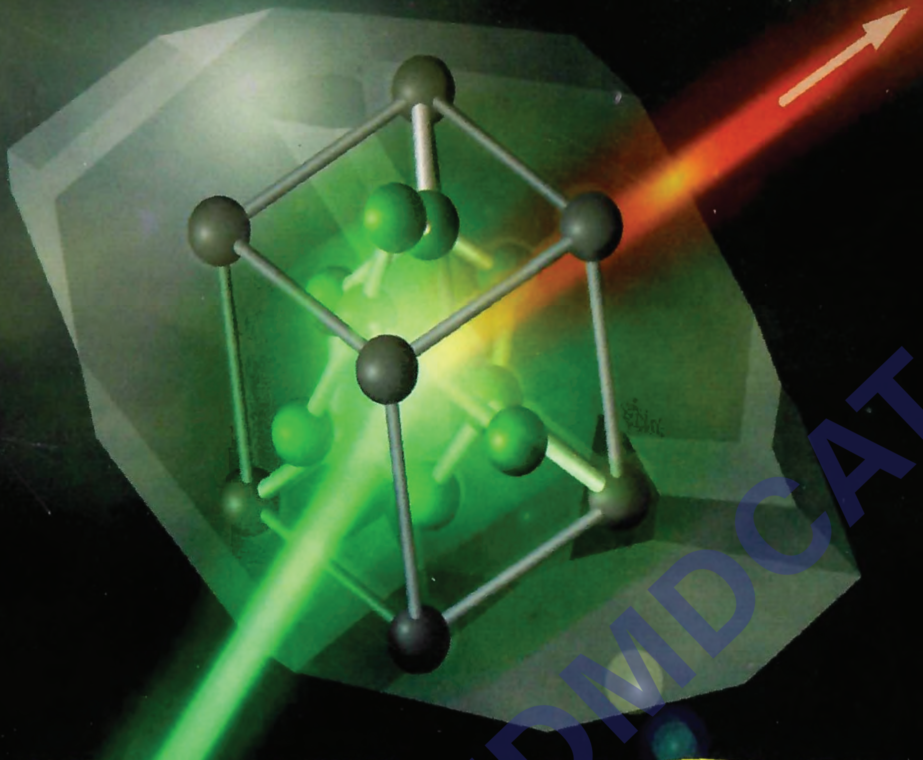
KIPS

ENTRY TESTS

SERIES

PREP BOOK



PHYSICS

NATIONAL MDCAT

- Topic-wise Complete Syllabus
- Comprehensive Course Revision
- Detailed Explanation of Topics
- Key Points, Tables, Flow Sheets & Diagrams
- Easy to Remember; Points to Ponder

A Kitab Dost Publication

CONTENT

TOPIC-1 FORCE AND MOTION

LEARNING OUTCOMES

COURSE CONTENT

- Displacement
- Velocity
- Displacement-time graph
- Acceleration
- Uniform acceleration
- Variable acceleration
- Graphical representation of acceleration with velocity time graph
- Newton's laws of motion
- Linear Momentum
- Law of conservation of momentum
- Collision
- Elastic collision
- Elastic collision in one dimension
- Elastic collision in one dimension under different cases
- Projectile motion
- Characteristics of projectile motion
- Time of flight
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- Effect of Air Resistance in Projectile Motion

DISPLACEMENT, VELOCITY

Rest and Motion

The concept of state of motion and rest are described relative to the observer.

- If a body does not change its state w.r.t surroundings, it is in the state of rest.
- If a body changes its state w.r.t surroundings, then it is in the state of motion.
- A moving body can possess both states of rest and motion, depending on the observer.

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g.. Motion of car on a straight road. Motion of freely falling body.	*e.g.* Motion of car on a circular turn. Motion of billiards ball.	*e.g.*. Motion of flying kite. Motion of flying insect.

Distance

The total actual path traversed by a body between its initial and final positions is called distance. It is a scalar quantity.

Displacement

The change in the position of a body in a certain direction from initial position to the final position is known as displacement. It is a vector quantity.

- Displacement may be positive, negative or zero but distance is always positive.

Speed

It is defined as distance traveled in one second. Speed is a scalar quantity.

Velocity

Time rate of change of displacement is called velocity. It is a vector quantity and may be positive or negative.

$$\vec{v} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta \vec{d}}{\Delta t}$$

$$\vec{v} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

Uniform Velocity

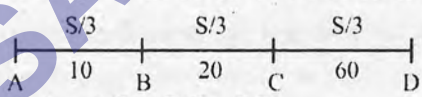
When a body covers equal displacements in equal interval of time, however small this time interval may be, then its velocity is said to be uniform.

- If a body travels with uniform velocity $\vec{v}_1$ for time t_1 and with uniform velocity v_2 for time t_2, then its average velocity will be

$$\vec{v} = \frac{\vec{s}_1 + \vec{s}_2}{t_1 + t_2} = \frac{\vec{v}_1 t_1 + \vec{v}_2 t_2}{t_1 + t_2}$$

Example

A car traveled the first third of distance S at speed of 10 m s^{-1}, the second third at a speed of 20 m s^{-1} and last third at a speed of 60 m s^{-1}. Determine average speed of car.



$$v_{av} = \frac{\text{Total distance traveled}}{\text{total time}} = \frac{AB + BC + CD}{t_1 + t_2 + t_3}$$

$$= \frac{S/3 + S/3 + S/3}{\frac{S/3}{10} + \frac{S/3}{20} + \frac{S/3}{60}}$$

$$= 18 \, \text{m s}^{-1}$$

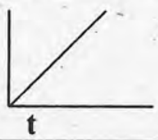
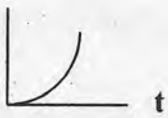
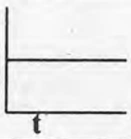
Instantaneous Velocity

The instantaneous velocity $\vec{v}$ of a body is defined as the limit of the ratio of change in position $\Delta \vec{d}$ (displacement) to the small-time interval Δt as Δt following an instant 't' approach zero:

$$\vec{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

DISPLACEMENT-TIME GRAPH

- The velocity at any instant is found by the slope of the displacement – time graph.
- For a body at rest, the displacement – time graph is a straight line parallel to the time axis.
- For a body moving with constant velocity, the displacement time graph is a straight line inclined to time axis.
- If slope increases, the body speeds up and if slope decreases, the body slows down.

Graph Shape	Slope	Velocity
	Constant positive	Velocity is constant
	positive	Velocity is increasing
	Constant (zero)	Velocity is zero
	negative	Velocity decreases

ACCELERATION

Rate of change of velocity is known as acceleration. It is a vector quantity. If $\Delta\vec{v}$ is the change in velocity in time Δt, then acceleration is given as

$$\vec{a}=\frac{\Delta\vec{v}}{\Delta t}$$

Acceleration may be positive or negative. Negative acceleration is termed as retardation. It indicates that the velocity of the body is decreasing with time.

Uniform Acceleration

If the velocity of a body changes by same amount in same interval of time, then the acceleration of the body is known as uniform acceleration.

- If a body travels with uniform acceleration a_1 for a time interval t_1 and with uniform acceleration a_2 for a time interval t_2, then the average acceleration $\vec{a}=\dfrac{a_1t_1+a_2t_2}{t_1+t_2}$

Instantaneous Acceleration

Acceleration of a body at a particular instant is known as instantaneous acceleration. It is obtained from average acceleration as Δt is made smaller and smaller till it approaches zero. Mathematically, instantaneous is the limit of the ratio of the change in velocity $\Delta\vec{v}$ to the time interval Δt as Δt following the instant t approaches zero.

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

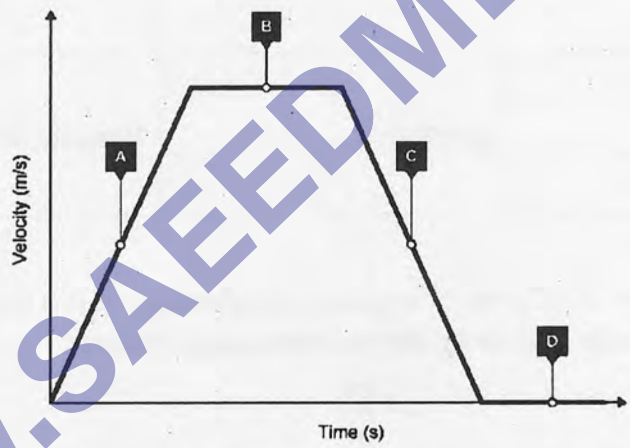
The change in velocity can occur due to change in speed or in direction or in both.

GRAPHICAL REPRESENTATION OF ACCELERATION WITH VELOCITY TIME GRAPH

- The velocity at any instant is found by the slope of the displacement time graph. The slope of velocity time graph gives acceleration i.e.

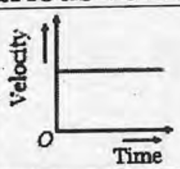
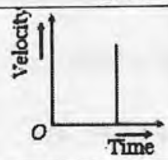
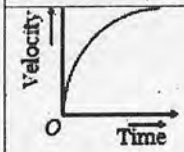
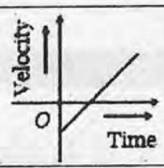
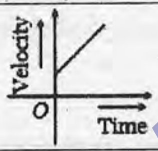
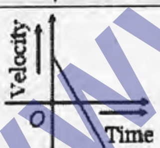
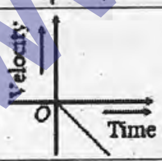
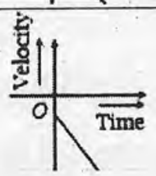
  $a = \frac{\Delta v}{\Delta t} = \tan\theta =$ Slope of graph.
- The area under the v-t graph gives the distance covered by object.
- For a body moving with constant velocity, the velocity – time graph is a straight line parallel to time axis.
- For a body moving with constant acceleration, the velocity – time graph is straight line inclined to time axis.

For Your Information:

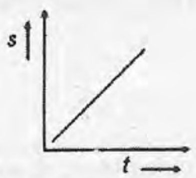
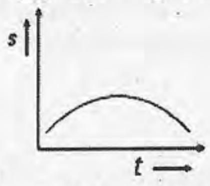
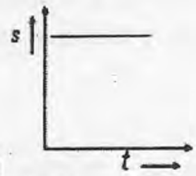
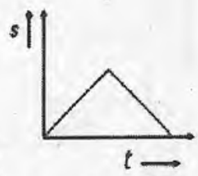


Section of graph	**Gradient**	**Velocity**	**Acceleration**
A	positive	increasing	positive
B	zero	constant	zero
C	negative	decreasing	negative
D	zero	zero (at rest)	zero

Various velocity – time graphs and their interpretation

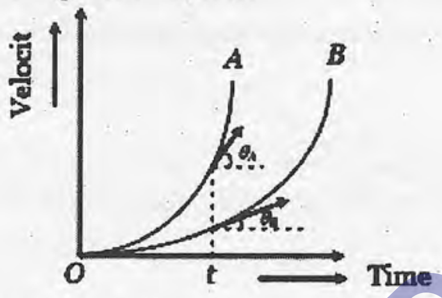
	$\theta = 0$, $a = 0$, v = constant i.e., line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90^o$, a = infinite, v = increasing i.e., line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible
	θ =constant, so a = constant and v is increasing uniformly with time i.e., line with constant slope represents uniform acceleration of the particle.
	θ increasing so acceleration increasing i.e., line bending towards velocity axis represent the increasing acceleration in the body.
	θ decreasing so acceleration decreasing i.e. line bending towards time axis represents the decreasing acceleration in the body.
	Positive constant acceleration because θ is constant and $< 90^o$ but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and $< 90^o$ but initial velocity of particle is positive
	Negative constant acceleration because θ is constant and $> 90^0$ but initial velocity of the particle is positive
	Negative constant acceleration because θ is constant and $> 90^o$ but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and $> 90^o$ but initial velocity of the particle is negative.

Example: Which of the following graph represents uniform motion

A)  B)  C)  D) 

Solution: A) When distance time graph is a straight line with constant slope than motion is uniform.

Example: Velocity-time graphs of two cars which start from rest at the same time, are shown in the figure. Graph shows, that



A) Initial velocity of A is greater than the initial velocity of B
B) Acceleration in A is increasing at lesser rate than in B
C) Acceleration in A is greater than in B
D) Acceleration in B is greater than in A

Solution: C) At a certain instant t slope of A is greater than B ($\theta_A > \theta_B$), so acceleration in A is greater than B

NEWTON'S LAWS OF MOTION

Newton's Laws of Motion

First Law of Motion (Law of Inertia): It states that everybody continues to be in state of rest or of uniform motion along a straight line unless it is compelled to change that state by an applied force.

- This law qualitatively defines the force.
- The inability of the body to change its state is called inertia. So, it is also known as the law of inertia of Galileo.
- Inertia resists change in the state of motion of the body.

Second Law of Motion (Force and Acceleration)

The effect of an applied force on a body is to cause it to accelerate in the direction of the force. The acceleration is in direct proportion to the force and is inversely proportional to the mass of the body.

OR

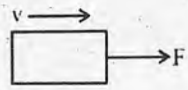
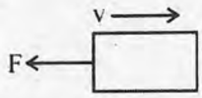
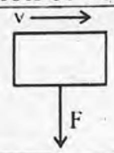
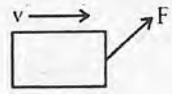
A force F acting on a body is equal to its change in momentum per second

$F \propto$ Mass of the body x Change in velocity per second $\propto$ ma

$\therefore$ $F = k\,ma$ (where k is a constant)

But $k = 1$ $\therefore$ $\mathbf{F = ma}$

- Weight: The weight of a body is equal to the force with which the body is attracted by the earth towards its centre.

When force acts in direction of velocity	When force acts opposite to direction of velocity	When force acts perpendicular to direction of velocity	When force acts at some angle to the direction of velocity
			
Speed increases and direction of motion remain same	Speed decreases and direction of motion remain same	Only direction changes and magnitude of velocity remain same	Both magnitude and direction of velocity changes

Third Law of Motion (Action and Reaction Forces)

It states that to every action, there is an equal and opposite reaction.

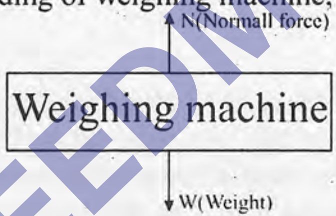
- Action-reaction forces always occur in pairs.
- If a body A exerts a force (action) on a body B, then B will exert an equal and opposite force (reaction) on A.

When a force acts on a body then the reaction acts normally to the surface of the body.

Example: A man of mass 60 kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10\ m/s^2$).

A) 600 N B) 400 N C) 200 N D) 100 N

Solution: A) For calculating the reading of weighing machine, we draw diagram.



$w = mg = 60 \times 10 = 600\,N$

LINEAR MOMENTUM

"The idea of linear momentum was introduced by Newton who defined it as product of mass and velocity of an object".

$\vec{p} = m\vec{v}$

- Linear momentum is a vector pointing along velocity
- Linear momentum depends upon

$p \propto v$ $p \propto m$

- $p = 0$ if $v = 0$, how massive the body may be.

SI unit of linear momentum are kg $m.s^{-1}$ or N s.

Dimension of momentum are $\left[MLT^{-1}\right]$

Newton's 2nd Law And Linear Momentum

Consider a body of mass m moving with an initial velocity $\vec{v}_i$ .Suppose an external force $\vec{F}$ acts upon it for time t after which velocity becomes $\vec{v}_f$. The acceleration $\vec{a}$ produced by this force is given by

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

By Newton's second law, the acceleration is given as

$$\vec{a} = \frac{\vec{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\vec{F}}{m} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

or

$$\vec{F} \times t = m\vec{v}_f - m\vec{v}_i \quad \text{.....................} \; \text{(i)}$$

Where $m\vec{v}_i$ is the initial momentum and $m\vec{v}_f$ is the final momentum of the body.

The equation (i) shows that change in momentum is equal to the product of force and the time for which force is applied. This form of the second law is more general than the form $\vec{F} = m\vec{a}$, because it can easily be extended to account for changes as the body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Equation (i) $\quad \vec{F} = \dfrac{m\vec{v}_f - m\vec{v}_i}{t}$

Thus, second law of motion can also be stated in terms of momentum as follows.
Time rate of change of momentum of a body equals the applied force.

Example: A player caught a cricket ball of mass 150g moving at a rate of 20 m/s. If the catching process is completed in 0.1s, the force of the blow exerted by the ball on the hand of the player is equal to:

A) 150 N B) 3 N C) 30 N D) 300 N

Solution: $F = \dfrac{mv}{t} = \dfrac{0.15 \times 20}{0.1}$

$F = 30\,N$

LAW OF CONSERVATION OF LINEAR MOMENTUM

In an isolated system $(p_i)_{total} = (p_f)_{total}$

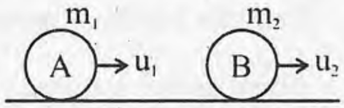
$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

The total linear momentum of an isolated system remains constant.

- Within the limits of experimental accuracy, it follows that the total momentum of A and B before collision = the total momentum after collision,
- If there is a system of particles free from external resultant force, $\vec{p}_1, \vec{p}_2, \vec{p}_3$..... being the linear momentum of its individual particle then

  $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... = \text{constant}$

  Thus, for a system of particles the total linear momentum cannot change, unless an external resultant force acts on the system.



$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

The principle of the conservation of linear momentum states that, if no external forces act on a system of colliding objects, the total momentum of the objects in a given direction **before collision = total momentum in same direction after collision**

Example: Two railway trucks of mass m and 3m move towards each in opposite directions with speeds 2v and v respectively. These trucks collide and stick together. What is the speed of the trucks after the collision?

A) $\frac{v}{4}$ B) $\frac{v}{2}$ C) v D) $\frac{5v}{4}$

Solution: A) Momentum before collision = momentum after collision

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$

$$m(2v) + 3m(-v) = (m + 3m)v'$$

$$-mv = 4mv'$$

$$v' = -\frac{1}{4}v$$

COLLISIONS

Collision is said to be taken place if some sort of interaction appears between bodies due to their closeness.

COLLISIONS

- Elastic collisions
  (K.E and linear momentum both are conserved)
- Inelastic collisions
  (only linear momentum is conserved)

Elastic Collision in One Dimension

- In every collision the law of conservation of momentum is always obeyed.
- The collisions during which the kinetic energy is conserved, together with the momentum are called elastic collisions.
- If the collisions are not elastic, they are said to be inelastic. During inelastic collision, some part of the kinetic energy is converted into other form of energy, such as, heat energy, sound energy etc.
- If the colliding particles stick together and move as a single unit after collision, the collision is said to be perfectly inelastic collision.

Perfectly Elastic Head–on–Collision

- Elastic collision in one dimension is that one, in which colliding bodies do not deviate from their line of motion, after the collision.
- In case of two bodies undergoing elastic collision in one dimension, we have

  $v_1 + v'_1 = v_2 + v'_2$ or $v_1 - v_2 = -(v'_1 - v'_2)$.

  Speed of approach = Speed of recession

  After collision,

  $$v'_1 = \frac{(m_1 - m_2)v_1}{(m_1 + m_2)} + \frac{2m_2v_2}{(m_1 + m_2)} \quad \textbf{or} \quad v'_2 = \frac{2m_1v_1}{(m_1 + m_2)} + \frac{(m_2 - m_1)v_2}{(m_1 + m_2)}$$

Elastic Collision In One Dimension Under Different Cases

Case I:	If	$m_1=m_2$	&	$v_2 \neq 0$
	then	$v'_1=v_2$	&	$v'_2=v_1$
Case II:	If	$m_1=m_2$	&	$v_2=0$
	then	$v'_1=0$	&	$v'_2=v_1$

In both cases I & II due to the same masses of the colliding bodies their velocities after collision got interchanged.

Case III:	If	$m_1<<m_2$	&	$v_2=0$
	then	$v'_1=-v$	&	$v'_2=0$
Case IV:	If	$m_1>>m_2$	&	$v_2=0$
	then	$v'_1=v_1$	&	$v'_2=2v_1$

Collision Between a Body and the Floor or Wall

- In this case, the floor or wall is considered as a body of infinite mass with zero velocity.
- If the body strikes the surface normally, it is returned also normally either with its initial speed or with reduced speed. If the speed is not reduced the collision is elastic.
  The change in momentum in this case is –2 mv

Examples:

(1) A particle of mass having velocity 'v' makes head on elastic collision with another particle of the same mass and initially at rest. The velocity of the first particle after the collision is

(a) v (b) –v

(c) $\frac{v}{2}$ (d) 0

Solution: If masses are same their velocities will alter, so velocity of 1st ball after collision = velocity of 2nd ball before collision = 0

(2) A particle of mass m moving with velocity v strikes a stationary particle of mass 2m and sticks to it. The speed of the system will be?

(a) $\frac{v}{2}$ (b) 2v

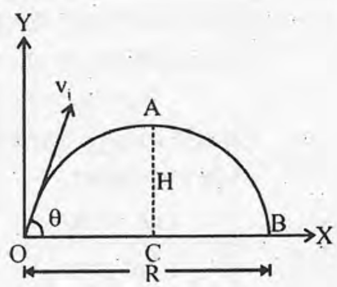
(c) $\frac{v}{3}$ (d) 3v

Solution: $mv+0=(m+2m)v'$

$$v'=\frac{mv}{3m}=\frac{v}{3}$$

PROJECTILE MOTION

Projectile Motion



- Projectile motion is a two-dimensional motion. The motion of the particle is constrained in a plane.
- When a particle is thrown obliquely near the earth's surface it moves in a parabolic path, provided the particle remains close to the surface of earth and the air resistance is negligible. This is an example of projectile motion.

Terms Used in Projectile Motion

Figure shows a particle projected from the point "O" with an initial velocity "v_i" at an angle θ with the horizontal.

- The point O is called the point of projection.
- The angle "θ" is called the "angle of projection".
- The distance OB is called the, horizontal range (R) or simply range, the vertical height AC is called maximum height (H) or Vertical Range.
- The total time taken by the particle in describing the path OAB is called the time of flight (T).
- Horizontal and vertical coordinates of projectile at time 't' are given as; $x = v_i \cos\theta\, t$ and $y = v_i \sin\theta\, t - \frac{1}{2}gt^2$
- Trajectory of projectile is parabola in the absence of air friction.
- If two identical balls are thrown simultaneously from same height, one vertically and other horizontally, then both falls to the earth simultaneously.

Time of flight:

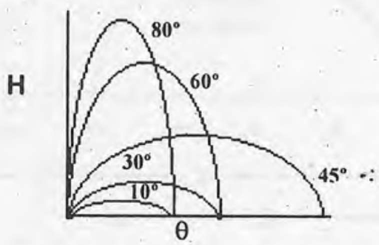
- Time to reach maximum height is given as; $t = \frac{v_i \sin\theta}{g}$
- Total time of flight is given as; $T = \frac{2v_i \sin\theta}{g}$

Maximum Height:

- Vertical range (height) is given as; $H = \frac{v_i^2 \sin^2\theta}{2g}$

Horizontal range:

- Range (horizontal) is a distance between point of projection and point at which it comes back to its level of projection. It is given as;
$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{v_i^2 (2\sin\theta\cos\theta)}{g}$$
- Maximum horizontal range is at angle **θ = 45°** and given as; $R_{max} = \frac{v_i^2}{g}$
- The relation between range and the height of the projectile is **R tanθ = 4H**
- With same initial velocity the range of projectile for two angles of projection will be equal if sum of the angles is equal to 90° i.e. $\theta_1 + \theta_2 = 90°$
- Variation in the range and height with angle of projection is shown with the following sketch for same speed of projectile.



- When angle of projection is 76° then range and maximum height of projectile are equal to each other.

Application to Ballistic Missile

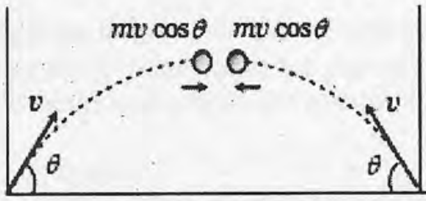
- An **un-powered** and **un-guided** missile is called ballistic missile
- Friction of air effects the horizontal and vertical motion of the missile
- Ballistic missiles are useful only for short ranges.

Powered and remote control guided missiles are used for long ranges and **precision.**

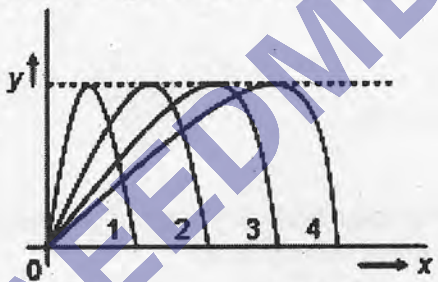
Example: Two equal masses (m) are projected at the same angle (θ) from two points separated by their range with equal velocities (v). The momentum at the point of their collision is

A) Zero B) 2 mv cosθ C) – 2 mv cosθ D) None of these

Solution: A) Both masses will collide at the highest point of their trajectory with equal and opposite momentum. So net momentum of the system will be zero



Example: Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first

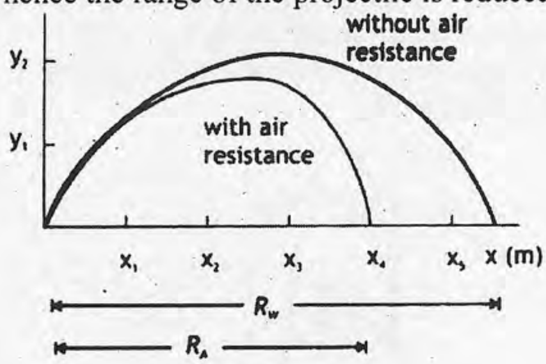


A) 1, 2, 3, 4 B) 2, 3, 4, 1 C) 3, 4, 1, 2 D) 4, 3, 2, 1

Solution: D) Range directly proportional to horizontal component of velocity. Graph 4 shows maximum range, so football possess maximum horizontal velocity in this case.

EFFECT OF AIR RESISTANCE IN PROJECTILE MOTION

Range with air resistance: Air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile is reduced.



Projectile motion

TOPIC-2 ⟫ WORK AND ENERGY

LEARNING OUTCOMES

COURSE CONTENT

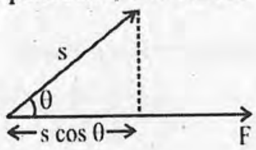
- Work
- Energy
- Kinetic energy
- Potential energy
- Gravitational potential energy
- Power
- Work Energy Principle
- Implications of energy losses in practical devices
- Interconversion of kinetic and potential energy.

WORK

The work done by a force in displacing an object is defined as the product of the displacement and the component of the force in the direction of the displacement is called work.

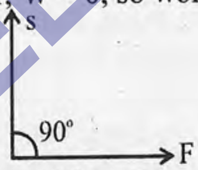
W = Force x Displacement in the direction of force

$W = F\,s\cos\theta$. F is force, s or d is displacement and θ is angle between F and s or d.

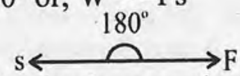


- If $\theta = 0^o$ Then $W = Fs\cos 0^o$ or $W = Fs$. It is the maximum value of work.

- If $\theta = 90^o$ Then $W = Fs\cos 90^o$ or, $W = 0$, so work is zero



- If $\theta = 180^o$ Then $W = Fs\cos 180^o$ or, $W = -Fs$

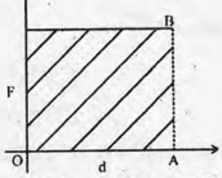


So, work is negative and is said to be done or, the force.

- In S.I the unit of work is joule (J) and in C.G.S the unit of work is erg.

Work Done by Constant Force:

Graphically work can be obtained from force displacement graph. The area under this graph is work done. Work is a **scalar** quantity.



Work done = Area of rectangle = (OA) (AB) = (F) (d)

Work Done by a Variable Force

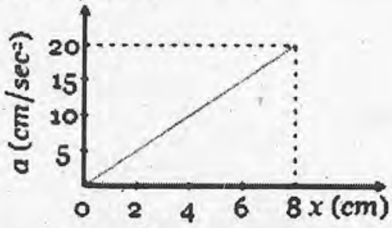
Total Area = $\lim\limits_{\Delta d\to 0}\sum\limits_{i=1}^{n} F_i\cos\theta_i\Delta d_i$ = work done

Example: A box of mass 1 kg is pulled on a horizontal plane of length 1 m by a force of 8 N then it is raised vertically to a height of 2m, the net work done is

A) 28 J B) 8 J C) 18 J D) None of above

Solution: A) Work done to displace it horizontally = F x s = 8 x 1 = 8 J Work done to raise it vertically F x s = mgh = 1 x 10 x 2 = 20 J hence, Net work done = 8 +20 = 28 J

Example: A 10 kg mass moves along x-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from x = 0 to x = 8cm



A) 8 J B) 8×10^{-2}J C) 16×10^{-2}J D) 4×10^{-3}J

Solution: Work done on the mass = mass x covered area between the graph and displacement axis on a-t graph

$$=10\times\frac{1}{2}\left(8\times10^{-2}\right)\times20\times10^{-2}=8\times10^{-2}\,\text{J}$$

Conservative Field

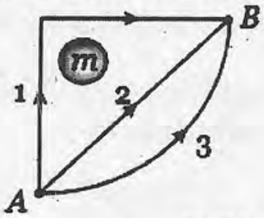
- If a field satisfies following two conditions, it is said to be conservative.

(i) Work done along a closed path is zero i.e. $W_{total} = 0$

(ii) Work done is independent of path followed by body but depends on final and initial position of a body.
Example: Electric field, Gravitational field.

- Frictional force is non-conservative force. Other non-conservative forces are propulsion force on rocket, force of a motor, tension in string etc.
- Spring force F = kx is conservative force.
- Conservative field and conservative force has the property of storing energy in the system. This energy is known as P.E of the system.

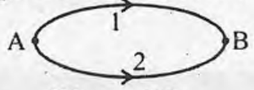
Example: If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation



A) $W_1 = W_2 = W_3$ C) $W_1 < W_2 < W_3$

B) $W_1 > W_2 > W_3$ D) none of these

Solution: A) As gravitational field is conservative in nature. So work done in moving a particle from A to B does not depends upon the path followed by the body. It always remains same.

	Conservative forces	Non-conservative forces
1.	The work done by these forces in carrying a particle around a closed path is zero i.e., $W_{total} = 0$.	The work done by these forces in carrying a particle around a closed path is not zero i.e., $W_{total} \neq 0$
2.	The work done by these forces in displacing a particle does not depend on the path along which the particle is displaced.  $W_{AB} = W_{BA}$	The work done by these forces depends upon the path along which the particle is displaced. In this case $(W_{AB}) \neq (W_{BA})$
3.	Under these forces the kinetic energy of the particle remains constant $K.E_i = K.E_f$ e.g., central forces, gravitational force elastic force, Lorentz force, electrostatic force, magnetic force etc.	Under these forces the kinetic energy of the particle changes $K.E_i \neq K.E_f$ e.g., frictional force, retarding force, viscous force, magnetic force due to an electric current etc. (all these are velocity dependent forces)

ENERGY

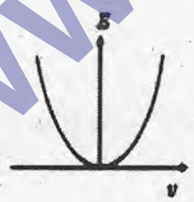
Energy of a body is defined as its ability to do work. Units of energy are the same as that of work. The SI unit of energy is joule. Other units of energy are erg, foot-pound and kilowatt-hour etc. energy occurs in many forms such as mechanical, electrical, chemical, nuclear, magnetic, heat and elastic energy etc. energy possessed by a body is basically of two types: Kinetic energy and Potential energy.

KINETIC ENERGY

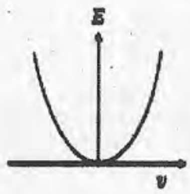
$$K.E = \frac{1}{2}mv^2$$

- Relation between linear momentum (p) and kinetic energy: $p^2 = 2m\,K.E$
- For two bodies having equal momentum $\frac{K.E_1}{K.E_2} = \frac{m_2}{m_1}$
- For two bodies having equal kinetic energies: $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

Graphs of kinetic Energy



$E \propto v^2$
m = constant



$E \propto p^2$
m = constant

$E \propto \frac{1}{m}$
p = constant

$p \propto \sqrt{E}$
m = constant

GRAVITATIONAL POTENTIAL ENERGY

Gravitational Potential Energy of a body of mass m at height h from surface of earth: P.E = mgh

Example: The work done in raising a mass of 15 gm from the ground to a table of 1 m height is

A) 15 J B) 152 J C) 1500 J D) 0.15 J

Solution: $W = mgh = 15 \times 10^{-3} \times 10 \times 1 = 0.15\,J$

POWER

Power (P)

$$\text{Power (P)} = \frac{\text{Work}}{\text{Time}} \quad \text{or} \quad P = \frac{W}{t}$$

- If the point of application of a force F moves with a velocity v, then Power $P = \vec{F}.\vec{v}$ or, $P = Fv\cos\theta$, θ is angle between $\vec{F}$ and $\vec{v}$.
- Instantaneous power is given as; $P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$. If $\langle P \rangle = P_{ins}$, then body does work at constant rate.
- Unit of power is watt defined as; 1 W = 1 J/1 sec
- Dimensionally power is equivalent to $[ML^2T^{-3}]$
- I h.p = 746 watt = 550 foot pound/sec
- Commercial unit of electricity is kW h since, 1kW h=3.6×10^6 J
- Kilowatt hour(kW h) is also known as B.O.T.U (Board of trade unit)

B.O.T.U. = 1 kW h = 3.6×10^6 joule

Example: From a water fall, water is falling at the rate of 100 kg/s on the blades of turbine. If the height of the fall is 100m then the power delivered to the turbine is approximately equal to

A) 100 kW C) 1 kW

B) 10 kW D) 1000 kW

Solution: A)

$$\text{Power} = \frac{\text{work done}}{t} = \frac{mgh}{t} = 100 \times 10 \times 100 = 10^5 \text{ watt} = 100\,\text{kW} \left[\text{As } \frac{m}{t} = 100\frac{\text{kg}}{\text{sec}} \text{(given)}\right]$$

POINT TO PONDER

A person having more power than other person does not means that he has more energy as well. Reverse is also true.

WORK ENERGY PRINCIPLE

Work done = Change in kinetic energy = $K.E_f - K.E_i$ **or** $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

- If a car stopped by applying brakes, then the stopping distance $s = \frac{mv^2}{2F} \Rightarrow s \propto v^2$

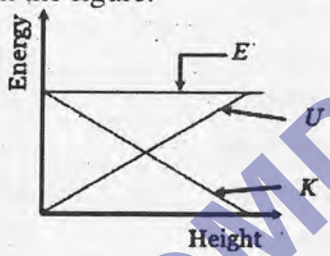
  Here F is the force applied by brakes and v is the velocity of car.

Elastic P.E.

E.P.E = ½ k x²

Energy height graph:

When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its potential energy(U) is zero. As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but throughout the complete motion total energy remains constant as shown in the figure.



POINT TO PONDER

If a spring is compressed then work done on it equals the increase in its elastic potential energy.

IMPLICATIONS OF ENERGY LOSSES IN PRACTICAL DEVICES

- Mechanical efficiency is the ratio of work output to work input.
- The efficiency of an ideal machine is 100 percent but an actual machine's efficiency will always be less than 100%.
- If a machine moves a load IV through a distance *h* then the useful work done by the machine is called output.

  Output= Load x distance *h* through which the load moves

  $= F_{out} \times D_{out}$
- If an effort F_{in} acts through a distance D_{in} then the work done on the machine is called input.

  In put= Effort force x Effort distance.

  In put= $F_{in}xD_{in}$
- The ratio of out put to the input of a machine is called its efficiency.

Mathematically

$$\text{Efficiency} = \frac{\text{out put work}}{\text{in put work}}$$

$$= \frac{\text{Load force} \times \text{Load distance}}{\text{Effort force} \times \text{Effort distance}}$$

$$\text{Efficiency}=\frac{F_{out}\times D_{out}}{F_{in}\times D_{in}}\ldots 4.6$$

The equation for percentage efficiency is

$$\text{Percentage Efficiency}=\frac{\text{Output work}}{\text{Input work}}\times 100\%=\frac{W_{out}}{W_{in}}\times 100\%$$

$$=\frac{F_{out}\times D_{out}}{F_{in}\times D_{in}}\times 100\%\ldots 4.7$$

- Even a very efficient device will waste some of its input energy in the form of heat due to the friction forces between different parts of machine.
- An incline is used as a simple machine. Which is a flate surface tilted at an angle. Which is commonly used to load truck, planes and trains.

Table

Practical devices	Efficiency
Petrol heat engine	(25-30)%
Diesel engine	(34-40)%
Steam locomotive	(35-40)%
Incandescent lamp	5%
Fluorescent lamp	20%
Steam turbine	(34-46)%
Air craft gas turbine	36%
Nuclear power plant	(30-35)%
Fossil fuel power plant	(30-40)%
Electric generator	(70-98)%
Electric motor	(50-92)%
Dry cell battery	90%
Battery	90%
'Home coal furnace	55%

FOR YOUR INFORMATION

Efficiency of some electrical equipments: LED light bulbs have been introduced to replace ordinary light bulbs, as they are much more efficient. Let's take a look at a standard 50-watt. The energy consumption to use a light bulb like this would cost about 1278 Rs in a year. An LED, running over the course of 1 year would cost only 26ORs to operate. Using these causes less energy to be wasted as heat. Recently developed, AC/DC fans can operate on less energy while producing a high airflow. In fact, they can cut down your power consumption by up to 65% and can operate on solar penal. AC/DC fans are designed to run on 12V and consume around 26-35W. Ordinary Fan consume 75watt while AC/DC fans consume about 35 to 40watt so AC/DC fans are more efficient.

Example

A machine needed 1000J of energy to raise a 10kg block at a distance of 6.0m. What is the machine efficiency?

(A) 40% (B) 49%

(C) 59% (D) 50%

Solution: First, find the work done to raise the block: $W = mgh$

$$= 10kg \times 9.8\text{m/s}^2 \times 6.0m = 588J$$

$$\text{Efficiency}=\eta = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{588}{1000} \times 100\% = 58.8\% = 59\%$$

Pulleys are machines used to lift heavy loads. Modern cranes are complicated form of pulley system.

η=59% Answer

INTERCONVERSION OF KINETIC ENERGY AND POTENTIAL ENERGY

Conservation of Energy

Energy cannot be destroyed. It can be transformed from one form into another, but total amount of energy remains constant.

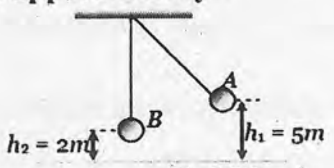
When a cup is dropped the P.E changes to K.E, but on striking the ground, the K.E changes to heat and sound but total energy at each instance is always **conserved.**

Kinetic energy + Potential energy = constant

Interconversion of Potential and Kinetic Energies

- If a body is dropped from height 'h' to earth's surface in **absence of air**, then;
  Loss in P.E. = Gain in K.E.
  $$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$
- If a body is dropped from height 'h' to earth's surface in **presence of air**, then;
  Loss in P.E = gain in K.E + work done against air.
  $$mgh = \frac{1}{2}mv^2 + fh$$
- If a body is thrown vertically upward in gravitational field in the **presence of air**, then;
  Loss in K.E = gain in P.E + work done against air.
  $$\frac{1}{2}mv^2 = mgh + fh$$

Example: A boy is sitting on a swing at a maximum height of 5m above the ground. When the swing passes through the mean position which is 2m above the ground its velocity is approximately



A) 7.6 m/s C) 6.26 m/s

B) 9.8 m/s D) None of these

Solution:

$$\Rightarrow mgh_1 = mgh_2 + \frac{1}{2}mv^2 \Rightarrow 9.8 \times 5 = 9.8 \times 2 + \frac{1}{2}v^2 \Rightarrow v^2 = 58.8 \therefore v = 7.6\,m\,s^{-1}$$

TOPIC-3 ROTATIONAL & CIRCULAR MOTION

LEARNING OUTCOMES

COURSE CONTENT

- Angular displacement (Revolution, Degree, Radian)
- Angular velocity
- Relation between linear and angular variables
- Relation between linear and angular displacements
- Relation between linear and angular velocities
- Relation between linear and angular accelerations
- Centripetal force (centripetal acceleration)

ANGULAR DISPLACEMENT (REVOLUTION, DEGREE, RADIAN)

Circular Motion

"Motion of bodies in circular path is called circular motion."

- During uniform circular motion, the direction of position vector changes continuously but the magnitude remains constant which is equal to r (radius of circular path)
- In circular motion, the direction of velocity change at every point but its magnitude remains constant.
- For one complete revolution, the angular displacement is 2π and time taken is T (time period). So angular velocity $\omega = \frac{2\pi}{T}$.
- Speed, kinetic energy and angular momentum remain constant in circular motion.

Angular Motion

"Circular motion of a body may also be called angular motion."

Angular Displacement

"It is the angle swept by the radial line during circular motion of a particle measured from some initial point to some final point."

- Angular displacement has direction along axis of rotation and can be determined by right hand rule

Radian

- SI unit of angular displacement is radian
- One radian is an angle made by an arc at the center, whose length is equal to the radius of circle.
- Definition of radian gives following useful relation $\mathbf{S = r\theta}$

  $1° = \frac{\pi}{180}$ rad = 0.0174 rad, 1 rad = 57.3°
- Angular displacement is angle in radian or degree covered by body having circular motion.
- Non S.I units are also used which are "degree" and "rev".

ANGULAR VELOCITY

"Rate of change of angular displacement is called angular velocity."

$\omega_{av} = \frac{\Delta\theta}{\Delta t}$, usually not a vector quantity or $\vec{\omega}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$, always vector quantity.

- Tangential and angular velocities are related as $v = \omega r$
- S.I unit of angular velocity is rad s^{-1}.
-
- The magnitude of an angular velocity is called the angular speed which is also represented by ω.

Example: The angular velocity of seconds hand of a watch will be

A) $\frac{\pi}{60} rad/\sec$ C) 60π rad/sec

B) 30π rad/sec D) $\frac{\pi}{30} rad/\sec$

Solution: D) We know that second's hand completes its revolution (2π) in 60 sec

$$\therefore \omega = \frac{\theta}{t} = \frac{2\pi}{60} = \frac{\pi}{30} \text{rad / sec}$$

Angular Acceleration.

- The rate of change of angular velocity is defined as angular acceleration. If particle has angular velocity ω_1 at time t_1 and angular velocity ω_2 at time t_2 then

  Angular acceleration $= \vec{\alpha} = \frac{\vec{\omega_2} - \vec{\omega_1}}{t_2 - t_1}$

- Instantaneous angular acceleration $= \vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$
- Unit: rad/sec^2
- Average angular acceleration $= \alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$

Rigid Body

- Nobody in the universe is perfectly rigid. However, the bodies in which the distance between any two particles does not change with respect to time are taken as rigid bodies.
- There are two types of motion of a rigid body.

(i) Translatory motion (Linear motion).

(ii) Rotatory Motion.

RELATION BETWEEN LINEAR AND ANGULAR VARIABLES

- Relation between linear and angular displacements: $s = r\theta$
  Vector form is given by $\vec{S} = \vec{\theta} \times \vec{r}$
- Relation between linear and angular velocities: $v = r\omega$
  Vector form is given by $\vec{v} = \vec{\omega} \times \vec{r}$
- Relation between linear and angular accelerations: $a = r\alpha$
  $\vec{a} = \vec{\alpha} \times \vec{r}$

Analogy Between Translatory Motion And Rotatory Motion

✦ Linear velocity, $\vec{v} = \dfrac{\Delta\vec{d}}{\Delta t}$	Angular velocity, $\vec{\omega} = \dfrac{\Delta\vec{\theta}}{\Delta t}$
✦ Linear displacement, $\vec{d}$	Angular displacement, $\vec{\theta}$
✦ Acceleration or linear acceleration, $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$	Angular acceleration, $\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t}$
✦ Mass, m	Moment of inertia, $I = mr^2$
✦ Linear momentum, $\vec{p} = m\vec{v}$	Angular momentum, $\vec{L} = I\vec{\omega}$
✦ Impulse, I or $J = \vec{F} \times \Delta t$	Angular impulse $= \vec{\tau} \times \Delta t$
✦ Force, $\vec{F} = m\vec{a} = \dfrac{\Delta\vec{p}}{\Delta t}$	Torque, $\vec{\tau} = I\vec{\alpha} = \dfrac{\Delta\vec{L}}{\Delta t}$
✦ Work, $W = \vec{F}.\vec{d} = \dfrac{1}{2}m\left(v_2^2 - v_1^2\right)$	Rotational work, $W_r = \tau\theta = \dfrac{1}{2}I\left(\omega_2^2 - \omega_1^2\right)$
✦ Kinetic energy, $K.E = \dfrac{1}{2}mv^2$	Kinetic energy of rotation, $K.E_r = \dfrac{1}{2}I\omega^2$
✦ **Newton's laws in linear motion First law** As $F = 0$ then v = constant or $v=0$ **Second Law** $\vec{F} = m\vec{a}$ **Third Law** $\vec{F}_{12} = -\vec{F}_{21}$	**Newton's laws in rotational motion First law** As $\tau = 0$ then ω = constant or $\omega=0$ **Second Law** $\vec{\tau} = I\vec{\alpha}$ **Third Law** $\vec{\tau}_{12} = -\vec{\tau}_{21}$

CENTRIPETAL FORCE (CENTRIPETAL ACCELERATION)

"The force required to bend a straight-line path of a body into the circular path is called centripetal force."

- If the centripetal force is removed from the rotating object it will follow a straight-line motion confined on the tangent to that circular path.
- In vector form, centripetal force and acceleration can be written as;

$$\vec{F}_c = -mr\omega^2\hat{r} = -m\vec{r}\omega^2 = -\left(\frac{mv^2}{r}\right)\hat{r} = -\left(\frac{mv^2}{r^2}\right)\vec{r} \quad \text{or} \quad \left|F_c\right| = \frac{mv^2}{r} = mr\omega^2$$

$$\vec{a}_c = -r\omega^2\hat{r} = -\vec{r}\omega^2 = -\left(\frac{v^2}{r}\right)\hat{r} = -\left(\frac{v^2}{r^2}\right)\vec{r} \quad \text{or} \quad \left|a_c\right| = \frac{v^2}{r} = r\omega^2$$

- Work done by centripetal force is zero.
- Centripetal and centrifugal forces form true action & reaction pair but they can't balance each other because they don't act on same body.

Example: An object of mass of 2 kg rotates at constant speed in a horizontal circle of radius 5 m. The time for one complete revolution is 3 s. What is the magnitude of the resultant force acting on the object?

A) $\frac{4\pi^2}{9}N$ B) $\frac{40\pi^2}{9}N$ C) $\frac{100\pi^2}{9}N$ D) $\frac{400\pi^2}{9}N$

Solution: B) $F = mr\omega^2$

$$= mr\left(\frac{2\pi}{T}\right)^2$$

$$= 2\times5\times\left(\frac{2\pi}{3}\right)^3 = \frac{40\pi^2}{9}N$$

TOPIC-4 WAVES

LEARNING OUTCOMES

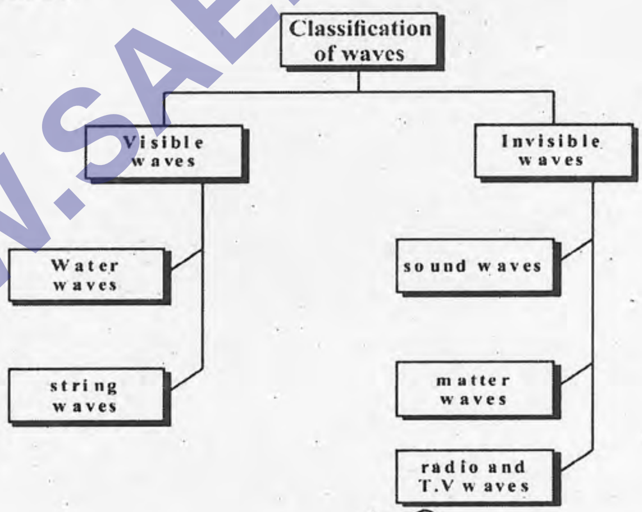
COURSE CONTENT

- Progressive wave
- Types of progressive waves (Transverse waves, Longitudinal waves)
- Periodic waves (Transverse periodic waves, Longitudinal periodic waves)
- Wave motion as illustrated by the vibrations in ropes, springs and ripple tank
- Speed of sound in air
- Principle of superposition/ superposition of sound waves
- Stationary waves/ standing waves
- Stationary waves in a stretched string/fundamental frequency and harmonics
- Doppler effect
- Simple harmonic motion (SHM)
- Characteristics of simple harmonic motion
- Instantaneous displacement
- Vibration
- Time period
- Frequency and Angular frequency
- Simple Pendulum
- Energy conservation in SHM

Introduction

- *Wave is due to disturbance created in a medium.*
- Waves transport energy without transporting matter.

Classification of Waves



POINT TO PONDER

In case of mechanical waves we deal with the cooperative motion of a collection of particles.

WAVES (on the basis of nature)

- **Mechanical waves** (reauire medium for propagation)
  - sound waves
  - string waves
  - water waves
- **Electromagnetic waves** (do not require medium for propagation)
  - Radio waves
  - Heat waves
  - Light waves

PROGRESSIVE/TRAVELING WAVES

- Progressive wave or Traveling wave is that which propagates or distributes its pulses in space along specific direction. e.g.

(i) Waves in a string
(ii) Waves on a water surface

Types Of Progressive Waves

- On the basis of vibration of the particle, waves are classified in two types
  (a) Transverse Waves **(b)** Longitudinal Waves

(a) Transverse Waves

- The particles of the medium vibrate at right angles to the direction of propagation of the wave.
- Crests and troughs are produced.
- It is because a liquid surface has property of surface tension which resists any deformation of shape.
- Transverse wave is not produced or possible in gases.

(b) Longitudinal Waves

- The particles of the medium vibrate along the direction of propagation of the wave.
- Compressions and rarefactions are produced.

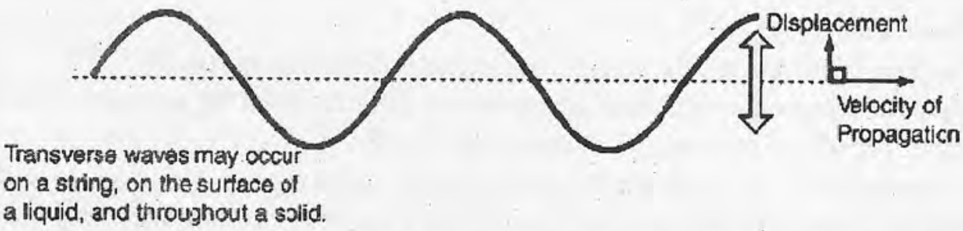
Longitudinal waves are possible in all media i.e., solid, liquid and gas.

PERIODIC WAVES

- Periodic waves are those, which are repeated in regular interval of time.
- Periodic wave may be transverse or longitudinal.

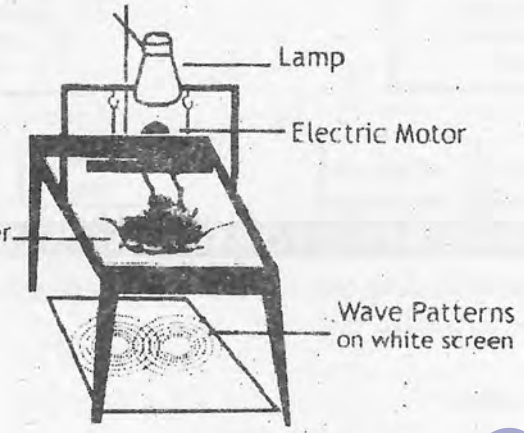
Transverse Periodic waves

- For transverse waves the displacement of the medium is perpendicular to the direction of propagation of the wave. A <u>ripple in a pond</u> and a <u>wave in a string</u> are easily visualized as transverse waves.



- Transverse waves cannot propagate in a gas or a liquid because there is no mechanism for driving motion perpendicular to the propagation of the wave.

i. circular periodic waves can be generated in a ripple tank when spherical dippers just touching the water surface. This is done by means of a mechanical arrangement driven by a small electric motor. The rate of dipping the rods is controlled by changing the speed of motor.



Waves in ripple tank

- In fluids, transverse waves die out very quickly and usually cannot be produced at all.

POINT TO PONDER

The waves transport both energy and momentum in a medium.

- In a transverse periodic wave time interval equal to time period, a particle in the wave travels a distance equal to **wavelength.**
- For all waves $v=f\lambda$

Characteristics of Wave Motion

(i) **Frequency (f)**
The number of waves which pass a point per unit time is called the frequency of the wave motion.

(ii) **Wavelength (λ)**
It is shortest distance between two consecutive points in the same phase.

(iii) **Time Period (T)**
Time taken to complete one vibration is called time period.

(iv) **Amplitude (A)**
The maximum displacement of a vibrating particle from mean position is called its amplitude.

(v) **Wave Velocity (v)**
The distance traveled by the wave in one second is defined as its velocity.

✦ The velocity of the particles of the medium is different from the velocity of the wave.
✦ Wave velocity = frequency x wavelength $\mathbf{v = f\lambda}$
✦ The relation $v = f\lambda$ holds good for any type of wave motion – transverse or longitudinal.
✦ When a given wave passes from one medium to the other, its frequency does not change.

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

- The particles in the wave separated by a distance which is integral multiple of λ i.e. $n\lambda$ are in phase with each other.
- The particles separated by a distance which is odd multiple of $\frac{\lambda}{2}$ i.e $\left(n+\frac{1}{2}\right)\lambda=(2n+1)\frac{\lambda}{2}$ are out phase to each other.

Example: A sound wave of frequency 400 Hz is travelling in a gas at a speed of 320 m s^{-1}. What is the phase difference between two points 0.2 m apart in the direction of travel?

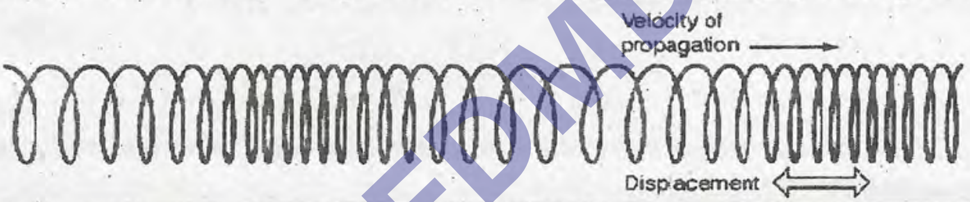
A) $\frac{\pi}{4}$ rad B) $\frac{\pi}{2}$ rad C) $\frac{2\pi}{5}$ rad D) $\frac{4\pi}{5}$ rad

Solution: B) Wavelength, $\lambda=\frac{320}{400}=0.8\,m$

Phase difference $=\frac{2\pi x}{\lambda}=\frac{0.2}{0.8}\times 2\pi=\frac{\pi}{2}$ rad

LONGITUDINAL PERIODIC WAVES

- In longitudinal waves the displacement of the medium is parallel to the propagation of the wave. A wave in a "slinky" is a good visualization. Sound waves in air are longitudinal waves.



SPEED OF SOUND IN AIR

SOUND

- A vibrating body produces sound waves ($\lambda \approx 1$m).
- Three things are essential for the detection of sound.
  - ✦ Vibrating body for production of sound
  - ✦ Medium for propagation of sound
  - ✦ Listener (ear) for the detection of sound
- Sound waves are longitudinal waves having three dimensional propagation in air.
- Longitudinal sound waves consist of compressions and rarefactions.
- Compression is a region where crowding of particles of medium is maximum.
- Rarefaction is region where crowding of particles of medium is minimum.
- Sound waves produce Reflection, Refraction, Diffraction, Interference but not polarization because sound waves are longitudinal.

Speed of sound in air

- **Newton's Formula:** Newton proved that when longitudinal waves (sound) move in elastic medium, the velocity is given by

$$v=\sqrt{\frac{E}{\rho}}$$ Where E is the modulus of elasticity of the medium and ρ is its density.

Note: *Wave velocity in a medium is fixed. Wave velocity is a material constant. It does not depend on wavelength, frequency and intensity.*

- **For Solids:** Modulus of elasticity

$$E = \text{Young's modulus of elasticity} = Y$$

$$\therefore \quad v = \sqrt{\frac{Y}{\rho}}$$

- **For liquids:** Modulus of elasticity

$$E = \text{Bulk modulus of elasticity} = B$$

$$\therefore \quad v = \sqrt{\frac{B}{\rho}}$$

- **For gases:** For a gaseous medium. Newton assumed that the propagation of longitudinal wave is an isothermal process (temperature remains constant).In this case, modulus of elasticity

$$E = \text{Pressure of the gas} = P$$

$$\therefore v = \sqrt{\frac{P}{\rho}}$$

Note:

(i) *The experimental results did not confirm to Newton's assumption. Laplace corrected the formula by arguing that sound waves travel adiabatically. Hence,*

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma KT}{m}}$$

where, T is absolute temperature of the gas, M is its molecular weight and R is universal gas constant.

$k = \frac{R}{N}$ = *Boltzmann constant, N = Avogadro number*

$m = \frac{M}{N}$ = *Mass of one molecule*

(ii) *Velocity of longitudinal wave in:* ***solid medium>liquid medium>gaseous medium***

Effects on the speed of sound in a Gas

- **Effect of pressure:** With the change of pressure, the velocity of sound in a gas remains unchanged, that is, there is no effect of pressure on the velocity of sound in gas.
- **Effect of temperature:** Velocity of sound $\propto \sqrt{T}$. Thus, the velocity of sound is directly proportional to the square root of the absolute temperature, i.e.,

$$\frac{v_t}{v_o} = \sqrt{\frac{T_t}{T_o}} \qquad = \sqrt{\frac{273+t}{273}}$$

$$v_t = v_o\left(1+\frac{t}{273}\right)^{\frac{1}{2}} \qquad \text{or} \qquad v_t = v_o + 0.61\,t$$

Effect of moisture: The presence of moisture in the air reduces the resultant density of air. The net result is that the speed of sound increases with humidity. Hence the velocity of sound in damp air is greater than its value in dry air.

Effect of density: The speed of sound in a gas varies inversely as the square of the density of gas.

Effect of Wind

If the air carrying sound waves, is itself moving i.e. there is wind. The speed of sound in the direction of wind relative to the ground is $(v+v_W)$ while against the wind is $(v-v_W)$, where v_W is the speed of wind and v is the speed of sound.

PRINCIPLE OF SUPERPOSITION/SUPERPOSITION OF SOUND WAVES

- If two or more waves propagate simultaneously in a medium then the resultant displacement is given by the vector sum of displacement due to individual waves.

  If the displacement given by the various waves to the particle are $\vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...\vec{y}_n$, then the resultant displacement of the particle is $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...\vec{y}_n$

Interference of Sound

Superposition (mixing up) of two identical sound waves while passing through same medium propagating along same direction is called their interference.

Conditions for interference

(i) coherent waves
(ii) same medium
(iii) same direction
(iv) identical waves
(v) Sources of sound should be close to each other

- In constructive interference, two interfering sound waves reinforce each other, so that the resultant is a louder sound.

Condition for Constructive Interference

Path difference = $n\lambda$ where $n = 0, +1, +2, ...$

- In destructive interference, two interfering sound cancel each other's effect, so that the resultant loudness of sound wave is become fainter.

Condition for Destructive Interference

Path difference $= \left(n + \frac{1}{2}\right)\lambda$ where $n = 0, +1, +2, ...$

- Echoing zone is region of constructive interference
- Silence zone is region of destructive interference
- Path difference is the difference between lengths of paths traveled by two waves in reaching the same point.

STATIONARY WAVES/STANDING WAVES

Superposition of Waves

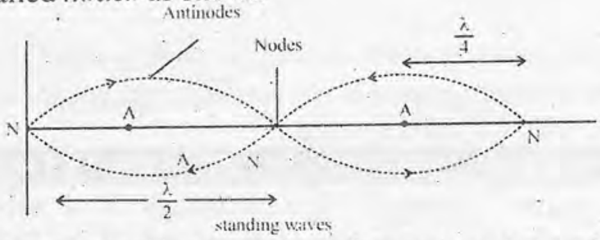
- If two or more waves propagate simultaneously in a medium then the resultant displacement is given by the vector sum of displacement due to individual waves.
- If the displacement given by the various waves to the particle are $\vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...\vec{y}_n$, then the resultant displacement of the particle is $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...\vec{y}_n$
- Different phenomenon due to principle of superposition are

  (a) Interference **(b)** Beats **(c)** Stationary waves

Stationary Waves

- Super position of two identical waves traveling opposite to each other in the same medium simultaneously, gives rise to stationary or standing waves

- Points of constructive interference are called *antinodes* while points of destructive interference are called *nodes* as shown



- Amplitude is maximum at antinodes and minimum (zero) at nodes.
- Nodes are stationary points whereas antinodes are points that vibrate with maximum amplitude.

  Two consecutive nodes or antinodes are separated by distance equal to $\lambda/2$ and an antinode and its consecutive node by $\lambda/4$.

Example: Progressive waves of frequency 300 Hz are superimposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart. What is the speed of the progressive waves?

A) 100 m s^{-1} B) 200 m s^{-1} C) 450 m s^{-1} D) 900 m s^{-1}

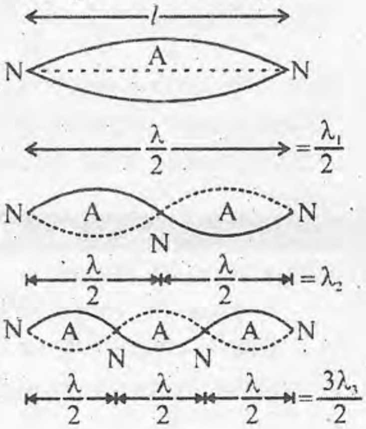
Solution: D) The distance 1.5 m corresponds to half of a wavelength, λ. The wavelength is thus given by $\lambda = 2(1.5) = 3.0\,\text{m}$

The speed of the wave, $v = f\lambda = (300)(3) = 900\ \text{m s}^{-1}$

STATIONARY WAVES IN A STRETCHED STRING/FUNDAMENTAL FREQUENCY AND HARMONICS

- The vibrations of a thin, long and perfectly elastic string are transverse stationary.
- On both the ends of string there are nodes, and an antinode is there in the middle.
- The speed of transverse wave in a stretched string is given by $v = \sqrt{\frac{T}{m}}$

  Where T and m are respectively the tension and mass per unit length of the string.
- Modes of vibration in a stretched string are as under



- If a string of length l vibrates in one loop then $\lambda_1 = 2l$ and frequency $f_1 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}$. This frequency is called the fundamental note or first harmonic.
- If the string vibrates in two loops, then $\lambda = l$ and $f_2 = \frac{v}{l} = 2f_1$. This frequency is called the first overtone or second harmonic.
- If the string vibrates in three loops, then $\frac{3\lambda_3}{2} = l \rightarrow \lambda_3 = \frac{2l}{3}$

  $\therefore\quad f_3 = \frac{3v}{2l} = 3f_1$. This frequency is called the second overtone or third harmonic.

  Both the odd and even harmonics are emitted from a stretched string. That is $f_n = nf_1$, where $n = 1, 2, 3 ....$

Example: The frequency of the fundamental mode of transverse vibration of a stretched wire 100 mm long is 256 Hz. When the wire is shortened to 400 mm at the same tension, what is the fundamental frequency?

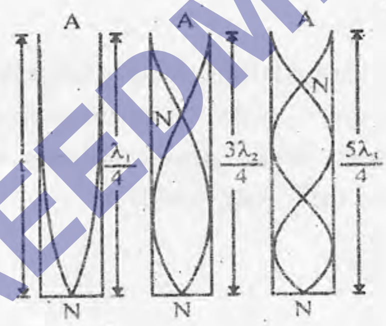
A) 162 Hz B) 312 Hz C) 416 Hz D) 640 Hz

Solution: D) The frequency of the fundamental mode of a stretched wire is $f=\frac{1}{2\ell}\sqrt{\frac{T}{m}}$

$$f \propto \frac{1}{\ell} \Rightarrow \frac{f_2}{f_1}=\frac{\ell_1}{\ell_2} \Rightarrow \frac{f_2}{256}=\frac{1000}{400} \Rightarrow f=640\text{ Hz}$$

STATIONARY WAVES IN AIR COLUMNS

- An organ pipe is a pipe that sets in vibration the air enclosed in it when the air is blown into it. As a result sound is produced in it.
- Organ pipes are of two types – closed end organ pipe and open end organ pipe.
- An open end organ pipe has both its ends open.
- A closed end organ Pipe has one of its ends closed and the other open.
- In a closed end pipe a node is always formed at the closed end and an antinode is formed at the open end.
- Longitudinal stationary waves are formed in an organ pipe.
- Various stages of resonance in a **CLOSED END** organ pipe are represented in the following diagrams.



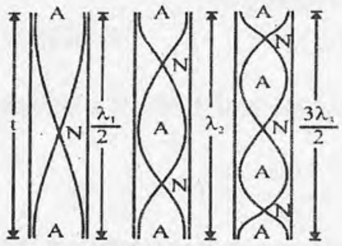
- If the length of the pipe $l=\frac{\lambda_1}{4}$ then $\lambda_1=4l$, $\therefore$ Frequency, $f_1=\frac{v}{\lambda_1}=\frac{v}{4l}$

  This frequency is called fundamental frequency or fundamental note or first harmonic.
- If $l=\frac{3\lambda_2}{4}$ then $\lambda_2=\frac{4l}{3}$, $\therefore$ Frequency, $f_2=\frac{v}{\lambda_2}=\frac{3v}{4l}=3f_1$

  This frequency is called third harmonic or first overtone.
- If $l=\frac{5\lambda_3}{4}$ then $\lambda_3=\frac{4l}{5}$, $\therefore$ Frequency, $f_3=\frac{v}{\lambda_3}=\frac{5v}{4l}=5f_1$

  This frequency is called fifth harmonic or second overtone.
- Only odd harmonics can be produced in a closed end organ pipe. That is $f_n=(2n-1)f_1$ where $n=1,3,5,......$
- Longitudinal stationary waves are formed in an **OPENEND ORGAN PIPE** too.
- The antinodes are formed at both the ends of an open pipe.

- Various stages of resonance in an open organ pipe have been represented in the following diagrams



- If $l = \frac{\lambda_1}{2}$ then $\lambda_1 = 2l$, $\therefore$ Frequency, $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$

  This frequency is called the fundamental frequency or the fundamental note or the first harmonic.

- If $l = \lambda_2$ then $f_2 = \frac{v}{\lambda_2} = \frac{v}{l} = 2f_1$

  This frequency is called the second harmonic or first overtone.

- If $l = \frac{3\lambda_3}{2}$ then $\lambda_3 = \frac{2l}{3}$, $\therefore$ Frequency, $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3f_1$

  This frequency is called the third harmonic are produced in an open-end organ pipe. That is $f_n = nf_1$, where $n = 1, 2, 3......$

- The sound emitted by an open-end organ pipe is musical.

  No. of harmonics in open pipe = 2 x No. of harmonics in closed pipe

Example: 2nd overtone of an open organ pipe resonates with 3rd harmonics of a closed organ pipe. The ratio of their length will be

A) $\frac{2}{1}$ B) $\frac{1}{2}$ C) $\frac{6}{5}$ D) $\frac{5}{6}$

Solution: A) $\frac{L_1}{L_2} = \frac{3\lambda/2}{3\lambda/4} = \frac{2}{1}$

DOPPLER'S EFFECT

Doppler Effect (Frequency Shift)

- Apparent change in pitch (frequency) of sound is due to relative motion of source and observer.
- Doppler's effect was discovered by Doppler, an Australian physicist, in 1845.
- Apparent frequency of sound heard by **stationary listener** due to source **moving towards** him at speed 'u' is given as;

$$f' = \left(\frac{v}{v - u_s}\right) f \qquad f' > f$$

- Apparent frequency of sound heard by **stationary listener** due to source **moving away** from him at speed 'u' is given as;

$$f' = \left(\frac{v}{v + u_s}\right) f \qquad f' < f$$

- Apparent frequency of sound heard by a **person moving towards a stationary source with** speed 'u' is given as;

$$f' = \left(\frac{v + u_o}{v}\right) f \qquad f' > f$$

- Apparent frequency of sound heard by a listener **moving away from a stationary source with** speed 'u' is given as;

$$f' = \left(\frac{v - u_o}{v}\right) f \qquad f' < f$$

- When both source and observer move towards each other.

$$f' = \left(\frac{v + u_o}{v - u_s}\right) f \qquad f' > f$$

Where v = true speed of sound u_o = speed of observer u_s = speed of source

- When both source and observer move away from each other.

$$f' = \left(\frac{v - u_o}{v + u_s}\right) f \qquad f' < f$$

Applications of Doppler's effect

(i) Ships and submarine (sonar devices)
(ii) Bats (for traveling)
(iii) Radar (for detection)
(iv) Determining velocity of a star w.r.t earth
(v) To monitor blood flow in major arteries.
(vi) When a star is moving away from Earth then wavelength of light increases and red shift of spectrum is observed.
(vii) When a star is moving towards the Earth then wavelength of light decreases and blue shift of spectrum is observed.

Example: A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed of sound = 330 m/s)

A) 409 C) 517
B) 429 D) 500

Solution: D)

$$f' = \left(\frac{v}{v - v_s}\right) f = 450\left(\frac{330}{330 - 33}\right) = 500 \text{ Hz}$$

SIMPLE HARMONIC MOTION

- Vibratory motion is that in which a body moves to and fro about a fixed position along same path. e.g. **(i)** Motion of simple pendulum **(ii)** Motion of molecules of a solid
- Simple harmonic motion (SHM) is a special type of vibratory motion in which;
  (i) $\vec{a} \propto -\vec{x}$ **(ii)** a is directed towards mean position.
- Restoring force is always directed towards mean position hence assigned negative sign.

Example: A simple harmonic oscillator has a time period of 10 seconds. Which equation relates its acceleration a and displacement x?

A) $a = -10x$ B) $a = -(20\pi)x$ C) $a = -(20\pi)^2 x$ D) $a = -(2\pi/10)^2 x$

Solution: D) $a = -\omega^2 x$ and $\omega = \frac{2\pi}{T}$

Where T is the period.

$\therefore a = \left(\frac{2\pi}{10}\right)^2 x$

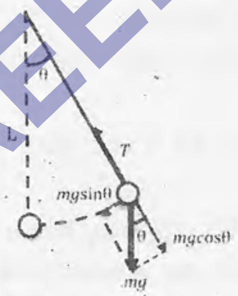
CHARACTERISTICS OF SIMPLE HARMONIC MOTION

- Instantaneous displacement is distance covered by body at any instant from mean position.
- Periodic motion is that which repeats itself after equal time intervals.
- Vibration is one complete round trip of a body about its mean position.
- Time period is defined as time taken by vibrating body to complete its one vibration and denoted by T.
- Frequency is number of vibrations per second and denoted by f $f = \frac{1}{T}$

  Its unit is Hz, other units are vibrations/s, cycle/s, rev/sec.
- Amplitude is maximum distance from mean position.
- Angular frequency is $\omega = 2\pi/T \Rightarrow \omega = 2\pi f$

SIMPLE PENDULUM

It consists of a heavy point mass suspended from a rigid support by means of almost weightless and inextensible string.



- Galileo invented simple pendulum.
- Motion of simple pendulum is S.H.M if there is no damping.
- Damping force reduces the amplitude of simple pendulum continuously and finally its motion is stopped.
- In absence of damping force, restoring force on simple pendulum is given as; $F_r = -mg\sin\theta$, and for small amplitude oscillations $F_r = -mg\theta$.
- Equation of acceleration of simple pendulum for small amplitude is: $a = -\left(\frac{g}{l}\right)x$

  Thus $\omega = \sqrt{\frac{g}{l}}$ for simple pendulum and does not depend on mass like the mass-spring system does.

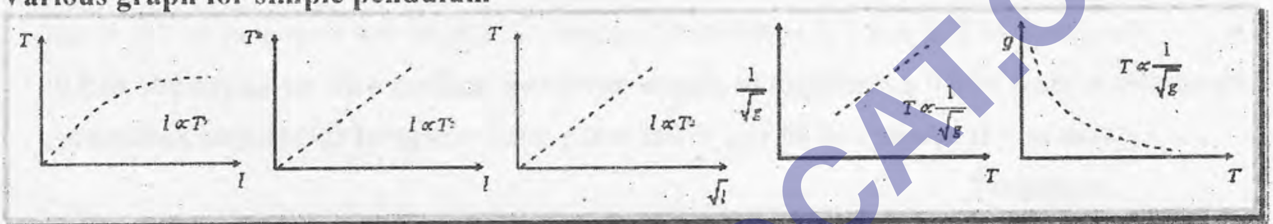
- Time period and frequency of simple pendulum are given as;

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{and} \qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

- If amplitude of simple pendulum is not small then, it has non-S.H.M as $a = -g\sin\theta$ and we know that $\sin\theta = 0$ only when θ is small.
- A second pendulum has following characteristics;

Time period	2 seconds
Frequency	0.5 Hz
Length	0.99 cm or 1 meter

Various graph for simple pendulum



Example: The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

A) first decrease and then increase to the original value.
B) first increase and then decrease to the original value.
C) increase towards a saturation value.
D) remain unchanged.

Solution: B) Centre of mass of combination of liquid and hollow portion (at position l), first goes down $(\text{to } \ell + \Delta\ell)$ and when total water is drained out, center of mass regain its original position $(\text{to } \ell)$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\therefore$ T first increases and then decreases to original value.

ENERGY CONSERVATION IN SHM

Energy Conservation in SHM

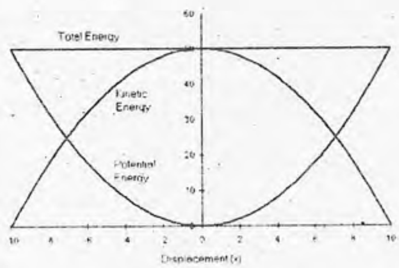
- Its K.E is given as;

$$K.E_{ins} = \frac{1}{2}k\,x_o^2\left(1-\frac{x^2}{x_o^2}\right)$$

$$(K.E)_{max} = \frac{1}{2}kx_o^2$$ It is at mean position.

$$(K.E)_{min} = 0$$ It is at extreme position.

$$K.E_{ins} = (K.E)_{max}\left(1-\frac{x^2}{x_o^2}\right)$$



- Its P.E is given as;

$$P.E_{ins} = \frac{1}{2}kx^2$$

$$(P.E)_{max} = \frac{1}{2}kx_o^2$$ It is at extreme position.

$(P.E)_{min} = 0$ It is at mean position.

- Total energy of system $= \frac{1}{2}kx_o^2$ energy remain conserve in SHM. In one vibration K.E. attains its maximum value twice.
- Frequency of K.E and P.E is twice of frequency of S.H.M But frequency of T.E is zero.

Example: A mass of 8.0 g oscillates in simple harmonic motion with an amplitude of 5.0 mm at a frequency of 40 Hz. What is the total energy of this simple harmonic oscillator?

A) 0.16 mJ C) 6.3 mJ

B) 13 mJ D) 640 mJ

Solution: C) Total energy $= \frac{1}{2}m\omega^2 x_o^2$

$$= \frac{1}{2}m(2\pi f)^2 x_o^2$$

$$= \frac{1}{2}(8.0\times10^{-3})(2\pi\times40)^2(5.0\times10^{-3})^2$$

$$= 6.3 \text{ mJ}$$

TOPIC-5 》 THERMODYNAMICS

LEARNING OUTCOMES

COURSE CONTENT

- First law of thermodynamics
- Isothermal process, Adiabatic process, Isobaric process and Isochoric process
- Specific heat and molar specific heat/ specific heat capacity.
- Relation $C_p - C_v = R$
- Internal Energy
- Heat and Work
- Thermodynamic system
- Second law of thermodynamics (Lord Kelvin statement)

FIRST LAW OF THERMODYNAMICS

When heat is transformed into other forms of energy total heat remains constant

$\Delta Q = \Delta U + \Delta W$ where ΔQ is positive when heat is added and vice versa.

ΔW is positive when work is done by system and vice versa.

Inferences from 1st Law of Thermodynamics: $\Delta U = \Delta Q - \Delta W$

$$\begin{pmatrix}\text{change in in internal}\\ \text{energy}\end{pmatrix} = (\text{Heat energy flowing in}) - \begin{pmatrix}\text{Heat energy flowing out}\\ \text{as mechanical work}\end{pmatrix}$$

- Internal energy is a state function. i.e. depends on initial and final states
- For a cyclic process, we have: $\Delta U = 0$, $U_i = U_f$ Then $\Delta Q = \Delta W$

ISOTHERMAL PROCESS, ADIABATIC PROCESS, ISOBARIC PROCESS AND ISOCHORIC PROCESS

Isothermal process is that in which temperature remains constant.

$\Delta Q = \Delta W$ as $\Delta U = 0$

POINT TO PONDER

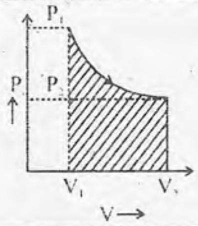
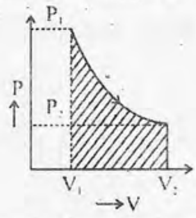
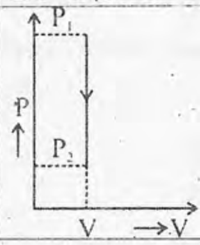
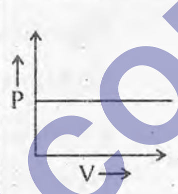
- Can any type of energy enters or leave a system during adiabatic change?

- Isochoric process is that in which volume remains constant.

  $Q = \Delta U$ as $\Delta W = 0$
- Isobaric process is that in which pressure remains constant. $\Delta Q = \Delta U + P\Delta V$
- Adiabatic process is that in which no heat enters or leaves the system such that temperature remain constants

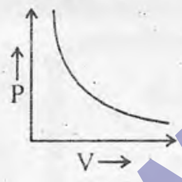
Note: *Cooling is produced when adiabatic expansion takes place and heating is produced during adiabatic compression.*

COMPARISON OF DIFFERENT THERMO DYNAMICAL PROCESSES

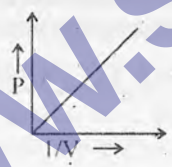
Sr.	Property	Isothermal	Adiabatic	Isometric (Isochoric)	Isobaric
1.	Condition	T=constant, $\Delta T=0$, $\Delta U=0$	Q=constant, $\Delta Q=0$	V=constant, $\Delta V=0$, $\Delta W=0$	P=constant; $\Delta P=0$
2.	Form of first Law	$\Delta Q=0+\Delta W=P\Delta V$	$0=\Delta U+\Delta W$ $\Delta W=-\Delta U$	$\Delta Q=\Delta U+0$, $\Delta U=n\,C_V\Delta T$	$\Delta Q=\Delta U+\Delta W$ $=nC_P\Delta T$
3.	P.V Diagram				
4.	Equation of state	PV=constant	PV^{γ}=constant	$\frac{P}{T}=$ constant	$\frac{V}{T}=$ constant
5.	Specific Heat	$c=\infty$	$c=0$	$c=C_V$	$c=C_P$
6.	Slope of P.V curve	$\frac{\Delta P}{\Delta V}=-\frac{P}{V}$	$\frac{\Delta P}{\Delta V}=-\gamma\frac{P}{V}$	$\frac{\Delta P}{\Delta V}=\infty$	$\frac{\Delta P}{\Delta V}=0$
7.	Example	**(i)** Isothermal expansion of ideal gas, **(ii)** Conversion of ice at 0°C to water of 0°C	**(i)** Burst of air tube **(ii)** Propagation of sound in air **(iii)** Refrigeration	**(i)** To supply heat at constant volume, **(ii)**Atmospheric changes, **(iii)**Explosion in gases	**(i)** Melting of ice, **(ii)** Boiling of water

Different Thermodynamic Process and Relative:

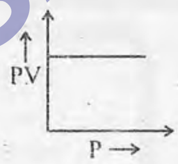
Thermodynamics Process



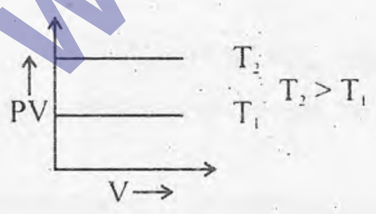
P–V curve

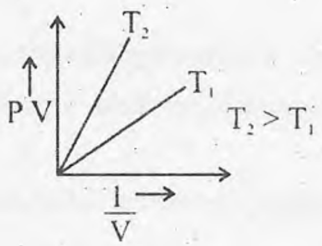


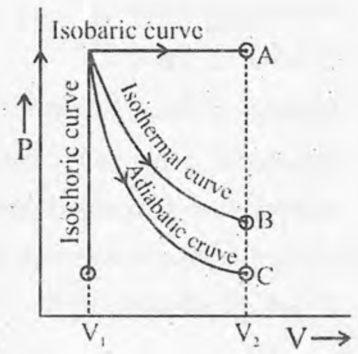
P–$\frac{1}{V}$ curve



P and PV curve







SPECIFIC HEAT AND MOLAR SPECIFIC HEAT/ SPECIFIC HEAT CAPACITY.

- Amount of heat required to raise the temperature of a substance through 1 K is called *heat capacity*, denoted by C.
  $Q = C\, m\, \Delta T$
- *Specific heat* is the amount of heat required to raise the temperature of unit mass through unit temperature.
  $Q = C\, \Delta T$

POINT TO PONDER

Molar specific heat of diatomic gas is greater than that of monatomic gas.

Molar Specific heat at Constant Volume

- Molar specific heat at constant volume is the amount of heat required to raise the temperature of one mole of the gas through 1K keeping volume constant.

Specific Heat at Constant Pressure

Molar specific heat at constant pressure is the amount of heat required to raised the temperature of one mole of the gas through 1K keeping pressure constant.

Relation $C_p - C_v = R$:

- $\Delta Q_V = n\, C_V\, \Delta T$ (Heat supplied at constant volume).
- $\Delta Q_P = n\, C_p\, \Delta T$ (Heat supplied at constant pressure).
- $C_P - C_V = R.$
  $\dfrac{C_p}{C_V} = \gamma$

POINT TO PONDER

- As we know that $C_P > C_V$ then can we say that ΔT_P is greater than ΔT_V?

Specific Heat The heat required to increase the temperature of unit mass of substance by 1^0C is called the specific heat of the substance. $\mathbf{Q = mc\Delta T}$ **or** $\mathbf{c = \dfrac{Q}{m\Delta T}}$.

TYPES OF GAS	C_v	C_p	$\gamma = \dfrac{C_p}{C_v}$
Monoatomic	$\frac{3}{2}R$	$\frac{5}{2}R$	1.67
Diatomic	$\frac{5}{2}R$	$\frac{7}{2}R$	1.40
Polyatomic	3R	4R	1.33

Example: In an experiment to determine the specific heat capacity of a liquid by an electrical method, a student obtained the following results.

Mass of liquid heated	1.5 kg
Initial liquid temperature	300 K
Final liquid temperature	357 K
Electrical power of heater	1.0 kW
Time of heating	180 s

What is the specific heat capacity of the liquid?

A) 2.1 J kg^{-1} K^{-1} C) 1800 J kg^{-1} K^{-1}
B) 18 J kg^{-1} K^{-1} D) 2100 J kg^{-1} K^{-1}

Solution: D) The specific heat capacity, $c = \dfrac{Q}{m\Delta T}$

$$= \frac{\text{Power} \times \text{time}}{\text{mass} \times \Delta T}$$

$$= \frac{(1.0\times10^{3})(180)}{(1.5)(357-300)} = 2100\ \text{J kg}^{-1}\text{K}^{-1}$$

INTERNAL ENERGY

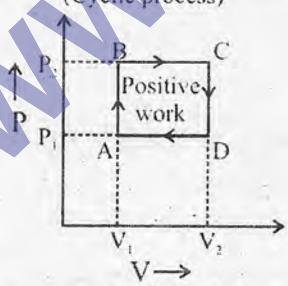
Internal Energy

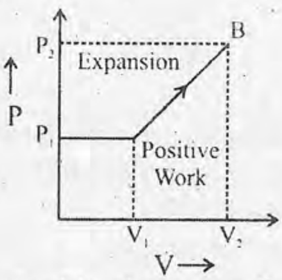
- The energy possessed by a system due to molecular motion and configuration is considered as its internal energy.
- Internal energy of a system = Kinetic energy of its constituents molecules + Potential energy of its constituents molecules.
- Internal energy is a state function which depends on pressure, temperature, volume etc. of the system.
- The change in the internal energy of a system depends only on initial and final states of the system and not on the path followed.
- For an ideal gas the potential energy of the molecules is zero. Here internal energy = Kinetic energy of molecules and it depends on temperature.

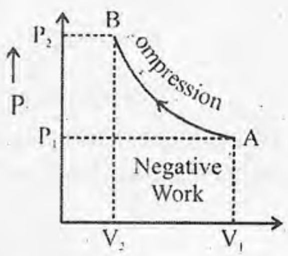
HEAT AND WORK

- "Heat is a form of energy which flows from the hotter body to the colder body till the temperatures of the two bodies become equal".
- Work can be defined as the energy that is transferred one body to the other owing to a force that acts between them. The amount of work done by a system as it expands or contracts is given by: **W = PΔV**
- Work is taken to be positive if the system expands against some external force. Work is taken to be negative if the system contracts because of some external force exerted by the surroundings.
- **Work calculation by indicator-diagram method:** It is positive if volume increases and negative if volume decreases.

(Cyclic process)

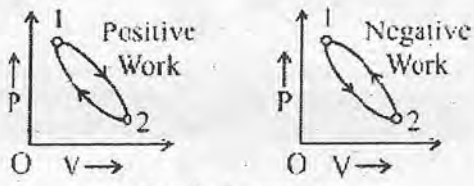






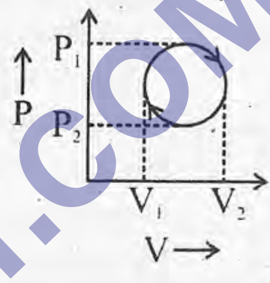
- If area under PV-diagram is traced in clockwise direction the work done will be positive (expansion) and will be negative (during compression) if the area is traced in anticlockwise direction.

- **Cyclic Process:** A cyclic process is one in which the thermodynamic variables periodically return to their original values. In other words, the initial and final states of the system coincide. The work done by a cyclic process is equal to the area enclosed by the loop.



Cyclic Processes

- **Positive work:** If the cycle is traversed in the clockwise sense.
- **Negative work:** If the cycle is traversed in the anticlockwise sense. If a cyclic process is represented by a circle on the P–V diagram as shown in the fig, then the work done is given by



$W = \frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$. The change in internal energy of a cycle process is zero $\Delta U = 0$. Therefore, from First Law, we may conclude $Q = W$.

THERMODYNAMIC SYSTEM

- "A system described in terms of thermodynamics variables such as pressure (P), volume (V), and temperature (T) is known as thermodynamic system".
- The quantity of matter or region of space under observation is called system.
- Everything other than the system in the universe is called the surrounding of the system.
- The system is separated from the surroundings by its boundary. In thermodynamics, the exchange of energy between the system and the surrounding can take place through the boundary by the performance of work or the flow of heat.
- "The system in which there is no transfer of mass across its boundary is called closed system".
- "The system in which there is a transfer of mass across its boundary is called an open system".
- "The system in which there is no transfer of mass and heat energy across its boundary is called an isolated system".
- These are macroscopic physical quantities like pressure (P); volume (V); and temperature (T) etc. which are used to describe the state of the system. The relation between these variables is called equation of state.

SECOND LAW OF THERMODYNAMICS

- The second law of thermodynamics deals with the direction in which any physical or chemical process involving energy changes take place.
- **Lord Kelvin Statement of Second Law:**
- It is impossible to construct a heat engine which, operating in a complete cycle, will extract heat from a single body and convert the whole of it to work, it simply means that it is impossible to convert all the heat extracted from a hot body into work.
- **Clausius Statement of Second Law:**
- It is impossible for a self-acting machine working in a cyclic process, unaided by any external agency, to transfer heat from a body at lower temperature to a body at a higher temperature. This statement implies that heat cannot be transferred from a cold body to a hot body without expenditure of work by an external energy.

TOPIC-6 ⟫ ELECTROSTATICS

LEARNING OUTCOMES

COURSE CONTENT

- Coulomb's Law (Coulomb's Law in material media)
- Electric field and its intensity
- Application of Gauss's law (Electric field intensity due to an infinite sheet of charge, Electric field intensity between two oppositely charged parallel plates)
- Electric potential
- Capacitor (Capacitance of a capacitor and its unit)
- Capacitance of a parallel plate capacitor
- Combination of capacitors
- Energy Stored in a Capacitor
- Charging and Discharging a Capacitor

COULOMB'S LAW (COULOMB'S LAW IN MATERIAL MEDIA)

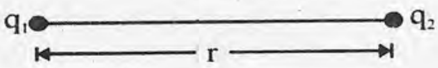
If two stationary point charges q_1 and q_2 are kept at a distance r, then it is found that force of attraction or repulsion between them is

$$F \propto \frac{q_1 q_2}{r^2} \qquad \text{i.e.,} \qquad F = k\frac{q_1 q_2}{r^2}$$

Vector form of Coulomb's Law

Vector form of Coulomb's law is

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad , \quad \vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$$



Where $\hat{r}_{12}$ is the unit vector from first charge to second charge along the line joining the two charges?

Note: Constant "k" depends upon system of units and medium between two charges.

(i) Effect of Units:

- In C.G.S. system for air k = 1, $F = \frac{q_1 q_2}{r^2}$ dyne.
- In S.I. for air $k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,Nm^2C^{-2}$, $F = \frac{1}{4\pi\varepsilon_o}.\frac{q_1 q_2}{r^2}$ Newton $\left(1N = 10^5 dyne\right)$

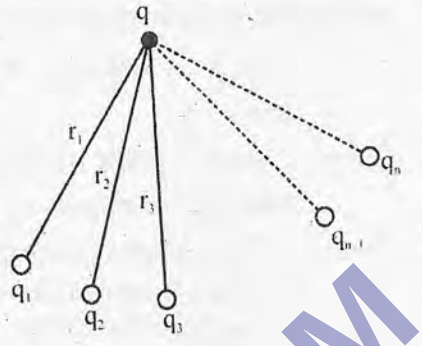
Example: The charge on the uranium nucleus is 1.5×10^{-17}C and the charge on the α-particle is 3.2×10^{-19}C.

A) 1.0×10^{-33} N B) 4.3×10^{-20} N C) 4.3×10^{-13} N D) 4.3 N

Solution: D) The electrostatic force F is given by

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o d^2} = \frac{\left(1.5\times10^{-17}\right)\left(3.2\times10^{-19}\right)}{4\pi\left(8.854\times10^{-12}\right)\left(1.0\times10^{-13}\right)^2} \Rightarrow 4.3\,N$$

(ii) Coulombs Law in Material Media



- When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of ε_r known as dielectric constant is also called relative permittivity ε_r of the medium. Hence in the presence of medium $F_m = \dfrac{F_{air}}{\varepsilon_r} = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}.\dfrac{q_1q_2}{r^2} \Rightarrow \varepsilon_r = \dfrac{F_{air}}{F_m}$

Principle of Superposition

According to the principle of superposition, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges. Consider number of charges $q_1, q_2, q_3 \ldots\ldots q_n$ are applying force on a charge q

- Net force on q will be

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots + \vec{F}_{n-1} + \vec{F}_n$$

ELECTRIC FIELD AND ITS INTENSITY

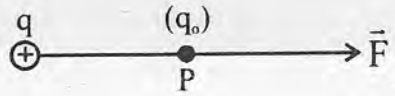
Electric Field

Electric field due to a point charge is the space surrounding it, within which electric force can be experienced by the another charge.

Electric field intensity $(\vec{E})$

The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point.

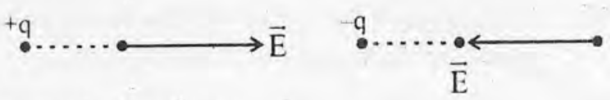
$$\vec{E} = \frac{\vec{F}}{q_o} = \frac{kq}{r^2}\hat{r}$$



Unit: NC^{-1}(S.I), Vm^{-1} and dyne/stat – C (CGS)

Direction of electric field

Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge.



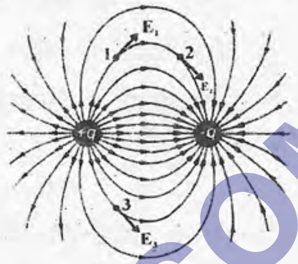
Relation between electric force and electric field

In an electric field $\vec{E}$ a charge (q) experiences a force F = qE. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field.

- The electric field of a continuous charge distribution at some point. $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$

Where q is the charge on one element of the charge distribution r is the distance from the element to the point under consideration. $\hat{r}$ is the unit vector directed from the position of elemented charge towards the point where electric field is to be found out?

- The path followed by a tiny positive charge in an electric field is called line of force.
- Electric lines of force are imaginary lines starting from positive charge and ending on negative charge.



Some of important properties are given below;

(i) Originate from positive charge.
(ii) End on negative charge.
(iii) Do not intersect
(iv) Contract longitudinally.
(v) Repel transversely.
(vi) No electric line is present inside the conductor.
(vii) Tangent drawn to electric lines gives the direction of electric intensity
(viii) Electric field is stronger where the electric lines are **closely packed**

Equipotential Surface

- If every point of a surface is at same potential, then it is said to be an equipotential surface.
- The direction of electric field is perpendicular to the equipotential surfaces or lines of force.
- The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.
- Equipotential surfaces can never cross each other.
- Work done in moving any charge over these surfaces is zero.

$E=\frac{\Delta V}{\Delta d}$ TO CALCULATE THE FIELD STRENGTH

We can relate electric potential difference and electric field intensity by following relation: $E=-\frac{\Delta V}{\Delta r}$

Where negative shows that $\vec{E}$ is along decreasing potential.

(i) We can call 'E' potential gradient because it represents the maximum rate of change of potential difference w.r.t displacement.

(ii) SI unit of E (N/C) is equivalent to V/m. $\frac{1N}{1C}=\frac{1V}{1m}$

Example: A thunder-cloud whose base is 500 m above the ground. The potential difference between the base of the cloud and the ground is 200 MV. A raindrop with a charge of 4.0×10^{-12} C is in the region between the cloud and the ground. What is the electrical force on the raindrop?

A) 1.6×10^{-6} N B) 8.0×10^{-4} N C) 1.6×10^{-3} N D) 0.40 N

Solution: A) $F=qE$

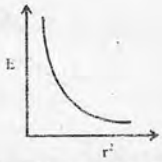
$$=q\left(\frac{V}{d}\right)$$

$$=\left(4.0\times10^{-12}\right)\left(\frac{200\times10^{6}}{500}\right)=1.6\times10^{-6}\,N$$

$E = \frac{Q}{4\pi\varepsilon_o r^2}$ FOR THE FIELD STRENGTH

- $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$
- $E \propto \frac{1}{r^2}$ (Inverse square law) $\Rightarrow \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2$

Graph:



GRAVITATIONAL FORCE AND ELECTRIC FORCE

Particulars	Gravitational force	Electrostatic force
Formula	$F_g = Gm_1m_2/r^2$	$F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$
Range	Infinite	infinite
Symbol of constant	G	$k = \frac{1}{4\pi\varepsilon_o}$
Value of constant	Very small	Fairly large
Nature	Always attractive	Attractive or repulsive
Dependence	Medium independent	Medium dependent
Relative strength	Weak: can be felt with massive object	Strong at close range

APPLICATION OF GAUSS'S LAW (ELECTRIC FIELD INTENSITY DUE TO AN INFINITE SHEET OF CHARGE, ELECTRIC FIELD INTENSITY BETWEEN TWO OPPOSITELY CHARGED PARALLEL PLATES)

- Electric intensity due to **infinite sheet of charge** is given as: $\vec{E} = \frac{\sigma}{2\epsilon_o}\hat{r}$
- Electric intensity between **two equal but oppositely charged** plates is given as. $\vec{E} = \frac{\sigma}{\epsilon_o}\hat{r}$

ELECTRIC POTENTIAL

Electric potential difference between two points is defined as:

"Work done per unit positive charge in moving it against electric field with uniform velocity. i.e. keeping the charge in electrostatic equilibrium."

$$\Delta V = \frac{W_{A\to B}}{q_o}$$

- It can be given in terms of potential energy as

$$\Delta V = \frac{\Delta U}{q_o} \qquad \because W_{A\to B} = \Delta U = \text{electrostatic P.E}$$

- SI unit of potential difference is volt.
  "If 1J of work is done on one coulomb charge between two points keeping the equilibrium, the potential difference is 1V"
  1V = 1J/1C
- **Absolute electric potential** at a distance 'r' from source is given as:
  $$V_{(r)} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Example: A conducting hollow sphere of radius 0.1 m is given a charge of 10 μC. The electric potential on the surface of sphere will be

A) zero B) 3×10^5V C) 9×10^5 V D) 9×10^9 V

Solution: C) $V_{surface} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = \frac{9\times10^9\times10\times10^{-6}}{10^{-1}} = 9\times10^5\,V$

- **Absolute potential** at a point due to collection of point charges is given as;
  $$V = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_i}{r_i}$$
- ECG records the voltage between points on human skin generated by electrical process in the heart while EEG records that by brain.

Electron Volt

The amount of energy acquired or lost by an electron as it traverses a potential difference of one volt.

- $1eV = 1.6x10^{-19}J$ and $1J = 6.25x10^{18}$ eV
  It is the unit of energy specially used for atomic particles

CAPACITOR (CAPACITANCE OF A CAPACITOR AND ITS UNIT)

Capacitor is a device used for **storing electric charge** and electrical energy.

- Charge stored by capacitor is given as; **Q = CV** Where C is **capacitance of** capacitor
- **Capacitance is defined as** Ability of a capacitor to store charge. **OR**
  "The ratio of charge stored to the potential difference between plates of capacitor."
  Mathematically, $C = \frac{Q}{V}$
- SI unit of capacitance is **Farad.**
- **1 Farad** It is defined as;
  The capacitance of a capacitor is one farad if a charge of one coulomb, given to one of the plates of a parallel plate capacitor, produces a potential difference of one volt between them **IF = 1C/1V**

CAPACITANCE OF PARALLEL PLATE CAPACITOR

- Capacitance of parallel plate capacitor with air between its plates is given as;
  $$C_{vac} = \varepsilon_o\frac{A}{d}$$

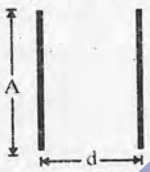
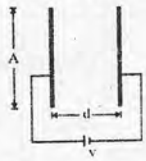
The above expression shows that-

- As we increase the surface area of plate the capacitance will increase.
- Decreasing the distance between plates will increase the capacitance.
- Introducing a dielectric between the plates will increase the capacitance of the capacitor.
- Capacitance of a parallel plate capacitor with dielectric between its plate is given as;

$$C_{med} = \varepsilon_r \varepsilon_o \frac{A}{d} = \varepsilon_r C_{vac} \qquad \text{so} \qquad C_{med} > C_{vac}$$

Dielectric co-efficient or Dielectric constant

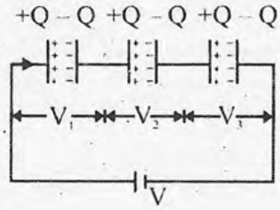
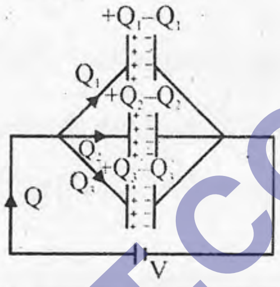
Definition The ration of the capacitance of a parallel plate capacitor with an insulating substance as medium between the plates to its capacitance with vacuum (or air) as medium between them. $\varepsilon_r = \dfrac{C_{med}}{C_{vac}}$

SEPARATION IS INCREASING		
Quantity	Battery is removed 	Battery remains 
Capacity	Decreases because $C \propto \frac{1}{d}$ i.e., $C_2 < C_1$	Decreases because $C \propto \frac{1}{d}$ i.e., $C_2 < C_1$
Charge	Remains constant because a battery is not present i.e., $q_1 = q_2$	Decreases because battery is present i.e., $q_2 < q_1$. Remaining charge $(q_1 - q_2)$ goes back to the battery.
Potential difference	Increases because $V = \frac{q}{c}$ $\Rightarrow \quad V \propto \frac{1}{C}$ i.e., $V_2 > V_1$	$V_2 = V_1$ (since battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\varepsilon_o} = \frac{q}{A\varepsilon_o}$ i.e., $E_2 = E_1$	Decrease because $E = \frac{q}{A\varepsilon_o}$ and $q_2 < q_1 \Rightarrow E \propto q$ i.e., $E_2 < E_1$
Energy	Increases because $U = \frac{q^2}{2C}$ Here $C_2 > C_1$ and $q_1 = q_2$ i.e., $U_2 > U_1$	Decreases because $U = \frac{1}{2}CV^2$ Here $C_2 < C_1$ but $V_2 = V_1$ i.e., $U_2 < U_1$

SEPARATION IS DECREASING		
Quantity	Battery is removed	Battery remains connected
Capacity	Increases because $C \propto \frac{1}{d}$ i.e., $C_2 > C_1$	Increases i.e., $C_2 > C_1$
Charge	Remains constant because battery is not present i.e., $q_2 = q_1$	Increase because battery is present i.e., $q_2 > q_1$. Remaining charge $(q_2 - q_1)$ supplied from the battery.
Potential difference	Decreases because $V = q/C$ $\Rightarrow$ $V \propto \frac{1}{C}$ and $C_2 > C_1$ i.e., $V_2 < V_1$	$V_2 = V_1$ (since battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\varepsilon_o} = \frac{q}{A\varepsilon_o}$ and $V_2 = V_1$ i.e., $E_2 = E_1$	Increases because $E = \frac{q}{A\varepsilon_o}$ And $q_2 > q_1$ i.e., $E_2 > E_1$
Energy	Decreases because $U = \frac{q^2}{2C}$ And in this case $q_2 = q_1$ and $C_2 > C_1$ i.e., $U_2 < U_1$	Increases because $U = \frac{1}{2}CV^2$ Here $C_2 > C_1$ but $V_2 = V_1$ i.e., $U_2 > U_1$

COMBINATIONS OF CAPACITORS

SERIES COMBINATION	PARALLEL COMBINATION
• Capacitors are said to be connected in series between two points when we can proceed from one point to the other only through one path. **OR** • In series grouping charge on each capacitor remains same and equals to the main charge supplied by the battery but potential difference across them may or may not be same.	• The capacitors are said to be connected in parallel between any two points if we proceed from one point to the other along different paths. **OR** • In parallel grouping potential difference across each capacitor remains same and equal to the applied potential difference while charge on them may or may not be same.

• Charge on each capacitor remains same and equals to the main charge supplied by the battery. **$V = V_1 + V_2 + V_3$**  	• Potential difference across each capacitor remains same and equal to the applied potential difference **$Q = Q_1 + Q_2 + Q_3$** 
• Equivalent capacitance $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ or $C_{eq} = \left(C_1^{-1} + C_2^{-1} + C_3^{-1}\right)$	$C_{eq} = C_1 + C_2 + C_3$
• In series combination potential difference and energy distribution is in the reverse ratio of capacitance, i.e., $V \propto \frac{1}{C}$ and P.E $\propto \frac{1}{C}$	• In parallel combination charge and energy distributes in the ratio of capacitance i.e. $Q \propto C$ and P.E $\propto C$
• If two capacitors having capacitance C_1 and C_2 are connected in series then $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	• If two capacitors having capacitance C_1 and C_2 are connected in series then $C_{eq} = C_1 + C_2$
• If n identical capacitors each having capacitances C are connected in series with supplied voltage V then equivalent capacitance $C_{eq} = \frac{C}{n}$ and Potential difference across each capacitor $V' = \frac{V}{n}$	• If n identical capacitors are connected in parallel Equivalent capacitance C_{eq} = nC and Charge on each capacitor $Q' = \frac{Q}{n}$

ENERGY STORED IN A CAPACITOR

- Charge on the plate of capacitor **possesses electrical potential energy** because of the work done to deposit the charge on the plates.
- $P.E = \frac{1}{2}qV$

  Energy in capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$
- Energy is stored in electric field between the plates.
- Energy $= \frac{1}{2}\left[\frac{A \epsilon_r \epsilon_o}{d}\right][Ed]^2$

  Energy Density =Energy/volume $= \frac{1}{2}\epsilon_r \epsilon_o E^2$

POINT TO PONDER

Can energy of a capacitor increase if insertion of dielectric takes place?

Example: A capacitor of capacitance 4 μF is charged to 80 V and another capacitor of capacitance 6 μF is charged to 30 V. when they are connected together, the energy lost by the 4 μF capacitor is

A) 7.8 mJ
B) 4.6 mJ
C) 3.2 mJ
D) 2.5 mJ

Solution: A)

$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = 50V$$

For 4 μF capacitor $E_i = \frac{1}{2}C_1V^2$, $E_f = \frac{1}{2}C_1v_C^2$

$$E_i - E_f = \frac{1}{2}C\left(v^2 - v_C^2\right) = 7.8\times10^{-3}\,J$$

CHARGING AND DISCHARGING A CAPACITOR

- D.C supply stores charges on the plate
- A.C supply does not store charge
- Charging and discharging time depends upon product of R & C

Time Constant

$$t=RC$$

Its unit is 's' i.e. second.

- Time constant is defined as the time required by a capacitor to charge up to 0.63 times the equilibrium charge on the capacitor.
- Charge reaches its equilibrium value **sooner** when time constant is smaller.
- Windshield wipers of car work by charging and discharging of capacitor.

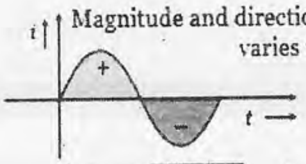
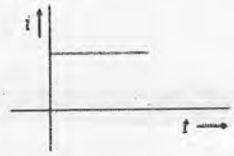
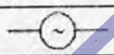
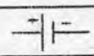
TOPIC-7 CURRENT ELECTRICITY

LEARNING OUTCOMES

COURSE CONTENT

- Ohm's Law
- Electrical resistance
- Specific resistance or resistivity
- Effect of temperature on resistance (Temperature coefficient of resistance or resistivity)
- Internal resistance of a supply
- Electric power (unit of electric power)
- Kilowatt-hours
- Kirchhoff's Rule (Kirchhoff's current law, Kirchhoff's voltage law)
- Potentiometer

Types of current:

Alternating current (ac)	Direct current (dc)
(i) Magnitude and direction both varies with time  ac → $\boxed{\text{Rectifier}}$ → dc	(i) (Pulsating dc)  (Constant dc)  dc → $\boxed{\text{Inverter}}$ → ac
(ii) Shows heating effect only	(ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is 	(iii) It's symbol is 

ELECTRIC CURRENT

- The rate of flow of charge in a circuit is defined as current.

  i.e. Current $= \dfrac{\text{Charge}}{\text{Time}}$ or $I = \dfrac{Q}{t}$ or $Q = It$

Example: The current in a resistor is 8.0 mA. What charge flows through the resistor in 0.020 s?

A) 0.16 mC B) 1.6 mC C) 4.0 mC D) 0.40 mC

Solution: B) $Q = It$

$$= (8.0\,\text{mA})(0.020\text{ s})$$
$$= 0.16\,\text{mC}$$

- Electric current is equal to charge that flows in one second.
- Electric current is a scalar quantity.
- Unit of current is ampere in M.K.S. system and stat ampere in C.G.S. system.

  and 1 ampere $= 3\times10^9$ stat ampere
- 1 ampere = 6.25×10^{18} electrons / second.
- Direction of current is from higher to lower potential. The direction of current is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

- In metals electric conduction takes place due to flow of free electrons only. But in gases and electrolytes, electric conduction takes place due to flow of both positive and negative ions.

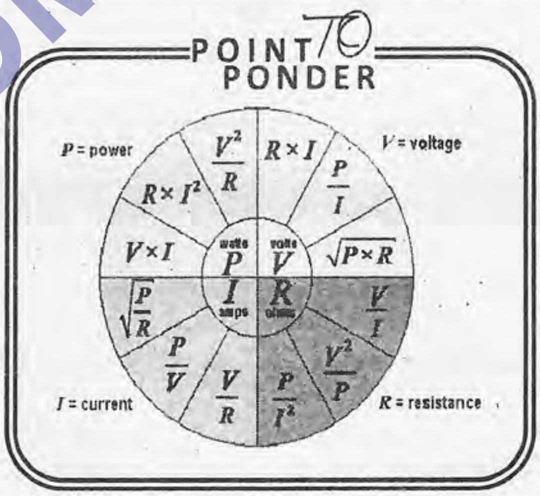
Conventional & Electronic Current

CONVENTIONAL CURRENT	ELECTRONIC CURRENT
Current due to positive charge is called conventional current.	Current due to electrons or negative charges is called electronic current.
It flows from higher to lower potential.	It flows from lower to higher potential.
It flows in direction along the direction of field.	It flows from lower to higher potential.

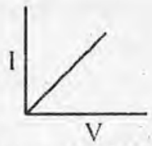
- Both conventional and electronic currents are flowing in a circuit, then total current will be given as; $I_{total} = I_{+ve} + I_{-ve}$
- **Drift velocity (V_d)** It is the average velocity attained by free electrons on applying external electric field. In conductors. $v_d = 10^{-3}\,ms^{-1}$
- **Current density (J)** The current flowing per unit normal area of cross section is defined as current density. i.e. $J = \frac{I}{A}\left(amp\,m^{-2}\right)$. Current density is a vector quantity. Its direction is from higher potential to lower potential or its direction is that of the flow of positive charge. $J = nev_d$ where v_d is the drift velocity of electrons.

OHM'S LAW

- If the physical conditions of the conductor (temperature etc.) remain constant, then the applied potential difference is directly proportional to the current flowing in it i.e.,
  $I \propto V$ or $V = IR$

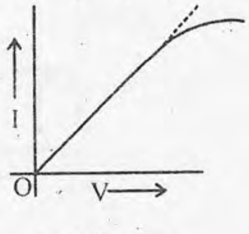


- R is constant called the resistance of the conductor.
- **Ohmic substances** The substances which obeys Ohm's law is called ohmic substances. Metals are ohmic substances. Graph for ohmic substances between V and I is a straight line.
- Ohm's law is valid only for metallic conductors.
- The conductor that does not obey Ohm's law is called **non-ohmic conductor.** Its V~I graph is not a straight line.

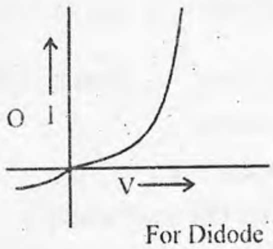


- **Non-ohmic resistances** Those resistances which do not obey Ohm's law are called non-ohmic resistances. Diode valve, triode valve, pentode, electrolytes, transistors, Torch bulb etc. are non-ohmic resistances. Graph between V and I for non-ohmic resistances is not a straight line but a curve. It is shown in figure.

Resistance at P, $R = \dfrac{V_2 - V_1}{I_2 - I_1}$ or $R = \dfrac{\Delta V}{\Delta I}$ it is called dynamic resistance.



For Filament



For Didode

- Slope of I-V graph is equal to conductance.
- Slope of I-V graph is equal to resistance.

ELECTRICAL RESISTANCE

- It is the property of a conductor, by virtue of which it opposes the flow of current in it.

$$R = \frac{V}{I} = \frac{\text{Potential difference between the end of conductor}}{\text{Current flowing in the conductor}}$$

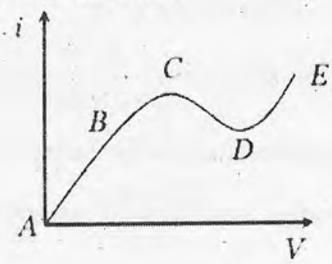
- Unit of R is ohm in M.K.S. system and stat-ohm in C.G.S. system and $1\,\text{ohm} = \dfrac{1}{9\times10^{11}}$ stat ohm
- **Unit ohm** The resistance of a wire, through which a current of 1 ampere flows on applying, a potential difference of 1 volt across its ends, is defined as one ohm.

Resistance depends upon

- The area of cross-section of the conductor $\left(R \propto \dfrac{1}{A}\right)$
- The length of the conductor $(R \propto L)$
- The nature of material of the conductor $\therefore \quad R = \rho\dfrac{L}{A}$

  Where ρ is known as the specific resistance or resistivity.
- The temperature of the conductors.

Example: **From the graph between current i & voltage V shown, identity the portion corresponding to negative resistance**



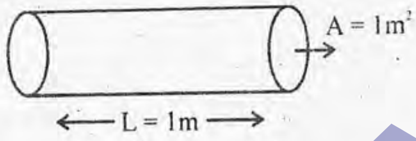
A) DE
B) CD
C) BC
D) AB

Solution: R=V/i in the graph CD has only negative slope. So in this portion R is negative

SPECIFIC RESISTANCE OR RESISTIVITY

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross-sectional area and unit length. $\rho = \frac{RA}{L}$

so, if L=1m and A = 1m^2 then ρ = R



- Unit of ρ is ohm x meter
- Resistivity depends on
  (a) nature of material **(b)** temperature.

Example: A wire l = 8m long of uniform cross-sectional area A = 8 mm^2 has a conductance of G = 2.45 Ω^{-1}. The resistivity of material of the wire will be

A) 2.1×10^{-7} Ωm B) 3.1×10^{-7} Ωm C) 4.1×10^{-7} Ωm D) 5.1×10^{-7} Ωm

Solution: C) $\rho = \frac{RA}{l} = \frac{A}{Gl} = \frac{8\times10^{-6}}{2.45\times8} = 4.1\times10^{-7}\Omega m$

- ρ does not depend on the size ad shape of the material because it is the characteristic property of the conductor material.
- Other of specific resistance $\rho_{alloy} > \rho_{semiconductor} > \rho_{conductor}$
- The value of specific resistance (ρ) is minimum for Ag and maximum for silver, so $\rho_{Al} > \rho_{Cu} > \rho_{Ag}$ Hence Ag is best conductor and Cu is second best conductor of electricity.
- Constantan, Nichrome, Maganine, Ureka (alloys) and tungsten have high value of ρ, hence resistance of wires of these material is also high.

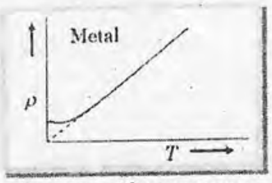
EFFECT OF TEMPERATURE ON RESISTANCE (TEMPERATURE CO-EFFICIENT OF RESISTANCE OR RESISTIVITY)

- Resistance of pure metal (e.g. copper, aluminum) increases with the increase of temperature. The change in resistance is fairly regular for normal range of temperatures.
- The resistance of electrolytes, insulators (e.g. glass, mica, rubber etc.) and semiconductors (e.g. germanium, silicon etc.) decreases with the increase in temperature.
- Resistance of constantan wire does not change with small change of temperature. (Since temperature coefficient of resistance of constantan is very small). Hence standard resistance used in resistance boxes and post office box is made of constantan.
- The reciprocal of resistivity is called conductivity.

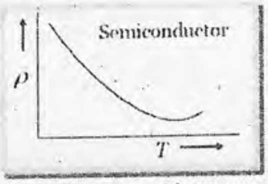
$\sigma = \frac{1}{\rho}$ and Unit of σ is $\frac{1}{\text{ohm}\times\text{metre}}$ or $\frac{\text{mho}}{\text{metre}}$

- **Temperature coefficient of resistance (α)** It is the increase in resistance per unit original resistance at 0^{o}C per unit rise (1^{o}B) in temperature. $\alpha = \frac{R_t - R_0}{R_0 t}$. Its unit is per ^{o}C.
- For conductors α is positive i.e., on increasing temperature, resistance increase.
  For semiconductors (Si, Ge) and insulators (diamond quartz) α is negative, i.e., on increasing temperature resistance decreases.

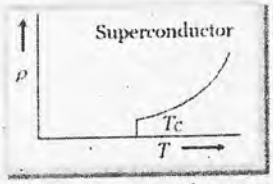
Variation of resistivity with temperature:



ρ increases with temperature



ρ decreases with temperature



ρ decreases with temperature and becomes zero at a certain temperature

INTERNAL RESISTANCE OF SUPPLY

emf of a source is defined as potential difference between its output terminals when either its internal resistance ***is zero or no current is being drawn from it.***

- When charge carriers flow through a conductor, they loose their electrical K.E. In doing work against resistance, loss of energy is compensated by source of **emf at same rate.**
- Every source of emf has its own resistance called ***internal resistance.***
- Smaller is the internal resistance of a battery, better it will be a source of emf
- **Terminal potential difference is a voltage between output terminals of a source of emf when current is drawn from it.**
- We can relate emf (Є), terminal potential (V_t) and internal resistance(r) by the following equation

  $V_t = \varepsilon - Ir$ **OR** $\varepsilon = V_t + Ir$

Example: A new flashlight cell of emf 1.5 volts gives a current of 15 amps, when connected directly to an ammeter of resistance 0.04 ohm. The internal resistance of cell is

A) 0.04 ohm C) 10 ohm

B) 0.06 ohm D) 0.10 ohm

Solution: B)

$$i = \frac{E}{R+r} \Rightarrow 15 = \frac{1.5}{0.04+r} \Rightarrow r = 0.06\Omega$$

ELECTRICAL POWER

Joule Thomson's Effect & Power Dissipation

- When current flows through conductors, then a part of energy of current carriers is transferred to ions on their way, due to which their amplitudes of vibration increases which result in rise of temperature. The above phenomenon is known as *Joule Thomson's effect*. Hence electrical energy is wasted that is given as;

$$W = Q V$$

$$P = \frac{W}{t} = \frac{QV}{t}$$

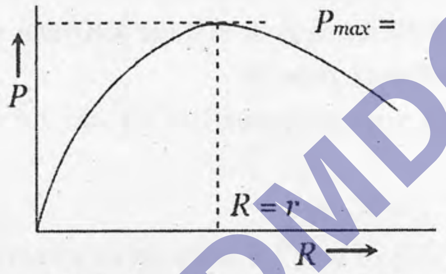
$$P = IV$$

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

Where P represents the power dissipation.

- Heat produced due to power dissipation is given as;
  $H = P \times t$
  $= I^2 R t$
  $= V I t$
  $= V^2 t / R$
- If 1J electrical energy is dissipated per second, then power dissipated will be 1 watt.
  1 watt = 1J /1s
- **Kilowatt-hour** is a commercial unit of electrical energy. It is defined as
  "When a power of 1kW is maintained through a circuit for 1 hour, then energy dissipated is 1kWh."
  1 KWh = 1000 W × 3600 sec
  $= 3.6 \times 10^6$ J
- J = 2.77×10^{-7} kWh
- Power delivered will be maximum when R = r , $P_{max} = E^2/r$

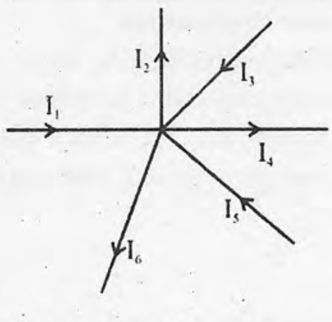


- Maximum power is delivered to a load R when the internal resistance of the source of emf is equal to the load resistance, also called maximum power transfer theorem.
  The value of the maximum output power is

$$(P_{out})_{max} = \frac{\varepsilon^2}{4r} = \frac{\varepsilon^2}{4R}$$

KIRCHHOFF'S RULE

Kirchhoff's Current Law

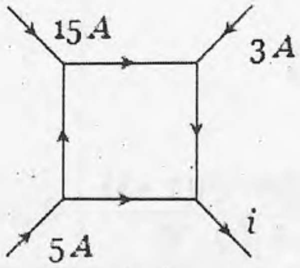


In an electric circuit, the algebraic sum of the currents meeting at any junction in the circuit is zero.

i.e., $I_1 - I_2 + I_3 - I_4 + I_5 - I_6 = 0$ or $\Sigma I = 0$

This law is based on the law of conservation of charge.

Example: The figure shows a network of currents. The magnitude of current is shown here. The current I will be



A) 23A B) 3A C) 13A D) -3A

Solution: I =15+13+5= 23A

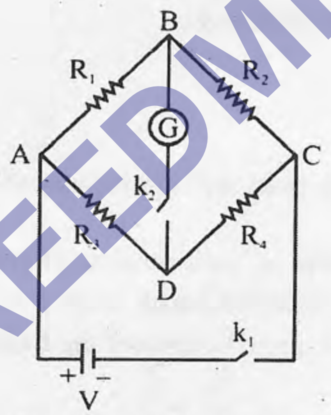
Kirchhoff's Voltage Law

The algebraic sum of potential changes in a closed circuit is zero.

- This law is based on the law of conservation of energy.
- $\Sigma V = 0$

Wheatstone Bridge

- W.S.B consists of 4 resistances R_1, R_2, R_3 and R_4 connected in loop or mesh of certain pattern.



- Wheatstone bridge is an accurate measuring instrument of resistance up to two decimal place.
- Wheatstone bridge is balanced when its galvanometer shows no deflection even when its keys are closed.
- Condition for balanced bridge is that current through galvanometer is zero. Potential difference is across galvanometer is zero.

Principle of Wheatstone Bridge

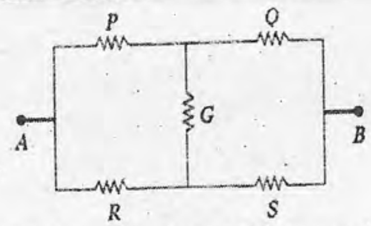
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

- Unknown resistance is placed at R_3 or R_4.

  Practical form of Whetstone Bridge is also called Meter Bridge.

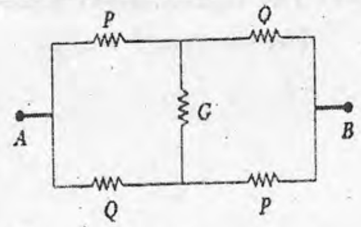
- **Equivalent resistance between points A and B in an unbalanced Wheatstone's bridge as shown in the diagram**

(i)



$$R_{AB} = \frac{PQ(R+S)+(P+Q)RS+G(P+Q)(R+S)}{G(P+Q+R+S)+(P+R)(Q+S)}$$

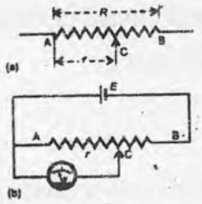
(ii)

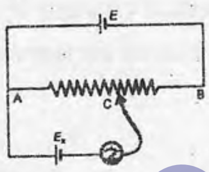


$$R_{AB} = \frac{2PQ+G(P+Q)}{2G+P+Q}$$

POTENTIOMETER

It is a device which is used to compare and find unknown emf in a circuit without drawing any current from the circuit.





Uses

- Accurate measurement of potential difference
- Measurement of emf of a battery
- Comparing emf's of batteries
- Internal resistance of a cell
- Wire of potentiometer is **4 m long**, made of high resistance wire **(eureka)**

Principle of Potentiometer

Voltage across any length of a wire of uniform area of cross section is directly proportional to its length when a constant **current flows through** it i.e. $v \propto l$ and current is **I = E/R**

- The ratio of the emf of two cells is equal to ratio of the **balancing lengths** i.e.

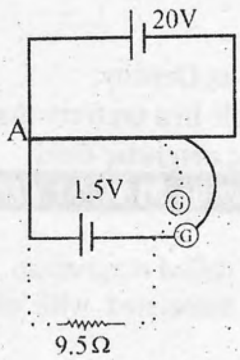
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Difference between Voltmeter and Potentiometer

VOLTMETER	POTENTIOMETER
It draws some current from source of emf	It does not draw any current from the source of unknown emf
The potential difference measured by it is lesser than the actual potential difference	The potential difference measured by it is equal to actual potential difference
It is a versatile instrument	It measures only emf or potential difference
It is based on deflection method	It is based on zero deflection method

Example:

Figure shows a 2.0 V potentiometer of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell



A) 1.7Ω C) 3Ω

B) 2.7Ω D) 1Ω

Solution:

Here $l_1 = 76.3\,\text{cm}\, l_2 = 64.8\,cm, R = 9.5\Omega$

The formula for the internal resistance of a cell by potentiometer method is

$$r = R\left(\frac{l_1 - l_2}{l_2}\right)$$

$$\therefore \qquad r = 9.5\left(\frac{76.3 - 64.8}{64.8}\right)\Omega$$

$$= \frac{9.5 \times 11.5}{64.8} = 1.686\Omega \sim 1.7\Omega$$

TOPIC-8 ELECTROMAGNETISM

LEARNING OUTCOMES

COURSE CONTENT

- Magnetic field
- Magnetic Flux and Magnetic Flux Density
- Force acting on a charged particle in a uniform magnetic field.
- Path followed by charge particle magnetic field

Introduction MAGNETIC FIELD

- Study of magnetic properties is called magnetism.
- Study of magnetic properties associated with electricity and laws relating to them is called electromagnetism.
- Iron ore magnetite (Fe_3O_4) was discovered as **early as 600 B.C** from Magnesia, a region in Asia Minor, situated **in modern** Turkey.
- Magnetic field arises due to moving charges.

Characteristics of a Magnet

- It has two poles.
- North Pole of magnet coincides with the South Pole of earth's magnet & vice versa.
- Freely suspended magnet sets itself along N-S direction.
- An un-magnetized magnetic material can be magnetized as follows;
  - ✦ By electric method (passing strong D.C)
  - ✦ By single touch or double touch methods.
- Magnet can be demagnetized as follows;
  - ✦ By passing A.C
  - ✦ By heating strongly
  - ✦ By striking a magnet again and again with a surface like that of earth. e.g by hammering.

Magnetic Effect of Current

When an electric current is passed through a conductor then a magnetic field is produced around the conductor. It is called the magnetic effect of current.

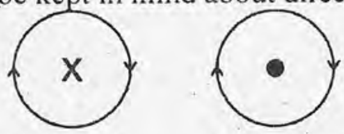
Magnetic field (B): The region of space around a magnet in which its magnetic effects are experienced, is defined as magnetic field.

- It is a vector quantity.
- Unit intensity of magnetic field (B) is weber m^{-2} or Newton/ampere x meter or tesla in M.K.S. system and Maxwell/cm^2 or Gauss or Oersted in C.G.S. system.

Right Hand Rule

If the wire is grasped in fist of right hand with the thumb pointing in the direction of the conventional current, the fingers of the hand will circle the wire in the direction of the magnetic field

- Following figures should be kept in mind about direction of magnetic field.



I-[Into paper]
B-Clockwise

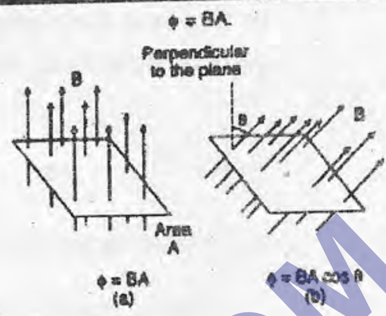
I-[Out of paper]
B- Anti-Clock wise

MAGNETIC FLUX AND MAGNETIC FLUX DENSITY

MAGNETIC FLUX

When a plane is held perpendicular to a uniform magnetic field, the product of magnetic field (B) and the area (A) of the plane is called the magnetic flux ϕ. Thus,

$$\phi = \vec{B}.\vec{A}.$$



If the magnetic field is not perpendicular to the plane, but makes an angle θ with the normal to the plane the flux linked with the plane will be **$\phi = (B\cos\theta)\ A = BA\cos\theta$**

Unit of Magnetic Flux:

The unit of magnetic flux ϕ is weber.

Magnetic flux density:

The magnetic induction B is the flux per unit area of a surface perpendicular to B is also called as flux density.

- **Formula:** $B = \dfrac{\phi}{A}$

Unit:

- weber metre^{-2}
- Newton/amp-meter.
- Unit of B is also called tesla. In C.G.S. system unit of magnetic flux density is gauss.

1 tesla = 10^4 gauss

Dimension: $= \left[\dfrac{\text{newton}}{\text{amp} - \text{metre}}\right] \times [A] = \left[\dfrac{MLT^2}{AL}\right] \times [L^2] = [ML^2T^{-2}A^{-1}]$

FORCE ACTING ON A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Magnetic Force on a Moving Charge

When a charged particle of charge q is moving with velocity v in a magnetic field B at an angle θ, then force acting on the particle:

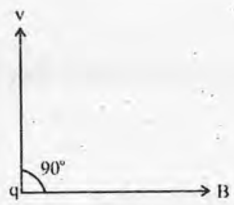
$\vec{F}_m = q(\vec{v} \times \vec{B})$ or $F = qvB\sin\theta$

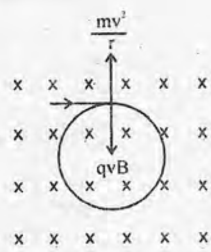
Where θ is angle between $\vec{v}$ and $\vec{B}$.

- Direction of force F is perpendicular to both v and B. this direction can be found out by "Fleming's left hand rule".

For electron $F = -e(\vec{v} \times \vec{B})$ and for proton $F = +e(\vec{v} \times \vec{B})$

PATH FOLLOWED BY CHARGE PARTICLE IN MAGNETIC FIELD





- If v = 0, then F = 0
- If $\theta = 0°$, then $F = qvB\sin\theta = 0$

So the particle will continue to move in straight line with the same speed.
- If $\theta = 180°$, then $F = qvB\sin 180$ or $F = 0$

Here also the particle will move in same direction in straight line with same speed.
- If $\theta = 90°$, then $F = qvB\sin 90$ or $F_{max} = qvB$

In this case the charged particle will move in circuilar path and the plane of the circle is perpendicular to B. Here, Lorentz force acts as centripetal force.

So

$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

R is called radius of circular path or cyclotron radius.

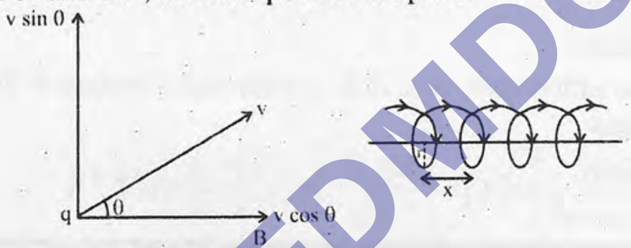
$\therefore$ Angular velocity or angular frequency of the particle: $\omega = \frac{v}{r} = \frac{qB}{m}$

So, $\omega = \frac{qB}{m}$ But $\omega = 2\pi f$ or $\mathbf{f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}}$

Also time period of particle $T = \frac{1}{f} = \frac{2\pi m}{qB}$ and kinetic energy of particle

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{rqB}{m}\right)^2 \quad \text{so} \quad \mathbf{K.E = \frac{r^2q^2B^2}{2m}}$$

If θ lies between 0° and 90°, then the path of the particle is helical or helix

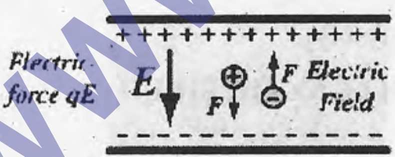


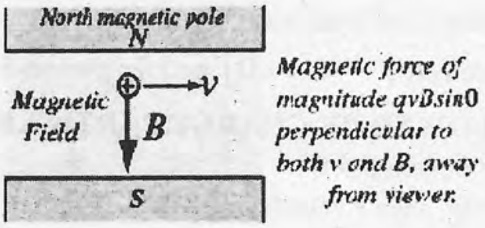
Here radius of helix $r = \frac{mv\sin\theta}{qB}$ and pitch of helix

$$x = v\cos\theta \times T = v\cos\theta \times \frac{2\pi m}{qB} \quad \therefore \quad T = \frac{2\pi m}{qB}$$

Force on Charge Particle in An Electric and Magnetic Field

As, $\vec{F_e} = q\vec{E}$, and $\vec{F_m} = q(\vec{v} \times \vec{B})$ or $\vec{F} = \vec{F_e} + \vec{F_m}$ or $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$





Where $\vec{F}$ is called **Lorentz force**. If charge is of mass 'm' in uniform electric field then

$$\vec{a} = \frac{\vec{F}}{m}$$

- Electric force does work
- Magnetic force does no work

Example: A particle of charge -16×10^{-18} C moving with velocity 10 m/s along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

A) 10^3 Wb m^{-2} B) 10^5 Wb m^{-2} C) 10^{16} Wb m^{-2} D) 10^{-3} Wb m^{-2}

Solution: A) $\vec{F}=\vec{F}_e+\vec{F}_m$, $\vec{F}=q\vec{E}+q\left(\vec{v}\times\vec{B}\right)=0$

$$\Rightarrow B=\frac{E}{v}=10^3\,\text{Wb m}^{-2}$$

Difference between Electric and Magnetic Forces

Sr.No.	Electric Force	Magnetic Force
1	Electric force on a charged particle is always collinear with the field.	Magnetic force on a charge particle is always perpendicular to the field.
2	Electric force (qE) is independent of the state of rest of the motion of the charged particle.	Magnetic force is velocity dependent i.e. acts only when the charged particle is in motion.
3	Electric force does work in displacing the charged particle.	Magnetic force does no work when the charged particle is displaced.
4.	Electric force, $\vec{F}=q\vec{E}$	Magnetic force, $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$ $=qvB\sin\theta$ When $\theta=0$ Force $=0$ When $\theta=90^\circ$ $F=qvB(\max)$
5.	Electric force may or may not be non-central.	Magnetic force is always non-central.

e/m FOR AN ELECTRON

- e/m of a charged particle can be given as; $\frac{e}{m}=\frac{v}{Br}$

Where R is determined by **Thomson's apparatus** while velocity is determined by following two methods:

Potential difference method: $v = \sqrt{2V_o \frac{e}{m}}$ so that $\frac{e}{m} = \frac{2V_o}{B^2r^2}$

Velocity selector method: $\mathbf{v} = \frac{\mathbf{E}}{\mathbf{B}}$ so that $\frac{e}{m} = \frac{E}{B^2r}$

The value of $\frac{e}{m}$ is equal to 1.7588×10^{11} C/kg

POINT TO PONDER

The e/m of a proton is greater or less than an electron?

Example: Four particles independently move at the same speed in a direction perpendicular to the same magnetic field. Which particle is deflected the most?

A) a copper ion
B) a helium nucleus
C) an electron
D) a proton

Solution: The force exerted on a particle by the magnetic field is given by Bqv. As this force provides the necessary centripetal force to deflect the particle.

$$Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq}$$

For a large deflection, r is small. Among the four particles, the electron has the lowest $\frac{m}{q}$ ratio.

TOPIC-9 ELECTROMAGNETIC INDUCTION

LEARNING OUTCOMES

COURSE CONTENT

- Electromagnetic induction
- Faraday's Law, Application in seismometer
- Motional emf
- Lenz's Law
- Mutual induction (Mutually Induced EMF)
- Self-induction
- Alternating Current Generator
- Transformers

ELECTROMAGNETIC INDUCTION

- "When the magnetic flux linking a conductor changes, an e.m.f is induced in the conductor, this phenomenon is known as electromagnetic induction". Induction is the change in flux linking the conductor (or coil)
- The product of number of turns (N) of the coil and the magnetic flux (Φ) linking the coil is called flux linkages i.e.
  Flux linkages = $N\Phi$

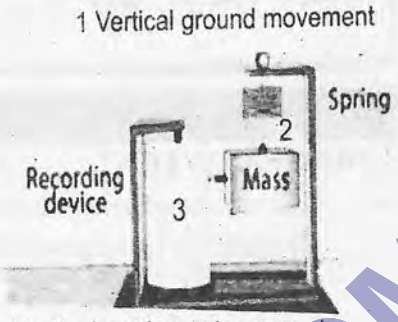
FARADAY'S LAW, APPLICATION IN SEISMOMETER

Faraday's Laws of Electromagnetic Induction

Faraday has given the following two laws regarding electromagnetic induction.

- Whenever there is a change in the magnetic flux linked with a circuit, an induced emf is produced in the circuit. If the circuit is closed, an induced current flows through it. The current flows only so long as the magnetic flux is changing.
- The induced emf is equal to the negative rate of change of magnetic flux. Thus, if $\Delta\phi$ be the change in flux in time interval Δt, then the induced emf in the circuit is
  $\varepsilon = -\frac{\Delta\phi}{\Delta t}$, N number of turns, $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$
- The negative sign shows the induced emf opposes the change in magnetic flux.
- If rate of change of magnetic flux be in weber/sec, the induced emf will be in volt.
- If the coil contains N turns $\varepsilon = -N\frac{\Delta\phi}{\Delta t} = \frac{\Delta(N\phi)}{\Delta t}$
- To measure the earthquakes, we use a device which is called seismometer.
- One kind of seismometer is called inertial because it is based on Newton's 1st Law. It consists of spring mass system which records the vibrations in the earth's surface and will pick up even the slightest vibration. This is recorded on a sheet of paper under the seismograph needle that writes it. When there are vibrations, the needle sways, causing bigger lines to be drawn.
  The other kind of seismometer works on the principle of electromagnetic induction. It transforms received vibration enemy into an electrical voltage.

The relative motion between a magnet and a coil (one of which is attached to the inertial mass and one is attached to the frame) induces an emf in the coil that is proportional to the velocity of the relative motion. The magnitude of emf is also proportional to the strength of the magnet used and the number of turns in the coil.
In practice either the magnet or the coil can be attached to the inertia] mass (in commercial systems the magnet is itself often used as the menial mass).



Vertical motion seismograph

•

Example: A flat circuit coil of 120 turns, each of area 0.70 m^2, is placed with its axis parallel to a uniform magnetic field. The flux density of the field is changed steadily form 80 mt to 20 mT over a period of 4.0 s.

A) 0 B) 130 mV C) 170 mV D) 500 mV

Solution: B) $\varepsilon = NA\dfrac{\Delta B}{\Delta t}$

$$= 120 \times 0.070 \times \frac{(80-20)\times 10^{-3}}{4.0}$$

$$= 0.126 \text{ V}$$

$$= 130 \text{mV}$$

Induced Current

Induced emf in a coil is given by $\varepsilon = -N\dfrac{\Delta\phi}{\Delta t}$. If the coil be closed and the total resistance of its circuit be R, then the induced current in the circuit is $I = \dfrac{\varepsilon}{R} = \dfrac{N}{R}\dfrac{\Delta\phi}{\Delta t}$.

Induced Charge

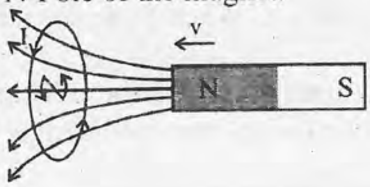
The charge flowing through the circuit in time interval Δt is

$$q = I \times \Delta t = \frac{N}{R}\frac{\Delta\phi}{\Delta t} \times \Delta t = \frac{N}{R}\Delta\phi.$$

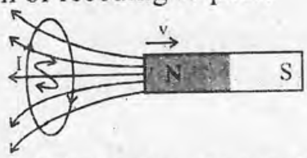
Lenz's Law

According to Lenz's rule the induced current produced in a closed circuit always flows in such a direction that it opposes the cause due to which it has been produced itself.

- In general, the direction of induced emf of current is determined by "Lenz's law", in case of motion of straight conductor in a magnetic field, it can be also be determined by the "Fleming's right hand rule.
- When the north pole of a magnet moves towards a stationary loop, an induced current I flow in the anticlockwise sense as seen from the side on which the magnet is located. The anticlockwise sense corresponds to the generation of north pole (N) which opposes the motion of the approaching N-Pole of the magnet.



- When the N-pole of the magnet is moved away from the loop, the current I flows in the clockwise sense which corresponds to the generation of south-pole (S). The induced south pole opposes the motion of receding N-pole.



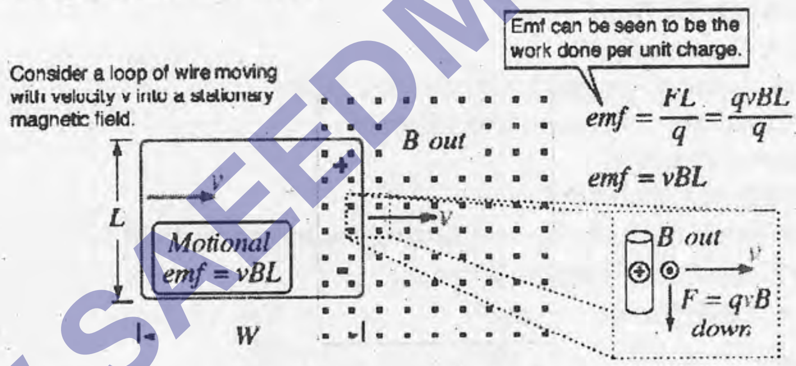
- Because in each case induced current opposes the motion of magnet hence some mechanical work has to be done on the system to move the magnet against this opposing force. According to law of conservation of energy, this work is obtained in the coil in the form of heat energy.
  If induced current was not to oppose the motion of magnet, then we could be getting electrical energy continuously without doing any work which is never possible. Hence, Lenz's rule is the necessity of law of conservation of energy.
  Motor.

MOTIONAL emf

The emf induced by the motion of a conductor across a magnetic field is called motional emf.

$$\epsilon_{(motional)} = -vBL$$

Where v = speed of the conductor
L = length of the conductor



- If there is some orientation of the coil to the magnetic field, then

$$\epsilon_{(motional)} = -vBL\sin\theta$$

Where θ is the angle between v & B.

- The unit of motional emf is volt.
- Magnetic field strength is considered to be uniform while deriving the above expression.

MUTUAL INDUCTION & SELF INDUCTION

- Mutual induction is a phenomenon in which a changing current in one **(primary)** coil induces an emf in another coil **(secondary)**

$$e.m.f_{sec} = -M \ \Delta I_P/\Delta t$$

Where $M = N_s\Phi_s/I_p$ called mutual inductance.

- Mutual inductance is defined as the ratio of back e.m.f in the secondary to the rate of change of current in primary coil.

$$M = \frac{-emf}{\Delta I_P/\Delta t}$$

- **Self-induction** is a phenomenon of generation of induced e.m.f in a coil due to change of current in itself.

$$e.m.f_{self} = -L\frac{\Delta I}{\Delta t}$$

Where L is called self-inductance. Also **NΦ = LI**

- Self-inductance is defined as the ratio of back e.m.f to rate of change of current in the same coil.

$$L = \frac{-emf_{self}}{\Delta I/\Delta t}$$

- SI unit of mutual and self-inductance is Henry (H).

(i) M = 1 H if $e.m.f_{sec} = 1V$ and $\Delta I_p/\Delta t = 1 A/s$

Mutual inductance is said to be 1 H if a current change in primary at a rate of $1As^{-1}$ produces e.m.f of 1 volt in secondary.

(ii) L = 1 H if e.m.f = 1V and $\Delta I/\Delta t = 1\ A\ s^{-1}$

Self-inductance is said to be 1H, if **current changing at the rate of 1 A/sec produces an e.m.f of 1V in the coil itself.**

ALTERNATING CURRENT GENERATOR

Current generator is a device, which converts mechanical energy into **electrical energy in the presence of magnetic field.**

- The principle of generator is that emf is induced across a coil rotating in a magnetic field due to change of **magnetic flux.**
- Main parts of A.C generator
  - ✦ Pole pieces (U-shape magnet) with concave poles.
  - ✦ Armature (assembly of coil on iron cylinder)
  - ✦ Slip rings (as connector)
  - ✦ Carbon brush (external supply)
- emf is induced by the side of loop intersecting the **magnetic** field.
- Total emf for N number of loops is given as;

$\varepsilon = N(2vB\ell Sin\theta)$ also $\varepsilon = N\,\omega\,AB\,Sin\,\omega t$

$\varepsilon_{max} = N\,\omega\,AB$ when $\theta = 90^o$

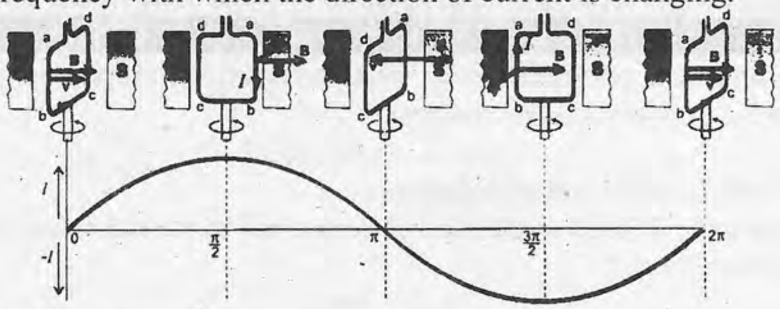
then $\varepsilon = \varepsilon_{max}\,Sin\,\omega t$ If $\omega = 2\pi f$, then

$\varepsilon = \varepsilon_{max}\,Sin 2\pi ft$

In terms of potential difference, $\mathbf{V = V_o\,Sin\,2\pi ft}$

In terms of current, $\mathbf{I = I_o\,Sin 2\pi ft.}$

where f is the frequency with which the direction of current is changing.

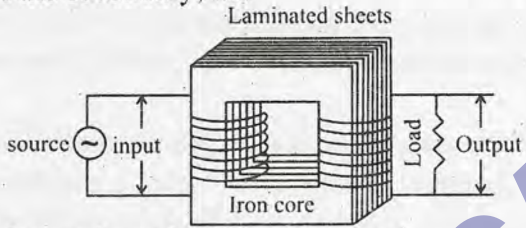


- In Pakistan f = 50Hz, it means 50 times in a second direction is changing.

TRANSFORMER

Transformer

It is a device which rises or lowers the voltage in A.C. circuits through mutual induction and consists of two coils wound on the same core. The coil which is connected to the source (i.e., to which input is applied) is called primary while the other which is connected to the load (i.e., from which output is taken) is called secondary. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary. As magnetic lines of force are closed curves, the flux per turn of the primary must be equal to the flux per turn of the secondary, i.e.



And as the number of turns in each coil is constant,

$$\frac{\phi_s}{N_s} = \frac{\phi_p}{N_p} \quad \text{or} \quad \frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} \quad \left[\text{as} \quad \varepsilon = \frac{\Delta\phi}{\Delta t}\right]$$

And as in an ideal transformer there is no loss of power, so $\varepsilon = V$ and $P = VI$ = constant,

So, $$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

Now there are two possibilities

- $N_s > N_p$ the transformer is said to be "step up" and it increases voltage and reduces current.
- $N_s < N_p$ the transformer is said to be "step down" and it reduces voltage and increases current.
- Transformer works on A.C. only and never on D.C.
- It can increase or decrease either voltage or current but not both simultaneously (as power = constant).

Example: The secondary coil of an ideal transformer delivers an r.m.s current of 2.5 A to a load resistor of resistance 8.0 Ω. The r.m.s current in the primary is 10 A. What is the r.m.s. potential difference across the primary coil?

A) 3.5 V B) 5.0 V C) 57 V D) 80 V

Solution: B) r.m.s potential difference across the secondary coil

$= (2.5)(8)$

$= 20$ V

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} \Rightarrow V_p = \frac{I_s}{I_p} \times V = \frac{2.5}{10} \times 20 = 5.0\text{V}$$

Losses in Actual Transformer

The losses in transformer occur in core and winding.

Types of Losses in Transformer

- **Copper Losses in Winding**

  Due to resistance of the windings of primary and secondary coils, some electrical energy is always lost in the form of heat energy.

- **Flux Losses**
  The coupling of the primary and secondary is never perfect and whole of the magnetic flux produced in the primary coil does not link the secondary coil. This results in some energy loss.
- **Iron Losses in Core**
  Iron losses are of two types: Eddy current loss and hysteresis loss.

(i) **Eddy Current Loss**
Due to the periodically varying nature of A.C. supplied in primary, the flux associated with core changes and, therefore, eddy currents are induced it.
Eddy currents induced in the core are undesirable as they heat the core and result in energy loss. To minimize the eddy current losses, core is laminated.

(ii) **Hysterisis Loss**
The alternating current flowing through the coils magnetizes and demagnetizes the iron core again and again. Therefore, during each cycle of magnetization, some energy is lost due to hysterisis. To minimize this loss we choose material of core of smaller hysterisis loss generally soft iron.

Efficiency of Transformer

Ideal transformer, efficiency is 100 % or 1. but in actual transformer output power is always less than input power, so efficiency also always less than 100%. In general efficiency of transformer is very high (and is of the order of 90%).

Efficiency is given by $\eta = \frac{\textbf{Output power}}{\textbf{Input power}} \times 100$. In terms of secondary and primary voltages and currents, $\eta = \frac{V_sI_s}{V_pI_p}$. Also since, **Input = Output + Losses, so**

$$\eta = \frac{\text{Output power}}{\text{Input power} + \text{Losses}}$$

TOPIC-10 ELECTRONICS

LEARNING OUTCOMES

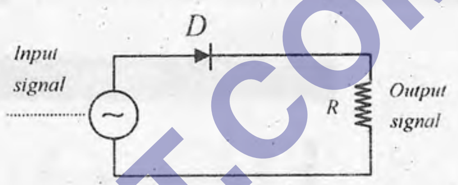
COURSE CONTENT

- Rectification

RECTIFICATION

Rectification

Conversion of alternating current into direct current is called rectification.



- There are two very common types of rectification.

Half-wave rectification (HWR)

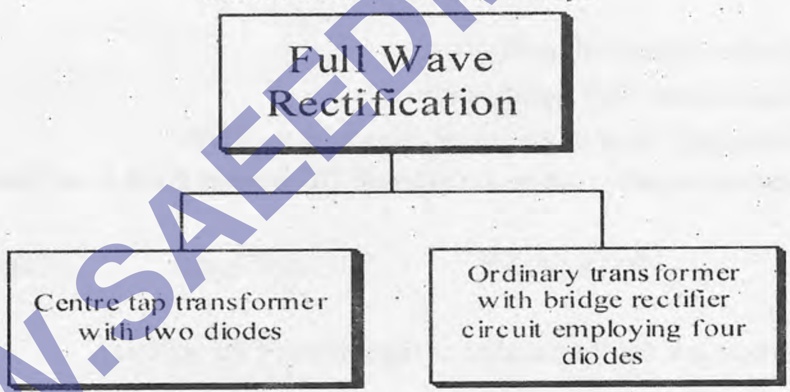
- One diode is used.
- For upper half or lower half is rectified.
- Can be used for charging battery.
- Output frequency is equal to input frequency.

Draw Backs

- More pulsations in the output wave form.
- Average value of the output DC signal drops.

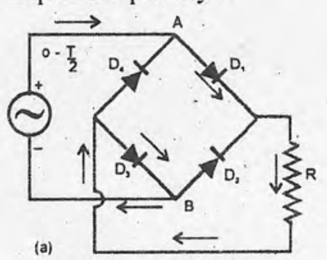
Full wave Rectification (FWR)

Can be carried out in two ways



Bridge Rectifier

- Four diodes are used.
- Two diodes remains ON (FWD Biased) in each half of the input cycle while other two remains off (Reverse Biased) in the same half of the cycle.
- Output frequency is twice of input frequency.



Summary

If input wave has "T" Time period then

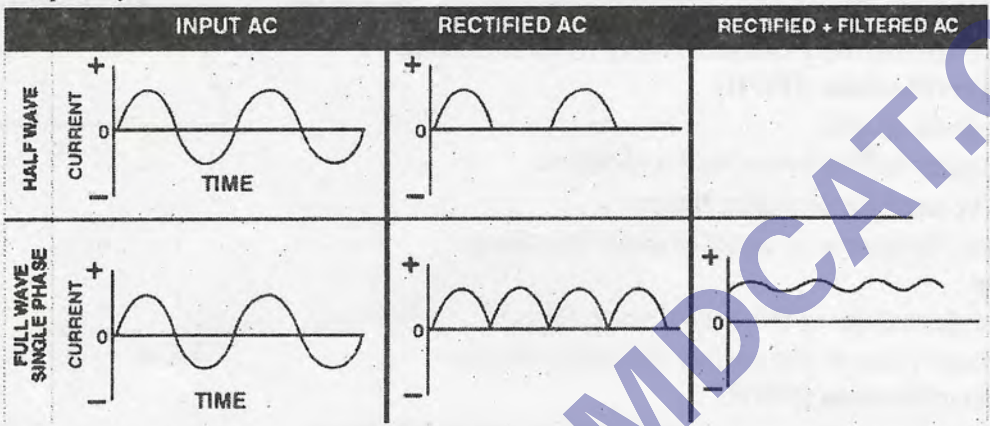
O ~ T/2 D_1 & D_3 – **Off and** D_2 & D_4 – **On**

T/2 ~ T D_1 & D_3 – **On** D_2 & D_4 – **Off**

Example: If a full wave rectifier circuit is operating from 50 Hz mains, the fundamental frequency in the ripple will be

A) 50 Hz B) 70.7 Hz C) 100 Hz D) 25 Hz

Solution: C) In full wave rectifier, the fundamental frequency in ripple is twice that of input frequency.



Advantages

- Very inexpensive method of rectification.
- Average values do not drop significantly.
- Lesser pulsations are there in the signal compared to HWR.

Example: The electrical circuit used to get smooth DC output from a rectifier circuit is called

A) Filter B) Oscillator C) Logic gates D) Amplifier

Solution: A)

'Filter' filters out the fluctuations in the output of the rectifier.

TOPIC-11 DAWN OF MODERN PHYSICS

LEARNING OUTCOMES

COURSE CONTENT

- Photon
- The wave nature of particles
- The wave-particle duality
- Electron microscope
- Uncertainty principle

PHOTON

The Photon

- Einstein presented the idea of light energy consisting of packets of **electromagnetic** energy.
- Max plank explained the emission and absorption by the atoms from a black surface is in the form of indivisible packets called **quanta.**
- Max plank put discontinuous **(granular)** nature of light.
- The beam of light with wavelength λ consists of stream of photons traveling at speed c and carrier energy hf.
- As $E = mc^2$ and $E = hf$
  then $mc^2 = hf$
  or $mcc = hf$
  $mc = hf/c$
  As $mc = p$ and $f = c/\lambda$ P = momentum
  then $p = h/\lambda$
- Emission or absorption of energy is applied to any **oscillating system.**

Example: An air station is broadcasting the waves of wavelength 300 m. If the radiating power of the transmitter is 10 kW, then the number of photons radiated per second is

A) 1.5×10^{29} B) 1.5×10^{31} C) 1.5×10^{33} D) 1.5×10^{35}

Solution: B) If n be the number of photons/s, then $E = \dfrac{nhc}{\lambda}$

$$\frac{nhc}{\lambda} = 10 \times 10^3 = n = \frac{10 \times 10^3 \times 300}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$n = \frac{3 \times 10^6}{6.6 \times 3 \times 10^{-26}} = \frac{1}{6.6} \times 10^{32} = 1.5 \times 10^{31}$$

THE WAVENATURE OF PARTICLE

De-Broglie's Hypothesis (Wave Particle Duality)

All the moving particles behave as waves called matter waves or particle waves. The wave length associated with moving particles is given by

$$\lambda = \frac{h}{mv}$$

mv = momentum of particles

$$\lambda = \frac{h}{p}$$

$$\lambda \propto \frac{1}{m}$$

$$\lambda \propto \frac{1}{v}$$

An object of large mass and ordinary speed has such a small wavelength that its wave effects such as interference and diffraction are negligible.

Davisson and Germer Experiment

- **Germer** and **Davisson** using low energy electron beam provided experimental confirmation of **de-Broglie's hypothesis**. They showed that electrons are diffracted from metal crystals in exactly the same manner as X – rays or any other wave.
- The electron beam of energy Ve is made incident on a nickel crystal. The beam diffracted from crystal surface. The wavelength associated with the moving electrons is given as

$$\lambda = \frac{h}{mv}$$

$$mv = \sqrt{2mVe}$$

$$\lambda = \frac{h}{\sqrt{2mVe}}$$ where V is accelerating potential

$V = 54V$

$\lambda = 1.66 \times 10^{-10} m$

- This beam of electrons diffraction from crystal surface was obtained for a glancing angle of 65^o. According to Bragg's equation $2d\sin\theta = m\lambda$
- For 1^{st} order diffracting $m = 1$
- For nickel $d = 0.91 \times 10^{-10} m$
- Which gives $\lambda = 1.65 \times 10^{-10} m$
- Prince Louis Victor de Broglie received noble prize in physics in 1929. Clinton Joseph Davisson and George Paget Thomson shared the Nobel Prize in 1937 for their experimental confirmation of the wave nature of particles.
- Electron microscope is a practical application of wave particle duality

Example: A tiny particle of mass 10^{-13} kg moving with a velocity of 10 cm s^{-1} is associated with a wave of wavelength

A) 6.62×10^{-18} cm B) 6.62×10^{-12} cm C) 6.62×10^{-2} cm D) 6.62×10^{-34} cm

Solution: A) $\lambda = \frac{h}{mv}$

$$= \frac{6.62\times10^{-34}}{10^{-13}\times10\times10^{-2}}\times100 \text{ cm}$$

$$= 6.62\times10^{-18} \text{ cm}$$

WAVE PARTICLE DUALITY

- In other words matter and radiation have a dual wave – particle nature and this new concept is known as wave particle duality.
- For complete description of radiation or matter, both wave and particle aspects are required.
- Only single aspect (either wave or particle) of radiation or matter can be revealed by an experiment.

ELECTRON MICROSCOPES

- Electron beam wave length is thousands time shorter than visible light.
- In stead of optical lenses, magnetic field is used to focus the electron beam.
- Resolving power of electron microscope is 0.5 to 1nm which in many times greater than optical microscope.

Three dimensional image is obtained by scanning electron microscope.

UNCERTAINTY PRINCIPLE

It states that following pairs of quantities can't be measured with perfect accuracy at same time.

- Linear momentum and position
- Energy and time

Such quantities are called conjugate quantities.
Mathematically,

$$\Delta p \approx \frac{h}{\lambda}$$

$$\Delta x \approx \lambda$$

$$\Delta p \cdot \Delta x = h$$

$$\Delta E \cdot \Delta t = h$$

- To increase accuracy in measuring position, we should use short wavelength.
- To increase accuracy in measuring momentum, we should use long wavelength.

TOPIC-12 ATOMIC SPECTRA

LEARNING OUTCOMES

COURSE CONTENT

- Atomic Spectra/Line Spectrum
- Production of X-rays and Characteristics X-rays
- Contineous X-rays (Braking X-rays)
- Properties and Uses of X-rays

ATOMIC SPECTRA / LINE SPECTRUM

Spectrum

Spectrum means set of frequencies absorbed or emitted by a substance.

Types

(a) **Emission Spectrum**

Set of frequencies emitted by atoms of a substance.

(b) **Absorption spectrum**

Set of frequencies absorbed by atoms of a substance.

- **Absorption spectrum** is caused by up **transition** of atomic electrons.
- Each element has characteristic emission spectra in its vapour state.

Emission spectra fall into following three categories

(a) **Continuous spectra**

It is emitted from condensed matter (solid or liquid). A **black body** spectrum is **continuous spectra**

(b) **Line spectra**

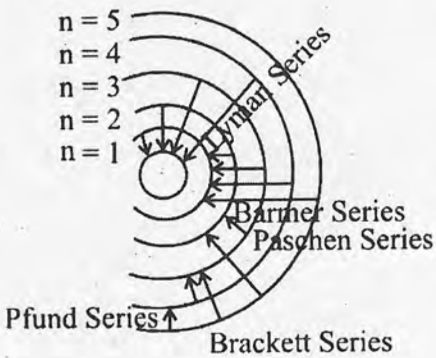
It is emitted by a gas or vapour state of element.

(c) **Band spectra**

Molecular spectra is **band spectra**

Spectrum of Hydrogen

Whenever an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two levels is emitted as a radiation of particular wavelength. It is called a spectral line. Depending upon the order of their wavelengths, these spectral lines can be grouped into a number of **spectral Series** as shown in figure.



SERIES	RELATIONS	REGION	LONGEST WAVELENGTH	SHORTEST WAVELENGT
LYMAN SERIES	$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$ $n=2,3,4.......\infty$	U–V	Put n=2 $\lambda=\left(\frac{4}{3R}\right)=1216Å$	Put $n=\infty$ $\lambda=\frac{1}{R}=912Å$
BALMER SERIES	$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$ $n=3,4,5....\infty$	Visible	Put n=3 $\lambda=\frac{36}{5R}=6563Å$	Put $n=\infty$ $\lambda=\frac{4}{R}=3648Å$
PASCHEN SERIES	$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$ $n=4,5,6....\infty$	Infrared	Put n = 4 $\lambda=\frac{144}{7R}=18761.1Å$	Put $n=\infty$ $\lambda=\frac{9}{R}=8208Å$
BRACKET SERIES	$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$ $n=5,6,7....\infty$	Infrared	Put n = 5 $\lambda=\frac{400}{9R}=40533.3Å$	Put $n=\infty$ $\lambda=\frac{16}{R}=14592Å$
PFUND SERIES	$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right)$ $n=6,7,8....\infty$	Infrared	Put n = 6 $\lambda=\frac{900}{11R}=74618.18Å$	Put $n=\infty$ $\lambda=\frac{25}{R}=22800Å$

Example: If the wavelength of photon emitted due to transition of electron from third orbit to first orbit in a hydrogen atom is λ, then the wavelength of photon emitted due to transition of electron from fourth orbit to second orbit will be

A) $\frac{128}{27}\lambda$ B) $\frac{25}{9}\lambda$ C) $\frac{36}{7}\lambda$ D) $\frac{125}{11}\lambda$

Solution: A)

$$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

$$\frac{1}{\lambda}=R\left[1-\frac{1}{9}\right]=\frac{8R}{9} \text{ or } \lambda=\frac{9}{8R}$$

Again $$\frac{1}{\lambda'}=R\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

$$\frac{1}{\lambda'}=R\left[\frac{1}{4}-\frac{1}{16}\right]$$

$$\frac{1}{\lambda'}=R\times\frac{3}{16}\Rightarrow\lambda'=\frac{16}{3}R$$

$$\frac{\lambda'}{\lambda} = \frac{16}{3R} \times \frac{8R}{9}$$

$$\lambda' = \frac{128}{27}\lambda$$

Main Points of Bohr's Atomic Model

- An electron can revolve only in those orbits in which their angular momentum is **integral multiple of** $h/2\pi$. So, $mv_n r_n = nh/2\pi$
- As long as an electron remains in an allowed orbit, its energy remains constant, in spite of fact that continuously it undergoes angular acceleration.
- Emission or absorption of energy from atom takes place only when its electron undergoes down or up transition between two allowed orbits. $\Delta E = E_n - E_p$ or $hf = E_n - E_p$
- Bohr's atomic model produced good results for H-atom and for one- electron atom.

Quantized Radii

- **Radius** of nth orbit of the atom is given as;

$$r_n = \frac{n^2h^2}{4\pi^2 m\, e^2 k} \quad \text{or} \quad r_n = r_o\, n^2$$

where $r_o = \dfrac{h^2}{4\pi^2 m\, e^2 k}$

If $n = 1$ then $r_1 = 0.53$ A°(Bohr's Radius)

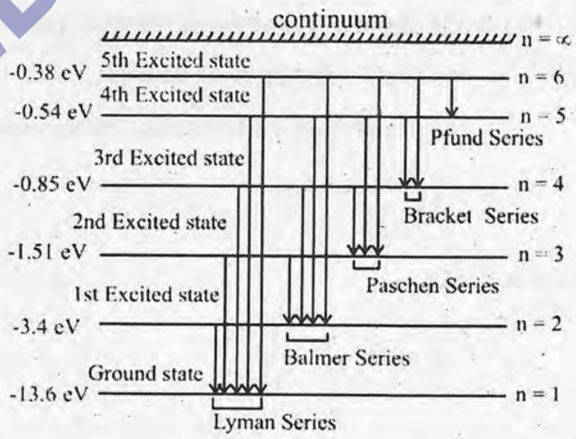
Similarly, $r_2 = 4r_1$ & $r_3 = 9\, r_1$ and so on.

It shows that radii are **quantized** and they occur in the following ratio

$r_1 : r_2 : r_3 ------ = 1:4:9 -----$

Quantized Energy

Energy of nth level of hydrogen atom is given by $E_n = -\dfrac{13.6}{n^2}\text{eV}$. Energy associated with first orbit of the hydrogen atom is $E_1 = \dfrac{-13.6}{1^2} = -13.6\text{ eV}$. It is called as **ground state energy of hydrogen atom.** Energy associated with second orbit is given by $E_2 = -\left(\dfrac{13.6}{2^2}\right) = -3.4\text{eV}$. It is called as energy of **first excited state of hydrogen atoms.**



The energy of second, third, fourth.........excited states of hydrogen atom are given by

$$E_3 = \frac{-13.6}{3^2} = -1.51\text{eV},\ E_4 = -\frac{-13.6}{4^2} = -0.85\text{eV} \text{ and when } n = \infty,\ E_\infty = \frac{-13.6}{\infty} = 0\text{eV}$$

We find that as n increases, the energy associated with a state becomes less negative and approaches closer and closer to the maximum value zero for n = ∞. The figure shows the energy level diagram of hydrogen and various spectral series emitted by it.

Excitation Potential

The potential required to raise the electron from ground state to higher state. e.g. for hydrogen,

1^{st} excitation potential = 10.2 eV and 2^{nd} excitation potential= 12.1eV

Ionization Potential

The potential required for ionization of atom e.g. for hydrogen, **13.6 V** is ionization potential.

Note

(i) There is a large no. of excitation potentials and only limited no. of **ionization potentials**.

(ii) Each next ionization potential is greater than its preceding one.

Example: If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from n = 2 is

A) 10.2 eV B) 0 eV C) 3.4 eV D) 6.8 eV

Solution: C) $\frac{13.6}{4}$ eV = 3.4 eV

Excitation Energy

The energy required to raise the atom from ground state to higher state.

Limitations of Bohr's Model

- It could not interpret the details of the optical spectra of atoms containing more than one electron.
- There was no theoretical basis for selecting the orbits which had an integral multiple of $\frac{h}{2\pi}$.
- It involves the concept of orbit which could not be checked experimentally.

It failed to explain the line structure of spectral lines

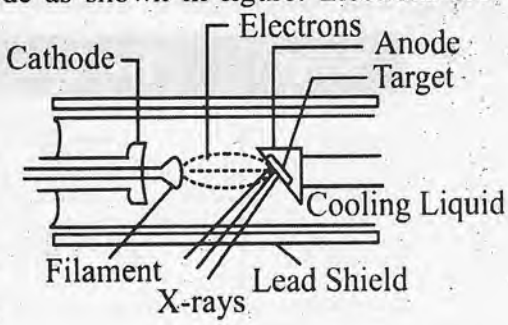
PRODUCTION OF X-RAYS

X-Rays

- X-rays were discovered in 1895 by a German physicist W.C. Roentgen who was honoured with Nobel prize for it.
- When highly energetic electrons are made to strike a metal target of high atomic weight and high melting point, electromagnetic radiation comes out. A large part of this radiation has wavelength of the order of 0.1 nm (≈1Å) and is known as X-rays.

Production of X-Rays

- These are produced in a highly evacuated tube (called Coolidge tube) which contains an anode and a tungsten filament connected to cathode as shown in figure. Electrons are obtained from the filament by thermionic emission and are accelerated to the anode by a **potential difference (~ 100 kV).**
- The anode is copper block inclined to the electron stream and having a small target of tungsten, or another metal of high melting point, on which electrons are focused by the concave cathode. The tube has a lead shield with a small window to allow the passage of the X-ray beam.



Coolidge Tube

Nature of X-Rays

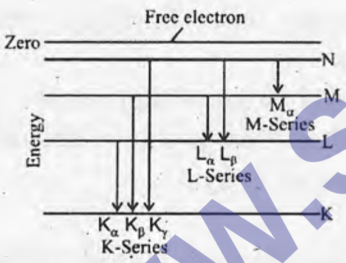
- X-rays are electromagnetic waves propagating through space with the velocity of light. Therefore, X-rays possess all the **basic properties of the electromagnetic radiations.**
- X-rays occupy the **frequency spectrum from 10^{17} to 10^{20} hertz.** From this, we can calculate their wavelength as $\lambda = \frac{c}{f} = \frac{3\times10^{8}}{10^{17}\ \text{to}\ 10^{20}}$ = from 30Å to 0.03Å.
- The energy of X-rays can be calculated as E = hf
  = From $6.62 \times 10^{-34} \times 10^{17}$ to $6.62 \times 10^{-34} \times 10^{20}$ J
  = From 6.62×10^{-17} to 6.62×10^{-14} Joule = From 400 eV to 40,000 eV.

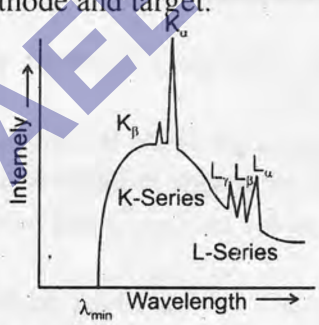
X-Rays Spectrum

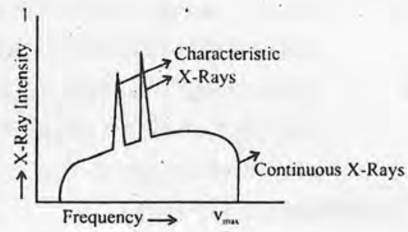
- X-ray spectrum consists of two parts.
  (i) characteristic x-ray spectrum. **(ii)** continuous x-ray spectrum

(i) Characteristic x-ray spectrum

- Characteristic x-rays produce line spectrum. It is because these x-rays are the result of electron jump from outer orbit to the inner orbit and hence have discrete energies.
- Just as the various energy levels in the hydrogen atom give rise to various series of spectral lines, so the various energy levels in other atoms also cause the emitted radiations to be grouped in series. These series are generally called by the name of the electron shell (K, L, M etc.) to which the electron jumps, e.g., if the vacancies are filled in K-shell by electrons from outer orbits, the spectral lines obtained are grouped **K-series X-rays.**
- Characteristic X-rays depend upon the material of the target but not on the potential difference applied between cathode and target.







Energy and wavelength of different lines

Series	Transition	Energy	Wavelength
$K\alpha$	$\underset{(2)}{L} \rightarrow \underset{(1)}{K}$	$E_L - E_K = hf_{K\alpha}$	$\lambda_{K\alpha} = \frac{hc}{E_L - E_K}$
$K\beta$	$\underset{(3)}{M} \rightarrow \underset{(1)}{K}$	$E_M - E_K = hf_{K\beta}$	$\lambda_{K\beta} = \frac{hc}{E_M - E_K}$
$L\alpha$	$\underset{(3)}{M} \rightarrow \underset{(2)}{L}$	$E_M - E_L = hf_{L\alpha}$	$\lambda_{L\alpha} = \frac{hc}{E_M - E_L}$

Note The wavelength of characteristic X-ray doesn't depend on accelerating voltage. It depends on the atomic number (*Z*) of the target material.

$\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$ and $f_{K\alpha} > f_{L\alpha} > f_{M\alpha}$

(ii). Continuous spectrum

- In a continuous spectrum of x-rays there exists a minimum wavelength or maximum frequency beyond which no ray is emitted. Their intensities are inter mixed such that no definite separating line can be drawn between two wavelengths.
- This spectrum does not depend on the nature of the target.
- Maximum frequency or minimum wavelength of this spectrum depends on the potential difference applied in the X-ray tube.

Such type of spectrum is produce in the following way.

✦ When a potential difference V volt is applied in X-ray tube, the minimum kinetic energy of the electrons $\frac{1}{2}mv^2 = eV$. Where e is the charge of the electron, and m is its mass. When this electron strikes the atom of the target, it looses some part of its energy. This energy appears in the form of photon of X-rays whose energy is hf. It is obvious that hf will be less than eV.

✦ The incident electron before coming to rest collides with many atoms of the target. In this process moving electron is retarded and gets deviated. This event is called Bremsstrahlung. Each retarded electron radiates electromagnetic waves. X-rays emitted in this way by the target will have continuous range of frequencies upto a maximum frequency f_{max}. $f_{max} = \frac{eV}{h} = 0.24 \times 10^{15} V$

✦ If the minimum wavelength (Cut-off wavelength) corresponding to the maximum frequency is λ_{min}. Then, $\lambda_{min} = \frac{c}{f_{max}} = \frac{hc}{eV} = \frac{12400}{V} \text{Å}$

Example: In certain element the K-shell electron energy is –18.525 keV and the L-shell electron energy is –3 keV. When an electron jumps from the L-shell to K shell, an x-ray photon is emitted. The wavelength of the emitted x-rays is

A) 0.8 Å B) 1 Å C) 0.6 Å D) 1.2 Å

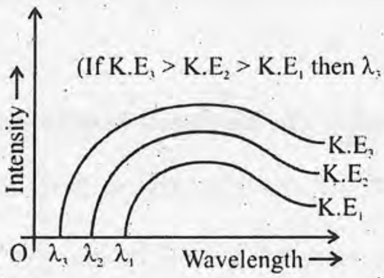
Solution: A)

$$E_L - E_K = -3 - (-18.525) = 15.525\,\text{keV}$$

$$\lambda = \frac{hc}{15.525\,\text{keV}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{15.525 \times 10^3 \times 1.6 \times 10^{-19}} = 0.8\,\text{Å}$$

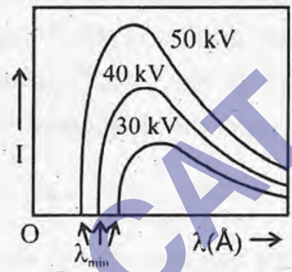
POINTS TO REMEMBER

- The cut-off wavelength is independent of the target material. It depends only on the kinetic energy of the electrons. The effect of K.E. of striking electrons on the cut-off wavelength is shown in figure.



- The cute-off wavelength is inversely proportional to the kinetic energy.

- $f_{max} \propto V$ (accelerating voltage)
- $\lambda_{min} \propto \frac{1}{V}$ (accelerating voltage)
- For different values of accelerating voltages V a continuous spectrum is obtained as shown in the figure.



Properties of x-rays

- X-rays are electromagnetic waves of very short wavelength (0.03 Å to 30 Å). Hence they carry high energy. It is to be noted here that wavelength of light rays ranges from 4000 Å to 7500 Å. This is the only difference between X-rays and light rays.
- They travel in straight lines.
- They are not deflected by electric or magnetic fields. Hence they are not charged particles.
- They can readily penetrate through matter. Soft X-rays only penetrate such object as flesh. Hard X-rays can penetrate much more solid matter.
- They can eject electrons from matter by the photoelectric effect. These ejected electrons are responsible for the following three effects.

(a) They ionize a gas, permitting it to conduct.
(b) They cause certain substance to fluorescence.
(c) They affect a photographic emulsion in a similar manner as light.

- When they are incident on a substance, they get absorbed. The absorption by the substance is given by $I = I_o e^{-\mu x}$. Where I_0 = Initial intensity, I = Intensity of X-rays after traversing a distance x, μ = Absorption coefficient of the substance.

Uses of X-Rays

(i) Fracture study. (Shadow Photography)

(ii) Medical diagnosis (such as **tuberculosis**).

(iii) Medical treatment (such as cancer).

(iv) Hidden things can be located (such as **weapons** and contraband things).

(v) Research field (such as X-ray diffraction for crystal structure).

(vi) Industrial uses (such as to locate **internal defects**)

(vii) **CAT-scanner** CAT stands for computerized axial tomography. The x-ray source produces a thin fan shaped beam that is detected on the opposite side of the subject by an array of several hundred detectors in a line. Tumors, and other anomalies much too small to be seen with older techniques can be detected.

(viii) X-rays cause damage to living tissues.

(ix) X-rays are used for destruction of cancer cells

(x) X-rays can cause cancer

(xi) X-rays can cause changes in the productive system thus damages the organism's offspring.

TOPIC-13 NUCLEAR PHYSICS

LEARNING OUTCOMES

COURSE CONTENT

- The atom to include protons, neutrons and electrons. (Atomic Nucleus)
- Spontaneous and random nuclear decay/ the Law of Radioactive Decay
- Half Life and rate of decay
- Biological effects of Radiation
- Biological and Medical Uses of Radiation

ATOMIC NUCLEUS

- The nucleus is very small part which exists at the centre of the atom.
- Nucleus was discovered by Rutherford through his α–scattering experiments.
- The whole positive charge and almost the whole mass of an atom resides inside the nucleus.
- The charge on the nucleus is (+ Ze). It is due to protons present in the nucleus.
- The radius of the nucleus is the order of 10^{-15} to 10^{-14}m.(fermi)
- If the nucleus is presumed to be spherical its radius $r = r_o A^{\frac{1}{3}}$ where $r_o = 1.2 \times 10^{-15}$ m and A is atomic number.
- The constituents of nucleus are neutrons and protons. In an atom electron equal in number to protons, revolve round the nucleus.
- In lighter nuclei the proton number equals the neutron number (N = Z) e.q. $_7N^{14}$ etc.
- In heavier nuclei the number of neutrons is greater than the number of protons (N > Z) e.g. $_{92}U^{238}$, $_{90}Th^{294}$ etc.
- The neutrons and protons present inside the nucleus taken together are known as nucleons.

Isotopes

The atoms of an element having same atomic number but different atomic mass number are called isotopes, e.g.

Hydrogen $_1H^1, _1H^2, _1H^3$ and Oxygen : $_8O^{16}, _8O^{17}, _8O^{18}$

- The mass numbers (i.e. number of nucleons) of all isotopes of an element are different. Hence their physical properties are not the same.
- Among isotopes of the same element, some may be stable and some radioactive. This is due to difference in their nuclear structure. For example $_6C^{12}$ is stable while $_6C^{14}$ is radioactive similarly $_{11}N^{23}$ is stable while $_{11}Na^{24}$ is radioactive.

Isobars

- The nuclei having same number of nucleons (A) but different number of protons (Z) are called isobars. They also have different number of neutrons. For example

  (a) $_1H^3$ and $_2He^3$ **(b)** $_6C^{14}$ and $_7N^{14}$ **(c)** $_8O^{17}$ and $_9F^{17}$
- They differ in chemical properties.
- Isobars differ in physical properties also.

- Nuclei of isobars belong to different elements.
- The daughter nucleus remaining after emission of β-particles is an isobar of the parent nucleus.

Isotones

- The nuclei having equal number of neutrons are called isotones. For them both Z and A are different but (A – Z) is same. For example

  (a) ${}_3Li^7$ and ${}_4Be^8$, $A-Z=4$ **(b)** ${}_1H^3$ and ${}_2He^4$, $A-Z=2$

  (c) ${}_{11}Na^{23}$ and ${}_{12}Mg^{24}$, $A-Z=12$

POINT TO PONDER

When a nucleus ejects a β-particle then what happen to the neutrons protons ratio?

RADIOACTIVITY AND EMISSION OF RADIATION

- Becquerel discovered **radioactivity**.
- Nuclei having Z > 82 are unstable and they emit α, β and γ rays; the phenomenon is called **radioactivity.**
- Some radioactive elements are Po (Z=84) Rd (Z=88) and U (Z=92) etc.

α-decay Charge no. (Z) decreases by 2 and mass no. (A) by 4

$${}_ZX^A \longrightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$$

e.g. $${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4 + 4.3MeV.$$

β-particle β-particle is electron or positron coming from nucleus. So it may be classified as;

β⁻ decay $${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0$$

Only charge no. (Z) **increases by 1**.

e.g. $${}_{11}Na^{24} \longrightarrow {}_{12}Mg^{24} + {}_{-1}e^0$$

Note *Weak interaction only appears in β-decay.*

γ-decay γ–rays are massless photons; their emission will cause no change either in A or Z of the parent nuclide. γ-decay is due to de-excitation of nucleus.

e.g. $$\left({}_ZX^A\right)^* \longrightarrow {}_ZX^A + \gamma$$

where * sign show excitation of atom

Example: A nucleus of the nuclide ${}^{241}_{94}Pu$ decays by emission of a $\beta-$particle followed by the emission of an $\alpha-$particle. Which of the nuclides shown is formed?

A) ${}^{239}_{93}Np$ B) ${}^{239}_{91}Pa$ C) ${}^{237}_{93}Np$ D) ${}^{237}_{92}U$

Solution: C) ${}^{241}_{94}Pu \rightarrow {}^{x}_{y}A + {}^{0}_{-1}\beta + {}^{4}_{2}He$

$$\therefore \begin{cases} 241 = x + 0 + 4 \Rightarrow x = 237 \\ 94 = y + (-1) + 2 \Rightarrow y = 93 \end{cases}$$

Type of radiation	Alpha particle	Beta particle	Gamma ray
Symbol	α or $_2He^4$	β or β^-	γ (can look different, depends on the font)
Mass (atomic mass units)	4	1/2000	0
Charge	+2	–1	0
Speed	slow	fast	very fast (speed of light)
Ionizing ability	high	medium	0
Penetrating power	low	medium	high
Stopped by	paper	aluminium	lead

SPONTANEOUS AND RANDOM NUCLEAR DECAY

- Radioactive elements disintegrates and emit α, β and γ radiations. This process is called transmutation by spontaneous disintegration.
- in the natural spontaneous disintegration of a radioactive material not all the atoms disintegrates at the same time.
- The process of disintegration takes place randomly, when a nucleus disintegrates, nobody knows.

HALF-LIFE AND RATE OF DECAY

- Radioactive decay is a random process so we get idea of **half-life**.
- The half-life $T_{1/2}$ of a radioactive element is that period in which half of the atoms decay.
- Half-life ($T_{1/2}$) depends on element and is not affected by any physical change or chemical change.
- Half-life may be classified as follows

(a) Extremely short half-life $T_{1/2}$ in micro or nano seconds

(b) Moderate half-life $T_{1/2}$ in minutes.

(c) Long half-life $T_{1/2}$ in years.

Half- life can be given as $T_{1/2} = 0.693/\lambda$

Where λ is called **decay constant** depends upon nature of material. Decay constant of any element is equal to the fraction of the decaying atoms per unit time. The unit of the decay constant is s^{-1}.

- The decay curve shows that radioactive element decay exponentially
- Half-life is used to identify an atom.

POINT TO PONDER

How many electrons are present in the nucleus of U_{92}^{235}? Can an electron be ejected from its nucleus?

Laws of Radioactivity

- $\Delta N/\Delta t \propto N_o$
- $\Delta N/\Delta t = -\lambda N_o$
- $N_t = e^{-\lambda t}N_o$
- $T_{1/2} \propto 1/\lambda$
- $\lambda = \dfrac{-\Delta N/\Delta t}{N}$
- Mean life = $T = 1/\lambda$

Example: The half-life of a certain radioactive element is such that 7/8 of a given quantity decays in 12 days. What fraction remains undecayed after 24 days?

A) 0 B) $\dfrac{1}{128}$ C) $\dfrac{1}{64}$ D) $\dfrac{1}{32}$

Solution: C) The radioactive element has 1/8 of a given quantity remains after 12 days. After 24 days, or in additional 12 days, the fraction remains undecayed is

$$\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) \text{ or } \frac{1}{64}$$

BIOLOGICAL EFFECTS OF RADIATION

- Radiations can damage **living tissues**. The degree of damage and kind of damage depend on type, energy and dose of radiation. Incident radiations ionize the body cell, and then change biochemistry of the cell. A damaged cell **may die** or **begins to work in wrong way** e.g. sometimes radiation changes the chemistry of cells in such a way hat they begin to reproduce rapidly leading to a **condition called cancer**.
- Damage to ozone layer is done through following sources: -

(i) **Chemical** industry

(ii) **Nuclear** tests

(iii) **CFC (chlorofluorocarbons)**

(iv) **Aerosol** sprays and plastic foam industry

- Some radiation in the environment is added by human activates like diagnostic x-ray, nuclear facilities, hospitals, research and industrial establishments, colour television, luminous watches and tobacco leaves.

  Effects of radiations are of two types:

(i) Somatic Effect:

These are direct effects on body e.g. **skin burns, loss of hair**, ulceration, stiffening of lungs, drop in white blood cells and contraction in eyes etc.

These effects can be cured if the dose is small.

(ii) Genetic Effects:

These affect (alter) the chemistry of genes and cause mutation e.g. **cancer, different syndromes etc.**

These effects are incurable and pass on to future generation.

BIOLOGICAL AND MEDICAL USES OF RADIATION

- C-14 is one of useful tracer that can help in the understanding of **photosynthesis.**
- I-131 and Na-24 are used to check **cracks in pipes**.
- Radiation therapy is a process of destruction of cancerous cells deep into body e.g. Co-60 & I-131 are **used to treat cancers**.
- Tumors are treated by **γ-rays.**
- Radio processes in space give information about **structure of stars**.
- When ethylene is radiated with γ-rays, it is polymerized into polyethylene, which is used to **produce soft and flexible products**.
- **Sterilization** is a process of killing of germs with β-particles or γ-rays.
- Radiation treatment can **preserve food** and other eatables.
- γ-gauges are used for preparing or measuring the thickness of high density and thicker materials such as **steel, Al & rubber**. In such gauges, Co-60 is used as γ-rays source.
- β-gauges are used for preparing or measuring the thickness of thin sheets of low density materials e.g. paper. In **such gauges, Sr - 90** is **used as a β-rays source**
- **Radiography** is employed to check cracks of cavities in metal casting, faults in welding & heavy machinery.
- Ratio of C-14 to C-12 found in dead matter is a measure of time span **since death.**
- **β particle is used for** superficial skin therapy.
- **α particle is used for deep** skin therapy.
- Activation analysis (γ-ray energy measurement) is applied to determine concentration of elements in a given sample and to estimate corrosion and wear of machinery.
- Technetium - 99 has given rise to positron emission tomography.
- The ratio of $_{62}Pb^{206}$ — $_{92}U^{238}$ is used to determine age of rocks.

Topic -1
Force and Motion

Displacement $(\vec{d})$:

$\Delta\vec{d} = \vec{r_2} - \vec{r_1}$

$\vec{d} = x\hat{i} + y\hat{j}$

$d = \sqrt{x^2 + y^2}$

$|\vec{d}| \leq s \quad , \quad \dfrac{|d|}{s} \leq 1$

Units:

metre (m)

Dimensions:

[L]

Velocity:

$\vec{v} = \dfrac{\Delta\vec{d}}{\Delta t}$

$v_{avg} = \dfrac{Total\ displacment}{Total\ time} = \dfrac{d}{t}$

$\vec{v}_{ins} = \lim\limits_{\Delta t \to 0}\left(\dfrac{\Delta\vec{d}}{\Delta t}\right)$

$v_{avg} = \dfrac{2v_1v_2}{v_1 + v_2}$ if $(s_1 = s_2)$

$v_{avg} = \dfrac{v_1 + v_2}{2}$ if $(t_1 = t_2)$

$v_1 = v_2 = v_{avg}$ if $(v_1 = v_2)$

Relative velocity:

$v_r = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$

$v_r = \sqrt{v_1^2 + v_2^2}$ $(\theta = 90^o)$

$v_r = v_1 - v_2$ $(\theta = 0^o)$

$v_r = v_1 + v_2$ $(\theta = 180^o)$

Units:

$m\,s^{-1}$

Dimensions:

$\left[LT^{-1}\right]$

Acceleration:

$\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$

$a_{avg} = \dfrac{v_1 + v_2}{t_1 + t_2} = \dfrac{a_1t_1 + a_2t_2}{t_1 + t_2}$

Units:

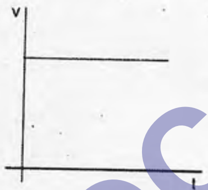
$m\,s^{-2}$

Dimensions:

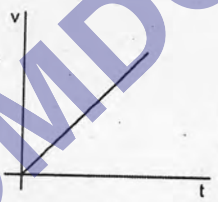
$\left[LT^{-2}\right]$

Graphical representation of acceleration with velocity time graph:

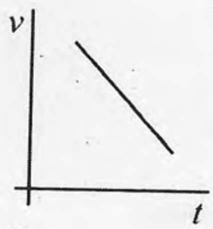
- Slope of graph = Acceleration
- Area under graph = Distance



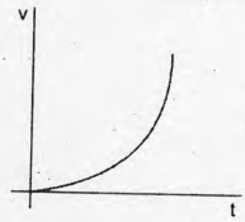
(Uniform Velocity) (a = 0)



(Uniform +ve acceleration)



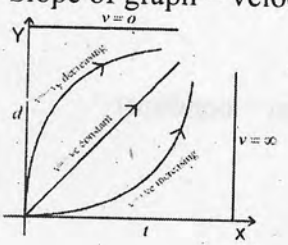
(Uniform –ve acc)

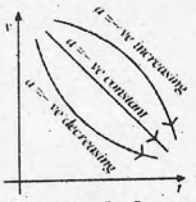


(Increasing +ve acceleration)

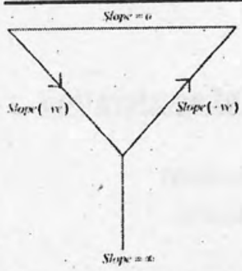
Displacement-time graph:

Slope of graph = velocity





General slope tree:



Newton's laws of motion:

$F = ma$

$\frac{F_1}{F_2} = \frac{m_1}{m_2}$ $(a_1 = a_2)$

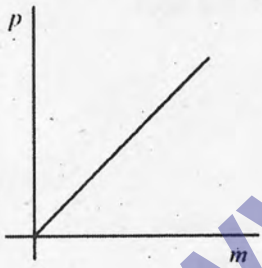
$\frac{F_1}{F_2} = \frac{a_1}{a_2}$ $(m_1 = m_2)$

$\frac{m_1}{m_2} = \frac{a_2}{a_1}$ $(F_1 = F_2)$

Linear Momentum:

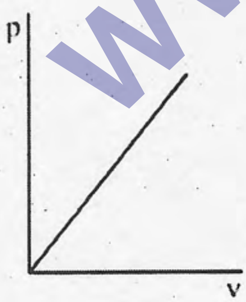
$\vec{p} = m\vec{v}$

$p \propto m$



(v = constant)

$p \propto v$

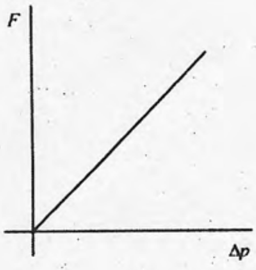


(m = constant)

When body dropped:

$p = mv = m\sqrt{2gh}$

$F \propto \Delta p$

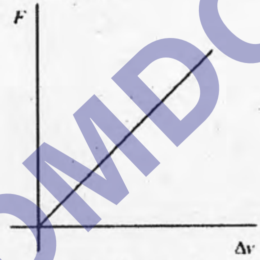


(Δt = constant)

$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}$

$F = \frac{m(v_f - vi)}{\Delta t}$

$F \propto \Delta v$



$F \times t = m(v_f - v_i) = impulse$

$\frac{F_1}{F_2} = \frac{\Delta p_1}{\Delta p_2}$

$P \propto m$

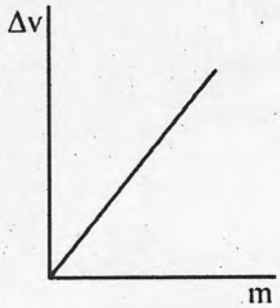
Law of conservation of momentum:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

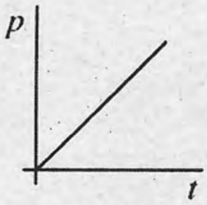
$\Delta p = 0$

$F_{ext} = \frac{\Delta p}{\Delta t} = 0$

$\Delta v \propto m$



Momentum - time graph:



$Slop = \frac{p}{t} = \frac{mv}{t} = Force = F$

Projectile Motion:

$v_{fx} = v_{ix} = v_i \cos\theta$

$v_{fy} = v_{iy} - gt$

$v_{fy} = v_i \sin\theta - gt$

$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$

$v_f = \sqrt{v_i^2 \cos^2\theta + v_i^2 \sin^2\theta + g^2t^2 - 2v_i \sin\theta(gt)}$

$v_f = \sqrt{v_i^2 + g^2t^2 - 2v_i \sin\theta gt}$

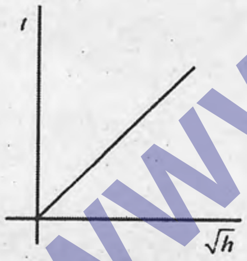
Vertical distance:

$Y = s = \frac{1}{2}gt^2 = 5t^2$

$t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{h}{5}}$

$t \propto \sqrt{h}$



Horizontal distance:

$x = v_x \times t = v_x\sqrt{\frac{2h}{g}}$

$x = v_i\sqrt{\frac{h}{5}}$

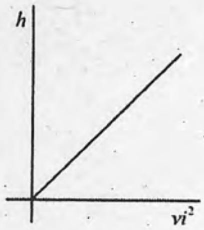
$x = v \times \sqrt{\frac{h}{5}}$

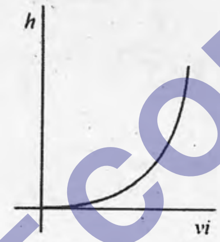
Maximum Height:

$h = \frac{v_i^2 \sin^2\theta}{2g} = \frac{v_{iy}^2}{2g}$

$h \propto \sin^2\theta$

$h \propto v_i^2$





$h_{max} = \frac{v_i^2}{2g}$

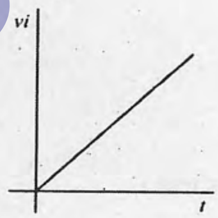
$h = h_{max}.\sin^2\theta$

Time of flight:

$t = \frac{2v_i \sin\theta}{g} = \frac{2v_{iy}}{g}$

$t \propto \sin\theta$

$t \propto v_i$



- Time taken by projectile from place of projection to maximum height.

  $t_1 = \frac{t}{2} = \frac{v_i \sin\theta}{g} = \frac{v_{iy}}{g}$

- Time taken by projectile from maximum height to place of landing.

  $t_2 = \frac{t}{2} = \frac{v_i \sin\theta}{g}$

  $t_{max} = \frac{2v_i}{g}$

  $t = t_{max}.\sin\theta$

Horizontal Range:

$R=(t)(v_{ix})$

$R=t\times v_i\cos\theta$

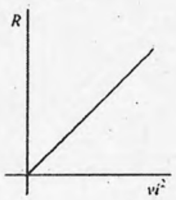
$R=\dfrac{v_i^2\sin^2 2\theta}{g}$

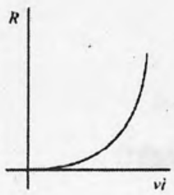
$R=\dfrac{2(v_i\sin\theta)(v_i\cos\theta)}{g}$

$R=\dfrac{2(v_{iy})(v_{ix})}{g}$

$R_{max}\quad(\theta=45^o)$

$R\propto v_i^2$





$R_{max}=\dfrac{v_i^2}{g}$

$R=R_{max}\sin^2\theta$

$R\propto\sin^2\theta$

For two angles, if

$\theta_1+\theta_2=90^o$ then $R_1=R_2$

(If "v_i" for both projectiles is same)

K.E at maximum height:

$K.E_h=KE_i\cos^2\theta$

P.E at maximum height:

$P.E=KE_i\sin^2\theta$

Momentum at maximum height:

$p_h=p_i\cos\theta$

$p_h=mv_i\cos\theta=mv_ix$

Relations between R, H and t:

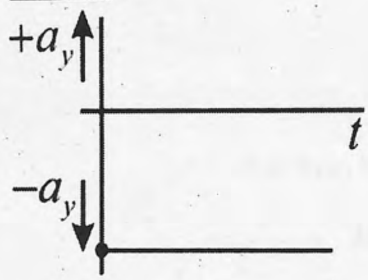
$R\tan\theta=4\text{H}$

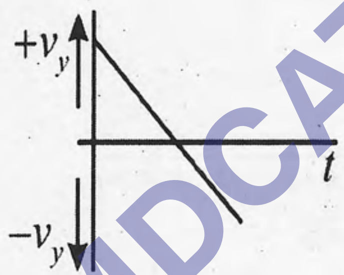
$R_{max}=4H\qquad(\theta=45^o)$

$H=\left(\dfrac{g}{8}\right)t^2$

$H\propto t^2$

Graph:





Topic - 2

Work and Energy

Work:

$W = \vec{F}.\vec{d}$

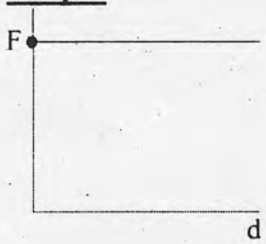
$W = Fd\cos\theta$

$W_{max} = +Fd \qquad (\theta = 0^o)$

$W_{max} = -Fd \qquad (\theta = 180^o)$

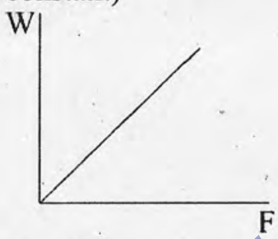
$W_{min} = 0 \qquad (\theta = 90^o)$

Graph:



(When 'F' is constant)

$W \propto F$ (When 'd' & 'θ' are constant)



$W \propto d$

(When 'F' is constant)

Area=Fd=Work

Units:

- joule (S.I unit)
- kilowatt hour (kW h)
- calories (cal)
- erg (c.g.s)
- electron volt (eV)
- 1kW h=3.6 MJ
- 1cal=4.18 J
- 1erg=10^{-7} J
- 1eV=1.6×10^{-19} J

Dimension:

$[W]=[M L^2 T^{-2}]$

Work done by Variables Force:

$W = F_1\Delta d_1\cos\theta_1 + F_2\Delta d_2\cos\theta_2 + ... + F_n\Delta d_n\cos\theta_n$

$W = \sum_{i=1}^{N} F_i\,\Delta d_i\cos\theta_i$

Power:

$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{\vec{F}.\Delta\vec{d}}{\Delta t} = \frac{mgh}{t} = \frac{P.E}{t} = \frac{K.E}{t} = \frac{1/2mv^2}{t} = \frac{mv^2}{2t}$

$P_{ins} = \lim_{\Delta t\to 0}\frac{\Delta W}{\Delta t}$

$P_{ins} = \vec{F}.\vec{v}$

$= Fv\cos\theta$

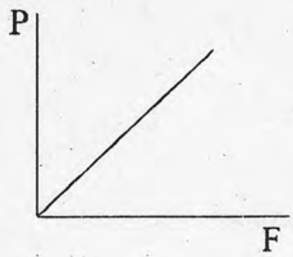
$P_{ins} = Fv \qquad (\theta = 0^o)$

$P_{ins} = -Fv \qquad \theta = (180^o)$

$P_{rot} = \vec{\tau}.\vec{\omega} = \tau\omega\cos\theta$

When 'v' & 'θ' are constant

$P \propto F$



Energy:

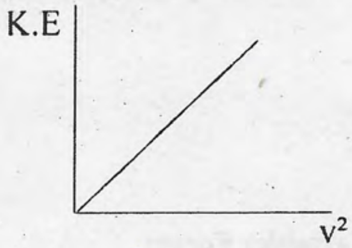
$G.P.E = mgh$

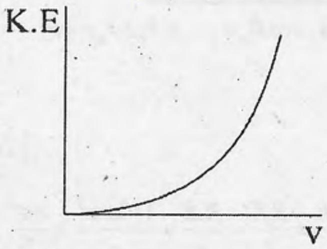
$Elastic\ P.E = \frac{1}{2}kx^2$

$Electric\ P.E = q\Delta V$

$K.E = \frac{1}{2}m(\vec{v}.\vec{v}) = \frac{1}{2}mv^2$

$K.E \propto v^2$ (when m = constant)



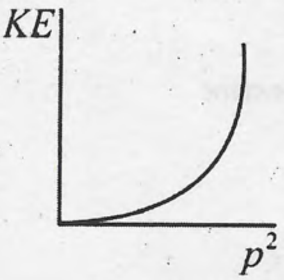


K.E and momentum:

$$K.E=\frac{p^2}{2m}$$

$$K.E=\frac{1}{2}pv$$

$$K.E\propto p^2$$



$$\frac{K.E_1}{K.E_2}=\left(\frac{m_2}{m_1}\right)\left(\frac{p_1}{p_2}\right)^2$$

$$\frac{K.E_1}{K.E_2}=\left(\frac{p_1}{p_2}\right)^2 \quad (m=\text{constant})$$

$$p=\sqrt{2m\,K.E}$$

$$\frac{p_1}{p_2}=\sqrt{\frac{K.E_1}{K.E_2}} \quad (m=\text{constant})$$

$$p\propto\sqrt{K.E}$$

Interconversion of K.E and P.E:

In the absence of Air;

Loss in P.E = Gain in K.E

$$mgh=\frac{1}{2}mv^2$$

$$\sqrt{2gh}=v$$

In the presence of air;

Loss in P.E = Gain in K.E + Work done against friction

$$mgh=\frac{1}{2}mv^2+fh$$

$$mgh-fh=\frac{1}{2}mv^2$$

Topic - 3

Rotational and Circular Motion

Angular displacement (Revolution, Degree, Radian):

$S = r\theta$

$r = S/\theta$

$\theta = S/r$

$1^{o} = 0.0174$ rad

$180^{0} = \pi$ rad	$1\ rad = 57.3^{0}$	$1\ \text{rev} = 2\pi$ rad

Angular Velocity:

$\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$

$\vec{v} = \vec{\omega} \times \vec{r} = \omega r \sin 90^{o} (\hat{n})$

$v = r\omega$

$r = \dfrac{v}{\omega}$

$\omega = \dfrac{v}{r}$

Units:

- degree/s
- radian/s (S.I unit)
- rev/min

$1\ \text{rev / mint} = \dfrac{2\pi}{60}\ \text{rad / s} = \dfrac{\pi}{30}\ rad / s$

Dimensions:

$[T^{-1}]$

Angular acceleration:

$\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t}$

$\vec{\alpha}_{ins} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{\omega}}{\Delta t}$

$\vec{a} = \vec{\alpha} \times \vec{r} = \alpha r \sin 90^{o} (\hat{n})$

$a = r\alpha$

$r = \dfrac{a}{\alpha}$

$\alpha = \dfrac{a}{r}$

Units:

- degrees/s^2
- rad/s^2
- rev/min^2
- $1\ rev / \min^2 = \dfrac{2\pi}{(60)^2}\ rad / s^2 = \dfrac{\pi}{1800}\ rad / s^2$

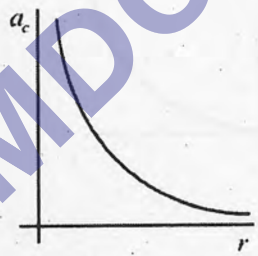
Dimensions:

$[T^{-2}]$

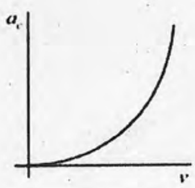
Centripetal acceleration a_c:

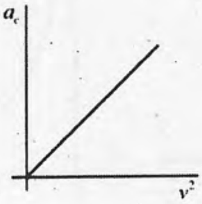
$a_c = \dfrac{v^2}{r} \Rightarrow \vec{a}_c = -\left(\dfrac{v^2}{r^2}\right)\vec{r} = -\omega^2 \vec{r}$

$a_c \propto \dfrac{1}{r}$ (v = Constant)



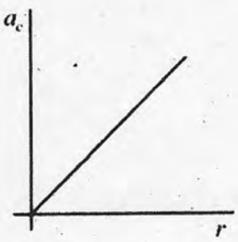
$a_c \propto v^2$ (r = Constant)



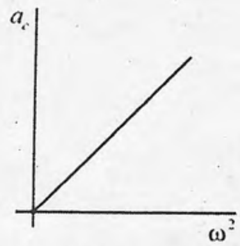


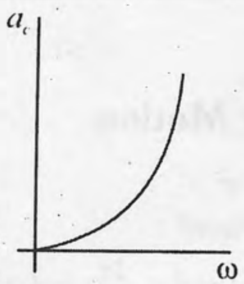
$a_c = r\omega^2$

$a_c \propto r$ (ω=Constant)



$a_c \propto \omega^2$ (r=Constant)

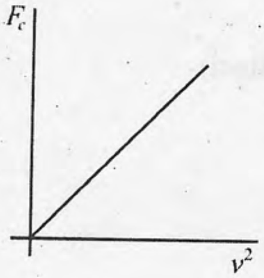


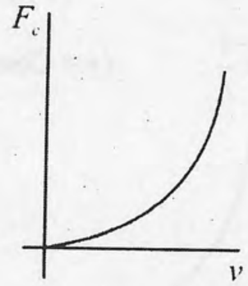


Centripetal force F_c:

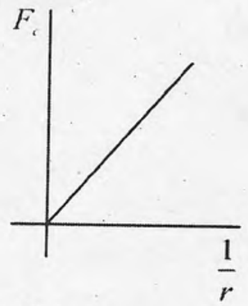
$$F_c = m\frac{v^2}{r} \rightarrow \vec{F}_c = -\left(\frac{mv^2}{r}\right)\hat{r} = -\left(\frac{mv^2}{r^2}\right)\vec{r}$$

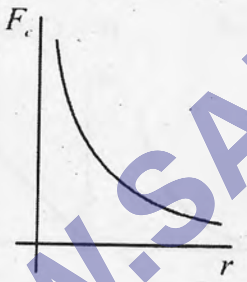
$F_c \propto v^2$ (r=Constant)





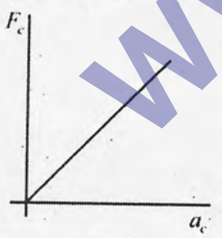
$F_c \propto \frac{1}{r}$ (v=Constant)





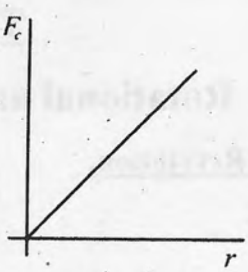
$F = ma_c$

$F_c \propto a_c$ (m=Constant)

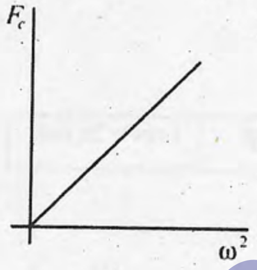


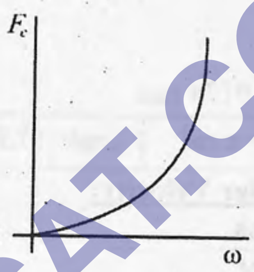
$F_c = mr\omega^2$ or $\vec{F}_c = -\left(m\omega^2\right)\vec{r}$

$F_c \propto r$ (ω = Constant)



$F_c \propto \omega^2$ (r=Constant)





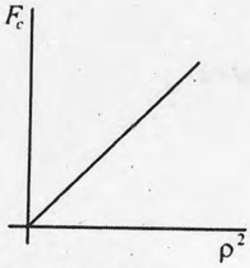
$$F_c = \frac{mv^2}{r}$$

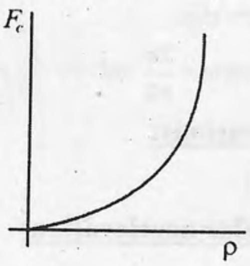
$$F_c = \frac{2mv^2}{2r}$$

$$F_c = \frac{2K.E}{r}$$

$$F_c = \frac{p^2}{mr}$$

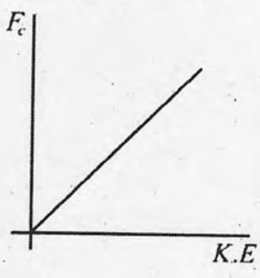
$F_c \propto p^2$ (mr = Constant)



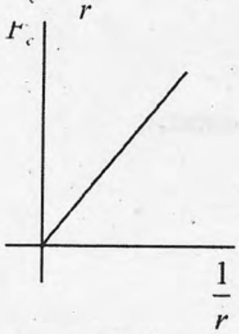


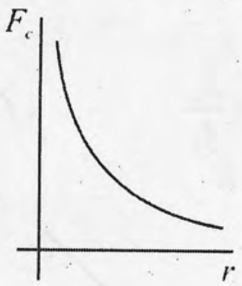
$$F_c = \frac{2K.E}{r}$$

$F_c \propto K.E$ (r = Constant)

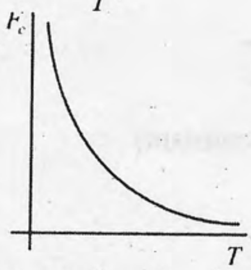


$F_c \propto \frac{1}{r}$ (K.E = Constant)

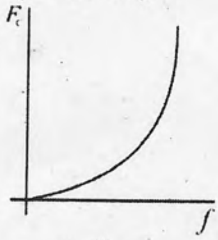




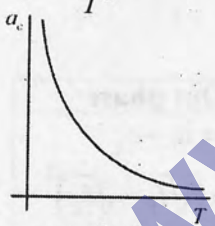
$F_c = \frac{mr4\pi^2}{T^2}$



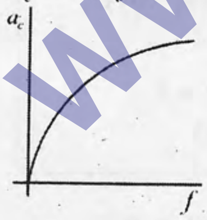
$F_c = 4\pi^2 mrf^2$



$a_c = \frac{4\pi^2 r}{T^2}$



$a_c = 4\pi^2 f^2 r$

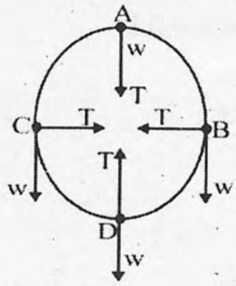


Vertical circle:

Non-uniform circular motion:

$T = F_c - w\cos\theta$

$v = \sqrt{3gr - 2gr\cos\theta}$



At A:

- $\theta = 0$
- $v = \sqrt{gr}$
- $T_{min} = F_c - w$

At B:

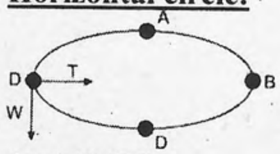
- $\theta = 90$
- $v = \sqrt{3gr}$
- $F_c = T$

At D:

- $\theta = 180$
- $v = \sqrt{5gr}$
- $Fc = T - w$

- **Note:**
- θ is angle between T *and* w

Horizontal circle:



At A,B,C,D

- $\theta = 90^\circ$
- $F_c = T$
- $v = \sqrt{3gr}$

Topic - 4

Waves

In-phase:

Path difference $= x = n\lambda$

Minimum path difference $= x = 0\lambda$

$n = 0,1,2,3,....$

Phase difference $= \phi = \dfrac{2\pi x}{\lambda}$

$\phi = 0, 2\pi, 4\pi, 6\pi, ...$

$\phi = 2n\pi$

$\phi_{min} = 0$

Out of phase:

Path difference $= x = \left(n + \dfrac{1}{2}\right)\lambda = (2n+1)\dfrac{\lambda}{2}$

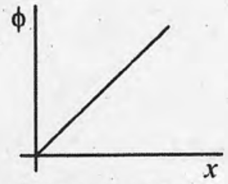
Minimum path difference$= = x = \dfrac{\lambda}{2}$

Phase difference$= \phi = \dfrac{2\pi x}{\lambda}$

$\phi = \pi, 3\pi, 5\pi, ...$

$\phi_{min} = \pi$

$\phi \propto x$



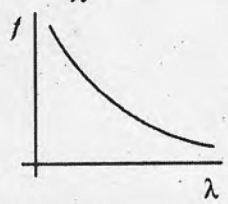
Wave speed v:

$v = f\lambda = \dfrac{\lambda}{T}$

For same medium,

$\dfrac{f_1}{f_2} = \dfrac{\lambda_2}{\lambda_1}$ (v = constant)

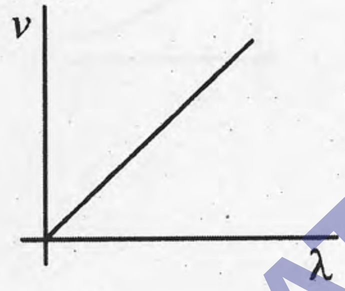
$f \propto \dfrac{1}{\lambda}$



For different medium,

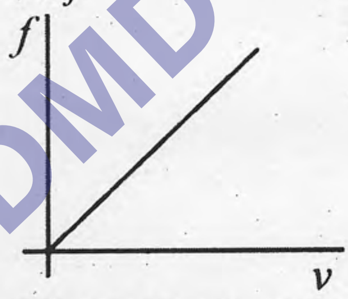
$\dfrac{v_1}{v_2} = \dfrac{\lambda_1}{\lambda_2}$ (f =constant)

$v \propto \lambda$



$\dfrac{v_1}{v_2} = \dfrac{f_1}{f_2}$ (λ =constant)

$v \propto f$



Principle of superposition:

Interference

$Y_o = y_1 + y_2 + y_3 + \cdots y_n$

In phase	Out phase
$A = a_1 + a_2$	$A = a_1 - a_2$
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$	$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$
$I_{max} = (a_1 + a_2)^2$	$I_{min} = (a_1 - a_2)^2$
If,	*If*,
$I_1 = I_2 = I$	$I_1 = I_2 = I$
$I_{max} = 4I$	$I_{min} = 0$

Beats:

Beat frequency $= n = \pm(f_1 - f_2) \le 10$

Beat period = T = $\dfrac{1}{f} = \dfrac{1}{n}$

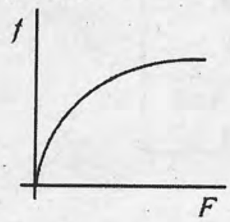
Stationary waves:

	Δx	$\Delta\phi$	Δt
1) N-N	$\frac{\lambda}{2}$	π	$\frac{T}{2}$
2) A-A	$\frac{\lambda}{2}$	π	$\frac{T}{2}$
3) N-A	$\frac{\lambda}{4}$	$\frac{\pi}{2}$	$\frac{T}{4}$

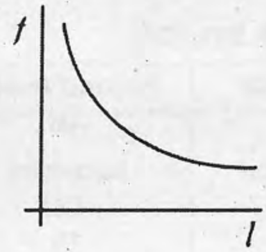
Stationary waves in a stretched string/fundamental frequency and harmonics:

$f = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{F}{m}}$ Where $m = \frac{mass}{length}$

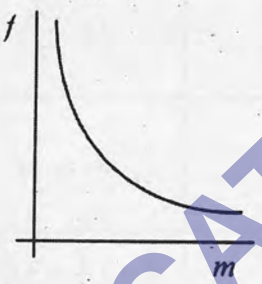
$f \propto \sqrt{F}$



$f \propto \frac{1}{l}$



$f \propto \frac{1}{\sqrt{m}}$



Stationary waves in stretched string or organ pipe open from both ends:

Frist mode	Second mode	Third mode	nth mode
Ist Harmonics Or Fundamental	2nd Harmonics Or 1stOvertone	3rd Harmonics Or 2nd Overtone	nth Harmonics Or (n-1)th Overtone
$l = \frac{\lambda_1}{2}$	$l = \lambda_2$	$l = \frac{3}{2}\lambda_3$	$l = n\frac{\lambda_4}{2}$
$\lambda_1 = 2l$	$\lambda_2 = l$	$\lambda_3 = \frac{2}{3}l$	$\lambda_n = \frac{2l}{n}$
$f_1 = \frac{v}{2l}$	$f_2 = \frac{v}{l} = 2f_1$	$f_3 = \frac{3}{2}\frac{v}{l} = 3f_1$	$f_n = nf_1$ $n = 1,2,3,....$
$2N,1A$	$3N,2A$	$4N,3A$	No. of Nodes>A

- No of nodes= no. of loops+1
- No of antinodes = no. of loops

Organ pipe closed from one end:

Frist mode	Second mode	3rd mode	nth mode
1st Harmonics Or Fundamental	2nd harmonics Or 1st Over tone	3rd Harmonics Or 2nd Over tone	nth Harmonics Or (n-1)th Over tone
$l = \frac{\lambda_1}{4}$	$l = \frac{3\lambda_3}{4}$	$l = \frac{5}{4}\lambda_5$	$l = n\frac{\lambda_n}{4}$ $n = 1,3,..$
$\lambda_1 = 4l$	$\lambda_3 = \frac{3}{4}l$	$\lambda_5 = \frac{4}{5}l$	$\lambda_n = \frac{4l}{n}$ $n = 1,3,..$
$f_1 = \frac{v}{4l}$	$f_3 = 3\frac{v}{4l} = 3f_1$	$f_5 = 5\frac{v}{4l} = 5f_1$	$f_n = nf_1$ $n = 1,3,..$
N, A	$2N, 2A$	$3N, 3A$	Nodes = Anti nodes

$$f_{open} = 2f_{closed}$$

$$f_{closed} = \frac{f_{open}}{2}$$

Doppler's effect:

General formula:

$$f' = \left(\frac{v \pm u_o}{v \pm u_s}\right)f \quad \text{or} \quad \lambda' = \left(\frac{v \pm u_s}{v \pm u_o}\right)\lambda$$

i. Direction of v always towards observer.
ii. If u_o and u_s along v, then u_o and u_s taken as negative.
iii. If u_o and u_s opposite to v, then u_o and u_s taken as positive.

1) **When observer moves towards stationary source:**

$$f' = \left(\frac{v + u_o}{v}\right)f$$

2) **When observer moves away from stationary source:**

$$f' = \left(\frac{v - u_o}{v}\right)f$$

3) **When source moves towards stationary observer:**

$$f' = \left(\frac{v}{v - u_s}\right)f$$

4) **When source moves away from stationary observer:**

$$f' = \left(\frac{v}{v + u_s}\right)f$$

5) **When source and observer are moving towards each other:**

$$f' = \left(\frac{v + u_o}{v - u_s}\right)f$$

6) **When source and observer are moving away from each other:**

$$f' = \left(\frac{v - u_o}{v + u_s}\right)f$$

7) **When observer follows source moving in same direction:**

$$f' = \left(\frac{v + u_o}{v + u_s}\right)f$$

8) **When source follows observer moving in same direction:**

$$f' = \left(\frac{v - u_o}{v - u_s}\right)f$$

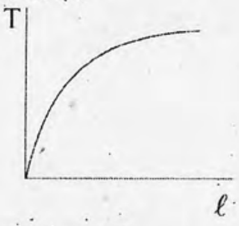
Simple Pendulum:

$$\omega = \sqrt{\frac{g}{\ell}}$$

$$F_r = -mg\ \sin\theta$$

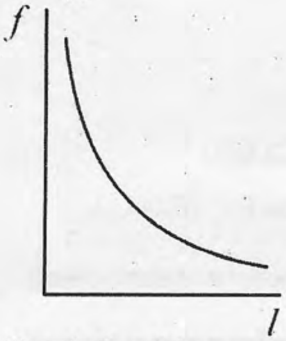
And $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T \propto \sqrt{\ell}$$



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$f \propto \frac{1}{\sqrt{\ell}}$$



Simple Pendulum,

$T = 1sec$

$f = 1Hz$

$\ell = 0.25m$

Second Pendulum:

$T = 2sec$

$f = 0.5Hz$

$\ell = 0.99m$

Lift cases:

Lift is at rest or moving with uniform **velocity:** $T = 2\pi\sqrt{\frac{\ell}{g}}$

Lift is moving upward with uniform acceleration: $T = 2\pi\sqrt{\frac{\ell}{g+a}}$

Lift is moving downward with uniform acceleration:

$$T = 2\pi\sqrt{\frac{\ell}{g-a}}$$

Lift is moving freely:

$$T = 2\pi\sqrt{\frac{\ell}{0}} = \infty$$

$$f = \frac{1}{T} = \frac{1}{\infty} = 0\,Hz$$

Energy Conservation in S.H.M:

P.E:

$$P.E_{ins} = \frac{1}{2}kx^2 = \frac{1}{2}\left(\omega^2 m\right)x^2 = \frac{1}{2}(2\pi f)^2\ mx^2 = \frac{1}{2}\left(4\pi^2 f^2 mx^2\right)$$

$$P.E_{ins} = T.E\frac{x^2}{x_o^2}$$

$P.E_{min} = 0 \qquad (x = 0)$

$P.E_{max} = \frac{1}{2}kx_o^2 \qquad (x = x_o)$

K.E:

$$K.E_{ins} = \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$K.E_{ins} = T.E\left(1 - \frac{x^2}{x}\right)$$

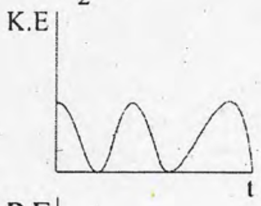
$K.E_{min} = 0 \qquad (x = x_o)$

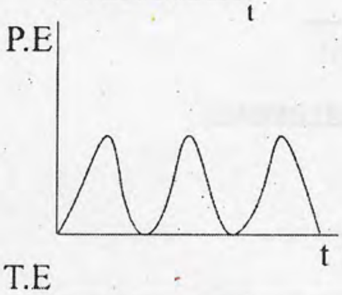
$K.E_{max} = \frac{1}{2}kx_o^2 \qquad (x = 0)$

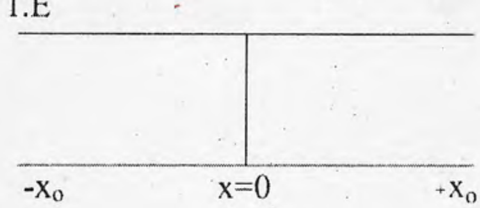
Total Energy:

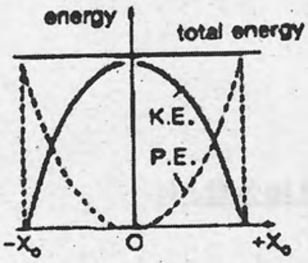
$$T.E = K.E + P.E$$

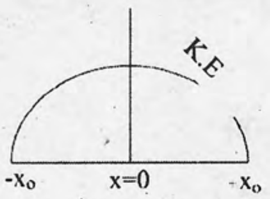
$$T.E = \frac{1}{2}kx_o^2$$

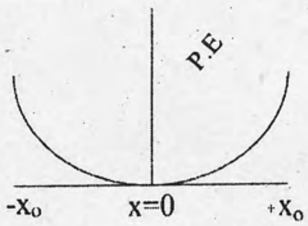












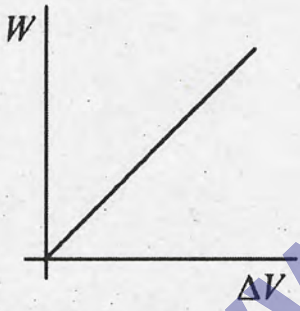
Topic - 5

Thermodynamics

Work and Heat:

$W = P\Delta V$

$W \propto \Delta V$



First law of thermodynamics:

$\Delta Q = \Delta U + W$

$\Delta U = \Delta Q - W$

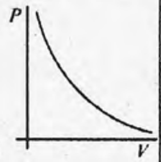
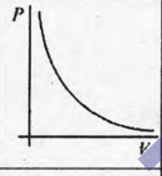
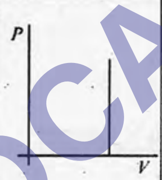
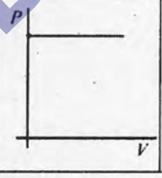
$W = \Delta Q - \Delta U$

Molar Specific Heat of a gas:

- C_p is greater in value than C_v ($C_p > C_v$)
- $\Delta Q_v = nC_v\Delta T$ (Heat supplied at constant volume)
- $\Delta Q_p = nC_p\Delta T$ (Heat supplied at constant pressure)
- $C_p - C_v = R$

• $\dfrac{C_p}{C_v} = \gamma$	• $C_v = \dfrac{3}{2}R$ (Monoatomic)
• $C_v = \dfrac{5}{2}R$ (Diatomic)	• $C_v = \dfrac{7}{2}R$ (Polyatomic)

Comparison of different thermodynamically process:

	Condition	Formula	1^{st}law	P-V diagram	Slope(m)	Specific heat(c)
Isothermal	$T=$constant $\Delta T=0$ $\Delta U=0$	PV=constant	$Q=\Delta U+W$ $\Delta U=0$ $Q=W$		Gradually Decreases	$\Delta Q=cm\Delta T$ $c=\frac{\Delta Q}{m\Delta T}$ $\Delta T=0$ $c=\infty$
Adiabatic	$\pm Q=0$	$PV^{\gamma}=$ constant $\gamma=\frac{C_P}{C_V}$	$Q=\Delta U+W$ $Q=0$ $0=\Delta U+W$ $-\Delta U=+W$		Rapidly decreases	$c=\frac{\Delta Q}{m\Delta T}$ $c=0$
Isochoric	$V=$constant $\Delta V=0$	$\frac{P}{T}=$constant	$Q=\Delta U+W$ $Q=\Delta U+P\Delta V$ $\Delta V=0$ $Q=\Delta U$ $\Delta U=nC_v\Delta T$		∞	$c=C_v$
Isobaric	P=constant $\Delta P=0$	$\frac{V}{T}=$constant $V\propto T$	$Q=\Delta U+W$ $Q=\Delta U+P\Delta V$ $Q=nC_p\Delta T$		0	$c=C_p$

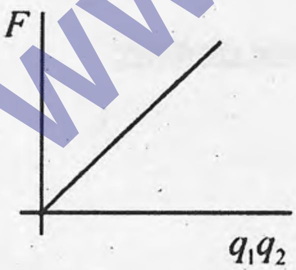
Topic - 6

Electrostatics

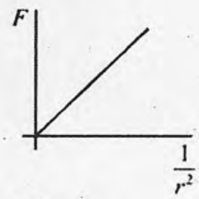
Coulomb's law:

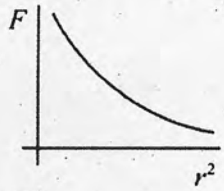
$F=k\frac{q_1q_2}{r^2}$

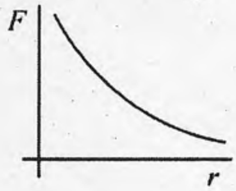
$F\propto q_1q_2$



$F\propto\frac{1}{r^2}$







$$k=\frac{1}{4\pi\varepsilon_0}=9\times10^{9}\,N\,m^{2}\,C^{-2}$$

$$\varepsilon_o=8.85\times10^{-12}\,C^{2}\,N^{-1}\,m^{-2}$$

$$F_{vac}=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}$$

$$F_{med}=\frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2}$$

$$\varepsilon_r=\frac{F_{vac}}{F_{med}}$$

$\varepsilon_r=1$ (For vacuum)

$\varepsilon_r=1.0006$ (for air)

$\varepsilon_r>1$ (always)

- ε_r has no units.

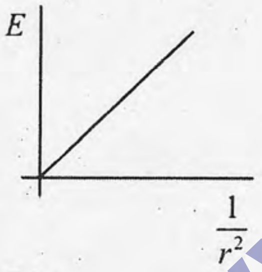
Vector Form of Coulomb's force:

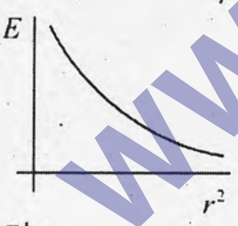
$$\vec{F}=\frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}$$

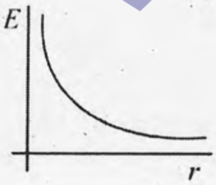
Electric field intensity $\vec{E}$:

$$\vec{E}=\frac{\vec{F}}{q}=K\frac{q}{r^2}\hat{r}$$

$$E\propto\frac{1}{r^2}$$







Units:

$E=NC^{-1}$ or Vm^{-1}

Potential difference ΔV:

$$\Delta V=\frac{\Delta W}{\Delta q}=\frac{\Delta U}{\Delta q}=\frac{K.E}{\Delta q}$$

$$\Delta W=\Delta U=K.E=q\Delta V$$

Units

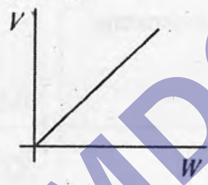
$$volt=\frac{joule}{coulomb}$$

$$v=j\,C^{-1}$$

Electric potential:

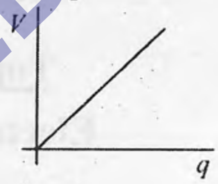
$$V=\frac{W}{q}$$

$$V\propto W$$

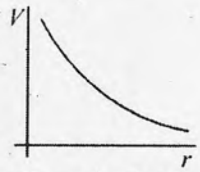


$$V=K\frac{q}{r}$$

$$V\propto q$$



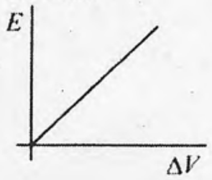
$$V\propto\frac{1}{r}$$



Electric field as potential gradient:

$$\Delta V=\frac{\Delta W}{q}=\frac{\vec{F}.\vec{d}}{q}$$

$$E\propto\Delta V$$

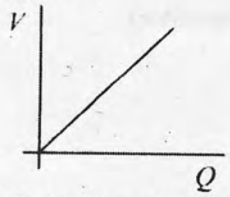


Capacitor:

$Q = CV$

$C = \frac{Q}{V}$

$Q \propto V$

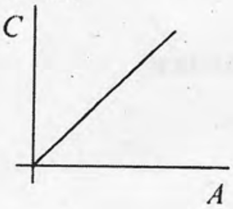


Capacitance of parallel plates capacitors:

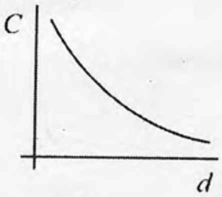
$C_{vac} = \frac{A\varepsilon_o}{d}$

$C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d}$

$C \propto A$



$C \propto \frac{1}{d}$



$\varepsilon_r = \frac{C_{med}}{C_{vac}}$

$\varepsilon_r = \frac{V_{vac}}{V_{med}}$

Units:

$c = \frac{coulomb}{volt}$

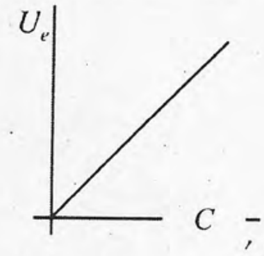
$c = farad\,(F)$

$F = CV^{-1}$

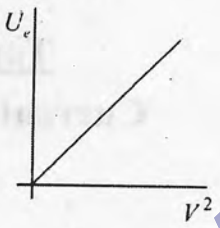
Energy stored in a capacitor:

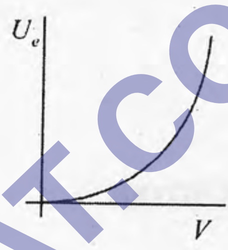
$U_e = \frac{1}{2}CV^2$

$U_e \propto C$



$U_e \propto V^2$





$U_e = \frac{1}{2}CV^2$

$U_e = \frac{1}{2}QV$

$U_e = \frac{Q^2}{2C}$

$Energy = \frac{1}{2}\varepsilon_o\varepsilon_r E^2 (Al)$

$Energy\ density = \frac{Energy}{Volume} = \frac{1}{2}\varepsilon_o\varepsilon_r E^2$

$Energy\ density = \frac{1}{2}\varepsilon_o\varepsilon_r \frac{V^2}{d^2}$

Charging & discharging:

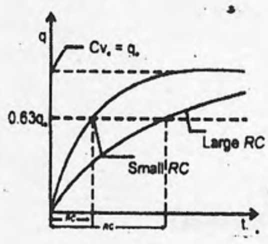
RC = constant

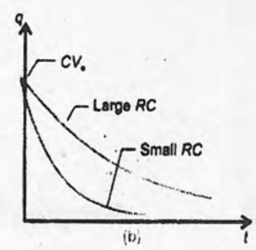
RC = time

RC = t

$ohm \times farad$ = second

$q = q_o\left(1 - e^{-\frac{t}{RC}}\right)$





Combination of capacitor:

Series combination:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

For two capacitor,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

For identical capacitor,

$$C_{eq} = \frac{C}{n}$$

Parallel combination:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

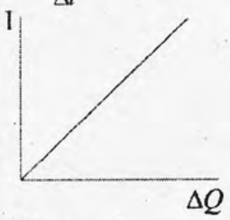
$C_{eq} = nC$ (for identical capacitor)

Topic - 7

Current Electricity

Electric Current:

$$I = \frac{\Delta Q}{\Delta t}$$



$Q = ne$

$$I = \frac{ne}{\Delta t}$$

Unit

$ampere\,(A)$

$A = Cs^{-1}$

Drift velocity:

$$v_d = \frac{I}{neA}$$

n = electron density

$v_d = 10^{-3}m$ (for metals)

Dimensions:

$[A]$

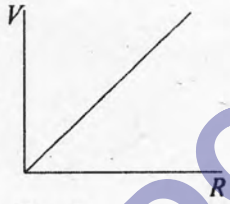
OHM's Law

$V = IR$

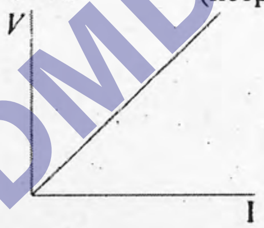
$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

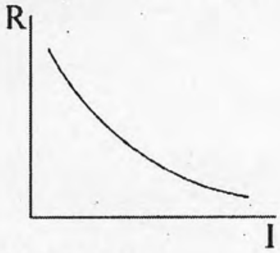
$V \propto R$ (keeping I constant)



$V \propto I$ (keeping R constant)



$R \propto \frac{1}{I}$ (keeping V constant)



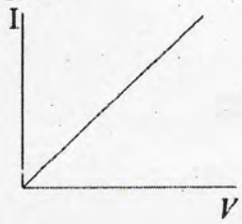
Unit of R:

Ohm (Ω)

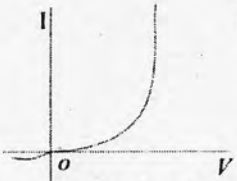
Ohmic conductors:

For metals,

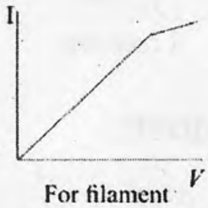
$I \propto V$



Non Ohmic conductors:

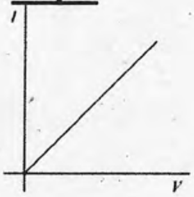


(For diode)

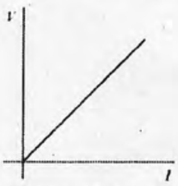


For filament

Slopes



$$\textit{Slope of } I-V \textit{ Graph } = \sigma = \frac{1}{\tan\theta} = \cot\theta$$

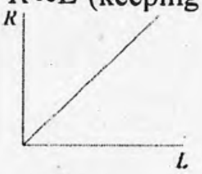


$$\textit{Slope of } V-I \textit{ Graph } = R = \tan\theta$$

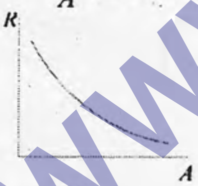
Specific resistance or resistivity:

$$R = \rho\frac{L}{A}$$

$R \propto L$ (keeping A constant)



$R \propto \frac{1}{A}$ (keeping L constant)



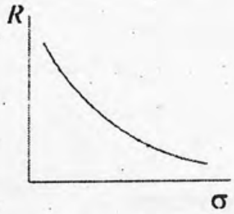
$$\rho = \frac{RA}{L}$$

Units:

$\Omega\, m$

$$\text{Resistance (R)} = \frac{1}{conductance} = \frac{1}{\sigma}$$

$$R \propto \frac{1}{\sigma}$$



$$\text{conductance } = \sigma = \frac{1}{resis\,\text{tance(R)}}$$

Units:

mho or Siemen or Ω^{-1} or $\mho$

$$conductivity \propto \frac{1}{\text{Resistivity}}$$

$$\sigma = \frac{1}{\rho}$$

Units:

$ohm^{-1}m^{-1}$ or mho m^{-1}

Temperature coefficient of resistance

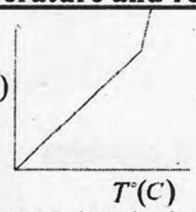
$$\alpha = \frac{Rt - R_o}{R_o t}$$

$$\alpha = \frac{\rho t - \rho_o}{\rho_o t}$$

Units:

K^{-1} or $\frac{1}{K}$

Temperature and resistivity:



Approximately ohmic

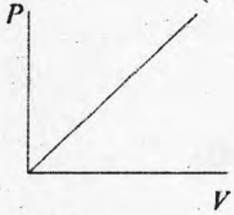
Electric power:

Work done by battery= $\Delta W = V \times \Delta Q$

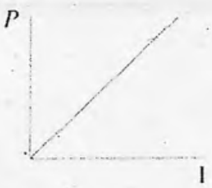
$$P = \frac{\Delta W}{t} = V\frac{\Delta Q}{\Delta t}$$

$$P = VI$$

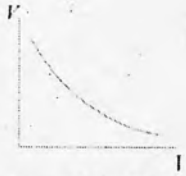
$P \propto V$ (keeping I constant)



$P \propto I$ (keeping V constant)



$V \propto \frac{1}{I}$ (keeping I constant)

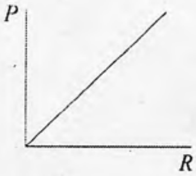


Possible formulas:

$P = I^2 R$

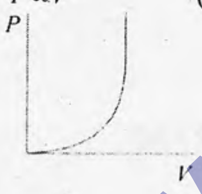
$P \propto I^2$ (keeping R constant)

$P \propto R$ (keeping I constant)

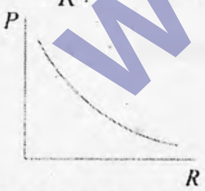


$P = \frac{V^2}{R}$

$P \propto V^2$ (keeping R constant)



$P \propto \frac{1}{R}$ (keeping V constant)



Internal resistance of supply:

Emf $E = \frac{\Delta W}{\Delta Q}$

Units

- volt
- $1\text{volt} = \frac{1\,joule}{1\,coulomb}$

Cases:

i. Open circuit:

$E = IR + Ir$

$E = IR = V_t \quad (I = 0)$

ii. Discharging:

$E = V_t + Ir$

$E > V_t$

iii. Charging:

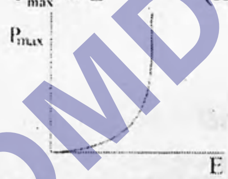
$E = V_t - Ir$

$E < V_t$

Maximum power output:

$\text{Pout}_{(max)} = \frac{E^2}{4r} = \frac{E^2}{4R}$

$P_{max} \propto E^2$ (keeping R constant)



Kirchhoff's rules:

Kirchhoff's current law:

$\sum I = 0$

$I_1 + I_2 + (-I_3) + (-I_4) = 0$

$I_1 + I_2 = I_3 + I_4$

Kirchhoff's Voltage law:

$\sum V = 0$

$E_1 - IR_1 - E_2 - IR_2 = 0$

Wheatstone Bridge:

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$ $\begin{pmatrix} \text{Balanced condition} \\ I_g = 0 \end{pmatrix}$

Let R_4 is unknown resistance,

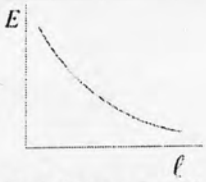
$R_x = \frac{R_2}{R_1} \times R_3$

Potentiometer:

$E_x = E\frac{r}{R} = E\frac{l}{L}$

$\frac{E_1}{E_2} = \frac{l_1}{l_2}$

$E \propto \frac{1}{l}$



Brightness of bulbs:

Series combination:

Brightness $\propto$ $P_{consumed}$

Brightness $\propto$ I^2R

Brightness $\propto$ R

$$P_{rated} = \frac{V^2_{rated}}{R}$$

$$P_{rated} \propto \frac{1}{R}$$

Parallel combination:

Brightness $\propto$ $P_{consumed}$

$$\text{Brightness} \propto \frac{V^2}{R}$$

$$\text{Brightness} \propto \frac{1}{R}$$

$$P_{rated} \propto \frac{1}{R}$$

Topic - 8
Electromagnetism

Magnetic flux & flux density:

$\phi_B = \vec{B}.\vec{A}$

$\phi_B = BA\cos\theta$

$\phi_{B_{max}} = BA \qquad (\theta = 0^o)$

$\phi_{B_{max}} = -BA \qquad (\theta = 180^o)$

$\phi_B = \frac{\phi_{max}}{2} = +\frac{BA}{2} \qquad (\theta = 60^o)$

$\phi_B = \frac{\phi_{max}}{\sqrt{2}} \qquad (\theta = 45^o)$

$\phi_B = \frac{\sqrt{3}BA}{2} \qquad (\theta = 30^o)$

$\phi_{B\,min} = 0 \qquad (\theta = 90^o)$

Flux density $= B = \frac{\phi_m}{A}$

Units:

$B = \frac{weber}{meter^2} = Wbm^{-2}$

$B = NA^{-1}m^{-1} = 1T$

Force acting on a charge particle in a uniform magnetic field:

$\vec{F} = q\left(\vec{v}\times\vec{B}\right) = \pm e\left(\vec{v}\times\vec{B}\right)$

$F = qvB\sin\theta$

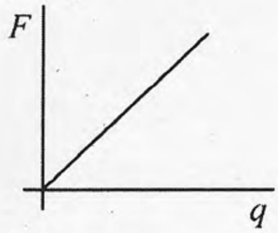
$B = \frac{F}{qv\sin\theta}$

$F_{max} = qvB \qquad \left(\theta = 90^o\right)$

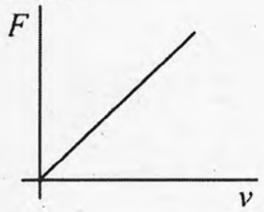
$F_{min} = 0 \qquad \left(\theta = 0^o\ or\ 180^o\right)$

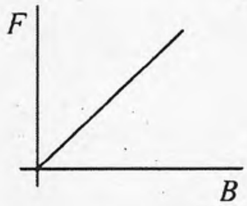
$F = \frac{F_{max}}{2} \qquad \left(\theta = 30^o\right)$

$F \propto q$



$F \propto v$





<u>Path followed by charge particle in magnetic field:</u>

If angle between $\vec{v}$ and $\vec{B}$.

$\theta = 0^\circ / 180^\circ$ path is straight line.

$\theta = 90^\circ$ path is circle.

$90 < \theta > 0^0$ path is helical.

$180^\circ < \theta > 90^\circ$ path is helical.

$F \propto B$

<u>Charged particle in electric field:</u>

$$\vec{a} = \frac{e}{m}\vec{E}$$

Lorentz force

$$\vec{F} = \vec{F}_E + \vec{F}_B$$

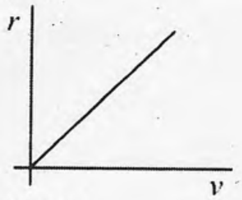
$$\vec{F} = q\vec{E} + q\left(\vec{v} \times \vec{B}\right)$$

$$\vec{F} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]$$

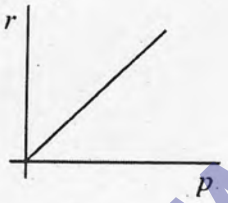
<u>Charge to mass ratio</u> $\frac{e}{m}$:

$$r = \frac{mv}{qB} = \frac{P}{qB}$$

$r \propto v$



$r \propto p$



$$\omega = \frac{q}{m}B$$

$$f = \frac{qB}{2\pi m}$$

$$T = \frac{2\pi m}{qB}$$

$$v = \frac{E}{B}$$

$$\frac{e}{m} = \frac{2v}{B^2r^2} = \frac{E}{B^2r}$$

$$\left(\frac{e}{m}\right)_{electron} = 1.75\times10^{-11} C\ kg^{-1}$$

$$\left(\frac{e}{m}\right)_{Proton} = 1.0\times10^{-8} C\ kg^{-1}$$

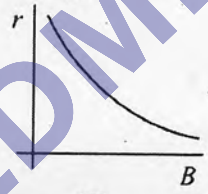
$$\left(\frac{e}{m}\right)_{Alpha} = \frac{1}{2}\left(\frac{e}{m}\right)_{proton} = 0.5\times10\ C\ kg^{-1}$$

$$\left(\frac{e}{m}\right)_{neuton} = 0$$

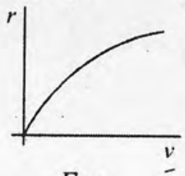
$$r^2 = \frac{2Vm}{B^2e}$$

$$r = \sqrt{\frac{2Vm}{B^2r}}$$

$r \propto \frac{1}{B}$ $(V = constant)$

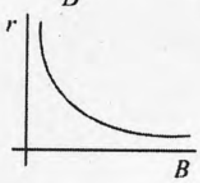


$r \propto \sqrt{V}$ (B=constant)

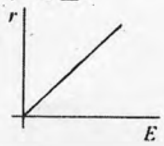


$$r = \frac{Em}{eB^2}$$

$r \propto \frac{1}{B^2}$ (E=constant)



$r \propto E$ (B=constant)



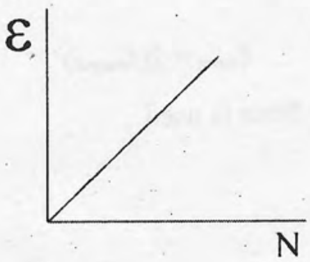
Topic - 9
Electromagnetic Induction

Faraday's Law:

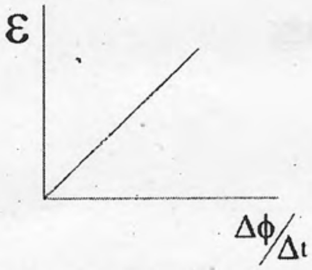
$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

$$\varepsilon = -\frac{NB\Delta A}{\Delta t} = \frac{NB\pi r^2}{\Delta t} \text{(for Circular Coil)}$$

$$\varepsilon \propto N$$



$$\varepsilon \propto \frac{\Delta\phi}{\Delta t}$$



Motional E.M.F ε :

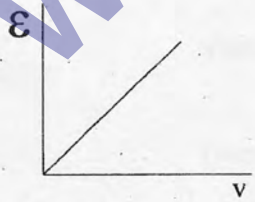
$$\varepsilon = -vBL\,Sin\theta$$

$$\varepsilon_{max} = -vBL \qquad (\theta = 90^o)$$

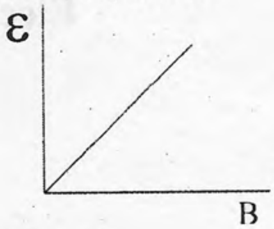
$$\varepsilon_{min} = 0 \qquad (\theta = 0^o \text{ or } \theta = 180^o)$$

$$\varepsilon = \frac{\varepsilon_{max}}{2} = -\frac{vBL}{2} \quad (\theta = 30^o)$$

$$\varepsilon \propto v$$



$$\varepsilon \propto B$$

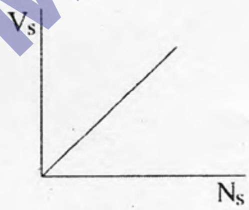


$$\varepsilon \propto L$$

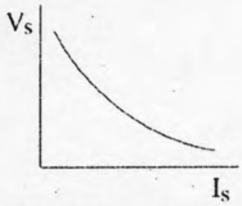
Transformer:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

$$V \propto N$$



$$V_s \propto \frac{1}{I_s}$$



Ideal Transformer:

$$P_{input} = P_{output}$$

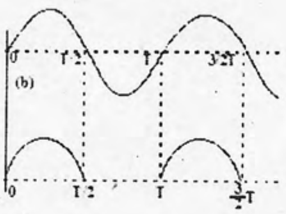
$$V_p I_p = V_s I_s$$

$$Effiency = \eta = \frac{output}{input} = \frac{P_o}{P_i} = \frac{V_s I_s}{V_p I_s} \times 100\%$$

Topic - 10
Electronics

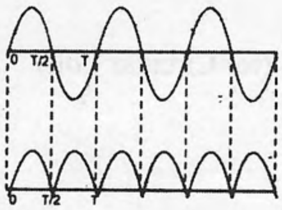
Rectification:

i. Half wave:



$f_{input} = f_{output}$

ii. Full wave:



$f_{input} = 2(f_{output})$

For filtration capacitor filter is used

Topic – 11
Dawn of Modern Physics

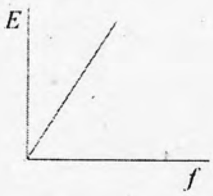
Photons:

$E = mc^2$

$E = pc$

$E = hf$

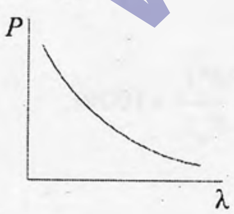
$E \propto f$



$E = \frac{hc}{\lambda}$

$p = \frac{h}{\lambda}$

$p \propto \frac{1}{\lambda}$



Wave nature of particle:

$\lambda = \frac{h}{p} = \frac{h}{mv}$

$p = \sqrt{2mE} = \sqrt{2meV}$

$\lambda = \frac{h}{\sqrt{2mVe}}$

$\lambda = \frac{h}{\sqrt{2mK.E}}$

$\lambda \propto \frac{1}{\sqrt{E}}$ $(m = constant)$

$\lambda \propto \frac{1}{\sqrt{m}}$ $(E = constant)$

$\lambda \propto \frac{1}{\sqrt{e}}$ $(m, V \text{ are constant})$

$\lambda \propto \frac{1}{\sqrt{V}}$ $(\text{for same charges})$

Determination of no. of photons

$$E_n = nhf$$

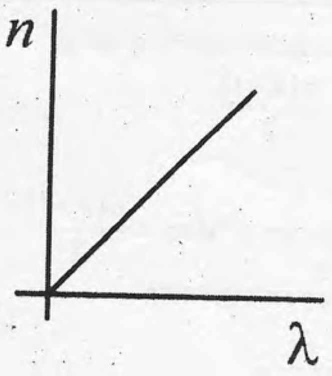
$$n = number\ of\ photons$$

$$n = \frac{E_n}{hf}$$

$$n = \frac{E\lambda}{hc} \because f = \frac{c}{\lambda}$$

$$\frac{E}{hc} = Constant$$

$$n \propto \lambda$$



Topic - 12

Atomic spectra

Spectral Series $\rightarrow \frac{1}{\lambda} = R_h\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$

Series	Formula	Region	Longest Wavelength	Shortest Wavelength
Lyman series	$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$ $n = 2,3,4,......\infty$	U-V	$Put\ n=2$ $\lambda = \frac{4}{3R_H}$	$Put\ n=\infty$ $\lambda = \frac{1}{R_H}$
Balmar series	$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n = 3,4,5,......\infty$	Visible	$Put\ n=3$ $\lambda = \frac{36}{5R_H}$	$Put\ n=\infty$ $\lambda = \frac{4}{R_H}$
Paschen series	$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ $n = 4,5,6,......\infty$	I.R	$Put\ n=4$ $\lambda = \frac{144}{7R_H}$	$Put\ n=\infty$ $\lambda = \frac{9}{R_H}$
Bracket series	$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$ $n = 5,6,7,......\infty$	I.R	$Put\ n=5$ $\lambda = \frac{400}{9R_H}$	$Put\ n=\infty$ $\lambda = \frac{16}{R_H}$
Pfund series	$\frac{1}{\lambda} R_H\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$ $n = 6,7,......\infty$	I.R	$Put\ n=6$ $\lambda = \frac{900}{11R_H}$	$Put\ n=\infty$ $\lambda = \frac{25}{R_H}$

R_H = Rydberg's constant$=1.0974\times10^{7}m^{-1}$

No. of transitions $= \dfrac{n(n-1)}{2}$

Or

No. of spectral series for n^{th} state $= \dfrac{n(n-1)}{2}$

$n =$ *higher energy* state

Ratio $= \dfrac{\lambda_{Longest}}{\lambda_{Shortest}} = \dfrac{n^2}{n^2-p^2}$

$p =$ lower energy state

$n = p+1$

Bohr's second postulate:

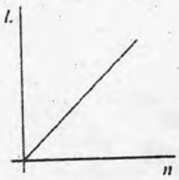
$$L = mvr = n\frac{h}{2\pi}$$

Principal Quantum number $= n = 1,2,3.....$

$L \propto n$

Bohar's third postulate:

Energy of photon $= E = E_2 - E_1$



$$hf = E_2 - E_1$$

$$\frac{hc}{\lambda} = E_2 - E_1$$

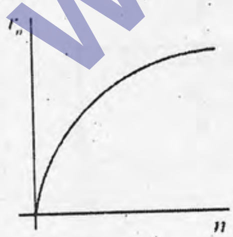
$$\lambda = \frac{hc}{E_2 - E_1}$$

Quantized radii:

$r_n = n^2 r_1$

$r_n = r_1, 4r_1, 9r_1 ....$

$r_n \propto n^2$

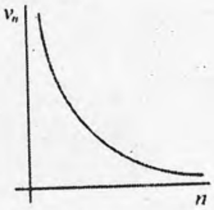


$$r_1 = \frac{h^2}{4\pi^2 kme^2}$$

$r_1 = 0.0053nm$

$$v_n = \frac{1}{n}\ \frac{2\pi ke^2}{h}$$

$$v_n \propto \frac{1}{n}$$



$$p \propto \frac{1}{n}$$

Quantized energy:

$$E_n = K.E + U \Rightarrow K.E = \frac{1}{2}mv_n^{\ 2}$$

$$U_e = -\frac{Ke^2}{2r_n}$$

$$E_n = -E_0, \frac{-E_0}{4}, \frac{-E_0}{9}, ...$$

$$E_n = \frac{-E_0}{n^2}$$

$$E_0 = \frac{2\pi^2 K^2 me^4}{h^2} = -13.6eV$$

$$E \propto \frac{1}{n^2}$$

Hydrogen Emersion spectrum:

$$\Delta E = E_2 - E_1$$

$$hf = E_n - E_p$$

$$hf = E_0\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\frac{hc}{\lambda} = E_0\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = \left(\frac{E_0}{hc}\right)\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$R_H = \frac{E_0}{hc} = 1.0974\times10^{7}m^{-1}$$

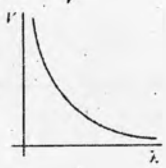
$$\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

X-rays:

$K.E_{max} = Ve$

$v = \sqrt{\frac{2Ve}{m}}$

$\lambda \propto \frac{1}{V}$



$\lambda_{min} = \frac{12400}{V} A^o$

$hf_{k\alpha} = E_L - E_k$

$\lambda_{k\alpha} = \frac{hc}{E_L - E_K}$

$hf_{k\beta} = E_M - E_k$

$\lambda_{k\beta} = \frac{hc}{E_M - E_k}$

Topic - 13
Nuclear Physics

To find no of neutron in a nucleus

$\Rightarrow N = A - Z$

Radius of nucleus:

$r = r_o A^{\frac{1}{3}} \Rightarrow r^3 \propto A$

$r_o = 1.2 fm$

Volume of nucleus:

$V = \frac{4}{3}\pi r^3$

$V \propto r^3$

$V \propto A$

Radioactivity:

$\propto -decay$	$\beta - decay$	$r - decay$
$X_Z^A \to ^{A-4}_{Z-2}Y + ^4_2He$ $^{226}_{88}Ra \to ^{222}_{86}Rn + ^4_2He$	$^A_ZX \to ^A_{z+1}Y + ^0_{-1}e$ $^{234}_{90}Th \to ^{234}_{91}Pa + ^0_{-1}e$	$^A_ZX^* \to ^A_zY + ^0_0\gamma$

Dependence:

Radioactivity is purely nuclear phenomenon, does not depend on temperature, pressure, electric field & magnetic field.

Half-life:

$\Delta N = -\lambda N \Delta t$

$\lambda = \frac{\frac{\Delta N}{N}}{\Delta t}$

Decay constant:

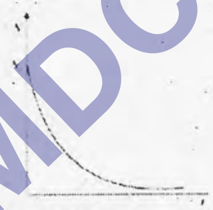
$N = N_o e^{-\lambda t} \Rightarrow \frac{N}{N_o} = e^{-\lambda t}$

$N = N_o\left(\frac{1}{2^n}\right)$

n= no of half-lives

$\frac{N}{N_o} = \frac{1}{2^n}$

$\Delta N \propto N$



Graph follows this equation $N = N_o e^{-\lambda t}$

It is exponential decay.

For decay rate (Activity)

$A = A_o e^{-\lambda t}$

Relation between λ&T½

$\lambda = \frac{0.693}{T_{1/2}}$

$T_{1/2} = \frac{0.693}{\lambda}$

This graph is for I-128

$\lambda T_{1/2} = 0.693$

Mean lifetime:

$T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ell n 2} = \frac{T_{1/2}}{0.693}$

$T_{mean} \propto T_{1/2}$

$T_{mean} \propto \frac{1}{\lambda}$

For carbon dating

$t = \frac{1}{\lambda}\ell n\left(\frac{N_o}{N}\right)$