \leftrightarrow \underline{Q}$
   $B \leftrightarrow \underline{R}$
   $AB = \underline{PQ}$ (given)
   $BC = \underline{QR} = 8$ cm
   $AC = \underline{PR} = 6$ cm
   $\therefore \triangle ABC \equiv \triangle \underline{PQR}$ ($\underline{\text{SSS}}$)
   (b) $D \leftrightarrow \underline{Z}$
   $E \leftrightarrow \underline{Y}$
   $F \leftrightarrow \underline{X}$
   $DE = \underline{ZY} = 3$ m
   $D\hat{E}F = \underline{Z\hat{Y}X} = \underline{70°}$
   $EF = \underline{YX} = \underline{5}$ m
   $\therefore \triangle DEF \equiv \triangle \underline{ZYX}$ ($\underline{\text{SAS}}$)
   (c) $L \leftrightarrow \underline{W}$
   $M \leftrightarrow \underline{V}$
   $N \leftrightarrow \underline{U}$
   $L\hat{M}N = \underline{W\hat{V}U} = \underline{30°}$
   $L\hat{N}M = \underline{W\hat{U}V} = \underline{70°}$
   $MN = \underline{VU} = \underline{7}$ cm
   $\therefore \triangle \underline{NML} \equiv \triangle UVW$ ($\underline{\text{AAS}}$)
   (d) $G \leftrightarrow \underline{U}$
   $H \leftrightarrow \underline{T}$
   $I \leftrightarrow \underline{S}$
   $G\hat{H}I = \underline{U\hat{T}S} = \underline{90°}$
   $GI = \underline{US} = 13$ mm
   $HI = \underline{TS} = \underline{5}$ mm
   $\therefore \triangle \underline{IHG} \equiv \triangle STU$ ($\underline{\text{RHS}}$)

3. **(a)** $A \leftrightarrow E$
   $B \leftrightarrow D$
   $C \leftrightarrow F$
   The 3 sides of $\triangle ABC$ are not equal to the 3 corresponding sides of $\triangle EDF$.
   $\therefore \triangle ABC$ is not congruent to $\triangle EDF$.
   (b) $X \leftrightarrow Q$
   $Y \leftrightarrow R$
   $Z \leftrightarrow P$
   $X\hat{Z}P = Q\hat{P}R = 40°$
   $YZ = RP = 6$ mm
   XZ is not equal to QP.
   $\therefore \triangle XYZ$ is not congruent to $\triangle QRP$.
   (c) $G \leftrightarrow U$
   $H \leftrightarrow T$
   $I \leftrightarrow S$
   $H\hat{G}I = 180° - 75° - 40°$ ($\angle$ sum of a $\triangle$)
   $= 65°$
   $T\hat{S}U = 180° - 55° - 40°$ ($\angle$ sum of a $\triangle$)
   $= 85°$
   $HI = ST = 5$ cm
   $G\hat{H}I = U\hat{T}S = 40°$
   $G\hat{I}H$ is not equal to $U\hat{S}T$ and $H\hat{G}I$ is not equal to $T\hat{U}S$.
   $\therefore \triangle GHI$ is not congruent to $\triangle UTS$.

(d) $M \leftrightarrow P$
$N \leftrightarrow Q$
$O \leftrightarrow R$
By Pythagoras' Theorem,
$PQ = \sqrt{12^2 - 5^2}$
$= 10.91$ cm (to 4 s.f.)
$M\hat{N}O = P\hat{Q}R = 90°$
$NO = QR = 5$ cm
MN is not equal to PQ and OM is not equal to RP.
$\therefore \triangle MNO$ is not congruent to $\triangle PQR$.

4. (a) $A \leftrightarrow C$
$B \leftrightarrow B$
$D \leftrightarrow D$
$AB = CB$ (given)
$AD = CD$ (given)
$BD = BD$ (common side)
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)

(b) $A \leftrightarrow C$
$B \leftrightarrow D$
$D \leftrightarrow B$
$AB = CD$ (given)
$AD = CB$ (given)
$BD = DB$ (common side)
$\therefore \triangle ABD \equiv \triangle CDB$ (SSS)

(c) $A \leftrightarrow E$
$B \leftrightarrow D$
$C \leftrightarrow C$
$AC = EC$ (given)
$CB = CD$ (given)
$A\hat{C}B = E\hat{C}D$ (vert. opp. $\angle$s)
$\therefore \triangle ABC \equiv \triangle EDC$ (SAS)

(d) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$BC = DA$ (given)
$AC = CA$ (common side)
$B\hat{C}A = D\hat{C}A$ (alt. $\angle$s)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

(e) $A \leftrightarrow C$
$D \leftrightarrow D$
$E \leftrightarrow B$
$AE = CB$ (given)
$A\hat{E}D = C\hat{B}D$ (given)
$E\hat{A}D = B\hat{C}D$ (given)
$\therefore \triangle ADE \equiv \triangle CDB$ (AAS)

(f) $B \leftrightarrow E$
$C \leftrightarrow F$
$D \leftrightarrow D$
$BC = EF$ (given)
$C\hat{B}D = F\hat{E}D$ (given)
$B\hat{D}C = E\hat{D}F$ (vert. opp. $\angle$s)
$\therefore \triangle BCD \equiv \triangle EFD$ (AAS)

(g) $A \leftrightarrow C$
$B \leftrightarrow B$
$D \leftrightarrow D$
$AD = CD$ (given)
$AB = CB$ (given)
$BD = BD$ (common side)
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)

(h) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$BC = DA$ (given)
$AC = CA$ (common side)
$A\hat{B}C = C\hat{D}A = 90°$
$\therefore \triangle ABC \equiv \triangle CDA$ (RHS)

5. (i) $R \leftrightarrow V$
$S \leftrightarrow U$
$T \leftrightarrow T$
$RT = VT$ (given)
$ST = UT$ (given)
$R\hat{T}S = V\hat{T}U$ (vert. opp. $\angle$)
$\therefore \triangle RST \equiv \triangle VUT$ (SAS)

(ii) Since $\triangle RST \equiv \triangle VUT$,
$UV = SR = 4$ cm

(iii) Since $\triangle RST \equiv \triangle VUT$,
$U\hat{V}T = S\hat{R}T = 80°$

(iv) RS is parallel to UV.

6. (i) $J \leftrightarrow G$
$I \leftrightarrow H$
$H \leftrightarrow I$
$JI = GH$ (given)
$JH = GI$ (given)
$IH = HI$ (common side)
$\therefore \triangle JIH \equiv \triangle GHI$ (SSS)

(ii) Since $\triangle JIH \equiv \triangle GHI$,
$I\hat{G}H = H\hat{J}I = 60°$
$G\hat{H}I = 180° - 60° - 40°$ ($\angle$ sum of a $\triangle$)
$= 80°$

7. (a) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$BC = DA$ (given)
$AC = CA$ (common side)
$B\hat{C}A = D\hat{A}C$ (alt. $\angle$s)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

(b) $E \leftrightarrow G$
$F \leftrightarrow H$
$G \leftrightarrow E$
$GF = EH$ (given)
$EG = GE$ (common side)
$E\hat{F}G = G\hat{H}E = 90°$
$\therefore \triangle EFG \equiv \triangle GHE$ (RHS)

(c) $I \leftrightarrow K$

$J \leftrightarrow L$

$K \leftrightarrow I$

$IJ = KL$ (given)

$JK = LI$ (given)

$IK = KI$ (common side)

$\therefore \triangle IJK \equiv \triangle KLE$ (SSS)

(d) $M \leftrightarrow O$

$N \leftrightarrow P$

$O \leftrightarrow M$

$MO = OM$ (common side)

$M\hat{N}O = O\hat{P}M = 90°$

$M\hat{O}N = O\hat{M}P$ (alt. $\angle$s)

$\therefore \triangle MNO \equiv \triangle OPM$ (AAS)

(e) $Q \leftrightarrow S$

$R \leftrightarrow T$

$S \leftrightarrow Q$

$QS = SQ$ (common side)

$R\hat{Q}S = T\hat{S}Q$ (alt. $\angle$s)

$Q\hat{S}R = S\hat{Q}T$ (alt. $\angle$s)

$\therefore \triangle QRS \equiv \triangle STQ$ (AAS)

(f) $U \leftrightarrow Q$

$V \leftrightarrow X$

$W \leftrightarrow U$

$VW = XU$ (given)

$UV = QX$ (given)

$U\hat{V}W = Q\hat{X}U = 90°$

$\therefore \triangle UVW \equiv \triangle QXU$ (RHS)

8. $A \leftrightarrow C$

$B \leftrightarrow D$

$C \leftrightarrow A$

$AB = CD$

$BC = DA$

$AC = CA$ (common side)

$\therefore \triangle ABC \equiv \triangle CDA$ (SSS)

$B\hat{A}C = D\hat{C}A$ (alt. $\angle$s)

$A\hat{C}B = C\hat{A}D$ (alt. $\angle$s)

$AC = CA$ (common side)

$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)

$BC = DA$

$AC = CA$ (common side)

$B\hat{A}C = D\hat{C}A$ (alt. $\angle$s)

$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

Exercise 11B

1. **(a)** Comparing triangle **(i)** and triangle **(iii)**,

The 2 angles of triangle **(i)** are equal to the 2 corresponding angles of triangle **(iii)**.

$\therefore$ The two triangles are similar (2 pairs of corr. $\angle$s equal).

Comparing triangle **(v)** and triangle **(vii)**,

The 2 angles of triangle **(v)** are equal to the 2 corresponding angles of triangle **(vii)**.

$\therefore$ The two triangles are similar (2 pairs of corr. $\angle$s equal).

(b) Comparing triangle **(ii)** and triangle **(vi)**,

$\frac{12}{6} = 2$

$\frac{8}{4} = 2$

$\frac{7.8}{3.9} = 2$

The 3 ratios of the corresponding sides of triangle **(ii)** and triangle **(vi)** are equal.

$\therefore$ The two triangles are similar (3 ratios of corr. sides equal).

(c) Comparing triangle **(iv)** and triangle **(viii)**,

$\frac{24}{6} = 4$

$\frac{18}{4.5} = 4$

The ratios of the corresponding sides of triangle **(iv)** and triangle **(viii)** are equal and the pair of included angles are also equal.

$\therefore$ The two triangles are similar (2 ratios of corr. sides and included $\angle$ equal).

2. **(a)** $S\hat{T}U = 180° - 70° - 50°$ ($\angle$sum of a $\triangle$)

$= \underline{60°}$

$A \leftrightarrow \underline{S}$

$B \leftrightarrow \underline{T}$

$C \leftrightarrow \underline{U}$

$B\hat{A}C = \underline{T\hat{S}U} = 70°$

$A\hat{B}C = \underline{S\hat{T}U} = \underline{60°}$

$\therefore \triangle ABC$ is similar to $\triangle \underline{STU}$ (<u>2</u> pairs of corr. <u>$\angle$s</u> equal)

(b) $X \leftrightarrow \underline{N}$

$Y \leftrightarrow \underline{M}$

$Z \leftrightarrow \underline{L}$

$\frac{XY}{NM} = \frac{24}{\underline{8}} = \underline{3}$

$\frac{XZ}{NL} = \frac{21}{\underline{7}} = \underline{3}$

$\frac{YZ}{ML} = \frac{15}{\underline{5}} = \underline{3}$

$\therefore \triangle XYZ$ is similar to $\triangle \underline{NML}$ (<u>3</u> ratios of corr. <u>sides</u> equal)

(c) $D \leftrightarrow \underline{G}$

$E \leftrightarrow \underline{I}$

$F \leftrightarrow \underline{H}$

$D\hat{E}F = \underline{G\hat{I}H} = \underline{90°}$

$\frac{DE}{GI} = \frac{6}{\underline{9}} = \frac{\underline{2}}{\underline{3}}$

$\frac{EF}{IH} = \frac{4}{\underline{6}} = \frac{\underline{2}}{\underline{3}}$

$\frac{DE}{GI} = \frac{\underline{EF}}{\underline{IH}}$

$\therefore \triangle DEF$ is similar to $\triangle \underline{GIH}$ (<u>2</u> ratios of corr. <u>sides</u> and <u>included</u> $\angle$ equal).

3. (a) In the smaller triangle,

$180° - 60° - 60° = 60°$

In the larger triangle,

$\frac{180° - 50°}{2} = 65°$

Two angles of the smaller triangle are not equal to the two corresponding angles of the larger triangle.

$\therefore$ The two triangles are not similar.

(b) $\frac{22.5}{45} = 0.5$

$\frac{15}{30} = 0.5$

$\frac{8}{15} = 0.5333$ (to 4 s.f.)

The 3 ratios of corresponding sides of both triangles are not equal.

$\therefore$ The two triangles are not similar.

(c) Included angle = 110°

$\frac{10}{25} = 0.4$

$\frac{7}{15} = 0.4667$ (to 4 s.f.)

The ratios of the corresponding sides of both triangles are not equal.

$\therefore$ The two triangles are not similar.

4. (a) $A \leftrightarrow E$

$B \leftrightarrow D$

$C \leftrightarrow C$

$A\hat{B}C = E\hat{D}C$ (alt. $\angle$s)

$A\hat{C}B = E\hat{C}D$ (vert. opp. $\angle$s)

$\therefore$ $\triangle ABC$ is similar to $\triangle EDC$ (2 pairs of corr. $\angle$s equal).

(b) $I \leftrightarrow I$

$J \leftrightarrow F$

$H \leftrightarrow G$

$I\hat{J}H = I\hat{F}G$ (corr. $\angle$s, JH // FG)

$J\hat{I}H = F\hat{I}G$ (common angle)

$\therefore$ $\triangle IJH$ is similar to $\triangle IFG$ (2 pairs of corr. $\angle$s equal).

(c) $P \leftrightarrow T$

$Q \leftrightarrow S$

$R \leftrightarrow R$

$P\hat{R}Q = T\hat{R}S$ (vert. opp. $\angle$s)

$\frac{PR}{TR} = \frac{3}{6} = 0.5$

$\frac{QR}{SR} = \frac{4}{8} = 0.5$

$\therefore$ $\triangle PQR$ is similar to $\triangle TSR$ (2 ratios of corr. sides and included $\angle$ equal).

(d) $U \leftrightarrow U$

$V \leftrightarrow X$

$W \leftrightarrow Y$

$V\hat{U}W = X\hat{U}Y$ (common angle)

$\frac{UV}{UX} = \frac{8}{8+4} = \frac{8}{12} = \frac{2}{3}$

$\frac{UW}{UY} = \frac{10}{10+5} = \frac{10}{15} = \frac{2}{3}$

$\therefore$ $\triangle UVW$ is similar to $\triangle UXY$ (2 ratios of corr. sides and included $\angle$ equal).

5. (a) $A \leftrightarrow A$

$B \leftrightarrow D$

$C \leftrightarrow E$

$A\hat{C}B = A\hat{E}D$ (corr. $\angle$s, BC // DE)

$B\hat{A}C = D\hat{A}C$ (common angle)

$\therefore$ $\triangle ABC$ is similar to $\triangle ADE$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ABC$ is similar to $\triangle ADE$, then

$$\frac{BC}{DE} = \frac{AC}{AE}$$

i.e. $\frac{x}{12} = \frac{12}{12+4}$

$$\therefore x = \frac{12}{16} \times 12$$
$$= 9$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

i.e. $\frac{y}{y+6} = \frac{12}{16}$

$$\therefore \frac{y}{y+6} = \frac{3}{4}$$
$$4y = 3(y+6)$$
$$4y = 3y + 8$$
$$y = 18$$

(b) $A \leftrightarrow E$

$B \leftrightarrow D$

$C \leftrightarrow C$

$B\hat{A}C = D\hat{E}C$ (corr. $\angle$s, AB // DE)

$A\hat{C}B = E\hat{C}D$ (common angle)

$\therefore$ $\triangle ABC$ is similar to $\triangle EDC$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ABC$ is similar to $\triangle EDC$, then

$$\frac{AC}{EC} = \frac{BC}{DC}$$

i.e. $\frac{x}{4} = \frac{6}{5}$

$$\therefore x = \frac{6}{5} \times 4$$
$$= 4.8$$

$$\frac{DE}{BA} = \frac{DC}{BC}$$

i.e. $\frac{y}{9} = \frac{5}{6}$

$$\therefore y = \frac{5}{6} \times 9$$
$$= 7.5$$

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(c) $A \leftrightarrow A$

$B \leftrightarrow E$

$C \leftrightarrow D$

$A\hat{B}C = A\hat{E}D$ (given)

$B\hat{A}C = E\hat{A}D$ (common angle)

$\therefore \triangle ABC$ is similar to $\triangle AED$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ABC$ is similar to $\triangle AED$, then

$$\frac{DE}{CB} = \frac{AE}{AB}$$

i.e. $\frac{x}{4} = \frac{6+3}{3}$

$\therefore x = \frac{9}{3} \times 4$

$= 12$

$$\frac{AB}{AE} = \frac{AC}{AD}$$

i.e. $\frac{3}{9} = \frac{6}{3+y}$

$\therefore 3(3 + y) = 54$

$3 + y = 18$

$y = 15$

(d) $A \leftrightarrow A$

$B \leftrightarrow E$

$C \leftrightarrow F$

$A\hat{B}C = A\hat{E}F$ (corr. $\angle$s, BC // EF)

$B\hat{A}C = E\hat{A}F$ (common angle)

$\therefore \triangle ABC$ is similar to $\triangle AEF$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ABC$ is similar to $\triangle AEF$, then

$$\frac{AC}{AF} = \frac{AB}{AE}$$

i.e. $\frac{8}{8+x} = \frac{10}{10+4}$

$\therefore \frac{8}{8+x} = \frac{10}{14}$

$\frac{8}{8+x} = \frac{5}{7}$

$56 = 40 + 5x$

$5x = 16$

$x = 3.2$

$A \leftrightarrow A$

$C \leftrightarrow F$

$D \leftrightarrow G$

$A\hat{C}D = A\hat{F}G$ (corr. $\angle$s, CD // FG)

$C\hat{A}D = F\hat{A}G$ (common angle)

$\therefore \triangle ACD$ is similar to $\triangle AFG$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ACD$ is similar to $\triangle AFG$, then

$$\frac{AD}{AG} = \frac{AC}{AF}$$

i.e. $\frac{y}{y+3} = \frac{8}{8+3.2}$

$\therefore \frac{y}{y+3} = \frac{8}{11.2}$

$11.2y = (8y + 3)$

$11.2y = 8y + 24$

$3.2y = 24$

$y = 7.5$

6. **(i)** $A \leftrightarrow D$

$B \leftrightarrow C$

$D \leftrightarrow B$

$\frac{AB}{DC} = \frac{5}{15} = \frac{1}{3}$

$\frac{BD}{CB} = \frac{6}{18} = \frac{1}{3}$

$\frac{AD}{DB} = \frac{2}{6} = \frac{1}{3}$

$\therefore \triangle ABD$ is similar to $\triangle DCB$ (3 ratios of corr. sides equal).

(ii) Since $\triangle ACD$ is similar to $\triangle AFG$, then

$D\hat{A}B = C\hat{D}B = 110.5°$

7. $\frac{3}{5}XY = 18$ cm

$XY = 30$ cm

$UY = \frac{2}{5} \times 30 = 12$ cm

$\triangle YUV$ is similar to $\triangle YXZ$ (2 pairs of corr. $\angle$s equal).

Since $\triangle YUV$ is similar to $\triangle YXZ$, then

$$\frac{XZ}{UV} = \frac{XY}{UY}$$

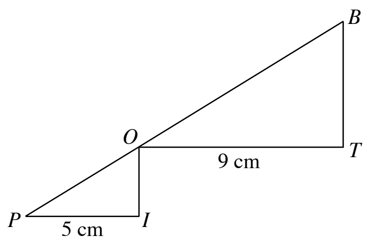
i.e. $\frac{18+WZ}{18} = \frac{30}{12}$

$\therefore \frac{18+WZ}{18} = \frac{5}{2}$

$18 + WZ = 45$

$WZ = 27$ cm

8.



$\triangle POI$ is similar to $\triangle OBT$.

$OI = 9 - 5 = 4$ cm

$$\frac{BT}{OI} = \frac{OT}{PI}$$

i.e. $\frac{BT}{4} = \frac{9}{5}$

$\therefore BT = \frac{9}{5} \times 4$

$= 7.2$ cm

Length side of square $BLUE$ $= 9 + 7.2$

$= 16.2$ cm

9. **(i)** $B \leftrightarrow D$

$A \leftrightarrow B$

$C \leftrightarrow C$

$A\hat{B}C = B\hat{D}C = 90°$

$B\hat{C} = D\hat{B}C$ (common angle)

$\therefore$ $\triangle BAC$ is similar to $\triangle DBC$ (2 pairs of corr. $\angle$s equal).

$B \leftrightarrow D$

$A \leftrightarrow A$

$C \leftrightarrow B$

$A\hat{B}C = A\hat{D}B = 90°$

$B\hat{A}C = D\hat{A}B$ (common angle)

$\therefore$ $\triangle BAC$ is similar to $\triangle DAB$ (2 pairs of corr. $\angle$s equal).

Hence $\triangle BAC$, $\triangle DBC$ and $\triangle DAB$ are similar.

(ii) By Pythagoras' Theorem,

$$AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ m}$$

Since $\triangle BAC$, $\triangle DBC$ and $\triangle DAB$ are similar, then

$$\frac{BC}{DB} = \frac{AB}{AD}$$

i.e. $$\frac{BC}{4} = \frac{5}{3}$$

$$\therefore BC = \frac{5}{3} \times 4 = 6\frac{2}{3} \text{ m}$$

By Pythagoras' Theorem,

$$CD = \sqrt{\left(6\frac{2}{3}\right)^2 - 4^2} = \sqrt{\frac{256}{9}} = 5\frac{1}{3} \text{ m}$$

10. **(i)** $\triangle QTU$ is similar to $\triangle QRS$ (2 pairs of corr. $\angle$s equal).

$$\frac{QT}{QR} = \frac{QU}{QS}$$

i.e. $$\frac{12 + RT}{12} = \frac{8 + 4}{8}$$

$$\therefore \frac{12 + RT}{12} = \frac{3}{2}$$

$$2(12 + RT) = 36$$
$$24 + 2RT = 36$$
$$2RT = 12$$
$$RT = 6 \text{ cm}$$

$\triangle QPU$ is similar to $\triangle SRU$ (2 pairs of corr. $\angle$s equal).

$$\frac{PU}{RU} = \frac{QU}{SU}$$

i.e. $$\frac{PR + 6}{6} = \frac{8 + 4}{4}$$

$$\therefore \frac{PR + 6}{6} = 3$$

$$PR + 6 = 18$$
$$PR = 12 \text{ cm}$$

(ii) $\triangle QPR$ is similar to $\triangle TUR$ (2 pairs of corr. $\angle$s equal).

$$\frac{PQ}{TU} = \frac{QR}{RT}$$

$$\therefore \frac{PQ}{TU} = \frac{12}{6} = 2$$

$\therefore$ The ratio $PQ : TU$ is $2 : 1$.

11. **(i)** Since A coincides with T when the triangle is folded, ANT is a straight line.

Hence MN is perpendicular to AT.

(ii) $A \leftrightarrow M$

$R \leftrightarrow N$

$T \leftrightarrow T$

$A\hat{R}T = M\hat{N}T = 90°$

$A\hat{T}R = M\hat{T}N$ (common angle)

$\therefore$ $\triangle ART$ is similar to $\triangle MNT$ (2 pairs of corr. $\angle$s equal).

(iii) By Pythagoras' Theorem,

$$AT = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ cm}$$

$$NT = \frac{1}{2} \times 10 = 5 \text{ cm}$$

Since $\triangle ART$ is similar to $\triangle MNT$, then

$$\frac{MN}{AR} = \frac{NT}{RT}$$

i.e. $$\frac{MN}{6} = \frac{5}{8}$$

$$MN = \frac{5}{8} \times 6 = 3.75 \text{ cm}$$

12. $A \leftrightarrow J$

$J \leftrightarrow A$

$D \leftrightarrow E$

Since $AO = JO$ and $EO = DO$,

$JD = AE$

Since $AO = JO$, $EO = DO$ and $J\hat{O}E = A\hat{O}E$ (vert opp. $\angle$s),

$AD = JE$

$AJ = JA$ (same side)

$\therefore$ $\triangle AJD \equiv \triangle JAE$ (SSS)

Exercise 11C

1. $A \leftrightarrow A'$

$B \leftrightarrow B'$

$C \leftrightarrow C$

$AC = A'C$ (given)

$BC = B'C$ (given)

$A\hat{C}B = A'\hat{C}B'$ (vert. opp. $\angle$s)

$\therefore$ $\triangle ABC \equiv \triangle A'B'C$ (SAS)

Hence, $AB = B'A'$.

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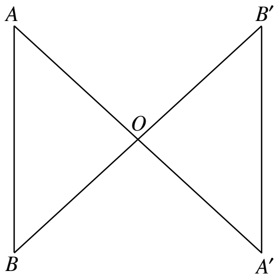
2. 

$A \leftrightarrow A'$
$O \leftrightarrow O$
$B \leftrightarrow B'$
$A\hat{O}B = A'\hat{O}B'$ (vert. opp. ∠s)
Since AA' and BB' are hinged halfway at O,
$OA = OA'$ and $OB = OB'$
$\therefore \triangle AOB \equiv \triangle A'OB'$ (SAS)
Hence, $AB = A'B'$.

3. $\triangle SOR$ is similar to $\triangle POQ$.
$$\frac{SR}{PQ} = \frac{OR}{OQ}$$
i.e. $\frac{SR}{4} = \frac{30}{15}$
$$\therefore SR = \frac{30}{15} \times 4 = 8 \text{ m}$$
Hence the height of the tree is 8 m.

4. $P \leftrightarrow Q$
$O \leftrightarrow O$
$C \leftrightarrow C$
$PO = QO$ (given)
$OC = OC$ (common side)
$O\hat{C}P = O\hat{C}Q = 90°$
$\therefore \triangle POC$ and $\triangle QOC$ are congruent (RHS Congruence Test).
Hence OC is the angle bisector of $A\hat{O}B$.

5. $A \leftrightarrow A$
$B \leftrightarrow B'$
$C \leftrightarrow C$
$AC = AC$ (common side)
$B\hat{C}A = B'\hat{C}A$ (same line of vision)
$BC = B'C$
$\therefore \triangle AOB \equiv \triangle A'OB'$ (SAS)
Hence $AB = AB'$.

6. Given that the image of the candle is 3 times the length of the candle,
$$\frac{AC}{DE} = \frac{1}{3}$$
$\triangle ABC$ is similar to $\triangle DBE$.
$$\frac{BD}{BA} = \frac{DE}{AC}$$
i.e. $\frac{x}{15} = \frac{3}{1}$
$\therefore x = 45$

7. $P \leftrightarrow Q$
$O \leftrightarrow O$
$M \leftrightarrow M$
$OP = OQ$ (given)
$OM = OM$ (common side)
$P\hat{M}Q = 90°$
$PM = QM$
$\therefore \triangle POM$ and $\triangle QOM$ are congruent (SSS Congruence Test).
Hence OM is the angle bisector of $A\hat{O}B$.

Review Exercise 11

1. **(a)** $A \leftrightarrow R$
$B \leftrightarrow P$
$C \leftrightarrow Q$
$AB = RP = 5$ m
$AC = RQ = 7$ m
$BC = PQ = 6$ m
$\therefore \triangle ABC \equiv \triangle RPQ$ (SSS)

(b) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$AB = PQ = 12$ cm
$AC = PR = 8.9$ cm
$B\hat{A}C = Q\hat{P}R = 80°$
$\therefore \triangle ABC \equiv \triangle PQR$ (SAS)

(c) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$BC = QR = 9$ cm
$A\hat{B}C = P\hat{Q}R = 30°$
AB is not equal to PQ.
$\therefore \triangle ABC$ and $\triangle PQR$ are not congruent.

(d) $P\hat{Q}R = 180° - 75° - 45°$ (∠ sum of a △)
$= 60°$
$A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$AB = PQ = 65$ mm $= 6.5$ cm
$BC = QR = 89$ mm $= 8.9$ cm
$A\hat{B}C = P\hat{Q}R = 60°$
$\therefore \triangle ABC \equiv \triangle PQR$ (SAS)

2. $J\hat{L}K = 180° - 55° - 50°$ (∠ sum of a △)
$= 75°$
$D \leftrightarrow J$
$E \leftrightarrow L$
$F \leftrightarrow K$
$D\hat{E}F = J\hat{L}K = 75°$
$D\hat{F}E = J\hat{K}L = 50°$
$EF = LK = 3$ cm
$\therefore \triangle DEF \equiv \triangle JLK$ (AAS)

3. (a) $A \leftrightarrow D$
$B \leftrightarrow E$
$C \leftrightarrow C$
$AB = DE$ (given)
$A\hat{B}C = D\hat{E}C$ (given)
$B\hat{A}C = E\hat{D}C$ (given)
$\therefore \triangle ABC \equiv \triangle DEC$ (AAS)
$A\hat{C}B = D\hat{C}E$
$BC = EC$
$AC = DC$

(b) $F \leftrightarrow F$
$G \leftrightarrow I$
$H \leftrightarrow J$
$GH = IJ$ (given)
$G\hat{H}F = I\hat{J}F$ (given)
$G\hat{F}H = I\hat{F}J$ (vert. opp. $\angle$s)
$\therefore \triangle FGH \equiv \triangle FIJ$ (SAS)
$F\hat{G}H = F\hat{I}J$
$FG = FI$
$FH = FJ$

(c) $K \leftrightarrow M$
$L \leftrightarrow N$
$N \leftrightarrow L$
$KN = ML$ (given)
$LN = LN$ (same side)
$L\hat{K}N = N\hat{M}L = 90°$
$\therefore \triangle KLN \equiv \triangle MNL$ (RHS)
$K\hat{L}N = M\hat{N}L$
$K\hat{N}L = M\hat{L}M$
$KL = MN$

(d) $S \leftrightarrow R$
$Q \leftrightarrow P$
$P \leftrightarrow Q$
$QP = PR$ (same side)
$Q\hat{S}P = P\hat{R}Q$ (given)
$S\hat{P}Q = R\hat{Q}P$ (given)
$\therefore \triangle SQP \equiv \triangle RPQ$ (AAS)
$S\hat{Q}P = R\hat{P}Q$
$SQ = RP$
$SP = RQ$

(e) $E \leftrightarrow E$
$B \leftrightarrow C$
$F \leftrightarrow D$
$BE = CE$ (given)
$B\hat{E}F = C\hat{E}D$ (vert. opp. $\angle$s)
$B\hat{F}E = C\hat{D}E$ (given)
$\therefore \triangle EBF \equiv \triangle ECD$ (AAS)
$E\hat{B}F = E\hat{C}D$
$BF = CD$
$EF = ED$

(f) $F \leftrightarrow F$
$H \leftrightarrow I$
$G \leftrightarrow J$
$GF = JF$ (given)
$G\hat{F}H = J\hat{F}I$ (vert. opp. $\angle$s)
$H\hat{G}F = J\hat{J}F$ (corr. $\angle$s, GH // IJI)
$\therefore \triangle FHG \equiv \triangle FIJ$ (AAS)
$\therefore \triangle FH \equiv \triangle FIJ$ (AAS)
$F\hat{H}G = F\hat{I}J$
$FH = FI$
$GH = JI$

4. (a) $A\hat{B}C = 180° - 75° - 40°$ ($\angle$ sum of a $\triangle$)
$= 65°$
$Q\hat{P}R = 180° - 65° - 40°$ ($\angle$ sum of a $\triangle$)
$= 75°$
$A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$A\hat{B}C = P\hat{Q}R = 65°$
$B\hat{A}C = Q\hat{P}R = 75°$
$B\hat{C}A = Q\hat{R}P = 40°$
$\therefore \triangle ABC$ is similar to $\triangle PQR$ (3 pairs of corr. $\angle$s equal).

(b) $A\hat{B}C = 180° - 120° - 25°$ ($\angle$ sum of a $\triangle$)
$= 35°$
$Q\hat{R}P = 180° - 120° - 45°$ ($\angle$ sum of a $\triangle$)
$= 15°$
$A \leftrightarrow Q$
$B \leftrightarrow P$
$C \leftrightarrow R$
$A\hat{B}C = Q\hat{P}R = 120°$
$B\hat{A}C$ is not equal to $P\hat{Q}R$ and $B\hat{C}A$ is not equal to $P\hat{R}Q$.
$\therefore \triangle ABC$ is not similar to $\triangle QPR$.

(c) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$$\frac{AB}{PQ} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{BC}{QR} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{AC}{PR} = \frac{5}{10} = \frac{1}{2}$$
$\therefore \triangle ABC$ is similar to $\triangle PQR$ (3 ratios of corr. sides equal).

(d) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$$\frac{AB}{PQ} = \frac{1}{2}$$
$$\frac{BC}{QR} = \frac{4}{8} = \frac{1}{2}$$
$$\frac{AC}{PR} = \frac{3.5}{7.5} = \frac{7}{15}$$
$\therefore$ Since the 3 ratios of corresponding sides are not equal, $\triangle ABC$ is not similar to $\triangle PQR$.

(e) $A \leftrightarrow P$

$B \leftrightarrow R$

$C \leftrightarrow Q$

$B\hat{A}C = Q\hat{P}R = 70°$

$\frac{AB}{PR} = \frac{6}{2} = 3$

$\frac{AC}{PQ} = \frac{5}{3} = \frac{5}{3}$

$\therefore$ Since the 2 ratios of corresponding sides are not equal, $\triangle ABC$ is not similar to $\triangle PRQ$.

(f) $A \leftrightarrow P$

$B \leftrightarrow Q$

$C \leftrightarrow R$

$A\hat{B}C = P\hat{Q}R = 90°$

$\frac{AC}{PR} = \frac{9}{4.5} = 2$

$\frac{BC}{QR} = \frac{7}{3.5} = 2$

$\therefore \triangle ABC$ is similar to $\triangle PQR$ (2 ratios of corr. sides and included $\angle$ equal).

5. **(i)** $O \leftrightarrow O$

$A \leftrightarrow B$

$D \leftrightarrow C$

$AO = BO$ (given)

$DO = CO$ (given)

$A\hat{O}D = B\hat{O}C$ (vert. opp. $\angle$s)

$\therefore \triangle OAD \equiv \triangle OBC$ (SAS)

(ii) Since $\triangle OAD \equiv \triangle OBC$,

then $O\hat{A}D = O\hat{B}C$ and $O\hat{D}A = O\hat{C}A$.

6. **(i)** $P \leftrightarrow S$

$Q \leftrightarrow R$

$R \leftrightarrow Q$

$PQ = SR$ (given)

$QR = RQ$ (common side)

$P\hat{Q}R = S\hat{R}Q$ (corr. $\angle$s, PQ // RS)

$\therefore \triangle PQR \equiv \triangle SRQ$ (SAS)

(ii) Since $\triangle PQR \equiv \triangle SRQ$,

then $QS = PR = 5$ cm

$Q\hat{P}R = Q\hat{S}R = 50°$

7. $P \leftrightarrow Q$

$O \leftrightarrow O$

$C \leftrightarrow C$

$OP = OQ$ (given)

$O\hat{P}C = O\hat{Q}C = 90°$

$OC = OC$ (common side)

$\therefore \triangle POC \equiv \triangle QOC$ (RHS)

Since $\triangle POC \equiv \triangle QOC$,

$P\hat{O}C = \hat{O}C$

$\therefore$ Hence OC is the angle bisector of $A\hat{O}B$.

8. **(a)** $\triangle ABC$ is similar to $\triangle ADE$ (2 pairs of corr. $\angle$s equal).

$$\frac{AE}{AC} = \frac{AD}{AB}$$

i.e. $\frac{7.4 + a}{7.4} = \frac{5 + 4}{5}$

$\therefore \frac{7.4 + a}{7.4} = \frac{9}{5}$

$5(7.4 + a) = 66.6$

$37 + 5a = 66.6$

$5a = 29.6$

$a = 5.92$

(b) $\triangle ABC$ is similar to $\triangle EDC$ (2 pairs of corr. $\angle$s equal).

$$\frac{EC}{AC} = \frac{CD}{CB}$$

i.e. $\frac{b}{10} = \frac{11}{7}$

$\therefore 7b = 110$

$b = 15\frac{5}{7}$

$$\frac{ED}{AB} = \frac{CD}{CB}$$

i.e. $\frac{c}{8} = \frac{11}{7}$

$\therefore 7c = 88$

$c = 12\frac{4}{7}$

(c) $\triangle PXQ$ is similar to $\triangle PAR$ (2 pairs of corr. $\angle$s equal).

$$\frac{PR}{PQ} = \frac{PA}{PX}$$

i.e. $\frac{6 + d}{6} = \frac{9 + 4}{9}$

$\therefore 54 + 9d = 78$

$9d = 24$

$d = 2\frac{2}{3}$

$\triangle PQY$ is similar to $\triangle PRB$ (2 pairs of corr. $\angle$s equal).

$$\frac{PY}{PB} = \frac{PQ}{PR}$$

i.e. $\frac{e}{e + 3} = \frac{6}{6 + 2\frac{2}{3}}$

$\therefore 8\frac{2}{3}e = 6e + 18$

$2\frac{2}{3}e = 18$

$e = 6\frac{3}{4}$

9. **(i)** $P \leftrightarrow R$

$Q \leftrightarrow Q$

$S \leftrightarrow P$

$P\hat{Q}S = R\hat{Q}P$ (common angle)

$Q\hat{S}P = Q\hat{P}R = 90°$

$\therefore \triangle PQS$ is similar to $\triangle RQP$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle PQS$ and $\triangle RQP$ are similar, then

$$\frac{QS}{QP} = \frac{QP}{QR}$$

i.e. $\frac{QS}{8} = \frac{8}{10}$

$10QS = 64$

$\therefore QS = 6.4$ cm

10. (a) (i) $B \leftrightarrow A$

$C \leftrightarrow C$

$D \leftrightarrow E$

$C\hat{B}D = C\hat{A}E$ (corr. $\angle$s, BD // AE)

$B\hat{C}D = A\hat{C}E$ (common angle)

$\therefore \triangle BCD$ is similar to $\triangle ACE$ (2 pairs of corr. $\angle$s equal).

$B \leftrightarrow G$

$C \leftrightarrow F$

$D \leftrightarrow E$

Since $C\hat{B}D = C\hat{A}E$, $C\hat{B}D = F\hat{G}E$ (corr. $\angle$s, AC // FG)

$B\hat{C}D = G\hat{F}E$ (corr. $\angle$s, BC // FC)

$\therefore \triangle BCD$ is similar to $\triangle GFE$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle BCD$ and $\triangle ACE$ are similar, then

$$\frac{BD}{AE} = \frac{BC}{AC}$$

i.e. $\frac{BD}{16} = \frac{6}{10+6}$

$\therefore 16BD = 96$

$BD = 6$ cm

(b) $BH = 18 + 6 = 24$ cm

$AG = BH = 24$ cm

$\therefore EG = 24 - 16 = 8$ cm

Since $\triangle BCD$ and $\triangle GFE$ are similar, then

$$\frac{FG}{CB} = \frac{EG}{DB}$$

i.e. $\frac{FG}{6} = \frac{8}{6}$

$\therefore FG = 8$ cm

$\therefore FH = 8 + 10 = 18$ cm

(c) $A \leftrightarrow H$

$C \leftrightarrow F$

$E \leftrightarrow D$

$A\hat{C}D = H\hat{F}D$ (corr. $\angle$s, AC // FH)

$A\hat{E}C = H\hat{D}F$ (corr. $\angle$s, DH // AE)

$\therefore \triangle ACE$ is similar to $\triangle HFD$ (2 pairs of corr. $\angle$s equal).

11. (a) (i) $P \leftrightarrow R$

$L \leftrightarrow L$

$Q \leftrightarrow N$

$Q\hat{P}L = N\hat{R}L$ (corr. $\angle$s, PQ // RN)

$P\hat{L}Q = R\hat{L}N$ (vert. opp. $\angle$s)

$\therefore \triangle PLQ$ is similar to $\triangle RLN$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle PLQ$ and $\triangle RLN$ are similar, then

$$\frac{LR}{LP} = \frac{LN}{LQ}$$

i.e. $\frac{LR}{4} = \frac{12+4}{8}$

$8LR = 64$

$\therefore LR = 8$ cm

(b) (i) $N \leftrightarrow N$

$Q \leftrightarrow M$

$R \leftrightarrow S$

$N\hat{R}Q = N\hat{S}M$ (corr.$\angle$s, SP // QR)

$Q\hat{N}R = M\hat{N}S$ (common angle)

$\therefore \triangle NQR$ is similar to $\triangle NMS$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle NQR$ and $\triangle NMS$ are similar, then

$$\frac{MS}{QR} = \frac{NM}{NQ}$$

i.e. $\frac{MS}{18} = \frac{12}{24}$

$\frac{MS}{18} = \frac{1}{2}$

$\therefore MS = 9$ cm

(c) $\triangle PLM$ is similar to $\triangle RLQ$ (2 pairs of corr. $\angle$s equal).

$\triangle PQM$ is similar to $\triangle SNM$ (2 pairs of corr. $\angle$s equal).

$\triangle PQM$ is similar to $\triangle RNQ$ (2 pairs of corr. $\angle$s equal).

12. (i) $S \leftrightarrow S$

$T \leftrightarrow T$

$R \leftrightarrow P$

$SR = SP = 9$ cm

$ST = ST$ (given)

$S\hat{T}R = S\hat{T}P = 90°$

$\therefore \triangle STR \equiv \triangle STP$ (RHS)

(ii) $R \leftrightarrow R$

$T \leftrightarrow P$

$U \leftrightarrow Q$

$R\hat{T}U = R\hat{P}Q$ (corr. $\angle$s, TU // PQ)

$T\hat{R}U = P\hat{R}Q$ (common angle)

$\therefore \triangle RTU$ is similar to $\triangle RPQ$ (2 pairs of corr. $\angle$s equal).

Since $\triangle STR \equiv \triangle STP$, $TP = RT$.

Since $\triangle RTU$ and $\triangle RPQ$ are similar, then

$$\frac{RQ}{RU} = \frac{RP}{RT}$$

i.e. $\frac{7+UQ}{7} = \frac{2}{1}$

$7 + UQ = 14$

$\therefore UQ = 7$ cm

$$\frac{PQ}{TU} = \frac{RP}{RT}$$

i.e. $\frac{PQ}{5} = \frac{2}{1}$

$\therefore PQ = 10$ cm

13. (i) $C \leftrightarrow P$

$A \leftrightarrow A$

$N \leftrightarrow N$

$AN = AN$ (common side)

$C\hat{A}N = P\hat{A}N$ (given)

Since CP is a straight line,

$A\hat{N}C = A\hat{N}P = 90°$

$\therefore \triangle CAN \equiv \triangle PAN$ (AAS)

(ii) $C \leftrightarrow C$

$M \leftrightarrow T$

$N \leftrightarrow P$

$M\hat{C}N = T\hat{C}P$ (common angle)

Since $\triangle CAN \equiv \triangle PAN$,

$CN = PN$

$$\frac{CN}{CP} = \frac{1}{2}$$

Since M is the midpoint of CT,

$$\frac{CM}{CT} = \frac{1}{2}$$

$\therefore \triangle CMN$ is similar to $\triangle CTP$ (2 ratios of corr. sides and included $\angle$ equal).

Since $\triangle CMN$ is similar to $\triangle CTP$, $C\hat{N}M = C\hat{P}T$ and MN is parallel to TA.

Hence $MTAN$ is a trapezium.

Challenge Yourself

1. *The following solution gives the shortest working. If the students do not manipulate algebra properly, this can lead to a long and tedious working.*

Let the height of $\triangle PST$ from P to ST be h'.

Using similar triangles,

$$\frac{h'}{h + h'} = \frac{a}{b}$$

i.e. $bh' = a(h + h')$

$bh' - ah' = ah$ — (1)

Area of trapezium $QRST$

= area of $\triangle PST$ – area of $\triangle PQR$

$= \frac{1}{2}b(h + h') - \frac{1}{2}ah'$

$= \frac{1}{2}(bh' - ah' + bh)$

$= \frac{1}{2}(ah + bh)$ (Substitute (1) into the equation)

$= \frac{1}{2}(a + b)h$

2. Given $PQ = QR = RS$, $\triangle PQU$, $\triangle VRU$ and $\triangle VST$ are similar (AA Similarity Test).

Let $QU = x$ cm and $VS = y$ cm.

Then $RU = (5 - x)$ cm and $VR = (5 - y)$ cm.

$$\frac{PQ}{QU} = \frac{VS}{ST}$$

$$\frac{5}{x} = \frac{y}{1}$$

$xy = 5$ — (1)

$$\frac{PQ}{QU} = \frac{VR}{RU}$$

$$\frac{5}{x} = \frac{5 - y}{5 - x}$$

$x(5 - y) = 5(5 - x)$

$5x - xy = 25 - 5x$

$10x - xy = 25$ — (2)

Substitute (1) into (2),

$10x - 5 = 25$

$10x = 30$

$x = 0$

$\therefore QU = 3$ cm

3. It is given that $AB = AC$, $CB = CE$ and $BD = BE$.

Since $CB = CE$ (given), then $DE = 9 - 5 = 4$ cm.

$\triangle CBE$ and $\triangle BDE$ are similar (AA Similarity Test since both triangles are isosceles and $\angle BEC = \angle DEB$).

$$\frac{BE}{BC} = \frac{DE}{DB}$$

$\frac{BE}{9} = \frac{4}{BE}$ since $BD = BE$ (given)

$BE^2 = 36$

$BE = 6$ cm (since $BE > 0$)

$\triangle ABC$ is also similar to $\triangle BDE$ (AA Similarity Test since both triangles are isosceles and $\angle ABC = \angle CBE$).

$$\frac{AC}{BC} = \frac{CE}{BE}$$

$$\frac{AC}{9} = \frac{9}{6}$$

$\therefore AC = \frac{3}{2} \times 9$

$= 13.5$ cm

Chapter 12 Area and Volume of Similar Figures and Solids

TEACHING NOTES

Suggested Approach

Teachers may bring some real-life samples of similar solids, i.e. shampoo and body bath bottles in original sizes and travel sizes, and ask students whether these samples are similar and draw a relationship between their length, height, area, volume and mass. Teachers may also highlight that scale drawings of structures and models are representations of the actual size of the structures. With this, teachers may ask students to give real-life examples of such scale drawings and models.

Section 12.1: Area of Similar Figures

Teachers may wish to recap with the students what they have learnt in Book 2 and previously in Chapter 11 on similar triangles and similarity tests before guiding them on how to find the relationship between the length of a figure and its area (see Investigation: Areas of Similar Figures). For problems involving area of similar figures, teachers can highlight to the students that it would be easier to write the unknown first to help in subsequent algebraic manipulation.

Section 12.2: Volume of Similar Solids

Now that students have learnt how to find the area of similar figures, teachers may guide students on how to find the relationship between the length of a solid and its volume, the height of a solid and its volume and the mass of a solid and its volume (see Investigation: Volume and Mass of Similar Solids). Teachers may highlight to students that the similarity ratio formula does have some real-life implications and ask them what some of these implications are (see Thinking Time on page 402).

Challenge Yourself

For Question 1, students will have to find the area of $\triangle BAT$ and square $BLUR$ in terms of x and y first before finding the ratio of the areas. For Question 2, formulate a pair of simultaneous equations involving the lengths of HM and MB to find the length of ST. For Question 3, let the length of the sides of the right-angled triangle be a_1, b_1 and c_1 corresponding to A_1, A_2 and A_3 respectively then use Pythagoras' Theorem.

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WORKED SOLUTIONS

Investigation (Areas of Similar Figures)

1. Since all the corresponding angles in the three squares are 90° and all the ratios of the corresponding sides of the three squares are equal, the three squares are similar.
2. Area of first square = 1 cm^2
   Area of second square = 4 cm^2
   Area of third square = 9 cm^2
3. **(a)** The area of the second square is 4 times that of the first square.
   (b) The area of the third square is 9 times that of the first square.
4. $\dfrac{A_2}{A_1} = \left(\dfrac{l_2}{l_1}\right)^2$
5. Area of second triangle = 4 square units
   Area of third triangle = 9 square units
6. **(a)** The area of the second triangle is 4 times that of the first triangle.
   (b) The area of the third triangle is 9 times that of the first triangle.
7. $\dfrac{A_2}{A_1} = \left(\dfrac{l_2}{l_1}\right)^2$

Investigation (Volume and Mass of Similar Solids)

1. Since all the ratios of the corresponding sides of the three cubes are equal, the three cubes are similar.
2. Volume of first cube = 1 cm^3
   Volume of second cube = 8 cm^3
   Volume of third cube = 27 cm^3
3. **(a)** The volume of the second cube is 8 times that of the first cube.
   (b) The volume of the third cube is 27 times that of the first cube.
4. $\dfrac{V_2}{V_1} = \left(\dfrac{l_2}{l_1}\right)^3$
5. $\dfrac{r_2}{r_1} = \dfrac{h_2}{h_1} = k$
6. **(a)** $$\begin{aligned} V_2 &= \pi r_2^2 h_2 \\ &= \pi \times (kr_1)^2 \times (kh_1) \\ &= \pi \times k^2 r_1^2 \times kh_1 \\ &= k^3(\pi r_1^2 h_1) \end{aligned}$$
   (b) $V_2 = k^3 V_1$
7. $\dfrac{V_2}{V_1} = k^3 = \left(\dfrac{h_2}{h_1}\right)^3 = \left(\dfrac{r_2}{r_1}\right)^3$
8. $\dfrac{m_2}{m_1} = \dfrac{V_2}{V_1}$

Thinking Time (Page 402)

1. Given that the two cones are similar, $\dfrac{A_1}{A_2} = \left(\dfrac{l_1}{l_2}\right)^2$
2. Consider a giant human 20 m tall and a man of height 2 m, the giant human is 10 times taller than the man. Using the similarity ratio formulae, if the mass of the man is 80 kg, then the mass of the giant is 80 000 kg. The mass of a man is supported by both his leg bones. If the cross-sectional area of *each* of the man's leg bones is 10 cm^2, 4 kg of mass is supported by 1 cm^2 of the man's leg bone. For *each* of the giant's leg bones, 40 kg of mass will be supported by 1 cm^2 of the giant's leg bone. As such, it is not possible for a human to be a giant with a height of about 20 m because the giant's leg bones will not be able to support his mass.

Teachers may ask students to search online for the Jolly Green Giant to learn more about some real-life implications of the similarity ratio formulae.

Journal Writing (Page 402)

These 5 Merlions in Singapore which are recognised by the Singapore Tourism Board are:

1. 8.6-m tall original statue at Merlion Park
2. 2-m tall cub statue at Merlion Park
3. 37-m tall gigantic replica at Sentosa Island
4. 3-m tall statue at Tourism Court
5. 3-m tall statue placed on Mount Faber's Faber Point

Assuming that all the 5 different Merlions are geometrically similar,

(a) The ratio of the height is:

8.6 : 2 : 37 : 3 : 3
86 : 2 : 370 : 30 : 30
43 : 1 : 185 : 15 : 15

(b) **(i)** The ratio of the total surface area is:

$8.6^2 : 2^2 : 37^2 : 3^2 : 3^2$
73.96 : 4 : 1369 : 9 : 9
7396 : 400 : 136 900 : 900 : 900
1849 : 100 : 34 225 : 225 : 225

(ii) The ratio of the volume of material used is:

$8.6^3 : 2^3 : 37^3 : 3^3 : 3^3$
636.056 : 8 : 50 653 : 27 : 27
636 056 : 8000 : 50 653 000 : 27 000 : 27 000
159 014 : 2000 : 12 663 250 : 6750 : 6750

Practise Now 1

(a) $$\begin{aligned} \frac{A_2}{A_1} &= \left(\frac{l_2}{l_1}\right)^2 \\ \frac{A_2}{32} &= \left(\frac{7}{4}\right)^2 \\ \frac{A_2}{32} &= \frac{49}{16} \\ \therefore A_2 &= \frac{49}{16} \times 32 \\ &= 98 \text{ cm}^2 \end{aligned}$$

(b) $\dfrac{A_1}{A_2} = \left(\dfrac{l_1}{l_2}\right)^2$

$\dfrac{A_1}{72} = \left(\dfrac{2.5}{6}\right)^2$

$\dfrac{A_1}{72} = \dfrac{25}{144}$

$\therefore A_1 = \dfrac{25}{144} \times 72$

$= 12.5 \text{ m}^2$

Practise Now 2

Since BC is parallel to DE, $\triangle ABC$ and $\triangle ADE$ are similar.

$$\left(\frac{AB}{AD}\right)^2 = \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ADE}$$

$$\left(\frac{8.4}{8.4 + BD}\right)^2 = \frac{49}{100}$$

$$\frac{8.4}{8.4 + BD} = \sqrt{\frac{49}{100}}$$

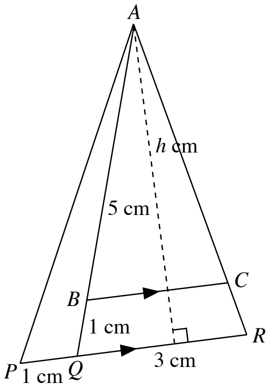
$$\frac{8.4}{8.4 + BD} = \frac{7}{10}$$

$$84 = 58.8 + 7BD$$

$$7BD = 25.2$$

$$BD = 3.6 \text{ m}$$

Practise Now 3



(i) Notice that $\triangle APQ$ and $\triangle AQR$ have a common height corresponding to the bases PQ and QR respectively.
Let the common height be h cm.

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle AQR} = \frac{\frac{1}{2} \times PQ \times h}{\frac{1}{2} \times QR \times h}$$

$$= \frac{PQ}{QR}$$

$$\frac{\text{Area of } \triangle APQ}{21} = \frac{1}{3}$$

$$\text{Area of } \triangle APQ = \frac{1}{3} \times 21$$

$$= 7 \text{ cm}^2$$

(ii) Since BC is parallel to QR, $\triangle ABC$ and $\triangle AQR$ are similar.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle AQR} = \left(\frac{AB}{AQ}\right)^2$$

$$\frac{\text{Area of } \triangle ABC}{21} = \left(\frac{5}{6}\right)^2$$

$$\text{Area of } \triangle ABC = \frac{25}{36} \times 21$$

$$= 14.6 \text{ cm}^2$$

Practise Now 4

1. $\dfrac{V_2}{V_1} = \left(\dfrac{l_2}{l_1}\right)^3$

$\dfrac{V_2}{16.2} = \left(\dfrac{10}{6}\right)^3$

$\dfrac{V_2}{16.2} = \dfrac{1000}{216}$

$\therefore V_2 = \dfrac{1000}{216} \times 16.2$

$= 75 \text{ cm}^3$

2. $\dfrac{V_1}{V_2} = \left(\dfrac{r_1}{r_2}\right)^3$

$\dfrac{2}{16} = \left(\dfrac{r_1}{1}\right)^3$

$r_1^3 = \dfrac{1}{8}$

$\therefore r_1 = \sqrt[3]{\dfrac{1}{8}}$

$= 0.5 \text{ m}$

Practise Now 5

1. Let m_1, V_1 and h_1 be the mass, volume and height of the smaller prism respectively, and m_2, V_2 and h_2 be the mass, volume and height of the larger prism respectively.

$\dfrac{m_2}{m_1} = \dfrac{V_2}{V_1}$

$= \left(\dfrac{h_2}{h_1}\right)^3$

$\dfrac{m_2}{80} = \left(\dfrac{8}{5}\right)^3$

$m_2 = \left(\dfrac{8}{5}\right)^3 \times 80$

$= 328$ (to the nearest integer)

$\therefore$ The mass of the larger prism is 328 g.

2. Let m_1, V_1 and h_1 be the mass, volume and height of the smaller statue respectively, and m_2, V_2 and h_2 be the mass, volume and height of the larger statue respectively.

$$\frac{m_2}{m_1} = \frac{V_2}{V_1}$$
$$= \left(\frac{h_2}{h_1}\right)^3$$
$$\frac{m_2}{3} = \left(\frac{200}{20}\right)^3$$
$$m_2 = \left(\frac{200}{20}\right)^3 \times 3$$
$$= 3000 \text{ kg}$$

$\therefore$ The mass of the statue made by Michael is 3000 kg.

Practise Now 6

(i) Let V_1 and h_1 be the volume and height of the smaller pyramid respectively, and V_2 and h_2 be the volume and height of the larger pyramid respectively.

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$
$$\frac{1}{6} = \left(\frac{h_1}{27}\right)^3$$
$$\frac{1}{6} = \frac{h_1^3}{19\ 683}$$
$$6h_1^3 = 19\ 683$$
$$h_1^3 = 3280.5$$
$$h_1 = 14.9 \text{ cm (to 3 s.f.)}$$

$\therefore$ The depth of the vegetable oil is 14.9 cm.

(ii) The top surface of the vegetable oil and that of the container are squares. Let l_1 and l_2 be the length of the smaller square and the bigger square respectively.

$$\frac{l_1}{l_2} = \frac{h_1}{h_2} = \frac{14.86}{27}$$
$$\frac{A_1}{A_2} = \left(\frac{14.86}{27}\right)^2$$
$$= \frac{1}{3.30}$$

$\therefore$ The ratio of the area of the top surface of the vegetable oil to the area of the top surface of the container is 1 : 3.30.

Exercise 12A

1. (a)
$$\frac{A_2}{64} = \left(\frac{2}{8}\right)^2$$
$$\frac{A_2}{64} = \frac{1}{16}$$
$$\therefore A_2 = \frac{1}{16} \times 64$$
$$= 4 \text{ cm}^2$$

(b)
$$\frac{A_1}{0.6} = \left(\frac{0.4}{0.2}\right)^2$$
$$\frac{A_1}{0.6} = 4$$
$$\therefore A_1 = 4 \times 0.6$$
$$= 2.4 \text{ m}^2$$

(c)
$$\frac{A_2}{125} = \left(\frac{6}{15}\right)^2$$
$$\frac{A_2}{125} = \frac{4}{25}$$
$$\therefore A_2 = \frac{4}{25} \times 125$$
$$= 20 \text{ cm}^2$$

(d)
$$\frac{A_1}{48} = \left(\frac{12}{8}\right)^2$$
$$\frac{A_1}{48} = \frac{9}{4}$$
$$\therefore A_1 = \frac{9}{4} \times 48$$
$$= 108 \text{ m}^2$$

(e)
$$\frac{A_2}{12} = \left(\frac{6}{4}\right)^2$$
$$\frac{A_2}{12} = \frac{9}{4}$$
$$\therefore A_2 = 9 \times 12$$
$$= 27 \text{ m}^2$$

(f)
$$\frac{A_2}{24} = \left(\frac{3p}{6p}\right)^2$$
$$\frac{A_2}{24} = \frac{1}{4}$$
$$\therefore A_2 = \frac{1}{4} \times 24$$
$$= 6 \text{ cm}^2$$

2. Let r_1 and A_1 be the radius and area of the smaller circle respectively, and r_2 and A_2 be the radius and area of the larger circle respectively.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$
$$= \left(\frac{4}{7}\right)^2$$
$$= \frac{16}{49}$$

$\therefore$ The ratio of the areas of the two circles is 16 : 49.

3. (i) Since ST is parallel to QR, $\triangle PST$ and $\triangle PQR$ are similar.

$$\frac{\text{Area of } \triangle PQR}{\text{Area of } \triangle PST} = \left(\frac{PR}{PT}\right)^2$$
$$\frac{\text{Area of } \triangle PQR}{24} = \left(\frac{10}{6}\right)^2$$
$$\frac{\text{Area of } \triangle PQR}{24} = \frac{100}{36}$$
$$\text{Area of } \triangle PQR = \frac{100}{36} \times 24$$
$$= 66\frac{2}{3} \text{ cm}^2$$

(ii) Area of $SQRT$ = Area of $\triangle PQR$ – Area of $\triangle PST$

$$= 66\frac{2}{3} - 24$$
$$= 42\frac{2}{3} \text{ cm}^2$$

4. **(a)** $\left(\frac{a}{3}\right)^2 = \frac{24}{6}$

$$\frac{a^2}{9} = 4$$
$$a^2 = 4 \times 9$$
$$= 36$$
$$\therefore a = 6$$

(b) $\left(\frac{b}{5}\right)^2 = \frac{90}{10}$

$$\frac{b^2}{25} = 9$$
$$b^2 = 9 \times 25$$
$$= 225$$
$$\therefore b = 15$$

(c) $\left(\frac{c}{5}\right)^2 = \frac{240}{15}$

$$\frac{c^2}{25} = 16$$
$$c^2 = 16 \times 25$$
$$= 400$$
$$\therefore c = 20$$

(d) $\left(\frac{d}{6}\right)^2 = \frac{12}{27}$

$$\frac{d^2}{36} = \frac{12}{27}$$
$$d^2 = \frac{12}{27} \times 36$$
$$= 16$$
$$\therefore d = 4$$

5. $\frac{\text{Area of smaller hexagon}}{200} = \left(\frac{8}{10}\right)^2$

$$\frac{\text{Area of smaller hexagon}}{200} = \frac{64}{100}$$
$$\text{Area of smaller hexagon} = \frac{64}{100} \times 200$$
$$= 128 \text{ m}^2$$

6. $\frac{\text{Area of } \triangle CAE}{9} = \left(\frac{4}{3}\right)^2$

$$\frac{\text{Area of } \triangle CAE}{9} = \frac{16}{9}$$
$$\text{Area of } \triangle CAE = \frac{16}{9} \times 9$$
$$= 16 \text{ cm}^2$$

Area of $ABDE$ = Area of $\triangle CAE$ – Area of $\triangle CBD$

$$= 16 - 9$$
$$= 7 \text{ cm}^2$$

7. $1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$

$$3250 \text{ cm}^2 = 3250 \times 10^{-4}$$
$$= 0.325 \text{ m}^2$$
$$\frac{\text{Actual land area}}{0.325} = \left(\frac{1.5}{0.03}\right)^2$$
$$\frac{\text{Actual land area}}{0.325} = 2500$$
$$\text{Actual land area} = 2500 \times 0.325$$
$$= 812.5 \text{ m}^2$$

8. Since HG is parallel to QR, $\triangle PHG$ and $\triangle PQR$ are similar.

$$\frac{\text{Area of } \triangle PHG}{\text{Area of } \triangle PQR} = \left(\frac{HG}{QR}\right)^2$$
$$= \left(\frac{p}{p+q}\right)^2$$
$$= \frac{p^2}{(p+q)^2}$$

9. **(i)** Height of larger cone = $1.5 \times 12 = 18$ cm

(ii) $\frac{\text{Surface area of larger cone}}{124} = \left(\frac{18}{12}\right)^2$

$$\frac{\text{Surface area of larger cone}}{124} = \frac{9}{4}$$
$$\text{Surface area of larger cone} = \frac{9}{4} \times 124$$
$$= 279 \text{ cm}^2$$

10. Since MN is parallel to YZ, $\triangle XMN$ and $\triangle XYZ$ are similar.

$$\left(\frac{XM}{XY}\right)^2 = \frac{\text{Area of } \triangle XMN}{\text{Area of } \triangle XYZ}$$
$$\left(\frac{6}{6+MY}\right)^2 = \frac{14}{14+22}$$
$$\frac{36}{MY^2 + 12MY + 36} = \frac{14}{36}$$
$$\frac{36}{MY^2 + 12MY + 36} = \frac{7}{18}$$
$$645 = 7MY^2 + 84MY + 252$$
$$7MY^2 + 84MY - 393 = 0$$
$$MY = \frac{-84 \pm \sqrt{84^2 - 4(7)(-393)}}{2(7)}$$
$$= \frac{-84 \pm \sqrt{18\ 060}}{14}$$
$$= 3.60 \text{ (3 s.f.)}, -15.6 \text{ (3 s.f.)}$$
(rejected since $MY > 0$)

$\therefore$ The length of MY is 3.60 cm.

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11. Since MN is parallel to QR, $\triangle PMN$ and $\triangle PQR$ are similar.

$$\left(\frac{MN}{QR}\right)^2 = \frac{\text{Area of } \triangle PMN}{\text{Area of } \triangle PQR}$$

$$\left(\frac{MN}{QR}\right)^2 = \frac{9}{9+16}$$

$$\left(\frac{MN}{QR}\right)^2 = \frac{9}{25}$$

$$\frac{MN}{QR} = \sqrt{\frac{9}{25}}$$

$$= \frac{3}{5}$$

$\therefore$ The ratio $MN : QR$ is 3 : 5.

12. Since BA is parallel to DE, $\triangle ABC$ and $\triangle EDC$ are similar.

$$\left(\frac{BC}{DC}\right)^2 = \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle EDC}$$

$$\left(\frac{BC}{BC+4.5}\right)^2 = \frac{25}{64}$$

$$\frac{BC}{BC+4.5} = \sqrt{\frac{25}{64}}$$

$$= \frac{5}{8}$$

$$8BC = 5BC + 22.5$$

$$3BC = 22.5$$

$$\therefore BC = 7.7 \text{ cm}$$

13. (i) Since XY is parallel to RS, $\triangle PXY$ and $\triangle PRS$ are similar.

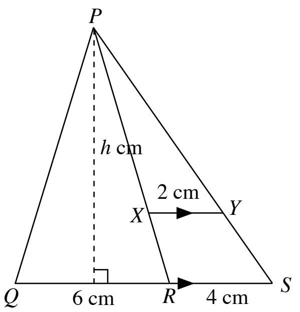
$$\frac{\text{Area of } \triangle PRS}{\text{Area of} \triangle PXY} = \left(\frac{RS}{XY}\right)^2$$

$$\frac{\text{Area of } \triangle PRS}{10} = \left(\frac{4}{2}\right)^2$$

$$\text{Area of } \triangle PRS = 4 \times 10$$

$$= 40 \text{ cm}^2$$

(ii)



Notice that $\triangle PQR$ and $\triangle PRS$ have a common height corresponding to the bases QS and RS respectively.

Let the common height be h cm.

$$\frac{\text{Area of } \triangle PQR}{\text{Area of } \triangle PRS} = \frac{\frac{1}{2} \times QR \times h}{\frac{1}{2} \times RS \times h}$$

$$= \frac{QR}{RS}$$

$$\frac{\text{Area of } \triangle PQR}{40} = \frac{6}{4}$$

$$\text{Area of } \triangle PQR = \frac{6}{4} \times 40$$

$$= 60 \text{ cm}^2$$

14. (i) Since AB is parallel to QD, $\triangle ABR$ and $\triangle QDR$ are similar.

$$\frac{AB}{QD} = \frac{AR}{QR}$$

$$\frac{AB}{7} = \frac{4}{3+4}$$

$$AB = \frac{4}{7} \times 7$$

$$= 4 \text{ cm}$$

Since AC is parallel to QP, $\triangle RAC$ and $\triangle RQP$ are similar.

$$\frac{AC}{QP} = \frac{AR}{QR}$$

$$\frac{AC}{7+4} = \frac{4}{3+4}$$

$$AC = \frac{4}{7} \times 11$$

$$= 6\frac{2}{7} \text{ cm}$$

$$\therefore BC = 6\frac{2}{7} - 4$$

$$= 6\frac{2}{7} \text{ cm}$$

(ii) Notice that $\triangle ARB$ and $\triangle BRC$ have a common height AR corresponding to the bases AB and BC respectively.

$$\frac{\text{Area of } \triangle ARB}{\text{Area of } \triangle BRC} = \frac{\frac{1}{2} \times AB \times AR}{\frac{1}{2} \times BC \times AR}$$

$$= \frac{AB}{BC}$$

$$= \frac{4}{2\frac{2}{7}}$$

$$= \frac{7}{4}$$

$\therefore$ The ratio of the area of $\triangle ARB$ to that of $\triangle BRC$ is 7: 4.

(iii)

$$\frac{\text{Area of } \triangle BRC}{\text{Area of } \triangle ARB} = \frac{4}{7}$$

$$\frac{\text{Area of } \triangle ARB}{\text{Area of } \triangle QRD} = \left(\frac{AR}{QR}\right)^2$$

$$= \left(\frac{4}{7}\right)^2$$

$$= \frac{16}{49}$$

$$\frac{\text{Area of }\triangle ARB}{\text{Area of }ABDQ} = \frac{16}{49-16} = \frac{16}{33}$$

$$\frac{\text{Area of }\triangle BRC}{\text{Area of }ABDQ} = \frac{\text{Area of }\triangle BRC}{\text{Area of }\triangle ARB} \times \frac{\text{Area of }\triangle ARB}{\text{Area of }ABDQ}$$

$$= \frac{4}{7} \times \frac{16}{33}$$

$$= \frac{64}{231}$$

$\therefore$ The ratio of the area of $\triangle BRC$ to that of $ABDQ$ is 64 : 231.

15. **(i)** Since PQ is parallel to AC, $\triangle PBQ$ and $\triangle ABC$ are similar.

$$\frac{\text{Area of }\triangle ABC}{\text{Area of }\triangle PBQ} = \left(\frac{BC}{BQ}\right)^2$$

$$\frac{\text{Area of }\triangle ABC}{8} = \left(\frac{10}{4}\right)^2$$

$$\text{Area of }\triangle ABC = \frac{100}{16} \times 8$$

$$= 50 \text{ cm}^2$$

(ii) Notice that $\triangle PQC$ and $\triangle PBQ$ have a common height corresponding to the bases QC and BQ respectively.

Let the common height be h cm.

$$\frac{\text{Area of }\triangle PQC}{\text{Area of }\triangle PBQ} = \frac{\frac{1}{2} \times QC \times h}{\frac{1}{2} \times BQ \times h}$$

$$\frac{\text{Area of }\triangle PQC}{\text{Area of }\triangle PBQ} = \frac{QC}{BQ}$$

$$\frac{\text{Area of }\triangle PQC}{8} = \frac{10-4}{4}$$

$$\text{Area of }\triangle PQC = \frac{6}{4} \times 8$$

$$= 12 \text{ cm}^2$$

(iii) Notice that $\triangle ABC$ and $\triangle AQC$ have a common height corresponding to the bases BC and QC respectively.

Let the common height be h cm.

$$\frac{\text{Area of }\triangle AQC}{\text{Area of }\triangle ABC} = \frac{\frac{1}{2} \times QC \times h}{\frac{1}{2} \times BC \times h}$$

$$\frac{\text{Area of }\triangle AQC}{\text{Area of }\triangle ABC} = \frac{QC}{BC}$$

$$\frac{\text{Area of }\triangle AQC}{50} = \frac{10-4}{10}$$

$$\text{Area of }\triangle AQC = \frac{6}{10} \times 50$$

$$= 30 \text{ cm}^2$$

Exercise 12B

1. **(a)** $\frac{V_1}{72} = \left(\frac{12}{6}\right)^3$

$\therefore V_2 = 8 \times 72$

$= 576 \text{ cm}^3$

(b) $\frac{V_2}{48} = \left(\frac{6}{4}\right)^3$

$\frac{V_2}{48} = \frac{27}{8}$

$\therefore V_2 = \frac{27}{8} \times 48$

$= 162 \text{ cm}^3$

(c) $\frac{V_1}{12} = \left(\frac{7.5}{2.5}\right)^3$

$\therefore V_1 = 27 \times 12$

$= 324 \text{ cm}^3$

(d) $\frac{V_1}{2464} = \left(\frac{4}{16}\right)^3$

$\frac{V_1}{2464} = \frac{1}{64}$

$\therefore V_1 = \frac{1}{64} \times 2464$

$= 38.5 \text{ m}^3$

(e) $\frac{V_2}{3.2} = \left(\frac{32}{64}\right)^3$

$\frac{V_2}{3.2} = \frac{1}{8}$

$\therefore V_2 = \frac{1}{8} \times 3.2$

$= 0.4 \text{ m}^3$

2. **(a)** Ratio of volume $= 10^3 : 8^3$

$= 1000 : 512$

$= 125 : 64$

(b) Ratio of volume $= 9^3 : 12^3$

$= 729 : 1728$

$= 27 : 64$

(c) Ratio of volume $= 4^3 : 6^3$

$= 64 : 216$

$= 8 : 27$

3. $\frac{\text{Capacity of Junior glass}}{540} = \left(\frac{6}{9}\right)^3$

$\frac{\text{Capacity of Junior glass}}{540} = \frac{8}{27}$

$\therefore$ Capacity of Junior glass $= \frac{8}{27} \times 540$

$= 160 \text{ cm}^3$

4. **(a)** $\left(\frac{a}{2}\right)^3 = \frac{48}{6}$

$a^3 = 8 \times 8$

$= 64$

$\therefore a = \sqrt[3]{64}$

$= 4$

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(b) $\left(\frac{b}{6}\right)^3 = \frac{54}{16}$

$$\frac{b^3}{216} = \frac{54}{16}$$

$$b^3 = \frac{54}{16} \times 216$$

$$= 729$$

$$\therefore b = \sqrt[3]{729}$$

$$= 9$$

(c) $\left(\frac{c}{7}\right)^3 = \frac{270}{10}$

$$\frac{c^3}{343} = \frac{270}{10}$$

$$c^3 = \frac{270}{10} \times 343$$

$$= 9261$$

$$\therefore c = \sqrt[3]{9261}$$

$$= 21$$

(d) $\left(\frac{d}{15}\right)^3 = \frac{20}{540}$

$$\frac{d^3}{3375} = \frac{20}{540}$$

$$d^3 = \frac{20}{540} \times 3375$$

$$= 125$$

$$\therefore d = \sqrt[3]{125}$$

$$= 5$$

5. (i) Let h_1 and A_1 be the height and the area of the base of the smaller cone respectively and h_2 and A_2 be the height and the area of the base of the larger cone respectively.

$$\left(\frac{h_1}{h_2}\right)^2 = \frac{A_1}{A_2}$$

$$\left(\frac{h_1}{h_2}\right)^2 = \frac{9}{16}$$

$$\frac{h_1}{h_2} = \sqrt{\frac{9}{16}}$$

$$\frac{h_1}{h_2} = \frac{3}{4}$$

$\therefore$ The ratio of the heights of the cones is 3 : 4.

(ii) Let V_1 and V_2 be the volume of the smaller cone and the larger volume respectively.

$$\frac{V_1}{448} = \left(\frac{3}{4}\right)^3$$

$$\frac{V_1}{448} = \frac{27}{64}$$

$$V_1 = \frac{27}{64} \times 448$$

$$= 189$$

$\therefore$ The volume of the smaller cone is 189 cm^3.

6. Let d_1, V_1 and m_1 be the diameter, volume and mass of the larger sphere respectively and d_2, V_2 and m_2 be the diameter, volume and mass of the smaller sphere respectively.

$$\frac{m_1}{m_2} = \frac{V_1}{V_2}$$

$$= \left(\frac{d_1}{d_2}\right)^3$$

$$\frac{640}{270} = \left(\frac{d_1}{d_2}\right)^3$$

$$\frac{d_1}{d_2} = \sqrt[3]{\frac{640}{270}}$$

$$= \frac{4}{3}$$

$\therefore$ The ratio of the diameters is 4 : 3.

7. Let h_1, V_1 and m_1 be the height, volume and mass of the larger bottle respectively and h_2, V_2 and m_2 be the height, volume and mass of the smaller bottle respectively.

$$\frac{m_1}{m_2} = \frac{V_1}{V_2}$$

$$= \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{750}{280} = \left(\frac{h_1}{15}\right)^3$$

$$\frac{750}{280} = \frac{h_1^3}{3375}$$

$$h_1^3 = \frac{750}{280} \times 3375$$

$$= 9040 \text{ (to 4 s.f.)}$$

$$h_1 = \sqrt[3]{9040}$$

$$= 20.8 \text{ cm (to 3 s.f.)}$$

8. (i) Let h_1 and A_1 be the height and surface area of the smaller candy cane respectively and h_2 and A_2 be the height and surface area of the larger candy respectively.

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$$

$$= \left(\frac{4}{7}\right)^2$$

$$= \frac{16}{49}$$

$\therefore$ The ratio of the total surface areas of the candy canes is 16 : 49.

(ii) Let m_1 and V_1 be the mass and volume of the smaller candy cane respectively and m_2 and V_2 be the mass and volume of the larger candy respectively.

$$\frac{10}{m^2} = \frac{V_1}{V_2}$$
$$= \left(\frac{4}{7}\right)^3$$
$$\frac{10}{m^2} = \frac{64}{343}$$
$$64m_2 = 3430$$
$$m_2 = 53.6 \text{ g (to 3 s.f.)}$$

$\therefore$ The mass of the larger candy cane is 53.6 g.

9. Let r_1 and V_1 be the radius and volume of the larger sphere respectively and r_2 and V_2 be the radius and volume of the smaller sphere respectively.

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{V_1}{V_2}$$
$$\left(\frac{r_1}{3}\right)^3 = \frac{4}{1}$$
$$\frac{r_1^3}{27} = 4$$
$$r_1^3 = 108$$
$$r_1 = 4.76 \text{ (to 3 s.f.)}$$

$\therefore$ The radius of the larger sphere is 4.76 cm.

10. Let h_1, V_1 and m_1 be the height, volume and mass of the smaller glass figurine respectively and h_2, V_2 and m_2 be the height, volume and mass of the larger glass figurine respectively.

$$\frac{m_1}{m_2} = \frac{V_1}{V_2}$$
$$= \left(\frac{h_1}{h_2}\right)^3$$
$$\frac{m_1}{500} = \left(\frac{4}{6}\right)^3$$
$$\frac{m_1}{500} = \frac{64}{216}$$
$$m_1 = \frac{64}{216} \times 500$$
$$= 148 \text{ (to 3 s.f.)}$$

$\therefore$ The mass of the similar glass figurine of height 4 cm is 148 g.

11. (i) Let l_1 and m_1 be the length and mass of the smaller train respectively and l_2 and m_2 be the length and mass of the larger train respectively.

$$\frac{m_1}{m_2} = \left(\frac{l_1}{l_2}\right)^3$$
$$\frac{m_1}{72\ 000} = \left(\frac{40}{1000}\right)^3$$
$$\frac{m_1}{72\ 000} = 6.4 \times 10^{-5}$$
$$m_1 = 4.61 \text{ (to 3 s.f.)}$$

$\therefore$ The mass of the model is 4.61 kg.

(ii) Let V_1 and V_2 be the capacity of the smaller train and the larger train respectively.

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$
$$\frac{0.85}{V_2} = \left(\frac{40}{1000}\right)^3$$
$$\frac{0.85}{V_2} = 6.4 \times 10^{-5}$$
$$6.4 \times 10^{-5}V_2 = 0.85$$
$$V_2 = 13\ 281 \text{ (to the nearest integer)}$$

$\therefore$ The capacity of the tank of the train is 13 281 litres.

12. Let l_1, A_1 and m_1 be the length, base area and mass of the smaller plastic box respectively and l_2, A_2 and m_2 be the length, base area and mass of the larger plastic box respectively.

$$\frac{m_1}{m_2} = \left(\frac{l_1}{l_2}\right)^3$$
$$\frac{4.29}{8.58} = \left(\frac{l_1}{l_2}\right)^3$$
$$\frac{l_1}{l_2} = \sqrt[3]{\frac{4.29}{8.58}}$$
$$= \sqrt[3]{0.5}$$
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$
$$\frac{A_1}{12.94} = (\sqrt[3]{0.5})^2$$
$$A_1 = (\sqrt[3]{0.5})^2 \times 12.94$$
$$= 8.15 \text{ (to 3 s.f.)}$$

$\therefore$ The base area of the second box is 8.15 m^2.

13. (i) Let d_1, A_1 and V_1 be the depth, top surface area and volume of the water respectively and d_2, A_2 and V_2 be the depth, top surface area and volume of the container respectively.
Given that the volume of water, $V_1 = 336 \text{ cm}^3$

$$\frac{1}{3} \times \text{Base area} \times \text{depth} = 336$$
$$\frac{1}{3} \times 28 \times \text{depth} = 336$$
$$\text{Depth } d_1 = \frac{336 \times 3}{28}$$
$$= 36 \text{ cm}$$

(ii)
$$\left(\frac{d_1}{d_2}\right)^2 = \frac{A_1}{A_2}$$
$$= \frac{28}{63}$$
$$= \frac{4}{9}$$
$$\frac{d_1}{d_2} = \sqrt{\frac{4}{9}}$$
$$= \frac{2}{3}$$

$\therefore$ The ratio of the depth of the water to the height of the container is 2 : 3.

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(iii) $\frac{V_1}{V_2} = \left(\frac{d_1}{d_2}\right)^3$

$\frac{336}{V_2} = \left(\frac{2}{3}\right)^3$

$\frac{336}{V_2} = \frac{8}{27}$

$8V_2 = 9072$

$V = 1134$

$\therefore$ The capacity of the container is 1134 cm^3.

14. (i) Let d_1 and V_1 be the depth and volume of mercury respectively and d_2 and V_2 be the depth and volume of the container respectively.

$\left(\frac{d_1}{d_2}\right)^3 = \frac{V_1}{V_2}$

$\left(\frac{d_1}{15}\right)^3 = \frac{8}{27}$

$\frac{d_1}{15} = \sqrt[3]{\frac{8}{27}}$

$d_1 = \frac{2}{3} \times 15$

$= 10$

$\therefore$ The depth of the mercury is 10 cm.

(ii) $\frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2$

$\frac{A_1}{45} = \left(\frac{10}{15}\right)^2$

$A_1 = \frac{100}{225} \times 45$

$= 20$

$\therefore$ The area of the mercury that is exposed to the air is 20 cm^2.

(iii) Capacity of container $= \frac{1}{3} \times 45 \times 15$

$= 225$ cm^3

15. Let h_1, V_1 and m_1 be the height, volume and mass of the smaller clay model respectively and h_2, V_2 and m_2 be the height, volume and mass of the larger clay model respectively.

$\frac{m_1}{m_2} = \frac{V_1}{V_2}$

$= \left(\frac{h_1}{h_2}\right)^3$

$\frac{x + 0.3}{x^2} = \left(\frac{20}{30}\right)^3$

$\frac{x + 0.3}{x^2} = \frac{8}{27}$

$27x + 8.1 = 8x^2$

$8x^2 - 27x - 8.1 = 0$

$x = \frac{27 \pm \sqrt{(-27)^2 - 4(8)(-8.1)}}{2(8)}$

$= \frac{27 \pm \sqrt{988.2}}{16}$

$= 3.65$ (to 3 s.f.) or -0.277 (to 3 s.f.) (rejected)

Review Exercise 12

1. (a) Ratio of area $= 3^2 : 5^2$

$= 9 : 25$

(b) Ratio of area $= 4.5^2 : 9^2$

$= 20.25 : 81$

$= 1 : 4$

(c) Ratio of area $= 2^2 : 3^2$

$= 4 : 9$

2. Let l_1 and A_1 be the perimeter and the area of the smaller triangle respectively and l_2 and A_2 be the perimeter and the area of the larger triangle respectively.

$\frac{A_1}{A_2} = \left(\frac{294}{336}\right)^2 = \frac{49}{64}$

$\therefore$ The ratio of the areas is 49 : 64.

3. (i) Let l_1 and A_1 be the length and the area of the smaller pentagon respectively and l_2 and A_2 be the length and the area of the larger pentagon respectively.

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$= \left(\frac{l_1}{2l_1}\right)^2$

$= \frac{1}{4}$

$A_2 = 4A_1$

$\therefore$ When the length of a regular pentagon is doubled, its area increases 4 times.

(ii) Let l_1 and A_1 be the length and the area of the smaller pentagon respectively and l_2 and A_2 be the length and the area of the larger pentagon respectively.

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{25}{A_2} = \left(\frac{l_1}{2l_1}\right)^2$

$\frac{25}{A_2} = \frac{1}{4}$

$A_2 = 100$

$\therefore$ The area of the enlarged pentagon is 100 cm^2.

(iii) Let l_1 and A_1 be the length and the area of the smaller polygon respectively and l_2 and A_2 be the length and the area of the larger polygon respectively.

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{A_1}{A_2} = \left(\frac{l_1}{3l_1}\right)^2$

$\frac{A_1}{A_2} = \frac{1}{9}$

$A_2 = 9A_1$

$\therefore$ When the length of a regular n-sided polygon is tripled, its area increases 9 times.

4. Let r_1 and V_1 be the base radius and volume of the smaller cylinder respectively and r_2 and V_2 be the base radius and volume of the larger cylinder respectively.

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{8}{27} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{r_1}{r_2} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$$

$\therefore$ The ratio of the base radii of the cylinders is 2 : 3.

5. **(i)** Let h_1 and V_1 be the height and volume of the smaller jug respectively and h_2 and V_2 be the height and volume of the larger jug respectively.

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{27}{64} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{h_1}{h_2} = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

$\therefore$ The ratio of the heights of the jugs is 3 : 4.

(ii) Let A_1 and A_2 be the total surface area of the smaller jug and the larger jug respectively.

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$\therefore$ The ratio of the total surface area of the jugs is 9 : 16.

6. Let h_1, V_1 and m_1 be the height, volume and mass of the larger marble statue respectively and h_2, V_2 and m_2 be the height, volume and mass of the smaller marble statue respectively.

$$\frac{m_1}{m_2} = \frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{12}{m_2} = \left(\frac{40}{320}\right)^3 = \frac{1}{512}$$

$$m_2 = 6144$$

Mass of statue = 6144 kg

= 6.144 tonnes

$\therefore$ The mass of the statue is 6.144 tonnes.

7. Let r_1 and A_1 be the radius and the area of the smaller sector respectively and r_2 and A_2 be the radius and the area of the larger sector respectively.

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$\frac{\text{Area of shaded region}}{\text{Area of sector } RUN} = \frac{25-9}{25} = \frac{16}{25}$$

$\therefore$ The ratio of the area of the shaded region to that of the area of sector RUN is 16 :25.

8. **(i)** Let l_1 and A_1 be the perimeter and surface area of the larger pond respectively and l_2 and A_2 be the perimeter and surface area of the smaller pond respectively.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{3l_2}{l_2}\right)^2 = \frac{9}{1}$$

$\therefore$ The ratio of the total surface areas of the ponds is 9 : 1.

(ii) Let V_1 and V_2 be the capacity of the larger and the smaller pond respectively.

$$\frac{10\ 800}{V_2} = \left(\frac{3l_2}{l_2}\right)^3$$

$$\frac{10\ 800}{V_2} = 27$$

$$27V_2 = 10\ 800$$

$$V_2 = 400$$

$\therefore$ The amount of water contained in the smaller pond is 400 l.

9. **(a)** $S \leftrightarrow Q$

$L \leftrightarrow L$

$P \leftrightarrow M$

$S\hat{P}L = Q\hat{M}L$ (alt. $\angle$s)

$P\hat{L}S = M\hat{L}Q$ (vert. opp. $\angle$s)

$\therefore \triangle SLP$ is similar to $\triangle QLM$ (2 pairs of corr. $\angle$s equal).

(b) **(i)** Since M is the midpoint of QR,

$$\frac{QM}{SP} = \frac{1}{2}$$

$$\frac{\text{Area of } \triangle QLM}{\text{Area of } \triangle SLP} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

(ii) $$\frac{LQ}{LS} = \frac{1}{2}$$

$$\frac{LS}{QS} = \frac{LS}{LQ+LS} = \frac{2}{2+1} = \frac{2}{3}$$

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(iii) $\triangle PLS$ and $\triangle PQS$ have a common height corresponding to the bases LS and QS respectively.

Let the common height be h cm.

$$\frac{\text{Area of } \triangle PLS}{\text{Area of } \triangle PQS} = \frac{\frac{1}{2} \times LS \times h}{\frac{1}{2} \times QS \times h} = \frac{LS}{QS} = \frac{2}{3}$$

10. (a) $\triangle LMN$ is similar to $\triangle LXY$ (2 pairs of corr. $\angle$s equal).

$$\frac{XY}{MN} = \frac{2}{5}$$

$$\frac{\text{Area of } \triangle LXY}{\text{Area of } \triangle LMN} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\frac{\text{Area of quadrilateral } XMNY}{\text{Area of } \triangle LMN} = \frac{25-4}{25} = \frac{21}{25}$$

(b) (i) $N \leftrightarrow X$

$O \leftrightarrow O$

$M \leftrightarrow Y$

$N\hat{M}O = X\hat{Y}O$ (alt. $\angle$s)

$N\hat{O}M = X\hat{O}Y$ (vert. opp. $\angle$s)

$\therefore$ $\triangle NOM$ is similar to $\triangle XOY$ (2 pairs of corr. $\angle$s equal).

(ii) $\frac{YO}{OM} = \frac{XY}{NM} = \frac{2}{5}$

(iii) $\triangle XOY$ and $\triangle XOM$ have a common height corresponding to the bases OY and OM respectively.

Let the common height be h cm.

$$\frac{\text{Area of} \triangle XOY}{\text{Area of} \triangle XOM} = \frac{\frac{1}{2} \times OY \times h}{\frac{1}{2} \times OM \times h} = \frac{YO}{OM} = \frac{2}{5}$$

$\triangle XOM$ and $\triangle MON$ have a common height corresponding to the bases OX and OM respectively.

Let the common height be d cm.

$$\frac{\text{Area of } \triangle XOM}{\text{Area of} \triangle MON} = \frac{\frac{1}{2} \times OX \times d}{\frac{1}{2} \times OM \times d} = \frac{YO}{OM} = \frac{2}{5}$$

Area of $\triangle XOY$	:	Area of $\triangle XOM$	:	Area of $\triangle MON$
2	:	5		
		2	:	5
4	:	10	:	25

11. (i) $P \leftrightarrow P$

$Q \leftrightarrow M$

$R \leftrightarrow L$

$R\hat{P}Q = L\hat{P}N$ (common angle)

$\frac{PQ}{PM} = \frac{11+4}{6} = \frac{5}{2}$

$\frac{PR}{PL} = \frac{10}{4} = \frac{5}{2}$

$\therefore$ $\triangle PQR$ is similar to $\triangle PML$ (2 ratios of corr. sides and included $\angle$ equal).

(ii) $\frac{\text{Area of } \triangle PQR}{\text{Area of } \triangle PML} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

(iii) $\frac{\text{Area of } \triangle PQR}{\text{Area of } \triangle PML} = \frac{25}{4}$

$\frac{\text{Area of } \triangle PQR}{6} = \frac{25}{4}$

Area of $\triangle PQR = \frac{25}{4} \times 6$

$= 37.5 \text{ cm}^2$

Area of $LMRQ = 37.5 - 6$

$= 431.5 \text{ cm}^2$

12. $\frac{XP}{PQ} = \frac{1}{2}$

$\frac{QY}{PQ} = \frac{1}{3}$

$XP : PQ : QY$

$3 : 6 : 2$

$\triangle QZX$ and $\triangle QYZ$ have a common height corresponding to the bases XQ and QY respectively.

Let the common height be d cm.

$$\frac{\text{Area of } \triangle QYZ}{\text{Area of } \triangle QZX} = \frac{\frac{1}{2} \times QY \times d}{\frac{1}{2} \times XQ \times d} = \frac{QY}{XQ} = \frac{2}{9}$$

$\triangle QRX$ and $\triangle QZX$ have a common height corresponding to the bases RZ and XZ respectively.

Let the common height be h cm.

$$\frac{\text{Area of } \triangle QRZ}{\text{Area of } \triangle QZX} = \frac{\frac{1}{2} \times RZ \times h}{\frac{1}{2} \times XZ \times h} = \frac{RZ}{XZ} = \frac{3}{5}$$

$$\frac{\text{Area of } \triangle QRZ}{\text{Area of } \triangle QYZ} = \frac{\text{Area of } \triangle QRZ}{\text{Area of } \triangle QZX} \times \frac{\text{Area of } \triangle QZX}{\text{Area of } \triangle QYZ}$$

$$= \frac{3}{5} \times \frac{9}{2}$$

$$= \frac{27}{10}$$

$\therefore$ The ratio of the area of $\triangle QRZ$ to that of $\triangle QYZ$ is 27 : 10.

13. (i) Total volume of paper weight $= \frac{1}{3} \times \pi \times 2^2 \times 8$

$= 33.5 \text{ cm}^3$ (to 3 s.f.)

(ii) $\frac{\text{Volume of wooden portion}}{\text{Volume of paper weight}} = \left(\frac{6}{8}\right)^3$

$\frac{\text{Volume of wooden portion}}{33.51} = \frac{27}{64}$

Volume of wooden portion $= \frac{27}{64} \times 33.51$

$= 14.1 \text{ cm}^3$ (to 3 s.f.)

(iii) Volume of lead portion $= 33.51 - 14.14$

$= 19.37 \text{ cm}^3$

Total mass of paper weight $= 19.37 \times 11.3 + 14.14 \times 0.9$

$= 232$ g (to 3 s.f.)

14. **(i)** $\dfrac{V_A}{V_{A+B}} = \left(\dfrac{a}{2a}\right)^3 = \dfrac{1}{8}$

$\dfrac{V_A}{V_B} = \dfrac{1}{8-1} = \dfrac{1}{7}$

$\therefore$ The ratio of the volume of A to that of B is 1 : 7.

(ii) $\dfrac{V_{A+B}}{V_{A+B+C}} = \left(\dfrac{2a}{3a}\right)^3 = \dfrac{8}{27}$

$\dfrac{V_B}{V_C} = \dfrac{7}{27-8} = \dfrac{7}{19}$

$\therefore$ The ratio of the volume of B to that of C is 7 : 19.

(iii) $\dfrac{V_{A+B+C}}{V_{A+B+C+D}} = \left(\dfrac{3a}{4a}\right)^3 = \dfrac{27}{64}$

$\dfrac{V_{A+B+C}}{V_D} = \dfrac{27}{64-27} = \dfrac{27}{37}$

$\therefore$ The ratio of the volume of A, B and C to that of D is 27 : 37.

Challenge Yourself

1. Let $BT = x$ and $BR = y$.

$$\begin{aligned}\text{Area of } \triangle BAT &= \frac{1}{2} \times AB \times BT \\ &= \frac{1}{2} \times 2x \times x \\ &= x^2\end{aligned}$$

$$\begin{aligned}\text{Area of square } BLUR &= y \times y \\ &= y^2\end{aligned}$$

In order to answer the question, we have to find $\dfrac{y}{x}$.

$\triangle LAU$ is similar to $\triangle BAT$.

$$\begin{aligned}\frac{AB}{AL} &= \frac{TB}{UL} \\ \frac{2x}{2x-y} &= \frac{x}{y} \\ 2xy &= 2x^2 - xy \\ x(3y-2x) &= 0 \\ x &= 0 \text{ (rejected) or } \frac{y}{x} = \frac{2}{3}\end{aligned}$$

$\therefore$ The ratio of the area of the square $BLUR$ to the area of $\triangle BAT$ is 4 : 9.

2. **(i)** $B \leftrightarrow B$

$M \leftrightarrow H$

$E \leftrightarrow T$

$B\hat{M}E = B\hat{H}T$ (corr. $\angle$s)

$M\hat{B}E = H\hat{B}T$ (common angle)

$\therefore$ $\triangle BME$ is similar to $\triangle BHT$ (2 pairs of corr. $\angle$s equal).

(ii) $\triangle EAS$ is similar to $\triangle THS$.

$\triangle TSA$ is similar to $\triangle BSH$.

(iii) Let $HM = a$ and $MB = b$.

Since $\triangle TSA$ is similar to $\triangle BSH$,

$$\begin{aligned}\frac{AT}{BH} &= \frac{ST}{BS} \\ \frac{a}{a+b} &= \frac{ST}{18+32} \\ \frac{a}{a+b} &= \frac{ST}{50} \qquad \text{— (1)}\end{aligned}$$

Since $\triangle BME$ is similar to $\triangle BHT$,

$$\begin{aligned}\frac{BM}{BH} &= \frac{BE}{BT} \\ \frac{b}{a+b} &= \frac{18}{50+ST} \qquad \text{— (2)}\end{aligned}$$

(1) + (2):

$$\begin{aligned}\frac{a}{a+b} + \frac{b}{a+b} &= \frac{ST}{50} + \frac{18}{50+ST} \\ \frac{a+b}{a+b} &= \frac{ST(50+ST)+18(50)}{50(50+ST)} \\ 1 &= \frac{50ST + ST^2 + 900}{50(50+ST)} \\ 2500 + 50ST &= 50ST + ST^2 + 900 \\ ST^2 &= 1600 \\ ST &= \sqrt{1600} \\ &= 40 \text{ cm (since } ST > 0)\end{aligned}$$

(iv) Area of $\triangle BME$: Area of $\triangle BHT$

BE^2 : BT^2

18^2 : 90^2

1 : 25

3. Let the length of the sides of the right-angled triangle be a_1, b_1 and c_1 corresponding to A_1, A_2 and A_3 respectively.

By Pythagoras' Theorem,

$a_1^2 = b_1^2 + c_1^2$

Since the 3 shaded figures are similar,

$A_1 : A_2 : A_3$

$a_1^2 = b_1^2 + c_1^2$

Hence $A_1 = A_2 + A_3$.

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Chapter 13 Geometrical Properties of Circles

TEACHING NOTES

Suggested Approach

Teachers may begin the topic by asking students how the centre of a circle can be determined, or how a circle can be constructed given only three points that pass through the circle. With this, teachers may introduce both the symmetric and angle properties of circles and build up on past knowledge, i.e. Pythagoras' Theorem, Basic Geometry, Triangles learnt by students when covering this chapter.

Section 13.1: Symmetric Properties of Circles

Teachers may recall with students what the perpendicular bisector of a chord is before introducing Circle Symmetric Property 1 through exploring three conditions and using a cut-out piece of circle to better visualise and understand the symmetric properties of circles (see Investigation: Circle Symmetric Property 1).

For Circle Symmetric Property 2, teachers may ask students what equal chords are and the properties of equal chords of a circle (see Investigation: Circle Symmetric Property 2).

For Circle Symmetric Property 3 and Circle Symmetric Property 4, these two properties involve the radius of a circle and tangent to a circle (see Investigation: Circle Symmetric Property 3 and Investigation: Circle Symmetric Property 4). It is important for students to know and identify a secant and a tangent.

Section 13.2: Angle Properties of Circles

Now that students have learnt the symmetric properties of circles, teachers may guide students on angle properties of circles. It is important for students to recognise which angle is subtended by which arc by looking at the shape of the arc (see Class Discussion: Identifying Angles at the Centre and at the Circumference) before they explore the relationship between the angle at the centre of a circle and the angle at the circumference subtended by the same arc (see Investigation: Circle Angle Property 1 and Thinking Time on page 434).

For Circle Angle Property 2, teachers can ask students what the angle at the circumference subtended by the arc is when the angle at the centre of the circle is 180° (see Investigation: Circle Angle Property 2 and Thinking Time on page 438).

Teachers should guide students along on identifying angles in the same or opposite segments of a circle before teaching Circle Angle Property 3 (see Class Discussion: Angles in Same or Opposite Segments, Investigation: Circle Angle Property 3 and Investigation: Circle Angle Property 4).

Teachers should note that students may have difficulty in recognising the properties of circles when the diagrams are not drawn and given in an obvious manner. As such, teachers may get students to observe the diagram from a different orientation or to add extra lines to the diagram so that they can better visualise and identify the properties of circles.

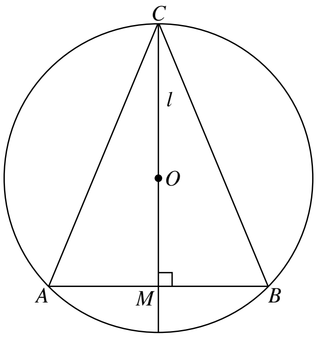
WORKED SOLUTIONS

Investigation (Circle Symmetric Property 1)

1. Conditions A and B
2. **(a)** $AM = MB$
   (b) M is the midpoint of AB.
3. Conditions A and C
4. $\angle AMO = \angle BMO = 90°$
5. It has an equal distance from all of the points on its circumference.
6. **(a)** Conditions B and C
   (b) Yes

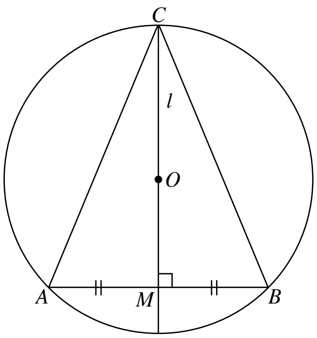
Thinking Time (Page 414)

1.



$A \leftrightarrow B$
$C \leftrightarrow C$
$M \leftrightarrow M$
$CM = CM$ (common side)
$AC = BC$
$A\hat{M}C = B\hat{M}C = 90°$
$\therefore \triangle ACM = \triangle BCM$ (SAS).
Hence $AM = MB$ and the line l bisects the chord AB.

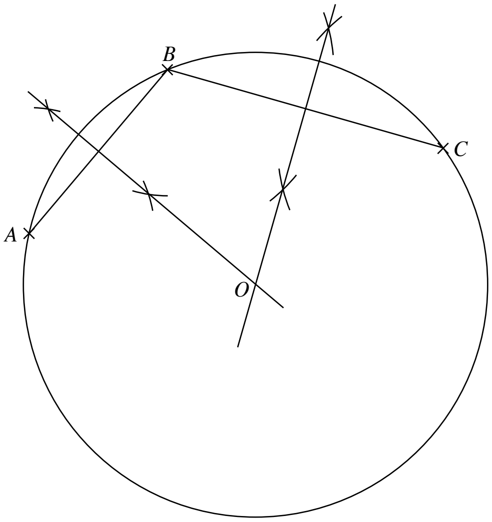
2.



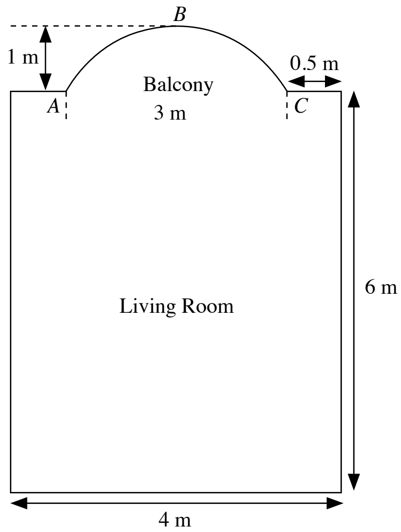
$A \leftrightarrow B$
$C \leftrightarrow C$
$M \leftrightarrow M$
$CM = CM$ (common side)
$AM = MB$
$AC = BC$
$\therefore \triangle ACM \equiv \triangle BCM$ (SSS).
Hence $A\hat{M}C = B\hat{M}C = 90°$ and the line l is perpendicular to the chord AB.

Class Discussion (Application of Circle Symmetric Property 1)

1. Construct the perpendicular bisector of AB.
Construct the perpendicular bisector of BC.
The point of intersection of the two perpendicular bisectors will be the centre O of the circle passing through the three points A, B and C.
Construct the circle with centre O and radius OA.



2.



Label the balcony ABC.
Draw a line segment AC of length 6 cm.
Construct the perpendicular bisector of AC.
Measure 2 cm from the midpoint of AC along the perpendicular bisector to get the point B.
Construct the perpendicular bisector of AB (or BC).
The point of intersection of the two perpendicular bisectors will be the centre O of the circlepassing through the three points A, B and C.
Construct the arc ABC of circle with centre O and radius OA (or ON or OC).

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Investigation (Circle Symmetric Property 2)

2. The distance of both chords from centre O is equal.
3. <u>equidistant</u>
5. The lengths of both chords are equal.
6. <u>equal</u>

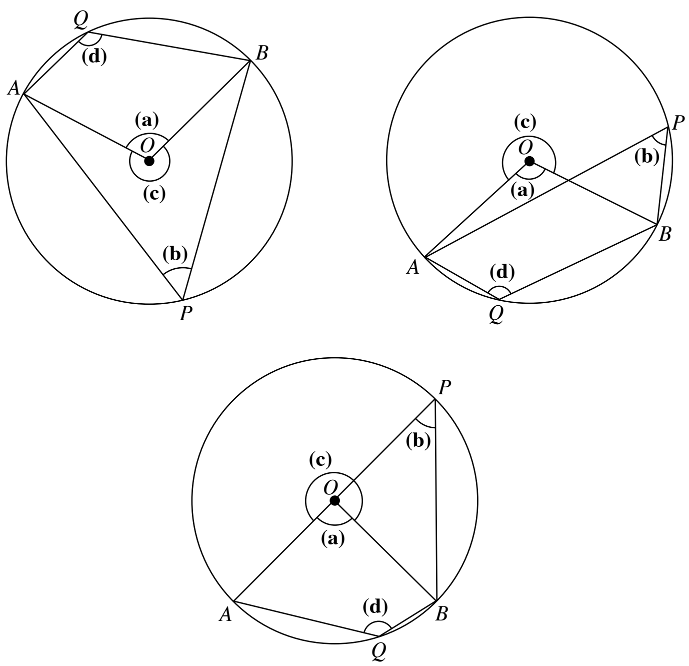
Investigation (Circle Symmetric Property 3)

3. **(a)** The secant is perpendicular to the radius.
(b) It is a right-angle.
4. <u>perpendicular</u>

Investigation (Circle Symmetric Property 4)

2. **(a)** $AP = BP$
(b) $\angle OPA = \angle OPB$
3. **(a)** <u>equal</u>
(b) <u>bisects</u>
4. $O \leftrightarrow O$
$A \leftrightarrow B$
$P \leftrightarrow P$
$OA = OB$ (radii of circle)
$O\hat{A}P = O\hat{B}P = 90°$
$OP = OP$ (common side)
$\therefore \triangle OAP \equiv \triangle OBP$ (SAS)
Hence, tangents from an external point are equal in length and the line from the centre of a circle to an external point bisects the angle between the two tangents.

Class Discussion (Identifying Angles at the Centre and at the Circumference)



Investigation (Circle Angle Property 1)

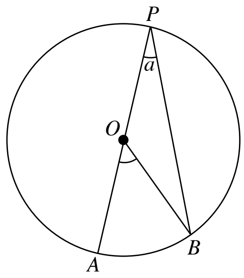
2.

$\angle AOB$	60°	90°	120°	145°	250°	320°
$\angle APB$	30°	45°	60°	72.5°	125°	160°
$\dfrac{\angle AOB}{\angle APB}$	2	2	2	2	2	2

3. $\angle AOB = 2\angle APB$
4. <u>twice</u>

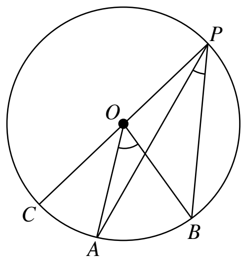
Thinking Time (Page 434)

Case 3



Let $\angle APB = a$.
Since $OP = OB$ (radii),
$\triangle BOP$ is an isosceles triangle.
Then $\angle OBP = \angle OPB = a$
$\therefore \angle AOB = \angle OBP + \angle OPB$ (ext. $\angle$ of $\triangle$)
$= 2a$
$= 2 \times \angle APB$ (proven)

Case 4



Let $\angle OPB = a$.
Since $OB = OP$ (radii),
$\triangle BOP$ is an isosceles triangle.
Then $\angle OBP = \angle OPB = a$
$\therefore \angle BOC = \angle OBP + \angle OPB$ (ext. $\angle$ of $\triangle$)
$= 2a$
$= 2 \times \angle OBP$
Let $\angle OPA = b$.
Since $OA = OP$ (radii),
$\triangle AOP$ is an isosceles triangle.
Then $\angle OAP = \angle OPA = b$
$\therefore \angle AOC = \angle OPA + \angle OAP$ (ext. $\angle$ of $\triangle$)
$= 2b$
$= 2 \times \angle OPA$

Since $\angle BOC = 2 \times \angle OBP$ and $\angle AOC = 2 \times \angle OPA$,
$\therefore \angle APB = 2 \times \angle OAB$ (proven)

Investigation (Circle Angle Property 2)

2. **(a)** $\angle APB = 90°$
   (b) Semicircle
3. <u>90°</u>
4. $\angle AOB = 180°$
   $\angle APB = \frac{1}{2} \times 180°$ ($\angle$ at centre = 2 $\angle$ at circumference)
   $= 90°$

Thinking Time (Page 438)

The diameter of a circle subtends a right angle to any point on the circle. The converse is also true, whereby a right angle on the circle must cut off a diameter. Hence, by finding two diameters PQ and RS, we can find the centre where they intersect.

Class Discussion (Angles in Same or Opposite Segments)

$\angle x$ and $\angle y$ are angles in the same segment (formed by the chord BE). $\angle w$ and $\angle z$ are angles in opposite segments (formed by the chord BF).

$\angle w$ and $\angle y$ are not angles in opposite segments because the two segments in which the two angles lie are not formed by the same chord, i.e. the two segments are just diffferent segments.

Investigation (Circle Angle Property 3)

2. $\angle APB = \angle AQB$
3. <u>equal</u>
4. $\angle AOB = 2 \times \angle APB$ ($\angle$ at centre = 2 $\angle$ at circumference)
   $\angle AOB = 2 \times \angle AQB$ ($\angle$ at centre = 2 $\angle$ at circumference)
   $\therefore \angle APB = \angle AQB$

Investigation (Circle Angle Property 4)

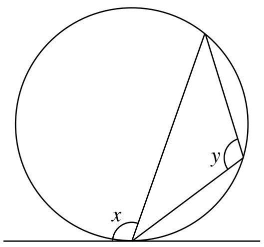
2. $\angle APB + \angle AQB = 180°$
3. <u>180°</u>
4. $\angle AOB = 2 \times \angle APB$ ($\angle$ at centre = 2$\angle$ at circumference)
   Reflex $\angle AOB = 2 \times \angle AQB$ ($\angle$ at centre = 2$\angle$ at circumference)
   Since $\angle AOB$ + reflex $\angle AOB = 360°$ ($\angle$s at a point), then $2 \times \angle APB + 2 \times \angle AQB = 360°$.
   $\therefore \angle APB + \angle AQB = 180°$, i.e. $\angle APB$ and $\angle AQB$ are supplementary.
   For the special case where both $\angle APB$ and $\angle AQB$ are in a semicircle, then $\angle APB = \angle AQB = 90°$ (rt. $\angle$ in semicircle) and so $\angle APB + \angle AQB = 180°$.

Investigation (Circle Angle Property 5)

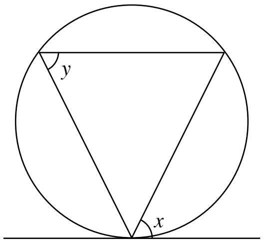
2. $\angle x = \angle y$
3. <u>equal</u>
4. $\angle AOB = 2 \times \angle APB$ (at centre = 2$\angle$ at circumference)
   $= 2 \times \angle y$
   Since $OA = OB$ (radii of circle),
   $\angle OAB = \frac{180° - 2\angle y}{2}$
   $= 90° - \angle y$
   Since $\angle OAT = 90°$ (tangent $\perp$ radius),
   $\angle OAB = 90° - \angle x$
   $\therefore \angle x = \angle y$

Thinking Time (Page 446)

(a)



(b)



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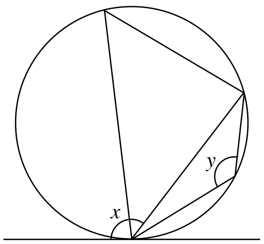
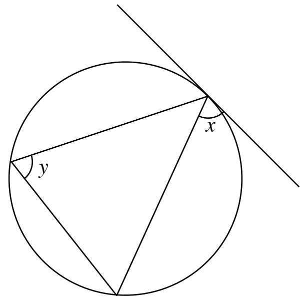


(d)



Class Discussion (Proof of Tangent-Chord Theorem)

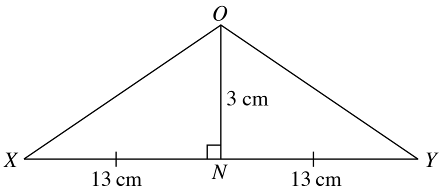
$\angle ABS$ = <u>90°</u> (rt. $\angle$ in a semicircle)
$\angle SAQ$ = <u>90°</u> (tangent $\perp$ radius)
$\angle ASB = \angle ACB$ ($\angle$s in same segment)
$\angle ASB + \angle SAB = 90°$ ($\angle$ sum of a $\triangle$)
$\angle SAB + \angle BAQ = 90$ (tangent $\perp$ radius)
$\therefore \angle ASB = \angle BAQ$
$\angle BAQ = \angle ACB$ (since $\angle ASB = \angle ACB$)

Practise Now 1

1. ON bisects XY (perpendicular bisector of chord).

$\therefore XN = NY = \dfrac{26}{2} = 13$ cm

Consider $\triangle ONX$.



By Pythagoras' Theorem,

$OX^2 = ON^2 + XN^2$
$= 3^2 + 13^2$
$= 178$

Since $OP = OX$ (radii of circle),
then $OP^2 = OX^2 = 178$

Consider $\triangle OMP$.

By Pythagoras' Theorem,

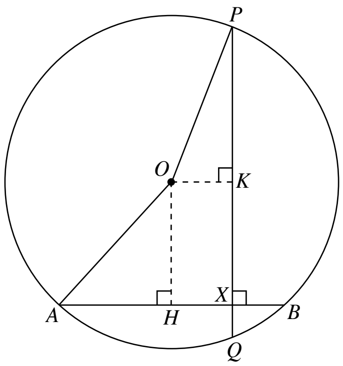
$OP^2 = OM^2 + MP^2$
$OX^2 = 8^2 + MP^2$ (since $OX = OP$, radii of circle)
$178 = 64 + MP^2$
$\therefore MP^2 = 178 - 64$
$= 114$

i.e. $MP = \sqrt{114}$ (since length $MP > 0$)

OM bisects PQ. (perpendicular bisector of chord)

$\therefore PQ = 2 \times MP$
$= 2 \times \sqrt{114}$
$= 21.35$ cm (to 2 d.p.)

2.



OH bisects AB (perpendicular bisector of chord).

$\therefore AH = BH = \dfrac{11}{2} = 5.5$ cm

Consider $\triangle OAH$.

By Pythagoras' Theorem,

$OH^2 = OA^2 - AH^2$
$= 7^2 - 5.5^2$
$= 18.75$

OK bisects PQ (perpendicular bisector of chord).

$\therefore PK = QK = \dfrac{13}{2} = 6.5$ cm

Consider $\triangle OKP$.

By Pythagoras' Theorem,

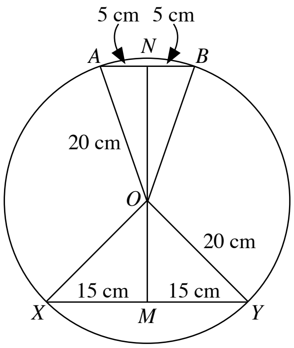
$OK^2 = OP^2 - PK^2$
$= 7^2 - 6.5^2$
$= 6.75$

$OX^2 = 18.75 + 6.75$
$= 25.5$

$\therefore OX = \sqrt{25.5}$ (since length $OX > 0$)
$= 5.05$ cm (to 3 s.f.)

Practise Now 2

There are two possible cases about the positions of the two chords AB and XY (equal chords).
Let $AB = 10$ cm and $XY = 30$ cm.



Case 1: The chords are on opposite sides of the centre O.

In $\triangle AON$,

By Pythagoras' Theorem,

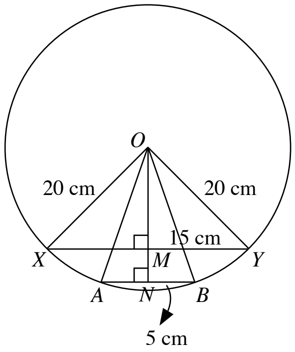
$ON^2 = 20^2 - 5^2$
$= 375$
$ON = \sqrt{375}$
$= 19.36$ cm (to 4 s.f.)

In $\triangle YOM$,

By Pythagoras' Theorem,

$OM^2 = 20^2 - 15^2$
$= 175$
$OM = \sqrt{175}$
$= 13.23$ cm (to 4 s.f.)

Distance between the chords $= MN$
$= NO + OM$
$= 19.36 + 13.23$
$= 32.6$ cm (to 3 s.f.)



Case 2: The chords are on the same side of the centre O.

Distance between the chords $= MN$

$= ON - OM$

$= 19.36 - 13.23$

$= 6.13$ cm (to 3 s.f.)

$\therefore$ The distance between the chords can either be 6.13 cm or 32.6 cm.

Practise Now 3

1. (i) $\angle OAP = 90°$ (tangent $\perp$ radius)

In $\triangle OAP$,

$$\tan \angle OPA = \frac{4.5}{10.5}$$

$$\angle OPA = \tan^{-1} \frac{4.5}{10.5}$$

$= 23.2°$ (to 1 d.p.)

(ii) By Pythagoras' Theorem,

$$OP^2 = OA^2 + AP^2$$

$$= 4.5^2 + 10.5^2$$

$$= 130.5$$

$OP = \sqrt{130.5}$ (since length $OP > 0$)

$= 11.4$ cm (to 3 s.f.)

(iii) Area of $\triangle OPA = \frac{1}{2} \times 4.5 \times 10.5$

$= 23.625$ cm^2

2. (i) $\angle OAB = 90°$ (tangent $\perp$ radius)

In $\triangle OAB$,

By Pythagoras' Theorem,

$$OB^2 = OA^2$$

$$(5 + x)^2 = x^2 + 8^2$$

$$x^2 + 10x + 25 = x^2 + 64$$

$$10x = 39$$

$$x = 3.9$$

(ii) $\tan \angle AOB = \frac{8}{3.9}$

$$\angle AOB = \tan^{-1} \frac{8}{3.9}$$

$= 64.0°$ (to 1 d.p.)

(iii) Area bounded = Area of $\triangle OAB$ – Area of sector OAC

$$= \frac{1}{2} \times 8 \times 3.9 - \frac{64.01°}{360°} \times \pi \times 3.9^2$$

$= 7.10$ cm^2 (to 3 s.f.)

Practise Now 4

1. (i) $\angle OBP = 90°$ (tangent $\perp$ radius)

$\angle OPB = 180° - 90° - 62°$ ($\angle$ sum of a $\triangle$)

$= 28°$

(ii) $\angle AOP = \angle BOP = 62°$

Since $OC = OA$ (radii of circle),

$\angle OAC = \angle OCA$

$= \frac{180° - 62°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 59°$

(iii) $\tan 62° = \frac{BP}{14}$

$BP = 14 \tan 62°$

$= 26.3$ cm (to 3 s.f.)

(iii) Area of quadrilateral $APBO$

= Area of $\triangle PBO$ + Area of $\triangle PAO$

$= 2\left(\frac{1}{2} \times 14 \times 26.33\right)$

$= 369$ cm^2 (to 3 s.f.)

2. $\angle PQO = 90°$ (tangent $\perp$ radius)

$\angle PQT = \angle PTQ$

$= \frac{180° - 64°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 58°$

$\angle SQT = 90° - 58°$

$= 32°$

Practise Now 5

1. (i) $\angle SOR = 2 \times \angle SPR$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 2 \times 28°$

$= 56°$

(ii) Reflex $\angle POR = 180° + 56° = 236°$

$\angle PQR = \frac{1}{2} \times \angle POR$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= \frac{1}{2} \times 236°$

$= 118°$

2. Since $OA = OB$ (radii of circle),

$\angle OAB = \angle OBA = 35°$ (base $\angle$s of isos. $\triangle$)

$\angle AOB = 180° - 35° - 35°$ ($\angle$ sum of $\triangle$)

$= 110°$

$\therefore x = 110°$

$\angle ACB = \frac{1}{2} \angle AOB$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= \frac{1}{2} \times 110°$

$= 55°$

$\therefore y = 55°$

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3. $\angle ABC = 180° - 73°$ (adj. $\angle$s on a str. line)
$= 107°$
Reflex $\angle AOC = 2 \times \angle ABC$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 2 \times 107°$
$= 214°$
Obtuse $\angle AOC = 360° - 214°$ ($\angle$s at a point)
$= 146°$

Practise Now 6

(i) $\angle POR = 180° - 50°$ (adj. $\angle$s on a str. line)
$= 130°$
Since $OP = OR$ (radii of circle),
$\angle OPR = \dfrac{180° - 130°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 25°$

(ii) $\angle OPQ = 50°$ (corr. $\angle$s, PR // OR)
$\angle QPR = 50° - 25° = 25°$
$\angle QOR = 2 \times \angle QPR$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 2 \times 25°$
$= 50°$

(iii) $\angle PQO = \angle QOR = 50°$ (alt. $\angle$s)
$\angle PXQ = 180° - 50° - 25°$ ($\angle$ sum of $\triangle$)
$= 105°$

Practise Now 7

1. **(i)** $\angle CDX = \angle CAB$ ($\angle$s in same segment)
$= 44°$
(ii) $\angle ABX = \angle DCX$ ($\angle$s in same segment)
$= 25°$
(iii) $\angle CXB = 25° + 44°$ (ext. $\angle$ of $\triangle$)
$= 69°$

2. $x = 2 \times 25°$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 50°$
$y = 25°$ ($\angle$s in same segment)

3. $\angle APB = 90°$ (rt. $\angle$ in semicircle)
$\angle PRB = 45°$ ($\angle$s in same segment)
$\angle APR = 25° + 45°$ (ext. $\angle$ of $\triangle$)
$= 70°$
$\angle BPR = 90° - 70°$
$= 20°$

Practise Now 8

1. **(i)** $\angle BAD = 180° - 21° - x°$ ($\angle$ sum of a $\triangle$)
$= (159 - x)°$
(ii) $\angle BCD = 180° - 31° - x°$ ($\angle$ sum of a $\triangle$)
$= (149 - x)°$
(iii) $\angle BAD + \angle BCD = 180°$ ($\angle$s in opp. segments)
$(159 - x)° + (149 - x)° = 180°$
$308° - 2x° = 180°$
$2x° = 128°$
$x = 64$
(iv) $\angle PAD = 21° + 64°$ (ext. $\angle$ of $\triangle$)
$= 85°$

2. $\angle BCD = 180° - 68° = 112°$ ($\angle$s in opp. segments)
Since $BC = CD$,
$\angle BDC = \angle CBD = \dfrac{180° - 112°}{2} = 34°$ (base $\angle$s of isos. $\triangle$)
$\angle BAC = \angle BDC = 34°$ ($\angle$s in same segment)

Practise Now 9

1. Let the centre of the circle be O.
$\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle CAT = 33°$ ($\angle$s in alt. segment)
$\angle OAC = \angle OCA$ (base $\angle$s of isos. $\triangle OAC$)
$= 90° - 33°$
$= 57°$
$\angle ATC = 57° - 33°$ (ext. $\angle$ of $\triangle$)
$= 24°$

2. **(i)** Since $\triangle ABC$ is an isos $\triangle$,
$\angle ABC = 180° - 2 \times 41°$
$= 98°$
$\angle CAT = 98°$ ($\angle$s in alt. segment)
$\angle DAT = 46°$ ($\angle$s in alt. segment)
$\therefore \angle CAD = 98° - 46°$
$= 52°$
(ii) $\angle ATC = 180° - 98° - 46°$ ($\angle$ sum of $\triangle$)
$= 36°$

Practise Now 10

Reflex $\angle AOC = 2 \times 114°$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 228°$
$\angle AOC = 360° - 228°$ ($\angle$s at a point)
$= 132°$
$\angle ABC = 180° - 132°$ ($\angle$s in opp. segments)
$= 48°$

Practise Now 11

(i) $A \leftrightarrow P$
$X \leftrightarrow X$
$Q \leftrightarrow B$
$A\hat{X}Q = P\hat{X}B = 90°$
$Q\hat{A}X = B\hat{P}X$ ($\angle$s in same segment)
$A\hat{Q}X = P\hat{B}X$ ($\angle$s in same segment)
$\therefore \angle AXQ$ is similar to $\angle PXB$ (3 pairs of corr. $\angle$s equal).

(ii) $\dfrac{AX}{PX} = \dfrac{XQ}{XB}$
$\dfrac{5}{3.4} = \dfrac{10.5}{XB}$
$5XB = 35.7$
$XB = 7.14$ cm

Exercise 13A

1. (a) By Pythagoras' Theorem,

$a^2 = 13^2 - 5^2$
$= 144$
$a = \sqrt{144}$
$= 12$ (since length $a > 0$)

$\cos b° = \dfrac{5}{13}$

$b = \cos^{-1} \dfrac{5}{13}$
$= 67.4$ (to 1 d.p.)

(b) By Pythagoras' Theorem,

$c^2 = 17^2 - 13^2$
$= 120$
$c = \sqrt{120}$
$= 11.0$ (to 3 s.f.) (since length $a > 0$)

$\cos d° = \dfrac{8}{17}$

$d = \cos^{-1} \dfrac{8}{17}$
$= 61.9$ (to 1 d.p.)

(c) Let the radius of the circle be r mm.

By Pythagoras' Theorem,

$r^2 = 5^2 + 6^2$
$= 61$

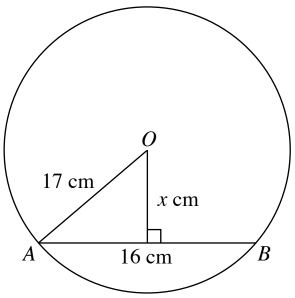
By Pythagoras' Theorem,

$e^2 = r^2 - 5^2$
$= 61 - 25$
$= 36$
$c = \sqrt{36}$
$= 6$ (since length $a > 0$)

$\tan f° = \dfrac{6}{5}$

$f = \tan^{-1} \dfrac{6}{5}$
$= 50.2$ (to 1 d.p.)

2.

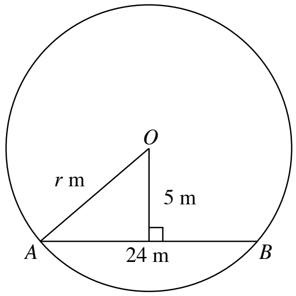


By Pythagoras' Theorem,

$x^2 = 17^2 - 8^2$
$= 225$
$x = \sqrt{225}$ (since length $x > 0$)
$= 15$

$\therefore$ The perpendicular distance from O to AB is 15 cm.

3.

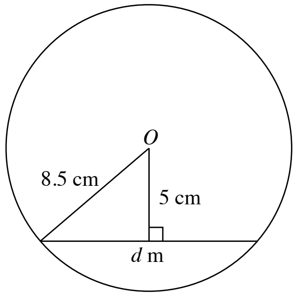


By Pythagoras' Theorem,

$r^2 = 5^2 + 12^2$
$= 169$
$r = \sqrt{169}$ (since length $r > 0$)
$= 13$

$\therefore$ The radius of the circle is 13 m.

4.



By Pythagoras' Theorem,

$\left(\dfrac{d}{2}\right)^2 = 8.5^2 - 5^2$

$\dfrac{d^2}{4} = 47.25$

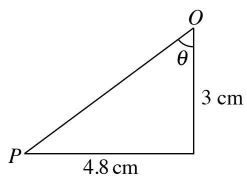
$d = \sqrt{189}$ (since length $d > 0$)
$= 13.7$ (to 3 s.f.)

$\therefore$ The length of the chord is 13.7 cm.

5. (a) $a = 12$
$b = 90$

(b) $x = \dfrac{22}{2} = 11$
$y = 90$

6.



By Pythagoras' Theorem,

$OP^2 = 3^2 + 4.8^2$
$= 32.04$
$OP = \sqrt{32.04}$ (since length $OP > 0$)
$= 5.660$ (to 4 s.f.)

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$$\tan\theta = \frac{4.8}{3}$$

$\theta = \tan^{-1} 1.6$

$= 57.99°$ (to 2 d.p.)

$\angle POQ = 2 \times 57.99° = 115.98°$

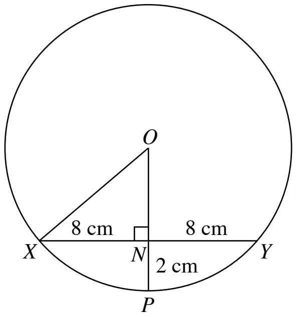
Area of cross section of pipe

= Area of sector POQ – Area of $\triangle POQ$

$$= \frac{115.98°}{360°} \times \pi \times 5.660^2 - \frac{1}{2} \times 9.6 \times 3$$

$= 18.0\text{ cm}^2$ (to 3 s.f.)

7.



Let the radius of the circle be r cm.

By Pythagoras' Theorem,

$OX^2 = ON^2 + XN^2$

$r^2 = (r - 2)^2 + 8^2$

$r^2 = r^2 - 4r + 4 + 64$

$4r = 68$

$r = 17$

$\therefore$ The radius of the circle is 17 cm.

8. **(i)** By Pythagoras' Theorem,

$OC^2 = 9^2 - 7^2$

$= 32$

$OC = \sqrt{32}$ (since length $OC > 0$)

$= 5.66$ cm (to 3 s.f.)

(ii) By Pythagoras' Theorem,

$OE^2 = 5.657^2 + (6 + 7)^2$

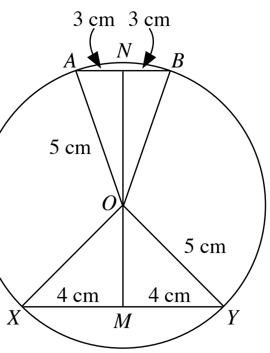
$= 201.0$ (to 4 s.f.)

$OE = \sqrt{201.0}$ (since length $OE > 0$)

$= 14.2$ cm (to 3 s.f.)

9. There are two possible cases about the positions of the two chords AB and XY (equal chords).

Let $AB = 6$ cm and $XY = 8$ cm.



Case 1: The chords are on opposite sides of the centre O.

In $\triangle AON$,

By Pythagoras' Theorem,

$ON^2 = 5^2 - 3^2$

$= 16$

$ON = \sqrt{16}$

$= 4$ cm

In $\triangle YOM$,

By Pythagoras' Theorem,

$OM^2 = 5^2 - 4^2$

$= 9$

$OM = \sqrt{9}$

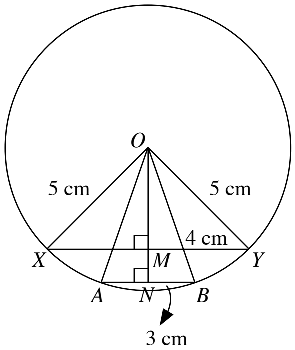
$= 3$ cm

Distance between the chords $= MN$

$= NO + OM$

$= 4 + 3$

$= 7$ cm



Case 2: The chords are on the same side of the centre O.

Distance between the chords $= MN$

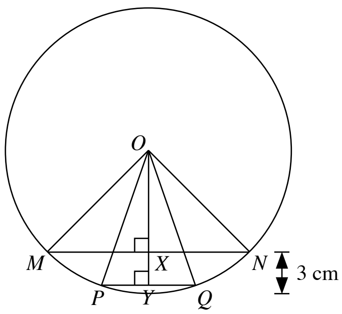
$= ON - OM$

$= 4 - 3$

$= 1$ cm

$\therefore$ The distance between the chords can either be 1 cm or 7 cm.

10.

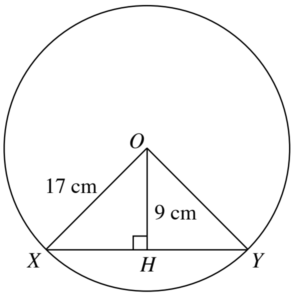


Let the radius of the circle be r cm and the length of OX be x cm.

Given $PQ = 7$ cm and $MN = 14$ cm,

In $\triangle OMX$,
By Pythagoras' Theorem,
$OM^2 = OX^2 + MX^2$
$r^2 = x^2 + 7^2$
$r^2 = x^2 + 49$ — (1)
In $\triangle OPY$,
By Pythagoras' Theorem,
$OP^2 = OY^2 + PY^2$
$r^2 = (x + 3)^2 + 3.5^2$
$r^2 = x^2 + 6x + 9 + 12.25$
$r^2 = x^2 + 6x + 21.25$ — (2)
Substitute (1) into (2):
$x^2 + 49 = x^2 + 6x + 21.25$
$6x = 27.75$
$x = 4.625$
$r = \sqrt{4.625^2 + 7^2}$
$= 8.39$ (to 3 s.f.)
$\therefore$ The radius of the circle is 8.39 cm.

11.



$\cos \angle HOX = \frac{9}{17}$
$\angle HOX = \cos^{-1} \frac{9}{17}$
$= 58.03°$ (to 2 d.p.)
$\angle XOY = 2 \times 58.03° = 115.98°$
By Pythagoras' Theorem,
$XH^2 = 17^2 - 9^2$
$= 208$
$XH = \sqrt{208}$ (since length $XH > 0$)
$= 14.42$ cm (to 4 s.f.)
$XY = 2 \times 14.42 = 28.84$ cm
Perimeter of minor segment = XY + minor arc XY
$= 28.84 + \frac{115.98°}{360°} \times 2\pi \times 17$
$= 63.3$ cm (to 3 s.f.)

Exercise 13B

1. $\angle OPB = 90°$ (tangent $\perp$ radius)
$OA = OP$ (radii of circle)
$\angle OAP = \angle OPA = 33°$ (base $\angle$s of isos. $\triangle$)
$\angle PBA = 180° - (33 + 90)° - 33°$ ($\angle$ sum of a $\triangle$)
$= 24°$

2. **(i)** $\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle ATB = 180° - 90° - 64°$ ($\angle$ sum of a $\triangle$)
$= 26°$
(ii) $OA = OB$ (radii of circle)
$\angle OAB = \angle OBA = \frac{64°}{2} = 32°$ (ext. $\angle$ of $\triangle$)
$\angle TAB = 90° + 32°$
$= 122°$

3. $\angle AOB = 180° - x - 90°$ ($\angle$ sum of a $\triangle$)
$= 90° - x$
$OB = OC$ (radii of circle)
$\angle OCD = \angle OBD = \frac{90° - x}{2} = 45° - \frac{x}{2}$ (ext. $\angle$ of $\triangle$)
$\angle COD = 180° - 90° - \left(45° - \frac{x}{2}\right)$ ($\angle$ sum of a $\triangle$)
$= 180° - 90° - 45° + \frac{x}{2}$
$= 45° + \frac{x}{2}$

4. **(a)** $a = 49$
$b = 14$
(b) $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)
$c° = 180° - 32° - 90° = 58°$ ($\angle$ sum of a $\triangle$)
$\therefore c = 58$
$d = 15$
(c) $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)
$OA = OB$ (radii of circle)
$\angle POA = \frac{112°}{2} = 56°$
$e° = 180° - 56° - 90° = 34°$ ($\angle$ sum of a $\triangle$)
$\therefore e = 34$
$\tan 56° = \frac{f}{10}$
$f = 10 \tan 56°$
$= 14.8$ (to 3 s.f.)
(d) $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)
$OA = OB$ (radii of circle)
$h° = 90° - 35° = 55°$
$\therefore h = 55$
$g° = 90° - 55° = 35°$
$\therefore g = 35$
(e) $\angle OBP = 90°$ (tangent $\perp$ radius)
By Pythagoras' Theorem,
$(5 + i)^2 = 5^2 + 12^2$
$i^2 + 10i + 25 = 169$
$i^2 + 10i - 144 = 0$
$(i - 8)(i + 18) = 0$
$i = 8$ (since length $i > 0$)
$\tan j° = \frac{12}{5}$
$j = \tan^{-1} \frac{12}{5}$
$= 67.4$ (to 1 d.p.)

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(f) $\angle OBP = 90^\circ$ (tangent $\perp$ radius)

By Pythagoras' Theorem,

$$(k + 7)^2 = k^2 + 15^2$$
$$k^2 + 14k + 49 = k^2 + 225$$
$$14k = 176$$
$$k = 12.6 \text{ (to 3 s.f.)}$$

$$\tan l^\circ = \frac{15}{12.57}$$
$$l = \tan^{-1} \frac{15}{12.57}$$
$$= 50.0 \text{ (to 1 d.p.)}$$

5. **(i)** $\angle OAT = 90^\circ$ (tangent $\perp$ radius)

$OA = OB$ (radii of circle)

$\angle OAB = \angle OBA = 46^\circ$

$\angle BAT = 90^\circ - 46^\circ = 44^\circ$ (base $\angle$s of isos. $\triangle$)

(ii) $\angle CAT = 180^\circ - 69^\circ$ (adj. $\angle$s on a str. line)

$= 111^\circ$

$\angle PTC = 180^\circ - 44^\circ - 111^\circ$ ($\angle$ sum of a $\triangle$)

$= 25^\circ$

6. **(i)** Let the radius of the circle be r cm.

$\angle OAC = 90^\circ$ (tangent $\perp$ radius)

By Pythagoras' Theorem,

$$(r + 12)^2 = r^2 + 18^2$$
$$r^2 + 24r + 144 = r^2 + 324$$
$$24r = 180$$
$$r = 7.5$$

$\therefore$ The radius of the circle is 7.5 cm.

(ii) $\tan \angle AOB = \frac{18}{7.5}$

$$\angle AOB = \tan^{-1} \frac{18}{7.5}$$
$$= 67.4^\circ \text{ (to 1 d.p.)}$$

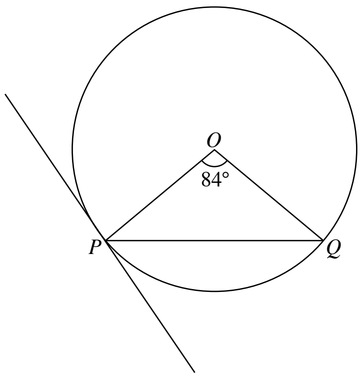
(iii) Area of shaded of region

$= \text{Area of } \triangle OAC - \text{Area of sector } OAB$

$= \frac{1}{2} \times 18 \times 7.5 - \frac{67.4^\circ}{360^\circ} \times \pi \times 7.5^2$

$= 34.4 \text{ cm}^2$ (to 3 s.f.)

7.



$OP = OQ$ (radii of circle)

$\angle OPQ = \angle OQP$

$= \frac{180^\circ - 84^\circ}{2}$ (base $\angle$s of isos. $\triangle$)

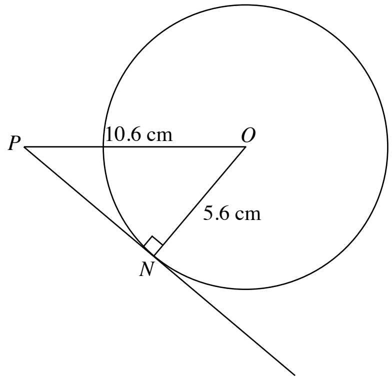
$= 48^\circ$

Obtuse angle between PQ and tangent at P

$= 90^\circ + 48^\circ$

$= 138^\circ$

8.



$\angle ONP = 90^\circ$ (tangent $\perp$ radius)

By Pythagoras' Theorem,

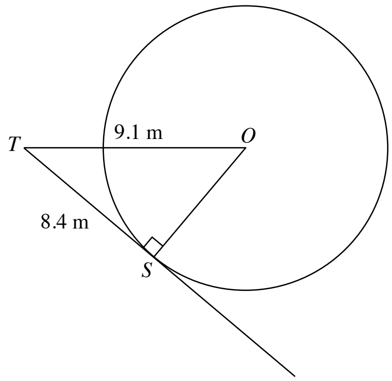
$PN^2 = 10.6^2 - 5.6^2$

$= 81$

$PN = \sqrt{81}$ (since length $PN > 0$)

$= 9$ cm

9.



$\angle OST = 90^\circ$ (tangent $\perp$ radius)

By Pythagoras' Theorem,

$OS^2 = 9.1^2 - 8.4^2$

$= 12.25$

$OS = \sqrt{12.25}$ (since length $OS > 0$)

$= 3.5$ m

Diameter of circle $= 2 \times 3.5$

$= 7$ m

10. $\angle ABO = \angle ACO = 90^\circ$ (tangent $\perp$ radius)

$\angle AOC = 180^\circ - 122^\circ$ (adj. $\angle$s on a str. line)

$= 58^\circ$

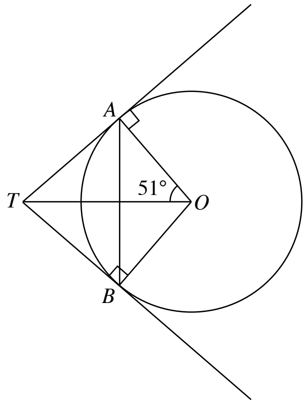
$\angle CAO = 180^\circ - 90^\circ - 58^\circ$ ($\angle$ sum of a $\triangle$)

$= 32^\circ$

$\angle BAC = 2 \times 32^\circ$

$= 64^\circ$

11.



$\angle OAT = \angle OBT = 90°$ (tangent $\perp$ radius)

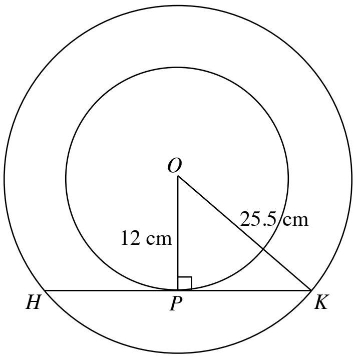
$\angle OAB = 180° - 90° - 51°$ ($\angle$sum of a $\triangle$)

$= 39°$

$\angle BAT = 90° - 39°$

$= 51°$

12.



$\angle OPK = 90°$ (tangent $\perp$ radius)

By Pythagoras' Theorem,

$PK^2 = 25.5^2 - 12^2$

$= 506.25$

$PK = \sqrt{506.25}$ (since length $PK > 0$)

$= 22.5$ cm

$HK = 2 \times 22.5$

$= 45$ cm

Exercise 13C

1. **(a)** $a° = 2 \times 40°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 80°$

$\therefore a = 80$

(b) $b° = \frac{1}{2} \times 60°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 30°$

$\therefore b = 30$

(c) $c° = 40°$ ($\angle$s in the same segment)

$\therefore c = 40$

(d) $d° = \frac{1}{2} \times 230°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 115°$

$\therefore d = 115°$

(e) $360° - 110° = 250°$ ($\angle$s at a point)

$e° = \frac{1}{2} \times 250°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 125°$

$\therefore e = 125$

(f) $\angle O = 2 \times 40°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 80°$

$f° = \frac{180° - 80°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 50°$

$\therefore f = 50$

(g) $g° = \angle$ at circumference (alt. $\angle$s)

$= \frac{1}{2} \times 70°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 35°$

$\therefore g = 35$

(h) $\angle$ at circumference

$= \frac{1}{2} \times 98°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 49°$

$\frac{180° - 98°}{2} = 41°$ (base $\angle$s of isos. $\triangle$)

$h° = 180° - 41° - 41° - 21° - 49°$ ($\angle$ sum of a $\triangle$)

$= 28°$

$\therefore h = 28$

2. **(a)** $a° = 180° - 40° - 90°$ (rt. $\angle$ in semicircle)

$= 50°$

$\therefore a = 50$

(b) $b° = \frac{180° - 90°}{2}$ (rt. $\angle$ in semicircle)

$= 45°$

$\therefore b = 45$

(c) $c° = 90° - 60°$ (rt. $\angle$ in semicircle)

$= 30°$

$\therefore c = 30$

(d) $d° = 90° - 30°$ (rt. $\angle$ in semicircle)

$= 60°$

$\therefore d = 60$

3. **(a)** $x° = 50°$ ($\angle$s in same segment, alt. $\angle$s)

$\therefore x = 50$

(b) $y° = 180° - 100° - 68°$ ($\angle$s in same segment, $\angle$ sum of a $\triangle$)

$= 12°$

$\therefore y = 12$

4. $\angle PTQ = \angle PSQ = 20°$ ($\angle$s in same segment)

$\angle PQT = 180° - 100° - 20°$ ($\angle$ sum of a $\triangle$)

$= 60°$

5. $\angle ADC = 180° - 65°$ (adj. $\angle$s on a str. line)

$= 115°$

$\angle ABC = 180° - 115°$ ($\angle$s in opp. segments)

$= 65°$

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6. **(a)** $180° - 110° = 70°$ ($\angle$s in opp. segments)

$x° = 180° - 70° - 70°$ ($\angle$ sum of a $\triangle$)

$= 40°$

$\therefore x = 40$

(b) $y° + 54° + y° + 54° = 180°$ ($\angle$s in opp. segments)

$2y° + 108° = 180°$

$2y° = 72°$

$y° = 36°$

$\therefore y = 36$

(c) $41° + 27° + 65° + x° = 180°$ ($\angle$s in same segment, $\angle$ sum of a $\triangle$)

$x° + 133° = 180°$

$x° = 47°$

$\therefore x = 47$

(d) $40° + 90° + (180 - y)° = 180°$

($\angle$s in same segment, rt. $\angle$ in semicircle)

$y° = 130°$

$\therefore y = 130$

7. **(i)** $\angle BAD = 180° - 80° - 30°$ ($\angle$ sum of a $\triangle$)

$= 70°$

(ii) $\angle ADC = 180° - 80°$ ($\angle$s in opp. segments)

$= 100°$

$\angle XCD = 100° - 30°$ (ext. $\angle$ of $\triangle$)

$= 70°$

8. $\angle PQR = 90°$ (rt. $\angle$ in semicircle)

$\angle PRS + \angle PTS = 180°$ ($\angle$s in opp. segments)

$\therefore \angle PQR + \angle PRS + \angle PTS = 90° + 180°$

$= 270°$

9. **(a)** $x° = 40°$ ($\angle$s in alt. segment)

$\therefore x = 40°$

(b) $y = 180° - 60° - 70°$ ($\angle$s in alt. segment, $\angle$ sum of $\triangle$)

$= 50$

$\therefore y = 50$

(c) $x = 180° - 44° - 59°$ ($\angle$s in alt. segment, $\angle$ sum of $\triangle$)

$= 77°$

$\therefore x = 77$

(d) $y = 180° - 81° - 43°$ ($\angle$s in alt. segment, $\angle$ sum of $\triangle$)

$= 56°$

$\therefore y = 56$

10. Reflex $\angle AOC = 360° - 144° = 216°$

$\angle ADC = \dfrac{1}{2} \times 216°$ ($\angle$ at centre $= 2\ \angle$ at circumference)

$= 108°$

$\angle APD = 180° - 145°$ (adj. $\angle$s on a str. line)

$= 35°$

$\angle BAD = 180° - 108° - 35°$ ($\angle$ sum of a $\triangle$)

$= 37°$

11. **(i)** $\angle BAC = 180° - 43° - 28°$ ($\angle$ sum of a $\triangle$)

$= 109°$

Reflex $\angle BOC = 2 \times 109°$ ($\angle$ at centre $= 2\ \angle$ at circumference)

$= 218°$

Obtuse $\angle BOC = 360° - 218° = 142°$

$\angle OBC = \angle OCB$

$= \dfrac{180° - 142°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 19°$

$\angle OBA = 19° + 43° = 62°$

(ii) $\angle OCA = 19° + 28°$

$= 47°$

12. **(i)** $\angle PWS = 90°$ (rt. $\angle$ in semicircle)

$\angle PWR = 90° - 26°$

$= 64°$

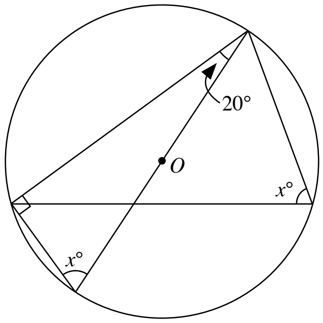
(ii) $OW = OS$ (radii of circle)

$\angle OSW = 26°$

$\angle SPW = 180° - 90° - 26°$ ($\angle$ sum of a $\triangle$)

$= 64°$

13.



$x° = 180° - 20° - 90°$ (rt. $\angle$ in semicircle, $\angle$ sum of a $\triangle$)

$= 70°$

$\therefore x = 70$

14. In $\triangle ABD$,

$\angle BAD = 90°$

If $\angle BAD$ is a right angle in a semicircle, then BD is the diameter of the circle.

In $\triangle BCD$,

$\angle BCD = 90°$ (rt. $\angle$ in semicircle)

By Pythagoras' Theorem,

$BD^2 = BC^2 + CD^2$

$= 6^2 + 8^2$

$= 100$

$BD = \sqrt{100} = 10$ cm

Area of circle $= \pi \times 5^2$

$= 78.5$ cm^2 (to 3 s.f.)

15. $\angle BXQ = 24°$ ($\angle$s in same segment)

$\angle AQB = 90°$ (rt. $\angle$ in semicircle)

$\angle BQP = 90°$

$\angle BQX = 180° - 24° - 35° - 90°$ ($\angle$ sum of a $\triangle$)

$= 31°$

16. $\angle ACB = 54°$ ($\angle$s in same segment)

$\angle BCP = 180° - 58° - 54°$ (adj. $\angle$s on a str. line)

$= 68°$

$\angle APD = 180° - 80° - 68°$ ($\angle$ sum of a $\triangle$)

$= 32°$

17. $\angle ADC = 90°$ (rt. $\angle$ in semicircle)

$\angle DCE = \angle DEC$

$= \dfrac{180° - 90°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 45°$

$\angle BCD = 180° - 45°$ (adj. $\angle$s on a str. line)

$= 135°$

$\angle BAD = 180° - 135°$ ($\angle$s in opp. segments)

$= 45°$

18. In $\triangle ABC$,

$\angle ACB = 90°$ (rt. $\angle$ in semicircle)

$\angle ABC = 180° - 90° - 35°$ ($\angle$ sum of a $\triangle$)

$= 55°$

$\angle ADC = 180° - 55°$ ($\angle$s in opp. segments)

$= 125°$

19. **(i)** $\angle ABD = 45°$ ($\angle$s in alt. segment)

$\angle ABC = \dfrac{1}{2} \times 130°$ ($\angle$ at centre $= 2 \angle$ at circumference)

$= 65°$

$\angle CBD = 65° - 45°$

$= 20°$

(ii) $\angle OAT = 90°$ (tangent $\perp$ radius)

$\angle AOT = \dfrac{130°}{2}$

$= 65°$

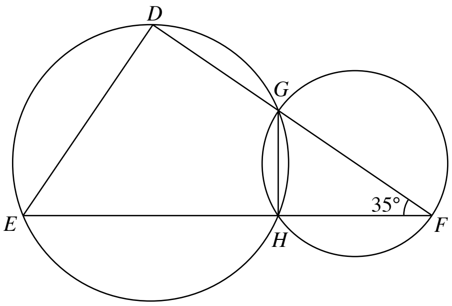
$\angle OTA = 180° - 65° - 45°$ ($\angle$ sum of $\triangle$)

$= 25$

$\angle ATC = 2 \times 25°$

$= 50°$

20.



(i) $\angle GHF = 90°$ (rt. $\angle$ in semicircle)

$\angle GHE = 90°$

$\angle EDG = 180° - 90°$ ($\angle$s in opp. segments)

$= 90°$

(ii) $\angle DEF = 180° - 90° - 35°$ ($\angle$ sum of a $\triangle$)

$= 55°$

21. Reflex $\angle ROQ = 2 \times 110°$ ($\angle$ at centre $= 2 \angle$ at circumference)

$= 220°$

Obtuse $\angle ROQ = 360° - 220°$

$= 140°$

$\angle QPS = 180° - 140°$ ($\angle$s in opp. segments)

$= 40°$

22. **(i)** $\angle PAX = 26°$ ($\angle$s in same segment)

$\angle YAQ = 26°$ (vert. opp. $\angle$s)

$\angle YBQ = 26°$ ($\angle$s in same segment)

$26° + 58° = 23° + 26° + \angle AQB$ (ext. $\angle$ of $\triangle$)

$\angle AQB = 35°$

(ii) $\angle YQA = 23°$ ($\angle$s in same segment)

$\angle AYQ = 180° - 23° - 26°$ ($\angle$ sum of a $\triangle$)

$= 131°$

23. **(a)** $A \leftrightarrow A$

$B \leftrightarrow C$

$O \leftrightarrow D$

$B\hat{A}O = C\hat{A}D$ (common angle)

$O\hat{B}A = D\hat{C}A = 90°$ (rt. $\angle$ in semicircle)

$A\hat{O}B = A\hat{D}C$

$\therefore$ $\triangle ABO$ is similar to $\triangle ACD$ (2 pairs of corr. $\angle$s equal).

(b) **(i)** OA = radius of larger circle

$= 2 \times 4$

$= 8$ cm

OC = radius of larger circle

$= 8$ cm

(ii) $\dfrac{OA}{DA} = \dfrac{OB}{CD}$

$\dfrac{8}{16} = \dfrac{4.5}{CD}$

$8CD = 72$

$CD = 9$ cm

24. **(i)** $P \leftrightarrow P$

$A \leftrightarrow B$

$D \leftrightarrow Q$

$A\hat{P}Q = B\hat{P}Q$ (common angle)

$A\hat{D}P = B\hat{Q}P$ (corr. $\angle$s, $BQ \,//\, AD$)

$P\hat{A}D = P\hat{B}Q$ (corr. $\angle$s, $BQ \,//\, AD$)

$\therefore$ $\triangle PAD$ is similar to $\triangle PBQ$ (3 pairs of corr. $\angle$s equal).

(ii) $P \leftrightarrow P$

$C \leftrightarrow A$

$B \leftrightarrow D$

$C\hat{P}B = A\hat{P}D$ (common angle)

$C\hat{P}B = 180° - A\hat{D}C$ ($\angle$s in opp. segments)

$A\hat{D}P = 180° - A\hat{D}C$ (adj. $\angle$s on a str. line)

$\therefore$ $C\hat{B}P = A\hat{D}P$

$\therefore$ $\triangle PCB$ is similar to $\triangle PAD$ (2 pairs of corr. $\angle$s equal).

25. $\angle ACD = 180° - 90° - 18°$ ($\angle$ sum of a $\triangle$)

$= 72°$

$\angle AEB = 72°$ ($\angle$s in same segment)

$\angle ABE = 90°$ (rt. $\angle$ in semicircle)

$\angle BAE = 180° - 90° - 72°$ ($\angle$ sum of a $\triangle$)

$= 18°$

26. $\angle CRQ = \angle QBC = \dfrac{70°}{2} = 35°$ ($\angle$s in same segment)

$\angle CRP = \angle PAC = \dfrac{50°}{2} = 25°$ ($\angle$s in same segment)

$\angle R = 35° + 25° = 60°$

$\angle PQB = \angle PAB = \dfrac{50°}{2} = 25°$ ($\angle$s in same segment)

$\angle RQB = \angle RCB = \dfrac{60°}{2} = 30°$ ($\angle$s in same segment)

$\angle Q = 25° + 30° = 55°$

$\angle P = 180° - 60° - 55°$ ($\angle$ sum of a $\triangle$)

$= 65°$

27. (i) $A \leftrightarrow C$

$C \leftrightarrow B$

$K \leftrightarrow K$

$A\hat{K}C = C\hat{K}B = 90°$

$A\hat{C}B = 90°$ (rt. $\angle$ in semicircle)

$C\hat{A}K = 180° - 90° - C\hat{B}K$ ($\angle$ sum of $\triangle$)

$B\hat{C}K = 180° - 90° - C\hat{B}K$ ($\angle$ sum of $\triangle$)

$\therefore C\hat{A}K = B\hat{C}K$

$\therefore \triangle ACK$ is similar to $\triangle CBK$ (2 pairs of corr. $\angle$s equal).

(ii) $\dfrac{CK}{BK} = \dfrac{AK}{CK}$

i.e. $\dfrac{10}{BK} = \dfrac{12}{10}$

$12BK = 100$

$BK = 8\dfrac{1}{3}$ cm

$AB = 12 + 8\dfrac{1}{3} = 20\dfrac{1}{3}$ cm

Radius of circle $= \dfrac{1}{2} \times 20\dfrac{1}{3}$

$= 10\dfrac{1}{6}$ cm

Review Exercise 13

1. (a) $x° = 25°$ ($\angle$s in alt. segment)

$y° = 90° - 65°$ (rt. $\angle$ in semicircle)

$= 25°$

$\therefore x = 25, y = 25$

(b) $\angle OAP = 90°$ (tangent $\perp$ radius)

$90° - 24° - 32° = 34°$

$x° = 34°$ ($\angle$s in same segment)

$y° = 90° + 24°$ (rt. $\angle$ in semicircle, $\angle$s in same segment)

$= 114°$

$\therefore x = 34, y = 114$

(c) $\angle OAP = 90°$ (tangent $\perp$ radius)

$\angle TOA = 180° - 90° - 33°$ ($\angle$ sum of $\triangle$)

$= 57°$

$x° = \dfrac{57°}{2}$ (ext. $\angle$ of $\triangle$, base $\angle$s of isos. $\triangle$)

$= 28.5°$

$y° = \dfrac{1}{2} \times 33°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 16.5°$

$\therefore x = 28.5, y = 16.5$

(d) $x° = 64°$ ($\angle$s in alt. segment)

$180° - 64° = 116°$ (adj. $\angle$s on a str. line)

$y° = 180° - 116° - 26°$ ($\angle$ sum of $\triangle$)

$= 38°$

$\therefore x = 64, y = 38$

(e) $\angle OAP = 90°$ (tangent $\perp$ radius)

$\angle AOP = 2 \times 32°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 64°$

$x° = 180° - 90° - 64°$ ($\angle$ sum of $\triangle$)

$= 26°$

$y° = 180° - 32°$ (adj. $\angle$s on a str. line)

$= 148°$

$\therefore x = 26, y = 148$

(f) $x° = 90°$ (rt. $\angle$ in semicircle)

$y° = 180° - (180° - 90° - 28°)$ ($\angle$ sum of $\triangle$, $\angle$s in opp. segments)

$= 118°$

$\therefore x = 90, y = 118$

2. (a) $\angle OAP = 90°$ (tangent $\perp$ radius)

$x° = 180° - 90° - 20°$ ($\angle$ sum of $\triangle$)

$= 70°$

$\dfrac{180° - 70°}{2} = 55°$ (base $\angle$s of isos. $\triangle$)

$y° = 90° - 55° = 35°$

$\therefore x = 70, y = 35$

(b) $\angle AOP = 180° - 90° - 22° = 68°$ ($\angle$ sum of $\triangle$, tangent $\perp$ radius)

$x° = \dfrac{68°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 34°$

$y° = 34° + 22° = 56°$

$\therefore x = 34, y = 56$

(c) $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)

$x° + 48° + 90° + 90° = 360°$ ($\angle$ sum of quadrilateral)

$x° + 228° = 360°$

$x° = 132°$

Reflex $\angle AOB = 360° - 132°$

$= 228°$

$y° = \dfrac{1}{2} \times 228°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 114°$

$\therefore x = 132, y = 114$

(d) $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)

Reflex $\angle AOB = 360° - 150°$

$= 210°$

$x° = \dfrac{1}{2} \times 210°$ ($\angle$ at centre = 2 $\angle$ at circumference)

$= 105°$

$y° + 150° + 90° + 90° = 360°$

$y° + 330° = 360°$

$y° = 30°$

$\therefore x = 105, y = 30$

(e) $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)

$\angle AOB = 360° - 90° - 90° - 50°$ ($\angle$ sum of quadrilateral)

$= 130°$

$y° + y° + 130° = 180°$ ($\angle$ sum of $\triangle$)

$2y° = 50°$

$y° = 25°$

In $\triangle OBP$,

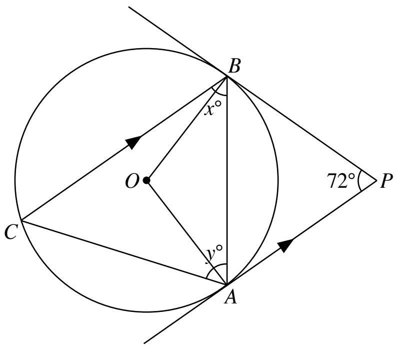
$$\tan 25° = \frac{3}{x}$$

$$x = \frac{3}{\tan 25°}$$

$= 6.43$ (to 3 s.f.)

$\therefore x = 6.43, y = 25$

(f)



$$\angle PAB = \frac{180° - 72°}{2} \text{ (base } \angle \text{s of isos. } \triangle)$$

$= 54°$

$x° = 54°$ (alt. $\angle$s)

$\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)

$\angle BOA = 360° - 72° - 90° - 90°$ ($\angle$ sum of quadrilateral)

$= 108°$

$\angle BCA = \frac{1}{2} \times 108°$ ($\angle$ at centre $= 2 \angle$ at circumference)

$= 54°$

$y° = 180° - 54° - 54° = 72°$ ($\angle$ sum of $\triangle$)

$\therefore x = 54, y = 72$

3. **(a)** $x° = \frac{1}{2} \times 124°$ ($\angle$ at centre $= 2 \angle$ at circumference)

$= 62°$

$y° = 180° - 62°$ ($\angle$s in opp. segments)

$= 118°$

$\therefore x = 62, y = 118$

(b) $x° = 2 \times 58°$ ($\angle$ at centre $= 2 \angle$ at circumference)

$= 116°$

$\frac{180° - 116°}{2} = 32°$ (base $\angle$s of isos. $\triangle$)

$y° = 180° - 12° - 32° - 32° - 58°$ ($\angle$ sum of $\triangle$)

$= 46°$

$\therefore x = 116, y = 46$

(c) $x° = \frac{1}{2} \times 230°$ ($\angle$ at centre $= 2 \angle$ at circumference)

$= 115°$

Reflex $\angle O = 360° - 230° = 130°$

$y° + y + 115° + 130° = 360°$

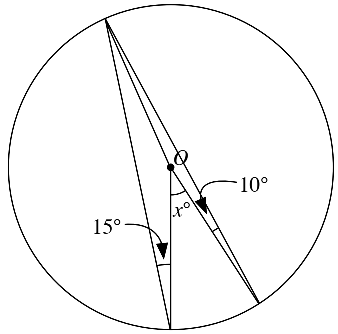
$2y° + 245 = 360°$

$2y° = 115$

$y = 57.5°$

$\therefore x = 115, y = 57.5$

(d)



$\angle$ at circumference $= 15° + 10° = 25°$

$x° = 2 \times 25°$ ($\angle$at centre $= 2 \angle$ at circumference)

$= 50°$

$\therefore x = 50$

(e) $x° = 90° - 18°$ (rt. $\angle$ in semicircle)

$= 72°$

$y° = 180° - 80° - 72°$ ($\angle$ sum of $\triangle$)

$= 28°$

$\therefore x = 72, y = 28$

(f) $x° = 180° - 90° - 42°$ (rt. $\angle$ in semicircle, $\angle$ sum of $\triangle$)

$= 48°$

$y° = 48° - 26°$ (ext. $\angle$ of $\triangle$)

$= 22°$

$\therefore x = 48, y = 22$

4. **(a)** $x° = 180° - 90° - 49°$ (rt. $\angle$ in semicircle, $\angle$s in same segment)

$= 41°$

$\therefore x = 41$

(b) $y° = 108° - 78° = 30°$ (ext. $\angle$ of $\triangle$)

$180° - 108° = 72°$ (supplementary $\angle$s)

$x° = 180° - 72° - 30°$ ($\angle$s in opp. segments)

$= 78°$

$\therefore x = 78, y = 30$

(c) $x° = 180° - 72°$ ($\angle$s in opp. segments)

$= 108°$

$y° = 180° - 36°$ ($\angle$s in opp. segments)

$= 144°$

$\therefore x = 108, y = 144$

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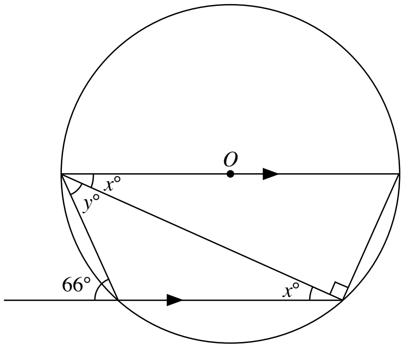
(d)



$180° - 66° = 114°$ (adj. ∠s on a str. line)

$180° - 114° = 66°$ (∠s in opp. segments)

$x° = 180° - 90° - 66°$ (∠ sum of △)

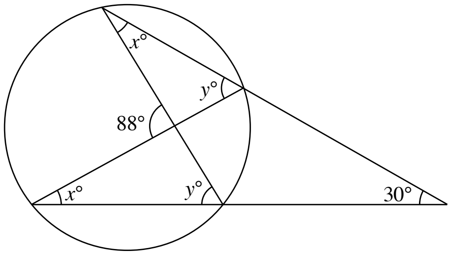
$= 24°$

$y° = 66° - 24°$ (alt. ∠s)

$= 42°$

$\therefore x = 24, y = 42$

(e)



$x° + y° = 88°$

$x° + (180° - y°) + 30° = 180°$ (∠ sum of △)

$y° = x° + 30°$

$x° + x° + 30° = 88°$

$2x° = 58°$

$x = 29°$

$y° = 29° + 30° = 59°$

$\therefore x = 29, y = 59$

(f) $y° = 180° - 90°$ (∠s in opp. segments)

$= 90°$

$x° = 180° - 90° - 48°$ (∠ sum of △)

$= 42°$

$\therefore x = 42, y = 90$

(g) $180° - 92° - 58° = 30°$ (adj. ∠s on a str. line)

$y° = 180° - 105° - 30°$ (∠s in same segment, adj. ∠s on a str. line)

$= 45°$

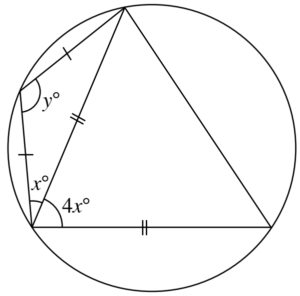
$x° = y° + 58°$ (ext. ∠ of △)

$= 45° + 58°$

$= 103°$

$\therefore x = 103, y = 45$

(h)



$x° + x° + y° = 180°$ (∠ sum of isos. △)

$2x° + y° = 180°$

$\dfrac{180° - 4x°}{2} = (90 - 2x)°$ (base ∠ of isos. △)

$x° + 4x° + x° + (90 - 2x)° = 180°$ (∠s in opp. segments)

$4x° + 90° = 180°$

$4x° = 90°$

$x° = 22.5°$

$2 \times 22.5° + y° = 180°$

$y° = 180° - 45°$

$= 135°$

$\therefore x = 22.5, y = 135$

5. $\angle PSR + x° + y° = 180°$ (∠ sum of a △)

$\angle PSR + \angle PQR = 180°$ (∠s in opp. segments)

$\angle PSR + \angle PQR = \angle PSR + x° + y°$

$\therefore \angle PQR = x° + y°$

6. $\angle BEF = 180° - 40°$ (adj. ∠s on a str. line)

$= 140°$

$\angle BAF = 180° - 140°$ (∠s in opp. segments)

$= 40°$

$\angle AFB = 90°$ (rt. ∠ in semicircle)

$\angle ABF = 180° - 90° - 40°$ (∠ sum of a △)

$= 50°$

$\angle ABE = 40° + 30° = 70°$

$\therefore \angle EBF = 70° - 50°$

$= 20°$

7. **(i)** $\angle AOC = 180° - 2 \times 66°$ (∠ sum of a △)

$= 48°$

$\angle CQA = \frac{1}{2} \times 48°$ (∠ at centre = 2 ∠ at circumference)

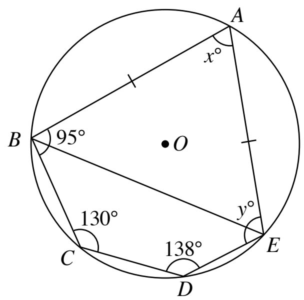
$= 24°$

(ii) $\angle AOQ = 66° + 32° = 98°$ (ext. ∠ of △)

$\angle QCA = \frac{1}{2} \times 98°$ (∠ at centre = 2 ∠ at circumference)

$= 49°$

8.



$\angle CBE = 180° - 138°$ ($\angle$s in opp. segments)
$= 42°$
$\angle ABE = 95° - 42° = 53°$
$\therefore x° = 180° - 53° - 53°$ ($\angle$ sum of an isos. $\triangle$)
$= 74°$
$\angle BED = 180° - 130°$ ($\angle$s in opp. segments)
$= 50°$
$\therefore y° = 50° + 53° = 103°$
$\therefore x = 74, y = 103$

9. (i) Let $\angle ADE = \angle DCA = x°$
$\angle ACB = 90°$ (rt. $\angle$ in semicircle)
$= \angle ADB$ ($\angle$s in same segment)
$\angle DAE = 180° - 90° - x°$ ($\angle$ sum of a $\triangle$)
$= (90 - x)°$
$\angle FEB = (90 - x)° + x°$
$= 90°$

(ii) $\angle EFC = 360° - 90° - 90° - 70°$
$= 110°$

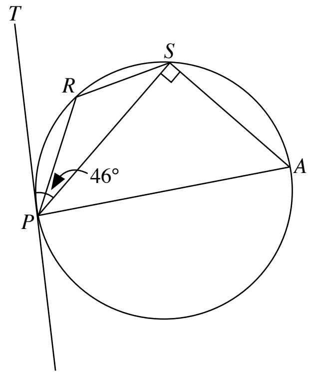
10. (i) $\triangle CEB$ is similar to $\triangle CFO$ (2 pairs of corr. $\angle$s equal).
$OF = OB = 2$ cm (radii of circle)
$$\frac{BE}{OF} = \frac{BC}{OC}$$
i.e. $\dfrac{BE}{2} = \dfrac{2}{4}$
$\therefore BE = \dfrac{1}{2} \times 2$
$= 1$ cm

(ii) $\angle BOF = \angle CBE = y°$
$\angle FAO = \dfrac{1}{2} y°$ (base $\angle$ isos. $\triangle$)

11. (i) Let the radius of the circle be r cm.
By Pythagoras' Theorem,
$OA^2 = AH^2 + OH^2$
$r^2 = 13^2 + OH^2$ — (1)
Pythagoras' Theorem,
$OP^2 = PK^2 + OK^2$
$r^2 = 11^2 + (4 + OH)^2$ — (2)
Substitute (1) into (2),
$13^2 + OH^2 = 11^2 + (4 + OH)^2$
$169 + OH^2 = 121 + OH^2 + 8OH + 16$
$169 = 137 + 8OH$
$8OH = 32$
$OH = 4$ cm

(ii) $r^2 = 13^2 + OH^2$
$= 13^2 + 4^2$
$= 185$
$r = \sqrt{185}$
$= 13.6$ (to 3 s.f.)
$\therefore$ The radius of the circle is 13.6 cm.

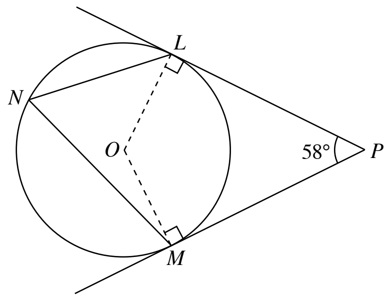
12.



(i) $\angle APT = 90°$ (tangent $\perp$ radius)
$\angle PSA = 90°$ (rt. $\angle$ in semicircle)
$\angle SPA = 90° - 46° = 44°$
$\angle PAS = 180° - 90° - 44°$ ($\angle$ sum of a $\triangle$)
$= 46°$

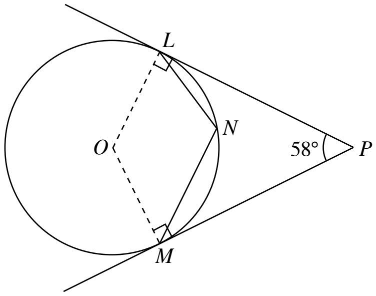
(ii) $\angle PRS = 180° - 46°$ ($\angle$s in opp. segments)
$= 134°$

13.



$\angle OLP = \angle OMP = 90°$ (tangent $\perp$ radius)
$\angle LOM = 360° - 90° - 90° - 58°$ ($\angle$ sum of quadrilateral)
$= 122°$
$\angle LNM = \dfrac{1}{2} \times 122°$ ($\angle$ at centre $= 2\ \angle$ at circumference)
$= 61°$

or



$\angle OLP = \angle OMP = 90°$ (tangent $\perp$ radius)

$\angle LOM = 360° - 90° - 90° - 58°$ ($\angle$ sum of quadrilateral)
$= 122°$

Reflex $\angle LOM = 360° - 122° = 238°$

$\angle LNM = \frac{1}{2} \times 238°$ ($\angle$ at centre = 2 $\angle$ circumference)
$= 119°$

14. (i) $\angle ABC = \frac{1}{2} \times 156°$ ($\angle$ at centre = 2 $\angle$ circumference)
$= 78°$

$\angle ADC = 180° - 78°$ ($\angle$s in opp. segments)
$= 102°$

$\angle PDC = 180° - 102$ (adj. $\angle$s on a str. line)
$= 78°$

(ii) $\angle PQC = 180° - 78°$ ($\angle$s in opp. segments)
$= 102°$

15. (i) $\angle APR = 180° - 64° - 54°$ ($\angle$ sum of a $\triangle$)
$= 62°$

(ii) $\angle BCD = 180° - 64°$ $\angle$s in opp. segments)
$= 116°$

$\angle DCQ = 180° - 62°$ ($\angle$s in opp. segments)
$= 118°$

$\angle BCQ = 360° - 118° - 116°$ ($\angle$s at a pt.)
$= 126°$

16. Since CA is the diameter of the circle,

$\angle ABC = 90°$ (rt. $\angle$ in semicircle)

$\angle ABE = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$

$\angle ADE = 180° - \angle ABE$ ($\angle$s in opp. segments)
$= 180° - 90°$
$= 90°$

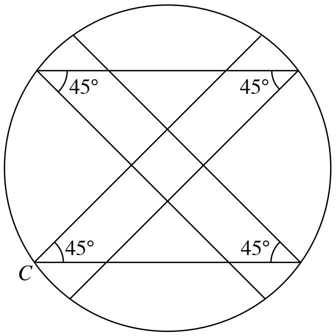
$\therefore$ $\angle ADE$ is a right angle.

Challenge Yourself

For the camera which scans an angle of 45°,

No. of cameras needed = $\frac{360°}{2 \times 45°} = 4$

3 more video cameras must be installed so that they will cover the entire hall. The 4 cameras can be placed at any point on the 4 different quadrants.



(a) For the camera which scans an angle of 35°,

No. of cameras needed = $\frac{360°}{2 \times 35°} \approx 5$

(b) For the camera which scans an angle of 60°,

No. of cameras needed = $\frac{360°}{2 \times 60°} = 3$

(c) For the camera which scans an angle of 90°,

No. of cameras needed = $\frac{360°}{2 \times 90°} = 2$

(d) For the camera which scans an angle of 100°,

No. of cameras needed = $\frac{360°}{2 \times 100°} \approx 2$

Revision Exercise D1

1. $P \leftrightarrow P$
$Q \leftrightarrow R$
$S \leftrightarrow T$
$PQ = PR$ (given)
Since $P\hat{S}T = P\hat{T}S$, $P\hat{T}R$ (adj. $\angle$s on a straight line).
$P\hat{Q}S = P\hat{R}T$ (base $\angle$s of isos. $\triangle$)
$\therefore \triangle PQS \equiv \triangle P\hat{R}T$ (ASA)

2. **(a)** $A \leftrightarrow Q$
$S \leftrightarrow R$
$Q \leftrightarrow A$
$A\hat{Q}S = A\hat{R}Q$ (opp. $\angle$s of a parallelogram)
$A\hat{Q}S = Q\hat{A}R$ (alt. $\angle$s, AR // SC)
$AQ = QA$ (common side)
$\therefore \triangle ASQ$ is congruent to $\triangle QRA$ (ASA)

(b) $A \leftrightarrow A$
$S \leftrightarrow B$
$Q \leftrightarrow C$
$A\hat{S}Q = A\hat{B}C$ (corr. $\angle$s, SQ // BC)
$S\hat{A}Q = B\hat{A}C$ (common angle)
$\therefore \triangle ASQ$ is similar to $\triangle ABC$ (2 pairs of corr. $\angle$s equal).

(c) Since $\triangle ABC$ is similar to $\triangle ADE$, then

$$\frac{AS}{AB} = \frac{SQ}{BC}$$

i.e. $\dfrac{2}{6} = \dfrac{SQ}{15}$

$6SQ = 30$
$\therefore SQ = 5$ cm

(d) $B \leftrightarrow P$
$C \leftrightarrow C$
$A \leftrightarrow Q$
$B\hat{C}A = P\hat{C}Q$ (common angle)
$A\hat{B}C = Q\hat{P}C$ (corr. $\angle$s, AB // QP)
$\therefore \triangle BCA$ is similar to $\triangle PCQ$ (2 pairs of corr. $\angle$s equal).
$R \leftrightarrow P$
$A \leftrightarrow C$
$Q \leftrightarrow Q$
$R\hat{Q}A = P\hat{Q}C$ (vert. opp. $\angle$s)
$A\hat{R}Q = C\hat{P}Q$ (alt. $\angle$s, AR // PC)
$\therefore \triangle RAQ$ is similar to $\triangle PCQ$ (2 pairs of corr. $\angle$s equal).

(e) **(i)** $\dfrac{\text{Area of } \triangle PCQ}{\text{Area of } \triangle ABC} = \left(\dfrac{10}{15}\right)^2 = \dfrac{4}{9}$

$$\text{Area } \triangle PCQ = \frac{4}{9} \times 36 = 16 \text{ cm}^2$$

(ii) Area of $\triangle BPQ = \dfrac{1}{2} \times \triangle PCQ$

$$= \frac{1}{2} \times 16 = 8 \text{ cm}^2$$

(iii) $\dfrac{\text{Area of } \triangle ASQ}{\text{Area of } \triangle ABC} = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

$$\text{Area of } \triangle ASQ = \frac{1}{9} \times 36 = 4 \text{ cm}^2$$

Area of quadrilateral $ASQR = 2 \times 4 = 8$ cm^2

3. **(i)** The scale used is 45 cm : 30 m, i.e. 45 cm : 3000 cm which is 3 : 200.

(ii) $\dfrac{\text{Area of model}}{\text{Actual area}} = \left(\dfrac{3}{200}\right)^2$

$$\frac{810}{\text{Actual area}} = \frac{9}{40\,000}$$

$$\text{Actual area} = \frac{810 \times 40\,000}{9} \text{ cm}^2 = 3\,600\,000 \text{ cm}^2 = 360 \text{ m}^2$$

(iii) $\dfrac{\text{Volume of model}}{\text{Actual volume}} = \left(\dfrac{3}{200}\right)^3$

$$\frac{162}{\text{Actual volume}} = \frac{27}{8\,000\,000}$$

$$\text{Actual volume} = \frac{162 \times 8\,000\,000}{27} \text{ cm}^3 = 48\,000\,000 \text{ cm}^3 = 48 \text{ m}^3$$

4. **(i)** $OB = OC$ (radii of circle)
$\angle BOC = 180° - 18° - 18°$ ($\angle$ sum of isos. $\triangle$)
$= 144°$
$\angle BAC = \dfrac{1}{2} \times 144°$ ($\angle$ at centre $= 2 \angle$ at circumference)
$= 72°$

(ii) $\angle ABP = \dfrac{180° - 48°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 66°$
$\angle OBP = 90°$ (tangent $\perp$ radius)
$\angle ABC = (90° - 66°) + 18° = 42°$

5. **(i)** $\angle BAE = 180° - 26° - 26°$ ($\angle$ sum of isos. $\triangle$)
$= 128°$

(ii) $\angle AEB = 26°$
$\angle BED = 118° - 26° = 92°$
$\angle BCD = 180° - 92°$ ($\angle$s in opp. segments)
$= 88°$

6. $\angle ABF = 90°$ (tangent $\perp$ radius)
$v = 180 - 90° - 20°$ ($\angle$ sum of $\triangle$)
$= 70°$
$\angle BDA = 90°$ (rt. $\angle$ in semicircle)
$\angle ABD = 180 - 90° - 20°$ ($\angle$ sum of $\triangle$)
$= 70°$
$u = 90° - 70° = 20°$
In $\triangle ABF$,
$90° + 20° + w° + 40° = 180°$ ($\angle$ sum of $\triangle$)
$150° + w° = 180°$
$w = 30°$

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$x = \angle ABD = 70°$

$y = 180° - 30° - 20°$ ($\angle$s in opp. segments)

$= 130°$

$z = 180° - 2(20° + 30°)$ ($\angle$ sum of isos. $\triangle$)

$= 80°$

$\therefore u = 20°, v = 70°, w = 30°, x = 70°, y = 130°$ and $z = 80°$

7. **(i)** $\angle RSQ = \angle RPQ = b$ ($\angle$s in same segment)

$\angle RQS = \angle RSQ = b$ (base $\angle$s of isos. $\triangle SRQ$)

(ii) $\angle PRS = b - a$ (ext. $\angle$ of $\triangle$)

(iii) $\angle SPR = b$

$\angle SPQ = b + b = 2b$

$\angle PST = 2b - a$

(iv) $\angle QRS = 180° - 2b$ ($\angle$ sum of $\triangle$)

(v) $\angle PSQ = 180° - b - (2b - a)$

$= 180° + a - 3b$

8. **(i)** By Pythagoras' Theorem,

$l^2 = 5^2 + 12^2$

$= 169$

$l = \sqrt{169}$

$= 13$ cm

Total surface area $= \pi \times 5^2 + \pi \times 5 \times 13$

$= 283$ cm^2 (to 3 s.f.)

(ii) Given that it takes 1 hour 40 minutes to burn the candle completely,

Volume of candle burnt after $12\frac{1}{2}$ minutes

$= \frac{12.5}{100} \times \frac{1}{3} \times \pi \times 5^2 \times 12$

$= 12\frac{1}{2}\pi$ cm^3

Let the radius of the candle burnt off be r cm.

Height of candle burnt off $= \frac{12}{5}r$ cm

$$\frac{1}{3} \times \pi \times r^2 \times \frac{12}{5}r = 12\frac{1}{2}\pi$$

$$\frac{4}{5}\pi r^3 = 12\frac{1}{2}\pi$$

$$r^3 = 15.625$$

$$r = \sqrt[3]{15.625}$$

$$= 2.5$$

Height of candle burnt off $= \frac{12}{5} \times 2.5$

$= 6$ cm

$\therefore h = 12 - 6 = 6$

(iii) By Pythagoras' Theorem,

$l^2 = 2.5^2 + 6^2$

$= 42.45$

$l = \sqrt{15.625}$

$= 6.5$ cm

Curved surface area of frustum $= \pi \times 5 \times 13 - \pi \times 2.5 \times 6.5$

$= 48.75\pi$ cm^2

Surface area of frustrum $= \pi \times 5^2 \times \pi \times 2.5^2 + 48.75\pi$

$= 251$ cm^2 (to 3 s.f.)

Revision Exercise D2

1. (i) $A \leftrightarrow B$
$P \leftrightarrow Q$
$C \leftrightarrow A$
$PA = QB$ (given)
$PC = QA$
$A\hat{P}C = B\hat{Q}A = 60°$
$\therefore \triangle APC \equiv \triangle BQA$ (ASA)

(ii) $C \leftrightarrow A$
$R \leftrightarrow P$
$B \leftrightarrow C$
$CR = AP = 4$ cm
$BR = CP = 12$ cm
$C\hat{R}B = A\hat{P}C = 60°$
$\therefore \triangle CRB \equiv \triangle APC \equiv BQA$ (ASA)
Since $R\hat{C}B = P\hat{A}C = Q\hat{B}A = R\hat{B}C = P\hat{C}A = Q\hat{A}B$,
$B\hat{A}C = A\hat{B}C = A\hat{C}B = 60°$.
$\therefore \triangle ABC$ is an equilateral triangle.

2. (i) $\triangle ABX$ is similar to $\triangle PQX$ (2 pairs of corr. $\angle$s equal).

$$\frac{PX}{AX} = \frac{PQ}{AB}$$

i.e. $\frac{PX}{3.6} = \frac{7.2}{4}$

$4PX = 25.92$

$\therefore PX = 6.48$ cm

$$\frac{QX}{BX} = \frac{PQ}{AB}$$

i.e. $\frac{QX}{4.2} = \frac{7.2}{4}$

$4QX = 30.24$

$\therefore QX = 7.56$ cm

(ii) $\dfrac{\text{Area of } \triangle ABX}{\text{Area of } \triangle PQX} = \left(\dfrac{4}{7.2}\right)^2 = \dfrac{25}{81}$

$\therefore$ The ratio of the area of $\triangle ABX$ to that of $\triangle PQX$ is 25 : 81.

3. (i) Let h_1 and A_1 be the height and the surface area of the smaller cup respectively and h_2 and A_2 be the height and the surface area of the larger cup respectively.

$$\frac{A_1}{A_2} = \frac{9}{64}$$

$$\left(\frac{h_1}{h_2}\right)^2 = \frac{9}{64}$$

$$\frac{h_1}{h_2} = \sqrt{\frac{9}{64}}$$

$$= \frac{3}{8}$$

$$\frac{25}{h_2} = \frac{3}{8}$$

$3h_2 = 200$

$h_2 = 66\frac{2}{3}$ cm

$\therefore$ The height of the larger cup is $66\frac{2}{3}$ cm.

(ii) Let V_1 be the volume of the smaller cup and V_2 be the volume of the larger cup.

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{2400}{V_2} = \left(\frac{3}{8}\right)^3$$

$$\frac{2400}{V_2} = \frac{27}{512}$$

$27V_2 = 1\ 228\ 800$

$V_2 = 45\ 511\frac{1}{9}$ cm^3

$\therefore$ The volume of the larger cup is $45\ 511\frac{1}{9}$ cm^3.

4. (i) $\triangle BCX$ is similar to $\triangle ZYX$ (2 pairs of corr. $\angle$s equal).

$$\frac{XY}{XC} = \frac{YZ}{CB}$$

i.e. $\frac{XY}{4.8} = \frac{15}{8}$

$8XY = 72$

$\therefore XY = 9$ cm

$\triangle ABC$ is similar to $\triangle AYZ$ (2 pairs of corr. $\angle$s equal).

$$\frac{AZ}{AC} = \frac{YZ}{BC}$$

i.e. $\frac{8.5 + CZ}{8.5} = \frac{15}{8}$

$68 + 8CZ = 127\frac{1}{2}$

$8CZ = 59\frac{1}{2}$

$\therefore CZ = 7\frac{7}{16}$ cm

(ii) $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle AYZ} = \left(\dfrac{8}{15}\right)^2 = \dfrac{64}{225}$

$$\frac{\text{Area of } \triangle AYZ}{\text{Area of trapezium } BCZY} = \frac{225}{225 - 64} = \frac{225}{161}$$

$\therefore$ The ratio of the area of $\triangle AYZ$ to that of the area of trapezium $BCZY$ is 225 : 161.

5. (i) $\angle OBE = 90°$ (tangent $\perp$ radius)
$\angle OBA = 90° - 50° = 40°$
$\angle AOB = 180° - 40° - 40°$ ($\angle$ sum of isos. $\triangle$)
$= 100°$

(ii) $\angle ACB = \frac{1}{2} \times 100°$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 50°$

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6. $\angle OAP = \angle OBP = 90°$ (tangent $\perp$ radius)

$\angle AOB = 360° - 56° - 90° - 90° = 124°$

$\angle OPB = \angle OPA = \dfrac{56°}{2} = 28°$

$\tan 28° = \dfrac{14}{PB}$

$PB = \dfrac{14}{\tan 28°}$

$= 26.33$ cm (to 4 s.f.)

Area of shaded region = Area of $AOBP$ – Area of sector AOB

$= 2 \times \dfrac{1}{2} \times 26.33 \times 14 - \dfrac{124°}{360°} \times \pi \times 14^2$

$= 157$ cm^2 (to 3 s.f.)

7. **(i)** $\angle ABC = 180° - 56° - 56°$ ($\angle$ sum of isos. $\triangle$)

$= 68°$

$\angle ACB = 10° - 42° - 68°$ ($\angle$ sum of $\triangle$)

$= 70°$

(ii) $\angle CQR = \dfrac{180° - 70°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 55°$

$\angle PQR = 180° - 55° - 56°$ (adj. $\angle$s on a str. line)

$= 69°$

(iii) $\angle ARP = \dfrac{180° - 42°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 69°$

$\angle PRQ = 180° - 55° - 69°$ (adj. $\angle$s on a str. line)

$= 56°$

$\angle RPQ = 180° - 56° - 69°$ ($\angle$ sum of $\triangle$)

$= 55°$

8. **(i)** Let h_1, t_1 and V_1 be the height of water, time taken to fill that amount and the volume of water respectively after 30 s; and h_2, t_2 and V_2 be the height of water, time taken to fill and the volume of water, when the pyramid is completely filled respectively.

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 = \frac{t_1}{t_2}$$

$$\left(\frac{h_1}{18}\right)^3 = \frac{0.5}{4}$$

$$= \frac{1}{8}$$

$$\frac{h}{18} = \sqrt[3]{\frac{1}{8}}$$

$$= \frac{1}{2}$$

$$2h = 18$$

$\therefore h = 9$ cm

(ii) Let A_1 be the surface area of the pyramid of water excluding the base and A_2 be the surface area of the pyramid containing the water excluding the base.

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{\text{Area in contact with water}}{\text{Area not in contact with water}} = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$$

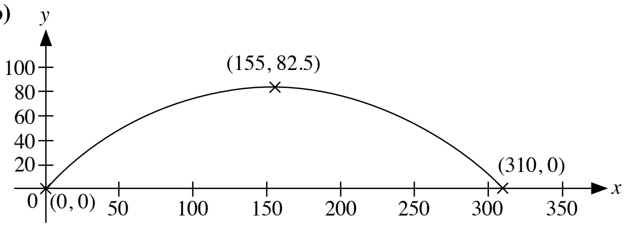
$\therefore$ The ratio is 1 : 3.

Problems in Real-World Contexts

1. **(a)** From Fig. **(b)**, we can state the coordinates of two more points on the dome since we know that the dome is 82.5 m tall and 310 m wide.

$\therefore$ The coordinates of two more points are the maximum point (155, 82.5) and the other minimum point (310, 0).

(b)



(c) **(i)** $y = ax^2 + bx + c$

At (0, 0),

$(0) = a(0)^2 + b(0)$

$c = 0$ —(1)

At (155, 82.5),

$82.5 = a(155)^2 + b(155)$ (since $c = 0$)

$82.5 = 155^2a + 155b$ —(2)

At (310, 0),

$0 = a(310)^2 + b(310)$ (since $c = 0$)

$0 = 310^2a + 310b$ —(3)

(ii) From (2),

$155b = 82.5 - 155^2a$

$$b = \frac{82.5 - 155^2 a}{155}$$

$$= \frac{33}{62} - 155a \qquad \text{—(4)}$$

Substitute (4) into (3),

$$310^2a + 31\left(\frac{33}{62} - 155a\right) = 0$$

$$310\left[310a + \left(\frac{33}{62} - 155a\right)\right] = 0$$

$$310a + \frac{33}{62} - 155a = 0$$

$$155a + \frac{33}{62} = 0$$

$$155a = -\frac{33}{62}$$

$$a = -\frac{33}{9610}$$

Substitute $a = -\frac{33}{9610}$ into (4),

$$b = \frac{33}{62} - 155\left(-\frac{33}{9610}\right)$$

$$= \frac{33}{31}$$

$\therefore a = -\frac{33}{9610}$, $b = \frac{33}{31}$ and $c = 0$

(iii) **Method 1**

Equation of the graph representing the dome:

$y = ax^2 + bx + \text{c}$

$\therefore y = -\frac{33}{9610}x^2 + \frac{33}{31}x$

Method 2

Since the coordinates where the curve cuts the x-axis are known, we can write the equation in the form $y = a(x - h)(x - k)$.

i.e. $y = ax(x - 310)$

Since the curve passes through the point (155, 82.5),

$82.5 = 155a(155 - 310)$

$$\frac{33}{62} = -155a$$

$$a = -\frac{33}{9610}$$

Equation of the graph representing the dome:

$$y = -\frac{33}{9610}x(x - 310)$$

Method 3

Since the coordinates of the maximum point of the curve are known, we can make use of 'completing the square' to find the equation,

i.e $y = a(x - 155)^2 + 82.5$

Since the curve passes through (0, 0),

$0 = a(0 - 155)^2 + 82.5$

$24\,025a = -82.5$

$$a = -\frac{33}{9610}$$

Equation of the graph representing the dome:

$$y = -\frac{33}{9610}(x - 155)^2 + 82.5$$

(d) One possibility is to centre the y-axis at the maximum height of the dome. We will then get the following three coordinates, (–155, 0), (0, 82.5) and (155, 0).

2. **(a)** Cost of 6-month road tax for a 1400 cc car

$= [250 + 0.375(1400 - 1000)] \times 0.782$

= S\$312.80

(b) Cost of 6-month road tax for a 3000 cc car

$= [475 + 0.75(3000 - 1600)] \times 0.782$

= S\$1192.55

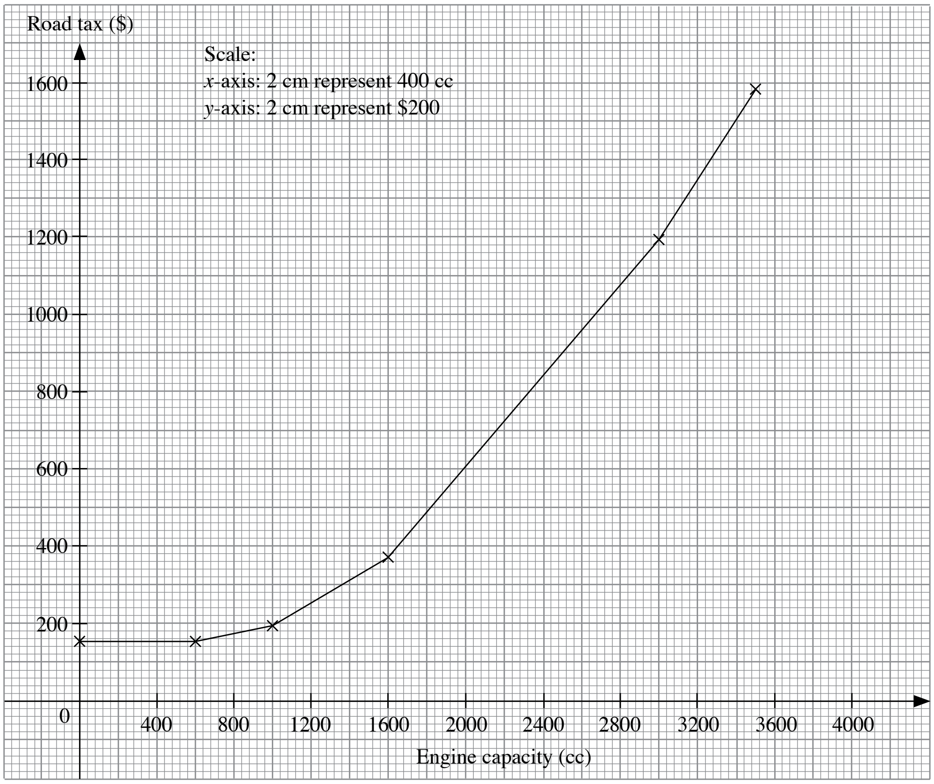
Road tax payable per annum

$= 2 \times 1192.55$

= S\$2385.10

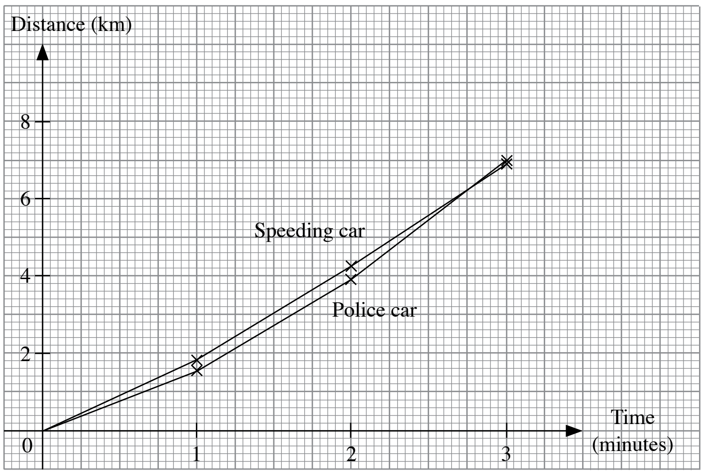
(c)

Engine capacity (cc)	0	600	1000	1600	3000	3500
Road tax ($)	156.40	156.40	195.50	371.45	1192.55	1583.55



3. (i)

	Speeding Car		Police Car	
Time, n^{th} minute	Distance travelled during the n^{th} minute (km)	Total distance travelled after n minutes (km)	Distance travelled during the n^{th} minute (km)	Total distance travelled after n minutes (km)
1	$110 \times \frac{1}{60} = 1.83$ (to 3 s.f.)	1.83	$95 \times \frac{1}{60} = 1.58$ (to 3 s.f.)	1.58
2	$145 \times \frac{1}{60} = 2.42$ (to 3 s.f.)	1.83 + 2.42 = 4.25	$140 \times \frac{1}{60} = 2.33$ (to 3 s.f.)	1.58 + 2.33 = 3.91
3	$160 \times \frac{1}{60} = 2.67$ (to 3 s.f.)	4.25 + 2.67 = 6.92	$185 \times \frac{1}{60} = 3.08$ (to 3 s.f.)	3.91 + 3.08 = 6.99



From the distance-time graph, the police car overtook the speeding car between the 2.5^{th} and the 3^{rd} second. As such, the police car will be able to overtake the speeding car and arrest the driver during the high-speed chase.

(ii) Assume that the flow of traffic on the expressway is smooth, i.e. both cars did not stop along the way and that the motions of both the police car and the speeding car are travelling along the same path.

4. (a) (i) <u>Polynomial function</u>

From the spreadsheet, the equation of the trendline is $y = 0.0037x^3 + 0.5184x^2 + 52.479x + 99.999$.

Using the equation of the trendline,

Amount of debt owed at the end of 1 year

$= 0.0037(12)^3 + 0.5184(12)^2 + 52.479(12) + 99.999$

$= \$810.79$ (to the nearest cent)

<u>Quadratic function</u>

From the spreadsheet, the equation of the trendline is $y = 0.557x^2 + 52.363x + 100.09$.

Using the equation of the trendline,

Amount of debt owed at the end of 1 year

$= 0.557(12)^2 + 52.363(12) + 100.09$

$= \$808.65$ (to the nearest cent)

<u>Linear function</u>

From the spreadsheet, the equation of the trendline is $y = 56.262x + 94.893$.

Using the equation of the linear trendline,

Amount of debt owed at the end of 1 year

$= 56.262(12) + 94.893$

$= \$770.04$ (to the nearest cent)

<u>Exponential function</u>

From the spreadsheet, the equation of the trendline is $y = 133.83e^{0.2057x}$.

Using the equation of the exponential trendline,

Amount of debt owed at the end of 1 year

$= 133.83e^{0.2057(12)}$

$= \$1579.67$ (to the nearest cent)

(ii) <u>Polynomial function</u>

From the spreadsheet, the equation of the trendline is $y = 0.0037x^3 + 0.5184x^2 + 52.479x + 99.999$.

Using the equation of the trendline,

Amount of debt owed at the end of 3 years

$= 0.0037(36)^3 + 0.5184(36)^2 + 52.479(36) + 99.999$

$= \$2833.72$ (to the nearest cent)

<u>Quadratic function</u>

From the spreadsheet, the equation of the trendline is $y = 0.557x^2 + 52.363x + 100.09$.

Using the equation of the trendline,

Amount of debt owed at the end of 3 years

$= 0.557(36)^2 + 52.363(36) + 100.09$

$= \$2707.03$

<u>Linear function</u>

From the spreadsheet, the equation of the trendline is $y = 56.262x + 94.893$.

Using the equation of the linear trendline,

Amount of debt owed at the end of 3 years

$= 56.262(36) + 94.893$

$= \$2120.33$ (to the nearest cent)

<u>Exponential function</u>

From the spreadsheet, the equation of the trendline is $y = 133.83e^{0.2057x}$.

Using the equation of the exponential trendline,

Amount of debt owed at the end of 3 years

$= 133.83e^{0.2057(36)}$

$= \$220\,085.27$ (to the nearest cent)

(b) (i) D at the end of 1 year $= 100(1.02)^{12} + 2550(1.02)^{12} - 2550$

$= \$810.84$ (to the nearest cent)

(ii) D at the end of 3 years $= 100(1.02)^{36} + 2550(1.02)^{36} - 2550$

$= \$2855.70$ (to the nearest cent)

(c) The polynomial trendline provides a better model for the estimation of values for the different periods of time as the polynomial curve is a better fit to the points than the rest of the trendlines.

Moreover, comparing the values obtained in **(a)** with those in **(b)**, we observe that the estimated values for D using the polynomial trendline are the closest to those values computed in **(b)**.

5. Assume that the quality of the watermelons is the same irrespective of their sizes and that all the watermelons are spherical in shape.

For the small watermelon of $r = 12$ cm,

Volume of the watermelon $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12)^3 = 2304\pi$ cm^3

Amount of watermelon that can be bought per dollar

$= \frac{2304\pi}{4.10}$

$= 1765$ cm^3 (to the nearest cm^3)

$\Rightarrow$ \$1 can buy approximately 1765 cm^3 of watermelon.

For the medium watermelon of $r = 14$ cm,

Volume of the watermelon $= \frac{4}{3}\pi(14)^3 = 3658\frac{2}{3}\pi$ cm^3

Amount of watermelon that can be bought per dollar

$= \frac{3858\frac{2}{3}\pi}{5.80}$

$= 1982$ cm^3 (to the nearest cm^3)

$\Rightarrow$ \$1 can buy approximately 1982 cm^3 of watermelon.

For the small watermelon of $r = 16$ cm,

Volume of the watermelon $= \frac{4}{3}\pi(16)^3 = 5461\frac{1}{3}\pi$ cm^3

Amount of watermelon that can be bought per dollar

$= \frac{5461\frac{1}{3}\pi}{7.60}$

$= 2256$ cm^3 (to the nearest cm^3)

$\Rightarrow$ \$1 can buy approximately 2256 cm^3 of watermelon.

$\therefore$ The large watermelon is the best buy.

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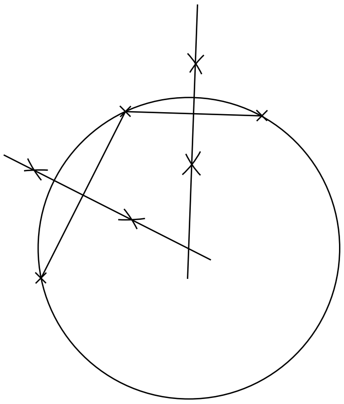
6. (i) Take any two points on the circumference of the circle and draw a line passing through the points. Then construct the perpendicular bisector of this line segment. Repeat for another two points. The point of intersection of the two perpendicular bisectors gives the centre of the circle. Hence, we are able to measure the radius to obtain the diameter.

(ii)



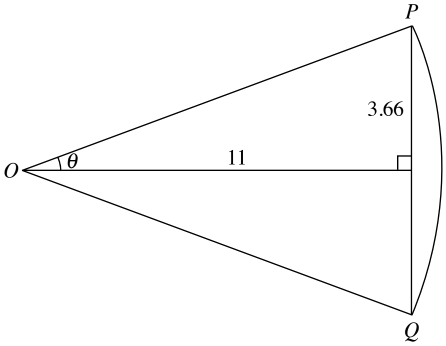
Radius of circle in scale drawing = 2 cm

Radius of plate = 2×4
= 8 cm

Diameter of plate = 8×2
= 16 cm

(iii) Area of original circular plate = $\pi(8)^2$
= 201 cm^2 (to 3 s.f.)

7. (i) For the penalty shootout, the ball is placed 11 m from the centre of the goal post.

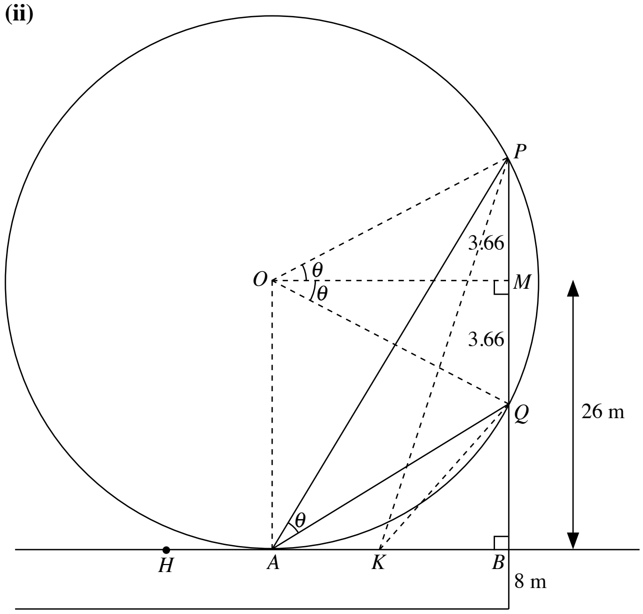


The widest angle for shooting is given by $\angle POQ$.

An arc PQ with centre O and radius OP is drawn.

$$\angle POQ = 2\left(\tan^{-1}\frac{3.66}{11}\right)$$
$$= 36.8° \text{ (to 1 d.p.)}$$

(ii)



The optimal position for the winger to shoot at the goal will be at the point A, which is the point of tangency to the circle passing through P, Q and A, and with radius equal to OA. In this case, the point O is such that

$OA = OP = 34 - 8 = 26$ m

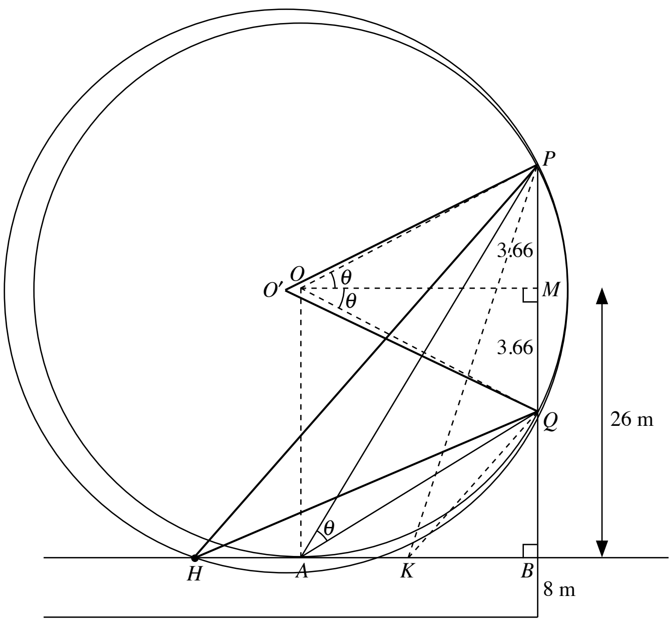
$$\angle PAQ = \frac{1}{2}\angle AOQ$$
$$= \angle POM \text{ (}\angle \text{ at centre} = 2\ \angle \text{ at circumference)}$$

$$\theta = \sin^{-1}\frac{3.66}{26} = 8.1° \text{ (to 1 d.p.)}$$

(iii) The size of the angle at the circumference of the circle, centre O and radius 26 m, will be equal to $\angle PAQ$.

Referring to the figure drawn in **(ii)**,

$$\angle PAQ = \frac{1}{2}\angle POQ \text{ (}\angle \text{ at centre} = 2\ \angle \text{ at circumference)}$$



However, if we consider a circle, centre O' and radius $O'P$, that passes through the point H,

$\angle PHQ = \frac{1}{2}\angle PO'Q < \frac{1}{2}\angle POQ = \angle PAQ$

$\therefore$ The angle for shooting is the largest when the circle passes through P and Q and HAB is a tangent to the circle at A.

8. **(i)** Total reliefs = $3000 + $2000 + $4000 + $23 000 + $1500
= $33 500

Chargeable income = $115 000 – $33 500
= $81 5000

			Tax
$81 500	→	First $80 000	: $3350
	→	Next $1500 at 11.5%	: $172.50

$\therefore$ Income tax payable = $3350 + $172.50
= $3522.50

(ii) Total reliefs
= $3000 + $5000 + 15% × $156 000 + 20% × $156 000 + $31 200
= $93 800

Chargeable income = $156 000 – $93 000
= $62 200

			Tax
$62 200	→	First $40 000	: $550
	→	Next $22 000 at 7%	: $1554

Income tax payable = $550 + $1554
= $2104

(iii) This is because people who earn very little should not be required to pay tax since they may find it difficult to make ends meet. On the other hand, people who earn more should be subjected to paying a higher amount of income tax as they have more disposable income.

(iv) This depends on the income tax systems of other countries. In some countries, people whose incomes fall within the lowest income bracket are taxed. Although this system may seem fair, as everyone is required to pay income tax, this may not be ideal as it creates a heavier financial burden on the poor.

Teachers may wish to discuss whether this type of income tax system benefits all the people in the country.

9. **(i)** Stamp duty = 3% × $500 000 – $5400
= $15 000 – $5400
= $9600

(ii) Stamp duty
= 1% × $180 000 + 2% × $180 000 + 3% × ($500 000 – $180 000 – $180 000)
= 3% × $180 000 + 3% × $140 000
= $5400 + $4200
= $9600

Yes, the answer obtained is the same as in **(i)**.

(iii) Let the selling price of the flat be $$x$.

Using the formula in **(i)**,

Stamp duty = 3% × $$x$ – $5400

Using the formula in **(ii)**,

Stamp duty
= 1% × $180 000 + 2% × $180 000 + 3% × $($x$ – 360 000)
= $5400 + 3% × $$x$ – 3% × $360 000
= $5400 + 3% × $$x$ – $10 800
= 3% × $$x$ – $5400

Hence, both formulae can be used to calculate the stamp duty that Mr Lee has to pay.

(iv) The minimum selling price of the flat is $360 000. For prices below this amount, the formula in **(i)** will not work because from **(iii)**, the stamp duty for the formula in **(ii)** is only equal to 3% × $$x$ – $5400 if $($x$ – 360 000) is greater than or equal to 0. If the selling price is less than $360 000 but more than $180 000, the way in which stamp duty is calculated will be different.

(v) Replace the 3% in both formulae with 4% for x 360 000.

Using the formula in **(i)**,

Stamp duty = 4% × $$x$ – $5400

Using the formula in **(ii)**,

Stamp duty
= 1% × $180 000 + 2% × $180 000 + 4% × $($x$ – 360 000)
= $5400 + 3% × $$x$ – $14 400
= 4% × $$x$ – $9000

Hence, it is just a coincidence that the formula in **(i)** contains $5400 which is the stamp duty for the first $360 000 in the formula in **(ii)**.

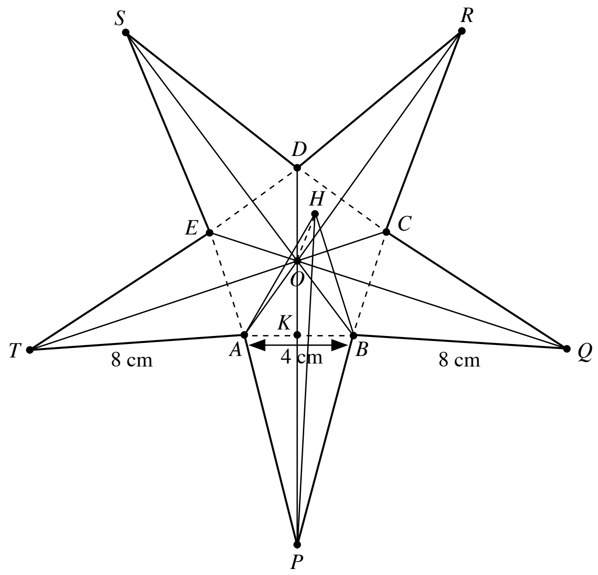
10. *Students can search on the Internet for 'paper star lantern' for step by step instructions on how to make a paper star lantern.*

<u>Guiding Questions</u>

1. Draw a regular pentagon $ABCDE$ with AB = 4 cm. Join AP, BP, BQ, CQ, CR, DR, DS, ES, ET and AT to form the 2-dimensional star.

Angle subtended at the centre of regular pentagon by each side is $\frac{360°}{5} = 72°$

Let AB and OP intersect at K, i.e. AK = 2 cm and $\angle AOK = 36°$.



$\sin 36° = \dfrac{AK}{OA}$

$= \dfrac{2}{OA}$

$OA = \dfrac{2}{\sin 36°}$

$= 3.402$ cm (to 4 s.f.)

$\tan 36° = \dfrac{BK}{OK}$

$= \dfrac{2}{OK}$

$OK = \dfrac{2}{\tan 36°}$

$= 2.752$ cm (to 4 s.f.)

Using Pythagoras' Theorem,

$KP^2 + BK^2 = BP^2$

$KP^2 + 2^2 = 8^2$

$KP^2 = 60$

$KP = \sqrt{60}$

$= 7.745$ cm (to 4 s.f.)

$OP = OK + KP$

$= 2.752 + 7.745$

$= 10.49$ cm (to 4 s.f.)

As H is 3 cm above O,

$AH = \sqrt{3^2 + 3.402^2}$

$= 4.536$ cm (to 4 s.f.)

$PH = \sqrt{10.49^2 + 3^2}$

$= 10.91$ cm (to 4 s.f.)

2. Consider $\triangle HAP$.

Using cosine rule,

$8^2 = 4.536^2 + 10.91^2 - 2(4.536)(10.91) \cos \angle AHP$

$\cos \angle AHP = 0.7638$ (to 4 s.f.)

$\angle AHP = 40.19°$ (to 2 d.p.)

Area of $\triangle HAP = \dfrac{1}{2} \times 4.536 \times 10.91 \times \sin 40.19°$

$= 15.96 \text{ cm}^2$ (to 4 s.f.)

Area of template in Fig. **(b)**, excluding flaps $= 4 \times 15.96$

$= 63.84 \text{ cm}^2$ (4 s.f.)

Assume that the width of each flap is 0.5 cm.

Area of 3 flaps $= 2 \times AH \times 0.5 \times AP \times 0.5$

$= 2 \times 4.536 \times 0.5 + 8 \times 0.5$

$= 8.536 \text{ cm}^2$ (to 4 s.f.)

Area of template in Fig. **(b)**, including flaps $= 63.84 + 8.536$

$= 72.37 \text{ cm}^2$ (to 4 s.f.)

Total area of surface required for a 3-dimensional star, including flaps $= 5 \times 72.37$

$= 361.9 \text{ cm}^2$ (to 4 s.f.)

Area of a sheet of A4-sized paper $= 21 \times 29.7$

$= 623.7 \text{ cm}^2$

Estimated number of templates in Fig. **(b)** obtainable from a sheet of A4-sized paper

$= \dfrac{623.7}{72.37}$

$= 8$ (round down to the nearest integer)

Mathematically, this is the maximum number of templates obtainable. However, the template is of an irregular shape and we may not be able to fit all 8 templates onto the rectangular A4-sized paper. We will have to physically arrange the templates on the sheet of paper to determine whether it is possible to fit all 8 templates on it. Alternatively, we could consider other sizes of paper such as A3-sized paper which may reduce wastage.

Students should check on the Internet for the possible sizes of paper and their respective costs. For example, a 40-sheet packet of A3-sized 120 gsm art paper of various colours costs between \$6 and \$7 while a 80-sheet packet of A3-sized 80 gsm art paper of various colours costs between \$6 and \$7. Students should consider the thickness of the paper for the paper star lantern to hold its shape as well as the level of transparency for light to pass through. Other considerations include how the templates should be fit onto the paper for optimal use of paper and how the lanterns should be packed for sale.

3. *No fixed answer.*

4. *No fixed answer.*